VERY PRELIMINARY AND INCOMPLETE

Technology Adoption and Endogenous Debt Maturity: Liquidity Risk vs. Default Risk*

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Abstract

We study the joint determination of a technology — as indexed by the expected duration of a low growth regime and the variability of the growth rate in the high growth regime— and the optimal debt financing —the face value of the debt, its coupon rate, and expected maturity— in an economy in which credit markets and decision makers are risk neutral and have the same discount rate.

We find that the optimal maturity of the debt balances the risk of default due to low output with the risk of strategic default. In a quantitative version of the model we find that, holding project characteristics constant, when potential output is low (and, hence, the probability of strategic default is high) firms choose to issue short term debt. More generally, we find a positive relationship between potential output and maturity. We also show that firms operating in more uncertain environments choose to issue shorter debt, while firms whose assets have higher resale value (e.g. face lower costs of fire sales) issue longer maturity debt.

We then study the simultaneous choice of the characteristics of a project (i.e. the technology) and the structure of debt. We find that when potential output is low firms choose relatively risky projects —that is, there is "gambling for redemption"— as well as projects that display longer periods of low growth, and they are financed using relatively short term bonds. Higher levels of potential output at the time that the debt is issued result in shorter periods of low growth and less variable growth rates (in the high growth regime), at the same time that the expected duration of the debt increases.

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1 Introduction

The endogenous determination of the maturity structure of debt is a topic that interests both financial and macro economists. In the case of business firms —the case mostly analyzed by financial economists— the early work of Diamond (1991) and Leland and Toft (1996) has been followed by substantial work on the optimal structure of debt. In the macro literature, Lucas and Stokey (1983) highlighted the role that debt maturity has in supporting an optimal policy. Angeletos (2002) and Buera and Nicolini (2004) study the case of non-contingent debt while Shin (2007) allows for a maturity structure that changes depending on the state of the economy. In all cases there is no default in equilibrium. In the case of sovereign debt, Arellano and Ramanarayanan (2012) develop a theory of debt maturity based on the relative costs of different types of debt that is consistent with the evidence that in low output periods countries issue shorter-term debt and with the presence of default. Aguiar, and Amador (2015) use the framework of Arellano and Ramanarayanan (2012) to argue that when a country must reduce debt under risk of default, it is optimal to remain passive in the long-term bond market (let those bonds mature) and use the short-term bond market to roll over part of such debt. Hatchondo et. al. (2015) quantify the impact of reducing the ability of a country to dilute existing debt on average maturity and find it substantial. Fernandez and Martin (2015) highlight the trade-off between illiquidity and maturity when the country chooses between two types of debt and there is an incentive for debt dilution. Similarly, Sanchez, Sarpiza and Yurdagul (2015) present a model of endogenous determination of sovereign debt maturity but focus on the term structure of interest rate spreads and the impact of alternative debt rescheduling arrangements.

In this paper we study a model of a risk neutral agent —which we interpret as a firm— that chooses a project/technology and the type of debt that it will issue to finance the initial investment. On the technological side, the agent faces a tradeoff between the expected duration of a low growth (but inexpensive) regime, the expected growth rate (in the high growth regime) and the variability of the growth rate (in the high growth regime). On the financing side, we allow for a general menu of noncontingent securities. We assume that the firm can choose the structure of its debt from a continuum of securities. These securities —that we view as bonds— are completely summarized by three parameters: the coupon rate, b, the face value (or the value that is due at maturity), K, and a Poisson parameter, η, that determines the stochastic maturity of the bond. The expected maturity of the bond is then given by $1/\eta$.[1] This flexible specification of the structure of debt allows us to study debt that can be either front or backloaded and it captures the tradeoff between expected maturity and face value of a bond. We assume that both the firm and the credit market are risk neutral and have the same discount rate. Thus, we abstract away from many issues studied in the sovereign debt literature.

[1] We assume that maturity is stochastic to keep the model stationary.

First, we study the optimal choice of investment and the specification of a technology by a firm that is 100% equity owned. Given a specification of the cost function that makes it costly to decrease risk, it is optimal for the risk neutral firm to choose the riskiest technology. We show that the expected duration of the low growth regime is a decreasing function of potential output.

We then consider the choice of the optimal debt structure when the characteristics of the technology are given. We view this as an interesting exercise on its own since our "project/technology" could be, in some applications, viewed as the outcome of a market equilibrium. In this case exogeneity of some features (e.g. variability of the growth rate of earnings) is a reasonable assumption. We study in detail a special case in which the optimal bond is a pure discount bond. We develop some theoretical results on how features of the technology affect the pricing kernel of bonds and influence the choice of maturity. In the simple case we assume no refinancing[2]. Even though this is somewhat restrictive it allows us to disentangle how the financial market penalizes different maturity bonds depending on the state of the firm and the chances of a regime change (from low to high growth). Throughout the paper we abstract away from the strategic aspects associated with issuing short and long term debt simultaneously since there is no risk of debt dilution.

In a quantitative version of the model we show that the structure of the debt depends both on the level of potential output and on features of the project/technology. Firms whose potential output (that is, output that would obtain if the project moved from the low growth regime to the high growth regime) is low —and hence more likely to default for strategic/solvency reasons— choose shorter term bonds. As potential output increases, the risk of strategic default decreases and the optimal bond is chosen so as to decrease the risk of default associated with the bond maturing in the low growth regime. This is accomplished by increasing expected maturity of the bond at the cost of a higher face value and, hence, of a higher premium paid for strategic default.

We find that firms that operate in more unstable environments choose shorter maturities and that the expected maturity-output relationship is flatter the higher the uncertainty about growth rates. Thus, the model not only implies that firms in high uncertainty environments borrow using more short term debt than similar firms in a high uncertainty environment, but also that the cross sectional dispersion of expected maturities is smaller in the high uncertainty case.

Our results show that the degree of salability of the firm's assets (or, alternatively the cost of fire sales) also influences the choice of the optimal financing structure. We find that higher post-default value of the assets results in longer expected maturities. The result is intuitive: A higher recovery rate —conditional on default— reduces the cost of strategic default and, hence its shadow price. The firm then chooses to reduce the higher price risk

[2]The limit to refinancing is an extreme form of rollover risk: the probability of rolling over the debt is effectively zero.

—the risk of illiquidity driven default— by choosing a bond with longer duration.

For the examples that we study, we find that higher levered firms (i.e. firms with higher financing needs) choose shorter maturities while, at the same time, selling bonds with similar face values. Thus, unlike other features of the environment, changes in leverage have a large impact on expected maturity and a small impact on the face value of the optimal bond.

Finally, we endogeneize **simultaneously** the choice of project/technology and the structure of the debt. We find that endogenously chosen riskiness is a decreasing function of potential output. Thus, in a sense, at low levels of output there is "gambling for redemption." At low levels of output firms also choose technologies that result in relatively long low growth periods. Thus, their technology choice exposes them to high shadow prices of both types of risk, low growth and strategic. This is the best choice available to the firm given that it makes the choice at a time that potential output is low. At high levels of output the choices are reversed: the low growth period is shorter and the projects are less risky. On the financing side we find that the low growth and risky projects —those chosen when potential output is low— are financed with relatively short term debt. This might seem counterintuitive but it is driven by the relative shadow prices of the two types of risk. If the firm issues longer term debt the price of low growth risk and the price of strategic default risk increase (the latter because a longer pure discount debt carries a higher face value) and the optimal (from the point of view of the firm) structure of debt minimizes the sum of those two costs subject to a financing requirement. As in the case when the characteristics of the technology are fixed, firms with higher levels of potential output issue debt with longer maturities. We also explore the role of leverage and find that lower levered firms issue longer debt.

The need to finance an investment project issuing debt results in some inefficiencies —relative to the private optimum— in the choice of technology. First, in general firms choose technologies that are too safe relative to the efficient choice. Second, for low levels of output firms select technologies that display an excessively long (expected) duration of the low growth phase. Finally, we do not find a simple relationship between leverage and distortions: Firms with lower levels of leverage choose technologies whose riskiness —for each level of potential output— is closer to the optimal. At the same time they choose technologies that result in long (expected) durations of the low growth regime, especially at low levels of potential output.

2 Model

In this section we describe the simplest version of the model. We study the case of a risk neutral firm that has to incur an irreversible cost to implement an investment project. We assume that the returns of the project are completely described by a few parameters that determine the associated stochastic processes for output (or earnings). This is our specification of the technology. To simplify we assume that, initially the project is in the

low growth regime and has constant earnings given by z. The amount of time spent in this regime is random and distributed exponential with parameter υ. Denote by T_υ the time spent in the low growth regime. Once the technology switches from the low growth to the **high growth regime** net earnings/output are given by a stochastic process $x_t + \theta z$, where $\theta \in (0, 1)$ and x_t satisfies,

$$dx_t = \mu x_t dt + \sigma x_t dW_t.$$

This formulation captures the idea that some managerial resources must be allocated to the x_t dimensions and, hence, the net output from the low growth activity decreases (as captured by $\theta < 1$). In what follows we view the technology as a triplet (υ, μ, σ).

We assume that x_t is known at all times. That is, the firm —and the market— know its potential output if the technology switched to the high growth regime. We restrict the feasible technological choices to lie in a set indexed by the current potential productivity, that is, $(\upsilon, \mu, \sigma) \in \Lambda(x)$. The choice of technology determines the cost of initial investment, $C(\upsilon, \mu, \sigma)$. We assume an extreme form of moral hazard: In the low growth regime no outsider can operate the project and, hence the firm cannot issue equity. In the high growth regime, the firm can issue equity and its market price would be the expected present value of earnings.

Since our interest is to understand the interplay between project/technology choice and financial structure we first study the choice of technology by a firm that is 100% equity owned (or that can finance the project) out of internal funds.

2.1 Perfect Capital Markets: Efficient Choice of Technology

Consider first the value of a 100% equity owned firm in the low growth (initial) state given output x.[3] It is

$$\bar{E}_0(x, z, \upsilon, \mu, \sigma) = E\left\{\int_0^{T_\upsilon} e^{-rt} z dt + e^{-rT_\upsilon} \bar{E}_1(x_{T_\upsilon}, \mu, \sigma) \mid x_0 = x\right\}$$

where

$$\bar{E}_1(x, z, \mu, \sigma) = E\left\{\int_t^{\infty} e^{-rs}(x_{t+s} + \theta z) ds \mid x_t = x\right\} = \frac{x}{r-\mu} + \frac{\theta z}{r}.$$

Thus,

$$\bar{E}_0(x, z, \upsilon, \mu, \sigma) = \frac{(r + \upsilon\theta)z}{r\,(r+\upsilon)} + \frac{\upsilon}{r+\upsilon-\mu}\frac{x}{r-\mu}.$$

In this specification $1/\upsilon$ is the expected duration of the time spent in the low growth regime. If

$$\frac{x}{z} > \left(\frac{r+\upsilon-\mu}{r+\upsilon}\right)^2 (1-\theta)$$

[3] Since in this state the firm can issue equity, it is also the market value of the assets owned by the firm.

the model implies that lower growth technologies (lower υ) are less valuable[4]. Finally, given the risk neutrality assumption the diffusion component (σ) has no impact on the value of the firm.

The optimal value of this project is given by

$$E^*(x) = \max\left\{\max_{(\upsilon,\mu,\sigma)\in\Lambda(x)} \bar{E}_0(x,z,\upsilon,\mu,\sigma) - C(\upsilon,\mu,\sigma,x), 0\right\}$$

where $C(\upsilon,\mu,\sigma,x)$ is the one time cost associated with choosing technology (υ,μ,σ). It follows that, depending on the particular form of the set $\Lambda(x)$ and the cost function $C(\upsilon,\mu,\sigma,x)$ the optimal choice of technology can be independent of the state of demand, x.

2.2 Debt Financing

In this section we assume that the firm has to finance either partially or totally the cost of the project issuing non-contingent debt. The firm has access to a risk neutral credit market that will price any debt issued by the firm using the same discount rate. Thus, none of our results are driven by either differences in discount factor or differences in the curvature of direct payoffs.

We study a market where the set of feasible debt instruments is completely described by three parameters (b, K, η), where b is the coupon rate, K is the payment due at maturity (which does not necessarily coincide with the initial value of the debt) and η is the parameter of a Poisson process that determines the maturity of the bond. Thus, maturity is stochastic and the maturity time is denoted T_η. The expected maturity of a bond is $1/\eta$.[5]

We assume that if the firm defaults on its debt in the low growth regime, bondholders receive a certain fraction of the value of the certain stream z, denoted by $(1-\delta_L)z/r$. Since the firm is still in the low growth regime we assume that outsiders cannot operate the technology in a way that it will transition to the high growth regime. If the firm defaults in the high growth regime bondholders receive a fraction $1-\delta_H$ of the value of the firm, while shareholders go empty handed. Thus, we interpret the δ_i as a measure of the cost of fire sales[6] or, alternatively, the degree of salability of the firm's assets or the level of bankruptcy costs.

Our simple version of the technology puts some restrictions on the type of bonds that the firm can issue. We discuss relaxing this constraint in the extensions. Since we assumed that the firm cannot issue equity and that earnings are z in the low growth regime, it follows that

[4] We will restirct our analysis to this case.

[5] We only consider bonds that make payments that are not contingent on the state of the firm. In particular, we rule out callable bonds.

[6] In this interpretation we view δ_i as a measure of the probability of extreme fire sales in each regime. To be precise, assume that with probability δ_i there is a fire sale and the value of the assets is zero, and with probability $1-\delta_i$ there is no fire sale and the value of the assets is the present discounted value.

it cannot commit to paying a coupon rate that exceeds this value. Thus, the only feasible choice is $b \leq z$.[7] In this case a bond is completely described by a triplet (b, η, K).

If the debt matures in the low growth regime it must be refinanced; otherwise the firm will default. If the firm cannot issue more debt that raises enough revenue to pay off the original debtholders then the payoff to bondholders is as described above and shareholders receive zero. The cost of refinancing (a fixed cost) is denoted C_F.

In the high growth regime, depending on the values of b and θz the firm has incentives to default strategically either before the debt matures (when x is low) or at maturity. Formally, this corresponds to the case in which in the high growth regime the firm can issue equity to pay its creditors. As it is standard in these models the existence of debt financing could potentially lead to the liquidation of the firm and this is inefficient given our assumptions about the stochastic process for earnings.

Thus, debt financing can potentially lead to two types of inefficiencies. First, as in Diamond (1991) and Leland (1994) it could result in inefficient liquidation. Second, in some sense related to the literature on debt overhang (see Diamond and He (2014)), it can bias the investment decision and the choice of technology

2.2.1 The Value of the Firm in the High Growth Regime

The market value of a productive firm that has issued a (b, η, K) bond, denoted by $T(x, z, b, \eta, K)$, is

$$\begin{aligned} T(x,z,b,\eta,K) = \Big\{ & E\int_0^{T_s\hat{}T_\eta} e^{-rs}\left(x_{t+s}+\theta z-b\right)ds \\ & + e^{-rT_s\hat{}T_\eta}\max\left(\frac{x_{T_\eta}}{r-\mu}+\frac{\theta z}{r}-K,0\right) \mid x_t = x \Big\}. \end{aligned}$$

Here $T_s = \inf\{t : x_t \leq x^*\}$ is an endogenously chosen stopping time that determines default before the debt matures. The first term is the value of net output until default or maturity, while the second term is the residual value of the firm. The optimal default rule at maturity is very simple: Default if and only

$$\frac{x_{T_\eta}}{r-\mu}+\frac{\theta z}{r} < K,$$

that is, if the value of the assets falls short of the face value of the outstanding debt.

Standard arguments show that $T(x)$ satisfies the following HJB equation (where the

[7]Later, we generalize the technology and allow for a low growth (but positive earnings) in the low growth state.

arguments are suppressed to keep the notation simple):

$$rT(x) = x + \theta z - b + \frac{\partial T}{\partial x}(x)\mu x + \frac{\partial^2 T}{\partial x^2}(x,)\frac{\sigma^2}{2}x^2 \qquad (1)$$
$$+ \eta[\max(\frac{x}{r-\mu} + \frac{\theta z}{r} - K, 0) - T(x)],$$

and the boundary conditions are

$$\lim_{x\to 0} T(x) = 0, \text{ and } \lim_{x\to\infty} \frac{T(x)}{x} = 0,$$

while the optimal default value satisfies

$$T(x^*) = 0, \text{ and } T'(x^*) = 0.$$

Let's denote the lowest value of x that triggers bankruptcy at maturity by $\bar{x}$. Thus,

$$\bar{x} = (r-\mu)\left(K - \frac{\theta z}{r}\right).$$

Of course, if the face value of the debt is sufficiently small and $\bar{x} < 0$ there is no default at maturity. Appendix A discusses the details of the solution to equation (1) but, in general, it is possible —and this depends on the choice of debt structure— that:

- The debt is safe. This implies no default at any time.
- The debt is safe until it matures. This means that the firm will choose to honor the coupon but potentially default at maturity.
- The debt is not safe in any state. In this case —depending on x— the firm will choose to default before the debt matures —standard strategic default— or at maturity (a type of solvency default).

2.2.2 The Value of the Debt in the High Growth Regime

The value of an (b, η, K) bond satisfies (as before we suppress the constants from the notation)

$$rB(x) = b + \frac{\partial B}{\partial x}(x)\mu x + \frac{\partial^2 B}{\partial x^2}(x)\frac{\sigma^2}{2}x^2 \qquad (2)$$
$$+ \eta[\Pi(x, K) - B(x, \eta, K)],$$

where

$$\Pi(x,K)=\left\{\begin{array}{ll} K & if\ x\geq\bar{x} \\ (1-\delta_H)\left(\frac{x}{r-\mu}+\frac{\theta z}{r}\right) & if\ x<\bar{x} \end{array}\right. .$$

Moreover, in the relevant regime,

$$B(x^*)=(1-\delta_H)\left(\frac{x^*}{r-\mu}+\frac{\theta z}{r}\right)$$

To rule out bubbles, the solution to the HJB equation must satisfy the following boundary conditions:

$$\lim_{x\to 0}B(x)=0,\ \text{and}\ \lim_{x\to\infty}\left[B(x)-\frac{b+\eta K}{r+\eta}\right]\leq 0.$$

This last condition says that the market value a the bond cannot exceed the risk free value of the promised payments.

2.2.3 Valuations in the Low Growth Regime

Since the firm makes technological and financing choices in the low growth regime we now describe the relevant payoffs. Let $V(x,b',\eta',K')$ be the value of the firm in the low growth regime when (potential) earnings are given by x and the firm has debt characterized by the triplet (b',η',K'). Let $L(x,b',\eta',K')$ be the market value of such a debt.

The value of a firm that needs to refinance a bond with face value K at the time it matures and that it chooses its debt contract optimally —within the class that we study— is given by

$$M(x,K)\equiv\max_{(b',\eta',K')}V(x,b',\eta',K') \tag{3}$$

subject to

$$L(x,b',\eta',K')\geq K. \tag{4}$$

Equation (4) summarizes credit market equilibrium. The function $L(x,b',\eta',K')$ is the pricing kernel for any (b',η',K') bond issued by a firm with (potential) output x. The constraint simply states that the firm can choose any bond that raises sufficient revenue to pay off the existing (maturing) bond.

As a matter of convention, we set $M(x,K)=0$ if the set

$$\Sigma(x,K)\equiv\{(b',\eta',K'):L(x,b',\eta',K')\geq K\}$$

is empty.

The value of the firm (again ignoring constants) is given by

$$V(x)=E\left\{\int_0^{T_v\wedge T_\eta}e^{-rt}(z-b)dt+e^{-r(T_v\wedge T_\eta)}M(x,K)\mathbf{X}_{[T_\eta<T_v]}+\right. \tag{5}$$
$$\left.T(x)\left(1-\mathbf{X}_{[T_\eta<T_v]}\right)]\mid x_0=x,\right\}$$

where T_η is the stopping time that corresponds to the maturity of the existing debt, while T_υ is the stopping time that indicates a switch from the low growth to the high growth regime. The term $\mathbf{X}_{[T_\eta<T_\upsilon]}$ is the indicator function of the event $\{T_\eta < T_\upsilon\}$.

The valuation of the firm satisfies:

$$\begin{aligned} rV(x) = z - b + \frac{\partial V}{\partial x}(x)\mu x + \frac{\partial^2 V}{\partial x^2}(x)\frac{\sigma^2}{2}x^2 \\ +\eta\left[\max\left(M(x,K) - C_F, 0\right) - V(x)\right] + \upsilon\left[T(x) - V(x)\right], \end{aligned} \tag{6}$$

where $M(x,K)$ is given by equation (3). In addition, the solution must satisfy the appropriate boundary conditions to rule out bubble solutions.

The value of a (b,η,K) bond in the low growth regime is given by

$$\begin{aligned} L(x) = E\left\{\int_0^{T_\upsilon\wedge T_\eta} e^{-rt}b dt + e^{-r(T_\upsilon\wedge T_\eta)}[\Lambda(x,K)\mathbf{X}_{[T_\eta<T_\upsilon]} + \right. \\ \left. B(x,\eta,K)\left(1 - \mathbf{X}_{[T_\eta<T_\upsilon]}\right)] \mid x_t = x\right\}. \end{aligned} \tag{7}$$

Standard arguments show that the solution to equation (7) also satisfies the following HJB equation:

$$\begin{aligned} rL(x) = b + \frac{\partial L}{\partial x}(x)\mu x + \frac{\partial^2 L}{\partial x^2}(x)\frac{\sigma^2}{2}x^2 \\ + \eta\left[\Lambda(x,K) - L(x)\right] \\ + \upsilon\left[B(x) - L(x)\right], \end{aligned} \tag{8}$$

where

$$\Lambda(x,K) = \begin{cases} K & \text{if } M(x,K) - C_F \geq 0 \\ (1-\delta_L)\frac{z}{r} & \text{otherwise} \end{cases}$$

and subject, as in the case of the firm, to the appropriate boundary conditions.

Finding the solution to equations (6) and (8) involves finding a fixed point in the space of functions, since the payoff term $M(x,K)$ depends on the functions $V(x)$ and $L(x)$. We postpone discussing the problem of existence until later.

2.3 Technology Adoption and Debt Structure

The problem faced by a firm that has to finance its investment issuing debt is similar to the problem faced by the 100% equity owned firm: It chooses a bond, (b,η,K), and a technology, (μ,υ,σ), given a potential productivity level, x, to solve

$$\max_{(\upsilon,\mu,\sigma,b,\eta,K)} V(x) - C(x,\upsilon,\mu,\sigma)$$

subject to

$$(\upsilon,\mu,\sigma) \in \Delta(x) \text{ and } L(x,b,\eta,K) \geq E,$$

2.4 Existence (Preliminary)

The logic of the argument is as follows:

1. Consider the one bond, no refinancing, case (discussed in the next section) and let the value functions have superindex 1. By construction, these functions are continuous in x and in all parameters (appeal to the maximum theorem). This gives a continuous functions Fix a continuous function $M^1(x, K)$.

2. Let $V^k(x, b, \eta, K)$ and $L^k(x, b, \eta, K)$ be the continuous functions if the firm is allowed to refinance at most k times. By a standard recursion argument these functions are also continuous

3. Define the mapping $H(V^k, L^k)(x, \eta, K)$ as follows

$$H(V^k, L^k)(x, b, \eta, K) = \max_{(\eta', K')} V^k(x, b', \eta', K'; M) \tag{9}$$

   subject to

$$L^k(x, b', \eta', K'; M) \geq K. \tag{10}$$

4. Given this mapping the same recursive argument can be used to define V^{k+1}, L^{k+1}.

5. Note that this sequence of functions correspond to decision problems with progressively fewer restrictions and hence that the functions are monotone increasing. Thus, the limit is well defined.

6. However, given that there is a fixed cost of refinancing we want to argue that, for any fixed x, there is a finite number of refinancing options $\bar{k}(x)$ after which the initial value of the problem is zero. (This is a delicate argument and it is the missing part.)

3 The One Bond Case

In order to highlight the interplay between technological parameters and the choice of financing we describe some results in a special case of the general model as it provides insights into the factors that influence the choice of maturity. in this section we assume that:

- Firms cannot refinance the initial debt.
- The low growth regime has $z = 0$. This implies that it is not only low growth but also illiquid. Moreover it implies that the only feasible debt is a pure discount bond. Formally, in this case $b = 0$.

3.1 Some Theoretical Results

To distinguish this special case —the no refinancing and zero output in the low growth regime case — from the general case we denote the appropriate valuation functions in the low growth regime $V^1(x, \eta, K; \upsilon, \sigma)$ and $L^1(x, \eta, K; \upsilon, \sigma)$. In this case the value of a riskless (η, K) bond is simply $\eta K/(r + \eta)$, and the market value of the bond is given by

$$L^1(x, \eta, K; \upsilon, \sigma) = \frac{\eta K}{r + \eta}\left[1 - P_L(x, \eta, K; \upsilon, \sigma) - P_H(x, \eta, K; \sigma)\right]$$

which shows that —depending on the endogenous choice of debt structure— the market reduces the amount of resources that an (η, K) bond can raise by apply two correction factors: One, that we label P_H, we interpret as the risk price in the high growth regime — and hence independent of υ. The other, P_L, captures both the risk that the debt will mature in the low output state as well as the solvency risk. The following proposition summarizes the properties of these risk prices as a function of the state of the firm (value of its assets) and the technology

Proposition 1 *Assume that*

$$\frac{\sigma^2}{2} > \mu,$$

then

1. *Changes in potential output:*

$$\frac{\partial P_H}{\partial x} < 0 \text{ and } \lim_{x\to\infty} P_H(x) = 0,$$
$$\frac{\partial P_L}{\partial x} > 0 \text{ and } \lim_{x\to\infty} P_L(x) = \frac{r + \eta}{r + \eta + \upsilon}$$

2. *Changes in technology:*

$$\frac{\partial P_H}{\partial \sigma}(x) > 0 \text{ and } \lim_{x\to\infty} \frac{\partial P_H}{\partial \sigma}(x) = 0,$$
$$\frac{\partial P_L}{\partial \sigma}(x) < 0 \text{ and } \lim_{x\to\infty} \frac{\partial P_H}{\partial \sigma}(x) = 0,$$
$$\frac{\partial P_H}{\partial \upsilon}(x) = 0 \text{ and } \frac{\partial P_L}{\partial \upsilon}(x) < 0(?)$$

Proof. : See Appendix A ■

The proposition highlights how potential output affects the two types of risk. Consider the impact of an increase in x. This implies that the probability that the firm will default in the high growth regime decreases. Consequently, the price that the market charges for that risk, $P_H(x, \eta, K; \sigma)$ decreases as well. The increase in potential output does not change the probability that the debt matures in the low growth regime—which in this example is also an illiquid state— and it makes this event even costlier. This is captured by bot the fact that $P_L(x, \eta, K; \upsilon, \sigma)$ is increasing in x and that this cost does not disappear as potential output grows without limit as the risk of the debt maturing in the low growth regime is independent of x.

Increases in σ unambiguously lower the value of the debt once the firm is in the high growth region. The impact on the price that the market charges for the risk of maturity in the low output regime is exactly the opposite: Higher variability in the growth rate reduces the shadow cost of maturity in the low growth regime. Finally, when potential output is arbitrarily high, variability does not matter: The market value of the debt converges to the value associated with default in the low growth regime and this is independent of potential output.

The following proposition describes how the value of the debt varies with the structure as captured by (η, K).

Proposition 2 *Assume that*

$$\frac{\sigma^2}{2} > \mu,$$

then

$$\lim_{K \to 0} \frac{\partial L^1}{\partial K} = \frac{\eta}{r + \eta} \frac{\upsilon}{r + \eta + \upsilon} \text{ and } \frac{\partial L^1}{\partial K} |_{K = \frac{x}{r - \mu}} < 0.$$

Proof. : See Appendix A ■

This result captures the standard intuition that as the face value of the debt increases its market value need not increase since the probability of default increases as well. Since the proposition shows that the market value of the bond as a function of its face value must attain a maximum in the region $K < x/(r - \mu)$, it follows that the firm will always choose the face value of debt in the region $[0, x/(r - \mu)]$. This implies that, if the debt were to mature instantaneously and the firm became productive, there would be no default. Note that the market is willing to buy debt that would be defaulted in that situation. However, the firm will never choose to be in that region.

3.2 Optimal Debt: Fixed Technology

In order to better understand the interaction between technology and cyclical elements we present some very preliminary numerical results. In the case that the technology is fixed, the optimal debt structure is the solution to the following problem:

$$\max_{(\eta, K)} V^1(x, \eta, K)$$

subject to

$$L^1(x, \eta, K) \geq L^0.$$

In this case our base specification sets $r = 0.05$, $\mu = 0.01$, $\sigma = 0.3$, $\upsilon = 0.25$, $\delta = 1$. We illustrate how the structure of the debt changes as we allow $(\mu, \sigma, \upsilon, \delta)$ to vary.

Maturity Figure 1 shows how expected maturity varies with technological parameters holding the market value of the bond equal to the cost of investment that varies as technological parameters are changed

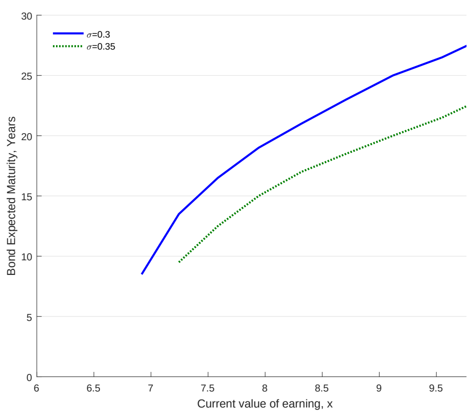

Figure 1.a: Changing σ

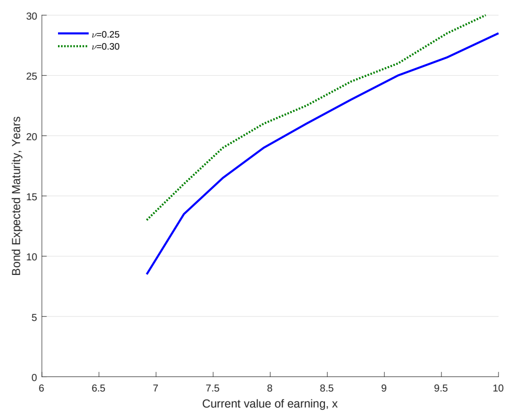

Figure 1.b: Changing μ

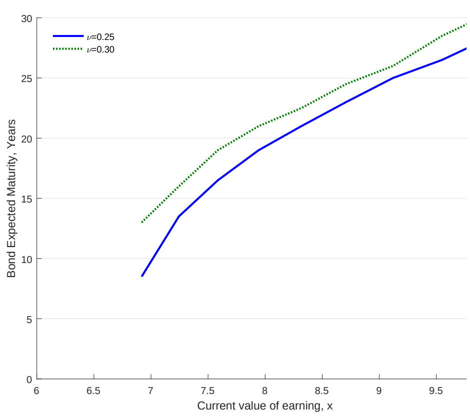

Figure 1.c: Changing υ

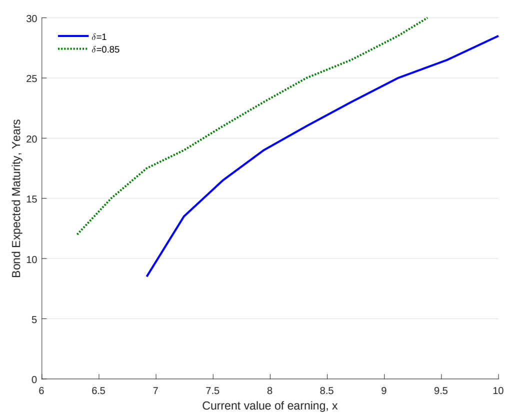

Figure 1.c: Changing δ

We find that in all experiments, expected duration increases with the level of potential output. We view this as evidence that firms that need to issue debt in "bad" times will choose to issue relatively short term debt. In all the cases that we study, the firm could have issued longer term debt —say with the expected duration that it chooses when x is higher— but facing a high cost of lowering expected maturity decides not to shorten the duration of debt.

The results also show that features of the technology influence the choice of projects and the structure of debt. For example, firms in sectors with more variable growth rates of earnings choose not to invest in some low x projects that would be carried out if the technology had a lower σ. In addition, increases in σ are associated with a shortening of the expected duration of the debt.[8] The reasons for this are complex but it is possible to show that in the high growth regime the market value of the debt decreases as σ increases. This, in turn implies that the shadow price of long run risk increases and the corresponding price of low growth risk decreases. As a consequence of this change in relative costs of the two risks the firm chooses to issue shorter term debt.

Figure 1.b shows that firms in markets characterized by higher expected growth choose lower maturities. The intuition is that higher μ is associated with a lower market market price of strategic default and this allows that firm to lower the price of default in the low growth regime by increasing maturity. As expected, projects that are associated with technologies that display longer expected low growth periods are financed with longer term debt, and higher post-default value of the assets (lower δ) is associated with longer maturities. As in the case of the changes in riskiness, projects/assets with higher resale value are associated with lower prices of strategic default and it is optimal for a firm to redesign its debt by lowering the implicit cost of illiquidity default and this is accomplished increasing average maturity. This is consistent with the findings of Benmelech (2009) who reports that American railroads that used rolling stock that could be more easily redeployed (higher resale value) tended to issue longer term bonds.

Figure 2 displays the behavior of the two risk prices, $P_L(x, \eta, K; \upsilon, \sigma)$ and $P_H(x, \eta, K; \sigma)$. Since these are "endogenous" prices they capture not only the change in x but the response of the firm —in the form of changes in (η, k)— as potential output changes. At low levels of output even though duration is short, the shadow cost of default in the low growth regime, P_L, is high relative to P_H. As x increases the firm increases maturity (and this lowers P_L) at the same time that it increases the face value of the bond, which results in higher levels of P_H.

[8] Even though we view differences in σ as different projects, it is possible to reinterpret them as arising from differences in policies or demand since the appropriate interpretation of x is that it captures earnings and not just output.

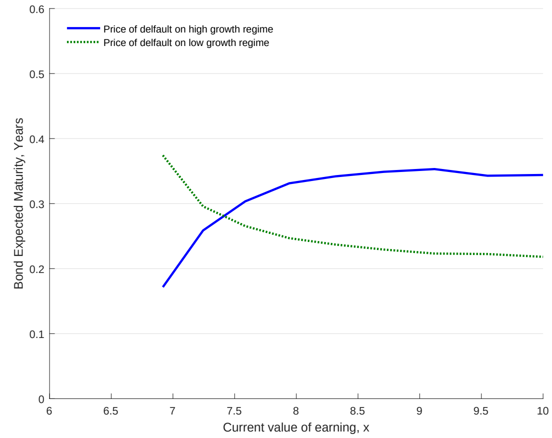

Figure 2: Risk Prices

Leverage Figure 3 shows how the structure of the debt varies with the financing needs of the firm —for a given level of potential earnings— as a function of the project characteristics. Since we are holding the cost of the project constant, higher value of L_0, correspond to higher leverage.

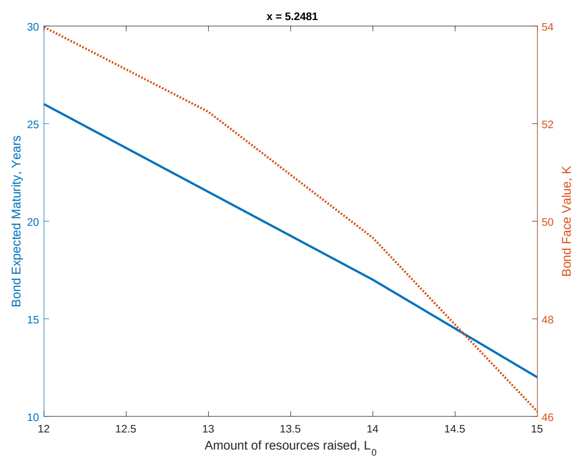

Figure 3: Duration and Face Value.
Leverage.

The model implies that higher levered firms issue shorter term bonds. In our base case increases in leverage result in significant changes in expected maturity with fairly small variations in the face value of the bond (note the difference in scales). Our numerical results (not shown) indicate that sectors with higher σ imply that the leverage-maturity relationship is pushed down relative to the base case, while sectors with lower δ move in the opposite direction.

To the extent that differences in technology correspond to different sectors the model implies that firms with similar values of expected earnings (under the no distortion case) would be issuing debt with different characteristics.

3.3 Optimal Debt: Endogenous Technology Adoption

How does the choice of technology depend on the state of potential output (x), the value of the firm in secondary markets (δ), and the degree of leverage when the debt is chosen optimally? In particular, there are two dimensions of the choice of technology that seem particularly interesting: the level of uncertainty in earnings growth chosen by the firm, σ, and the (expected) length of the period of low growth, υ.

We view this preliminary exercise as a first cut at understanding if firms that have low potential output will choose to "gamble for redemption," that is, if they will choose to pick risky projects so that, if the firm is lucky, they will generate a high payoff to shareholders. This intuition is a priori plausible in our model as firms are risk neutral and, in the presence of strategic default the value of the firm —as a function of x— is convex.

To keep the presentation simple we fix μ and we let the firm choose (υ, σ) as well as the structure of the debt. Formally the problem is

$$\max_{(\upsilon,\sigma,\eta,K)} V^1(x, \eta, K)$$

subject to

$$L^1(x, \eta, K) \geq C(\upsilon, \mu, \sigma, x)$$

We assume that the cost function is given by

$$C(\upsilon, \mu, \sigma, x) = c + \xi\upsilon + \varsigma\mu + \psi(\bar{\sigma} - \sigma)^2$$

where $c = 0$, $\xi = 50, \varsigma = 20$ and $\psi = 50, \bar{\sigma} = 0.55$. We consider —as in the previous section— the case $r = 0.05$ and, for this exercise, we fixed $\mu = 0.01$. Thus, the firm chooses the scale of the project —as reflected in the cost— and, on the technological side, the expected duration of the low growth phase, $1/\upsilon$, and the riskiness of the technology, σ. We restrict the feasible choices to the following intervals:

$$\sigma \in [0.1, 0.6], 1/\upsilon \in [1, 30], 1/\eta \in [1, 100], \text{ and } K \in [0.1, 200]$$

These values are not meant to capture any particular investment environment. Rather, they are designed so we can highlight that implications of the model.

In addition, as in the previous section, the firm is choosing the pair (η, K) that determine the expected maturity and the face value of the bond.

In the case of a 100% equity owned firm, the trivial choice given risk neutrality is $\sigma = \bar{\sigma}$ which we set at 0.55.

Figure 4 a-d shows the optimal choices of the two technological parameters and the total amount invested for both the bond financed and the equity financed case. We also display the expected maturity of the debt,

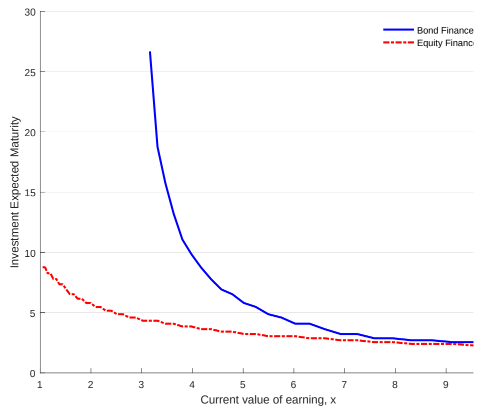

Figure 4.a: Duration - Low Growth

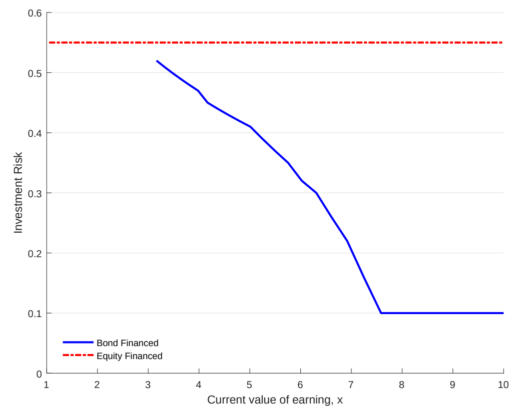

Figure 4.b: Variability - High Growth

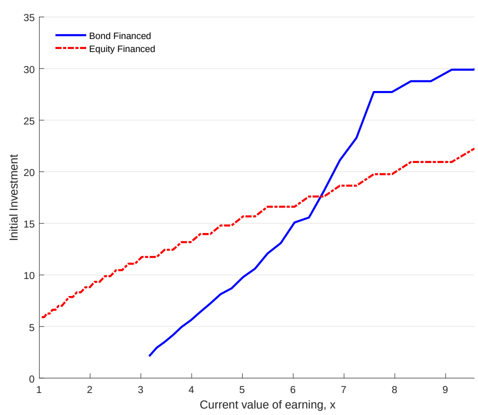

Figure 4.c: Investment

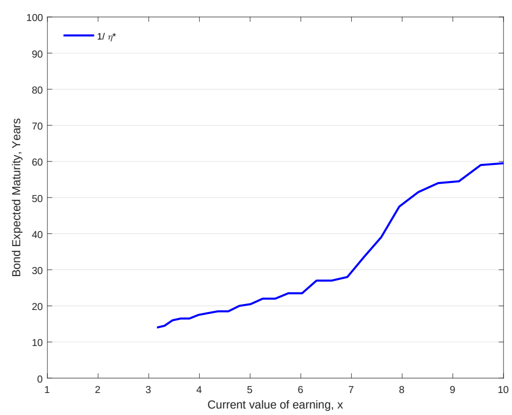

Figure 4.d: Maturity

There is a clear pattern that emerges: At low levels of potential output firms choose technologies that display relatively long periods (compared with the optimum) of low growth and are risky. This behavior can be interpreted as a form of “gambling for redemption” although, in this case, the reason is that those choices result in (inefficiently) lower cost of investment and this, in turn, reduces the financing needs.

As potential output increases, the choice of duration of the low growth regime converges to the optimum but the choice of σ drifts away from the efficient value. This last result simply reflects the property that there is an inverse relationship between the value of the debt and σ. Low σ is chosen to lower the cost of financing.[9]

As in the case of exogenous technology there is a positive relationship between potential output and maturity: Firms with low present value of earnings choose shorter term debt, while "similar" firms with higher potential output in the high growth regime choose to reduce the time it takes to switch regimes and, at the same time, they decrease the risk of maturity in the low output/illiquid state by issuing long term bonds.

Finally, we note that, relative to the optimum there could be either under investment —as when x is low— or over investment but in all cases the need to finance the project results in inefficient choice of technology.

Leverage In this section we assume that the firm only needs to finance 70% of the (endogenous) investment.[10] Figure 5 summarizes the results. For completeness we also display the efficient choice of technology.

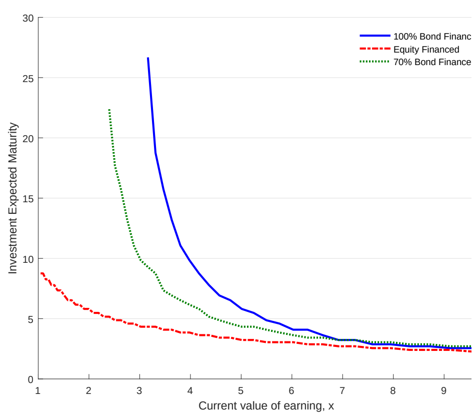

Figure 5.a: Duration - Low Growth

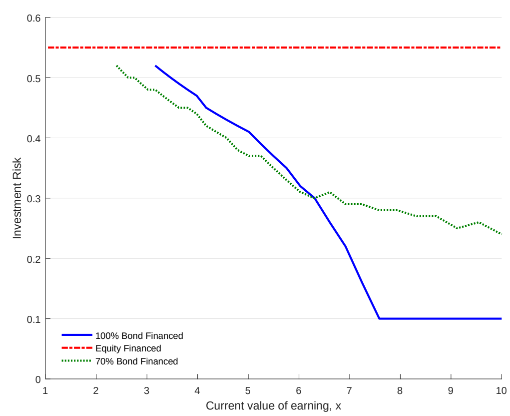

Figure 5.b: Riskiness - High Growth

[9]One can show that as $x \to \infty$ the endogenously chosen σ converges to the efficient one.
[10]In Appendix B we show the results when the firm has a fixed amount of resources

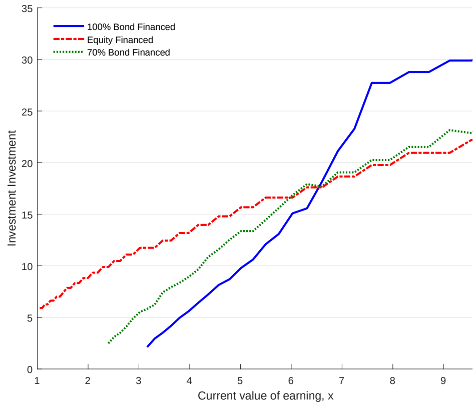

Figure 5.c: Investment

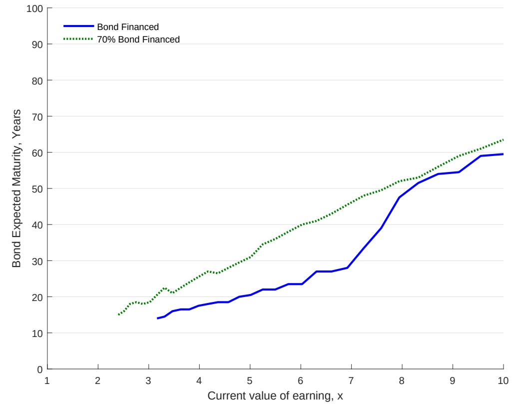

Figure 5.d: Maturity

The overall picture is that reducing the financing needs moves the choice of technology closer to the optimum. However, even at this level of leverage there are significant distortions in terms of riskiness and total investment for relatively high levels of potential output. As in the exogenous case firms that display lower leverage issue longer term bonds as the lower borrowing needs reduce the duration of the low growth regime and decrease the price of strategic risk (this is due to lower face values of the bond)

Asset Salability Our last sensitivity results explores the impact on the choice of technology and on the optimal structure of the debt of an increase in the post-default value of the firm's assets. The results are in Figure 6

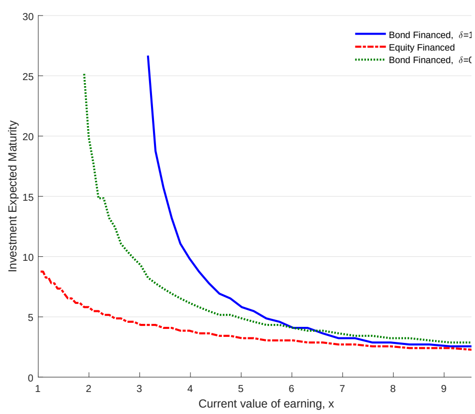

Figure 6.a: Duration - Low Growth

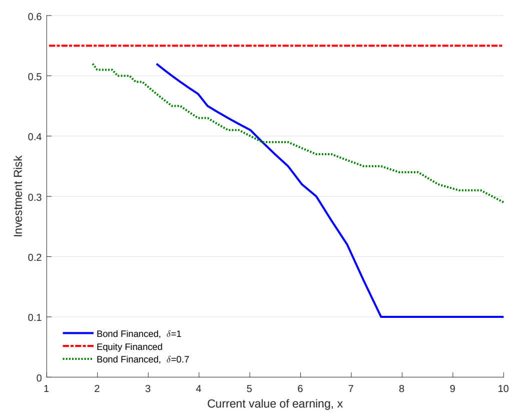

Figure 6.b: Riskiness - High Growth

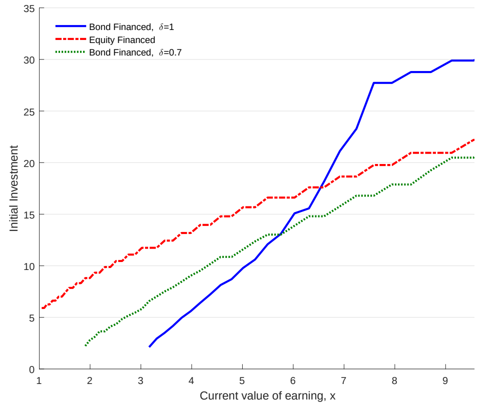

Figure 6.c: Investment

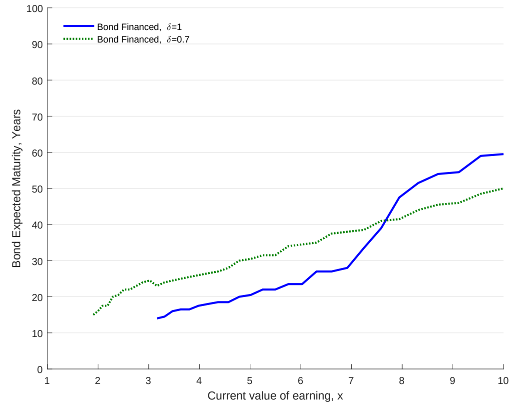

Figure 6.d: Maturity

As in the case of leverage, a lower value of δ reduces the price of high growth default risk. For low values of x, this allows the firm to trade off that risk against the risk of illiquidity default. The result is that, on the technological side, the duration of the low growth regime is lower and the maturity of the debt is longer. For relatively high values of x the low δ firm issues shorter term debt.

The effect of a lower δ on the choice of σ and on the level of investment is not monotonic in x. For low levels of potential output, the firm chooses a less risky technology and a higher level of investment (which is the cost) due to the lower σ and the lower $1/\upsilon$.

4 Extensions:

4.1 One Bond Case: More General Technology

In this section we describe the valuations of debt and equity in a slightly more general model. We assume, as before, that the firm's initial state is illiquid (although this may not be a good label any more). During this initial phase the firm has earnings given by $z > 0$. At the time, say t, —driven as before by a Poisson with parameter υ— of the switch to the high growth regime earnings of the firm are

$$x_t = (1+\gamma)x_{t^-} + \theta z, \;\; \gamma > 0 \text{ and } \theta \in [0,1].$$

Thus, relative to the old x_t process the switch to the high growth regime implies that earnings increases by γ%. Of course, we can make $\gamma = 0$. The other feature is that there is a tradeoff as the safe component of earnings decrease from z to θz. The reason for this is that now in the low growth regime the firm can issue a bond that promises to pay $b \leq z$ and, in this region, there is still no strategic default. However, if $b > \theta z$ then once the technology switches to the high growth regime there is true strategic motive for default is x_t is sufficiently low.

4.2 More Productive Technology

The model described above —and, in particular, the one bond version that we used for the preliminary numerical results— has some limitations. In this section we briefly discuss how to generalize the model.

In the previous section it was optimal (in the strong sense of being the only feasible option) for the firm to issue a pure discount bond. This was driven by our assumption that output is zero in the illiquid phase. A simple alternative is to view the technology state zero as associated with a low —but not zero— stochastic process for output and the switch to the technology state one as a growth option.

One possible such a specification is the following: In technology state zero output satisfies,

$$dx_t = \mu_0 x_t dt + \sigma_0 x_t dZ_t.$$

Since output is nonnegative bonds with positive coupon rate are possible and, in this case, the model accommodates strategic default in the sense of Leland (1996). In this case the gross value of a 100% equity financed firm is

$$E(x, \mu_0, \sigma_0, \upsilon, \mu_1, \sigma_1) = \frac{r + \upsilon - \mu}{r + \upsilon - \mu_0} \frac{x}{r - \mu},$$

which is similar to our finding before.

4.3 Cash Hoarding

So far we have assumed that the firm pays out as dividends its earnings. This, of course increases the chance that a bad shocks at the time that the bond matures will result in bankruptcy. In this section we extend the model to allow the firm to accumulate earnings. We assume that this alternative "liquid asset" has a lower return, that is there is a carry cost that we denote ψ but, unlike expected earnings, can be used to repay the bond. Formally, this liquid asset earns a return $r - \psi$. We also assume that the firm has another line of business that generates income. To simplify we assume that this income is constant and equal to $m > 0$. Let the stock of the liquid asset be denoted z. It evolves according to

$$dz_t = (r - \psi) z_t dt + m dt + dc_t - dn_t$$

where c_t is cumulative cash hoarding. We assume that c_t is a nondecreasing process and, since we need to allow for jumps, it need not have the form $dc_t = c_t dt$ (although it will in some regions). We denote by n_t the cumulative cash disbursals, which is also a nondecreasing process. In this setting we do not assume that the coupon b is zero. Finally, we restrict the stock of liquid assets to be nonnegative, that is $z_t \geq 0$. Payments to shareholders are given by

$$x_t - b - dc_t + dn_t \geq \pi_L,$$

where π_L (if negative) is the maximal contribution that shareholders are willing to make in any given period.

The relevant HJB equation describing the value of the firm in the high growth regime is

$$rT(x,z;\eta,b,K) = \max_{dc_t,dn_t}\{x_t - b - dc_t + dn_t + \frac{\partial T}{\partial x}(x,z;\eta,b,K)\mu x \tag{11}$$
$$+\frac{\partial^2 T}{\partial x^2}(x,z;\eta,b,K)\frac{\sigma^2}{2}x^2 + \frac{\partial T}{\partial z}(x,z;\eta,b,K)[(r-\psi)z + m + dc - dn]$$
$$+\eta[\frac{m}{r} + \max(z + \frac{x}{r-\mu} - K, 0) - T(x,z;\eta,b,K)],$$

In this formulation, we are implicitly assuming (essentially for convenience at this point) that just before declaring bankruptcy the shareholders can spin-off the cash project. Thus, they get to keep m/r, the present value of the cash project that is not included in bankruptcy proceedings. At the same time, we assume that the stock of liquid assets, z, is included in the bankruptcy.

In this case, we conjecture that the firm will choose to default whenever its income fall below a certain level that depends on its cash holdings[11]. Thus, the firm chooses to default the first time that $x_t \leq x_L(z)$, for some function $x_L(z)$.

$$\lim_{x\to x_L(z)} T(x,z;\eta,K) = 0, \text{ and } \lim_{x\to x_L(z)} \frac{\partial T}{\partial x}(x,z;\eta,b,K) = 0$$
$$\lim_{x\to\infty} \frac{T(x,z;\eta,K)}{x} < \infty \text{ and } \lim_{z\to\infty} \frac{T(x,z;\eta,K)}{z} < \infty$$

We assume that, as before, when the bond matures the firm can sell shares and raise $x/(r-\mu)$. In addition it has liquid assets worth z. Thus, the value of an ongoing firm is the sum of all three values minus the amount that it has to pay to bondholders (K).

5 Conclusion (preliminary)

We study the optimal choice of project characteristics —duration of the low growth regime and riskiness of the growth rate in the high growth regime— and the structure of the debt by a risk neutral agent. At this point, the results from a simple version of the model show that:

- Optimal maturity (from the point of view of the borrower) increases with the level of potential output. Put it differently, firms that need to issue debt that would have

[11]The literature on two dimensional optimal stopping problems is very limited and restricted to special cases. For this reason, some of the arguments in this section are very speculative.

a higher probability of default given their current (potential) output, choose shorter term debt. This is driven by features of the environment and calls into question policy recommendations that implicitly take the maturity structure as exogenous. In our model, firms with poor prospects choose short term debt, not the other way around.

- The optimal maturity strikes a balance between two risks: the risk of default in the low growth (illiquid) regime and the risk of strategic default (in the high growth regime). As the level of (potential) output increases the shadow price of the risk of strategic default goes down and the cost of default in the low growth regime goes up and, hence, it is optimal to lower the risk of default in the low growth (initial) regime by extending the maturity of the debt.

- The degree of uncertainty in the economic environment influences the choice of financial structure. In the examples that we study we find that higher uncertainty about growth rates is associated with shorter term debt. Moreover, across borrowers with different levels of output (and, hence, different default probabilities) the differences in endogenously chosen expected debt maturity are smaller than in low uncertainty environments.

- The need to use debt markets to finance a project has an impact on the characteristics of the projects that are implemented. In general, the better the prospects of the firm (in the form of higher potential output) the shorter the chosen duration of the low growth regime and the safer the project. Moreover, as prospects improve, those projects are financed with longer term debt.

- Project characteristics are distorted relative to the choices made by firms that do not need external financing. The need to issue debt results in less risky choices in the high growth regime but in longer duration of the low growth period.

- Investment projects with higher value (to the lenders) upon default (e.g. higher resale value or lower cost of fire sales) and projects with lower leverage are financed with longer term debt.

In ongoing work we are extending the quantitative results to the case of refinancing and computing the optimal choice of technology so that the impact of financial structure on real variables can be quantified.

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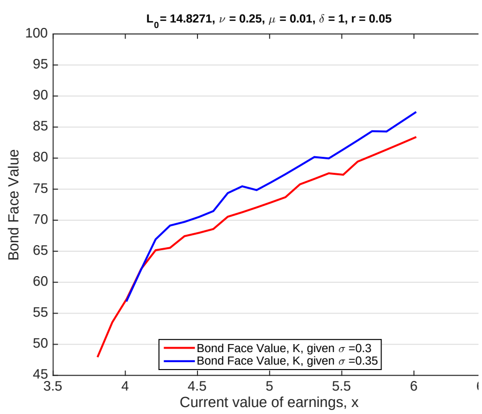
6 Appendix A: Proofs and Theoretical Additional Results

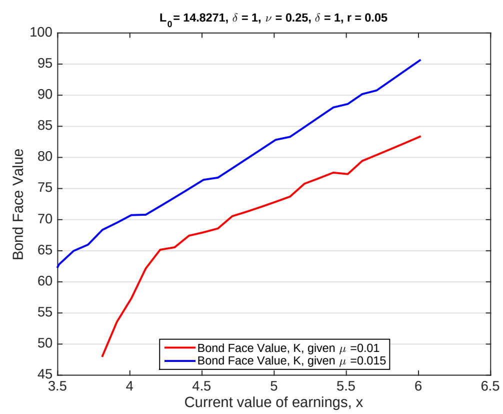
7 Appendix B: One Bond Case: Additional Graphs

7.1 Exogenous Technology

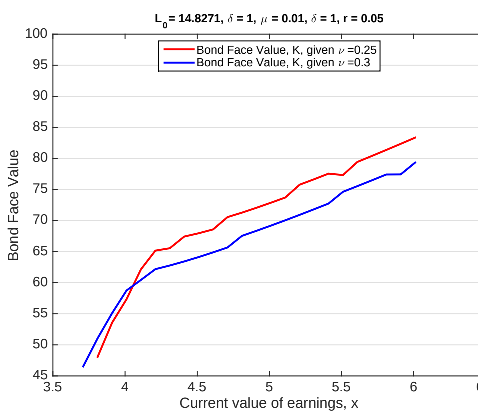
Face Value



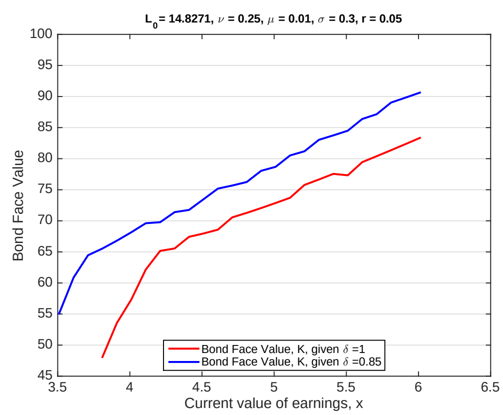
Face Value and σ



Face Value and μ

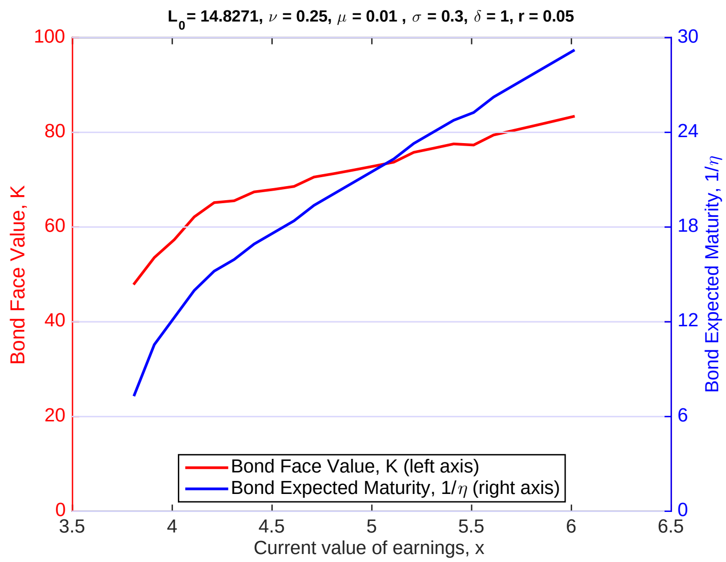


Face Value and υ



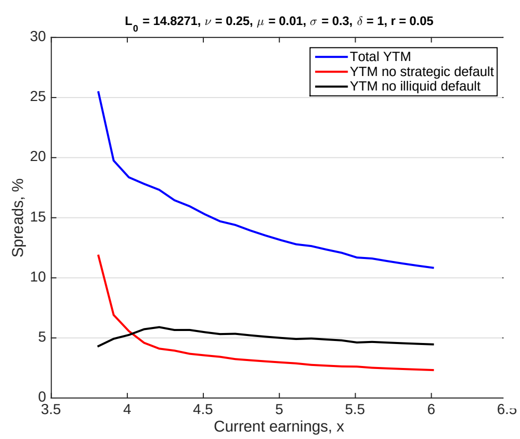
Face Value and δ

Structure of Debt

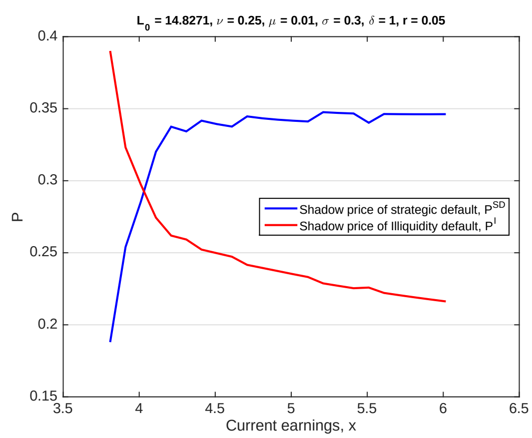


Debt Structure and (Potential) Output

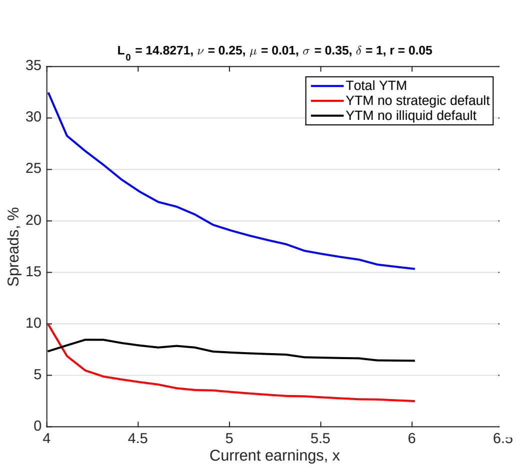
Risk Prices



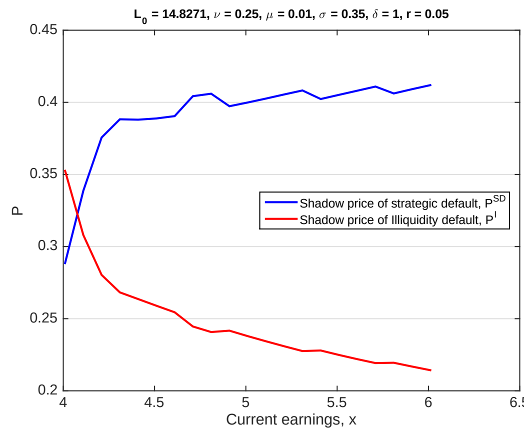
Yield to Maturity (Baseline)



Prices of Illiquid and Strategic Risk (Baseline)



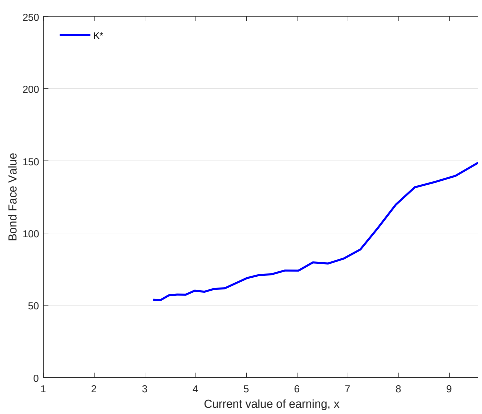
Yield to Maturity (High σ)

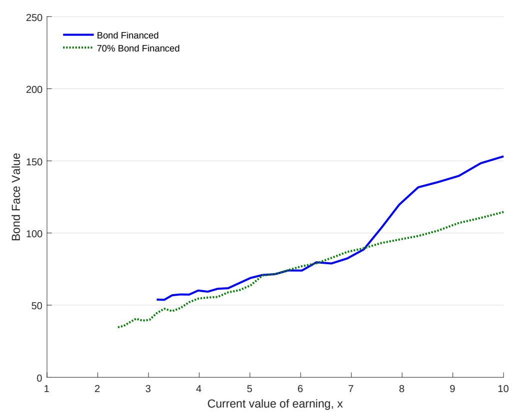


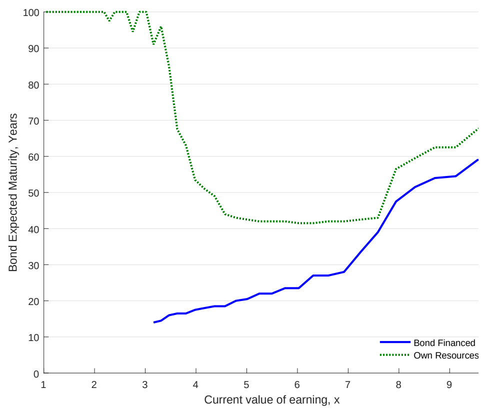
Prices of Low Growth and Strategic Risk (High σ)

7.2 Endogenous Technology Choice

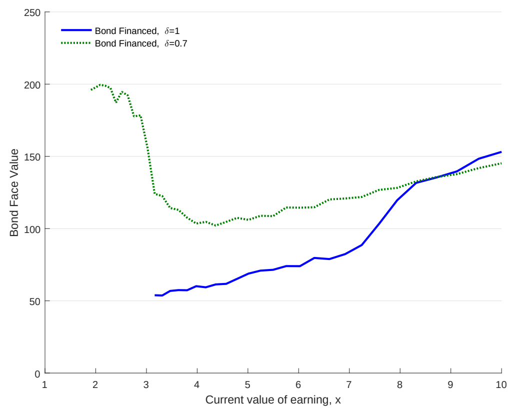
Face Value







Bond Expected Maturity, Years
Current value of earning, x
Bond Financed
Own Resources



Bond Face Value
Current value of earning, x
Bond Financed, δ=1
Bond Financed, δ=0.7