Directed search with phantom vacancies

Jim Albrecht, Bruno Decreuse, Susan Vroman

This version, February 2015

Abstract

Information persistence about already filled vacancies implies that old vacancies are likely to be obsolete or phantom vacancies. When ads for jobs stipulate the vacancy creation date, job-seekers apply for the different jobs so as to equalize matching odds across vacancy age. This search behavior leads them to over-apply to young vacancies. Finding a job of a given age creates phantom vacancy and a negative informational externality that affects all cohorts of vacancies after this age. Thus the magnitude of the externality decreases with the age of the vacancy filled. We calibrate the model on US labor market data. The magnitude of the externality is small, despite the fact that the contribution of phantom vacancies to overall frictions is large.

JEL codes:

Keywords: Directed search; information persistence; vacancy vintage

1 Introduction

> I am currently on the job hunt and I had a question about applying to jobs online. You know how most websites will tell you the job has been posted 1 day ago, 28 days ago, etc. For some reason, I have concluded that I need to apply to a job the first week they post the position to have the best chances of being hired. Although I heard that it can take up to a month for the company to hire anyone for the position, I feel that applying to a job that was posted 3 weeks ago isn't that promising. What is your take on this situation?
>
> AskaManager.com

The purpose of this paper is to provide a theoretical answer to this question, both from the individual and the collective perspective. Our model has two key ingredients. First, it contains a reason why the question makes sense. Namely, a number of advertised jobs are already filled: they are phantom vacancies, or phantoms for short. No one wants to lose time courting phantoms. Second, the search market is segmented by vacancy age. Job-seekers observe the age of job listings, and figure out how likely it is that the corresponding job is still available. Thus search is directed to particular vacancy ages.

Due to phantoms, workers may decide to limit their search to the most recent vacancies. However, trying older ages is also an option on the ground that many fewer job seekers search such segments. In equilibrium, workers direct their search by vacancy age so that the matching odds are the same. We first study a situation where recruiters cannot renew their vacancies (Section 2). Thus vacancies age with calendar time. We deduce the corresponding unemployment rate, and the distributions by vacancy age of job seekers, phantom, and nonphantom vacancies. We also compare this allocation to the random search allocation where workers randomly pick a vacancy from the whole distribution.

Directed search is not constrained efficient: workers over-search young vacancies (Section 3). Finding jobs creates a negative informational externality. Match formation fuels the phantom stock, thereby contributing to weaken the efficiency of the matching process. Under random search, this externality is the same whatever the vacancy age when the match is formed. When the search market is segmented by vacancy age, the magnitude of the externality decreases with vacancy age. Phantoms only hurt the job-seekers who search for older vacancies. Consider a match formed with a vacancy created one month ago. This generates a one-month old phantom vacancy. This phantom will only affect the job-seekers who search job listings that are older than one month. Conversely, a match formed with a newly created vacancy generates a phantom that will affect all job-seekers during its lifetime.

Quantitatively, the directed search allocation does not differ much from the (almost) efficient allocation. The reason is that efficient search methods generate more matches, thereby creating more phantoms. The overall effect is small, despite the fact that the nuisance caused by information obsolescence can be rather large. We provide an example grounded on a calibration for the US labor market. In this example, the unemployment rate difference between the worst allocation (random search) and the (almost) constrained efficient allocation is only 0.5 percentage point. In this example, the withdrawal of all phantoms would decrease the unemployment rate by one third, and information obsolescence amounts to two thirds of overall frictions.

We then examine what happens when job listings are renewed at random intervals of time (Section 4). Vacancy renewal means that the job listing is replaced by a new one. This practice concentrates the distribution of advertised jobs in young ages, i.e. more frequent renewal implies younger vacancies on average. Job-seekers react by searching for more recent vacancies compared with the allocation without renewal. The qualitative conclusion stays the same: directed search is not constrained efficient. Quantitatively, the spread between the random search allocation and the efficient allocation grows: it reaches 1.2 percentage point. Meanwhile, the contribution of phantom vacancies to overall frictions decreases to 40%.

2 The model

We focus on the stationary state of a continuous time model. Calendar time is denoted by t. At each time, there is a fixed number of jobs K that can be either vacant or occupied. The number of vacancies is $v(t)$. They differ in age $a \geq 0$. There is also a continuum of workers of total size one. Each worker can be either unemployed or employed. The number of unemployed is $u(t)$. By construction, we have $v(t) + 1 - u(t) = K$.

Employed workers and jobs separate with flow probability λ. Newly unemployed workers join the pool of unemployed and immediately start searching for job. Newly destroyed jobs join the pool of vacancies.

All jobs produce the flow output $y \equiv 1$ and pay the same wage w. The search market is segmented by vacancy age a. In each sub-market a, $u(a,t)$ unemployed workers try to match with $v(a,t)$ vacancies. The matching process is frictional. On top of the usual search frictions, information persistence about former vacancies creates an additional source of matching frictions. The flow number of matches is

$$M(a,t) = \frac{v(a,t)}{v(a,t) + p(a,t)} m(u(a,t), v(a,t) + p(a,t)), \tag{1}$$

where $p(a,t)$ is the number of phantom vacancies.

We define $\theta(a,t) \equiv (v(a,t)+p(a,t))/u(a,t)$ as the sub-market tightness. The numerator is composed of the total number of vacancies, thus including phantom and nonphantom vacancies.

The function m is strictly concave, has constant returns to scale, and is such that $m(0,v) = m(u,0) = 0$, $\lim_{u\to 0} m_u(u,v) = \lim_{v\to 0} m_v(u,v) = \infty$. In Section 3 below, we present the model assuming that the meeting function is Cobb-Douglas, i.e., $m(x_1,x_2) = m_0 x_1^{1-\alpha} x_2^{\alpha}$, $\alpha \in (0,1)$. The number of matches has two components: the number of contacts m multiplied by the proportion of contacts where an active vacancy is involved. Phantom vacancies hurt the search process: by increasing p at given v and u, M falls.

Unemployed workers spread themselves over the different sub-markets with $u(t) = \int_0^\infty u(a,t)da$. Unemployment obeys the law of motion:

$$du(t)/dt = -\int_0^\infty M(a,t)da + \lambda(1-u). \tag{2}$$

The stocks of active and phantom vacancies evolve according to the following laws of motion:

$$\begin{aligned} \partial v(a,t)/\partial a + \partial v(a,t)/\partial t &= -M(a,t), & (3)\\ \partial p(a,t)/\partial a + \partial p(a,t)/\partial t &= \beta M(a,t) - \delta p(a,t), & (4) \end{aligned}$$

with $\beta, \delta \geq 0$, $v(0,t) = \lambda(1-u(t))$, $p(0,t) = 0$, and $v(t) = \int_0^\infty v(a,t)da$. The number of vacancies decreases across age by the number of matches. Matches fuel the phantom stock, whereas phantoms depreciate at rate δ.

The vacancy motion implies that vacancies cannot be renewed or refreshed. Thus a job that became available at date t and which is still available at date $t+a$ is identified as having age a, and nothing can be done to signal that this vacancy is still active and must be distinguished from the other members of its cohort. Section 4 analyzes the case where vacancies can be renewed.

Let $\pi(a,t) \equiv v(a,t)/[v(a,t)+p(a,t)]$ be the nonphantom proportion. This evolves according to

$$\partial\pi(a,t)/\partial a + \partial\pi(a,t)/\partial t = -\frac{M(a,t)}{v(a,t)}\pi(a,t)[1-(1-\beta)\pi(a,t)] + \delta\pi(a,t)(1-\pi(a,t)), \tag{5}$$

with $\pi(0,t) = 1$.

In steady state, calendar time does not affect the different variables. Thus $du/dt = dv/dt = 0$, $\partial v(a,t)/\partial t = \partial p(a,t)/\partial t = 0$, $u(a,t) = u(a)$, and $M(a,t) = M(a)$. Hereafter we refer to variables without mentioning time again. A dot over some variable x denotes the derivative of x with respect to age a, i.e. $\dot{x} \equiv x'(a)$.

For later use, we define the elasticity of the meeting function with respect to phantom and nonphantom vacancies $\alpha(\theta) = \theta m_2(1,\theta)/m(1,\theta)$, the job-finding rate by vacancy age $\mu(a) = M(a)/u(a)$ and the rate of filling vacancies by vacancy age $\eta(a) = M(a)/v(a)$. Finally, ϕ_u, ϕ_v, and ϕ_j denote the density functions of unemployment, nonphantom vacancies and total number of vacancies by age.

2.1 Random search allocation

We start with the allocation obtained when workers do not observe vacancy age, or, equivalently, when they do not take this information into account. Thus search is random and the ratio of phantom and nonphantom vacancies to job-seekers is constant over age.

For all $a \geq 0$,

$$\theta(a) = (v(a) + p(a))/u(a) = (v + p)/u = \theta. \tag{6}$$

The numbers of vacancies and phantom vacancies follow

$$\begin{aligned} \dot{v} &= -v(a)\frac{m(1,\theta)}{\theta}, &(7)\\ \dot{p} &= \beta v(a)\frac{m(1,\theta)}{\theta} - \delta p(a), &(8)\end{aligned}$$

with $v(0) = v_0 = \lambda(1 - u)$ and $p(0) = 0$. This gives two ordinary differential equations in v and p.

The resolution gives

$$\begin{aligned} v(a) &= v_0 \exp(-am(1,\theta)/\theta), &(9)\\ p(a) &= \sigma v_0 \left[\exp(-\delta a) - \exp(-am(1,\theta)/\theta)\right], &(10)\end{aligned}$$

with

$$\sigma = \frac{\beta m(1,\theta)/\theta}{m(1,\theta)/\theta - \delta}. \tag{11}$$

Using the random search property (6), the number of job-seekers by vacancy age follows

$$u(a) = \frac{v_0}{\theta}\left[\sigma \exp(-\delta a) + (1 - \sigma)\exp(-am(1,\theta)/\theta)\right]. \tag{12}$$

The total number of unemployed is $u = \int_0^\infty u(a)da$. Using the fact that $v_0 = \lambda(1-u)$, we obtain

$$u = \frac{\lambda + \lambda(\beta/\delta)m(1,\theta)/\theta}{m(1,\theta) + \lambda + \lambda(\beta/\delta)m(1,\theta)/\theta}. \tag{13}$$

This equation is the Beveridge curve. On top of the usual decreasing relationship between unemployment and vacancies, equation (13) features the role played by phantom vacancies. Holding tightness constant, unemployment increases with the ratio β/δ. Phantoms create a negative informational externality that reduces the efficiency of the matching process. As this externality is entirely due to job creation, its magnitude increases with the rate at which vacancies are filled $m(1,\theta)/\theta$. Thus it decreases with tightness.

To close the model, we use the resource constraint $1 - u + v = K$.

Proposition 1 The random search allocation is characterized by the unique θ_{rs} that solves

$$\frac{m(1,\theta_{rs})+\lambda\theta_{rs}}{m(1,\theta_{rs})+\lambda+\lambda(\beta/\delta)m(1,\theta_{rs})/\theta_{rs}}=K. \tag{14}$$

Proof. We have $v=\int_0^\infty v(a)da$. Thus $v=\lambda(1-u)\theta/m(1,\theta)$. The resource constraint $1-u+v=K$ implies that $(1-u)(1+\lambda\theta/m(1,\theta))=K$. When combined with (13), this equation gives (14). To prove existence and uniqueness, let $\psi:[0,\infty)\rightarrow(-\infty,\infty)$ be such that

$$\psi(\theta)=m(1,\theta)(1-K)+\lambda\theta-K\lambda\frac{\beta}{\delta}\frac{m(1,\theta)}{\theta}-K\lambda. \tag{15}$$

The steady-state tightness, if any, is such that $\psi(\theta_{rs})=0$. The properties of the function m imply that $\lim_{\theta\rightarrow 0}\psi(\theta)<0$ and $\lim_{\theta\rightarrow\infty}\psi(\theta)>0$. By continuity, there is $\theta_{rs}>0$ such that $\psi(\theta_{rs})=0$. Moreover,

$$\psi'(\theta)=\frac{1}{\theta}\left[\alpha(\theta)m(1,\theta)(1-K)+\lambda\theta+K\lambda\frac{\beta}{\delta}(1-\alpha(\theta))m(1,\theta)\right]. \tag{16}$$

Using the fact that $\psi(\theta_{rs})=0$, it becomes

$$\psi'(\theta_{rs})=\frac{1}{\theta_{rs}}\left[\alpha(\theta_{rs})K\lambda+(1-\alpha(\theta_{rs}))\lambda\theta_{rs}+K\lambda\frac{\beta}{\delta}m(1,\theta_{rs})/\theta_{rs}\right]>0. \tag{17}$$

It follows that θ_{rs} is unique. ■

There is a unique random search allocation. In this allocation, tightness increases with the phantom birth rate to death rate ratio, i.e. $d\theta_{rs}/d(\beta/\delta)>0$. This ratio governs the quantitative impact of the negative informational externality that affects the matching process. When the ratio increases, the phantom stock increases as well, other things equal. Thus the ratio of overall jobs, including phantoms, to unemployment goes up.

The corollary is that unemployment increases with β/δ. To see this, let us write the resource constraint as follows:

$$u=1-K/(1+\lambda\theta_{rs}/m(1,\theta_{rs})). \tag{18}$$

This equation is particularly convenient because the ratio β/δ only affects the right-hand side through changes in θ_{rs}. An increase in β/δ implies that $m(1,\theta_{rs})/\theta_{rs}$ goes down and so unemployment must rise.

In the random search allocation, the rate at which vacancies are filled is $\eta(a)=m(1,\theta_{rs})/\theta_{rs}$ for all $a\geq 0$. It is constant over vacancy age. Thus the chance of being filled does not change with vacancy ageing. Conversely, the job-finding rate varies with the nonphantom proportion. The job-finding rate is $\mu(a)=m(1,\theta_{rs})\pi(a)$. The proportion $\pi(a)$ decreases over age, and so the job-finding rate goes down.

Formally, the nonphantom proportion follows

$$\dot{\pi} = -\pi(a)[m(1,\theta_{rs})/\theta_{rs} - \delta - \pi(a)((1-\beta)m(1,\theta_{rs})/\theta_{rs} - \delta)]. \tag{19}$$

We have $\dot{\pi}(0) < 1$. Then two cases are possible. If $m(1,\theta_{rs})/\theta_{rs} \geq \delta$, then $\pi(a)$ monotonically converges towards 0. It does not reach 0 in finite age. Thus the exit rate $\mu(a)$ strictly decreases from $m(1,\theta_{rs})$ to 0. If $m(1,\theta_{rs})/\theta_{rs} < \delta$, then $\pi(a)$ monotonically converges towards $\pi_{rs} = (\delta - m(1,\theta_{rs})/\theta_{rs})/\ (\delta - (1-\beta)m(1,\theta_{rs})/\theta_{rs})$. The value π_{rs} is reached in a finite age. Thus the exit rate strictly decreases from $m(1,\theta_{rs})$ to $m(1,\theta_{rs})\pi_{rs}$, and stays constant afterwards.

The density functions of nonphantom vacancies, total vacancies, and job-seekers by vacancy age are

$$\phi_v(a) = \frac{\exp(-am(1,\theta_{rs})/\theta_{rs})}{m(1,\theta_{rs})/\theta_{rs}}, \tag{20}$$

$$\phi_j(a) = \frac{(1-\sigma_{rs})\exp(-am(1,\theta_{rs})/\theta_{rs}) + \sigma_{rs}\exp(-\delta a)}{(1-\sigma_{rs})m(1,\theta_{rs})/\theta_{rs} + \sigma_{rs}/\delta}, \tag{21}$$

$$\phi_u(a) = \frac{(1-\sigma_{rs})\exp(-am(1,\theta_{rs})/\theta_{rs}) + \sigma_{rs}\exp(-\delta a)}{(1-\sigma_{rs})m(1,\theta_{rs})/\theta_{rs} + \sigma_{rs}/\delta}. \tag{22}$$

Nonphantom vacancies are distributed according to an exponential law. This distribution obtains in standard matching models. Thus it also holds when there are phantom vacancies and search is random. The density functions of total vacancies and of job-seekers coincide. This is a direct implication of the random search assumption whereby the ratio of total vacancies to job-seekers does not change across vacancy age. These density functions are composed of two exponential terms. Since σ_{rs} is not necessarily positive, these terms can evolve in opposite directions.

When there are no phantoms, which happens when $\beta = 0$ or δ tends to infinity, unemployment is minimized. Both $\eta(a)$ and $\mu(a)$ are constant over vacancy age, and all distributions follow the same exponential law. Phantoms increase unemployment and distort the distribution of vacancies and unemployed towards older vacancy vintages.

2.2 Directed search allocation

Workers now observe vacancy age and choose which market to enter. A sub-market a is open if and only if $u(a) > 0$. We denote by Ω the set of open sub-markets. All open sub-markets must yield the same utility. Thus the job-finding rate $\mu(a) = M(a)/u(a)$ must be the same across open sub-markets. Given that $v(a) > 0$ for all $a \geq 0$, the properties of the meeting function m imply that $u(a) > 0$. If not, a worker who would enter a non-open sub-market would immediately find a job. Thus all sub-markets are open in equilibrium and so $\Omega = \mathbb{R}_+$.

That $\mu(a)$ is constant across age implies for all $a \geq 0$ that $M(a)/u(a) = m(1, \theta(0))$. Using the definition of M and the definition of $\pi(a)$, we obtain

$$m(1, \theta(a))\pi(a) = m(1, \theta(0)). \tag{23}$$

This equation defines a strictly decreasing relationship between $\theta(a)$ and $\pi(a)$. the relationship is parameterized by $\theta(0)$. Unlike the random search allocation, tightness is not constant in the directed search allocation. It strictly increases with $\theta(0)$ and strictly decreases with $\pi(a)$.

Differentiating (23) with respect to age gives:

$$-\alpha(\theta)\frac{\dot{\theta}}{\theta(a)} = \frac{\dot{\pi}}{\pi(a)}. \tag{24}$$

Using (23) and (24), we can characterize the motions of these variables as follows:

$$\begin{aligned} \dot{\pi} &= -\frac{m(1, \theta(0))}{f(\pi(a), \theta(0))}((1 - \pi(a)) + \beta\pi(a)) + \delta\pi(a)(1 - \pi(a)), & (25) \\ -\alpha(\theta)\dot{\theta} &= \left[1 - \frac{m(1, \theta(0))}{m(1, \theta(a))}\right](\delta\theta - m(1, \theta(a)) - \beta m(1, \theta(0)), & (26) \end{aligned}$$

where $f(\pi(a), \theta(0))$ is defined by $m(1, f(\pi(a), \theta(0)))\pi(a) = m(1, \theta(0))$. Both differential equations can be solved given $\pi(0) = 1$ and $\theta(0)$. Note, however, that this latter value can only be found once the equilibrium is solved.

The stationary number of unemployed balances inflows and outflows. That the exit rate from unemployment does not vary with vacancy age considerably simplifies the calculation. Indeed, outflows are $\int_0^\infty M(a)da = m(1, \theta(0))u$. Thus

$$u = \frac{\lambda}{\lambda + m(1, \theta(0))}. \tag{27}$$

The number of vacancies by age follows

$$\dot{v} = -\frac{m(1, \theta(a))}{\theta(a)}v(a)$$

Therefore

$$v(a) = v(0)\exp\left[-\int_0^a \frac{m(1, \theta(b))}{\theta(b)}db\right]. \tag{28}$$

The resource constraint finally writes

$$1 - u + \int_0^\infty v(a)da = K. \tag{29}$$

Proposition 2 The directed search allocation is characterized by the function $\theta_{ds}(a)$ that solves

$$-\alpha(\theta_{ds})\dot{\theta}_{ds} = \left[1 - \frac{m(1, \theta_{ds}(0))}{m(1, \theta_{ds}(a))}\right][\delta\theta_{ds} - m(1, \theta_{ds}(a)] - \beta m(1, \theta_{ds}(0)), \tag{30}$$

$$\frac{m(1, \theta_{ds}(0))}{\lambda + m(1, \theta_{ds}(0))}\left\{1 + \lambda\int_0^\infty \exp\left[-\int_0^a \frac{m(1, \theta_{ds}(b))}{\theta_{ds}(b)}db\right]da\right\} = K. \tag{31}$$

Proof. A directed search allocation must be characterized by these two equations. Once the function $\theta_{ds}(a)$ is known, we can compute $\pi(a)$, $v(a)$, $p(a)$, and $u(a)$. We now prove existence and uniqueness. At given $\theta(0) = \theta_0$, equation (30) determines a unique function $\theta(a, \theta_0)$. This function strictly increases with a. Let $\xi : [0, \infty) \to (-\infty, \infty)$ be such that

$$\xi(\theta_0) = \frac{m(1, \theta_0)}{\lambda + m(1, \theta_0)} \left\{ 1 + \lambda \int_0^{\infty} \exp\left[- \int_0^a \frac{m(1, \theta(b, \theta_0))}{\theta(b, \theta_0)} db \right] da \right\} - K. \tag{32}$$

We have $\lim_{\theta \to 0} \xi(\theta) = -K$, whereas $\lim_{\theta \to \infty} \xi(\theta) = +\infty$. Thus there is $\theta_0 > 0$ such that $\xi(\theta_0) = 0$. The function ξ is strictly increasing. Therefore θ_0 is unique. ■

The differential equation (30) describes tightness by vacancy age at given initial condition $\theta(0)$. The resource constraint (31) then determines $\theta(0)$.

The rate at which vacancies are filled is $\eta(a) = m(1, \theta_{ds}(0))/(\pi(a)\theta_{ds}(a))$. It decreases over vacancy age. The job-finding rate is $\mu(a) = m(1, \theta_{ds}(0))$. It is constant over vacancy age. This property naturally arises in a context where there are no phantoms. Then, the ratio of vacancies to unemployed stays constant, and the resulting job-finding rate must be the same at all vacancy ages. Accounting for phantoms modifies this scenario though the result is the same. Workers who take into account obsolete information apply to younger vacancies. Thus tightness increases over vacancy age and the hazard rate of filling vacancies decreases with age.

The density functions of nonphantom vacancies, total vacancies, and job-seekers by vacancy age are

$$\phi_v(a) = \frac{\exp\left[- \int_0^a m(1, \theta_{ds}(b))/\theta_{ds}(b) db \right]}{\int_0^{\infty} \exp\left[- \int_0^c m(1, \theta_{ds}(b))/\theta_{ds}(b) db \right] dc}, \tag{33}$$

$$\phi_j(a) = \frac{m(1, \theta_{ds}(a)) \exp\left[- \int_0^a m(1, \theta_{ds}(b))/\theta_{ds}(b) db \right]}{\int_0^{\infty} m(1, \theta_{ds}(c)) \exp\left[- \int_0^c m(1, \theta_{ds}(b))/\theta_{ds}(b) db \right] dc}, \tag{34}$$

$$\phi_u(a) = \frac{(m(1, \theta_{ds}(a))/\theta_{ds}(a)) \exp\left[- \int_0^a m(1, \theta_{ds}(b))/\theta_{ds}(b) db \right]}{\int_0^{\infty} (m(1, \theta_{ds}(c))/\theta_{ds}(c)) \exp\left[- \int_0^c m(1, \theta_{ds}(b))/\theta_{ds}(b) db \right] dc}. \tag{35}$$

Unlike the random search allocation, the density of job-seekers by vacancy age ϕ_u differs from the density of overall jobs ϕ_j. The former strictly decreases over age, whereas the latter may increase. When there are no phantoms, the directed search allocation coincides with the random search allocation. Thus exit rates are constant across vacancy ages, and the distributions of job-seekers and vacancies by age follow exponential laws.

2.3 Efficient allocation

The planner observes vacancy age and decides on the allocation of unemployed people to the different sub-markets. To justify our focus on steady-state allocations, we assume the discount rate is equal to zero. Thus the efficient allocation minimizes the unemployment rate.

The efficient allocation solves the following optimal control problem:

$$\max_{u(.)} -\int_0^\infty u(a)da \tag{$*$}$$

subject to

$$\begin{aligned}
\dot{v} &= -m(\pi(a)u(a), v(a)), && (c1)\\
v(0) &= \lambda\left(1-\int_0^\infty u(a)da\right), && (c2)\\
\dot{p} &= \beta m(\pi(a)u(a), v(a)) - \delta p(a), && (c3)\\
p(0) &= 0, && (c4)\\
\lambda\left(1-\int_0^\infty u(a)da\right) &= \int_0^\infty m(\pi(a)u(a), v(a))da, && (c5)\\
1-\int_0^\infty u(a)da + \int_0^\infty v(a)da &= K. && (c6)
\end{aligned}$$

The planner is subject to the evolution of vacancies over age $(c1)$–$(c2)$, the evolution of phantoms over age $(c3)$–$(c4)$, the inflow-outflow constraint $(c5)$, and the resource constraint $(c6)$.

Proposition 3 In the efficient allocation tightness has the following form:

$$\begin{aligned}
\left[\alpha(\theta) - \frac{\theta\alpha'(\theta)}{1-\alpha(\theta)}\right]\dot{\theta} &= -\pi m(1,\theta)\left[\alpha(\theta) + b_0(1-\alpha(\theta))\theta\right]\\
&\quad -\delta(1-\pi)\theta + \beta\pi m(1,\theta)\left[\alpha(\theta)+\delta y(1-\alpha(\theta))\theta\right],
\end{aligned} \tag{36}$$

where b_0 is a constant term and the function $y : [0,\infty) \to (-\infty,\infty)$ is such that

$$y(a) = e^{-\delta a}\left[y_0 + \int_0^a e^{\delta r}\theta(r)^{-1}dr\right]. \tag{37}$$

Proof. We transform the constraints $(c2)$, $(c5)$ and $(c6)$ into differential equations. We define the following functions:

$$\begin{aligned}
\phi(a) &= \int_0^a [m(\pi(b)u(b), v(b)) + \lambda u(b)]db, && (38)\\
\psi(a) &= \int_0^a [v(b) - u(b)]db, && (39)\\
\chi(a) &= -\lambda\int_0^a u(b)db. && (40)
\end{aligned}$$

The optimization program $(*)$ is equivalent to

$$\max_{u(.)} -\int_0^\infty u(a)da \tag{$*$}$$

subject to

$$\begin{aligned}
\dot{v} &= -m(\pi(a)u(a), v(a)), && (41)\\
v(0) &= v_0, && (42)\\
\phi'(a) &= m(\pi(a)u(a), v(a)) - \lambda u(a), && (43)\\
\phi(0) &= 0, && (44)\\
\phi(\infty) &= \lambda, && (45)\\
\dot{\psi} &= v(a) - u(a), && (46)\\
\psi(0) &= 0, && (47)\\
\psi(\infty) &= K - 1, && (48)\\
\dot{\chi} &= -\lambda u(a), && (49)\\
\chi(0) &= 0, && (50)\\
\chi(\infty) &= v_0 - \lambda. && (51)
\end{aligned}$$

The Lagrangian is

$$\mathcal{L}(.) = -u - \sigma_1 m(\pi u, v) + \sigma_2[\beta m(\pi u, v) - \delta p] + \sigma_3(m(\pi u, v) + \lambda u) + \sigma_4(v - u) - \sigma_5 \lambda u \quad (52)$$

The optimality conditions are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial u} &= -1 + \pi m_1(-\sigma_1 + \beta\sigma_2 + \sigma_3) + \sigma_3\lambda - \sigma_4 - \sigma_5\lambda = 0, && (53)\\
\frac{\partial \mathcal{L}}{\partial v} &= \left(\frac{u}{v}\pi(1-\pi)m_1 + m_2\right)(-\sigma_1 + \beta\sigma_2 + \sigma_3) + \sigma_4 = -\dot{\sigma}_1, && (54)\\
\frac{\partial \mathcal{L}}{\partial p} &= -\frac{u}{v}\pi^2 m_1(-\sigma_1 + \beta\sigma_2 + \sigma_3) - \sigma_2\delta = -\dot{\sigma}_2, && (55)\\
\frac{\partial \mathcal{L}}{\partial \phi} &= \frac{\partial \mathcal{L}}{\partial \psi} = \frac{\partial \mathcal{L}}{\partial \chi} = 0 = \dot{\sigma}_3 = \dot{\sigma}_4 = \dot{\sigma}_5. && (56)
\end{aligned}$$

Equations (56) imply that σ_3, σ_4 and σ_5 are constant over age a.

From equation (53), we have

$$\pi m_1(-\sigma_1 + \beta\sigma_2 + \sigma_3) = 1 + \sigma_4 + (\sigma_5 - \sigma_3)\lambda \equiv t. \quad (57)$$

We also know that

$$\begin{aligned}
m_1 &= (1-\alpha)m(1, \theta), && (58)\\
m_2 &= \alpha m(1, \theta)/\theta, && (59)\\
\frac{u}{v} &= \frac{u}{v+p}\frac{v+p}{v} = \frac{1}{\theta\pi}. && (60)
\end{aligned}$$

We replace (57)–(60) in equations (54) and (55). We obtain

$$\begin{aligned}
\pi(1-\alpha)m(1,\theta)(-\sigma_1+\beta\sigma_2+\sigma_3) &= t, &(61)\\
\dot{\sigma}_1 &= -\frac{1}{1-\alpha}\frac{t}{\theta}\left(\alpha+\frac{p}{v}\right)-\sigma_4, &(62)\\
\dot{\sigma}_2 &= \frac{t}{\theta}-\delta\sigma_2. &(63)
\end{aligned}$$

Differentiating (61) with respect to a, we get

$$-\frac{t\dot{\alpha}}{1-\alpha}+\alpha t\frac{\dot{\theta}}{\theta}+t\frac{\dot{\pi}}{\pi}+(1-\alpha)\pi m(1,\theta)\left[-\dot{\sigma}_1+\beta\dot{\sigma}_2\right]=0. \tag{64}$$

We now assume that $t \neq 0$. Using (61)–(63), and dividing by t, we obtain

$$\begin{aligned}
&-\frac{\dot{\alpha}}{1-\alpha}+\alpha\frac{\dot{\theta}}{\theta}-\beta\frac{m(1,\theta)}{\theta}\pi+\left[\delta-\frac{m(1,\theta)}{\theta}\right](1-\pi) &(65)\\
&+(1-\alpha)\pi\frac{m(1,\theta)}{\theta}\left[\frac{1}{1-\alpha}\left(\alpha+\frac{p}{v}\right)+\beta+\frac{\sigma_4-\beta\delta\sigma_2}{t}\theta\right]=0. &(66)
\end{aligned}$$

Rearranging terms, noting that $\dot{\alpha}=\alpha'(\theta)\dot{\theta}$ and $p/v=(1-\pi)/\pi$, and denoting $b\equiv\sigma_4/t$ and $y\equiv\sigma_2/t$, we finally obtain (36). To get (37), we integrate the following differential equation:

$$\dot{y}=\frac{1}{\theta}-\delta y, \tag{67}$$

with $y(0)=y_0$. ■

Proposition 3 only gives the shape of the efficient allocation. Formally, the efficient tightness function depends on two parameters, b_0 and y_0, which still need to be found. However, having the shape of the solution allows us to eliminate potential candidates.

Proposition 4 The following statements hold:

(i) The random search allocation differs from the efficient allocation if and only if $\beta > 0$;

(ii) The directed search allocation generically differs from the efficient allocation when $\beta > 0$.

Proof. (i) Let $\beta > 0$ and suppose that both allocations coincide. Then $\dot{\theta}(a)=0$ and $\theta(a)=\theta_{rs}$ for all $a \geq 0$. Equation (36) implies that

$$\begin{aligned}
b_0 &= \alpha\frac{\beta-1}{(1-\alpha)\theta_{rs}}, &(68)\\
\frac{1-\pi}{\pi} &= \beta(1-\alpha)my. &(69)
\end{aligned}$$

The term $(1-\pi)/\pi=p/v$ can be computed using equations (9) and (10). The variable $y(a)$ can be computed from (37). Replacing $(1-\pi)/\pi$ and $y(a)$ by the computed values, we obtain

$$e^{-\delta a}(\sigma-\beta(1-\alpha)my_0+\beta(1-\alpha)m/(\delta\theta))-\sigma e^{-\eta a}-\beta(1-\alpha)m/(\delta\theta)=0. \tag{70}$$

This equation must hold for all $a \geq 0$. This is impossible unless $\beta = 0$. In this case, $\sigma = 0$ and equation (70) is always satisfied.

(ii) For convenience, we rewrite the differential equations that describe the directed search and the efficient allocations:

$$
\begin{aligned}
[\alpha - g(\theta)]\dot{\theta}_e &= -\pi m_e(\alpha + (1-\alpha)b_0\theta_e) - \delta(1-\pi)\theta_e + \beta\pi m_e(\alpha + (1-\alpha)\delta y\theta_e), && (71)\\
\alpha\dot{\theta}_{ds} &= -\delta(1-\pi)\theta_{ds} + m_{ds} - (1-\beta)m_0, && (72)
\end{aligned}
$$

with $m_0 \equiv m(1,\theta_0)$, $m_e \equiv m(1,\theta_e)$, $m_{ds} \equiv m(1,\theta_{ds})$ and $g(\theta) \equiv \theta\alpha'(\theta)/(1-\alpha(\theta))$. Both allocations coincide iff $\dot{\theta}_{ds}(a) = \dot{\theta}_e(a)$ for all $a \geq 0$ and $\theta_{ds}(0) = \theta_e(0) = \theta_0$. We now neglect the indices ds and e. As $\pi m = m_0$ in the directed search allocation, we have

$$
\begin{aligned}
[\alpha - g(\theta)]\dot{\theta} &= -m_0(\alpha + (1-\alpha)b_0\theta) - \delta(1-m_0/m)\theta + \beta m_0(\alpha + (1-\alpha)\delta y\theta), && (73)\\
\alpha\dot{\theta} &= -\delta(1-m_0/m)\theta + m - (1-\beta)m_0. && (74)
\end{aligned}
$$

Thus

$$
(1-\alpha)m_0\beta\delta y\theta = h(\theta)[-\delta(1-m_0/m)\theta + m - (1-\beta)m_0] + m_0(\alpha + (1-\alpha)b_0\theta) + \delta(1-m_0/m)\theta - \beta\alpha m_0, \quad (75)
$$

with $h(\theta) = 1 - g(\theta)/\alpha(\theta)$. Equation (75) provides a first relationship between y and θ. This relationship must hold for all $a \geq 0$. This implies that its differential with respect to a must be equal to 0. Hence,

$$
\begin{aligned}
& \dot{\theta}\{h'(\theta)[-\delta(1-m_0/m)\theta + m - (1-\beta)m_0] \\
& +(h(\theta)-1)[-\delta(1-m_0/m) + \delta m_0 m'_\theta/m^2] + h(\theta)m'_\theta + m_0(1-\alpha)b_0\} && (76)\\
= \; & (1-\alpha)m_0\beta\delta(\theta\dot{y} + y\dot{\theta}). && (77)
\end{aligned}
$$

Using (67) and (74), we can replace $\dot{y}$ and $\dot{\theta}$ by functions of y and θ. This gives

$$
\begin{aligned}
(1-\alpha)m_0\beta\delta y\theta \;=\; & (\dot{\theta}/\theta - \delta)^{-1} \times \\
& \{\dot{\theta}[h'(-\delta(1-m_0/m)\theta + m - (1-\beta)m_0) + (h-1)(-\delta(1-m_0/m) + \delta m_0 m'/m^2) \\
& + hm' + (1-\alpha)mb_0] - (1-\alpha)m_0\beta\delta\}. && (78)
\end{aligned}
$$

This gives a second relationship between y and θ. We can eliminate $(1-\alpha)m_0\beta\delta y\theta$, which finally gives

$$
\begin{aligned}
& [-\delta(1-m_0/m)\theta + m - (1-\beta)m_0] \times \\
& \{h'[-\delta(1-m_0/m)\theta + m - (1-\beta)m_0] + (h-1)[-\delta(1-m_0/m) + \delta m_0 m'/m^2] \\
& + hm' + (1-\alpha)m_0 b_0\} - (1-\alpha)m_0\beta\delta \\
= \; & \{h[-\delta(1-m_0/m)\theta + m - (1-\beta)m_0] + m_0(\alpha + (1-\alpha)b_0\theta) + \delta(1-m_0/m)\theta - \beta\alpha m_0\} \\
& \times[(-\delta(1-m_0/m)\theta + m - (1-\beta)m_0)/\theta - \alpha\delta], && (79)
\end{aligned}
$$

which has the form $f(\theta) = 0$. The directed search and the efficient allocation coincide only if this equation is satisfied for all $\theta(a)$, $a \geq 0$. When $\beta > 0$, θ is continuously increasing in a, and so the equation must hold for the whole subset $[\theta_0, \infty)$. This is generally not the case, as the properties of f depend on those of h, which is a primitive of the model. Equation (79) actually defines a differential equation in h. This differential equation may have a solution, and this may be compatible with the requirement $0 \leq \alpha(\theta) \leq 1$ for all $\theta \in [0, \infty)$. We do not know whether this case is possible or not. However, h is exogenous and so it has no reason to coincide with this particular function, if it exists. Consider for instance the Cobb-Douglas case with $\alpha(\theta) = \alpha$ for all $\theta \geq 0$. This gives $h(\theta) = 1$ and $h'(\theta) = 0$. After simplification, equation (79) is now

$$\begin{aligned} & -(2+\alpha(1-\beta))\delta m_0 + \alpha\delta m_0 b_0 \theta + \delta m/(1-\alpha) + 2(1-\beta)m_0 m/\theta \\ & -m^2/\theta + (1-\beta)\delta m_0^2/m - (1-\beta)^2 m_0^2/\theta \\ = \; & 0 \end{aligned} \tag{80}$$

This equation has a finite number of solutions in $\theta \in [\theta_0, \infty)$ – none is also a possibility. However, it does not hold for all $\theta \in [\theta_0, \infty)$.

Finally, when $\beta = 0$, equation (79) is equivalent to $b_0\theta_0 = -\alpha/(1-\alpha)$. This condition is also the one that ensures $\theta_e(a) = \theta_{ds}(a) = \theta_{rs}$ for all $a \geq 0$. ■

We prove Proposition 4 by contradiction. We assume in turn that each of the proposed allocation is efficient, and we reach a formal impossibility. For the random search allocation, we show that $\theta(a) = \theta_0$ for all $a \geq 0$ cannot solve equations (36) and (37). Thus randomizing over the different vacancies without accounting for their age is not efficient. This result is not surprising. Still, it differs from standard search models where applying for the different jobs with equal probability is actually efficient. The reason here is of course that older vacancies have a higher probability of being phantom vacancies.

For the directed search allocation, we suppose that there are two functions $\theta(a)$ and $y(a)$ that solve equations (30) on the one hand and (36) and (37) on the other hand. We show that this generally requires that θ takes a fixed number of discrete values, which violates the fact that θ takes a continuum of values in the directed search allocation. Thus, and this is more surprising than in the random search case, the directed search allocation is also inefficient. Equalizing payoffs across vacancy ages conveys a negative externality. We explain it with a parameterized version of the model.

3 The Cobb-Douglas case

The purpose of this section is to compare the different allocations. The meeting technology is Cobb-Douglas, i.e. $m(u, v+p) = m_0 u^{1-\alpha}(v+p)^\alpha$, $\alpha \in (0,1)$. As usual, m_0 is the scale parameter that governs total factor productivity. The elasticity α belongs to $(0,1)$.

Proposition 1 allows us to find the random search allocation. Tightness is $\theta(a) = \theta_{rs}$ for all $a \geq 0$, where θ_{rs} results from

$$m_0\theta_{rs}^{\alpha}(1-K) + \lambda\theta_{rs} - K\lambda(\beta/\delta)m_0\theta_{rs}^{\alpha-1} - K\lambda = 0. \tag{81}$$

As for the directed search allocation, Proposition 2 implies that we need to find the function $\theta_{ds}(a)$ such that

$$-\alpha\dot{\theta}_{ds} = \delta\theta_{ds} - m_0\theta_{ds}^{\alpha} - \delta\theta_0^{\alpha}\theta_{ds}^{1-\alpha} + (1-\beta)m_0\theta_0^{\alpha}, \tag{82}$$

$$\frac{m_0\theta_0}{\lambda + m_0\theta_0}\left\{1 + \lambda\int_0^{\infty}\exp\left[-\int_0^{a} m_0\theta(b)^{\alpha-1}db\right]da\right\} = K. \tag{83}$$

Let $\varepsilon \in [0,1]$ and define the function $\theta_{\varepsilon} : [0,\infty) \to [0,\infty)$ such that

$$-\alpha\dot{\theta}_{\varepsilon} = \varepsilon[\delta\theta_{\varepsilon} - m_0\theta_{\varepsilon}^{\alpha} - m_0\theta_0^{\alpha}\theta_{\varepsilon}^{1-\alpha} + (1-\beta)m_0\theta_0^{\alpha}]. \tag{84}$$

For a given ε, we can solve this equation together with the resource constraint $1-u+v = K$ to compute the unemployment rate u_{ε}, the job-finding rate by vacancy age $\mu_{\varepsilon}(a)$, and the different distributions ϕ_u^{ε}, ϕ_v^{ε}, and ϕ_j^{ε}. Appendix A explains the solving algorithm.

When $\varepsilon = 0$, $\theta_{\varepsilon}(a) = \theta_{rs}$ for all $a \geq 0$. When $\varepsilon = 1$, $\theta_{\varepsilon}(a) = \theta_{ds}(a)$ for all $a \geq 0$. As ε goes from 0 to 1, we obtain allocations that go from the random search allocation to the directed search allocation. We know that θ_{ds} strictly increases over age, whereas θ_{rs} is flat. By changing the value of ε, we modify the slope of θ_{ε}. More precisely, the slope increases with ε, whereas θ_0 decreases with ε.

Let $E = \arg\min_{\varepsilon\in[0,1]} u_{\varepsilon}$ be the set of almost efficient allocations. This set strongly constrains tightness by vacancy age and thus is very unlikely to contain the efficient allocation. However, E is much easier to find than the efficient allocation.

We use JOLTS and BLS data for the period 2000-2008. Over this period, the monthly probability of finding a job was about $\mu_m = 0.4$. Thus $1-\exp(-\mu) = 0.4$ and $\mu = -\ln(1-0.4) \approx 0.5$. The monthly job loss probability was $\lambda = 0.03$. This gives $\lambda = -\ln(1-0.03) = 0.03$. In the JOLTS dataset, firms declare how many vacancies they have. Thus the dataset provides a measure of v. The mean ratio of vacancies to unemployed was about $x = v/u = 0.5$. As for the phantom parameters, we assume that each match gives birth to a new phantom and so $\beta = 1.0$. We also assume that phantoms reach one month with probability 0.5. Thus $\delta_m = 0.5$ and so $\delta = -\ln(1-0.5) \approx 0.7$, i.e. phantoms live for 1.4 months on average. As for α, the aggregate matching technology is $M = m_0\int_0^{\infty}\pi(a)u(a)\theta(a)^{\alpha}da$. The elasticity of this function with respect to the overall tightness depends on the nonphantom proportion at each age and on the allocation of job-seekers across age. Intuitively, this elasticity should be larger than α:

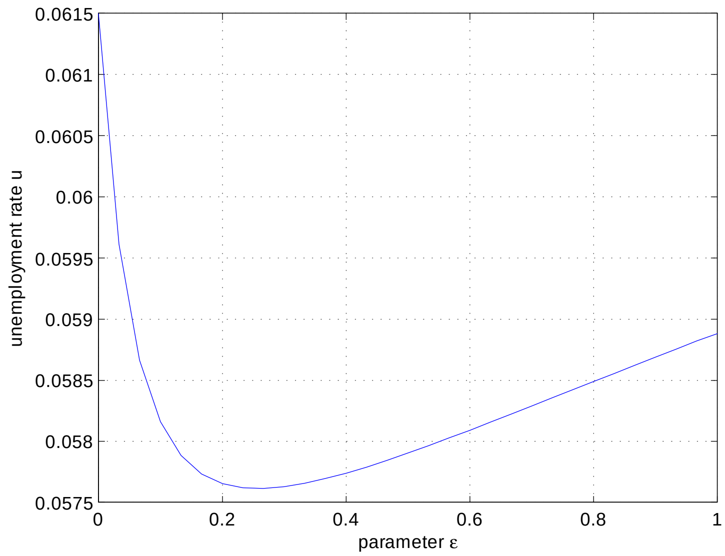

Figure 1: Unemployment rate as a function of ε. The random search allocation results when ε=0, whereas the directed search allocation results when ε=1.

having more vacancies reduces the nonphantom proportion, thereby increasing the number of matches. The value used by Shimer (2005) is 0.34. So we choose $\alpha = 0.2$. The final parameter is m_0. We set it so that the mean job-finding rate of the directed search allocation is roughly equal to μ. It turns out that $m_0 = 1.0$ gives a very close value and so we set $m_0 = 1.0$.

Table 1 gives the set of parameters.

m_0	α	β	δ	λ	K	μ_{ds}	u_{ds}
1.0	0.2	1.0	0.7	0.03	0.972	0.487	0.059

Table 1: Parameter values

Figure 1 depicts the unemployment rate, our measure of inefficiency, as a function of ε. It displays a U-shaped curve, with the random search allocation at the extreme left, and the directed search allocation at the extreme right. It shows two results. First, the directed search allocation is more efficient than the random search allocation. In the directed search allocation, the job-finding rate is 0.487 and the corresponding unemployment rate is 0.059. In the random search allocation, the corresponding figures are 0.465 and 0.062. That the job-seekers take into account the information vehicled by vacancy age is thus not only good for themselves but also for the society as a whole. Second, as expected from Proposition 4, the directed search allocation is not the most efficient one. The unemployment rate is minimized for $\varepsilon = 0.25$. Then, the job-finding rate is 0.498 and the unemployment rate is 0.058.

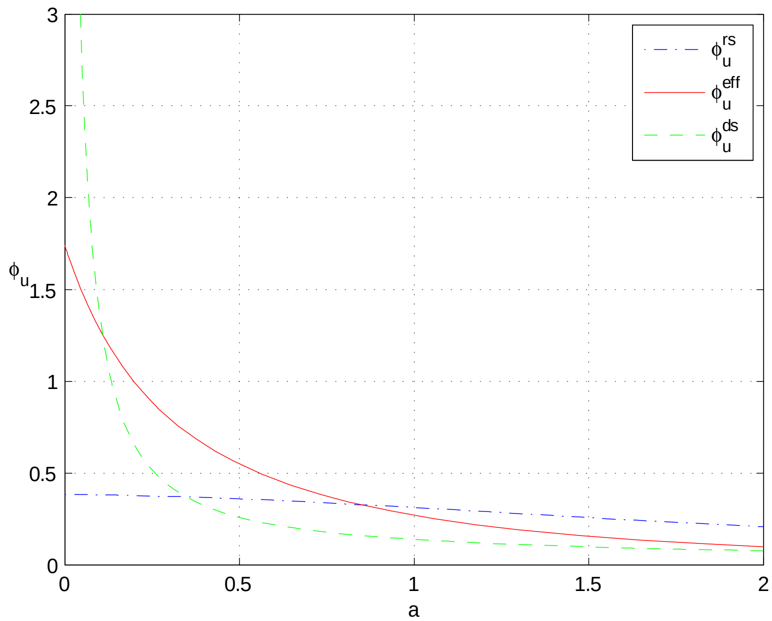

Figure 2: Density of job-seekers by vacancy age (in months)

Figure 2 displays the density of job-seekers by vacancy age in three different allocations: random search ($\varepsilon = 0$), directed search ($\varepsilon = 1$) and the allocation that minimizes the unemployment rate displayed by Figure 1 ($\varepsilon = .25$). Figure 2 shows spectacular differences in terms of search behavior. In the random search allocation, the density slightly decreases over vacancy age: it halves in two months. In the directed search allocation, the density has a quasi spike for newly created vacancies. Then it strongly decreases. The almost efficient allocation reveals an intermediate behavior: the spike is much less pronounced for new vacancies. Figure 3 shows the resulting tightness by vacancy age in the three allocations. Tightness is constant under random search, whereas it strictly increases with vacancy age in the other cases. Job-seekers who care for phantom vacancies search more at low vacancy ages, and thus the ratio of advertised jobs to seekers increases over age.

To understand why directed search is inefficient, Figure 4 depicts the job-finding rate, whereas Figure 5 shows the nonphantom-job proportion. The job-finding rate is by definition constant under directed search, and strictly decreases over age in the other allocations. In these latter cases, reallocating individual search effort towards recent vacancies would increase the individual odds of finding a job. This individual gain would also lead to a social gain in the random search allocation. However, it would deteriorate welfare in the almost efficient allocation. Figure 5 tells why. That job-seekers search more for recently created jobs implies that the nonphantom

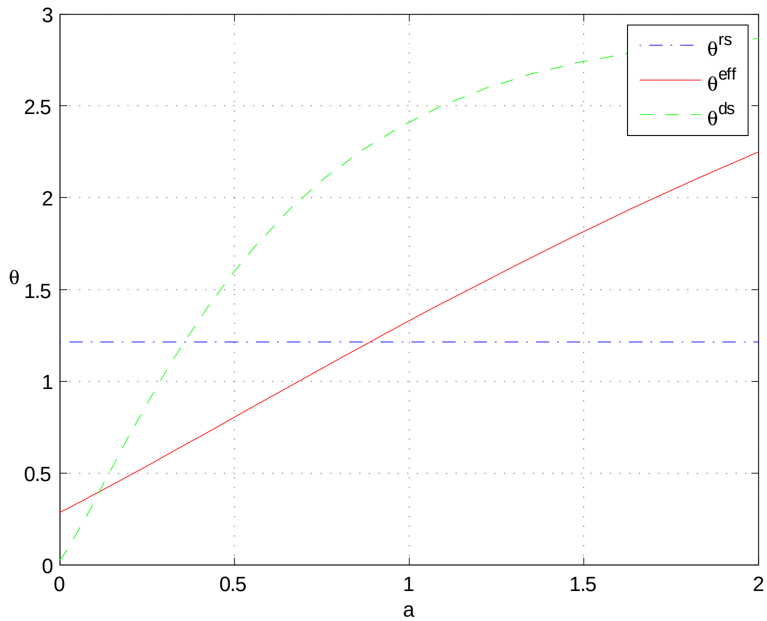

Figure 3: Tightness by vacancy age (in months)

proportion decreases very fast with vacancy age. As phantoms haunt the search place for some time, such a strong decline in the nonphantom proportion lasts and affects all vacancy ages. The almost efficient allocation accounts for this effect. Thus the pace of phantom creation is smoother over vacancy age, and the nonphantom proportion declines less rapidly.

Directed search leads the job-seekers to over-apply to young vacancies. Reallocating some of the job-seekers from young vacancies to older ones can achieve efficiency gains. The reason for this outcome is that directed search implies a negative informational externality that goes through different vacancy ages. Searching for jobs of a given age, say a, means finding jobs of such age. Phantoms result and they haunt the search market. However, because of the nature of directed search, phantoms only hurt agents who search for jobs older than a. It follows that the magnitude of this externality decreases with vacancy age. When workers search for newly created vacancies, they affect all the other agents through phantom creation. When they search for much older vacancies, they affect almost none.

Quantitatively, efficiency gains achieved by the almost efficient allocation are rather modest. The magnitude of the unemployment rate is only slightly affected by the way agents search for jobs. This is because any search strategy that produces more matches also leads to an increase in the stationary phantom stock. One way to see this is to compute the mean nonphantom proportion in the three allocations. We obtain $\pi_{rs} = 44.7\%$, $\pi_{ds} = 42.7\%$, $\pi_{eff} = 47.1\%$. This,

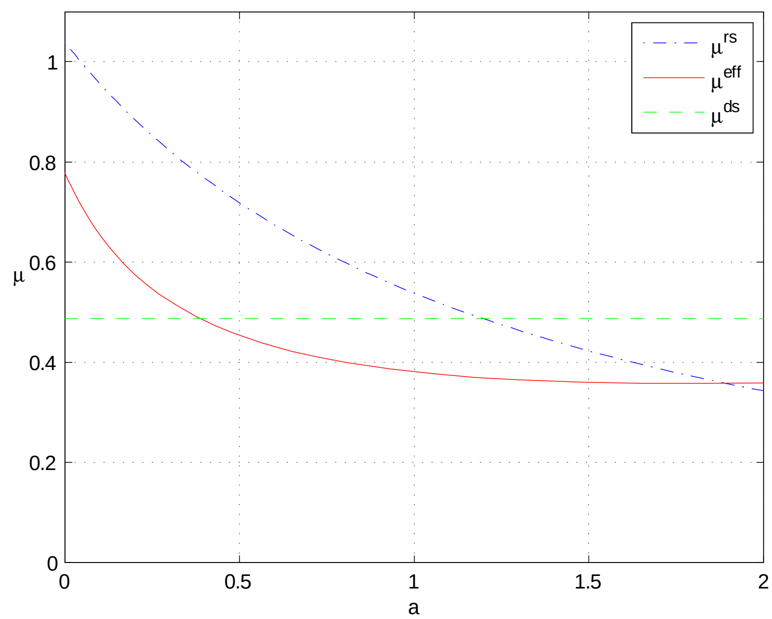

Figure 4: Job-finding rate by vacancy age (in months)

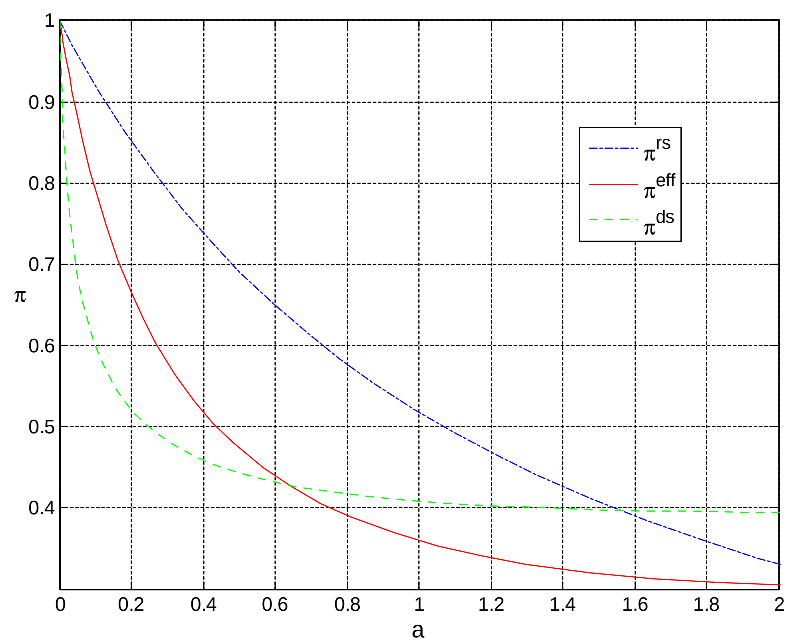

Figure 5: Nonphantom proportion by vacancy age (in months)

however, does not mean that phantom vacancies do not affect the matching function. In this calibration, removing all phantoms from the market leads to massive employment gains. The corresponding allocation obtains with $\beta = 0$. The job-finding rate is 0.813 and the corresponding unemployment rate is 0.036.

We can compute the contribution of information obsolescence to overall frictions. In the absence of frictions, the unemployment rate would be $\min(1-K, 0)$. In our calibration, $K = 0.972 < 1$. Thus the nonfrictional unemployment rate is equal to 0.028. Under directed search the unemployment rate is $u_{ds} = 0.059$. It follows that matching frictions account for half the unemployment rate. Without information obsolescence, the unemployment rate decreases to 0.038. Hence, phantom vacancies contribute to $0.021/0.031 \approx 68\%$ of overall frictions and $0.021/0.056 \approx 37\%$ of unemployment.

4 Renewing offers

Websites offer the possibility to refresh offers. In exchange for a cost, the vacancy age is reset to 0. If the previous ad disappears, each vacancy is characterized by a single ad. If it does not, then a given vacancy has several ads at a time, each one corresponding to a specific age. The former effect is the pure effect of renewing offers. The latter effect mixes vacancy renewal with search intensity. We here focus on the former effect.

Exogenous renewal.—To begin with, we assume that nonphantom vacancies are renewed at exogenous rate γ. This modifies the dynamics of vacancies as follows:

$$\dot{v} = -M(a) - \gamma v(a), \tag{85}$$

$$v(0) = \lambda(1-u) + \gamma v. \tag{86}$$

At each age a, the number of vacancies decreases by the flow of matches formed with such jobs, and by the flow of vacancies that are renewed. The inflow of new vacancies is then equal to newly destroyed employment relationships $\lambda(1-u)$ plus the total number of renewed vacancies γv.

The rest of the model is unchanged. We thus proceed as in the previous section. We first derive the directed search allocation, and then construct the other possible allocations by manipulating the differential equation that governs tightness.

Under directed search, the allocation of job-seekers across the different vacancy ages leads to equalize the job-finding rate at the different ages. Thus $\mu(a) = \pi(a)m(1, \theta(a)) = m(1, \theta(0))$. The nonphantom proportion is now affected by vacancy renewal:

$$\frac{\dot{\pi}}{\pi} = -(1-\pi)(m(1,\theta)/\theta - \delta) - \beta\pi m(1,\theta)/\theta - \gamma(1-\pi). \tag{87}$$

The additional term $-\gamma(1-\pi)$ reveals that vacancy renewal tends to speed up the decline of the nonphantom proportion with vacancy age. Renewing a vacancy means destroying a nonphantom vacancy to replace it by another one with a shorter age. This behavior atrophies the nonphantom proportion by age.

We deduce the dynamics of tightness:

$$\begin{aligned}-\alpha(\theta_{ds})\frac{\dot{\theta}_{ds}}{\theta_{ds}} &= \left[1-\frac{m(1,\theta_{ds}(0))}{m(1,\theta_{ds}(a))}\right][\delta\theta_{ds}-m(1,\theta_{ds})]-\beta m(1,\theta_{ds}(0)) \\ &\quad -\gamma\left[1-\frac{m(1,\theta_{ds}(0))}{m(1,\theta_{ds}(a))}\right]. \end{aligned} \tag{88}$$

Vacancy renewal increases the slope of the tightness function. The consecutive decline in the nonphantom proportion must be compensated by a stronger increase in the job-to-unemployed ratio.

We proceed as in Section 3. We alter equation (88) as follows:

$$\begin{aligned}-\alpha(\theta_{\varepsilon})\frac{\dot{\theta}_{\varepsilon}}{\theta_{\varepsilon}} &= \varepsilon\times\left\{\left[1-\frac{m(1,\theta_{\varepsilon}(0))}{m(1,\theta_{\varepsilon}(a))}\right][\delta\theta_{\varepsilon}-m(1,\theta_{\varepsilon})]-\beta m(1,\theta_{\varepsilon}(0))\right. \\ &\quad \left.-\gamma\left[1-\frac{m(1,\theta_{\varepsilon}(0))}{m(1,\theta_{\varepsilon}(a))}\right]\right\}. \end{aligned} \tag{89}$$

To find $\theta_{\varepsilon}(0)$, we use the resource constraint $1-u+v=K$ together with the motions for $v(a)$ and $p(a)$ and the definition of tightness $u(a)=(v(a)+p(a))/\theta(a)$.

We calibrate the model as in Section 3. On top of the different parameters already discussed there, we need to set γ, the renewal rate. As a benchmark we set $\gamma=2.0$: job listings are renewed every two weeks on average. The parameter m_0 adjusts so that the job-finding rate stays the same. Table 2 shows the parameter values of the baseline calibration. We only report the unemployment rate in the different allocations. The distribution of job-seekers, the pattern of tightness and job-finding rate, and the nonphantom proportion by vacancy age are very similar to Figures 2 to 5.

m_0	α	β	δ	λ	K	γ	μ_{ds}	u_{ds}
0.75	0.2	1.0	0.7	0.03	0.972	2.0	0.488	0.059

Table 2: Parameter values

Figure 6 shows that the results obtained in Section 3 are qualitatively unchanged. There is a U-shaped curve between the unemployment rate and parameter ε. The random search allocation is dominated by the directed search allocation. Moreover, the unemployment rate is minimized for $\varepsilon=0.7<1$, i.e. the directed search allocation is not constrained efficient.

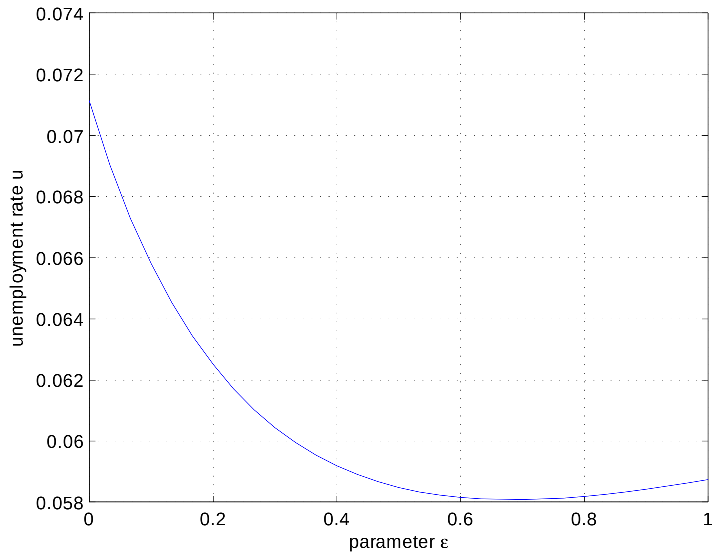

Figure 6: Unemployment rate as a function of ε. The random search allocation results when $\varepsilon = 0$, whereas the directed search allocation results when $\varepsilon = 1$.

Vacancy renewal increases the magnitude of unemployment rate differences between the random search and the directed search allocations. The unemployment rate is 7.1% in the former case, against 5.9% in the latter one. Thus the difference is 1.2 percentage points, against 0.2-0.3 percentage points when $\gamma = 0$. Random search is more costly with renewal: as explained below equation (87), the nonphantom proportion declines more rapidly with age.

The difference between the directed search allocation and the almost efficient allocation is small, about 0.1 percentage point. However, information obsolescence still plays an important role in the magnitude of unemployment. The nonfrictional unemployment rate is unchanged and equal to 0.028. Thus the contribution of matching frictions to unemployment stays the same, slightly above 50%. By setting $\beta = 0$ or $\delta = \infty$, the unemployment rate reaches 0.047. It follows that phantom vacancies explain $1.2/5.9 \approx 20\%$ of unemployment, $1.2/3.1 \approx 39\%$ of overall matching frictions.

We finally examine the role played by the rate of vacancy renewal. Figure 7 shows the unemployment rate in the directed search allocation as a function of γ. It strictly decreases with γ. As γ becomes very large, the unemployment rate tends to 0.047, i.e. information obsolescence plays no role.

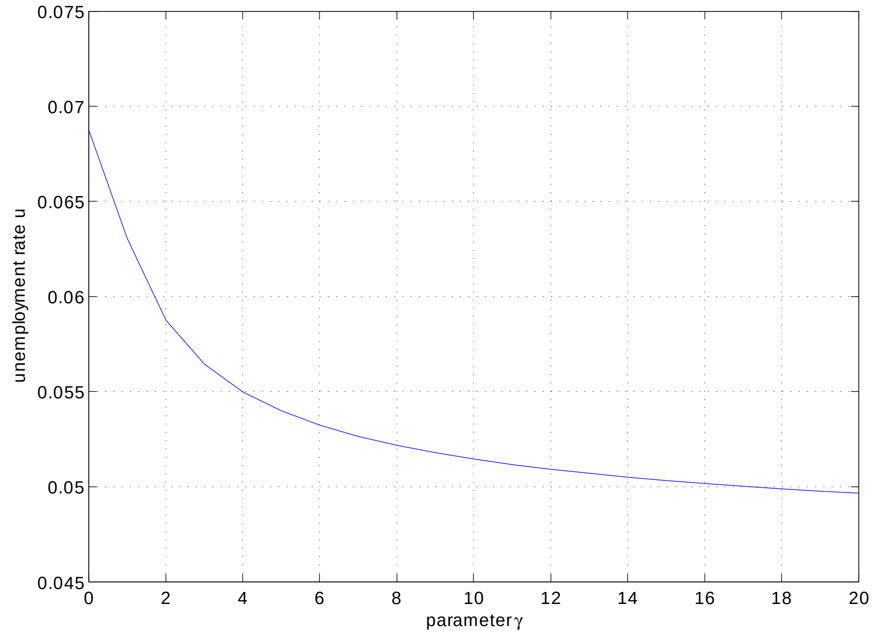

Figure 7: Unemployment rate in the directed search allocatiobn as a function of parameter γ

Endogenous renewal.— **(To Be Done)**

With endogenous renewal, we have another margin of decision, and, therefore, another source of inefficiency. Intuition suggests that firms will renew their job ads too often. Maybe multiple equilibria?

Basic idea: Firms choose parameter γ when they have a new vacancy. Each time they refresh their offer, they pay C. The value of a vacancy V and the value of a filled job J are defined as follows:

$$rV(a,\gamma) = \eta(a)[J - V(a,\gamma)] + \gamma[V(0,\gamma) - V(a,\gamma) - C] + V_a(a,\gamma), \quad (90)$$

$$rJ = y + \lambda[V(0,\bar{\gamma}) - J] \quad (91)$$

The value of γ is chosen so that $\hat{\gamma} \in \arg\max rV(0,\gamma)$. The main advantage is that we can use the previous framework to compute the different allocations. A generalization would be

$$rV(a,\gamma) = \eta(a)[J - V(a,\gamma)] + \gamma[V(0,\gamma) - V(a,\gamma)] + V_a(a,\gamma), \quad (92)$$

with $\hat{\gamma} \in \arg\max_\gamma V(0,\gamma) - C(\gamma)$. The cost function C accounts for the global expenses paid for recruitment activities.

Sophisticated idea: firms choose to renew their offer in real time:

$$rV(a) = \eta(a)[J - V(a)] + \max\{V(0) - V(a) - C, 0\} + V'(a). \quad (93)$$

Under directed search, η decreases with a. Thus $V(A) = V(0) - C$ defines the optimal age of renewal. The hazard rate $\gamma(a) = 0$ until A where $\gamma(A) = \infty$.

If firms are homogenous, then they all renew their offer at the same age. The distribution of vacancies by age is uniform and truncated in A. The main advantage is tractability: simulations are easy, and finding the efficient A consists of trying as many A as we want and pick the one with the lowest unemployment rate. The main drawback is the irrealistic nature of the distribution of vacancies.

If firms differ in C or J, then we have a distribution for A. This distribution may mimic the case of constant γ. Of course, it is much harder to simulate, and finding the efficient $A(C, J)$ will be difficult.

Other sophisticated idea: firms pay a cost for monitoring their offer, on top of the direct cost C:

$$rV(a) = \eta(a)[J - V(a)] + \gamma(a)[V(0) - V(a) - C] - c(\gamma(a)) + V'(a), \tag{94}$$

$$c'(\gamma(a))) = V(0) - V(a) - C. \tag{95}$$

Remarks:

- Under random search, firms never renew vacancies. The reason is η is constant across vacancy age.

- Under directed search, η decreases over time. Thus $V'(a) < 0$. This explains why there is an optimal renewal age A. This also predicts that the hazard rate $\gamma(a)$ should increase with age.

5 Conclusion

(To Be Added)

References

[1] (To Be Added)

APPENDIX

A Numerical simulations

Let ε vary from 0 to 1. For each ε, we find the function $\theta_\varepsilon : [0, \infty) \to [0, \infty)$ with typical element $\theta_\varepsilon(a)$ that solves

$$-\alpha\dot{\theta}_\varepsilon = \varepsilon[\delta\theta_\varepsilon - m_0\theta_\varepsilon^\alpha - m_0\theta_0^\alpha\theta_\varepsilon^{1-\alpha} + (1-\beta)m_0\theta_0^\alpha]. \tag{A}$$

At given θ_0, the differential equation (A) defines a unique function $\theta_\varepsilon(a) \equiv \theta_\varepsilon(a, \theta_0)$. Then $\theta(0)$ solves the resource constraint $1 - u + v = K$.

For a given $\theta_\varepsilon(a)$, we numerically solve

$$\begin{aligned} d\tilde{v}/da &= -\tilde{v}(a)m_0\theta_\varepsilon(a)^{\alpha-1}, & (96)\\ d\tilde{p}/da &= \beta\tilde{v}(a)m_0\theta_\varepsilon(a)^{\alpha-1} - \delta\tilde{p}(a), & (97)\\ d\tilde{u}/da &= (\tilde{v}(a) + \tilde{p}(a))/\theta_\varepsilon(a) & (98) \end{aligned}$$

with $\tilde{v}(0) = 1$, $\tilde{p}(0) = 0$ and $\tilde{u}(0) = 0$. Then we compute $u = \lambda(1-u)\int_0^\infty \tilde{u}(a)da$. This gives

$$u = \frac{\lambda\int_0^\infty \tilde{u}(a)da}{1 + \lambda\int_0^\infty \tilde{u}(a)da}. \tag{99}$$

The resource constraint is

$$\frac{1 + \lambda\int_0^\infty \tilde{v}(a)da}{1 + \lambda\int_0^\infty \tilde{u}(a)da} = K, \tag{100}$$

which is equivalent to

$$1 - K + \lambda\int_0^\infty (\tilde{v}(a) - \tilde{u}(a))da = 0. \tag{101}$$

Let $\psi : [0, \infty) \to [0, \infty)$ be such that

$$\psi(x) = 1 - K + \lambda\int_0^\infty (\tilde{v}(a) - \tilde{u}(a))da.$$

The algorithm of resolution is as follows. Fix a lower bound θ_- and an upper bound θ_+ to θ_0. Fix also $a_{\max}$, the approximation for $+\infty$, and $\tau > 0$ a (small) tolerance parameter.

Step 1. Pick $x = (\theta_- + \theta_+)/2$, and solve the differential equation (A) from $a = 0$ to $a = a_{\max}$.

Step 2. Compute $\psi(x)$. If $|\psi(x)| < \tau$, go to Step 4. If not, go to Step 3.

Step 3. If $\psi(x) < 0$, set $\theta_+ = x$. Else, set $\theta_- = x$. Go to Step 1.

Step 4. Set $\theta_0 = x$ and compute the different functions used in the text.

Proposition 3 describes the efficient tightness as follows:

$$\alpha\dot{\theta} = -\pi m_0 \theta^{\alpha-1} \left[\alpha + b_0(1-\alpha)\theta\right]$$
$$+\delta(1-\pi) - \beta\pi m_0 \theta^{\alpha-1} \left[\alpha + \delta y(1-\alpha)\theta\right], \tag{A'}$$
$$\dot{y} = 1/\theta - \delta y, \tag{A''}$$

where parameters b_0 and y_0 are still to be found. The initial parameter θ_0 must be such that the resource constraint $1 - u + v = K$ holds.

We proceed as follows. We determine a closed subset $S \subset [0, \infty) \times [0, \infty)$ of possible values of b_0 and y_0. We then set a grid within this subset, where each point of the grid is an element of S, and we go from an element to the other by means of two indices, one for the line and the other for the column. For each pair (b_0, y_0), we find $\theta(a)$ with the previous algorithm where we replace equation (A) by equations (A')-(A"). Then, we pick the lowest unemployment rate of the grid, and the corresponding allocation is the efficient one.