

Theory of College, Student Loans, and Education Policy

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Abstract

This paper analyzes the effectiveness of three different types of education policies: tuition subsidies (broad based, merit based, and flat tuition), grant subsidies (broad based and merit based), and loan limit restrictions. We develop a quantitative theory of college within the context of general equilibrium overlapping generations economy. College is modeled as a multi-period risky investment with endogenous enrollment, time-to-degree, and dropout behavior. Tuition costs can be financed using federal grants, student loans, and working while at college. We show that our model accounts for the main statistics regarding education (enrollment rate, dropout rate, and time to degree) while matching the observed aggregate wage premiums. Our model predicts that broad based tuition subsidies and grants increase college enrollment. However, due to the correlation between ability and financial resources most of these new students are from the lower end of the ability distribution and eventually dropout or take longer than average to complete college. Merit based education policies counteract this adverse selection problem but at the cost of a muted enrollment response. The importance of loan availability critically depends on the underlying distribution of abilities.

Keywords: Student Loans, Education Subsidies, Higher Education

J.E.L. classification codes: E0, H52, H75, I22, J24

1 Introduction

Public policy as it relates to the subsidizing of higher education has been a focal point of empirical and theoretical economists for some time. In developing economies, young individuals lack adequate resources to access to higher education. Without government intervention only individuals with access to sufficient resources would be able to pursue higher education. In developed economies the access to education is even more challenging. These observations have driven macroeconomists to understand the role education subsidies play in reducing economic inequality and promote social mobility.

While the relationship between inequality and education subsidies is important, this project integrates outcomes in an environment where consumer decisions determine the effectiveness of various policies. For instance, offering full scholarships may enable more students to attend college, but those students may not be highly able and hence may be unlikely to finish. It might thus be inefficient to fund students with a low probability of finishing rather than perhaps allocating greater funding to more promising students. Performance based subsidies can be more efficient than general based policies. By formulating college as a complex, multi-period investment we are able to delve deeper into understanding the trade-offs of many types of policy proposals, not just one. Understanding how different policies affect the enrollment and completion decisions of students is essential for drawing conclusions of how and why education subsidies affect economic equity.

Dynamic general equilibrium models are arguably the best suited for studying national policy initiatives that have aggregate effects, although the empirical econometric approach is by far the most popular. This is because aggregate effects in-turn impacts the response to the policy itself (e.g. national student loan program). Heckman, Lochner, and Taber (1998) point out that most empirical studies neglect the general equilibrium effects on wages and taxes. Thus, it is misleading to extrapolate the results from a local policy change to the national level. Using a general equilibrium overlapping generations model, Heckman, Lochner, and Taber (1998) find that neglecting the general equilibrium effects on wages and taxes overestimates the enrollment response to a tuition subsidy by more than ten times. Their model allows for the decomposition of welfare effects for students affected by the policy. Those induced into college after the tuition subsidy or those that stay in college after the change are better off, but those that would not go to college with or without the subsidy or those that do not enroll because of the policy are worse off. This is because taxes must be raised to finance the subsidy and this reduces after-tax wages.

The quantitative theory of college behavior and financial aid features endogenous enrollment, time-to-degree, and dropout decisions made by individuals that differ in their innate ability and initial wealth. College is modeled as a multi-period risky investment that requires a commitment of both physical resources and time in order to complete. Effectively students learn their college ability after enrolling in college and the realization of this uncertainty induces some students to dropout. This risk is calibrated according to micro-level education data. The same data is used to account for the empirical correlation between innate ability and available financial resources. Students learn their college ability after enrolling in college but before dropping out. This implies that there is an

option value embedded in college. A key feature of this framework is that models the three major college decisions (enrollment, time to graduation and dropout) as the result of optimal decision making on the part of rational individuals. Allowing for such intricate college behavior is necessary for studying policy proposals designed to specifically alter these three behavioral margins.

The labor supply of both full-time workers and college students in the model is endogenous. While allowing for college students to work during their college years greatly increases the computational complexity of our model, it is essential for understanding the influence of borrowing constraints. If borrowing constraints begin to bind labor income becomes a viable financing alternative, but only at the cost of a reduction in the amount of time remaining for studying. In addition, college students may choose to work only a few hours in order to reduce the burden associated with large student loan payments in the event of dropping out. In the absence of this mechanism low-income students would be forced to rely solely on grants and loans to finance the cost of education. This has the potential to overestimate the sensitivity to proposed subsidy policies.

A key driver of the results is the correlation between ability and wealth. According to Cameron and Heckman (1998) the failure to account for ability heterogeneity leads to the biased conclusion that policy interventions are effective at increasing the skill level even at the later stages of the education process. The correlation between innate ability and an agent's initial financial asset is absent in Gallipoli, Meghir, and Violante (2006). Another important dimension is the endogenous response to the dropout rate to education policies. This dimension is also absent in the literature (Caucutt and Kumar (2003) or Akyol and Athreya (2005)). To our knowledge, no other study so far has accounted for the time-to-degree dimension of the college investment process.

The model is closed embedding the college investment decision in an overlapping generations production economy. Closing the model is important because it relates the returns to skill to college outcomes. For example, the expansion of college access increases the supply of college graduates in the labor market and lowers the returns to a college degree. Closing the model allows the integration of these general equilibrium effects, but also the distortionary income taxes used to finance education.

Financing of education policies. In addition to consider education policies, the framework allows exploring different financing options. The financing choices of education programs will have important fiscal and distributional considerations. In principle, it could appear that offering full scholarships makes higher education more progressive. However, if the scholarships are funded by consumption taxes, that may not be the case. Moreover, even in the case of full scholarships the students most likely to finish are those that are most academically ready at the end of high school – students that, at least in the developed world, mostly come from high SES families. Thus, a full-scholarship system might be quite regressive. Even if the system is regressive in the short run, it might be progressive in the long run. This is because to the extent that more students graduate from college and obtain higher-paying jobs than those they would have obtained otherwise, they will pay taxes. These tax revenues, in turn, can be spent in social programs that favor low-income individuals.

The current baseline has been calibrated for the US (baseline) and for two Colombia and Brazil

as two distinct education systems in the LAC (=Latin America and the Caribbean) countries. These different models account for the main statistics regarding education such as enrollment rate, dropout rate, and time to degree while matching the observed aggregate wage premiums consistent with the labor and macro literature. The main findings suggest that universal education programs, such as free tuition or grants, have a positive effect on college enrollment but reduce the cutoff of ability. Given that the marginal student is not as productive and it is subject to a higher risk of completing the education, the result is that drops out and the taxpayers end up bearing the cost of the program expansion. Two alternatives that provide incentives: 1) non defaultable loans: The students have less incentives to dropout because they will have to pay the cost of college in the future, 2) performance-based programs: The policy only subsidizes individuals that complete a certain number of credits.

Separating the policy from performance generates pervasive incentives. We find that non-incentive based programs cannot simultaneously increase enrollment and reduce dropouts because students incentivized to enroll are from the lower end of the ability and wealth distribution. These results are consistent with those of Cameron and Heckman (1998) who find that failure to account for ability heterogeneity leads to the biased conclusion that policy interventions late in the life-cycle are effective at raising skill levels. This type of adverse selection is also present in Akyol and Athreya (2005). Contrary, merit based programs serve as a screening mechanism. As such, they are successful at significantly reducing dropouts but only marginally improving enrollment.

2 The Determinants of College Decisions

2.1 Basic Framework

Scholars often argue that education is important and provides a lot of (non-pecuniary) value. In this section we want to explore the determinants of going to college from a financial perspective using a very simple and stylized framework. In this model the benefit associated to college are captured by the skill premium and the cost is captured by tuition expenditures and foregone income during college. An individual decides to enroll in college if the benefits exceed the costs to go to college.

The goal of education policy is to reduce the cost and increase college enrollment. One of the challenges is that the payback period for college can be quite large and it might not make sense for some students.

Let's consider the path associated to enroll full-time in college (4-years)

$$v^c = \sum_{i=1}^4 \frac{-T_i}{(1+r)^{i-1}} + \sum_{i=5}^I \frac{w_i^c}{(1+r)^{i-1}}.$$

The alternative after completing high school is to join the labor force in the market place. The life-time payoff is

$$v^{hs} = \sum_{i=1}^I \frac{w_i^{hs}}{(1+r)^{i-1}}.$$

An students finds optimal to enroll when

$$\Delta v = v^c - v^{hs} \geq 0.$$

or more specifically

$$\Delta v = \sum_{i=1}^4 \frac{-T_i - w_i^{hs}}{(1+r)^{i-1}} + \sum_{i=5}^I \frac{w_i^c - w_i^{hs}}{(1+r)^{i-1}}.$$

We can explore the effects of different education policies that try to reduce the barriers to college.

1. **Free tuition:** Consider an extreme case of free tuition, $T_i = 0$.

$$\Delta v = \sum_{i=1}^4 \frac{-w_i^{hs}}{(1+r)^{i-1}} + \sum_{i=5}^I \frac{w_i^c - w_i^{hs}}{(1+r)^{i-1}}.$$

If the skill premium is not large enough, some individual might still not choose to go to college.

2. **Work during the college years:** Students could work part-time ($w_i^{pt} < w_i^{hs}$) and reduce the opportunity cost,

$$\Delta v = \sum_{i=1}^{\tau} \frac{w_i^{pt} - T_i - w_i^{hs}}{(1+r)^{i-1}} + \sum_{i=\tau+1}^I \frac{w_i^c - w_i^{hs}}{(1+r)^{i-1}},$$

working during college increases the time-to-degree and delays the payback year (τ). The structural model emphasizes this trade-off.

2.2 College Risk: Dropping Out

There are some additional challenges that education policy has to confront. The data shows that a significant fraction of the enrolled college students do not graduate (i.e., US 40%, Columbia, 50%, Brazil, 52%). Low ability students are expose to the risk of paying the cost of college but not receive the benefit (skill premium). We expand the previous framework to consider the risk of dropping out. We define $\pi_{\tau}(\theta)$ as the probability that an student of ability θ drops out in year τ . Without loss of generality we can assume a decreasing probability in ability, $\pi_{\tau}(\theta) > \pi_{\tau}(\theta')$ is $\theta < \theta'$.

Next, we show that the timing of the risk revelation, τ , matters for the enrollment decision, $\Delta v^{\tau}(\theta)$. Consider dropping out at the end of 1st year of college.

$$\Delta v^1(\theta) = \sum_{i=1}^1 -T_i - w_i^{hs} + \sum_{i=2}^I \frac{(1 - \pi_1(\theta))(w_i^c - w_i^{hs})}{(1+r)^{i-1}}.$$

Then, consider the risk of dropping out at the end of the 4th year:

$$\Delta v^4(\theta) = \sum_{i=1}^4 \frac{-T_i - w_i^{hs}}{(1+r)^{i-1}} + \sum_{i=5}^I \frac{(1 - \pi_4(\theta))(w_i^c - w_i^{hs})}{(1+r)^{i-1}}.$$

The presence of risk can significantly reduce the benefit of college education. This is important an important mechanism in our quantitative model.

2.3 Can Education Subsidies Pay by Itself?

In this subsection we explore whether college policies can pay by it self? To answer this question we consider an extreme case of where we give free college tuition, but we want to recover the cost via future taxes on higher human capital. To keep the analysis very tractable we assume $r = 0$, and we consider lump-sum taxation.

In the absence of the policy the student needs to finance the college tuition for 4 years.

$$\Delta v^T = \underbrace{\sum_{i=1}^4 -T_i - w_i^{hs}}_{\text{Cost Coll}} + \underbrace{\sum_{i=5}^I (w_i^c - w_i^{hs})}_{\text{Skill Prem}}.$$

The alternative policy fully waves the tuition, but expects to collect total cost of education as future taxes

$$\Delta v^{FT} = \underbrace{\sum_{i=1}^4 -w_i^{hs}}_{\text{Opp. Cost Coll}} + \underbrace{\sum_{i=5}^I (w_i^c - w_i^{hs})}_{\text{Skill Prem}} - \underbrace{\sum_{i=1}^4 T_i}_{\text{Future Taxes}}.$$

When does college pay by itself? From the expressions, it is direct to see that when the individuals do not face credit constraints and credit markets are fully operational both expressions are equivalent, $\Delta v^{FT} = \Delta v^T$. For college to pay by itself, we need to individuals to face severe credits constraints that impede their ability to borrow. In this case the timing of repayment matters, $\Delta v^{FT} > \Delta v^T$. An important assumption is the absence of distortionary taxes. If the collection of tax revenue is distortionary that reduces the benefit of going to college and can reduce the inequality in the credit constraint case, $\Delta v^{FT} \approx \Delta v^T$.

These are some important takeaways that will be present in our model: 1) For some individuals the opportunity costs can be too high, even with free tuition. 2) Working during the college years reduces the opportunity cost, but postpones the education payoff and increases the risk of dropping out. 3) For low ability individuals college is extremely risky. 4) For individuals subject to borrowing constraints, college can pay by itself. Our model includes these various trade-offs, but also take into account the importance of general equilibrium effects (feedback education decision into labor market compensation).

3 Education Outcomes and Income Inequality

One of the goals of education policy is to increase the fraction of the skill labor force in the economy. A larger number of workforce with college graduates is likely to reduce the skill premium and income inequality. The general equilibrium model discussed in Section 4 relates the outcome of education policies and income inequality. Using the aggregate production function specified in the quantitative model, one can explore the effects on inequality without having to solve the model. This section explores the dynamic effects of changes in education outcomes and income inequality, measured in terms of education premium.

To perform these simulations, we modify the production function used by Card and Lemieux (2001) to the study changes in the skill premium across age groups by incorporating capital as a factor of production and assuming access to international capital markets.¹ We assume a Cobb-Douglas constant returns to scale that depends on aggregate capital K and labor N .

$$Y_t = f(K_t, N_t) = AK_t^\alpha N_t^{1-\alpha},$$

The skill premium is determined using CES sub-aggregates of high school educated labor H_t and college educated labor G_t . Specifically, output is determined according to:and the aggregator for labor inputs is defined by

$$N_t = (A_H H_t^\rho + A_G G_t^\rho)^{\frac{1}{\rho}},$$

where A_H and A_G represent the technology efficiency parameters of high school and college graduates, respectively. The labor input from high school and college graduates is computed using CES sub-aggregators that satisfy

$$\begin{aligned} H_t &= (\nu_o H_{o,t}^\varphi + \nu_y H_{y,t}^\varphi)^{1/\varphi}, \\ G_t &= (\psi_o G_{o,t}^\varphi + \psi_y G_{y,t}^\varphi)^{1/\varphi}. \end{aligned}$$

where ν_j and ψ_j are the efficiency parameters for age group j high school educated workers $H_{j,t}$ and college educated workers $G_{j,t}$, respectively. The technology separates the return for young workers (24-35 years old) from experienced/old workers (36-60 years old). The parameters ρ and φ are functions of the elasticity of substitution between high school and college workers σ_E , and between different aged workers within education groups σ_A , respectively. Specifically, the relationships are $\rho = 1 - 1/\sigma_E$ and $\varphi = 1 - 1/\sigma_A$.

The assumption of perfect access to international markets imply that the domestic interest rate is determined by

$$r^* = f_K(K_t, N_t) = \alpha A \left(\frac{N_t}{K_t} \right)^{1-\alpha} - \delta.$$

¹They argue that this form of production function is consistent with two observations: First, the gap in average earnings between workers with a college degree and those with only high school diploma rose from approximately 25 percent in the mid 1970's to 40 percent in 1998. The second one is that most of the rise can be attributed to the increase in the college wage premium of the younger cohorts.

From the international capital markets, firms will choose the optimal level of capital and employment/hours.

$$\frac{K_t}{N_t} = \left(\frac{\alpha A}{r^* + \delta} \right)^{\frac{1}{1-\alpha}}$$

Perfect competition requires workers and capital to be paid their marginal products. The implied equilibrium factor prices are:

$$\begin{aligned} w_{o,t}^h &= \nu_o (1 - \alpha) A A_H \left(\frac{\alpha A}{r^* + \delta} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{N_t}{H_t} \right)^{1-\rho} \left(\frac{H_t}{H_{o,t}} \right)^{1-\varphi}, \\ w_{y,t}^h &= \nu_y (1 - \alpha) A A_H \left(\frac{\alpha A}{r^* + \delta} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{N_t}{H_t} \right)^{1-\rho} \left(\frac{H_t}{H_{y,t}} \right)^{1-\varphi}, \\ w_{o,t}^g &= \psi_o (1 - \alpha) A A_G \left(\frac{\alpha A}{r^* + \delta} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{N_t}{G_t} \right)^{1-\rho} \left(\frac{G_t}{G_{o,t}} \right)^{1-\varphi}, \\ w_{y,t}^g &= \psi_y (1 - \alpha) A A_G \left(\frac{\alpha A}{r^* + \delta} \right)^{\frac{\alpha}{1-\alpha}} \left(\frac{N_t}{G_t} \right)^{1-\rho} \left(\frac{G_t}{G_{y,t}} \right)^{1-\varphi}. \end{aligned}$$

To distinguish between the wages of workers with different education levels the superscripts h and g are used to identify high school educated workers and college educated workers, respectively.

Consider an economy with a population characterized by the number of individuals within each class of education types $\{H_{o,t}, H_{y,t}, G_{o,t}, G_{y,t}\}$. For a given parametrization, we can use the previous wage equations to derive the dynamics of wages for each type. We have used the estimation technique of Card and Lemieux (2001) to determine the relevant elasticities of substitution for Brazil and Colombia. Table 1 summarizes the key elasticity parameters.

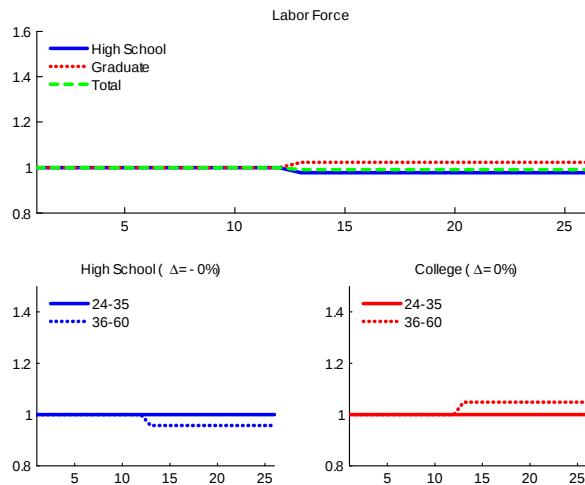
Table 1: Parametrization for Colombia and Brazil

Education	USA	Colombia	Brazil
Labor income share, α	0.33	0.36	0.36
HS vs College $\rho = 1 - \frac{1}{\sigma_E}$	0.60	0.69	0.64
Young vs Old $\varphi = 1 - \frac{1}{\sigma_A}$	0.80	0.87	0.86
Skill Prem	1.87	2.7	2.77
Coll.Age Prem	1.55	1.29	1.61
HS.Age Prem	1.27	1.32	1.44

The estimates show that Colombia and Brazil have a higher elasticity of substitution between college and high school and young vs. old when compared to the case of the United States. The Table also shows that the skill premium in Colombia and Brazil are very similar around 2.7 and about 42 percent times higher than the one in the United States. The next graph explores the dynamic evolution of the labor market as the fraction of skill and unskilled converge to a stationary

distribution by assuming that the long-run trends are determined by the current levels for high school and college graduates ($H_y = H_{y,t}, G_y = G_{y,t}$).

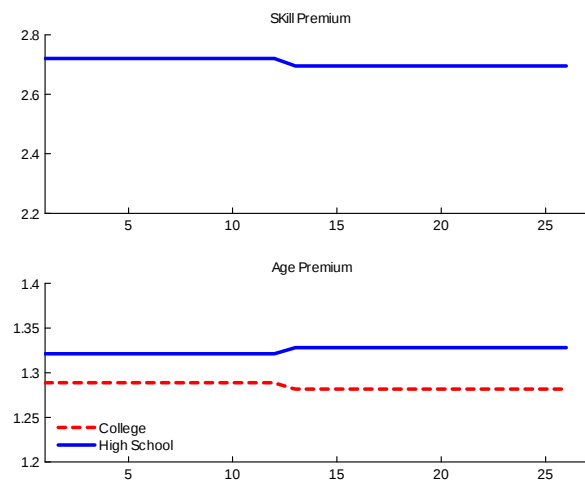
Figure 1: Dynamics of Labor Market Skills (Colombia)



Source: Author's calculations

The data in Figure 1 shows that the total level of employment will remain essentially unchanged, despite the fact the the fraction of high school and college graduates will adjust in 12 years. The resulting dynamics of the skill and education premiums are summarized in Figure 2

Figure 2: Dynamics of Education Premiums (Colombia)



Source: Author's calculations

As we can see in Figure 2, the adjustments in the labor force will have some medium term effects reducing the skill premium, but the size of the reduction is very small. The future cohort of college graduates (24-35) is larger than the existing one (36-60) and this is why the skill premium

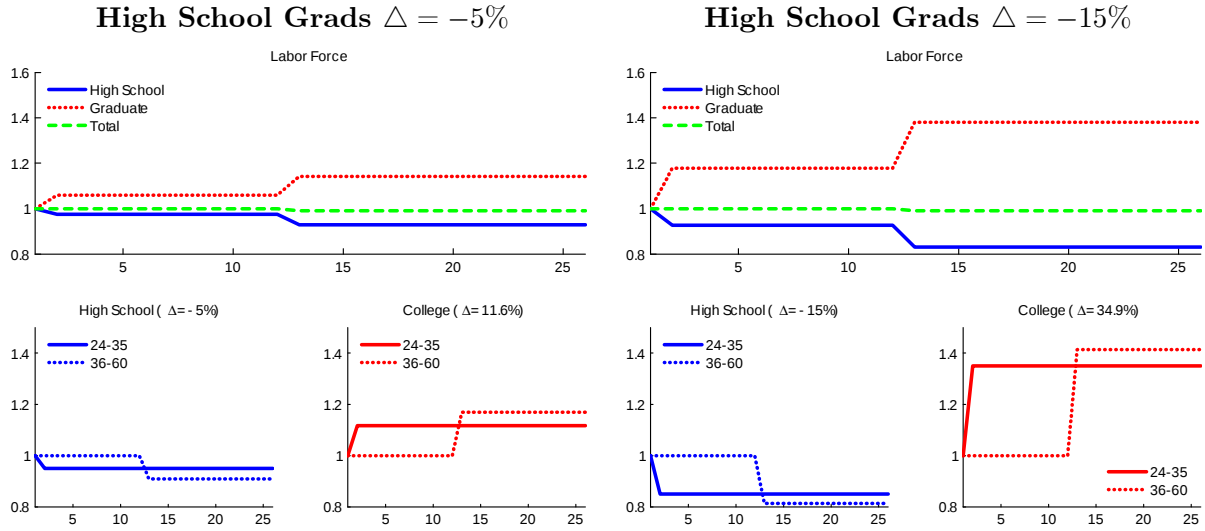
and the college age premium will decline. There is a benefit for those that do not attend college, due to a relative shortage of high school workers that increase their compensation.

We can use the model to explore the effect of changes (permanent and temporary) in the composition of skills in the labor force $\{H_{o,t}, H_{y,t}, G_{o,t}, G_{y,t}\}_{t=2}^T$. In particular, we want to emphasize permanent changes as opposed to transitory. Consider a permanent increase in the number of young college graduates, $eH_{y,t}$, where $e \geq 0$ that simultaneously reduce the number of high school graduates, $eH_{y,t}$. Formally,

$$\begin{aligned} H_{y,t+1} &= (1 - e)H_{y,t}, \\ G_{y,t+1} &= G_{y,t} + eH_{y,t}. \end{aligned}$$

The objective is to identify the rate at which the education programs have to graduate students for the skill premium to decline in a meaningful way. The first experiment considers an reduction in the number of high school graduates of 5 percent, and the second one is 15 percent. The effects in the dynamics in the labor force are summarized in Figures 3.

Figure 3: Education Policy and Labor Market (Colombia)

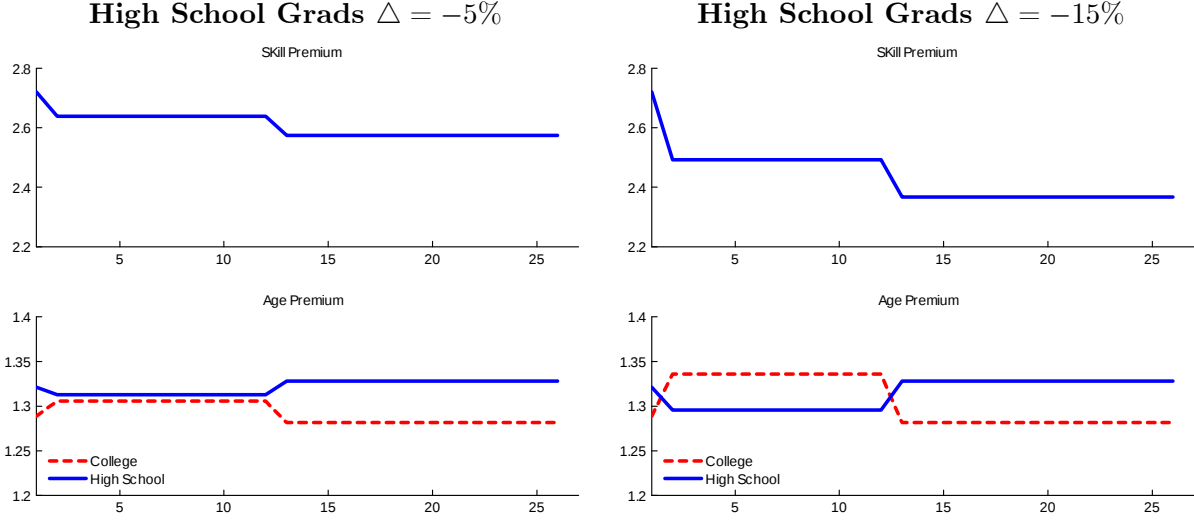


Source: Author's calculations

Both policies have significant effects in the composition of the labor market. There is an initial short-run effect, and there is a medium-run that results from adjusting the demographic structure of the labor force. The reduction of the number of high school workers increases their relative

compensations reducing the college premium as can be seen in Figure 4.

Figure 4: Education Policy and Education Premium (Colombia)



Source: Author's calculations

4 Quantitative Model

4.1 General Description

The economy is populated by overlapping generations of individuals that are economically active up to period J at which time they enter retirement. At the beginning of the first period of life each individual draws an innate ability and asset position from a joint distribution. With this information individuals decide to enroll in college or enter the work force as a full-time high school educated worker. The option to enroll in college is only available during the first period of life. To graduate from college a student must successfully complete a fixed minimum number of credits within three periods.

After enrolling in college, a new student decides on the number of credits to register for and the amount of effort to exert in turning registered credits into completed credits. Students fund their purchases of registered credits and per-period consumption by drawing on four resources: labor income earned from endogenously supplying labor, student loans, initial assets, and government provided grants. The total cost of obtaining an education is a function of the number of credits registered for.

At the beginning of the second period, each college student draws a new college ability from a conditional distribution. Upon learning their new college ability each student decides to continue their education, or drop out of college and enter the work force. Dropping out is a nonreversible decision and the return to a partial education is uncertain. Students that decide to continue in college face the same problem as first period students, but particular students in the second period may differ in their college ability, the number of credits they have completed, and their current

asset position. There is no more uncertainty over ability after the second period.

Students that have satisfied the minimum college degree requirement in two periods begin the third period as college educated workers. Students that have not completed the required minimum number of credits face the same problem as an agent beginning the second period. After making their dropout/continuation decision students choose registered credits and consumption expenditures, as well as how much to borrow and work. Should a student fail to complete their degree by the end of the third period they are effectively a dropout.

Upon entering the labor market by either forgoing college, dropping out, or graduating, workers choose how much labor to supply at the given education and age specific wage rate, how much to consume, and tomorrow's asset position. Earnings are subject to nondistortionary taxation. We assume that the repayment of student loans begins immediately after leaving school and that only a fraction of debt incurred in school may be rolled-over each period. Thus, because no agents in our model begin life with a negative asset position, those individuals that never attend college are subject to a strict borrowing constraint. Extending the credit limits in this manner allows us to summarize the idea that more skilled agents usually face looser credit constraints without having to endogenize borrowing constraints. A similar approach can be found in Akyol and Athreya (2005).

At each date there is a single output good produced in the economy using a constant returns to scale production technology that is a function of aggregate capital and labor. Aggregate labor is comprised of age and education specific labor inputs. The government runs a balanced budget tax and transfer educational grant program. Our analysis only focuses on a stationary equilibrium where all the aggregates and prices are time invariant.

4.2 Demographics

The economy is populated by overlapping generations of individuals that are indexed by their age, $j \in \mathcal{J} = \{1, 2, \dots, J\}$. Each agent is economically active until age $J - 1$, after which they enter retirement at age J . Consumers are considered "young" from birth up to age j_o , and thereafter until retirement they are characterized as "old." There is no survival uncertainty.² For convenience the total measure of agents in the economy is normalized to unity. We assume that each newborn population grows relative to the previous generation at a constant rate η each period. The cohort shares $\{\mu_j\}_{j=1}^J$ are computed as $\mu_j = \mu_{j-1}/(1 + \eta)$, where $\sum_{j=1}^J \mu_j = 1$.

4.3 Firms and Wages

The model is closed using an aggregate production function based on the formulation of Card and Lemieux (2001). The details are described in Section 3.

Since we explicitly model the college dropout decision we must to assign a wage rate for the students pursuing this option. Kane and Rouse (1995) find that, on average, those that attended two year colleges earned approximately 10 percent more than those with just a high school education.

²The survival probabilities for individuals of age 65 and less are sufficiently close to one that we may abstract from modelling mortality risk and the structure of annuity markets.

To capture this partial return to completing some higher education the wages of college dropouts are modeled as a linear combination of high school educated workers and college educated workers:

$$w_i^d = \chi w_i^h + (1 - \chi) w_i^g, \quad i = o, y, \quad (1)$$

where $\chi \in (0, 1)$ dictates the return to partial education.

4.4 Consumers

Consumers preferences are defined over consumption c , leisure l , and retirement assets a_J , according to the following expected, discounted utility function:

$$E \left\{ \sum_{j=1}^{J-1} \beta^{j-1} u(c, l) + \phi(a_J) \right\},$$

where β is the subjective discount factor and the function $\phi(\cdot)$ is the agent's value function upon retirement. Because there is no uncertainty after the final period, or more generally that all uncertainty is *iid*, the use of a terminal value function is valid.³ The partial derivatives of the utility function $u : \mathbb{R}_2 \rightarrow \mathbb{R}$ satisfy $u_i > 0$, $u_{ii} < 0$, and $u_{ij} > 0$, and are consistent with the Inada conditions. The retirement value function $\phi : \mathbb{R} \rightarrow \mathbb{R}$ is C^2 and strictly concave. Specific functional forms for the per-period utility function and retirement value function are discussed in the parameterization section.

Upon first entering the economy new high school graduates are differentiated by their initial asset position and innate ability (a_0, θ_h) which are drawn from a joint probability distribution $\Omega(a_0, \theta_h)$. The manner in which initial assets and ability are determined is an extremely important feature of the model. In abstracting away from the pre-college portion of a student's life we have neglected important socioeconomic influences that invariably determine the college preparedness of an agent, as well as the financial resources available to potentially college bound students. For example, wealthier families may be able to invest more heavily in their child's secondary education which leads to a correlation between family wealth and college preparedness. Restuccia and Urruria (2004) used a quantitative model of intergenerational human capital transmission and found that approximately one-half of the intergenerational correlation in earning can be accounted for by the parents investment in early education. In addition, wealthier families may offer more financial support to their child to go to college. The potential correlation between wealth and ability, and then wealth and financial support for college implies a correlation between a student's college financial resources and their ability. The joint probability distribution allows us account for this correlation, which effectively summarizes the socioeconomic influences prior to college. The estimation of this distribution is discussed later when we parameterize the benchmark economy.

In the first period of life newborns are offered the opportunity to enroll in college or enter the labor market with a high school education. As a result of this decision we can classify each agent

³See Merton (1971).

as being in one of two categories: student, or a full-time worker. We present the problem of the college student first followed by the problem of the worker.

4.5 College Student Problem

College is modeled as a multi-period risky investment that requires a student to successfully complete a minimum of credits $\underline{x}$ within three periods to graduate. Students progress through college by combining their ability $\theta \in \Theta$, effort e , and registered credits $\tilde{x}$, using an education technology $Q(\theta, e, \tilde{x})$. The education technology is a non-linear function dictating the production of completed credits x , according to:

$$x = Q(\theta, e, \tilde{x}) = \theta \tilde{x} e^\gamma, \quad 0 < \gamma < 1.$$

We choose to model progression through college in terms of credits instead of human capital in order to more accurately incorporate the cost of education into the model using empirical data. The specified technology is multiplicative in ability, registered credits, and effort. In addition, the marginal returns to investment in education are constant in the first two factors and diminishing in the third. The multiplicative structure implies that students with higher ability are more productive at the margin in terms of completing all college credits, and is not simply a scaling factor.

Students can affect the production of completed credits by choosing the number of registered credits and/or supplying more effort. For example, a student with lower ability $\theta_i < \theta_j$, can choose to register for a larger number of credits $\tilde{x}_i > \tilde{x}_j$, and obtain the same return (in terms of completed credits) as a higher ability student. Although the cost in terms of tuition will be higher. The assumption that higher-ability types are more productive is common in the human capital literature(see, for example, Becker (1993)). A low ability student could also increase effort in school, but there is an associated utility cost as an increase in effort reduces the time available for leisure and work. The education technology also exhibits diminishing returns to effort following the work of Ben-Porath (1967).⁴ Despite the apparent differences, the college credit function is closely related to a version of the frequently used human capital accumulation equation, where the stock of human capital is replaced with the agent's credit stock.

As mentioned earlier, allowing the labor supply of college students to be determined endogenously addresses a previously neglected interaction with the student's choice of debt. It also serves another important function related to the riskiness of college. In the presence of uncertainty over the ability to complete college students may choose to hedge the risk by substituting labor income for debt. This further increases the chances of failure as time spent working may be drawn away from school. Students from the lower end of the asset distribution are particularly vulnerable because we correlate ability with initial assets.

The structure of the model allows us to exploit the recursive nature of the consumer's problem. In addition, we can break the agent's optimization problem into distinct time periods in order to

⁴Ben-Porath assumes that the human capital technology exhibits diminishing returns in effort and the stock of human capital, $f(h, e, \theta) = \theta(he)^\gamma$. The curvature of the production function allows one to characterize interior solutions and also bounds the stock of human capital. In our model, we formalize the acquisition of education through credits that are bounded by the minimum number of credits required to graduate.

make explicit how the agent's information set and trade-offs evolve. Each agent has a total of five state variables: assets a , current ability θ , completed college credits x , age j , and education indicator s . The education indicator state lies in the set $\mathcal{S} = \{h, d, c, g\}$ where h refers to a high school educated worker, d a college dropout, c an enrolled college student, and g a college graduate. Let $v_j^s(a, x, \theta)$ be the value function of an age j agent with education level s , assets a , completed college credits x , and schooling ability θ .⁵

First Period of College: Given initial assets and ability, an agent that decides to enroll in college must choose consumption c , registered credits $\tilde{x}$, effort e , leisure l , labor supply n , and tomorrow's asset position a' . A freshman student has an initial endowment of college credits, $x = 0$. The first period college problem may be written as:

$$v_1^c(a, x, \theta_h) = \max_{c, \tilde{x}, a', e, l, n} \left\{ u(c, l) + \beta E_{\theta_c, w_y^d} \max \left[v_2^c(a', x', \theta'_c), v_2^d(a', x', \theta'_c) \right] \right\} \quad (2)$$

subject to

$$\begin{aligned} c + \mathcal{T}\tilde{x} + a' &\leq w_y^h n + a + y \\ x' &= \theta_h \tilde{x} e^\gamma \\ a' &\geq a_{2c} \\ l + e + n &= 1 \\ \tilde{x} &> 0, \quad x' &\leq \tilde{x} \end{aligned}$$

The total education expenditure depends on the number of registered credits $\tilde{x}$ and the price per-credit $\mathcal{T}$. In order to finance their education, students may draw on their initial assets and three additional resources. First, students may work while in school earning a young high school graduates wage w_y^h . Second, the government provides all students with a per-period college grant y . Students also have access to the financial market where they are permitted to take a negative position in the only financial asset $a' \in \mathcal{A}$ up to the borrowing constraint a_{2c} . We allow the per-period loan limit to vary in each period of college as indicated by the time indexing. Each agent has a time endowment normalized to the unity. During college years this endowment can be allocated between work, effort in school, and leisure. The last two constraints simply states that students must register for positive credits, and that completed credits may not be greater than registered credits.

The continuation value functions for a first-year student depends on whether the student continues with their education in the following period, $v_2^c(\cdot)$, or drops from school and joins the labor force as a full-time worker, $v_2^d(\cdot)$. The expectation in the continuation value is the result of two sources of risk associated with obtaining an education. First, we assume that after the first period of college each student's college ability θ_c is randomly drawn from the conditional distribution $\Pi(\theta_h, \theta_c)$. Once the agent's college ability is determined there is no further uncertainty over ability.⁶ Second,

⁵ Writing the value function as $v_j^s(a, x, \theta)$ rather than $v(a, x, \theta, s, j)$ keeps the notation compact and saves space.

⁶ As we discuss in greater detail when we outline our parameterization of the model, we estimate the conditional

should a student choose to dropout they receive a high school graduate's wage with probability p and a college dropout's wage with probability $(1 - p)$. The uncertainty over wages enables us to easily incorporate the documented partial return to college. Thus, the expectation in the value function is with respect to next periods college ability θ_c , and the wage a dropout will receive, w_y^d .

Second Period of College: At the beginning of the period each student draws a new college ability type $\theta_c \sim \Pi(\theta_h \mid \theta_c)$. After learning their new ability the student decides to dropout or continue on with college. The second period college problem is similar to that of the first period. However, the borrowing constraint in the second period is relaxed with respect to the previous period. In addition, the student now has to weigh the option of completing enough credits to graduate at the end of the period. The student now solves:

$$v_2^c(a, x, \theta_c) = \max_{c, \tilde{x}, a', e, l, n} \left\{ u(c, l) + \beta E_{w_y^d} \max \left[v_3^c(a', x', \theta'_c), v_3^d(a', x', \theta'_c), v_3^g(a', x', \theta'_c) \right] \right\} \quad (3)$$

subject to

$$\begin{aligned} c + \mathcal{T}\tilde{x} + a' &\leq w_y^h n + y + (1 + r)a \\ x' &= x + \theta_c \tilde{x} e^\gamma \\ a' &\geq a_{3c} \\ l + e + n &= 1 \\ \tilde{x} &> 0, \quad x' \leq \tilde{x}, \quad x' \geq \underline{x}, \end{aligned}$$

where $v_3^g(a', x', \theta'_c)$ is the value of entering the labor market in the third period as a college graduate. The law of motion for completed credits now includes the stock of completed credits from the previous college year x . To satisfy the graduation requirement a college student must complete $x' \geq \underline{x}$ college credits. Note that the production function of credits depends only on the realized value of college ability and is therefore independent of past abilities. A college student is always allowed to borrow as least as much in the second period as in the third period. This assumption allows the agent to at least roll over the previous periods debt if $a_{3c} = a_{2c}$, and increase accumulated student loan debt if $a_{3c} < a_{2c}$. If the credit constraint were not to be relaxed a college student at the borrowing limit during the first-period of college would be forced to repay the principal and accrued interest $(1 + r)a_{2c}$ in the third period, while only relying on labor income and grants to fund their education. Because all ability uncertainty is resolved before the student makes any decisions, the expectation operator is only defined over the wage rate of dropouts.

Third Period of College: Students that extend their time in school into the third period solve a slightly different problem than in the second period. Should a student not be able to complete $\underline{x}$ credits in the final period they are automatically classified as dropouts as there is no further college periods. As in the second period, we allow the borrowing constraint to change although we do not

probability distribution to match empirical data that indicates that successful high school students are more likely to be successful college students.

require that it allow for an increased level of debt.⁷ The problem in the final period of college is

$$v_3^c(a, x, \theta_c) = \max_{c, \tilde{x}, a', e, l, n} \left\{ u(c, l) + \beta E_{w_y^d} \max \left[v_4^d(a', x', \theta_c), v_4^g(a', x', \theta_c) \right] \right\} \quad (4)$$

subject to

$$\begin{aligned} c + \mathcal{T}\tilde{x} + a' &\leq w_y^h n + y + (1 + r) a \\ x' &= x + \theta_c \tilde{x} e^\gamma \\ l + e + n &= 1 \\ a' &\geq a_{4c} \\ \tilde{x} &> 0, \quad x' \leq \tilde{x}, \quad x' \geq \underline{x} \end{aligned}$$

4.6 College Enrollment Decision

A newborn high school graduate with innate ability θ_h , initial assets a_0 , and no college credits ($x = 0$) will choose to go to college when the expected discounted utility of doing so is as least as great as the utility gain from entering the workforce as a high school educated worker. This cut-off may be summarized in terms of the agent's value function as:

$$v_1^c(a, 0, \theta_h) \geq v_1^h(a, 0, \theta_h) \quad (5)$$

To compute the initial value function it is necessary to solve the model using backward recursion from the last period followed by the workers problem. We turn into these problems next.

4.7 Workers

All workers solve the same general problem regardless of their path to the workforce: forgoing college ($s = h$), dropping out ($s = d$), or graduating ($s = g$). After leaving school the laws of motion for credits and ability for college students are trivially $x' = x$ and $\theta' = \theta$, respectively, and all the relevant educational information is summarized by the college status s , age j , and asset position a . Workers choose consumption, tomorrow's asset position, and how much labor to supply at the given education and age specific wage rate. All income is subject to a lump-sum tax τ . The problem of a worker in the period immediately preceding retirement is complicated by our use of a terminal value function to model post-retirement. We present the problem of workers aged $j < J - 1$ first and postpone the aged $J - 1$ worker's problem to the next section. For ages $j < J - 1$ the worker's

⁷When we estimate the benchmark economy we specify $a_{4c} < a_{3c} < a_{2c}$ so the agent may continually increase borrowing while in school. However in our policy experiments we investigate how restricted debt in the third period of college affects time-to-degree.

wage rate is age dependent

$$w_j^s = \begin{cases} w_y^s & \text{if } j < j_o \\ w_o^s & \text{if } j_o \leq j < J - 1 \end{cases}$$

Notice that this specification differs from the standard formulation where the profile of earning changes over the life-cycle according to some hump-shaped profile of exogenously specified efficiency units of labor. In the current specification the age and education heterogeneity, as well as the evolution of the asset distribution are responsible for changes in the labor supply. Full-time workers allocate their time endowment between leisure and work as effort in school is no longer required. The worker's optimization problem may be written as:

$$v_j^s(a) = \max_{c,l,n,a'} \{u(c, 1 - n) + \beta v_{j+1}^s(a')\} \quad (6)$$

subject to

$$\begin{aligned} c + a' &\leq w_j^s n + (1 + r) a - \tau, \\ a' &\geq \min[0, \kappa a], \quad \kappa \in (0, 1). \end{aligned}$$

Our borrowing constraint is nonstandard and requires some discussion due to the restrictions we impose on student loan repayment. We assume that repayment of student loans begins immediately after leaving school and that only a fraction $\kappa \in (0, 1)$ of outstanding loans may be rolled-over each period. This prevents us from adding an additional state variable while simultaneously approximating the repayment time period currently placed on many student loans.⁸ Agents are not permitted to hold negative assets beyond what they enter the workforce with in the form of student loans. Thus, tomorrow's asset decision must satisfy $a' \geq \min[0, \kappa a]$. Since all agents begin life with a non-negative asset position, it is clear that forgoing college results in a hard borrowing constraint. This specification is equivalent to an education dependent borrowing constraint where $a'_s \geq \kappa a$ when $s \neq h$, and $a'_h \geq 0$.

4.8 Retirement

Compulsory retirement occurs at age J . Because agents have utility defined over terminal assets, the period $J - 1$ worker problem is slightly different than the standard worker problem. The problem in the period immediately preceding retirement is:

$$v_{J-1}^s(a) = \max_{c,l,n,a_{J+1}} \{u(c, l) + \phi(a_J)\}, \quad (7)$$

⁸Under the federal student loan program the standard repayment option for Stafford loans is 10 years. Matching this repayment length exactly would require adding the number of repayment periods remaining as a state variable.

subject to $a_J > 0$ and the old worker's budget constraint. Here, $\phi(\cdot)$ determines the value retirees place on assets. This allows us to abstract away from post retirement behavior which we feel is appropriate as we are concerned with behavior extremely early in the economic life-cycle. This is a convenient adaptation of the method used in Roussanov (2004) and Akyol and Athreya (2006).

4.9 Government

The government runs a tax and transfer education grant program. All workers not in college are taxed a lump-sum tax τ which is redistributed to college students in the form of grants y . Our balanced budget assumption implies that in equilibrium the government's tax revenue must equal total grant expenditures. The lump-sum tax that balances the education budget can be written as:

$$\tau = y \frac{\int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=c} \times \mathcal{J}} \mu_j \Gamma(da \times d\theta \times dx \times ds \times dj)}{\int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s \neq c} \times \mathcal{J}} \mu_j \Gamma(da \times d\theta \times dx \times ds \times dj)}, \quad (8)$$

where $\Gamma(\cdot)$ represents the measure of households over the state space. The government budget constraint needs to be modified when we consider tuition subsidies, or merit based programs. However, we defer these discussions to the results section.

It can be argued that compared with a marginal income tax, our assumption of a lump-sum tax may not accurately capture the distortionary effect taxes have on the incentive to pursue a college education. However, given that only a small mass of the population is receiving grants, the per-capita tax burden in this economy is likely not to have a significant affect on the return to education. In a subsequent paper we plan to investigate this proposal by examining the optimal tax instrument to finance a publicly provided higher education subsidy program.

4.10 College Sector

There is an extensive literature on the supply side of education. The objective of the paper is to focus on the demand side by specifying a simple college sector that produces the credits. We assume a competitive education sector with constant returns to scale, or linear cost structure. Free entry in the sector ensures that profits will be zero and the price per credit equals the marginal cost of producing credits. The advantage of this formulation is that allows to parameterize the cost of college education as fraction of average income and it simplifies an already complex model.

5 Stationary Equilibrium

To define the notion of stationary equilibrium it is useful to introduce some additional notation. For an individual of a given age $j \in \mathcal{J} = (1, 2, \dots, J) \subset I$ and education status $s \in \mathcal{S} = (h, d, g, c)$, the relevant state vector in the recursive representation is denoted by $\Lambda_j^s = (a, x, \theta)$. Let $a^s \in \mathcal{A}^s \equiv \mathcal{A}$, $\theta \in \Theta$, $x \in \mathcal{X} \subset I$. Notice that the set of asset holding is conditioned by the education status as a result of the education specific borrowing constraint. We also define $\Lambda = (a, x, \theta, s, j)$ to be

the state vector including the education status and age, and $\Gamma(\Lambda)$ represents the distribution of individuals over the entire state space.

A *stationary recursive equilibrium* for this economy is a collection of: (i) individual value functions $\{v_j^s(\Lambda_j^s), \phi(\Lambda_j^s)\}$, (ii) individual decision rules for college students $\{c_j^s(\Lambda_j^s), a_{j+1}^s(\Lambda_j^s), n_j^s(\Lambda_j^s), e_j^s(\Lambda_j^s), \tilde{x}_j^s(\Lambda_j^s), s_j(\Lambda_j^s)\}$; (iii) individual decision rules for workers and retirees $\{c_j^s(\Lambda_j^s), a_{j+1}^s(\Lambda_j^s), n_j^s(\Lambda_j^s)\}$; (iv) a college enrollment decision $I_1^c(a, 0, \theta_h)$; (v) aggregate capital and labor inputs $\{K, H_y, H_o, G_y, G_o\}$; (vi) price vector $\{r, w_y^g, w_o^g, w_y^h, w_o^h, w_y^d, w_o^d\}$; (vii) education policy $\pi = \{\tau, y\}$; and (viii) a stationary population distribution $\{\mu_j\}$, and an invariant distribution $\Gamma(\Lambda)$ of individuals over the entire state space such that:

1. Given prices $\{r, w_y^g, w_o^g, w_y^h, w_o^h, w_y^d, w_o^d\}$ and tax and grant policy π , the individual decision rules $\{c_j^s(\Lambda_j^s), a_{j+1}^s(\Lambda_j^s), l_j^s(\Lambda_j^s), e_j^s(\Lambda_j^s), \tilde{x}_j^s(\Lambda_j^s), s_j(\Lambda_j^s)\}$ solve the college student's problem specified in (13)-(15). The decision rules $\{c_j^s(\Lambda_j^s), a_{j+1}^s(\Lambda_j^s), l_j^s(\Lambda_j^s)\}$ solve the worker's problem summarized by (17) and (18). And the college enrollment decision $I_1^c(a, 0, \theta_h)$ is consistent with (16).
2. Given prices $\{r, w_y^g, w_o^g, w_y^h, w_o^h, w_y^d, w_o^d\}$, the representative firm maximizes profits (i.e., conditions (7)-(12) are satisfied).
3. The labor markets clear:

$$\begin{aligned}
H_y &= \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=h,c} \times \mathcal{J}_{j < j_o}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda) + \chi \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=d} \times \mathcal{J}_{j < j_o}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda), \\
H_o &= \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=h} \times \mathcal{J}_{j < j_o < J}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda) + \chi \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=d} \times \mathcal{J}_{j < j_o < J}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda), \\
G_y &= \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=g} \times \mathcal{J}_{j < j_o}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda) + (1 - \chi) \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=d} \times \mathcal{J}_{j < j_o}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda), \\
G_o &= \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=g} \times \mathcal{J}_{j < j_o < J}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda) + (1 - \chi) \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=d} \times \mathcal{J}_{j < j_o < J}} \mu_j n_j^s(\Lambda_j^s) d\Gamma(\Lambda),
\end{aligned}$$

where $d\Gamma(\Lambda) \equiv \Gamma(da \times d\theta \times dx \times ds \times dj)$.

4. The government budget constraint is satisfied:

$$\tau \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s \neq c} \times \mathcal{J}} \mu_j \Gamma(da \times d\theta \times dx \times ds \times dj) = y \int_{\mathcal{A} \times \Theta} \sum_{\mathcal{X} \times \mathcal{S}_{s=c} \times \mathcal{J}} \mu_j \Gamma(da \times d\theta \times dx \times ds \times dj),$$

5. Letting T be an operator which maps the set of distributions into itself, aggregation requires

$$\Gamma(a', \theta', x', s', j+1) = T(\Gamma),$$

and T be consistent with individual decisions.

There are two remarks about the definition of equilibrium. First, the labor market conditions are slightly more complex due to the existence of college dropouts. Recall that there is uncertainty over the exact wage a college dropout will receive; a fraction will receive the high school wage while the rest will receive the dropout wage. The labor supply of dropouts earning the high school wage is aggregated into the high school labor supply. The aggregation of the labor supply for the dropout wage earners is carried out in-line with how the dropout wage is determined. Because we calculate the college dropout's wages as a linear combination of the wages of high school educated workers and college educated workers we must aggregate a fraction of their labor into both education group's labor supply. We weight the labor supply of college dropouts according to the fraction of the wage which is determined by high school and college education workers.

Second, market clearing in the asset market is determined at the point where the quantity of capital demanded by firms is equal to net resources provided by households in the form of savings. While the loan programs are usually funded by the government, this form of lending is equivalent to the issue of government debt that is then purchased by the households. The total magnitude of debt to issue should be equivalent to the aggregate level of outstanding college debt.

6 Benchmark Economy

To solve the benchmark economy we must first specify our demographic assumptions, pick functional forms for the per-period utility function and retirement value function, assign initial assets and ability, pin down all parameters associated with the education process and aggregate production function, as well as estimate the joint ability and asset distribution, and conditional ability distribution. The time period that we choose to use in benchmarking our model is crucial. Beginning with the 1993-94 school year the federal student loan program changed in three significant ways due to the 1992 Reauthorization of the Higher Education Act (HEA92): the federal need based formula changed to allow less needy students to qualify for need-based aid; there was a widespread increase in the availability of unsubsidized student loans; and the nominal aggregate student loan limits increased approximately 33 percent for dependent students and 23 percent for independent students. Data availability limitations force us to benchmark our model to the pre-HEA92 period of the early 1990s.

We parameterize the benchmark economy in three steps. First, a number of common parameters in the model are taken directly from the literature. Second, we estimate production efficiency parameters, initial assets, borrowing limits, and the ability transition matrix using data from the pre-HEA92 period. Third, given the parameters we found in the previous two steps, we choose the remaining parameters so that our model replicates the economic and educational environment as close as possible to that of the early 1990s in the U.S., while at the same time satisfying the market clearing conditions.

6.1 Demographics

A period in this model is two years. Agents begin life at age 18. They are considered young until age 36 (j_o) at which time they become old. All agents enter retirement at age 66 (J). The population growth rate η is set to an annual rate of 1.20 percent.

6.2 Preferences

Preferences come from the CRRA family of utility functions. Speciality, the per-period utility function is

$$u(c, l) = \frac{(c^\lambda l^{1-\lambda})^{1-\sigma}}{1-\sigma},$$

and the retirement value function is of the form

$$\phi(a_J) = \beta_R \frac{(a_J)^{1-\sigma}}{1-\sigma}.$$

The per-period utility function was chosen to allow for consumption and leisure to be complements, a potentially important feature of the college experience. In-line with preference parameter values found in the life-cycle literature we set the curvature parameter $\sigma = 4.0$, the utility weight of consumption is chosen to be $\lambda = 0.33$, and the agent's subjective discount factor is $\beta = 0.98$. The agent's coefficient of relative risk aversion is $-\frac{cu_{cc}}{uc} = 1 - \gamma(1 + \sigma) \approx 2$. The remaining parameter, β_R , is determined in the estimation of the benchmark economy.

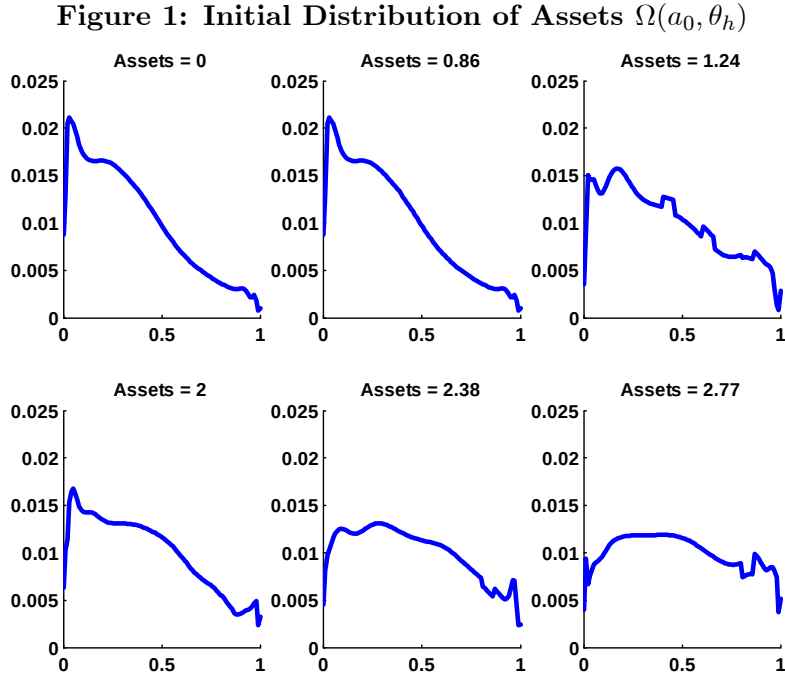
6.3 Initial Ability, Initial Assets, and the Ability Transition Matrix

Black, Devereux, and Salvanes (2003) contend that the correlation between educational attainment of parents and their children are most likely due to inherited ability and family characteristics such as the resources to invest in education when the child is young. As previously discussed, the initial asset and ability state pair (a_0, θ_h) essentially summarize all socioeconomic characteristics of the agent that have been determined prior to making the college/work decision. Thus, the parametrization of the joint probability distribution $\Omega(a_0, \theta_h)$ should be in-line with the empirical facts on the relationship between initial assets of young agents and their schooling ability. Previous work by Cameron and Heckman (1998) and Black, Devereux, and Salvanes (2003) suggests that long-term family resources have a strong influential affect on a student's ability as measured by standardized testing at the end of their high school years. In addition, Keane and Wolpin (2001) find that parental transfers are a monotonically increasing function of the parent's education. Given the positive relationship between parental education and parental wealth, this implies that our parameterization of the probability distribution $\Omega(a_0, \theta_h)$ should capture a correlation between initial assets available to agents and their innate high school ability.

We use the 1993 National Postsecondary Student Aid Survey (NPSAS:93) and High School and Beyond Sophomore Cohort (HS&B) data to carry out our estimation of the joint probability distribution $\Omega(a_0, \theta_h)$ using a two-step procedure that naturally correlates initial assets and innate

ability. Neither data set contains a sufficiently complete record of family income, parental contributions, and high school ability to allow us to estimate $\Omega(a_0, \theta_h)$ with one set of data. However the HS&B data does contain high school and college GPA data, along with other control variables that enables us to estimate the ability transition matrix $\Pi(\theta_h, \theta_c)$ with a single data set.

The probability distribution $\Omega(a_0, \theta_h)$ is estimated as follows. In the first step we use the NPSAS:93 data and partition family income into quintiles. For each income quintile, initial assets are estimated as the discounted average four year family contribution for students attending four-year institutions. In the second step, we take the HS&B data and partition family socioeconomic status into quintiles. A kernel density of cumulative high school GPAs, normalized to lie in the unit interval, is then estimated for each socioeconomic quintile. Under the assumption that income and socioeconomic status are sufficiently correlated, this procedure provides us with an estimate of the distribution of initial ability (as measured by high school GPA) and initial assets (as measured by family contributions). Therefore, we have naturally correlated assets and ability to match their empirical counterparts. Because we normalize our measure of mass to one, the kernel density estimate provides us with the joint probabilities for $\Omega(a_0, \theta_h)$. We transform these estimates of initial assets into model units by expressing them as a fraction of the wage of young college graduates in the benchmark model according to the corresponding ratio calculated using annual wage data from the March CPS supplements. Figure 1 summarizes this bivariate distribution for some discrete cuts of assets.



To completely characterize the ability learning process we must also estimate the ability transition matrix $\Pi(\theta_h, \theta_c)$. It would be inappropriate to simply assume that one's college ability is independent of their high school ability. A more reasonable assumption is that better performing high school students are more likely to perform well in college. That is, there is persistence in ability

going from high school to college. Such an assumption, however, does not give us any guidance on how persistent ability is. We therefore free ourselves from making arbitrary assumption about $\Pi(\theta_h, \theta_c)$ by estimating the ability transition matrix using data on high school and college GPAs.

The estimation of the ability transition matrix is more straight forward. Using the HS&B data we normalize high school and college GPA to lie in the unit interval, and then partition the two into quintiles. We then estimate an ordered probit to obtain the probability of moving from high school ability quintile q_i to college ability quintile q_j . Our probit model controls for numerous personal and institutional characteristics affecting college GPA, including: selectivity of college, type of college (public or private), degree expectations, race, high school region, mother and father's education, and income. Finally, we average the predicted transition probabilities across individuals in each respective high school GPA quintile. The probability transition matrix values are

$$\Pi(\theta_h \mid \theta_c) = \begin{bmatrix} 0.41 & 0.24 & 0.18 & 0.12 & 0.05 \\ 0.29 & 0.23 & 0.21 & 0.17 & 0.10 \\ 0.19 & 0.20 & 0.23 & 0.22 & 0.16 \\ 0.12 & 0.16 & 0.22 & 0.25 & 0.25 \\ 0.05 & 0.10 & 0.17 & 0.26 & 0.42 \end{bmatrix}.$$

where π_{ij} represents the probability of a new high school graduate in the i th ability quintile drawing a college ability in the j th quintile.

6.4 College

On average, a bachelor's degree in the U.S. requires a student to complete 120 credit hours. For computational purposes, we scale down the credit choice set so that one model credit corresponds to 10 credit hours. Thus, the graduation credit requirement is $\underline{x} = 12$. The returns to effort in terms of credits produces is dictated by the curvature of the credit production technology. Because our credit accumulation function is analogous to the human capital accumulation function we set $\gamma = 0.70$, a standard parameterization in the literature. The remaining college parameters are related to the financial aspect of higher education.

In the 1992-93 school year, tuition and fees represented only 40 percent of the estimated total annual cost at public 4-year institutions. It is important to accurately measure the total direct cost of college as the influence of credit constraints may be biased downward otherwise. Therefore, our measure of tuition is average tuition, fees, and room and board charges at public four-year institutions during the 1992-3 school year as reported by College Board (2006) measured on a per-credit basis consistent with our model units. This measure of tuition more completely reflects the total direct cost of attending college than simply using tuition. As with assets, we express the per-credit cost of college $\mathcal{T}$ as a fraction of young college graduates' wage.⁹ We find that the per-credit cost of college is approximately 7 percent of a young college graduate's wage calculated

⁹To account for the fact that we have scaled the 120 credit hour degree requirement down by twelve, we also adjust the per-credit cost of college before expressing in terms of wages.

using the 1994 March CPS supplement. In the benchmark estimation we set $\mathcal{T} = 0.07w_y^g$.

Per-period grants y are estimated using the 1992-93 National Postsecondary Student Aid Study. Our measure of grants is designed to account for the wide range of financial aid available to students in addition to student loans. Grants are computed as the average sum of all federal, state, institutional, and other grants and scholarships. As a percent of a young college graduate's wage, we find total grants to be 5.18 percent. Therefore, in the benchmark estimation $y = 0.0518w_y^g$ per period.

Table 1: 1992-1993 Stafford Loan Annual Loan Limits

Class Level	Loan Limit (Data)	Period	Loan Limit (Model)
First & Second Year	\$2,625	First	$-0.17w_y^c$
Third & Fourth Year	\$4,000	Second	$-0.42w_y^c$
Fifth Year	\$4,000	Third	$-0.67w_y^c$

Per-period borrowing constraints are chosen to match as closely as possible the Federal Stafford Loan Program as it existed during the 1992-93 academic school year. Since our model of college corresponds to a maximum of six years, but students may only participate in the federal loan program for up to five years, we restrict the amount a student may borrow if they take three periods. As with tuition and grants we must convert the empirical loan limits into model units. Again, this is done by expressing benchmark loan limits as a fraction of the young college graduates wage. In Table (1) we present the loan limits used in the model along with their empirical counterparts.

Under the federal program students repayment does not begin until after leaving school. To mimic this feature of the student loan program we have adjusted the loan limits to allow for cumulative debt a to be rolled over each period. The final student loan parameter relates to the repayment of debt after leaving school. The standard repayment plan under the federal program is ten years which is approximated by setting the fraction of debt that maybe rolled over each period after entering the workforce κ to be 0.50.

7 Estimation and Model Evaluation

Using a minimum distance approach we estimate the model's remaining parameters. The parameters are chosen so that the following three aggregate economic statistics produced in the baseline economy comes as close as possible to those of the educational environment of the early 1990s in the U.S. while at the same time respecting the market clearing conditions.

Enrollment and dropout rates: The first two statistics that we target are the college enrollment rate and college dropout rate. Enrollment and dropout rates are nontrivial to calculate because of the many ways to define them. Our model is best used to study first-time college students considering a four-year college path. To keep our targets in-line with our model, we choose to target the enrollment and dropout rates corresponding to these students. According to the BLS, 62 percent of recent high school graduates enrolled in college (broadly defined) in 1993. Nearly two-thirds

(64 percent) of these students enrolled in 4-year institutions. Thus, we choose the enrollment rate target to be 40 percent. Gladieux and Perna's (2005) use the Beginning Postsecondary Students data (BPS) to estimate the college dropout rate for first time students. The authors classify anyone who has not graduated by the end of the sixth year of the study as a dropout. In reality, a good portion of those students still in school after six years will eventually graduate. Using the tables provided in their paper a range of 24%-39% can be placed on the real dropout rate depending on the graduation/dropout assumption of students still in school at the end of the study. We chose a benchmark dropout target rate of 26%, well within in the range of the real dropout rate.

Time-to-Degree: The third target is time-to-degree at four-year colleges and universities. We rely on the recent empirical work of Bound, Lovenheim, and Turner (2006) that focuses on this often neglected topic. Similar to the problem with accurately calculating dropout rates, time-to-degree estimates suffer from the data's failure to track every entering student until they complete their degree. Bound, Lovenheim, and Turner (2006) use the National Educational Longitudinal Study of 1988 to estimate the time-to-degree for the high school class of 1992 that first enrolled at non-top 50 4-year institutions at 5.23 years. This value is most likely lower than the actual time to degree because they condition on having received a B.A. within eight years of entering. However this horizon is sufficiently long that their estimate should not be far from the true mean. Thus, the benchmark time-to-degree target is set to 5.23 years.

Production function: The production function has a total of ten parameters. In addition, we must specify the depreciation rate which we choose as $\delta = 0.06$. Capital's share of income α is set to 0.36. We normalize the aggregate productivity parameter A to unity. The elasticity of substitution between high school and college workers σ_E and the elasticity of substitution between young and old workers σ_A pin down the parameters ρ and φ . Card and Lemieux (2001) estimated the elasticity of substitution between high school and college educated workers to be about 2.5, and the elasticity of substitution between young and old workers to be around 5. Using these estimates and the fact that $\rho = 1 - 1/\sigma_E$ and $\varphi = 1 - 1/\sigma_A$, we set $\rho = 0.60$ and $\varphi = 0.80$.

The remaining productivity parameters ($\nu_y, \nu_o, \psi_y, \psi_o, A_H, A_G$) were estimated as follows. First note that we can invert the equilibrium wage equations and form the following ratios

$$\frac{A_C}{A_H} = \frac{w_C}{w_H} \left(\frac{G}{H} \right)^{1-\rho} \quad (9)$$

$$\frac{\psi_o}{\psi_y} = \frac{w_{Co}}{w_{Cy}} \left(\frac{G_o}{G_y} \right)^{1-\varphi} \quad (10)$$

$$\frac{\nu_o}{\nu_y} = \frac{w_{Ho}}{w_{Hy}} \left(\frac{H_o}{H_y} \right)^{1-\varphi} . \quad (11)$$

The ratio of productivity units determines the the various age and skill premiums in the economy. Because the premiums are relative we can set $\psi_y = \nu_y = A_H = 1$. Thus, given our choices for ρ and φ , and values for average wages and aggregate labor supply which we calculate from the 1994 March CPS supplement using the method of Card and Lemieux (2001), equations(9) – (11) provide us with estimates for the three remaining parameters.

In Table (2) we summarize the benchmark estimation results. The top panel shows how well the model performs compared to our chosen targets, while the middle panel presents the corresponding parameter estimates.

Table 2: Estimation of the Model (Annualized Values)

Statistic	Target	Model
Enrollment Rate	40.0%	39.6%
Dropout Rate	26.0%	27.8%
Time-to-Degree	5.2 years	5.4 years
College Skill Premium 18-65	1.87	1.82
College Age Premium 24-65	1.55	1.52
High School Age Premium 18-35	1.27	1.25

Variable	Parameter	Value
Return to partial education	χ	0.86
Prob. of return to partial education	p	0.92
Continuation value factor	β_R	3.68
Efficiency parameter old with College	ν_o	1.29
Efficiency parameter old with HS	ψ_o	1.68
Technology efficiency College	A_C	1.37

Overall, the model performs well relative to the educational statistics calculated from the data. In the baseline economy the enrollment rate is 39.6 percent, whereas the empirical counterpart at four-year institutions is 40.0 percent. The model's predicted dropout rate is slightly higher than the target in the data, 27.8 percent versus 26.0 percent. However, the obtained figure is within the range of estimates of Gladioux and Perna's (2005). With respect to our last estimation target we find that the model is consistent with an average time-to-degree of within 2 months and 12 days of that calculated by Bound, Lovenheim, and Turner (2006). To put this difference in perspective, consider that the benchmark time-to-degree is within the terms of a regular academic quarter of that found in Bound, Lovenheim, and Turner (2006). The parameters controlling the dropout wage rate reveal that the college dropout wages is determined as a weighted average of the high school wage (86 percent) and college wage (14 percent). However, the model's predicted probability of receiving the dropout wage is only 8 percent. These two parameters combined are consistent with the fact that the return to partial education is much closer to that of high school graduates wage than to a college graduates wage.

Since we use a production function with heterogenous labor inputs it is interesting to explore the implied wage premiums that result from the parameterized baseline model. Wage rates, or

more specifically wage premium, are a key factor determining enrollment and dropout rates. The wage rates in the model do not represent an agent's net return to education. Calculating the return to education is complicated by other factors that vary across individuals such as the initial assets and the agent's financing choice. For individuals that have sufficient resources at hand the cost of education is lower than those that need to use student loans which require interest payments. Table 2 reports the skill and age premiums produced by the model with those found in the data: college to high school skill premium w^g/w^h , the college age premium w_o^c/w_y^c , and the high school age premium w_o^h/w_c^h . The wage premiums implied by the model are consistent with their empirical counterparts calculated using the March CPS. For example the model predicts a college skill premium of 1.82 compared to an empirical skill premium of 1.87.¹⁰ The college age premium is determined by the ratio of wages of college graduates that are between ages 36 and 65 and those between ages 24 and 35. This feature captures the upward slopping profile of earnings over the life-cycle. This upward trend in wages over time within education groups is also found in high school graduates as indicated by a high school age premium of 1.27 in the model.

The model also makes prediction for the distribution of hours worked by college students. The model only considers the extensive margin ignoring the participation decision. As a result all students supply some amount of labor in the market. Conditional on participating, it is possible to compare the distribution of hours worked in the model with the data. Figure 2 compares the model distribution with the CPS October supplement for 1992 using only full-time students between age 18 and 26.

Figure 2: Distribution of Hours Worked by College Students

The data suggests that the majority of the college (65 percent) students work 20 hours a week or less. Only a small fraction of students (8.5 percent) work 40 hours a week or more. Along the extensive margin, the model is consistent the asymmetric pattern observed in the data, but the model distribution of is more skewed towards part-time with 75 percent of the students working part-time or less and 4.5 percent working 40 hours a week or more.

The model makes prediction of the labor supplied by college students. Table 3 summarizes the in-school labor supply for the individual enrolled in college in period 1 and period 2 (ages 18 and 20). The baseline model predicts that individuals in the lowest ability group (Q1) do not choose to enroll in college. The model makes interesting predictions about the labor supply across ability,

¹⁰Because there are two age specific labor inputs for each education level, the skill premium is the ratio average college wage w^g to average high school wage w^h . These were calculated as $w^g = \frac{dY}{dN} \frac{dN}{dG}$ and $w^h = \frac{dY}{dN} \frac{dN}{dH}$, respectively.

income types, and school years.

Table 3: In-School Labor Supply

		Ability θ				
	Age=18	Q1	Q2	Q3	Q4	Q5
C	Q1	0.0000	0.3017	0.3133	0.3642	0.3748
A	Q2	0.0000	0.2048	0.1395	0.1552	0.2281
S	Q3	0.0000	0.1959	0.0996	0.0887	0.1421
H	Q4	0.0000	0.0439	0.0497	0.1080	0.1121
(x)	Q5	0.0000	0.0699	0.0545	0.0709	0.1142

		Ability θ'/θ				
	Age=20	Q1	Q2	Q3	Q4	Q5
C	Q1	0.1059	0.1642	0.0573	0.1024	0.1257
A	Q2	0.0579	0.1293	0.0531	0.0508	0.0805
S	Q3	0.0224	0.1182	0.0160	0.0148	0.0203
H	Q4	0.0593	0.0932	0.0317	0.0494	0.0105
(x)	Q5	0.0570	0.1697	0.0560	0.0228	0.0434

8 Education Policy

In this section we explore the impact of various education policies on the incentives to enroll in college, extend time in school, and drop out. Predicting the affects of education policy in our model is complicated by several features. First, the structure of the financial aid package offered interacts with each agent’s existing resources, borrowing constraints, ability, and time endowment which must be allocated optimally among effort required, labor supply, and leisure. Second, we model college as a multi-period lumpy investment. The restriction on students that credits may only be chosen in a discrete fashion prevents them from adjusting their investment decision in a continuous manner in response to policies. In turn, this requires that policy interventions be of sufficient size to affect behavior across the various behavioral margins. Third, the correlation between ability and financial resources is likely to impact the behavior of lower income students who are also from the bottom end of the ability distribution. At the same time there general equilibrium effects in the labor markets that encourage targeted students to enter the labor market without obtaining a college education.

It is our contention that the complexity of the model is what makes it ideal for studying the role of education policy intervention. The majority of our analysis is focused on policies that affect the cost of tuition and change the size of government provided grants. In contrast with

Abbot, Gallipoli, Meghir, and Violante (2006) who assume a single arbitrary change in the size the education program, we consider the impact of tuition and grant policies across a range education program sizes. The advantage of this approach is that allows to determine the effectiveness of each program to impact the decisions of college students across the various behavioral margins.

9 Conclusions

In this paper we developed a quantitative theory of college education which is embedded within the context of general equilibrium overlapping generations economy. We depart from the standard human capital literature and model college as a multi-period risky investment with endogenous enrollment, time-to-degree, and dropout behavior. The tuition expenditures required to complete college were allowed to be funded using federal grants, student loans, and working while in college. We used the model to test the effectiveness of three distinct education policies: tuition subsidies (broad based, merit based, and flat tuition), grant subsidies (broad based and merit based), and loan limit restrictions (with and without endogenous in-school labor supply). Our model predicts that broad based tuition subsidies and grants increase college enrollment. However, due to the correlation between ability and financial resources most of these new students are from the lower end of the ability distribution and eventually dropout or take longer than average to complete college. Merit based education policies counteract this adverse selection problem but at the cost of a muted enrollment response. We find that tuition programs perform marginally better with respect to enrollment, time to degree, and total cost while grant based programs improves slightly upon dropouts.

The final policy experiment highlights an important interaction between borrowing constraints and the labor supply of college students. The baseline model is consistent with the findings of Cameron and Heckman (1998, 1999) and Keane and Wolpin (2001) that find short term liquidity constraints play no significant role in college attendance decisions. Nevertheless, a significant decrease in enrollment is found to occur only when borrowing constraints are severely tighten and the option to work while in school is removed. This result suggests that previous models that have ignored the student's labor supply when analyzing borrowing constraints may be lacking and insufficient for understanding the impact of education policy.

In a situation where the government has no information about student ability or college performance, we find that a significant adverse selection problem that prevents broad-based education policies (tuition subsidies and grant) from simultaneously increasing enrollment and reducing the number of dropouts and time to degree. However, there may exist merit based programs that would eliminate the apparent trade-off between enrollment and dropout rates of the uniform education policies. We leave the study of all these policies for future research.

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