

Monetary Shocks and Bank Balance Sheets

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PRELIMINARY

June 2015

Abstract

We propose a model to explain why banks' balances sheets are exposed to interest rate risk despite the existence of markets where that risk can be hedged. A rise in nominal interest rates raises the opportunity cost of holding currency; since bank liabilities are close substitutes of currency, demand for bank liabilities rises and banks earn higher spreads. If risk aversion is higher than 1, the optimal dynamic hedging strategy is to sustain capital losses when nominal interest rates rise and, conversely, capital gains when they fall. A traditional bank balance sheet with long duration nominal assets achieves that.

Keywords: Monetary shocks, bank deposits, interest rate risk

JEL codes: E41, E43, E44, E51

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1 Introduction

Monetary policy can have major redistributive effects. One of the channels of redistribution is through banking system. Typically, banks hold long duration nominal assets such as fixed-rate mortgages and therefore sustain capital losses (in mark-to-market terms) when nominal interest rates rise. One could conjecture that a maturity-mismatched balance sheet is inherent to the banking business and the resulting interest rate risk is an inevitable side effect. However, there exist deep and liquid markets for interest rate derivatives where banks could hedge against interest rate changes if they wanted. Furthermore, Begenau et al. (2013) show that, if anything, banks tend to use interest rate derivatives to increase rather than reduce their exposure to interest rate risk. Why, then, do they choose this exposure?

We argue that banks choose to bear interest rate risk as part of optimal dynamic hedging. We model a flexible price, complete markets, monetary economy, with three key ingredients. First, the economy consists of banks and households, who are identical except that banks can issue deposits which are close substitutes to currency, up to a leverage limit. Second, there are indeed monetary shocks which move nominal interest rates. Third, risk aversion is high, with a CRRA coefficient greater than 1. In this economy, banks optimally choose to be exposed to interest rate risk.

The mechanism works as follows. Because deposits provide liquidity services, banks earn the spread between the nominal interest rate on bonds and the lower interest rate on deposits. If nominal interest rates rise, the opportunity cost of holding currency rises so, given that currency and deposits are substitutes, demand for deposits rises. This drives up the spread between the nominal interest rate and the interest rate on deposits, increasing banks' return on wealth. This has both income and substitution effects. Because risk aversion is higher than 1, the income effect dominates and banks want to transfer wealth from states of the world with high-return-on-wealth to states of the world with low-return-on-wealth. They are willing to take capital losses when interest rates rise because spreads going forward will be high, and want to make gains when interest rates fall because spreads going forward will be low. Choosing a portfolio of long-duration nominal assets is a way to achieve this exposure and they do not want to undo it even if complete markets allow them to do so.

The fact that bank deposit rates move less than one-for-one with market interest rates has been observed before. Hannan and Berger (1991) and Driscoll and Judson (2013) attribute it to a form of price stickiness; Drechsler et al. (2014) attribute it to imperfect competition among bank branches. Nagel (2014) makes a related observation: the premium on other near-money assets (besides banks deposits) also co-moves with interest rates. He attributes

this, as we do, to the substitutability between money and other liquid assets. Krishnamurthy et al. (2015) document a negative correlation between the supply of publicly issued liquid assets and the supply of liquid bank liabilities, also consistent with their being substitutes. Relative to this literature, the contribution of our work is to derive the implications for equilibrium risk management in a model where the underlying risk is modeled explicitly. Landier et al. (2013) shows cross-sectional evidence that exposure to interest rate risk has consequences for bank lending.

2 The Model

Preferences and technology. Time is continuous. There is a fixed capital stock k which can be used to produce a flow of consumption goods with a linear technology $y_t = ak$. There are two types of agents in the economy: households and bankers, a continuum of each. Both have identical preferences:

$$U(c, m) = \mathbb{E} \left[\int_0^\infty e^{-\rho t} \frac{x(c_t, m_t)^{1-\gamma}}{1-\gamma} dt \right] \quad (1)$$

where

$$m(h, d) = \left(\alpha^{\frac{1}{s}} h^{\frac{s-1}{s}} + (1-\alpha)^{\frac{1}{s}} d^{\frac{s-1}{s}} \right)^{\frac{s}{s-1}} \quad (2)$$

$$x(c, m) = \left(\beta^{\frac{1}{\eta}} c^{\frac{\eta-1}{\eta}} + (1-\beta)^{\frac{1}{\eta}} m^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \quad (3)$$

(1) is a money-in-the-utility-function specification. c is consumption; h are real currency holdings and d are real holdings of deposits.¹ m represents the liquidity services of real money holdings, which are made up of a CES aggregate of currency and deposits, with elasticity of substitution s ; since we think of currency and deposits as substitutes, we focus on $s \geq 1$. x is a CES aggregate of consumption and liquidity services, with elasticity of substitution η .

¹Throughout, uppercase letters denote nominal variables and their corresponding lowercase letter are real variables. Hence $h \equiv \frac{H}{p}$ and $d \equiv \frac{D}{p}$ where p is the price of consumption goods in terms of currency, which we take as the numeraire.

Currency supply. The government issues a nominal amount of currency H . We take monetary policy as exogenously given by the following stochastic process

$$\frac{dH_t}{H_t} = \mu_{H,t}dt + \sigma_{H,t}B_t$$

where B is a standard Brownian motion in a probability space $(\Omega, P, \mathcal{F})$ with the usual conditions, and equipped with the filtration $\mathbb{F}$ generated by B . The process B drives equilibrium dynamics.

The government distributes or withdraws currency to and from agents through lump-sum transfers or taxes.

Markets. There are complete markets. We denote the real price of capital by q , the nominal interest rate by i , the real interest rate by r , and the price of risk by π (so an asset with exposure σ to the process B will pay an excess return $\sigma\pi$). All these processes are contingent on the history of shocks B .

The total real wealth of private agents in the economy includes the value of the capital stock qk , the real value of outstanding currency $h \equiv \frac{H}{p}$ and the net present value of future government transfers and taxes, which we denote by g . Total household wealth is denoted by w and total bankers' wealth is denoted by n , so

$$n + w = qk + h + g \tag{4}$$

Notice that with complete markets it is not necessary to specify who receives government transfers when the supply of currency changes: all those transfers are priced in and included in the definition of wealth. We denote by $z \equiv \frac{n}{n+w}$ the share of the aggregate wealth that is owned by bankers.

The only difference between households and bankers is that bankers may issue deposits. These pay a nominal interest rate i^d and also enter the utility function according to equation (2).²

The amount of deposits bankers can issue is subject to a leverage limit. A banker whose individual wealth is n can issue deposits d^S up to

$$d^S \leq \phi n \tag{5}$$

²Note that even though deposit contracts are specified in nominal terms, nothing prevents a banker and a deposit holder from also trading securities to allocate risk in any way they want.

where ϕ is an exogenous parameter. Constraint (5) may be interpreted as either a regulatory constraint or a level of capitalization required for deposits to actually have the liquidity properties implied by (3).

Monetary policy. As is standard, monetary policy can be described in terms of the supply of currency or in terms of the nominal interest rate. We assume that the government chooses a path for H such that i follows the Cox et al. (1985) stochastic process:³

$$di_t = -\lambda(i_t - \bar{i})dt + \sigma\sqrt{i_t}dB_t \quad (6)$$

Shocks to B are our representation of monetary shocks.

There is more than one stochastic process H that will result in (6). Let

$$\frac{dp_t}{p_t} = \mu_{p,t}dt + \sigma_{p,t}dB_t$$

be the stochastic process for the price level. We assume that the government implements the unique process H such that (6) holds and $\sigma_{p,t} = 0$. Informally, this means that monetary shocks affect the rate of inflation μ_p but the price level moves smoothly.

3 Equilibrium

Households' problem. Starting with some initial nominal wealth W_0 , each household solves a standard portfolio problem:

$$\begin{aligned} & \max_{W,c,h,d,\sigma_W} U(c,m) \\ s.t. \quad & \frac{dW_t}{W_t} = \left(i_t + \sigma_{W,t}\pi_t - \hat{c}_t - \hat{h}_t i_t - \hat{d}_t (i_t - i_t^d) \right) dt + \sigma_{W,t}dB_t \\ & m_t = \left(\alpha^{\frac{1}{s}} h_t^{\frac{s-1}{s}} + (1-\alpha)^{\frac{1}{s}} d_t^{\frac{s-1}{s}} \right)^{\frac{s}{s-1}} \\ & W_t \geq 0 \end{aligned} \quad (7)$$

(a hat denotes the variable is normalized by wealth, i.e. $\hat{c} = \frac{pc}{W} = \frac{c}{w}$). The household obtains a nominal return i_t on its wealth but incurs an opportunity cost i_t on its holdings of currency

³This is a square root process. It is always nonnegative and if $2\lambda\bar{i} \geq \sigma^2$ then it is strictly positive almost surely and has a stationary distribution.

and an opportunity cost $(i_t - i_t^d)$ on its holdings of deposits. Furthermore, the household chooses its exposure σ_W to the monetary shock and obtains the risk premium $\pi\sigma_W$ in return.

Constraint (7) can be rewritten in real terms as

$$\frac{dw_t}{w_t} = \left(r_t + \sigma_{w,t}\pi_t - \hat{c}_t - \hat{h}_t i_t - \hat{d}_t (i_t - i_t^d) \right) dt + \sigma_{w,t} dB_t \quad (8)$$

where $r_t = i_t - \mu_{p,t}$ is the real interest rate.

Bankers' problem. Each banker solves:⁴

$$\begin{aligned} & \max_{n,c,h,d,d^S,\sigma_n} U(c, m) \\ s.t. \quad & \frac{dn_t}{n_t} = \left(r_t + \sigma_{n,t}\pi_t - \hat{c}_t - \hat{h}_t i_t + \left(\hat{d}_t^S - \hat{d}_t \right) (i_t - i_t^d) \right) dt + \sigma_{n,t} dB_t \\ & m_t = \left(\alpha^{\frac{1}{s}} h_t^{\frac{s-1}{s}} + (1 - \alpha)^{\frac{1}{s}} d_t^{\frac{s-1}{s}} \right)^{\frac{s}{s-1}} \\ & \hat{d}_t^S \leq \phi \\ & n_t \geq 0 \end{aligned} \quad (9)$$

This problem differs from the household's problem in that bankers can issue deposits (denoted d^S) up to the leverage limit. They earn the spread $i - i^d$ on the deposits they issue.

Equilibrium definition Given an initial distribution of wealth between households and bankers z and an interest rate process i , a competitive equilibrium is

- a process for the supply of currency H
- processes for prices p, i^d, q, g, r, π
- a plan for the household: $w, c^h, m^h, h^h, d^h, \sigma_w$
- a plan for each banker: $n, c^b, m^b, h^b, d^b, d^S, \sigma_n$

such that

- Households and bankers optimize taking prices as given and $w_0 = (1 - z)(q_0 k + h_0 + g_0)$ and $n_0 = z(q_0 k + h_0 + g_0)$

⁴In the numerical section we will introduce random retirement for bankers in order for the wealth distribution to remain stationary.

- The goods, deposit and currency markets clear:

$$\begin{aligned}c_t^h + c_t^b &= ak \\d_t^h + d_t^b &= d_t^S \\h_t^h + h_t^b &= h_t\end{aligned}$$

- Capital and government transfers and nominal claims are priced by arbitrage:

$$\begin{aligned}q_t &= \mathbb{E}_t^Q \left[a \int_t^\infty \exp \left(- \int_t^s r_u du \right) ds \right] \\g_t &= \mathbb{E}_t^Q \left[\int_t^\infty \exp \left(- \int_t^s r_u du \right) \frac{dH_s}{p_s} \right]\end{aligned}$$

where Q is the equivalent martingale measure implied by r and π .

- Monetary policy is consistent⁵

$$\begin{aligned}i_t &= r_t + \mu_{p,t} \\ \sigma_{p,t} &= 0\end{aligned}$$

4 Equilibrium Characterization

Hamilton-Jacobi-Bellman equations and FOCs. We study the banker's problem first. It can be separated into a static problem (choosing c , m , h and d given x) and a dynamic problem (choosing x and σ_n).

Consider the static problem first. Given the form of the CES aggregators $m(h, d)$ and $x(c, m)$, we immediately get that the minimized cost of one unit of liquidity services is given by ι :

$$\iota(i, i^d) = \left(\alpha i^{1-s} + (1 - \alpha) (i - i^d)^{1-s} \right)^{\frac{1}{1-s}} \quad (10)$$

the minimized cost of one unit of the aggregator x is given by χ :

$$\chi(i, i^d) = \left(\beta + (1 - \beta) \iota(i, i^d)^{1-\eta} \right)^{\frac{1}{1-\eta}} \quad (11)$$

⁵This condition can be interpreted as arbitrage pricing for nominal bonds. The nominal interest rate takes this form because we assumed $\sigma_p = 0$.

and the static choices of c , m , h and d are given by:

$$\begin{aligned}\frac{c}{x} &= \beta \chi^\eta \\ \frac{m}{x} &= (1 - \beta) \left(\frac{\chi}{\iota} \right)^\eta \\ \frac{h}{m} &= \alpha \left(\frac{\iota}{i} \right)^s \\ \frac{d}{m} &= (1 - \alpha) \left(\frac{\iota}{i - i^d} \right)^s\end{aligned}$$

Turn now to the dynamic problem. In equilibrium it will be the case that $i^d < i$ so bankers' leverage constraint will always bind. This means that (9) reduces to

$$\frac{dn_t}{n_t} = \underbrace{\left(r_t + \sigma_{n,t} \pi_t - \chi(i_i, i_t^d) \hat{x}_t + \phi(i_t - i_t^d) \right)}_{\equiv \mu_{n,t}} dt + \sigma_{n,t} dB_t$$

Given the homotheticity of preferences and the linearity of budget constraints the problem of the banker has the value function:

$$V_t^b(w) = \frac{(\xi_t w)^{1-\gamma}}{1-\gamma}$$

ξ_t captures the value of the banker's investment opportunities, i.e. his ability to convert units of wealth into units of lifetime utility, and follows the law of motion

$$\frac{d\xi_t}{\xi_t} = \mu_{\xi,t} dt + \sigma_{\xi,t} dB_t$$

ξ_t is endogenous but exogenous to each individual banker. The associated Hamilton-Jacobi-Bellman equation is

$$\rho V_t^b = \max_{x, \sigma_n, \mu_n} \frac{x^{1-\gamma}}{1-\gamma} + \mathbb{E}_t [dV_t^b]$$

Using Ito's lemma we obtain

$$\begin{aligned}\rho \frac{(\xi_t n)^{1-\gamma}}{1-\gamma} &= \max_{x, \sigma_n, \mu_n} \frac{\hat{x}^{1-\gamma}}{1-\gamma} n^{1-\gamma} + \xi_t^{1-\gamma} n^{1-\gamma} \left(\mu_n + \mu_{\xi,t} - \frac{\gamma}{2} \sigma_n^2 - \frac{\gamma}{2} \sigma_{\xi,t}^2 + (1-\gamma) \sigma_{\xi,t} \sigma_n \right) \\ s.t. \quad &\mu_n = r_t + \sigma_n \pi_t + \phi(i_t - i_t^d) - \hat{x} \chi_t\end{aligned}$$

With FOC:

$$\hat{x}^b = \xi_t^{\frac{\gamma-1}{\gamma}} \chi_t^{-\frac{1}{\gamma}} \quad (12)$$

$$\sigma_n = \frac{\pi_t}{\gamma} + \frac{1-\gamma}{\gamma} \sigma_{\xi,t} \quad (13)$$

Equation (13) has the following interpretation. The first term relates the banker's exposure to B to the risk premium π ; this is the myopic motive for choosing risk exposure. The second term captures the dynamic hedging motive, which depends on an income and a substitution effect. If the agent is sufficiently risk averse ($\gamma > 1$), then the income effect dominates. The agent will want to have more wealth when his investment opportunities (captured by ξ) are worse.

The household's problem is similar. The only difference is that the term $\phi(i_t - i^d)$ is absent from the budget constraint. The value function has the form

$$V_t^h(w) = \frac{(\zeta_t w)^{1-\gamma}}{1-\gamma}$$

where

$$\frac{d\zeta_t}{\zeta_t} = \mu_{\zeta,t} dt + \sigma_{\zeta,t} dB_t$$

and the HJB equation is

$$\begin{aligned} \rho \frac{(\zeta_t w)^{1-\gamma}}{1-\gamma} &= \max_{x, \sigma_w, \mu_w} \frac{\hat{x}^{1-\gamma}}{1-\gamma} w^{1-\gamma} + \zeta_t^{1-\gamma} w^{1-\gamma} \left(\mu_w + \mu_{\zeta,t} - \frac{\gamma}{2} \sigma_w^2 - \frac{\gamma}{2} \sigma_{\zeta,t}^2 + (1-\gamma) \sigma_{\zeta,t} \sigma_w \right) \\ \text{s.t.} \quad &\mu_w = r_t + \sigma_w \pi_t - \hat{x} \chi_t \end{aligned} \quad (14)$$

The FOCs are:

$$\hat{x}^h = \zeta_t^{\frac{\gamma-1}{\gamma}} \chi_t^{-\frac{1}{\gamma}} \quad (15)$$

$$\sigma_w = \frac{\pi_t}{\gamma} + \frac{1-\gamma}{\gamma} \sigma_{\zeta,t} \quad (16)$$

Aggregate state variables. There are two state variables for this economy: the interest rate i (which is exogenous) and the bankers' share of aggregate wealth z (which is endogenous).

Definition 1. A recursive equilibrium is a set of value functions $\xi(i, z)$ and $\zeta(i, z)$, and policy functions $(\hat{x}^b(i, z), \sigma_n(i, z))$ and $(\hat{x}^h(i, z), \sigma_w(i, z))$; prices $q(i, z)$, $g(i, z)$, $h(i, z)$, $r(i, z)$,

$\pi(i, z)$, $i^d(i, z)$, $\mu_p(i, z)$; and a law of motion for the endogenous aggregate state variable $dz_t = \mu_z(i, z)z_t dt + \sigma_z(i, z)z_t dB_t$ such that

1. ξ and ζ , and the corresponding policy functions solve the HJB equations of bankers and households respectively, taking prices as given.

2. Markets clear:

(a) for goods:

$$[\hat{x}^h(1-z) + \hat{x}^b z] \beta \chi^\eta = a \frac{k}{qk + h + g}$$

(b) For deposits:

$$[\hat{x}^h(1-z) + \hat{x}^b z] (1-\alpha)(1-\beta) \left(\frac{\chi}{\iota}\right)^\eta \left(\frac{\iota}{i-i^d}\right)^s = \phi z$$

(c) For currency:

$$[\hat{x}^h(1-z) + \hat{x}^b z] \alpha(1-\beta) \left(\frac{\chi}{\iota}\right)^\eta \left(\frac{\iota}{i}\right)^s = \frac{h}{qk + h + g}$$

3. Arbitrage pricing:

(a) For capital:

$$\frac{a}{q} + \mu_q - r = \pi \sigma_q$$

(b) For government transfers

$$(\mu_h + \mu_p) h + \mu_g - r g = (\sigma_h h + \sigma_g) \pi$$

4. Monetary policy

$$i = r + \mu_p$$

5. The law of motion of z satisfies:

$$\mu_z = (1-z) \left((\sigma_n - \sigma_w) \pi + \phi(i - i^d) - (\hat{x}^b - \hat{x}^h) \chi + \sigma_w(\sigma_w - \sigma_n) \right) - \frac{z}{1-z} \sigma_z^2 \quad (17)$$

$$\sigma_z = (1-z) (\sigma_n - \sigma_w) \quad (18)$$

Aggregate risk sharing. From the FOC for σ_n and σ_w we obtain the following expression for σ_z

$$\sigma_z = (1 - z) \frac{1 - \gamma}{\gamma} (\sigma_\xi - \sigma_\zeta) \quad (19)$$

This expression has the following interpretation. Bankers are always relatively better off than households per unit of wealth because they are able to issue deposits and earn the spread $i - i^d$, i.e. $\xi_t > \zeta_t$ always. The gap in investment opportunities depends on aggregate conditions. The term $\sigma_\xi - \sigma_\zeta$ captures the sensitivity of this gap with respect to aggregate shock. Risk sharing is governed by an income and substitution effects. If agents are not very risk averse ($\gamma < 1$), then they will prefer to shift aggregate wealth to bankers after aggregate shocks that improve their investment opportunities relative to households', i.e. $\frac{\xi}{\zeta}$ goes up. This is the substitution effect. In contrast, if agents are more risk averse ($\gamma > 1$) they will prefer to shift aggregate wealth to bankers after shocks that worsen their investment opportunities relative to households', i.e. $\frac{\xi}{\zeta}$ goes down.

We can use Ito's lemma to obtain an expression for $\sigma_\xi - \sigma_\zeta$:

$$\sigma_\xi - \sigma_\zeta = \left(\frac{\xi_z}{\xi} - \frac{\zeta_z}{\zeta} \right) \sigma_z z + \left(\frac{\xi_i}{\xi} - \frac{\zeta_i}{\zeta} \right) \sigma \sqrt{i} \quad (20)$$

We can see that the response of relative investment opportunities to aggregate shocks, depends on aggregate risk sharing decisions as captured by σ_z . This is because in equilibrium investment opportunities depend on the distribution of wealth z . We can put (19) and (20) together to obtain

$$\sigma_z = \frac{(1 - z) \frac{1 - \gamma}{\gamma} \left(\frac{\xi_i}{\xi} - \frac{\zeta_i}{\zeta} \right)}{1 - z(1 - z) \frac{1 - \gamma}{\gamma} \left(\frac{\xi_z}{\xi} - \frac{\zeta_z}{\zeta} \right)} \sigma \sqrt{i} \quad (21)$$

5 Numerical Results

We solve for the recursive equilibrium by mapping it into a system of partial differential equations for the equilibrium objects. We solve them numerically using a finite difference scheme. In order to obtain a stationary wealth distribution we add a random retirement for bankers which arrives with Poisson intensity θ . After retirement, bankers become households. Appendix A explains the numerical procedure in detail.

Parameter values. We choose parameter values so that the model economy matches some key features of the US economy. Our choice of parameters is shown on Table 1.

Parameter	Value	Target
a	1	Normalization
k	1	Normalization
γ	3	Standard value
$\bar{i}$	0.0453	Interest rate process
λ	1.05	
σ	0.173	
ρ	0.02	Average real interest rates.
α	0.17	Currency-to-M2 ratio
β	0.3	M2-to-wealth ratio
ϕ	2.01	M2-to-bank-equity ratio
θ	0.06	Bank-equity-to-wealth ratio
η	0.3	Average bank spreads and sensitivity to interest rates
s	20	

Table 1: Parameter values

Our measure of the interest rate is the 6-month LIBOR rate in US dollars. We choose the parameters $\bar{i}$, λ , σ to match the mean, variance and slope of the autocorrelation function of interest rates for the period 1986-2012. ρ is set to match an average real interest rate of 2%. The rest of the parameters are calibrated jointly, but some moments are especially sensitive to each of them.

We take the amount of currency in circulation to be our measure of h and M2 minus the stock of currency to be our measure of d . The aggregator m in the model does not correspond exactly to M2 because currency and deposits are not perfect substitutes. However, it is still possible to construct an analogue of M2 by simply adding $h + d$. The parameter α (the weight of currency in the m aggregator) is chosen to match the observed currency-to-M2 ratio. The parameter β (the weight of liquidity services in the x aggregator) is chosen to match the M2-to-wealth ratio, where our measure of wealth is Total Wealth of the Private Sector, taken from the Flow of Funds.

The leverage parameter ϕ is chosen to match the ratio of M2 to the equity of the financial sector. This raises a number of issues. First, many liabilities of the financial system are not included in M2; the leverage ratio we target is not based on total liabilities but on liabilities that are included in M2. In the model, there is a sharp distinction between “deposits” (bank liabilities that provide some liquidity services) and any other liability, which does not, and the leverage limit only applies to the former. In practice, not every bank liability included in M2 provides the same liquidity service. For instance, checking deposits provide more liquidity than time deposits. Conversely, some bank liabilities not included in M2, such as

short term wholesale borrowing, provide some liquidity service as well. Thus, the choice of M2 as a cutoff for what constitutes a liquid liability is somewhat arbitrary. Second, there is the question of what institutions to include in the definition of “banks”. We choose a broad definition, which includes commercial banks, savings and loans institutions, credit unions broker dealers and hedge funds. We take the aggregate net equity of these institutions from the Flow of Funds, and take that as the counterpart for n . The retirement intensity θ is chosen to match the average bank-equity-to-total-wealth ratio.

The elasticities of substitution η and s are key for determining the level of bank spreads and how sensitive these are to interest rates. Drechsler et al. (2014) report average interest rates for checking deposits, savings deposits and time deposits paid by US banks. We weight them by their aggregate total from the Flow of Funds to construct a measure of the interest rate paid on the average deposit and construct spreads as the difference between the average deposit rate and the 6-month LIBOR rate. We choose η and s to match two moments of the joint distribution of interest rates and spreads: the average level of spreads and the regression coefficient obtained by regressing spreads on interest rates.

Table summarizes the moments and how closely the model matches them:

Moment	Data	Model
Average real interest rates	0.02	0.0198
Currency-to-M2 ratio	0.102	0.153
M2-to-wealth ratio	0.112	0.107
M2-to-bank-equity ratio	2.01	2.01
Bank-equity-to-wealth ratio	0.0560	0.0449

Table 2: Data and Model Moments

Steady state. The model has a stochastic steady state. Figure 1 shows the joint steady state distribution of the state variables and the drift of z . The wealth share of bankers spends most of the time near its average value of 0.05. However, low interest rates are associated with a downward rift in z , so the economy also spends some time in low i states, low- z states.

Aggregate risk sharing. Figure 2 shows σ_z is negative throughout. This means that after a monetary shock that increases the interest rate i , banks’ share of aggregate wealth goes down. Figure 3 shows the same effect, rescaled so that the units are more easily interpretable. The quantity $\frac{dz/z}{di} = \frac{\sigma_z}{\alpha_i}$ represents how much z changes⁶ if interest rates rise 1%. The size

⁶In percentage terms, not percentage point terms.

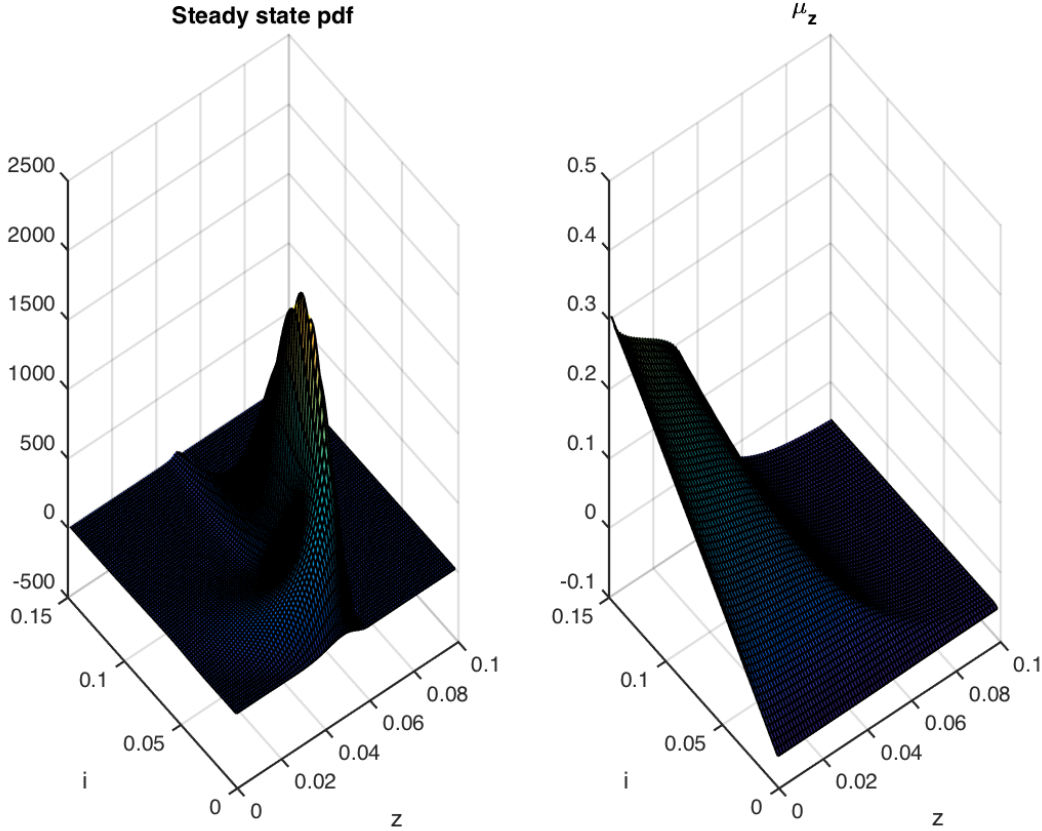


Figure 1: Steady state distribution and drift of z .

of this effect is different in different parts of the state space, and averages 0.46%, i.e. a 1 percentage point rise in interest rates leads to a 0.46% fall in banker's share of total wealth.

Exploring the mechanism. A higher interest rate i makes holding currency more costly for agents. Since currency and deposits are close substitutes ($s = 20$) this increases the demand for deposits, other things being equal. This is shown in Figure 4 for the currency-to-deposit ratio.

Since the supply of deposits is constrained by banks' leverage constraint, the spread $i - i^d$, which is the opportunity cost of holding deposits, must rise to clear the deposit market, as shown by Figure 5.

Since banks earn this spread and households don't, a monetary shock that raises interest rates improves the relative investment opportunities of bankers. Figure 6 shows the ratio $\frac{\xi}{\zeta}$ is increasing in i .

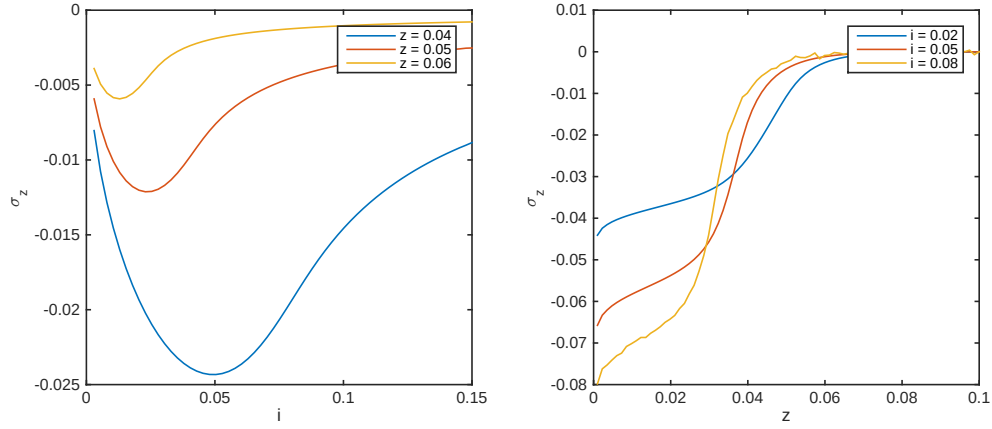


Figure 2: Aggregate risk sharing as captured by σ_z .

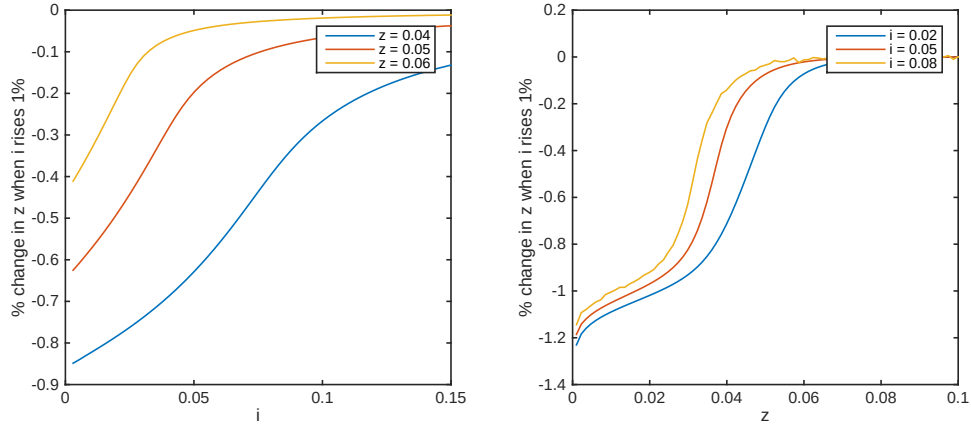


Figure 3: Bank's exposure to interest rate risk.

Since $\gamma > 1$, this means that the right hand side of (19) is negative: bankers' share of aggregate wealth goes down after interest rates go up. Since bankers benefit from a monetary shock that raises interest rates (via higher spreads) it makes sense that they are willing to sustain financial losses relative to households in that state.

In addition, there is an amplification effect. Since banks' share of aggregate wealth goes down, the supply of deposits goes down too. In order to clear the market the currency-to-deposit ratio rises, and so do spreads $i - i^d$, as seen in Figures 4 and 5. This further improves bankers' relative investment opportunities, as seen in Figure 6, which shows that the ratio $\frac{\xi}{\zeta}$ rises when z falls. As a result, this amplifies the differential risk exposure of banks and households. Mathematically, since $\frac{\xi_z}{\xi} - \frac{\zeta_z}{\zeta} < 0$, the denominator in (21) is smaller than 1.

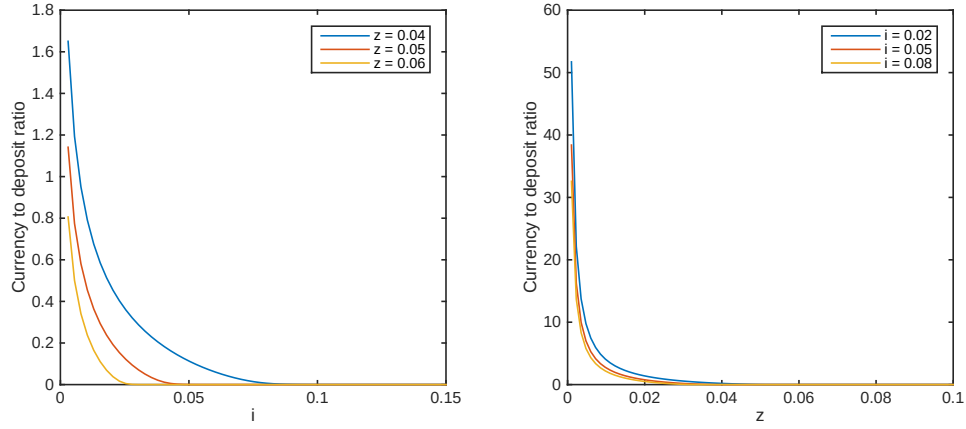


Figure 4: Currency-to-deposit ratio.

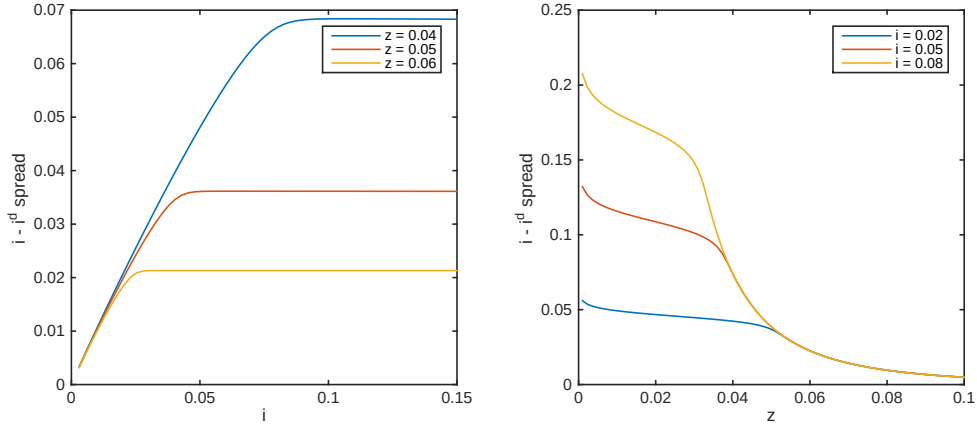


Figure 5: The interest rate spread $i - i^d$.

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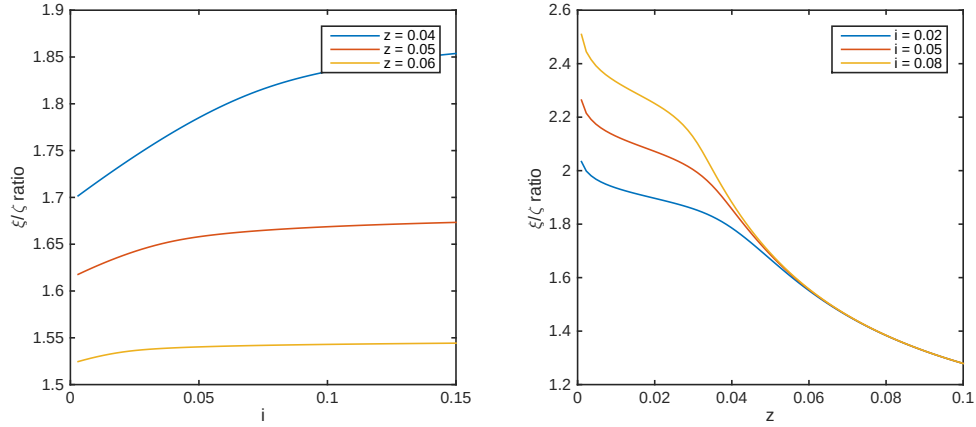


Figure 6: Relative investment opportunities as captured by the ratio $\frac{\xi}{\zeta}$.

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Appendix A: Solution Method

Retirement In order to have a stationary distribution for z we assume that bankers retire randomly with Poisson intensity θ . Upon retirement, they keep their wealth but lose their

ability to issue deposits, effectively becoming households. The HJB equation then becomes:⁷

$$\begin{aligned} \rho \frac{(\xi_t \tilde{n})^{1-\gamma}}{1-\gamma} &= \max_{x, \sigma_n, \mu_n} \frac{\hat{x}^{1-\gamma}}{1-\gamma} \tilde{n}^{1-\gamma} + \xi_t^{1-\gamma} \tilde{n}^{1-\gamma} \cdot \\ &\quad \left(\mu_n + \mu_{\xi,t} - \frac{\gamma}{2} \sigma_n^2 - \frac{\gamma}{2} \sigma_{\xi,t}^2 + (1-\gamma) \sigma_{\xi,t} \sigma_n + \theta \left(\frac{(\xi_t \tilde{n})^{1-\gamma}}{1-\gamma} - \frac{(\xi_t \tilde{n})^{1-\gamma}}{1-\gamma} \right) \right) \\ \text{s.t.} \quad \mu_n &= r_t + \sigma_n \pi_t + \phi(i_t - i_t^d) - \hat{x} \chi_t \end{aligned}$$

Replacing the first order conditions (12) and (13) (which are unaffected), we obtain:

$$\frac{\rho + \theta}{1-\gamma} = \frac{\gamma}{1-\gamma} \xi_t^{\frac{\gamma-1}{\gamma}} \chi_t^{\frac{\gamma-1}{\gamma}} + r_t + \frac{\gamma}{2} \sigma_{n,t}^2 + \phi(i_t - i_t^d) + \mu_{\xi,t} - \frac{\gamma}{2} \sigma_{\xi,t}^2 + \theta \frac{\left(\frac{\xi_t}{\xi_t}\right)^{1-\gamma}}{1-\gamma} \quad (22)$$

Similarly, replacing (15) and (16) in (14) we obtain the following HJB equation for households:

$$\frac{\rho}{1-\gamma} = \frac{\gamma}{1-\gamma} \zeta_t^{\frac{\gamma-1}{\gamma}} \chi_t^{\frac{\gamma-1}{\gamma}} + r_t + \frac{\gamma}{2} \sigma_w^2 + \mu_{\zeta,t} - \frac{\gamma}{2} \sigma_{\zeta,t}^2 \quad (23)$$

Overview of the solution procedure The solution method finds endogenous objects as functions of state variables. We'll divide the equilibrium objects into two groups. Denote the first group of variables by $X = \{\xi(i, z), \zeta(i, z), q(i, z), h(i, z), g(i, z), i^d(i, z)\}$. We'll express these as a system of differential equations and solve it backwards. Denote the second group of variables by $Y = \{\hat{x}^b, \hat{x}^h, \sigma_z, \sigma_n, \sigma_w, \pi, r\}$. These variables can be solved statically for every possible value of X .

Solving for Y given X Suppose we had found all the variables in X as functions of (i, z) . By Ito's Lemma it follows that the law of motion of any of these variables X is:

$$dX(i, z) = \mu_X(i, z) dt + \sigma_X(i, z) dB \quad (24)$$

⁷Introducing retirement implies that there is a distinction between the net worth of an individual banker and the collective net worth of all bankers, since the group of individuals who are bankers keeps shrinking. We retain the notation n to refer to the collective net worth and denote the net worth of an individual banker by $\tilde{n}$.

where the drift and volatility are

$$\begin{aligned}\mu_X(i, z) &= X_z(i, z) \mu_z(i, z) + X_i(i, z) \mu_i(i) \\ &\quad + \frac{1}{2} [X_{zz}(i, z) \sigma_z^2(i, z) z^2 + X_{ii}(i, z) \sigma_i^2(i) + 2X_{zi}(i, z) \sigma_i(i) z \sigma_z(i, z)] \\ \sigma_X &= X_z(i, z) \sigma_z(i, z) z + X_i(i, z) \sigma_i(i)\end{aligned}$$

or, in geometric form:

$$\frac{dX(i, z)}{X(i, z)} = \mu_X(i, z) dt + \sigma_X(i, z) dB \quad (25)$$

where the drift and volatility are

$$\begin{aligned}\mu_X(i, z) &= \frac{X_z(i, z)}{X(i, z)} \mu_z(i, z) + \frac{X_i(i, z)}{X(i, z)} \mu_i(i) \\ &\quad + \frac{1}{2} \left[\frac{X_{zz}(i, z)}{X(i, z)} \sigma_z^2(i, z) z^2 + \frac{X_{ii}(i, z)}{X(i, z)} \sigma_i^2(i) + 2 \frac{X_{zi}(i, z)}{X(i, z)} \sigma_i(i) z \sigma_z(i, z) \right] \\ \sigma_X &= \frac{X_z(i, z)}{X(i, z)} \sigma_z(i, z) z + \frac{X_i(i, z)}{X(i, z)} \sigma_i(i)\end{aligned}$$

Hence if we know the functions X and their derivatives, we know their drifts and volatilities at every point of the state space. Numerically, we approximate the derivatives with finite-difference matrices such for any set of values of X on a grid, the values of the derivatives on the grid are:

$$\begin{aligned}X_i &\approx D_i X \\ X_z &\approx X D_z \\ X_{ii} &\approx D_{ii} X \\ X_{zz} &\approx X D_{zz} \\ X_{iz} &\approx D_i X D_z\end{aligned}$$

The variables in Y can be found as follows. $\iota(i, z)$ and $\chi(i, z)$ are immediate from (10) and (11). $\hat{x}^b(i, z)$ and $\hat{x}^h(i, z)$ follow from the first order conditions (12) and (15). $\sigma_z(i, z)$ follows from (21).

By definition,

$$z = \frac{n}{qk + h + g}$$

which implies:⁸

$$\sigma_z = \sigma_n - \frac{qk\sigma_q + h\sigma_h + \sigma_g}{qk + h + g} \quad (26)$$

Knowing g , q , h and their volatilities, as well as σ_z , $\sigma_n(i, z)$ can be obtained from (26).

$\pi(i, z)$ can then be obtained from the FOC (13). $\sigma_w(i, z)$ follows from the FOC (16). $r(i, z)$ follows from (23).

Solving for X The remaining equilibrium conditions are:

$$[\hat{x}^h(1 - z) + \hat{x}^b z] \beta \chi^\eta = a \frac{k}{qk + h + g} \quad (27)$$

$$[\hat{x}^h(1 - z) + \hat{x}^b z] (1 - \alpha)(1 - \beta) \left(\frac{\chi}{\iota}\right)^\eta \left(\frac{\iota}{i - i^d}\right)^s = \phi z \quad (28)$$

$$[\hat{x}^h(1 - z) + \hat{x}^b z] \alpha(1 - \beta) \left(\frac{\chi}{\iota}\right)^\eta \left(\frac{\iota}{i}\right)^s = \frac{h}{qk + h + g} \quad (29)$$

$$\frac{\gamma}{1 - \gamma} \xi^{\frac{\gamma-1}{\gamma}} \chi^{\frac{\gamma-1}{\gamma}} + r + \frac{\gamma}{2} \sigma_n^2 + \phi(i_t - i_t^d) + \mu_\xi - \frac{\gamma}{2} \sigma_\xi^2 + \theta \frac{\left(\frac{\xi}{\chi}\right)^{1-\gamma}}{1 - \gamma} = \frac{\rho + \theta}{1 - \gamma} \quad (30)$$

$$\frac{a}{q} + \mu_q - r = \pi \sigma_q \quad (31)$$

$$(\mu_h + \mu_p) h + \mu_g - rg = (\sigma_h h + \sigma_g) \pi \quad (32)$$

Equation (27) is the market clearing condition for the goods market; (28) is a market clearing condition for the deposits market; (29) is a market clearing condition for the currency market; (30) is the banker's HJB equation; (31) is an arbitrage-pricing condition for capital and (32) is an arbitrage-pricing condition for government transfers.

We find the functions X by differentiating equations (27)-(32) with respect to time and finding X such that the time derivatives are equal to zero. Differentiating yields the following system of differential equations:

$$A \cdot \begin{pmatrix} \dot{\xi} \\ \dot{\zeta} \\ \dot{q} \\ \dot{h} \\ \dot{g} \\ \dot{i}^d \end{pmatrix} = B \quad (33)$$

⁸ g is expressed in absolute terms using (24) but n , q , h are expressed in geometric terms using (25)

where A is a 6×6 matrix with entries:

$$\begin{aligned}
a_{11} &= \beta \chi^\eta z \frac{\gamma-1}{\gamma} \xi^{-\frac{1}{\gamma}} \chi^{-\frac{1}{\gamma}} \\
a_{12} &= \beta \chi^\eta (1-z) \frac{\gamma-1}{\gamma} \zeta^{-\frac{1}{\gamma}} \chi^{-\frac{1}{\gamma}} \\
a_{13} &= a \frac{k^2}{(qk+h+g)^2} \\
a_{14} &= a \frac{k}{(qk+h+g)^2} \\
a_{15} &= a \frac{k}{(qk+h+g)^2} \\
a_{16} &= \left[- \left(z \xi^{\frac{\gamma-1}{\gamma}} + (1-z) \zeta^{\frac{\gamma-1}{\gamma}} \right) \frac{1}{\gamma} \chi^{-\frac{1+\gamma}{\gamma}} \beta \chi^\eta + [\hat{x}^h(1-z) + \hat{x}^b z] \beta \eta \chi^{\eta-1} \right] a_\chi
\end{aligned}$$

$$\begin{aligned}
a_{21} &= (1-\alpha)(1-\beta) \left(\frac{\chi}{\iota} \right)^\eta \left(\frac{\iota}{i-i^d} \right)^s z \frac{\gamma-1}{\gamma} \xi^{-\frac{1}{\gamma}} \chi^{-\frac{1}{\gamma}} \\
a_{22} &= (1-\alpha)(1-\beta) \left(\frac{\chi}{\iota} \right)^\eta \left(\frac{\iota}{i-i^d} \right)^s (1-z) \frac{\gamma-1}{\gamma} \zeta^{-\frac{1}{\gamma}} \chi^{-\frac{1}{\gamma}} \\
a_{23} &= 0 \\
a_{24} &= 0 \\
a_{25} &= 0 \\
a_{26} &= [\hat{x}^h(1-z) + \hat{x}^b z] (1-\alpha)(1-\beta) (i-i^d)^{-s} \iota^{s-\eta} \chi^\eta \left[\eta \chi^{-1} a_\chi + (s-\eta) \iota^{-1} a_\iota + s (i-i^d)^{-1} \right] \\
&\quad - \left[\zeta^{\frac{\gamma-1}{\gamma}} (1-z) + \xi^{\frac{\gamma-1}{\gamma}} z \right] \frac{1}{\gamma} \chi^{-\frac{1+\gamma}{\gamma}} (1-\alpha)(1-\beta) \left(\frac{\chi}{\iota} \right)^\eta \left(\frac{\iota}{i-i^d} \right)^s a_\chi
\end{aligned}$$

$$a_{31} = \alpha(1 - \beta) \left(\frac{\chi}{\iota}\right)^\eta \left(\frac{\iota}{i}\right)^s z \frac{\gamma - 1}{\gamma} \xi^{-\frac{1}{\gamma}} \chi^{-\frac{1}{\gamma}}$$

$$a_{32} = \alpha(1 - \beta) \left(\frac{\chi}{\iota}\right)^\eta \left(\frac{\iota}{i}\right)^s (1 - z) \frac{\gamma - 1}{\gamma} \zeta^{-\frac{1}{\gamma}} \chi^{-\frac{1}{\gamma}}$$

$$a_{33} = \frac{hk}{(qk + h + g)^2}$$

$$a_{34} = -\frac{qk + g}{(qk + h + g)^2}$$

$$a_{35} = \frac{h}{(qk + h + g)^2}$$

$$a_{36} = [\hat{x}^h(1 - z) + \hat{x}^b z] (\alpha)(1 - \beta) \iota^{s-\eta} \chi^\eta i^{-s} [\eta \chi^{-1} a_\chi + (s - \eta) \iota^{-1} a_\iota] + \\ - \left[\zeta^{\frac{\gamma-1}{\gamma}} (1 - z) + \xi^{\frac{\gamma-1}{\gamma}} z \right] \frac{1}{\gamma} \chi^{-\frac{1+\gamma}{\gamma}} (\alpha)(1 - \beta) \left(\frac{\chi}{\iota}\right)^\eta \left(\frac{\iota}{i}\right)^s a_\chi$$

$$a_{41} = -\frac{1}{\xi}$$

$$a_{42} = \frac{1}{\zeta}$$

$$a_{43} = 0$$

$$a_{44} = 0$$

$$a_{45} = 0$$

$$a_{46} = 0$$

$$a_{51} = 0$$

$$a_{52} = -\frac{1}{\zeta}$$

$$a_{53} = -\frac{1}{q}$$

$$a_{54} = 0$$

$$a_{55} = 0$$

$$a_{56} = 0$$

$$\begin{aligned}
a_{61} &= 0 \\
a_{62} &= -\frac{g+h}{\zeta} \\
a_{63} &= 0 \\
a_{64} &= -1 \\
a_{65} &= -1 \\
a_{66} &= 0
\end{aligned}$$

and B is a 6×1 vector with entries

$$\begin{aligned}
b_1 &= 0 \\
b_2 &= 0 \\
b_3 &= 0 \\
b_4 &= \frac{\gamma}{1-\gamma} \xi^{\frac{\gamma-1}{\gamma}} \chi^{\frac{\gamma-1}{\gamma}} + \frac{\rho}{1-\gamma} - \frac{\gamma}{1-\gamma} \zeta^{\frac{\gamma-1}{\gamma}} \chi^{\frac{\gamma-1}{\gamma}} - \frac{\gamma}{2} \sigma_w^2 - \tilde{\mu}_\zeta + \frac{\gamma}{2} \sigma_\zeta^2 + \frac{\gamma}{2} \sigma_n^2 + \phi(i_t - i_t^d) + \tilde{\mu}_\xi \\
&\quad - \frac{\gamma}{2} \sigma_\xi^2 + \theta \frac{\left(\frac{\zeta}{\xi}\right)^{1-\gamma}}{1-\gamma} - \frac{\rho+\theta}{1-\gamma} \\
b_5 &= \frac{a}{q} + \tilde{\mu}_q - \frac{\rho}{1-\gamma} + \frac{\gamma}{1-\gamma} \zeta^{\frac{\gamma-1}{\gamma}} \chi^{\frac{\gamma-1}{\gamma}} + \frac{\gamma}{2} \sigma_w^2 + \tilde{\mu}_\zeta - \frac{\gamma}{2} \sigma_\zeta^2 - \pi \sigma_q \\
b_6 &= (\tilde{\mu}_h + i)h + \tilde{\mu}_g - \left(\frac{\rho}{1-\gamma} - \frac{\gamma}{1-\gamma} \zeta^{\frac{\gamma-1}{\gamma}} \chi^{\frac{\gamma-1}{\gamma}} - \frac{\gamma}{2} \sigma_w^2 - \tilde{\mu}_\zeta + \frac{\gamma}{2} \sigma_\zeta^2 \right) (g+h) - (\sigma_h h + \sigma_g) \pi
\end{aligned}$$

where for any variable X , $\tilde{\mu}_X$ is defined as

$$\tilde{\mu}_X \equiv \mu_X - \frac{\dot{X}}{X}$$

The algorithm for finding X is as follows.

1. Guess values for X at every point in the state space
2. Compute the derivatives with respect to i and z by a finite difference approximation
3. Compute Y at every point in the state space given the guess for X .
4. Compute $\dot{X}$ at every point in the state space using (33)
5. Take a time-step backwards to define a new guess for X

6. Repeat steps 1-5 until $\dot{X} \approx 0$.

The condition $\dot{X} = 0$ is equivalent to saying that equilibrium conditions (27)-(32) hold.

Finding the steady state Once we solve for the equilibrium, this defines drifts and volatilities for the two state variables: $\mu_i(i, z)$, $\sigma_i(i, z)$, $\mu_z(i, z)$, $\sigma_z(i, z)$. The density $f(i, z)$ of the steady state distribution is the solution to the stationary Kolmogorov Forward Equation:

$$0 = -\frac{\partial}{\partial i} [\mu_i(i, z) f(i, z)] - \frac{\partial}{\partial z} [\mu_z(i, z) f(i, z)] \quad (34)$$

$$+ \frac{1}{2} \left(\frac{\partial^2}{\partial i^2} [\sigma_i(i, z)^2 f(i, z)] + \frac{\partial^2}{\partial z^2} [\sigma_z(i, z)^2 f(i, z)] + 2 \frac{\partial^2}{\partial i \partial z} [\sigma_i(i, z) \sigma_z(i, z) f(i, z)] \right)$$

We solve this equation by rewriting it in matrix form.⁹ The first step is to discretize the state space into a grid of $N_i \times N_z$ points and then convert it to a $N_i N_z \times 1$ vector. Let $vec(\cdot)$ be the operator that does this conversion. We then convert the differentiation matrices so that they are properly applied to vectors:

$$\begin{aligned} D_i^{vec} &\equiv I_{N_i} \otimes D_i \\ D_{ii}^{vec} &\equiv I_{N_i} \otimes D_{ii} \\ D_z^{vec} &\equiv M' (I_{N_z} \otimes D_z) M \\ D_{zz}^{vec} &\equiv M' (I_{N_z} \otimes D_{zz}) M \\ D_{iz}^{vec} &\equiv D_i^{vec} D_z^{vec} \end{aligned}$$

where $\otimes$ denotes the Kreonecker product and M is the vectorized transpose matrix such that $Mvec(A) = vec(A')$.

Now rewrite (34):

$$-D_i^{vec} \cdot (diag(vec(\mu_i)) vec(f)) - D_z^{vec} (diag(vec(\mu_z)) vec(f))$$

$$+ \frac{1}{2} \left[D_{ii}^{vec} \cdot (diag(vec(\sigma_i^2)) vec(f)) + D_{zz}^{vec} (diag(vec(\sigma_z^2)) vec(f)) \right. \\ \left. + 2D_{iz}^{vec} (diag(vec(\sigma_i)) diag(vec(\sigma_z)) vec(f)) \right] = 0$$

and therefore

$$Avec(f) = 0 \quad (35)$$

⁹See Achdou et al. (2014) for details on this procedure.

where

$$A = -D_i^{vec} \cdot diag(vec(\mu_i)) - D_z^{vec} \cdot diag(vec(\mu_z)) \\ + \frac{1}{2} [D_{ii}^{vec} \cdot diag(vec(\sigma_i^2)) + D_{zz}^{vec} \cdot diag(vec(\sigma_z^2)) + 2D_{iz}^{vec} \cdot (diag(vec(\sigma_i)) diag(vec(\sigma_z)))]$$

Equation (35) defines an eigenvalue problem. We solve it by imposing the additional condition that f integrates to 1.