

The Tail that Wags the Economy: Belief-Driven Business Cycles and Persistent Stagnation*

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Abstract

In the wake of the “great recession,” many economists explored new sources of business cycle fluctuations, such as news, sentiment or uncertainty shocks. But these theories have difficulty explaining why post-recession output would remain below trend long after many commonly used measures of uncertainty recovered to their pre-crisis levels. We propose a business cycle model where new information has persistent effects on real output. In our model, firms do not know the true distribution of economic shocks. Each period, they observe a new shock realization and re-estimate its distribution, just as an econometrician would. Tails of the distribution are difficult to estimate. So estimated tails risk can fluctuate greatly as new data is observed. Shocks have persistent effects because they permanently change beliefs about future realizations. Since debt payoffs are affected disproportionately by tail risk, changes to beliefs lead to large changes in the cost of issuing debt and therefore, incentives to invest. Thus, the combination of belief revisions and debt financing can amplify shocks and generate large, persistent fluctuations in investment and output.

JEL Classifications:

Keywords:

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1 Introduction

The recent “great recession” caused economists to question standard theories of business cycles. Many have explored shocks to beliefs that can trigger downturns, such as news, sentiment or uncertainty shocks. However, post- great recession output remained below trend for a prolonged period of time. This is troubling for belief-driven theories, most of which have no internal propagation mechanisms that would deliver such persistent effects¹. Their effects are only as persistent as their shocks are assumed to be. Many measures of news and uncertainty recovered quickly as the financial crisis passed. Yet, output did not follow suit. Even more importantly, if persistence is a direct result of the assumptions about the shock process, the theory doesn’t teach us anything about why some downturns are more persistent than others. So, if beliefs matter for business cycles, why did output remain depressed, long after the bad news had passed?

We propose a belief-driven business cycle model where some shocks have large, persistent effects on real output. In our model, firms do not know the true distribution of shocks to capital quality. Each period, they observe a new shock realization, add it to their data set, use a standard kernel-density estimator to re-estimate the shock distribution, and then choose investment and debt. The reason shocks have persistent effects is that once a shock is observed, it is in the agents’ data set forever after. The shock itself may pass quickly. But the observation of that event permanently alters the estimated probability of extreme events. Some types of shocks have small permanent effects and others have large permanent effects on beliefs. When the permanent effect on beliefs is large relative to the transitory direct effect of the shock, the business cycle effect is more persistent.

When firms finance capital investments with debt, it amplifies the effect of belief revisions. Debt is an asset whose payoffs are insensitive to output in most of the state space, but are very sensitive in the case of a left-tail outcome that triggers default. Thus, firms’ use of debt financing makes the economy sensitive to changes in estimated tail risk. The problem is that tail risk is incredibly difficult to estimate. The fact that we have so few tail observations makes our estimates of tail risk noisy and thus sensitive to new data. So the cost of issuing debt depends on estimated disaster risk, which in turn, is sensitive to new data. Since real investment depends on the cost of credit, the combination of debt financing and re-estimating tail risks amplifies small shocks and allows them to generate large fluctuations in investment and output.

This theory of time-varying disaster risk builds on existing models that trace out the macro consequences of an exogenous increase in disaster risk, e.g. Gourio (2012) and is similar in

¹See Backus et al. (2015) for a formal analysis of propagation in business cycle models with belief shocks.

many respects to the existing theories of shocks to beliefs that drive business cycles². Our approach based on re-estimating tails risks offers two advantages. First, our belief shocks are not exogenous. Without any discipline on the possible time-series of beliefs, many macroeconomic outcomes are rationalizable. Our agents' beliefs are the outcome of a standard kernel-density estimation using real-world data series on capital values. The second advantage is that our model explains why some shocks are more persistent than others. It helps us understand why many recessions have rapid recoveries and yet, some do not. Events that trigger larger revisions in our perception of tail risk will have more persistent effects.

The model features a continuum of firms that produce output with capital. Each period, the firms' capital is hit by aggregate and idiosyncratic shocks to the capital quality. Because the complete distribution of idiosyncratic shocks is observed every period, it is not uncertain and is common knowledge to our agents. But only one realization of the aggregate shock is observed each period. As a result, agents learn about the distribution from which the aggregate shocks are drawn slowly, one observation at a time. Given beliefs about the distribution of future capital quality shocks, each firm chooses its capital investment. That capital investment can be financed with debt. Issuing debt to finance investment earns firms a tax advantage, but also subjects them to a cost if they cannot service the debt and need to declare bankruptcy. The cost of issuing debt (the credit spread or risk premium) depends on the probability of default, which in turn, depends on the probability of an extremely low capital quality shock. Thus, when the probability of a left tail event rises, the credit spread rises, debt issuance and real investment fall, and output declines. This mechanism is not novel - see, for example, Gourio (2013). The contribution of the paper lies in the way in which it gets tail risks to fluctuate, the endogenous persistence of tail risk fluctuations and the tools for tying these fluctuations to observable data.

To determine the size and persistence of our tail risk shocks, we parameterize the model and estimate each period's perceived distribution of capital quality shocks using data on output and on the replacement and market value of the non-financial capital stock from the flow of funds data. Following the negative shock to capital values in 2008-'09, agents revise their estimates of the probability of similar shocks. This increase in tail risk triggers a decline in leverage and investment. Firms' debt load falls by about 15% in the 5 years following the financial crisis while capital falls by a cumulative 15-20%. With less capital, output falls about 10% below its pre-crisis peak. These effects are economically large, but perhaps more importantly, persistent. Despite the fact that realized defaults spike and then recover quickly, the investment and output

²Papers on news driven business cycles include papers on news shocks, such as, Beaudry and Portier (2004), Lorenzoni (2009), Nimark (ming), Veldkamp and Wolfers (2007), papers on uncertainty shocks, such as Jaimovich and Rebelo (2006), Bloom et al. (2014), Nimark (ming) and papers on higher-order belief shocks, such as Angeletos and La'O (2013) or Huo and Takayama (2015).

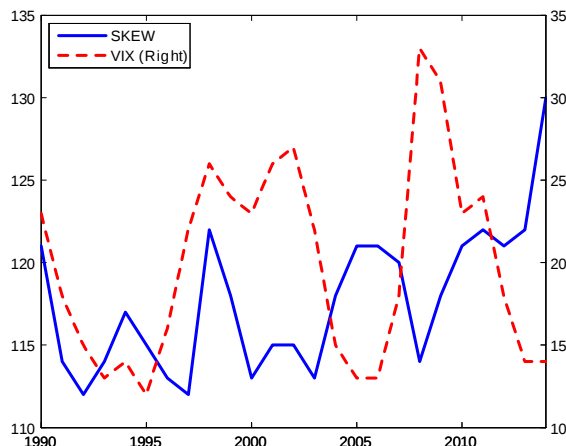
effects show almost no rebound since the crisis.

The real effects of the financial crises, as predicted by our model, are similar in magnitude to the real effects observed in the data. Hall (2014) estimates that the U.S. capital stock and U.S. real GDP are each 13% lower than where they would be if the economy had continued to grow at its pre-crisis rate of trend growth. He argues that the depressed rate of business capital formation was the single largest contributor to the persistent depressed output (often referred to as secular stagnation) in the post-crisis period. Our model explains why a short episode of unusually low capital returns can produce a protracted period of low investment and therefore output and gets the magnitude of the decline about right as well. If anything, our model overpredicts the capital and output effects of the crisis. Perhaps this over-estimate reflects the fact that policies such as government guarantees, successfully reduced the perceived risk of tail events and these policies are not incorporated in our model estimates.

To understand which assumptions are responsible for which results, we compare our economy to two benchmarks: one in which the learning mechanism is shut down and the other where learning is active, but investments are financed entirely with equity. In the first benchmark, we estimate the distribution of capital quality shocks, given all the data we have, and endow our agents with the knowledge of that distribution right from the start. This is a standard rational expectations approach. The econometrician estimates a distribution of shocks and the agents are assumed to know that distribution and regard it as truth. When we turn off belief revisions, we turn off most of the fluctuations and all of the internal propagation in the model. As a result, the contraction in economic activity brought about by a tail event is very transitory. In fact, this version of our model predicts that, despite the substantial negative shock it experienced over the great recession, the US economy should have returned to its pre-crisis level within a couple of years.

In the second model, we turn off the debt channel by turning off the tax benefit of debt. Since debt involves a risk of bankruptcy cost and has no benefit over equity, firms choose not to issue it. In this setting, an increase in estimated tail risk still decreases the expected return to capital investment and persistently reduces output. But without debt, the economy is not sensitive to this persistent tail risk effect. Output responds mostly to the transitory, direct effect of the capital quality shock. Since most of the economic response is transitory, the model predicts a quick recovery. We learn that when firms are financed with equity instead of debt, they are not sufficiently sensitive to tail risk shocks for macro outcomes to exhibit much persistence.

A small number of uncertainty-based theories of business cycles also deliver persistent effects from transitory shocks. In Straub and Ulbricht (2013) and Van Nieuwerburgh and Veldkamp (2006), a negative shock to output raises uncertainty, which feeds back to lower output, which in



turn creates more uncertainty. To get even more persistence, Fajgelbaum et al. (2014) combine this mechanism with an irreversible investment cost. The combination can generate multiple steady-state investment levels. But this uncertainty-based explanation leaves two questions unanswered. First, why were credit markets and investment the hardest hit and the most persistently impaired after the crisis? Second, why did the depressed level of economic activity continue long after the VIX and other measures of uncertainty had recovered? Our theory is based on tail risk. Like uncertainty, tail risk is a moment of the perceived distribution of outcomes. But the value of debt is particularly sensitive to this moment. While a rise in tail risk will often increase conditional variance (uncertainty), the two do not always move in lock step. Figure 1 plots the values of the VIX and the SKEW indices from 1990-2014. These are an implied volatility index and a 2-standard-deviation tail risk index, both constructed using option prices by the Chicago Board of Options. The indices show that while post-crisis uncertainty measures (VIX) recovered quickly, the tail risk (SKEW) index did not. It started rising as the VIX was peaking and continued to rise throughout the post-crisis period. Thus, financial data suggests that the effects of uncertainty-based theories should have dissipated by now. Even if some initial shocks had persistent effects, if those effects work through uncertainty, when the uncertainty has passed, the effects should have as well. The same is true of our tail risk effects. The difference is that the data reveal that tail risk has lingered, making it a better candidate for explaining continued depressed output.

Finally, this model uses a similar belief formation process to the parameter learning models by Johannes et al. (2014), Cogley and Sargent (2005) and Orlik and Veldkamp (2014), but none of these papers has a production economy. This paper adds a story about how parameter learning can generate large and persistent real effects in a standard macroeconomic setting with production and investment.

2 Model

Preferences and technology: An infinite horizon, discrete time economy has a representative household with period utility over consumption given by:

$$u(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$$

The economy is also populated by a unit measure of firms, indexed by j and owned by the representative household. Capital held by firms is subject to ‘quality’ shocks, or equivalently depreciation shocks, as in, for example, Gertler and Karadi (2011) and Gourio (2013). A firm which enters the period with capital $\hat{k}_{jt}$ and is subject to aggregate and idiosyncratic shocks (denoted ϕ_t and ϕ_{jt} respectively) has an effective capital stock of k_{jt} according to:

$$k_{jt} = \phi_t^a \phi_{jt} \hat{k}_{jt}$$

Both components are assumed to be i.i.d across time and are drawn according to cumulative distribution functions G^a and G respectively. The idiosyncratic shock is also iid across firms and satisfies

$$\int \phi_{jt} \, dj = 1$$

Information Sets: The distribution for idiosyncratic shocks, G , is assumed to be common knowledge, but agents do not know the true aggregate shock distribution G^a , but estimate it from the realized history of ϕ_t^a . The details of this estimation process are described in the following subsection. At each t , they use the empirical distribution of ϕ_t^a as the best estimate for the true distribution³. We use $\hat{G}_t^a$ to denote this estimated distribution.

Our equilibrium concept employs the anticipated utility approach, widely used in the learning literature⁴. Under this notion, agents are myopic with respect to changes in their beliefs but otherwise make optimal decisions. In other words, at each t , agents act as if the true distribution is $\hat{G}_t^a$. We maintain this assumption throughout our analysis.

Effective capital yields output according to a production function $f(k) = k^\alpha$ and depreciates at the rate δ . The beginning-of-period capital in period $t + 1$ is governed by a standard law of motion:

$$\hat{k}_{jt+1} = k_{jt}(1 - \delta) + x_{jt}$$

where x_{jt} represents new investment.

³In our numerical implementation, we fit a smooth density function to the empirical distribution. We also studied some flexible parametric specifications, which yielded similar results.

⁴See, for example, Cogley and Sargent (2008).

Credit markets: Firms have access to a competitive non-contingent debt market, where lenders offer bond price schedules. Anticipating a recursive formulation, we let $q(\hat{k}', b', S, \hat{G})$ denote the bond price as function of the firm's capital choice (at the end of the current period, i.e. before the realization of quality shocks in the following period) $\hat{k}'$, debt choice (i.e. promised future payment) b' and a vector S which collects all the relevant aggregate state variables. This notation also makes explicit the dependence of the bond price on perceived distribution $\hat{G}^a$.

There is a tax advantage associated with debt, modeled as a one-time subsidy $\chi > 1$, i.e. a firm which promises to pay b' in the following period (and chooses $\hat{k}'$) in aggregate state S receives a net payment today of $\chi q(\hat{k}', b', S, \hat{G}^a) b'$. This tax advantage creates incentives to issue debt (i.e. breaks the Modigliani-Miller capital structure irrelevance result) and is traded off against the deadweight losses associated with default to determine the optimal level of debt.

The dividend payout of firm j is

$$d_j = k_j^\alpha + k(1 - \delta) - \hat{k}' + \chi q' b' - b$$

Importantly, we do not restrict dividends to be positive, i.e. we allow firms to (costlessly) raise equity. In recursive form, the firm's problem becomes:

$$\begin{aligned} V(k, b, S, \hat{G}^a) &= \max \left[0, \max_{\hat{k}', b'} \lambda(S) d + \beta \mathbb{E} V(k', b', S', \hat{G}^a) \right] \\ \text{s.t.} \quad d &= k^\alpha + (1 - \delta) k - \hat{k}' + \chi q(\hat{k}', b', S, \hat{G}) b' - b \\ k' &= \phi^\alpha \phi \hat{k}' \\ S' &= T(S) \end{aligned} \tag{1}$$

where $\lambda(S)$ is the state price (which, in equilibrium, is the marginal utility of the representative household). The aggregate state S and the associated transition function, $T(\cdot)$, will be described later in this section. The first max operator in the objective is the firm's default decision. Let $r(k, b, S, \hat{G}^a) \in \{0, 1\}$ denote the induced default function.

Default: If the firm defaults, debt holders take over the firm. The the productive resources of the firm are sold to an identical new firm at a discount, but at a discounted price. Let $\tilde{V}(k, S, \hat{G}^a)$ denote the value of the productive resources to the new firm⁵. Creditors receive only a fraction $\theta < 1$ of this value. Since credit markets are competitive, lenders make zero

⁵Formally, $\tilde{V}(k, S, \hat{G}^a) = V(k, 0, S, \hat{G}^a)$, i.e. it is the value of an identical firm without any outstanding debt.

profits in expectation, which means the bond price function has to satisfy

$$q\left(\hat{k}', b', S, \hat{G}^a\right) = \mathbb{E} \left[r\left(k', b', S', \hat{G}^a\right) \lambda\left(S', \hat{G}^a\right) + \left(1 - r\left(k', b', S', \hat{G}^a\right)\right) \theta \frac{\tilde{V}\left(k', S', \hat{G}^a\right)}{b'} \right] \quad (2)$$

where the expectation is taken over the aggregate and idiosyncratic shocks.

Belief Updating: Finally, we describe the belief updating process. As mentioned earlier, all agents in the economy (both households and firms) do not know true distribution of the aggregate capital quality shock, ϕ_t^a . Formally, at every date t , agents construct the following kernel density estimator using the available data:

$$\hat{g}_t^a(\phi) = \frac{1}{n_t h} \sum_{s=0}^{n_t-1} \Omega\left(\frac{\phi - \phi_{t-s}^a}{h}\right)$$

where $\Omega(\cdot)$ is the standard normal density function, h is the bandwidth parameter and n_t is the number of available observations of at date t . As new data arrives, agents update their estimates, generating a sequence of beliefs $\{\hat{G}_t^a\}$.

Equilibrium: A number of features help simplify the equilibrium characterization. First, we can show that the aggregate state can be summarized by a single variable, the economy-wide capital stock $S = K = \int k_j dj$. Second, since the idiosyncratic shocks are iid, capital and debt policies are the same for all firms, i.e. $\hat{k}(k, b, S, \hat{G}^a) = \hat{k}(K, \hat{G}^a)$ and $b'(k, b, S, \hat{G}^a) = b'(K, \hat{G}^a)$.

Our anticipated utility equilibrium concept implies that allocations at each date are the outcome of a rational expectations equilibrium under the assumption that the estimated distribution at that point is the true distribution. For a given $\hat{G}_t^a$, such an equilibrium can be described in recursive form as a set of functions for (i) firm policies $r(k, b, K, \hat{G}^a)$, $\hat{k}(K, \hat{G}^a)$ and $b'(K, \hat{G}^a)$ (ii) consumption and the resulting stochastic discount factor of the representative household $C(K, \hat{G}^a)$ and $\lambda(K, \hat{G}^a)$ (iii) a bond price function $q(\hat{k}', b', K, \hat{G}^a)$ such that the policies in (i) solve (1), taking as given (ii) and (iii), the bond price function in (iii) satisfies (2) and the aggregate function in (ii) is consistent with individual choices, i.e. $C(K, \hat{G}^a) = K^\alpha + K(1 - \delta) - \hat{k}(K, \hat{G}^a)$ and $\lambda(K, \hat{G}^a) = C(K, \hat{G}^a)^{-\sigma}$.

3 Numerical Results

Our main questions are about how large and how persistent the real effects of belief shocks are. We need a quantitative model to answer these questions. In this section, we parameterize

and numerically solve the model described in the previous section. We then subject this model economy to the realized time series of capital quality shocks from US post-war data and evaluate the implications of changes in beliefs for aggregate investment, output, consumption etc.

We make a few simplifying assumptions, primarily for tractability. First, we assume risk-neutral households, i.e. $\sigma = 0$, which implies that the stochastic discount factor, λ , is constant. While this does rule out general equilibrium forces acting through interest rates, it allows for a sharp demonstration of the main forces at work. We also assume zero depreciation⁶. The values for the other parameters are assigned as follows. A period is interpreted as a year and accordingly the discount factor β is set to 0.9. The curvature of the production, α , is set to 0.33. The value for χ , the tax advantage of debt, is 1.05 and the recovery rate, θ , is set to 0.70. The idiosyncratic capital quality shocks, ϕ_{jt} are assumed to be lognormally distributed, i.e. $\ln \phi_{jt} \sim N\left(-\frac{\hat{\sigma}^2}{2}, \hat{\sigma}^2\right)$ with $\hat{\sigma}^2 = 0.001$. Table 3 contains a summary of this parameterization.

	Value	Description
β	0.9	Discount factor
α	0.33	Returns to scale
χ	1.05	Tax advantage of debt
θ	0.70	Recovery rate

Table 1: Parameter values

Measuring capital quality shocks: The next step is to construct a time series of $\{\phi_t^a\}$. We use annual data on non-financial assets of non-financial corporations in the US economy. The Flow of Funds reports published by the Federal Reserve contain two such series - one evaluated at historical cost and the other at replacement cost or market value. We interpret the latter as corresponding to effective capital. Letting P_t^k denote the nominal price of capital in t , we can then map these two series into model objects as follows⁷:

$$\begin{aligned} NFA_t^{HC} &= \text{Historical cost of non-financial assets in } t = P_{t-1}^k X_{t-1} + (1 - \delta) NFA_{t-1}^{HC} \\ NFA_t^{RC} &= \text{Replacement cost of non-financial assets in } t = P_t^k K_t \end{aligned}$$

Now,

$$\begin{aligned} \frac{NFA_t^{RC}}{NFA_{t-1}^{RC}} - 1 &\approx \ln NFA_t^{RC} - \ln NFA_{t-1}^{RC} \\ &= \ln P_t^k - \ln P_{t-1}^k + \ln K_t - \ln K_{t-1} \end{aligned}$$

⁶Our work in progress relaxes both these assumptions, which allows for a more careful calibration exercise.

⁷The measurement equations shown here are for the general case with non-zero depreciation, even though in the results presented for the special case of $\delta = 0$.

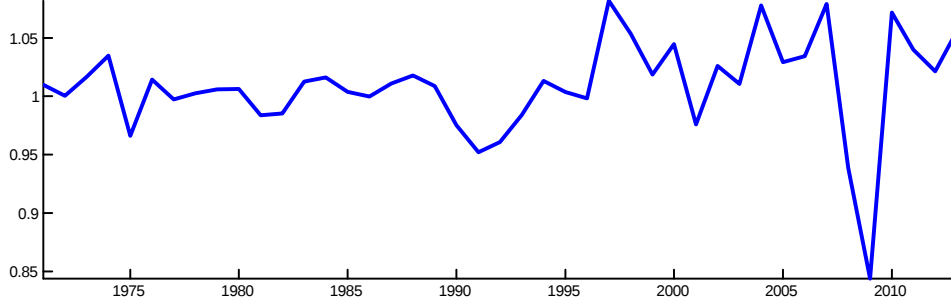


Figure 1: Time series of capital quality shocks, ϕ_t^a .

We then use the change in price index for the non-residential investment from the National Income and Product Accounts (denoted $\ln PINDX_t^{Inv}$) as a proxy for the change in the replacement cost or market price, i.e.

$$\ln P_t^k - \ln P_{t-1}^k = \ln PINDX_t^{Inv} - \ln PINDX_{t-1}^{Inv}$$

Substituting and rearranging, we get

$$\frac{K_t}{K_{t-1}} \approx 1 + \ln K_t - \ln K_{t-1} = 1 + \ln \frac{NFA_t^{RC}}{NFA_{t-1}^{RC}} - \ln \frac{PINDX_t^{Inv}}{PINDX_{t-1}^{Inv}} \quad (3)$$

Finally, from the law of motion for $K_t = \phi_t^a (X_{t-1} + K_{t-1} (1 - \delta))$,

$$\begin{aligned} \frac{K_t}{K_{t-1}} &= \phi_t^a \left(\frac{X_{t-1}}{K_{t-1}} + 1 - \delta \right) = \phi_t^a \left(\frac{P_{t-1}^k X_{t-1}}{P_{t-1}^k K_{t-1}} + 1 - \delta \right) \\ &= \phi_t^a \left(\frac{NFA_t^{HC} - (1 - \delta) NFA_{t-1}^{HC}}{NFA_{t-1}^{RC}} + 1 - \delta \right) \end{aligned}$$

Combining with (3), we get

$$\phi_t^a = \frac{\frac{K_t}{K_{t-1}}}{\frac{NFA_t^{HC} - (1 - \delta) NFA_{t-1}^{HC}}{NFA_{t-1}^{RC}} + 1 - \delta} = \frac{1 + \ln \frac{NFA_t^{RC}}{NFA_{t-1}^{RC}} - \ln \frac{PINDX_t^{Inv}}{PINDX_{t-1}^{Inv}}}{\frac{NFA_t^{HC} - (1 - \delta) NFA_{t-1}^{HC}}{NFA_{t-1}^{RC}} + 1 - \delta} \quad (4)$$

Using (4) as a measurement equation, we construct an annual time series for capital quality shocks for the US economy over the last few decades. The last panel (bottom row, right) of Figure 3 plots the resulting series. For most of the sample period, the shock realizations are

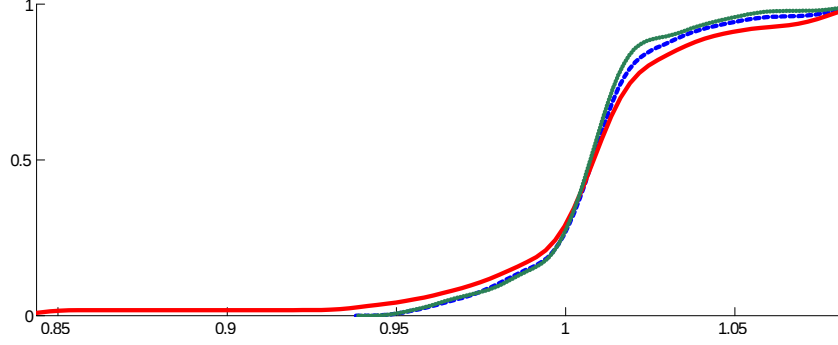


Figure 2: Estimated distribution functions $\{\hat{G}_t^a\}$. Solid = 2010, Dashed = 2005, Dotted = 2000.

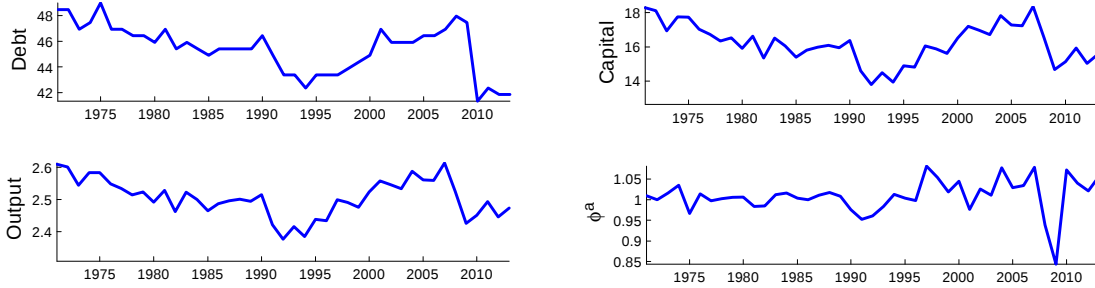


Figure 3: Time series of aggregate variables generated by the model.

in a relatively tight range around 1, but at the onset of the recent Great Recession, we saw an extreme negative realization, of about 0.85.

Model Results We then apply the kernel density estimation procedure to this time series to construct a sequence of beliefs $\{\hat{G}_t^a\}$. Figure 2 plots the estimated cumulative distribution functions from three years - 2000, 2005 and 2010. It clearly shows the significant increase in the perceived tail risk - the CDFs for 2000 and 2005 imply almost zero mass below 0.95, but after the Great Recession, agents revise their estimates and now attach a non-trivial probability to significantly worse realizations.

Next, for each year starting from 1970⁸, we solve for the value/policy functions using (1) and the associated $\{\hat{G}_t^a\}$. These are then used to construct a time series for the aggregate variables of interest, which are plotted in Figure 3.

The figure shows a gradual increase in debt, capital and output through the 1990s and early

⁸In order to avoid problems stemming from very small sample sizes, we start our numerical simulations in 1970.

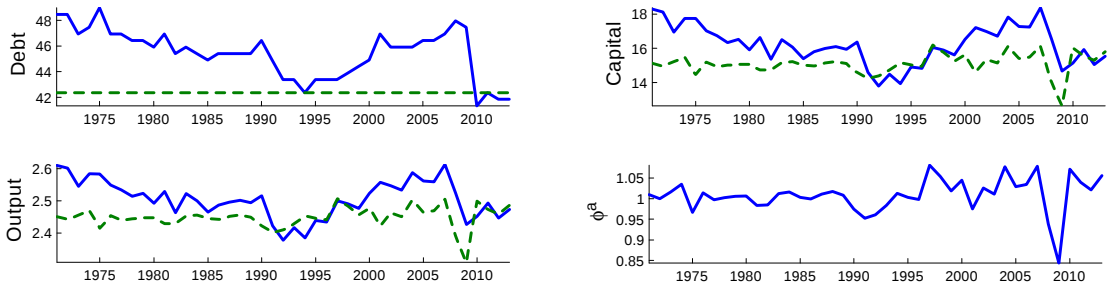


Figure 4: With fixed beliefs, investment and output recover quickly. Belief updating makes large shocks persistent. The solid line is our baseline estimate with belief updating. The dashed line is the model with fixed beliefs.

2000s, with a sharp reversal during the recent crisis. Following the large negative shock in 2009 and the associated change in beliefs, firms reduce both their debt burden and capital by about 15%. The resulting fall in output is slightly smaller, about 10%. More importantly, these effects are very persistent, with all 3 variables remaining at their new depressed levels even 5 years after the shock.

Both belief revisions and the presence of debt financing are crucial to these results. To isolate their roles, we compare our results to two alternative versions. In the first, we turn off belief revisions and in the second, we look at an economy without debt.

The Role of Belief Updating For our first counterfactual economy, we assume that, throughout the period under study, the perceived distribution of shocks is constant and equal to the one that is estimated using the entire sample. This corresponds to a standard rational expectations approach, where agents are assumed to know the true distribution. The econometrician estimates this distribution using all the available data. Figure 4 compares the outcome of this specification (the dashed line) to our baseline (the solid line). We see that the time series are much less volatile (debt, in fact, shows no variation at all). Capital (and therefore, output) remains more or less constant throughout the 90s and the early 00s, whereas the baseline model showed a gradual increase over that period. Similarly, at the onset of the Great Recession, the available stock of capital drops sharply but rebounds just as quickly. In other words, in the absence of learning, even extreme negative shocks, such as the one experienced during the recent recession, generate only very transitory effects on economic activity⁹.

⁹In fact, without curvature in the stochastic discount factor, this adjustment occurs immediately.

The Role of Debt In our second experiment, we focus on the role of debt by comparing our results to an identical economy where all investment is financed through equity. Formally, we set the tax advantage of debt to 0, which eliminates any incentive for firms to issue debt¹⁰. In this case, optimality implies the following intertemporal condition that pins down capital choice:

$$\hat{k}_t = \left(\frac{1 - \beta \mathbb{E}_t [\phi_{t+1}^a]}{\beta \alpha \mathbb{E}_t [(\phi_{t+1}^a)^\alpha]} \right)^{\frac{1}{\alpha-1}} \quad (5)$$

Note that the expectation operator is indexed by t , to reflect revisions in light of new data. In other words, even in the absence of debt, changes in beliefs affect the level of economic activity. Figure 5 plots the time path for aggregate variables for this variant of our model. The graph for capital (top, right) shows some of the qualitative features of our baseline model in Figure 3 - the gradual increase in the runup to the Great Recession followed by the sharp reversal. However, there are important differences. For one, the magnitudes of these fluctuations¹¹ are smaller. Second, in the absence of debt, the negative shock realizations of 2008-09 change beliefs, and through equation (5), reduce the optimal level of capital. However, relative to the size of the negative shock, this reduction is modest, so we see a quick rebound in the capital stock. The predicted 2014 capital stocks do not look very different from those in 2005.

Thus, in the absence of debt, the effect of the great recession shock cannot be more persistent than the shock itself. In Figure 5, the time series of output and capital look just like the capital quality shock itself. The reason for this lack of additional persistence is that most of the revisions in beliefs occur in the tails. Without debt, real economic variables are not very sensitive to tail probabilities. So, even though there are permanent changes in tail probabilities, they have small effects on the real economy, which are swamped by the transitory direct effect of the capital quality shock. This exercise reveals why increases in perceived tail risk alone cannot generate large, persistent contractions of the sort that we saw in the US economy over the last 6 years- for that, we need both belief revisions and debt financing.

4 Conclusion

No one knows the true distribution of shocks to the economy. Economists typically assume that agents in their models do know this distribution as a way to discipline beliefs. But assuming that agents do the same kind of real-time estimation that an econometrician would do is equally disciplined and more plausible. For many applications, assuming full knowledge has little effect

¹⁰A specification with non-defaultable debt leads to identical outcomes.

¹¹We are referring to percentage changes here - since the ‘steady-states’ in the two cases are quite different, levels of variables are not directly comparable.

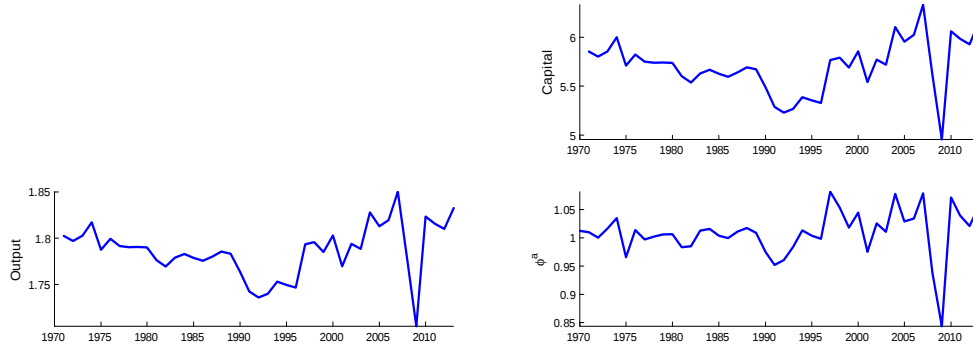


Figure 5: Time series of aggregate variables in the absence of debt.

on outcomes and offers tractability. But for outcomes that are sensitive to tail probabilities, the difference between knowing these probabilities and estimating them with real-time data can be large. The estimation error can be volatile and can introduce new, persistent dynamics into a model with otherwise transitory shocks. The essence of the persistence mechanism is this: Once observed, a shock (a piece of data) stays in one's data set forever and therefore permanently affects belief formation.

When firms finance investments with debt, they make investment and output sensitive to tail risk. Debt is an asset whose payoffs are flat throughout most of the state space, but very sensitive to the state for left-tail, default events. Therefore, the cost of debt depends precisely on the probabilities of a tail event, which are hardest to estimate and whose estimates fluctuate greatly. When debt (leverage) is low, the economy is not very sensitive to tail risk, and economic shocks will be more transitory. The combination of high debt levels and a shock that is a negative outlier makes tail risk surge, investment fall and depresses output in a persistent way.

When we quantify this mechanism and use capital price and quantity data to directly estimate beliefs, our model's predictions resemble observed macro outcomes in the wake of the great recession. These results suggests that perhaps persistent stagnation arose because, after seeing how fragile our financial sector is, market participants will never think about tail risk in the same way again.

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