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The Interaction of Entry Barriers and Financial Frictions in Growth*

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ABSTRACT

This paper studies the interaction between financial frictions and firm entry barriers on growth. We construct a model in which aggregate growth is driven by the continual entry of new firms that face barriers to entry and financial frictions. We find that reforms to financial frictions and entry barriers are substitutes — once a country has enacted one type of reform, the percentage increase in GDP from the other reform decreases. We also show that economies with more severe financial frictions and entry costs have lower levels of output along the balanced growth path, even though all economies grow at the same constant rate. The model generates sharp predictions regarding entry barriers, financial frictions, and output levels, which are borne out in the cross country data.

Keywords: Economic growth; Barriers to entry; Financial frictions

JEL classification: D53; E27; O16

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1. Introduction

Economies often have numerous policies in place that impede growth. The existing literature has provided insight about the growth effects associated with removing individual distortions, yet, when countries undergo the kinds of reforms typically advocated for by economists — such as liberalizing capital markets or decreasing the costs of doing business — the results vary. In this paper, we argue that it is important to think about the interaction of policy reforms within a framework with multiple distortions.

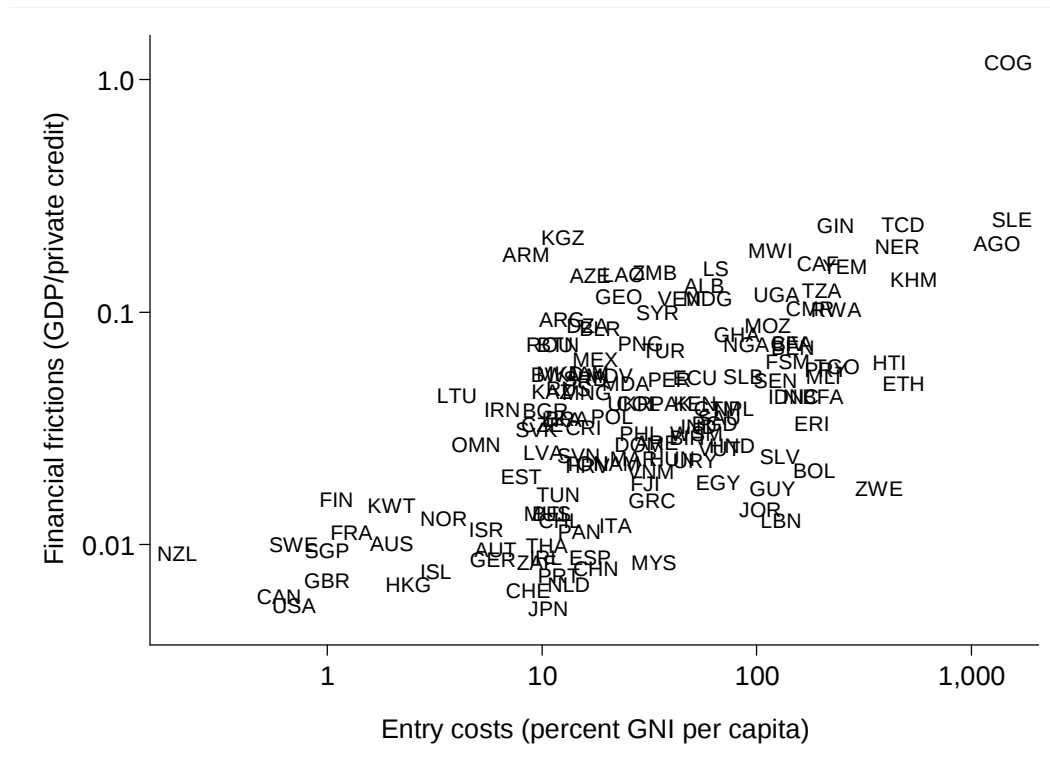
We construct a model in which the entry of new firms leads to long run growth. We use this model to study the impact of economic reform in an economy distorted by two policies: poor contract enforcement, which decreases the efficiency of the financial system, and policy-driven barriers that increase the entry cost of new firms. In the model, improving the financial system or decreasing firm entry costs increases output. These results are consistent with the previous literature that has studied these distortive policies in isolation. Our focus here, however, is on the interaction of the two policies. In the model, reforms are substitutes: Improving the financial system has a larger impact on output when entry costs are large, and reducing entry costs has a larger impact on output when it is difficult to borrow.

We have chosen to study the interaction between the distortions caused by entry costs and poorly functioning financial markets because they are particularly relevant: Countries with high costs of entry tend to be countries in which financial development is lagging. In figure 1 we plot a measure of the costs of starting a firm against a measure of financial development. Entry costs come from the World Bank's *Doing Business* surveys and consist of all expenses required to start a business (as a percentage of gross national income per capita). The measure of financial sector distortions is the *inverse* of private credit as a percentage of GDP, a commonly used measure of financial usage in quantitative papers on economic development and finance. It is clear from the figure that the countries with larger regulatory costs of starting a business are also the countries in which financing is harder to obtain.

Our model is populated by firms that are heterogeneous in productivity, and whose managers must make entry decisions. Potential entrants draw their productivity from a distribution that is improving over time: Aggregate growth is driven by the continual entry of

new firms that are, on average, more productive than the previous cohort.¹ After observing their productivity, potential entrants can choose to pay the entry cost to create a new firm or they can exit immediately. When entry costs are large, the marginal entrant will need to borrow to pay the entry cost; when financial markets do not function well, the distortions that arise from entry costs increase.

Figure 1: Entry costs and inverse of financial usage



Distortions in the financial system are the result of limited contract enforcement: A firm's manager may default on the firm's debt and abscond with a fraction of the firm's profits. The enforcement constraint distorts the entry margin in the model because some firms will be unable to finance the entry cost. In the balanced growth path, the strength of contract enforcement will be a factor in determining the cutoff productivity for new entrants.

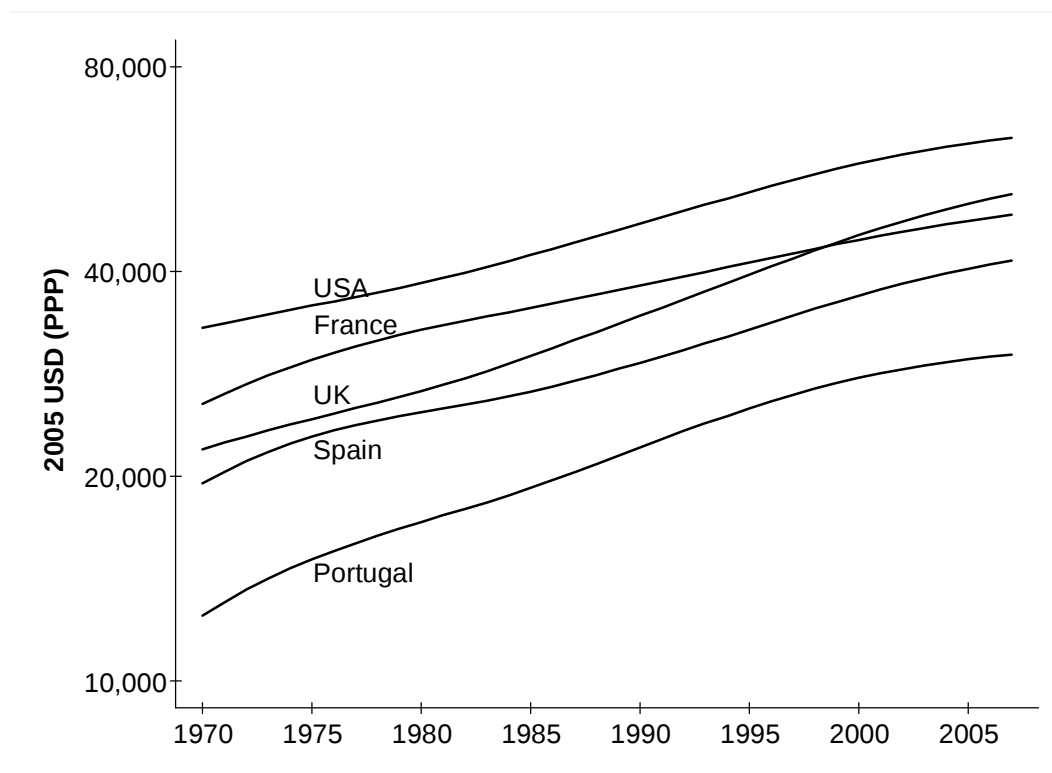
Given the complexity of the model — heterogeneous firms, firm entry and exit, and occasionally binding enforcement constraints — the model admits a surprisingly tractable

¹ This is consistent with empirical literature which finds that entry of new firms is important to the growth process. See Bartelsman et al. (2009), Bartelsman and Doms (2000), and Foster et al. (2001). Brandt et al. (2012) studies the case of China.

balanced growth path. The extreme tractability of this model makes it easy to see how the parameters that govern the distortions in the model affect output.

In the model's balanced growth path, all countries — regardless of the severity of the distortions — grow at the same rate. The rate of growth of the economy is equal to the rate of growth of the productivity distribution of potential entrants. In this way, the model captures the empirical regularity that countries grow at similar rates but on different levels. Figure 2, for example, shows economies growing at a rate of close to two percent per year since the 1970s, although at different levels. Our model is consistent with Parente and Prescott (2002) and Kehoe and Prescott (2007), who hypothesize that the levels of gross domestic product are determined by the policies and institutions of that country.

Figure 2: Real GDP per working age population



Notes: Hodrick–Prescott filtered GDP per working age person with the smoothing parameter set to 400. Working age is 15–64.

The model predicts that output is decreasing in the size of firm entry costs and the size of the distortion in the financial system, and that the two distortions amplify each other: Entry costs are more distortionary when financial markets do not function well. In section 5, we show that

these implications of the model are borne out in the data. We find that aggregate output is negatively correlated with entry barriers and financial frictions, a result that is consistent with previous studies. Moreover, we find that the interaction term between entry barriers and financial frictions is negative and statistically significant: If one friction is reduced, then the other becomes less related to income, consistent with our theoretical findings. Our estimates imply that GDP per capita in countries with poorly functioning financial markets such as Mexico is almost twice as sensitive to changes in entry costs than is output in countries with better functioning financial markets such as the United States.

A recent set of empirical and quantitative papers highlight the importance of financial frictions and entry costs in determining income levels. Amaral and Quintin (2010) find that economies in which all production is self-financed have income levels that are one-third that of an economy with financial markets similar to the United States.² Other papers have studied the effects of entry barriers through the suboptimal entry of new firms.³ These literatures have studied particular frictions in isolation: Our goal is to understand how the interaction of different frictions influences output.

Our work is complementary to the quantitative works of Buera et al. (2009) and Fang (2010). Buera et al. (2009) construct a model in which potential entrepreneurs face entry costs that vary by industry. This entry cost is interpreted as technological in nature, resulting in entrepreneurs operating at different scales across sectors. The paper finds that sectors that have higher entry costs are also the most sensitive to financial frictions. Fang (2010) also constructs a model in which firms have both entry costs and imperfect financial markets. Financial distortions are modeled as working capital constraints, which impinge on the entry of new firms and keep existing firms from achieving optimal scale. While these studies are focused on providing a quantitative assessment of the distortions, we are focused on building a rich but tractable framework that can be used to theoretically study multiple distortions. Our paper is also related to the work of Erosa and Hidalgo Cabrillana (2008), who study how financial

² See also Buera and Shin (2013), Greenwood et al. (2009), Greenwood et al. (2012), and Moll (2012). Our work is also complementary to Cole et al. (2012), which focuses on the impact of financial frictions on the technological choices of firms. Levine (2005) surveys the empirical literature.

³ Barseghyan (2008), Barseghyan and DiCecio (2011), Herrendorf and Teixeira (2009) conduct quantitative exercises and find that entry costs can significantly affect output. Nicoletti and Scarpetta (2003) empirically study the effects of entry costs. Djankov et al. (2002) documents the high level of entry costs across many countries.

frictions along with different entry costs across industries affect the allocation of resources in the economy.

Other quantitative papers have studied interaction among policies within the context of a model with heterogeneous firms. D’Erasmus et al. (2011) study the effects of changing financial frictions and the cost of operating in the formal sector. Bergoeing et al. (2011) investigate the joint outcomes of changing entry costs and the strength of bankruptcy laws. Moscoso Boedo et al. (2012) study the combined effects of reducing entry regulations and firing costs.⁴

The paper is organized as follows. Section 2 presents the model and defines the equilibrium. Section 3 characterizes the balanced growth path. Section 4 shows comparative statics of changes in entry barriers and financial frictions. Section 5 documents the main facts about aggregate output, entry costs, and financial frictions that are relevant to our theory. Section 6 concludes.

2. Model

We study an economy that is closed to foreign trade and capital flows. The production side of the economy is comprised of a representative final good producer and a continuum of monopolistically competitive intermediate goods producers. The intermediate good firms face endogenous borrowing constraints that arise from the limited enforcement of contracts as in Kehoe and Levine (1993) and Albuquerque and Hopenhayn (2004).

2.1. Households

The representative household is endowed with a unit of labor, which is inelastically supplied to firms. The problem of the household is

$$\begin{aligned} & \max_{C_t, B_{t+1}} \sum_{t=0}^{\infty} \beta^t \log C_t \\ & \text{s.t. } P_t C_t + q_{t+1} B_{t+1} = w_t + D_t + B_t \\ & C_t \geq 0, B_t \geq -g^t \bar{B}, B_0 \text{ given,} \end{aligned} \tag{1}$$

⁴ Our work is also related to past literature of the proper sequencing of reforms. See McKinnon (1973) for an example of early work in this field. Surveys of this literature include Edwards (1990) and Funke (1993). Fischer and Gelb (1991) discuss the sequencing of reforms for socialist economies in transition.

where $\beta \in (0,1)$ is the discount factor, C_t is consumption of the final good, P_t is the price of the final good, q_{t+1} is the price of a one-period bond, B_{t+1} is the face value of one-period debt purchased, w_t is the wage rate, D_t is the aggregate dividend paid by firms in the economy, and $g \geq 1$ is the growth factor of the economy, which we will characterize below. The condition $B_t \geq -g^t \bar{B}$, where $\bar{B}$ is large, rules out Ponzi schemes but otherwise does not bind in equilibrium. We normalize the price of the final good so that $P_t = 1$ in each period.

2.2. Final good producers

We model perfectly competitive final good firms that purchase intermediate goods, and assemble them to produce the final good. The representative final good firm minimizes costs and earns zero profits:

$$\begin{aligned} \min_{y_t(i)} \int_0^{\eta_t} p_t(i) y_t(i) di \\ \text{s.t. } \left(\int_0^{\eta_t} y_t(i)^\rho di \right)^{\frac{1}{\rho}} = Y_t, \end{aligned} \quad (2)$$

where $p_t(i)$ and $y_t(i)$ are the price and quantity of intermediate good i , η_t is the measure of intermediate goods available, $1/(1-\rho)$, for $0 < \rho < 1$, is the elasticity of substitution between intermediate goods, and Y_t is real aggregate output. Solving the final good producer's problem, we obtain the demand function for good i ,

$$y_t(i) = p_t(i)^{\frac{-1}{1-\rho}} P_t^{\frac{1}{1-\rho}} Y_t, \quad (3)$$

and the aggregate price index

$$P_t = \left(\int_0^{\eta_t} p_t(i)^{\frac{-\rho}{1-\rho}} di \right)^{\frac{1-\rho}{-\rho}}. \quad (4)$$

2.3. Intermediate goods producers

There is a continuum of heterogeneous intermediate good firms. A firm can produce for a maximum of two periods. In each period, a measure μ of potential entrants draw their marginal

productivities from a distribution and enter the market if it is profitable to do so. The firm producing good i uses labor to produce according to

$$y_t(i) = x(i)\ell_t(i), \quad (5)$$

where $x(i)$ is the productivity of firm i . Conditional on choosing to produce, a firm chooses its price to maximize profits,

$$\pi_{jt}(i) = \max_{p_t(i)} p_t(i)y_t(i) - w_t \left(\frac{y_t(i)}{x(i)} + \kappa_j \right), \quad (6)$$

where κ_j is the fixed cost of operating. This fixed cost is denominated in units of labor and is conditional on the age, j , of the firm. We consider a simple structure for the fixed costs. All firms face entry cost κ and all existing firms pay the continuation cost, f . We assume that $\kappa > f$ so that entry is more costly than continuing to produce. We interpret the entry cost to be made up of technological entry costs (for example, purchases of new equipment) and regulatory entry costs. These regulatory costs are the outcomes of policy and potentially can vary across different economies. Consequently, part of the parameter κ is our measure of the distortion generated by policy-driven barriers to entry.

The solution to the profit-maximization problem (6) yields the standard markup over marginal cost pricing,

$$p_t(i) = \frac{w_t}{\rho x(i)}. \quad (7)$$

Notice that every firm with productivity x chooses the same price. In what follows, we no longer characterize a good by its label i but by the productivity x of the firm that produces it.

The problem of an existing firm (i.e., a firm in its second period of life), with productivity x , can be written as

$$\begin{aligned} V_{2t}(b_t, x) &= \max \{d_{2t}(x), 0\} \\ \text{s.t. } V_{2t}(b_t, x) &\geq \theta \pi_{2t}(x) \\ d_{2t}(x) &\equiv \pi_{2t}(x) - b_t \geq 0 \end{aligned} \quad (8)$$

where b_t denotes the firm's debt holdings and d_{2t} is the firm's dividend payment. The first term in the maximand is the value of the firm if it continues to produce. The second term is the value

of exiting. Once a firm has exited, it cannot return to produce in a later period; the value of exit is zero. The manager of the firm can abscond with a fraction θ of the period's profits. We interpret this possibility as the result of poor contract enforcement. The first constraint is the enforcement constraint, which ensures that the firm's manager never prematurely exits: The value of honoring a firm's debt commitments must be greater than the value of leaving with fraction θ of the current period's profits. The parameter θ is our measure of the distortion generated by poor contract enforcement. The second constraint states that dividend payments, defined as profits net of debt payments, must be non-negative.

The firm's exit decision can be summarized by cutoff rules that characterize the minimum productivity necessary to operate. This cutoff rule does not depend on b_t since no firm with positive debt will exit in equilibrium. Let $\hat{x}_{jt}$ be the minimum productivity of all firms of age j that operate in period t . For existing firms there are two possible cases characterizing $\hat{x}_{2t}$. The first is the case when some firms choose to exit. In this case $\hat{x}_{2t}$ describes the firm that is indifferent between producing and exiting, given by the zero profit condition:

$$\hat{x}_{2t} = \max \left\{ f^{\frac{1-\rho}{\rho}} \left(\frac{1}{1-\rho} \frac{w_t}{Y_t} \right)^{\frac{1-\rho}{\rho}} \frac{1}{\rho} w_t, \hat{x}_{1,t-1} \right\}. \quad (9)$$

In the second case, all of the existing firms operate. In this case, $\hat{x}_{2t}$ is equal to $\hat{x}_{1,t-1}$.

2.4. Entry decision

In each period t , a measure μ of potential entrants draw their productivities from a Pareto distribution,

$$F_t(x) = 1 - \left(\frac{x}{g^t} \right)^{-\gamma}, \text{ for } x \geq g^t$$

which is characterized by a mean that grows at rate $g-1$. We require the standard condition that $\gamma(1-\rho)-\rho > 0$, which is necessary for the distribution of profits to have a finite mean. The continual improvement of the technologies available to new firms drives the long-run

aggregate growth in the model: Older firms exit and are replaced by new entrants who are, on average, more productive.⁵

The problem of a potential entrant can be written recursively as

$$\begin{aligned} V_{1t}(x) &= \max_{b_{t+1}} \{d_{1t}(x) + q_{t+1}V_{2,t+1}(b_{t+1}, x), 0\} \\ \text{s.t. } V_{1t}(x) &\geq \theta\pi_{1t}(x) \\ d_{1t}(x) &\equiv \pi_{1t}(x) + q_{t+1}b_{t+1} \geq 0. \end{aligned} \quad (10)$$

Conditional on entry, the entrant's problem is similar to problem (8), but a new entrant does not have an existing stock of debt and faces the larger entry cost, κ , embedded in π_{1t} . The firm's entry decision is summarized by a minimum cutoff productivity, $\hat{x}_{1t}$. All potential entrants with marginal productivity less than $\hat{x}_{1t}$ immediately exit, and all potential entrants with marginal productivity greater than $\hat{x}_{1t}$ will enter and produce.

Since we assume that $\kappa > f$, first period profits for some entrants can be negative. In this case, these firms borrow the amount needed to finance the entry cost, $-\pi_{1t}(x)/q_{t+1}$. These firms use future profits to pay down the debt before distributing any dividend payments. The minimum productivity among entering firms is characterized by the second-period enforcement constraint,

$$V_{2,t+1}\left(-\frac{\pi_{1t}(\hat{x}_{1t})}{q_{t+1}}, \hat{x}_{1t}\right) = \theta\pi_{2,t+1}(\hat{x}_{1t}). \quad (11)$$

This is because if the second-period enforcement constraint is violated, the firm would exit prematurely without honoring its debt commitments, which cannot be an equilibrium outcome. Note that the first-period enforcement constraint does not bind since the profits are negative in the first period for firms that borrow. Given our characterization of entry and exit, we can write the measure of firms producing at time t as

$$\eta_t = \mu \sum_{i=1}^2 \left(1 - F_{t-i+1}(\hat{x}_{i,t-i+1})\right). \quad (12)$$

⁵ Many papers focus on endogenizing the growth rate. For example Alvarez et al. (2014) and Lucas and Moll (2013) model the diffusion of ideas to generate the endogenous growth of the productivity distribution. In this paper, we take the growth rate of the productivity distribution as given, and instead focus on distortions that determine the level of output along a balanced growth path.

2.5. Equilibrium

We focus on balanced growth paths, but before defining a balanced growth path, we first define an equilibrium. To define an equilibrium, we need to provide as initial conditions the measure of firms operating in period 0 with ages $j = 1, 2$, given by minimum productivities of operating firms $\hat{x}_{j0}$, the bond holdings by households B_0 , and the bond holdings of firms $b_{j0}(x)$, for all $x \geq \hat{x}_{j0}$ and $j = 1, 2$.

Definition: Given the initial conditions, an *equilibrium* is sequences of entry-exit threshold values $\{\hat{x}_{jt}\}_{t=0}^{\infty}$, for $j = 1, 2$, prices and allocations for intermediate firms

$\{p_t(x), y_t(x), \ell_t(x), b_{j,t+1}(x)\}_{t=0}^{\infty}$, for all $x \geq \hat{x}_{jt}$ and $j = 1, 2$, prices $\{w_t, q_{t+1}\}_{t=0}^{\infty}$, aggregate dividends and final good output $\{D_t, Y_t\}_{t=0}^{\infty}$, and household consumption and bond holdings $\{C_t, B_{t+1}\}_{t=0}^{\infty}$, such that:

1. Given $\{w_t, D_t, q_{t+1}\}_{t=0}^{\infty}$, $\{C_t, B_{t+1}\}_{t=0}^{\infty}$ solve the household's problem

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2. Given $\{w_t, Y_t, q_{t+1}\}_{t=0}^{\infty}$, $\{p_t(x), \ell_t(x), b_{j,t+1}(x)\}_{t=0}^{\infty}$ solve the problem of the intermediate good firm with productivity x and age j in (6), (8), and (10) for all $x \geq \hat{x}_{jt}$ and $j = 1, 2$.
3. Given $\{Y_t, p_t(x)\}_{t=0}^{\infty}$, $\{y_t(x)\}_{t=0}^{\infty}$ solves the final good firm problem (2).
4. The labor market clears for all $t \geq 0$,

$$1 = \mu \int_{\hat{x}_{1t}}^{\infty} (\ell_t(x) + \kappa) dF_t(x) + \mu \int_{\hat{x}_{2t}}^{\infty} (\ell_t(x) + f) dF_{t-1}(x). \quad (13)$$

5. Entry-exit thresholds satisfy conditions (9) and (11) for all $j = 1, 2$ and $t \geq 0$.
6. The bond market clears for all $t \geq 0$,

$$B_{t+1} = \mu \int_{\hat{x}_{1t}}^{\infty} b_{1,t+1}(x) dF_t(x). \quad (14)$$

7. Dividend payments satisfy for all $t \geq 0$,

$$D_t = \mu \sum_{i=1}^2 \int_{\hat{x}_{it}}^{\infty} d_{it}(x) dF_{t-i+1}(x). \quad (15)$$

3. Balanced growth path

In this section, we prove that the model has a balanced growth path, and characterize the behavior of its key variables.

The consumer's income is the sum of labor income and net capital income. Net capital income, A_t , is the sum of firm profits,

$$A_t = \mu \sum_{i=1}^2 \int_{\hat{x}_{it}}^{\infty} \pi_{jt}(x) dF_{t-i+1}(x). \quad (16)$$

In equilibrium, net capital income is equal to the sum of aggregate dividends and net debt income,

$$A_t = D_t + B_t - q_{t+1} B_{t+1}. \quad (17)$$

Definition: A *balanced growth path* is sequences of final good output $\{Y_t\}_{t=0}^{\infty}$, household consumption $\{C_t\}_{t=0}^{\infty}$, wages $\{w_t\}_{t=0}^{\infty}$, net capital income $\{A_t\}_{t=0}^{\infty}$, minimum productivities of operating firms $\{\hat{x}_{jt}\}_{t=0}^{\infty}$ for $j=1,2$, and one-period bond prices $\{q_{t+1}\}_{t=0}^{\infty}$, such that $Y_{t+1}/Y_t = C_{t+1}/C_t = w_{t+1}/w_t = A_{t+1}/A_t = g$ for all $t \geq 0$, $\hat{x}_{j,t+1}/\hat{x}_{jt} = g$ for all $t \geq 0$ and $j=1,2$, and $q_{t+1} = \beta/g$ for all $t \geq 0$.

On the balanced growth path, growth in the economy is driven by the continual entry of new firms that are, on average, more productive than the previous cohorts. The growth rate of output, consumption, and both components of income grow at the rate $g-1$, which is the rate at which the mean of the productivity distribution of potential entrants grows.

Existing firms on the balanced growth path will exit if they are not profitable. If some firms exit then the lowest productivity among the cohort aged 2, $\hat{x}_{2t}$, is given by the zero profit condition in the left term of the maximand in (9). If no firm exits then $\hat{x}_{2t}$ is given by the right term of the maximand in (9).

We now characterize the entry decisions of new firms. We focus on parameterizations in which the entry cost, κ , is high relative to the fixed continuation cost, f , which implies that some entrants have to borrow to pay the entry cost. These firms will borrow in the first period of

life to pay the entry cost and will earn profits in the second period, out of which they will pay back the debt incurred in the first period.

When an entrant borrows to finance its entry costs in the first period, the enforcement constraint when the firm is age 2 determines the marginal entrant. The condition also helps illustrate how entry costs and limited contract enforcement restrict the entry of firms. We can re-write (11) as

$$\pi_{1t}(\hat{x}_{1t}) + q_{t+1}\pi_{2,t+1}(\hat{x}_{1t}) = q_{t+1}\theta\pi_{2,t+1}(\hat{x}_{1t}). \quad (18)$$

The left-hand side of (18) is the present value of the firm after entry costs are paid. If $\theta = 0$, then a firm will enter the market if its present value is greater than zero. As θ increases, the present value of the firm must be higher for that firm to enter. Furthermore, as κ increases, the present value of the firm before entry costs are paid must be higher in order to justify entering.

Notice that the enforcement constraint does not distort the intensive margin of the firm: Conditional on producing, the firm is of the optimal size. The enforcement constraint affects the extensive margin, keeping some firms from producing who would have otherwise produced if $\theta = 0$. Lemma 1 characterizes the cutoff productivity of the marginal entrant.

Lemma 1. *Let $\hat{n}(\kappa, \theta)$ be the number of periods that the marginal entrant operates, that is, the time-to-exit of the marginal entrant. On the balanced growth path, the cut-off productivity of an entrant is characterized by*

$$\hat{x}_{1t} = \tilde{\kappa}(\kappa, \theta)^{\frac{1-\rho}{\rho}} \left(\frac{1}{1-\rho} \frac{w_t}{Y_t} \right)^{\frac{1-\rho}{\rho}} \frac{1}{\rho} w_t, \quad (19)$$

where $\tilde{\kappa}(\kappa, \theta) = \kappa$ when $\hat{n}(\kappa, \theta) = 1$ and

$$\tilde{\kappa}(\kappa, \theta) = \frac{\kappa + \beta(1-\theta)f}{1 + \beta(1-\theta)g^{\frac{-\rho}{1-\rho}}} \leq \kappa \quad (20)$$

when $\hat{n}(\kappa, \theta) = 2$.

Proof: See Appendix.

If $\hat{n}(\kappa, \theta) = 1$, the marginal entrant lives for one period. In this case, no firm will borrow in equilibrium and the minimum productivity is given by the standard zero profit condition in

(19) where $\tilde{\kappa}(\kappa, \theta) = \kappa$. If $\hat{n}(\kappa, \theta) = 2$, the marginal entrant will incur a loss in the first period, which it finances by borrowing and repaying in the second period. Notice that the minimum productivity condition in (19) looks very similar to the standard zero-profit condition, except the *effective entry cost*, $\tilde{\kappa}(\kappa, \theta)$, is an expression of the entry cost, the enforcement parameter, the discount factor, and the growth rate of the economy. With finance, the entrant is able to spread the entry cost, κ , over both periods, thereby lowering the effective entry cost.

Proposition 1. *A balanced growth path exists.*

Proof: On the balanced growth path, the aggregate variables are

$$w_t = g^t \rho \left[\frac{\gamma(1-\rho)}{\gamma(1-\rho)-\rho} \mu \right]^{\frac{1}{\gamma}} (1-\rho)^{\frac{\gamma(1-\rho)-\rho}{\gamma\rho}} \omega(\kappa, \theta)^{\frac{1}{\gamma}} (1-\xi(\kappa, \theta))^{\frac{\gamma(1-\rho)-\rho}{-\gamma\rho}} \quad (21)$$

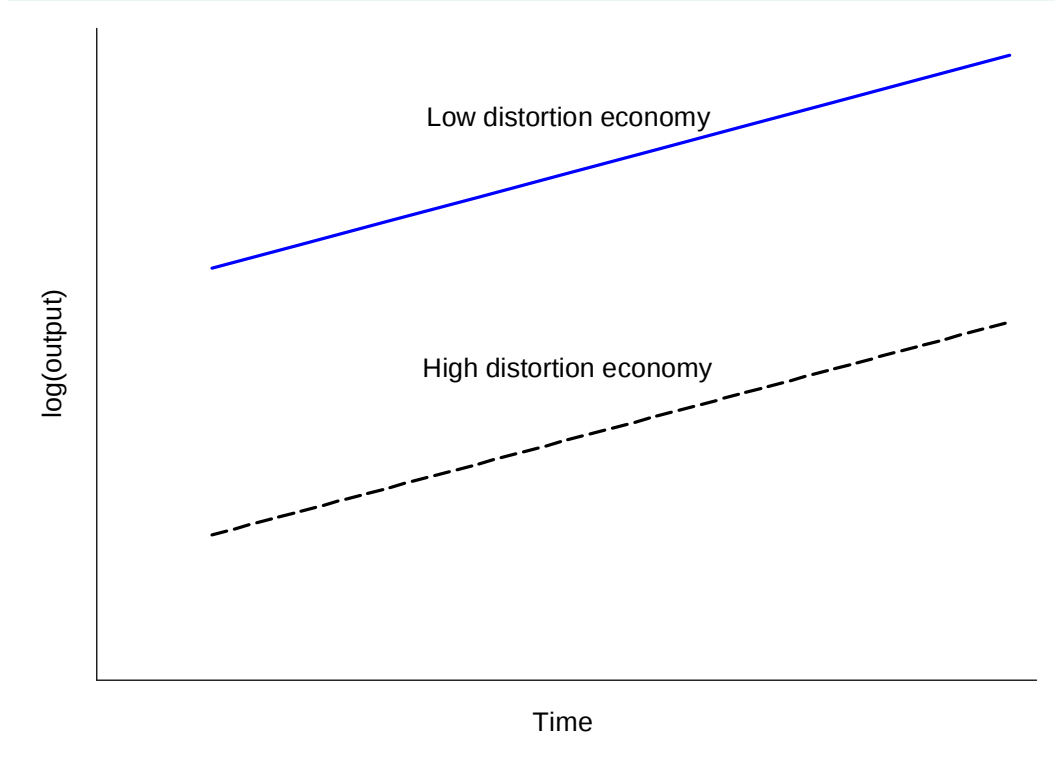
$$A_t = \frac{\xi(\kappa, \theta)}{1-\xi(\kappa, \theta)} w_t \quad (22)$$

$$C_t = Y_t = w_t + A_t \quad (23)$$

where $\xi(\kappa, \theta)$, the ratio of net capital income to total output, and $\omega(\kappa, \theta)$ are positive constants. From the above equations we see that w_t , A_t , C_t , and Y_t grow at rate $g-1$ and satisfy the equilibrium conditions. Furthermore, the cutoffs $\hat{x}_{jt}$, for $j=1,2$, given by (9) and (19), also grow at rate $g-1$. Finally, from the first-order condition of the household and applying the balanced growth path conditions, we obtain $q_{t+1} = \beta/g$. See the appendix for details. $\square$

The balanced growth path of the model is tractable, despite the potential complications arising from the limited enforcement constraint. If we consider two economies that differ only in θ and κ , the two economies will both grow at the same rate, $g-1$. As we show in the next section, the difference between the two economies is in the level of their balanced growth paths. Figure 3 illustrates that an economy with smaller regulatory entry costs or better contract enforcement will be on a balanced growth path that delivers higher levels of consumption and output.

Figure 3: Output in high and low distortion economies



Before we discuss the relationship between the underlying distortions in the economy and the balanced growth path, we further characterize the dynamics of firm entry and exit on the balanced growth path.

An existing firm remains in the market as long as its period profits are positive. This implies that

$$\hat{x}_{2,t+1} = \max \left\{ \tilde{\kappa}(\kappa, \theta)^{\frac{1-\rho}{\rho}} \left(\frac{1}{1-\rho} \frac{w_t}{Y_t} \right)^{\frac{1-\rho}{\rho}} \frac{w_t}{\rho}, f^{\frac{1-\rho}{\rho}} \left(\frac{1}{1-\rho} \frac{w_{t+1}}{Y_{t+1}} \right)^{\frac{1-\rho}{\rho}} \frac{w_{t+1}}{\rho} \right\}, \quad (24)$$

where the first term of the maximand is the cutoff productivity of the marginal entrant in period t (no exit), and the second term is the minimum productivity necessary to satisfy the second period zero profit condition (exit). Using the balanced growth path conditions that $A_{t+1} = gA_t$ and $w_{t+1} = gw_t$, we have

$$\hat{x}_{2,t+1} = \max \left\{ \tilde{\kappa}(\kappa, \theta)^{\frac{1-\rho}{\rho}}, gf^{\frac{1-\rho}{\rho}} \right\} \left(\frac{1}{1-\rho} \frac{w_t}{Y_t} \right)^{\frac{1-\rho}{\rho}} \frac{w_t}{\rho}. \quad (25)$$

The time-to-exit of marginal entrants is given by

$$\hat{n}(\kappa, \theta) = \begin{cases} 1 & \text{if } 1 \leq \frac{\tilde{\kappa}(\kappa, \theta)}{f} < g^{\frac{\rho}{1-\rho}} \\ 2 & \text{if } g^{\frac{\rho}{1-\rho}} \leq \frac{\tilde{\kappa}(\kappa, \theta)}{f}. \end{cases} \quad (26)$$

Since we focus on equilibria in which marginal entrants use finance, we assume that

$$g^{\frac{\rho}{1-\rho}} \leq \frac{\tilde{\kappa}(\kappa, \theta)}{f}. \quad (27)$$

4. Comparative Statics

In this section we investigate how changes in distortions affect the level of the balanced growth path. First, we prove that output levels are decreasing in both financial distortions and barriers to entry. Next, we study the interaction between these two distortions in determining output levels. We show that reforms to entry barriers and financial distortions are substitutes, meaning that reducing one friction has a larger effect when the other friction is large.

Proposition 2. *In any balanced growth path, real output is decreasing in entry costs and financial distortions.*

Proof: See Appendix.

Proposition 2 comes from the fact that an increase in financial distortions and entry costs block firms that would otherwise enter the market, which reduces the number of intermediate goods producers and aggregate output.

The relationship between output and entry costs, or financial distortions, has been studied separately in the previous literature. As do we, the literature finds that larger entry costs, or greater financial distortions, decrease output. The main focus of this paper, however, lies in the interaction of financial distortions and entry costs. We summarize this relationship in the following proposition.

Proposition 3. *In any balanced growth path,*

$$\frac{d^2 \log Y_t(\kappa, \theta)}{d\kappa d\theta} < 0. \quad (28)$$

Proof: See Appendix.

Proposition 3 implies that reforms to entry costs and financial distortions are substitutable. The proposition indicates that as contract enforcement increases (low θ), the gains from reducing entry costs decrease ($d \log Y_t(\kappa, \theta) / d\kappa$ becomes less negative). As the cost of entry decreases (low κ), the gains from reducing financial distortions decrease ($d \log Y_t(\kappa, \theta) / d\theta$ becomes less negative). In other words, if one distortion decreases, changes in the other distortion have less of an impact on output.

The proof of proposition 3 shows the intuition for the result quite clearly. One of the sufficient conditions for this proposition is that $d^2 \log \tilde{\kappa}(\kappa, \theta) / d\kappa d\theta > 0$. This condition requires the mixed derivative of the effective entry cost of the marginal entrant to be positive. This means that if entry barriers increase, then increasing financial frictions will have a larger percentage change on the effective entry cost (and vice versa).

5. Evidence

In this section, we document two patterns in the data that are consistent with our theory. Our theory indicates that: first, as entry costs and financial frictions increase, output decreases and, second, changes in entry costs have a stronger effect on output as financial frictions worsen, and vice-versa.

We use a panel of 169 countries from 2003 to 2010. Our measure of output is GDP per working age person (WAP).⁶ Entry costs are measured as the cost of start-up procedures (as a percentage of GNI per capita) from the World Bank's *Doing Business* surveys. The costs include official fees and expenses for professional services (such as accountants and lawyers) that are legally required to start a business. Our measure of financial frictions is the inverse of the private credit-GDP ratio, as is commonly used in quantitative papers on development and finance. (e.g., Amaral and Quintin 2010, Buera et al. 2009, Buera and Shin 2013, and Moll 2012). We estimate

⁶ We think about the fixed costs paid by firms in our model as intermediate inputs and not investment. If we consider the fixed costs to be investment, then output is Y_t plus fixed costs paid by firms.

$$\log(\text{output}_{it}) = \alpha_0 + \alpha_1 \log(\text{financialfrictions}_{it}) + \alpha_2 \log(\text{entrycosts}_{it}) + \alpha_3 \log(\text{financialfrictions}_{it}) \times \log(\text{entrycosts}_{it}) + F_i + F_t + \varepsilon_{it}. \quad (29)$$

where F_i represents country fixed effects and F_t represents time dummies. The first column in table 1 reports the negative relationship between financial frictions and output: A 10 percent increase in financial frictions is associated with a 0.82 percent decrease in output. The second column in table 1 shows that entry costs are negatively associated with output. A 10 percent increase in entry costs is associated with 0.36 percent decrease in output. These two results are consistent with previous work on the negative effects of both financial frictions and entry barriers on output.

Table 1: Time and country fixed effect regressions

	$\log(\text{output})$	$\log(\text{output})$	$\log(\text{output})$	$\log(\text{output})$
$\log(\text{financialfrictions})$	-0.082*** (0.010)		-0.076*** (0.010)	-0.048*** (0.014)
$\log(\text{entrycosts})$		-0.036*** (0.005)	-0.032*** (0.005)	-0.056*** (0.010)
interaction				-0.007*** (0.003)
observations	1255	1255	1255	1255
R^2	-0.082***		-0.076***	-0.048***

Note: OLS estimation of (29), which includes time and country fixed effects. *** and ** denote significance at 1 percent and 5 percent level, respectively.

Our focus is on the interaction between financial frictions and entry costs. Column 3 in table 1 shows that both financial frictions and entry costs remain negative and statistically significant even when we consider them jointly. Column 4 shows that the interaction term is negative and statistically significant.

To get a sense of the importance of the interaction term, we report the interacted coefficient on entry costs, $\alpha_2 + \alpha_3 \log(\text{financialfrictions}_i)$, for the United States, which is at the bottom of our sample in terms of financial frictions, and Mexico, which is near the top, in table 2. The new coefficient on entry costs for Mexico is almost twice as large as that of the United States.

Table 2: Example of interaction effect

	$\alpha_2 + \alpha_3 \log(\text{financialfrictions}_i)$
United States	-0.020
Mexico	-0.037

6. Conclusion

The goal of this paper is to understand the joint effect of financial frictions and entry costs on growth. Theoretically, we have built a tractable model that allows us to study the two distortions jointly. In this model, growth is driven by the entry of new firms that face financial frictions and entry costs. We find that policies that reform financial markets and entry costs are substitutable. This is because without access to finance, potential entrants cannot spread the costs of entry across future profitable periods, increasing the impact of entry costs on entry. Empirically, we show that the predictions of the model are borne out in the data.

Our findings shed light on the varying growth results experienced by countries enacting policy reforms. The stark implication is that a country will experience the strongest growth after the first set of reforms (either to reduce financial frictions or entry costs) and then experience modest growth with a subsequent reform. This outcome is particularly important given the high correlation between these two distortions in the data. We have studied two distortions for which reforms are substitutable; other reforms might be complementary. It would be useful to study these distortions jointly to guide policymakers in thinking about the sequencing of reforms. We leave this interesting line of research for future work.

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Appendix

Proof of Lemma 1:

Equation (11) can be written as

$$\pi_{2,t+1}(\hat{x}_{1t}) - b_{1,t+1}(\hat{x}_{1t}) = \theta \pi_{2,t+1}(\hat{x}_{1t}). \quad (30)$$

Substituting $q_{t+1}b_{1,t+1}(\hat{x}_{1t}) = -\pi_{1t}(\hat{x}_{1t})$, we obtain

$$\pi_{1t}(\hat{x}_{1t}) + q_{t+1}(1-\theta)\pi_{2,t+1}(\hat{x}_{1t}) = 0. \quad (31)$$

Substituting the expression for profits, we obtain

$$\begin{aligned} & w_t \left[(1-\rho)(w_t + A_t) w_t^{\frac{-1}{1-\rho}} (\hat{x}_{1t} \rho)^{\frac{\rho}{1-\rho}} - \kappa \right] \\ & + q_{t+1}(1-\theta) w_{t+1} \left[(1-\rho)(w_{t+1} + A_{t+1}) w_{t+1}^{\frac{-1}{1-\rho}} (\hat{x}_{1t} \rho)^{\frac{\rho}{1-\rho}} - f \right] = 0 \end{aligned} \quad (32)$$

Applying the balanced growth path conditions, $w_{t+1} = gw_t$, $A_{t+1} = gA_t$, $q_{t+1} = \beta/g$, we obtain

$$\begin{aligned} & (1-\rho)(w_t + A_t) w_t^{\frac{-1}{1-\rho}} (\hat{x}_{1t} \rho)^{\frac{\rho}{1-\rho}} + \beta(1-\theta)(1-\rho)(w_t + A_t) w_t^{\frac{-1}{1-\rho}} g^{\frac{-\rho}{1-\rho}} (\hat{x}_{1t} \rho)^{\frac{\rho}{1-\rho}} \\ & = \kappa + \beta(1-\theta)f. \end{aligned} \quad (33)$$

Rearranging terms yields the desired result. $\square$

Proof of Proposition 1:

The proof of proposition 1 involves guessing and verifying the existence of an equilibrium with a balanced growth path.

From the first order condition of the consumer and applying the balanced growth path conditions, we obtain $q_{t+1} = \beta/g$.

Next, using **Error! Reference source not found.** and **Error! Reference source not found.**, we can derive:

$$w_t^{\frac{\rho}{1-\rho}} = \rho^{\frac{\rho}{1-\rho}} \frac{\gamma(1-\rho)}{\gamma(1-\rho)-\rho} \mu \sum_{i=1}^2 g^{\gamma(t-i+1)} \hat{x}_{it}^{\frac{\rho-\gamma(1-\rho)}{1-\rho}}. \quad (34)$$

Using (16) we find that

$$\frac{A_t}{w_t} = (1-\rho) \left(1 + \frac{A_t}{w_t} \right) - \mu \sum_{i=1}^2 g^{\gamma(t-i+1)} \hat{x}_{it}^{-\gamma} \kappa_i. \quad (35)$$

Using the expression for cutoffs from equation **Error! Reference source not found.** and lemma 1 which is given by,

$$\hat{x}_{it} = \begin{cases} \tilde{\kappa}(\kappa, \theta)^{\frac{1-\rho}{\rho}} \left[(1-\rho) \left(1 + \frac{A_{t-i+1}}{w_{t-i+1}} \right) \right]^{\frac{1-\rho}{-\rho}} \frac{1}{\rho} w_{t-i+1} & \text{if } i \leq \hat{n}(\kappa, \theta) \\ f^{\frac{1-\rho}{\rho}} \left[(1-\rho) \left(1 + \frac{A_t}{w_t} \right) \right]^{\frac{1-\rho}{-\rho}} \frac{1}{\rho} w_t & \text{if } i > \hat{n}(\kappa, \theta), \end{cases} \quad (36)$$

and the balanced growth path conditions, $w_t = gw_{t-1}$ and $A_t = gA_{t-1}$, we obtain

$$w_t^\gamma = \rho^\gamma \frac{\gamma(1-\rho)}{\gamma(1-\rho)-\rho} \mu g^{\gamma t} \left[(1-\rho) \left(1 + \frac{A_t}{w_t} \right) \right]^{\frac{\gamma(1-\rho)-\rho}{\rho}} \omega(\kappa, \theta), \quad (37)$$

and

$$\frac{A_t(\kappa, \theta)}{w_t(\kappa, \theta)} = \frac{\xi(\kappa, \theta)}{1 - \xi(\kappa, \theta)}, \quad (38)$$

where

$$\xi(\kappa, \theta) = 1 - \rho - \frac{\gamma(1-\rho) - \rho}{\gamma} \frac{\tilde{\xi}(\kappa, \theta)}{\omega(\kappa, \theta)}, \quad (39)$$

$$\omega(\kappa, \theta) = \tilde{\kappa}(\kappa, \theta)^{\frac{1-\rho}{\rho} \gamma \hat{n}(\kappa, \theta)} \sum_{i=1}^{(1-i)\frac{\rho}{1-\rho}} g^{(1-i)\frac{\rho}{1-\rho}} + f^{\frac{1-\rho}{\rho} \gamma} \sum_{i=\hat{n}(\kappa, \theta)+1}^2 g^{\gamma(1-i)}, \quad (40)$$

$$\tilde{\xi}(\kappa, \theta) = \tilde{\kappa}(\kappa, \theta)^{-\frac{1-\rho}{\rho} \gamma \hat{n}(\kappa, \theta)} \sum_{i=1}^{\frac{1-\rho}{\rho} \gamma \hat{n}(\kappa, \theta)} \kappa_i + f^{\frac{1-\rho}{\rho} \gamma} \sum_{i=\hat{n}(\kappa, \theta)+1}^2 g^{\gamma(1-i)}. \quad (41)$$

Finally, by substituting (38) into (37) we obtain

$$w_t = g^t \rho \left[\frac{\gamma(1-\rho)}{\gamma(1-\rho)-\rho} \mu \right]^{\frac{1}{\gamma}} (1-\rho)^{\frac{\gamma(1-\rho)-\rho}{\gamma\rho}} \omega(\kappa, \theta)^{\frac{1}{\gamma}} (1 - \xi(\kappa, \theta))^{\frac{\gamma(1-\rho)-\rho}{-\gamma\rho}} \quad (42)$$

Thus, our guess has been verified and all optimality conditions are satisfied. $\square$

Proof of Proposition 2:

Real output is given by

$$Y_t(\kappa, \theta) = \rho \left[\frac{\gamma(1-\rho)}{\gamma(1-\rho)-\rho} \mu \right]^{\frac{1}{\gamma}} g^t (1-\rho)^{\frac{\gamma(1-\rho)-\rho}{\gamma\rho}} \omega(\kappa, \theta)^{\frac{1}{\gamma}} (1 - \xi(\kappa, \theta))^{\frac{\gamma-\rho}{-\gamma\rho}}. \quad (43)$$

(i) First, we will show that $d \log Y_t(\kappa, \theta) / d\theta < 0$. We can write

$$\frac{d \log Y_t(\kappa, \theta)}{d\theta} = \frac{1}{\gamma} \frac{\frac{d\omega(\kappa, \theta)}{d\theta}}{\omega(\kappa, \theta)} Q(\kappa, \theta) \quad (44)$$

where

$$Q(\kappa, \theta) = 1 - \frac{\gamma - \rho}{\gamma} \frac{\gamma(1 - \rho) - \rho}{\rho} \frac{\frac{d\tilde{\xi}(\kappa, \theta)}{d\theta} \omega(\kappa, \theta) - \tilde{\xi}(\kappa, \theta)}{\rho \omega(\kappa, \theta) + \frac{\gamma(1 - \rho) - \rho}{\gamma} \tilde{\xi}(\kappa, \theta)}. \quad (45)$$

Since $d\omega(\kappa, \theta) / d\theta < 0$, it suffices to show $Q(\kappa, \theta) > 0$. Equivalently,

$$\frac{\gamma(1 - \rho) - \rho}{\rho} \tilde{\xi}(\kappa, \theta) > \left[\frac{\gamma - \rho}{\gamma} \frac{\gamma(1 - \rho) - \rho}{\rho} \frac{\frac{d\tilde{\xi}(\kappa, \theta)}{d\theta}}{\frac{d\omega(\kappa, \theta)}{d\theta}} - \rho \right] \omega(\kappa, \theta). \quad (46)$$

Substitute equations (40) and (41) and their respective derivatives to obtain

$$\begin{aligned} & \frac{\gamma(1 - \rho) - \rho}{\rho} \frac{\tilde{\kappa}(\kappa, \theta)^{-1} \sum_{i=1}^2 \kappa_i}{\sum_{i=1}^2 g^{(1-i)\frac{\rho}{1-\rho}}} \\ & > \frac{(\gamma - \rho)(1 - \rho)}{\rho} \frac{\tilde{\kappa}(\kappa, \theta)^{-1} \sum_{i=1}^2 \kappa_i}{\sum_{i=1}^2 g^{(1-i)\frac{\rho}{1-\rho}}} - \rho. \end{aligned} \quad (47)$$

This is true if and only if $\tilde{\kappa}(\kappa, \theta) > \sum_{i=1}^2 \kappa_i / \sum_{i=1}^2 g^{(1-i)\rho/(1-\rho)}$, which is true.

(ii) We want to show that $d \log(Y_t(\kappa, \theta)) / d\kappa < 0$. Similarly, we can write

$$\frac{d \log Y_t(\kappa, \theta)}{d\kappa} = \frac{1}{\gamma} \frac{\frac{d\omega(\kappa, \theta)}{d\kappa}}{\omega(\kappa, \theta)} S(\kappa, \theta) \quad (48)$$

where

$$S(\kappa, \theta) = 1 - \frac{\gamma - \rho}{\gamma} \frac{\gamma(1 - \rho) - \rho}{\rho} \frac{\frac{d\tilde{\xi}(\kappa, \theta)}{d\kappa} \omega(\kappa, \theta) - \tilde{\xi}(\kappa, \theta)}{\rho \omega(\kappa, \theta) + \frac{\gamma(1 - \rho) - \rho}{\gamma} \tilde{\xi}(\kappa, \theta)}. \quad (49)$$

Since $d\omega(\kappa, \theta) / d\kappa < 0$, it suffices to show $S(\kappa, \theta) > 0$. Equivalently,

$$\frac{\gamma(1 - \rho) - \rho}{\rho} \tilde{\xi}(\kappa, \theta) > \left[\frac{\gamma - \rho}{\gamma} \frac{\gamma(1 - \rho) - \rho}{\rho} \frac{\frac{d\tilde{\xi}(\kappa, \theta)}{d\kappa}}{\frac{d\omega(\kappa, \theta)}{d\kappa}} - \rho \right] \omega(\kappa, \theta). \quad (50)$$

Substitute equations (40) and (41) and their respective derivatives to obtain

$$\begin{aligned}
& \frac{\gamma(1-\rho)-\rho}{\rho} \frac{\tilde{\kappa}(\kappa, \theta)^{-1} \sum_{i=1}^2 \kappa_i}{\sum_{i=1}^2 g^{(1-i)\frac{\rho}{1-\rho}}} \\
& > \frac{(\gamma-\rho)(1-\rho)}{\rho} \frac{\tilde{\kappa}(\kappa, \theta)^{-1} \sum_{i=1}^2 \kappa_i}{\sum_{i=1}^2 g^{(1-i)\frac{\rho}{1-\rho}}} - \frac{\gamma-\rho}{\gamma} \frac{1}{\frac{d\tilde{\kappa}(\kappa, \theta)}{d\kappa} \sum_{i=1}^2 g^{(1-i)\frac{\rho}{1-\rho}}} - \rho.
\end{aligned} \tag{51}$$

Equivalently,

$$\rho \left(1 - \frac{\tilde{\kappa}(\kappa, \theta)^{-1} \sum_{i=1}^2 \kappa_i}{\sum_{i=1}^2 g^{(1-i)\frac{\rho}{1-\rho}}} \right) + \frac{\gamma-\rho}{\gamma} \frac{1}{\frac{d\tilde{\kappa}(\kappa, \theta)}{d\kappa} \sum_{i=1}^2 g^{(1-i)\frac{\rho}{1-\rho}}} > 0, \tag{52}$$

which holds since $\tilde{\kappa}(\kappa, \theta) > \sum_{i=1}^2 \kappa_i / \sum_{i=1}^2 g^{(1-i)\rho/(1-\rho)}$. $\square$

Proof of Proposition 3:

Using (44), we obtain

$$\begin{aligned}
\frac{d^2 \log Y_t(\kappa, \theta)}{d\theta d\kappa} &= \frac{1}{\gamma} \frac{\omega(\kappa, \theta) \frac{d^2 \omega(\kappa, \theta)}{d\theta d\kappa} - \frac{d\omega(\kappa, \theta)}{d\theta} \frac{d\omega(\kappa, \theta)}{d\kappa}}{\omega(\kappa, \theta)^2} Q(\kappa, \theta) \\
&+ \frac{1}{\gamma} \frac{\frac{d\omega(\kappa, \theta)}{d\theta}}{\omega(\kappa, \theta)} \frac{dQ(\kappa, \theta)}{d\kappa}.
\end{aligned} \tag{53}$$

Since $Q(\kappa, \theta) > 0$ and $d\omega(\kappa, \theta) / d\theta < 0$, it suffices to show

$$\begin{aligned}
(i): & \omega(\kappa, \theta) \frac{d^2 \omega(\kappa, \theta)}{d\theta d\kappa} - \frac{d\omega(\kappa, \theta)}{d\theta} \frac{d\omega(\kappa, \theta)}{d\kappa} < 0 \\
(ii): & \frac{dQ(\kappa, \theta)}{d\kappa} \geq 0.
\end{aligned} \tag{54}$$

(i) Substitute the derivatives of equation (40) to obtain

$$\begin{aligned}
& \omega(\kappa, \theta) \left(\frac{1-\rho}{\rho} \gamma - \tilde{\kappa}(\kappa, \theta) \frac{\frac{d^2 \tilde{\kappa}(\kappa, \theta)}{d\kappa d\theta}}{\frac{d\tilde{\kappa}(\kappa, \theta)}{d\kappa} \frac{d\tilde{\kappa}(\kappa, \theta)}{d\theta}} \right) \\
& < \left(\frac{1-\rho}{\rho} \gamma - 1 \right) \tilde{\kappa}(\kappa, \theta)^{1-\frac{1-\rho}{\rho}\gamma} \sum_{i=1}^2 g^{(1-i)\frac{\rho}{1-\rho}}
\end{aligned} \tag{55}$$

Substitute (40) to obtain

$$\frac{1-\rho}{\rho} \gamma - \tilde{\kappa}(\kappa, \theta) \frac{\frac{d^2 \tilde{\kappa}(\kappa, \theta)}{d\kappa d\theta}}{\frac{d\tilde{\kappa}(\kappa, \theta)}{d\kappa} \frac{d\tilde{\kappa}(\kappa, \theta)}{d\theta}} < \left(\frac{1-\rho}{\rho} \gamma - 1 \right). \quad (56)$$

Equivalently,

$$\tilde{\kappa}(\kappa, \theta) \frac{\frac{d^2 \tilde{\kappa}(\kappa, \theta)}{d\kappa d\theta}}{\frac{d\tilde{\kappa}(\kappa, \theta)}{d\kappa} \frac{d\tilde{\kappa}(\kappa, \theta)}{d\theta}} > 1. \quad (57)$$

Note that this is equivalent to showing $d^2 \log \tilde{\kappa}(\kappa, \theta) / d\kappa d\theta > 0$. Next, substitute the derivatives of (20) to obtain

$$\frac{\tilde{\kappa}(\kappa, \theta)}{\tilde{\kappa}(\kappa, \theta) - fg^{\frac{\rho}{1-\rho}}} > 1, \quad (58)$$

which is true since $\tilde{\kappa}(\kappa, \theta) / f > g^{\frac{\rho}{1-\rho}}$.

(ii) We can rewrite $Q(\kappa, \theta)$ as

$$Q(\kappa, \theta) = 1 - \frac{\gamma - \rho}{\gamma} \frac{\frac{\gamma(1-\rho) - \rho}{\rho} \frac{\frac{d\tilde{\xi}(\kappa, \theta)}{d\theta}}{\frac{d\omega(\kappa, \theta)}{d\theta}} + \frac{\gamma}{\rho} (\xi(\kappa, \theta) - (1-\rho))}{1 - \xi(\kappa, \theta)} \quad (59)$$

Then, we need to show

$$\frac{d}{d\kappa} \left(\frac{\frac{\gamma(1-\rho) - \rho}{\rho} \frac{\frac{d\tilde{\xi}(\kappa, \theta)}{d\theta}}{\frac{d\omega(\kappa, \theta)}{d\theta}} + \frac{\gamma}{\rho} (\xi(\kappa, \theta) - (1-\rho))}{1 - \xi(\kappa, \theta)} \right) < 0. \quad (60)$$

Equivalently,

$$\begin{aligned}
& (1 - \xi(\kappa, \theta)) \frac{d \left(\frac{\gamma(1-\rho) - \rho}{\rho} \frac{\frac{d\tilde{\xi}(\kappa, \theta)}{d\theta}}{\frac{d\omega(\kappa, \theta)}{d\theta}} + \frac{\gamma}{\rho} (\xi(\kappa, \theta) - (1-\rho)) \right)}{d\kappa} \\
& \leq \left(\frac{\gamma(1-\rho) - \rho}{\rho} \frac{\frac{d\tilde{\xi}(\kappa, \theta)}{d\theta}}{\frac{d\omega(\kappa, \theta)}{d\theta}} + \frac{\gamma}{\rho} (\xi(\kappa, \theta) - (1-\rho)) \right) \frac{d(1 - \xi(\kappa, \theta))}{d\kappa},
\end{aligned} \tag{61}$$

which can be written as

$$\begin{aligned}
& (1 - \xi(\kappa, \theta)) \left(\frac{\gamma(1-\rho) - \rho}{\rho} \frac{\frac{d^2\tilde{\xi}(\kappa, \theta)}{d\kappa d\theta} \frac{d\omega(\kappa, \theta)}{d\theta} - \frac{d\tilde{\xi}(\kappa, \theta)}{d\theta} \frac{d^2\omega(\kappa, \theta)}{d\kappa d\theta}}{\frac{d\omega(\kappa, \theta)}{d\theta}^2} + \frac{\gamma}{\rho} \frac{d\xi(\kappa, \theta)}{d\kappa} \right) \\
& \leq \left(\frac{\gamma(1-\rho) - \rho}{\rho} \frac{\frac{d\tilde{\xi}(\kappa, \theta)}{d\theta}}{\frac{d\omega(\kappa, \theta)}{d\theta}} + \frac{\gamma}{\rho} (\xi(\kappa, \theta) - (1-\rho)) \right) \left(-\frac{d\xi(\kappa, \theta)}{d\kappa} \right).
\end{aligned} \tag{62}$$

Equivalently,

$$\begin{aligned}
& (1 - \xi(\kappa, \theta)) \left(\frac{\gamma(1-\rho) - \rho}{\rho} \frac{\frac{d^2\tilde{\xi}(\kappa, \theta)}{d\kappa d\theta} \frac{d\omega(\kappa, \theta)}{d\theta} - \frac{d\tilde{\xi}(\kappa, \theta)}{d\theta} \frac{d^2\omega(\kappa, \theta)}{d\kappa d\theta}}{\frac{d\omega(\kappa, \theta)}{d\theta}^2} \right) \\
& \leq - \left(\frac{\gamma(1-\rho) - \rho}{\rho} \frac{\frac{d\tilde{\xi}(\kappa, \theta)}{d\theta}}{\frac{d\omega(\kappa, \theta)}{d\theta}} + \gamma \right) \frac{d\xi(\kappa, \theta)}{d\kappa}.
\end{aligned} \tag{63}$$

Substitute equation (39) and its derivative to obtain

$$\begin{aligned}
& \left(\frac{\gamma(1-\rho) - \rho}{\gamma} \frac{\frac{d\tilde{\xi}(\kappa, \theta)}{d\theta}}{\frac{d\omega(\kappa, \theta)}{d\theta}} + \rho \right) \left(\frac{\tilde{\xi}(\kappa, \theta) \frac{d\omega(\kappa, \theta)}{d\kappa} - \omega(\kappa, \theta) \frac{d\tilde{\xi}(\kappa, \theta)}{d\kappa}}{\omega(\kappa, \theta)^2} \right) \\
& \leq \left(\frac{\gamma(1-\rho) - \rho}{\gamma} \frac{\tilde{\xi}(\kappa, \theta)}{\omega(\kappa, \theta)} + \rho \right) \left(\frac{\frac{d\tilde{\xi}(\kappa, \theta)}{d\theta} \frac{d^2\omega(\kappa, \theta)}{d\theta d\kappa} - \frac{d\omega(\kappa, \theta)}{d\theta} \frac{d^2\tilde{\xi}(\kappa, \theta)}{d\theta d\kappa}}{\frac{d\omega(\kappa, \theta)}{d\theta}^2} \right).
\end{aligned} \tag{64}$$

Substitute equations (40) and (41) and their respective derivatives to obtain

$$\begin{aligned}
& \left(\frac{\gamma(1-\rho)-\rho}{\gamma} \frac{\gamma^{\frac{1-\rho}{\rho}}}{\gamma^{\frac{1-\rho}{\rho}}-1} \frac{\tilde{\kappa}(\kappa, \theta)^{-1} \sum_{i=1}^2 \kappa_i}{\sum_{i=1}^2 g^{(1-i)\frac{\rho}{1-\rho}}} + \rho \right) \\
& \times \frac{\tilde{\kappa}(\kappa, \theta)^{-1-\frac{1-\rho}{\rho}\gamma} \left(\sum_{i=1}^2 \kappa_i \frac{d\tilde{\kappa}(\kappa, \theta)}{d\kappa} - \tilde{\kappa}(\kappa, \theta) \right)}{\tilde{\kappa}(\kappa, \theta)^{1-\frac{1-\rho}{\rho}\gamma} \sum_{i=1}^2 g^{(1-i)\frac{\rho}{1-\rho}}} \\
& \leq \left(\frac{\gamma(1-\rho)-\rho}{\gamma} \frac{\tilde{\kappa}(\kappa, \theta)^{-1} \sum_{i=1}^2 \kappa_i}{\sum_{i=1}^2 g^{(1-i)\frac{\rho}{1-\rho}}} + \rho \right) \\
& \times \frac{\left(-\frac{1-\rho}{\rho} \gamma \right) \frac{d\tilde{\kappa}(\kappa, \theta)}{d\theta} \tilde{\kappa}(\kappa, \theta)^{-\frac{1-\rho}{\rho}\gamma-2} \left(\frac{d\tilde{\kappa}(\kappa, \theta)}{d\kappa} \sum_{i=1}^2 \kappa_i - \tilde{\kappa}(\kappa, \theta) \right)}{\left(1 - \frac{1-\rho}{\rho} \gamma \right) \tilde{\kappa}(\kappa, \theta)^{-\frac{1-\rho}{\rho}\gamma} \frac{d\tilde{\kappa}(\kappa, \theta)}{d\theta} \sum_{i=1}^2 g^{(1-i)\frac{\rho}{1-\rho}}}.
\end{aligned} \tag{65}$$

which is true if and only if

$$\begin{aligned}
& \left(\frac{\gamma(1-\rho)-\rho}{\gamma} \frac{\gamma^{\frac{1-\rho}{\rho}}}{\gamma^{\frac{1-\rho}{\rho}}-1} \frac{\tilde{\kappa}(\kappa, \theta)^{-1} \sum_{i=1}^2 \kappa_i}{\sum_{i=1}^2 g^{(1-i)\frac{\rho}{1-\rho}}} + \rho \right) \\
& \leq \left(\frac{\gamma(1-\rho)-\rho}{\gamma} \frac{\tilde{\kappa}(\kappa, \theta)^{-1} \sum_{i=1}^2 \kappa_i}{\sum_{i=1}^2 g^{(1-i)\frac{\rho}{1-\rho}}} + \rho \right) \frac{\gamma^{\frac{1-\rho}{\rho}}}{\gamma^{\frac{1-\rho}{\rho}}-1},
\end{aligned} \tag{66}$$

which is true. $\square$