

Long-Run and Short-Run Effects of Money Injections*

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Abstract

We construct a tractable model of monetary exchange with search and bargaining that features a non-degenerate distribution of money holdings in which one can study the short-run and long-run effects of changes in the money supply. While money is neutral in the long run, an unanticipated, one-time, money injections in a centralized market with flexible prices and unrestricted participation generates an increase in aggregate real balances and aggregate output, a decrease in the rate of return of money, and a redistribution of output and consumption levels across agents in the short run. Moreover, the initial impact on the price level is non-monotonic with the size of the money injection, e.g., small injections can lead to a deflation followed by inflation. We also study repeated money injections and show they can lead to higher output and higher welfare.

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1 Introduction

We develop a tractable model of monetary exchange with random matching and bargaining that features a non-degenerate distribution of money holdings and a non-degenerate distribution of prices. Our model builds on the framework of Lagos and Wright (2005) and Rocheteau and Wright (2005) in which agents trade alternatively in decentralized markets with search and bargaining and in centralized markets with competitive pricing. The model is made tractable by adopting preferences that eliminate wealth effects in order to keep the distribution of money holdings degenerate. Our model provides a simple and natural departure from this environment leading to ex-post heterogeneity in money holdings while preserving tractability. In order to illustrate some of the new insights arising from this generalization we revisit a classical question in monetary economics: the short-run and long-run effects of changes in the money supply.

We adopt the off-the-shelves monetary model of Lagos and Rocheteau (2005), LR thereafter, with a single change: we impose a finite bound, $\bar{h}$, on agents' endowment of labor.¹ We will show that whenever the feasibility constraint on agents' labor supply, $h \leq \bar{h}$, binds, wealth effects become relevant: individual real balances depend on past trading histories, value functions are strictly concave in money holdings, and the distribution of money holdings is non-degenerate. The model remains tractable and can be solved in closed form—including distributions and value functions—for a large class of equilibria. When it cannot be solved in closed form the equilibrium has a simple recursive structure allowing it to be easily computed.

The key ingredients for the tractability of the model are as follows. First, following Rocheteau and Wright (2005), there is heterogeneity in terms of agents' role in pairwise meetings: some agents are always buyers in bilateral matches in the decentralized goods market while other agents are always sellers. Sellers, who are risk-neutral in terms of their centralized-market consumption, do not hold any real balances. As a result, buyers effectively trade with a representative seller in the decentralized market. Second, the terms of trade in pairwise meetings are determined by take-it-or-leave-it offers by buyers, which simplifies the fixed-point problem when searching for an equilibrium. Third, the buyer's disutility of work in the centralized market is linear, which simplifies the policy rule for the accumulation of real balances.

We first characterize steady-state equilibria where the distribution of real balances remains constant over time. In such equilibria agents have a target for their real balances which increases with their degree of patience and the frequency of trading opportunities in the decentralized goods market. If their labor endowment is sufficiently large, agents can reach this target in a single period and, as a result, the distribution of real balances across buyers at the beginning of each period is degenerate. This special case is the one the literature has been focusing on. In contrast if the labor endowment is sufficiently small it takes $N \geq 2$ periods, where N is endogenous and depends on preferences and endowments, for an agent to reach his targeted real balances. As a result of this slow adjustment of real balances toward a target the distribution of money holdings is non-degenerate. For most of the paper we will focus on a class of tractable equilibria

¹Lagos and Wright (2005, p.467) do introduce an upper bound on hours of work, $\bar{H}$, but they restrict their attention to equilibria where $H \leq \bar{H}$ does not bind.

where buyers deplete their money holdings in full whenever they are matched with a seller. Under such trading pattern the distribution of real balances is a truncated geometric distribution with $N \in \mathbb{N}$ mass points. This heterogeneity in wealth generates a distribution of output levels and prices across matches. Equilibria where buyers do not deplete their money holdings in full in a match are not solvable in closed form but can easily be solved numerically due to the recursive structure of the equilibrium. We provide examples of such equilibria and we show that the distribution can combine mass points and continuous intervals.

We then study the transitional dynamics for allocations and prices following a one-time money injection in the centralized market. If agents can reach their targeted real balances in a single period, as in the New Monetarist model, such a money injection has no real effect since the price level adjusts proportionally to the money supply and the economy returns to its steady state instantly. Our model, by contrast, features non-trivial transitional dynamics. For tractability we start the economy at a steady-state equilibrium where the distribution of money holdings has two mass points at the beginning of the period: there are agents with low real balances, below their target, and agents holding their targeted money holdings. We show that at the time of the money injection aggregate real balances increase, i.e., the price level does not increase as much as the money supply and appears to be "sticky". The economy returns to its steady state in the following centralized market, i.e., the transition only lasts one period.

The reason for why the value of money fails to reach its new steady-state value instantly is because the centralized market cannot reshuffle the units of money among buyers in a way that preserves neutrality. Indeed, the buyers entering the competitive market with no money are constrained by their labor endowment in the laissez-faire equilibrium. Hence, if they receive a lump-sum transfer they will hold onto it in order to increase their real balances. If the value of money attains its new steady state instantly, then unconstrained buyers accumulate their steady-state target and aggregate real balances are larger than their steady-state value—a contradiction. Consequently, the value of money does not adjust in proportion to the increase in the money supply and the anticipated rate of return of money following the money injection is negative, which drives the target for real balances down. The distribution of real balances becomes less disperse in the following decentralized goods market, which raises aggregate output if the seller's production cost is strictly convex and leads to higher welfare. The effects of the money injection on individual labor supplies and output in the centralized market depend on the elasticity of the target with respect to the rate of return of money—and hence on preferences. We provide conditions under which the injection of money triggers a deflation in the short run—the value of money rises above its initial-steady-state value—and aggregate output increases. Sufficiently large money injections are always inflationary and they make the distribution of money holdings degenerate in the short run.

If the initial steady state features a richer heterogeneity, at least three mass points in the distribution of real balances, $N \geq 3$, then the real effects of a one-time money injection are long-lasting. First, the transfer of money has a direct effect on a buyer's real balances that lasts up to $N - 1$ periods, until he has reached his

target. Second, this effect is propagated through time by the fact that individual real balances depend on the past rates of return of money. For small money injections we obtain the difference equation for the value of money in closed form and we compute examples where the money injection generates several periods of positive inflation followed by a long-lasting deflation.

Finally, we study repeated money injections leading to a constant money growth rate. An increase in the money growth rate reduces the rate of return of money but it also improves risk-sharing by raising the real balances of the poorest agents. Under some conditions the positive risk-sharing effect dominates and social welfare increases. The model also has new predictions for the output effect of anticipated inflation. We show that a positive money growth rate leads to higher aggregate output when buyers are sufficiently risk averse in terms of their decentralized market consumption.

1.1 Literature review

The earlier search-theoretic models of monetary exchange by Shi (1995) and Trejos and Wright (1995) were kept tractable by restricting money holdings to $\{0, 1\}$. The model was extended by Camera and Corbae (1999) to allow for a more general support for the distribution of money holdings and it was solved numerically by Molico (2006) under the assumption that buyers make take-it-or-leave-it offers to sellers. Zhu (2005) provides an existence result for monetary steady states. Green and Zhou (1998) and Zhou (1999) study a similar environment where goods are indivisible and prices are posted by sellers. They characterize analytically a class of equilibria where all transactions occur at the same price. Menzio, Shi, and Sun (2013) consider a related environment with directed search and price posting.

Closer to what we do, Chiu and Molico (2010, 2011) adopt the alternating-market structure of Lagos and Wright (2005) where some trades take place in competitive markets and others through search and bargaining. They relax the assumption of quasi-linear preferences in order to obtain distributional effects. While the results in Chiu and Molico are numerical, we obtain a tractable model with closed-form solutions. The main two differences between our approach and the one in Chiu and Molico are as follows. First, we assume ex ante heterogeneity between buyers and sellers as in Rocheteau and Wright (2005) and LR. As a result, the only relevant distribution of money holdings is the one across buyers and this distribution affects the buyer's problem only through its first moment. Second, we adopt the fully linear specification for preferences over the centralized market good, as in LR, but we add an upper bound on labor supply. This specification implies that buyers supply their full endowment of labor until they reach their targeted real balances. Moreover, it allows us to have the LR model with a degenerate distribution and linear value function as a particular case. Zhu (2008) constructs a model with alternating market structures and general preferences and achieves tractability by assuming that agents from overlapping generations can trade at most once in the decentralized market with search and bargaining.

This model is related to our earlier work in Rocheteau, Weill, and Wong (2014) with important differences. Rocheteau, Weill, and Wong (2014) describe a competitive economy in continuous time populated with ex-ante identical agents where the idiosyncratic uncertainty takes the form of preferences shocks for

lumpy consumption. In contrast, we study a discrete-time economy with search and bargaining and ex-ante heterogeneous agents and idiosyncratic uncertainty due to random matching. These ingredients make our model more easily comparable to the New-Monetarist framework. Moreover, the use of discrete time allows us to harness the ex-post heterogeneity, thereby facilitating the study of transitional dynamics, which is a main focus of our paper. Finally, Rocheteau, Weill, and Wong (2014) provide a general characterization of their model under general preferences without restricting equilibria to the case of full depletion.

Berentsen, Camera, and Waller (2005) generalize the Lagos-Wright model by assuming two rounds of trade before agents can readjust their money holdings. This assumption generates a non-degenerate distribution of money holdings at the start of the second decentralized market. In contrast to our environment, any money injection in the centralized market is neutral. Moreover, our model generates a rich distribution of money holdings with a single round of pairwise meetings—the distribution can have any number of mass points as well as continuous intervals. Finally, the Friedman rule is optimal in Berentsen, Camera, and Waller while it is not feasible in our model. Williamson (2006) obtains short-run non-neutralities in the Lagos-Wright model by introducing limited participation while Faig and Li (2009) achieve a similar objective by adopting the Lucas signal extraction problem. In our model all agents can participate in the centralized market in all periods and information about changes in the money supply is perfect.

Wallace (1997) considers an unanticipated change of the money supply in a random matching model with $\{0, 1\}$ money holdings and shows that the short-run effects are predominantly real while the long-run effects are predominantly nominal. Jin and Zhu (2014) generalize the model by assuming a large upper bound on money holdings and by allowing lotteries to set of the terms of trade in order to overcome the indivisibility of money. They show through numerical examples that a money injection can have a persistent effect on output and price adjustments are sluggish. Chiu and Molico (2014) study a closely related model with divisible money and no upper bound on money holdings—as in our setting—and show numerically that unanticipated inflation shocks can have persistent effects on output, prices, and welfare. Finally, Scheinkman and Weiss (1986) in the context of a Bewley economy with competitive markets and alternating endowments show that one-time money injections generate output and price effects that depend on the state of the economy.

2 Environment

Time is discrete and the horizon infinite. Each period has two stages. In the first stage agents trade in a decentralized market (DM) with pairwise meetings and bargaining. In the second stage they trade in a centralized market (CM). The DM and CM consumption goods are perishable and the CM good is taken as the numéraire.

The economy is populated by a measure two of infinitely-lived agents divided evenly between *buyers* and *sellers*, where these labels refer to an agent’s role in the DM. In the first stage buyers want to consume but cannot produce while sellers are able to produce but do not wish to consume. The existence of intertemporal gains from trade that cannot be exploited due to agents’ inability to commit is what generates an essential

role for money.

The period-utility function of a buyer is

$$u(y) - h, \tag{1}$$

where y is DM-consumption and h the quantity of labor supplied by the agent in the CM. We allow h to be negative in which case the buyer consumes the numéraire good. We assume that u is strictly concave with $u(0) = 0$, $u'(0) = \infty$, and $u'(+\infty) = 0$. The technology to produce the CM good is linear so that h units of labor generate h units of numéraire. The buyer's endowment of labor is $\bar{h}$. This upper bound on the labor supply will play a key role in our analysis as we will consider equilibria where the feasibility constraint, $h \leq \bar{h}$, binds for some agents allowing us to depart from a quasi-linear environment with degenerate distributions of money holdings.² We also impose a lower bound on h , $h \geq \underline{h}$ where $-\underline{h} \geq 0$ can be interpreted as a satiation level for CM consumption. The period-utility function of a seller is

$$-v(y) + c, \tag{2}$$

where $v(y)$ is the disutility of producing y units of the DM good in a pairwise meeting and c is the linear utility of consuming the numéraire. The discount factor across periods, $\beta \in (0, 1)$, is common to all agents.

Agents cannot commit and there is no monitoring. Hence, unsecured credit arrangements are not incentive-feasible. There is an intrinsically useless, perfectly divisible and storable asset called money. We use M_t to denote the money supply in the first subperiod of period t . The price of money in terms of the numéraire is ϕ_t . The gross rate of return of money is denoted $R_t \equiv \phi_t/\phi_{t-1}$.

2.1 Full insurance

Suppose that buyers have the ability to commit in order to repay their debt and insure themselves against DM idiosyncratic uncertainty. Buyers pool their labor endowments and agree to an insurance contract according to which they supply h units of labor every period in exchange for a consumption level y in the (observable) event they are matched in the DM. The expected utility of the buyer in each period is $\alpha u(y) - h$. The total CM output produced by buyers, h , is promised to the α sellers who are matched in the DM, $c = h/\alpha$. Sellers are willing to go along with this allocation if $c \geq v(y)$, i.e., their consumption is greater than their disutility of production. Hence, the insurance contract among buyers, (h, y) , solves the following maximization problem:

$$\max_{y, c, h \leq \bar{h}} [\alpha u(y) - h] \quad \text{s.t.} \quad c = \frac{h}{\alpha} \geq v(y).$$

The solution is $y = y^*$, where $u'(y^*) = v'(y^*)$, if $\bar{h} \geq \alpha v(y^*)$. Otherwise, $y = v^{-1}(\bar{h}/\alpha) < y^*$.

²Instead of this linear specification with an upper bound we could assume a convex disutility of work. See. e.g., Rocheteau, Weill, and Wong (2014) in a continuous-time version with competitive pricing. Typically in this case it takes an infinite number of periods for the buyer to reach his targeted real balances. In contrast, with our current specification the buyer can reach his target in a finite number of periods. This result will simplify the analysis of the transitional dynamics.

3 Equilibrium

We characterize an equilibrium in three steps. First, we study the decision problem of a buyer taken as given the sequence of rates of return for currency, $\{R_t\}_{t=1}^{+\infty}$. Second, given the buyer's saving rate in the CM and his spending decision in the DM, we will characterize the law of motion for the distribution of real balances. Third, we will clear the money market in order to obtain the value of money, ϕ_t , and hence its rate of return.

Value functions Consider first the problem of a buyer at the beginning of the CM of period t holding z real balances (money balances expressed in terms of the period- t CM good). The value function of a buyer solves:

$$W_t(z) = \max_{h, z'} \{-h + \beta V_{t+1}(z')\} \quad (3)$$

$$\text{s.t. } z' = R_{t+1}(h + z) \quad (4)$$

$$z' \geq 0, \quad h \in [\underline{h}, \bar{h}]. \quad (5)$$

According to (3) the buyer chooses his supply of labor, h , and next-period real balances, z' , in order to maximize his discounted continuation value in $t + 1$ net of the disutility of work. The budget identity, (4), specifies that the next-period real balances are equal to the sum of the current wealth and labor income multiplied by the gross rate of return of money. From (5), the buyer's problem is subject to a non-negativity constraint for real balances and a feasibility constraint on the labor supply. The value functions are indexed by time as the gross rate of return of money, R_t , might vary over time.

In the DM each matched buyer makes a take-it-or-leave-it offer, (y, p) , to a seller, where y is the DM output to be produced by the seller and p is the payment in terms of real balances by the buyer.³ This payment must satisfy the feasibility constraint $p \leq z$ (since buyers' IOUs are not accepted by sellers due to lack of commitment and lack of monitoring). It must also satisfy the individual rationality constraint of the seller according to which the payment must be at least equal to the disutility of production, $-v(y) + p \geq 0$. Indeed, sellers get no surplus in the DM and hence they have no motive for carrying real balances from one period to the next provided that $R_{t+1}\beta < 1$. Consequently, sellers spend all the money they accumulate in the DM in the following CM.⁴ It is clear that the seller's participation constraint will hold at equality, $v(y) = p$, as otherwise the buyer would have an incentive to reduce the size of the payment for the same output level. Hence the lifetime expected discounted utility of a buyer at the beginning of the DM is:

$$V_t(z) = \alpha \max_{y \leq v^{-1}(z)} \{u(y) + W_t[z - v(y)]\} + (1 - \alpha)W_t(z). \quad (6)$$

With probability α the buyer is matched in the DM in which case he chooses an output level, y , in exchange

³The model remains tractable under competitive pricing. See Rocheteau, Weill and, Wong (2014) for a related model in continuous-time where households trade in a competitive market.

⁴This is the place where the assumption of ex-ante heterogeneous buyers and sellers borrowed from Rocheteau and Wright (2005) and Lagos and Rocheteau (2005) plays a crucial role to keep the model tractable. If instead agents could be both buyers and sellers in the DM, as in Lagos and Wright (2005), then the outcome of the bargaining would depend on both the money holdings of the buyer and the money holdings of the seller in the match and the distribution of money holdings would be a state variable in the agent's problem. For such a model, see Chiu and Molico (2011).

for $v(y)$ units of real balances. With probability $1 - \alpha$ the buyer is unmatched and enters the next CM with z real balances.

We now prove the existence, continuity, and differentiability of the value functions, W_t . From (3)-(5) and (6) we can define W_t in standard recursive fashion as follows:

$$W_t(z) = \max_{y, z'} \left\{ z - \frac{z'}{R_{t+1}} + \beta \alpha \{u(y) + W_{t+1}[z' - v^{-1}(y)]\} + \beta(1 - \alpha)W_{t+1}(z') \right\} \\ \text{s.t. } z' \in [R_{t+1}(z + \underline{h}), R_{t+1}(z + \bar{h})] \text{ and } y \leq v^{-1}(z'). \quad (7)$$

Lemma 1 *There exists a unique W_t solution to (7) in the space of continuous and bounded functions. Moreover, W_t is concave and differentiable.*

Choice of real balances In the following we focus on equilibria such that the constraint $h \geq \underline{h}$ never binds. Let $\xi_t(z)$ denote the Lagrange multiplier associated with $h \leq \bar{h}$. Substituting $h = z'/R_{t+1} - z$ from (4) into the objective we can rewrite the buyer's problem as:

$$W_t(z) = z + \max_{z' \geq 0} \left\{ -\frac{z'}{R_{t+1}} + \beta V_{t+1}(z') + \xi_t \left(\bar{h} - \frac{z'}{R_{t+1}} + z \right) \right\}. \quad (8)$$

Equation (8) is analogous to (9) in LR except for the last term in the maximization problem, the feasibility constraint on labor. If $\xi_t = 0$, then the second term on the right side of (8) is independent of z , the choice of next-period real balances is independent of current wealth, and W_t is linear. However, if the feasibility constraint on labor binds, $\xi_t > 0$, then the choice of real balances is no longer independent of current wealth and W_t is no longer linear—the two key ingredients of the tractability of the Lagos-Wright model. The envelope theorem applied to (8) gives:

$$W'_t(z) \equiv \lambda_t(z) = 1 + \xi_t. \quad (9)$$

The first-order condition for the choice of real balances is

$$-\lambda_t(z) + R_{t+1}\beta V'_{t+1}(z') \leq 0, \quad \text{"=" if } z' > 0. \quad (10)$$

The cost of accumulating real balances is measured by $\lambda_t = 1 + \xi_t$. The marginal benefit of real balances is the discounted marginal value of the buyer in the next DM times the gross rate of return of real balances, $R_{t+1}\beta V'_{t+1}(z')$. We define the buyer's targeted real balances for $t + 1$, z_{t+1}^* , as the solution to (10) when $\xi_t(z) = 0$, i.e.,

$$R_{t+1}\beta V'_{t+1}(z_{t+1}^*) = 1. \quad (11)$$

The buyer reaches this target if he is unconstrained by his labor endowment, i.e., $z + \bar{h} \geq z_{t+1}^*/R_{t+1}$. Similarly, the constraint $h \geq \underline{h}$ does not bind if $z + \underline{h} \leq z_{t+1}^*/R_{t+1}$.

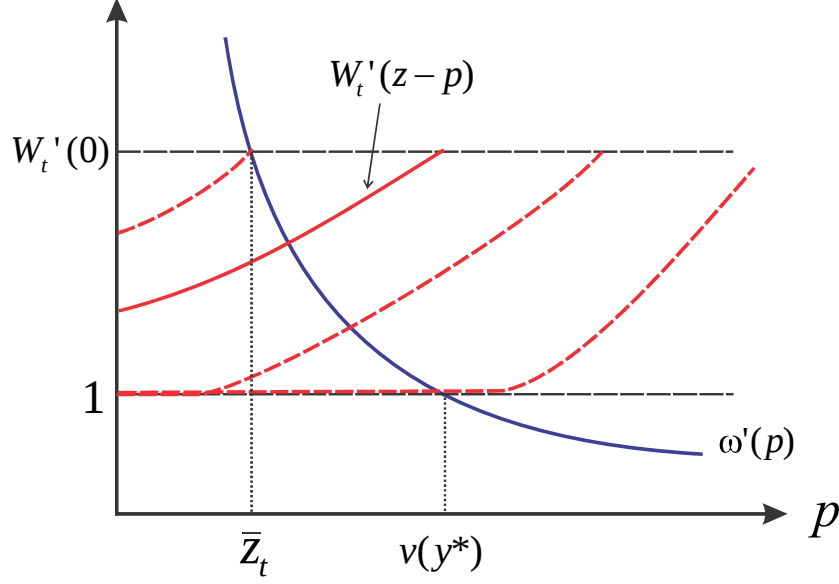


Figure 1: Bargaining outcome

Terms of trade in the DM. The solution to the maximization problem on the right side of (6) is $p = v(y)$ and

$$\omega'(p) = W'_t(z - p) \text{ if } \omega'(z) < W'_t(0) \quad (12)$$

$$p = z \text{ otherwise,} \quad (13)$$

where $\omega(p) \equiv u \circ v^{-1}(p)$. According to (12) the buyer equalizes his marginal utility from spending a unit of real balances in the DM, $u'(y)/v'(y)$, with the marginal value of real balances in the CM as measured by W'_t . We represent (12) in Figure 1 where the left side is the blue downward-sloping curve and the right side is the red upward-sloping curve. From the concavity of W it follows that $W'_t(z - p)$ is non-increasing in z . Hence, as z increases the red upward-sloping curve moves downward. (See the two dashed curves located underneath the plain upward-sloping curve. Those curves are horizontal for low values of p because buyers enter the next CM with enough real balances to reach their targeted real balances.) As a result, p and y are non-decreasing in the buyer's real balances. Similarly, if we denote post-trade real balances by $\tilde{z} = z - p$ then $\omega'(z - \tilde{z}) = W'_t(\tilde{z})$. Hence, post-trade real balances are non-decreasing with pre-trade real balances.

From (13) if the marginal utility from spending real balances in the DM is larger than the marginal value of money in the CM when money holdings are depleted in full, $\omega'(z) \geq W'_t(0)$, then the buyer spends all his real balances. We denote by $\bar{z}_t$ the threshold below which there is full depletion of real balances. It solves

$$\omega'(\bar{z}_t) = W'_t(0). \quad (14)$$

In Figure 1 we represent $W'(\bar{z}_t - p)$ by a dashed curve located above the plain upward-sloping curve. It intersects the horizontal line given by $W'_t(0)$ and $\omega'(p)$ when $p = \bar{z}_t$. There is another threshold for real

balances, $\hat{z}_t$, above which buyers are unconstrained by their labor endowment in the following CM given their post-trade real balances, in which case $W'_t(z - p) = 1$. From (12) it follows that for all $z \geq \hat{z}_t$, $y = y^*$ and $p = v(y^*)$. Hence, the threshold $\hat{z}_t$ is

$$\hat{z}_t = v(y^*) + \frac{z_{t+1}^*}{R_{t+1}} - \bar{h}. \quad (15)$$

In order to be unconstrained in the following CM the buyer must hold enough real balances so that he can pay for the first-best level of output in the DM, which requires $v(y^*)$, and he can supplement his labor endowment to finance his next-period targeted real balances, $z_{t+1}^*/R_{t+1} - \bar{h}$. We summarize the outcome of the bargaining problem in the following proposition and Figure 2.

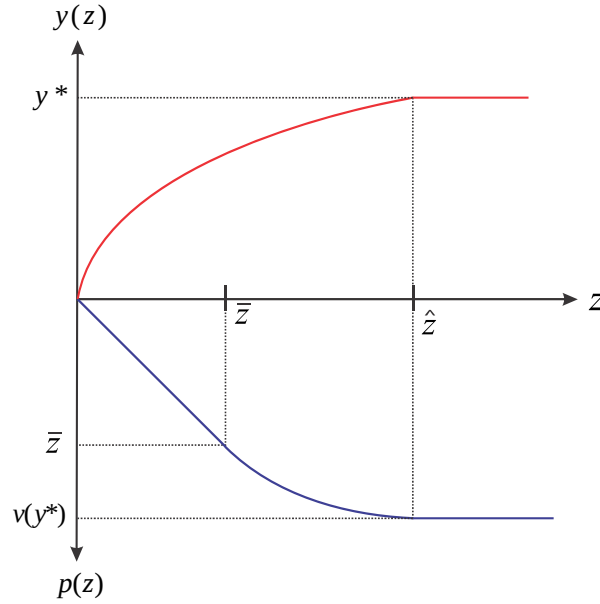


Figure 2: Terms of trade in DM pairwise meetings

Proposition 1 (Bargaining outcome) Assume $W'_t(0) > 1$, i.e., $\bar{h} < z_{t+1}^*/R_{t+1}$. There are two thresholds for real balances, $\bar{z}_t < \hat{z}_t$ defined in (14) and (15), such that the outcome of the buyer's take-it-or-leave-it bargaining game satisfies:

1. For all $z \leq \bar{z}_t$, $p = z$ and $y = v^{-1}(z)$.
2. For all $z \in (\bar{z}_t, \hat{z}_t)$, $p < z$, and $y = v^{-1}(p)$ are increasing with real balances.
3. For all $z \geq \hat{z}_t$, $p = v(y^*)$, and $y = y^*$.

The marginal value of real balances at the beginning of the DM is

$$V'_t(z) = \alpha \omega' [p_t(z)] + (1 - \alpha) \lambda_t(z), \quad (16)$$

where $\omega(p) = u'[v^{-1}(p)]/v'[v^{-1}(p)]$ and $p_t(z)$ is the solution to the bargaining problem, (12)-(13). The marginal value of real balances in the DM is equal to the marginal utility of DM consumption with probability α (a match occurs) and the marginal utility of real balances in the following CM with probability $1 - \alpha$ (the buyer is unmatched). We substitute $V'_{t+1}(z')$ by its expression given by (16) into (10) to obtain:

$$\lambda_t(z) = R_{t+1}\beta \{ \alpha \omega' [p_{t+1}(z')] + (1 - \alpha) \lambda_{t+1}(z') \}. \quad (17)$$

Distribution of real balances. Let us turn to the law of motion for the distribution of real balances. We denote $F_t(z)$ the distribution of real balances at the beginning of period t . The distribution at time $t + 1$ is given by:

$$F_{t+1}(z) = \int \alpha \mathbb{I}_{\{z_{t+1}[x-p_t(x)] \leq z\}} + (1 - \alpha) \mathbb{I}_{\{z_{t+1}(x) \leq z\}} dF_t(x), \quad (18)$$

where $z_{t+1}(x)$ is the policy function derived from (12)-(13) that specifies the choice of real balances in $t + 1$ given the real balances at the beginning of the CM in t . It is given by:

$$z_{t+1}(x) = \min\{(x + \bar{h})R_{t+1}, z_{t+1}^*\}.$$

Either the buyer can reach his target, z_{t+1}^* , or he supplies all his labor so that his total wealth is composed of his initial wealth and his labor endowment, $x + \bar{h}$. This wealth is capitalized according to the gross real rate of return of fiat money. The first term underneath the integral on the right side of (18) represents the measure α of buyers who are matched in the DM and enter the CM with $x - p_t(x)$ real balances, where x is their pre-trade real balances. In the CM they accumulate $z_{t+1}[x - p_t(x)]$ for period $t + 1$. The second term represents the unmatched buyers.

Value of money Finally, the value of money is determined by the following money market clearing condition:

$$\phi_t M_t = \int x dF_t(x). \quad (19)$$

Hence, the rate of return of money is

$$R_{t+1} = \frac{\phi_{t+1}}{\phi_t} = \frac{M_t}{M_{t+1}} \frac{\int x dF_{t+1}(x)}{\int x dF_t(x)}. \quad (20)$$

Definition 1 An equilibrium is a sequence, $\{F_t, \phi_t, R_{t+1}\}_{t=0}^{+\infty}$, that solves (18), (19), and (20).

4 Money in the long run

We characterize steady-state equilibria where aggregate real balances are constant over time. Given that the money stock is constant, the gross rate of return of money is $R_t = \phi_{t+1}/\phi_t = 1$. We focus on equilibria where it takes $N > 1$ consecutive rounds of CM trades for a buyer with depleted money holdings to rebuild his targeted real balances if he remains unmatched in all DMs. (The case $N = 1$ is the LR model.) Moreover, we

will be focusing on equilibria where buyers deplete all their real balances in the DM, $z \leq \bar{z}$ and $y(z) = v^{-1}(z)$ for all z in the support of the distribution.⁵

Targeted real balances Once a buyer has reached his targeted real balances the feasibility constraint on labor ceases to bind, $\xi(z) = 0$. From (17) his choice for next-period real balances is $z^* \in ((N-1)\bar{h}, N\bar{h}]$ solution to

$$\omega'(z^*) \equiv \frac{u'[y(z^*)]}{v'[y(z^*)]} = 1 + \frac{r}{\alpha}, \quad (21)$$

where $z^* = v(y^*)$. At the targeted real balances the ratio of the marginal utility of DM consumption to the marginal disutility of DM production is equal to the marginal disutility of labor in the CM, one, plus the average holding cost of real balances. This cost is equal to the rate of time preference multiplied by the average period length until a match in the DM occurs, $1/\alpha$. As buyers become more patient, or as the frequency of matches increases, the targeted real balances increase. The condition $z^* \in ((N-1)\bar{h}, N\bar{h}]$ can be reexpressed as

$$(N-1)\bar{h} < \omega'^{-1}\left(1 + \frac{r}{\alpha}\right) \leq N\bar{h}. \quad (22)$$

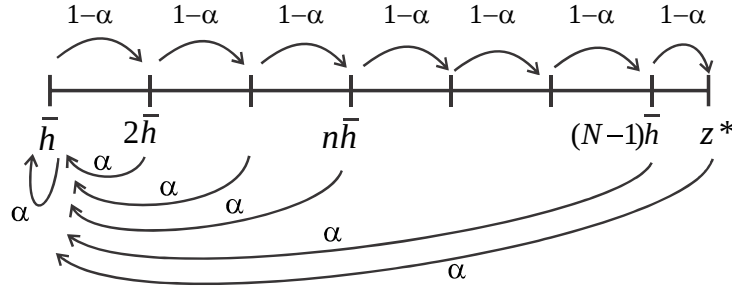


Figure 3: Support of the distribution of real balances

Distribution of real balances The support of the distribution of real balances across buyers at the beginning of a period is $\{\bar{h}, 2\bar{h}, \dots, (N-1)\bar{h}, z^*\}$. Indeed, buyers increase their real balances by the size of their labor endowment, $\bar{h}$, until they reach their target. See Figure 3. The distribution F is composed of N mass points, $\{\mu_n\}_{n=1}^N$, where μ_n is the measure of buyers holdings $n\bar{h}$ for all $n \in \{1, \dots, N-1\}$ and μ_N is the measure of buyers holding their target, z^* . We have:

$$\mu_1 = \alpha \quad (23)$$

$$\mu_n = (1-\alpha)\mu_{n-1} \text{ for all } n \in \{2, N-1\} \quad (24)$$

$$\alpha\mu_N = (1-\alpha)\mu_{N-1}. \quad (25)$$

According to (23) each buyer is matched with a seller with probability α , in which case he spends all his real balances (since we are focusing on equilibria with full depletion). By the Law of Large Numbers the measure

⁵Equilibria with partial depletion of money holdings are not tractable analytically but they can be easily computed numerically. Indeed, the agent's problem does not depend on the distribution of real balances and it can be solved by a standard value function iteration since W is the fixed point of a contraction mapping.

of buyers entering the CM with depleted money balances is α . Those buyers supply their full endowment of labor in order to start the following period with $z_1 = \bar{h}$ real balances. According to (24) the measure of agents holding $z_n = n\bar{h} \in (z_1, z^*)$ is equal to the measure of buyers holding z_{n-1} , μ_{n-1} , times the probability that they were unmatched in the last DM round, $1 - \alpha$, so that such buyers add $\bar{h}$ to their existing real balances. Finally, the measure of agents holding the targeted real balances is determined such that the flow of buyers with the targeted real balances who are matched in the DM, $\alpha\mu_N$, is equal to the flow of buyers holding z_{N-1} who are unmatched in the DM and reach z^* in the next CM. It is easy from (23)-(25) to solve for the distribution of real balances in closed form:

$$\mu_n = \alpha(1 - \alpha)^{n-1} \text{ for all } n = 1, \dots, N - 1 \quad (26)$$

$$\mu_N = (1 - \alpha)^{N-1}. \quad (27)$$

From (26)-(27) the distribution of real balances is a truncated geometric distribution.⁶

The model also features a distribution of prices in the DM. The unit price of the DM output for a buyer holding $z_n = n\bar{h}$ real balances is $z_n/v^{-1}(z_n)\phi$, which is increasing in z_n if v is strictly convex. So the richest agents in the DM pay a higher price to compensate sellers for their convex disutility of production. The fraction of the transactions taking place at that price is μ_n .

Value of money and prices. Aggregate real balances are $\phi M = \sum_{n=1}^N \mu_n z_n$. From (26)-(27), and after some calculation, this gives

$$\phi M = \bar{h} \frac{\{1 - (1 - \alpha)^{N-1} [(N - 1)\alpha + 1]\}}{\alpha} + (1 - \alpha)^{N-1} z^*. \quad (28)$$

Aggregate real balances do not depend on the nominal money supply and hence money is neutral in the long run. For a given N the value of money increases with the buyer's labor endowment, $\bar{h}$, and it decreases with the rate of time preference, r .

Marginal value of real balances Next, we determine the marginal value of real balances, $\lambda(z) = 1 + \xi(z)$, recursively. Suppose $z \in (z^* - \bar{h}, z^*)$. If the buyer can reach his targeted real balances by supplying less than $\bar{h}$, then the feasibility constraint on labor is slack, $\xi(z) = 0$. As a result, $\lambda(z) = 1$ and $W(z)$ is linear. From (17),

$$\lambda(z) = \beta [\alpha \omega'(z + \bar{h}) + (1 - \alpha)\lambda(z + \bar{h})], \text{ for all } z \leq z^* - \bar{h}. \quad (29)$$

If a buyer enters the CM with $z \leq z^* - \bar{h}$ real balances then he supplies his endowment of labor and enters the next period with $z + \bar{h}$. With probability α the buyer is matched and spends all his real balances. The marginal value of a unit of money is then $\omega'(z + \bar{h})$. With probability $1 - \alpha$ the buyer is unmatched and enters the CM with $z + \bar{h}$, in which case the marginal value of money is $\lambda(z + \bar{h})$. The difference equation

⁶Green and Zhou (1998), Zhou (1999), and Rocheteau (2000) also find geometric distributions of money holdings in search model with price posting and indivisible goods. However, the dynamics of individual real balances are different as individual accumulate and deplete real balances one unit at a time.

(29) can be solved in closed form to give:

$$\lambda(z) = 1 + \alpha \sum_{j=1}^n \beta^j (1 - \alpha)^{j-1} [\omega'(z + j\bar{h}) - \omega'(z^*)]^+, \quad (30)$$

where $[x]^+ = \max\{x, 0\}$. The marginal value of money is equal to one, the marginal disutility of work, plus the discounted sum of the differences between the marginal utility of lumpy consumption given the buyer's real balances at a point in time and his marginal utility of consumption at the targeted real balances. It is easy to check that $\lambda(z) = W'(z)$ is decreasing in z (from the concavity of $u \circ v^{-1}(z)$) and continuous.

Given $\lambda(z)$ we can obtain the value function, $W(z)$, in closed form. At his targeted real balances the lifetime expected utility of a buyer is

$$W(z^*) = \beta \{\alpha [\omega(z^*) + W(0)] + (1 - \alpha)W(z^*)\}. \quad (31)$$

The buyer does not need to readjust his real balances, and hence he incurs no cost in the CM. In the following DM he is matched with probability α in which case he depletes all his money balances. If he is unmatched he enters the subsequent CM with his targeted real balances. Multiplying both sides of (31) by β^{-1} and using that $W(z^*) - W(0) = \int_0^{z^*} \lambda(x)dx$, $W(z^*)$ can be rewritten as

$$rW(z^*) = \alpha \left[\omega(z^*) - \int_0^{z^*} \lambda(x)dx \right]. \quad (32)$$

Given $W(z^*)$ we obtain $W(z)$ as follows:

$$W(z) = W(z^*) - \int_z^{z^*} \lambda(x)dx = \frac{\alpha}{r} \left[\omega(z^*) - \int_0^{z^*} \lambda(x)dx \right] - \int_z^{z^*} \lambda(x)dx. \quad (33)$$

The condition for full depletion of real balances is $\omega'(z^*) \geq \lambda(0)$. The marginal utility that the buyer gets from spending his last unit of real balances, $\omega'(z^*)$, must be greater than the marginal utility from holding onto this unit of money, $\lambda(0)$. From (30) the condition for full depletion can be rewritten as

$$\omega'(z^*) - 1 = \frac{r}{\alpha} \geq \alpha \sum_{j=1}^{+\infty} \beta^j (1 - \alpha)^{j-1} [\omega'(j\bar{h}) - \omega'(z^*)]^+, \quad (34)$$

We represent the condition (34) by a grey area in Figure 4. The dotted lines represent the conditions in (22). The case studied in LR, $N = 1$, requires the endowment in labor, $\bar{h}$, to be large so that the buyer can readjust his money balances in a single period. If the endowment is such that $\omega'(\bar{h}) > 1 + r/\alpha$ then it will take more than one period for the buyer to reach his targeted real balances. In the Appendix A2 we provide numerical examples to illustrate how N varies with fundamentals, α , $\bar{h}$, and r .

We can now define a steady state equilibrium as follows.

Definition 2 *A steady-state, monetary equilibrium with full depletion of real balances is a list, $(N, z^*, \phi, \{\mu_n\}_{n=1}^N)$, that solves (21), (22), (26)-(27), (28), and (34).*

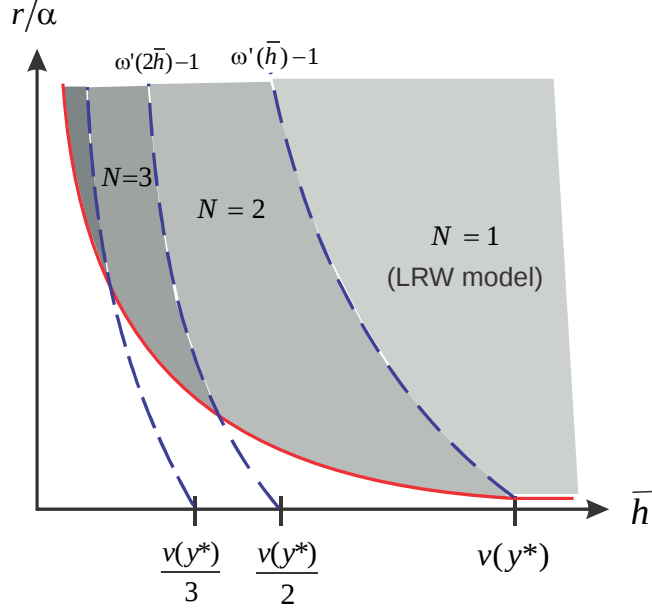


Figure 4: Existence of equilibria with full depletion of real balances

Provided that the condition for full depletion, (34), holds one can construct a steady-state equilibrium as follows. From (21) one determines the targeted real balances, z^* . We use (22) to compute the number of periods it takes to reach the target, N . Given N and z^* the steady-state distribution of real balances is obtained from (26)-(27). Finally, the value of money is obtained from (28).

Proposition 2 (*Existence of steady-state monetary equilibria with full depletion.*) *If (34) holds, then there exists a steady-state monetary equilibrium with full depletion. Moreover, an equilibrium with full depletion approaches the first best if and only if $\bar{h} \geq v(y^*)$ and r approaches 0.*

Provided that the buyer's labor endowment is not too low and agents are not too patient, there exists an equilibrium where buyers deplete their money holdings in full whenever they are matched in the DM. The first best requires that agents are infinitely patient and they have a sufficiently large labor endowment to reach their targeted real balances in a single period. In contrast, if $\bar{h} < v(y^*)$ then there is no equilibrium with full depletion when r approaches 0. Indeed, if agents are infinitely patient but they cannot accumulate $v(y^*)$ in a single period, then they will want to achieve full insurance by accumulating very large real balances and they will only spend a fraction of their real balances when a match occurs.

4.1 A numerical example with partial depletion of real balances

Equilibria that feature partial depletion of real balances cannot be solved in closed form. However, the model is still very tractable numerically. Indeed, $W(z)$ is the fixed point of a contraction mapping, (7), that is independent of the distribution of real balances. Once the policy function is obtained (by iterations of the Bellman equation) we can use it to compute the distribution of real balances. (The algorithm is detailed in

the appendix.) In the following we show a numerical example obtained with the following parameter values: $\bar{h} = 0.5$, $\alpha = 0.8$, $u(y) = 3y^{1/3}$, $r = 0.04$. The left panel of Figure 5 plots the distribution of real balances. The middle panel plots the value function, and the right panel plots post-trade real balances.

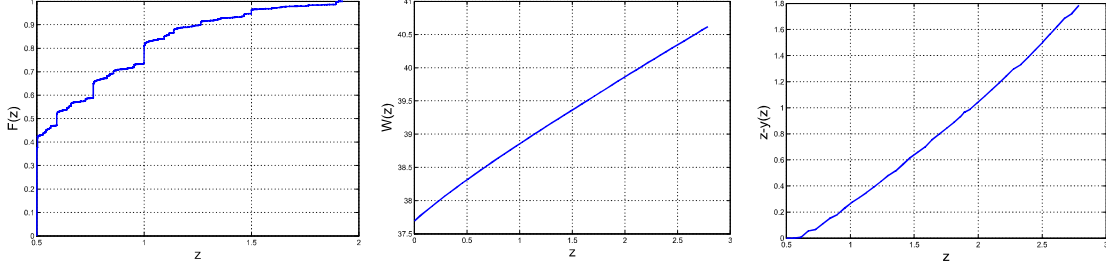


Figure 5: Numerical example of an equilibrium with partial depletion

The left panel shows that there is a mass of buyers, about 40%, holding real balances equal to their endowment, $\bar{h} = 0.5$. This mass is less than the frequency of a trading opportunity in the DM, $\alpha = 0.8$, because the equilibrium features partial depletion, i.e., not all buyers who are matched in the DM deplete their money holdings in full. According to the right panel the threshold for full depletion is about $\bar{z}_1 \approx 0.6$ which is less than the target, $z^* \approx 2.7$. Moreover, it takes $N = 6$ periods for a buyer with depleted money holdings to accumulate the targeted real balances. One can also see that the distribution, F , has multiple mass points. For instance, there is about 10 percent of buyers holding $2\bar{h} = 1$. Buyers with 1 unit of real balances leave a DM match with $z - y(z) \approx 0.25$ and then they accumulate $\bar{h}$ in the following CM. As a result there is also a mass of buyers at $z \approx 0.75$.

5 Money in the short run

We now study the short-run effects of a money injection. We focus first on equilibria with $N = 2$ as these equilibria can be characterized in closed form even for out-of-steady-state dynamics. From (22) and (27) such an equilibrium exists if

$$\omega'(2\bar{h}) \leq 1 + \frac{r}{\alpha} < \omega'(\bar{h}) \leq \left(1 + \frac{1+r}{\alpha}\right) \left(1 + \frac{r}{\alpha}\right). \quad (35)$$

The economy starts at a steady state at the beginning of $t = 0$. Before entering the DM there is a measure α of buyers holding $m_\ell = \bar{h}M / [\alpha\bar{h} + (1-\alpha)z^*]$ units of money and a measure $1 - \alpha$ holding $m_h = z^*M / [\alpha\bar{h} + (1-\alpha)z^*]$. At the beginning of the CM of $t = 0$, after a round of DM trades, the distribution of money balances across buyers has three mass points: there is a measure α of buyers holding no money (the buyers who were matched in the previous DM), a measure $\alpha(1 - \alpha)$ holding m_ℓ and a measure $(1 - \alpha)^2$ holding m_h . This distribution is illustrated in Figure 6. The monetary authority injects $(\gamma - 1)M$, with $\gamma > 1$, in a lump-sum fashion to all buyers at the time they enter the CM.⁷ The change in the money

⁷We assume that an agent's type is observable so that the monetary authority can distinguish between buyers and sellers. However, it cannot make transfers contingent on the buyer's money balances. The same assumption is made in LR and

supply is common knowledge among all agents.

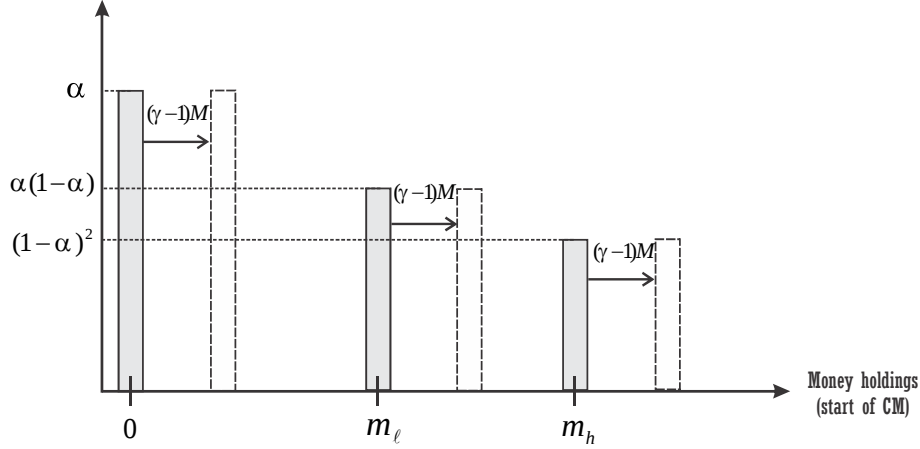


Figure 6: Distribution of money holdings at the start of the CM of $t = 0$

We will construct equilibria such that the economy returns to a steady state in the CM of $t = 1$, i.e.,

$$\phi_t = \phi_1 = \frac{\alpha \bar{h} + (1 - \alpha) z^*}{\gamma M} \quad \text{for all } t \geq 1. \quad (36)$$

The short run corresponds to the CM of $t = 0$ and the DM of $t = 1$. We will distinguish small money injections that do not affect the number of mass points in the distributions of real balances from larger money injections such that the distribution F_t is degenerate at $t = 1$.

5.1 Small injection

We consider first the case where γ is close to 1 so that the feasibility constraint on labor, $h \leq \bar{h}$, binds only for the α buyers with no money balances at the beginning of the CM of $t = 0$. We will characterize the distribution of real balances at $t = 1$ and the rate of return of money. We will provide conditions for the proposed equilibrium to exist, and we will study the effects of the money injection on CM and DM output and individual labor supplies.

Distribution of real balances. Consider first buyers who enter the CM of $t = 0$ with no money. Their real balances after transfers and CM trades expressed in the CM good of $t = 1$ are

$$z_1^0 = R_1 \bar{h} + (\gamma - 1) \phi_1 M. \quad (37)$$

The buyer supplies $\bar{h}$ units of labor in the CM of $t = 0$, which yields $R_1 \bar{h}$ real balances at $t = 1$, and he receives a lump-sum transfer of money of size $(\gamma - 1)M$ valued at the price ϕ_1 .

We conjecture that the buyers holding m_ℓ and m_h at the beginning of the CM of $t = 0$ accumulate their targeted real balances for period 1, z_1^* , which requires $\xi_0(\phi_0 m_\ell) = \xi_0(\phi_0 m_h) = 0$. We also conjecture that

Rocheteau and Wright (2005). Alternatively, transfers could be made to both buyers and sellers in which case each buyer receives $(\gamma - 1)M/2$. See Appendix 5.

$h \leq \bar{h}$ does not bind for buyers holding z_1^* in the CM of $t = 1$, i.e., $\xi_1(z_1^*) = 0$. From (17) z_1^* solves

$$\omega'(z_1^*) = 1 + \frac{1 + r - R_1}{\alpha R_1}. \quad (38)$$

To summarize, the distribution of real balances at the beginning of $t = 1$ has two mass points: α buyers hold z_1^0 and $1 - \alpha$ buyers hold z_1^* . Hence aggregate real balances at $t = 1$ are $\phi_1 \gamma M = \alpha z_1^0 + (1 - \alpha) z_1^*$. Substituting z_1^0 by its expression given by (37) and solving for aggregate real balances we obtain:

$$\phi_1 \gamma M = \frac{\alpha R_1 \bar{h} + (1 - \alpha) z_1^*}{1 - \alpha \left(1 - \frac{1}{\gamma}\right)}. \quad (39)$$

Aggregate real balances at $t = 1$ are a linear combination of the capitalized labor endowment, $R_1 \bar{h}$, of buyers with depleted money balances and the targeted real balances, z_1^* , of all other buyers. The denominator in (39) takes into account that the α buyers with depleted money balances at $t = 0$ hold onto their transfer of real balances which is equal to a fraction $1 - \gamma^{-1}$ of aggregate real balances.

Rate of return and value of fiat money. Next, we determine the gross real rate of return of fiat money from $t = 0$ to $t = 1$. Our guess that the economy reaches a steady state in the CM of $t = 1$ pins down aggregate real balances, $\phi_1 \gamma M = \alpha \bar{h} + (1 - \alpha) z^*$, and hence the rate of return of money in the initial period. Indeed, from (39),

$$\frac{\alpha R_1 \bar{h} + (1 - \alpha) z_1^*}{1 - \alpha \left(1 - \frac{1}{\gamma}\right)} = \alpha \bar{h} + (1 - \alpha) z^*. \quad (40)$$

The left side of (40) is increasing in R_1 : it is equal to 0 when $R_1 = 0$ and it is greater than $\alpha \bar{h} + (1 - \alpha) z^*$ (because $1 > \alpha(1 - \gamma^{-1})$) when $R_1 = 1$. Hence, there is a unique R_1 solution to (40) and it is such that $R_1 < 1$ and $\phi_0 > \phi_1$.

Figure 7 illustrates the determination of the equilibrium value for R_1 , denoted R_1^e , where the left side of (40) is represented by the upward-sloping red curve. As γ increases this curve shifts upward and, as a result, R_1^e decreases. Moreover, $\lim_{\gamma \downarrow 1} R_1(\gamma) = 1$. So despite prices being flexible and all agents having access to the centralized market where money is injected, the value of money does not adjust instantly to its new steady-state value and money is not neutral in the short run.⁸

Given R_1 we determine the value of money at the time of the money injection, $\phi_0 = \phi_1 / R_1$. Using the expression for ϕ_1 given by (36) we obtain:

$$\phi_0 = \frac{\alpha \bar{h} + (1 - \alpha) z^*}{R_1 \gamma M}. \quad (41)$$

The value of money at $t = 0$ decreases with $R_1 \gamma$.

Let $\phi_{-1} = [\alpha \bar{h} + (1 - \alpha) z^*] / M$ denote the value of money at the initial steady state. The ex-post rate of return of money at $t = 0$ is $R_0 = \phi_0 / \phi_{-1} = 1 / (R_1 \gamma)$. From (38) and (40) it can be checked that R_0 is not necessarily monotone in γ . To see this we determine the condition under which a small increase in γ above 1 raises R_0 above one. In that case the money injection raises the value of money at the time of

⁸Results are qualitatively similar if the money supply increases through transfers to all agents in the economy.

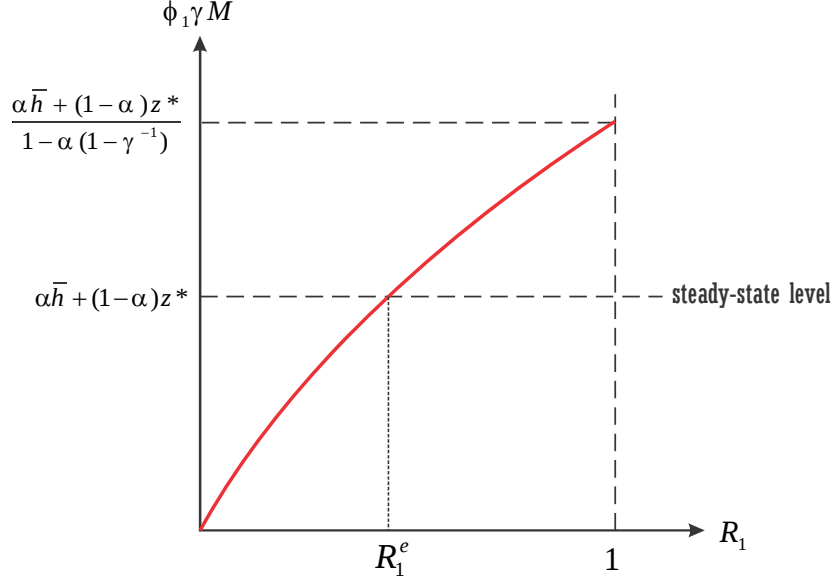


Figure 7: Determination of R_1

the money injection, ϕ_0 , above its initial steady-state value, $\gamma\phi_1$, i.e., there is a deflation in the short run. Differentiating R_1 defined in (40) with respect to γ we show that

$$\left. \frac{dR_1/R_1}{d\gamma/\gamma} \right|_{\gamma=1} < -1 \Leftrightarrow \frac{-z^* \omega''(z^*)}{\omega'(z^*)} > \frac{z^*}{(z^* - \bar{h}) \beta \alpha (\alpha + r)}. \quad (42)$$

If the elasticity of buyers' targeted real balances with respect to R_1 is sufficiently low, which happens when buyers are sufficiently risk averse, then ϕ_0 increases with the size of the money injection. In order to illustrate this possibility we consider a numerical example with the following functional forms and parameter values: $v(y) = y$, $\beta = 0.25$, $\alpha = 0.8186$, and $\bar{h} = 0.0942$, and

$$u(c) = \frac{A}{1-\eta} \left[\left(\frac{c+b}{A} \right)^{1-\frac{1}{\eta}} - \left(\frac{b}{A} \right)^{1-\frac{1}{\eta}} \right],$$

where $A = 0.1028$, $\eta = 0.0280$, $b = 10^{-10}$. In Figure 8 we plot ϕ_0 and $\gamma\phi_1$ as a function of the size of the money injection, γ . For low values of γ ($\gamma < 1.5$) the increase in the money supply at time $t = 0$ generates an increase in R_0 (a deflation). In contrast, for large values of γ , R_0 decreases so that the value of money adjusts partially to the increase in the money supply. Finally, there is a value for γ ($\gamma \approx 1.5$) such that $\phi_0 = \gamma\phi_1$ and $R_0 = 1$. The price level in $t = 0$ is the same than the one at the initial steady state. So prices are "sticky" in that they do not adjust at all to a change in the money supply.

Output and labor-supply effects. The output levels in the DM of $t = 1$ are $y_{\ell,1} = v^{-1}(z_1^0) > v^{-1}(\bar{h})$ and $y_{h,1} = v^{-1}(z_1^*) < v^{-1}(z^*)$. Hence, the money injection reduces the dispersion of output and consumption levels across matches. Aggregate output is

$$Y_1 = \alpha y_{\ell,1} + (1-\alpha) y_{h,1} \geq Y^{ss} = \alpha v^{-1}(\bar{h}) + (1-\alpha) v^{-1}(z^*),$$

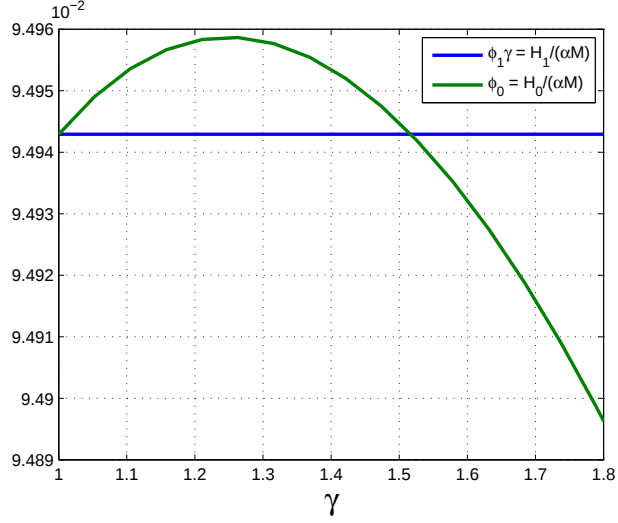


Figure 8: Example where the price level falls below its initial steady-state value in the short-run.

with a strict inequality if v is strictly convex. So DM aggregate output increases relative to its steady-state value, Y^{ss} . We summarize these results in Figure 9 by plotting the trajectories for aggregate real balances, $\phi_t \gamma M$, and DM output levels.

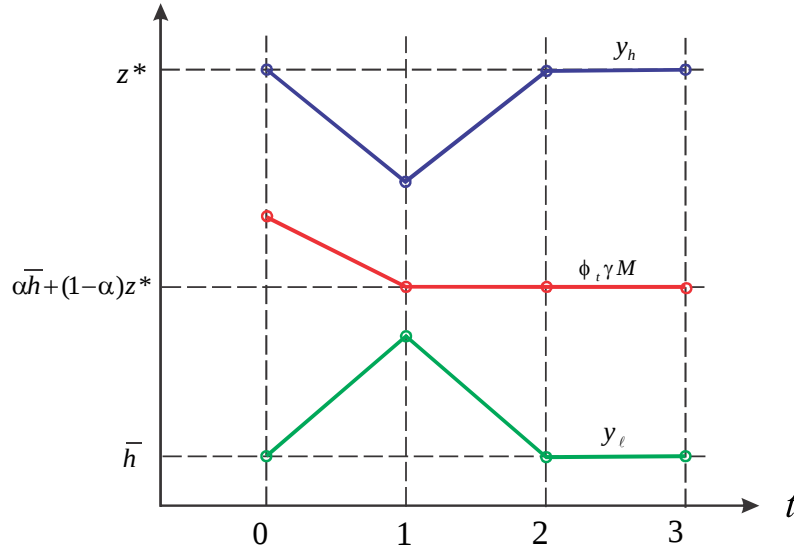


Figure 9: Effects of a small money injection

Next, we turn to CM aggregate output, denoted $H_t \equiv \int \max\{h_t(i), 0\} di$, where $h_t(i)$ is the choice of h at time t by buyer i . Recall that $h_t < 0$ corresponds to consumption of the CM good by the buyer, hence we only count $h_t(i)$ when it is positive. Summing the buyers' budget constraints in the CM of $t = 0$, (4), we find:

$$H_0 = \alpha \bar{h} + \alpha(1 - \alpha)h_0^+(m_\ell) + (1 - \alpha)^2 h_0^+(m_h), \quad (43)$$

with

$$h_0^+(m_\ell) = \left\{ \frac{z_1^*}{R_1} - \phi_0 [m_\ell + (\gamma - 1)M] \right\}^+ \quad (44)$$

$$h_0^+(m_h) = \left\{ \frac{z_1^*}{R_1} - \phi_0 [m_h + (\gamma - 1)M] \right\}^+. \quad (45)$$

The first term on the right side of (43) corresponds to the labor supply of the buyers with depleted money holdings: those buyers supply their labor endowment. The second term corresponds to the labor supply of buyers with m_ℓ units of money, $h_0^+(m_\ell)$, and the third term is the labor supply of buyers with m_h units of money, $h_0^+(m_h)$. From (44) and (45) buyers holding m_ℓ and m_h accumulate their targeted real balances, z_1^*/R_1 , and their wealth is composed of their initial real balances and the real transfer of money. By definition of the money holdings at the initial steady state, $\phi_0 m_\ell = \bar{h}/(R_1\gamma)$ and $\phi_0 m_h = z^*/(R_1\gamma)$. Moreover, aggregate real balances are $\phi_0 M = [\alpha\bar{h} + (1 - \alpha)z^*]/(\gamma R_1)$. Substituting these expressions into (44)-(45) and using (40) to express z_1^* as a function of γ and R_1 , i.e.,

$$\gamma z_1^* = \frac{[(1 - \alpha)\gamma + \alpha][\alpha\bar{h} + (1 - \alpha)z^*] - \alpha\gamma R_1 \bar{h}}{1 - \alpha}, \quad (46)$$

we obtain the following individual labor supplies:

$$h_0^+(m_\ell) = \frac{\{\alpha\bar{h}(1 - \gamma R_1) + (1 - \alpha)(z^* - \bar{h})\}^+}{(1 - \alpha)\gamma R_1} \quad (47)$$

$$h_0^+(m_h) = \frac{\alpha\bar{h}\{1 - \gamma R_1\}^+}{(1 - \alpha)\gamma R_1}. \quad (48)$$

From (47) and (48) individual labor supplies are decreasing in $\gamma R_1 = \gamma\phi_1/\phi_0$. Moreover, when $\gamma R_1 = 1$ labor supplies are at their steady-state levels. Hence, when $\gamma R_1 < 1$ —the value of money at $t = 0$ is larger than the one at the initial steady state—aggregate CM output is larger than its steady-state value. Conversely, when $\gamma R_1 > 1$ it is lower than its steady-state value. So high CM output is associated with deflation while low CM output is associated with inflation. Notice also that when $\gamma R_1 < 1$ all buyers supply some labor in the CM. Since sellers sell all their units of money in the CM, it follows that $H_0 = \alpha\phi_0 M$. In contrast, from (48), when $\gamma R_1 \geq 1$ buyers holding m_h do not supply any labor.

From (42) the case where CM output increases occurs when buyers are sufficiently risk averse in terms of their DM consumption. Equivalently, their targeted real balances are inelastic with respect to the rate of return of currency. In that case a money injection leads buyers holding m_ℓ and m_h units of money to supply more labor in the short run (since those buyers are unconstrained by their labor endowment) in order to maintain their targeted real balances, thereby generating a fall in the price level. Indeed, those buyers bid for the αM units of money held by sellers, but this residual money supply might not be sufficient to allow buyers to reach their target. In that case the value of money in $t = 0$, ϕ_0 , rises above its initial steady-state value, $\gamma\phi_1$, in order to clear the money market.

Conditions for the proposed equilibrium. First, we check that $z_1^0 < z_1^*$. From (37)

$$R_1 \bar{h} + \left(1 - \frac{1}{\gamma}\right) [\alpha\bar{h} + (1 - \alpha)z^*] < z_1^*. \quad (49)$$

As γ approaches 1 the left side tends to $\bar{h}$ while the right side tends to z^* . Hence, by continuity (49) holds for low values of γ . Substituting z_1^* by its expression given by (46) this condition can be rewritten as:

$$\gamma R_1 < \frac{\alpha \bar{h} + (1 - \alpha) z^*}{\bar{h}}. \quad (50)$$

According to (50) the ex-post rate of return of money at time $t = 0$, $R_0 = 1/(R_1 \gamma)$, must be above a threshold less than one. Second, we check that buyers holding m_ℓ units of money at the beginning of the CM of $t = 0$ (before transfers) can accumulate z_1^* , i.e., $h_0^+(m_\ell) \leq \bar{h}$. From (47) this condition can be rewritten as:

$$(1 - \alpha)(z^* - \bar{h}) \leq (\gamma R_1 - \alpha) \bar{h}. \quad (51)$$

The right side tends to $(1 - \alpha)\bar{h}$ as γ goes to 1. Hence, under the condition $z^* \leq 2\bar{h}$, (51) holds for values of γ close to 1. Finally, we check that the $1 - \alpha$ measure of agents with money balances at the beginning of the CM of $t = 1$ can accumulate z^* since $z_1^0 > \bar{h}$. Indeed, from the clearing condition of the money market, $\alpha z_1^0 + (1 - \alpha)z_1^* = \alpha \bar{h} + (1 - \alpha)z^*$, i.e.,

$$z_1^0 - \bar{h} = \left(\frac{1 - \alpha}{\alpha} \right) (z^* - z_1^*) > 0.$$

An equilibrium following a small money injection is described by short-run allocations and prices, (ϕ_0, R_1, z_1^*) , that solve (38), (40), (41), (49), and (51), followed by a steady state as characterized above.

We summarize the results of this section in the following proposition.

Proposition 3 (Small money injection.) *Suppose the economy is initially at a steady state with $N = 2$. A one-time money injection, $(\gamma - 1)M$, in the CM of $t = 0$ that satisfies (49)-(51) has the following consequences:*

1. *It raises aggregate real balances, $\phi_0 \gamma M$, above their steady-state value, and reduces the gross rate of return of money, R_1 , below one.*
2. *If (42) holds, then $\phi_0 > \gamma \phi_1$, i.e., there is deflation, and CM output increases, $H_0 > H_1$.*
3. *It generates a mean-preserving reduction in the distribution of real balances in the DM of $t = 1$, an increase in aggregate DM output if $v'' > 0$, and an increase in society's welfare.*
4. *The economy returns to its steady state in the CM of $t = 1$.*

5.2 Large injections

We now consider the case of a large money injection. Suppose that all buyers enter the DM of period 1 with z_1^* real balances where z_1^* solves (38), i.e., the distribution of real balances is degenerate. Assuming that the economy has reached its steady state in the CM of $t = 1$ it follows that

$$z_1^* = \alpha \bar{h} + (1 - \alpha) z^*. \quad (52)$$

The right side of (52), z_1^* , tends to 0 as R_1 approaches 0 and it is equal to z^* when $R_1 = 1$. Hence, (52) determines a unique $R_1 < 1$ which is independent of γ . So an increase in the size of the money injection affects current and future prices in the same proportion so as to keep their ratio, ϕ_1/ϕ_0 , constant. As before, the money injection generates a mean-preserving decrease in the spread of the distribution of real balances across buyers. Aggregate output in the DM of $t = 1$ is

$$Y_1 = v^{-1} [\alpha \bar{h} + (1 - \alpha)z^*] \geq Y^{ss} = \alpha v^{-1}(\bar{h}) + (1 - \alpha)v^{-1}(z^*).$$

It is independent of the size of the money injection, but it is larger than the steady-state value provided that $v'' > 0$.

We need to check that the buyers who enter the CM of $t = 0$ with no money balances are not constrained by their endowment of labor. This will be the case if $R_1 \bar{h} + \left(1 - \frac{1}{\gamma}\right) \phi_1 \gamma M > z_1^*$, i.e.,

$$\gamma R_1 > \frac{\alpha \bar{h} + (1 - \alpha)z^*}{\bar{h}}. \quad (53)$$

So an equilibrium with a degenerate distribution of real balances at $t = 1$ exists provided that the size of the transfer is sufficiently large. Moreover, from (53) the rate of return of money is $R_1 > \gamma^{-1}$. So for large money injections it is always the case that $\phi_0 < [\alpha \bar{h} + (1 - \alpha)z^*]/M$, prices increase relative to their initial steady-state value. As a result, from (48), buyers holding m_h do not supply any labor. Buyers holding no money supply

$$\begin{aligned} h_0^+(0) &= \frac{z_1^*}{R_1} - (\gamma - 1)\phi_0 M \\ &= \frac{\alpha \bar{h} (1 - \gamma R_1) + (1 - \alpha)z^*}{(1 - \alpha)\gamma R_1}. \end{aligned} \quad (54)$$

From (47) and (54) both $h_0^+(0)$ and $h_0^+(m_\ell)$ decrease with γR_1 . Using that $\gamma R_1 > 1$ it follows that aggregate output in the CM of $t = 0$ is lower than its steady-state value.

Proposition 4 (*Large money injection.*) *Suppose the economy is initially at a steady state with $N = 2$. A large one-time money injection, $(\gamma - 1)M$, in the CM of $t = 0$ such that (53) holds has the following consequences:*

1. *It raises aggregate real balances, $\phi_0 \gamma M$, above their steady-state value, and reduces the gross rate of return of money, R_1 , below one. Moreover, $\phi_0 < \gamma \phi_1$ and $H_0 < H_1$.*
2. *The distribution of real balances is degenerate in the DM of $t = 1$ and DM aggregate output is higher than its steady-state value if $v'' > 0$.*
3. *The economy returns to its steady state in the CM of $t = 1$.*

6 Long-lasting effects of a one-time money injection

So far we have restricted our attention to equilibria with $N = 2$ because such equilibria generate simple, one-period transitions to a steady state. We now consider steady states with $N \geq 3$ and we assume a small

injection of money so that the distribution of real balances preserves N mass points with probabilities given by (26)-(27). We will focus on equilibria with full depletion.

Consider period $t \geq N$. Buyers who have not been matched in any DM since the money injection in $t = 0$ had the possibility to reach their targeted real balances. Hence, aggregate real balances in period t are defined by

$$\phi_t \gamma M = \sum_{j=1}^{N-1} \mu_j \bar{h} \left(\sum_{n=1}^j \frac{\phi_t}{\phi_{t-n}} \right) + \mu_N z_t^*, \text{ for all } t \geq N. \quad (55)$$

In order to explain the right side of (55) recall that we can index buyers by the last time they had a DM encounter and depleted their money balances. There is a measure μ_1 of buyers who entered the CM of $t - 1$ with depleted money balances. Those buyers accumulated $\bar{h}$ real balances and entered the period t with $R_t \bar{h}$ real balances. There is a measure μ_2 of buyers who entered the CM of $t - 2$ with no money balances and who were unmatched in the DM of $t - 1$. Such buyers accumulated $\bar{h}$ in the CM of $t - 2$ and the CM of $t - 1$ so that their real balances at the beginning of t are $R_{t-1} R_t \bar{h} + R_t \bar{h}$. We have the same reasoning for buyers who entered the CM of $t - n$ with no money and did not trade in all subsequent DMs until the beginning of t , where $n \leq N - 1$. Finally, there is a measure μ_N of buyers who entered the CM of $t - N$ with no money and did not get matched in the DMs of $t - N + 1$ until the DM of $t - 1$. Those buyers whose labor supply is unconstrained accumulate z_t^* solution to

$$\omega'(z_t^*) = 1 + \frac{1 - \beta R_t}{\alpha \beta R_t}. \quad (56)$$

Let us turn to periods $t < N$. Buyers hold onto the transfer of money they received in the CM of $t = 0$ until they reach their target. Hence, aggregate real balances are

$$\begin{aligned} \phi_t \gamma M &= \sum_{j=1}^{t-1} \mu_j \bar{h} \left(\sum_{n=1}^j \frac{\phi_t}{\phi_{t-n}} \right) + \mu_N z_t^* \\ &\quad + \sum_{j=t}^{N-1} \mu_j \left\{ \phi_t [m_{j-t} + (\gamma - 1)M] + \bar{h} \sum_{n=1}^t \frac{\phi_t}{\phi_{t-n}} \right\}, \end{aligned} \quad (57)$$

for all $t \leq N - 1$ where

$$m_k = \frac{\alpha k \bar{h} M}{\bar{h} \{1 - (1 - \alpha)^{N-1} [(N - 1)\alpha + 1]\} + \alpha(1 - \alpha)^{N-1} z^*}.$$

The first term on the right side of (57) corresponds to buyers who have been matched since the money transfer took place. There is a measure μ_j of buyers who had their last match in the DM of $t - j \geq 1$. Such buyers accumulated $\bar{h}$ real balances in every CM from $t - j$ to $t - 1$. The value of the real balances accumulated in the CM of $t - n$ are $\bar{h}(R_{t-n+1} \times \dots \times R_t)$. The second term on the right side of (57) corresponds to buyers who have reached their target. The novelty relative to (55) is the third term on the right side of (57). For all j between t and $N - 1$ there is a measure μ_j of buyers who entered the CM of $t = 0$ with $m_{j-t} = (j - t)\bar{h}/\phi$ units of money, where ϕ is the initial steady-state value given by (28). Such buyers have not reached their targeted real balances. So they kept their initial wealth, composed of the initial m_j units of money and the transfer $(\gamma - 1)M$, and they accumulated $\bar{h}$ real balances in the following t consecutive periods.

A candidate equilibrium is a sequence $\{\phi_t\}_{t=0}^{+\infty}$ that satisfies (55)-(57). In order to guarantee that we have an equilibrium we check that $h \leq \bar{h}$ binds for the buyer of type $N - 1$ but not for the type N . These conditions can be written as

$$\phi_t m_{N-1-t} < z_t^* - \bar{h} \sum_{n=1}^t \frac{\phi_t}{\phi_{t-n}} - (\gamma - 1)\phi_t M \leq \phi_t m_{N-t}, \quad \forall t \leq N - 1 \quad (58)$$

$$\bar{h} \sum_{n=1}^{N-1} \frac{\phi_t}{\phi_{t-n}} < z_t^* \leq \bar{h} \sum_{n=1}^N \frac{\phi_t}{\phi_{t-n}}, \quad \forall t \geq N. \quad (59)$$

We must also check that buyers have incentives to deplete their money holdings in full at all point in time, i.e.,

$$1 + \frac{1 - \beta R_t}{\alpha \beta R_t} \geq \lambda_t(0) \quad \text{for all } t, \quad (60)$$

where, from (17),

$$\begin{aligned} \lambda_t(0) &= \sum_{n=1}^{N-1} \alpha(1 - \alpha)^{n-1} \beta^n \prod_{s=1}^n R_{t+s} \omega' \left[\sum_{u=1}^n \bar{h} \prod_{j=0}^{u-1} R_{t+n-j} \right] \\ &\quad + (1 - \alpha)^{N-1} \beta^{N-1} \prod_{s=1}^{N-1} R_{t+s}. \end{aligned} \quad (61)$$

It can be seen from (55) and (57) that the transition to a steady state is long lived because the real balances of a buyer whose last match was in $t - j$, with $j < N - 1$, depends on the sequence of past rates of return, R_{t-j+1} to R_t .

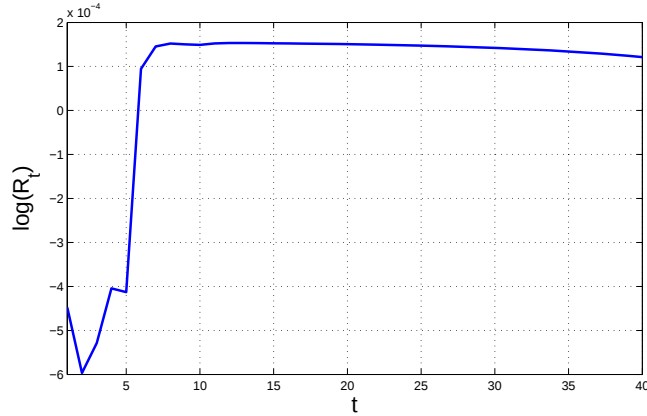


Figure 10: Transitional dynamics following a one-time money injection

In Figure 10 we provide a numerical example for the transitional dynamics of the rate of return of money, R_t , following a one-time money injection. We adopt the following parameter values: $\bar{h} = 0.095$, $\alpha = 0.1$, $u(c) = 2\sqrt{c}$, $r = 0.04$, $\gamma = 1.05$. For such parametrization it takes $N = 6$ periods for buyers with depleted money balances to reach the targeted real balances. For the first six periods following the increase in the money supply the rate of return of money is negative, i.e., the value of money declines over time. The fall in the value of money overshoots its new steady-state value. As a result for all $t \geq 7$ the rate of return

of money is positive (but small). So the initial periods of inflation are followed by long-lasting periods of deflation. The economy returns to its steady state asymptotically.

7 Constant money growth

So far we have described a one-time, unanticipated change in the quantity of money.⁹ We now turn to the case where the money injection is repeated every period, which leads to anticipated inflation. Suppose that the government increases the money supply in every period by injecting $M_{t+1} - M_t = (\gamma - 1)M_t$ at the beginning of each CM of period t . We focus on steady-state equilibria where aggregate real balances are constant and $N = 2$. The gross rate of return of money is $R = \gamma^{-1}$. Hence, the targeted real balances are

$$\omega'(z_\gamma^*) = 1 + \frac{\gamma - \beta}{\beta\alpha}. \quad (62)$$

From (62) the targeted real balances decrease with the rate of growth of money supply. At the beginning of period t the distribution of real balances has two mass points. There is a measure α of buyers holding $z_\ell \equiv \gamma^{-1}\bar{h} + (\gamma - 1)\phi_t M_{t-1}$. Those buyers depleted their money holdings in the previous DM, they supplied all their labor in the CM, and they kept the money transfer. There is a measure $1 - \alpha$ of buyers holding their targeted real balances, z_γ^* . Aggregate real balances are

$$Z \equiv \phi_t M_t = \alpha \left[\frac{\bar{h}}{\gamma} + (1 - \gamma^{-1})\phi_t M_t \right] + (1 - \alpha)z_\gamma^*. \quad (63)$$

Solving for aggregate real balances we obtain:

$$Z = \frac{\alpha\gamma^{-1}\bar{h} + (1 - \alpha)z_\gamma^*}{\alpha\gamma^{-1} + 1 - \alpha}. \quad (64)$$

From (64) aggregate real balances (and CM output) are a weighted average of the buyer's labor endowment and his targeted real balances where the weights vary with the money growth rate. As γ increases the weight on $\bar{h}$ decreases, which tends to raise real balances, but the targeted real balances decrease, which tends to reduce aggregate real balances. The overall effect of a small increase in the money growth rate above $\gamma = 1$ is given by:

$$\left. \frac{\partial Z}{\partial \gamma} \right|_{\gamma=1} = (1 - \alpha) \left[\alpha (z^* - \bar{h}) + \frac{1}{\beta\alpha\omega''(z^*)} \right].$$

A small inflation raises aggregate real balances if (42) holds. If ω' is very elastic to a change in z then buyers will not want to change their target much as the cost of real balances changes because the insurance provided by real balances is very valuable. However, the real balances of the poorest buyers increase due to the lump-sum transfer. In this case there is an increase in the mean of the distribution of real balances and a decrease in its dispersion. Hence, CM output and welfare increase.

⁹More precisely, we have assumed that agents do not assign a positive probability to a change in the money supply ahead of time. However, when the change in the money supply happens it is common-knowledge. In the Appendix A6 we consider the case where a one-time money injection is announced one period ahead. The results are essentially unaffected except for the fact that the early announcement triggers real effects ahead of the actual money injection.

In order to characterize the distribution of real balances we now compute the real balances of the poorest buyers, z_ℓ . From (64) it is

$$z_\ell \equiv \frac{\gamma^{-1}\bar{h} + (1 - \gamma^{-1})(1 - \alpha)z_\gamma^*}{\alpha\gamma^{-1} + 1 - \alpha}. \quad (65)$$

Differentiating z_ℓ we obtain

$$\left. \frac{dz_\ell}{d\gamma} \right|_{\gamma=1} = (1 - \alpha)(z^* - \bar{h}) > 0.$$

For low inflation rates z_ℓ increases with γ while z_γ^* decreases with γ . This effect corresponds to the redistributive role of inflation. Moreover, from (65) $z_\ell \leq z_\gamma^*$ if and only if $\gamma \leq \bar{\gamma}$ where $z_{\bar{\gamma}}^* = \bar{h}$. Below the threshold $\bar{\gamma}$ the distribution of real balances has two mass points, z_ℓ and z_γ^* . The upper bound, z_γ^* , decreases with inflation. The lower bound, z_ℓ , is first increasing and then decreasing. See Figure 11.

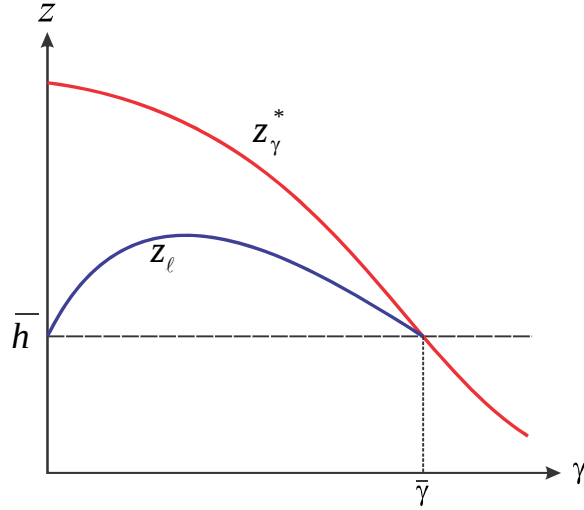


Figure 11: Output levels and inflation

We now study the welfare effect of anticipated inflation. We define society's welfare at a steady state with money growth rate γ by

$$\mathcal{W}_\gamma = \alpha^2 [\omega(z_\ell) - z_\ell] + (1 - \alpha)\alpha [\omega(z_\gamma^*) - z_\gamma^*]. \quad (66)$$

The first term on the right side of (66) corresponds to matches between a buyer holding z_ℓ and a seller. There is a measure α of such buyers and each of them has a probability α of being matched, so the total number of matches is α^2 . Similarly, the second term of the welfare function corresponds to matches between buyers holding their targeted real balances—there is a measure $1 - \alpha$ of such buyers—and sellers. We differentiate $\mathcal{W}_\gamma$ in the neighborhood of a constant money supply to obtain:

$$\left. \frac{d\mathcal{W}_\gamma}{d\gamma} \right|_{\gamma=1} = (1 - \alpha) \left\{ \alpha^2 [\omega'(\bar{h}) - 1] (z^* - \bar{h}) + \frac{r}{\omega''(z^*)\beta\alpha} \right\}.$$

Hence, inflation is welfare improving if

$$\frac{-z^*\omega''(z^*)}{\omega'(z^*)} > \frac{r}{(\alpha + r)\beta\alpha^2 [\omega'(\bar{h}) - 1]} \left(\frac{z^*}{z^* - \bar{h}} \right). \quad (67)$$

Condition (67) is satisfied if r is small so that the output consumed by the richest agents, $v^{-1}(z^*)$, is close to the first-best level, y^* , and the elasticity of ω' is large so that risk sharing is very valuable. Recall that the condition for a laissez-faire equilibrium with $N = 2$ is (35). In Figure 12 we plot the conditions (35) and (67) for $\alpha = 0.9$ and $u(c) = c^{1-1/\gamma_c} / (1 - 1/\gamma_c)$. It shows that the set of parameter values for which an equilibrium with $N = 2$ exists and a positive money growth rate is socially optimal has a positive measure. For given $\bar{h}$ and γ_c the rate of time preference cannot be too low or too high. Similarly, for given r there is an interval of values for $\bar{h}$ consistent with an equilibrium and a role for inflation.

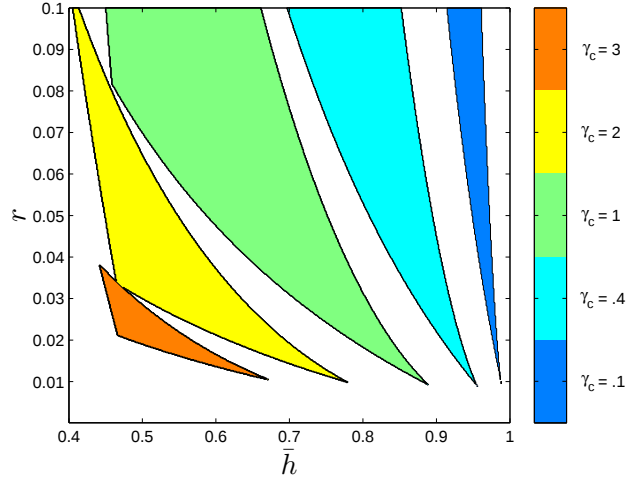


Figure 12: Conditions on parameters for an equilibrium with full depletion and welfare-improving inflation

Proposition 5 (Steady money growth.) *Consider a laissez-faire equilibrium with $\gamma = 1$ and $N = 2$. A small and constant money growth rate raises CM output if (42) holds and it raises social welfare if (67) holds.*

So far we have restricted ourselves to a pure currency economy with no enforcement technology. In such an economy a deflation implemented with lump-sum taxes is not feasible. We now relax this assumption and allow for deflation. In the New-Monetarist model the Friedman rule is optimal (unless there are search externalities), and under buyers-take-all bargaining it implements the first best allocation. We will see that allowing the constraint on buyers' labor supply to bind, thereby departing from quasi-linear preferences, the Friedman rule is not longer feasible.

Suppose that the monetary authority can engineer a rate of return for currency equal to $R_t = \phi_t / \phi_{t-1} = \beta^{-1}$ by contracting the money supply at the gross growth rate $\gamma = \beta$. The contraction of M is achieved by imposing a lump-sum tax on all buyers equal to $(1 - \beta)\phi_t M_t$. First, we establish that if $R_t = \beta^{-1}$ then at a steady state $y = y^*$ in all DM matches. From (62) the DM consumption at the targeted real balances, denoted z_β^* , is $y(z_\beta^*) = y^*$. From (12) the marginal value of real balances after a DM trade is $\lambda [z_\beta^* - v(y^*)] = 1$, i.e., following a DM trade the buyer is unconstrained by his labor endowment in the

following CM and hence he is able to get back to his target. Consequently, in any steady state all buyers should hold their target. (Note that the target might not be unique.)

Second, we check whether it is feasible for the buyer to remain at his target given his endowment of labor. Denote $\underline{z} = \inf\{\text{supp}(F)\}$. By definition $\underline{z} \leq Z$. Consider a buyer holding $\underline{z}$ at the beginning of the DM. Suppose he is matched in the CM and spends $p = v(y^*)$ in order to finance y^* . His next-period real balances are

$$\begin{aligned} z' &= (1+r) [\underline{z} - v(y^*) + h - (1-\beta)\phi_t M_t] \\ &= (1+r) [\underline{z} - v(y^*) + h] - rZ. \end{aligned}$$

In a steady-state equilibrium $z' \in \text{supp}F$ and hence $z' \geq \underline{z}$. It follows that

$$r(Z - \underline{z}) + (z' - \underline{z}) = (1+r) [h - v(y^*)].$$

The first two terms on the left side are non-negative while the term on the right side is negative whenever $\bar{h} < v(y^*)$. Hence, there does not exist an equilibrium at the Friedman rule unless $\bar{h} \geq v(y^*)$. Intuitively, if the rate of return of money is equal to the rate of time preference, buyers should accumulate real balances in order to achieve full insurance. If real balances are finite, they must have enough labor endowment to rebuild their real balances following a DM trade, in which case $N = 1$. Alternatively, aggregate real balances could be infinite in which case the lump-sum tax required to engineer a deflation at rate r is not feasible given buyer's finite labor endowment.

8 Conclusion

The objective of this paper was to construct a tractable of monetary exchange with search and bargaining featuring a non-degenerate distribution of real balances that can be used to study the short-run and long-run effects of monetary policy. We added a single change to the New-Monetarist environment of Lagos and Rocheteau (2005): agents have a finite endowment of labor. When this constraint on labor supply binds, value functions are no longer linear and the equilibrium distribution of real balances is non-degenerate. The model remains solvable in closed form for a large set of parameter values and it can easily be solved numerically for other parameter values. The model generates new insights for short-run and long-run effects of money. A one-time injection of money in a centralized market with flexible prices and unrestricted participation leads to higher aggregate real balances in the short run and, under some conditions, higher aggregate output. The effects on the rate of return of money and prices are non-monotone with the size of the money injection. We provided examples where money non-neutralities are long lived and non-monotone over time. Finally, a constant money growth rate can lead to higher aggregate real balances and output if agents are sufficiently risk averse, and it can raise society's welfare.

Our model is highly tractable and it can be extended in many directions. So far we have considered a simple departure from quasi-linear preferences by introducing a binding endowment for labor. Alternatively,

we could assume general preferences for buyers in the CM, $U(c, \bar{h} - h)$, as shown in Rocheteau, Weill, and Wong (2014) in the context of a continuous-time model. One can also endogenize the entry of sellers in order to relate the frequency of trades to the distribution of wealth across buyers. One can introduce multiple assets that differ in their liquidity properties and rates of return in order to study how individuals' portfolios change as they accumulate wealth. In the presence of monitoring our model can be used to study unsecured credit and the dynamics of debt accumulation.

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Appendix A1: Proofs of Lemmas and Propositions

Proof of Lemma 1 From (7) we define a mapping $T : \mathcal{B}(\mathbb{R}_+ \times \mathbb{N}) \rightarrow \mathcal{B}(\mathbb{R}_+ \times \mathbb{N})$, where $\mathcal{B}(\mathbb{R}_+ \times \mathbb{N})$ is the complete metric space of bounded functions (endowed with the sup metric), as:

$$Tf(z, t) = \max_{y, z'} \left\{ z - \frac{z'}{R_{t+1}} + \beta \alpha \{u(y) + f[z' - v^{-1}(y), t + 1]\} + \beta(1 - \alpha)f(z', t + 1) \right\},$$

subject to $z' \geq 0$, $z'/R_{t+1} - z \in [\underline{h}, \bar{h}]$ and $y \leq v^{-1}(z')$. The mapping T satisfies the Blackwell's monotonicity and discounting sufficient conditions for a contraction with modulus β (Theorem 3.3 in Stokey and Lucas, 1989). From Banach fixed point theorem it admits a unique fixed point. By the Theorem of the Maximum (Theorem 3.6 in Stokey and Lucas) if f is continuous in z so it Tf since the constraint set is a continuous correspondence. By the same reasoning Tf preserves the concavity of f with respect to z since the constraint set of a convex correspondence (see Theorem 4.7 in Stokey and Lucas, 1989). The differentiability follows from the Benveniste and Scheinkman theorem (Theorem 4.10 in Stokey and Lucas, 1989). ■

Proof of Proposition 2 The first part of the proposition follows from the discussion in the text. Consider a convergent sequence of discount rates, $\{r_i\}_{i=0}^{\infty}$, such that for all r_i there exists an equilibrium with full depletion, i.e., (34) holds. The equilibrium obtained at the limit when $i \rightarrow \infty$ implements the first best if the following is true. For all $\varepsilon > 0$, there is a $I \in \mathbb{N}$ such that for all $i \geq I$ and for all $j \in [0, \alpha]$, $|y_i(j) - y^*| < \varepsilon$, where $y_i(j)$ is the output level in a DM match j when $r = r_i$ and $[0, j]$ represent the set of all matches. A necessary condition is $\bar{h} \geq v(y^*)$ since otherwise the output level in matches where buyers hold $\bar{h}$ real balances is bounded away from y^* for all r_i . From (21) a second necessary condition is $r_i \rightarrow 0$ since otherwise the targeted real balances are bounded away from $v(y^*)$ and output levels are bounded away from y^* . If these conditions hold, then for all $I \in \mathbb{N}$ the equilibrium features $N = 1$. Moreover, from (21) as $i \rightarrow \infty$, $y(z^*) \rightarrow y^*$. ■

Appendix A2: Map of equilibria

In the following we illustrate the set of equilibria for our economy for different values for $\bar{h}$, r , and α . We adopt the following functional forms: $u(y) = 2\sqrt{y}$ and $v(y) = y$. In Figure 13 we set $r = 0.04$. The colored area corresponds to equilibria with full depletion of real balances. The white area is when the equilibrium features partial depletion. Equilibria with full depletion exist when $\bar{h}$ is above a threshold. Moreover, as $\bar{h}$ increases N decreases since buyers can reach their targeted real balances in a smaller number of periods.

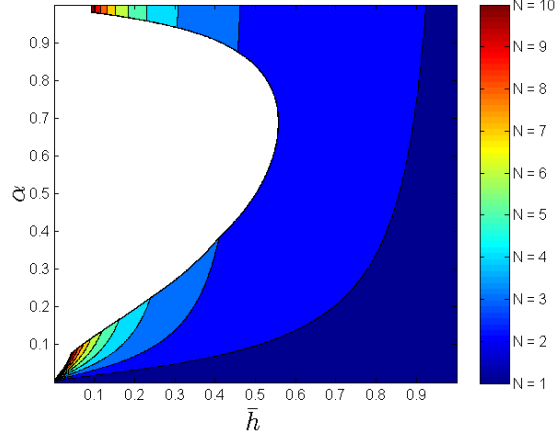


Figure 13:

In Figure 14 we allow r to vary and we set $\alpha = 0.1$. This figure is analogous to 4. For a given labor endowment N decreases as r decreases since more patient buyers have higher targeted real balances. When r is sufficiently low the equilibrium features partial depletion of real balances.

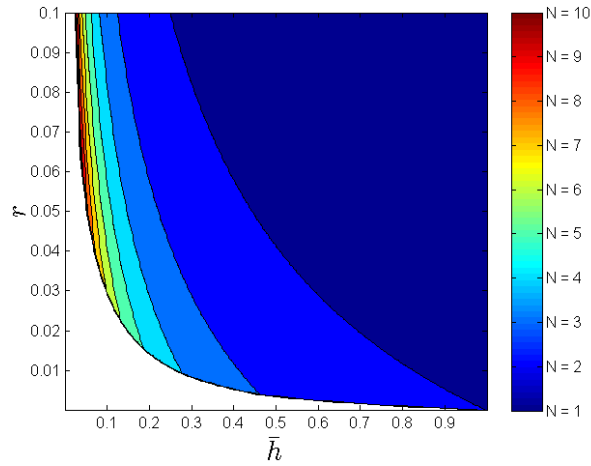


Figure 14:

Appendix A3: Algorithm to compute value functions and stationary distributions

The algorithm to compute the value function, $W(z)$, and the distribution of real balances, F , at a steady-state equilibrium is composed of four steps:

Step 0. Fix some initial guess of $W_0(z)$ for the domain $z \in [0, \bar{z}]$, where $\bar{z}$ should be reasonably large. For example set $W_0(z)$ to be the closed-form case with $N = 2$, and $\bar{z} = 3(u')^{-1}(1 + r/\alpha)$.

Step 1. Iterate the following k -th value function

$$W_{k+1}(z) = z + \max_{\substack{h \leq \bar{h}, \\ y \leq \bar{h} + z}} \{-h + \beta [\alpha [u(y) + W_k(z + h - y)] + (1 - \alpha) W_k(z + h)]\},$$

until it reaches some tolerance level, for example $\|W_{k+1}(z) - W_k(z)\| \leq 10^{-6}$. Obtain the value function $W(z) = W_k(z)$.

Step 2. Obtain the policy function $h(z)$ and $y(z)$. Initiate $s = 1, 2, \dots, 10^6$ sample points with $z_1^s = 0$.

Step 3. For each s , generate a uniform random variable U_s . Set $z_{t+1}^s = z_t^s + h(z_t^s) - I_{U_s \leq \alpha} y(z_t^s)$.

Step 4. Iterate z_t^s until $t = T = 10^5$. Obtain the stationary distribution $F(z)$ as the empirical CDF of z_T^s .

Appendix A4. Numerical Algorithm to Compute the Transitional Dynamics

Step 0. Fix a parameter that features a full-depletion stationary equilibrium with N mass points, for example we set $N = 4$ in the numerical example. Set a large $T > N$ number of periods of the transitional dynamics $\mathbf{R} = \{R_1, R_2, \dots, R_T\}$ and $\phi = \{\phi_0, \phi_1, \dots, \phi_T\}$, for example we set $T = 100$ in the numerical example.

Step 1. Define the constraints: for all $t \geq N$

$$\phi_t \gamma M = \sum_{j=1}^{N-1} \mu_j \left[\sum_{n=1}^j \bar{h} \prod_{s=1}^n R_{t+1-s} \right] + \mu_N z_t^* \quad (68)$$

and for all $t \leq N-1$

$$\phi_t \gamma M = \frac{\sum_{j=1}^{t-1} \mu_j \left[\sum_{n=1}^j \bar{h} \prod_{s=1}^n R_{t+1-s} \right] + \sum_{j=t}^{N-1} \mu_j \sum_{n=1}^t \bar{h} \prod_{s=1}^n R_{t+1-s} + \mu_N z_t^*}{1 - \frac{1}{\gamma} \sum_{j=t}^{N-1} \mu_j [m_j + (\gamma - 1)]} \quad (69)$$

where

$$m_j = \frac{\alpha(j-t)\bar{h}}{\bar{h} \{1 - (1-\alpha)^{N-1} [(N-1)\alpha + 1]\} + \alpha(1-\alpha)^{N-1} z^*}.$$

The real balances of the $N-1$ th agent is

$$z_{N-1,t} = \begin{cases} (N-1)hR_0, & \text{if } t=0 \\ h \left[(N-1-t) \prod_{s=1}^{t+1} R_{t+1-s} + \sum_{j=1}^t \prod_{s=1}^j R_{t+1-s} \right], & \text{if } t \in \{1 \dots N-2\} \\ \sum_{n=1}^{N-1} h \prod_{s=1}^n R_{t+1-s}, & \text{otherwise} \end{cases}$$

The marginal value of real balances is

$$\lambda_t(0) = \sum_{j=1}^{N-1} \left\{ \mu_j \beta^j \omega' \left(\sum_{n=1}^j \bar{h} \prod_{s=1}^n R_{t+j-s+1} \right) \prod_{s=1}^j R_{t+j-s+1} \right\} + \beta^{N-1} \mu_N \prod_{j=1}^{N-1} R_{t+N-j}$$

The sufficient condition for full depletion is

$$1 + \frac{(1+r)/R_t - 1}{\alpha} \geq \lambda_t(0). \quad (70)$$

The sufficient condition for N mass point is

$$\omega'(z_{N-1,t}) > 1 + \frac{(1+r)/R_t - 1}{\alpha} \geq \omega'(z_{N,t}) \quad (71)$$

Step 2. Solve the standard constrained minimization problem: $\varepsilon \equiv \min_{\phi, \mathbf{R}} (R_T - 1)^2$ subject to $R_t = \phi_t / \phi_{t-1}$, (68), (69), (70), and (71). Accept the transitional dynamics $\mathbf{R}$ if the error ε is less than some tolerance level, says 10^{-6} : after T period R_T is in the neighborhood of its steady state level 1. Notice that the algorithm does not always guarantee to reach a solution $\mathbf{R}$ for any parameter values, as the economy may feature partial depletion or the number of mass point changes along the transition dynamics.

Appendix A5. Money transfers to both buyers and sellers

In the main text we assumed that only buyers receive a lump-sum transfer following a one-time money injection. Suppose now that both buyers and sellers receive a transfer. Since buyers represent half of the population the size of the transfer is $(\gamma - 1)M/2$. We will focus on the case of a small money injection, γ is close to 1.

The real balances at the beginning of $t = 1$ of the buyers holding $m = 0$ at the time of the money injection are

$$z_1^0 = R_1 \bar{h} + (\gamma - 1)\phi_1 \frac{M}{2}. \quad (72)$$

The only difference with respect to (37) is the second term on the right side where M is replaced with $M/2$. Buyers holding m_ℓ and m_h accumulate z_1^* solution to (38). Aggregate real balances at the beginning of $t = 1$ are $\phi_1 \gamma M = \alpha z_1^0 + (1 - \alpha)z_1^*$. Assuming that the economy returns to its steady state in the CM of $t = 1$ we have:

$$\phi_1 \gamma M = \frac{\alpha R_1 \bar{h} + (1 - \alpha)z_1^*}{1 - \alpha(1 - \gamma^{-1})/2} = \alpha \bar{h} + (1 - \alpha)z^*. \quad (73)$$

The last term on the right corresponds to aggregate real balances at the steady state. By the same reasoning as in the text (73) determines a unique R_1 and it such that $R_1 < 1$. The value of money in the CM of $t = 0$, ϕ_0 , is less than its steady-state value. Differentiating the second equality on the right of (73) we obtain

$$\left. \frac{dR_1}{d\gamma} \right|_{\gamma=1} = \frac{-\frac{\alpha}{2} [\alpha \bar{h} + (1 - \alpha)z^*]}{\alpha \bar{h} - (1 - \alpha)\frac{(1+r)}{\omega''(z^*)\alpha}}.$$

Hence,

$$\left. \frac{dR_1}{d\gamma} \right|_{\gamma=1} < -1 \Leftrightarrow -\frac{(1 - \alpha)(1 + r)}{\omega''(z^*)\alpha} < \frac{\alpha}{2} [(1 - \alpha)(z^* - \bar{h}) - \bar{h}].$$

Under the assumption $z^* < 2\bar{h}$ the right side of the inequality is negative for all $\alpha \in (0, 1]$ while the left side is positive since $\omega'' < 0$. So the inequality never holds. As a consequence, following a small money injection $R_1 \gamma > 1$, i.e., ϕ_0 is lower than its initial steady-state value. This result differs from the case where only buyers receive the transfer. Indeed, we showed that if buyers were sufficiently risk averse a small money injection could raise the value of money relative to its initial steady state.

The output levels in the DM of $t = 1$ are determined as in the text: $y_{\ell,1} = v^{-1}(z_1^0) > v^{-1}(\bar{h})$ and $y_{h,1} = v^{-1}(z_1^*) < v^{-1}(z^*)$. Aggregate DM output is

$$Y_1 = \alpha y_{\ell,1} + (1 - \alpha)y_{h,1} \geq Y^{ss} = \alpha v^{-1}(\bar{h}) + (1 - \alpha)v^{-1}(z^*),$$

with a strict inequality if v is strictly convex. So DM aggregate output increases relative to its steady-state value, Y^{ss} .

Let us turn to CM aggregate output. At $t = 0$ it is determined by (43) where individual labor supplies

are $h_0^+(0) = \bar{h}$ and

$$h_0^+(m_\ell) = \left\{ \frac{z_1^*}{R_1} - \phi_0 \left[m_\ell + (\gamma - 1) \frac{M}{2} \right] \right\}^+ \quad (74)$$

$$h_0^+(m_h) = \left\{ \frac{z_1^*}{R_1} - \phi_0 \left[m_h + (\gamma - 1) \frac{M}{2} \right] \right\}^+. \quad (75)$$

Substituting $\phi_0 m_\ell = \bar{h}/(R_1 \gamma)$, $\phi_0 m_h = z^*/(R_1 \gamma)$, $\phi_0 M = [\alpha \bar{h} + (1 - \alpha) z^*] / (\gamma R_1)$ into (74)-(75) and using the following expression for z_1^* obtained from (73),

$$\gamma z_1^* = \frac{[\gamma - \alpha(\gamma - 1)/2] [\alpha \bar{h} + (1 - \alpha) z^*] - \alpha \gamma R_1 \bar{h}}{1 - \alpha},$$

we obtain the following individual labor supplies:

$$h_0^+(m_\ell) = \frac{\{(\gamma + 1)(1 - \alpha)(z^* - \bar{h}) + (\gamma - 1 + 2\alpha)\bar{h} - 2\alpha\gamma R_1 \bar{h}\}^+}{2(1 - \alpha)\gamma R_1} \quad (76)$$

$$h_0^+(m_h) = \frac{(\gamma + 1)\alpha \bar{h} + (\gamma - 1)(1 - \alpha)z^* - 2\alpha\gamma R_1 \bar{h}}{2(1 - \alpha)\gamma R_1}. \quad (77)$$

One can check that $h_0^+(m_\ell)$ and $h_0^+(m_h)$ are decreasing functions of γR_1 . Given that $\gamma R_1 > 0$ for small money injections, it follows CM aggregate output decreases in $t = 0$.

Proposition 6 (Small money injection to buyers and sellers.) *Suppose the economy is initially at a steady state with $N = 2$. A small one-time money injection, $(\gamma - 1)M$, in the CM of $t = 0$ has the following consequences:*

1. *It raises aggregate real balances, $\phi_0 \gamma M$, above their steady-state value, and reduces the gross rate of return of money, R_1 , below one.*
2. *There is inflation in the short run, $\phi_0 < \gamma \phi_1$, and CM output decreases, $H_0 < H_1$.*
3. *It generates a mean-preserving reduction in the distribution of real balances in the DM of $t = 1$, an increase in aggregate DM output if $v'' > 0$, and an increase in society's welfare.*
4. *The economy returns to its steady state in the CM of $t = 1$.*

Long-lasting effects of a money injection We now consider equilibria with $N \geq 3$ and we assume that both buyers and sellers receive a lump-sum transfer. For all $t \geq N$ aggregate real balances are defined by (55). For all $t < N$ aggregate real balances are

$$\begin{aligned} \phi_t \gamma M &= \sum_{j=1}^{t-1} \mu_j \bar{h} \left(\sum_{n=1}^j \frac{\phi_t}{\phi_{t-n}} \right) + \mu_N z_t^* \\ &\quad + \sum_{j=t}^{N-1} \mu_j \left\{ \phi_t \left[m_{j-t} + (\gamma - 1) \frac{M}{2} \right] + \bar{h} \sum_{n=1}^t \frac{\phi_t}{\phi_{t-n}} \right\}, \end{aligned}$$

where

$$m_k = \frac{\alpha k \bar{h} M}{\bar{h} \{1 - (1 - \alpha)^{N-1} [(N - 1)\alpha + 1]\} + \alpha(1 - \alpha)^{N-1} z^*}.$$

The novelty with respect to (57) is the size of the money transfer that is now equal to $(\gamma - 1)M/2$.

Constant money growth rate. We now consider the case where the money supply grows at a constant rate and all agents (buyers and sellers) receive a lump-sum transfer in the CM of period t equal to $(\gamma - 1)M_t$. At the beginning of period t the distribution of real balances has two mass points. There is a measure α of buyers holding $z_\ell \equiv \gamma^{-1}\bar{h} + (\gamma - 1)\phi_t M_{t-1}/2$. There is a measure $1 - \alpha$ of buyers holding their targeted real balances, z_γ^* , solution to (62). Aggregate real balances are

$$Z \equiv \phi_t M_t = \alpha \left[\frac{\bar{h}}{\gamma} + (1 - \gamma^{-1})\phi_t \frac{M_t}{2} \right] + (1 - \alpha)z_\gamma^*. \quad (78)$$

Solving for aggregate real balances we obtain:

$$Z = \frac{\alpha\gamma^{-1}\bar{h} + (1 - \alpha)z_\gamma^*}{1 - \alpha(1 - \gamma^{-1})/2}. \quad (79)$$

Differentiating (79) with respect to γ we find:

$$\left. \frac{\partial Z}{\partial \gamma} \right|_{\gamma=1} = \frac{\alpha}{2} [(1 - \alpha)(z^* - \bar{h}) - \bar{h}] + (1 - \alpha) \frac{1}{\omega''(z^*)\beta\alpha} < 0.$$

So inflation has a negative effect on aggregate real balances. In contrast, when transfers can be directed to buyers only a small inflation can raise aggregate real balances provided that buyers are sufficiently risk averse.

Substituting Z given by (79) into the expression for z_ℓ we obtain:

$$z_\ell \equiv \frac{\gamma^{-1}\bar{h} + \frac{1}{2}(1 - \gamma^{-1})(1 - \alpha)z_\gamma^*}{1 - \alpha(1 - \gamma^{-1})/2}.$$

Differentiating z_ℓ we obtain

$$\left. \frac{dz_\ell}{d\gamma} \right|_{\gamma=1} = \frac{(1 - \alpha)(z^* - \bar{h}) - \bar{h}}{2} < 0.$$

An increase in γ reduces real balances of all buyers, including those who are constrained by their labor endowment. This result is also in contrast to the one obtained when transfers are received by buyers only. As a result inflation is welfare worsening. This result illustrates the fact that the effects of inflation can be highly dependent on the way money is introduced in the economy, something that is ignored in models with degenerate distributions.

A6. One-time, anticipated money injection

At the beginning of the CM in $t = 0$ agents are informed that the money supply will increase in the CM of $t = 1$ by $(\gamma - 1)M$, where $\gamma - 1$ is small. As in the previous section lump-sum money transfers are directed to buyers only. We focus on equilibria where the economy returns to its steady state one period after the money injection, in the CM of $t = 2$, and we solve the equilibrium by backward induction.

Aggregate real balances in $t = 2$ are at their steady-state level, $\gamma M \phi_2 = \alpha \bar{h} + (1 - \alpha)z^*$, i.e., a measure α of buyers hold $\bar{h}$ real balances, which corresponds to their labor endowment, while a measure $1 - \alpha$ hold their target, z^* . Aggregate real balances before the money injection are

$$\phi_1 M = \frac{\alpha \bar{h} + (1 - \alpha)z^*}{\gamma R_2}, \quad (80)$$

where $R_2 = \phi_2 / \phi_1 < 1$ solves an equation analogous to (40),

$$\frac{\alpha R_2 \bar{h} + (1 - \alpha)z_2^*}{1 - \alpha \left(1 - \frac{1}{\gamma}\right)} = \alpha \bar{h} + (1 - \alpha)z^*, \quad (81)$$

where z_2^* solves $\omega'(z_2^*) = 1 + (1 + r - R_2) / (\alpha R_2)$. In order to understand (81) recall that α agents enter period 2 with $R_2 \bar{h} + (\gamma - 1)\phi_2 M$ real balances while $1 - \alpha$ agents enter with their targeted real balances, z_2^* . From (81) the rate of return of money, R_2 , is unaffected by the announcement made in the initial period. The reason is that even though the announcement might affect the rate of return, R_1 , and the distribution of real balances at the beginning of period 1, it does not affect the target for period 2.

Let us turn to the rate of return of money in period 1. Aggregate real balances in $t = 1$ are $\phi_1 M = \alpha R_1 \bar{h} + (1 - \alpha)z_1^*$. From (80) R_1 solves

$$\alpha R_1 \bar{h} + (1 - \alpha)z_1^* = \frac{\alpha \bar{h} + (1 - \alpha)z^*}{\gamma R_2}. \quad (82)$$

The left side of (82) is increasing in R_1 and it is greater than the right side when $R_1 = 1$ if $\gamma R_2 > 1$. Hence, if $R_2 > \gamma^{-1}$ then $R_1 < 1$, i.e., $\phi_0 > \phi_1$. The value of money declines over time until it reaches its new steady-state value in the CM of $t = 2$. Because $R_1 < 1$ the real balances of all buyers in the DM of $t = 1$ are lower. Hence, aggregate DM output falls in $t = 1$, before the money injection. It increases in $t = 2$ following the money injection if $v'' > 0$ because of the mean-preserving redistribution of real balances. In the left panel of Figure 15 we represent the trajectories for aggregate DM output and the value of money.

Consider next the case where $\gamma R_2 < 1$ which implies $R_1 > 1$. The path for the value of money is such that $\phi_0 < \phi_1$ and $\phi_1 > \phi_2$. The value of money increases first and then decreases. In the DM of $t = 1$ all consumption levels increase whereas in the DM of $t = 2$ the consumption of the richest decreases while the consumption of the poorest increases. Welfare increases in both periods. In the right panel of Figure 15 we represent the trajectories for aggregate DM output and the value of money. Finally, in the knife-edge case where $\gamma R_2 = 1$ then $R_1 = 1$ so that prices stay constant between $t = 0$ and $t = 1$. The output levels in the DM of $t = 1$ are unchanged.

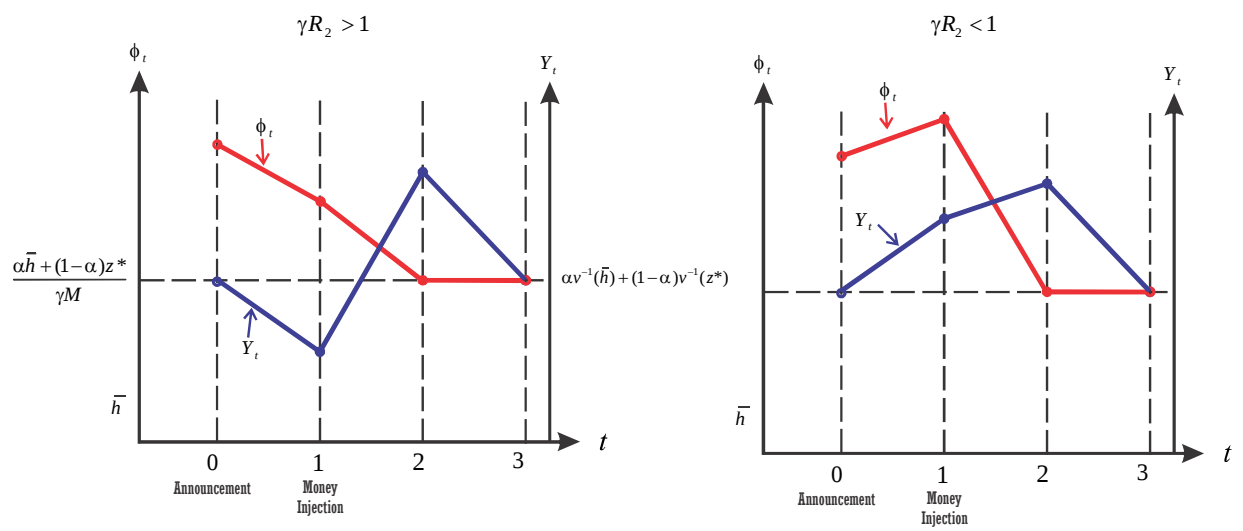


Figure 15: Small, anticipated, money injection