Stress Testing in a Structural Model of Bank Behavior

Dean Corbae[*], Pablo D'Erasmo[†], Sigurd Galaasen[‡], Alfonso Irarrazabal[§], Thomas Siemsen[¶]

Preliminary Version. Do Not Distribute.

January 29, 2016

Abstract

We develop a structural banking model for microprudential stress testing. We model a single bank that optimally chooses portfolio allocation, dividend policy and exit, facing regulatory and technological constraints. In our calibrated model, the bank has an incentive to hold a buffer stock of capital even in excess of regulatory requirements to protect its charter value. We explore optimal behavior during severe macroeconomic stress. We employ bank's endogenous exit choice as a novel metric for counterfactual stress outcomes. Finally, we discuss implications for current stress testing framework.

JEL classifications: C63, G11, G17, G21, G28

Key words: Stress testing, structural banking model, microprudential regulation

[*] Department of Economics, University of Wisconsin-Madison
[†] Federal Reserve Bank of Philadelphia, Research Department
[‡] Norges Bank, Research Department
[§] Department of Economics, BI Norwegian School of Economics
[¶] Department of Economics, Ludwig-Maximilians-University Munich

1 Introduction

State-of-the-art models for micro- and macroprudential stress tests derive capital shortfalls during counterfactual scenarios relying on a combination of exogenous, behavioral rules and reduced-form relationships extrapolated from historical data. This approach is susceptible to breakdowns in these relationships due to financial innovations, regulatory changes and large shocks and it is prone to the Lucas critique. This paper makes a first step towards a microfounded stress test.

To this end we propose a quantitative banking model for microprudential stress testing based on Corbae and D'Erasmo (2014). Our model can be summarized according to four features. First, we consider a single bank's optimization problem in a partial equilibrium environment à la De Nicolo, Gamba, and Lucchetta (2014). To permit quantitative results, the model is closed by exogenous bank-specific loan demand derived from a model of discrete choice. Second, the bank rationally anticipate the likelihood of stress, affecting normal times behavior. Third, given expectations, the bank can choose to exit the market by liquidating assets. Fourth, the bank conducts maturity transformation between demandable external funding and term loans. We calibrate the model using balance sheet information from a Norwegian bank and track behavior during counterfactual stress scenarios, including endogenous exit decision.

Our results are threefold: First, we show that the bank has an incentive to hold a buffer stock of capital even in excess of regulatory requirements to reduce the likelihood of exit. We show that this incentive for self-insurance is decreasing in the capital requirement, such that for a high enough requirement excess capital holdings are zero and loan supply decreases in the requirement. In our calibrated model for a 13 % capital ratio the banks does not hold any excess capital. However, when we counterfactually set the capital requirement to 0 % the bank still holds a 8.8 % capital ratio. Second, we use the endogenous exit probability as a novel, forward looking stress test metric when assessing the sufficiency of a bank's equity holdings under counterfactual scenarios. We show that looking only at equity shortfalls below an exogenous equity threshold ('hurdle rate') to measure bank health can be misleading if the bank prefers to exit above this threshold. Since exit leads to full loss of equity for the financial institution, stress testing frameworks that do not allow for endogenous exit underestimate equity losses during stress. In our calibrated model, we find that under the current capital requirement of 13 %, the large bank has a 3 % probability of exit for a probabilistic stress scenario and does not exit for stress horizons of three years with different severity. However, for lower capital requirements the exit probability goes up to 22 % at the minimum Basel III capital requirement of 4.5 %. Third, we use our model to study quantitative implication of deviations from optimality in current state-of-the-art stress test models. We show that under the macroprudential stress testing assumption of no deleveraging during stress, the fraction of banks exiting increases from 20 % under unconstrained behavior to 64 %. This extensive margin effect may reduce ex-post aggregate loan supply, despite 118 % higher loan supply on the intensive margin.

Related Literature. We contribute to two strands of literature: the literature on structural banking model and on microprudential stress testing. Our model is related to partial equilibrium model of banking such as Allen and Gale (2004); Boyd and Nicolo (2005); De Nicolo, Gamba, and Lucchetta (2014); Bianchi and Bigio (2014). We extend these models with a calibrated bank-specific loan demand equation to allow for quantitative results. In industrial organization there is a long tradition of estimating firm-specific demand using discrete choice models (see for example Berry, Levinsohn, and Pakes, 1995). In banking, Dick (2008) and Egan, Hortacsu, and Matvos (2015) apply this approach to the market for deposits. Our approach is also related to the work of Elizalde and Repullo (2007) by quantifying the wedge between regulatory and economic bank capital.

Our major contribution is to the microprudential stress testing literature. To the best of our knowledge we are the first to employ a structural model to project key bank variables under counterfactual scenarios. State-of-the-art stress testing frameworks use a combination of reduced-form dependencies (Acharya, Engle, and Pierret, 2014; Covas, Rump, and Zakrajcek, 2014) and exogenous behavioral rules (Burrows, Learmonth, and McKeown, 2012; Board of Governors of the Federal Reserve System, 2013; Hirtle, Kovner, Vickery, and Bhanot, 2014; European Banking Authority, 2011, 2014) to map aggregate economic conditions to bank-specific variables.[1] These frameworks do not identify deep parameters, which makes them prone to the Lucas critique and limits their area of application to counterfactual scenarios in endogenous macro variables.[2] Our model replaces backward looking and exogenous rules by optimizing forward looking behavior based on first principals. Thereby the policy functions that describe bank behavior become explicit function of exogenous

[1]For a survey on state-of-the-art stress testing models see for example Foglia (2009); Borio, Drehmann, and Tsatsaronis (2012).

[2]For example these framework can not conduct stress test under counterfactual capital requirements or risk weights as the estimated parameters are only implicit functions of these parameters.

states and deep parameters. This offers a flexible laboratory for stress testing as a battery of counterfactual scenarios can be considered without having to extrapolate from observed conditions. In addition, we contribute by providing an optimal behavior benchmark to analyze the quantitative implications of exogenous behavioral rules as imposed in current stress testing frameworks.

The remainder of the paper is structured as follows: Section 2 lays out the model, Section 3 shows calibration and Section 4 sheds light on the exit decision. Section 5 conducts stress testing exercises and compares structural with reduced-form stress test outcomes. Finally, Section 6 concludes.

2 Model

The setup is a partial equilibrium model of a single bank's dynamic program. We extend the framework of Corbae and D'Erasmo (2014) along two dimensions: first, we introduce maturity mismatch. The bank provides term loans using demandable funding. Sector specific maturity is taken to be exogenous. However, given endogenous allocation of loan supply to sectors, the model features endogenous aggregate loan portfolio maturity. Second, we introduce heterogeneous loan demand from different sectors of the economy, for example retail and commercial loan demand, to increase balance sheet granularity and to study portfolio reallocation motives during stress.

Time is discrete, indexed by t and infinite. Each period is dividend into two subperiods: beginning of period (bop) and end of period (eop). The bank supplies risky term loans to sector $s \in \mathcal{S}$. Funding supply d_t is stochastic and follows a Markov process with transition matrix $\Delta(d_{t+1}, d_t)$. The bank is exposed to an aggregate Markov shock z_t with transition matrix $\mathcal{F}(z_{t+1}, z_t)$, which affects loan demand and non–performing loans.

2.1 Demand for loans

To derive bank i- and sector s-specific loan demand we employ a discrete choice model à la Berry, Levinsohn, and Pakes (1995). This way, we can derive idiosyncratic loan demand, L_{ist}, as a function of i's own interest rate and the interest rates charged by all other banks in the loan market. In the absence of an industry equilibrium, we impose exogenous behavior on all other banks' interest rate choice, and study i's optimal rate setting choices conditional on these assumptions.

Let I_s denote the universe of all credit suppliers to sector s. Subscript i denotes bank i variables and subscript $-i$ denote corresponding variable vectors of all other credit suppliers, such that $i + (-i) = I_s$. There is a mass M_s of consumer choosing among I_s credit suppliers in each sector s. The mass of consumers changes and depends on aggregate interest rate r^L_{st} and the state of the economy z_t, $M_s(r^L_{st}, z_t)$. To derive bank i- and sector s-specific loan demand curve $L_{ist} = L(r^L_{ist}, r^L_{-ist}, z_t)$, we use the following decomposition

$$L(r^L_{ist}, r^L_{-ist}, z_t) = s(r^L_{ist}, r^L_{-ist}) \times M_s(r^L_{st}, z_t), \tag{2.1}$$

where $s(r^L_{ist}, r^L_{-ist})$ denotes bank i's share in the market for total credit to sector s as a function of its own loan rate and the loan rate of all other suppliers of credit. $M_s(r^L_{st}, z_t)$ denotes aggregate sector-specific demand for total credit as a function of sector-specific aggregate interest rate, r^L_{st}, and aggregate state of the world z_t.

To derive the relationship $s_{ist} = s(r^L_{ist}, r^L_{-ist})$ we use a discrete choice demand model (for a more thorough exposition see for example Egan, Hortacsu, and Matvos, 2015, among others). A loan with interest rate r^L_{ist} received from bank i, generates utility αr^L_{ist} for a borrower j. Moreover, borrower j also receives non-interest utility $\delta_{is} + \varepsilon_{jist}$ when borrowing from group i, where the δ_{is} captures time–invariant but group–specific factors, such as service quality in sector s, and the i.i.d shock ε_{jist} captures any borrower–specific bank preferences. We assume that ε follows an extreme value distribution, $\exp(-\exp(-\varepsilon))$. Therefore, when receiving a loan from bank i, total utility of borrower j in sector s and period t is given by

$$u_{jist} = \alpha r^L_{ist} + \delta_{is} + \varepsilon_{jist}$$

Given the assumption on the distribution of ε, demand for group i's loans relative to all other groups is given by

$$s_i(r^L_{ist}, r^L_{-ist}) = \frac{\exp(\alpha r^L_{ist} + \delta_{is})}{\sum_{k=1}^{I_s} \exp(\alpha r^L_{kst} + \delta_{ks})} \tag{2.2}$$

In addition, this framework naturally induces a mapping between the aggregate loan rate r^L_{st} and idiosyncratic loan rates $\{r^L_{ist}\}_{I_s}$:

$$r^L_{st} = \sum_{k=1}^{I_s} s_{kst} \times r^L_{kst}\,, \tag{2.3}$$

The second dependency in Equation (2.1), $M_s(r^L_{st}, z_t)$, will be imposed externally.

2.2 Bank environment

Time is discrete, indexed by t and infinite. Each period is dividend into two subperiods: beginning of period (bop) and end of period (eop). The bank supplies risky term loans to sector $s \in \mathcal{S}$. Funding supply d_t is stochastic and follows a Markov process with transition matrix $\Delta(d_{t+1}, d_t)$. The bank is exposed to an aggregate Markov shock z_t with transition matrix $\mathcal{F}(z_{t+1}, z_t)$, which affects loan demand and non-performing loans.

2.2.1 Timing

Beginning of period At the beginning of period t there are two endogenous state variables: stock of securities a_t, and heritage loans $\{\ell_{st}\}_{\mathcal{S}}$. In addition there are two exogenous states: aggregate state z_t and external funding stock d_t. Bop equity is given by

$$e_t = a_t + \sum_{\mathcal{S}} \ell_{st} - d_t$$

Given these states, the bank makes beginning-of-period portfolio choices. The liability side is pre-determined through state d_t. On the asset side, the bank chooses sector-specific loan supply L_{st} and security holdings A_t. We follow De Nicolo, Gamba, and Lucchetta (2014) and assume that loans have an exogenous maturity $1/(1 + m_s)$ such that each period a constant fraction m_s of loans L_{st} matures. While sector-specific maturity is exogenous, the fact that the bank endogenously chooses its loan exposure to the different sectors induces an endogenous aggregate loan portfolio maturity. The bank can decide, however, to reduce loan exposure faster than at rate m_s. In this case it must pay quadratic adjustment costs on disinvestment $L_{ts} - \ell_{ts} < 0$

$$\Psi_s(L_{st}) = \mathbb{I}(L_{st} < \ell_{st})\psi_s[L_s - \ell_{st}]^2 \ , \forall s \in \mathcal{S} \ , \tag{2.4}$$

where $\mathbb{I}(\cdot)$ denotes the indicator function and ψ_s is the cost coefficient. Marginal adjustment costs are increasing in $|L_{st} - \ell_{st}|$ to reflect increasing reductions on loan face value if a large fraction of the loan stock has to be liquidated and sold off. These costs can capture both liquidation costs that arise when loans are sold off and fire sale costs due to sudden and large reductions in the loans stock. In contrast, increasing the loan exposure by choosing $L_{st} \geq \ell_{st}$ does not generate adjustment costs. This induces the flow-of-funds constraint

$$a_t - A_t = \sum_{\mathcal{S}} \left[(L_{st} - \ell_{st}) + \Psi(L_{st}) \right] , \tag{2.5}$$

which states that given external funding supply d_t, the change in security investment and the change in loan investment (including adjustment costs) must equalize. Bank's portfolio choice is subject to a regulatory minimum capital constraint

$$\varphi \left(\sum_{\mathcal{S}} w_s L_{ts} + w_A A_t \right) \leq e_t \tag{2.6}$$

where φ is the regulatory common equity Tier 1 capital ratio requirement and w_k, $k \in \{s, A\}$, are regulatory risk-weights. We model the regulatory capital requirement as a hard constraint, i.e. it will never be violated on the equilibrium path.[3]

Securities pay a safe interest of r^a and successful bank loans generate an interest payment of r^L_{st}. However, a fraction $(1 - p_{st+1})$ of loans is non-performing. In that case it pays no interest and a fraction λ_s has to be written down, reducing next period loan stock ℓ_{st+1}. We assume that $p_{st+1} = p(r^L_{st}, z_t, z_{t+1})$.

End of period Eop is initiated with the realization of the new aggregate shock z_{t+1} and the new funding supply shock d_{t+1}.[4] The aggregate shock determines the fraction of non–performing loans $(1 - p(r^L_{ts}, z_t, z_{t+1}))$ in the bank's loan portfolio. At this stage, bank's cash flow is given by

$$\pi_{t+1} = \sum_{\mathcal{S}} \left[p_{st+1}(m_s + r^L_{st}) L_{st} - c_0 L^2_{st} \right] + r^a A_t - r^d d_t + (d_{t+1} - d_t) - \kappa \, , \tag{2.7}$$

where $c_0 L^2_{st}$ captures convex non-interest expenses of loan providence such as screening and monitoring costs and κ are fixed costs of operating in the loan market. We assume that loan interest

[3] One can think of this as the bank having to pay a prohibitively high regulatory fine if it violates this constraint, such that it would prefer to exit the market in the previous period than entering a period were the constraint cannot be satisfied. Modeling the minimum capital requirement has a hard constraint is in line with the BIS view, which motivates the counter-cyclical capital buffer as a way of giving banks a capital cushion, which can be eaten into before hitting the minimum requirement (Basel Committee on Banking Supervision, 2010).

[4] We use the timing convention that all variables that are determined after the realization of the aggregate shock z_{t+1} have time index $t + 1$.

rates are floating. This is reflected in the fact that contemporaneous interest rate r^L_{ts} applies to all loans L_{ts}, including the loan stock ℓ_{st}. This assumption reduces the state space, as we do not need to keep track of the whole history of loan providence. Non–performing loans do not pay any interest. Exogenous funding supply induces fluctuations in cash flow. If $d_{t+1} > d_t$ the bank receives an eop cash inflow and vice versa.

The bank now decides on its dividend policy, $\mathcal{D}_{t+1}$. It can distribute the cash flow to equity holder or retain earnings. Moreover, it has access to a short-run liquidity market in which it can borrow liquidity at net cost r^b. Let $B_{t+1} < 0$ denote retained earnings and $B_{t+1} > 0$ denote short–run borrowing. Then, dividends are determined as

$$\mathcal{D}_{t+1} = \pi_{t+1} + B_{t+1} \tag{2.8}$$

The bank is constrained in its dividend policy:

$$\mathcal{D}_{t+1} \in [\underline{\sigma}\mathcal{D}_t,\, \bar{\sigma}\mathcal{D}_t]\,, \tag{2.9}$$

that is, we assume that contemporaneous dividend payments cannot deviate more than a factor $\bar{\sigma}$ ($\underline{\sigma}$) above (below) previous period dividend. Constraint (2.9) can account for factors outside the model that induce sluggish dividend adjustment during crises (see Acharya, Gujral, Kulkarni, and Shin, 2011, for the recent financial crisis). A reason for this can be asymmetric information between banks, such that dividends act as signaling device to peer institutions and the market. In this case we would have $\underline{\sigma} < 1$ and $\bar{\sigma} = +\infty$.[5] If $\underline{\sigma} = 0$ and $\bar{\sigma} = +\infty$, the constraint is equivalent to ruling out seasoned equity offering, as dividends are constrained below at zero. Also, this constraint can capture the Basel III conservation requirement, which prohibits dividend payments if Tier 1 equity is below a threshold. In this case $\underline{\sigma} = \bar{\sigma} = 0$.

Equations (2.8) and (2.9) together imply that if the bank wants to stay in the market despite contemporaneous negative cash flow, it has to tap the short–term liquidity market ($B_{t+1} > 0$) to not violate constraint (2.9) and loose its continuation value. In contrast, if cash flow is high, the bank may not want to pay everything out as dividends but rather wants to retain some earnings ($B_{t+1} < 0$) to raise next period's initial resources, as shown below. Short–term borrowing requires

[5]For now, we don't consider signally issues since we work in a model with perfect information.

collateral in form of securities, in the sense that gross repayment of short–term borrowing must not exceed contemporaneous security holdings:

$$(1 + r^b)B_{t+1} \leq A_t \,, \tag{2.10}$$

with $r_b = 0$ if $B_{t+1} \leq 0$. If banks do not have enough securities for covering a negative cash flow, they are forced to exit the market as Constraint (2.9) is violated. Constraint (2.10) also reflects the assumption that loans on the balance sheet cannot be used as collateral for short-term borrowing.

Each period a fraction m_s of loans exogenously matures at the beginning of each period. Non-performing loans are written down immediately with λ_s. Therefore, beginning of period $t+1$ heritage loans are given by

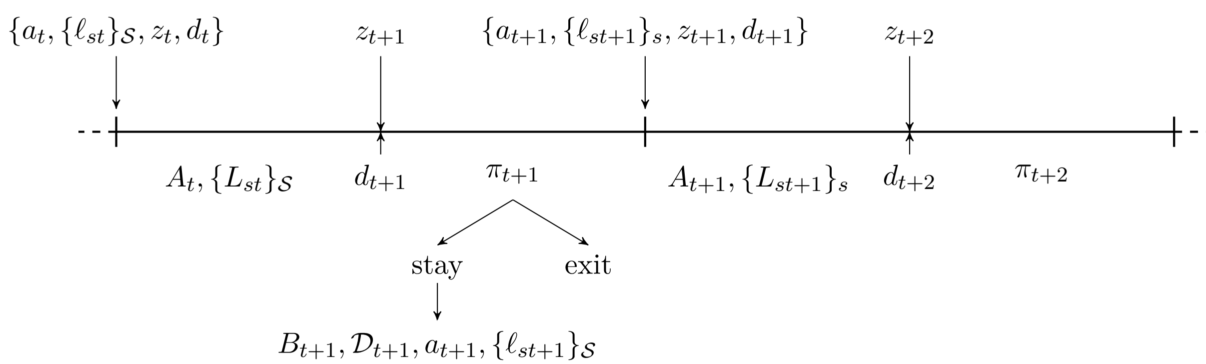
$$\ell_{t+1s} = [1 - m_s]p_{st+1}L_{st} + (1 - p_{st+1})[1 - \lambda_s]L_{st} \,, \forall s \in \mathcal{S} \tag{2.11}$$

Also, at the beginning of period $t+1$, before any choice is made, the short–term liquidity market is clears, i.e. B_{t+1} is repaid. Thus, beginning of next periods securities a_{t+1} are given by

$$a_{t+1} = A_t - (1 + r^b)B_{t+1} \geq 0 \tag{2.12}$$

As discussed above, retained earnings ($B_{t+1} < 0$) raises a_{t+1} and thus resources the beginning of the next period, which can be invested into either loans or securities. The figure below summarizes the timing.

Figure 1: Timing Assumption



2.3 Bank's dynamic programming problem

Due to the recursive nature of the bank's problem, we can drop time subscripts. Let $x_t = x$ and $x_{t+1} = x'$. The bank's objective is to maximize expected franchise value,

$$\mathbb{E}_t \sum_{k=t+1}^{+\infty} \beta^k \mathcal{D}_k \ , \tag{2.13}$$

where β is equity holders' discount factor. The value of the bank at the beginning of the period is given by

$$\begin{aligned}
&V(a, \{\ell_s\}_{\mathcal{S}}, z, d) = \max_{A, \{L_s\}_{\mathcal{S}}} \beta\, \mathbb{E}_{z'|z} W(A, \{L_s\}_{\mathcal{S}}, z', d') \ \ s.t. \\
&e = a + \sum_{\mathcal{S}} \ell_s - d \\
&a - A = \sum_s \left[(L_s - \ell_s) + \Psi(L_s)\right] \\
&\varphi \left(\sum_s w_s L_s + w_A A \right) \leq e \\
&L_s = L_s^d \ , \forall s \in \mathcal{S}
\end{aligned} \tag{2.14}$$

The last constraint requires bank–specific loan market clearing, where L_s^d is bank–specific loan demand from sector s. The eop value is given by

$$W\left(A, \{L_s\}_{\mathcal{S}}, z', d'\right) = \max_{x \in \{0,1\}} \left\{ W^{x=0}(A, \{L_s\}_{\mathcal{S}}, z', d')\, , W^{x=1}(A, \{L_s\}_{\mathcal{S}}, z', d) \right\} \ ,$$

where $x = 1$ denotes exit, and $x = 0$ denotes continuation. The exit value is given by

$$\begin{aligned}
W^{x=1}(A, \{L_s\}_{\mathcal{S}}, z', d) = \max \Bigg\{ 0\, , \sum_{\mathcal{S}} \Big[(m_s + r_s^L) p_s' L_s - c_0 L_s^2 + \ell_s' - \Psi(\ell_s') \Big] + (1 + r^a) A \\
- (1 + r^d) d - \kappa \Bigg\}
\end{aligned} \tag{2.15}$$

Upon exit the bank receives eop cash flow plus the principal on liquid securities. It liquidates the entire loan portfolio subject to adjustment costs, repays principal to external creditors and does not accept new external debt. If cash flow is sufficiently low, such that after liquidation of assets external creditors cannot be fully repaid, limited liability kicks in. The continuation value is given by

$$
\begin{aligned}
& W^{x=0}(A, \{L_s\}_{\mathcal{S}}, z', d') = \max_{B' \leq \frac{A}{1+r^b}} \{\mathcal{D}' + V(a', \{\ell'_s\}_{\mathcal{S}}, z', d')\} \quad s.t. \\
& \pi' = \sum_s \left[\left\{ p'_s \left(m_s + r^L_s \right) \right\} L_s - c^s_0 L^2_s \right] + r^a A - r^d d + (d' - d) - \kappa \\
& \mathcal{D}' = \pi' + B' \geq 0 \qquad (2.16) \\
& a' = A - (1 + r^b) B' \geq 0 \\
& \ell'_s = [1 - m_s] p'_s L_s + (1 - p'_s)[1 - \lambda_s] L_s, \quad \forall s \in \mathcal{S}
\end{aligned}
$$

2.4 Equilibrium Definition

Given parameters $\{\varphi, \{w_s\}_{\mathcal{S}}, w_A, r^a, r^b, r^d\}$ and stochastic processes $\{z_t, d_t\}$ a pure strategy Markov-perfect equilibrium is defined as a sequence of bank's policy rules $\{V_t, A_t, \{L_{st}\}, x_{t+1}, B_{t+1}, \mathcal{D}_{t+1}\}$ such that given loan demand $L^d(r^L_t, z_t)$ bank's choices of $\{V_t, A_t, \{L_{st}\}, x_{t+1}, B_{t+1}, \mathcal{D}_{t+1}\}$ are consistent with the two-stage optimization problem in Section 2.3.

3 Calibration

One period corresponds to a quarter. The bank in the model corresponds to a banking group. A banking group is the consolidated retail banking unit and associated credit companies, which emerged in Norway in 2007 and have since become an important funding source for banking groups (see Raknerud and Vatne, 2013). We allow for two sectors $s \in \mathcal{S}$ ={retail, C&I}. The data is taken from the Norges Bank ORBOF database, which provides information about individual Norwegian banks' balance sheets, income statements and interest rates. All parameters are in real terms. We deflate using total CPI index.

For expositional purposes, in this paper we calibrate the model to one big Norwegian banking group.

3.1 Loan Demand Calibration

To derive bank i- and sector s-specific loan demand curve $L_{ist} = L(r^L_{ist}, r^L_{-ist}, z_t)$, we estimate the two terms on the right hand side of Equation (2.1) separately.

Market Share Estimation. We estimate Equation (2.2) using interest rate and loan volume data for all Norwegian banking groups. We define each group's market share by sector, s_{it}, as gross lending to sector s relative to total credit to sector s.[6] Since we do not observe interest rates and loan volumes for all other loan suppliers except the banking groups (e.g. financial companies, shadow banks), we treat those sources for credit as an unobservable outside good, which we index by 0. Let ζ_{ist} denote the time–varying non–interest component of bank–specific utility and normalize non–interest utility of the outside good to zero, $\delta_{0s} + \zeta_{0st} = 0$. Dividing s_{ist} by s_{0st} and taking logs, we get

$$\log s_{ist} = \alpha r^L_{ist} + \delta_{is} + \varpi_{st} + \zeta_{ist}\,, \tag{3.1}$$

where r^L_{ist} denotes credit rate, δ_{is} is a firm- and sector-fixed effect, $\varpi_{st} \equiv \log s_{0st} - \alpha r^L_{0st}$ is a sector- and time-fixed effect and ζ_{ist} is an iid distortion. To identify the demand curve, we use the Libor interest rate as a supply shifter. Table 1(a) shows the estimation results.

[6] Data source for total credit by sector is SSB, Table 09560: Financial corporations. Loans, by borrower sector.

Table 1: Estimation Results: share and aggregate loan regression

(a) Loan share regression			
$\log s_{ist}$	Parameter	(I) Retail	(II) C&I
r^L_{ist}	α	-0.0514***	-0.0276*
r^L_{ist} elasticity		-0.1845	-0.1072
obs		255	255
R^2(within)		0.52	0.21

(b) Aggregate Credit Regression			
$\log L_t$	Parameter	Retail	C&I
r^L_t	β_1	-3.648***	-3.600**
$\log \tilde{z}_t$	β_2	2.050***	1.385***
r^L_t- elasticity		-13.788	-13.754
dummy 2008		X	–
obs		51	51

Notes: Panel (a): Dependent variable is log market share in total credit to sector s. The panel is balanced with quarterly observations from 2001Q1 to 2014Q2 for five Norwegian banking groups. Bank– and sector–specific interest rate instrumented with Libor. All variables are deflated with Norwegian CPI. Panel (b): Dependent variable is log total credit to sector s. Data from 2001Q1 to 2014Q2. Aggregate loan rate instrumented with Libor. Due to a structural break in the time series for retail credit after 2008, we include an additional dummy variable. All variables are deflated with Norwegian CPI. ***p<0.01, **p<0.05, *p<0.10

Aggregate Level Estimation. To derive the second relationship on the right hand side of equation (2.1), $M_s(r^L_{st}, z_t)$, we estimate the following dependency

$$\log L_{st} = c + \beta_1 r^L_{st} + \beta_2 \log \tilde{z}_t + \epsilon_{st} \,, \tag{3.2}$$

where $\log L_{st}$ denotes log HP-filtered ($\lambda = 400,000$)[7] total credit to sector s, r^L_{st} is average interest rate for total lending to sector and $\log \tilde{z}_t$ denotes log, HP-filtered ($\lambda = 3000$) real GDP.[8] Due to a lack of data, we do not observe r^L_{st} directly. Therefore, we approximate it using the average loan rate charged by all Norwegian banking groups, which is a good proxy, given that banking groups have an average market share of 73 % and 76 % in total C&I and retail credit, respectively. To identify credit demand, we use the Libor rate as supply shifter. Since we work with a normalization

[7]The choice of parameter is owned to the fact that credit cycles are about four times longer than business cycles and follows Borio and Lowe (2002).

[8]Data source for GDP is SSB, Table 09190: Gross domestic product Mainland Norway, market values, sa, 2011 prices.

in our model ($z_G = 1$), the estimated constant c is not relevant. Instead, we recalibrate c to match average credit over GDP in sector s conditional on average loan rate and $z_t = z_G$. Table 1(b) shows estimation results.

Mapping to the Model. Given that we consider a single bank's decision problem, we assume that the interest rates of all other credit suppliers, except for the bank under consideration, remain constant: $r^L_{-ist} = \bar{r}^L_s$, such that equation (2.3) simplifies to

$$r^L_{st} = s_{ist} \times r^L_{ist} + (1 - s_{ist}) \times \bar{r}^L_s \,, \tag{3.3}$$

where, s_{ist} is given by Equation (2.2). In the model, we identify the continues GDP-measure $\tilde{z}_t$ with the discretized aggregate process z_t. In the data 'normal times' GDP corresponds to GDP on trend, i.e. $\tilde{z}_t = 0$. In the model, 'normal times' corresponds to $z_t = z_G = 1$. We must therefore adjust the constant to reflect this normalization. Given the estimated dependencies (3.1), (3.2) and (3.3), group-i and sector s-specific loan demand is given by

$$\begin{aligned}
L(r^L_{ist}, \bar{r}^L_s, z_t) = &\quad s(r^L_{ist}, \bar{r}^L_s) \times M_s(r^L_{st}, z_t) \\
= &\quad s(r^L_{ist}, \bar{r}^L_s) \times \exp(c_s + \beta_{1s} r^L_{st} + \beta_{2s} z_t) \\
= &\quad s(r^L_{ist}, \bar{r}^L_s) \times \exp(c_s + \beta_{1s}[s(r^L_{ist}, \bar{r}^L_s) r^L_{ist} + \{1 - s(r^L_{ist}, \bar{r}^L_s)\}\bar{r}^L_s] + \beta_{2s} z_t)
\end{aligned}$$

We set $\bar{r}^L_{retail}$ and $\bar{r}^L_{C\&I}$ equal to the average quarterly sectoral lending rate for total credit, approximated by Norwegian bank lending rate.

3.2 Non-Performing Loans Estimation

In our model the fraction of loans that turns out to be non–performing ex-post, $[1 - p_s(r^L_{st}, z_t, z_{t+1})]$, is a function of loan rate, r^L_{st}, at the beginning of the period aggregate state, z_t, and end of period aggregate state, z_{t+1}. For normal business cycle times, we derive this dependency from Norwegian banking data, while for crisis times, we assign fraction of non-performing loans to the exogenous stress scenario.

For normal business cycle we estimate the following panel equation

$$(1 - p_{ist}) = c + \gamma_1 r^L_{ist} + \gamma_2 \log \tilde{z}_t + \gamma_3 \log \tilde{z}_{t-1} + \varpi + \epsilon_{ist}\,, \tag{3.4}$$

where $(1-p_{ist})$ denotes non–performing loans as a fraction of gross lending of group i in sector s and quarter t, r^L_{ist} is the corresponding lending rate and $\log \tilde{z}_t$ is HP-filtered (λ=3000) log real GDP. To account for seasonal patterns in the non-performing loans data, e.g. due to windows dressing, the regression also includes quarter dummies, ϖ. We account for time–invariant heterogeneity between banking groups by adding firm fixed effects. Table 2 shows the estimation results.

Table 2: Estimation Results: non–performing loans

$(1 - p_{ist})$	(I) Retail	(II) C&I
c	0.4585***	0.7739***
r^L_{ist}	0.0752***	0.1835***
$\log \tilde{z}_t$	-0.0479***	-0.1872***
$\log \tilde{z}_{t-1}$	-0.0308**	-0.0828*
ϖ	X	X
obs	241	241
R^2(within)	0.38	0.21

Notes: Dependent variable is non-performing loans in sector s. Data from 2001Q1 to 2014Q2 from ORBOF data base. Regression includes quarter dummies and firm fixed effects. All variables are deflated with Norwegian CPI. ***p<0.01, **p<0.05, *p<0.10

3.3 Aggregate Shock Calibration

We assume that the aggregate shock, z_t, follows a four state Markov process $z \in Z = [z_H\, z_L\, z_C\, z_R]$. We need to calibrate the state vector Z and the transition matrix $\mathcal{F}(z', z) \in \mathbb{R}^{4\times 4}$. In our model, z is the only source of aggregate fluctuations. Therefore, it captures normal business cycle fluctuations, as well as the aggregate component of the stress scenario. We allow for two states to capture normal fluctuations: a high state, z_H, and a low state, z_L. These states and their transition probabilities are calibrated to capture the normal Norwegian business cycle. There is one crisis state, z_C, and one recovery state, z_R, which captures a smooth transition out of crises. This section lays out the calibration of parameters that are not part of the stress scenario. We calibrate the Markov process using the Barro and Ursua (2008) data set, which captures boom–bust cycles for 36 countries between 1870 and 2008. We extend the data until 2013 and identify GDP peaks and troughs using the method suggested in Barro (2006).[9] The average contraction from a business cycle peak to a non–crises trough is $-2.58\,\%$ in Norway. We normalize z_H to unity and set $z_L = z_H - 0.0258 = 0.9742$ to match average business cycle contraction.

Consider transition probabilities next. Let p_{ij} denote the probability of switching from state i to j. For the transition matrix $\mathcal{F}(z', z)$ we impose the following zero restrictions:

$$\mathcal{F}(z', z) = \begin{bmatrix} p_{HH} & p_{HL} & 0 & 0 \\ p_{LH} & p_{LL} & p_{LC} & 0 \\ 0 & 0 & p_{CC} & p_{CR} \\ 0 & p_{RL} & 0 & p_{RR} \end{bmatrix},$$

i.e. from z_H only z_L can be reached, the only way into a crisis is through z_L, the recovery state z_R can only be reached from the crisis state and from the recovery state only z_L can be reached. To derive the switching probabilities between normal times state we follow Barro and Ursua (2008) and estimate these probabilities as the ratio of normal times Norwegian boom–bust cycles (13) over normal time years (118). Then, $p_{HL} = p_{LH} = 13/118 = 0.1102$ and $p_{HH} = 1 - 0.1102 = 0.8898$. We transform these annual probabilities to quarterly probabilities, p_{ij}^Q, through $p_{ij} = \left(p_{ij}^Q\right)^4$.

Calibrating the crisis states. Our framework offers a flexible laboratory to analyze counterfactual stress dynamics, since potentially all parameters can depend on the aggregate state z_t. The scenario we provide here is to illustrate the mechanics of our model. We consider a stress scenario in which a strong reduction in GDP depresses loan demand and induces a jump in non–performing loans. One can think of this scenario as a credit crisis.

The stress scenario requires calibration of aggregate states $\{z_C, z_R\}$, the corresponding transition probabilities $\{p_{LC}, p_{CC}, p_{CR}, p_{RR}, p_{RL}\}$ and fraction on non-performing loans during crisis states $\{p(z_L z_C), p(z_C, z_C), p(z_C, z_R), p(z_R, z_R), p(z_R, z_L)\}$.

In the aggregate shock process, z_t, there is one crisis state, z_C, and one recovery state, z_R, which captures a smooth transition out of crises. Theses states can be calibrated to reflect any exogenous stress scenario. Since crises observations in Norway are limited, we derive the crisis

[9] Extended data is taken from WDI database.

calibration from the average of 177 international crises observations in the Barro and Ursua (2008) data set. They define a crisis as a GDP contraction larger 9.5 %. The average GDP contraction from peak to crisis trough is $-20.56\,\%$. Since the normal business cycle peak is identified by z_H we set $z_C = z_H - 0.2056 = 0.7944$. To calibrate the recovery state z_R, we measure the average recovery time from crisis trough back to GDP trend. We find that it takes on average 2.95 years to recover back to trend. We identify z_R as the average GDP contraction after half the recovery time: $z_R = 0.9455$.

The probability of leaving normal times and entering a crisis is the ratio of crises observations over normal time years of all 36 countries. In our data set we have 5440 yearly observations including 515 crises years, during 177 crises, and 4925 normal time years. Then $p_{LC} = 177/4925 = 0.0359$ and $p_{LL} = 1 - p_{LH} - p_{LC} = 0.8539$. Along the same line, the probability of leaving a crisis and starting a recovery is estimated as the ratio of crises observations over crises year, i.e. $p_{CR} = 177/515 = 0.3437$. Thus, $p_{CC} = 1 - p_{CR} = 0.6563$. This implies an expected crisis duration from peak to trough of 2.9 years. Finally, we calibrate the recovery persistence to match the average recovery duration (trough to trend) of 2.95 years in the data. Since the expected recovery duration is given by $1/(1 - p_{RR})$, we have $p_{RR} = 0.6600$ and $p_{RL} = 0.3400$.

Consider non–performing loans next. We assume that on crisis impact the fraction of non–performing loans jumps to 14 % of total loans independent of interest rate, $1 - p(r^L_{st}, z_L, z_C) = 0.14\,, \forall s$. This value is taken from the Laeven and Valencia (2012) banking crisis data set and corresponds to mean peak non–performing loans. When staying in a crisis for multiple periods, non-performing loans are assumed to be 50 % below impact non–performing loans: $1 - p(r^L_{st}, z_C, z_C) = 0.14 \times 0.5$. When leaving the crisis trough and entering a recovery non-performing loans are $1 - p(r^L_{st}, z_C, z_R) = 1 - 0.04$. For the remaining state combinations involving z_R we let non–performing loans follow the process estimated in Section 3.2.

3.4 External Funding Shock Calibration

The external idiosyncratic funding shock d_t is calibrated by estimating the following dynamic model on the banking-group level for the period 1987Q4-2014Q2:

$$\log(d_t) = (1 - \rho)k_0 + \rho \log(d_t) + k_1 t + k_2 t^2 + u_t\,,$$

where d_t is the sum of outstanding deposits, bonds and commercial papers, t is a linear time trend and $u_t \sim N(0, \sigma^2)$. Using the estimates for $\hat{\rho} = 0.8695$ and $\hat{\sigma} = 0.0365$, we discretize the process with the method of Tauchen and Hussey (1991) into a three states Markov representation $d_t = [d_L \, d_N \, d_H]$ and to obtain the transition matrix $\Delta(d_{t+1}, d_t)$. Since the aggregate state is normalized ($z_H = 1$), the estimated mean k_0 is not relevant in our model. Instead we calibrate the mean of the finite state Markov process such that, given our sectoral demand equation, the ratio $[L_{retail}(\bar{r}^L_{retail}, z_H) + L_{C\&I}(\bar{r}^L_{C\&I}, z_H)]/d_N$ corresponds to the average total lending of external finance ratio for this banking group.

3.5 Remaining Parameter Calibration

Consider parametric interest rates first. All rates are calibrated using 1987Q1-2014Q2 variable averages. The marginal external funding cost parameter, r^d, is calibrated as the ratio of interest charges on deposits and bonds over the total stock of deposits and bonds. In this preliminary calibration we set $r^a = r^b = r^d$.

Due to a lack of data, we cannot calibrate loss-given default, λ, by sector but instead assume that it is identical between retail and commercial sector. We calibrate λ to target total loss on lending of a banking group in the data. In the model total loss on lending is given by $\sum_s (1 - p_{st+1}) L_{st} \lambda$. To calibrate λ consistently, we first derive a time series measure for $(1 - p_{st+1})$ as the ratio of new non–performing loans by sector over gross lending by sector.[10] We then calibrate λ as the ratio of average total loss on lending over average total non–performing loans, $(\sum_s (1 - p_s) L_s)$.

In Norway, the average original maturity of mortgages is 20 years. We assume a uniform distribution of mortgage age structure, such that the average maturity of mortgages outstanding is 10 years. ORBOF database provides a time series of average (across sectors) remaining loan maturity for each banking group. We assume that retail loans are equal to mortgages and trace out the average maturity for C&I loans using total maturity $= \frac{L_{retail}}{L_{retail} + L_{C\&I}}$(retail maturity)$+ \frac{L_{C\&I}}{L_{retail} + L_{C\&I}}$(C&I maturity). This yields an average C&I maturity of 4 years.

Risk weights, $(w_{retail}, w_{C\&I}, w_A)$, are calibrated based on the suggested risk weight revision

[10]ORBOF only provides data on new non–performing loans from 2010Q4 on. Therefore, we impute a time series going back until 1997 by computing the fraction of new non–performing loans in the stock of non–performing loans for the quarters available, take time average and then assume this fraction to be the same for the quarters where no data is available.

discussed in Basel Committee on Banking Supervision (2015). In our model securities are safe and collateralizable assets (e.g. Triple-A rated sovereigns bonds), which have a risk weight of 0 %. We think of retail loans mainly as longer–term mortgages, which have a 100 % risk weight. We also assume that loans to corporate firms have a risk weight of 100 %, which corresponds for example to small to medium revenue firms with leverage ratios between 1 and 5.

We calibrate β such that the annual cost of capital for the bank is 4 %. Finally, we calibrate fixed cost κ, non–interest expense parameter $\{c_0^s\}_{\mathcal{S}}$ and adjustment cost parameters $\{\psi_s\}_{\mathcal{S}}$ internally. For κ we target average return on equity of the banking group. For c_0^s we target average net interest margins and for ψ_s we target average volatility of the gross lending to sector s relative to GDP during the Norwegian banking crisis of 1988 to 1993.

We allow for a counter-cyclical capital requirement:

$$\varphi(z) = \begin{cases} \bar{\varphi} & , \text{ if } z \in z_H, z_L \\ 4.5\,\% & , \text{ if } z \in z_C, z_R \end{cases},$$

where $\bar{\varphi}$ is the normal times capital requirement, which we vary in the exercises below. Under suggested Basel III regulation, $\bar{\varphi} = 13\,\%$ corresponds to the maximum capital requirement targeted , with a 4.5 % minimum requirement, a 2.5 % conservation requirement, a 3.5 % systemic requirement and a 2.5 % counter-cyclical requirement. In our baseline calibration we set $\bar{\varphi} = 13\,\%$. Table 3 summarizes our preliminary calibration.

Table 3: Parameter Calibration for a large Norwegian Banking Group

Parameter		Calibration	Target
z_G	good state	1	normalization
z_B	bad state	0.9742	Norwegian business cycle
z_C	crisis state	0.7944	avg. Barro crisis
z_R	recovery state	0.9455	
d_H	high funding state	0.0689	external funding process
d_N	normal funding state	0.0635	avg. total lending/external funding
d_L	low funding state	0.0585	external funding process
r^a	security return	r^d	Corbae and D'Erasmo (2014)
r^b	borrowing costs	r^a	Corbae and D'Erasmo (2014)
r^d	funding costs	0.0040	avg. deposit and bond cost
$c_0^{retail,C\&I}$	non–interest expenses	0.02	net interest margin
$\psi_{retail,C\&I}$	adjustment costs	4.0	$Var(L_{st}/\tilde{z}_t)$
λ	loss given default	0.1786	loss on lending
$\chi_{retail,C\&I}$	maturity parameter	(1/17, 1/41)	avg. Norwegian maturity structure
β	discount factor	0.9901	cost of capital
κ	fixed costs	4.96×10^{-5}	avg. RoE
$w_{retail,C\&I}$, w_A	risk weights	[1, 1 , 0]	Basel III
$\bar{\varphi}$	normal times cap. req.	13 %	Basel III
$[\underline{\sigma}\,, \bar{\sigma}]$	dividend constraint	$[0, +\infty]$	no SEO

Calibrated normal times balance sheet. Given our preliminary calibration, Table 4 shows targeted and non–targeted moments for the banking group.

Table 4: Comparing model and simulated moments

Moment	Model avg. normal times	Data
	targeted	
RoE	0.10	0.12
NIM (retail)	0.026	0.024
NIM (C&I)	0.024	0.023
	non-targeted	
core capital ratio	0.130	0.136
loans/total assets	0.66	0.66
lending rate	0.042	0.036

Notes: Data moments are 2001Q4–2014Q2 averages, except for RoE and core capital ration, which are 2014Q2 observations.

4 Analysis of the bank's exit decision

Before moving to our stress test analysis, it is worth to take a closer look at the exit decision of a single bank. We show that the bank will choose exit if its charter value is sufficiently low, which - for our calibrated bank - only occurs during crises.[11] Therefore, we do not observe exit during normal times. We further look at the key determinants of exit decision: stress duration and initial equity position. Throughout this section, we assume a counterfactual capital requirement of $\bar{\varphi} = 4.5\,\%$, since for this requirement the capital constraint is not binding and the bank holds excess capital (see Table 5 below). This allows us to counterfactually reduce equity below the equilibrium level while still not violating the regulatory requirement.

4.1 Exit trade-off

In standard reduced-form stress tests, the passing of a stress test is measured against an exogenous equity threshold, often referred to as hurdle rate. If the equity projections of a bank drop below this threshold, the bank fails the test and may have to raise additional capital. In contrast to this approach, a structural setup with endogenous exit choice offers a novel stress test metric. Instead of an exogenous threshold, the forward-looking optimizing behavior of the bank induces an endogenous threshold through bank's charter value, $V(a, \ell, z, d)$.

When facing the choice of whether to exit or not, the bank trades off the cost and benefit of staying. When in crisis state z_C, bank profit is negative and hence equity falls over time. The gains from staying are associated with the profitability of bank operations in normal times. In order to get there, however, the bank must survive the crisis state. If the bank decides to stay, and the crisis state persists, equity will eventually turn negative. The bank is then forced to exit under limited liability (with value zero). In contrast, if the bank chooses to exit with positive equity, it receives the liquidation value of assets net of external debt. Hence, the cost of staying is the possible loss of liquidation value if the crisis persists. At high levels of equity, the probability of surviving the crisis is large and the bank prefers to stay and have the option to lend once the economy returned to normal times. At low levels of equity, the probability of surviving is small and the bank prefers to exit and take the liquidation value.

[11] Throughout, we use the terms "stress" and crisis" interchangeably. Both are defined as a consecutive episode of z_C and z_R states.

Figure 2(a) shows the exit value, $W^{x=1}(A, L, z', d)$, the continuation value, $W^{x=0}(A, L, z', d')$, and dividend payments in the crisis state z_C as a function of bop equity e. The heritage loan stock state ℓ is fixed at zero for expositional purposes. The exit value is increasing in e with slope $1 + r^a$. The reason is that during crisis state return on lending is negative and thus any additional bop resources are invested into riskless securities A, which, ceteris paribus, increases the liquidation value of assets (see Equation (2.15)).

The continuation value is the present discounted value of future dividend payments (see Equation (2.16)). During crisis state, but also when switching from crisis state to the recovery state, dividend payments are zero. Therefore, the crisis continuation value is solely driven by expected future dividend payments once the economy returned to normal times. In the exit region, left to the vertical line in Panel (a), the continuation value is smaller than the exit value. The reason they are nearly identical is that in the counterfactual case of no exit, the bank will exit the following period if the crisis persists. In the continuation region the slope of the continuation value is steeper than $1 + r^a$. The reason is that a higher bop equity raises the probability of surviving the stress episode, as equity losses can be sustained longer. This can be seen when tracing a given initial equity position over time. Suppose we start off with an equity level in the continuation region in Panel (a). The bank stays, and enters the next period with a lower equity level. This is shown in Panel (b), where the policy function for e' is below the 45 degree line. The bank moves closer to the exit threshold, and these dynamics continue until either the recovery state is reached or the bank exits. Thus, a higher equity level enables the bank to sustain more crisis state periods, which raises the continuation value.

Figure 2: Exit Decision

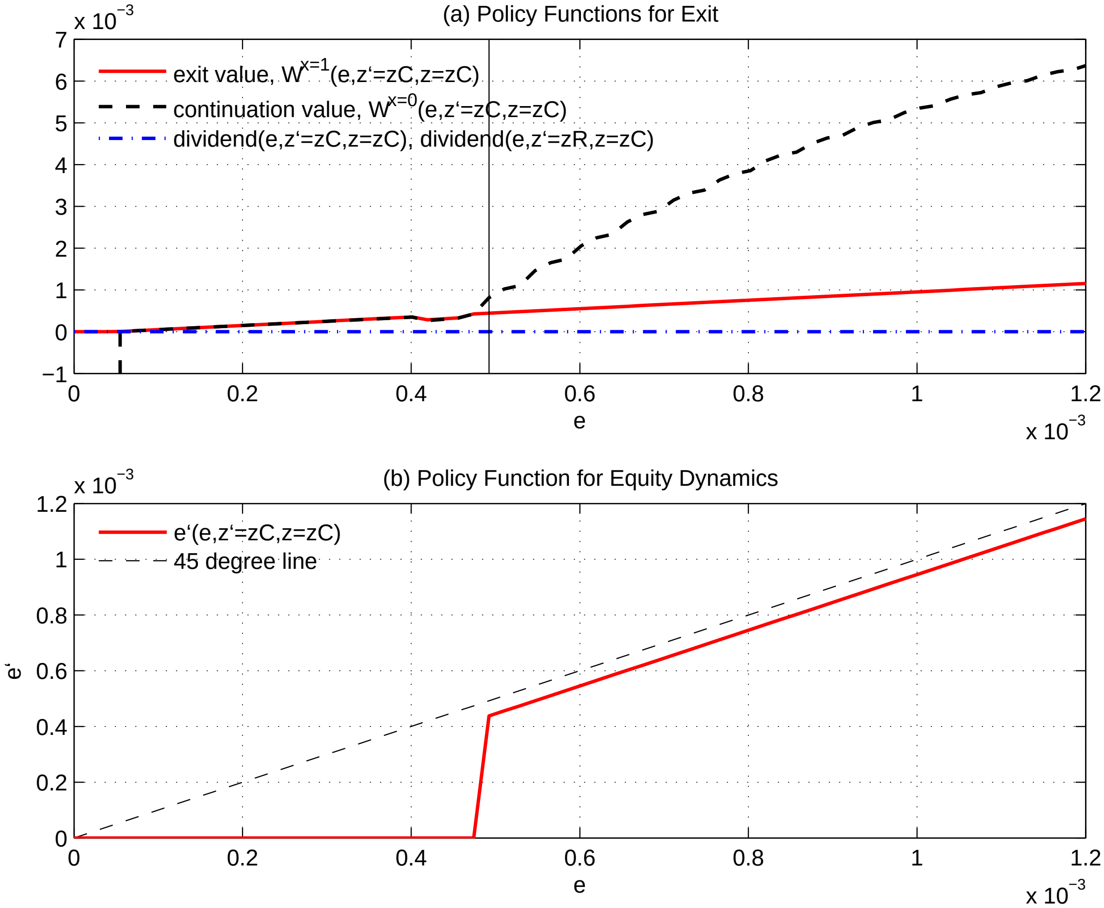
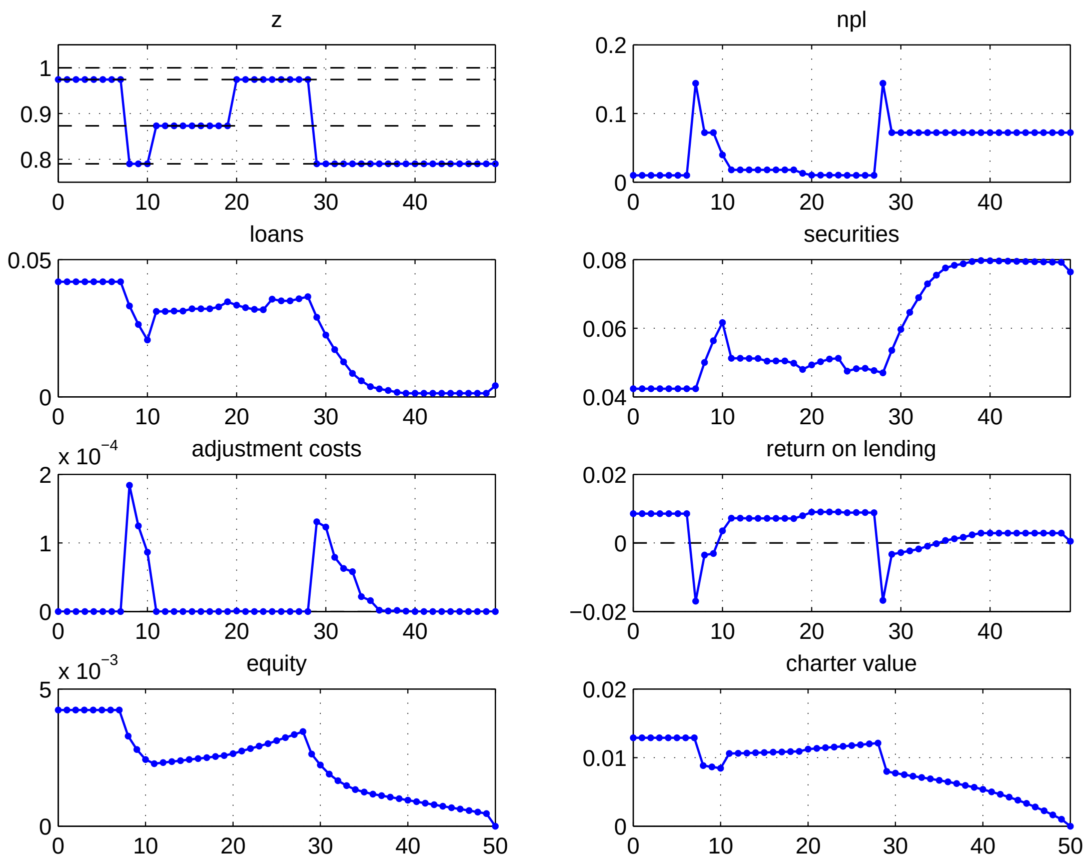


Figure 3 shows an example crisis simulation path that leads to exit. During z_C, the fraction of non–performing loans is high, inducing negative return on lending. The bank changes its portfolio by sharply reducing loan exposure, generating adjustment costs according to Equation (2.4), and increases security holdings. Since the return on securities is not high enough to compensate fixed costs, the bank suffers equity losses even if loan exposure is zero. Low return on safe securities, negative return on lending and fixed cost κ deplete bank equity and hence reduce its charter value. As long as the charter value is high enough, the bank chooses to stay in the market. Since the charter value captures the present value of all future dividend payments, it is forward-looking beyond the contemporaneous stress, into periods where return on lending is positive again and higher dividends can be paid. Thus, exiting implies the loss of option to participate in the market once it recovers. However, if stress persists long enough, the continuation value falls below the exit value. The first stress episode is brief enough, such that the bank stays in the market. Once the first episode of crisis states is left and the recovery state is entered, equity is gradually rebuilt through retained earnings. However, when the second stress episode hits, equity is still below normal times level. The second crisis turns out to be much more persistent, such that equity and charter value are increasingly depressed and in period 49 the bank decides to exit the market and liquidate the remaining charter value.

Figure 3: Exit Behavior, $\bar{\varphi} = 4.5\,\%$



4.2 Determinants of exit decision

Bank's equity level is the key determinant for the exit decision. The equity level during stress is determined by two factors: (1) stress duration and (2) initial equity upon stress entry.

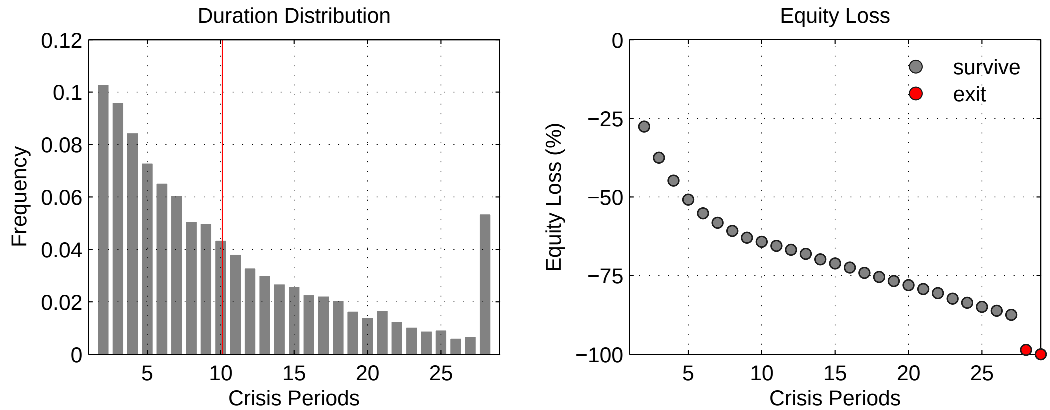
Stress duration. To separate the effect of stress duration from heterogeneity in initial equity position, we only consider crises into which the bank enters with the same initial balance sheet composition, in particular with same initial equity. Thus, the only source of heterogeneity in stress outcomes is crisis duration. Figure 4(a) shows the distribution of crisis state duration given our probabilistic stress scenario and its impact on stress outcome. Everything else equal, crisis state duration maps directly into the bank's equity losses during stress. The longer the crisis state episode, the higher is the corresponding equity loss. After 9 z_C periods, the bank is almost fully invested into securities. Since $r^a = r^d$ and external funding is stable, cash flow is approximately given by $\pi' = -\kappa$ (see Equation (2.7)). Therefore, the bank needs to borrow short-term, which reduces next period's equity at a constant rate (see Equation (2.12)). With a 4.5 % capital requirement, the equilibrium equity cushion is sufficiently thick to whether long (but unlikely) crisis state episodes of up to 28 quarters. The bank hangs on as the charter value is reduced period after period, expecting to leave the crisis state soon. However, if the economy does not leave the crisis state within the 28 periods, the charter value is sufficiently reduced for the bank to decide to exit. This leads to a discontinuity in the duration distribution at 28 crisis state periods, as no duration larger than 28 periods is observed.

Initial equity. The second key factor in bank's exit decision is the initial equity position upon stress entry. For a normal times capital requirement of 4.5 % the bank chooses to hold a z_L steady state capital ratio of 10 % (see Table 5). We now force this bank to enter the same crisis with lower initial equity. In particular, we force the bank to enter a crisis with a capital ratio of 7 %.

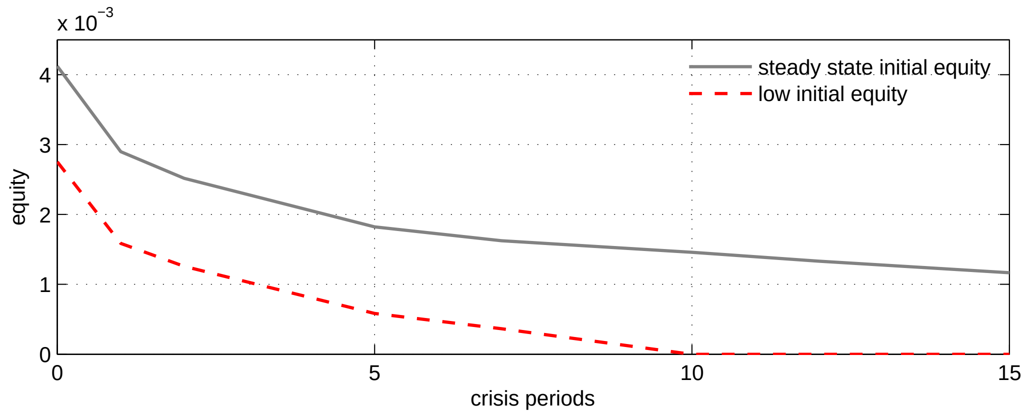
Figure 4(b) shows the equity paths for the different initial equity positions. When the bank enters the crisis with its equilibrium equity position, it is robust and can sustain 27 z_C periods. However, with counterfactually low initial equity the bank decides to exit after 9 crisis states. The reason is that with higher initial leverage, equity is lower and depleted faster, as the loss due to non-performing loans relative to equity is higher. Therefore, bank's initial balance sheet composition affects equity losses during stress and thus exit probability.

Figure 4: Determinants of Stress Outcomes

(a) Stress Duration, $\bar{\varphi} = 4.5\,\%$



(b) Initial Equity, $\bar{\varphi} = 4.5\,\%$



Notes: Panel (a): The only source of heterogeneous crisis outcomes derives from different crises durations, i.e. the bank enters each crisis with same balance sheet composition (from z_L steady state). Red vertical line indicates mean. Panel (b): "steady state initial equity" corresponds to to z_L steady state equity holdings upon crisis entry. "low initial equity" corresponds to initial equity 30 % below optimum, while loan stock remains unchanged.

5 Stress testing analysis

In this section we perform stress tests in our quantitative model using balance sheet information from a Norwegian bank. We first study bank resilience to a probabilistic crisis scenario for different counterfactual capital requirements and elaborate on the incentives for excess capital holdings. We then quantitatively analyze bank behavior during three stress scenarios that feature different degrees of severity and explore the effect of exit. Finally, we quantify the impact of the macroprudential assumption of constant crisis loan supply.

5.1 Capital regulation and bank resilience

Current stress test analysis derives capital shortfalls during stress relying on a combination of exogenous assumptions and reduced-form relationships and can not account for expectations and bank exit during stress. Bank behavior is only an implicit function of deep parameters. Therefore, current stress testing can not analyze counterfactual policy stance, such as capital requirements or risk weights, but is limited to stress scenarios capturing counterfactual dynamics in aggregate macroeconomic variables. In contrast, our structural approach offers a flexible laboratory to study a wider variety of counterfactual scenarios, since policy functions are explicit functions of exogenous state variables and deep parameters. This allows us to study the impact of counterfactual capital regulation on stress test results.

5.1.1 Expectations and precautionary capital motive

Capital regulation affects stress outcomes through the effect on bank's precautionary equity choice during normal times. Precautionary equity has two functions: First, it self-insures the bank from exit during stress and loosing the charter value. Second, it self-insures the bank from facing a binding capital constraint after a crisis, when equity is still low but return on lending is positive again.

Expecting the crisis state. We conduct the same stress exercise as in Section 4.1, seeding the bank in z_L steady state and exposing it to a probabilistic crisis path over a time frame of 120 periods to analyze resilience of this bank to a variety of adverse economic conditions. Table 5 illustrates how regulation affects bank robustness (measured as the likelihood of exit during a crisis) under two alternative assumptions on bank rationality. In the benchmark case (Panel a) of rational expectations, the bank internalizes the crisis state probability, and thus has an incentive to self-insure by accumulating capital. In the alternative case (Panel b) the bank believes that there is zero probability of going from normal times to a crisis state. This mutes the incentive to hold capital to protect the charter value as the bank believes there is a zero probability of exit.

Absent any regulatory requirements, the bank still chooses to hold positive equity. For $\bar{\varphi} = \varphi = 0$, the unregulated bank endogenously accumulates a capital ratio of 8.8 % to shield its charter value from crises. However, the bank does not find it optimal to accumulate sufficient capital to eliminate exit probability completely, and exits on average in 57 % of crises. When the bank does not expect to enter a crisis, the self-insurance motive disappears and capital holdings are zero. In this case, exit happens immediately upon being hit by a crisis. The intuition behind this result is that, absent crises, the marginal benefit of capital accumulation is to raise next period dividends with $(1 + r^a)$ at the marginal cost of -1 lower dividends today. Given our parameter values, we have that the discounted marginal benefit of retaining earnings exceeds the cost, $\beta(1 + r^a) < 1$. When the bank takes crises events into account, however, there is an additional gain from capital accumulation due to its insurance value. This explains the interior solution for capital holdings for the rational expectations bank.

With regulation, the rational expectation bank (Panel a) will for, low enough capital requirement, choose to hold a buffer above the regulatory level. For a capital requirement in normal times of $\bar{\varphi} = 4.5\,\%$ the bank actually holds a 10 % capital ratio, which is even higher than what the unregulated bank holds. This is explained by an interaction effect between regulation and capital accumulation, i.e. the second role of the precautionary buffer: the bank accumulates additional capital to reduce the likelihood of facing a binding capital constraint after a crisis, when equity is still low but return on lending is positive again (see below). Excess capital is, however, decreasing monotonically in $\bar{\varphi}$, such that for high enough requirements the precautionary capital holding disappears and the capital constraint is binding. For a normal times capital requirement of $\bar{\varphi} = 13$ the bank holds no excess capital.

Finally, Table 5 illustrates that tighter regulation indeed makes the bank more robust to crises, as can be seen by the negative relationship between normal times capital requirements and exit rates. Panel (b) highlights the role of expectations. When the bank is not expecting crises it chooses to be exactly at the capital requirement, i.e. it does not hold any voluntary excess capital. This implies high ex-post exit rates, especially for low capital requirements.

Table 5: Effect of Capital Regulation and Crisis Expectation

Capital requirement				
$\bar{\varphi}$ normal (%)	13.0	7.0	4.5	0
φ stress (%)	4.5	4.5	4.5	0
(a) Expected				
capital ratio (%)	13.0	10.7	10.0	8.8
excess capital (%)	0	53	122	–
$\mathbb{E}$[exit] (%)	3	11	22	57
(b) Non expected				
capital ratio (%)	13.0	7.0	4.5	0
excess capital (%)	0	0	0	–
$\mathbb{E}$[exit] (%)	3	98	98	100

Notes: Probabilistic stress scenario within a 120 periods time frame. Exit probability reflects heterogeneity in initial equity position and crisis duration. Excess capital computed as percentage deviation of capital ratio from regulatory requirement.

The role of recovery. To elaborate on the second role of precautionary equity, we consider a situation where the bank initially enters the crisis with an equity level below the z_L steady state level and track equity and loan supply dynamics during the crisis. The bank understands that once the economy fully recovers from a crisis, lending supply can be constrained by the counter-cyclical capital requirement. This induces a positive shadow value on equity in normal times.

In Figure 5 we consider two scenarios for the $\bar{\varphi} = 7\,\%$ capital requirement, for which the bank does hold excess capital (see Table 5): (1) the bank enters the crisis with z_L steady state balance sheet composition corresponding to a capital ratio of 10.7 % and (2) the bank enters the crisis with a capital ratio of 7 %, such that given same initial heritage loan stock equity is lower. The bank enters a stress episode of 4 quarters consisting of two crisis and two recovery states.[12] Panel (a) shows the equity paths and Panel (b) lending behavior and capital constraints.

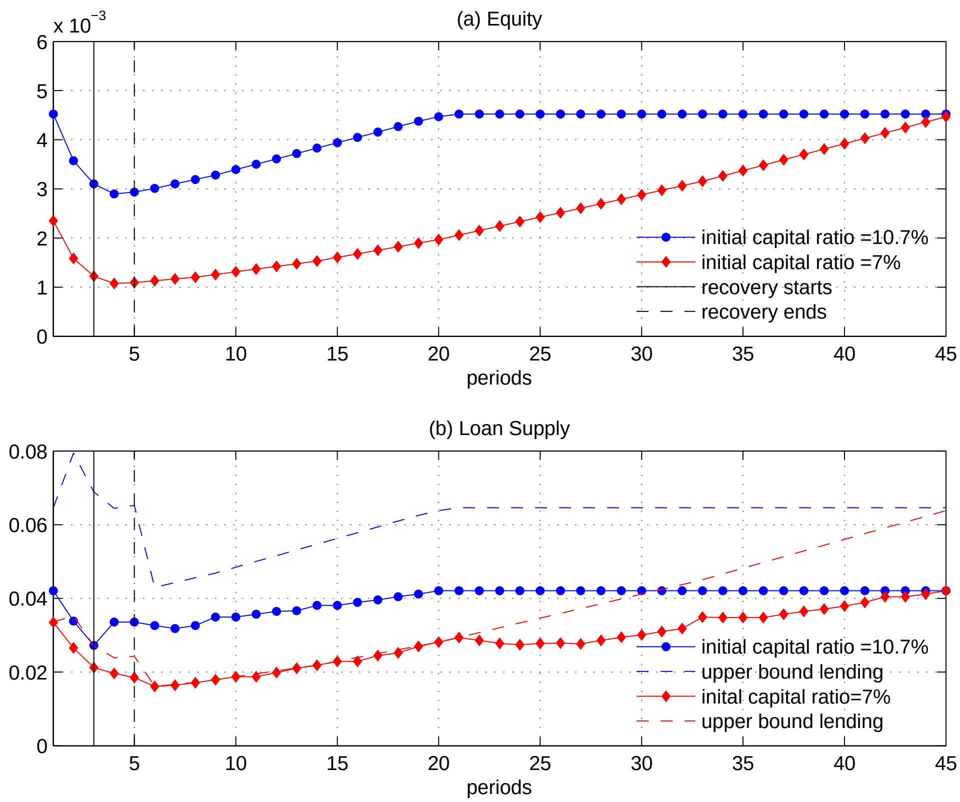
In scenario (1), the capital constraint is never binding. The endogenous buffer chosen by the bank is sufficiently large to allow the bank to reach its optimal lending level once the recovery is over and the capital requirement jumps back to 7%. In contrast, in scenario (2) the equity build-up after the crisis is more sluggish, taking almost twice as long as with a 10.7 % capital ratio. The reason is that with a lower capital ratio of 7 %, loan supply is constrained for the first 20 period

[12]During a crisis (z_C and z_R) the capital requirement is reduced to the Basel III minimum requirement of 4.5 %. In state z_C return on lending is negative and equity is depleted. In contrast, during recovery state z_R return on lending is positive such that equity can be rebuilt using retained earnings while the capital requirement remains low.

after the crisis (see Panel (b)). This triggers an adverse dynamic multiplier effect once the crisis state is left: since equity is low the capital constraint is binding and loan supply is low. The low loan supply slows down equity build-up as return on lending is positive after the crisis, which makes the capital constraint bind longer.

All in all, the recovery state gives the bank leeway to restore equity before the tighter, normal times capital requirement is back in place. However, if the recovery phase is short, the tighter capital requirement may be in place before the bank has restored sufficient equity, thereby constraining loan supply. In this case it can not fully benefit from higher returns on lending. To insure against this possibility, the bank has an incentive to hold higher equity in normal times, such that, for the expected crisis duration and for the expected ratio of z_C/z_R, equity is sufficiently high to avoid being capital constrained during normal times when lending is highly profitable.

Figure 5: Recovery Expectation, $\bar{\varphi} = 7\,\%$

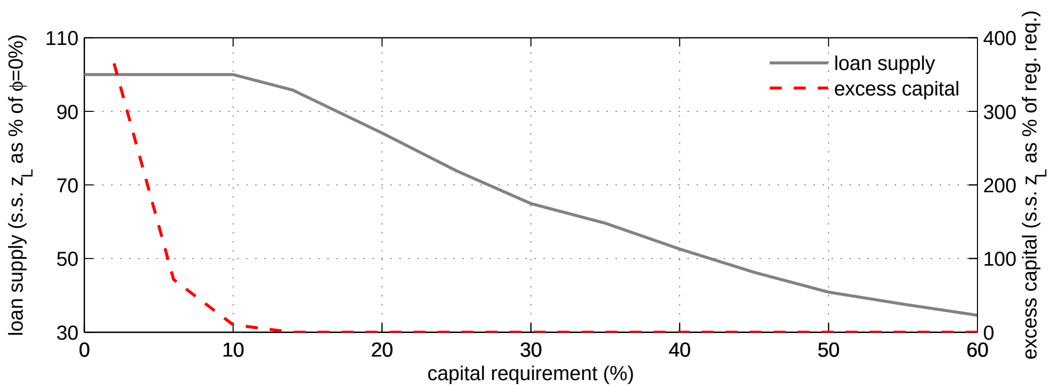


5.1.2 Capital regulation and equilibrium loan supply

The reduction of excess capital holdings in response to tighter capital requirements $\bar{\varphi}$ has also implication for bank's optimal equilibrium loan supply.

Figure 6 shows z_L steady state loan supply (relative to $\bar{\varphi} = 0\,\%$ s.s. loan supply) and excess capital holdings (relative to capital requirement) as a function of capital requirement $\bar{\varphi}$. All other parameters remain at their baseline calibration (Table 3). As discussed above excess capital holdings are decreasing monotonically in $\bar{\varphi}$. Up to $\bar{\varphi} = 11\,\%$ the bank hold excess capital, such that equilibrium loan supply is independent of $\bar{\varphi}$. For $\bar{\varphi} > 11\,\%$ the capital requirement is binding such that equilibrium loan supply decreases in $\bar{\varphi}$. Therefore, the calibrated bank is sufficient impatient such that it does not find it optimal to accumulate enough equity to keep equilibrium loan supply constant at high $\bar{\varphi}$. As a result, in our model the effect of capital regulation on loan supply is not strictly monotonic. For low $\bar{\varphi}$ the bank's self-insurance motive induces excess capital holdings. As long as the requirement is not binding, an increase in $\bar{\varphi}$ reduces excess equity holdings but has no effect on loan supply.

Figure 6: Lending and Capital Regulation



Notes: Loan supply and excess capital correspond to z_L steady state values.

5.2 Stress test in a structural model

A single bank. We start by conducting a stress test for a single bank using the calibrated parameters from Table 3. The aim is to study bank robustness to a fixed and calibrated stress scenario, as in standard stress tests, which do not consider probabilistic stress scenarios. We consider three different stress scenarios. Each scenario has a duration of three years and we fix the path of the aggregate state z to a deterministic sequence of z_C and z_R in the simulation.[13] We consider a mild scenario with one year in crisis state z_C and two years of recovery z_R, a severe scenario with

[13]Note that fixing a z-path does not imply that the bank has perfect foresight about this path. It will still hold rational expectations given the switching probabilities $\mathcal{F}(z', z)$. However, for a fixed crisis path there is no exit probability. A given crisis path either leads to exit or not.

two years in crisis and one year of recovery and a double-dip scenario with 6 quarters in state z_C, one quarters recovery z_R and one quarter in z_L and again 4 quarters in z_C. For each scenario the bank enters the stress from the stationary equilibrium of z_L.

We find that the calibrated bank does not exit in neither of the three scenarios. Figure 7 shows the predicted equity and loan supply paths for the three scenarios. Under the mild and severe scenario, equity reaches is trough during the last crisis state z_C and gradually starts increasing again as soon as the recovery state is reached. Under the double-dip scenario, equity recovers marginally during the brief recovery, but is then depleted again as the economy return to crisis. Our model predicts the bank to be very robust. It would only choose to exit if the single crisis would last for 40 quarters, or - under the double dip-scenario - the second crisis would last for 20 quarters.

Figure 7: Stress Test Projections

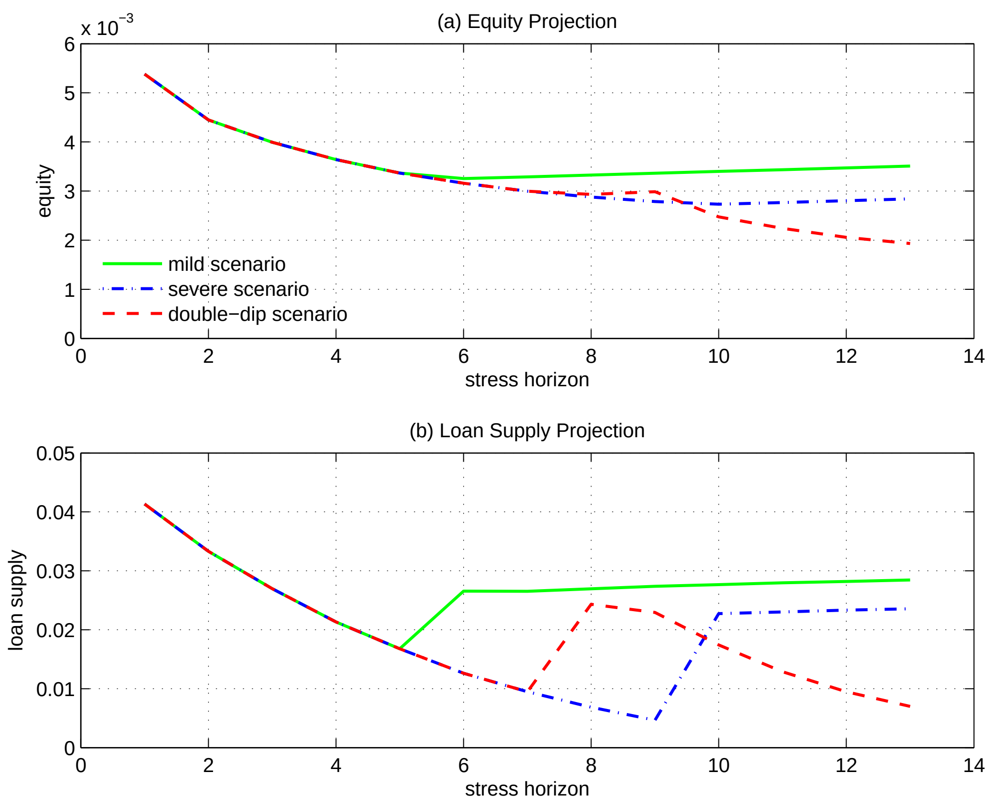


Table 6(a) summarizes further key results. Given that non-performing loans increase strongly to 14 % in the crisis state, even under the mild scenario the bank suffers substantial equity losses, which can only partially recovered during the recovery phase. Crucially, the bank deleverages during stress.

Therefore, the capital ratio increases during the crisis state, such that the crisis capital requirement is not binding. Only after the crisis state is left the bank levers up again to rebuild equity through retained earnings from return on lending.

To highlight the flexibility of our structural stress testing framework relative to state-of-the-art approaches, Table 6(b) shows stress test results for the same bank but for a counterfactual capital requirement of $\bar{\varphi} = 4.5\,\%$. The looser regulated bank is less robust to stress. Equity losses a higher for all three scenarios, as the bank is leveraged higher than under the $\bar{\varphi} = 13\,\%$ requirement. In the double-dip scenario the bank chooses to exit during the second quarter of the second crisis, leading to a full loss of equity for the financial institution. After the crisis state is left, the bank can lever up faster as it only has to satisfy a 4.5 % normal times requirement.

Table 6: Stress Test Results

	(a) $\bar{\varphi} = 13\,\%$			(b) $\bar{\varphi} = 4.5\,\%$		
stress scenario	mild	severe	double-dip	mild	severe	double-dip
(i) Equity reduction						
maximum	−39.5 %	−51.3 %	−64.1 %	−52.2 %	−64.4 %	−100 %
end of scenario	−34.8 %	−50.6 %	−64.1 %	−46.1 %	−61.8 %	−100 %
(ii) Loan reduction						
maximum	−59.4 %	−94.6 %	−83.0 %	−62.1 %	−90.1 %	−100 %
end of scenario	−31.2 %	−45.6 %	−83.0 %	−25.9 %	−17.7 %	−100 %
(iii) Capital Ratio						
at trough	20.1 %	59.8 %	31.6 %	13.6 %	39.7 %	20.4 %
end of scenario	12.3 %	12.1 %	27.6 %	7.3 %	4.7 %	–
Exit	No	No	No	No	No	Yes

Heterogeneous banks. As highlighted in Sections 4 heterogeneity in balance sheets upon stress entry affects bank behavior during stress. To see how this affects outcomes in a multi-bank stress test, we now consider a group of heterogeneous, independent banks.

We conduct a stress test on a group of 55 disjunct banks. Banks are identical except for their discount factor β, which we take to be equidistantly spaced from the interval [0.9785, 0.9955].[14] We

[14]Note that except for the bank with the baseline $\beta = 0.9901$, the other banks are not calibrated to Norwegian Banking Groups. Therefore, our panel is not representative for the Norwegian banking industry and should only be

solve for the policy functions for each β but for simplicity maintain the assumption that each bank supplies loan conditional on the assumption that all other banks maintain a constant lending rate (see Equation (3.3)). Thus, we abstract from any strategic interactions and general equilibrium effects. The different discount factors induce heterogeneity in risk–taking behavior and in stress dynamics. To illustrate the role of exit on the stress outcomes, we choose a longer stress horizon than for the single bank: the group of banks enters a stress scenario of four years in the crisis state z_C.

The top row of Figure 8 shows the ex-post equity and equity loss distribution. Panel (a) shows that 22 % of banks will choose to exit during the stress horizon (12 out of 55 banks), while 78 % of banks remain in the market, suffering equity losses between 39 % and 59 % (conditional on survival the mean equity loss is 53 %). The banks that exit are low β-banks, which put less weight on future dividend payments when considering the exit option. In addition, these banks are more likely to be ex-ante capital constrained, as their costs of capital are higher. In contrast, the patient high β-banks hold excess capital even at the a capital requirement of $\bar{\varphi} = 13\,\%$ (the bank with $\beta = 0.9955$ holds a z_L capital ratio of 17 %). The vertical black line in Panel (a) indicates the endogenous equity threshold (induced by the ratio of continuation value to exit value) below which the banks prefer to exit. For the unconditional distribution the mean equity loss is 63 % (as indicated by the red line in Panel (b)).

To highlight the effects of exit on stress outcomes, the bottom row of Figure 8 shows ex-post equity distributions for a simulation where we counterfactually ignore exit. Whenever the bank would choose to exit, we ignore this and update the security state a according to $b' = -\pi' \Rightarrow a' = A + (1 + r^b)\pi'$, i.e. assuming zero dividend. Consequently, banks continue independently of their equity position, which, as in standard stress tests, could potentially turn negative.

Endogenous exit induces an extensive margin to stress testing, as banks drop out of the market, liquidate assets and cease providing loans. This has implications for the predicted consequences of stress for the real economy, the bank and the regulator. First, for the real economy, bank exit reduces the number of banks that supply loans during and immediately after the stress episode. In our numerical example the simulations without exit predict a reduction of aggregate loan supply immediately after the crisis of 52 %, whereas the simulations with exits predict a reduction of 59 %.

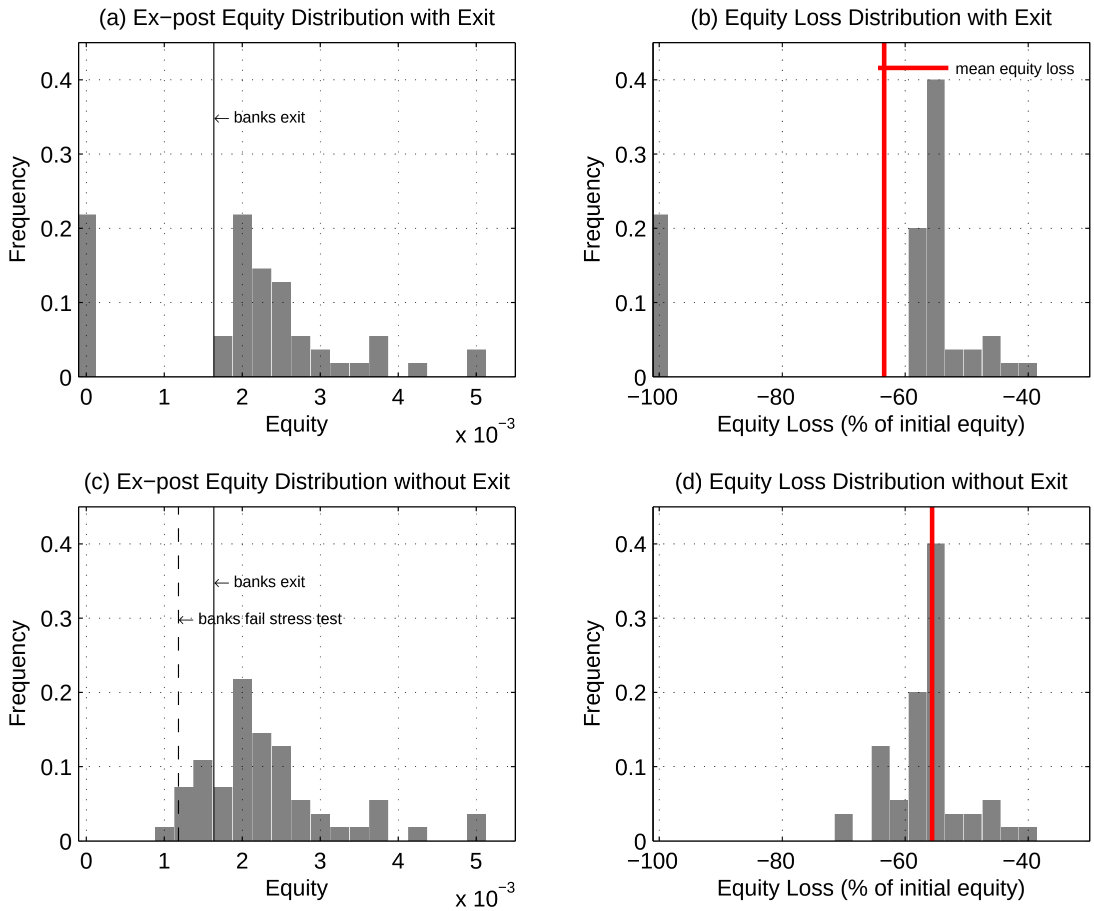
considered an illustrative example.

Conditional on survival the reduction is only 51%. Therefore, in the framework with endogenous exit 14 % of the loan supply reduction is attributed to the extensive margin of bank exit.[15]

Second, for the bank, exit affects the shape of the ex-post equity distribution. Upon exit, bank assets are liquidated and paid to equity holders. This leads to a complete loss of equity for the institution. The model without endogenous exit cannot account for the full loss of equity since it exogenously iterates equity forward over the stress horizon. As a result, equity losses biased downwards. This can be seen in Panels (b) and (d) of Figure 8. In our numerical example the unconditional mean equity loss with exit is 63 % and 55 % in the model without exit. Therefore, not allowing for exit underestimates the expected equity losses for institutions that would prefer to exit.

[15] Our partial equilibrium model neglects strategic interacts between banks. In the long run surviving banks can take over customers from the failing banks. However, if banks engage in relationship lending, taking over customers induces costs on borrowers due to switching costs (see for example Kim, Kliger, and Vale, 2003).

Figure 8: Effect of Endogenous Exit



Notes: Stress test for a panel of 55 banks with heterogeneous discount factor β and four year stress scenario of 16 z_C periods. Solid black line indicates endogenous equity threshold at which banks exit. Dashed black line indicates exogenous threshold at which bank are labeled as failing stress test.

Third, for regulators, exit induces an endogenous equity threshold below which banks exit and thus fail the test.[16] In contrast, state-of-the-art stress tests assume an exogenous hurdle rate. This harbors the possibility of committing an error of second type if the null that a bank is well capitalized is falsely accepted. To see this, consider Panels (a) and (c) in Figure 8. The solid vertical line indicates the lowest equity position with which a bank did not exit. The dashed line in Panel (c) is our model analogue to an exogenous threshold below which we label a bank as failing the stress test. We set this threshold to correspond to the equity level necessary for the bank in the group with the lowest z_L steady state loan supply, $\bar{L}^{z_L}$, to be able to maintain this

[16]Note that if the bank could not satisfy the crisis capital requirement it would face a penalty. Thus, on equilibrium path the capital requirement is never violated as the bank exits prior to violating the constraint. Therefore, no exit always implies satisfying the capital constraint.

loan supply at the crisis capital requirement of 4.5 %, i.e. hurdle rate $= 0.045 \times \min_{i\in\mathcal{I}}\{\bar{L}_i^{z_L}\}$. This allows banks with higher ex-ante loan supply to deleverage, while imposing an upper bound on aggregate deleveraging.[17] Banks with equity levels below this threshold are not able to maintain the minimum normal times loan supply such that they would have to deleverage stronger than deemed appropriate, e.g. because of macroeconomic concerns such as spillover effects to the real economy.

Given this exogenous threshold, only 2 % of banks in the panel (1 out of 55) would fail the stress test. The equity distribution in Panels (a) and (c) to the right of the solid vertical line are identical, since in this equity region no banks exit. If, as in this example, the exogenous threshold lies below the endogenous threshold, there is a region in which banks are passing the stress test, which would in fact have preferred to exit if possible. According to the exogenous exit threshold 12 banks decide to exit, but only 1 banks is labeled as failing the stress test according to the exogenous threshold, which leaves 11 banks (20 %) labeled as passing the test that would have preferred to exit, if they were allowed to. Table 7 summarizes the quantitative stress outcomes for the simulations with and without endogenous exit choice.

Table 7: Exit Decision and Stress Test Results

Fraction of banks		Δ
. . . (a) endogenously exiting	22 %	
. . . (b) failing stress test	2 %	−91 %
. . . (c) passing but exiting	20 %	
Ex-post aggregate equity		
. . . (a) exit	−63 %	
. . . (b) no exit	−55 %	−13 %
Ex-post aggregate loan supply		
. . . (a) exit	−59 %	
. . . (b) no exit	−52 %	−12 %

5.3 Comparison with a stylized non-structural stress test

Often regulators focus on credit sustainability during stress to limited the spill-over of financial turmoil to borrowers through deleveraging (for a recent example see Bank of England, 2015). These

[17] The bank with the highest ex–ante loan supply is allowed to deleverage by 51 % conditional on this threshold.

scenarios are designed to be conservative in the sense that they reverse-engineer the ex-ante equity level necessary for banks not drop below the exogenous equity threshold under the constant loan supply assumption despite negative return on lending. For the 2011 and 2014 euro-area-wide stress test banks' portfolio size was set to remain constant during the stress horizon (European Banking Authority, 2011, 2014). Hirtle, Kovner, Vickery, and Bhanot (2014) assume that banks' balance sheets continue growing by 1.25 % per quarter and that portfolio shares remain constant under stress. Since there is no regulatory constraint (yet) that prevents banks from deleveraging, these stress scenarios overestimate equity losses.

As our model is based on first principals, it can be used as a benchmark against which the quantitative implications of exogenous behavioral assumptions used in state-of-the-art stress testing can be compared. In particular, we use our model to quantify how conservative the assumption of constant stress loan supply is in terms of exit decision, equity and loan supply. This is important for regulators as it contributes to quantify potential burdens of raising costly equity when banks are required to re-capitalized based on these conservative stress test results.

To this end, we use the same group of banks as in Section 5.2, which are heterogeneous in their discount factor β. We solve for their policy functions, once for our baseline model and once for the case where we impose an additional constraint on lending at the bop optimization problem (2.14): during the stress states z_C and z_R loan supply must not drop below unconstrained z_L steady state lending. This rules out deleveraging during crisis relative to unconstrained z_L steady state. Note that this way banks anticipate that once a crisis hits they will not be able to cut lending. The group of independent banks is send into a three quarter crisis of three consecutive crisis states.[18]

Table 8 shows key stress test results for unconstrained and constrained lending behavior. First, it is evident that the exogenous assumption affect ex-post equity distribution strongly (Panel (a)). That is, it not only affect exit decision but also the equity distribution conditional on staying in the market. Under unconstrained behavior 20 % of the banks choose to exit. In contrast, under constrained behavior exit triples to 64 % during the stress horizon. The reason is that without being able to deleverage, the banks experience faster depletion of equity as return on lending is negative. As discussed in Section 4 this leads to faster exit. In addition, knowing that they are

[18]Sending the panel of banks into the same 16 quarter stress scenario as in Section 5.2 would lead to 100 % exit under constrained lending as banks are exogenously required to suffer high losses on non-performing loans.

not able to deleverage during the entire crisis period depresses to continuation value, as if the crisis persists next period, equity will be much lower than today such that exiting next period is much less worthwhile. This also increases exit probability. Under constrained behavior the unconditional (conditional) mean of equity losses increases by 86 % and 72 %, respectively.

Second, the effect of constant loan supply assumption on aggregate lending is ambiguous (Panel (b)). On one hand, banks that stay in the market are supplying more loans under constrained than under unconstrained behavior. However, since more banks are exiting the market, total aggregate lending during and immediately after the stress episode may be lower than under unconstrained deleveraging behavior. Which effect dominates depends on the degree of optimal deleveraging. If optimal deleveraging is low, the assumption of constant loan supply raises exit rates only weakly, such that the first effect dominates. If, however, banks strongly deleverage under unconstrained behavior, exit rates also increase strongly due to the exogenous behavioral assumption and the effect of the extensive margin dominates. For our numerical example optimal deleveraging is such that aggregate loan supply is almost the same under unconstrained and constrained behavior. However, on the intensive margin there is substantial difference: under constrained behavior mean loan supply conditional on staying in the market is 118 % above unconstrained loan supply. The effect on the intensive margin is compensated by a strong reduction of loan supply on the extensive margin: the fraction of banks exiting increases under constrained behavior by 218 %.

Table 8: Quantitative Implications of Constant Loan Supply

	unconstrained behavior	constrained behavior	Δ
Exit	20.0 %	63.6 %	+218 %
(a) Equity reduction (mean)			
unconditional	−44.4 %	−82.7 %	+86 %
conditional on staying	−30.5 %	−52.4 %	+72 %
(b) Loan Supply			
aggregate	0.8552	0.8462	−1 %
mean conditional on staying	0.0194	0.0423	+118 %

6 Conclusion

We have proposed a structural banking model for microprudential stress testing. We derived the behavior of banks during stress as the endogenous outcome of a bank's dynamic optimization problem, including an endogenous exit decision. The bank is subject to regulatory and technological occasionally binding constraints and aggregate uncertainty.

We have established three main results: First, we showed that in this environment the bank has an incentive to hold a buffer stock of capital even in excess of regulatory requirements to reduce the likelihood of exit and being capital constrained. Excess capital holdings are decreasing in the capital requirement $\bar{\varphi}$. In our calibrated model banks are holding capital slightly above the capital requirement of 13%. However, when we counterfactually set the capital requirement to 0 % banks would still hold 8.8 % in capital. We also showed that this incentives for self-insurance is decreasing in the capital requirement, such that for high enough requirement excess capital holdings are zero and loan supply becomes decreasing in the requirement.

Second, we used the endogenous exit probability as a novel, forward looking stress test metric when assessing the sufficiency of banks equity holdings under counterfactual scenarios. For a big Norwegian bank the exit probability is 3 % for the probabilistic stress scenario and can increase to 22 % for the minimum capital requirement of 4.5 %. The bank does not exit during fixed-duration three year stress scenarios with different severity. We showed that looking only at equity shortfalls below an exogenously set capital threshold to measure bank health can be misleading if the bank prefers to exit above the equity threshold. Since exit leads to full loss of equity for the financial sector, stress testing frameworks that does not allow for endogenous exit may lead to bias biased projections of ex-post equity distributions.

Third, we have used our structural model as a benchmark to evaluate the implications of a standard assumption often imposed in state-of-the-art stress test: constant loan supply during stress. We showed for a panel of heterogeneous banks that this deviation from optimal deleveraging behavior increases exit rates from 20 % to 64 %. In addition, under this assumption ex-post aggregate loan supply can be lower than under optimal deleveraging since more banks will exit during the stress episode.

The project is ongoing research. The results presented here stem from a one-sector version of our model without external funding fluctuations. Although this is sufficient for outlining the basic mechanisms at work in a structural stress test and for contrasting to reduced-form stress test, it still misses some features we deem important for stress testing. In the full-fledged two-sector version, with heterogeneous retail and C&I sector regarding loan demand and maturity, the bank has an additional dimension along which it can optimize: the loan portfolio composition. Moreover, although we take maturity to be parametric, since bank chooses loan portfolio composition optimally, the aggregate portfolio maturity will be endogenous. This allows us to stress test banks with different portfolio maturities and to study optimal loan portfolio adjustment during stress. In addition the full-fledged model allows us to study optimal bank responses to changes in risk-weights between sectors. Idiosyncratic fluctuations of external funding allow us to add an additional dimension to the stress scenario. For example, we can study robustness of banks, which rely to different degrees on external funding.

References

Acharya, V., R. Engle, and D. Pierret (2014): "Testing Macroprudential Stress Tests: The Risk of Regulatory Risk Weights," *Working Paper, NYU Stern School of Business.*

Acharya, V., I. Gujral, N. Kulkarni, and H. S. Shin (2011): "Dividends and Bank Capital in the Financial Crisis of 2009-2009," *NBER Working Paper 16896.*

Allen, F., and D. Gale (2004): "Competition and Financial Stability," *Journal of Money, Credit and Banking*, pp. 453–480.

Bank of England (2015): "The Bank of England's approach to stress testing the UK banking system," October, Bank of England.

Barro, R. (2006): "Rare Disasters and Asset Markets in the Twentieth Century," *Quarterly Journal of Economics*, 121(3), 823–866.

Barro, R., and J. Ursua (2008): "Macroeconomic Crises since 1870," *Brookings Papers on Economic Activity.*

Basel Committee on Banking Supervision (2010): "Guidance for national authorities operating the countercyclical capital buffer," *Bank for International Settlements Communications.*

——— (2015): "Revisions to the Standardised Approach for credit risk," *Consultative Document.*

Berry, S., J. Levinsohn, and A. Pakes (1995): "Automobile Prices in Market Equilibrium," *Econometrica*, 63(4), 841–890.

Bianchi, J., and S. Bigio (2014): "Banks, Liquidity Management and Monetary Policy," *Working Paper.*

Board of Governors of the Federal Reserve System (2013): "Dodd-Frank Act Stress Test 2013: Supervisory Stress Test Methodology and Results," Discussion paper, Board of Governors of the Federal Reserve System.

Borio, C., M. Drehmann, and K. Tsatsaronis (2012): "Stress-tesing macro stress testing: does it love up to expectations?," *BIS Working Papers No. 369.*

Borio, C., and P. Lowe (2002): "Assessing the Risk of Banking Crises," *BIS Quarterly Review*.

Boyd, J., and G. D. Nicolo (2005): "The Theory of Bank Risk Taking and Competition Revisited," *Journal of Finance*, 6(1), 1329–1343.

Burrows, O., D. Learmonth, and J. McKeown (2012): "RAMSI: a top-down stress-testing model," *Bank of England Financial Stability Paper No. 14*.

Corbae, D., and P. D'Erasmo (2014): "Capital Requirements in a Quantitative Model of Banking Industry Dynamics," *Working Paper, University of Wisconsin-Madison*.

Covas, F. B., B. Rump, and E. Zakrajcek (2014): "Stress-Testing U.S. Bank Holding Companies: A Dynamic Panel Quantile Regression Approach," *International Journal of Forecasting*.

De Nicolo, G., A. Gamba, and M. Lucchetta (2014): "Microprudential Regulation in a Dynamic Model of Banking," *The Review of Financial Studies*.

Dick, A. (2008): "Demand Estimation and Consumter Welfare in the Banking Industry," *Journal of Banking & Finance*, 32, 1661–1676.

Egan, M., A. Hortacsu, and G. Matvos (2015): "Deposit Competition and Financial Fragility: Evidence from the US Banking Sector," *Working Paper*.

Elizalde, A., and R. Repullo (2007): "Economic and Regulatory Capital. What is the difference?," *International Journal of Central Banking*, 3(3), 87–117.

European Banking Authority (2011): "Overview of the EBA 2011 banking EU-wide stress test," Discussion paper.

——— (2014): "Methodological Note EU-wide Stress Test 2014," Discussion paper.

Foglia, A. (2009): "Stress Testing Credit Risk: A Survey of Authorities' Approaches," *International Journal of Central Banking*, 5(3), 9–45.

Hirtle, B., A. Kovner, J. Vickery, and M. Bhanot (2014): "The Capital and Loss Assessment under Stress Scenarios (CLASS) Model," *Federal Reserve Bank of New York Staff Report No. 663*.

Kim, M., D. Kliger, and B. Vale (2003): "Estimating Switching Costs: The Case of Banking," *Journal of Financial Intermediation*, 12(1), 25–56.

Laeven, L., and F. Valencia (2012): "Systemic Banking Crises Database: An Update," *IMF Working Paper 12/163*.

Raknerud, A., and B. H. Vatne (2013): "The relations between bank-funding costs, retail rates, and loan volumes-Evidence form Norwegian microdata," *SSB Discussion Papers No. 742*.

Tauchen, G., and R. Hussey (1991): "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset pricing Models," *Econometrica*, 59(2), 317–396.