Fiscal Multipliers and Search Frictions in the Goods Market*

Nicolas Petrosky-Nadeau[†]
Tepper School of Business
Carnegie Mellon University

Etienne Wasmer[‡]
Sciences Po Paris

August 2014 (first version May 2014)

Preliminary, please do not circulate

Abstract

The literature on the effect of fiscal policy and the size of fiscal multipliers focusses mostly on price rigidities and, in particular, on the labor-efficiency wedge. This paper explores an alternative rationale for fiscal policy: a stochastic disequilibrium between demand and supply of goods modeled by a matching process between consumers and firms in the goods market. Our model is flexible enough to introduce endogenous search margins, various degrees of price and wage rigidities, along with frictions in multiple markets. The equilibrium features both unemployment and endogenous capacity utilization. We investigate several properties of fiscal multipliers. First, their magnitude. Second, the timing of their effects on aggregate output and other outcomes. Third, whether they vary within a business cycle, across types of business cycles. Fourth, the ingredients necessary for large multipliers to arise, such as tighter financial markets or the source of amplification in the goods market.Our paper indicates that, with pooling of resources across employment types but frictions in the goods market and conventional values for relative risk aversion, multipliers tend to be small, around 1.05, and reach 1.15 to 1.25 when relative risk aversion and the persistence of government spendings are higher (5 for CRRA, 0.90 to 0.95 for the monthly persistence of government spendings).

*Financial support from ANR-2010 Evalpolpub is gratefully acknowledged. We thank seminar participants at U. of Mainz, at the Search Frictions in the Goods Market conference in Paris, at the CORE Economic Theory seminar, at the Bank of Chile - U Computensa in Santiago de Chile and Tel Aviv University, and our discussant Jim Albrecht at SAM Konstanz.

[†]Tepper School of Business, Carnegie Mellon University, 5000 Forbes Ave., Pittsburgh, PA, 15213. E-mail: npn@cmu.edu.

[‡]Sciences Po, Paris, Departement d'Economie, 28 Rue des Saint-Pères, 75007 Paris, France. E-mail: etienne.wasmer@sciences-po.fr.

1 Introduction

In the recent financial crisis, state and central bank interventions have been massive in most Western economies, and this has called for considerable additional research on fiscal multipliers. A fiscal multiplier, in the simplest sense is the GDP impact of an additional unit of public spendings. It is essentially a dynamic concept : while the GDP impact is in part instantaneous (within period change in GDP), it has also a lagged impact of the spendings. In particular, the long term cumulated GDP impact divided by the cumulated spendings is of interest. Absent any distortion, the long-run multiplier should be 1. If public spendings or transfers crowd out the private sector, it may be less than 1 or even negative. If it amplifies the economic activity, it may be larger than 1, at least temporarily.

A recent debate has focussed on how the size of multipliers vary over the cycle. The empirical debate is suggestive of strong multipliers effects in recessions. Auerbach and Gorodnichenko (2012a,b) have found that fiscal multipliers in the US may be as large as 2.5 in a recession and close to zero in an expansion; and in the same range in a panel of OECD countries including France, Germany, Italy and Canada. These conclusions have also been reached in several IMF reports or papers (World Economic Outlook 2012, Callegari et al., 2012; Baum et al. (2012), Blanchard and Leigh (2013)).

Two main theoretical channels exist. One is in the interaction between monetary policy and fiscal policy. As the economy approaches the zero lower bound on nominal interest rates, central banks may not avoid deflationary effects of fiscal consolidation and therefore a fiscal stimulus, in avoiding deflation, may have sizable impact on economic activity. Christiano et al. (2011) find very large fiscal multipliers (more than 3) in a DSGE setting, while Hall (2009) argues that multipliers may be around 1.7 in a similar situation. The second channel, as put clearly in Woodford (2011), is related to price and wage rigidities and has been summarized as a labor-efficiency wedge in the labor market clearing equilibrium condition. In most environments, be them competitive or under monopolistic competition, the fiscal multipliers must be smaller than 1. Any additional demand requires higher supply and thus additional supply effort which raises the implicit price of consuming.[1] The existence of a *constant* price markup $m \geq 1$ does not change the picture, it only changes the equality of marginal utility for leisure and the marginally utility of consumption[2]. Hence monopolistic competition on its own has no effect on the fiscal multiplier. It is thus apparent that to get around a fiscal multiplier larger than 1, one needs to have time varying markups and, as put by Woodford, "*the key to obtaining a larger multiplier is an endogenous decline in the labor efficiency wedge*", of which the

[1]Following Woodford, start from a competitive economy with output Y_t, a function f of hours worked H_t, is consumed in equilibrium by households (C_t) or government (G_t). A log linearizing this output condition $Y_t = f(H_t) = C_t + G_t$ leads to $dY_t/dG_t = \eta_u/(\eta_u + \eta_v)$ where $\eta_u = -Cu"/u' \geq 0$ and $u(C_t)$ is utility from consumption, and η_v is the elasticity of the disutility from supplying one additional unit of output. This multiplier is smaller than 1. In equilibrium, with $v'(H_t) = u'(C_t)$, where $v(H_t)$ is disutility for hours worked.

[2]$mv'(H_t) = u'(C_t)$

price markup is an important element. More generally, the discussion on fiscal multipliers must include the existence of financial frictions. Indeed, under the assumption that a monetary policy can stabilize the real interest rate r_t to a constant, that is to say, away from a zero interest rate trap and more importantly away from financial imperfections, optimal intertemporal behaviour of households leads to $\frac{u'(C_t)}{\beta u'(C_{t+1})} = 1 + r_t$ or, under a constant real interest rate, to constant consumption with curved utility. Hence, the desire - and ability - to smooth consumption leads multipliers to be one: any additional public spending is smoothed away in future consumption.

Our paper provides a framework to think about fiscal multipliers. We address these questions by developing a dynamic general equilibrium environment in which the demand and supply of goods do not match in an equilibrium sense, exactly as in the early Keynesian and Neo-Keynesian literature (Benassy, 1993). The disequilibrium between demand and supply of goods arises in a goods market with matching process between consumers and firms, very much like the approach in the macro-labor literature following the pioneering work by Diamond (1982), Mortensen (1982a,b) and Pissarides (1984). In these markets the number of trading agents is governed by a smooth and well-behaved aggregate matching function, the inputs of which are the number of agents willing to trade.

Our model is flexible enough to introduce endogenous search margins from both consumers (search effort) and firms (advertising effort), to introduce various degrees of price and wage rigidities and frictions in labor markets markets, leading both to unemployment and endogenous capacity utilization. We therefore have a dynamic model that allows us to focus the response of the economy to fiscal stimulus and hence to define and quantify fiscal multipliers with the following questions in mind : i) whether these multipliers are larger or smaller than 1 ; ii) whether they vary across macroeconomic states ; iii) what is the timing of their effects.

The important point here, somewhat overlooked in the literature, is that a fiscal expansion may increase aggregate consumption through a channel that is distinct from the pure rigidity of price, although it is partly connected with price changes. In goods markets with search-matching frictions, an increase in the purchasing capacity of agents will raise the demand for goods produced by firms, hence to an increase in price; this will lead firms to raise advertising efforts and in turn, given the increase in consumption surplus, to raise households' consumption effort, thereby reducing unused or underutilized capacities in the economy. The key frictions here are therefore the congestion externalities in the goods market. The economy moves further away from the constrained efficient allocation when demand weakens during a recession. This gives scope for fiscal policy to have welfare improving interventions.

We build upon previous research (Petrosky-Nadeau and Wasmer, 2011, Petrosky-Nadeau and Wasmer, 2014) where we investigated the dynamic responses of an economy with three search and matching frictional market, credit, labor and goods, to productivity innovations. We use and expand this convenient framework to further investigate the role of fiscal shocks. In particular, we relax the assumption of linear utility, which is necessary to obtain non-trivial effects of public spendings on consumption. We also introduce bilateral search effort in the goods market from the start, and study the limiting case where these effort functions are inelastic.

We mostly focus on the effect of transfers to individuals as their fluctuations are quantitively larger than direct purchases. Hall (2010) reports the evolution of spendings since 2007:Q4 and find that purchases from the Federal Government increased by 60 billions dollars in 2010:Q1, but that State and Local Spendings decreased by almost 80 billions dollars, with a net negative impact of 20 billions dollars, while extra benefits to individuals rose by almost 250 billions dollars. In other words, the largest share of public spendings we want to analyze arises from transfers, and further transfers to poorer individuals, suggesting that risk-aversion and curvature of utility functions matter both in terms of the dynamic impact of the fiscal shocks and its welfare consequences. On the latter point, we also need to discuss the effect of the policy in a framework where a well-defined social welfare function can be quantified. See e.g. Petrosky-Nadeau and Wasmer, 2014 for an analysis of the welfare implications of such a search economy.

The matching approach has been the subject of a renewed interest in the goods market, 30 years after Diamond (1982). Wasmer (2009) studied a steady-state matching economy (steady-state with linear utility) and finds closed form recursive solutions including a constant tightness of the goods market in the steady-state; Petrosky-Nadeau and Wasmer (2011) studied the dynamic implications of search frictions in the goods market under linear utility and endogenous search margins (consumer effort and search margins) and obtain large amplification and persistence of productivity shocks that match better the US data than any of the alternative matching models where goods market frictions are shut down ; independently, Bai et al. (2011) used a matching approach in a DSGE to define demand shocks and radically change the perspective of quantitative macroeconomics models, in arguing that most fluctuations arise from demand and not productivity shocks which represent 18% of total fluctuations. Kaplan and Menzio (2013) introduce a Burdett-Judd type of matching frictions and obtain countercyclical search effort : as the value of time goes down and the value of consumptions goes up, consumers spend more time and effort to find cheaper goods ; opposite to that countercyclicality of search margins, Hall (2012) finds that firms's advertising behavior is rather pro-cyclical; Gourio and Rudanko (2013) interpret advertising as evidence of frictions and find important effects on the volatility of investment and Tobin's q ; Michaillat and Saez (2014) also use search frictions in the goods market to introduce demand side effects, with money and wealth in the utility function; they obtain a characterization of the equilibrium interpreted as IS, LM, AS and AD curves and discuss policy implications ; den Haan (2013) captures with search frictions the behavior of inventories and obtain amplification of shocks, however claims that goods-market frictions are quantitatively not very important ; overall, there has been a strong and renewed interest in search and matching frictions in the goods market, which act as a complementary tool to Neo-Keynesian price setting mechanisms[3].

In **Section 2**, we define our main concepts of fiscal multipliers. In **Section 3**, we introduce the setup of the model with perfect financial markets and the main building blocks, such as entry of agents. In **Section 4**, we study optimal search efforts and pricing and define the equilibrium of the model, which is also extended to financial market imperfections following Wasmer and Weil (2004). We also explore alternative assumptions on the pooling of resources. In **Section 5** we run a number of impulse responses to fiscal expansions in a variety of contexts. In **Section 6**, we study how the welfare properties of the model help understand the impact of policies in this context.

2 Government sector

2.1 Government spending

There are some issues associated with fiscal policies. First, the nature of public spendings matters. It may affect the economy in several ways: i) either through untargeted transfers to consumers, e.g. general tax rebates ; ii) or targeted transfers e.g. unemployment insurance, food stamps ; iii) through direct purchases of goods and services, e.g. military spendings ; or iv) through new infrastructure - in the latter case it may also be improve the productivity of private investments and the overall effect must decompose the two sources of stimulus. In what follows we will restrict the benchmark analysis to the first type - untargeted transfers, but the framework can easily be adapted to study direct spendings or targeted transfers.

Denote by G_t the real amount of such government spending, expressed in units of a numeraire and affecting the disposable income of consumers. We will throughout assume that fiscal policies exhibit some persistence known to economic agents and denote by ρ_G the first order auto-correlation of the policy, where the time period is typically a month. Hence, for a given impulse policy G_{t_0} at time t_0, we denote by $G_{t_0}^{cum}(t)$

[3] As argued in Wasmer (2009), the way the early neo-keynesian literature defines the aggregate function of signals of agents (Benassy, 1993) is very reminiscent of the way Pissarides (1990, chapter 1) rationalizes the aggregate matching function: he sees it as a technology that makes consistent the desired demand and desired supply side of markets. One can reinterpret Bénassy's signals of required consumption/supply as being the elements of the "vacancy" side of markets (that is how many and by how much agents are willing to trade) to link the neo-keynesian tradition and the general equilibrium matching approach.

the cumulated spending at horizon t, and

$$G_{t_0}^{cum}(t) = \sum_{k=0}^{t} G_{t_{0+k}} = \sum_{k=0}^{t} \rho_G^k G_{t_0}$$
$$G_{t_0}^{cum}(\infty) = \sum_{k=0}^{\infty} \rho_G^k G_{t_0} = \frac{G_{t_0}}{1-\rho_G}$$

while its present-discounted cost at time t_0 for the public authority is, given a discount factor $\beta_G < 1$ which may or not be the same as that of private agents :

$$Cost = \sum_{k=0}^{\infty} \beta^k \rho_G^k G_{t_0} \tag{1}$$

2.2 Government revenue and debt

Two other issues considerably matter for the analysis. The first one is whether the debt is owned by residents or non-residents. If the debt was only owned by residents (e.g. a closed economy), it would simply act as a transfer between the households, who would consume less and the government who would consume more, with unclear aggregate effects. Therefore, we will think here of a small open economy where the fiscal stimulus is financed by borrowing on international financial markets and the debt is owned by non-residents. It follows that interest payment and reimbursement of the government will not be part of consumption. The second one is the timing of repayment, as taxes will come in deduction to consumption of agents. As our previous discussion alluded to, net transfers are positive at the impulse of a fiscal expansion ; while they progressively become negative as the accumulated debt is reimbursed.

However, to disentangle more clearly between the effect of the fiscal stimulus and its cost in terms of future taxes, we will assume that the government faces an interest rate r_G that differs from that of private agents. Denote by $r_G = 1/\beta_G - 1$ the interest rate faced by the government. The direct effect of additional public spendings is the GDP impact of the stimulus if r_G was zero. The second part is obtained as the difference in the response of the economy with a positive r_G and the response of the economy with $r_G = 0$. It is necessarily contractionnary to the extent that repayments are out of the economy, that is, if the debt is financed by non-residents.

Having r_G different from the discount rate of agents also allows us to vary it and run comparative static exercises on its value, to feature times of tensions on sovereign debts, when r_G is higher than in the steady-state ; or when the government instead faces lower interest rate than agents. Finally, when $r_G = 0$, the fiscal stimulus is equivalent to the effect of a net foreign transfer (e.g.. dividends from abroad, direct foreign aid) ; or to interpret G_{t_0} as the effect of a rise of world prices of a natural resource extracted in the country.

Denote by D_t the government's current net debt position in units of numeraire. Debt evolves according to the following law of motion:

$$D_{t+1} \quad = (1+r_G)D_t + G_t - T_t$$

where T_t are current period tax revenue in units of numeraire and G_t the transfer to consumers. Denote by $\tau_t = G_t - T_t$ the net transfers to consumers that will affect consumption, while $r_G D_t$ will not affect consumption.

A number of alternative assumptions on the timing of the financing may be relevant. However, the essential aspect of a fiscal expansion that these questions of financing raise, for our purposes, is that a fiscal stimulus aims to raise purchasing power in the short-run, and will then reduce it progressively as the stimulus must be repaid. The transversality condition states that $D_t e^{-rt}$ tends to zero as t goes to infinity but we want here to have a benchmark with a more severe constraint where any new spendings has to be fully financed.

One possibility is to assume that agents repay a constant share of the total present discounted value of the cost of a stimulus program. If the current debt at time t_0 is D_{t_0}, given our previous assumption on financing of government expenditure, we have that the stream of government revenue calculated at time t_0, denoted by $T_t - G_t = -\tau_t$ must be calculated as a constant value $-\tau(D_{t_0})$ for all future periods that must solve:

$$D_{t_0} \quad = \quad \sum_{i=0}^{\infty} \left\{ \left(\frac{1}{1+r_G} \right)^i [-\tau(D_{t_0})] \right\} = [-\tau(D_{t_0})] \sum_{i=0}^{\infty} (\beta_G)^i = \frac{-\tau(D_{t_0})}{1-\beta_G}$$

that is, the government levies taxes such that current debt is entirely reimbursed in the limit with equal payments. For the debt to decay at the rate $1/(1+r_G)$, the government need to tax households at rate:

$$T_t = \frac{r_G}{1+r_G} D_t$$

We will assume a stochastic process for government spendings, such as

$$G_t \quad = \quad \rho_G G_{t-1} + \sigma_G \epsilon_t^G$$

where $\rho_G \in (0,1)$ and ϵ_t^G is drawn from a standard normal distribution.

2.3 Fiscal Multipliers

Denote by Y_t the per-period output of the economy, and by Y_{t_0-1} the value of output in the instant before the fiscal stimulus. Denote by ΔY_t the deviation from Y_{t_0-1} due to the fiscal policy, and by $\Delta^{cum} Y_t$ its cumulated value. Hence, we can define a GDP-multiplier at time horizon t the quantity M_G where G stands both for government spending in this instance and for the goods market elsewhere:

$$M_G(t) = \frac{\sum_{k=0}^{t} \Delta^{cum} Y_{t_0+k}}{\sum_{k=0}^{t} G_{t_0+k}} \tag{2}$$

From this convenient expression we have the short-run and the long-run multipliers quite readily as:

$$M_G(0) = \frac{Y_{t_0} - Y_{t_0-1}}{G_{t_0}}$$
$$M_G(\infty) = \frac{\sum_{k=0}^{\infty} \Delta^{cum} Y_{t_0+k}}{\sum_{k=0}^{\infty} G_{t_0+k}}$$

Further, the short-run multiplier itself, in a rational expectation setup, depends on the future sequence of spendings and how these spendings are anticipated by agents. Hence, an interesting special case is one-shot fiscal impulse on the economy, i.e., when $\rho_G = 0$. This isolates the direct effect of government spending from the dynamic effects of spending, and changes in taxes, at different time horizons on the fiscal multiplier function (2).

Finally, the multiplier might be artificially higher due to a possible effect of prices. Hence we need to compare a nominal multiplier to a real multiplier where both the Y_{t_0} (but not Y_{t_0-1}) and G_t in the denominator must be deflated by an index of prices.

3 A simple setup with imperfections in the goods, labor and credit market

We extend here the discrete time set up of Petrosky-Nadeau and Wasmer (2011). There are frictions in credit, labor and goods market, augmented with a government sector. In order to derive the main mechanism faster, we first simplify the financial block and assume perfect credit markets for firms until the end of Section 4 where we borrow the structure from Wasmer and Weil (2004). In particular, the quantitative analysis will take into such imperfections.

There are four types of goods: production goods indexed by 0 and 1, a numeraire fully convertible into good 0 (see below) and finally leisure.

- Good 1 (search good) is subject to search frictions. It can either be interpreted as services or manufactured goods such as cars, housing and consumers goods). It is produced inelastically by firms in quantity x_t (common to all firms but potentially time varying with technological innovations) and its consumption by individual households is indivisible. Its price is denoted by $\mathcal{P}_t$.

- Good 0 (frictionless essential good) is accessible with no frictions. We choose to normalize its price to 1. It can be thought as a set of inferior goods (food, basic utility, standard goods, services). It is produced by consumers in fixed quantity $\bar{y}_0$, as discussed in the consumption block section. It cannot be stored. Its marginal utility may vary with its consumption. In that case, it is not a numeraire and a conversion technology between the numeraire and the good is available to households.

- The government may decide to inject a true numeraire through transfers G_t or withdraw it through taxation of households T_t. This numeraire can be transformed into consumption of the essential good 0 with a linear technology, as if the government was producing services or building infrastructures. An alternative assumption is that good 0 is supplied in exchange for the numeraire. Each of these alternative assumptions insures the same set of equations.

- Consumers and workers (and in the imperfect credit extension of the last part of Section 4, entrepreneurs) will all search for a trading partner in their relevant market. Consumers search a selling firm, workers search for a vacancy (and under imperfect financial markets, entrepreneurs search for a banker). They sacrifice leisure against search effort.

- Selling firms search for a consumer and pay effort in terms of the numeraire. The vacancy searches for a worker at a numeraire cost. In the extension, the banker will also pay a search cost in terms of the numeraire.

At the end of the day, since the essential good 0 cannot be stored, it is fully consumed by the household until exhaustion of its individual supply available to the household and until exhaustion of the numeraire. Note however that this result does not rely on that assumption: instead, it relies on the assumption of indivisibility of good 1 and its inelastic supply. Even if good 0 could be stored or saved, that would not increase the utility of consuming the search good, it would simply lead to a higher price (see below the price section). The marginal utility of saving good 0 is therefore zero at the end of the period. This useful property of the model is perfectly consistent with the night-and-day trading markets in the money-search literature (Lagos and Wright 2004).

3.1 Consumer block

At any point in time in this economy there are matched and unmatched consumers. Matched consumers and firms may separate (change in tastes) with probability $s^G \in (0,1)$. In the goods market, we denote by $\Xi_{M,t}$ and $\Xi_{U,t}$ the number of matched and unmatched consumers, and by $\mathcal{N}_{\pi,t}$ and $\mathcal{N}_{G,t}$ the number of selling and searching firms respectively, where a firm is a single product produced by a a single worker. Thus the total number of employed workers is

$$1 - \mathcal{U}_t = \mathcal{N}_{\pi,t} + \mathcal{N}_{G,t} \tag{3}$$

We assume that each firm can serve only one consumer, which implies:

$$\Xi_{M,t} \equiv \mathcal{N}_{\pi,t} \tag{4}$$

3.1.1 Random matching in the goods market with endogenous bilateral search efforts

Unmatched consumers $\Xi_{U,t}$, exert an average search effort $\bar{e}_{g,t}$, to find unmatched goods, $\mathcal{N}_{G,t}$, through a process summarized by a constant return-to-scale function. Symmetrically, firms exert an average advertising effort $\bar{e}_{A,t}$, expressed in terms of the numeraire, to find unmatched consumers $\Xi_{U,t}$. The number of matching per unit of time in the goods market is given by the increasing, constant returns to scale function $\Omega_G(\bar{e}_{g,t}\Xi_{U,t}, \bar{e}_{A,t}\mathcal{N}_{G,t})$. The first argument, $\bar{e}_{c,t}\Xi_{U,t}$, can be thought of as the effective demand for new goods. Conversely, $\bar{e}_{A,t}\mathcal{N}_{G,t}$ is a measure of the effective supply of news/unmatched goods.

Following Pissarides (2000), the aggregate meeting rates between consumers and firms are given by

$$\begin{aligned}
\lambda_t &= \frac{\Omega_G(\bar{e}_{g,t}\Xi_{U,t}, \bar{e}_{A,t}\mathcal{N}_{G,t})}{\mathcal{N}_{G,t}} = \lambda(\xi_t, \bar{e}_{g,t}, \bar{e}_{A,t}) \quad \text{with} \quad \lambda'(\xi_t) > 0 \\
\check{\lambda}_t &= \frac{\Omega_G(\bar{e}_{g,t}\Xi_{U,t}, \bar{e}_{A,t}\mathcal{N}_{G,t})}{\Xi_{U,t}} = \check{\lambda}_t(\xi_t, \bar{e}_{g,t}, \bar{e}_{A,t}) \quad \text{with} \quad \check{\lambda}_t{}'(\xi_t) < 0,
\end{aligned}$$

where

$$\xi_t = \Xi_{U,t}/\mathcal{N}_{G,t} \tag{5}$$

is a measure of tightness in the goods market from the consumers' point of view. $\tilde{\lambda}_t$, the probability that an unmatched consumer finds a suitable firm from which to buy goods, is decreasing in goods-market tightness. Conversely, the greater ξ_t, the greater the demand from consumers relative to the goods waiting to find consumers and the shorter the producer's search. This is precisely what creates the feedback from the goods market to the labor market: the returns to hiring a worker are greater when it is easier to find customers.

3.1.2 Size of the consumer pool

An important modeling choice here is to fix the total number of consumers. Denote the total number by

$$\bar{\Xi} = \Xi_U + \Xi_M \tag{6}$$

There are three alternative assumptions: i) the total number of consumers could either be the total number of agents: workers (mass 1) to which we may want to add firms (mass $\mathcal{N}_G + \mathcal{N}_\pi$) ; ii) if we assume that profits are redistributed lump-sum to workers, the total number of consumers would be $\bar{\Xi}$=1 ; iii) if we assume that only employed workers can consume the frictional good, then the total number of consumers would be $\bar{\Xi} = 1 - \mathcal{U}$. We differ taking a stand on the value of $\bar{\Xi}$ for the moment.

3.2 Firms' side

Time is discrete. Firms have for the moment perfect access to financial markets. They have, as in a standard search model, to finance the recruitment of a worker and pay a per period cost γ to maintain an active job vacancy. With probability q_t, the firm is successful in hiring a worker, with complementary probability, it remains in this stage (denoted by v for *vacancy*). The firm offers a wage w_t to the worker as long as the firm is active.

Now endowed with a worker, the firm could start producing x_t units of output from this particular project and attempt to sell it on the goods market, but it has no customers. We call this stage g for *goods* market. The firm needs to spend resources to find customers which can be thought of as the cost of maintaining a shop, the cost of advertising and, more generally, all retail sector inputs. This cost of advertising effort is specified *in numeraire* as $\sigma_{A,t} = (\chi_A/\eta_A)\, e_{A,t}^{\eta_A}$ where individual firm advertising effort is $e_{A,t}$. The consumer arrives meet this good with probability proportional to the effort an individual firm relative to the average effort by other firms, over and above an aggregate term depending on the aggregate tightness of the goods market. That is, an individual firm's probability of meeting a consumer in the goods market is given by $e_{A,t}\lambda_t/\bar{e}_{A,t}$.

In the final stage, the firm is matched with a consumer and its output x_t is sold at price $\mathcal{P}_t$. Matched consumers will absorb the entire production.[4] Assuming that production involves an operating cost C_q over and above the wage and that the good cannot be stored, the firm chooses not to produce in stage g. With revenue $\mathcal{P}_t x_t$, the firm pays the worker w_t, the operating cost C_q, and enjoys the difference. We denote this stage by π, standing for profit. In addition, the consumer may stop consuming the particular good produced by this project with probability s^G, in which case the project returns to the previous stage

[4]An alternative interpretation of x_t is a quality shock affecting consumers' utility when the good produced is indivisible.

g to search for another consumer. Workers may separate from the firm, which occurs each period with probability s^L.

Let the value of a firm for each of the stages be denoted by $J_{j,t}$, with j indicating the corresponding stage v, g, π for respectively vacancy, consumer search (goods market) and profits. This results in the following Bellman equations:

$$J_{v,t} = -\gamma_t + \frac{1}{1+r}\mathbb{E}_t\left[q_t J_{g,t+1} + (1-q_t)J_{v,t+1}\right] \tag{7}$$

$$J_{g,t} = -w_t - \sigma_{A,t} + \frac{1}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left[\frac{e_{A,t}\lambda_t}{\bar{e}_{A,t}}{}_t\, J_{\pi,t+1} + (1-\frac{e_{A,t}\lambda_t}{\bar{e}_{A,t}})J_{g,t+1}\right] + s^L J_{v,t+1}\right] \tag{8}$$

$$J_{\pi,t} = \mathcal{P}_t x_t - w_t - C_q + \frac{1}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left[(1-s^G)J_{\pi,t+1} + s^G J_{g,t+1}\right] + s^L J_{v,t+1}\right] \tag{9}$$

The formulation the labor-market stage in equation (7) describes the value of a job vacancy as a flow cost γ and an expected gain from hiring a worker, valued at S_g. The value of J_g in equation (8) takes into account the labor cost w_t, and the fact that the firm may or may not become profitable in the next period. This depends on meeting customers with probability $e_{A,t}\lambda_t/\bar{e}_{A,t}$, and whether the worker remains with the firm with probability $\left(1-s^L\right)$. If the worker leaves, the firm returns to the earlier vacancy stage v. As we will discuss in detail, the presence of a frictional goods market fundamentally alters the dynamics of J_g compared to the standard framework (e.g., Pissarides, 2000, Shimer, 2005) through the dynamics of the goods-market meeting rate, $e_{A,t}\lambda_t/\bar{e}_{A,t}$, and the price, $\mathcal{P}_t$. Finally, in stage π, equation (9) shows that the match produces revenue $\mathcal{P}_t x_t$, pays a wage and other production costs, and may return to earlier stages depending on the occurrence of consumer change of taste, labor turnover or bankruptcy.

3.3 Income, GDP and demand side

3.3.1 Income and GDP

Total net profits in this economy, Π_t, are:

$$\Pi_t = (\mathcal{P}_t x_t - C_q)\mathcal{N}_{\pi,t} - w_t\mathcal{N}_t - \sigma_A(e_{A,t})\mathcal{N}_{G,t} - \gamma\mathcal{V}_t,$$

where $\mathcal{N}_t = \mathcal{N}_{\pi,t} + \mathcal{N}_{G,t}$ is the sum of the number of firms matched with a consumer, $\mathcal{N}_{\pi,t}$, and of the number of firms in stage g, $\mathcal{N}_{G,t}$. In the equation above, the first term is revenue generated by firms in the profit stage, net of operating costs C_q . The second term represents wage payments in the economy. The remaining terms represent the search costs in goods and labor markets, the latter being the cost of maintaining $\mathcal{V}_t$ vacancies in the economy.

The profits net of search costs are pooled and distributed as lump sums to consumers. All consumers also have some free time to produce a number $\bar{y}^0$ of units of numeraire, which adds up to these profits to comprise disposable income. Consumers therefore receive $\Pi_t/\bar{\Xi}$ per person (recall that $\bar{\Xi}$ is the total mass of unmatched and matched consumers) and per period as a transfer. Resources, in our benchmark specification, are pooled across categories of workers, as in Merz (1995) and Andolfatto (1996). We relax this assumption in Section 6. The average disposable income of a representative consumer, I^d, is

$$\begin{aligned} I_t^d &= (\Pi_t + \mathcal{N}_t w_t + G_t - T_t)/\bar{\Xi} + \bar{y}^0 \\ &= \left[(\mathcal{P}_t x_t - C_q)\mathcal{N}_{\pi,t} - \gamma \mathcal{V}_t - \sigma_A(e_{A,t})\mathcal{N}_{G,t} + G_t - T_t\right]/\bar{\Xi} + \bar{y}^0 \end{aligned}$$

where G_t is a transfer and T_t is a tax as defined in Section 2.

GDP is defined as the sum of consumptions, investment and government expenditures:

$$Y_t = \bar{\Xi}\bar{y}^0 + (\mathcal{P}_t x_t - C_q)\mathcal{N}_{\pi,t} + G_t$$

3.3.2 Value Functions for Consumers

Individuals want to consume manufactured goods, but cannot buy them before prospecting on the goods market. Let us denote by $D_{U,t}$ and $D_{M,t}$ the values for a consumer of being unmatched and matched, respectively. The generic utility of consuming both goods (recall that good 1 produced by firms and good 0 which is produced by consumers) is denoted by

$$v(c_t^1, c_t^0) - \sigma_g(e_{g,t}).$$

where the last part σ_g is the sacrifice of leisure from attempting to consume in exerting search effort $e_{g,t}$. We further assume $\sigma_g(0) = 0$. Denote by $c_{U,t}^0$ and $c_{M,t}^0$ the consumption pattern of this essential good 0 by matched and unmatched consumers in the goods market. The viability of this economy requires consumers to prefer the frictional good in order to avoid a degenerate equilibrium where no search takes place. This implies that an equilibrium must be such that:

$$v(x, c_{M,t}^0) > v(0, c_U^0)$$

where

$$c_M^0 = c_U^0 - \mathcal{P}x$$

Matched consumers will always absorb x_t units of good 1 at the expense $\mathcal{P}_t x_t$ and consume what is left, $I_t^d - \mathcal{P}_t x_t$, on $c_{0,t}$. In other words, the per-period utility of unmatched consumer is $v(0, I_t^d)$ and the per-period utility of matched consumer will be $v(x_t, I_t^d - \mathcal{P}_t x_t)$.

Unmatched consumers search for a good at an effort cost $\sigma_g(e_g)$, with $\sigma_g'(e_g) > 0$, $\sigma_g''(e_g) \geq 0$, $\sigma_g(0) = 0$, and elasticity with respect to effort $\eta_{\sigma_g} > 0$. They perceive their search effort as influencing their effective finding rate, $e_{g,t}\check{\lambda}_t/\bar{e}_{g,t}$, where $\bar{e}_{g,t}$ is defined as the average search effort by consumers in the goods market. Consequently, we have

$$D_{U,t} = v(0, c_{0,t}) - \sigma_g(e_{g,t}) + \frac{1}{1+r}\mathbb{E}_t\left[\frac{e_{g,t}\check{\lambda}_t}{\bar{e}_{g,t}} D_{M,t+1} + \left(1 - \frac{e_{g,t}\check{\lambda}_t}{\bar{e}_{g,t}}\right) D_{U,t+1}\right] \tag{10}$$

$$D_{M,t} = v(c_{1,t}, c_{0,t}) + \frac{1}{1+r}\mathbb{E}_t\left[\left(1 - s^L\right)\left[s^G D_{U,t+1} + \left(1 - s^G\right) D_{M,t+1}\right] + s^L D_{U,t+1}\right] \tag{11}$$

3.4 Matching in the Labor Market

Matching in the labor market is governed by a function $\Omega_L/(\mathcal{V}_t, \mathcal{U}_t)$, where $\mathcal{U}_t$ is the rate of unemployment. $\mathcal{V}_t$, already defined as the number of firms in stage l. The function is assumed to have constant returns to scale. Hence, the rate at which firms fill vacancies is a function of the ratio

$$\mathcal{V}_t/\mathcal{U}_t = \theta_t \tag{12}$$

where θ_t is a measure of the tightness of the labor market. This vacancy filling rate, $q(\theta_t)$, is given by

$$q(\theta_t) = \frac{\Omega_L(\mathcal{V}_t, \mathcal{U}_t)}{\mathcal{V}_t} \text{ with } q'(\theta_t) < 0.$$

Conversely, the rate at which the unemployed find a job is

$$\frac{\Omega_L(\mathcal{V}_t, \mathcal{U}_t)}{\mathcal{U}_t} = \theta_t q(\theta_t) = f(\theta_t) \text{ with } f'(\theta_t) > 0.$$

Once employed, workers earn a wage w_t specified later on.

4 Optimality conditions and equilibrium

4.1 Optimal consumer effort and advertising expenditures

The optimal search effort of consumers equalizes the marginal cost of effort and the discounted expected benefit yielded by that a marginal unit of effort:

$$\bar{e}_{g,t}\sigma_g'(e_{g,t}^*) = \frac{\check{\lambda}_t}{1+r}\mathbb{E}_t\left[D_{M,t+1} - D_{U,t+1}\right]. \tag{13}$$

Equation (13) implies that consumer search effort is increasing in the expected capital gain from consuming the manufactured good 1. All consumers exert the same effort thanks to the assumption of resource pooling for the essential good 0:

$$e_{g,t}^* = \bar{e}_{g,t}.$$

The first order condition for advertising effort is

$$\bar{e}_{A,t}\sigma_A'(e_{A,t}^*) \quad = \quad \frac{\lambda_t\left(1-s^L\right)}{1+r}\mathbb{E}_t\left[J_{\pi,t+1} - J_{g,t+1}\right] \tag{14}$$

and again, in a symmetric equilibrium, all firms do the same advertising effort:

$$e_{A,t}^* = \bar{e}_{A,t}$$

The search efforts of both sides of the goods market are procyclical as they follow the marginal expected surplus from the goods market match. This procyclicality of advertising is empirically confirmed by Hall (2012) the U.S. The procylicality in search for goods was discussed in Petrosky-Nadeau and Wasmer (2011).

4.2 Pricing rule in the Nash bargaining case

Consistent with the search literature, we postulate that the price $\mathcal{P}_t$ is bargained between a consumer and a firm. The total surplus to the consumption relationship is $S_t^G = (J_{\pi,t} - J_{g,t}) + (D_{M,t} - D_{U,t})$. The good's price is determined as

$$\mathcal{P}_t = \text{argmax}\left(J_{\pi,t} - J_{g,t}\right)^{1-\alpha_G}\left(D_{M,t} - D_{U,t}\right)^{\alpha_G}$$

, where $\alpha_G \in (0,1)$ is the share of the goods surplus, S_t^G, going to the consumer. Under linear utility, the rule leads to:

$$(1-\alpha_G)\left(D_{M,t} - D_{U,t}\right) = \alpha_G\left(J_{\pi,t} - J_{g,t}\right). \tag{15}$$

However, the sharing rule with non-linear utility is different. To fix ideas, consider that the utility in consumption of both goods is concave:

$$v(c_t^1, c_t^0) = b_1(c_t^1) + b_0(c_t^0) + b_{01} d_0(c_{0,t}) d_1(c_{1,t}) \tag{16}$$

with $b_1(0) = 0; \nu_1(0) = 0$; and with $b_1', b_0', d_0', d_1' \geq 0$, $b_1'' = 0$ and $b_0'' \leq 0$, $d"_0 \leq 0$, $d"_1 \leq 0$. The last term b_{01} is an interaction term reflecting possible complementarities in the two types of consumption.

If we assume that utility is not additively separable and concave, as specified in equation (16), the sharing rule is, denoting by $v_{0,t}' = b_{0,t}' + b_{01}\nu_1(x_t) d_{0,t}'$.

$$(1 - \alpha_G)(D_{M,t} - D_{U,t}) = \alpha_G v_{0,t}' (J_{\pi,t} - J_{g,t}).$$

This amounts to an effective bargaining power that depends on marginal utilities at date t and, thus, on time. We denote this time-varying share $\tilde{\delta}_t$ with:

$$\tilde{\delta}_t = \frac{\alpha_G v_{0,t}'}{1 - \alpha_G + \alpha_G v_{0,t}'} = \frac{1}{\alpha_G / v_{0,t}' + 1 - \alpha_G}$$

The effective bargaining power of consumers is equal to α_G with linear and separable utility ($b_{0,t}' = 1$ and $b_{01} = 0$). It is above α_G if $v_{0,t}' > 1$, and converges to 1 if $v_{0,t}' \to \infty$. It is below α_G if $v_{0,t}' < 1$ and converges to 0 if $v_{0,t}' = 0$.

Denote by $\sum(x_t, \mathcal{P}_t) = v(x, I_t^d - \mathcal{P}_t x_t) - v(0, I_t^d) + \mathcal{P}_t x_t$ the flow surplus associated with the consumption of x_t units of goods.

- Under linear and additively separable utility (with $b_{01} = 0$ and $b_0' \equiv 1$,), that is if the essential good c_0 enters linearly in utility with marginal utility $b_0' = 1$ (then it becomes a numeraire good), $\sum$ only depends on x_t and not on prices:

$$\sum(x_t, \mathcal{P}_t) = b_1(x_t)$$

- Under non linear utility:

$$\sum(x_t, \mathcal{P}_t) = b_1(x) + b_0(I_t^d - \mathcal{P}_t x_t) - b_0(I_t^d) + \mathcal{P}_t x_t + b_{01} d_0(c_{0,t}) d_1(c_{1,t})$$

In the case $b_{01} = 0$, a marginal increase of I_t^d by, say, $G_t = \Delta I_t^d$ leads to in increase in c_0 that has higher marginal value for matched consumers, since those have to pay for the manufactured good.

Hence, the flow surplus of the consumer, at a constant price, increases with income: the utility value of consuming good 1 is unchanged, but a non-linear utility for good 0 actually implies that the cost of good 1 in terms of good 0 decreases with income.

4.3 Procyclicality and complementarity of search efforts on both sides of the goods market

Equations (13) and (14) can be rewritten to include the surplus from consumption, leading to:

$$\bar{e}_{g,t}\sigma_g'(e_{g,t}) = \frac{\check{\lambda}_t}{1+r}\mathbb{E}_t\tilde{\delta}_{t+1}S_{t+1}^G. \tag{17}$$

$$\bar{e}_{A,t}\sigma_A'(e_{A,t}) \quad = \quad \frac{\lambda_t}{1+r}\left(1-s^L\right)\left(1-s^C\right)\mathbb{E}_t(1-\tilde{\delta}_{t+1})S_{t+1}^G \tag{18}$$

Hence, to the extent that S_t^G is procyclical - it indeed increases with productivity (see Petrosky-Nadeau and Wasmer 2011) as well as with G_t as will be shown, search efforts of both sides of the goods market would also be procyclical. A partial confirmation can be found in Hall (2012) who shows the procyclicality of advertising costs.

Further, combining the optimality conditions for the intensive search margins with bargaining, we have:

$$\frac{\bar{e}_{A,t}\sigma_A'(e_{A,t})}{\bar{e}_{g,t}\sigma_g'(e_{g,t})} = \xi_t \times \frac{\mathbb{E}_t(1-\tilde{\delta}_{t+1})S_{t+1}^G}{\mathbb{E}_t\tilde{\delta}_{t+1}S_{t+1}^G}\left(1-s^L\right)\left(1-s^C\right) > 0$$

Since effort of one side increases the returns to effort of the other side, we have a strategic complementarity arising from bilateral search effort. For instance, if both cost functions σ_g and σ_A had the same elasticity, say quadratic, and in a symmetric equilibrium where all consumers and all firms provide, respectively, the same effort level, it follows that the effort of one side would be equiproportional to the effort in the other side. In the general case of convex but non identical costs functions, the effort of each side are still strategic complement. To sum up, both search efforts in the goods market are individually procyclical and this procyclicality is strengthened by the strategic complementarity.

4.4 Solutions to the price equation

We first rewrite the consumption surplus as (see Appendix):

$$
\begin{aligned}
S_t^G = & \sum(x_t, \mathcal{P}_t) + \sigma_A(1-\eta_{\sigma_A}) + \sigma_g(1-\eta_{\sigma_g}) - C_q \\
& + \frac{\left(1-s^L\right)\left(1-s^C\right)(1-s^G)}{1+r}\mathbb{E}_t S_{t+1}^G
\end{aligned}
\tag{19}
$$

Using the pricing rule, we obtain an implicit equation for the price:

$$
\tilde{\delta}_t\left[\mathcal{P}_t x_t - C_q + \sigma_A(1-\eta_{\sigma_A})\right] = (1-\tilde{\delta}_t)\left\{v(x_t, I_t^d - \mathcal{P}_t x_t) - v(0, I_t^d) + \sigma_g(1-\eta_{\sigma_g})\right\} \tag{20}
$$

$$
+\Lambda\mathbb{E}_t\left[(1-\tilde{\delta}_t)S_{t+1}^G\right] - \Lambda\mathbb{E}_t\left\{(1-\tilde{\delta}_{t+1})S_{t+1}^G\right\} \tag{21}
$$

The special case with linear and additive utility comes quite simply: we have in this case $\tilde{\delta}_t \equiv \alpha_G$. In that case, $v(x, I_t^d - \mathcal{P}_t x_t) - v(0, I_t^d) = b_1(x_t) - \mathcal{P}_t x_t$ and the price simplifies to:

$$
\mathcal{P}_t x_t = \alpha_G\left[C_q - \sigma_A(1-\eta_{\sigma_A})\right] + (1-\alpha_G)\left[b_1(x_t) + \sigma_g(1-\eta_{\sigma_g})\right] \tag{22}
$$

In that case, the price is a weighted average of production and advertising costs on the one hand, and of the utility of consumptio of good 1 plus the effort associated with consumption. Further, and this is essential for the paper, an increase in I_t^d has no impact on prices: any additional income is absorbed by the numeraire good for both matched and unmatched consumers. However, with a concave utility for good 0 (essential good), there are interesting effects to take into account.

In the case of concave utility the pricing rule is more complex and include intertemporal terms that do not cancel, due to the fact that future bargaining share evolve in time due to fluctuations in consumption. To simplify, one could either neglect such fluctuations, or choose a solution to the price problem that is simpler, the Kalai solution, that delivers immediately:

$$
\tilde{\delta}_t\left[\mathcal{P}_t x_t - C_q + \sigma_A(1-\eta_{\sigma_A})\right] = (1-\tilde{\delta}_t)\left\{b_1(x_t) + b_0(I_t^d - \mathcal{P}_t x_t) + b_{01}\nu_0(I_t^d - \mathcal{P}_t x_t)\nu_1(x_t) - b_0(I_t^d) + \sigma_g(1-\eta_{\sigma_g})\right\} \tag{23}
$$

A Taylor-expansion of $b_0(I_t^d - \mathcal{P}_t x_t) = b_0(I_t^d) - b_0'(I_t^d)\mathcal{P}_t x_t$ leads to:

$$
\left(\tilde{\delta}/\alpha_G\right)\mathcal{P}_t x_t = (1-\tilde{\delta}_t)\left\{b_1(x_t) + b_{01}\nu_0(I_t^d - \mathcal{P}_t x_t)\nu_1(x_t) + \sigma_g(1-\eta_{\sigma_g})\right\} + \tilde{\delta}_t\left[C_q - \sigma_A(1-\eta_{\sigma_A})\right] \tag{24}
$$

using $\tilde{\delta} + b_0'(I_t^d)(1 - \tilde{\delta}_t) = \tilde{\delta}/\alpha_G$. This ratio is equal to 1 only with linear utility, and is either smaller or larger than 1 depending on v_0' ; remember that $\tilde{\delta}$ is above α_G if $v_{0,t}' > 1$, and is below α_G if $v_{0,t}' < 1$. Hence, the greater the marginal utility for the essential good, the lower the price of good 1, *ceteris paribus.*

4.5 Wages and a price deflator

For simplicity, we assume a wage rule that takes the functional form

$$w_t = \chi_w(\mathcal{P}_t x_t)^{\eta_w}, \tag{25}$$

where η_w is the elasticity of wages to the marginal production of labor in terms of the essential good $\mathcal{P}_t x_t$. This simple rule, adapted from in Blanchard and Galí (2010), allows us to focus on the role played by the elasticity of wages to productivity for propagation. That is, for a given elasticity of wages, we can evaluate the propagation of shocks to the economy, and does not includes a full pass-through of prices on to wages. Petrosky-Nadeau and Wasmer (2011) explore wage variants such as Nash-bargaining. This is also calculated in Appendix and lead to two different wage equations, one for which the firm is in stage g, the other when the firm is in stage π.

In the spirit of search models, one may want to have a wage schedule as the outcome of Nash-bargaining between the firm and the worker. This is the second approach in which there is not one, but two wage schedules depending on whether or not the firm is currently selling its production. This will add analytical complications, but makes only a small quantitative difference compared to the simple wage rule.[5] The wage rule in this case is

$$w_t = \begin{cases} \text{argmax}\,(J_{g,t} - J_{v,t})^{1-\alpha_L}\,(W_{g,t} - W_{u,t})^{\alpha_L} & \text{if the firm is in stage } g \\ \text{argmax}\,(J_{\pi,t} - J_{v,t})^{1-\alpha_L}\,(W_{\pi,t} - W_{u,t})^{\alpha_L} & \text{if the firm is in stage } \pi, \end{cases} \tag{26}$$

where W_g and W_π are the asset values of employment to a worker, and W_U is the value of unemployment. These Bellman equations, along with the derivations of the wage rules, are presented in Appendix E.

A price deflator is the weighted share of price of good 1 and the price of good 0 (which is 1) where the shares are the respective consumption shares.

[5] A second complication, which we do not fully address here and leave to future work, arises from the number of bargaining parties. This can lead to several complexities depending on the assumptions on timing and bargaining structure.

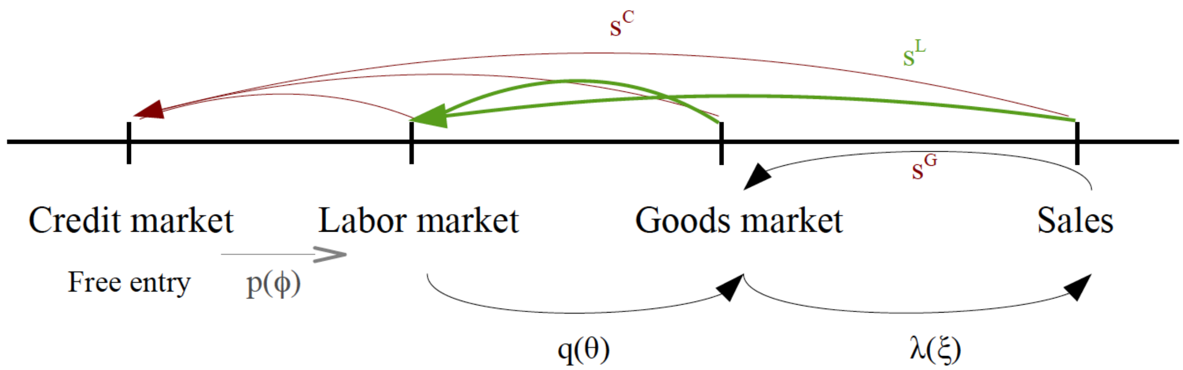

Figure 1: Timing of the Model: Transitions of the Firm

4.6 Extension to the credit market

We now introduce a first stage before the vacancy posting stage. There are therefore four stages instead of three. The firm is now an entity made up of an entrepreneur and of an investor (or a bank) who match in a third matching market. This part follows Wasmer and Weil (2004), Petrosky-Nadeau and Wasmer (2013) and Petrosky-Nadeau and Wasmer (2011). The new step introduces a new shock s^Cwhich dissolves all matches. Figure illustrates.

4.6.1 Matching in the credit market

Denote by p the rate at which a project meets the creditor, and by $\check{p}$ the rate at which the creditor has met - and implicitly screened and granted - the project. As before, there is an identity in the credit market between the total number of matches, Ω_C, the number of matched projects and the number of matched creditors in a unit of time:

$$\Omega_C \quad = p\mathcal{N}_C = \quad \check{p}\mathcal{B}_C \tag{27}$$

We denote by

$$\phi = \frac{\mathcal{N}_C}{\mathcal{B}_C} \tag{28}$$

is a measure of conditions in the credit market called *credit market tightness*. The credit market is tight when there are many projects looking for financing relative to the number of available creditors. In this situation, just as with the concept of labor market tightness, credit market tightness is from the perspective of the firm.

If follows from (27) that p and $\check{p}$ are necessarily linked through ϕ:

$$\breve{p} = \phi p \tag{29}$$

As in the case of the labor market, the transition rates p and $\breve{p}$ cannot be considered as exogenous simultaneously, at least on of them must depend on ϕ. It follows that a general matching function in the credit market encompasses all special cases of exogeneity of these two rates. We assume a constant returns to scale matching function with the mass of investment projects and creditors searching in the credit market as arguments:

$$\Omega_C(\mathcal{N}_C, \mathcal{B}_C) \tag{30}$$

$$\text{with} \quad \partial \log \Omega_C / \partial \log \mathcal{N}_C \quad = \eta_C(\phi) \tag{31}$$

where $\eta_C(\phi) = -\phi p'(\phi)/p(\phi)$ is the elasticity of matching in the credit market with respect to searching investment projects. The transition rates for investment projects and creditors are given by:

$$\frac{\Omega_C(\mathcal{N}_C, \mathcal{B}_C)}{\mathcal{N}_C} = \quad p(\phi) \quad \text{with} \;\; p'(\phi)<0 \tag{32}$$

$$\frac{\Omega_C(\mathcal{N}_C, \mathcal{B}_C)}{\mathcal{B}_C} = \quad \breve{p}(\phi) = \phi p(\phi) \quad \text{with} \;\; \breve{p}'(\phi) > 0 \tag{33}$$

The rate at which investment projects are matched with a creditor, $p(\phi)$, is decreasing in credit market tightness ϕ. It is relatively difficult for an investment project to meet a creditor when there is abundant demand from other projects in the market relative to the amount of creditors in the market. Just as in the labor market, the entry of an additional investment project generates a congestion externality on those projects already in the market searching for a creditor. The entry of the project, at the same time, increases the meeting rate for creditors. That is, the meeting rate $\breve{p}(\phi)$ is increasing in credit market tightness. However, entry of additional creditors causes a bottle neck for existing creditors, increasing their duration of search.

4.6.2 Entry stage in the credit market and repayment to the creditor

The firm is then able, in the profit stage, to produce and sell in the goods market, which generates a flow profit $x_t - w_t - \psi_t$ where x_t is the marginal product, w_t is the wage, and ψ_t is the flow repayment to the creditor.

The asset values of the entrepreneur in the three stages described above are denoted by $E_{j,t}$ with

$j = c, v, g$ or π , standing, respectively, for the credit, vacancy, goods or profits. We therefore have the following Bellman equation in the credit stage (for the next stages, the equations are described in Appendix) :

$$E_{c,t} = -\kappa_I + p_t E_{v,t} + (1 - p_t) \frac{1}{1+r} \mathbb{E}_t E_{c,t+1} \tag{34}$$

where $\mathbb{E}_t$ is an expectations operator over productivity.

Symmetrically, the bank's asset values are denoted by $B_{j,t}$, $j = c, v, g$ or π for each of the stages: in the first stage, the bank screens projects, in the second stage it finances the posting of a vacancy, and in the third stage it enjoys the repayment ψ_t from the project. We denote by κ_B the screening cost per unit of time incurred by banks in the first stage, and by $\check{p}_t$ the probability with which a bank finds a project to be financed. We have:

$$B_{c,t} = -\kappa_B + \check{p}_t B_{v,t} + (1 - \bar{p}_t) \frac{1}{1+r} \mathbb{E}_t B_{c,t+1} \tag{35}$$

As in Pissarides (2000), all profit opportunities are exhausted by new entrants such that the value of the entry stages are always driven to zero. In the case of the credit market, this implies that

$$E_{c,t} = B_{c,t} \equiv 0 \tag{36}$$

at all times, which is also the continuation value following the credit destruction shock s^C that can occur at the end of every stage. This results in the following Bellman equations:

$$E_{c,t} = B_{c,t} \equiv 0 \quad \Leftrightarrow \tfrac{\kappa_B}{\hat{p}_t} + \tfrac{\kappa_I}{p_t} = J_{v,t} \tag{37}$$

The rents from implementing a project, $S_{l,t}$, are divided by bargaining about ρ, upon meeting. Calling $\alpha_C \in (0, 1)$ the bargaining power of the creditor, the Nash bargaining condition,

$$(1 - \alpha_C) B_{v,t} = \alpha_C E_{v,t}, \tag{38}$$

states that with $\beta = 1$ the creditor receives all the surplus. Note that the rule for Ψ_t is determined at the time of the meeting but paid a few periods after the negotiation, when the firm becomes profitable. We assume that there is no commitment problem, as in Wasmer and Weil (2004), so that any new realization of aggregate productivity will not undo the financial contract and there is no renegotiation.

The equilibrium value of ϕ_t, denoted by ϕ^*, is obtain by combining (B-1), (B-5) and (38), with the definition of $\widehat{p}$ in (29):

$$\phi^* = \frac{\kappa_B}{\kappa_I}\frac{1-\alpha_C}{\alpha_C}\forall t. \tag{39}$$

Free-entry of both creditors and projects to credit markets implies a constant credit-market tightness over time, even out of the steady state. Going forward, all the information pertaining to the credit market is contained in the total transaction costs paid by both firms and creditors in stage c,

$$K \equiv \frac{\kappa_B}{\phi^* p(\phi^*)} + \frac{\kappa_I}{p(\phi^*)}, \tag{40}$$

where K is simply a constant of the parameters given the equilibrium value of ϕ determined in equation (39).

4.6.3 Continuation value

Summing up the asset values of the next stages,

$$J_k = E_k + B_k$$

with $k = c, c, g, \pi$, the derivation of these asset values is exactly identical to equations (7), (8) and (9) which are unchanged except that an additional shock s^C leading to destruction of the bank-entrepreneur relation may be introduced and affect the continuation value as in Appendix equations (B-9), (B-10) and (B-11). The free-entry condition in the vacancy stage is simply replaced by equation (37) where the value of ϕ in the free-entry equilibrium is determined by (39).

4.6.4 Consumers

Finally, the net disposable income includes additional expenditures:

$$\begin{aligned} I_t^d &= (\Pi_t + \mathcal{N}_t w_t + G_t - T_t)/\bar{\Xi} + \bar{y}^0 \\ &= \left[(\mathcal{P}_t x_t - C_q)\mathcal{N}_{\pi,t} - \gamma \mathcal{V}_t - \sigma_A(\bar{e_A}) - \kappa_B \mathcal{B}_{C,t} + G_t - T_t\right]/\bar{\Xi} + \bar{y}^0 \end{aligned}$$

Other differences arise from the existence of a new term in the demand side : we need to take account of the destruction of firms at rate s^C for both firms and for matched consumers.

$$D_{M,t} = v(c_{1,t}, c_{0,t}) + \frac{1}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left(1-s^C\right)\left[s^G D_{U,t+1} + \left(1-s^G\right)D_{M,t+1}\right] + \left[\left(1-s^C\right)s^L + s^C\right]D_{U,t+1}\right] \tag{41}$$

4.7 Definition of the dynamic equilibrium

Given initial conditions notably on the stocks of agents in each stage, the equilibrium is a set of policy and value functions for the consumers $\{D_{M,t}, D_{U,t}, \bar{e}_{g,t}\}$ and firms $\{J_{c,t}, J_{v,t}, J_{g,t}, J_{\pi,t}, \bar{e}_{A,t}\}$; a set of prices rules in goods, labor and credit; and stocks, measures of tightness in the markets for goods, labor and credit $\{\mathcal{B}_{C,t}, \mathcal{N}_{C,t}, \mathcal{V}_t, \mathcal{N}_{G,t}, \mathcal{N}_{\pi,t}, \Xi_{U,t}, \Xi_{M,t}, \mathcal{U}_t\}$ and $\{\xi_t, \theta_t, \phi\}$, overall 22 variables, such that 22 equations are satisfied:

1. Consumers' value follows functions (10) and (11), with the search-effort optimality conditions for consumers (13).

2. Firm's value functions (B-9), (B-10) and (B-11) in the vacancy stage, in the consumer stage and in the profit stage, together with the optimal advertising condition (14).

3. Three definitions of credit, labor and goods market tightness as a ratio of the searching stocks (5), (12) and (28)

4. A credit block with free-entry in the credit market for both projects and banks (2 equations) leading to (37).

5. Prices in the goods, labor and credit markets are determined by Nash bargaining given by conditions (15), (25) and (38)

6. Stock in the goods and labor markets follow seven law of motions (F-23) to (F-28), a fixed total number of consumers, finally two identities between the number of firms and the number of employed workers on the one hand, and between the number of matched firms and the number of matched consumers on the other hand, in equations (F-29) and (F-30).

4.8 Dynamic job creation condition and volatility

Fluctuations in the labor market are driven by the decision of firms to create jobs in response to any type of shock. Combining equations (37) and (7), and calling $o_t \equiv \frac{(1-q_t)(r+s^C)}{q_t(1+r)}$, a quantitatively negligible term, this job-creation condition is[6]

$$\underbrace{K(\phi^*)(1+o_t)}_{\text{Cost of credit frictions}} \underbrace{+\frac{\gamma}{q(\theta_t)}}_{\text{Cost of labor frictions}} \underbrace{=\frac{1-s^C}{1+r}\mathbb{E}_t J_{g,t+1}}_{\text{Expected profits}} \tag{42}$$

[6]With our calibration values o_t is approximately equal to 0.01.

which equates the average cost of creating a job – the left-hand side, equal to the financial costs properly discounted, K, and the expected costs of search on the labor market, $\gamma/q(\theta_t)$ – to the discounted expected value of a worker to the firm in the goods market stage (the right-hand side).

The relationship between labor-market tightness and the expected value of a filled vacancy to a firm becomes very transparent when looking at a log-linear approximation of this job-creation condition around the deterministic steady state:

$$\underbrace{\hat{\theta}_t}_{\text{Amplification from:}} = \underbrace{\frac{1}{\eta_L}}_{\text{Labor frictions}} \times \underbrace{\frac{J_g}{J_g - K}}_{\text{Credit frictions}} \times \underbrace{\mathbb{E}_t \hat{J}_{g,t+1}}_{\text{Goods frictions}} \tag{43}$$

where η_L is the elasticity of the job-filling rate with respect to labor-market tightness and "hatted" variables indicate proportional deviations from the steady state. Over and above the amplification of changes in J_g from frictions in the labor market, measured as the inverse of the elasticity of the labor-matching function, credit-market frictions create an amplifying factor of $\frac{J_g}{J_g - K}$.

The additional effects of goods-market frictions on the dynamics of the labor market work through their impact on the dynamics of the expected value of a filled vacancy, $\mathbb{E}_t \widehat{J}_{g,t+1}$. This term can be calculated as deviation around the steady-state as a function of log deviations in the price and in the total goods surplus. Using equations (C-12) and (C-13) in Appendix, and differentiating with respect to a change in G_t, one can isolate the several channels of the effect of the fiscal shock through respectively wages, advertising effort, consumption tightness and total surplus of consumption :

$$dJ_{g,t} = -dw_t - d\sigma_A(e_{A,t}) + \frac{1 - s^C}{1 + r} \mathbb{E}_t \left[\left(1 - s^L\right) \left[d\lambda(\xi_t, \bar{e}_{g,t}, \bar{e}_{A,t}) \Delta J_{t+1} + \lambda_t d\Delta J_{t+1} + dJ_{g,t+1} \right] \right]$$

A fiscal shocks, by raising prices, will lead to higher wages - which mitigates the response of tightness. It will raise advertising efforts, hence reduce the value of stage g, but eases the match formation for consumers, hence raise their effort. This produces a positive effect on λ_t . Finally, the response is also amplified through expectations of future surpluses - as the fiscal shock propagates with some autocorrelation, hence producing an amplification of the order of magnitude of

$$\frac{1}{1 - \frac{1 - s^C}{1 + r} \rho_G}$$

Finally, goods market tightness itself affects the dynamics though variations in ξ_t in $\lambda(\xi_t, \bar{e}_{g,t}, \bar{e}_{A,t})$. Goods market tightness is countercylical (that is, decreases after the fiscal impulse) as it reflects the relative evolution of consumers over selling firms, the number of which itself procyclical (that is, increases after the fiscal impulse). However, the net effect on the transition rate of firms $\lambda(\xi_t, \bar{e}_{g,t}, \bar{e}_{A,t})$ remains procyclical.

5 Simulated impact of the dynamics of fiscal shocks

5.1 Calibration Strategy

The basic unit of time is a month. The matching functions in the labor, goods and credit markets take the functional form proposed in den Haan et al. (2000). All parameter values are reported in Table 1.

Credit market parameters: r, s^C, α_C, ν_C κ_I, κ_B

The risk free rate r is set to an annualized 4%. The separation rate s^C is set for a 1% quarterly firm exit rate, as per the evidence reported in Carlstrom and Fuerst (1997). The remaining parameters follow the calibration strategy in Petrosky-Nadeau and Wasmer (2013), to which we refer for details. That is, K is set for a target unemployment rate of 5.80%, the average rate in the U.S. post-war sample. From the fact that the share of the financial sector in GDP in the model is a strictly increasing function of the bargaining weight α_C, we set $\alpha_C = 0.6$ to target a 2.5% share in U.S. data. The curvature of the credit matching function $\mathcal{N}_C\mathcal{B}_C / \left(\mathcal{N}_C^{\nu_C} + \mathcal{B_C}^{\nu_C}\right)^{1/\nu_C}$ is set such that the average duration of search in the credit market by creditors is four months, resulting in $\nu_C = 1.35$. Finally, search flow costs are obtained as $\kappa_I = \alpha_C K p(\phi)$ and $\kappa_B = (1 - \alpha_C)\, K p \phi(\phi)$.

Labor market parameters: s^L,ν_L, γ, χ_w and η_w

The five parameters related to the labor market, s^L,ν_L, γ, χ_w and η_w, are set as follows. The rate of labor separation s^L is set such that the aggregate rate of job separation, $s^C + (1 - s^C)s^L$, equals 0.045 per month, consistent with the estimate based on JOLTS data in Davis et al. (2010). Given a value of $s^C = 0.01/3$, we set $s^L = \left(0.045 - s^C\right)/(1 - s^C) = 0.043$. For the curvature parameter in the matching function $\mathcal{VU}/\left(\mathcal{V}^{\nu_L} + \mathcal{U}^{\nu_L}\right)^{1/\nu_L}$, we follow den Haan et al. (2000) set it to target an average job filling rate $q(\theta)$ of 0.40 per month. This results in a value $\nu_L = 1.25$, which is close to the value in the referenced work. The unit vacancy costs are set to $\gamma = 0.14$ to be consistent with evidence in Silva and Toledo (2009) that the average cost of recruiting is about 40% of a monthly wage. Finally, the parameters of the wage rule are set as follows. The level parameter χ_w is set to 0.80 or such that the wage equals 0.75 of labor productivity on average. The elasticity parameter is set to $\eta_w = 0.65$ such that the elasticity of wages to changes in productivity in the model is about 0.60. This elastic wage is in the upper range of estimates on U.S. aggregate data. Hagedorn and Manovskii (2008), for instance, estimate an elasticity of 0.49, while Haefke et al. (2013) argue for a value of 0.79.

Goods market parameters: s^G, ν_G, χ_σ, η_σ, α_G, $b_1(x)$, C_q.

Broda and Weinstein (2010) report a four-year product exit rate of 0.46, and a median 1 year exit rate of 0.24. From the model equivalent rate, $s^C + (1 - s^C)(1 - s^L)s^G$, targeting the mid-range of these exits rates implies a monthly goods separation rate due to changes in tastes of $s^G = 0.01$. Broda and Weinstein (2010) also report a rate of product entry of 0.25 on an annualized basis. This implies an average transition rate for consumers in the goods market of $\tilde{\lambda} = 0.20$, which we target with the curvature parameter in the goods market matching function $e_G \Xi_U e_A \mathcal{N}_G / \left((e_g \Xi_U)^{\nu_G} + (e_A \mathcal{N}_G)^{\nu_G} \right)^{1/\nu_G}$.[7] This results in $\nu_G = 1.40$. We rely on the Bureau of Labor Statistics' time-use survey calibrate the search costs in the goods market. This survey reports that households spend on average half an hour a day purchasing goods and services (0.4 hour for men, 0.6 hour for women). Of course, this is not necessarily time spent searching and comparing goods before making a choice. Nor does it include travel related to these activities. Assuming an individual works on average seven hours a day, spread over a week, the cost of time searching in the goods market corresponds to approximately 7% of wage income. That is, we target an average value for $\sigma(\bar{e}_g)/w \simeq 0.07$. Assuming a cost function $\sigma_g(e_g) = \left(\chi_{\sigma_g}/\eta_{\sigma_g}\right) e^{\eta_{\sigma_g}}$, we obtain $\chi_{\sigma_g} = 0.50$. advertising cost function is quadratic, and with $\sigma_A(e_A) = \left(\chi_{\sigma A}/\eta_{\sigma_A}\right) e_A^{\eta_{\sigma_A}}$, we obtain $\chi_{\sigma_A} = 0.75$ to match a value of 2% of advertising costs over GDP (Tremblay 2012).

Our baseline parameterization specifies the costs as quadratic with $\eta_\sigma = 2$, and we investigate the sensitivity of the results to the degree of curvature later. We target an expenditure share of the essential good in order to determine share of the goods market surplus accruing to the consumer, α_G. Data from the Household Consumption Expenditure Survey reveal that average annual expenditure on food consumed at home, plus utilities, over the period 1984 to 2009 amounts to 10 to 15% of total annual expenditures. We denote by $C^1 = x\Xi_M$ the aggregate consumption of goods 1; and by $C^0 = \Xi_U c_U^0 + \Xi_M c_M^0$ the aggregate consumption of goods 1, as a sum of individual consumptions c_K^0, $K = M, U$. The share in the model is defined as $C^0 / \left[\mathcal{P}C^1 + C^0\right]$. This results in $\alpha_G = 0.30$. The marginal utility $b(c_1)$ is set to 1.15 for an average markup over marginal cost of 10%, in the lower-range of values reported in Basu and Fernald (1997) and Nekarda and Ramey (2011). The cost of production parameter is assumed negligible and set to $C_q = 0$, avoiding any amplification that could arise from the introduction of an additional fixed element in the profit flow during the sales stage.

5.2 Dynamics in the benchmark case

We now impose, starting from a steady-state, a fiscal impulse that lasts on average about 12 months (Figure 2). The red curve is a fiscal expansion, the red curve is a fiscal consolidation, as a perfectly symmetrical

[7] At a steady state the annual entry rate of 0.25 implies a monthly consumer finding rate in the goods market of $\tilde{\lambda} = 0.25\mathcal{C}_1\tau / \left[\mathcal{C}_0 \left(1 - (1-\tau)^{12}\right)\right] = 0.20$ given an average share of matched consumers of $\mathcal{C}_1 = 0.90$ that results from our calibration. We verify in simulations that the average product exit rate is consistent with the empirical evidence reported above.

Table 1: Baseline Monthly Parameter Values

		Value		Sources or Target:
Labor market:				
job-separation rate	s^L	0.043	$\rightarrow$	Davis et al. (2006)
matching curvature	ν_L	1.25	$\rightarrow$	Den Haan et al. (2000)
vacancy cost	γ	0.14	$\rightarrow$	Silva and Toledo (2007)
wage elasticity	η_w	0.65	$\rightarrow$	Wage elasticity
wage level parameter	χ_w	0.80	$\rightarrow$	Wage to productivity ratio
Goods market:				
goods exit rate	s^G	0.01	$\rightarrow$	Broda and Weinstein (2010)
matching curvature	ν_G	1.40	$\rightarrow$	Goods market transition rate
cost function level parameter consumers	$\chi_{\sigma c}$	0.4	$\rightarrow$	American Time Use Survey
cost function level parameter advertising	$\chi_{\sigma c}$	0.75	$\rightarrow$	Tremblay (2012)
cost function elasticity (consumer)	η_{σ_c}	2	$\rightarrow$	Quadratic cost
cost function elasticity (advertising)	η_{σ_A}	2	$\rightarrow$	Quadratic cost
consumer bargaining weight	α_G	0.30	$\rightarrow$	Share of expenditure on essential good
marginal utility of c_1	$b_1(x)$	1.15	$\rightarrow$	Price Markup
utility of c_0	$b_0(c_0)$	log	$\rightarrow$	No good justification
production cost	C_q	0^+		
Credit market:				
separation rate	s^C	0.01/3	$\rightarrow$	Bernanke et al (1999)
bank bargaining weight	α_C	0.62	$\rightarrow$	Petrosky-Nadeau and Wasmer (2013)
search costs	$\kappa_B = \kappa_I$	0.1	$\rightarrow$	Petrosky-Nadeau and Wasmer (2013)
risk-free rate	r	0.01/3	$\rightarrow$	3 month U.S. T-bill
Government:				
persistence parameter	ρ_G	$0.75^{(1/3)}$	$\rightarrow$	(0.75 quarterly) $\simeq$ 1 year shock
standard deviation	σ_G	0.05	$\rightarrow$	No justification at all

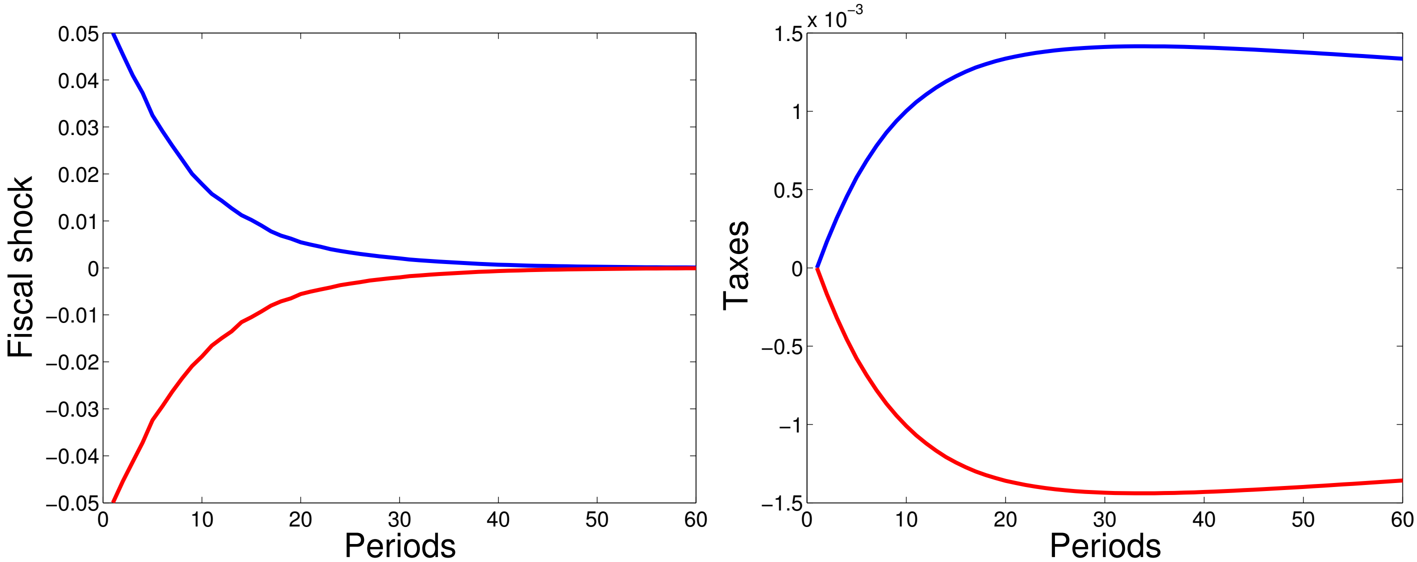

Figure 2: Impulse Responses to a Positive or Negative Fiscal Shock: Spendings and Taxes

shock. The right panel is taxation as a proportion of the current debt. Since repayment is over an infinite horizon, repayment is much lower than the fiscal impulse for the first part of the sample.

5.2.1 Main results

The evolution of debt as a state variable is displayed in the left panel of 3. The right panel is the net transfer $G_t - T_t$: as the stimulus vanishes, repayment makes the net transfer negative for a fiscal stimulus and positive for a consolidation.

The main channel of the impact of the fiscal expansion is through prices : price of good 1 goes up through the concavity of the utility function, as the sacrifice of buying good 1 is lower in terms of good 0. Hence, the surplus from consumption goes up. See Figure 4.

Next, we represent the GDP and unemployment impact. GDP goes up as expected, while unemployment goes down ant the number of profitable firms also increases, goes through a peak and goes down slowly to the steady state.

Next, we represent the GDP and unemployment impact. GDP goes up as expected, while unemployment goes down ant the number of profitable firms also increases, goes through a peak and goes down slowly to the steady state. The effect operates through higher entry of firms (more vacancies) and through search effort of consumers and firms in the goods market, as displayed in Figure 6. Goods market tightness operates countercyclically, as more firms enter.

We can now calculate the dynamic fiscal multiplier as the cumulated impact on GDP over the cumulated spendings. The multiplier is larger than 1 since spendings are transformed into production; the additional

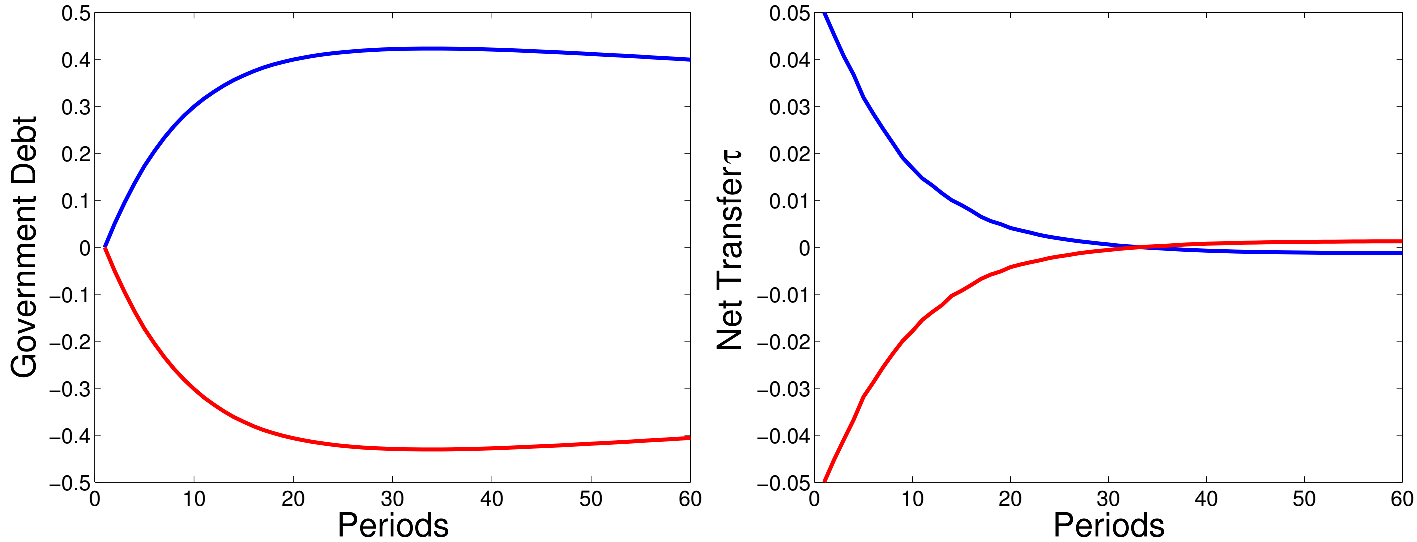

Figure 3: Impulse Responses to a Positive or Negative Fiscal Shock: Debt and Net Transfers

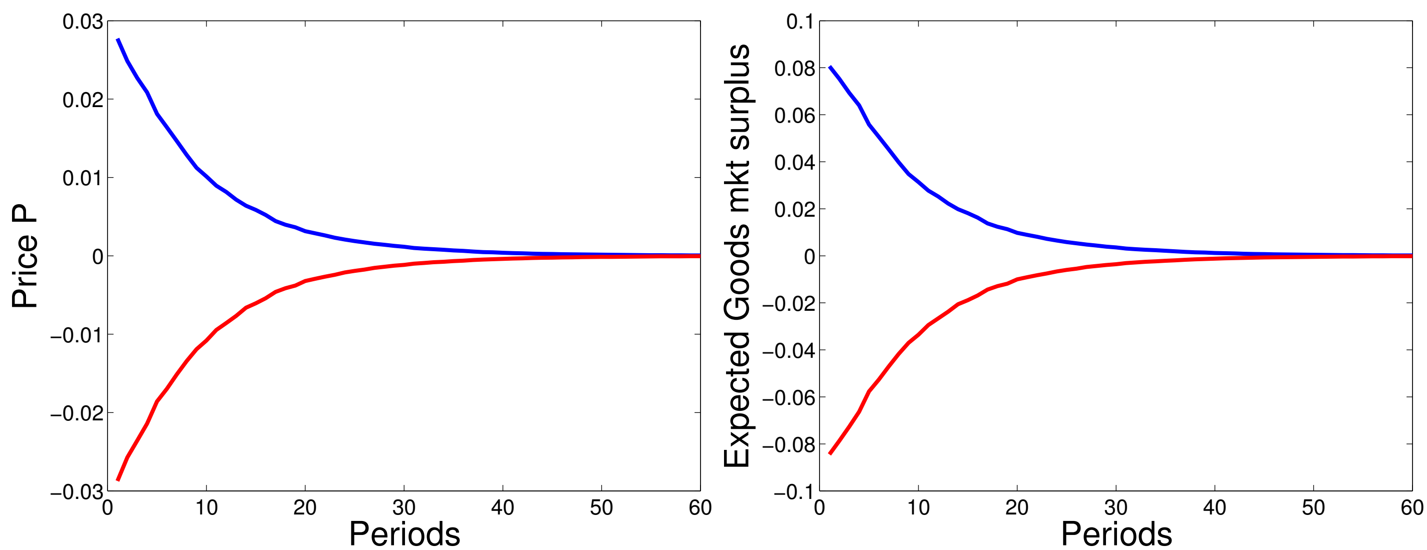

Figure 4: Impulse Responses to a Positive or Negative Fiscal Shock: Price and Expected Consumption Surplus

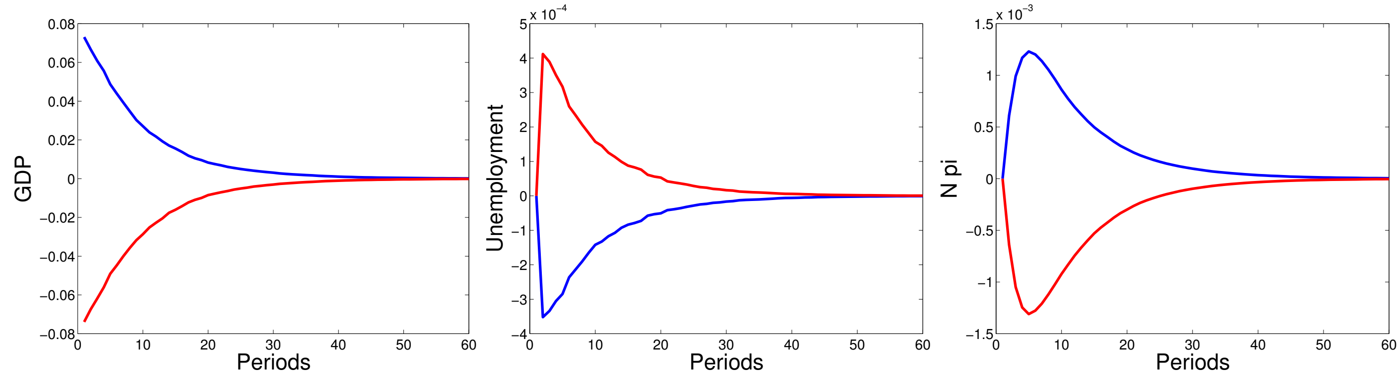

Figure 5: Impulse Responses to a Positive or Negative Fiscal Shock: GDP, Unemployment and # of Selling Firms, Benchmark Case

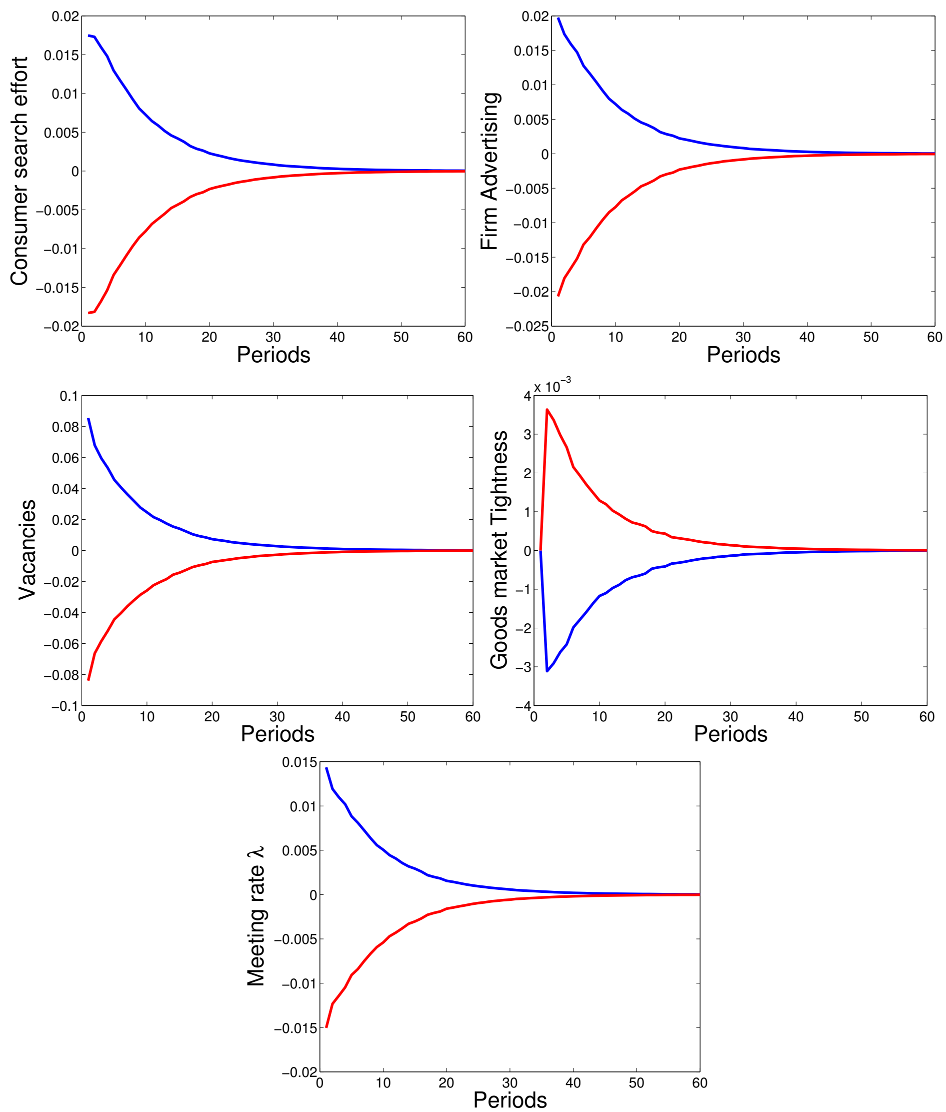

Figure 6: Impulse Responses to a Positive or Negative Fiscal Shock: Consumer effort, Advertising Effort (top panel) ; Vacancies and Consumption Tightness (intermediate panel) ; and Transition rate from Unmatched to Matched for Firms, Benchmark Case

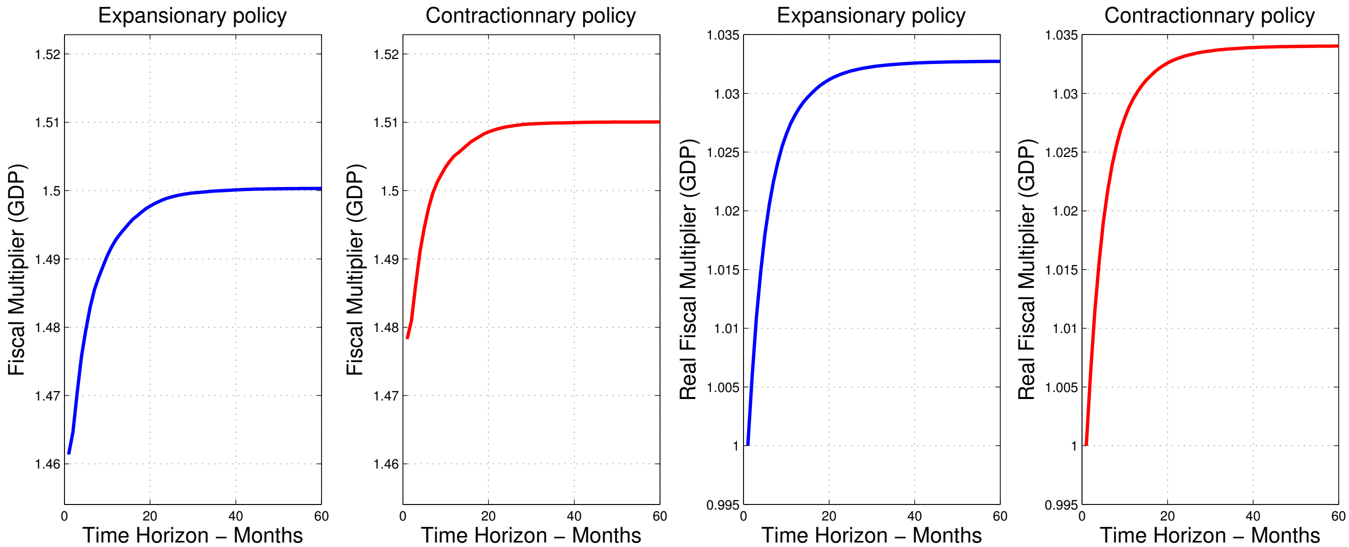

Figure 7: Impulse Responses to a Positive or Negative Fiscal Shock: Multiplier and "Real" Multiplier, Benchmark Case

amplification comes from the other endogenous variables described above. It is evolving between 1.445 on impact due to the price effect to 1.485 in the longer run, as the amplification develops through more entry of firms. In the short run, as production does not react to the impulse, the fiscal shock produces inflation on the price of good 1. As time goes, price returns to the steady state value; a fiscal consolidation, in turns, produces deflation.

For a fiscal consolidation, the fiscal multiplier is larger, which corresponds to the intuition.

5.2.2 The amplification from search efforts

As could be seen, a part of the amplification arises from search efforts, which are procyclical and also increases the matching probabilities. One way to sterilize this effect is to raise the elasticities of the costs of effort of both sides (consumers and firms) from quadratic to quartic.

5.3 Can we get higher multipliers?

5.3.1 Curvature of the utility function

A large part of the effect comes from the curvature of the utility for good 0. A more curved utility function delivers larger effects through larger swings in prices. But there is not much gain See Figure 9. Conversely, a linear utility function neutralizes all the effects and the multiplier is consistently 1. Finally, a moderetaly high coefficient of relative risk aversion of 5 leads to a rela mutiplier that goes beyond 1.2, still with a persistence parameter of 0.75.

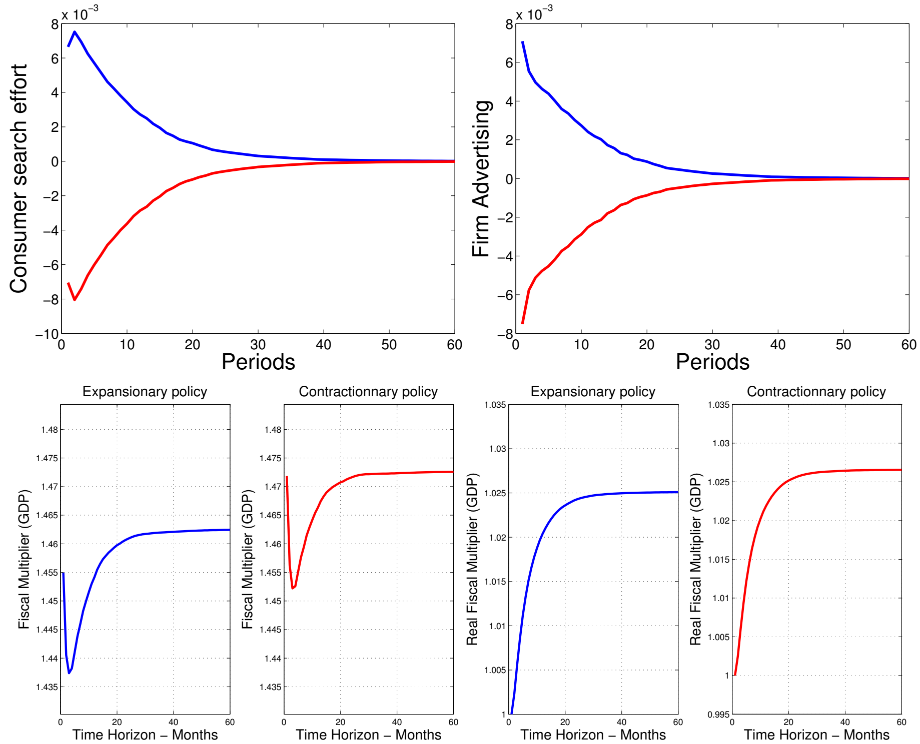

Figure 8: Impulse Responses to a Positive or Negative Fiscal Shock: Search efforts, Multipliers

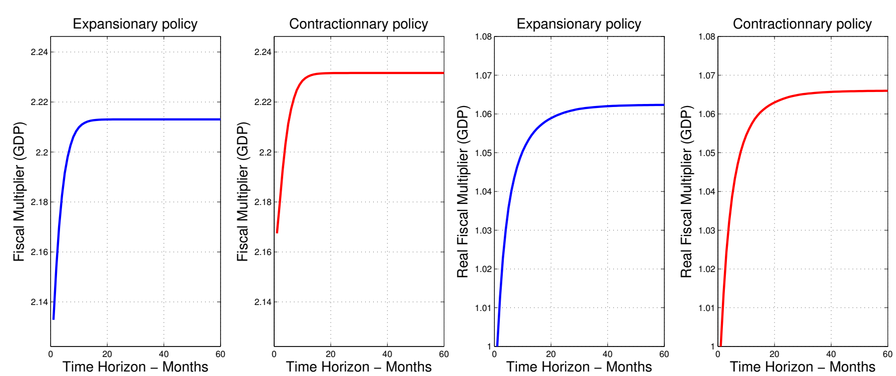

Figure 9: Impulse Responses to a Positive or Negative Fiscal Shock: GDP, Multiplier and “Real” Multiplier with CRRA=2

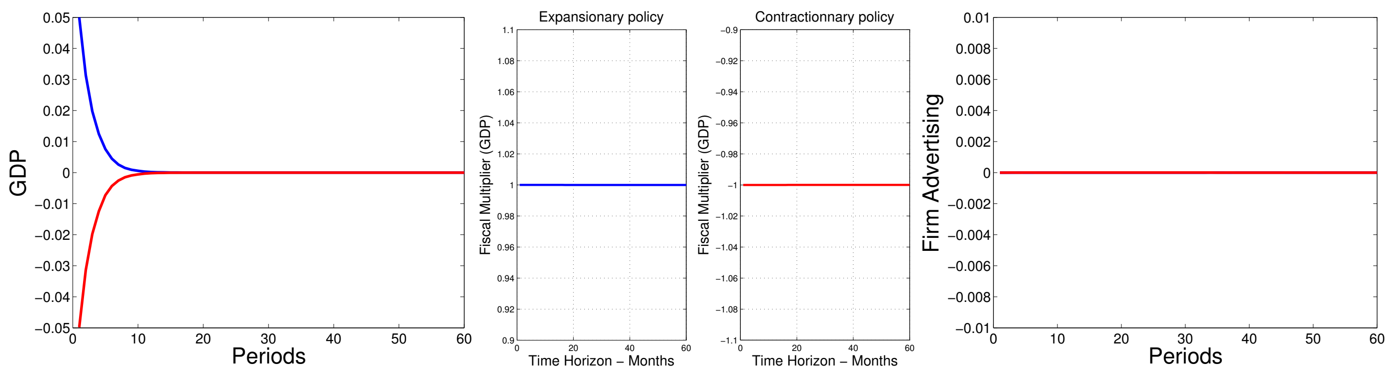

Figure 10: Impulse Responses to a Positive or Negative Fiscal Shock: GDP, Multiplier and Advertising with linear utility

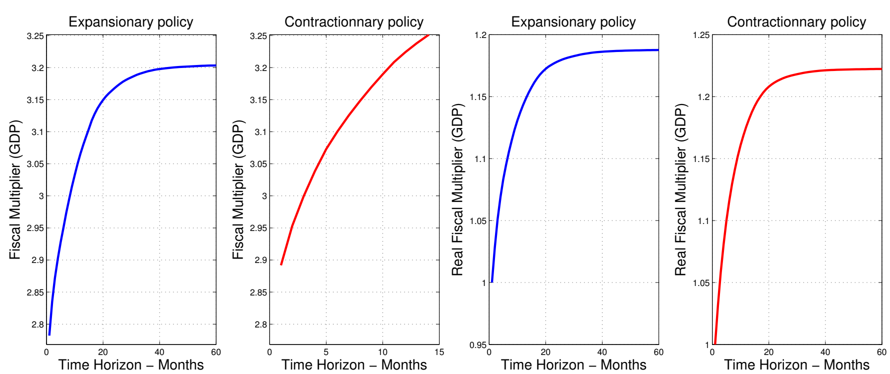

Figure 11: Impulse Responses to a Positive or Negative Fiscal Shock: GDP, Multiplier and "Real" Multiplier with CRRA=5

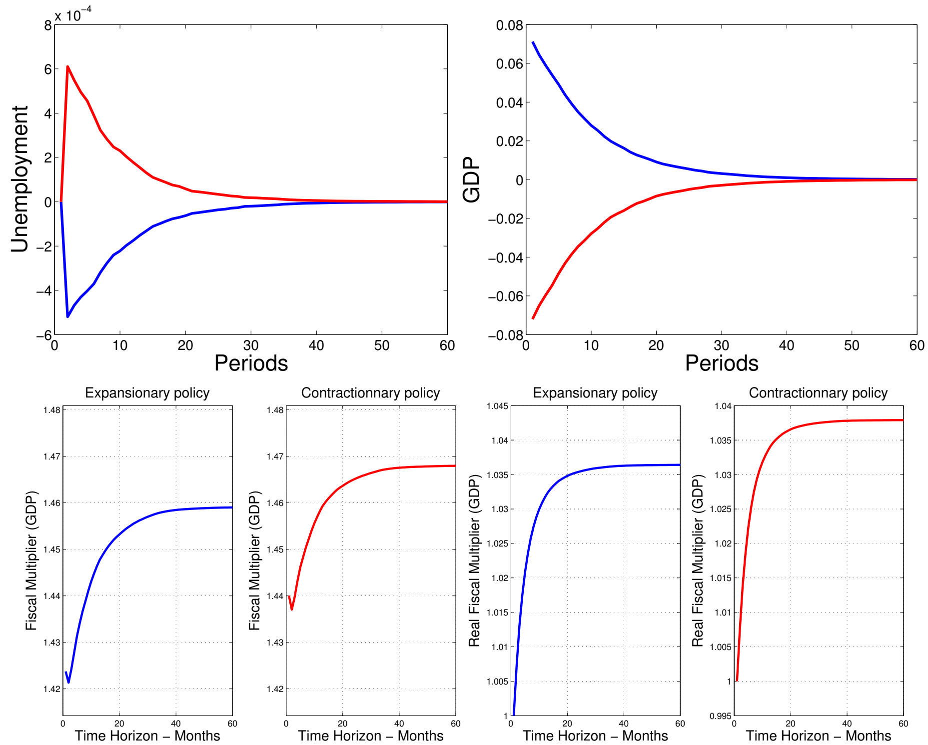

Figure 12: Impulse Responses to a Positive or Negative Fiscal Shock: Unemployment, GDP, Multiplier and Advertising Out-of-the-Steady-State (higher initial unemployment)

5.3.2 Are multipliers higher out of the steady-state?

The next exercise is to start from different initial values, out of the steady-state. We compare the initial steady-state of 5,6% unemployment to a large recession where we start from a larger 10%.

5.3.3 Do the causes of unemployment matter?

In the recent financial crisis, the presumption is that large empirical multipliers were observed. Our model captures financial recessions as an increase in the search cost of banks κ_B : banks are less likely to enter, financial tightness goes up and firms face more expensive credit, both in terms of time and future repayment. In this exercise, we raise κ_B so that steady-state unemployment increases from 5,8 to 6,7%. In this case, we obtain more amplification as K and the financial multiplier are larger. The effect is represented in Figure 13. This will therefore increase the effect of the fiscal stimulus. Since in the recession in the US, unemployment peaked at 10%, if all the effects were linear, the actual effect of a higher κ_B to reach these 10% would need to be multiplied by a factor (10-5.8)/(6.7-5.8)=3.78, or a multiplier going up to 1.045 for a fiscal stimulus. The financial crisis increases the fiscal multiplier, but not by that much.

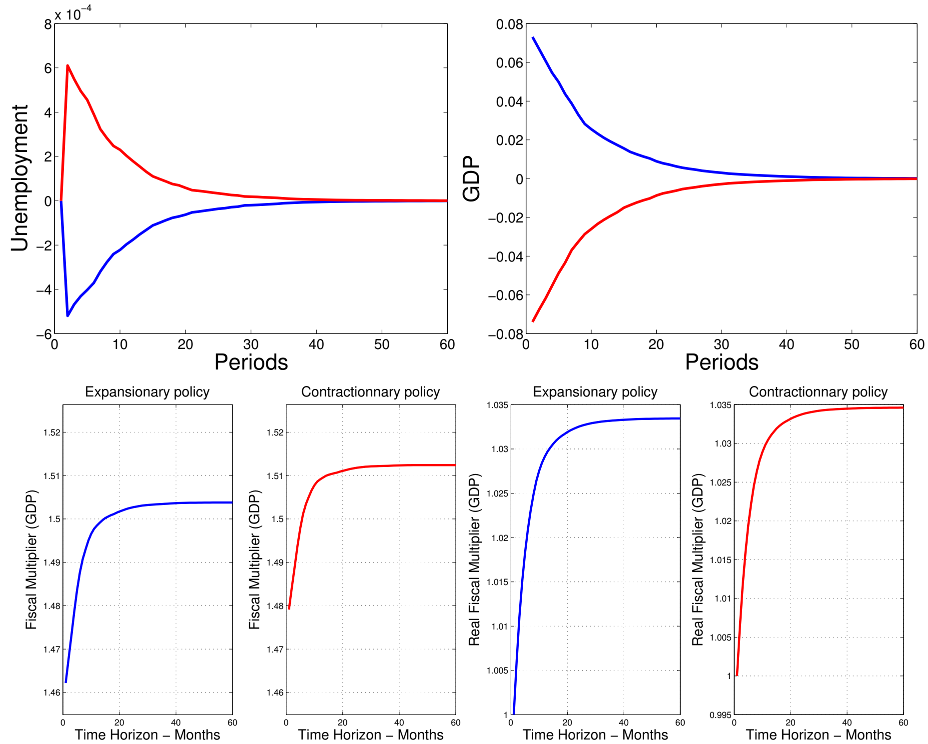

Figure 13: Impulse Responses to a Positive or Negative Fiscal Shock: GDP, Multiplier and "Real" Multiplier of a moderate increase in financial frictions

5.3.4 Do more persistent fiscal policy raise multipliers?

If we raise the persistence parameter of fiscal policy to 0.95 monthly, the effects are more dramatic. Indeed, as Figure 14 shows, a higher persistence, by raising the anticipation effect, will boost search efforts of both sides and lead to larger multipliers in the short run, about 1.08 in the long-run to 1.1 for a fiscal consolidation, which almost triples the size of the effect, as compared to the benchmark case real multiplier of 1.035. With a greater CRRA (2) and a slightly less persistent fiscal policy we reach the same conclusion.

5.3.5 Do higher nominal wage rigidity matter?

As Figure 16 shows, a higher degree of nominal rigidity, by increasing the correlation between fiscal shocks and firm's surplus from sales, raises the procyclicality of firm's profits. The multiplier is around 1.06 with the benchmark degree of persistence of fiscal policy (0.75), and reaches 1.23 to 1.25 for a higher degree of persistence (0.90).

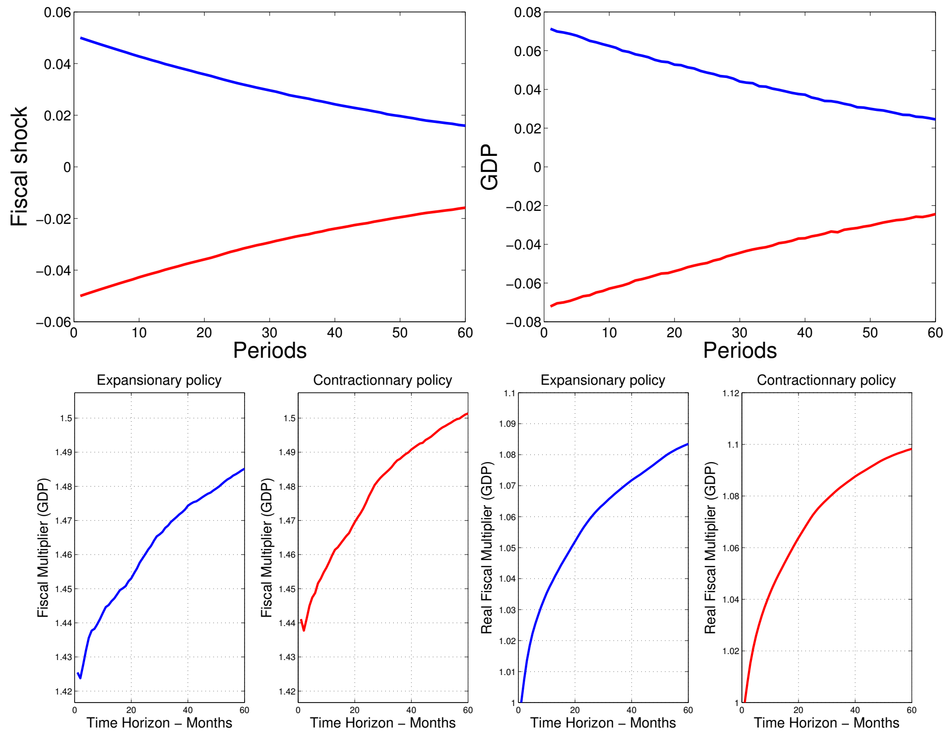

Figure 14: Impulse Responses to a Positive or Negative Fiscal Shock: GDP, Multiplier and "Real" Multiplier of a persistent fiscal policy ($\rho_G = 0.95$, CRRA=1)

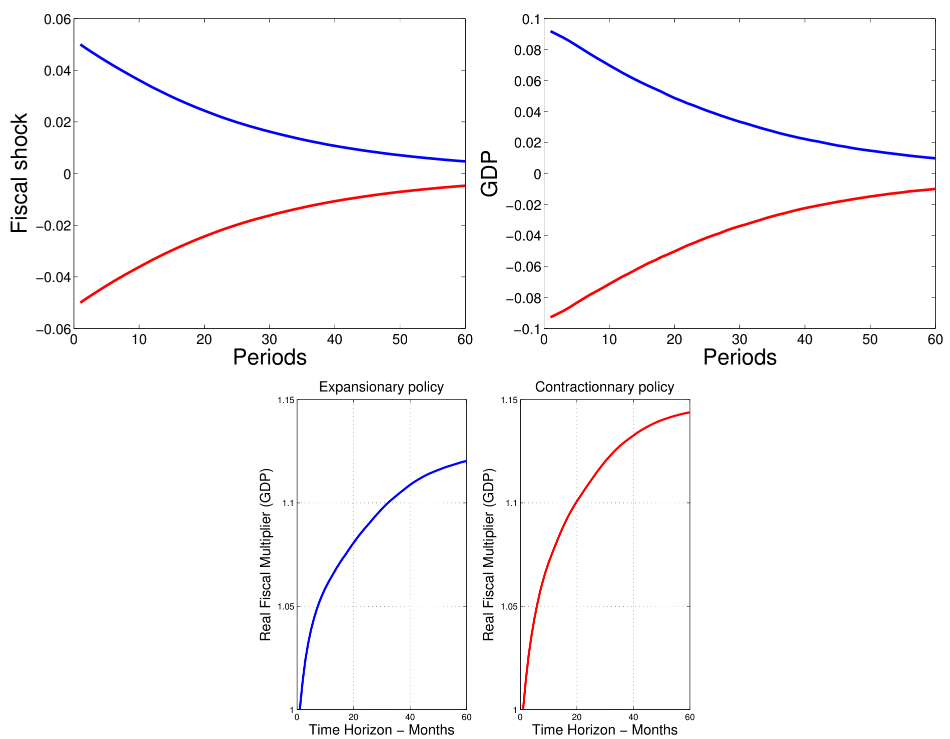

Figure 15: Impulse Responses to a Positive or Negative Fiscal Shock: GDP, Multiplier and "Real" Multiplier of a persistent fiscal policy ($\rho_G = 0.90$, CRRA=2)

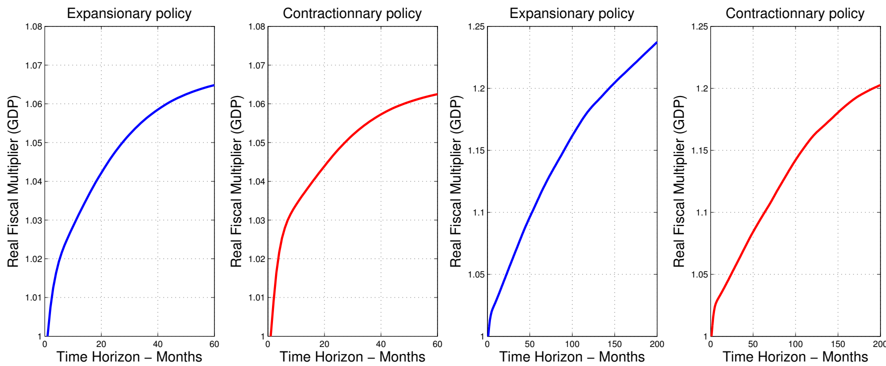

Figure 16: Impulse Responses to a Positive Shock: Lower value of η_W (higher degree of nominal rigidities). Left chart : $\rho_G = 0.75$, CRRA=2 ; Right chart ($\rho_G = 0.90$, CRRA=2)

6 Larger multipliers: imperfect pooling and imperfect savings

6.1 Savings and pooling, two faces of the same coin

As we discussed in introduction, fiscal multipliers are inexistant if agents have the ability to perfectly smooth consumption across states along the following dimensions:

1. either across time (savings) in response to aggregate fluctuations, that is, the ability to transfer the quasi-numeraire (good 0) across time periods ;
2. across employment states (pooling) in response to idiosyncratic income shocks generated by the matching and separation process in the labor market, that is, the ability to transfer the quasi-numeraire (good 0) across households of different employment status ;
3. across consumption states in response to idiosyncratic shocks generated by the matching and separation process in the goods market, that is,
   (a) either the ability to transfer the search goods (good 1) from matched to unmatched consumers
   (b) or the ability to transfer the search goods (good 1) from matched times to unmatched times;

In that sense, savings and pooling are the two faces of the same coin, both for search good and for the numeraire.

So far, we had kept two channels for fiscal multipliers, the third one, that is, the inability to smooth consumption across consumption states, and the first one, the inability to save the quasi-numeraire across states. We had kept channel 2, that is, the ability to transfer the numeraire across households of different employment status. This leads to large income flows in terms of the quasi-numeraire. This lead to the simplifying assumption that all consumers are ex ante identical from the firm's perspective, differing only ex post by the random matching process in the goods market, that is whether they are ex post matched with firms.

Hence in our benchmark setup, the motive for a fiscal stimulus is to provide more incentives to consume and raise consumption effort, through the curvature of the utility function for good zero.

The goal of this part is to relax these assumptions. We first maintain the no-saving assumption, and relax the assumption of perfect pooling of good 0 that is, let the disposable income to depend on the current labor market status. In the next sub-section, we explore the polar case, the introduction of savings while keeping the pooling assumption as in the benchmark case explored in Sections 3 to 5.

6.2 No savings, no pooling

6.2.1 Theory

This exercise requires to define two distinct consumption surpluses and thefore two different classes of consumers that firms have to take into account. Denote by $K = 0, 1$ an indicator for the employment status (0 if unemployed, 1 if employed) that will therefore be a subscript for value functions, policy functions and stock variables. We still assume that profits of firms and banks are redistributed equally to consumers. Allowing for G or T that are employment-status-specific, the disposable income$I^d_{t,K}$, is

$$\begin{aligned} I^d_{t,K} &= (\Pi_t + G_{t,K} - T_{t,K})/\bar{\Xi} + \bar{y}^0 + Kw \\ &= \left[(\mathcal{P}_t x_t - C_q)\mathcal{N}_{\pi,t} - \mathcal{N}_t w_t - \gamma \mathcal{V}_t - \sigma_A(e_{A,t})\mathcal{N}_{G,t} - \kappa_B \mathcal{B}_{C,t} + G_{t,K} - T_{t,K}\right]/\bar{\Xi} + \bar{y}^0 + Kw \end{aligned}$$

One implication of this setup is that consumption surplus of good 1 may vary according to the employment status. We assume that the initial price is negotiated one and for all at the time of the meeting: it then depends on the surplus but hen remains rigid ex post. Consequently, we have four value functions for unmatched and matched consumers, on for each employment status $K = 0$ or 1:

$$\begin{aligned} D_{U,t,K} &= v(0, I^d_{t,K}) - \sigma_g(e_{g,t,K}) + \frac{1}{1+r}\mathbb{E}_t(1 - d_{K,K'})\left[\frac{e_{g,t,K}\check{\lambda}_t}{\bar{e}_{g,t}} D_{M,t+1,K} + \left(1 - \frac{e_{g,t,K}\check{\lambda}_t}{\bar{e}_{g,t}}\right) D_{U,t+1,K}\right] + d_{K,K} \\ D_{M,t,K}(\mathcal{P}_{t,K}) &= v(x, I^d_{t,K} - \mathcal{P}_{t,K} x_t) + \frac{1}{1+r}\mathbb{E}_t(1 - d_{K,K'})\left[\left(1 - s^L\right)\left[s^G D_{U,t+1,K} + \left(1 - s^G\right) D_{M,t+1,K}(\mathcal{P}_{t,K})\right] + s^L D\right. \\ &\quad + \frac{1}{1+r}\mathbb{E}_t d_{K,K'} D_{M,t+1,K'}(\mathcal{P}_{t,K'}) \end{aligned}$$

where $K' = 1$ if $K = 0$ and vice-versa, and $d_{K,K'}$ is the transition probability from employment state $K = 0$ or 1 to the other employment state $K' = 0$ or 1 (e.g. either the job finding probability or the separation probability). Note that the price bargained is in principle dependent of the employment statuts and was denoted by $\mathcal{P}_{t,K}$. which adds therefore a new asset value for consumption utility for people having changed status within the consumption relationship.[8]

The first order conditions on optimal consumer search effort in the goods market is the same as before, but it delivers different efforts across employment statuses. The matching rate is similar again,

$$\lambda_t = \frac{\Omega_G(\bar{e}_{g,t}\Xi_{U,t}, \bar{e}_{A,t}\mathcal{N}_{G,t})}{\mathcal{N}_{G,t}} = \lambda(\xi_t, \bar{e}_{g,t}, \bar{e}_{A,t}) \quad \text{with} \quad \lambda'(\xi_t) > 0$$
$$\check{\lambda}_t = \frac{\Omega_G(\bar{e}_{g,t}\Xi_{U,t}, \bar{e}_{A,t}\mathcal{N}_{G,t})}{\Xi_{U,t}} = \check{\lambda}_t(\xi_t, \bar{e}_{g,t}, \bar{e}_{A,t}) \quad \text{with} \quad \check{\lambda}_t{}'(\xi_t) < 0,$$

but average search efforts in the consumer's population takes into account the different efforts across employment status:

$$\bar{e}_{g,t} = e_{g,t,K=0}\mathcal{U}_t + (1-\mathcal{U}_t)e_{g,t,K=1} \tag{47}$$

Goods market tightness is still defined in the same way:$\xi_t = \Xi_{U,t}/\mathcal{N}_{G,t}$.

In turn, firms may face different types of consumers: employed ($K = 1$) or unemployed ($K = 0$). They thus form expectations based on the observed proportions of employed and unemployed workers ; denote by

$$J^e_{\pi,t+1} = \frac{e_{g,t,K=0}\mathcal{U}_t J_{\pi,t+1,K=0} + e_{g,t,K=1}(1-\mathcal{U}_t)J_{\pi,t+1,K=1}}{e_{g,t,K=0}\mathcal{U}_t + e_{g,t,K=1}(1-\mathcal{U}_t)}$$

the expected value at time $t+1$ of being matched with a random consumer. The Bellman equations of firms are therefore:

$$J_{g,t} = -w_t - \sigma_A(e_{A,t}) + \frac{1-s^C}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left[\lambda_t\frac{e_{A,t}}{\bar{e}_{A,t}}J^e_{\pi,t+1} + (1-\lambda_t\frac{e_{A,t}}{\bar{e}_{A,t}})J_{g,t+1}\right] + s^L J_{v,t+1}\right] \tag{48}$$
$$J_{\pi,t,K} = \mathcal{P}_{t,K}x_t - w_t - C_q + \frac{1-s^C}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left[(1-s^G)J_{\pi,t+1,K} + s^G J_{g,t+1}\right] + s^L J_{v,t+1}\right]. \tag{49}$$

To simplify, we will assume here that the wage paid by firms is a weighted average of nominal income

[8]The symmetrical asset value is therefore:

$$D_{M,t,K'}(\mathcal{P}_{t,K}) = v(x, I^d_{t,K'} - \mathcal{P}_{t,K'}x_t) + \frac{1}{1+r}\mathbb{E}_t(1-d_{K',K})\left[\left(1-s^L\right)\left[s^G D_{U,t+1,K'} + \left(1-s^G\right)D_{M,t+1,K'}(\mathcal{P}_{t,K})\right] + s^L D_{U,t+1,K'}\right]$$
$$+ \frac{1}{1+r}\mathbb{E}_t d_{K',K} D_{M,t+1,K}(\mathcal{P}_{t,K})$$

of the firms:

$$w_t = \chi_w \left[\mathcal{P}_{t,K=0}\mathcal{U}_t + \mathcal{P}_{t,K=1}(1-\mathcal{U}_t)\right]^{\eta_w} x_t^{\eta_w}, \tag{50}$$

The calibration will therefore attempt to capture: different effort by employed and the unemployed, higher for the unemployed workers ; different price levels for the goods paid by the employed and the unemployed; lower for the unemployed.

6.2.2 Calibration targets and results

We now have more endogenous variables: consumer search effort is different for the employed and the unemployed. Our calibration targets are:

- $\bar{e}_{g,t}$ defined in equation (47) is such that the cost of time searching in the goods market corresponds to approximately 7% of wage income as in Section 5;
- $e_{g,t,K=0}/e_{g,t,K=1} = 1.25$ as in Kaplan and Menzio (2013).
- An alternative target would be to choose the relative level of actual expenses of the employed relative to the unemployed, that is, denoting by $\Xi_{U,K}$ and $\Xi_{M,K}$ the actual number of consumers in each employment state $K = 0, 1$, calculated from equations (F-31) and (F-32) in Appendix:

$$\frac{Expenditures_{unempl}}{Expenditures_{employed}} = \frac{G_{t,K=0} - T_{t,K=0}}{w_t + G_{t,K=1} - T_{t,K=1}} = \frac{\Xi_{U,K=0}c^0_{U,K=0} + \Xi_{M,K=0}(c^0_{M,K=0} + \mathcal{P}_{t,K=0}x_t)}{\Xi_{U,K=1}c^0_{U,K=1} + \Xi_{M,K=1}(c^0_{M,K=1} + \mathcal{P}_{t,K=1}x_t)}$$

-

6.3 Perfect pooling and some saving s_t

Savings is a transfer across periods. Unemployment insurance is a transfer across employment status and across time. One way to address the issue of savings is to recognize that agents want to save against idiosyncratic shocks in the labor market (employment loss) and against aggregate shocks.

Denote by s_t the savings diverted from consumption of the disposable income. The new budget constraint per period is:

$$\mathcal{P}_t c_{1,t} + c_{0,t} + s_t = I_t^d + s_{t-1}(1 + r_s),$$

where r_s is the rate of return on savings, assumed to be less than r

If the consumer expects a drop in price next period, he may not want to consume everything today in order to consume more of the numeraire next period. Hence, we have

$$\mathcal{P}_t c_{1,t} + s_t = \mathcal{P}_t x_t.$$

The Bellman equations for unmatched and matched consumers are

$$\begin{aligned}
D_{0,t} &= c_{0,t} + s_{t-1}(1 + r_s) - \sigma_g(e_{gt}) + \frac{1}{1+r}\mathbb{E}_t\left[\tilde{\lambda}_t D_{1,t+1}(s_t) + (1 - \tilde{\lambda}_t) D_{0,t+1}(s_t)\right] \\
&= I_t^d + s_{t-1}(1 + r_s) - \sigma_g(e_{gt}) + \frac{1}{1+r}\mathbb{E}_t\left[\tilde{\lambda}_t D_{1,t+1}(s_t) + (1 - \tilde{\lambda}_t) D_{0,t+1}(s_t)\right] \\
D_{1,t} &= v(c_{1,t}) + I_t^d - \mathcal{P}_t c_{1,t} + s_{t-1}(1 + r_s) \\
&\quad + \left(\frac{1 - s^c}{1 + r}\right)\mathbb{E}_t(1 - s^L)[\tau D_{0,t+1}(s_t) + (1 - s^G) D_{1,t+1}(s_t)] \\
&\quad + \frac{s^C + (1 - s^C)s^L}{1 + r}\mathbb{E}_t D_{0,t+1}(s_t) \\
&= v(x_t - s_t/\mathcal{P}_t) + I_t^d - \mathcal{P}_t x_t + s_{t-1}(1 + r_s) \\
&\quad + \left(\frac{1 - s^C}{1 + r}\right)\mathbb{E}_t(1 - s^L)[s^G D_{0,t+1}(s_t) + (1 - s^G) D_{1,t+1}(s_t)] \\
&\quad + \frac{s^C + (1 - s^C)s^L}{1 + r}\mathbb{E}_t D_{0,t+1}(s_t).
\end{aligned}$$

The solution for the optimal choice of savings is a trade-off between sacrificing consumption today, which costs

$$\frac{v'(x_t - s_t/\mathcal{P}_t)}{\mathcal{P}_t},$$

and increasing consumption opportunities tomorrow, which yields additional utility depending on the future state. In particular, in state 0, we have

$$\frac{\mathrm{d}\mathbb{E}_t D_{0,t+1}(s_t)}{\mathrm{d}s_t} = 1 + r_s$$

and

$$\frac{\mathrm{d}\mathbb{E}_t D_{1,t+1}(s_t)}{\mathrm{d}s_t} = 1 + r_s$$

since the consumer is constrained by the production tomorrow for next period's consumption. Hence, without calculation, we know that the consumer does not want to save: What is gained tomorrow is only to consume more numeraire, which by definition costs a fixed price. So at best, the gain is some more numeraire tomorrow bringing $(1+r_s)/(1+r) < 1$ but sacrificing $\frac{v'(x_t - s_t/\mathcal{P}_t)}{\mathcal{P}_t}$, which is larger than 1. Therefore, the price must be below the marginal utility here.

A slightly different reasoning applies if the agent could store good 1 for a return to stage 0. In this case the agent may have an interest to smooth consumption. But good 1 was assumed to be non-storable or equivalently, non-divisible.

By the same logic, one may want to know whether the agents could borrow in both stages at rate r_b, to consume more of the numeraire. Again, this is not possible given $(1+r_b)/(1+r) > 1$. The key insight here is that borrowing or savings is of no help because the agents are constrained to buy all the available goods.

Finally, a last possibility would be to have unconstrained agents, who would consume, when matched, both good 1 and the numeraire. In this case, they may want to reduce consumption of good 1 in periods of high prices and raise it in period of low price. However even in this case, this is not profitable: Postponing consumption of good 1 today entails a risk of loosing the good with probability s^G. The gain is to have some more consumption of the good, by a marginal quantity equal to deflation. In the numerical exercise, this marginal quantity has to be discounted by $(1-s^G)/(1+r)$ and compared to deflation. Given that deflation is typically at most 0.5% quarterly in our simulation and that s^G is 1%, the consumer, in this case, would still not want to sacrifice consumption of good 1 today to buy more good 1 tomorrow.

7 Constrained efficiency with goods markets frictions

So far we have assessed, quantitatively, the impact of fiscal shocks. We have not attempted to derive the optimal response of government. We address the issue of efficiency here. To ease the intuitions, we will discuss each component of the social optimum of this economy separately. First we will analyze the traditional hold-up arising from endogenous search efforts of consumers and firms. Second, we will focus on extensive search margins and analyze the constrained efficient solution in a continuous time environment in the static case. Lastly we extend the analysis to a dynamic environment. We assume that the flow of utility of the non-employed workers z is not a transfer in the form of unemployment insurance, but instead a pure utility component.

7.1 Inefficiency arising from the holdup problem of consumers and firms

Returning to the first order conditions on effort, we have: we have:

$$\bar{e}_{g,t}\sigma_g'(e_{g,t}) = \frac{\check{\lambda}_t}{1+r}\mathbb{E}_t\tilde{\delta}_{t+1}S_{t+1}^G. \tag{51}$$

$$\bar{e}_{A,t}\sigma_A'(e_{A,t}) \;=\; \frac{\lambda_t}{1+r}\left(1-s^L\right)\left(1-s^C\right)\mathbb{E}_t(1-\tilde{\delta}_{t+1})S_{t+1}^G \tag{52}$$

The social planner solution, on the other hand, and if other margins were optimal, would need to equalize the marginal costs of efforts of the left-hand sides of these two equations with the full marginal returns, that is:

$$\bar{e}_{g,t}\sigma_c'(e_{g,t}) = \frac{\check{\lambda}_t}{1+r}\mathbb{E}_tS_{t+1}^G. \tag{53}$$

$$\bar{e}_{A,t}\sigma_A'(e_{A,t}) \;=\; \frac{\lambda_t}{1+r}\left(1-s^L\right)\left(1-s^C\right)\mathbb{E}_tS_{t+1}^G \tag{54}$$

The social planner therefore requires stronger efforts from both consumers and firms, the decentralized equilibrium is affected by a classical hold-up problem leading to underinvestment.

7.2 The efficiency of other margins, static case

To simplify, we now analyze the welfare of the economy where search efforts are constant. Stocks evolve as follows:

$$\frac{\partial \mathcal{N}_\pi}{\partial t} = \Omega_G - (s^L+\tau)\mathcal{N}_\pi$$
$$\frac{\partial \mathcal{N}_G}{\partial t} = \Omega_L - M_G + \mathcal{N}_\pi s^L$$

We have $M_G(\Xi_U, \mathcal{N}_G)$ with

$$\frac{\partial \Omega_G}{\partial \Xi_U} = \eta_G \check{\lambda}(\xi)$$
$$\frac{\partial \Omega_G}{\partial \mathcal{N}_G} = (1-\eta_G)\lambda(\xi)$$

Hence, in a steady-state, we have:

$$(s^L + \tau)\mathcal{N}_\pi = \Omega_G(\Xi_U, \mathcal{N}_g) = \Omega_G(1-\mathcal{N}_\pi, 1-\mathcal{U}-\mathcal{N}_\pi)$$

which implies that $\mathcal{N}_\pi$ is an implicit function of $\mathcal{U}$. More precisely, since:

$$\frac{\partial \Omega_G}{\partial \Xi_U} = \eta_G \check{\lambda}(\xi)$$
$$\frac{\partial \Omega_G}{\partial \mathcal{N}_G} = (1-\eta_G)\lambda(\xi)$$

we have that

$$\frac{\partial \mathcal{N}_\pi}{\partial \mathcal{U}} = \frac{-(1-\eta_G)\lambda(\xi)}{1+(1-\eta_G)\lambda(\xi)+\eta_G\check{\lambda}(\xi)}$$

Finally, at a steady-state, we have:

$$\frac{\partial \mathcal{U}}{\partial \theta} = \frac{-\mathcal{U}(1-\mathcal{U})(1-\eta_L)}{\theta} \tag{55}$$

An increase in tightness of the labor market reduces unemployment.

It follows that :

$$\frac{\partial \Xi_U}{\partial \theta} = \frac{\partial \Xi_U}{\partial \mathcal{N}_\pi}\frac{\partial \mathcal{N}_\pi}{\partial \mathcal{U}}\frac{\partial \mathcal{U}}{\partial \theta} = -\frac{(1-\eta_G)\lambda(\xi)}{1+(1-\eta_G)\lambda(\xi)+\eta_G\check{\lambda}(\xi)}\frac{\mathcal{U}(1-\mathcal{U})(1-\eta_L)}{s^L} < 0 \tag{56}$$
$$\frac{\partial \Xi_M}{\partial \theta} = \frac{\partial \Xi_M}{\partial \mathcal{N}_\pi}\frac{\partial \mathcal{N}_\pi}{\partial \mathcal{U}}\frac{\partial \mathcal{U}}{\partial \theta} = -\frac{\partial \Xi_U}{\partial \theta} > 0$$

7.2.1 Steady-state social planner program

The social planner maximizes utilities of individual consumers. We have, under the assumption of separable utility in consumption and leisure z:

$$\max_{c_M^0, c_U^0, \theta} W^{SP} = \Xi_U v(0, c_U^0) + \Xi_M v(x, c_M^0) + \mathcal{U} z$$

subject to the resource constraint of goods 0:

$$\bar{y} \geq \Xi_U c_U^0 + \Xi_M c_M^0 - \gamma\theta\mathcal{U}$$

Denote by Ψ_c the multiplier associated with this constraint and by v_M and v_U the utility of the matched and unmatched, we have:

$$\begin{aligned}
\partial c_M^0 &: \partial v_M' / \partial c_M^0 = \Psi_c \\
\partial c_U^0 &: \partial v_U' / \partial c_U^0 = \Psi_c \\
\partial \theta &: \frac{\partial \Xi_U}{\partial \theta}(v_U - \Psi_c c_U^0) + \frac{\partial \Xi_M}{\partial \theta}(v_M - \Psi_c c_M^0) + \Psi_c \gamma \frac{\partial(\theta\mathcal{U})}{\partial \theta} + z\frac{\partial \mathcal{U}}{\partial \theta} = 0
\end{aligned}$$

or, replacing, These equations are easy to interpret. The first two state that the marginal utility of the consumption of good 0 of each type of agent (matched and unmatched) is equal and equal to the multiplier. The last equation says that the optimal level of θ is such that a marginal increase in θ :

- will reduce the number of unmatched agents by $\frac{\partial \Xi_U}{\partial \theta}$, hence reduce each time the social planner function by their utility v_U but saves resources evaluated in terms of marginal consumption $\Psi_c c_U^0$;
- will raise the number of matched agents by $\frac{\partial \Xi_M}{\partial \theta}$, hence raise each time the social planner function by their utility v_M but saves resources evaluated in terms of marginal consumption $\Psi_c c_M^0$;
- necessitates to reduce the consumption of good 0 to pay for search expenditures (evaluated in utility terms) $\gamma\Psi_c \frac{\partial(\theta\mathcal{U})}{\partial \theta}$;
- and finally reduces the leisure consumed by the unemployed.

7.2.2 Special cases with no goods market frictions

In this case, we have λ and $\check{\lambda}$ going to infinity and $\mathcal{N}_g = 0; \mathcal{N}_\pi = \Xi_M = 1 - \mathcal{U}; \Xi_U = \mathcal{U}$; in this case, the social planner solution becomes:

$$\frac{\partial \mathcal{U}}{\partial \theta}(v_U - \Psi_c c_U^0) - \frac{\partial \mathcal{U}}{\partial \theta}(v_M - \Psi_c c_M^0) + \Psi_c \gamma \frac{\partial(\theta\mathcal{U})}{\partial \theta} + z\frac{\partial \mathcal{U}}{\partial \theta} = 0$$

Under linear utility $v(x, c^0) = \Phi x + c^0$, we have $\Psi_c = 1$ and the equations further simplifies to:

$$\frac{\partial \mathcal{U}}{\partial \theta}(0) - \frac{\partial \mathcal{U}}{\partial \theta}(\Phi x) + \gamma \frac{\partial(\theta\mathcal{U})}{\partial \theta} + z\frac{\partial \mathcal{U}}{\partial \theta} = 0$$

or

$$\Phi x - z = \gamma \frac{\frac{\partial(\theta\mathcal{U})}{\partial\theta}}{\frac{\partial\mathcal{U}}{\partial\theta}}$$

which leads, using (55) and after straightforward calculations, to the standard social planner's solution in Pissarides (2000):

$$\frac{(\Phi x - z)(1-\eta_L) + \gamma\theta\eta_L}{s} = \frac{\gamma}{q(\theta)}$$

With non linear utility, we obtain a simple generalization:

$$\frac{(v_M - \Psi_c c_M^0 - (v_U - \Psi_c c_U^0) - z)(1-\eta_L) + \gamma\theta\eta_L}{s} = \frac{\gamma}{q(\theta)}$$

7.2.3 With goods market frictions

Under linear utility, one obtains:

$$\frac{\partial\mathcal{N}_\pi}{\partial\mathcal{U}}\frac{\partial\mathcal{U}}{\partial\theta}(0) - \frac{\partial\mathcal{N}_\pi}{\partial\mathcal{U}}\frac{\partial\mathcal{U}}{\partial\theta}(\Phi x) + \Psi_c\gamma\frac{\partial(\theta\mathcal{U})}{\partial\theta} + z\frac{\partial\mathcal{U}}{\partial\theta} = 0$$

or

$$(\frac{\partial\mathcal{N}_\pi}{\partial\mathcal{U}}\Phi x - z) = \Psi_c\gamma\frac{\frac{\partial(\theta\mathcal{U})}{\partial\theta}}{\frac{\partial\mathcal{U}}{\partial\theta}}$$

or finally using (56):

$$\frac{\left(\frac{(1-\eta_G)\lambda(\xi)}{1+(1-\eta_G)\lambda(\xi)+\eta_G\check{\lambda}(\xi)}\Phi x - z\right)(1-\eta_L) + \gamma\theta\eta_L}{s} = \frac{\gamma}{q(\theta)}$$

In the general case, the entry condition becomes:

$$\frac{\left(\frac{(1-\eta_G)\lambda(\xi)}{1+(1-\eta_G)\lambda(\xi)+\eta_G\check{\lambda}(\xi)}\left(v_M - \Psi_c c_M^0 - (v_U - \Psi_c c_U^0)\right) - z\right)(1-\eta_L) + \gamma\theta\eta_L}{s} = \frac{\gamma}{q(\theta)}$$

7.3 Dynamic programming

We treat here the simpler case without financial frictions.

7.3.1 Intermediate results: in the steady-state:

All stocks of workers depend on the variable of choice of θ - while in the general case the social planner only indirectly control θ through the entry of banks and of projects in the credit stage. The equation are however formally identical. Hence, fixing $K = 0$ will not change our results. The social planner will here have to anticipate the consequences of the variations of its control variable θ on the stock of consumers in each state. We can show that:

$$\begin{aligned}
\frac{\partial \Xi_U}{\partial \theta} &= \frac{\partial \Xi_U}{\partial \mathcal{N}_\pi}\frac{\partial \mathcal{N}_\pi}{\partial \mathcal{U}}\frac{\partial \mathcal{U}}{\partial \theta} = -\frac{(1-\eta_G)\lambda(\xi)}{1+(1-\eta_G)\lambda(\xi)+\eta_G\check{\lambda}(\xi)}\frac{\mathcal{U}(1-\mathcal{U})(1-\eta_L)}{s^L} < 0 \\
\frac{\partial \Xi_M}{\partial \theta} &= \frac{\partial \Xi_M}{\partial \mathcal{N}_\pi}\frac{\partial \mathcal{N}_\pi}{\partial \mathcal{U}}\frac{\partial \mathcal{U}}{\partial \theta} = -\frac{\partial \Xi_U}{\partial \theta} > 0
\end{aligned} \tag{57}$$

7.3.2 Dynamics of stocks

We have:

$$\begin{aligned}
\frac{\partial \mathcal{N}_\pi}{\partial t} &= \Omega_G(\Xi_U, \mathcal{N}_G) - (s^L + s^G)\mathcal{N}_\pi \\
\frac{\partial \mathcal{N}_g}{\partial t} &= \Omega_L(\mathcal{U}, \mathcal{V}) - \Omega_G(\Xi_U, \mathcal{N}_G) + \mathcal{N}_\pi s^G \\
\frac{\partial \Xi_M}{\partial t} &= \Omega_G(\Xi_U, \mathcal{N}_G) - (s^L + s^G)\Xi_M \\
\frac{\partial \Xi_U}{\partial t} &= -\Omega_G(\Xi_U, \mathcal{N}_G) + (s^L + s^G)\Xi_M \\
\frac{\partial \mathcal{U}}{\partial t} &= -\Omega_L(\mathcal{U}, \mathcal{V}) + s^L(1-\mathcal{U})
\end{aligned}$$

Hence, the social planner controls $\mathcal{V}$ and the other stock variables are state-variables.

7.3.3 The social planner's program with $K = 0$

We have:

$$\max_{c_M^0, c_U^0, \mathcal{V}} W^{SP} = \int_0^\infty e^{-rt}[\Xi_U v(0, c_U^0) + \Xi_M v(x, c_M^0) + \mathcal{U}z]dt$$

The Hamiltonian of the problem, denoting Ψ_i the co-states variables, is:

$$\begin{aligned}
H = e^{-rt}&[\Xi_U v(0, c_U^0) + \Xi_M v(x, c_M^0) + \mathcal{U}z] \\
&+ \Psi_{\mathcal{U}}[-\Omega_L(\mathcal{U}, \mathcal{V}) + s(1-\mathcal{U})] \\
&+ \Psi_{\Xi}[\Omega_G(\Xi_U, \mathcal{N}_G) - (s^L + s^G)\Xi_M] \\
&+ \Psi_{c^0}[\bar{y} + G_t - \Xi_U c_U^0 - \Xi_M c_M^0 - \gamma\mathcal{V} - iD_t] \\
&+ \Psi_\pi \left[\Omega_G(\Xi_U, \mathcal{N}_G) - (s^L + s^G)\mathcal{N}_\pi\right] \\
&+ \Psi_g \left[\Omega_L(\mathcal{U}, \mathcal{V}) - \Omega_G(\Xi_U, \mathcal{N}_G) + \mathcal{N}_\pi s^G\right]
\end{aligned}$$

The optimality conditions for vacancies and consumption are: respectively with $\mathcal{V}$, c_M^0 and c_U^0, and dropping by convenience the parenthesis and tightness after the elasticities $\eta_L(\theta)$:

$$\partial/\partial\mathcal{V}{=}0\Leftrightarrow\Psi_{\mathcal{U}}\left[-q(\theta)(1-\eta_L)\right] - \Psi_{c^0}\gamma + \Psi_g\left[q(\theta)(1-\eta_L)\right] = 0 \tag{58}$$

$$\partial/\partial c_M^0 = 0 \Leftrightarrow e^{-rt}[\Xi_M v_M'] - \Psi_{c^0}\Xi_M = 0 \tag{59}$$

$$\partial/\partial c_U^0{=}0\Leftrightarrow e^{-rt}[\Xi_U v_U'] - \Psi_{c^0}\Xi_U = 0 \tag{60}$$

$$\partial/\partial G_t{=}0\Leftrightarrow \left(1 - i\dot{G}_t/G_t\right)\Psi_{c^0} = 0 \tag{61}$$

$$\begin{aligned}
\partial/\partial\mathcal{U} = 0 \Leftrightarrow\ & e^{-rt}[z] + \Psi_{\mathcal{U}}[-\theta q(\theta)\eta_L - s] + \Psi_g[\theta q(\theta)\eta_L] = \dot{\Psi}_{\mathcal{U}} \\
\partial/\partial\Xi_M = 0 \Leftrightarrow\ & e^{-rt}[v_M - v_U] + \Psi_\Xi[-\partial\Omega_G/\partial\Xi_U + (s^L + s^G)] \\
&+ \Psi_{c^0}[c_U^0 - c_M^0] + \Psi_\pi\left[-\partial\Omega_G/\partial\Xi_U\right] + \Psi_g\left[\partial\Omega_G/\partial\Xi_U\right] = \dot{\Psi}_\Xi \\
\partial/\partial\mathcal{N}_\pi = 0 \Leftrightarrow\ & \Psi_\pi\left[-(s^L + s^G)\right] + \Psi_g[s^G] = \dot{\Psi}_\pi \\
\partial/\partial\mathcal{N}_G = 0 \Leftrightarrow\ & \Psi_\Xi[\partial\Omega_G/\partial\mathcal{N}_G] + \Psi_\pi[\partial\Omega_G/\partial\mathcal{N}_G] \\
&+ \Psi_g\left[-\partial\Omega_G/\partial\mathcal{N}_G\right] = \dot{\Psi}_g \\
\partial/\partial D_t = 0 \Leftrightarrow\ & -i\Psi_{c^0} = \dot{\Psi}_D
\end{aligned}$$

$$\text{if } \Psi_{c^0} \neq 0 \Rightarrow G_t + \bar{y} = \Xi_U c_U^0 + \Xi_M c_M^0 + \gamma\mathcal{V} + iD_t \tag{62}$$

8 Conclusion

Credit and goods market imperfections amplify the response of labor market tightness to shocks, both for productivity shocks or for fiscal shocks. Only goods market imperfections provide persistence, and these results are quite robust to changes in goods market specification. This work has provided a simple framework to deal with aggregate fiscal stimulus or consolidation. A fiscal multiplier is larger, the larger the concavity of utility: multipliers are higher when more desire of smoothing and more severe financial constraints. However, in normal times, a multiplier of 1.1 to 1.2 is a robust conclusion. The value remains to be re-calculated with more extreme parameters.

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Appendix

A Data Used in Calibration

(...)

B Value Functions of the Investment Project with Imperfect Credit

The Bellman equations of the investment project, which faces a discount rate r and assuming that transitions from the credit to the labor market stages occur within a single period, are, if we further introduce a credit destruction shock leading to bankruptcy with probability s^C leading back to the credit stage, that is to zero:

$$\begin{aligned}
E_{c,t} &= 0 = -\kappa_I + p_t E_{v,t} && \text{(B-1)} \\
E_{v,t} &= \frac{1-s^C}{1+r}\mathbb{E}_t\left[q_t E_{g,t+1} + (1-q_t)E_{v,t+1}\right] && \text{(B-2)} \\
E_{g,t} &= \frac{1-s^C}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left[\lambda_t\frac{e_{A,t}}{\bar{e}_{A,t}}E_{\pi,t+1} + (1-\lambda_t\frac{e_{A,t}}{\bar{e}_{A,t}})E_{g,t+1}\right] + s^L E_{v,t+1}\right] && \text{(B-3)} \\
E_{\pi,t} &= \mathcal{P}_t x_t - w_t - \psi_t - C_q \\
&\quad + \frac{1-s^C}{1+r}\mathbb{E}_t\left[\left(1-s^G\right)\left[(1-\tau)E_{\pi,t+1} + s^G E_{g,t+1}\right] + s^L E_{v,t+1}\right]. && \text{(B-4)}
\end{aligned}$$

The corresponding Bellman equations for the creditor are

$$\begin{aligned}
B_{c,t} &= 0 = -\kappa_B + \check{p}_t B_{v,t} && \text{(B-5)} \\
B_{v,t} &= -\gamma + \frac{1-s^C}{1+r}\mathbb{E}_t\left[q_t B_{g,t+1} + (1-q_t)B_{v,t+1}\right] && \text{(B-6)} \\
B_{g,t} &= -w_t - \sigma_A(e_{A,t}) + \frac{1-s^C}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left[\lambda_t\frac{e_{A,t}}{\bar{e}_{A,t}}B_{\pi,t+1} + (1-\lambda_t\frac{e_{A,t}}{\bar{e}_{A,t}})B_{g,t+1}\right] + s^L B_{v,t+1}\right] && \text{(B-7)} \\
B_{\pi,t} &= \psi_t + \frac{1-s^C}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left[(1-s^G)B_{\pi,t+1} + s^G B_{g,t+1}\right] + s^L B_{l,t+1}\right]. && \text{(B-8)}
\end{aligned}$$

The sum of the value of the investment project and the bank's value leads back to equation (7) to (9):

$$J_{v,t} = -\gamma + \frac{1-s^C}{1+r}\mathbb{E}_t\left[q_t J_{g,t+1} + (1-q_t)J_{v,t+1}\right] \tag{B-9}$$

$$J_{g,t} = -w_t - \sigma_A(e_{A,t}) + \frac{1-s^C}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left[\lambda_t\frac{e_{A,t}}{\bar{e}_{A,t}}J_{\pi,t+1} + (1-\lambda_t\frac{e_{A,t}}{\bar{e}_{A,t}})J_{g,t+1}\right] + s^L J_{v,t+1}\right] \tag{B-10}$$

$$\begin{aligned} J_{\pi,t} &= \mathcal{P}_t x_t - w_t - C_q \\ &+ \frac{1-s^C}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left[(1-s^G)J_{\pi,t+1} + s^G J_{g,t+1}\right] + s^L J_{v,t+1}\right]. \end{aligned} \tag{B-11}$$

C Determinations of Price $\mathcal{P}_t$

We treat price determination in the most general case with credit frictions.

Consumer Surplus

We have that, in a symmetric equilibrium on efforts and simplifying the notations (with v_M, v_U the consumptions in each state and Δ the difference operator across matched and unmatched states:

$$\begin{aligned} D_{U,t} &= v(0,c_{0,t}) - \sigma_g(e_{g,t}) + \frac{1}{1+r}\mathbb{E}_t\left[\frac{e_{g,t}\check{\lambda}_t}{\bar{e}_{g,t}}D_{M,t+1} + \left(1-\frac{e_{g,t}\check{\lambda}_t}{\bar{e}_{g,t}}\right)D_{U,t+1}\right] \\ &= v_U - \sigma_g + \frac{1}{1+r}\mathbb{E}_t\left[\check{\lambda}_t\Delta D_{t+1} + D_{U,t+1}\right] \end{aligned}$$

$$\begin{aligned} D_{M,t} &= v(c_{1,t},c_{0,t}) + \frac{1}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left(1-s^C\right)\left[s^G D_{U,t+1} + \left(1-s^G\right)D_{M,t+1}\right] + \left[(1-s^C)s^L + s^C\right]D_{U,t+1}\right] \\ &= v_M + \frac{1}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left(1-s^C\right)\left[(1-s^G)\Delta D_{t+1} + D_{U,t+1}\right] + \left[(1-s^C)s^L + s^C\right]D_{U,t+1}\right] \end{aligned}$$

Hence,

$$\begin{aligned} \Delta D_t &= \Delta v + \sigma_g + \frac{1}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left(1-s^C\right)\left[(1-s^G)\Delta D_{t+1} + D_{U,t+1}\right] + \left[(1-s^C)s^L + s^C\right]D_{U,t+1} - \check{\lambda}_t\Delta D_{t+1} - D_{U} \right. \\ &= \Delta v + \sigma_g + \frac{1}{1+r}\mathbb{E}_t\left\{\left[\left(1-s^L\right)(1-s^C)(1-s^G) - \check{\lambda}_t\right]\Delta D_{t+1} + \left[\left(1-s^L\right)\left(1-s^C\right) + (1-s^C)s^L + s^C - 1\right]D_{U,t} \right. \\ &= \Delta v + \sigma_g + \frac{1}{1+r}\mathbb{E}_t\left\{\left[\left(1-s^L\right)(1-s^C)(1-s^G) - \check{\lambda}_t\right]\Delta D_{t+1}\right\} \end{aligned}$$

Firm Surplus

We similarly have:

$$J_{g,t} = -w_t - \sigma_A(e_{A,t}) + \frac{1-s^C}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left[\lambda_t \Delta J_{t+1} + J_{g,t+1}\right] + s^L K\right] \tag{C-12}$$

$$\begin{aligned} J_{\pi,t} &= \mathcal{P}_t x_t - w_t - C_q \\ &\quad + \frac{1-s^C}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left[(1-s^G)J_{\pi,t+1} + s^G J_{g,t+1}\right] + s^L K\right]. \end{aligned} \tag{C-13}$$

$$= \mathcal{P}_t x_t - w_t - C_q \tag{C-14}$$

$$+ \frac{1-s^C}{1+r}\mathbb{E}_t\left[\left(1-s^L\right)\left[(1-s^G)\Delta J_{t+1} + J_{g,t+1}\right] + s^L K\right] \tag{C-15}$$

Hence the surplus from consumption for the firm is:

$$\Delta J_t = \mathcal{P}_t x_t - C_q + \sigma_A + \frac{\left(1-s^C\right)\left(1-s^L\right)}{1+r}\mathbb{E}_t\left\{(1-s^G)\Delta J_{t+1} - \lambda_t \Delta J_{t+1}\right\} \tag{C-16}$$

Total consumption surplus

Denote by $\sum(x_t, \mathcal{P}_t) = \Delta v + \mathcal{P}_t x_t = v(x, I_t^d - \mathcal{P}_t x_t) - v(0, I_t^d) + \mathcal{P}_t x_t$ the flow surplus associated with the consumption of x_t units of goods.

We have the total intertemporal surplus S_t^G defined as :

$$\begin{aligned} S_t^G &= \Delta J_t + \Delta D_t \\ &= \Delta v + \sigma_g + \frac{1}{1+r}\mathbb{E}_t\left\{\left[\left(1-s^L\right)(1-s^C)(1-s^G) - \check{\lambda}_t\right]\Delta D_{t+1}\right\} \\ &+ \mathcal{P}_t x_t - C_q + \sigma_A + \frac{\left(1-s^C\right)\left(1-s^L\right)}{1+r}\mathbb{E}_t\left\{(1-s^G)\Delta J_{t+1} - \lambda_t \Delta J_{t+1}\right\} \\ &= \sum(x_t, \mathcal{P}_t) + \sigma_A + \sigma_g - C_q \\ &+ \frac{\left(1-s^L\right)(1-s^C)(1-s^G)}{1+r}\mathbb{E}_t S_{t+1}^G \\ &- \frac{1}{1+r}\mathbb{E}_t\left\{\check{\lambda}_t \Delta D_{t+1} + \lambda_t (1-s^C)\left(1-s^L\right)\Delta J_{t+1}\right\} \end{aligned}$$

We can further simplify the surplus in using the first order conditions on effort (13) and (14) to get:

$$\begin{aligned} S_t^G &= \sum(x_t, \mathcal{P}_t) + \sigma_A(1-\eta_{\sigma_A}) + \sigma_g(1-\eta_{\sigma_g}) - C_q \\ &+ \frac{\left(1-s^L\right)(1-s^C)(1-s^G)}{1+r}\mathbb{E}_t S_{t+1}^G \end{aligned} \tag{C-17}$$

where $\eta_{\sigma_j} = e_j \sigma_j' / \sigma_j, j = g, A$.

D Determinations of price $\mathcal{P}_t$

Remember that we use $v(c_{1t}, c_{0,t}) = b_1(x_t) + b_0(x_t)$

Sharing rule with linear and separable utility $b_0(c_{0,t}) = c_{0,t}$

$$\mathcal{P}_t = \underset{\mathcal{P}_t}{\operatorname{argmax}} \left(J_{\pi,t} - J_{g,t}\right)^{1-\alpha_G} \left(D_{M,t} - D_{U,t}\right)^{\alpha_G}$$

. Since $\partial(J_{\pi,t} - J_{g,t})/\partial\mathcal{P}_t = x_t$ and $\partial(D_{M,t} - D_{U,t})/\partial\mathcal{P}_t = -x_t$, the sharing rule is

$$(1 - \alpha_G)\left(D_{M,t} - D_{U,t}\right) = \alpha_G \left(J_{\pi,t} - J_{g,t}\right).$$

Sharing rule with non-linear utility

Still with

$$\mathcal{P}_t = \underset{\mathcal{P}_t}{\operatorname{argmax}} \left(J_{\pi,t} - J_{g,t}\right)^{1-\alpha_G} \left(D_{M,t} - D_{U,t}\right)^{\alpha_G}$$

we now have, given that $\partial(J_{\pi,t} - J_{g,t})/\partial\mathcal{P}_t = x_t$ and $\partial(D_{M,t} - D_{U,t})/\partial\mathcal{P}_t = b_0'(I_t^d - P_t x_t)x_t$ the sharing rule is, using the simpler notation $v_{0,t}' = b_0'(I_t^d - P_t x_t) + b_{01}\nu_0'(c_{0,t})\nu_1(c_{1,t})$

$$(1 - \alpha_G)\left(D_{M,t} - D_{U,t}\right) = \alpha_G v_{0,t}' \left(J_{\pi,t} - J_{g,t}\right).$$

This amounts to an effective bargaining power

$$\tilde{\delta} = \frac{\alpha_G v_{0,t}'}{1 - \alpha_G + \alpha_G v_{0,t}'} = \frac{1}{\alpha_G / v_{0,t}' + 1 - \alpha_G}$$

The effective bargaining power of consumers is equal to α_G with linear separable utility ($b_{0,t}' = 1$), it is above α_G if $v_{0,t}'$>1and converges to 1 if $v_{0,t}' \to \infty$; and is below α_G if $v_{0,t}' < 1$ and converges to 0 if $v_{0,t}' = 0$.

Price determination

Use (C-16) and (C-17) together with (14) and $\Delta J_t = (1 - \tilde{\delta}_t)S_t^G$ to obtain using the convenient notation

$$\Lambda = \frac{\left(1 - s^C\right)\left(1 - s^L\right)\left(1 - s^G\right)}{1 + r}$$

:

$$\mathcal{P}_t x_t - C_q + \sigma_A(1-\eta_{\sigma_A}) + \Lambda \mathbb{E}_t \left\{(1-\tilde{\delta}_{t+1}) S^G_{t+1}\right\} \tag{D-18}$$
$$= (1-\tilde{\delta}_t)\left\{v(x, I^d_t - \mathcal{P}_t x_t) - v(0, I^d_t) + \mathcal{P}_t x_t + \sigma_A(1-\eta_{\sigma_A}) + \sigma_g(1-\eta_{\sigma_g}) - C_Q + \Lambda \mathbb{E}_t S^G_{t+1}\right\}$$

or finally:

$$\tilde{\delta}_t \left[\mathcal{P}_t x_t - C_q + \sigma_A(1-\eta_{\sigma_A})\right] = (1-\tilde{\delta}_t)\left\{v(x_t, I^d_t - \mathcal{P}_t x_t) - v(0, I^d_t) + \sigma_g(1-\eta_{\sigma_g})\right\} \tag{D-19}$$
$$+ \Lambda \mathbb{E}_t \left[(1-\tilde{\delta}_t) S^G_{t+1}\right] - \Lambda \mathbb{E}_t \left\{(1-\tilde{\delta}_{t+1}) S^G_{t+1}\right\} \tag{D-20}$$

The special case with linear separable utility comes quite simply: we have in this case $\tilde{\delta}_t \equiv \alpha_G$ and $b_{01} = 0$. In that case, $v(x, I^d_t - \mathcal{P}_t x_t) - v(0, I^d_t) = b_1(x_t) - \mathcal{P}_t x_t$ and the price simplifies to:

$$\mathcal{P}_t x_t = \alpha_G \left[C_q - \sigma_A(1-\eta_{\sigma_A})\right] + (1-\alpha_G)\left\{b_1(x_t) + \sigma_g(1-\eta_{\sigma_g})\right\} \tag{D-21}$$

E Wage Bargaining

This section provides the details for the alternative wage rule, which is the solution to

$$w_t = \begin{cases} \text{argmax}\,(J_{g,t} - J_{v,t})^{1-\alpha_L}\,(W_{g,t} - W_{u,t})^{\alpha_L} & \text{if the firm is in stage } g \\ \text{argmax}\,(J_{\pi,t} - J_{v,t})^{1-\alpha_L}\,(W_{\pi,t} - W_{u,t})^{\alpha_L} & \text{if the firm is in stage } \pi, \end{cases} \tag{E-22}$$

where $\alpha_L \in (0,1)$ is the worker's bargaining weight, $W_{u,t}$ is the value to a worker of being in the unemployment state, and $W_{g,t}$ and $W_{\pi,t}$ are the values of being employed at firms in stages g and π, respectively. These value are

$$\begin{aligned} W_{g,t} &= w^g_t - + \left(1-s^L\right)\frac{1-s^C}{1+r}\mathbb{E}_t\left[(1-\lambda_t)W_{g,t+1} + \lambda_t W_{\pi,t+1}\right] + \frac{s^C + (1-s^C)s^L}{1+r}\mathbb{E}_t W_{u,t+1} \\ W_{\pi,t} &= w^\pi_t + \left(1-s^L\right)\frac{1-s^C}{1+r}\mathbb{E}_t\left[(1-s^G)W_{\pi,t+1} + s^G W_{g,t+1}\right] + \frac{s^C + (1-s^C)s^L}{1+r}\mathbb{E}_t W_{u,t+1} \\ W_{u,t} &= z + \frac{1}{1+r}\mathbb{E}_t\left[f_t W_{g,t+1} + (1-f_t)\,W_{u,t+1}\right]. \end{aligned}$$

The sharing rules out of the Nash-bargaining problems are

$$\begin{aligned}\alpha_L\left[J_{g,t}-J_{v,t}\right] &= (1-\alpha_L)\left[W_{g,t}-W_{u,t}\right]\\ \alpha_L\left[J_{\pi,t}-J_{v,t}\right] &= (1-\alpha_L)\left[W_{\pi,t}-W_{u,t}\right].\end{aligned}$$

Note that $\alpha_L\left[J_{\pi,t}-J_{g,t}\right]=(1-\alpha_L)\left[W_{\pi,t}-W_{g,t}\right]$. Consider now the derivation of the wage rate in stage g. The labor surplus in this case, S_g^L, is

$$\begin{aligned}S_{g,t}^L &= -z-\frac{f(\theta_t)}{1+r}\mathbb{E}_t\left[W_{g,t+1}-W_{u,t+1}\right]-\frac{\left(1-s^C\right)\left(1-s^L\right)}{1+r}\mathbb{E}_tW_{u,t+1}+\frac{1-s^C}{1+r}s^LK-K\\ &\quad -\sigma_A(e_{A,t})+\frac{\left(1-s^C\right)\left(1-s^L\right)}{1+r}\mathbb{E}_t\left[(1-\lambda_t)\left(J_{g,t+1}+W_{g,t+1}\right)+\lambda_t\left(W_{\pi,t+1}+J_{\pi,t+1}\right)\right]\\ &= -z-\alpha_L\frac{f(\theta_t)}{1+r}\mathbb{E}_tS_{g,t+1}^L+\frac{\left(1-s^C\right)\left(1-s^L\right)}{1+r}\mathbb{E}_t\left[\lambda_tS_{\pi,t+1}^L+(1-\lambda_t)S_{g,t+1}^L\right]\\ &\quad -\sigma_A(e_{A,t})-K+s^L\frac{1-s^C}{1+r}K.\end{aligned}$$

From the Bellman equation for the firm in stage g, we also have

$$\begin{aligned}(1-\alpha)S_{g,t}^L &= -w_t^g-\sigma(e_{A,t})-K+s^L\frac{1-s^C}{1+r}K\\ &\quad +(1-\alpha_L)\frac{\left(1-s^C\right)\left(1-s^L\right)}{1+r}\mathbb{E}_t\left[\lambda_tS_{\pi,t+1}^L+(1-\lambda_t)S_{g,t+1}^L\right].\end{aligned}$$

Combining these two equations, along with the result that $(1-\alpha_L)\mathbb{E}_tS_{g,t+1}^L/(1+r)=\left(\gamma+\frac{r+s^C}{1+r}K\right)/q(\theta_t)$, yields the wage rule

$$w_t^g=\alpha_L\theta_t\left(\gamma+\frac{r+s^C}{1+r}K\right)-\alpha_L\left(1+s^L\frac{1-s^C}{1+r}\right)K+(1-\alpha_L)z-\alpha_L\sigma_A(e_{A,t}).$$

Consider now the derivation of the wage rate in stage π. The labor surplus in this case is

$$\begin{aligned}S_{\pi,t}^L &= \mathcal{P}_tx_t-C_q-z-K+s^L\frac{1-s^C}{1+r}K-\frac{\alpha_Lf(\theta_t)}{1+r}\mathbb{E}_tS_{g,t+1}^L\\ &\quad +\frac{\left(1-s^C\right)\left(1-s^L\right)}{1+r}\mathbb{E}_t\left[(1-s^G)S_{\pi,t+1}^L+s^GS_{g,t+1}^L\right].\end{aligned}$$

From the Bellman equation for the firms in stage π, we also have

$$\begin{aligned}(1-\alpha_L)S_{\pi,t}^L &= \mathcal{P}_tx_t-w_t^\pi-C_q-K+s^L\frac{1-s^C}{1+r}K\\ &\quad +(1-\alpha_L)\frac{\left(1-s^C\right)\left(1-s^L\right)}{1+r}\mathbb{E}_t\left[(1-s^G)S_{\pi,t+1}^L+s^GS_{g,t+1}^L\right].\end{aligned}$$

Combining these two equations yields the wage rule in stage π:

$$w_t^\pi = \alpha_L \left(\mathcal{P}_t x_t - C_q - \left[1 + s^L \frac{1-s^C}{1+r} \right] K \right) + (1-\alpha_L) z + \alpha_L \theta_t \left(\gamma + \frac{r+s^C}{1+r} K \right).$$

F Stock-Flow Equations

F.1 With perfect pooling

We have the following laws of motion.

$$\Xi_{U,t+1} = (1-\check{\lambda}_t)\Xi_{U,t} + \left[s^C + \left(1-s^C\right)\left(s^L + \left(1-s^L\right) s^G\right)\right] \Xi_{M,t} \tag{F-23}$$

$$\Xi_{M,t+1} = \left(1-s^C\right)\left(1-s^L\right)(1-s^G)\Xi_{M,t} + \check{\lambda}_t \Xi_{U,t} \tag{F-24}$$

There is an inflow $q(\theta_t)\mathcal{V}_t$ into the stock of firms searching in the goods market every period, where V_t is the number of vacancies at time t. To this, $\left(1-s^C\right)\left(1-s^L\right) s^G \mathcal{N}_{\pi,t}$ firms separated from consumers lead the stocks $\mathcal{N}_G$ and $\mathcal{N}_\pi$ to evolve according to

$$\mathcal{N}_{G,t+1} = \left(1-s^C\right)\left(1-s^L\right)\left[(1-\lambda_t)\mathcal{N}_{G,t} + s^G \mathcal{N}_{\pi,t}\right] + q(\theta_t)\mathcal{V}_t \tag{F-25}$$

$$\mathcal{N}_{\pi,t+1} = \left(1-s^C\right)\left(1-s^L\right)\left[(1-s^G)\mathcal{N}_{\pi,t} + \lambda_t \mathcal{N}_{G,t}\right]. \tag{F-26}$$

Finally, the dynamics of aggregate unemployment and employment are then given by

$$\mathcal{U}_{t+1} = \left[s^C + \left(1-s^C\right) s^L \right](1-\mathcal{U}_t) + (1-f(\theta_t))\,\mathcal{U}_t \tag{F-27}$$

With imperfect credit markets, we further have:

$$\begin{aligned} \mathcal{V}_{t+1} &= p(\phi)\mathcal{E}_{C,t} + (1-q(\theta_t))\mathcal{V}_t + s^L \mathcal{N}_{t+1} \\ &= \phi p(\phi)\mathcal{B}_{C,t} + (1-q(\theta_t))\mathcal{V}_t + s^L \mathcal{N}_{t+1}. \end{aligned} \tag{F-28}$$

Finally two identities:

$$1 - \mathcal{U}_t = \mathcal{N}_{G,t} + \mathcal{N}_{\pi,t} = \mathcal{N}_{t+1} \tag{F-29}$$

$$\mathcal{N}_{\pi,t} = \Xi_{M,t} \tag{F-30}$$

F.2 With imperfect pooling

In that case, given that transition probabilities in the goods market differ across employment status, we need to calculate each stock separately with the new laws of motion and take into account the transitions across employment states $d_{K,K'}$ and $d_{K',K}$:for each employment state $K = 0, 1$, that is, for the unemployed and the employed respectively, we have:

$$\Xi_{U,t+1,K} = (1 - d_{K,K'}) \left\{(1 - \check{\lambda}_{t,K})\Xi_{U,t,K} + \left[s^C + \left(1 - s^C\right)\left(s^L + \left(1 - s^L\right) s^G\right)\right] \Xi_{M,t,K}\right\} + d_{K',K}\Xi_{U,t+1,K'} \tag{F-31}$$

$$\Xi_{M,t+1,K} = (1 - d_{K,K'}) \left\{\left(1 - s^C\right)\left(1 - s^L\right)(1 - s^G)\Xi_{M,t} + \check{\lambda}_{t,K}\Xi_{U,t,K}\right\} + d_{K',K}\Xi_{M,t+1,K'} \tag{F-32}$$

Stocks $\mathcal{N}_G$ and $\mathcal{N}_\pi$ still evolve according to equations (F-25) and (F-26), where λ_t is defined as before, but with different average effort for consumers. The dynamics of aggregate unemployment and employment is still are then given by (F-27) and (F-28) and the two identities (F-29) and (F-30) remain valid. Other identities are introduced:

$$\Xi_{U,t,K=0} + \Xi_{D,t,K=0} = \mathcal{U}_t$$
$$\Xi_{U,t,K=1} + \Xi_{D,t,K=1} = 1 - \mathcal{U}_t$$
$$\Xi_{M,t,K=0} = \mathcal{N}_{\pi,t,K=0}$$
$$\Xi_{M,t,K=1} = \mathcal{N}_{\pi,t,K=1}$$

G Social efficiency

G.1 The general program with financial frictions

Social output is the present discounted value of consumption and leisure net of all search costs. Under the assumption of separable utility in consumption and leisure z:

$$W^{SP} = \int_0^\infty e^{-rt}[\Xi_U v(0, c_U^0) + \Xi_M v(x, c_M^0) + \mathcal{U}z]dt$$

.

The control variables are now $\mathcal{N}_c$ and $\mathcal{B}_c$ while $\mathcal{U}$ and $\mathcal{V}$ are state variables, as well as $\mathcal{B}_c$, and $\mathcal{N}_c$.

The first is associated with a classical law of motion:

$$\dot{\mathcal{U}} = s(1-\mathcal{U}) - q(\theta)\mathcal{V} = s(1-\mathcal{U}) - \theta q(\theta)\mathcal{U}$$

while the second has a law of motion as between vacancy creation and the number of vacancy filled

$$\begin{aligned}\dot{\mathcal{V}} &= \mathcal{B}_c\phi p(\phi) - q(\theta)\mathcal{V} = \Omega_C(\mathcal{N}_c, \mathcal{B}_c) - \Omega_L(\mathcal{U}, \mathcal{V}) \\ &= \mathcal{N}_c p(\phi) - q(\theta)\mathcal{V}\end{aligned}$$

We also have

$$\partial\Omega_C/\partial\mathcal{B}_c = \phi p(\phi)\eta_C(\phi)$$

and

$$\partial\Omega_C/\partial\mathcal{N}_c = p(\phi)(1-\eta_C(\phi))$$

, and similarly,

$$\partial\Omega_L/\partial\mathcal{U} = \theta q(\theta)\eta_L(\theta)$$

and

$$\partial\Omega_L/\partial\mathcal{V} = q(\theta)(1-\eta_L(\theta))$$

and finally, if $\xi\lambda'(\xi)/\lambda(\xi) = (1-\eta_G)$ e.g. $\Omega_G = a\Xi_U^{\eta_G}\mathcal{N}_G^{1-\eta_G}$

$$\partial\Omega_G/\partial\Xi_U = \check{\lambda}(\xi)\eta_G(\xi) = (\lambda(\xi)/\xi)\,\eta_G(\xi)$$

=and

$$\partial\Omega_G/\partial\mathcal{N}_G = \lambda(\xi)(1-\eta_G(\xi))$$

There is a constraint on the resources of goods $0:$ denote by G_t the net deficit (which could be negative in the case of a fiscal surplus), and by i the interest rate on the debt, with $i \lesseqgtr r$. We have:

$$\bar{y} + G_t \geq \Xi_U c_U^0 + \Xi_M c_M^0 + \gamma\theta\mathcal{U} + \kappa_B\mathcal{B}_c + \kappa_I\mathcal{N}_c + iD_t$$

with either $G_t \equiv 0$ (no deficit is allowed), or there is an optimal fiscal policy controlled by G_t with D_t is a new state variable and $\dot{\mathcal{D}}_t = G_t$. We follow the second approach here.