

Inequality and Aggregate Demand

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Extremely preliminary

Abstract

We explore the quantitative effects of transitory and persistent increases in income inequality on equilibrium interest rates and output. Our starting point is a Bewley-Huggett-Aiyagari model featuring rich heterogeneity and earnings dynamics as well as downward nominal wage rigidities. A temporary rise in inequality, if not accommodated by monetary policy, has an immediate effect on output that can be quantified using the empirical covariance between income and marginal propensities to consume. A permanent rise in inequality can lead to a permanent Keynesian recession, which is not fully offset by monetary policy due to a lower bound on interest rates. We show that the magnitude of the real interest rate fall and the severity of the steady-state slump can be approximated by simple formulas involving quantifiable elasticities and shares, together with two parameters that summarize the effect of idiosyncratic uncertainty and real interest rates on aggregate savings. For plausible parametrizations the rise in inequality can push the economy into a liquidity trap and create a deep recession. Capital investment and deficit-financed fiscal policy mitigate the fall in real interest rates and the severity of the slump.

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1 Introduction

There is an old idea that the distribution of income is an important determinant of aggregate economic activity, with higher income inequality translating into lower aggregate demand and employment. This idea is reflected in modern discussions on the consequences of income inequality. For example, the 2012 Economic Report of the President ([Council Of Economic Advisers \(2012\)](#)) argues that

some of the recent patterns in aggregate spending and saving behavior—including the sluggish growth in consumer spending—may reflect the sharp rise over the past 30 years in the inequality in the income distribution in the United States. [...] For example, the rise in income inequality may have reduced aggregate demand, because the highest income earners typically spend a lower share of their income—at least over intermediate horizons—than do other income groups.

Higher income inequality might, everything else equal, lower aggregate consumption for two related reasons. The first reason, highlighted in the quote above, is that richer households tend to have higher marginal propensities to save than poorer households.¹ Hence *transitory* increases in income inequality, which can be seen as the outcome of a transfer of resources from poor to rich agents, tend to reduce current aggregate consumption.

Permanent increases in income inequality, on the other hand, may also reduce aggregate consumption if they are the consequence of higher earnings risk perceived by households, and therefore increase desired precautionary savings. Much recent research has in particular argued that the increase in income inequality is likely due—at least in part—to an increase in the persistent component of earnings risk (see for example [Kopczuk, Saez and Song \(2010\)](#) and [Debacker, Heim, Panousi, Ramnath and Vidangos \(2013\)](#)), which can make this effect particularly strong.

Hence partial equilibrium reasoning suggests that widening inequality can reduce aggregate consumption and increase aggregate savings. Whether this effect can in turn result in a decline in aggregate GDP depends on macroeconomic equilibrating mechanisms. General equilibrium models differ on the details of these mechanisms, but

¹This is old idea (for example [Pigou \(1920\)](#), [Keynes \(1936\)](#), and [Kaldor \(1955\)](#)) has received empirical support in much of the recent empirical literature (see [Dynan, Skinner and Zeldes \(2004\)](#), [Johnson, Parker and Souleles \(2006\)](#), [Parker, Souleles, Johnson and McClelland \(2013\)](#), [Jappelli and Pistaferri \(2014\)](#), or [Misra and Surico \(2014\)](#)). It is also a prediction of a broad class of macroeconomic models with incomplete markets and precautionary savings.

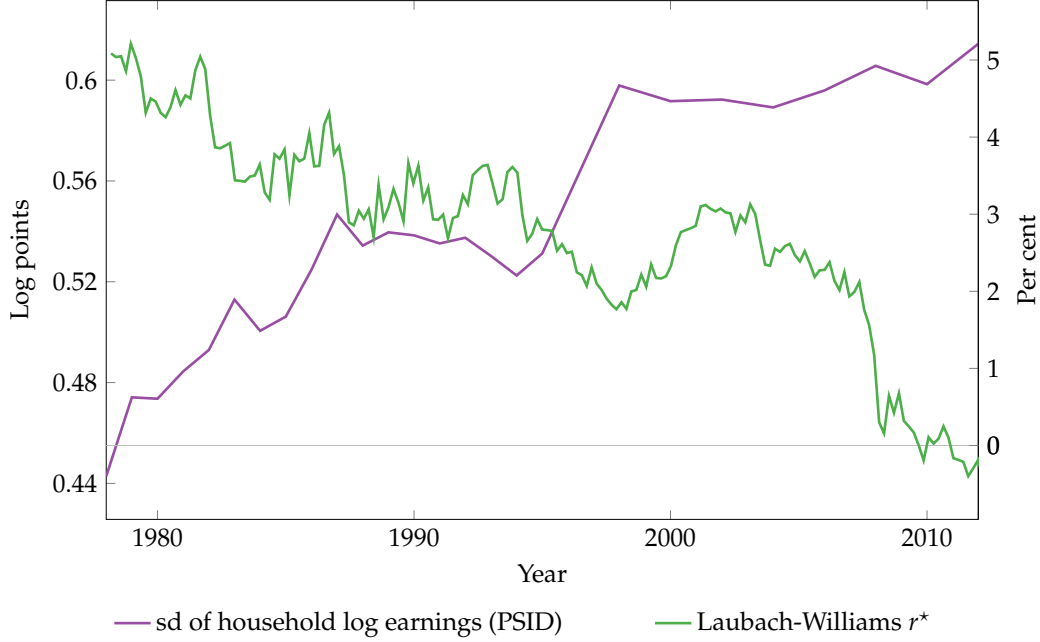


Figure 1: U.S. income inequality and the equilibrium real interest rate

most of them would predict a fall in equilibrium real interest rates. And indeed, broad trends in income inequality and real interest rates are consistent with such a causal effect of inequality. Figure 1 plots the Laubach-Williams measure of the equilibrium U.S. real interest rate against the standard deviation of household log earnings in the PSID from 1978 to 2012. The purple line shows that household income inequality, after rising rapidly in the 1980s and slowing down at the beginning of the 1990s (a trend documented in [Blundell, Pistaferri and Preston \(2008\)](#), whose sample selection we follow exactly), continued to rise in the late 1990s. The trend prolonged itself in the 2000s, and inequality in the most recent wave is the highest recorded. During the same period, the U.S. real interest rate fell, as illustrated by the green line, a widely-used measure of the equilibrium rate ([Laubach and Williams \(2003\)](#)). Both trends have been widely documented. The 17 point rise in the standard deviation of log earnings, and the 5 percentage point fall in the equilibrium real interest rate over this 35-year period are also illustrative of the magnitudes that the literature has found, and we take them as reasonable benchmarks.

Motivated by these trends, in this paper we investigate the quantitative effects of transitory and persistent increases in income inequality on equilibrium interest rates and output. We build a Bewley-Huggett-Aiyagari model with rich heterogeneity and earnings dynamics, allowing us to capture both the heterogeneity in marginal propen-

sities to consume by income group and the precautionary savings motive against earnings risk. To incorporate a channel of transmission from lower aggregate consumption to reduced output, we augment the standard model with downward nominal wage rigidities. This allows us to study the model in a Keynesian regime, in which real interest rates are fixed as inequality rises, due to the combination of unresponsive monetary policy—possibly because of a lower bound on nominal interest rates—and fixed nominal prices. Using this laboratory we investigate the transmission of inequality to output and the role played by monetary policy, fiscal policy, and private investment. We ask in particular the extent to which the observed rise in income inequality since the early 1980s might have depressed real interest rates and reduced the level of output.

2 Model

2.1 Environment

Households We consider a population made of ex-ante identical households who face idiosyncratic, but no aggregate risk. In each period t , household i is in idiosyncratic state $s_{it} \in \mathcal{S}$. s_{it} follows a Markov process with transition matrix Λ . We assume that at all times, the mass of households in each idiosyncratic state s is equal to the probability $\lambda(s)$ of s in the ergodic distribution induced by Λ .

The household maximizes $\mathbb{E} [\sum (\prod_{\tau \leq t} \beta_\tau (s_{i\tau})) u(c_{it})]$ subject to the period budget constraints

$$\begin{aligned} c_{it} + \frac{a_{i,t+1}}{1+r_t} &= y_t(s_{it}) + a_{it} \\ a_{i,t+1} &\geq 0 \end{aligned}$$

where u is a common CRRA period utility function, $u(c) = \frac{c^{1-\frac{1}{\psi}} - 1}{1-\frac{1}{\psi}}$.

Assets $a_{it} = b_{it} + ((1-\delta)q_t + d_t)v_{it}$ are made of bonds b_{it} and shares v_{it} in a capital-producing firm. Each share costs q_t at time t and delivers $1-\delta$ shares at time $t+1$ together with a dividend d_{t+1} . With perfect foresight over q_t , d_t , and the real interest rate r_t , the arbitrage equation

$$1 + r_t = \frac{(1-\delta)q_{t+1} + d_{t+1}}{q_t} \tag{1}$$

holds at all times, and bonds are perfect substitutes for capital. Hence, for a given

individual only the level of assets a , and not its components b and v , is pinned down.

We assume that post-tax income y_{it} follows:

$$y_t(s_{it}) = v_t(s_{it}) - T_t(v_t(s_{it})) \quad (2)$$

In (2), T_t represents the government tax function, mapping pre-tax income v_{it} to tax liability. In turn, pre-tax income is given by the product of the real wage $\frac{W_t}{P_t}$ and the amount of endowment that households are able to supply:

$$v_{it} = \frac{W_t}{P_t} \cdot \left(e_t(s_{it}) \cdot L_t \cdot \gamma(s_{it}, L_t) \right) \quad (3)$$

where the γ function satisfies $\gamma(s, L = 1) = 1$ for all s . Households' full idiosyncratic labor endowment is e_{it} , and this is the amount that they supply in case of full employment ($L_t = 1$). As per the standard formulation in the literature, their pre-tax income is then given by $\frac{W_t}{P_t} e_{it}$. Since our focus is on inequality, we assume that the aggregate endowment of labor is constant and equal to 1: $\sum \lambda(s) e_t(s) = 1$. By contrast, we will let other moments of the distribution of $\{e_t(s)\}$ change over time to capture "fundamental" increases in inequality.

Households may not be able to supply their full endowment if there is a labor demand shortfall, $L_t < 1$. In that case, a household in idiosyncratic state s is constrained to supply the fraction $L_t \cdot \gamma(s, L_t)$ of his full endowment, with L_t describing the effect of aggregate employment conditions and γ the distributional impact of these conditions. We make this aggregate/distributional distinction precise by assuming that γ satisfies

$$\sum \lambda(s) e_t(s) \gamma(s, L) = 1 \quad \forall L \leq 1, \forall t$$

and hence $\sum \lambda(s) v_t(s) = \frac{W_t}{P_t} L_t$ at all times. When $\gamma(s, L_t) = 1$, for all s , all households are equally rationed. By contrast, when $\gamma(s, L_t) \neq 1$ for some s , labor demand shortfalls can be a source of endogenous increase in inequality. We will see that this can give rise to a phenomenon we call the *inequality multiplier*.

Our notation embeds two possible views of the labor market, each of which is associated with a different equilibrium determination of the real wage $\frac{W_t}{P_t}$ and the employment rate L_t :

- a) A classical view, where firms and households enter a spot labor market each period, and the real wage $\frac{W_t}{P_t}$ adjusts to the point where labor demand and the ag-

aggregate endowment are equal. In this case $L_t = 1$ at all times.

- b) A Keynesian (or disequilibrium) view, which modifies the classical view by imposing that the nominal market wage W_t cannot fall from period to period:

$$W_t \geq W_{t-1} \quad (4)$$

and that if labor demand falls short of the aggregate endowment because the constraint (4) is binding, households are rationed ($L_t < 1$) and are each constrained to supply the fraction $L_t \gamma(s_{it}, L_t)$ of their labor endowment.

Final goods firms Perfectly competitive firms operate the constant returns to scale production function:

$$Y_t = F(K_t, L_t)$$

These firms maximize profits $PF(K, L) - WL - RK$ in each period, leading to the factor demand equations

$$\frac{R_t}{P_t} = F_K\left(\frac{K_t}{L_t}, 1\right) \quad (5)$$

$$\frac{W_t}{P_t} = F_L\left(\frac{K_t}{L_t}, 1\right) \quad (6)$$

Capital producing sector A representative, competitive firm owns the capital stock K_t . Capital depreciates at rate δ and delivers a nominal rent of R_t in period t . Each period the firm rents out the capital stock and decides how much to invest for next period. Net investment $K_{t+1} - K_t = I_t - \delta K_t$ is subject to convex adjustment costs worth $\zeta\left(\frac{K_{t+1} - K_t}{K_t}\right) K_t$. The ζ function satisfies:

$$\zeta(0) = 0 \quad \zeta'(0) = 0 \quad \zeta''(\cdot) \geq 0$$

The firm discounts the future at rate r_t . Hence its value in period t is

$$J_t(K_t) = \max_{I_t} \left\{ \frac{R_t}{P_t} K_t - I_t - \zeta\left(\frac{I_t}{K_t} - \delta\right) K_t + \frac{1}{1 + r_t} J_{t+1}(K_{t+1}) \right\} \quad (7)$$

Guess and verify that this value function is linear in K_t , and define current period q as

$$q_t \equiv \frac{1}{1 + r_t} \frac{J_{t+1}(K_{t+1})}{K_{t+1}}$$

From the firm's optimal investment decision follows the Q -theory relationship:

$$1 + \zeta' \left(\frac{I_t}{K_t} - \delta \right) = q_t \quad (8)$$

Defining the dividend paid out by the firm at time t per unit of capital as the current MPK plus the implicit rents on capital

$$d_t = \frac{R_t}{P_t} + \left(\frac{I_t}{K_t} \zeta' \left(\frac{I_t}{K_t} - \delta \right) - \zeta \left(\frac{I_t}{K_t} - \delta \right) \right) \quad (9)$$

it is easy to see that the cost of a share to the household and the marginal value of capital to the firm are equal at all times, as anticipated in the notation.

Government The government supplies a quantity of bonds $\bar{B}_t$ at time t , spends G_t , and collects taxes on labor income through the tax function T_t

$$\sum \lambda(s) T_t \left(\frac{W_t}{P_t} \cdot e_t(s) \cdot L_t \cdot \gamma(s, L_t) \right) + \frac{\bar{B}_{t+1}}{1 + r_t} = G_t + \bar{B}_t$$

We will consider various fiscal rules.

The central bank controls the nominal interest rate i_t . Perfect foresight implies that the real interest rate is

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}} \quad (10)$$

where π_{t+1} is the inflation rate, $1 + \pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$.

2.2 Equilibrium

Definition 1. Given K_0 and an initial joint distribution $\Psi_0(s, a)$ over idiosyncratic states and assets, a *classical closed-economy equilibrium* is a set of aggregate quantities $\{C_t, I_t, K_t, Y_t, L_t, V_t, B_t\}$, prices $\{i_t, r_t, P_t, R_t, W_t\}$, and government policy $\{T_t, \bar{B}_t\}$, individual decision rules $\{c_t(s, a), b_{t+1}(s, a), v_{t+1}(s, a), a_{t+1}(s, a)\}$ and joint distributions $\Psi_t(s, a)$, such that households maximize utility subject to their budget constraint, firms maximize profits, the government follows its fiscal rules, the Fisher equation (10) holds, the distribution of households is consistent with the exogenous law of motion

and the decision rules, and all markets clear:

$$\begin{aligned}
L_t &= 1 \\
V_{t+1} &\equiv \int v_{t+1}(s, a) d\Psi_t(s, a) = K_{t+1} \\
B_{t+1} &\equiv \int b_{t+1}(s, a) d\Psi_t(s, a) = \bar{B}_{t+1} \\
C_t + I_t + G_t + \zeta \left(\frac{K_{t+1} - K_t}{K_t} \right) K_t &= Y_t
\end{aligned}$$

Definition 2. A *classical open-economy* equilibrium is the same as above except that r_t is exogenous, and asset markets are no longer required to clear.

Definition 3. A *Keynesian closed-economy* equilibrium is the same as the classical closed economy, except that $W_t \geq W_{t-1}$, and labor market clearing is replaced by $L_t \leq 1$, with $W_t = W_{t-1}$ whenever $L_t < 1$.

2.3 Experiments

We now take the steady-state of a closed-economy neoclassical equilibrium as a starting point for two classes of experiments. Both address the consequence of *exogenous increases in inequality*. Specifically, we imagine that at $t = 0$, households learn about a change that affects the distribution of post-tax incomes going forward. These changes could stem from changes in fiscal policy $T_t(v)$ or in the distribution of labor endowments $\{e_t(s)\}$ itself. In section 3, we study temporary changes, and analyze the transition dynamics in the Keynesian regime back to the steady-state, with a focus on the impact effect on GDP. We then move on to the consequence of a permanent increase in inequality. Since the economy then converges towards a new steady-state, we first characterize steady-state equilibria in all three regimes in section 4, and then study in the quantitative effect of the increase in inequality under all three regimes, both in the transition and in the steady-state in section 5.

3 Temporary increases in inequality

In this section, we consider the effect of *temporary* increase in inequality. We therefore assume that an exogenous change in the post-tax income distribution takes place at $t = 0$ only. One natural source of increased inequality is a balanced-budget change in tax policy $T_0(v)$ that makes the tax system less progressive in that period, but we

could also imagine a change in the distribution of $e_0(s)$ that holds the mean constant. We obtain approximations for the immediate change in consumption that depend on sufficient statistics, and see how well these sufficient statistics perform in quantitative experiments.

We consider a flexible parametrization of the distributional change, such that the distribution of household post-tax incomes (3) is affected as follows:

$$\begin{aligned} dy_0(s) &= g(s) d\eta \\ \sum_s \lambda(s) g(s) &= 0 \end{aligned} \tag{11}$$

We focus in particular on two g functions satisfying the constraint in (11), defined as

$$\begin{aligned} g^{s'}(s) &\equiv \frac{1_{\{s=s'\}}}{\lambda(s')} - 1 \\ g^\tau(s) &\equiv e(s') - 1 \end{aligned}$$

$g^{s'}$ is a redistributive change that favors group s' at the expense of every other group—for example, a fiscal policy rebate to households in state s' , financed with an equal lump-sum tax on everyone. g^τ is a change that affects households in proportion to their pre-tax income. An example is an decrease in the progressivity of the tax system, lowering all marginal income tax rates by the same amount and financing this change by a lump-sum tax.

We look at equilibria where monetary policy maintains the real interest rate r_t fixed throughout the transition of this economy back to steady-state. This can capture either liquidity trap conditions, where monetary policy is constrained by a bound on nominal interest rates, or a situation where monetary policy is *inattentive* to the change in the income distribution—for example, dismissing it due to its temporary nature.

3.1 Partial equilibrium analysis

Define the time- t MPC of an agent in state (s, a) as their expected change in consumption at time t from an increase in current liquidity a :

$$MPC_t(s, a) \equiv \frac{\partial \mathbb{E}[c_t | s_0 = s, a_0 = a]}{\partial a}$$

We write, in particular, $MPC \equiv MPC_0$. A large empirical literature estimates those MPCs in cross-sections, and reports them by income group (see for example [Johnson et al. \(2006\)](#), [Parker et al. \(2013\)](#) or [Jappelli and Pistaferri \(2014\)](#)). The next proposition shows how to use this information to compute the *partial-equilibrium effect* (holding both r_t and L_t fixed at all times) of the redistribution on aggregate consumption at $t = 0$.

Proposition 1. *The immediate, first-order, aggregate partial equilibrium consumption response at time 0 of any considered change $g(s)$ is given by*

$$dC_0^{PE} = \text{Cov}(\overline{MPC}(s), g(s)) d\eta \quad (12)$$

where $\overline{MPC}(s)$ is the average MPC across all households with income s . In particular $g^{s'}$ yields

$$dC_0^{PE,s'} = (\overline{MPC}(s') - \overline{MPC}) d\eta$$

where $\overline{MPC}(s)$ is the average MPC across all households, while g^τ yields

$$dC_0^{PE,s'} = \text{Cov}(\overline{MPC}(s), e(s)) d\eta$$

Proof. It is clear that the first-order time- t partial equilibrium consumption response to the redistribution induced by $g(s)$ is

$$dc_t^{PE}(s, a) = MPC_t(s, a) g(s) d\eta$$

Aggregating across agents, we obtain the aggregate *partial equilibrium effect*:

$$dC_t^{PE} = \text{Cov}(\overline{MPC}_t(s), g(s)) d\eta$$

where $\overline{MPC}_t(s)$ is the average MPC_t across all households with income s . In particular, the time-0 effect on aggregate consumption is given by (12). $\square$

Proposition 1 gives simple sufficient statistics formulas to evaluate the effect of the redistribution on aggregate consumption, as in [Auclert \(2015\)](#). The impact of a distributional change $g^{s'}$ that favors households in state s' is, intuitively, given by the difference between the average MPC of households in this state and average MPC overall. The impact of a fall in tax rates $d\eta$ financed by a lump-sum tax increase is given by the product of the covariance between MPC and pretax incomes and $d\eta$. These formulas

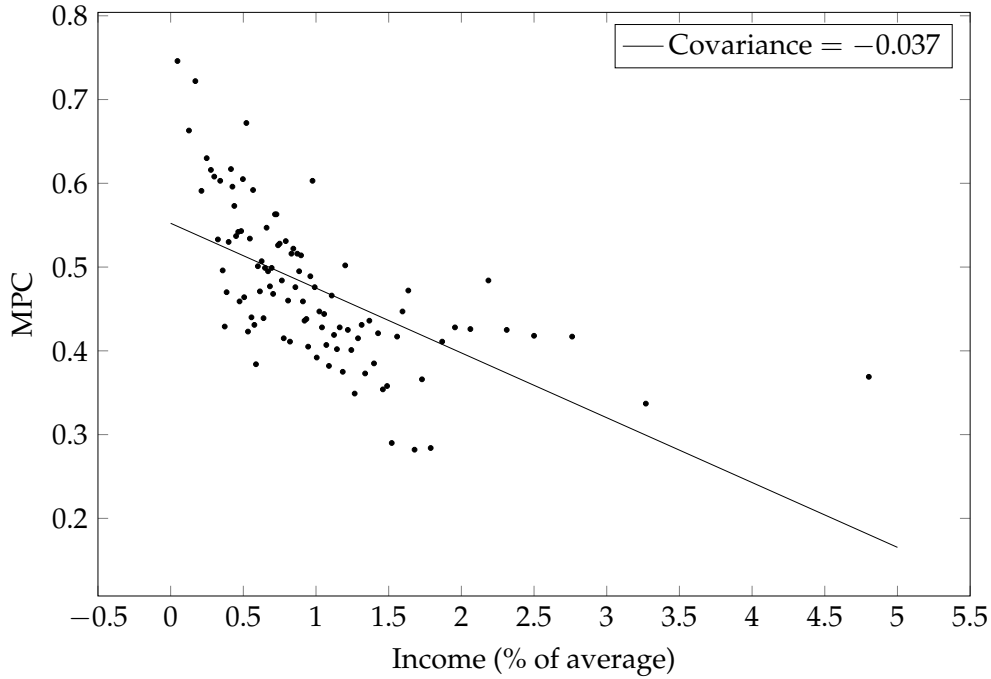


Figure 2: Average MPC by income group centile in the 2010 SHIW

are helpful to get a sense of the *aggregate magnitudes* involved by such changes, since the redistribution acts as an “impulse effect” for macroeconomic effects, as we discuss in more detail below.

Figure 2 plots the level of average MPC by income group in the 2010 Italian Survey of Household Income and Wealth (SHIW), analyzed in detail in Jappelli and Pistaferri (2014). The covariance between MPCs and normalized income in that survey is -0.037 , implying that a 10 percentage point balanced-budget fall in the income tax rate (which redistributes towards the richer agents) would lower aggregate consumption by 0.37%. This is not a very large number. Some of this may be due to measurement error in the Italian survey, so that more sources of data are needed to get a full sense of the magnitudes involved.

The other reason why the formula in (12) is not a complete answer is because it is only a partial equilibrium response. General equilibrium effects are likely to amplify the response given here, for three reasons: because of a Keynesian feedback between consumption and income, because income increase may endogenously reduce inequality and therefore have a multiplier effect, and because of the endogenous response of investment. We investigate these effects in turn, attempting to maintain the generality of the approach developed here, even though investigating general equilibrium neces-

sarily implies additional assumptions.

3.2 A benchmark case with no capital

We first consider a special class of economies: a Huggett economy with no capital, so that $C_t = Y_t = L_t$, which further satisfies equiproportional distribution of income $\gamma(s, L) = 1, \forall s, L$.

In this case, the date-0 consumption change is given by

$$dc_0^{GE}(s, a) = MPC(s, a) \left\{ g(s) d\eta + \frac{y(s)}{Y_0} dY_0 \right\} + \sum_{t \geq 1} (\dots) dY_t + \sum_{t \geq 0} (\dots) dT_t$$

If we suppose that dT_t and dY_t for $t \geq 1$ are negligible second-order effects, we can solve this equation for dY_0 as

$$dY_0^{GE0} \equiv \frac{\text{Cov}(\overline{MPC}(s), g(s))}{1 - \overline{MPC}^y} d\eta \quad (13)$$

where $\overline{MPC}^y$ is the income-weighted MPC. Formula (13) is an approximation to the full equilibrium response of the economy. Its benefit is that it is also computable using survey information. For example, in the Italian survey mentioned in the previous section, the income-weighted MPC is $\overline{MPC}^y = 0.47$. This implies that an approximation to the general equilibrium effect of a 10pp balanced-budget fall in the tax rate is a fall of $\frac{0.37\%}{1-0.47} = 0.69\%$ in aggregate GDP.

How well does (13) perform in approximating the full equilibrium response of the economy? We can investigate this question in the context of a calibrated model. Preliminary results show that the approximation is very good across a broad class of simulations. [to be completed]

3.3 The inequality multiplier

We now relax the assumption of proportional distribution of income from labor demand shortfalls (ie $\gamma \neq 1$). When $\gamma \neq 1$, the exogenous increase in inequality can result in a labor demand shortfall and induce endogenous increases in inequality, further reducing labor demand. We call this phenomenon the *inequality multiplier*. We can investigate the quantitative importance of this phenomenon using a calibrated model together with quantitative estimates of the distributional effect of exogenous in GDP.

[to be completed]

3.4 The role of investment

We now move on to the study of investment, relaxing the assumption of no capital. We maintain our assumption of fixed r_t , making investment behavior sensitive to the future level of GDP. Hence, to the extent that the redistribution creates persistent effects on aggregate consumption, investment will tend to *amplify* the initial response. We can investigate the strengths of these effects in a calibrated model under various assumptions about the sensitivity of investment to Q . [to be completed]

4 Steady-state analysis

We now illustrate how our Keynesian closed economy equilibrium concept compares to the usual classical concepts, by characterizing the steady states of all our equilibria. We will see that our Keynesian equilibrium accommodates the possibility of a steady-state slump. We then discuss the role of fiscal policy in this environment.

The following equations characterize the steady states of all three regimes. On the investment side, $I = \delta K$ implies $q = 1$ from (8), so that (1) and (9) together imply $r + \delta = \frac{R}{P}$. Hence the factor demand conditions (5) and (6) read

$$r + \delta = F_K \left(\frac{K}{L}, 1 \right) \quad (14)$$

$$\frac{W}{P} = F_L \left(\frac{K}{L}, 1 \right) \quad (15)$$

Given a depreciation rate δ , equation (14) defines a mapping between r and the capital-labor ratio $\frac{K}{L} = \kappa(r)$. Equation (15) then implies a mapping between r and the real wage $\frac{W}{P} = w(r) \equiv F_L(\kappa(r), 1)$. Using the production function, we obtain mappings from r to output per capita $\frac{Y}{L} = \iota(r)$, the capital-output ratio $\frac{K}{Y} = k(r)$ and the labor share $\frac{WL}{PY} = \alpha(r)$. The functions κ, k, w and ι are all decreasing in r while α is declining in r if the elasticity of substitution ϵ between capital and labor is strictly below 1.²

In steady state, the government maintains a constant level of spending G and bonds outstanding $\bar{B}$. Enough labor income taxes are raised through the tax schedule T to pay

²Appendix A makes these mappings explicit for the case where F has constant elasticity of substitution between capital and labor.

Table 1: Three steady-state regimes

Steady-state regime	Key endogenous variable	Exogenous variables
Neoclassical closed-economy	r	$B = \bar{B}, L = 1$
Neoclassical open-economy	B	$r = r^*, L = 1$
Keynesian closed-economy	L	$r = r^*, B = \bar{B}$

for spending and interest on the debt:

$$\sum \lambda(s) T \left(\frac{W}{P} \cdot e(s) \cdot L \cdot \gamma(s, L) \right) = G + \frac{r}{1+r} \bar{B} \quad (16)$$

The household problem leads to a steady-state policy function $a' \left(s, a; L, \frac{W}{P}, T, r \right)$ for assets. Aggregating over the steady-state distribution, we obtain a demand function for liquid assets $A \left(L, \frac{W}{P}, T, r \right) \equiv \int a' \left(s, a; L, \frac{W}{P}, T, r \right) d\Psi(s, a)$. Asset market clearing requires:

$$A \left(L, \frac{W}{P}, T, r \right) = B + (1+r) \kappa(r) L \quad (17)$$

Equation (17) is the only restriction on this economy's steady states. Given the implicit dependence of $\frac{W}{P}$ and T on r and L , it characterizes the triplets (r, B, L) that are consistent with a steady-state equilibrium in both our neoclassical regimes, as well as in our Keynesian regime. A *neoclassical closed-economy* steady-state can be deduced by solving (17) for r when $L = 1$ and $B = \bar{B}$. The procedure is familiar from the literature, see for example [Huggett \(1993\)](#) and [Aiyagari \(1994\)](#). An *open-economy steady-state* solves (17) for B given r and $L = 1$. That procedure is also familiar from the literature, see [Mendoza, Quadrini and Ríos-Rull \(2009\)](#). By contrast, our *Keynesian closed-economy steady-state* solves for L given $B = \bar{B}$ and r , taking into account the implicit dependence of T on L . Hence the main difference between the regimes is the equilibrating variable that ensure that equation (17) holds. This discussion is summarized in table 1.

Note that our Keynesian steady-state may feature a permanent slump, with the employment rate L away from its natural level of 1 in steady-state. What justifies our assumption of a fixed real interest rate in this case? We have in mind a situation where nominal interest rates face a lower bound $i \geq \underline{i}$ due to the existence of cash, and inflation is subject to an upper bound $\pi \leq \bar{\pi}$ due to the monetary policy regime. Then, if the neoclassical closed-economy equilibrium real interest rate r^n is lower than $\frac{\underline{i} - \bar{\pi}}{1 + \bar{\pi}}$, the central bank is constrained to maintain $i = \underline{i}$ even if the economy is in a slump. The slump implies in particular a binding (4) constraint, and hence through (15) a zero rate

of price inflation, implying a fixed real interest rate $r = \dot{i}$.³

4.1 A special case

We consider a useful benchmark case under which a further characterization of the steady-state is possible. The intuitions that we will obtain from this analysis will carry over to more general cases. This special case features proportional taxation and equal incidence, in other words:

$$T(v) = \tau v \quad (18)$$

$$\gamma(s, L) = 1 \quad \forall s, L \quad (19)$$

Under these conditions, the steady-state government budget constraint (16) simply reads

$$\tau \frac{W}{P} L = G + \frac{r}{1+r} \bar{B} \quad (20)$$

The following lemma allows us to considerably simplify the analysis of steady-state equilibria.

Lemma 1 (Homotheticity of policy functions). *In any steady state, the policy functions scale with post-tax labor income; in particular*

$$a' \left(s, a; L, \frac{W}{P}, \tau, r \right) = (1 - \tau) \frac{W}{P} L \tilde{a}'(s, a; r)$$

This lemma is a consequence of our assumptions that preferences are homothetic, the fact that the borrowing limit scales with income, proportional taxes, and our assumption of proportional distribution of aggregate income to individual incomes.

Define the *normalized asset demand function* $\hat{a}(r)$ as aggregate asset demand normalized by aggregate post-tax wage income:

$$\hat{a}(r) \equiv \frac{A \left(L, \frac{W}{P}, \tau, r \right)}{(1 - \tau) \frac{W}{P} L}$$

Lemma 1 shows that $\hat{a}(r)$ does not depend on any object equilibrium object except for

³Note that if wages were allowed to fall at some predetermined rate, the steady-state would be associated with price deflation and therefore an even higher constant real interest rate.

r . It is the canonical function from the Bewley-Huggett-Aiyagari class of models, corresponding to aggregate steady-state savings for a partial-equilibrium economy with an interest rate of r , and in which individual income follows the process $e(s)$, so that aggregate income is simply equal to 1. As we will see, $\hat{a}(r)$ embodies all the consequences of idiosyncratic uncertainty for aggregate savings in the general equilibrium economies that satisfy (18) and (19). We can now rewrite equation (17) as

$$(1 - \tau) \frac{W}{P} L \hat{a}(r) = B + (1 + r) \kappa(r) L \quad (21)$$

Given these equilibrium relationships, there exists a simple equation characterizing equilibrium, as given by Proposition 2.

Proposition 2. *In economies that satisfy (18) and (19), the steady-state equilibria of all three regimes satisfy, for all r, B, L, G and $\bar{B}$*

$$\hat{a}(r) = \frac{\frac{B}{L} + (1 + r) \kappa(r)}{w(r) - \left(\frac{r}{1+r} \frac{\bar{B}}{L} + \frac{G}{L} \right)} \equiv l(r, B, L, G, \bar{B}) \quad (22)$$

Proof. Collect equations (21), (20) and use the steady-state factor demand condition $\frac{W}{P} = w(r)$ $\square$

By stripping out general equilibrium effects from the determination of asset demand, equation (22) delivers important insights into the role played by equilibrating variables in our three regimes. It also gives a simple way to compute equilibria and perform comparative statics.

As mentioned, the $\hat{a}$ function is the partial-equilibrium aggregate savings function, a classic object in the Bewley-Huggett-Aiyagari literature. The l function is clearly increasing in $B, \bar{B}, G$ and decreasing in L . Consider *any* shock that shifts $\hat{a}$ up at a fixed level of r —for example, an increase in earnings risk pushing up desired precautionary savings. In the neoclassical open-economy regime, this results in an increase in the steady-state net foreign asset position (higher B), as in Mendoza et al. (2009). Likewise, in our Keynesian regime, this results in a *fall* in L . As we will see shortly, this fall can be prevented through fiscal policy through debt issuance (higher $\bar{B} = B$), government spending (higher G), or both.

Figure 3 illustrates the determination of equilibrium by plotting $\hat{a}$ and l against r . Two cases are considered, depending on the slope of l with respect to r . The sign of this slope depends on the specification of production and fiscal policy. Consider

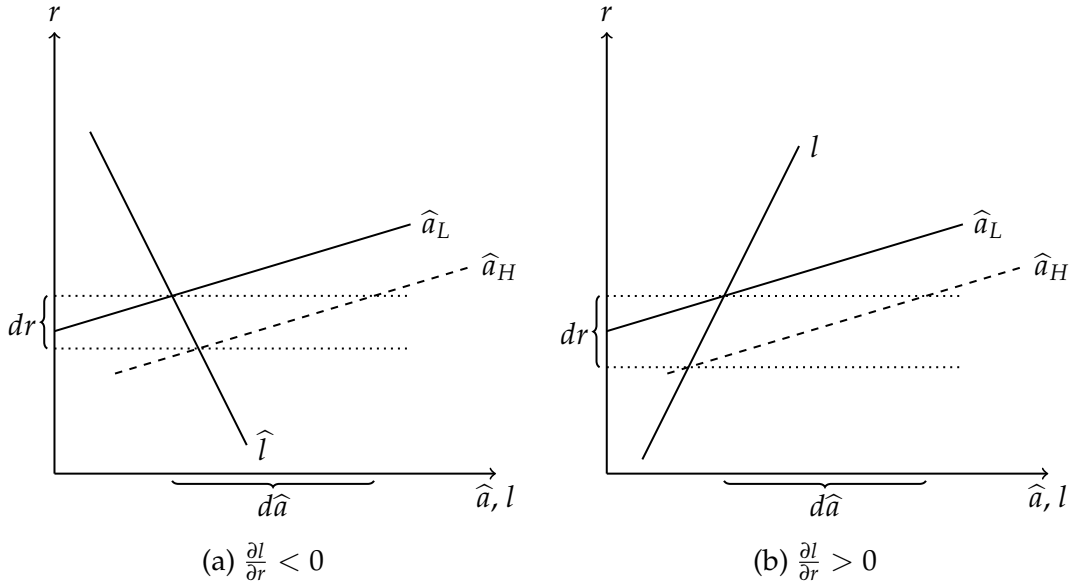


Figure 3: Equilibrium determination and comparative statics

for example the model of [Aiyagari \(1994\)](#), who is a closed economy model with no government ($B = \bar{B} = G = 0$). Then $l(r) = (1+r) \frac{\kappa(r)}{w(r)} = (1+r) \frac{k(r)}{\alpha(r)}$, the capital-output ratio divided by the labor share. Then the l curve slopes down if the elasticity of substitution between capital and labor exceeds the capital share (the left panel, which is the case with Cobb-Douglas production)⁴ and up otherwise. Similarly, in a Huggett model with no capital, $\kappa(r) = 0$, $w(r) = 1$ so $l(r) = \frac{B}{L - (\frac{r}{1+r}\bar{B} + G)}$ is increasing in r , as in the right panel of figure 3.⁵

The intersection of the solid lines in figure 3 represents an initial equilibrium r^* with relatively low desired aggregate savings $\hat{a}_L(r^*)$. Suppose that a change in fundamentals leads to a shift in the desired savings curve to $\hat{a}_H$. The closed neoclassical economy responds with a fall in the equilibrium real interest rate and no change in employment—in fact aggregate GDP goes up because the output-labor ratio rises, due to the higher level of steady-state capital. By contrast, if r is maintained fixed then equilibrium is restored by movements in B , $\bar{B}$, G or L ensuring $dl = d\hat{a}$. Unless fiscal policy responds, this implies a fall in employment L in the Keynesian regime. Because the output-labor ratio is fixed in this regime, this implies a proportional fall in Y .

⁴Note that our way of conducting the analysis shows a simple way to find multiple equilibria in the [Aiyagari \(1994\)](#) model, which has so far proved elusive in the literature. With low elasticities of substitution ϵ , multiple equilibria can arise. By contrast, provided one can show that the *partial equilibrium* savings function satisfies $\frac{\partial \hat{a}}{\partial r} > 0$, unique equilibrium follows whenever $\epsilon > 1 - \alpha$.

⁵Given the large elasticity of $\hat{a}$ with respect to r in most typical calibrations, the relative slopes are almost always as depicted. See section 5.1.

Note that proposition 2 was derived under very few assumption about the nature of idiosyncratic risk—it notably accommodates arbitrary earnings processes and heterogeneity in discount factors, which both affect the $\hat{a}$ function, but not the l function. While assumptions (18) and (19) are restrictive and can be relaxed in quantitative applications, they do illustrate the role of the employment rate L as an equilibrating variable in the Keynesian regime. The key intuition is that L takes on a symmetric role as the liquidity variables B and $\bar{B}$: when desired savings go up relative to labor income and monetary policy does not respond, *labor income falls* until desired savings is sated with the supply of available assets. Note that fixed real interest rates mean that capital supply does not play an equilibrating role, because the desired capital stock shrinks with L , so it is only the *outside supply of assets* (ie, government debt) that ends up restoring equilibrium. We make this point clearer by exploring government spending and liquidity multipliers in the model.

4.2 Government spending and liquidity multipliers in recessions

As noted above, proposition 2 has implications for the role of fiscal policy in Keynesian slumps. Consider a closed economy with $B = \bar{B}$. Manipulating equation (22) we obtain:

$$\left(w(r) - \frac{(1+r)\kappa(r)}{\hat{a}(r)} \right) L = G + \bar{B} \frac{c(r)}{\hat{a}(r)} \quad (23)$$

where $c(r) \equiv 1 + \frac{r}{1+r}\hat{a}(r)$ is aggregate consumption normalized by post-tax wage income. Recall that in a Keynesian slump, r is fixed and $L < 1$. We therefore obtain immediately:

Proposition 3. *Fiscal policy can entirely mitigate the demand shortfall $L < 1$ of a closed economy Keynesian steady-state equilibrium by simultaneously increasing the stock of outstanding government bonds and government spending by the fraction $\frac{1}{L}$.*

The increase in government debt pays entirely for itself since output and therefore tax revenue increase in proportion. This crowding-in effect of government liquidity on output and capital has been documented in models of financial frictions (Woodford (1990)). Importantly, here, liquidity does not facilitate production: it facilitates consumption. This is an effect that has been known to exist in the context of Bewley-Huggett-Aiyagari models, with a literature examining the benefits of public liquidity on welfare (Aiyagari and McGrattan (1998)). Relative to this literature, our Keynesian equilibria feature a direct link between consumption and output, so that the increase

in liquidity can be purely self-sustaining. At the same time, we abstract away from features such as distortionary taxation, which limit the extent to which higher public debt could be beneficial. Note that in our context, the intervention suggested in proposition 3 restores output to its first-best level and does not require any tax rate increase.

We can go further and characterize the government spending multiplier in this model, as well as the *government liquidity multiplier*. Indeed, combining (23) with the fact that $Y = (w(r) + (r + \delta)\kappa(r))L$ we obtain:

$$Y = \frac{w(r) + (r + \delta)\kappa(r)}{w(r) - \frac{(1+r)\kappa(r)}{\hat{a}(r)}} \left(G + \bar{B} \frac{c(r)}{\hat{a}(r)} \right) \quad (24)$$

Equation (24) shows clearly the symmetric role of government spending G and government debt $\bar{B}$ on steady-state output while r is fixed. In particular, we have

Proposition 4. *Throughout the Keynesian regime, holding government debt $\bar{B}$ fixed, each unit of government spending raises output by $\frac{dY}{dG} \geq 1$, with a crowding-in effect ($\frac{dY}{dG} > 1$) in any economy with capital ($\kappa > 0$). Holding government debt spending fixed at G , each extra unit of government debt raises output by $\frac{C}{A} \frac{dY}{d\bar{B}}$, where $\frac{C}{A}$ is the consumption-to-asset ratio in the initial steady-state.*

When the economy has no capital ($\kappa = 0$), increases in government spending G leave post-tax labor income unaffected, as can be seen by considering (21) with $B = \bar{B}$ fixed. Hence consumption is unaffected as well, and the government spending multiplier is 1. More generally, the increase in labor income *crowds in* capital, and the spending multiplier is above 1. The new steady-state features higher consumption, higher government spending and higher investment. These results contrast with neoclassical analyses of the steady-state government spending multiplier (eg Baxter and King (1993)), where the spending multiplier is typically below one, and complement those in the New-Keynesian literature with a zero lower bound (Christiano, Eichenbaum and Rebelo (2011), Farhi and Werning (2013)) by highlighting a different mechanism that works through the endogenous creation of liquid assets.

Proposition 4 also uncovers the *government liquidity multiplier* that the model features: in the Keynesian regime, increases in government debt increase steady-state output by providing liquidity, relaxing financial frictions, and crowding in additional liquid assets. This works much like a government spending multiplier except for the direct effect that government purchases have on output.

5 Permanent increases in inequality

Suppose that φ parametrizes the earnings process: $e_t(s) = e(s, \varphi_t)$. φ_t at once affects cross-sectional income inequality and the level of idiosyncratic uncertainty that households face. The model provides us with a map from inequality, to uncertainty, to desired savings at a given level of interest rates. In the neoclassical regime this translates into a change in the equilibrium real interest rate; in the Keynesian regime into a change in capacity utilization and therefore output. We now explore the quantitative strength of these mappings.

We first ask the extent to which the steady-state is altered. To understand the intuitions about the effects of adjustments on various margins, we first go back to our special case and explore the quantitative implications of Proposition 2 by highlighting the key quantitative magnitudes that are important for equilibrium adjustment under a small shock to φ . We then conduct our main quantitative experiment, looking at the steady-state effect and the effect on the transition.

5.1 A simple approach to computing magnitudes

Proposition 2 highlights the crucial importance of the elasticities of the $\hat{a}$ function in determining the macroeconomic outcomes in our three regimes, as the following corollary illustrates. Let ϵ be the elasticity of substitution between capital and labor in production and α be the labor share in an initial steady state.

Corollary 1. *To first order, across steady-states, level changes dr , $d\varphi$ and proportional changes $\hat{B}$, $\hat{\bar{B}}$, $\hat{G}$, $\hat{L}$ are linked by the following expression:*

$$(\epsilon_{ar} - \epsilon_{lr}) dr + \eta_L \hat{L} - \eta_B \hat{B} = -\epsilon_{a\varphi} d\varphi + \eta_{\bar{B}} \hat{\bar{B}} + \eta_G \hat{G} \quad (25)$$

Where $\epsilon_{ar} > 0$ and $\epsilon_{a\varphi} > 0$ are the steady-state semielasticities of savings to the real interest rate r and the level of idiosyncratic risk parameter, and the other magnitudes only depend on α , ϵ and observable magnitudes at steady-state.

Proof. The proof, in Appendix B, takes a log-linear approximation of (22) around an initial steady-state with $B = \bar{B}$. $\square$

Corollary 1 highlights the quantitative implications of equation (22) for equilibrium adjustments in the three regimes. We have $\eta_L > 0$ and $\eta_B > 0$ except in the limit case of an Aiyagari model in which $\bar{B} = 0$ and hence these quantities are zero. Provided

Table 2: Effects of inequality increases on aggregates in different regimes

Neoclassical	$r \downarrow$	$dr = -\frac{\epsilon_{a\varphi}}{\epsilon_{ar} - \epsilon_{lr}} d\varphi$
Open-economy	$B \uparrow$	$\widehat{B} = \frac{\epsilon_{a\varphi}}{\eta_B} d\varphi$
Keynesian	$L \downarrow$	$\widehat{L} = -\frac{\epsilon_{a\varphi}}{\eta_L} d\varphi$

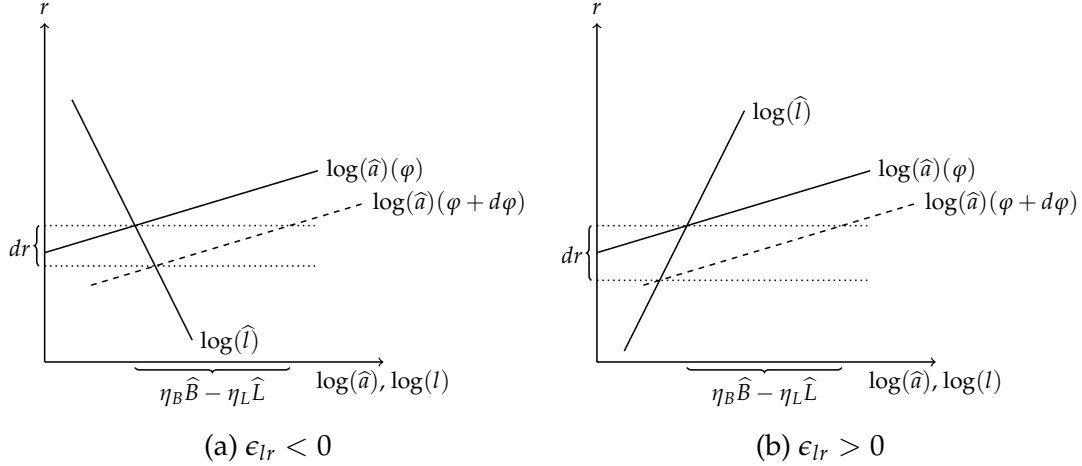


Figure 4: Equilibrium adjustment in log space, assuming $\bar{B}$ and G are constant

$\epsilon_{ar} - \epsilon_{lr} > 0$ (which is true in most quantitative applications where the semielasticity of aggregate savings to r is large), the model therefore generates predictions for the response of macroeconomic aggregates in response to an increase in idiosyncratic earnings risk summarized in Table 2.

Figure 4 illustrates this corollary by repeating figure 3 in log space. We see that the vertical distance $d \log \hat{a}$ quantitatively relates to $\hat{B}$ in the open-economy regime and $\hat{Y} = \hat{L}$ in the Keynesian regime. Higher steady-state inequality due to earnings risk $d\varphi$ moves the $\log \hat{a}$ curve by $\epsilon_{a\varphi} d\varphi$, explaining the magnitudes in the two bottom rows of table 2.

5.2 Quantitative analysis: steady-state [preliminary]

We now consider the quantitative implications of our model. We take a simple calibration where all households have a common annual discount factor β and face the standard earnings process

$$\log e_{it} = \rho \log e_{i,t-1} + \sigma \sqrt{1 - \rho^2} \epsilon_{i,t} \quad \epsilon \sim \mathcal{N}(0, 1)$$

			Value, SS1	Value, SS2
	ν	EIS	0.5	—
	σ	Baseline std of log earnings	0.51	0.71
	ρ	AR coefficient (annual rate)	0.9136	—
		Borrowing limit	0	—
Huggett	$\bar{B}$	Broad Liquidity	0.92	—
	β	Annual discount factor	0.8974	—
Aiyagari	ϵ	Substitution elasticity in production	1	—
	α	Labor share	0.64	—
	δ	Annual depreciation rate	0.08	—
	$\bar{B}^a$	Government Liquidity	0.62	—
	β	Annual discount factor (4% r , no trap)	0.94332	—
	β	Annual discount factor (1% r , trap)	0.977	—

Table 3: Tentative calibration parameters

This provides us with the simplest possible mapping between income inequality and idiosyncratic risk: in this model $\varphi = \sigma$ is the cross-sectional standard deviation of log earnings. In ongoing research, we generalize this earnings process so that it matches the moments of the earnings change distributions documented in [Guvenen, Ozkan and Song \(2014\)](#), as in [Goloso, Troshkin and Tsyvinski \(2016\)](#) and [McKay \(2015\)](#). Table 3 summarizes our calibration parameters. We consider two economies with an initial steady-state that feature an initial steady-state real interest rate of 4% and a standard deviation of log earnings of 0.51, which is close to the 1978 number from figure 1. We then consider the steady-state effect, in both economies, of increasing inequality by 20 basis points when all the inequality increase stems from an increase in the persistent component of earnings risk σ .

Huggett economy. Our first economy is a Huggett model with no capital. We first calibrate this economy as follows. Production is $Y = L = 1$, so that in a neoclassical equilibrium, $C = 1$ as well. The parameters left to calibrate are β and the level of liquidity $\bar{B}$. We follow the recent quantitative literature calibrating Huggett models ([Guerrieri and Lorenzoni \(2015\)](#), [McKay, Nakamura and Steinsson \(2015\)](#)) and broadly interpret $\bar{B}$ as capturing liquid assets: time and savings deposits, treasury, municipal, agency and corporate bonds, shares in mutual funds and money market mutual funds as well as corporate equities. Taking the average ratio to GDP in the years 1977-1983,

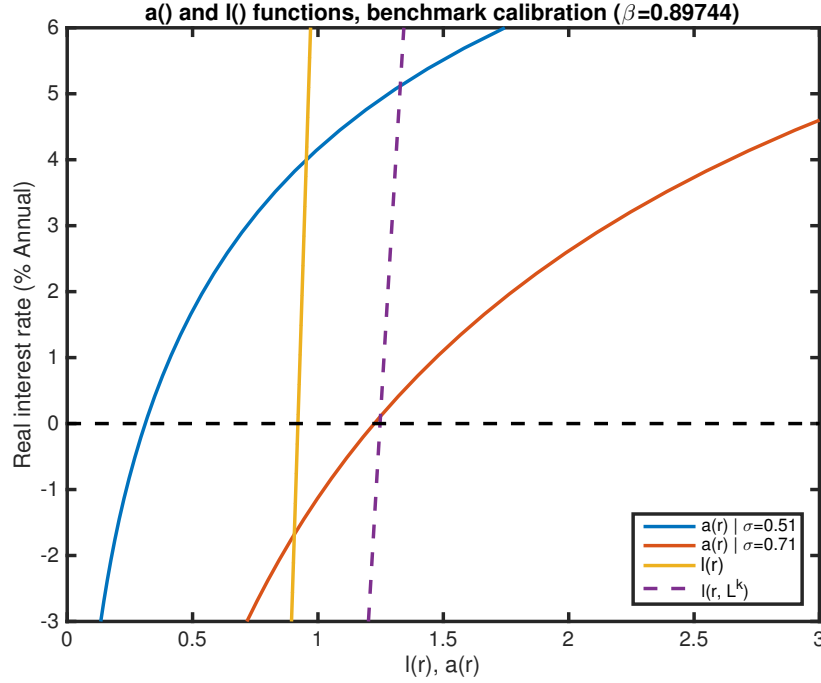


Figure 5: Steady-state analysis: Huggett model

we obtain $\bar{B} = 0.92$.⁶ We finally calibrate β so that the neoclassical equilibrium interest rate matches the level of real interest rates prevailing around that time, an annual rate of $r = 4\%$. This requires $\beta = 0.89$ at an annual rate.

Figure 5 shows the result of increasing our inequality parameter σ by 20 log points. The intersection of the blue and the yellow curve shows the initial level of the real interest rate, $r = 4\%$. The new neoclassical steady-state has a real interest rate that is below -1% . Hence this model can explain the entire quantitative decline in the real interest rate observed in figure 1. While a model without capital is fairly unrealistic (and we will see that capital does mitigate the fall in real interest rates), there are of course other forces affecting the equilibrium interest rate that we abstract away from. Hence it is interesting to see that the benchmark magnitude for dr in this model can be so large.

Figure 5 illustrates what can happen when the real interest rate hits 0: the horizontal distance between the yellow and red line at $r = 0$ must now be covered by a fall in labor income. In this case the fall is $L = 0.75$, a steady-state Keynesian recession with output 25% below potential.

⁶Source: Flow of Funds Accounts of the United States. A tighter interpretation of liquidity, excluding corporate equity, yields $\bar{B} = 0.62$

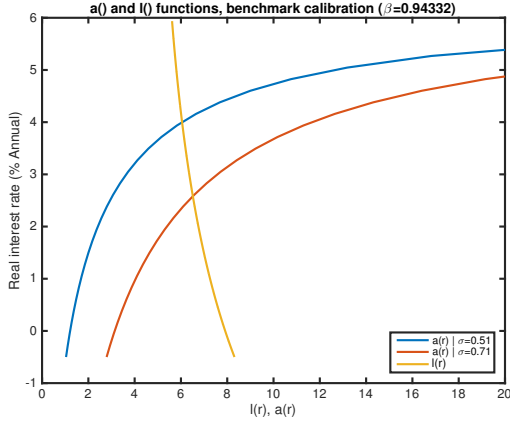


Figure 6: $r_{SS1} = 4\%$

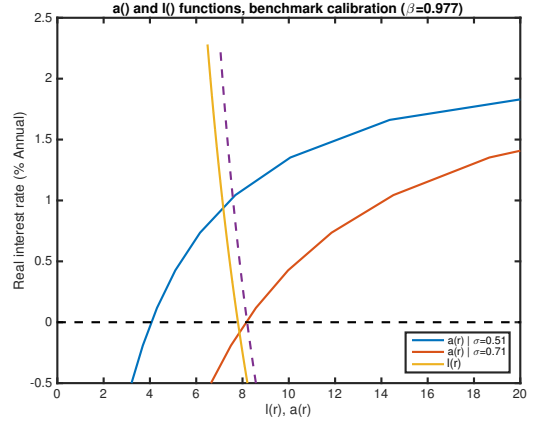


Figure 7: $r_{SS1} = 1\%$

Table 4: Values of elasticities from corollary 1 in calibrations

Calibration		Huggett	Aiyagari 1
Interest semielasticity of savings	ϵ_{ar}	30.2	62.1
Interest semielasticity of liquidity	ϵ_{lr}	0.9	-6.6
Inequality semielasticity of savings	$\epsilon_{a\sigma}$	5.10	3.26
Elasticity of employment gap	η_L	0.96	0.21
Neoclassical (bp per log point)	$dr/d\sigma$	-17.5	-4.7
Keynesian (pp per log point)	$\hat{L}/d\sigma$	4.9	15

Aiyagari economy. Our second economy is an Aiyagari economy, following a standard calibration summarized in table 3. Figure 6 shows that the increase in idiosyncratic risk that prevails in this case delivers a fall in r of around 1.5 percentage point—not enough to push the economy in a liquidity trap. As table 4 illustrates, this is mostly due to the large steady-state semielasticity of *savings* with respect to r , which is a typical feature of this class of models, rather than to the mitigating effect of capital investment, though both do play a role. By contrast, figure 7 shows that the same magnitude increase in inequality starting from a level of real interest rates of 1% is enough to deliver a fall in the equilibrium real interest rate below 1%. With a zero lower bound, the magnitude of the fall in employment is then extremely large, because, as we already stressed, K contracts with L and therefore does not provide an equilibrating mechanism, which instead entirely relies on the fixed supply of government bonds $\bar{B}$. In an ongoing quantitative analysis, we relax several of the extreme assumptions made here, including the lack of responsiveness of fiscal policy.

5.3 Quantitative analysis: transition dynamics

[to be completed]

6 Conclusion

[to be completed]

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A CES production function

Consider a CES production function

$$F(K, L) = A \left[(1 - a) K^{\frac{\epsilon-1}{\epsilon}} + a L^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \quad (26)$$

where A represents total factor productivity. We establish the relationship between r and all scaled quantities, as indicated in table 5, and calculate semielasticities. Our strategy is to establish an expression for the labor share $\alpha = \frac{W}{P} \frac{L}{Y}$ and to express all other quantities in terms of that share. We treat separately the Cobb-Douglas case where $\epsilon = 1$ and $F(K, L) = AK^{1-\alpha}L^\alpha$

Table 5: Quantity ratios for the CES production function in (26)

Notation	Definition	Expression for $\epsilon \neq 1$	Cobb-Douglas case $\epsilon = 1$	Semielasticity $\epsilon_{.,r}$
$\alpha(r)$	$\frac{W}{P} \frac{L}{Y}$	$1 - A^{\epsilon-1} (1 - a)^\epsilon (r + \delta)^{1-\epsilon}$	a	$\frac{1-\alpha}{\alpha} (\epsilon - 1) \frac{1}{r+\delta}$
$k(r)$	$\frac{K}{Y}$	$\frac{1}{A} \left(\frac{1-\alpha(r)}{1-a} \right)^{\frac{\epsilon}{\epsilon-1}}$	$\frac{1-\alpha}{r+\delta}$	$-\frac{\epsilon}{r+\delta}$
$\kappa(r)$	$\frac{K}{L}$	$\left(\frac{a}{1-a} \frac{1-\alpha(r)}{\alpha(r)} \right)^{\frac{\epsilon}{\epsilon-1}}$	$\left(\frac{1}{A} \frac{r+\delta}{1-a} \right)^{-\frac{1}{\alpha}}$	$-\frac{1}{\alpha} \frac{\epsilon}{r+\delta}$
$w(r)$	$\frac{W}{P}$	$a^{\frac{\epsilon}{\epsilon-1}} A \left(\frac{1}{\alpha(r)} \right)^{\frac{1}{\epsilon-1}}$	$\alpha A^{\frac{1}{\alpha}} \left(\frac{r+\delta}{1-a} \right)^{-\frac{1-\alpha}{\alpha}}$	$-\frac{1-\alpha}{\alpha} \frac{1}{r+\delta}$
$y(r)$	$\frac{Y}{L}$	$A \left(\frac{a}{\alpha(r)} \right)^{\frac{\epsilon}{\epsilon-1}}$	$A^{\frac{1}{\alpha}} \left(\frac{r+\delta}{1-a} \right)^{-\frac{1-\alpha}{\alpha}}$	$-\frac{1-\alpha}{\alpha} \frac{\epsilon}{r+\delta}$

The first order condition (14) is

$$r + \delta = (1 - a) AK^{\frac{-1}{\epsilon}} \left[(1 - a) K^{\frac{\epsilon-1}{\epsilon}} + a L^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{1}{\epsilon-1}} = (1 - a) A^{\frac{\epsilon-1}{\epsilon}} \left(\frac{K}{Y} \right)^{-\frac{1}{\epsilon}} \quad (27)$$

from which we immediately obtain the capital-output ratio

$$k(r) = \frac{K}{Y} = A^{\epsilon-1} (1 - a)^\epsilon (r + \delta)^{-\epsilon}$$

Hence a semielasticity of k to r of

$$\epsilon_{kr} = \frac{k'(r)}{k(r)} = -\epsilon \frac{1}{r + \delta} < 0$$

Moreover, we have

$$(r + \delta) \frac{K}{Y} = A^{\epsilon-1} (1 - a)^\epsilon (r + \delta)^{1-\epsilon}$$

so that the labor share, $\frac{WL}{PY} = 1 - (r + \delta) \frac{K}{Y}$, is

$$\alpha(r) = 1 - A^{\epsilon-1} (1 - a)^\epsilon (r + \delta)^{1-\epsilon} \quad (28)$$

Provided $\epsilon \neq 1$, from (27) we also have an alternative expression for $k(r)$ as a function of $\alpha(r)$

$$1 - \alpha(r) = (1 - a) A^{\frac{\epsilon-1}{\epsilon}} (k(r))^{\frac{\epsilon-1}{\epsilon}}$$

so

$$k(r) = \frac{1}{A} \left(\frac{1 - \alpha(r)}{1 - a} \right)^{\frac{\epsilon}{\epsilon-1}} \quad (29)$$

Hence a semielasticity of the labor share to r of

$$\epsilon_{\alpha r} = \frac{1 - \alpha}{\alpha} (\epsilon - 1) \frac{1}{r + \delta} \begin{matrix} \leq \\ \geq \end{matrix} 0$$

depending on $\epsilon \begin{matrix} \leq \\ \geq \end{matrix} 1$.

Next, since the first order condition for labor (15) is

$$\frac{W}{P} = a A^{\frac{\epsilon-1}{\epsilon}} \left(\frac{L}{Y} \right)^{-\frac{1}{\epsilon}} \quad (30)$$

we have

$$\alpha(r) = a A^{\frac{\epsilon-1}{\epsilon}} \left(\frac{L}{Y} \right)^{\frac{\epsilon-1}{\epsilon}}$$

so that output per worker solves

$$\frac{Y}{L} = y(r) = A \left(\frac{a}{\alpha(r)} \right)^{\frac{\epsilon}{\epsilon-1}} \quad (31)$$

With semielasticity

$$\epsilon_{yr} = -\frac{\epsilon}{\epsilon - 1} \times \epsilon_{\alpha r} = -\frac{1 - \alpha}{\alpha} \epsilon \frac{1}{r + \delta} < 0$$

Note also from (30) that

$$w(r) = \frac{W}{P} = aA^{\frac{\epsilon-1}{\epsilon}} \left(\frac{Y}{L} \right)^{\frac{1}{\epsilon}} = a^{\frac{\epsilon}{\epsilon-1}} A(\alpha(r))^{-\frac{1}{\epsilon-1}}$$

so that

$$\epsilon_{w,r} = -\frac{1-\alpha}{\alpha} \frac{1}{r+\delta} < 0$$

And finally that

$$\kappa(r) = \frac{K}{L} = k(r) y(r) = \left(\frac{a}{1-a} \frac{1-\alpha(r)}{\alpha(r)} \right)^{\frac{\epsilon}{\epsilon-1}}$$

so that

$$\epsilon_{\kappa r} = \left(\frac{\epsilon}{\epsilon-1} \right) \left(\frac{-1}{1-\alpha} \right) \times \epsilon_{\alpha r} = -\frac{1}{\alpha} \frac{\epsilon}{r+\delta} < 0$$

In the Cobb-Douglas case we use the fact that (14) implies

$$r+\delta = (1-\alpha) \frac{Y}{K} = (1-\alpha) A \left(\frac{K}{L} \right)^{-\alpha}$$

to find

$$\kappa(r) = \left(\frac{1}{A} \frac{r+\delta}{1-\alpha} \right)^{-\frac{1}{\alpha}}$$

and that (15) implies

$$\frac{W}{P} = \alpha A \kappa(r)^{1-\alpha} = \alpha A^{\frac{1}{\alpha}} \left(\frac{r+\delta}{1-\alpha} \right)^{-\frac{1-\alpha}{\alpha}}$$

Finally, $\alpha = \frac{W}{P} \frac{L}{Y}$ implies $\frac{Y}{L} = \frac{1}{\alpha} \frac{W}{P}$.

Calibration given δ, ϵ and a target α^* and r^* and y^* We first obtain the capital-output ratio

$$k^* = \frac{1-\alpha^*}{r^*+\delta}$$

When $\epsilon \neq 1$, we combine (29) and (31) in steady-state, $k^* = \frac{1}{A} \left(\frac{1-\alpha^*}{1-a} \right)^{\frac{\epsilon}{\epsilon-1}}$ and $y^* = A \left(\frac{a}{\alpha^*} \right)^{\frac{\epsilon}{\epsilon-1}}$ to get

$$k^* y^* = \left(\frac{a}{1-a} \frac{1-\alpha^*}{\alpha^*} \right)^{\frac{\epsilon}{\epsilon-1}}$$

which we can use to solve for

$$a = \frac{\alpha^* h}{1 - \alpha^* + \alpha^* h}$$

where $h \equiv (k^* y^*)^{\frac{\epsilon-1}{\epsilon}}$. Using (31) again, we obtain

$$A = y^* \left(\frac{\alpha^*}{a} \right)^{\frac{\epsilon}{\epsilon-1}}$$

When $\epsilon = 1$, instead, we have $a = \alpha$ and $y^* = A (k^* y^*)^\alpha$ so

$$A = (y^*)^{1-\alpha} (k^*)^\alpha$$

Using these expressions for a and A , we can determine all quantities in terms of our calibration targets. The expressions are collected in table 6.

Table 6: Calibrated quantity ratios for the CES production function in (26)

Notation	Definition	Expression for $\epsilon \neq 1$	Cobb-Douglas $\epsilon = 1$	$\alpha^* = 1$	Semielasticity ϵ_r
$\alpha(r)$	$\frac{W}{P} \frac{L}{Y}$	$1 - (1 - \alpha^*) \left(\frac{r+\delta}{r^*+\delta} \right)^{1-\epsilon}$	α^*	1	$\frac{1-\alpha}{\alpha} (\epsilon - 1) \frac{1}{r+\delta}$
$k(r)$	$\frac{K}{Y}$	$\frac{1-\alpha^*}{r^*+\delta} \left(\frac{1-\alpha(r)}{1-\alpha^*} \right)^{\frac{\epsilon}{\epsilon-1}}$	$\frac{1-\alpha^*}{r^*+\delta}$	0	$-\frac{\epsilon}{r+\delta}$
$\kappa(r)$	$\frac{K}{L}$	$\frac{1-\alpha^*}{r^*+\delta} y^* \left(\frac{\alpha^*}{1-\alpha^*} \frac{1-\alpha(r)}{\alpha(r)} \right)^{\frac{\epsilon}{\epsilon-1}}$	$\frac{1-\alpha^*}{r^*+\delta} y^* \left(\frac{r+\delta}{r^*+\delta} \right)^{-\frac{1}{\alpha}}$	0	$-\frac{1}{\alpha} \frac{\epsilon}{r+\delta}$
$w(r)$	$\frac{W}{P}$	$\alpha^* y^* \left(\frac{\alpha^*}{\alpha(r)} \right)^{\frac{1}{\epsilon-1}}$	$\alpha^* y^* \left(\frac{r+\delta}{r^*+\delta} \right)^{-\frac{1-\alpha}{\alpha}}$	y^*	$-\frac{1-\alpha}{\alpha} \frac{1}{r+\delta}$
$y(r)$	$\frac{Y}{L}$	$y^* \left(\frac{\alpha^*}{\alpha(r)} \right)^{\frac{\epsilon}{\epsilon-1}}$	$y^* \left(\frac{r+\delta}{r^*+\delta} \right)^{-\frac{1-\alpha}{\alpha}}$	y^*	$-\frac{1-\alpha}{\alpha} \frac{\epsilon}{r+\delta}$

For a calibration with $\alpha^* = 0.6$, $r^* = 6\%$ and $\delta = 4\%$, we obtain a capital-output ratio of $k^* = 4$. If we further target $y^* = 1$ (normalizing both Y^* and L^* to 1) and look at a plausible range of elasticities of substitution, we obtain the graph in Figure 8.

B Proof of Corollary 1

Take the total log-differential of (22) to find

$$\begin{aligned} & \frac{1}{1-\tau} \epsilon_{wr} dr - \frac{\tau}{1-\tau} \left(v \left(\frac{1}{(1+r)r} dr + \widehat{B} \right) + (1-v) \widehat{G} - \widehat{L} \right) + \epsilon_{ar} dr + \epsilon_{a\varphi} d\varphi \\ & = \omega \left(\widehat{B} - \widehat{L} \right) + (1-\omega) \epsilon_{kr} dr \end{aligned}$$

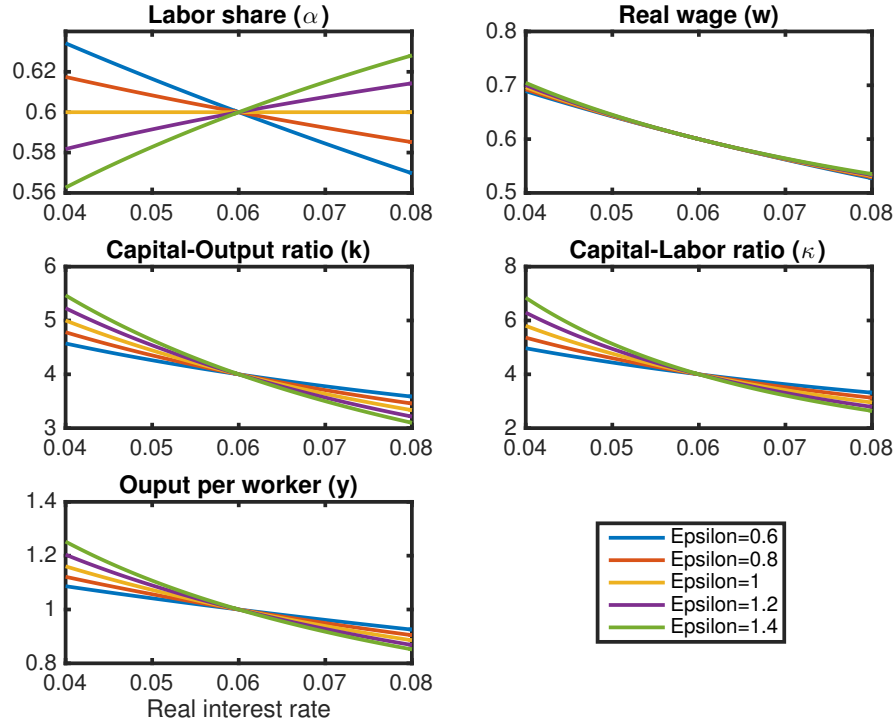


Figure 8: Functions α , w , k , κ and y for various values of ϵ

where $\omega \equiv \frac{\bar{B}}{\bar{A}}$ is the share of bonds in total assets in the initial steady state, $v = \frac{r\bar{B}}{G+r\bar{B}}$ the share of interest expenditures in government spending, and ϵ_{wr} , $\epsilon_{\kappa r}$ are the semielasticities given in table 6. Note that all are observable quantities except for the key objects ϵ_{ar} and $\epsilon_{a\varphi}$ which depend on the particular model of consumption, but not on any general equilibrium mechanism. Grouping terms, we obtain equation (25).