# Segmentation of Information and the Credit Market

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## 1 Introduction

The market for unsecured credit and, in particular, credit card debt has undergone dramatic changes in the last few decades.[1] Since the 1970s, credit card balances and transaction volume have grown at annual rates exceeding 10%, outpacing the growth of the US economy by a large margin (Evans and Schmalensee (2005)). The importance of revolving credit suggests that credit cards have been increasingly used as a means of borrowing, rather than just a means of payment.[2] At the same time, the average interest rate has dropped, while the variance of interest rates on offer has increased (Livshits, MacGee, and Tertilt (2014)).

One of the widely cited reasons for the rapid growth of the credit card industry has been the information technology revolution. Improvements in data collection, storage and processing have led to improvements in the efficiency of processing applications (Mester (1997)) and facilitated the widespread adoption of the use of automated credit scoring models in evaluation of the riskiness of potential borrowers.[3] These new techniques made it easier to evaluate risk and to target loans appropriately to households.

During the same time period, however, the US also experienced dramatic increases in bankruptcy and delinquency. Since the 1990s the bankruptcy rate (bankruptcy relative to

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[1] For all practical purposes, unsecured debt is the same as credit card debt. Livshits, MacGee, and Tertilt (2010) estimate that credit card debt constituted over 95% of all unsecured borrowing by 1995.

[2] Revolving credit (mostly outstanding balances on credit cards) relative to disposable income in the US reached a record-high 9% mark in the early 2000s and continued to grow throughout most of the decade.

[3] See Livshits, MacGee, and Tertilt (2014) and references therein.

working age population) has doubled and the *chargeoff rate* (the fraction of debt charged off due to delinquency) has substantially increased.[4]

In this paper we examine theoretically and quantitatively the relationship between, on the one hand, the greater ease of risk evaluation and, on the other, the increase in unsecured credit, bankruptcy rates and charge-off rates. Our theoretical model has three key features. First, borrowers are heterogeneous with respect to their repayment probability. Second, to identify borrowers of a particular type, a lender has to pay an up-front screening cost. Third, the market for unsecured credit is segmented by borrower type and imperfectly competitive. In more detail, each segment consists of a particular type of borrower and the lenders who have paid the screening cost to identify that type. Within the market lenders compete by offering a bundle which consists of the loan size and interest rate. The competition among lenders is modeled to occur as in a directed search (or, competitive search) market.

These features lead to natural separation of the market into borrowers who have access to unsecured credit and those who do not. Furthermore, the amount of credit that is available and the terms of trade are endogenous and depend on how costly it is for lenders to enter each borrower's market. In equilibrium, there is greater entry of lenders in the segment with high-repayment borrowers, which leads to more inter-lender competition, higher loan sizes and lower interest rates. For borrowers with lower repayment probability, loan sizes are smaller and interest rates higher and there is a marginal borrower type below which the credit market is inactive. In other words, the screening cost cannot be recouped when lending to borrowers with worse lower repayment probability than the marginal type and therefore no lender enters these markets.

The model predicts that, as the screening cost declines, credit increases both at the intensive and the extensive margin. Every market that was previously active experiences increased entry of lenders which leads to greater loan sizes, lower interest rates and lower ex post profit margins for lenders. In addition, some previously inactive markets become profitable, leading to loan to risky borrowers who were previously excluded from the unsecured credit market. Therefore, the decline in screening costs leads to a large increase in the outstanding amount of credit and also in an increase of the proportion of borrowers who fail to repay their loans. Furthermore, the model is consistent with an increase in the dispersion of interest rates: the rates of good borrowers fall while risky borrowers gain access to the market at high rates. These predictions are qualitatively consistent with the data.

We believe that the model's qualitative predictions make it a promising way of interpret-

[4]These and more detailed statistics have been widely documented in the literature. See, for example, Moss and Johnson (1999) or Sullivan, Warren, and Westbrook (2000).

ing the developments in the market for unsecured credit. We are in the process of assessing these predictions quantitatively.

The paper is organized as follows. Section 2 introduces our basic framework and Section 3 provides a full characterization of the case where loan sizes are fixed. Section 4 characterizes the equilibrium for the general case where loan size is endogenous. Section 5 sketches our plans for further work.

# 2 The Model

The economy is populated by borrowers and lenders. Time is discrete and there are two periods.

Each borrower has income $y_1$ in period 1 (which we think of as being "low") and stochastic income in period 2: his income is $y_H$ with probability $p$ and $y_L$ with $1-p$. Lenders can access funds at the risk-free rate $r$ and can make loans to borrowers. A borrower cannot commit to future actions and if he defaults on a loan he incurs utility loss which is equivalent to losing proportion $d \in (0,1)$ of his income.

The income levels and $d$ are common knowledge. $p$ is distributed according to $F$.

There are $B_p$ borrowers of type $p$. There is a large (unbounded) number of potential lenders who can pay cost $k(p)$ to enter the market for $p$-borrowers where, for simplicity, we assume that $k(p) = k\ \forall p$. The number of lenders who enter the market for $p$-borrowers is denoted by $L_p$ and the borrower-lender ratio is $\theta_p = \frac{B_p}{L_p}$.

The lender entry cost represents the investment in a screening technology to identify borrowers of a particular type. In other words, lenders who have not paid the entry cost cannot distinguish borrowers' types and are swamped by bad borrowers (frauds). Therefore, such lenders cannot compete in the market for type-$p$ borrowers.

Lenders offer loan contracts to borrowers. A contract $C$ is a one period loan which consists of the size of the loan that the borrower receives in period 1, $x_1$, and the amount to be repaid in period 2, $x_2$.[5] The gross interest rate is given by $R = \frac{x_2}{x_1}$. Competition is imperfect and is modeled as a directed search market: every lender posts a contract and then every borrower decides which contract to apply for, after having observed all posted contracts.

[5] In other words, the borrower and the lender cannot write contracts contingent on the borrower's period 2 income. This can be supported by assuming that income realizations are not observable by the lender and there are only two periods.

Let $L_p(C)$ denote the measure of lenders that offer contract $C$ to type-$p$ borrowers, where $L_p(C) \leq L_p$. Let $B_p(C)$ denote the type-$p$ borrowers that apply for contract $C$ which leads to a borrower-lender ratio at contract $C$ of $\theta_p(C) \equiv \frac{B_p(C)}{L_p(C)}$. The resulting probability that the borrower receives a loan and the lender makes a loan are given by $\alpha_B(\theta)$ and $\alpha_L(\theta)$, respectively, where $\alpha_L(\theta) = \theta\alpha_B(\theta)$.

We assume that $\alpha_B(\theta)$ is decreasing and convex in $\theta$ and that $\alpha_L(\theta)$ is increasing and concave in $\theta$. Denote the elasticity of $\alpha_L(\theta)$ with respect to $\theta$ by:

$$\epsilon_L(\theta) \quad = \quad \frac{\theta\alpha_L'(\theta)}{\alpha_L(\theta)} \geq 0$$

and assume that $\epsilon_L'(\theta) < 0$ with $\epsilon_L(0) = 1$ and $\lim_{\theta\to\infty} \epsilon_L(\theta) = 0$.

The expected profit to a lender from offering contract $C$ to type-$p$ borrowers is given by the probability he makes a loan times the profit of the loan, net of the probability of non-repayment:

$$\Pi_p(C) \quad = \quad \alpha_L(\theta_p(C))\Big( - x_1 + \frac{x_2 * p}{1+r}\Big) \tag{1}$$

where we have already included the equilibrium outcome that lenders only offer contracts that are repaid in the high state:

$$u(y_H - x_2) \quad \geq \quad u\big(y_H(1-d)\big) \tag{2}$$

The borrower-lender ratio at each contract is determined by an indifference condition for borrowers:

$$\alpha_B(\theta_p(C))\Big(u(y_1 + x_1) + pu(y_H - x_2) + (1-p)u\big(y_L(1-d)\big) - \overline{U}_p\Big) \quad = \quad U_p \tag{3}$$

for all $C$, where $U_p$ is the market utility of type-$p$ borrowers and is an equilibrium object and $\overline{U}_p$ is the autarky utility:

$$\overline{U}_p \quad = \quad u(y_1) + pu(y_H) + (1-p)u(y_L)$$

A lender chooses $C$ to maximize (1) subject to constraints (2) and (3).

# 3 A simple case: fixed loan size

We start with the simple case where the size of a loan is fixed. We also simplify (with some loss of generality) by assuming that borrowers' second period utility is linear. Section 4 considers the general case.

## 3.1 Simplified notation

Assume that $x_1 = 1$ which implies that $R = x_2$. The buyer's utility conditional on receiving a loan is:

$$\begin{aligned} u(y_1+1) &+ p(y_H - R) + (1-p)\big(y_L(1-d)\big) - \big(u(y_1) + py_H + (1-p)y_L\big) \\ &= u(y_1+1) - u(y_1) - pR - (1-p)y_L d \end{aligned}$$

Define

$$\begin{aligned} u &\equiv u(y_1+1) - u(y_1) \\ G_p &\equiv u - (1+r) - (1-p)y_L d \end{aligned}$$

Therefore $u$ is the first period utility gain of the borrower from receiving the loan and $G_p$ is the surplus that is created by a loan to a type-$p$ borrower. We assume that $G_p > k$ for all $p$ so that all loans would be made under commitment. This assumption implies that $u > y_L d$ for all $p$.

## 3.2 Equilibrium for given borrower-lender ratio

In this subsection, we take as given the borrower-lender ratio $\theta_p$ in the market for type-$p$ borrowers and solve for the equilibrium. The contract that a lender offers to a type-$p$ borrower is completely characterized by the gross interest rate. The lender's profits of offering $R$ to type-$p$ borrowers are given by:

$$\Pi_p(R) = \alpha_L(\theta_p(R))\Big(-1 + \frac{Rp}{1+r}\Big) \tag{4}$$

and the constraints become:

$$R \leq y_L d \tag{5}$$

$$U_p = \alpha_B(\theta_p(R))\Big(u + p(-R) + (1-p)(-y_L d)\Big) \tag{6}$$

The Proposition describes the equilibrium interest rates, and accompanying lender profits and borrower utility, taking the borrower-lender ratio $\theta_p$ as given.

**Proposition 1** *Consider the market for p-borrowers with fixed borrower-lender ratio $\theta_p$.*

1. *The interest rate $R_p(\theta_p)$, profits for lenders $\Pi_p(\theta_p)$ and market utility for borrowers $U_p(\theta_p)$ are uniquely determined.*

2. *For $p > \frac{1+r}{y_L d}$, define $\overline{\theta}_p$ by:*

$$\epsilon_L(\overline{\theta}_p) = \frac{u-d}{u-1-r-(1-p)y_L d}$$

*We have:*

- *If $p > \frac{1+r}{y_L d}$ and $\theta_p < \overline{\theta}_p$ then in equilibrium:*

$$\begin{aligned} R_p(\theta_p) &= \frac{1}{p}\Big(\big(u-(1-p)y_L d\big)\big(1-\epsilon_L(\theta)\big)+\epsilon_L(\theta)\Big) \\ \Pi_p(\theta_p) &= \frac{\alpha_L(\theta_p)G_p}{1+r}\big(1-\epsilon_L(\theta_p)\big) \\ U_p(\theta_p) &= \alpha_B(\theta_p)G_p\epsilon_L(\theta_p) \end{aligned}$$

- *If $p > \frac{1+r}{y_L d}$ and $\theta_p \geq \overline{\theta}_p$ or if $p \leq \frac{1+r}{y_L d}$ then in equilibrium:*

$$\begin{aligned} R_p(\theta_p) &= y_L d \\ \Pi_p(\theta_p) &= \frac{\alpha_L(\theta_p)}{1+r}\Big(py_L d-1-r\Big) \\ U_p(\theta_p) &= \alpha_B(\theta_p)\big(u-y_L d\big) \end{aligned}$$

**Proof.** We first calculate the equilibrium interest rate in the market for type-$p$ borrowers, $R_p(\theta_p)$. Rearrange equation (6):

$$Rp = u-(1-p)y_L d - \frac{U_p}{\alpha_B(\theta_p(R))}$$

and introduce it into the equation that describes the lender's profits (equation (4)):

$$\begin{aligned} \Pi_p(R) &= \frac{\alpha_L(\theta_p(R))}{1+r}\big(-1-r+u-(1-p)y_L d-\frac{U_p}{\alpha_B(\theta_p(R))}\big) \\ &= \frac{1}{1+r}\Big(\alpha_L(\theta_p(R))G_p-\theta_p(R)U_p\Big) \end{aligned}$$

The lender's problem can be reformulated so that he chooses $\theta$ to maximize

$$\tilde{\Pi}_p(\theta) \;=\; \frac{1}{1+r}\Big(\alpha_L(\theta)G_p - \theta U_p\Big) \tag{7}$$

subject to $R \leq y_L d$.

The first and second derivative of profits is:

$$\begin{aligned} \tilde{\Pi}'_p(\theta) &= \frac{1}{1+r}\Big(\alpha'_L(\theta)G_p - U_p\Big) \\ \tilde{\Pi}''_p(\theta) &= \frac{1}{1+r}\Big(\alpha''_L(\theta)G_p\Big) < 0 \end{aligned} \tag{8}$$

The lender's problem is strictly concave in $\theta$. Therefore every lender takes the same action $R_p(\theta_p)$ and the borrower-lender ratio at the optimal interest rate is equal to $\theta_p$. As a corollary, profits and market utility are uniquely determined.

The optimal interest rate is characterized either by the first order conditions or by the constraint $R \leq y_L d$. Let $\hat{R}_p(\theta_p)$ denote the interest rate that solves the unconstrained lender's problem (i.e. ignoring the constraint that $R \leq y_L d$). Therefore:

$$R_p(\theta_p) \;=\; \min[y_L d, \hat{R}_p(\theta_p)]$$

To characterize $\hat{R}_p(\theta_p)$, set the derivative of lenders' profits (equation (8)) to zero:

$$U_p \;=\; \alpha'_L(\theta)G_p \tag{9}$$

Introduce equation (9) inside equation (6) and rearrange:

$$\begin{aligned} \hat{R}_p(\theta) &= \frac{1}{p}\Big(u - (1-p)y_L d - \frac{\alpha'_L(\theta)G_p}{\alpha_B(\theta)}\Big) \\ &= \frac{1}{p}\Big(u - (1-p)y_L d - \frac{\theta\alpha'_L(\theta)G_p}{\alpha_L(\theta)}\Big) \\ &= \frac{1}{p}\Big(u - (1-p)y_L d - G_p\epsilon_L(\theta)\Big) \end{aligned} \tag{10}$$

We now check whether the borrowers' incentive compatibility condition is binding. Recalling that $\lim_{\theta\to\infty}\epsilon_L(\theta) = 0$ we have

$$\lim_{\theta\to\infty}\hat{R}_p(\theta) \;=\; \frac{u-(1-p)y_L d}{p}$$

which is greater than $y_L d$ because $u > y_L d$ by assumption. Therefore

$$\lim_{\theta \to \infty} R_p(\theta) = y_L d$$

At the other extreme, $\epsilon_L(0) = 1$ so $\hat{R}_p(0) = \frac{1+r}{p}$. As a result:

$$\begin{aligned} R_p(0) &= \frac{1+r}{p}, && \text{if } p > \frac{1+r}{y_L d} \\ R_p(0) &= y_L d, && \text{if } p \leq \frac{1+r}{y_L d} \end{aligned}$$

Furthermore, recall that $\epsilon'_L(\theta) < 0$ and therefore

$$\hat{R}'_p(\theta) = -\frac{G_p \epsilon'_L(\theta)}{p} > 0$$

Taking stock, for borrowers with $p \leq \frac{1+r}{y_L d}$ the interest rate is always equal to the constraint $y_L d$ regardless of $\theta_p$. In other words, the interest rate that lenders would prefer to offer if there were no repayment constraints is so high that borrowers would prefer to default rather than repay and therefore the market is constrained. Of course, being at the constraint implies that lenders make negative profits per loan which will deter entry (see the next section).

For borrowers with $p > \frac{1+r}{y_L d}$ there is a cutoff $\overline{\theta}_p$ where $\hat{R}_p(\overline{\theta}_p) = y_L d$ such that the interest rate is $y_L d$ if there are many borrowers per lender ($\theta_p \geq \overline{\theta}_p$) and $\hat{R}_p(\theta_p)$ otherwise. The cutoff is defined by:

$$\epsilon_L(\overline{\theta}_p) = \frac{u - y_L d}{u - 1 - r - (1-p) y_L d}$$

For future reference, notice that

$$\frac{d\overline{\theta}_p}{y_L dp} > 0$$

Finally, we calculate the profits of lenders $\Pi_p(\theta_p)$ and market utility of borrowers $U_p(\theta_p)$. The lenders' profits and borrowers' market utility in constrained markets are given by:

$$\begin{aligned} \Pi_p(\theta_p) &= \alpha_L(\theta_p)\big(\frac{p y_L d}{1+r} - 1\big) = \frac{\alpha_L(\theta_p)}{1+r}\Big(p y_L d - 1 - r\Big) \\ U_p(\theta_p) &= \alpha_B(\theta_p)\big(u - y_L d\big) \end{aligned}$$

Using (9), in unconstrained markets the lenders' equilibrium profits are given by:

$$\begin{aligned}\Pi_p &= \frac{G_p}{1+r}\big(\alpha_L(\theta_p) - \theta_p\alpha_L'(\theta_p)\big)\\ &= \frac{\alpha_L(\theta_p)G_p}{1+r}\big(1-\epsilon_L(\theta_p)\big)\end{aligned}$$

The borrower's utility can be written in a similar way:

$$\begin{aligned}U_p &= \alpha_L'(\theta_p)G_p\\ &= \frac{\alpha_B(\theta_p)\theta}{\alpha_L(\theta_P)}\alpha_L'(\theta_p)G_p\\ &= \alpha_B(\theta_p)G_p\epsilon_L(\theta_p)\end{aligned}$$

Notice that in unconstrained markets there is a symmetry between borrowers' utility and lenders' profits. They are both equal to the surplus times the probability of a trade times the share captured by their side of the market: the elasticity of the probability of matching by lenders ($\epsilon_L(\theta_p)$) or its converse.

This completes the characterization of a type-$p$ market for an arbitrary number of lenders.

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## 3.3 Entry of lenders

This section endogenizes the number of lenders in the market for $p$-borrower, $L_p$, which leads to the equilibrium borrower-lender ratio $\theta_p^*$. For entry to occur, the lenders' expected profits must be weakly greater than $k$. The Proposition summarizes the results.

**Proposition 2** *No lender enters the market for borrowers with $p \leq \frac{1+r}{y_L d}$: $L_p = 0$.*

*For markets with $p > \frac{1+r}{y_L d}$ define $\hat{\theta}_p$ by the solution to*

$$\frac{\alpha_L(\hat{\theta}_p)G_p}{1+r}\big(1-\epsilon_L(\hat{\theta}_p)\big) = k$$

*and $\tilde{\theta}_p$ by the solution to*

$$\frac{\alpha_L(\tilde{\theta}_p)}{1+r}\Big(py_L d - 1 - r\Big) = k$$

*The free entry borrower-lender ratio $\theta_p^*$ is given by*

$$\begin{aligned} \hat{\theta}_p \geq \overline{\theta}_p \quad &\Rightarrow \quad \theta_p^* = \hat{\theta}_p \\ \hat{\theta}_p < \overline{\theta}_p \quad &\Rightarrow \quad \theta_p^* = \tilde{\theta}_p \end{aligned}$$

**Proof.** In markets with $p \leq \frac{1+r}{y_L d}$ the interest rate is always constrained and the lender faces a negative margin in every loan:

$$\Pi_p(\theta_p) \quad = \quad \frac{\alpha_L(\theta_p)}{1+r}\Big(py_L d - 1 - r\Big) < 0$$

Therefore $L_p = 0$ for $p \leq \frac{1+r}{y_L d}$.

In markets with $p > \frac{1+r}{y_L d}$, profits per loan are positive and entry costs can be recouped because $G_p > k$. Let $\hat{\theta}_p$ denote the the borrower-lender ratio that allows the recouping of entry costs in an unconstrained market:

$$\frac{\alpha_L(\hat{\theta}_p)G_p}{1+r}\big(1 - \epsilon_L(\hat{\theta}_p)\big) \quad = \quad k$$

If $\hat{\theta}_p > \overline{\theta}_p$ then the market is constrained. The borrower-lender ratio that allows the recouping of entry costs is $\tilde{\theta}_p$ and corresponds to offering the constrained interest rate $y_L d$:

$$\frac{\alpha_L(\tilde{\theta}_p)}{1+r}\Big(py_L d - 1 - r\Big) \quad = \quad k$$

Therefore, the free entry borrower-lender ratio $\theta_p^*$ is:

$$\begin{aligned} \hat{\theta}_p \geq \overline{\theta}_p \quad &\Rightarrow \quad \theta_p^* = \hat{\theta}_p \\ \hat{\theta}_p < \overline{\theta}_p \quad &\Rightarrow \quad \theta_p^* = \tilde{\theta}_p \end{aligned}$$

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## 3.4 Comparative Statics

We now consider some comparative statics.

#### 3.4.1 Reduction of $k$

A drop in $k$ does not change the set of markets that are active: markets with $p \leq \frac{1+r}{y_L d}$ receive no entry of lenders.

The markets with $p > \frac{1+r}{y_L d}$ remain active but $\theta_p^*$ drops, meaning that that more lenders enter regardless of whether the market is constrained or not (both $\hat{\theta}_p$ and $\tilde{\theta}_p$ fall). Furthermore, the set of markets which are constrained decreases since $\overline{\theta}_p$ is not affected.

For unconstrained markets, the increase in the number of lenders leads to a drop in interest rates.

The increase in the number of lenders leads to more loans. The charge-off rate increases moderately because most of the increase in lending occurs at the relatively low-$p$ markets (this is heuristic).

#### 3.4.2 Reduction of $r$

A reduction in $r$ increases the set of markets that are served by lenders. It also leads to an increase in the number of lenders in existing markets and a drop in interest rates in unconstrained markets. The charge off rates increase.

# 4 Endogenous loan size

We turn to the case of endogenous loan size. The lender in a type-$p$ market solves:

$$\begin{aligned}
\max_{x_1,x_2} \Pi_p(x_1,x_2) &= \alpha_L(\theta(x_1,x_2))\Big(-x_1+\frac{px_2}{1+r}\Big) \\
\text{s.t.} \qquad U_p &= \alpha_B(\theta(x_1,x_2))\Big(u(y_1+x_1)+pu(y_H-x_2)+(1-p)u\big(y_L(1-d)\big)-\overline{U}_p\Big) \\
\text{and} \quad u\big(y_H(1-d)\big) &\leq u(y_H-x_2)
\end{aligned}$$

The second constraint corresponds to $x_2 \leq y_H d$. The main difference with Section 2 is that the lender chooses two numbers ($x_1$ and $x_2$) rather than just one ($R$).

To solve this problem, we first map each two-dimensional loan $(x_1, x_2)$ into a one-dimensional utility $v$ for the borrower and profit $\pi(v)$ for the lender. We rewrite the lender's problem as:

$$\begin{aligned}
\max_{v} \Pi_p(v) &= \alpha_L(\theta(v))\pi_p(v) \\
\text{s.t.} \qquad U_p &= \alpha_B(\theta(v))v
\end{aligned}$$

We then provide a characterization of the equilibrium outcomes.

## 4.1 The loan in one dimension

We derive $x_1^*(v)$ and $x_2^*(v)$ that maximize the lender's profits while delivering utility $v$ to the borrower. This will allow us to define the lender's profits directly as a function of the borrower's utility rather than as a function of the first and second period payments. The proposition summarizes the results.

**Proposition 3** *Consider a contract that delivers utility $v$ to a type-$p$ borrower. Define $\hat{x}_1(v)$ and $\hat{x}_2(v)$ to be the solutions to:*

$$\begin{aligned}
T_1(x_1,x_2,v) &= u(y_1+x_1)+pu(y_H-x_2)+(1-p)u\big(y_L(1-d)\big)-v-\overline{U}_p=0 \\
T_2(x_1,x_2,v) &= (1+r)u'(y_H-x_2)-u'(y_1+x_1)=0
\end{aligned}$$

*and $\overline{v}$ to be the solution to*

$$\hat{x}_2(\overline{v}) = y_H d$$

*If $v > \overline{v}$ then the optimal payments are given by:*

$$\begin{aligned} x_1^*(v) &= \hat{x}_1(v) \\ x_2^*(v) &= \hat{x}_2(v) \end{aligned}$$

*If $v \leq \overline{v}$ then the optimal payments are given by:*

$$\begin{aligned} x_1^*(v) &= u^{-1}\Big[v + \overline{U}_p - \Big(pu\big(y_H(1-d)\big) + (1-p)u\big(u_L(1-d)\big)\Big)\Big] - y_1 \\ x_2^*(v) &= y_H d \end{aligned}$$

*The lender's profits when meeting a borrower are given by*

$$\pi_p(v) \;=\; -x_1^*(v) + \frac{x_2^*(v)}{1+r}$$

*and are decreasing and concave in $v$.*

**Proof.** The lender solves the following problem:

$$\begin{aligned} \max_{x_1,x_2} \tilde{\pi}_p(x_1, x_2; v) &= -x_1 + \frac{px_2}{1+r} \\ \text{s.t. } \; v &\leq u(y_1 + x_1) + pu(y_H - x_2) + (1-p)u\big(y_L(1-d)\big) - \overline{U}_p \\ \text{and } \; x_2 &\leq y_H d \end{aligned}$$

Notice that the first constraint always binds in the solution. The second constraint might bind (or not).

The Lagrangian of this problem is:

$$\begin{aligned} \mathcal{L} \;=\; & -x_1 + \frac{px_2}{1+r} + \lambda\Big[u(y_1 + x_1) + pu(y_H - x_2) + (1-p)u\big(y_L(1-d)\big) - \overline{U}_p - v\Big] \\ & +\mu\Big[y_H d - x_2\Big] \end{aligned}$$

The first order conditions are:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x_1} &= -1 + \lambda u'(y_1 + x_1) \\ \frac{\partial \mathcal{L}}{\partial x_2} &= \frac{p}{1+r} - \lambda p u'(y_H - x_2) - \mu \end{aligned}$$

Since the first constraint always binds, we know that $\lambda > 0$ in any solution. Set the first order conditions to zero and combine them to get:

$$\frac{u'(y_H - x_2)}{u'(y_1 + x_1)} = \frac{1}{1+r} - \frac{\mu}{p} \tag{11}$$

Notice that the probability of repayment $p$ does not affect the loan when the second period constraint does not bind ($\mu = 0$). The probability of repayment matters when the second period constraint binds and an increase in $\mu$ leads to a drop in both $x_1$ and $x_2$. Furthermore, the drop is larger for borrowers with lower repayment probability.

We ignore the second constraint (i.e. we set $\mu = 0$) and solve for $\hat{x}_1(v)$ and $\hat{x}_2(v)$. Afterwards check whether the constraint binds (whether $\hat{x}_2(v) > y_H d$).

The solution to the following two equations gives $\{\hat{x}_1(v), \hat{x}_2(v)\}$:

$$\begin{aligned} T_1(x_1, x_2, v) &= u(y_1 + x_1) + pu(y_H - x_2) + (1-p)u\big(y_L(1-d)\big) - v - \overline{U}_p = 0 \\ T_2(x_1, x_2, v) &= (1+r)u'(y_H - x_2) - u'(y_1 + x_1) = 0 \end{aligned}$$

Essentially, the first equation guarantees that the lender leaves no money on the table (the first constraint binds) and the second guarantees that the lender optimizes in the combination of first and second period payments.

This system can be solved by hand, although it's a bit tedious. Letting $\mathbf{x} = (x_1, x_2)$ and $\mathbf{T} = (T_1, T_2)$, we can use the implicit function theorem on $\mathbf{T}(\mathbf{x}, v) = 0$ to find $\mathbf{x}(v)$. The derivative is $\mathbf{x}'(v) = -\mathbf{T}_{\mathbf{x}}^{-1}\mathbf{T}_v$.

We have:

$$\mathbf{T}_{\mathbf{x}} = \begin{pmatrix} \frac{\partial T_1}{\partial x_1} & \frac{\partial T_1}{\partial x_2} \\ \frac{\partial T_2}{\partial x_1} & \frac{\partial T_2}{\partial x_2} \end{pmatrix} = \begin{pmatrix} u'(y_1 + x_1) & -pu'(y_H - x_2) \\ -u''(y_1 + x_1) & -(1+r)u''(y_H - x_2) \end{pmatrix}$$

which implies that

$$\mathbf{T}_{\mathbf{x}}^{-1} = -\frac{1}{DET} \begin{pmatrix} -(1+r)u''(y_H - x_2) & pu'(y_H - x_2) \\ u''(y_1 + x_1) & u'(y_1 + x_1) \end{pmatrix}$$

where

$$DET = -(1+r)u''(y_H - x_2)u'(y_1 + x_1) - pu'(y_H - x_2)u''(y_1 + x_1)$$

Furthermore:

$$\mathbf{T}_v = \begin{pmatrix} \frac{\partial T_1}{\partial v} \\ \frac{\partial T_2}{\partial v} \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

Putting thing together, we have:

$$\begin{aligned} \hat{x}_1'(v) &= \frac{1}{u'(y_1+x_1) + \frac{pu'(y_H-x_2)u''(y_1+x_1)}{(1+r)u''(y_H-x_2)}} > 0 \\ \hat{x}_2'(v) &= -\frac{1}{(1+r)\frac{u''(y_H-x_2)u'(y_1+x_1)}{u''(y_1+x_1)} + pu'(y_H-x_2)} < 0 \end{aligned}$$

If $\hat{x}_2(v) < y_H d$ then the second constraint does not bind and therefore $x_1^*(v) = \hat{x}_1(v)$ and $x_2^*(v) = \hat{x}_2(v)$. If $\hat{x}_2(v) \geq y_H d$ then the second constraint binds and therefore $x_2^* = y_H d$ and

$$x_1^* = u^{-1}\Big[v + \overline{U}_p - \Big(pu\big(y_H(1-d)\big) + (1-p)u\big(u_L(1-d)\big)\Big)\Big] - y_1$$

Define $\overline{v}$ by $\hat{x}_2(\overline{v}) = y_H d$. The the second constraint binds when $v < \overline{v}$.

Finally, the lender's profits as a function of the utility he provides to borrowers are given by:

$$\pi_p(v) = -x_1^*(v) + \frac{x_2^*(v)}{1+r}$$

From the above, it is clear that profits decrease in $v$. They are also concave in $v$ (to be completed). ■

## 4.2 Lender competition and entry

**Proposition 4** *Consider the market for p-type borrowers with given $\theta_p$.*

1. *The expected utility of borrowers $U_p(\theta_p)$ is given by the solution to:*

$$\alpha_L'(\theta_p)\pi_p\Big(\frac{\theta_p U_p}{\alpha_L(\theta_p)}\Big) + \pi_p'\Big(\frac{\theta_p U_p}{\alpha_L(\theta_p)}\Big)U_p\Big(1 - \epsilon_L(\theta_p)\Big) = 0$$

2. *The utility of the contract offered to borrowers is given by:*

$$v(\theta_p) = \frac{U_p(\theta_p)}{\alpha_B(\theta_p)}$$

*3. A lender's profits from contract with utility $v$ are given by:*

$$\pi_p(v) \quad = \quad -x_1^*(v) + \frac{px_2^*(v)}{1+r}$$

*where $x_1^*(v)$ and $x_2^*(v)$ are determined in the previous section.*

**Proposition 5** *The equilibrium borrower-lender ratio $\theta_p^*$ is determined by the solution to*

$$\alpha_L(\theta_p^*)\pi_p\Big(\frac{\theta_p^* U_p(\theta_p^*)}{\alpha_L(\theta_p^*)}\Big) \quad = \quad k$$

**Proof.** The lender's problem can be rewritten as follows:

$$\begin{aligned}\max_v \Pi_p(v) \quad &= \quad \alpha_L(\theta(v))\pi_p(v) \\ \text{s.t.} \quad & \quad \alpha_B(\theta(v))v = U_p\end{aligned}$$

As in the Section 2, we rearrange the constraint

$$v \quad = \quad \frac{U_p}{\alpha_B(\theta(v))}$$

Introduce the constraint inside the firm's profits maximize over $\theta$ (instead of $v$):

$$\max_\theta \tilde{\Pi}_p(\theta) \quad = \quad \alpha_L(\theta)\pi_p\Big(\frac{\theta U_p}{\alpha_L(\theta)}\Big)$$

The first order conditions are:

$$\begin{aligned}\tilde{\Pi}_p'(\theta) \quad &= \quad \alpha_L'(\theta)\pi_p\Big(\frac{\theta U_p}{\alpha_L(\theta)}\Big) + \alpha_L(\theta)\pi_p'\Big(\frac{\theta U_p}{\alpha_L(\theta)}\Big)U_p\Big(\frac{\alpha_L(\theta) - \theta\alpha_L'(\theta)}{(\alpha_L(\theta))^2}\Big) \\ &= \quad \alpha_L'(\theta)\pi_p\Big(\frac{\theta U_p}{\alpha_L(\theta)}\Big) + \pi_p'\Big(\frac{\theta U_p}{\alpha_L(\theta)}\Big)U_p\Big(1 - \epsilon_L(\theta)\Big)\end{aligned}$$

The second order conditions are:

$$\begin{aligned}\tilde{\Pi}_p'' \quad = \quad & \alpha_L''(\theta)\pi_p\Big(\frac{\theta U_p}{\alpha_L(\theta)}\Big) + \frac{\alpha_L'(\theta)}{\alpha_L(\theta)}\pi_p'\Big(\frac{\theta U_p}{\alpha_L(\theta)}\Big)U_p\Big(1 - \epsilon_L(\theta)\Big) - \pi_p'\Big(\frac{\theta U_p}{\alpha_L(\theta)}\Big)U_p\epsilon_L'(\theta) \\ & + \pi_p''\Big(\frac{\theta U_p}{\alpha_L(\theta)}\Big)\frac{U_p^2}{\alpha_L(\theta)}\Big(1 - \epsilon_L(\theta)\Big)^2 < 0\end{aligned}$$

Every term above is negative, which means that a lender's expected profits are strictly concave in $\theta$ and therefore there is a unique solution.

Setting $\theta = \theta_p$ determines $U_p(\theta_p)$ as a solution to setting the lenders' first order conditions to zero:

$$\alpha_L'(\theta_p)\pi_p\Big(\frac{\theta_p U_p}{\alpha_L(\theta_p)}\Big) + \pi_p'\Big(\frac{\theta_p U_p}{\alpha_L(\theta_p)}\Big)U_p\Big(1 - \epsilon_L(\theta_p)\Big) = 0$$

which in turn determines the value of the contract offered to type-$p$ borrowers $v(\theta_p)$.

Finally, free entry of lenders determines the equilibrium borrower-lender ratio $\theta_p^*$:

$$\alpha_L(\theta_p^*)\pi_p\Big(\frac{\theta_p^* U_p(\theta_p^*)}{\alpha_L(\theta_p^*)}\Big) = k$$

■

### 4.3 Implications

The amount defaulted in a $p$-market are equal to

$$D_p = (1-p)x_{2p}^*$$

The total amount defaulted in the market is:

$$D = \int (1-p)x_{2p}^* dF(p)$$

## 5 Further Work

In work to date, we analyze a stylized model in order to fully flesh out the intuition behind our mechanism.

For the next step, we are working to incorporate our screening friction in a general equilibrium incomplete market environment in the spirit of Eaton and Gersovitz (1981). The main innovation is to forego the Bertrand competition assumption in that setup and instead use our friction to model the microstructure of how loans are granted to consumers. The main exercise in the model would be modeling the effects of increased competition and extensive margin expansion of banks due to lowering the cost of screening borrowers.

The qualitative predictions of the model suggest that the fully quantitative economy with our friction has a chance of simultaneously accounting for the trends in credit use and bankruptcy that have been a challenge in the literature. First, our friction implies that information technology progress results in a 'fanning out' of the interest rates. While overall greater competition leads to better terms, the extensive margin for high risk means that the highest interest rates will be higher, consistent with evidence for the U.S of a decreasing mean interest rate but increasing coefficient of variation.[6] Second, the extensive margin expansion into lower quality borrowers gives the model a chance of accounting for a growth in the charge off rate, which has been particularly challenging for existing models.[7] The main quantitative tradeoff in getting charge off rates to go up while at the same time seeing a drop in mean interest rates is to go down is the interplay between the intensity of competition at the top (best risk borrowers) and the expansion at the extensive margin at the bottom (worst risks expansion has to dominate). The theoretical model incorporates both effects, so it remains a quantitative question of whether the model can deliver the result (although we *know* the standard framework cannot).

The second quantitative exercise we plan to perform is exploring geographical segmentation of information due to heavy banking regulation in the U.S. in the 1970s and lack of information technology int eh 1980s.

This exercise is motivated by the observation that, concurrent with the trends described before, we have seen a geographical expansion in terms of the customer base of banks and other financial institutions in terms of their loan portfolios. Amel, Kennickell, and Moore (2008) report evidence from the Survey of Consumer Finances on the distance of households' home to providers of different financial services. They find that although the deposit account services remained largely local over the years, the fraction of households using non-local financial institutions grew from 35% in 1992 to almost 57% in 2004 (23% to 43% for those using the institutions to get loans). Our view of these data is that over the years, the consumer finance market became less segmented into geographical submarkets, with more banks now competing for customers in each area. This is consistent with the evidence on a rapid growth in mail solicitations for credit cards in the U.S., as well as plummeting response rate to these solicitations on the side of consumers.[8]

We think that our friction is uniquely well equipped to explain such trends. In an extension of the benchmark, single market model, we study the effect of better information availability on the entry and competition for customers in initially segmented sub-markets. We view initial segmentation as a result of *informational segmentation*. We study an initial situation of the credit card market in the U.S. in the 1980s: there are no legal barriers to entry

[6] Reported, for example in Livshits, MacGee, and Tertilt (2014).

[7] For an evaluation of the effects of several leading explanations for the bankruptcy rise in a relatively standard framework, see Livshits, MacGee, and Tertilt (2010).

[8] Between 1990 and 2004, the number of mail solicitations rose from 1 to over 5 billion annually (source: Mail Monitor).

(post 1978 Supreme Court Marquette decision), but in each geographical area, some banks have an informational advantage in the knowledge of their prior customer base. Hence, competition is endogenously limited by the fact that non-incumbent financial institutions have poorer quality information about the local custmers. We then model the introduction of the credit score as leveling the playing field between the incumbents and the non-incumbents. The idea is that the introduction of a standardized credit score (like the FICO score), allows non-incumbents to judge the riskiness of potential borrowers which were not their prior customers, and also to compare borrowers *between* geographical areas – with new potential for geographical diversification.

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