Benchmarks in Search Markets*

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Abstract

We analyze the role of benchmarks in over-the-counter markets subject to search frictions. The publication of a benchmark can, under conditions, raise total social surplus by *(i)* increasing the volume of beneficial trade, *(ii)* reducing total search costs, and *(iii)* facilitating more efficient trade matching between dealers and customers. Although the improvement in market transparency caused by benchmarks may lower dealer profit margins on each trade, dealers may nevertheless introduce a benchmark such as LIBOR in order to encourage greater market participation by investors. In some cases, the lowest-cost dealers may introduce a benchmark in order to increase their market share through reducing entry by high-cost dealers, a further source of efficiency gain.

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1 Introduction

An enormous quantity of transactions in over-the-counter (OTC) markets are negotiated on the basis of benchmark prices. This paper explains how benchmarks affect pricing and trading behavior by reducing market opaqueness, characterizes the associated welfare impact of benchmarks, and shows how the incentives of regulators and dealers to support benchmarks depend on market structure.

Trillions of dollars in loans are negotiated at a spread to LIBOR or EURIBOR, benchmark interbank borrowing rates.[1] The WM/Reuters daily fixings are the dominant benchmarks in the foreign exchange market, which covers over $5 trillion per day in transactions.[2] There are popular benchmarks for a range of commodities including silver, gold, oil, and natural gas, among others.[3] Benchmarks are also used to provide price transparency for manufactured products such as pharmaceuticals.[4]

We model the impact of benchmarks on the efficiency of markets that are impeded by search frictions, such as OTC financial markets. In our model, benchmarks serve the primary role of improving market transparency. Benchmarks mitigate the impact of search frictions through a reduction in the informational asymmetries between dealers and their "buy-side" customers. Adding a benchmark to an opaque OTC market can improve efficiency by encouraging entry by customers, which increases realized gains from trades.

We consider a market for an asset in which dealers offer price quotes to customers who are relatively uninformed about the typical cost to dealers of providing the asset.

[1] LIBOR is the London Interbank Offered Rate. EURIBOR is the Euro Interbank Offered Rate. For U.S. dollar Libor alone, the Market Participants Group on Reference Rate Reform (2014) (chaired by one of the authors of this paper) reports that over 3 trillion dollars in syndicated loans and over 1 trillion dollars in variable-rate bonds are negotiated relative to LIBOR. The MPG report lists many other fixed-income products that are negotiated at a spread to the "interbank offered rates" known as LIBOR, EURIBOR, and TIBOR, across five major currencies. As of the end of 2013, Bank for International Settlements (2014) report a total notional outstanding of interest rate derivatives of 583 trillion U.S. dollars, the vast majority of which reference LIBOR or EURIBOR. These swap contracts and many other derivatives reference benchmarks, but are not themselves benchmark products. Other extremely popular benchmarks for overnight interest rates include SONIA, the Sterling OverNight Index Average, and EONIA, the Euro OverNight Index Average.

[2] See Foreign Exchange Benchmark Group (2014), which reports that 160 currencies are covered by the WM/Reuters benchmarks. These benchmarks are fixed at least daily, and by currency pair within the 21 major "trade" currencies.

[3] The London Bullion Market Association provides benchmarks for gold and silver. Platts provides benchmarks for oil, refined fuels, and iron ore (IODEX). Another major oil price benchmark is ICE Brent. ICIS Heren provides a widely used price benchmark for natural gas.

[4] For a discussion of the Average Wholesale Price (AWP) drug-price benchmarks, see Gencarelli (2005).

Customers may seek quotes from more than one dealer. The potential for costs and delays associated with this search lowers the incentives of these customers to participate in the market. Some gains from trade may, as a result, be lost.

If, however, there is a published benchmark for dealers' costs of providing the asset, we show that customers suffer fewer costly delays in their search, a source of welfare improvement. Under natural conditions, the market transparency provided by the benchmark causes dealers to offer more competitive price quotes. This promotes greater market participation by customers and thus greater total gains from trade, a second source of welfare improvement. Finally, again under natural conditions, we show that the publication of a benchmark can improve matching efficiency, making it easier for customers to locate those dealers who are most cost-effective at handling customers' demands.

Dealers, however, may or may not prefer to introduce a benchmark, even if they are able to coordinate among themselves to do so. Typically, by reducing market opaqueness, a benchmark also reduces the local monopoly power of a dealer when facing a customer, and thus decreases the dealer's average profit margin. Dealers prefer to introduce a benchmark only when the resulting reduction in profit margin is more than offset by increased volume of trade. We provide supporting conditions on model parameters. Given the improvement in matching efficiency caused by a benchmark, the most cost-efficient dealers have the greatest incentives to introduce a benchmark. If they do, we show that under certain conditions less cost-efficient dealers may exit the market, further improving market efficiency.

Recent scandals involving the manipulation of interest-rate benchmarks such as LIBOR and EURIBOR, as well as currency price fixings provided by WM/Reuters, have shaken investor confidence in financial benchmarks. Major banks are also now more reluctant to support these benchmarks in the face of potential regulatory penalties and private litigation. For example, of the 44 banks contributing to EURIBOR before the initial reports of manipulation, 18 have already dropped out of the participating panel.[5] Regulators have responded not only with sanctions,[6] but also by taking action to support more robust benchmarks. The Financial Stability Board has set up several international working groups charged with recommending reforms to interest-rate and foreign-exchange benchmarks that would reduce their susceptibility to manipula-

[5] See Brundsen (2014).
[6] See Finch and Larkin (2014).

tion while maintaining their usefulness in promoting market efficiency.[7] The United Kingdom is preparing a comprehensive regulatory framework for benchmarks.[8] Our results offer a theoretical foundation for public-policy support of transparent financial benchmarks.

Our model works roughly as follows. In an over-the-counter market with a finite number of dealers and a continuum of investors that we call "traders," the cost to a dealer of providing the asset to a trader is the sum of a dealer-specific (idiosyncratic) component and a component that is common to all dealers. The existence of a benchmark is taken to mean that the common cost component is publicly announced. Each trader also observes, privately, whether her search cost is high or low. In practice the clients of financial intermediaries may be buying or selling the asset. We take the case in which traders wish to buy. (The opposite case is effectively the same, up to sign changes.) Traders are searching for a good price, and dealers offer price quotes to traders that depend endogenously on the presence of a benchmark. Each dealer posts an offer price, available for execution by any trader, anonymously. Traders, who have a commonly known value for acquiring the asset, contact the dealers sequentially, expending a costly search effort, or costly delay, with each successive dealer contacted. At each point in time the trader, given all of the information available to her at that time (including past price offers and, if published, the benchmark) decides whether to buy, keep searching, or exit the market. All market participants maximize their conditional expected net payoffs, at all times, in a perfect Bayesian equilibrium.

Under natural parameter assumptions, which vary with the specific result, we show that publishing the benchmark is socially efficient because of the following effects. First, publication of the benchmark can improve the information available to traders about the likely price terms they will face. This assists traders in deciding whether to participate in the market, based on whether there is a sufficiently large conditional expected gain from trade. As a result, entry is more efficient, and some wasteful search is avoided. Second, the existence of a benchmark causes dealers to compete more aggressively in their price quotes. This increases expected market participation by traders, thus increasing expected realized gains from trade. In this sense, the benchmark is a commitment device that mitigates the hold-up problem arising from the incentive of dealers to quote less attractive prices once the search costs of traders have been sunk. Finally,

[7] See Official Sector Steering Group (2014), Market Participants Group on Reference Rate Reform (2014), and Foreign Exchange Benchmark Group (2014).

[8] See Bank of England (2014).

the benchmark can improve matching efficiency, leading to proportionately more sales by low-cost dealers.

Our theory also reveals that when benchmarks improve market efficiency, dealers may not always have an incentive to introduce them. Thus, there may be scope for regulators to introduce benchmarks in order to improve market efficiency. There are also cases in which introducing a benchmark can lower welfare, but in this case it is natural to expect that dealers would not want to introduce benchmarks, and of course there is no reason in this case for regulators to promote benchmarks.

Indeed, in the simplest version of our model in which dealers have homogeneous costs, we characterize dealer and regulatory preferencestraders for benchmarks according to three regions for search costs. In the highest region of search costs, benchmarks are socially optimal and are also beneficial for dealers. In the middle region, benchmarks are socially optimal but dealers would not introduce them willingly, and there is scope for policy intervention. In the lowest region, benchmarks are not socially optimal and dealers do not want to introduce them.

Our analysis draws upon techniques first used in search-based models of labor markets, in a literature surveyed by Rogerson, Shimer and Wright (2005). The framework that we consider features endogenous pricing (as in Stahl 1989) and uncertainty about the distribution of prices (as in Rothschild 1974). Our model builds on Janssen, Pichler and Weidenholzer (2011), with the crucial difference that we allow endogenous entry and heterogeneity in dealers' costs. This enables us to study a range of issues related to welfare and matching efficiency. Benabou and Gertner (1993) consider a model with a general cost structure, but only two dealers. They analyze the influence of inflationary uncertainty on welfare and the split of surplus between consumers and firms.[9] Aside from the limitation of having only two dealers, their model does not endogenize the entry decisions of customers.

The remainder of the paper is organized as follows. Section 2 states the model. In Section 3 we analyze the role of benchmarks in markets with relatively high search costs, focusing on how benchmarks encourage market participation by traders. Section 4 tackles the case in which search costs are relatively small, and characterizes the effect of a benchmark on matching efficiency. In Section 5 we show that dealers may have a

[9]Inflation uncertainty in their model is similar in spirit to cost uncertainty in ours when there is no benchmark. The relation between their approach and ours with regard to uncertainty can be described as "local" versus "global." Benabou and Gertner (1993) analyze the marginal effect on welfare when uncertainty is reduced slightly, while the introduction of a benchmark in our setting reduces this source of uncertainty significantly.

total-profit incentive to commit to a benchmark, despite the adverse impact of higher market transparency on the profit margin of each trade. Section 6 discusses several points related to benchmark manipulation. Section 7 concludes. All proofs are relegated to appendices, which also contain supplementary supporting results and examples.

2 Model

This section describes our search-based model of an over-the-counter market, beginning with the primitive definitions of the market participants and then turning to the trading protocol and definition of market equilibrium.

Our market participants include a finite number $N \geq 2$ of dealers and an infinite number of "traders," distributed uniformly on the interval $[0, 1]$. For concreteness, we suppose that a trading encounter is one in which a dealer sells and a trader buys. The model can be equivalently formulated with the buying and selling roles reversed. The important distinction between the two types of agents is that dealers make markets by offering executable price quotes, whereas traders contact dealers sequentially and accept their quotes or not, in a manner to be described.

All trades are for a unit amount of a given asset. Dealer i can supply the asset at at a per-unit cost of $c_i = c + \epsilon_i$, where c is common to all dealers and ϵ_i is idiosyncratic. The common cost component c has a cumulative distribution function G with support[10] $[\underline{c}, \bar{c}]$, for some $\underline{c} \geq 0$, with $\underline{c} < \bar{c} < \infty$. The idiosyncratic component ϵ_i determines the type of a dealer. High-cost dealers are those whose outcome for ϵ_i is a constant $\Delta > 0$. Low-cost dealers are those with $\epsilon_i = 0$. The common probability of a low-cost type is $\gamma > 0$. The cost components $c, \epsilon_1, \ldots, \epsilon_N$ are independent. Dealer i observes c and ϵ_i, but does not observe the cost type ϵ_j of any other dealer j.

In a primary goods market, one may view Δ as a production cost disadvantage of high-cost dealers. For applications to financial markets, one could also view Δ as the incremental intermediation cost of a high-cost dealer. In that sense of market making, if we were to consider the parallel set of encounters in which traders with a low valuation for the asset wish to sell to a dealer, a high-cost dealer's value for acquiring the asset from a trader would be $c - \Delta$.

All traders assign a commonly known constant value $v > 0$ to acquiring the asset. Trader $j \in [0, 1]$ incurs a search cost of s_j for making each contact with a new dealer.

[10] Many of our results do not depend on the full-support assumption, as can be seen from the proofs in the appendix.

For tractability, we suppose that $s_j = 0$ with some probability p_0 in $(0, 1)$, and that $s_j = s$ with probability $1 - p_0$, for some constant $s > 0$. Search costs are independent across almost every pair of traders. By the exact law of large numbers of Sun (2006), p_0 is also the fraction of traders with zero search cost, almost surely.[11] The presence of some traders with zero search cost overcomes the usual Diamond paradox.[12]

Because search costs in practice often arise from delay costs, we refer for simplicity and concreteness to traders with zero search cost as "fast traders," and to those with non-zero search cost as "slow traders."[13] Traders have no information concerning which dealers are low-cost and which are high-cost.

For the reader's convenience, Appendix F provides a glossary of the primitive model parameters $(N, G, \Delta, \gamma, p_0, s, v)$ and the key derived variables.

The presence of a benchmark is taken to mean the publication of the common component c of the dealers' costs. We postpone our discussion of how dealers could coordinate a revelation mechanism for the benchmark. We will compare two market designs: the benchmark case (c is published at the outset, for all traders to observe) and the no-benchmark case (c is observed only by dealers).

The game proceeds as follows. If there is a benchmark, its value c is first revealed. Each dealer i posts a price p_i that constitutes a binding offer to sell one unit of the asset at this price to any trader. This offer price is observed only by those traders who contact the dealer. In OTC financial markets, this trade protocol is sometimes called "click to trade."

Then each trader (without yet having observed the quotes of any dealers) makes a entry decision. A failure to enter the market ends the game for the trader. With entry, a trader contacts one of the dealers, with equal likelihood across the N dealers.

Upon observing a dealer's offer, the trader can accept that offer or the offer of any previously contacted dealer, in which case the corresponding transaction is made and the trader leaves the market. A trader may alternatively continue searching by contacting another randomly selected dealer, again with the uniform distribution over

[11] We adopt throughout Sun's construction of the agent space and probability space, and the measurable subsets of the product of these two spaces, so as to allow without further comment various applications of the exact law of large numbers for a continuum of essentially pairwise-independent random variables.

[12] The Diamond paradox (Diamond 1971) is a common issue in search models and implies that all dealers charge the monopoly price in a unique equilibrium with no search.

[13] In the search literature the two types are often referred to as "shoppers" and "non-shoppers." See, for example, Janssen, Pichler and Weidenholzer (2011). These labels do not fit as well to OTC financial markets.

the yet-to-be-visited dealers.[14]

A trader may exit the market at any point without trading, even after having contacted all N dealers. The order of dealer contacts is independent across traders. Dealers observe neither the price offers posted by other dealers, nor the order in which the traders contact dealers.

A strategy for dealer i is a measurable function mapping the dealer's cost type ϵ_i and the common cost component c to a probability distribution over price offers. In the absence of a benchmark, a strategy for trader j maps the trader's search cost s_j and any prior history of observed offers to a choice from: *(i)* accept one of the available history of observed offers, *(ii)* continue searching, or *(iii)* exit. (If the trader has not visited any dealer, the decision to continue searching is equivalent to the decision to enter the market.) In the presence of a benchmark, the strategy of a trader may also depend on the benchmark, that is, the common cost component c.

The payoff of dealer i is $(p_i - c_i)Q_i$, where Q_i is the total quantity of sales[15] by dealer i. If trader j successfully conducts a purchase, say from dealer i, then her payoff is $v - p_i - s_j K_j$, where K_j is the number of dealers that she contacted. If she does not purchase the asset, then her payoff is $-s_j K_j$.

An equilibrium is a collection of strategies for the respective agents, possibly mixed (that is, allowing randomization), with the property that each agent's strategy maximizes at each time that agent's expected payoff conditional on the information available to the agent at that time, and given the strategies of the other agents. We focus on symmetric perfect Bayesian equilibria.

As is conventional in the literature for search-based markets, we restrict attention to reservation-price equilibria unless otherwise indicated. These are equilibria in which a trader's decision to continue searching can be based at any time on a cutoff for the best offer to that point. Other equilibria are known to be inherently intractable in a setting with learning and equilibrium pricing. We also assume, essentially without loss

[14] A uniform distribution of contacts over the yet-to-be-visited dealers is the standard assumption of undirected search that is used in the search literature. If, instead, certain dealers are more likely to be visited than others, the infrequently-visited dealers would update their belief about the frequently-visited dealers' offers upon the contacts by the traders. This asymmetric, strategic inference is modeled by Zhu (2012) in a setting with one investor and multiple dealers. In our model with multiple traders and entry decisions, this asymmetry is orthogonal to the effect of benchmarks we model, and adding it would substantially complicate the analysis. That said, in Section 5.2 we consider the incentives of low-cost dealers to introduce a benchmark and drive their high-cost competitors out of the market. This effectively tilts the traders' order flows toward the low-cost group of dealers.

[15] That is, $Q_i = \int_0^1 1_{(i,j)}\, dj$, where $1_{(i,j)}$ has outcome 1 if trader j accepts the offer of dealer i, and otherwise has outcome 0.

of generality, that fast traders play their weakly dominant strategy of always entering the market and contacting all dealers.[16]

In many over-the-counter financial markets, traders are not anonymous and dealers' quotes are good only when offered. Zhu (2012) shows that under the good-only-when-offered protocol, when a dealer recognizes a trader who has contacted the dealer for a second time, the dealer gives the trader a less attractive quote. The good-only-when-offered market-making protocol, along with a loss of anonymity of traders, would make the analysis of our model far too complicated, and we have no reason to suspect that it changes the implications of benchmarks that we model in this paper. In the concluding section, we further discuss the implications of the good-only-when-offered protocol.

3 With High Search Costs, Benchmarks Improve Entry Efficiency

This section considers how benchmarks affect the efficiency of entry by traders. We thus focus on cases in which search costs are relatively high compared to gains from trade.[17] In particular, we maintain throughout this section, unless otherwise indicated, that gains from trade we may fail to exist for sufficiently high cost realizations, in that $\bar{c} \geq v$.

The results include conditions under which having the benchmark dominates the no-benchmark case in terms of total social surplus, defined as the sum of the payoffs of all agents, both dealers and traders, net of costs. Our welfare comparisons are based on expected social surplus, a reasonable social welfare function given that all agents preferences are based on expected net payoffs.

In order to simplify and isolate the effect of a benchmark for entry decisions, we also assume throughout this section that $\gamma = 1$, that is, all dealers have supply cost c. An extension in Appendix E formulates and proves the main result of this section without that assumption. The general case adds technical complications but does not offer any additional insights when it comes to entry. Later, in Section 4, we consider the general case in which the dealers' costs are heterogeneous and explain how the introduction

[16] This assumption is without loss of generality in that for every equilibrium in which fast traders do not play this strategy, there exists a payoff-equivalent equilibrium in which they do, with the only exception of the degenerate Diamond-paradox equilibrium (in which all dealers quote the price v, each fast traders contacts no more than one dealer, and slow traders do not enter).

[17] A note on terminology: When we say "gains from trade" we refer to the random variable $(v-c)^+ \equiv \max(v-c, 0)$, representing the "potential" or "maximal" gains from trade. Unless the meaning is clear from the context, we use the term "realized gains from trade" to refer to gains that are generated in the actual equilibrium of the game.

of a benchmark can improve the efficiency with which traders are matched to low-cost dealers.

3.1 The benchmark case

We first characterize equilibrium in the benchmark case. A considerable part of the analysis here draws upon the model of Janssen, Pichler and Weidenholzer (2011). We sketch the equilibrium construction below, leaving technical details to Appendix A.

In the event that $c > v$, there are no gains from trade, and in light of the benchmark information, slow traders do not enter. Obviously, there can be no trade in equilibrium. If $v - s \leq c \leq v$, because dealers never quote prices below their costs, slow traders still do not enter. Fast traders enter and buy from the dealer that offers the lowest price. It is easy to show that the only equilibrium is one in which all dealers quote a price of c, amounting to Bertrand competition among dealers. From this point we therefore concentrate on the interesting case, the event in which $c < v - s$.

We fix some candidate probability λ_c of entry by slow traders, to be determined in equilibrium. Conditional on entry, a slow trader faces what is known as the Pandora Problem,[18] whose solution is characterized by Weitzman (1979). The optimal policy for a slow trader is to search until she contacts a dealer whose offer is no higher than a certain cutoff r_c, which depends neither on the history of received offers nor on the number of yet-to-be-visited dealers. A simple argument shows that the only possible equilibrium response of dealers is a mixed strategy in which offers are drawn from a continuous distribution with upper limit r_c. This in turn implies that, on the equilibrium path, a slow trader buys from the first dealer that she contacts. The details of this argument are shown in Appendix A.

In order to determine the probability distribution of dealers' offers, we first note that upon being contacted, a dealer updates his belief that he faces a fast trader. Given the search behavior of traders, the probability of facing a trader who is fast is

$$q(\lambda_c) = \frac{Np_0}{Np_0 + \lambda_c(1 - p_0)}. \tag{3.1}$$

Because, in equilibrium, dealers must be indifferent between all price offers in the

[18] Pandora has N boxes to open. Each box contains a reward of unknown value coming from a known distribution. The values of rewards are independent. Pandora opens the boxes sequentially, paying a fixed cost each time she opens a new box. The problem consists in finding the optimal stopping rule.

support of the distribution, and because fast traders visit all dealers, the equilibrium cumulative distribution function $F_c(\,\cdot\,)$ of a dealer's price offer must solve

$$\left[1 - q(\lambda_c) + q(\lambda_c)\left(1 - F_c(p)^{N-1}\right)\right](p - c) = \left[1 - q(\lambda_c)\right](r_c - c) \tag{3.2}$$

for every p in the support of F_c, which is $[\underline{p}_c,\, r_c]$, for $\underline{p}_c = \varphi(\lambda_c)r_c + (1 - \varphi(\lambda_c))c$ and

$$\varphi(\lambda_c) = \frac{\lambda_c(1 - p_0)}{Np_0 + \lambda_c(1 - p_0)}.$$

The solution of this equation is

$$F_c(p) = 1 - \left[\frac{\lambda_c(1 - p_0)}{Np_0}\frac{r_c - p}{p - c}\right]^{\frac{1}{N-1}}. \tag{3.3}$$

We can now calculate the optimal reservation price $r_c^\star$ of traders. Because traders value the asset at v, it must be that $r_c^\star \leq v$. Using the optimality condition that after observing a quote of $p = r_c^\star$ a trader must be indifferent between immediately accepting the offer and continuing to search, we arrive at the equation

$$v - r_c^\star = -s + v - \int_{\underline{p}_c}^{r_c^\star} p\; dF_c(p). \tag{3.4}$$

Plugging in the previously computed solution for $F_c(p)$, a change of variables yields

$$r_c^\star = c + \frac{1}{1 - \alpha(\lambda_c)}s, \tag{3.5}$$

where

$$\alpha(\lambda_c) = \int_0^1 \left(1 + \frac{Np_0}{\lambda_c(1 - p_0)}z^{N-1}\right)^{-1} dz < 1. \tag{3.6}$$

The expected offer, conditional on c, is $(1 - \alpha(\lambda_c))c + \alpha(\lambda_c)r_c^\star$, as shown in Lemma 3, found in Appendix A.

Equation (3.5) states that the maximum price that a slow trader is willing to accept is the cost of the dealer plus a profit margin equal to the trader's search cost s multiplied by a proportionality factor that reflects an entry externality, represented through the function α. The entry externality arises as follows. If the slow-trader entry probability λ_c is low, the market consists mainly of fast traders, and competition among dealers pushes the expected profit margins of dealers to zero, in that $\lim_{\lambda\to 0}\alpha(\lambda) = 0$ (the

trading protocol converges to an auction run by fast traders). On the other hand, if λ_c is close to 1, then slow traders constitute a considerable part of the market, and the existence of search frictions allows dealers to exert their local monopoly power and sell at prices bounded away from their costs.

To complete the description of equilibrium, we must specify the optimal entry decisions of slow traders. Holding the entry probability λ_c fixed, the expected payoff of a slow trader conditional on c and on entry is

$$\pi(\lambda_c) = v - s - \int_{\underline{p}_c}^{r_c^\star} p \; dF_c(p) = v - \frac{1}{1-\alpha(\lambda_c)} s - c.$$

It can be verified that $\pi(\lambda_c)$ is strictly decreasing in λ_c through the role of $\alpha(\lambda_c)$.

If $\pi(\lambda_c)$ is strictly positive at $\lambda_c = 1$, then the equilibrium slow-trader entry probability $\lambda_c^\star$ must be 1. Because α is maximized at $\lambda_c = 1$, this happens if and only if

$$c \leq v - \frac{1}{1-\bar{\alpha}} s,$$

where

$$\bar{\alpha} = \alpha(1) = \int_0^1 \left(1 + \frac{N p_0}{1-p_0} z^{N-1}\right)^{-1} dz. \tag{3.7}$$

If the profit $\pi(\lambda_c)$ is negative at $\lambda_c = 0$, then there can be no entry by slow traders, that is, $\lambda_c^\star = 0$. Since $\alpha(0) = 0$, this happens when $c > v - s$.

Finally, if $c \in (v-s,\, v-s/(1-\bar{\alpha}))$, then we have "interior entry," in that $\lambda_c^\star \in (0,\, 1)$ is uniquely determined by the equation

$$s = (1 - \alpha(\lambda_c^\star))(v - c). \tag{3.8}$$

We summarize these results in a proposition whose proof is found in Appendix A.

Proposition 1. *In the benchmark case, the equilibrium payoffs are unique, and there exists a reservation-price equilibrium in which the following properties hold.*

1. *Entry. In the event that $c \geq v - s$, no slow traders enter. If*

$$v - \frac{s}{1-\bar{\alpha}} \; < c < \; v - s,$$

then slow traders enter with the conditional probability $\lambda_c^\star \in (0,\, 1)$ determined by equation (3.8). If $c \leq v - s/(1-\bar{\alpha})$, then slow traders enter with conditional

probability 1.

2. *Prices. In the event that $c > v$, dealers quote arbitrary offers no lower than c. If $c \in [v - s, v]$, then dealers quote offers equal to c. If $c < v - s$, then every dealer quotes offers drawn with the conditional probability distribution function F_c given by (3.3).*

3. *Traders' reservation prices. In the event that $c < v - s$, conditional on entry, a slow trader's reservation price $r_c^\star$ is given by (3.5).*

4. *Social surplus. The conditional expected total social surplus given c is*

$$\lambda_c^\star(1 - p_0)\,(v - c - s) + p_0(v - c)^+,$$

where $(v - c)^+ \equiv \max(v - c, 0)$, and the conditional expected profit of each dealer is

$$\frac{\lambda_c^\star(1 - p_0)}{N}\frac{s}{1 - \alpha(\lambda_c^\star)}.$$

An immediate implication of Proposition 1 is that entry is not fully efficient. In the event that $c \in (v - s/(1 - \bar{\alpha}),\, v - s)$, the gains from trade for slow traders are larger than the search cost, but we do not observe full entry. This inefficiency can be understood as a hold-up problem. Once traders enter, search costs are sunk and dealers make higher-than-efficient price offers. Because of the negative externality that slow traders exert on other traders by entering, the equilibrium level of entry must be low enough to keep the profits of slow traders from being negative.

Even before analyzing the no-benchmark case, we can isolate one source of welfare advantage of a benchmark. We define the expected gain from trade

$$X = G(v)\left[v - \mathbb{E}\left(c \,|\, c \leq v\right)\right], \tag{3.9}$$

that is, the probability of a trade multiplied by the expected gain given a trade.

Proposition 2. *If $v - \underline{c} > s \geq X$, then the expected social surplus is strictly larger with the benchmark than in any equilibrium without the benchmark.*

Proposition 2 describes an important role of a benchmark as a device to signal gains from trade. In markets with relatively low expected gains from trade and relatively high search costs (i.e. $s \geq X$), the absence of a benchmark leads to a total failure by

slow traders to participate in the market. Although this situation is extreme, it may be considered realistic for markets in an early stage of development. The introduction of a benchmark in this setting encourages the participation of slow traders by providing them with information about the conditional expected gains from trade. Entry is now possible when the realization of cost c is low, and market efficiency is improved.

In order to further develop our comparison of the benchmark and no-benchmark cases, we now characterize equilibria in the absence of a benchmark.

3.2 The no-benchmark case

Without the benchmark, the task of characterizing equilibria of the game is more difficult. When the common component c is not observed, traders make complicated Bayesian inferences based on the observed price offers in order to assess the attractiveness of these offers. To keep the model tractable we restrict attention to equilibria in which traders, when on the equilibrium path, follow a reservation-price strategy. That is, in the k-th round of search a slow trader has a reservation price of the form $r_{k-1}(p_1, p_2, \ldots, p_{k-1})$, where $(p_1, p_2, \ldots, p_{k-1})$ is the history of prior price offers. According to this reservation-price strategy, any offer $p_k > r_{k-1}(p_1, p_2, ..., p_{k-1})$ is not immediately accepted and any offer $p_k < r_{k-1}(p_1, p_2, \ldots, p_{k-1})$ is immediately accepted. An offer $p_k = r_{k-1}(p_1, p_2, ..., p_{k-1})$ is accepted with some (mixing) probability that is determined in equilibrium. For simplicity, from this point we describe an offer that is not immediately accepted as "rejected," bearing in mind that the trader retains the option to later accept the offer.

We first characterize reservation-price equilibria, assuming one exists. Then we provide conditions under which a reservation-price equilibrium does exist. The following lemma, whose proof is found in Appendix A, is an important step in characterizing a reservation-price equilibrium.

Lemma 1. *In every reservation-price equilibrium in which slow traders enter with strictly positive probability, (i) the first-round reservation price $r_0^\star$ is equal to v and (ii) for each outcome of c strictly below v, the upper limit of the support of the conditional distribution of price offers is v.*

Without the benchmark the first-round reservation price of a slow trader cannot depend on c. The trader's ignorance about the common component c of dealers' costs makes it more difficult for her to evaluate the attractiveness of price offers. Lemma 1

states that this information asymmetry must result in the slow trader accepting any price offer below her value v for the asset, in a reservation-price equilibrium. Thus, only two things can happen if a positive mass of slow traders enter. If $c \leq v$, a slow trader buys from the first dealer that she contacts. If $c > v$, however, then a slow trader will observe a price offer above her value for the asset, conclude that there is no gain from trade or further search, and exit the market. This outcome—in which slow traders enter only to find out that there is no gains from trade—is a waste of costly search that would be avoided if there were a benchmark. With a benchmark, as seen in Proposition 1, slow traders do not enter unless the conditional expected gain from trade exceeds the cost s of entering the market (i.e. making the first contact with a dealer).

Using Lemma 1, we can describe the reservation-equilibrium without the benchmark, analogously with Proposition 1.

Proposition 3. *In the no-benchmark case, if a reservation-price equilibrium exists, it must satisfy the following properties:*

1. *Entry. If $s \geq X$, no slow traders enter, that is, $\lambda^\star = 0$. If $s \in ((1-\bar{\alpha})X,\, X)$, the fraction $\lambda^\star$ of entering slow traders solves*

$$s = (1 - \alpha(\lambda^\star))X. \tag{3.10}$$

   If $s \leq (1-\bar{\alpha})X$, all slow traders enter with probability $\lambda^\star = 1$.

2. *Prices. In the event that $c > v$, dealers quote an arbitrary price offer no lower than c. If $c \leq v$, dealers quote offers drawn with a cumulative distribution function F_c given by*

$$F_c(p) = 1 - \left[\frac{\lambda^*(1-p_0)}{Np_0}\frac{v-p}{p-c}\right]^{\frac{1}{N-1}}, \tag{3.11}$$

   for

$$p \in \left[\frac{\lambda^\star(1-p_0)v + Np_0c}{\lambda^\star(1-p_0) + Np_0},\ v\right].$$

3. *Traders' reservation prices. Conditional on entry, a slow trader has a reservation price of v at her first dealer contact. If this first dealer's price offer is no more than v, the slow trader accepts it. Otherwise the slow trader rejects it and exits the market.*

4. *Surplus. The expected total social surplus is* $\lambda^\star(1-p_0)(X-s)+p_0X$*, and the expected profit of each dealer is* $\lambda^\star(1-p_0)X/N$*.*

Propositions 1 and 3 share some common features. In both, dealers' strategies depend on the realization of the benchmark c, and slow traders never contact more than one dealer, on the equilibrium path. The distribution of quoted prices and the entry probability of slow traders are characterized by functions whose forms are similar with and without a benchmark.

That said, there are two crucial differences in the behavior of the market with a benchmark (Proposition 1) and without (Proposition 3). First, slow traders' entry decisions in the presence of the benchmark depend on the realization (through publication of the benchmark) of the gains from trade. By contrast, without a benchmark, entry depends only on the (unconditional) expected gain from trade. Second, with the benchmark, the reservation price of slow traders (and hence the upper limit of the price-offer distribution) depends on the realization of the benchmark c. Absent the benchmark, however, a slow trader's reservation price is always v, so that an offer of v is in the support of price offers regardless of the outcome of c.

Existence of reservation-price equilibria in the no-benchmark case

Before comparing welfare with and without the benchmark, it remains to characterize conditions under which a reservation-price equilibrium exists without the benchmark. Providing general conditions for the existence of reservation-price equilibria with learning and equilibrium pricing seems a challenging task. While significant progress has been made by Janssen, Pichler and Weidenholzer (2011), their results are not applicable in our setting because they assume a sufficiently large trader value[19] v. Benabou and Gertner (1993) also provide partial results for the case of two dealers.

Appendix A provides necessary and sufficient condition for the existence of reservation-price equilibrium in the case of two dealers, and a sufficient condition for existence with $N > 2$ dealers.[20] The main conclusion is summarized in the following proposition, demonstrated in Appendix A.

[19] Janssen, Pichler and Weidenholzer (2011) assume that v is large enough (sufficiently larger than $\bar{c}$) that it does not influence the equilibrium at all. We cannot make this assumption because it is not separable from our assumptions on the size of gains from trade (which play a key role in the analysis of entry).

[20] Using the formulas that we provide, one can verify existence "ex-post" in numerical examples.

Proposition 4. *There exists some $\underline{s} < X$ such that for any search cost s greater than $\underline{s}$, a reservation-price equilibrium in the no-benchmark case exists, and is payoff-unique.*

Proposition 4 states that the equilibrium described in Proposition 3 exists if the search cost is sufficiently large. The condition $\underline{s} < X$ ensures that there exists an equilibrium with strictly positive probability of entry by slow traders. If $s \geq X$ there exists a trivial reservation-price equilibrium in which slow traders do not enter.

3.3 Welfare comparison

We now show that if search costs are sufficiently high relative to the expected gain from trade, then introducing the benchmark raises the social surplus by encouraging the entry of slow traders.

As noted above, entry may be inefficiently low under search frictions due to the hold-up problem and the negative externality in the entry decisions of slow traders. Because search costs are sunk when a slow trader visits a dealer, a dealer can use its local monopoly power to charge high prices. Expecting this outcome, slow traders may refrain from entry despite the positive expected gains from trade. The hold-up problem is more severe when more slow traders enter (because this raises the posterior belief of a dealer that he faces a high-search-cost trader). These effects are present both with and without the benchmark. Our concern is whether benchmarks alleviate or exacerbate this situation.

The presence of the benchmark affects the entry probability of slow traders (and hence social surplus) in two ways. First, conditional on entry, the benchmark changes the split of surplus between dealers and traders. Without the benchmark, a slow trader is not sure whether an unexpectedly high price offer is due to a high outcome for the common cost c of dealers, or is due to an unlucky draw from the dealer's offer distribution. Dealers can exploit this informational advantage. If their local monopoly power is strengthened enough by high search costs, slow traders are forced to accept all prices below their value, as in Proposition 3. As a result, the expected payoffs of slow traders are (typically strictly) higher in the benchmark setting. Second, the presence of a benchmark allows slow traders to make their entry decisions contingent on the magnitude of gains from trade given the benchmark. It is not obvious (or even true in general) that this increased transparency before entry is always good for social surplus. Without the benchmark information, traders incentives to enter are pooled. The net

effect of the benchmark for entry efficiency is not obvious and must be analyzed with a model.

We now state the main result of this section, comparing welfare with and without the benchmark.

Theorem 1. *Suppose that (i) $s \geq (1-\bar{\alpha})(v-\underline{c})$ or (ii) $s \geq (1-\psi)X$ holds, where $\psi \in (0, \bar{\alpha})$ is a constant that depends explicitly*[21] *on p_0 and N. Then a reservation-price equilibrium in the no-benchmark case (if it exists) yields a lower social surplus than the equilibrium in the benchmark case. Condition (i) holds if there are sufficiently many dealers or if the fraction p_0 of fast traders is small enough.*

The proof of the theorem hinges upon a key lemma showing that, under assumption *(i)*, the expected social surplus in the benchmark case is a convex function of the realized gain from trade.[22] The expected welfare in the no-benchmark case is equal to the social surplus function in the benchmark case evaluated at the expected gain from trade. These two observations allow us to apply Jensen's Inequality to finalize the proof.

The crucial intuition behind Theorem 1 consists of two effects. First, if the benchmark is observed, entry is high precisely when the gain from each trade is large. This explains why convexity of the social surplus function plays an important role in the proof. The hold-up problem is especially harmful for traders when gains from trade turn out to be small. Publication of the benchmark raises surplus because it informs traders of the size of gains from trade, thus increasing entry by slow traders conditional on high gains from trade. Second, benchmarks reduce information asymmetry between dealers and traders. By providing additional information about dealers costs, benchmarks give more bargaining power to slow traders, which further encourages entry and thus adds to total realized gains from trade.[23] We examine these two effects in more detail in the next subsection.

The condition of Theorem 1 that gains from trade are not too large relative to search costs cannot be dispensed with. Without the benchmark, slow traders' incentives to enter are "pooled" as they calculate an expectation over all possible realizations of

[21] We have $\psi = \frac{1}{2}\left[\sqrt{(1-\bar{\alpha}+\bar{\alpha}\beta)^2 + 4\bar{\alpha}(1-\bar{\alpha})} - (1-\bar{\alpha}+\bar{\alpha}\beta)\right]$, where $\beta = Np_0/(1-p_0)$, and $\bar{\alpha}$ is defined by equation (3.7).

[22] Assumption *(ii)* ensures subdifferentiability of the function at X.

[23] The profit of a slow trader in an interior-entry equilibrium is zero, but what matters is the off-equilibrium-path profit, as this profit determines the probability of entry, according to equations (3.8) and (3.10).

dealers' costs and of their gains from trade. If their expected gains from trade are sufficiently large relative to their search costs, then without the benchmark all of the slow traders may enter. In the presence of the benchmark, however, slow-trader entry may be low in the event of high realizations of c (but still allowing gains from trade). In other words, adding the benchmark need not increase welfare if the entry of slow traders is already nearly efficient without the benchmark.

The following proposition emphasizes our point about pooling the incentives to enter by giving conditions under which introducing a benchmark can actually reduce welfare.

Proposition 5. *Suppose that the equilibrium described by Proposition 3 exists. If (i)* $(1-\bar{\alpha})(v-\bar{c}) \leq s$*, (ii)* $s \leq (1-\bar{\alpha})X$*, and (iii)* $G(v-s)$ *is sufficiently close to one, then the expected social surplus is higher without the benchmark than with the benchmark.*

The three conditions of Proposition 5 are strong but none can be discarded. The condition $s \leq (1-\bar{\alpha})X$ ensures that there is full entry without the benchmark. (By Theorem 1, this condition fails if p_0 is small enough or N is large enough.) The condition that $s \geq (1-\bar{\alpha})(v-\bar{c})$ ensures that there are cost realizations for which we do not have full entry with the benchmark. Finally, the condition that $G(v-s)$ is close to one ensures that the entry of slow traders is indeed socially desirable for *nearly all* cost realizations.

Section A.8 of Appendix A contains a numerical example illustrating the considerations outlined above.

3.4 Separating the two entry-promoting roles of a benchmark

As argued in our discussion of Theorem 1, introducing a benchmark encourages entry through two channels: *(i)* signaling when gains from trade are high and *(ii)* increasing the slow traders' share of gains from trade by reducing the asymmetry of their information regarding the asset supply cost, with respect to the information of dealers. In order to distinguish between these two effects, we study an intermediate "costly-benchmark-observation" setting in which traders observe the benchmark only upon making their first contact with a dealer (after making the entry decision but before accepting or rejecting an offer). Essentially, this means that slow traders must pay the search cost s to learn the outcome of the benchmark. This artificial costly-benchmark-observation setting allows us to characterize in the next theorem the specific role of entry effect *(ii)* of benchmarks, while keeping the other entry effect *(i)* "switched off." The proof can be found in Appendix A.

Theorem 2. *A reservation-price equilibrium always exists (and is payoff-unique) in the costly-benchmark-observation setting. Moreover, under the proviso that* $(1-\bar{\alpha})X < s < X$*, the equilibrium in the costly-benchmark-observation setting has a strictly higher expected social surplus than that of the reservation-price equilibrium without the benchmark.*

The theorem states that channel *(ii)*, reducing information asymmetry between dealers and traders, always works in favor of introducing a benchmark. By providing slow traders with information about the market-wide cost of the asset to dealers, the presence of a benchmark increases traders' expected payoffs off the equilibrium path, thus encouraging their entry and raising total social surplus on the equilibrium path.

The next result, proven in Appendix A, states that role *(i)* of a benchmark, signaling when there are high gains from trade, is also relevant.

Proposition 6. *There exists* $\underline{s} < X$ *such that for all* $s \in (v - \underline{c}, \underline{s})$ *the expected social surplus is strictly higher in the benchmark case than in the costly-benchmark-observation case.*

4 When Do Benchmarks Improve Matching?

In this section we explore the matching-efficiency role of benchmarks in search-based markets. For this purpose, we must analyze the full-fledged model in which dealers' costs are heterogeneous. So, from this point, we assume that the probability γ that a dealer has a low cost for providing the asset is in (0, 1). Throughout this section we maintain the following two assumptions.[24]

Assumption A.1. *Search is socially optimal, that is,* $s < \gamma\Delta$.

Assumption A.2. *Gains from trade exist with probability 1, that is,* $\bar{c} < v - \Delta$.

Together, these conditions imply full entry by slow traders in equilibrium, which allows us to separately identify the welfare effect of matching efficiency. Assumption A.2 is adopted for expositional purposes only. We give generalized statements (weakening Assumption A.2) of the results of this section in Appendix B. We will show that if search costs are relatively low then adding a benchmark raises social surplus by making

[24] Appendix B provides the supporting analysis when Assumption A.1 fails. In that case, there will be no search in the equilibrium with the benchmark. While the absence of search is socially optimal in this case, it is not the case in which we are most interested.

it easier for traders to find efficient (that is, low-cost) dealers. Having a low search cost is important for traders because contacting a low-cost dealer is socially optimal only if the search cost is lower than the potential improvement in efficiency, that is, under Assumption A.1.

In the presence of a benchmark, the key intuition for the equilibrium construction from Section 3 carries over to this setting, but the supporting arguments are more complicated, and several cases need to be considered. For that reason, we focus here on parameter regions that are relevant for social-surplus comparisons, and relegate a full characterization to Appendix B.

In the no-benchmark case, we face additional difficulties. If search costs are large enough, we can easily generalize the equilibrium construction from Proposition 3.[25] However, with idiosyncratic components in dealers' costs, Lemma 1 ceases to be true. For a low enough search cost s, there may exist reservation-price equilibria with reservation prices lower than v. These equilibria are much harder to characterize because slow traders search multiple times with strictly positive probability, and their optimal strategy depends at any point on the entire history of offers received to that point, as they successively update their beliefs about c. Given these difficulties, our objective is to show that, under natural conditions, welfare in the benchmark case is higher than in *any* equilibrium of the no-benchmark case.

We begin with a result that is demonstrated in Appendix B.

Proposition 7. *In the presence of a benchmark, the equilibrium is payoff-unique and slow traders use a reservation-price strategy.*

Proposition 7 is not surprising given the analysis of Section 3.1. There is, however, one key difference. Under a reservation-price strategy, a trader is typically indifferent between accepting and continuing to search when she sees an offer equal to her reservation price. In the setting of Section 3 it does not matter whether traders accept such an offer or not because this event has zero probability. With idiosyncratic costs, however, there are parameter regions in which the only equilibrium involves mixing between accepting and continue-searching by traders facing an offer at their reservation price. The mixing probabilities are important when there is an atom in the probability distribution of offers located at a trader's reservation price. In equilibrium, these atoms may arise if high-cost dealers make offers equal to the reservation price of slow traders.

[25] That is, we can prove that there exists a reservation-price equilibrium with first-round reservation price equal to v if search costs are large enough.

This affects the inference made by dealers when they calculate the probability of facing a fast trader.

To account for heterogeneity in dealers' costs, we need to adjust the probability that a dealer's counterparty is fast (as opposed to slow), from that given by equation (3.1). This probability now depends on both the entry probability λ_c and the probability that a slow trader rejects an offer from a high-cost dealer, which we denote by θ_c.[26] (As θ_c gets larger, slow traders search more, and the posterior probability that a dealer is facing a fast trader falls.) We will denote by $q(\lambda_c, \theta_c)$ the probability that a contacting trader is fast. Accordingly, the definition of the function $\alpha(\lambda_c)$ from equation (3.6) is generalized to a two-argument function $\alpha(\lambda_c, \theta_c)$ with values in $(0, 1)$. Explicit formulas are provided by equations (B.5) and (B.6) in Appendix B. The role of $\alpha(\lambda_c, \theta_c)$ is analogous to that of $\alpha(\lambda_c)$ in Section 3. Here, $\alpha(\lambda_c, \theta_c)$ is strictly increasing in both arguments and related to the level of price offers in the market. As λ_c and θ_c increase, the probability that a counterparty is slow rises, leading dealers to quote higher prices in equilibrium. The constant $\alpha(1, 1)$ is an analogue of $\bar{\alpha}$ in Section 3, and bounds $\alpha(\lambda_c, \theta_c)$ from above. For the sake of simplifying upcoming expressions, we denote

$$\hat{\alpha} = \alpha(1, 1).$$

With these technical preliminaries, we now state one of the main results of this section, whose proof is found in Appendix B.

Proposition 8. *If $s \leq (1 - \hat{\alpha})\gamma\Delta$, then equilibrium in the benchmark case leads to efficient matching. That is, slow traders always enter, and all traders buy from a low-cost dealer, in the event that there is at least one such dealer present in the market. Additionally, if $s \geq \kappa(1 - \hat{\alpha})\gamma\Delta$, where $\kappa < 1$ is a constant depending explicitly[27] on γ, p_0, and N, the equilibrium with the benchmark achieves the second best, in the sense that each slow trader buys from the first low-cost dealer that she contacts, thus minimizing search costs subject to matching efficiency.*

To understand how the benchmark leads to efficient matching and second best, consider first the case in which the search cost s is in the interval

$$(\kappa(1 - \hat{\alpha})\gamma\Delta,\ (1 - \hat{\alpha})\gamma\Delta).$$

[26] Note that θ_c is not part of the trader's strategy, as traders do not observe the types of dealers. Nevertheless, θ_c is determined uniquely by players' strategies.

[27] We have $\kappa = (1 - \gamma)^{N-1} / \left[p_0(1 - \gamma)^{N-1} + (1 - p_0)[1 - (1 - \gamma)^N]/(N\gamma)\right]$.

In equilibrium, slow traders follow a reservation-price strategy with a reservation price $r_c^\star$ that is below $c + \Delta$. Low-cost dealers quote prices according to a continuous probability distribution whose support is below that reservation price. Thus, if there are any low-cost dealers in the market, slow traders buy from the first low-cost dealer that they contact. In the unlikely event that there are only high-cost dealers in the market (which happens with probability $(1 - \gamma)^N$), slow traders search the entire market and then trade with one of the high-cost dealers. This second-best equilibrium outcome is therefore fully efficient at matching.

The key role of the benchmark in this case is to introduce enough transparency to permit traders to distinguish between efficient and inefficient dealers. The benchmark not only ensures that traders ultimately transact with the "right" sort of counterparty, but also ensures that no search cost is wasted while looking for this transaction. This last conclusion is true under the weaker condition that $s \geq \kappa(1 - \hat{\alpha})\gamma\Delta$.

If $s < \kappa(1 - \hat{\alpha})\gamma\Delta$, however, slow traders may search inefficiently long, for reasons that we explain below. Adding the benchmark makes it possible for slow traders to identify low-cost dealers but if the search cost is too small, then low-cost dealers simply do not *want* to be identified. If search costs are very small, the equilibrium reservation-price $r_c^\star$ also gets very low. Low-cost dealers are forced to quote very low prices if they want to sell at the first contact of any slow trader. Because of their cost advantage, low-cost dealers always have the "outside option" of trying head-on competition by quoting a price above the reservation price (and just below $c + \Delta$), hoping that all other dealers have high costs (in which case low-cost dealers win the resulting effective auction, making positive profits). It turns out that low-cost dealers prefer this strategy when $s < \kappa(1 - \hat{\alpha})\gamma\Delta$. The resulting equilibrium has the following feature: Low-cost dealers randomize their offers within two disjoint intervals.[28] The lower interval has the upper limit $r_c^\star$, and the higher interval has the upper limit $c + \Delta$. If a low-cost dealer quotes a price above $r_c^\star$ with a positive probability, then a slow trader might not buy from him on the first visit, implying extra search costs. As a result, in equilibrium, matching remains efficient but we do not achieve the second best, given the unnecessary expenditure of search costs.

The intuition sketched above indicates that a low-cost dealer's incentive to quote a high price should disappear as the number N of dealers gets large. Indeed, as N becomes large the probability that all other dealers have high costs goes to zero quickly. We

[28] The lower part of the distribution of offers is continuous on $[\underline{p}_c^l, r_c^\star]$. The upper part is continuous on $[\hat{p}_c^l, c + \Delta]$, where $\hat{p}_c^l > r_c^\star$.

confirm this in Appendix B.3, where we show that an upper bound on the potential surplus loss (compared to first best) goes to zero exponentially fast with N when $s < \kappa(1-\hat{\alpha})\gamma\Delta$. In sharp contrast, surplus losses are potentially unbounded in N when s is close to $(1-\hat{\alpha})\gamma\Delta$. Hence, for practical purposes, it is natural to focus on the case $s \geq \kappa(1-\hat{\alpha})\gamma\Delta$.

We now show that without the benchmark, it is impossible to achieve the second best.

Proposition 9. *In the absence of a benchmark, if $\bar{c} > \underline{c} + \Delta$ there does not exist an equilibrium that achieves the second best.*

The proof, found in Appendix B, explores the simple idea that when there is no benchmark for traders to observe, they cannot recognize a low-cost dealer when they contact one. In the absence of a benchmark, traders can only rely on Bayesian inference based on the observed price quotes. However, Bayesian inference can be relatively ineffective. With low realizations of the common cost component c, high-cost dealers may make offers that "imitate" the offers that low-cost dealers would make at higher realizations of c. The benchmark introduces enough transparency to allow traders to distinguish between high offers at low costs and low offers at high costs.

As a direct consequence of Propositions 8 and 9, we obtain the following main result, providing conditions under which adding a benchmark improves welfare.

Theorem 3. *If (i) $\kappa(1-\hat{\alpha})\gamma\Delta \leq s \leq (1-\hat{\alpha})\gamma\Delta$ and (ii) $\bar{c} > \underline{c} + \Delta$ both hold, then the equilibrium in the benchmark case yields a strictly higher expected social surplus than that of any equilibrium in the no-benchmark case.*

The theorem provides a sufficient condition and does not cover the entire search-cost space. We discuss the remaining cases in Section B.6 of Appendix B, where we show in particular that the second best cannot be achieved without the condition $s \leq (1-\hat{\alpha})\gamma\Delta$. Nonetheless, with a benchmark, if search costs are not too large, partial efficiency applies to the matching of traders to low-cost dealers. The (unique) equilibrium supporting this outcome has an interesting structure. High-cost dealers post a price $c+\Delta$ equal to the reservation price $r_c^\star$ of slow traders. Slow traders accept that price with some nontrivial (mixing) probability that is determined in equilibrium.

The analysis of the no-benchmark case in this section is not based on an explicit construction of an equilibrium. In Section B.7 of Appendix B we specialize to the case of two dealers ($N = 2$), which permits a sharp characterization of reservation-price

equilibria. In particular, we show that the first-round reservation price must be either $\underline{c} + \Delta$ or v, a generalization of Lemma 1. We describe equilibrium pricing strategies, and provide conditions for existence. Furthermore, the explicit equilibrium construction yields additional conclusions for the case of two dealers.

First, we demonstrate that matching is always more efficient with a benchmark than without, regardless of search costs, provided that traders use a reservation-price strategy in equilibrium. Second, for the equilibrium with reservation price $\underline{c} + \Delta$, we show that the inefficiency of not having a benchmark might take the form of excessive search costs. Without the benchmark, traders fail to recognize a low-cost dealer upon the first visit and engage in wasteful search that would not arise if there were a benchmark. We show that the resulting welfare losses may be quite significant depending on parameters, and may constitute a considerable fraction of total welfare losses arising from search frictions.

5 Incentives of Dealers to Introduce a Benchmark

So far in this paper we have taken the existence of a benchmark as given. In practice, benchmarks are often introduced by market participants, such as dealers in OTC financial markets. In this section we explicitly explore the incentives of dealers to introduce a benchmark. Section 5.1 provides conditions under which all dealers have a joint incentive to introduce the benchmark to encourage the entry of slow traders. Section 5.2 demonstrates conditions under which low-cost dealers have an incentive to introduce a benchmark in order to drive their high-cost competitors out of business, an additional potential source of welfare related to the availability of benchmarks.

5.1 Introducing benchmarks to encourage entry

As we have seen in previous sections, the introduction of a benchmark reduces the informational advantage of dealers relative to traders, and increases the expected payoffs of slow traders (whether on or off the equilibrium path). It might superficially seem that dealers have no incentive to introduce the benchmark.

In this section we show that the contrary can be true: Under certain conditions dealers want to introduce a benchmark in order to increase their volume of trade. We assume that dealers are able to commit to a mechanism leading to truthful revelation of c, so the question of whether they prefer to have the benchmark boils down to comparing

dealers' profits with and without the benchmark. We address the implementability of adding a benchmark in Section 6.

For simplicity of exposition we concentrate on the effects of entry on dealers' profits in the setting in which dealers have homogeneous costs for supplying the asset, that is, $\gamma = 1$. We discuss in Section C.3 of Appendix C and show formally in Appendix E that the same conclusions hold if dealers' costs are heterogeneous.[29] We show that although the benchmark reduces the expected profit of a dealer on each trade, the transparency afforded by the benchmark may increase trading volume so much that dealers are better off collectively with a benchmark. This requires search costs to be relatively large.

The following theorem, proven in Appendix C, is similar to Theorem 1, and demonstrates that dealers indeed benefit from the introduction of a benchmark if search costs are relatively high.

Theorem 4. *Suppose that (i)* $s \geq (1 - \bar{\alpha})(v - \underline{c})$ *or (ii)* $s \geq (1 - \eta)X$ *holds, where* $\eta \in (0, \bar{\alpha})$ *is a constant that depends only on* N *and* p_0*. If all dealers have the same supply cost (that is,* $\gamma = 1$*), then a reservation-price equilibrium in the no-benchmark case*[30] *yields a lower expected profit for dealers than in the setting with the benchmark. Condition (i) holds if there are sufficiently many dealers or if the fraction* p_0 *of fast traders is small enough.*

The benchmark raises the profits of dealers by encouraging the entry of slow traders. If search costs are large relative to gains from trade (assumption *(i)* or *(ii)* of Theorem 4), dealers benefit from increased volume of trade after introducing the benchmark. Entry by slow traders must be sufficiently low without the benchmark, for otherwise the dealers' gain in trade volume does not compensate for the drop in profit margin on each trade.

A benchmark can be viewed as a commitment device, by which dealers promise higher expected payoffs to traders in order to encourage entry. In particular, a benchmark partially solves the hold-up problem by reducing market opaqueness and hence by giving more bargaining power to traders.

It can be shown that the conclusion of Theorem 4 implies the conclusion of Theorem 1. That is, whenever dealers would opt for the benchmark, it must be the case that the introduction of the benchmark raises social surplus. The opposite is not true. There

[29] Just as in Section 3, the heterogeneity of dealers' costs does not "interact" with the effects of entry.

[30] Whenever it exists, see Proposition 4 and Section 3.

generally exists a range of search costs in which the benchmark raises social surplus but dealers would have no incentive to introduce the benchmark.

Theorem 4 and the above discussion are illustrated with a numerical example found in Section C.2 of Appendix C.

5.2 Low-cost dealers may compete by introducing a benchmark

This subsection analyzes the incentives of low-cost dealers to introduce a benchmark on their own—despite the opposition from high-cost dealers—as a powerful device to compete for business. We show that under certain conditions the collective decision of low cost dealers to add a benchmark drives high-cost dealers' profits to zero and forces them out of the market. As a result, low-cost dealers make more profits, and the market becomes more efficient overall. This analysis provides another natural economic rationale for why volume of trade in a given asset may shift toward those dealers who make markets based on a benchmark.

To explain how "benchmark clubs" may emerge, we augment our search-market game of the previous sections with an earlier stage in which dealers decide whether to introduce a benchmark and, after calculating their expected profits, whether to enter the market themselves. To provide a simple framework for studying endogenous introduction of a benchmark, we assume that there are two types of "environments" with different degrees of competitiveness measured by the number of low-cost dealers. Let the random variable L be the number of low-cost dealers. Suppose that $L \geq 2$ ("competitive environment") with probability $\tilde{\gamma} \in (0, 1)$ and $L = 0$ ("uncompetitive environment") with probability $1 - \tilde{\gamma}$.[31] A formal description of the game follows:

1. Pre-trade stage: the introduction of a benchmark and entry by dealers.

   (a) Nature chooses the dealer-cost environment, whose outcome is not observed. With probability $1 - \tilde{\gamma}$, all dealers have high costs. With probability $\tilde{\gamma}$, the number L of low-cost dealers is drawn from a truncated binomial distribution with parameters (N, γ), where the truncation restricts the support to the set $\{2, 3, ..., N\}$.[32] The idiosyncratic component ϵ_i of dealer i is the private

[31] To simplify the analysis, we ruled out the case that there is exactly one low-cost dealer in the market. When Δ is high, such a seller effectively becomes a monopolist which complicates the analysis.

[32] Conditional on L, the identities of dealers with low costs are drawn independently of L and uniformly at random. Of course, in that case $\{\epsilon_i\}$ are no longer i.i.d. For our results to hold, we can allow a more general distribution of types, relaxing the binomial distribution assumption from

information of dealer i.

(b) Any (non-trivial) coalition of dealers may commit to introducing a benchmark. We model this as a voting game in which dealers simultaneously vote whether they prefer the benchmark or not. If there are at least two votes in favor, the benchmark is introduced by those in favor, that is, c becomes common knowledge as soon as it is drawn from the distribution. If there is one or no votes in favor, the benchmark is not introduced, and c is observed only by dealers.

(c) Dealers make entry decisions. For simplicity, we adopt a tie-breaking rule that dealers enter if and only if their expected profits are strictly positive.[33]

(d) After dealers' entry decisions, the number of dealers that enter, denoted M, becomes common knowledge among dealers and traders.

2. Trading stage. The game proceeds according to the baseline model described in Section 2, but with N replaced by M.[34,35]

Let

$$X_\Delta = G(v-\Delta)\mathbb{E}\left(v - c - \Delta \,|\, c \leq v - \Delta\right)$$

be the expected gains from trade when all dealers have high costs. We assume throughout that $X_\Delta > s$, that is, expected gains from trade between slow traders and high-cost dealers are positive after accounting for search costs. We will say that Δ is (sufficiently) large, when $X_\Delta - s$ is (sufficiently) small and $\bar{c} + \Delta > v$.[36]

previous sections. Namely, the only required restriction on the unconditional distribution of L is that the events $L = 0$ and $L \geq 2$ both have positive probability, and the event $L = 1$ has zero probability.

[33]This is equivalent to assuming that there is a sufficiently small but strictly positive fixed cost c_f of entry.

[34]The trading stage above can be extended to multiple "rounds" of trading. In each round the one-shot baseline game of Section 2 repeats itself, with an i.i.d. draw of the common cost component c. This extension is natural because the introduction of a benchmark is a long-term or permanent decision, whereas the realization of common cost components are time-varying as market conditions change. What is important, though, is that the idiosyncratic costs are sufficiently persistent, so that the benefit to low-cost dealers of introducing a benchmark is unlikely to "flip" to a cost in the near future. For the simplicity of exposition but at no cost of economic intuition, we have assumed that the idiosyncratic cost types are permanent and that the search game is played only "once."

[35]To be conservative, we continue to assume that dealers remains anonymous, that is, the identities of dealers who voted for introducing the benchmark are unobserved by the traders, and traders cannot "direct" their search to those dealers. If dealers who vote for benchmarks were able to direct search to themselves, the incentives for introducing the benchmark would likely be stronger.

[36]If Δ were allowed to be arbitrarily large, the analysis would not be interesting as high-cost dealers

The following theorem establishes conditions that are sufficient to induce low-cost dealers to collectively introduce the benchmark and drive their high-cost competitors out of the market.

Theorem 5. *Suppose that the dealer cost difference Δ is sufficiently large, and the search cost s is strictly lower than $(1-\bar{\alpha})(v-\bar{c})$. Then the following hold.*

- *There exists an equilibrium of the extended game in which low-cost dealers always vote in favor of the benchmark, and high-cost dealers always vote against it. Moreover, there are no profitable group deviations in the voting stage.*[37]
- *If the environment is competitive (that is, $L \geq 2$), the benchmark is introduced, all high-cost dealers stay out of the market, all low-cost dealers enter the market, and all traders enter the market.*
- *If the environment is uncompetitive ($L = 0$), the benchmark is not introduced, and all dealers enter the market.*

A proof of the theorem is provided in Appendix C. Here, we explain the economic intuition behind the result.

The benchmark serves as a signaling device for low-cost dealers to announce to traders that the environment is competitive. The signal is credible because traders, expecting low prices conditional on introducing the benchmark, set a low reservation price in equilibrium. Therefore, high-cost dealers cannot imitate low-cost dealers by deviating and announcing the benchmark. Instead, they prefer to trade in opaque markets without the benchmark and low participation by slow traders (which allows them to make positive profits). This may explain why emergent "benchmark clubs" are often able to quickly attract the bulk of trades in some OTC markets, as was the case with LIBOR.

Low-cost dealers have two distinct incentives to add the benchmark. First, adding the benchmark encourages the entry of slow traders. In addition to the intuition conveyed in Section 3, in the setting of this section the benchmark plays the additional role

simply cannot trade, regardless of the details of the trading environment. Our definition of high Δ says that expected gains from trade between slow traders and high-cost dealers are small, and that high-cost dealers cannot trade at the highest cost realizations.

[37] In the usual Nash equilibrium of the voting game, if everyone is voting against or in favor, the individual vote of a dealer is never pivotal, and thus each outcome may be supported in equilibrium. By showing that group deviations are not profitable we make sure that in equilibrium every type of dealers votes in favor of the setting that yields higher profits for that type.

of signaling the types of active dealers—because the benchmark is added endogenously. On the equilibrium path, once a benchmark is introduced, slow traders believe with probability one that all active dealers have low costs. If a benchmark is not introduced, slow traders believe that all dealers have high costs. As a consequence, the (correctly) perceived gains from trade by slow traders goes up considerably if a benchmark is added. This channel encourages entry. In particular, the assumption $s < (1-\bar{\alpha})(v-\bar{c})$ ensures full entry by traders if the benchmark is introduced.

Second, low-cost dealers capture additional volumes of trades by adding the benchmark. With a large enough cost difference Δ the expected gains from trade are small if the benchmark is not introduced. As a result, we show that slow traders who enter will set a reservation price $r^\star = v$ in the trading-stage subgame, and high-cost dealers inevitably capture a large proportion of trades with slow traders. If, however, the benchmark is introduced, a sufficiently large Δ makes high-cost dealers' quotes highly uncompetitive, which drives trades to low-cost dealers. Thus, although low-cost dealers' profit per trade may be lower with the benchmark, they capture additional volume of trade. In fact, in equilibrium, if the environment is sufficiently competitive, high-cost dealers drop out completely because they cannot make any profit, and low-cost dealers handle all of the trades.

6 Benchmark Manipulation and Implementation

So far we have assumed that dealers have the ability to credibly commit to the truthful revelation of c. In this section we briefly address manipulation of the benchmark.

Major scandals surrounding the manipulation of interest rate and foreign currency price benchmarks have placed the robustness of benchmarks to manipulation into the spotlight of international investigators, policymakers, and researchers. Serious manipulation problems or allegations have been reported for a number of other major benchmarks, including those for term swap rates, gold, silver, and oil.[38] Gencarelli (2002) discusses the manipulation of benchmarks in the market for pharmaceuticals. In light of concerns over manipulation, a cross-agency "Fair and Effective Markets Review" published by the Bank of England (2014) provided a list of over-the-counter-market benchmarks "that should be brought into the regulatory framework originally

[38] See, respectively, Patterson and Burne (2013), Vaughn (2014), Hurtado (2014), and Scheck and Gross (2013).

implemented in the wake of the LIBOR misconduct scandal."[39]

In our model, dealers have an incentive to manipulate traders' conditional distributions of the common cost component c. In this section, we suppose that dealers have homogeneous costs ($\gamma = 1$).

Suppose that customers "trust" the announced benchmark and do not anticipate manipulation by the dealers. We show in Proposition 13 of Appendix D that if dealers can collude and announce any desired level $\hat{c}$ as the supposed benchmark, they would announce $\hat{c} = v - s/(1-\bar{\alpha})$, regardless of the true c. Although dealers want to report the same benchmark outcome $\hat{c}$ regardless of c, the nature of manipulation is quite different depending on whether the actual common-cost benchmark c is larger or smaller than $v - s/(1-\bar{\alpha})$. If $c < v - s/(1-\bar{\alpha})$, the manipulation is aimed at increasing profit margins. If the announced benchmark were the correct common cost component c, slow traders would not accept prices above $r_c^\star < v$. With the false benchmark, they incorrectly believe that costs are higher, and accept all prices below v. When $c > v - s/(1-\bar{\alpha})$, the manipulation is instead aimed at encouraging entry. Announcing $\hat{c} = v - s/(1-\bar{\alpha})$ guarantees full entry of slow traders. Somewhat surprisingly, this is welfare-enhancing, but slow traders make negative profits on average.

Specifying and solving an equilibrium model of manipulation with correct beliefs by traders concerning the manipulation, while desirable, is beyond the scope of this paper. Instead, we close our discussion of manipulation by considering the following mechanism design problem.

Suppose that there exists a benchmark administrator who can design an arbitrary "benchmark announcement" mechanism with transfers. Here, a mechanism is a pair (M, g), where $M = (M_1 \times \cdots \times M_N)$ is the product of the message spaces of the N respective dealers, and where $g : M \to [\underline{c}, \bar{c}] \times \mathbb{R}^N$. The function g maps the dealers' messages $(m_1, \ldots, m_N)$ to an announced benchmark $\tilde{c}$ and to transfers $t_1, \ldots, t_N$ from the respective dealers to the mechanism designer. Each mechanism induces a game in which dealers first submit messages. The second stage of the game is the trading game presented in Section 2 of this paper, in which traders assume that the announced benchmark $\tilde{c}$ is a truthful report of the actual cost c.

[39] See page 3 of this report. A table listing the benchmarks that are recommended for regulatory treatment is found on page 15. In addition to LIBOR, which is already regulated in the United Kingdom, these are the overnight interest rate benchmarks known as SONIA and RONIA, the ISDAFix interest-rate-swap index, the WM/Reuters 4pm closing foreign exchange price indices (which cover many currency pairs), the London Gold Fixing, the LBMA Silver Price, and ICE Brent (a major oil price benchmark).

In this setting, "Nash implementability" means that there exists a mechanism whose associated game has a Nash equilibrium in which the announced benchmark $\tilde{c}$ is the true cost c. "Full implementability" adds the requirement that this is the unique equilibrium of the mechanism-induced game.

Proposition 10. *Truthful revelation of c is Nash implementable, but is not fully Nash-implementable.*

The proposition states that each dealer wants to report a message supporting the announcement of a benchmark that is the true cost c, provided that he believes that all other dealers report in this manner. However, for the mechanism that we construct in the proof of this result, there is also an equilibrium in which all dealers report the same, but a false, common cost level, say $\hat{c} = v - s/(1 - \bar{\alpha})$. The second part of Proposition 10 asserts that this cannot be avoided. That is, there does not exist a mechanism that leads to truthful revelation of c as a *unique* Nash equilibrium. Informally, this means that the benchmark is not robust to collusion.

Coulter and Shapiro (2014) solve a mechanism design problem with transfers in a setting that incorporates important incentives to manipulate that are absent from our model. They reach a similar conclusion in that it is possible to implement a truthful benchmark, but their mechanism can also be "rigged" for false reporting through collusion by dealers. Duffie and Dworczak (2014) consider a different model of benchmark design and manipulation, showing that, without transfers, an optimizing mechanism designer will not in general implement truthful reporting. Instead, considering a restricted class of mechanisms, they characterize a robust benchmark that minimizes the variance of the "garbling," meaning the difference between the announced benchmark and the true cost level.

7 Concluding Remarks

Benchmarks underly a significant fraction of transactions in financial and non-financial markets, particularly those with an over-the-counter structure that rules out a common trading venue and a publicly announced market-clearing price. This paper provides a theory of the effectiveness and endogenous introduction of benchmarks in search-based markets that are opaque in the absence of a benchmark. Our focus is the role of benchmarks in improving market transparency, lowering the informational asymmetry between dealers and their customers regarding the true cost to dealers of providing the

underlying asset. Other potential benefits of benchmarks are discussed by Duffie and Stein (2014).

In the absence of a benchmark, traders have no information other than their own search costs and what they learn individually by "shopping around" for an acceptable quote. Dealers exploit this market opaqueness in their price quotes. Adding a benchmark alleviates information asymmetry between dealers and their customers. We provide conditions under which the publication of a benchmark raises total social surplus by encouraging greater market participation by the customers of dealers through an associated reduction in their search costs, improvement in their price terms caused by greater effective competition among dealers, and improvement in the efficiency of matching. In some cases, dealers have an incentive to introduce benchmarks despite the associated loss of local-monopoly advantage, because of a more-than-offsetting increase in the volume of trade achieved through greater customer participation.

It follows that which of our markets have a benchmark is not an accident of chance, but rather is likely to be an outcome of a conscious decision of dealers, case by case, when trading off the costs and benefits of the additional market transparency afforded by a benchmark. We show that in some cases there is a public-welfare role for regulators regarding which markets should have a benchmark, and also over the potential for benchmark manipulation.

We also show that in some cases benchmarks are introduced by the subset of dealers who are most cost-effective, because the additional transparency afforded by a benchmark improves their market share, potentially driving out higher-cost dealers.

Our model assumes that dealers' price offers remain valid at any time, the "recall" assumption. The equilibrium path that we demonstrated in most cases does not actually involve recall by slow traders, and would remain the same even if slow traders could not execute at previously obtained price offers.[40] If, however, fast traders could not recall early offers, for example because they are not anonymous, then their search process would no longer be equivalent to an auction, and effective search costs would arise for them as a consequence of search-induced adverse selection, as modeled by Zhu (2012). No recall by fast traders would also lead to technical challenges associated with the Diamond paradox. In general, the absence of recall would make the analysis of benchmarks intractable, based at least on currently available modeling approaches.

[40]For example, in the equilibria of Section 3, a slow trader always accepts the first offer. If anything, the no-recall assumption would make slow traders even more willing to accept the first price quote.

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Appendix

A Proofs for Section 3

A.1 Proof of Proposition 1

We fill in the gaps in the derivation of the equilibrium in the benchmark case. We focus on the non-trivial case $\underline{c} \leq c < v - s$.

As argued in Section 3, regardless of the price distribution that dealers use in a symmetric equilibrium, slow traders play a reservation-price strategy with some reservation price r_c. Fast traders play their weakly dominant strategy of searching the entire market. (Thus, if the trader is a fast trader, the dealers are essentially participating in a first-price auction.) Given this strategy of traders, the following Lemma establishes the properties of the equilibrium response of dealers.

Lemma 2. *If slow traders enter with a strictly positive probability, the equilibrium price distribution cannot have atoms or gaps, and the upper limit of the distribution is equal to r_c.*

Proof. Suppose there is an atom at some price p in the distribution of prices $F_c(\,\cdot\,)$ for some cost level $c \in (\underline{c},\, v - s)$. Suppose further that $p > c$. In this case a dealer quoting p can profitably deviate to a price $p - \epsilon$, for some small $\epsilon > 0$ (because slow traders play a reservation-price strategy, the probability of trade jumps up discontinuously). Because dealers never post prices below their costs, we must have $p = c$. But that is also impossible, because a dealer could then profitably deviate to r_c (clearly, $r_c \geq c + s$ in equilibrium). Thus, there are no atoms in the distribution.

Second, suppose that $\bar{p}_c > r_c$. In this case the dealer posting $\bar{p}_c$ makes no profits, so she could profitably deviate to r_c. On the other hand, if $\bar{p}_c < r_c$, a dealer can increase profits by quoting r_c instead of $\bar{p}_c$ as this does not effect the probability of selling. Thus $\bar{p}_c = r_c$.

Third, suppose that there is an open gap in the support of the distribution of prices conditional on some cost level c, that is, an interval $(p_1,\, p_2) \subset [\underline{p}_c,\, \bar{p}_c] \setminus \operatorname{supp}(F_c(\,\cdot\,))$. Take this interval to be maximal, that is, such that p_1 is infimum and p_2 is a supremum, both subject to being in the support of $F_c(\,\cdot\,)$. Then we get a contradiction because the probability of selling is the same whether the dealer posts p_1 or p_2. □

Given Lemma 2, we know that in a symmetric equilibrium dealers must be indifferent between prices in the interval $[\underline{p}_c,\, \bar{p}_c]$.[41] Using Bayes' rule, the posterior probability that a trader is a fast trader conditional on a visit (while fixing the probability of entry by slow traders at λ_c) is given by $q(\lambda_c)$ (as given by equation (3.1)). Thus, the indifference condition is expressed in equation (3.2) and gives us the equilibrium cdf (3.3).

[41] Strictly speaking, the above arguments only imply that they need to be indifferent between prices in some set that is dense in $[\underline{p}_c,\, \bar{p}_c]$. But since any cdf is right-continuous, it is uniquely defined by its values on a dense set of the support.

Lemma 3. *The expected value of the distribution given by equation (3.3) is* $(1-\alpha(\lambda_c))c + \alpha(\lambda_c)r_c$*, where* $\alpha(\,\cdot\,)$ *is defined by (3.6).*

Proof. The proof is also provided in Janssen, Pichler and Weidenholzer (2011), so we only sketch the argument. We have

$$\int_{\underline{p}_c}^{\bar{p}_c} p\, dF_c(p) = c + \int_{\underline{p}_c}^{\bar{p}_c} (p-c)\, dF_c(p).$$

Consider a change of variables $z = 1 - F_c(p)$. We then have

$$(p-c) = \frac{r_c - c}{1 + \frac{Np_0}{\lambda_c(1-p_0)} z^{N-1}},$$

and thus

$$\int_{\underline{p}_c}^{\bar{p}_c} p\, dF_c(p) = c + (r_c - c) \int_0^1 \left(1 + \frac{Np_0}{\lambda_c(1-p_0)} z^{N-1}\right)^{-1} dz.$$

□

Lemma 3 implies equation (3.5). Finally, equation (3.8) has indeed a unique solution for $c \in (v-s,\, v - \frac{s}{1-\bar{\alpha}})$ because $\alpha(\lambda_c)$ is strictly increasing and continuous in λ_c.

A.2 Proof of Proposition 2

Because dealers never quote prices below their costs, X provides an upper bound on the expected payoff of a slow trader after entry. Thus, since $s \geq X$, slow traders cannot enter in any equilibrium.[42] The social surplus is at most $p_0 X$. With the benchmark, whenever $v - c > s$, which has positive probability if $v - \underline{c} > s$, there is a positive probability of entry by slow traders, by Proposition 1. Thus, the expected surplus is strictly higher than $p_0 X$.

A.3 Proof of Lemma 1

Let $r_0^\star$ be the equilibrium first-round reservation price for slow traders. Note that, unlike in the benchmark case, $r_0^\star$ is a number, not a function of c.

We take $c < r_0^\star$. Such a c exists because $r_0^\star \geq \underline{c} + s$. Suppose that the upper limit of the support of the distribution F_c of offer prices, $\bar{p}_c$, is (strictly) larger than $r_0^\star$. Since traders follow a reservation-price strategy, and because fast traders visit all dealers, there can be no atoms in the distribution of prices (otherwise a dealer could profitably deviate by quoting a price just below the atom). Thus, a dealer setting the price $\bar{p}_c$ never sells in equilibrium, and hence makes zero profit. However, she could

[42] Even if $s = X$, we can still show that there are no equilibria with positive entry by slow traders. Indeed, if the probability of entry is positive, since slow traders have to accept prices below $\underline{c} + s$, dealers will quote prices above c with positive probability.

make positive profit by setting a price equal to $r_0^\star$. Thus, $\bar{p}_c \leq r_0^\star$. Because we took an arbitrary $c < r_0^\star$, it follows that whenever $c < r_0^\star$, traders do not observe prices above $r_0^\star$ on the equilibrium path.

Suppose that $r_0^\star < v$. Whenever the realization of c lies above $r_0^\star$, the offer in the first round must be rejected by a slow trader (dealers cannot offer prices below their costs). In particular, a slow trader must reject the price $p^\star \in \operatorname{supp}(F_c(\,\cdot\,))$ with $r_0^\star < p^\star \leq \inf\{p \in \operatorname{supp}(F_c(\,\cdot\,) : c > r_0^\star\} + \delta < v$, for a sufficiently small $\delta > 0$.[43] This is a contradiction. Indeed, by the previous paragraph, conditional on observing a price $p > r_0^\star$ in the first round, the trader believes that c must lie above $r_0^\star$ with probability 1. But in this case, the price $p^\star$ is within δ of the best possible price that the trader can ever be offered, so this offer should be accepted by a slow trader (if $\delta < s$), contrary to $p^\star > r_0^\star$. This shows that $r_0^\star = v$.

Finally, suppose that $\bar{p}_c < v$ for some $c < v$. Then a dealer quoting the price $\bar{p}_c$ could profitably deviate by posting a price v (the probability of trade is unaffected). This justifies the second claim.

A.4 Proof of Proposition 3

Fix a fraction λ of slow traders that enter. By Lemma 1 and the arguments used in the derivation of equilibrium prices in the benchmark case, the cdf of offered prices must be

$$F_c(p) = 1 - \left[\frac{\lambda(1-p_0)}{Np_0}\frac{v-p}{p-c}\right]^{\frac{1}{N-1}} \tag{A.1}$$

with support $[\underline{p}_c, v]$, where $\underline{p}_c = \varphi(\lambda)v + (1-\varphi(\lambda))c$ and

$$\varphi(\lambda) = \frac{\lambda(1-p_0)}{Np_0 + \lambda(1-p_0)}.$$

We note that the only difference with the equilibrium pricing under the benchmark is that the reservation price and probability of entry are constants, not functions of c.

We can now calculate the expected profits of slow traders if they choose to enter:

$$\pi(\lambda) = -s + \int_{\underline{c}}^{v}\left[\int_{\underline{p}_c}^{v}(v-p)\,dF_c(p)\right]dG(c) = -s + (1-\alpha(\lambda))X,$$

where

$$X = G(v)\left[v - \mathbb{E}\left[c|\, c \leq v\right]\right]$$

is the expected gains from trade. By reasoning analogous to that in the benchmark case, we determine that:

- If $s \leq (1-\bar{\alpha})X$, there must be full entry by slow traders ($\lambda^\star = 1$).

[43] Such $p^\star$ exists. As long as $c < v$, in equilibrium dealers must be posting prices below v with positive probability.

- If $s \geq X$, there cannot be entry by slow traders ($\lambda^\star = 0$).
- If $s \in ((1-\bar{\alpha})X,\, X)$, then the entry of slow traders is interior, with probability $\lambda^\star$ determined uniquely by the equation (3.10).

A.5 Proof of Proposition 4

Given Proposition 3, in order to prove existence in our setting we need only show that a slow trader does not want to search after observing a price $p \leq v$ in the first round. After observing a price p, the slow trader forms a posterior probability distribution of c, given by the cdf

$$G(c \,|\, p) = \frac{\int_{\underline{c}}^{c} f_y(p)\, dG(y)}{\int_{\underline{c}}^{\bar{c}_p} f_y(p)\, dG(y)},$$

where $f_c(p)$ denotes the density of the distribution defined by the cdf (3.11), and

$$\bar{c}_p = \frac{1}{1-\varphi(\lambda^\star)}p - \frac{\varphi(\lambda^\star)}{1-\varphi(\lambda^\star)}v$$

is the upper limit of the support of the posterior distribution.

With two dealers, it is easy to provide a sufficient and necessary condition for existence. A price p is accepted in the first round if and only if

$$v - p \geq -s + \int_{\underline{c}}^{\bar{c}_p} \left[\int_{\underline{p}_c}^{p} (v-\rho) f_c(\rho)\, d\rho + (v-p)(1-F_c(p)) \right] dG(c \,|\, p),$$

or

$$s \geq \frac{\int_{\underline{c}}^{\bar{c}_p} \int_{\underline{p}_c}^{p} F_c(\rho) d\rho (v-c)(p-c)^{-2}\, dG(c)}{\int_{\underline{c}}^{\bar{c}_p} (v-c)(p-c)^{-2}\, dG(c)}. \tag{A.2}$$

Thus, a reservation-price equilibrium with two dealers exists if and only if inequality (A.2) holds for all $p \in (\underline{p}_c,\, v)$.The condition can be easily verified, as the expression on the right hand side of (A.2) is directly computable.

With more than two dealers, an additional difficulty arises because it is not easy to calculate the continuation value when an offer p is rejected in the first round. We can nevertheless provide a sufficient condition based on the following argument. Suppose that after observing p and forming the posterior belief about c, the slow trader is promised to find, in the next search, an offer equal to the lower limit of the price distribution. This provides an upper bound on the continuation value; thus, if the trader decides not to search in this case, she would also not search under the actual continuation value. Thus, a sufficient condition for existence is that

$$s \geq (p-v) + (1-\varphi(\lambda^\star)) \frac{\int_{\underline{c}}^{\bar{c}_p} (v-c)^2 (p-c)^{-\frac{N}{N-1}}\, dG(c)}{\int_{\underline{c}}^{\bar{c}_p} (v-c)(p-c)^{-\frac{N}{N-1}}\, dG(c)}, \tag{A.3}$$

for all $p \in (\underline{p}_{\underline{c}}, v)$. Again, inequality (A.3) can be directly computed and verified.

The last step in the proof is to show that inequality (A.3) holds for s in some range below X. To this end, we analyze the behavior of the posterior distribution of costs $G(c \mid p)$ after a price p is observed by a slow trader in the first round when probability of entry $\lambda^\star$ is small. As $\lambda^\star \searrow 0$, conditional on p, the upper limit of the support of the posterior cost distribution, $\bar{c}_p$, converges to p. Thus $G(c \mid p)$ converges pointwise to 0 for $c < p$ and to 1 for $c > p$. By one of the (equivalent) definitions of weak* convergence of probability measures, the posterior distribution converges in distribution to an atom at p. Thus, in the limit, inequality (A.3) becomes

$$s \geq (p - v) + (1 - \varphi(0))(v - p) = 0,$$

and is thus vacuously satisfied. By continuity of the right-hand side of inequality (A.3), the inequality holds if $\lambda^\star$ is smaller than some $\underline{\lambda} > 0$. Recall that $\lambda^\star$ is determined uniquely by equation (3.10). Moreover, it is continuous and strictly decreasing in s for $s \in ((1 - \bar{\alpha})X, X)$, and equal to zero at $s = X$. Thus, there exists $\underline{s} < X$ such that for all $s > \underline{s}$, $\lambda^\star$ is smaller than $\underline{\lambda}$.

A.6 Proof of Theorem 1

We first outline the main steps of the argument, and leave the technical details for the two lemmas that follow.

In order to make the proof concise, we make a change of variables by defining $x = (v - c)^+ \equiv \max(v - c, 0)$ as the realized gain from a trade given the common cost c.

Note first that conditions *(i)* and *(ii)* both imply that $s > (1 - \bar{\alpha})X$. Since the case $s \geq X$ is already covered by Proposition 2, in this proof we focus on the range $(1 - \bar{\alpha})X < s < X$, within which Proposition 3 implies interior entry in the absence of the benchmark.

The total expected surplus in the no-benchmark case is

$$W_{nb} \equiv \left[\lambda^\star(1 - p_0) + p_0\right] X - \lambda^\star(1 - p_0)s.$$

With the benchmark, we let $\lambda(x)$ denote the probability of entry by slow traders conditional on a realized gain from trade of x. By Proposition 1,

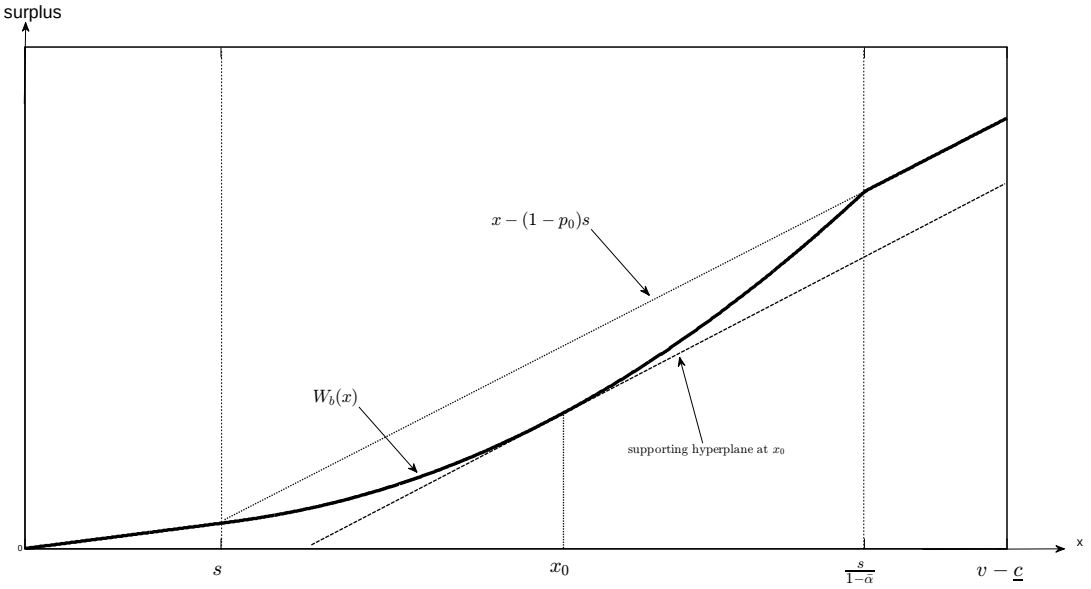
$$\lambda(x) \quad \begin{cases} = 0, & \text{if } x \leq s, \\ \text{solves } s = (1 - \alpha(\lambda(x)))x, & \text{if } s < x < \frac{s}{1-\bar{\alpha}}, \\ = 1, & \text{if } x \geq \frac{s}{1-\bar{\alpha}}. \end{cases}$$

The conditional expected social surplus in the benchmark case conditional on x is

$$W_b(x) \equiv \left[\lambda(x)(1 - p_0) + p_0\right] x - \lambda(x)(1 - p_0)s.$$

The crucial observation, demonstrated in Lemma 4 below, is that W_b is a convex function on $[0,\ s/(1-\bar{\alpha})]$. Figure A.1 depicts a typical shape of that function.

Fig. A.1: Conditional expected social surplus given the realized gain x from each trade.



Under condition *(i)*, W_b is convex on its entire domain. (This corresponds to cutting off the part of the domain that upsets convexity, as shown in Figure A.1.) We can thus apply Jensen's Inequality to obtain

$$\mathbb{E}\left[W_b(x)\right] \geq W_b\left[\mathbb{E}\left(x\right)\right] = W_b\left(\int_{\underline{c}}^{\bar{c}} (v-c)^+ \, dG(c)\right) = W_b(X) = W_{nb}.$$

To justify the last equality, one notes that $\lambda^\star$ is precisely $\lambda(X)$, by equations (3.8) and (3.10). (This inequality is actually strict because G is a non-degenerate distribution and because $\lambda(x) > 0$ with positive probability under G.)

Under condition *(ii)*, W_b may fail to be convex on its entire domain. However, an inspection of the proof of Jensen's Inequality shows that all that is required to achieve the inequality is that the function W_b is subdifferentiable[44] at $\mathbb{E}(x)$. The slope of W_b is increasing on $[0,\ s/(1-\bar{\alpha})]$ and equal to 1 on $(s/(1-\bar{\alpha}),\ v-\underline{c}]$. Thus, a sufficient condition for existence of a supporting hyperplane of W_b at X is that $W_b'(X) \leq 1$. We thus want to solve the equation $W_b'(x_0) = 1$ for $x_0 \in (s,\ s/(1-\bar{\alpha}))$ and impose $X \leq x_0$. (See Figure A.1.) An explicit solution is not available, so instead we show in Lemma 5 below (by approximating the functions α and λ) that this condition is implied by $s \geq (1-\psi)X$.

[44] A function $f : [a,b] \to \mathbb{R}$ is said to be subdifferentiable at x_0 if there exists a real number ξ such that, for all x in $[a,b]$, we have $f(x) - f(x_0) \geq \xi(x - x_0)$. If W_b is convex, then it is subdifferentiable on the interior of its domain, by the Separating Hyperplane Theorem.

Finally, a simple application of the Lebesgue Dominated Convergence Theorem shows that $\bar{\alpha}$ converges (monotonically) to 1 when either $N \to \infty$ or $p_0 \to 0$. Thus, condition *(i)* holds if N is large enough or if p_0 is small enough.

Lemma 4. *$W_b(x)$ and $\lambda(x)$ are convex functions on $[0,\, s/(1-\bar{\alpha})]$.*

Proof. First we prove that $\lambda(x)$ and $W_b(x)$ are convex on $(s,\, s/(1-\bar{\alpha})]$. By the Implicit Function Theorem λ is twice differentiable on this interval and we have

$$\frac{\partial \lambda}{\partial x} = \frac{(1-\alpha(\lambda))}{\alpha'(\lambda)x} > 0,$$

and

$$\frac{\partial^2 \lambda}{\partial x^2} = \frac{-\alpha'(\lambda)(1-\alpha(\lambda)) - (1-\alpha(\lambda))\left[\alpha'(\lambda) + \alpha''(\lambda)\frac{(1-\alpha(\lambda))}{\alpha'(\lambda)}\right]}{\left[\alpha'(\lambda)x\right]^2}.$$

Hence, $\frac{\partial^2 \lambda}{\partial x^2} \geq 0$ for all $x \in (s,\, s/(1-\bar{\alpha}))$ if and only if, for all $\lambda \in (0,\, 1)$,

$$2\left[\alpha'(\lambda)\right]^2 + \alpha''(\lambda)(1-\alpha(\lambda)) \leq 0. \tag{A.4}$$

Taking $\beta = Np_0/(1-p_0)$, we have

$$\alpha'(\lambda) = \int_0^1 \frac{\beta z^{N-1}}{\left(\lambda + \beta z^{N-1}\right)^2}\, dz,$$

$$\alpha''(\lambda) = -2\int_0^1 \frac{\beta z^{N-1}}{\left(\lambda + \beta z^{N-1}\right)^3}\, dz.$$

The inequality (A.4) can therefore be written as

$$\left(\int_0^1 \frac{\beta z^{N-1}}{\left(\lambda + \beta z^{N-1}\right)^2}\, dz\right)^2 \leq \left(\int_0^1 \frac{\beta z^{N-1}}{\left(\lambda + \beta z^{N-1}\right)^3}\, dz\right)\left(\int_0^1 \frac{\beta z^{N-1}}{\lambda + \beta z^{N-1}}\, dz\right).$$

Hölder's Inequality states that for all measurable and square-integrable functions f and g,

$$\|fg\|_1 \leq \|f\|_2 \|g\|_2.$$

By letting

$$f(z) = \sqrt{\frac{\beta z^{N-1}}{\left(\lambda + \beta z^{N-1}\right)^3}} \quad \text{and} \quad g(z) = \sqrt{\frac{\beta z^{N-1}}{\lambda + \beta z^{N-1}}},$$

we have proven inequality (A.4) and thus the convexity of $\lambda(x)$.

Now it becomes a straightforward to check that $W_b(x)$ is convex on $[s,\, s/(1-\bar{\alpha})]$. Notice that $W_b(x)$ and $\lambda(x)$ are trivially convex on $[0,\, s]$ (because, on this interval, $\lambda(x)$ is identically zero and $W_b(x)$ is affine). Therefore, to finish the proof it is enough to make sure that $\lambda(x)$ and $W_b(x)$ are differentiable at s. We can verify this by computing

the left and right derivatives:

$$\partial_- W_b([s]) = p_0 = \partial_+ W_b([s]),$$

$$\partial_- \lambda([s]) = 0 = \partial_+ \lambda([s]).$$

□

Lemma 5. *If* $x \le \frac{s}{1-\psi}$, *where* $\psi = \frac{1}{2}\left[\sqrt{(1-\bar\alpha+\bar\alpha\beta)^2 + 4\bar\alpha(1-\bar\alpha)} - (1-\bar\alpha+\bar\alpha\beta)\right]$ *and* $\beta = \frac{Np_0}{1-p_0}$, *then* $W_b'(x) \le 1$.

Proof. The claim is true for $x \le s$, and since $\psi \le \bar\alpha$, we can focus on the region where $\lambda(x)$ is defined as the solution to the equation (3.8) which can be written as

$$\alpha(\lambda(x)) = 1 - \frac{s}{x}.$$

Since $\alpha(\,\cdot\,)$ is a strictly increasing function, if we replace $\alpha(\,\cdot\,)$ in the above equation by a lower bound, any solution of the new equation will be an upper bound on $\lambda(x)$. Because $W_b(x)$ is convex in the relevant part of the domain (by Lemma 4), to make sure that $W_b'(x) \le 1$, it's enough to require that $x \le x_0$, where x_0 solves $W_b'(x_0) = 1$ (such x_0 exists and is unique). We have

$$W_b'(x_0) = p_0 + \lambda'(x_0)(1-p_0)(x_0 - s) + \lambda(x_0)(1-p_0) = 1. \tag{A.5}$$

We cannot solve this equation explicitly, so we will provide a lower bound on the solution. Because $W_b'(x)$ is increasing, we need to bound $W_b'(x)$ from above. Since $\alpha(\lambda) \ge \lambda\bar\alpha$, by the above remark, the solution of the equation

$$\bar\alpha\bar\lambda(x) = 1 - \frac{s}{x}$$

provides an upper bound on $\lambda(x)$. That is,

$$\lambda(x) \le \bar\lambda(x) = \frac{1}{\bar\alpha} - \frac{s}{\bar\alpha}\frac{1}{x}.$$

Moreover,

$$\lambda'(x) = \frac{1}{\alpha'(\lambda(x))}\frac{s}{x^2},$$

and we have, for all $\lambda \in [0,\,1]$,

$$\alpha'(\lambda) = \int_0^1 \frac{\beta z^{N-1}}{(\lambda+\beta z^{N-1})^2}\,dz \ge \frac{1}{\lambda+\beta}\int_0^1 \left(\frac{\lambda+\beta z^{N-1}}{\lambda+\beta z^{N-1}} - \frac{\lambda}{\lambda+\beta z^{N-1}}\right)dz$$

$$= \frac{1}{\lambda+\beta}(1-\alpha(\lambda)) \ge \frac{1-\bar\alpha}{\lambda+\beta}.$$

Plugging all of these estimates into equation (A.5) and rearranging, we obtain

$$\frac{\beta + \frac{1}{\bar{\alpha}} - \frac{s}{\bar{\alpha}}\frac{1}{x_0}}{1-\bar{\alpha}}\frac{s}{x_0}(1-\frac{s}{x_0}) + \frac{1}{\bar{\alpha}}\left[1-\frac{s}{x_0}\right] = 1.$$

Denoting $y = 1 - s/x_0$, estimating the left hand side from above one more time, and rearranging, we get

$$y^2 + (1 - \bar{\alpha} + \bar{\alpha}\beta)y - \bar{\alpha}(1-\bar{\alpha}) = 0.$$

The relevant solution is ψ. □

A.7 Proof of Proposition 5

This result follows directly from Propositions 1 and 3.

A.8 A numerical example for Section 3.3

To illustrate the intuition of Theorem 1 and Proposition 5, Figure A.2 shows how the surplus and the entry probability of slow traders depend on the search cost s in the benchmark case (shown with a thick solid line) and in the no-benchmark case (shown with a thick dotted line). Here, we take the distribution G of dealer costs to be uniform on $[0, 1]$, a trader asset valuation of $v = 1$, a fraction $p_0 = 0.25$ of slow traders, and $N = 2$ dealers. The reservation-price equilibrium in the no-benchmark case exists if $s \geq 0.063$.

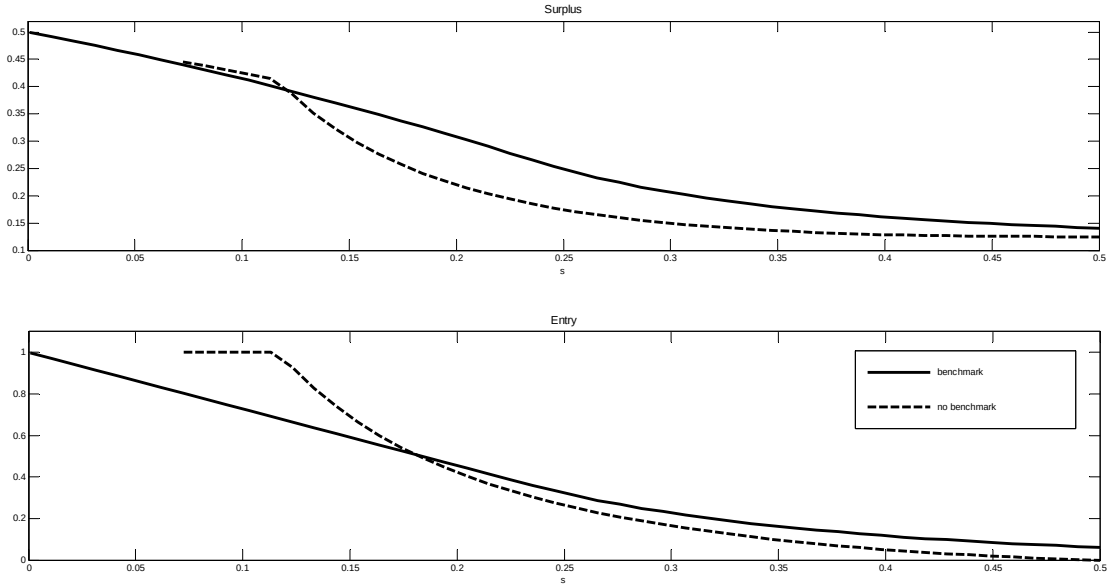
The benchmark case dominates the no-benchmark case in terms of social surplus for all $s \geq 0.12$. We note that the unconditional probability of entry is larger without the benchmark when $s \leq 0.18$. In the range $s \in (0.12, 0.18)$, the benchmark nevertheless raises social surplus because it encourages more entry when gains from trade are large. In the range $s \in (0.063, 0.12)$ the pooling effect of not observing the benchmark cost c overcomes the other benefits of the benchmark, resulting in a higher social surplus without the benchmark.

A.9 Proof of Theorem 2

Using the same arguments used in the derivation of equilibrium from Proposition 1 we can show that in the costly-benchmark case there exists a reservation-price equilibrium, and that equilibrium payoffs are unique. Fixing the probability of entry at λ (and noting that it is independent of c), we compute the reservation price

$$r_c^{cb} = \min\left\{v,\, c + \frac{1}{1-\alpha(\lambda)}s\right\}.$$

Fig. A.2: The dependence of expected social surplus and entry on the slow-trader search cost s.



A slow trader buys from the first contacted dealer if $c \leq v$. The profit of a slow trader who enters, conditional on c, can be shown to be

$$\pi_c^{cb}(\lambda) = \max\left\{v - \frac{1}{1-\alpha(\lambda)}s - c,\ -s + (1-\alpha(\lambda))(v-c)\right\}$$

if $c \leq v$, and $-s$ if $c > v$. When $s \geq X$, there can be no entry in equilibrium. If the equilibrium probability of entry λ^{cb} is interior, then it must be determined by the indifference condition, analogous to (3.8) and (3.10), given by

$$\mathbb{E}\pi_c^{cb}(\lambda^{cb}) = 0. \tag{A.6}$$

The solution to that equation exists and is unique if $X \geq s \geq (1-\bar{\alpha})X + \phi$, where

$$\phi = \frac{\bar{\alpha}}{1-\bar{\alpha}} \int_{\underline{c}}^{v-\frac{s}{1-\bar{\alpha}}} \left[(1-\bar{\alpha})(v-c) - s\right] dG(c) \geq 0.$$

When $s < (1-\bar{\alpha})X + \phi$, we must have entry with probability one.

To show that surplus is higher in the costly-benchmark case than in the no-benchmark case, it is enough to show that entry is higher. Because the function max is convex, we can apply Jensen's Inequality to conclude that, for all λ,

$$\mathbb{E}\pi_c^{cb}(\lambda) \geq -s + (1-\alpha(\lambda))X = \pi^{nb}(\lambda),$$

that is, the expected profit is always higher in the costly-benchmark setting (and is strictly higher provided that $(1-\bar{\alpha})X < s < X$). It follows that equilibrium entry of slow traders must also be higher (from equations (3.10) and (A.6)).

A.10 Proof of Proposition 6

By Theorem 1 we know that when s is higher than $(1-\psi)X$, surplus under the benchmark is higher than in the reservation-price equilibrium of the no-benchmark case.[45] It is easy to observe that the difference in surpluses is bounded away from zero as a function of s (under the assumption that $v - \underline{c} > s > (1-\psi)X$). Given Theorem 2, it suffices to show that the surplus of the costly-benchmark case converges to the surplus of the no-benchmark case as s goes to X (when $s \geq X$, they coincide). It is enough to prove that λ^{cb}, the solution of equation (A.6), converges to $\lambda^{\star}$, the solution of equation (3.10), as $s \to X$. Because the solution of equation (A.6) is continuous in s and equal to 0 at $s = X$, λ^{cb} converges to 0, and so does $\lambda^{\star}$.

B Proofs and Supporting Contents for Section 4

This appendix provides proofs for results in Section 4, and additional results cited in Section 4.

B.1 Proof of Proposition 7 and equilibrium characterization in the benchmark case

Because the distribution of costs is i.i.d. across dealers conditional on observing the benchmark, slow traders must follow a reservation-price strategy with some reservation price r_c. A stationary[46] reservation-price strategy of slow traders will now be characterized by three numbers: λ_c, the probability of entry; r_c, the reservation price; and $\hat{\theta}_c$, the probability of rejecting an offer equal to the reservation price r_c. Fixing the strategy of the dealers and the reservation price r_c, the rejection probability $\hat{\theta}_c$ determines the probability θ_c that a slow trader rejects an offer from a high-cost dealer, and vice versa. Given the one-to-one correspondence between θ_c and $\hat{\theta}_c$, for convenience we will abuse the notation for the strategy of a slow trader, denoting it by the triple $(r_c, \lambda_c, \theta_c)$. Again without loss of generality, we can assume that fast traders play their

[45] Even if the latter equilibrium does not exist, the comparison between surpluses is valid, and that is all we need for the proof.

[46] Requiring stationarity, that is, the same mixing probability at every search round, simplifies the exposition and is without loss of generality. Without stationarity, there is an indeterminacy in specifying the probability of rejecting the reservation price in equilibrium. Traders can use different mixing probabilities in every search round, as long as they lead to the same posterior beliefs of dealers. This indeterminacy does not change expected equilibrium payoffs, so without loss of generality we get rid of it by requiring stationarity.

weakly dominant strategy of always entering and visiting all dealers. We also ignore the issue of off-equilibrium beliefs, as it is fairly trivial to deal with.

Fixing c and a strategy $(r_c, \lambda_c, \theta_c)$ we will characterize the equilibrium best-response of dealers. We start with two technical lemmas.

Lemma 6. *In equilibrium, conditional on c (for $c < v$), if dealers of a certain type (high-cost or low-cost) make positive expected profits, then the probability distribution of price offers for that type is atomless. If high-cost dealers make zero expected profits, then in equilibrium they must quote a price equal to their cost, provided that $c + \Delta < v$.*

Proof. The first part of the Lemma can be proven using the argument from the proof of Lemma 2. To prove the second part, suppose that, for some $c < v - \Delta$, a price above $c + \Delta$ is in the support of the equilibrium strategy of high-cost dealers. The probability of selling at that price (or some lower price above $c + \Delta$) must be positive since with probability $(1 - \gamma)^N$ only high-cost dealers are present in the market. Thus, we get a contradiction with the assumption that high-cost dealers make zero expected profits. □

Lemma 7. *In equilibrium, conditional on c (and on the event $c < v$), for any equilibrium price p_l of a low-cost dealer, and any equilibrium price p_h of a high-cost dealer, we have $p_l \leq p_h$.*

Proof. The claim is true by a standard "revealed-preference" argument. Suppose that $p_l > p_h$. Fix an equilibrium, and let $\varrho(p)$ (for some fixed $c \leq v$) be the probability that a dealer sells the asset when posting the price p. Since dealers are optimizing in equilibrium, we must have

$$\varrho(p_l)(p_l - c) \geq \varrho(p_h)(p_h - c), \tag{B.1}$$

$$\varrho(p_h)(p_h - c - \Delta) \geq \varrho(p_l)(p_l - c - \Delta). \tag{B.2}$$

We have, if $\varrho(p_h) \neq 0$,

$$\varrho(p_h)(p_h - c - \Delta) < \varrho(p_h)(p_l - c - \Delta).$$

If $p_l > c + \Delta$, then $\varrho(p_h) > \varrho(p_l)$. From inequality (B.1),

$$\varrho(p_l)(p_l - c) + \Delta(\varrho(p_h) - \varrho(p_l)) > \varrho(p_h)(p_h - c)$$

or

$$\varrho(p_l)(p_l - c - \Delta) > \varrho(p_h)(p_h - c - \Delta)$$

which contradicts inequality (B.2).

We are left with two cases. First, suppose that $p_l \leq c + \Delta$. Then $p_h < c + \Delta$ which is impossible in equilibrium. Second, suppose that $\varrho(p_h) = 0$. Then it must be the case that $\varrho(p_l) = 0$ as well, which is a contradiction if $c < v$. □

The Lemma simply says that in equilibrium high-cost dealers always post higher prices than low-cost dealers.

Finally, we prove a lemma about the possibility of gaps in the distribution of prices. Let $\underline{p}_c^i$ and $\bar{p}_c^i$ denote the lower and upper limit of the support of the distribution of prices for dealer of type $i \in \{l, h\}$.

Lemma 8. *In equilibrium, conditional on c (for $c < v$), there can be no gaps in the distribution of prices except for the case in which the support of the distribution of prices of low-cost dealers consists of two intervals, $[\underline{p}_c^l, r_c]$ and $[\hat{p}_c^l, \min\{c+\Delta, v\}]$, and in which either (i) high-cost dealers post $c+\Delta$, or (ii) $c > v - \Delta$.*

Proof. Suppose that there is a gap in the distribution of prices conditional on some cost level c for some type of dealers, that is, an interval $(p_1, p_2) \subset [\underline{p}_c^i, \bar{p}_c^i] \setminus \operatorname{supp}(F_c^i(\cdot))$, $i \in \{l, h\}$. We take this interval to be maximal, that is, such that p_1 and p_2 are in the support of $F_c^i(\cdot)$. It must be the case that probability of selling is strictly larger at p_1 than at p_2, and thus, in a reservation-price equilibrium, $p_1 \leq r_c \leq p_2$ (we made use here of Lemma 7). It cannot be that $p_1 < r_c$ because then the dealer posting p_1 could profitably deviate to r_c. Thus $p_1 = r_c$.

By Lemma 7, $\bar{p}_c^h$ is the highest price that can be observed on equilibrium path, and it lies above r_c. It follows, using Lemma 6, that high-cost dealers make zero expected profits (if the price distribution for high-cost dealers were atomless, the probability of selling at the price $\bar{p}_c^h > r_c$ would be zero). Moreover, either (i) high-cost dealers post $c+\Delta$, or (ii) $c > v - \Delta$. In either case we can conclude that $i = l$, i.e. the gap occurs in the price distribution of low-cost dealers.

By the above, if there is a gap, then the support of the distribution for low-cost dealers consists of two intervals, the first of which must be $[\underline{p}_c^l, r_c]$. To prove that $\bar{p}_c^l = \min\{c+\Delta, v\}$, we use the fact that $\bar{p}_c^l > r_c$, and thus if $\bar{p}_c^l < \min\{c+\Delta, v\}$, the dealer quoting $\bar{p}_c^l$ would want to deviate to $\min\{c+\Delta, v\}$. □

Using the above observations, we can now show, case by case, that the equilibrium pricing strategies are uniquely pinned down when there are gains from trade. (We assume throughout that $c < v$; the opposite case is trivial.) We let $F_c^l(p)$ denote the cdf of prices for low-cost dealers, and $F_c^h(p)$ the cdf of prices for high-cost dealers. In most cases it is a routine exercise to rule out the possibility of a gap in the distribution, using Lemma 8. We will therefore only comment on this possibility explicitly in the two cases when a gap actually occurs in equilibrium.

Case 1: $\lambda_c = 0$. When $\lambda_c = 0$, only fast traders enter. In this case, we have a standard first-price auction between dealers. There are two subcases.

When $c > v - \Delta$, high-cost dealers cannot sell in equilibrium, and the specification of their strategy is irrelevant (they can choose any price above $c + \Delta$). In this case low-cost dealers randomize according to a distribution $F_c^l(p)$ that solves the equation

$$\left[\sum_{k=0}^{N-1}\binom{N-1}{k}\gamma^k(1-\gamma)^{N-1-k}\left(1-F_c^l(p)\right)^k\right](p-c)=(1-\gamma)^{N-1}(v-c).$$

Let us define the function

$$\Phi(z)=\frac{1}{1-(1-\gamma)^{N-1}}\sum_{k=1}^{N-1}\binom{N-1}{k}z^k\gamma^k(1-\gamma)^{N-1-k}, \tag{B.3}$$

which can be viewed as a generalization of the function z^{N-1} that appears in the definition (3.6). It is easy to see that $\Phi(z)$ is a (strictly) increasing polynomial with $\Phi(0)=0$, $\Phi(1)=1$, and $\Phi(z)=z^{N-1}$ when $\gamma=1$. Moreover, using the binomial identity, we can write $\Phi(z)$ alternatively as

$$\Phi(z)=\frac{(z\gamma+1-\gamma)^{N-1}-(1-\gamma)^{N-1}}{1-(1-\gamma)^{N-1}}. \tag{B.4}$$

Using definition (B.3), we can write

$$F_c^l(p)=1-\Phi^{-1}\left(\frac{(1-\gamma)^{N-1}}{1-(1-\gamma)^{N-1}}\frac{v-p}{p-c}\right)$$

with upper limit $\bar{p}_c^l=v$, and lower limit

$$\underline{p}_c^l=(1-\gamma)^{N-1}v+\left(1-(1-\gamma)^{N-1}\right)c.$$

When $c\leq v-\Delta$, high-cost dealers can sell in equilibrium, but a standard result from auction theory (see for example Fudenberg and Tirole, 1991) says that in the unique equilibrium they will make zero profit by bidding $c+\Delta$. In this case, the distribution $F_c^l(p)$ solves

$$\left[\sum_{k=0}^{N-1}\binom{N-1}{k}\left(1-F_l(p|\,c)\right)^k\gamma^k(1-\gamma)^{N-1-k}\right](p-c)=(1-\gamma)^{N-1}\Delta,$$

and thus we get

$$F_c^l(p)=1-\Phi^{-1}\left(\frac{(1-\gamma)^{N-1}}{1-(1-\gamma)^{N-1}}\frac{(c+\Delta)-p}{p-c}\right)$$

with upper limit $\bar{p}_c^l=c+\Delta$, and lower limit

$$\underline{p}_c^l=(1-\gamma)^{N-1}(c+\Delta)+\left(1-(1-\gamma)^{N-1}\right)c=c+(1-\gamma)^{N-1}\Delta.$$

Case 2: $\lambda_c > 0$. From now on, we assume $\lambda_c > 0$, that is, slow traders enter with positive probability. There are again two subcases.

When $c > v - \Delta$ (case 2.1), high-cost dealers cannot sell in equilibrium, and the specification of their strategy is irrelevant. Low-cost dealers mix according to a continuous distribution $F_c^l(p)$ on an interval with upper limit $\bar{p}_l^c = r_c$, or on a union of two intervals as in Lemma 8.

When $c \leq v - \Delta$ (case 2.2), using Lemmas 6, 7, 8, and the argument from the proof of Lemma 2, we can show that only two subcases are possible:

- If $r_c \leq c + \Delta$, (case 2.2.1), high-cost dealers make zero profit; they post a price $c + \Delta$ with probability 1, while low-cost dealers mix according to a continuous distribution on an interval with upper limit $\bar{p}_c^l = r_c$, or on a union of two intervals as in Lemma 8.

- If $r_c > c + \Delta$ (case 2.2.2), high-cost dealers make positive profits, and in equilibrium both low-cost and high-cost dealers mix according to continuous distributions with adjacent supports ($\bar{p}_c^l = \underline{p}_c^h$), and with r_c being the upper limit of the distribution of prices of high-cost dealers ($\bar{p}_c^h = r_c$).

Below we analyze these cases in detail and characterize the optimal search behavior of slow traders. We first define some key functions that generalize their equivalents from Section 3 to the case of idiosyncratic component in the costs. Let $q(\lambda_c, \theta_c)$ be the posterior probability that a customer is a fast trader, conditional on a visit, given the strategy $(r_c, \lambda_c, \theta_c)$. That is,

$$q(\lambda_c, \theta_c) = \frac{Np_0}{Np_0 + \frac{1-\theta_c^N(1-\gamma)^N}{1-\theta_c(1-\gamma)}\lambda_c(1-p_0)}. \tag{B.5}$$

This definition generalizes formula (3.1). We also generalize the definition of the function α from equation (3.6), which now becomes a function of two arguments:

$$\alpha(\lambda_c, \theta_c) = \int_0^1 \left(1 + \frac{q(\lambda_c, \theta_c)\left(1-(1-\gamma)^{N-1}\right)}{1-q(\lambda_c, \theta_c)\left(1-(1-\gamma)^{N-1}\right)}\Phi(z)\right)^{-1} dz, \tag{B.6}$$

where $\Phi(z)$ is defined in formula (B.3). Finally, we let

$$\hat{\alpha} = \alpha(1, 1),$$

which corresponds to formula (3.7).

To emphasize the point that we will now deal with equilibrium rather than just best response of dealers to some generic strategy of traders, we add star superscripts to symbols denoting the strategy of traders.

Case 2.1: $\lambda_c^\star > 0$, $c > v - \Delta$. In this case, we clearly have $\theta_c^\star = 1$. We first suppose that the support of the distribution for low-cost dealers is an interval. Then $F_c^l(p)$ must

satisfy

$$\left[1 - q(\lambda_c^\star,\, 1) + q(\lambda_c^\star,\, 1) \sum_{k=0}^{N-1} \binom{N-1}{k} \left(1 - F_c^l(p)\right)^k \gamma^k (1-\gamma)^{N-1-k}\right](p - c)$$

$$= \left[1 - q(\lambda_c^\star,\, 1) + q(\lambda_c^\star,\, 1)(1-\gamma)^{N-1}\right](r_c^\star - c).$$

Solving for $F_c^l(p)$, we obtain

$$F_c^l(p) = 1 - \Phi^{-1}\left(\frac{1 - q(\lambda_c^\star,\, 1)\left(1 - (1-\gamma)^{N-1}\right)}{q(\lambda_c^\star,\, 1)\left(1 - (1-\gamma)^{N-1}\right)} \frac{r_c^\star - p}{p - c}\right)$$

with $\bar{p}_c^l = r_c^\star$, and lower limit

$$\underline{p}_c^l = \left[1 - q(\lambda_c^\star,\, 1)\left(1 - (1-\gamma)^{N-1}\right)\right] r_c^\star + \left[q(\lambda_c^\star,\, 1)\left(1 - (1-\gamma)^{N-1}\right)\right] c.$$

We can determine $r_c^\star$ in this case from the fact that it must solve the following equation (specifying that the trader must be indifferent at $r_c^\star$ between buying and searching), analogous to equation (3.4),

$$v - r_c^\star = -s + \gamma\left[v - \int_{\underline{p}_c^l}^{r_c^\star} p dF_c^l(p)\right] + (1-\gamma)(v - r_c^\star).$$

Using a direct generalization of Lemma 3, we can transform this equation into the form

$$s = \gamma\left[r_c^\star - \int_{\underline{p}_c^l}^{r_c^\star} p\, dF_c^l(p)\right] = (1 - \alpha(\lambda_c^\star,\, 1))\gamma(r_c^\star - c).$$

Thus we have

$$r_c^\star = c + \frac{s}{(1 - \alpha(\lambda_c^\star,\, 1))\gamma}.$$

The last thing to determine is the probability $\lambda_c^\star$ of entry by slow traders. The profit of a slow trader conditional on entry is equal to

$$\pi_c = \left(1 - (1-\gamma)^N\right)(v - \alpha(\lambda_c^\star,\, 1) r_c^\star - (1 - \alpha(\lambda_c^\star,\, 1))c) - \left(\sum_{k=1}^{N} (1-\gamma)^{k-1}\gamma k + (1-\gamma)^N N\right) s$$

$$= \left(1 - (1-\gamma)^N\right)\left[v - c - \frac{s}{(1 - \alpha(\lambda_c^\star,\, 1))\gamma}\right].$$

When profit is strictly positive, we must have entry with probability one. That is, we have $\lambda_c^\star = 1$ if

$$c \leq v - \frac{s}{(1 - \alpha(1,\, 1))\gamma}.$$

When profit is strictly negative, we must have entry with probability zero, meaning that $\lambda_c^\star = 0$ if

$$c \geq v - \frac{s}{(1-\alpha(0,\,1))\gamma}.$$

This takes us back to case 1 analyzed before. Finally, if

$$v - \frac{s}{(1-\alpha(1,\,1))\gamma} < c < v - \frac{s}{(1-\alpha(0,\,1))\gamma},$$

then we must have interior entry $\lambda_c^\star \in (0,\,1)$, where $\lambda_c^\star$ is the unique solution of the equation

$$s = (1-\alpha(\lambda_c^\star,\,1))\gamma(v-c).$$

In this case, slow traders have zero profits and we have $r_c^\star = v$.

To check whether the above strategies constitute an equilibrium, we need to verify that the support of price offers by low-cost dealers is indeed an interval, that is, these dealers cannot profitably deviate from posting prices in the range $[\underline{p}_c^l,\, r_c^\star]$. The only deviation that we need to check is bidding v in the case $r_c^\star < v$.[47] This leads to the condition

$$\left[p_0(1-\gamma)^{N-1} + (1-p_0)\frac{1-(1-\gamma)^N}{N\gamma}\right]\frac{s}{(1-\alpha(1,\,1))\gamma} \geq (1-\gamma)^{N-1}(v-c),$$

where the left hand side is the expected profit from bidding $r_c^\star$, and the right hand side is the expected profit from bidding v (a dealer quoting v can only sell if all other dealers have high costs). We define

$$\kappa = \frac{(1-\gamma)^{N-1}}{p_0(1-\gamma)^{N-1} + (1-p_0)\frac{1-(1-\gamma)^N}{N\gamma}}. \tag{B.7}$$

Thus, we have an equilibrium when

$$c \geq v - \frac{s}{\kappa(1-\alpha(1,\,1))\gamma}.$$

Note that $\kappa < 1$, and thus

$$v - \frac{s}{\kappa(1-\alpha(1,\,1))\gamma} < v - \frac{s}{(1-\alpha(1,\,1))\gamma}.$$

When $c < v - \frac{s}{\kappa(1-\alpha(1,\,1))\gamma}$, by Lemma 8, we must have an equilibrium in which the support for low-cost dealers consists of two intervals: $[\underline{p}_c^l,\, r_c^\star]$ and $[\hat{p}_c^l,\, v]$. Let ζ_c be the conditional probability that a low-cost dealer posts a price in the lower interval. Then, in particular, the dealer must be indifferent between $r_c^\star$ and v which pins down ζ_c, in

[47] If there is a profitable deviation, this one is the most profitable.

that

$$\left[p_0(1-\gamma\zeta_c)^{N-1} + (1-p_0)\frac{1-(1-\gamma\zeta_c)^N}{N\gamma\zeta_c}\right](r_c^\star - c) = (1-\gamma)^{N-1}(v-c). \tag{B.8}$$

We define

$$\vartheta(\zeta_c) = \frac{(1-\gamma)^{N-1}}{p_0(1-\gamma\zeta_c)^{N-1} + (1-p_0)\frac{1-(1-\gamma\zeta_c)^N}{N\gamma\zeta_c}}. \tag{B.9}$$

Note that $\vartheta(1) = \kappa$. Then, equation (B.8) becomes

$$r_c^\star = (1-\vartheta(\zeta_c))c + \vartheta(\zeta_c)v. \tag{B.10}$$

We can now determine the exact distribution of prices. In the upper interval we must have

$$\left[\sum_{k=0}^{N-1}\binom{N-1}{k}\gamma^k(1-\gamma)^{N-1-k}\left(1-F_c^l(p)\right)^k\right](p-c) = (1-\gamma)^{N-1}(v-c),$$

so we get

$$F_c^l(p) = 1 - \Phi^{-1}\left(\frac{(1-\gamma)^{N-1}}{1-(1-\gamma)^{N-1}}\frac{v-p}{p-c}\right).$$

In the lower interval, the distribution must satisfy

$$\left[p_0\sum_{k=0}^{N-1}\binom{N-1}{k}(\gamma\zeta_c)^k(1-\gamma\zeta_c)^{N-1-k}\left(1-\frac{F_c^l(p)}{\zeta_c}\right)^k + \frac{1-p_0}{N}\frac{1-(1-\gamma\zeta_c)^N}{\gamma\zeta_c}\right](p-c)$$

$$= \left[p_0(1-\gamma\zeta_c)^{N-1} + \frac{1-p_0}{N}\frac{1-(1-\gamma\zeta_c)^N}{\gamma\zeta_c}\right](r_c^\star - c),$$

which gives

$$F_c^l(p) = \zeta_c - \zeta_c\Phi^{-1}\left(\frac{(1-\gamma)^{N-1}}{1-(1-\gamma\zeta_c)^{N-1}}\frac{1}{p_0\vartheta(\zeta_c)}\frac{r_c^\star - p}{p-c};\ \zeta_c\right),$$

where

$$\Phi(z;\ \zeta_c) = \frac{1}{1-(1-\gamma\zeta_c)^{N-1}}\sum_{k=1}^{N-1}\binom{N-1}{k}z^k(\gamma\zeta_c)^k(1-\gamma\zeta_c)^{N-1-k}.$$

That is, $\Phi(z;\ \zeta_c)$ is the analogue to $\Phi(z)$ when replacing γ with $\gamma\zeta_c$.

Finally, the reservation price is determined by

$$v - r_c^\star = -s + \gamma\zeta_c\left[v - \int_{\underline{p}_c^l}^{r_c^\star} p\ d\left(\frac{F_c^l(p)}{\zeta_c}\right)\right] + (1-\gamma\zeta_c)(v - r_c^\star). \tag{B.11}$$

Using the substitution from the proof of Lemma 3, this time substituting $z = (\zeta_c - F_c^l(p))/\zeta_c$, we obtain

$$\int_{\underline{p}_c^l}^{r_c^\star} p \; d\left(\frac{F_c^l(p)}{\zeta_c}\right) = c + (r_c^\star - c)\tilde{\alpha}(\zeta_c),$$

where

$$\tilde{\alpha}(\zeta_c) = \int_0^1 \left(1 + \frac{1-(1-\gamma\zeta_c)^{N-1}}{(1-\gamma)^{N-1}} p_0 \vartheta(\zeta_c)\Phi(z;\,\zeta_c)\right)^{-1} dz.$$

Note that $\tilde{\alpha}(1) = \alpha(1,\,1)$. From this we can calculate the optimal reservation price, determined by equation (B.11), as

$$r_c^\star = c + \frac{s}{(1-\tilde{\alpha}(\zeta_c))\gamma\zeta_c}. \tag{B.12}$$

Equations (B.10) and (B.12) together pin down $r_c^\star$ and ζ_c. Combining them, we get a single equation that pins down ζ_c, in the form

$$s = \vartheta(\zeta_c)(1-\tilde{\alpha}(\zeta_c))\gamma\zeta_c(v-c).$$

A unique solution $\zeta_c^\star \in (0,\,1)$ exists if and only if $0 < s < \kappa(1-\alpha(1,\,1))\gamma(v-c)$ which is precisely our assumption for that case.

Note that in this range the equilibrium level $\zeta_c^\star$ will be close to 1 when s is close to $\kappa(1-\alpha(1,\,1))\gamma(v-c)$ and will converge to 0 as s goes to 0.

Case 2.2.1: $c \le v - \Delta$, $r_c^\star \le c + \Delta$. In this case, high-cost dealers offer the price $c + \Delta$. We have two cases to consider, and call them (a) and (b).

Case (a). When $r_c^\star < c + \Delta$, we must have $\theta_c^\star = 1$. Suppose that low-cost dealers mix on an interval. Then the distribution of prices is

$$F_c^l(p) = 1 - \Phi^{-1}\left(\frac{1-q(\lambda_c^\star,\,1)\left(1-(1-\gamma)^{N-1}\right)}{q(\lambda_c^\star,\,1)\left(1-(1-\gamma)^{N-1}\right)}\frac{r_c^\star - p}{p-c}\right),$$

just as in the previous case. What differs from the previous case is the profit of a slow trader conditional on entry. In the event that there are no low-cost dealers in the market, a trader buys from a high-cost dealer instead of exiting. Accordingly, the profit now becomes

$$\pi_c = v - c - (1-\gamma)^N\Delta - \left(1-(1-\gamma)^N\right)\frac{s}{(1-\alpha(\lambda_c^\star,\,1))\gamma}.$$

We can have strictly positive entry by slow traders only if

$$v \ge c + (1-\gamma)^N\left[\Delta - \frac{s}{(1-\alpha(\lambda_c^\star,\,1))\gamma}\right] + \frac{s}{(1-\alpha(\lambda_c^\star,\,1))\gamma}. \tag{B.13}$$

Recall that we have

$$r_c^\star = c + \frac{s}{(1-\alpha(\lambda_c^\star,\,1))\gamma}.$$

Thus, given that we assumed $r_c^\star < c+\Delta$, we have an equilibrium with positive entry if inequality (B.13) holds and

$$\Delta > \frac{s}{(1-\alpha(\lambda_c^\star,\,1))\gamma}.$$

Notice that we have

$$v - c - (1-\gamma)^N\Delta - \left(1-(1-\gamma)^N\right)\frac{s}{(1-\alpha(\lambda_c^\star,\,1))\gamma}$$

$$> v - c - (1-\gamma)^N\Delta - \left(1-(1-\gamma)^N\right)\Delta = v - c - \Delta \geq 0,$$

which means that profits are always strictly positive in this case. Thus we must have full entry, meaning $\lambda_c^\star = 1$, and this can be an equilibrium only if $s < (1-\alpha(1,\,1))\gamma\Delta$.

Finally, we verify the supposition that low-cost dealers mix on an interval. We need to check the deviation to (just below) $c+\Delta$, analogous to deviation to v in the previous case. We require

$$\left[p_0(1-\gamma)^{N-1} + (1-p_0)\frac{1-(1-\gamma)^N}{N\gamma}\right]\frac{s}{(1-\alpha(1,\,1))\gamma} \geq (1-\gamma)^{N-1}\Delta.$$

Thus, the above strategies are an equilibrium if

$$s \geq \kappa(1-\alpha(1,\,1))\gamma\Delta.$$

In the case $s < \kappa(1-\alpha(1,\,1))\gamma\Delta$, we will have an equilibrium with low-cost dealers mixing on two intervals $[\underline{p}_c^l,\, r_c^\star]$ and $[\hat{p}_c^l,\, c+\Delta]$. The analysis is analogous to the one in the previous case 2.1 so we skip some details. First, the indifference condition between $r_c^\star$ and $c+\Delta$[48] is

$$(r_c^\star - c) = \vartheta(\zeta_c)\Delta. \tag{B.14}$$

The upper part of the distribution is given by

$$F_c^l(p) = 1 - \Phi^{-1}\left(\frac{(1-\gamma)^{N-1}}{1-(1-\gamma)^{N-1}}\frac{c+\Delta-p}{p-c}\right),$$

while the lower part is

$$F_c^l(p) = \zeta_c - \zeta_c\Phi^{-1}\left(\frac{(1-\gamma)^{N-1}}{1-(1-\gamma\zeta_c)^{N-1}}\frac{1}{p_0\vartheta(\zeta_c)}\frac{r_c^\star-p}{p-c};\,\zeta_c\right).$$

[48]Note that $c+\Delta$ is the upper limit of the support but prices posted by a low-cost dealer are below $c+\Delta$ with probability one. Thus, when we say that the dealer must be indifferent between posting $r_c^\star$ and $c+\Delta$, we really mean $c+\Delta-\epsilon$ for arbitrarily small $\epsilon \to 0$ which leads to the formula below.

The reservation price is determined by equation (B.11). Simplifying as before, we obtain

$$r_c^\star = c + \frac{s}{(1-\tilde{\alpha}(\zeta_c))\gamma\zeta_c}.$$

Combining with equation (B.14) ζ_c is pinned down by the equation

$$s = \vartheta(\zeta_c)(1-\tilde{\alpha}(\zeta_c))\gamma\zeta_c\Delta.$$

The equation does not depend on c, so neither does the solution. That is, $\zeta_c^\star$ is independent of c and solves

$$s = \vartheta(\zeta)(1-\tilde{\alpha}(\zeta))\gamma\zeta\Delta.$$

This equation has a unique solution in $(0,\,1)$ precisely when $0 < s < \kappa(1-\alpha(1,\,1))\gamma\Delta$, which was our assumption for this case.

Case (b). We now look at the second possibility: $r_c^\star = c + \Delta$. We can now have $\theta_c^\star \in (0,\,1)$, and this will matter for equilibrium pricing through the impact on the posterior beliefs of dealers. The probability $F_c^l(p)$ of an offer of p or less by a low-cost dealer solves

$$\left[1 - q(\lambda_c^\star,\,\theta_c^\star) + q(\lambda_c^\star,\,\theta_c^\star)\sum_{k=0}^{N-1}\binom{N-1}{k}\left(1-F_c^l(p)\right)^k\gamma^k(1-\gamma)^{N-1-k}\right](p-c)$$

$$= \left[1 - q(\lambda_c^\star,\,\theta_c^\star) + q(\lambda_c^\star,\,\theta_c^\star)(1-\gamma)^{N-1}\right](r_c^\star - c).$$

The profit of a slow trader is the same as in the previous case. The condition $r_c^\star = c+\Delta$ means that we must have

$$\frac{s}{(1-\alpha(\lambda_c^\star,\,\theta_c^\star))\gamma} = \Delta.$$

This implies that we must again have entry with probability one. Thus, we have an equilibrium with full entry and the probability of rejecting an offer of $r_c^\star$ given by $\theta_c^\star$ that solves

$$s = (1-\alpha(1,\,\theta_c^\star))\gamma\Delta.$$

Note that $\theta_c^\star = \theta^\star$ (the equation, and hence the solution, is independent of c). An interior solution exists if and only if

$$(1-\alpha(1,\,1))\gamma\Delta < s < (1-\alpha(1,\,0))\gamma\Delta.$$

Notice that $\theta^\star$ is close to 1 when s is close to $(1-\alpha(1,1))\gamma\Delta$, and close to 0 when s is close to $(1-\alpha(1,\,0))\gamma\Delta$.

Case 2.2.2: $c \le v - \Delta$, $r_c^\star > c + \Delta$. This is the case when high-cost dealers make positive profits and mix according to a continuous distribution $F_c^h(p)$ with upper limit

$r_c^\star$. We must have $\theta_c^\star = 0$. The cdf $F_c^h(p)$ solves

$$\left[1 - q(\lambda_c^\star,\, 0) + q(\lambda_c^\star,\, 0)(1-\gamma)^{N-1}(1 - F_c^h(p))^{N-1}\right](p - c - \Delta) = \left[1 - q(\lambda_c^\star,\, 0)\right](r_c^\star - c - \Delta).$$

Simplifying, we obtain

$$F_c^h(p) = 1 - \left(\frac{1 - q(\lambda_c^\star,\, 0)}{q(\lambda_c^\star,\, 0)(1-\gamma)^{N-1}} \frac{r_c^\star - p}{p - c - \Delta}\right)^{\frac{1}{N-1}}$$

with upper limit $\bar{p}_c^h = r_c^\star$, and lower limit

$$\underline{p}_c^h = \frac{1 - q(\lambda_c^\star,\, 0)}{1 - q(\lambda_c^\star,\, 0)\left(1 - (1-\gamma)^{N-1}\right)} r_c^\star + \frac{q(\lambda_c^\star,\, 0)(1-\gamma)^{N-1}}{1 - q(\lambda_c^\star,\, 0)\left(1 - (1-\gamma)^{N-1}\right)}(c + \Delta).$$

To simplify notation, let us denote

$$\phi(\lambda_c^\star) = \frac{1 - q(\lambda_c^\star,\, 0)}{1 - (1 - (1-\gamma)^{N-1})\, q(\lambda_c^\star,\, 0)}. \tag{B.15}$$

Next, $F_c^l(p)$ must solve

$$\left[1 - q(\lambda_c^\star,\, 0) + q(\lambda_c^\star,\, 0) \sum_{k=0}^{N-1} \binom{N-1}{k} \left(1 - F_c^l(p)\right)^k \gamma^k (1-\gamma)^{N-1-k}\right](p - c)$$

$$= \left[1 - q(\lambda_c^\star,\, 0) + q(\lambda_c^\star,\, 0)(1-\gamma)^{N-1}\right](\underline{p}_c^h - c).$$

Solving for $F_c^l(p)$ we get

$$F_c^l(p) = 1 - \Phi^{-1}\left(\frac{1 - q(\lambda_c^\star,\, 0)\left(1 - (1-\gamma)^{N-1}\right)}{q(\lambda_c^\star,\, 0)\left(1 - (1-\gamma)^{N-1}\right)} \frac{\underline{p}_c^h - p}{p - c}\right),$$

with $\bar{p}_c^l = \underline{p}_c^h$ and lower limit

$$\underline{p}_c^l = \left[1 - q(\lambda_c^\star,\, 0)\left(1 - (1-\gamma)^{N-1}\right)\right] \underline{p}_c^h + \left[q(\lambda_c^\star,\, 0)\left(1 - (1-\gamma)^{N-1}\right)\right] c.$$

We need to define one more function, analogous to $\alpha(\lambda,\, \theta)$, and corresponding to the distribution of prices used by high-cost dealers. Let

$$\alpha_h(\lambda) = \int_0^1 \left(1 + \frac{q(\lambda,\, 0)(1-\gamma)^{N-1}}{1 - q(\lambda,\, 0)} z^{N-1}\right)^{-1} dz.$$

Then, using the familiar change of variables, we get

$$\int p\, dF_c^h(p) = (1 - \alpha_h(\lambda_c^\star))(c + \Delta) + \alpha_h(\lambda_c^\star) r_c^\star,$$

and

$$\int p\, dF_c^l(p) = (1-\alpha(\lambda_c^\star,\, 0))c + \alpha(\lambda_c^\star,\, 0)\underline{p}_c^h.$$

As always, $r_c^\star$ is determined by the indifference condition

$$v - r_c^\star = -s + \gamma\left[v - \int_{\underline{p}_c^l}^{\bar{p}_c^l} p\, dF_c^l(p)\right] + (1-\gamma)\left[v - \int_{\underline{p}_c^h}^{\bar{p}_c^h} p\, dF_c^h(p)\right].$$

From this we can obtain

$$r_c^\star = c + \Delta + \frac{s - (1-\alpha(\lambda_c^\star,\, 0))\gamma\Delta}{\gamma(1-\phi(\lambda_c^\star)\alpha(\lambda_c^\star,\, 0)) + (1-\gamma)(1-\alpha_h(\lambda_c^\star))}.$$

Next, we consider entry decision of slow traders. The profit conditional on entry is simply $v - r_c^\star$. Thus, we have entry with probability one if and only if

$$c < v - \Delta - \frac{s - (1-\alpha(1,\, 0))\gamma\Delta}{\gamma(1-\phi(1)\alpha(1,\, 0)) + (1-\gamma)(1-\alpha_h(1))}.$$

Since we have assumed that $r_c^\star > c + \Delta$, we additionally require

$$s > (1-\alpha(1,\, 0))\gamma\Delta.$$

Interior entry requires $\lambda_c^\star$ to solve

$$v = c + \Delta + \frac{s - (1-\alpha(\lambda_c^\star,\, 0))\gamma\Delta}{\gamma(1-\phi(\lambda_c^\star)\alpha(\lambda_c^\star,\, 0)) + (1-\gamma)(1-\alpha_h(\lambda_c^\star))}. \tag{B.16}$$

An interior solution exists if and only if

$$\frac{s - (1-\alpha(0,\, 0))\gamma\Delta}{\gamma(1-\phi(0)\alpha(0,\, 0)) + (1-\gamma)(1-\alpha_h(0))} < v - c - \Delta < \frac{s - (1-\alpha(1,\, 0))\gamma\Delta}{\gamma(1-\phi(1)\alpha(1,\, 0)) + (1-\gamma)(1-\alpha_h(1))}. \tag{B.17}$$

Noticing that $\alpha_h(0) = 0$ and $\phi(0) = 0$, we can simplify the first inequality to

$$s - (1-\alpha(0,\, 0))\gamma\Delta < v - c - \Delta$$

Finally, since we have assumed that $r_c^\star > c + \Delta$, we require

$$s > (1-\alpha(\lambda_c^\star,\, 0))\gamma\Delta.$$

This condition is satisfied vacuously when equation (B.16) holds.

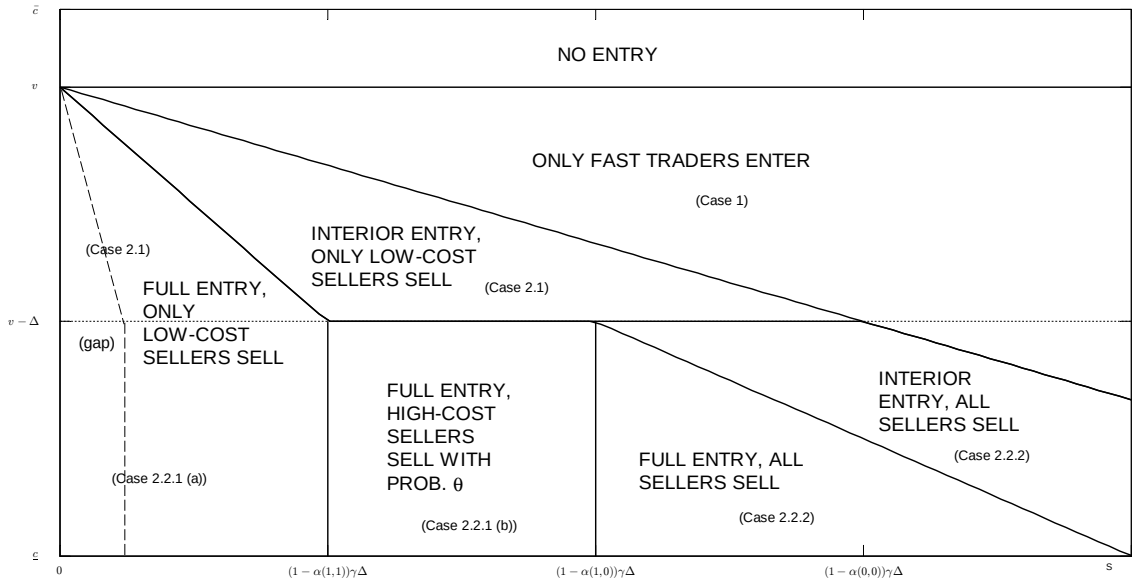
When

$$s - (1-\alpha(0,\, 0))\gamma\Delta \geq v - c - \Delta,$$

we must have entry with probability zero which brings us back to case 1.

This concludes the analysis of all cases. By direct inspection, we check that for any given pair (s, c), there is exactly one equilibrium (up to payoff-irrelevant changes in equilibrium strategies). Figure B.1 summarizes our conclusions by depicting the equilibrium correspondence in the (s, c) space. "Full entry" means that $\lambda_c^\star = 1$ in the relevant range. "Interior entry" means that $\lambda_c^\star \in (0, 1)$. When we say that "only low-cost dealers sell," we mean that if there is at least one low-cost dealer in the market and that all customers trade with low-cost dealers. When we say that "all dealers sell" or that "high-cost dealers sell with probability θ," we refer to the probability of selling to a slow trader upon a visit. Finally, the trapezoidal area denoted by "(gap)" corresponds to the case in which low-cost dealers have a gap in the support of the distribution of their offer distribution.

Fig. B.1: The benchmark case - equilibrium correspondence



B.2 Proof of Proposition 8

Generalized statement (without assuming A.2): *If* $s \leq (1-\hat{\alpha})\gamma \min\{\Delta, v-c\}$, *the equilibrium in the benchmark case leads to efficient matching. That is, slow traders always enter, and all traders buy from a low-cost dealer as long as there is at least one in the market. Additionally, if* $s \geq \kappa(1-\hat{\alpha})\gamma \min\{\Delta, v-c\}$, *where* $\kappa < 1$,[49] *the*

[49] $\kappa = (1-\gamma)^{N-1} / \left[p_0(1-\gamma)^{N-1} + (1-p_0)[1-(1-\gamma)^N]/(N\gamma)\right]$.

equilibrium with the benchmark achieves the second best. That is, slow traders buy from the first low-cost trader that they encounter.

Proof. The theorem follows directly from the derivation above (cases 2.1 and 2.2.1 (a)). When

$$\kappa(1-\hat{\alpha})\gamma \min\{\Delta,\, v-c\} \leq s \leq (1-\hat{\alpha})\gamma \min\{\Delta,\, v-c\},$$

we are in the region in which the equilibrium achieves the second best. Slow traders always enter, and search until they find the first low-cost dealer (low-cost dealers always post prices below the reservation price, and high-cost dealers always post prices above the reservation price). If there are no low-cost dealers in the market and $c > v-\Delta$, then traders exit without trading. When $c < v-\Delta$, they buy from a high-cost dealer. When $s < \kappa(1-\hat{\alpha})\gamma\Delta$, low-cost dealers post prices below the reservation price with probability $\zeta_c^\star \in (0,\, 1)$. Because high-cost dealers still post prices above the reservation-price (and above the prices posted by low-cost dealers), the matching of traders to low-cost dealers is efficient. □

B.3 Lemma 9

Here, we show that a low-cost dealer's incentive to quote a high price disappears as the number N of dealers gets large. Indeed, as N becomes large the probability that all other dealers have high costs goes to zero quickly.

Lemma 9. *Letting* $\bar{s}(N) = (1-\hat{\alpha})\gamma\Delta$ *and* $\underline{s}(N) = \kappa(1-\hat{\alpha})\gamma\Delta$, *we have*

$$\lim_{N\to\infty} N\bar{s}(N) = \infty$$

and

$$\lim_{N\to\infty} N\underline{s}(N) = 0,$$

where the convergence to 0 *is exponentially fast.*

The quantity Ns is the upper bound on the search costs incurred by a slow trader. If slow traders adopted the sub-optimal strategy of searching the entire market, we would get the fully efficient outcome of a centralized exchange, before considering the search costs. Thus, $(1-p_0)Ns$ is an upper bound on the potential welfare loss in our setting. Lemma 9 says that the case $s < \kappa(1-\hat{\alpha})\gamma\Delta$ in which case the benchmark fails to achieve the second best can be safely ignored for practical purposes, given that the (rough) upper bound of possible inefficiency goes to 0 exponentially fast[50] with N. On the other hand, the search-cost range $(\kappa(1-\hat{\alpha})\gamma\Delta,\, (1-\hat{\alpha})\gamma\Delta)$ is much more important, as the potential welfare gains or losses are unbounded in this region (if we allow v to get large).

[50] For example, if $p_0 = \gamma = \frac{1}{2}$, $\kappa \approx 0.019$ for just $N = 10$, and $\kappa \approx 1.5 * 10^{-6}$ for $N = 25$.

To prove the first claim, we show that $(1-\hat{\alpha})$ converges to zero (as $N \to \infty$) more slowly than $\log(N)/N$. (That $(1-\hat{\alpha})$ converges to 0 follows from Lebesgue Dominated Convergence Theorem.) We have

$$1-\hat{\alpha} = \int_0^1 \frac{a_N \Phi(z)}{1+a_N\Phi(z)}\, dz,$$

where

$$a_N = \frac{Np_0\left(1-(1-\gamma)^{N-1}\right)}{\frac{1-(1-\gamma)^N}{\gamma}(1-p_0)+Np_0(1-\gamma)^{N-1}}.$$

Clearly,

$$1-\hat{\alpha} \geq \int_{\Phi^{-1}(\frac{1}{N})}^1 \frac{a_N\Phi(z)}{1+a_N\Phi(z)}\, dz \geq \left(1-\Phi^{-1}(\frac{1}{N})\right)\frac{a_N}{N+a_N}.$$

The term $a_N/(N+a_N)$ has a finite and strictly positive limit. It is therefor enough to show that

$$\lim_{N\to\infty}\frac{N}{\log N}\left(1-\Phi^{-1}(\frac{1}{N})\right) > 0.$$

Using equation (B.4) to invert Φ, and applying d'Hospital rule a few times to simplify the expression, we obtain

$$\lim_{N\to\infty}\frac{N}{\log N}\left(1-\Phi^{-1}(\frac{1}{N})\right) = \lim_{N\to\infty}\frac{N}{\log N}\left(\left(\frac{1}{N}\right)^{\frac{1}{N}}-1\right)$$

$$= \lim_{N\to\infty}\frac{N}{\log N}\left(\exp\left(\frac{\log N}{N}\right)-1\right) = \lim_{K\to\infty} K\left(e^{\frac{1}{K}}-1\right) = 1.$$

To prove the second claim, recall that

$$N\kappa = \frac{N(1-\gamma)^{N-1}}{p_0(1-\gamma)^{N-1}+(1-p_0)\frac{1-(1-\gamma)^N}{N\gamma}} = \frac{1}{\frac{p_0}{N}+(1-p_0)\frac{1-(1-\gamma)^N}{N^2\gamma(1-\gamma)^{N-1}}}.$$

The above expression goes to 0 as quickly as $N^2(1-\gamma)^{N-1}$, that is, exponentially.

We note that we did not use assumption A.2. Thus the result is true in the generalized setting without assuming A.2.

B.4 Proof of Proposition 9

Generalized statement (not assuming A.2): *In the no-benchmark case, if* $\min\{v, \bar{c}\} > \underline{c}+\Delta$*, there does not exist an equilibrium achieving the second best.*

Proof. In an equilibrium in which the second-best is achieved under the condition $s < \gamma\Delta$, high-cost dealers can only sell when there are no low-cost dealers in the market, and the slow trader searched the entire market. Thus, if an equilibrium like this exists,

high-cost dealers quote prices as if they participated in an auction with all other high-cost dealers. A standard result in auction theory says that in this case they must bid their costs, that is, they must offer to sell for $c + \Delta$.

Consider a situation when a slow trader enters and the first dealer has low costs, for some $c < v$. If the second-best is achieved, that offer needs to be accepted by a slow trader. Under the assumption of the Proposition, we can find a $c^\star$ that satisfies $v > c^\star > \underline{c} + \Delta$. By the above observation, (almost) all prices in the support of the distribution of the low-cost dealer at $c = c^\star$ must be accepted by a slow trader in the first search round. This leads to a contradiction. Since high-cost dealers post a price of $c + \Delta$ conditional on c, they make zero profits. They can profitably deviate at $c = \underline{c}$ by quoting a price in the support of the distribution of a low-cost dealer at $c = c^\star$. □

B.5 Generalized statement of Theorem 3 (not assuming A.2)

Theorem: If (i) $\kappa(1-\hat{\alpha})\gamma \min\{\Delta, v - \underline{c}\} \leq s \leq (1-\hat{\alpha})\gamma \min\{\Delta, v - \bar{c}\}$, and (ii) $\bar{c} > \underline{c} + \Delta$, then the equilibrium in the benchmark case yields a strictly higher social surplus that any equilibrium in the no-benchmark case.

B.6 Supporting Contents for Section 4

In this subsection we complete the analysis of the matching efficiency by commenting on cases not covered in the main text. We maintain the same assumptions as in Section 4.

We begin by stating the result about partial matching efficiency.

Proposition 11. *Suppose that* $(1-\hat{\alpha})\gamma\Delta < s < (1-\alpha(1,0))\gamma\Delta$. *Then the equilibrium in the benchmark case has the following properties.*

1. *Slow traders enter with probability one.*
2. *High-cost dealers always quote the price* $c + \Delta$, *and low-cost dealers make offers in an interval whose upper limit is* $c + \Delta$.
3. *Slow traders set a reservation price of* $r_c^\star = c + \Delta$. *The price* $r_c^\star$ *is rejected by a slow trader with probability* θ, *where* $\theta \in (0, 1)$ *solves the equation* $s = (1 - \alpha(1, \theta))\gamma\Delta$. *An offer strictly below* $r_c^\star$ *is accepted by a slow trader with probability one.*

Proof. This follows directly from the derivation in the proof of Proposition 7 in Section B.1 (case 2.2.1 (b)). □

Under the parameter restrictions of the Proposition, the equilibrium resembles that of Proposition 8, but has an unexpected twist. Slow traders follow a reservation-price strategy with $r_c^\star = c + \Delta$, and high-cost dealers always offer to sell at $c + \Delta$. Upon seeing a price offer of $r_c^\star$, slow traders randomize between accepting and rejecting. The equilibrium rejection probability θ does not depend on c and changes continuously

from 1 to 0 as s grows from $(1-\hat{\alpha})\gamma\Delta$ to $(1-\alpha(1,0))\gamma\Delta$. Surprisingly, the unique reservation-price equilibrium involves non-trivial randomization at the reservation price. As a consequence, we get partial efficiency in the matching of slow traders to low-cost dealers.

If $s > (1-\alpha(1,0))\gamma\Delta$, then all dealers, including high-cost dealers, sell the asset in a benchmark-based equilibrium. (See the analysis in Section B.1 and Figure B.1.) Thus, a sufficiently low search cost is necessary to match traders to low-cost dealers.

If $s < \kappa(1-\hat{\alpha})\gamma\Delta$, the equilibrium in the benchmark case is difficult to analyze. Because the benchmark does not achieve the second best in this setting, Proposition 9 alone is insufficient to provide a welfare comparison. Moreover, as shown in Section B.7, a reservation-price equilibrium fails to exist in the no-benchmark case if search costs are very low. Given Lemma 9 and the intractability of non-reservation-price equilibria, we leave open the question of welfare comparison in this parameter range.

B.7 Equilibrium in the no-benchmark case with two dealers

In this subsection we explore the special case of two dealers ($N = 2$). Otherwise, we maintain the same assumptions as in Section 4. Proofs of results are provided in the next subsection.

If there are two dealers, traders update their beliefs at most once, so the continuation conditional expected payoff of a slow trader after rejecting the first dealer's offer is easy to calculate. Benabou and Gertner (1993) characterize reservation-price equilibria in their model with two dealers. Our model has a different cost structure for dealers; thus, their results, although fairly general, cannot be applied.

The following result is an analogue of Lemma 1 for the case of two dealers with idiosyncratic costs, ϵ_1 and ϵ_2, for supplying the asset.

Lemma 10. *With no benchmark and two dealers, if there exists a reservation-price equilibrium, then the first-round reservation price is either* $\underline{c}+\Delta$ *or* v. *If*

$$s < (1-\alpha(1,0))\gamma\Delta,$$

then the first-round reservation price is $\underline{c}+\Delta$.

We briefly characterize pricing strategies of dealers in the two equilibria. Remaining details can be found in the proof of the Lemma in Section B.8.1.

In the equilibrium with $r^\star = \underline{c}+\Delta$, because $c+\Delta$ exceeds $r^\star$, high-cost dealers cannot sell immediately when they are the first dealer contacted by a trader. As a consequence, it can be shown that they post a price offer equal to their cost $c+\Delta$ and make zero profits. Low-cost dealers always use a continuous distribution, but there are three regions for the outcome of the cost c that lead to qualitatively different offer distributions. (See the three diagrams (counting from the right) in Figure B.2.) When the cost c is very low, all offers lie below $r^\star$. For the middle range, the support consists of two intervals, one below $r^\star$, and one above $r^\star$. Thus, low-cost dealers sell to a

slow trader upon the trader's first contact with some probability equal to the (mixing) probability that a offer from the lower interval is drawn. Finally, when c is high, all offers lie above $r^\star$. Conditional on this outcome of c, traders never buy from the dealer on their first contact.

In the equilibrium with $r^\star = v$, we can distinguish two qualitatively different regions of cost c. (See the two diagrams (counting from the left) in Figure B.2.) When $c < v - \Delta$, both high- and low-cost dealers make positive profits, and quote prices according to continuous distributions with adjacent supports lying below $r^\star$. In the opposite case, there are no gains from trade between traders and high-cost dealers. Low-cost dealers use a continuous distribution with upper limit $r^\star$.

Although the proof of Lemma 10 assumes $N = 2$, the equilibrium with $r^\star = v$ is easy to characterize even if N is arbitrary. Because slow traders do not search in equilibrium, the only difference is that we need to correctly adjust the posterior probability of a fast trader. Using arguments along the lines of the proof of Proposition 4, we can also show existence if search costs are sufficiently high.

In line with the objective of Section 4, analyzing matching efficiency when search costs are low, in the remainder we focus on the equilibrium with $r^\star = \underline{c} + \Delta$.

Let us define a function $\Psi(p;\, r^\star)$ that returns the expected benefit from search (that is, the expected benefit from visiting the second dealer, excluding the search costs) after a price offer of p was observed, fixing the reservation price $r^\star$. In the proofs of Lemma 10 and Proposition 12 we give a closed-form expression for $\Psi(p;\, r^\star)$ and show that $\Psi(p;\, r^\star)$ is continuous in p on $[\underline{p}^l_{\underline{c}},\, r^\star)$ and on $(r^\star,\, v]$. Moreover, for $r^\star = \underline{c} + \Delta$, $\Psi(p;\, r^\star)$ jumps up at $p = r^\star$. This ensures that the incentives to search go in the right direction "locally" around $r^\star$. Define $\underline{s} = \sup\left\{\Psi(p;\, r^\star) : p \in [\underline{p}^l_{\underline{c}},\, r^\star)\right\}$ and $\bar{s} = \inf\{\Psi(p;\, r^\star) : p \in (r^\star,\, v]\}$. We have the following result.

Proposition 12. *Suppose that $s < (1 - \alpha(1,\, 0))\gamma\Delta$. Then, a reservation-price equilibrium in the no-benchmark case exists if and only if the search cost s is in the interval $[\underline{s},\, \bar{s}]$. Moreover, $\bar{s} \leq (1 - \alpha(1,\, 1))\gamma\Delta$.*[51]

While the constants $\underline{s}$ and $\bar{s}$ are directly computable, the associated analytic formulas are complicated. It can be shown, however, that the interval $[\underline{s},\, \bar{s}]$ is not empty if the distribution G of costs is uniform and γ is not too large. It is also easy to find examples (similar to the one considered by Janssen, Pichler and Weidenholzer 2011) when the interval is empty.

Proposition 12, taken together with Lemma 10, implies that there is no reservation-price equilibrium when the search cost s is in the interval

$$((1 - \alpha(1,\, 1))\gamma\Delta,\, (1 - \alpha(1,\, 0))\gamma\Delta)\,.$$

[51] We interpret the interval $[a,\, b]$ as the empty set when $a > b$, and as the singleton $\{a\}$ when $a = b$. The proof that we provide also implies that for $s \leq (1 - \alpha(1,\, 1))\gamma\Delta$ we cannot have a reservation-price equilibrium with $r^\star = \underline{c} + \Delta$ (even if we allow $s \geq (1 - \alpha(1,\, 0))\gamma\Delta$).

Moreover, and perhaps surprisingly, a reservation-price equilibrium fails to exist when the search cost s is sufficiently small.

The above analysis implies the following Corollary.

Corollary 1. *Fix a search cost $s \geq \kappa(1-\hat{\alpha})\gamma\Delta$, and suppose that in all equilibria (with and without the benchmark) there is full entry. If introducing the benchmark does not increase welfare, it must be the case that the equilibrium in the no-benchmark case is not a reservation-price equilibrium.*

Proof. This follows directly from Proposition 8, Lemma 10, and Proposition 12. □

The Corollary can be interpreted as saying that the no-benchmark setting cannot lead to a better matching efficiency than the benchmark setting if slow traders are using reservation-price strategies in equilibrium.

With two dealers and a search cost s in the interval $[\underline{s}, \bar{s}]$, the social cost of not having a benchmark does not arise from inefficient matching between slow traders and high-cost dealers in equilibrium. Rather, the inefficiency is caused by having slow traders engage in superfluous search. Indeed, unless the realization of the dealer's common cost c is very small, low-cost dealers make offers above the reservation price $\underline{c}+\Delta$, and slow traders might not trade with the their first encountered low-cost dealers. The Corollary below expresses the welfare gain from introducing a benchmark by comparing it to the gain that would be achieved if the market was organized as a centralized exchange with no search costs.

Corollary 2. *With two dealers and for $s \in [\max\{\kappa(1-\hat{\alpha})\gamma\Delta, \underline{s}\}, \bar{s}]$, introducing a benchmark eliminates at least a fraction*

$$\frac{(1-G(\underline{c}+\gamma\Delta))\,\gamma}{2-\gamma+(1-G(\underline{c}+\gamma\Delta))\,\gamma}$$

of the total loss in social surplus that is induced by search frictions.

B.8 Proofs for Section B.7

B.8.1 Proof of Lemma 10

Outline for the main steps of the proof. The proof is long and tedious, unlike the proof of Lemma 1. We outline the main steps below. First, we characterize the equilibrium response of dealers as a function of the first-round reservation price $r^\star$. We obtain four regions of cost c with different qualitative pricing strategy of dealers (see Figure B.2). When costs are low, both low-cost and high-cost dealers post price offers according to continuous distributions with adjacent supports below the reservation price $r^\star$ and make positive profits. When c is in the lower-middle region, low-cost dealers continue to mix below $r^\star$, while high-cost dealers bid $c+\Delta > r^\star$ and make zero profits. In the upper-middle region, the support of the distribution of low-cost dealers consists

of two disjoint intervals, $[\underline{p}^l_c, r^\star]$ and $[\hat{p}^l_c, c+\Delta)$, where $\hat{p}^l_c > r^\star$. Finally, when costs are highest, low-cost dealers bid exclusively in the upper interval that lies above $r^\star$. Second, we analyze the optimal search policy of slow traders. The proof proceeds by finding a contradiction when $r^\star \notin \{v, \underline{c}+\Delta\}$. Clearly, $r^\star$ cannot be larger than v. When $r^\star \in (\underline{c}+\Delta, v)$, we show that the posterior distribution of costs conditional on observing a price p converges to an atom at $r^\star - \Delta$ as p converges to $r^\star$. Because search behavior of traders is different on the two sides of $r^\star$, there is a discontinuity in expected price as c crosses the level $r^\star - \Delta$ (the price distribution is impacted through the posterior probability of a fast trader). As a result, the benefits from search for a slow trader jump down discontinuously at $r^\star$, a contradiction. Finally, when $r^\star < \underline{c} + \Delta$, we can show that upon observing a price just above $r^\star$, a slow trader believes with probability one that this is (nearly) the best price that she can get, and thus wants to accept, a contradiction.

The proof. Let the reservation price (in the first round of search) be $r^\star$ and probability of entry be $\lambda^\star$.[52] We first characterize the equilibrium response of dealers. We focus on the case $c < v - \Delta$, as the other case requiring only minor modifications.[53] As the derivation of dealers' strategies is similar to the benchmark case, we skip some of the details. We also summarize the conclusions in Lemma 11 and Figure B.2 below for the convenience of the reader.

First, consider $c < r^\star - \Delta$. In that case high-cost dealers make positive profits and we have a situation analogous to case 2.2.2 with the benchmark. The cdf $F^h_c(p)$ for high-cost dealers must satisfy

$$\left[1 - q(\lambda^\star, 0) + q(\lambda^\star, 0)(1-\gamma)(1 - F^h_c(p))\right](p - c - \Delta) = \left[1 - q(\lambda^\star, 0)\right](r^\star - c - \Delta).$$

Solving, we obtain

$$F^h_c(p) = 1 - \left(\frac{\lambda^\star(1-p_0)}{2p_0(1-\gamma)}\frac{r^\star - p}{p - c - \Delta}\right),$$

with upper limit $\bar{p}^h_c = r^\star$, and lower limit

$$\underline{p}^h_c = \frac{\lambda^\star(1-p_0)}{2p_0(1-\gamma) + \lambda^\star(1-p_0)} r^\star + \frac{2p_0(1-\gamma)}{2p_0(1-\gamma) + \lambda(1-p_0)}(c+\Delta).$$

Then, $F^l_c(p)$ must satisfy

$$\left[\lambda^\star(1-p_0) + 2p_0\left[(1-\gamma) + \left(1 - F^l_c(p)\right)\gamma\right]\right](p - c) = \left[\lambda(1-p_0) + 2p_0(1-\gamma)\right](\underline{p}^h_c - c).$$

[52] When the benchmark is not present, there is no need to consider the parameter θ that was a relevant part of the strategy in the benchmark case.

[53] The main difference is that when $c > v - \Delta$, the upper limit of the distribution of prices for low-cost dealers will be v in all cases when it was $c + \Delta$.

Solving for $F_c^l(p)$ we get

$$F_c^l(p) = 1 - \frac{\lambda^\star(1-p_0) + 2p_0(1-\gamma)}{2\gamma p_0} \frac{\underline{p}_c^h - p}{p-c},$$

with upper limit $\bar{p}_c^l = \underline{p}_c^h$ and lower limit

$$\underline{p}_c^l = \frac{\lambda^\star(1-p_0) + 2p_0(1-\gamma)}{\lambda^\star(1-p_0) + 2p_0} \underline{p}_c^h + \frac{2\gamma p_0}{\lambda^\star(1-p_0) + 2p_0} c.$$

Second, consider the case $c > r^\star - \Delta$. Now high-cost dealers cannot make positive profits, so they post a deterministic offer price equal to $c + \Delta$. It is easy to show (using arguments familiar from previous derivations) that the upper limit of the distribution of prices for low-cost dealers must be either $r^\star$ or $c + \Delta$. Thus, the support of the distribution is either (i) an interval with upper limit $r^\star$, (ii) an interval above $r^\star$ with upper limit $c + \Delta$, or (iii) a sum of intervals from (i) and (ii). We analyze these possibilities below.

If low cost dealers bid on an interval with upper limit $r^\star$, then the distribution $F_c^l(p)$ must solve

$$\begin{aligned}&\left[1 - q(\lambda^\star,\, 1) + q(\lambda^\star,\, 1)\left[(1-\gamma) + \left(1 - F_c^l(p)\right)\gamma\right]\right](p-c)\\ &\qquad = \left[1 - q(\lambda^\star,\, 1) + q(\lambda^\star,\, 1)(1-\gamma)\right](r^\star - c),\end{aligned}$$

and is thus given by

$$F_c^l(p) = 1 - \frac{\lambda^\star(2-\gamma)(1-p_0) + 2p_0(1-\gamma)}{2\gamma p_0} \frac{r^\star - p}{p-c},$$

with upper limit $\bar{p}_c^l = r^\star$, and lower limit

$$\underline{p}_c^l = \frac{\lambda^\star(2-\gamma)(1-p_0) + 2p_0(1-\gamma)}{\lambda^\star(2-\gamma)(1-p_0) + 2p_0} r^\star + \frac{2\gamma p_0}{\lambda^\star(2-\gamma)(1-p_0) + 2p_0} c.$$

The profit of a low-cost dealer conditional on a contact is

$$\frac{\lambda^\star(2-\gamma)(1-p_0) + 2p_0(1-\gamma)}{\lambda^\star(2-\gamma)(1-p_0) + 2p_0}(r^\star - c).$$

To verify optimality of the pricing strategy, we need to check that a low-cost dealer cannot improve upon the above profit by bidding just below $c + \Delta$ (this is the most profitable deviation). The expected profit (conditional on a visit) under that deviation can get arbitrarily close to

$$\frac{2(1-\gamma)(p_0 + \lambda^\star(1-p_0))}{\lambda^\star(2-\gamma)(1-p_0) + 2p_0}\Delta.$$

Comparing the two expressions, we conclude that the above price distribution constitutes an equilibrium if and only if

$$c < r^\star - \frac{2(1-\gamma)(p_0 + \lambda^\star(1-p_0))}{\lambda^\star(2-\gamma)(1-p_0) + 2p_0(1-\gamma)}\Delta.$$

We will denote[54]

$$\kappa = \frac{2(1-\gamma)(p_0 + \lambda^\star(1-p_0))}{\lambda^\star(2-\gamma)(1-p_0) + 2p_0(1-\gamma)}.$$

Clearly, $r^\star - \kappa\Delta > r^\star - \Delta$.

When the cost c is above $r^\star - \kappa\Delta$, we must have one of the cases (ii) or (iii). We explore the possibility of case (iii) below.

The two intervals in the support of the price distribution of low-cost dealers will be denoted by $[\underline{p}^l_c,\, r^\star]$ and $[\hat{p}^l_c,\, c+\Delta)$. Let ζ_c be the probability of bidding in the lower of the two intervals. Note that under such price distribution we need to adjust the posterior probability that a visiting trader is a slow trader. Moreover, the fact of being visited is informative of the price posted by a competing dealer.

If the support consists of two intervals, then the dealer must be indifferent between posting $r^\star$ and $c + \Delta - \epsilon$ (for $\epsilon \to 0$) which gives us the condition

$$\left[p_0(1-\gamma\zeta_c) + \lambda^\star(1-p_0)(\frac{1}{2} + \frac{1}{2}(1-\gamma\zeta_c))\right](r^\star - c) = (1-\gamma)(p_0 + \lambda^\star(1-p_0))\Delta.$$

Solving for ζ_c we obtain

$$\zeta_c = \frac{2p_0 + 2\lambda^\star(1-p_0)}{2p_0 + \lambda^\star(1-p_0)}\left[1 - \frac{1-\gamma}{\gamma}\frac{c+\Delta-r^\star}{r^\star - c}\right].$$

Because ζ_c must lie in $[0,\, 1]$, we can determine the maximal interval of costs for which the conjectured price distribution might arise in equilibrium. Simple calculation shows that this interval is $[r^\star - \kappa\Delta,\, r^\star - (1-\gamma)\Delta]$. (The interval is always non-empty because $\kappa > 1-\gamma$.)

Given the structure of the support, the upper part of the distribution $F^l_c(p)$ must satisfy

$$\left[1 - \gamma + \gamma(1 - F^l_c(p))\right](p - c) = (1-\gamma)\Delta,$$

for all $p \in [\hat{p}^l_c,\, c+\Delta)$, so that

$$F^l_c(p) = 1 - \frac{1-\gamma}{\gamma}\frac{c+\Delta-p}{p-c}.$$

[54] This coincides with the previous definition of κ when $\lambda^\star = 1$.

To determine the cutoff $\hat{p}_c^l$, we use the fact that $F_c^l(\hat{p}_c^l) = \zeta_c$ to obtain the equation

$$1 - \frac{1-\gamma}{\gamma}\frac{c+\Delta-\hat{p}_c^l}{\hat{p}_c^l - c} = \zeta_c,$$

which gives

$$\hat{p}_c^l = c + \frac{1-\gamma}{1-\gamma+\gamma(1-\zeta_c)}\Delta.$$

Note that when $c = r^\star - \kappa\Delta$, so that $\zeta_c = 1$, we get $\hat{p}_c^l = c + \Delta = r^\star + (1-\kappa)\Delta$. On the other hand, when $c = r^\star - (1-\gamma)\Delta$, we have $\zeta_c = 0$, and $\hat{p}_c^l = r^\star$. As c increases, the lower limit of the upper interval converges to $r^\star$ from above. This observation will be important in a later part of the proof.

To solve for the lower part of the distribution $F_c^l(p)$, we write down the indifference condition between bids in $[\underline{p}_c^l, r^\star]$:

$$\begin{aligned}&\left[2(1-\gamma) + (2-\zeta_c)\lambda^\star(1-p_0)\gamma + 2p_0\gamma(1-F_l(p))\right](p-c)\\ &= \left[2p_0(1-\gamma\zeta_c) + (2-\zeta_c\gamma)\lambda^\star(1-p_0)\right](r^\star - c).\end{aligned}$$

This gives us

$$F_c^l(p) = \zeta_c - \frac{2p_0(1-\gamma\zeta_c) + \lambda^\star(1-p_0)(2-\zeta_c\gamma)}{2p_0\gamma}\,\frac{r^\star - p}{p-c},$$

with the lower limit

$$\underline{p}_c^l = \frac{(\lambda^\star(1-p_0)+2p_0)(1-\gamma)\Delta}{2p_0(r^\star-c)+\lambda^\star(1-p_0)(1-\gamma)\Delta}r^\star + \frac{2p_0(r^\star-c)-2p_0(1-\gamma)\Delta}{2p_0(r^\star-c)+\lambda^\star(1-p_0)(1-\gamma)\Delta}c.$$

Finally, we consider the case $c > r^\star - (1-\gamma)\Delta$. Since we have shown that neither of cases (i) and (iii) is possible, we explore case (ii), that is, we solve for the distribution $F_c^l(p)$ with support that lies above $r^\star$. The usual indifference condition is

$$\left[1-\gamma+\gamma(1-F_c^l(p))\right](p-c) = (1-\gamma)\Delta,$$

so we get

$$F_c^l(p) = 1 - \frac{1-\gamma}{\gamma}\,\frac{c+\Delta-p}{p-c},$$

with lower limit

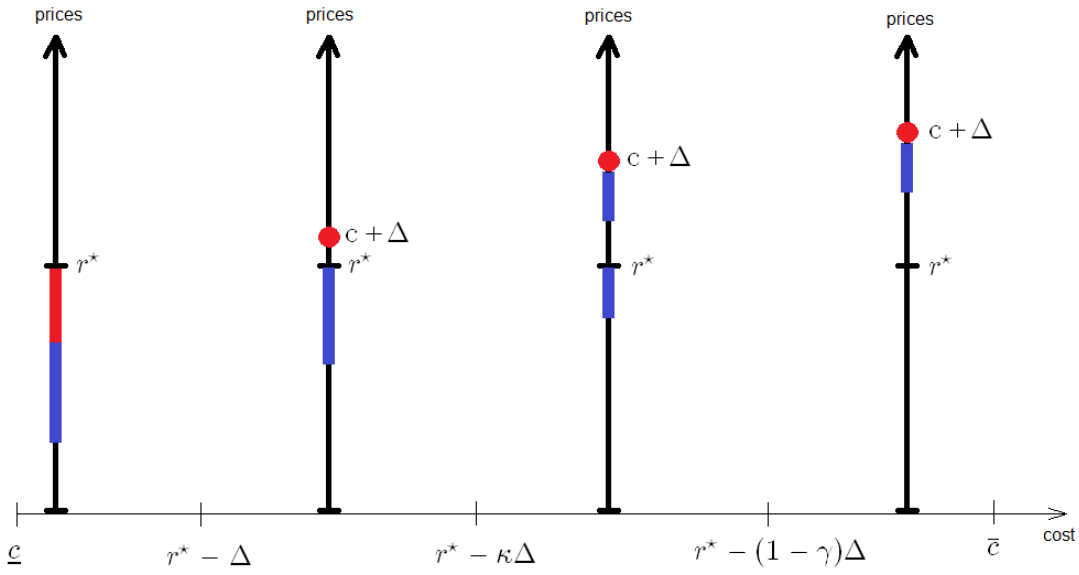
$$\underline{p}_c^l = c + (1-\gamma)\Delta.$$

By comparing the profit of a low-cost dealer under this price distribution to the profit from a deviation to $r^\star$ (which is the most profitable deviation), we conclude that we have an equilibrium best response precisely when $c \geq r^\star - (1-\gamma)\Delta$.

This concludes the characterization of the best-response of dealers to a reservation-price strategy $r^\star$. We summarize the most important observations in the Lemma below.

Figure B.2 depicts the qualitative features of the supports for different cost ranges.

Fig. B.2: Supports of the distributions of prices (as a function of c)
(low-cost dealers: blue; high-cost dealers: red; a dot denotes an atom)



Lemma 11. *The equilibrium response of dealers to slow traders playing a reservation-price strategy $(r^\star, \lambda^\star)$ is payoff-unique. When $c < r^\star - \Delta$, low-cost and high-cost dealers use continuous distributions of price offers with adjacent supports that lie below $r^\star$. When $c > r^\star - \Delta$, high-cost dealers offer $c + \Delta$, and low-cost dealers offer according to a continuous distribution. When $c \in (r^\star - \Delta, r^\star - \kappa\Delta)$ the support is $[\underline{p}^l_c, r^\star]$. For $c \in (r^\star - \kappa\Delta, r^\star - (1-\gamma)\Delta)$ the support is $[\underline{p}^l_c, r^\star] \cup [\hat{p}^l_c, c + \Delta]$. Finally, for $c > r^\star - (1-\gamma)\Delta$ the support is $[\underline{p}^l_c, c + \Delta]$ with $\underline{p}^l_c > r^\star$.*

Having determined the equilibrium pricing policy of dealers, we can turn to the analysis of the search behavior of slow traders. Our goal is to exclude the possibility that $r^\star \notin \{\underline{c} + \Delta, v\}$.

The posterior distribution of cost c conditional on observing a price p in the first search is

$$G(c \,|\, p) = \frac{\int_{\underline{c}}^{c} \left[\gamma f^l_y(p) + (1-\gamma) f^h_y(p)\right] \, dG(y)}{\int_{\underline{c}}^{\bar{c}} \left[\gamma f^l_y(p) + (1-\gamma) f^h_y(p)\right] \, dG(y)},$$

where $f^i_c(p)$ denotes the density corresponding to $F^i_c(p)$, for $i \in \{l, h\}$. Here, whenever $F^h_c(p)$ is a step function with jump at $c + \Delta$, $f^h_c(p) = \delta_{c+\Delta}(p)$, that is, we interpret the density as a dirac delta measure at $c+\Delta$. As in Section B.7, we define $\Psi(p; r^\star)$ to be the benefit from search after observing price p. The second argument $r^\star$ emphasizes that

the whole function changes with $r^\star$ because the distribution of prices changes. Since there are only two dealers, we can calculate $\Psi(p;\, r^\star)$ explicitly:

$$\Psi(p;\, r^\star) = \int_{\underline{c}}^{\bar{c}} \left[\int_{\underline{p}_c^l}^{p} (v-p) \left[\gamma f_c^l(p) + (1-\gamma) f_h(p) \right] dp \right] dG(c \,|\, p)$$

$$-(v-p) \int_{\underline{c}}^{\bar{c}} \left[\gamma F_c^l(p) + (1-\gamma) F_c^h(p) \right] dG(c \,|\, p).$$

Because the function $\Psi(p;\, r^\star)$ does not need to be continuous in general, we can no longer without loss of generality use the condition that $\Psi(r^\star;\, r^\star) = s$, that is, the indifference of a slow trader between buying and searching at $p = r^\star$. Rather, a necessary and sufficient condition for a reservation-price strategy $r^\star$ to be optimal is that

$$\begin{aligned} s &\geq \Psi(p;\, r^\star), \quad p \leq r^\star, \\ s &\leq \Psi(p;\, r^\star), \quad p \geq r^\star. \end{aligned} \tag{B.18}$$

The strategy for the rest of the proof is to show that the condition (B.18) fails when $r^\star \notin \{\underline{c} + \Delta,\, v\}$.

Clearly, we cannot have $r^\star > v$. The lemma below deals with the case $r^\star \in (\underline{c}+\Delta,\, v)$.

Lemma 12. *When $r^\star \in (\underline{c}+\Delta,\, v)$, the posterior distribution of costs $G(\,\cdot\,|\,p)$ converges to an atom at $r^\star - \Delta$ as p converges to $r^\star$.*

Proof. First, suppose that p converges to $r^\star$ from the left. By the above derivation, the support of the posterior distribution must be contained in $[r^\star - \Delta,\, r^\star - (1-\gamma)\Delta]$ in the limit. We have

$$\lim_{p \to r^\star} G(c \,|\, p) = \lim_{p \to r^\star} \frac{\int_{\underline{c}}^{c} \left[\gamma f_y^l(p) + (1-\gamma) f_y^h(p) \right] dG(y)}{\int_{\underline{c}}^{\bar{c}} \left[\gamma f_y^l(p) + (1-\gamma) f_y^h(p) \right] dG(y)}. \tag{B.19}$$

Letting

$$\phi(p) = \frac{p - \phi(\lambda^\star) r^\star}{1 - \phi(\lambda^\star)} - \Delta$$

and

$$\phi(\lambda^\star) = \frac{\lambda^\star (1 - p_0)}{2p_0(1-\gamma) + \lambda^\star (1 - p_0)},$$

we note that the integral

$$\int_{\underline{c}}^{\bar{c}} f_y^h(p)\, dG(y) = \frac{\lambda^\star (1-p_0)}{2p_0(1-\gamma)} \int_{\underline{c}}^{\phi(p)} \frac{r^\star - y - \Delta}{(p - y - \Delta)^2}\, dG(y),$$

diverges to ∞ as $p \nearrow r^\star$. Thus, the numerator in expression (B.19) is going to ∞. For

any $\epsilon > 0$, the integral

$$\int_{r^\star-\Delta+\epsilon}^{\bar{c}} \left[\gamma f_y^l(p) + (1-\gamma) f_y^h(p)\right] \, dG(y)$$

is finite. Therefore, $G(c \,|\, p) \to \mathbf{1}_{\{c \geq r^\star - \Delta\}}$.

The intuition for this result is simple: As c gets closer to $r^\star - \Delta$, the distribution of prices for high-cost dealer gets "squeezed" on a very small interval below $r^\star$, and the density explodes. This is never the case for low-cost dealers. Thus, upon observing a price offer p just below $r^\star$, a trader believes that it is much more likely that it has been posted by a high-cost dealer in which case the cost must be close to $p - \Delta$ (and exactly $r^\star - \Delta$ in the limit).

Second, consider the case when p converges to $r^\star$ from the right. In this case, if the offer is posted by a high-cost dealer, then the cost must be equal to $p - \Delta$. The numerator in expression (B.19) is finite and bounded away from zero. Moreover, we have, for any $\epsilon > 0$,

$$\begin{aligned}\int_{\underline{c}}^{\bar{c}} f_y^l(p) \, dG(y) = &\underbrace{\int_{r^\star-\Delta}^{r^\star-(1-\gamma)\Delta-\epsilon} f_y^l(p) \, dG(y)}_{I_1} \\ &+ \underbrace{\int_{r^\star-(1-\gamma)\Delta-\epsilon}^{r^\star-(1-\gamma)\Delta+\epsilon} f_y^l(p) \, dG(y)}_{I_2} + \underbrace{\int_{r^\star-(1-\gamma)\Delta+\epsilon}^{\bar{c}} f_y^l(p) \, dG(y)}_{I_3}. \quad \text{(B.20)}\end{aligned}$$

As $p \searrow r^\star$, integrals I_1 and I_3 become zero at some point, because p falls out of the support of $f_c^l(p)$. And because $f_c^l(p)$ (as a function of c) is bounded in the neighborhood of $r^\star - (1-\gamma)\Delta$, integral I_2 can be made arbitrarily small. It follows once again that $G(c \,|\, p) \to \mathbf{1}_{\{c \geq r^\star - \Delta\}}$.

The intuition this time is a little more tricky. The key observation is that (i) prices just above $r^\star$ are in the support of the distribution of low-cost dealers only when c is close to $r^\star - (1-\gamma)\Delta$, and (ii) even when $c = r^\star - (1-\gamma)\Delta$, prices very close to $r^\star$ are unlikely (density is bounded). For high-cost dealers, prices just above $r^\star$ are in the support only when c is close to $r^\star - \Delta$, but conditional on $c = r^\star - \Delta$, the price is $r^\star$ with probability one. □

Lemma 13. *When $r^\star \in (\underline{c} + \Delta, v)$, $\Psi(p; r^\star)$ jumps down discontinuously at $p = r^\star$.*

Proof. By Lemma 12, as the observed price p converges to $r^\star$, the posterior distribution of costs converges to an atom at $r^\star - \Delta$. Moreover, by the inspection of the proof, when p converges to $r^\star$ from the left, the probability mass converges to an atom at $r^\star - \Delta$ from the left, and when p converges to $r^\star$ from the right, the probability mass converges to an atom at $r^\star - \Delta$ from the right.[55] The price distribution of high-cost

[55] We do not formalize what we mean by "probability mass converges from the left/ right" although

dealers is continuous (for example, in the Lévy-Prokhorov metric) in c. The lemma will be thus proven if we can show that the price distribution of low-cost dealers changes discontinuously at $c = r^\star - \Delta$ in such a way that expected benefits from search jump down at $p = r^\star$. By the above derivation and direct calculation, we show that $F_c^l(p)$ for c in the right neighborhood of $r^\star - \Delta$ strictly first-order stochastically dominates $F_c^l(p)$ for c in the left-neighborhood of $r^\star - \Delta$, and the difference between the cdfs is bounded away from zero. This means that expected continuation value of search jumps down at $c = r^\star - \Delta$. □

The intuition behind Lemma 13 is as follows. When $c < r^\star - \Delta$, high-cost dealers sell when contacted by a slow trader. Thus, low-cost dealers attach a higher probability to the trader being a fast trader and as a result they quote smaller prices. When $c > r^\star - \Delta$, high-cost dealers do not sell when they are visited by a slow trader. Thus, slow traders search more, the posterior probability of a fast trader falls, and low-cost dealers quote higher prices. Thus, there is a discontinuity in expected price, which jumps up at $c = r^\star - \Delta$. When a slow trader sees p just below $r^\star$, she thinks that c is just below $r^\star - \Delta$ and prices are low. When a slow trader sees a price just above $r^\star$, she thinks that c is just above $r^\star - \Delta$ and prices are high. The value of taking the offer is almost the same in both cases, but the value of search is clearly more attractive in the first. Thus, if a slow traders does not want to search at p below $r^\star$, she definitely does not want to search for p just above $r^\star$. As a result, condition (B.18) must fail, that is, we cannot have an equilibrium for $r^\star \in (\underline{c} + \Delta, v)$.

Now we deal with the case $r^\star < \underline{c} + \Delta$. Because $r^\star < c + \Delta$ for all c, all prices p below $r^\star$ observed on equilibrium path must be posted by low-cost dealers (in equilibrium there has to be positive probability of observing a price below $r^\star$). Moreover, by the characterization above, prices $p \in (r^\star, r^\star + \epsilon)$ for small $\epsilon < \underline{c} + \Delta - r^\star$ can only be quoted by low-cost dealers when c is close to $r^\star - (1 - \gamma)\Delta$. In other words, upon observing $p = r^\star + \epsilon$ for small $\epsilon > 0$, a slow trader believes that c is within ϵ of $r^\star - (1 - \gamma)\Delta$. But in this case, price p is within ϵ of the best possible price given the beliefs. Therefore, the benefit from search drops to zero as p crosses $r^\star$ from left to right. Clearly, this contradicts existence of an equilibrium.

We have thus shown that $r^\star = \underline{c} + \Delta$ or $r^\star = v$ in a reservation-price equilibrium.

To conclude the proof of Lemma 10, we argue that when $s < (1 - \alpha(1, 0))\gamma\Delta$, we cannot have an equilibrium with $r^\star = v$. We prove this by showing that in this case a slow trader wants to search when observing a price offer at or slightly below v. By the argument used in Lemma 12, we show that the posterior cost distribution converges to an atom at $v - \Delta$ as p converges to v. We can then calculate the benefit from search explicitly using the price distribution derived above. We obtain $\Psi(v; v) = (1 - \alpha(1, 0))\gamma\Delta$, which is also the right limit of $\Psi(p; v)$ as $p \nearrow v$. This produces a contradiction with condition (B.18).

this could be done easily. The point is that the probability mass is centered around $p - \Delta$ for p close to $r^\star$.

B.8.2 Proof of Proposition 12

The proposition follows directly from what has been shown above. We know that when $s < (1 - \alpha(1, 0))\gamma\Delta$, we cannot have an equilibrium with $r^\star = v$. Thus, we can only have $r^\star = \underline{c} + \Delta$. The condition $s \in [\underline{s}, \bar{s}]$ is equivalent to condition (B.18). High-cost dealers quote $c+\Delta$, and low-cost dealers use a continuous distribution corresponding to the cases $c \in [r^\star - \Delta, r^\star - \kappa\Delta]$, $c \in [r^\star - \kappa\Delta, r^\star - (1-\gamma)\Delta]$, and $c \in [r^\star - (1-\gamma)\Delta, \bar{c}]$. Entry can be analyzed in the same way as in previous equilibrium constructions (it can be shown that slow traders enter with probability one under the assumptions that we have imposed).

When $r^\star = \underline{c} + \Delta$, continuity of $\Psi(p; r^\star)$ at all points p in the support other than $r^\star$ is easy to show by direct inspection. We prove that $\Psi(p; r^\star)$ jumps up at $p = r^\star$.

When p converges to $r^\star$ from the right, the posterior distribution of costs converges to an atom at $r^\star - \Delta = \underline{c}$. In that case, we can calculate $\lim_{p \searrow r^\star} \Psi(p; r^\star)$ explicitly. We have

$$\lim_{p \searrow r^\star} \Psi(p; r^\star) = (1 - \alpha(1, 1))\gamma\Delta.$$

This means that $\bar{s} \leq (1 - \alpha(1, 1))\gamma\Delta$.

When p converges to $r^\star$ from the left, the benefit from search must converge to a number that is strictly lower than $(1 - \alpha(1, 1))\gamma\Delta$. The reason is that in this case the trader believes that the offer has been posted by a low-cost dealer, and thus the posterior distribution of costs will be atomless with support $[\underline{c}, r^\star - (1 - \gamma)\Delta]$. Since prices are increasing with costs, the trader expects that prices are higher than in the case in which the cost c is equal to $\underline{c}$. Unfortunately, a closed form solution for $\lim_{p \nearrow r^\star} \Psi(p; r^\star)$ is hard to obtain because the expected price becomes non-linear in c for $c \in [r^\star - \kappa\Delta, r^\star - (1 - \gamma)\Delta]$.

B.8.3 Proof of Corollary 2

Under the assumption $s \geq \kappa(1 - \hat{\alpha})\gamma\Delta$, the equilibrium with the benchmark achieves the second best. In the no-benchmark case, when $s \in [\underline{s}, \bar{s}]$, we have a reservation-price equilibrium with $r_0^\star = \underline{c} + \Delta$. By the derivation of equilibrium pricing strategies from the proof of Lemma 10, whenever $c > \underline{c} + \gamma\Delta$, low-cost dealers quote prices above $\underline{c}+\Delta$. With probability γ a slow trader visits a low-cost dealer in the first search round. Therefore, the expected surplus loss in the no-benchmark case relative to the second best is at least

$$(1 - G(\underline{c} + \gamma\Delta))(1 - p_0)\gamma s. \tag{B.21}$$

On the other hand, the surplus gain from moving from the second best to centralized exchange is $(1-p_0)(2-\gamma)s$. Dividing (B.21) by the sum of (B.21) and $(1-p_0)(2-\gamma)s$ we conclude the proof of the Corollary.

C Supporting Contents for Section 5

C.1 Proof of Theorem 4

The proof of Theorem 4 is very similar to the proof of Theorem 2, so we skip some of the details. Denote the expected profits of a dealer in the benchmark case conditional on x (where $x = (v - c)_+$) by $\chi_b(x)$ and in the case with no benchmark by χ_{nb}. Recall from Propositions 1 and 3 that

$$\chi_b(x) = \frac{\lambda(x)(1 - p_0)}{N} \frac{s}{1 - \alpha(\lambda(x))}$$

and $\chi_{nb} = X\lambda^\star(1 - p_0)/N$.

Assume that condition (i) holds. Then, using the fact that $\lambda(x)$ is given by $s = (1-\alpha(\lambda(x)))x$ in the relevant range, we can write $\chi_b(x) = (1 - p_0)\lambda(x)x/N$. By Lemma 4, $\lambda(x)$ is increasing and convex, so $\chi_b(x)$ is also convex. Therefore, applying Jensen's Inequality we get

$$\mathbb{E}\left[\chi_b(x)\right] \geq \chi_b\left(\mathbb{E}\left[x\right]\right) = \chi_b(X) = \chi_{nb}.$$

Now assume that condition (ii) holds. As in the proof of Theorem 2 we want to find a condition on X that would guarantee that the profit function χ_b is subdifferentiable at X. Using the reasoning from the proof of Theorem 2, we can establish existence of a constant $\eta \in (0, \hat{\alpha})$, that depends only on p_0 and N, and such that $X \leq s/(1 - \eta)$ guarantees existence of a supporting hyperplane at X (thus allowing us to apply Jensen's Inequality).

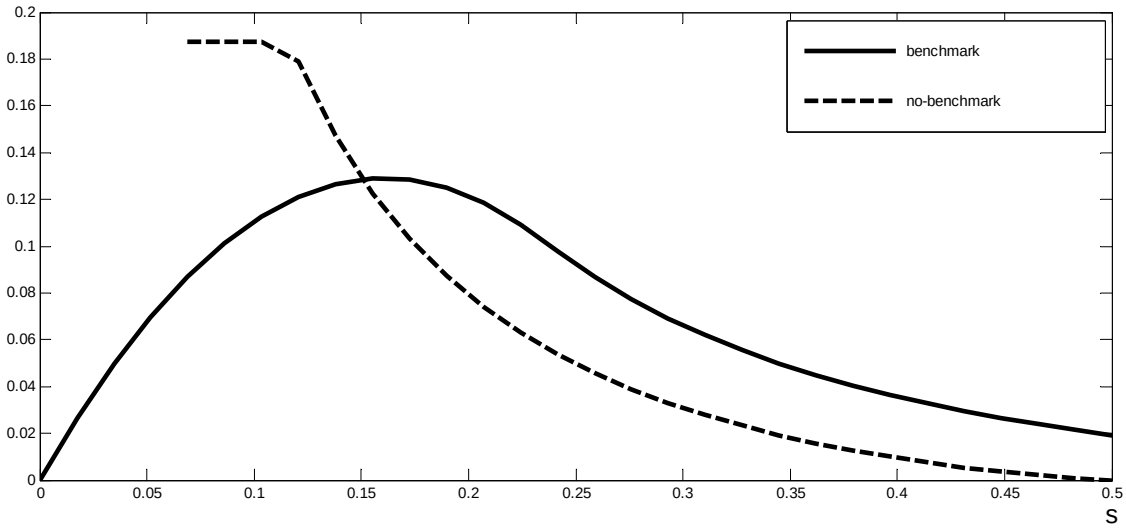
C.2 A numerical example for Section 5.1

We revisit the numerical example of Section A.8. We recall that the reservation-price equilibrium in the no-benchmark case exists if $s \geq 0.063$, and that social surplus is higher in the benchmark case for $s \geq 0.12$. Figure C.1 depicts the total profit of a dealer as a function of the search cost s in the benchmark case (with a thick solid line) and in the no-benchmark case (with a thick dotted line). The profits of dealers are higher in the benchmark case if $s \geq 0.15$. For search costs in the interval $(0.12, 0.15)$, social surplus rises with the introduction of a benchmark, but dealers' profits are reduced.

C.3 Completion of the analysis of Section 5.1—What changes if dealers' costs are heterogeneous?

If $\gamma \in (0, 1)$, a reservation-price equilibrium with $r^\star = v$ exists in the no-benchmark setting if search costs are sufficiently large. (See the discussion under Lemma 10 in Section B.7.) Here, we might use this equilibrium to ask if it is possible that the introduction of the benchmark benefits one type of the dealers but not the other (through the entry channel, that is, when search costs are relatively high).

Fig. C.1: The dependence of dealers' expected profits on the slow-trader search cost s.



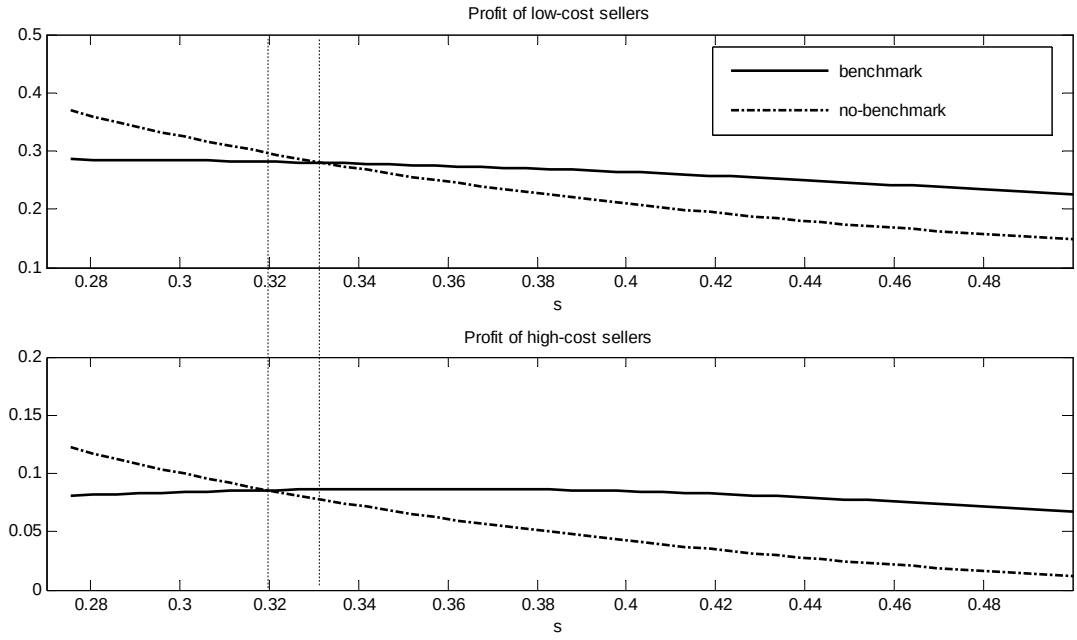
The answer turns out to be generally no.[56] When the benchmark increases entry sufficiently, both types of dealers benefit. When the benchmark fails to encourage entry (for example because we have already full entry without the benchmark), profits of both types of dealers are harmed. This is intuitive. Under parameter restrictions that guarantee full entry and existence, reservation-price equilibria will have no search in both cases, so the volumes of trade remain the same for both types of dealers. Prices generally decrease. Thus, if entry does not increase, introducing a benchmark acts as a transfer of surplus from dealers to traders.

There is however one case in which high-cost dealers would opt for a benchmark while low-cost dealers would not. Just as in the homogeneous-cost case, dealers prefer to trade under the benchmark if the search cost of slow traders exceeds a certain cutoff. The threshold for high-cost dealers will be slightly lower that for low-cost dealers. To understand this observation, recall from Section 3 that the benchmark has the effect of increasing entry especially in the case when gains from trade are large. Also in this case (that is, when gains from trade are large) high-cost dealers trade in equilibrium. In the opposite case (when $c > v - \Delta$), high-cost dealers cannot trade anyway, and are thus not harmed by relatively smaller entry in the benchmark case for high cost realizations.

Figure C.2 illustrates the above point. We take the same numerical example as in Section 4. When search costs are relatively small, the profits of dealers are larger when there is no benchmark. When s gets bigger, the positive effect of benchmarks on entry gets strong enough for dealers to benefit from increased volume of trade. Once s crosses 0.32, high-cost dealers would like to introduce the benchmark. For low-cost dealers, the corresponding threshold is slightly above 0.33.

[56] For a formal result, see Appendix E.

Fig. C.2: The profits of low-cost and high-cost dealers (in reservation-price equilibria for intermediate and large s)



C.4 Proof of Theorem 5

To prove the Theorem, we first describe the equilibrium path, and then show the optimality of dealers' strategies.

If the environment is efficient, the benchmark is introduced, only low-cost dealers enter and we have a reservation-price equilibrium in the trading-stage subgame described in Section 3.1 (with the exception that N is now replaced by M, which is equal to L in equilibrium). Under the assumption that $s < (1-\bar{\alpha})(v-\bar{c})$, we have full entry in this case, and the reservation price of slow-traders is

$$r_c^\star = c + \frac{s}{1-\bar{\alpha}_L},$$

where the subscript L in $\bar{\alpha}_L$ indicates that N is replaced by L in the definition of $\bar{\alpha}$ given by equation (3.7).

If the environment is inefficient, the benchmark is not introduced, and all dealers enter. In this case, we have the equilibrium described in Section 3.2 with the exception that c is replaced by $c+\Delta$ (that is, X is replaced by X_Δ). High-cost dealers make positive expected profits. Existence follows directly from Proposition 4 by the assumption that Δ is sufficiently large.

On the equilibrium path in the pre-play stage, low-cost dealers vote in favor of the benchmark, and always enter (regardless of whether the benchmark is introduced

or not). High-cost dealers vote against the benchmark and enter if and only if the benchmark is not introduced.

We now verify the optimality of these dealer strategies.

First, we show that a high-cost dealer does not want to deviate and enter when the benchmark is introduced. Indeed, when the benchmark is observed, slow traders follow a reservation-price strategy with

$$r_c^\star = c + \frac{s}{1-\bar{\alpha}_M} \leq c + \frac{s}{1-\bar{\alpha}},$$

using the fact that $\bar{\alpha}_M$ is increasing in M.[57] Since $v - \bar{c} - \Delta < 0$ and $s < (1-\bar{\alpha})(v-\bar{c})$, we conclude that $c + \Delta > r_c^\star$. Thus, using familiar arguments from previous sections, we show that a high-cost dealer cannot make positive profits after entering the market, regardless of the identities of other dealers in the market.[58]

Second, we show that a high-cost dealer does not want to deviate and stay out of the market when the benchmark is not introduced. By the remark above, high-cost dealers make strictly positive profits on the equilibrium path.

Third, low-cost dealers always make positive profits under the assumptions of the theorem, and thus they cannot deviate by not entering, regardless of whether there is a benchmark or not.

Fourth, we show that any coalition of high-cost dealers does not want to deviate by voting in favor of the benchmark. By what we established above, if the benchmark is introduced, a high-cost dealer finds it optimal not to enter. When there is no benchmark, she makes positive expected profits.

Fifth, we show that any coalition of low-cost dealers does not want to deviate by voting against the benchmark. Note that $L \geq 2$ is common knowledge among low-cost dealers. In equilibrium, the benchmark is introduced, high-cost dealers stay out, and the low cost dealer's expected profit is equal to

$$\frac{1-p_0}{L}\frac{s}{1-\bar{\alpha}_L} > 0,$$

and it does not depend on Δ. If the benchmark is not introduced, slow traders believe with probability one that only high-cost dealers are present in the market. If Δ is large enough (that is, $X_\Delta - s$ is small enough), the equilibrium probability of entry by slow traders is arbitrarily small (see the analysis in Section 3.2). Because $L \geq 2$ the expected profits of low-cost dealers in this case converge to zero as the posterior probability of meeting a slow trader approaches zero. Because the profit on equilibrium path is bounded away from zero, we conclude that the low-cost dealers do not want to

[57] This is shown in Janssen and Moraga-González (2004).

[58] Note that off-equilibrium path traders may observe offers above their reservation price, something that never happens on equilibrium path. We specify off-equilibrium beliefs of traders by saying that this off-equilibrium event does not change the belief of any trader about the types of active dealers. This is consistent with a perfect Bayesian equilibrium.

deviate in this way.

D Proofs for Section 6

D.1 Proposition 13

Proposition 13. *In the equilibrium setting with a benchmark described in Proposition 1, if customers do not anticipate manipulation, the best deviation for dealers is to announce that the benchmark outcome is* $\hat{c} = v - s/(1-\bar{\alpha})$ *regardless of the actual outcome of the benchmark* c.

Proof. The proof follows directly from Proposition 1. Assuming that customers do not anticipate manipulation and simply play their equilibrium strategy believing that the outcome of c is $\hat{c}$, the expected profits of dealers can be shown to be

$$\frac{\lambda_{\hat{c}}(1-p_0)}{N}\left(r_{\hat{c}} - c\right).$$

(Dealers use the offer price distribution that arises when slow traders follow a reservation-price strategy with reservation price $r^\star_{\hat{c}}$ and when the true cost is c). Maximizing the above expression over $\hat{c}$ conditional on c, we obtain the conclusion of the Proposition. □

D.2 Proof of Proposition 10

To prove the first part of the Proposition, consider the following direct revelation mechanism, in which the message space of every dealer is the support of the distribution of c. The mechanism designer asks each dealer to report the true market-wide cost c. Conditional on the report $\hat{c}_i$ of dealer i, and the reports of other dealers $\hat{c}_{-i}$, the transfer from dealer i to the mechanism designer is

$$t_i(\hat{c}_i,\, \hat{c}_{-i}) = \begin{cases} 0 & \text{if } \hat{c}_i = \frac{1}{N-1}\sum_{j\neq i}\hat{c}_j \\ t & \text{otherwise.} \end{cases}$$

Then, reporting the true cost c is a Nash equilibrium if t is large enough (the benefit from misreporting is bounded). Given truthful reporting by dealers, the mechanism designer can reveal c to customers by announcing $N^{-1}\sum_{i=1}^{N}\hat{c}_i$.

The second part follows from the fact that choice rule to be implemented is not monotonic. (See Maskin 1999 for the definition of monotonicity and the relevant result.)

E Generalization of Theorems 1 and 4

This appendix generalizes Theorems 1 and 4 by relaxing the assumption that $\gamma = 1$.

Theorem 6. *Consider the model with heterogeneous dealers' costs.*[59] *Suppose that (i)* $s \geq \gamma\Delta$ *and (ii)*

$$\frac{s - (1 - \alpha(1,\, 0))\gamma\Delta}{\gamma\,(1 - \phi(1)\alpha(1,\, 0)) + (1 - \gamma)\,(1 - \alpha_h(1))} \geq v - \underline{c} - \Delta.$$

Then, a reservation-price equilibrium in the no-benchmark case (if it exists) yields a lower social surplus than in the setting with the benchmark. Moreover, if $\bar{c} \leq v - \Delta$*, then the expected profits of both high-cost and low-cost dealers are higher in the setting with the benchmark. Condition (ii) holds if there are sufficiently many dealers or if the fraction* p_0 *of fast traders is small enough.*

Remark. Notice that when $\gamma \to 1$, condition (ii) boils down to

$$\frac{s}{(1 - \alpha(1,\, 0))} \geq v - \underline{c}.$$

Because $\alpha(1,\, 0) = \bar{\alpha}$, this is exactly condition (i) from Theorem 1 and Theorem 4.

Proof. Because the logic of the proof is the same as in the case of homogeneous dealers' costs, we sketch the main arguments and omit most calculations. Unless stated otherwise, the symbols that we use have the same meaning as in the proof of Theorem 1. We begin by describing the welfare and profits in the two settings. Without loss of generality we assume that $\bar{c} \leq v$.[60]

Benchmark setting. By the equilibrium characterization from the proof of Proposition 7 in Appendix B (cases 1 and 2.2.2), under the parameter restrictions of the Theorem we can have two types of equilibria with the benchmark. When

$$s - (1 - \alpha(0,\, 0))\gamma\Delta \geq x - \Delta,$$

we have no entry of slow traders (that is, $\lambda(x) = 0$). Therefore, social welfare is equal to $p_0\left(x - (1 - \gamma)^N\Delta\right)$. High-cost dealers make no profits, and low-cost dealers have expected profits equal to $(1 - \gamma)^{N-1}\min\{\Delta,\, x\}$. When

$$s - (1 - \alpha(0,\, 0))\gamma\Delta < x - \Delta, \tag{E.1}$$

there is interior entry of slow traders, determined by the equation

$$s = \gamma\left[\left(1 - \phi\left(\lambda\right)\alpha\left(\lambda,\, 0\right)\right)x - \alpha\left(\lambda,\, 0\right)\left(1 - \phi\left(\lambda\right)\right)\Delta\right] + (1 - \gamma)\left[\left(1 - \alpha_h\left(\lambda\right)\right)\left(x - \Delta\right)\right], \tag{E.2}$$

[59] That is, $\gamma \in (0,\, 1)$, $\Delta > 0$.

[60] In the opposite case, the comparison can only be even more favorable for the benchmark case.

suppressing from the notation the argument x of $\lambda(x)$. Because slow traders buy from the first dealer, social welfare (as a function of gains from trade) is

$$W_b(x) = p_0\left(x - (1-\gamma)^N\Delta\right) + \lambda(x)(1-p_0)\left(x - (1-\gamma)\Delta - s\right). \tag{E.3}$$

(We note that by equation (E.1), $c \leq v - \Delta$). The expected profits of low-cost dealers are

$$\chi_b^l(x) = \left[\frac{\lambda(x)(1-p_0)}{N} + p_0(1-\gamma)^{N-1}\right]\left[\phi\left(\lambda(x)\right)(x-\Delta) + \Delta\right]. \tag{E.4}$$

The expected profits of high-cost dealers are

$$\chi_b^h(x) = \frac{\lambda(x)(1-p_0)}{N}(x-\Delta). \tag{E.5}$$

No-benchmark setting. In the no-benchmark setting we concentrate on the reservation-price equilibrium with a reservation price (of slow traders) equal to v. (See the comment below Lemma 10 in Section B.7 for details.[61]) This equilibrium exists if search costs are sufficiently large. Under the parameter restrictions of the Theorem we can have two types of equilibria without the benchmark.

First, consider the case $c < v - \Delta$, in which we have an equilibrium analogous to that under the benchmark. In particular, conditional on cost realization $x > \Delta$, social surplus is

$$W_{nb}(x) = p_0\left(x - (1-\gamma)^N\Delta\right) + \lambda^\star(1-p_0)\left(x - (1-\gamma)\Delta - s\right),$$

where $\lambda^\star$ denotes the equilibrium probability of entry of slow traders (in this case a constant, not a function of x). Low-cost dealers make conditional expected profits

$$\chi_{nb}^l(x) = \left[\frac{\lambda^\star(1-p_0)}{N} + p_0(1-\gamma)^{N-1}\right]\left[\phi\left(\lambda^\star\right)(x-\Delta) + \Delta\right],$$

and high-cost dealers earn

$$\chi_{nb}^h(x) = \frac{\lambda^\star(1-p_0)}{N}(x-\Delta).$$

Second, consider $c \geq v - \Delta$. Now high-cost dealers cannot make positive profits, so they post a deterministic price equal to $c + \Delta$. Social surplus conditional on x is given by

$$W_{nb}(x) = p_0\left(1 - (1-\gamma)^N\right)x + \lambda^\star(1-p_0)(\gamma x - s).$$

[61] As explained in Section B.7, although $N = 2$ is assumed throughout that subsection, the characterization of equilibrium with $r^\star = v$ is valid for an arbitrary N.

Low-cost dealers have a conditional expected profit of

$$\chi_{nb}^{l}(x) = \left[\frac{\lambda^\star(1-p_0)}{N} + p_0(1-\gamma)^{N-1}\right] x.$$

We now turn our attention to traders. When $x > \Delta$, conditional on x and entry, a slow trader has an expected profit of

$$-s + \gamma\left[(1-\phi(\lambda^\star)\alpha(\lambda^\star,\,0))\,x - \alpha(\lambda^\star,\,0)(1-\phi(\lambda^\star))\Delta\right] + (1-\gamma)\left[(1-\alpha_h(\lambda^\star))\,(x-\Delta)\right].$$

When $x \leq \Delta$, the corresponding expected profit is

$$-s + \gamma(1-\alpha(\lambda^\star,\,0))x.$$

Because slow traders do not observe c when there is no benchmark, their entry decision is determined by taking an expectation with respect to the distribution of x. Thus, $\lambda^\star$ solves

$$s = G(v-\Delta)\mathbb{E}\left[\Lambda(\lambda^\star, x)\,|\,x > \Delta\right] + (1-G(v-\Delta))\,\mathbb{E}\left[\gamma(1-\alpha(\lambda^\star,\,0))x\,|\,x \leq \Delta\right], \quad \text{(E.6)}$$

where

$$\Lambda(\lambda^\star, x) = \gamma\left[(1-\phi(\lambda^\star)\alpha(\lambda^\star,\,0))\,x - \alpha(\lambda^\star,\,0)(1-\phi(\lambda^\star))\Delta\right] + (1-\gamma)\left[(1-\alpha_h(\lambda^\star))\,(x-\Delta)\right].$$

A few steps to simplify the problem. First, we notice that the comparison of welfare can only be more favorable for the no-benchmark case when we condition on $c \leq v-\Delta$. As for the profits of dealers, we have $\bar{c} \leq v-\Delta$ by assumption. Thus from now on, we assume without loss of generality that $\bar{c} \leq v-\Delta$. This simplifies the formulas in the no-benchmark setting. Unconditional expected welfare can now be written as

$$W_{nb} = p_0\left(X - (1-\gamma)^N\Delta\right) + \lambda^\star(1-p_0)\left(X - (1-\gamma)\Delta - s\right). \quad \text{(E.7)}$$

The expected profit of low-cost dealers is

$$\chi_{nb}^{l} = \left[\frac{\lambda^\star(1-p_0)}{N} + p_0(1-\gamma)^{N-1}\right]\left[\phi\left(\lambda^\star\right)(X-\Delta) + \Delta\right]. \quad \text{(E.8)}$$

The expected profit of high-cost dealers is

$$\chi_{nb}^{h} = \frac{\lambda^\star(1-p_0)}{N}(X-\Delta). \quad \text{(E.9)}$$

Finally, the key equation (E.6) determining entry in the no-benchmark setting simplifies to

$$s = \gamma\left[(1-\phi(\lambda^\star)\alpha(\lambda^\star,0))\,X - \alpha(\lambda^\star,0)(1-\phi(\lambda^\star))\Delta\right] + (1-\gamma)\left[(1-\alpha_h(\lambda^\star))\,(X-\Delta)\right]. \tag{E.10}$$

Note that, by equations (E.2) and (E.10), $\lambda(X) = \lambda^\star$.

The crucial (and most tedious) step in the proof is to show the following lemma, which generalizes Lemma 4.

Lemma 14. *The function $\lambda(x)$ is convex (for x in the range permitted by the assumptions of the Theorem).*

Before we show the proof of Lemma 14, we analyze its consequences. It is easy to observe that if $\lambda(x)$ is convex, then also $W_b(x)$, $\chi_b^l(x)$ and $\chi_b^h(x)$ are convex.[62] Hence, we can apply Jensen's Inequality to these three functions (just as in the proofs of Theorem 1 and Theorem 4). The observation made above that $\lambda(X) = \lambda^\star$, and direct inspection of formulas (E.3) and (E.7), (E.4) and (E.8), (E.5) and (E.9), complete the proof.

Proof of Lemma 14. The full proof is long and tedious, and thus some of the details are omitted.

We rewrite equation (E.2), suppressing from the notation the dependence of λ on x, as

$$s \equiv \gamma\left[(1-\alpha_l(\lambda))\,x - (1-\tilde{\alpha}_l(\lambda))\Delta\right] + (1-\gamma)\left[(1-\alpha_h(\lambda))\,x - (1-\alpha_h(\lambda))\Delta\right], \tag{E.11}$$

where

$$\alpha_l(\lambda) = \phi(\lambda)\alpha(\lambda,0)$$

and

$$\tilde{\alpha}_l(\lambda) = 1 - \alpha(\lambda,0)(1-\phi(\lambda)).$$

A real-valued function $f(x)$ is strictly increasing and convex if and only if $f^{-1}(x)$ is strictly increasing and concave. Thus, to show that the solution $\lambda(x)$ of equation (E.11) is convex, it is enough to prove that the function $\lambda \mapsto x(\lambda)$, defined by

$$x(\lambda) = \frac{s + (\gamma(1-\tilde{\alpha}_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda)))\,\Delta}{\gamma(1-\alpha_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda))},$$

is concave. We have

$$\begin{aligned} x(\lambda) &= \frac{s}{\gamma(1-\alpha_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda))} + \Delta + \frac{\alpha_l(\lambda) - \tilde{\alpha}_l(\lambda)}{\gamma(1-\alpha_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda))}\gamma\Delta \\ &= \frac{s-\gamma\Delta}{\gamma(1-\alpha_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda))} + \Delta + \frac{1+\alpha_l(\lambda) - \tilde{\alpha}_l(\lambda)}{\gamma(1-\alpha_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda))}\gamma\Delta. \end{aligned}$$

[62] In case of $\chi_b^l(x)$ this requires some calculation that we omit.

Because a sum of concave functions is concave, and due to $s \geq \gamma\Delta$, a sufficient condition for concavity of $x(\lambda)$ is that

$$\frac{1}{\gamma(1-\alpha_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda))} \tag{E.12}$$

and

$$\frac{1+\alpha_l(\lambda)-\tilde{\alpha}_l(\lambda)}{\gamma(1-\alpha_l(\lambda)) + (1-\gamma)(1-\alpha_h(\lambda))} \tag{E.13}$$

are both concave in λ. We show formally the concavity of the first of these functions and omit a similar proof of concavity of the second one.

Because the function (E.12) is twice continuously differentiable, concavity is implied by the second derivative being non-positive. The second derivative of the function (E.12) is non-positive if and only if (omitting for simplicity the notational dependence on λ)

$$\left[\gamma\alpha_l'' + (1-\gamma)\alpha_h''\right]\left(\gamma(1-\alpha_l) + (1-\gamma)(1-\alpha_h)\right) + 2\left(\gamma\alpha_l' + (1-\gamma)\alpha_h'\right)^2 \leq 0.$$

Expanding this inequality, we can obtain a sufficient condition by requiring that each of the terms multiplied by γ^2, $\gamma(1-\gamma)$, and $(1-\gamma)^2$, accordingly, is non-positive:

$$\alpha_l''(1-\alpha_l) + 2\left(\alpha_l'\right)^2 \leq 0, \tag{E.14}$$

$$\alpha_h''(1-\alpha_h) + 2\left(\alpha_h'\right)^2 \leq 0, \tag{E.15}$$

$$\alpha_l''(1-\alpha_h) + \alpha_h''(1-\alpha_l) + 4\alpha_l'\alpha_h' \leq 0. \tag{E.16}$$

Inequalities (E.14) and (E.15) are proven in exactly the same way as inequality (A.4) in the proof of Lemma 4. We show how to prove inequality (E.16). Using the definitions of functions α_l and α_h, and after a tedious calculation of the first and second derivatives, we can express the inequality equivalently as

$$2\left(\int_0^1 \frac{\beta(1-\gamma)^{N-1} + \beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)}{\left(\lambda + \beta(1-\gamma)^{N-1} + \beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)\right)^2}\,dz\right)\left(\int_0^1 \frac{\beta(1-\gamma)^{N-1}z^{N-1}}{\left(\lambda+\beta(1-\gamma)^{N-1}z^{N-1}\right)^2}\,dz\right)$$

$$\leq \underbrace{\left(\int_0^1 \frac{\beta(1-\gamma)^{N-1} + \beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)}{\left(\lambda + \beta(1-\gamma)^{N-1} + \beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)\right)^3}\,dz\right)}_{a}\underbrace{\left(\int_0^1 \frac{\beta(1-\gamma)^{N-1}z^{N-1}}{\lambda+\beta(1-\gamma)^{N-1}z^{N-1}}\,dz\right)}_{b}$$

$$+ \underbrace{\left(\int_0^1 \frac{\beta(1-\gamma)^{N-1}z^{N-1}}{\left(\lambda+\beta(1-\gamma)^{N-1}z^{N-1}\right)^3}\,dz\right)}_{c}\underbrace{\left(\int_0^1 \frac{\beta(1-\gamma)^{N-1} + \beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)}{\lambda + \beta(1-\gamma)^{N-1} + \beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)}\,dz\right)}_{d}.$$

By Hölder's Inequality applied twice to the two integrals on the left-hand side, we get

$$2\left(\int_0^1 \frac{\beta(1-\gamma)^{N-1}+\beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)}{\left(\lambda+\beta(1-\gamma)^{N-1}+\beta\left(1-(1-\gamma)^{N-1}\right)\Phi(z)\right)^2}\,dz\right)\left(\int_0^1 \frac{\beta(1-\gamma)^{N-1}z^{N-1}}{\left(\lambda+\beta(1-\gamma)^{N-1}z^{N-1}\right)^2}\,dz\right)$$

$$\leq 2\sqrt{ad}\sqrt{bc}.$$

Thus, we have to prove that for any positive constants a, b, c and d, we have

$$2\sqrt{ad}\sqrt{bc} \leq ab+cd.$$

This follows immediately from the fact that $(ab-cd)^2 \geq 0$.

F Glossary of symbols

Symbol	Definition	Remarks
Primitive parameters		
N	number of dealers	$N \geq 2$
G	cdf of common dealer cost c	with support $[\underline{c}, \bar{c}]$
Δ	cost differential between high-cost and low-cost dealers	idiosyncratic cost ϵ_i is 0 or Δ
γ	ex-ante probability of a low-cost dealer	$P(\epsilon_i = 0) = \gamma$
p_0	fraction of fast traders	$p_0 \in (0, 1)$
s	search cost of slow traders	$s > 0$
v	traders' asset valuation	a constant
Derived quantities		
λ	slow-trader entry probability	An additional subscript 'c' indicates that the quantity depends on c (in the benchmark case); the superscript * denotes the quantity in equilibrium.
r	reservation price of slow traders	
θ	probability that a slow trader buys from a high-cost dealer	
F_c^i	cdf of dealer offers, $i \in \{l, h\}$	lower limit: $\underline{p}_c^i$; upper limit: $\bar{p}_c^i$
$q(\lambda, \theta)$	$\frac{Np_0}{Np_0 + \frac{1-\theta^N(1-\gamma)^N}{1-\theta(1-\gamma)}\lambda(1-p_0)}$	probability that a contacting trader is fast
$\alpha(\lambda, \theta)$	$\int_0^1 \left(1 + \frac{q(\lambda,\theta)\left(1-(1-\gamma)^{N-1}\right)}{1-q(\lambda,\theta)(1-(1-\gamma)^{N-1})}\Phi(z)\right)^{-1} dz$	values between 0 and 1, strictly increasing in both arguments
$\Phi(z)$	$\frac{\sum_{k=1}^{N-1}\binom{N-1}{k} z^k \gamma^k (1-\gamma)^{N-1-k}}{1-(1-\gamma)^{N-1}}$	strictly increasing polynomial, $\Phi(0) = 0$, $\Phi(1) = 1$.
$\hat{\alpha}$	$\alpha(1, 1)$	upper bound on $\alpha(\lambda, \theta)$; strictly below 1
κ	$\frac{(1-\gamma)^{N-1}(p_0 + \lambda^\star(1-p_0))}{p_0(1-\gamma)^{N-1} + \lambda^\star(1-p_0)\frac{1-(1-\gamma)^N}{N\gamma}}$	Special cases: $\frac{(1-\gamma)^{N-1}}{p_0(1-\gamma)^{N-1} + (1-p_0)\frac{1-(1-\gamma)^N}{N\gamma}}$ when $\lambda^\star = 1$, $\frac{2(1-\gamma)(p_0+\lambda^\star(1-p_0))}{\lambda^\star(2-\gamma)(1-p_0)+2p_0(1-\gamma)}$ when $N = 2$.
Special case of homogeneous dealers' cost ($\gamma = 1$)		
$q(\lambda)$	$\frac{Np_0}{Np_0 + \lambda(1-p_0)}$	equal to $q(\lambda, 0)$
$\alpha(\lambda)$	$\int_0^1 \left(1 + \frac{Np_0}{\lambda(1-p_0)} z^{N-1}\right)^{-1} dz$	equal to $\alpha(\lambda, 0)$
$\bar{\alpha}$	$\alpha(1)$	upper bound on $\alpha(\lambda)$; strictly below 1
X	$G(v)\left[v - \mathbb{E}\left[c \middle\vert c \leq v\right]\right]$	expected gain from trade
$\varphi(\lambda)$	$\frac{\lambda(1-p_0)}{Np_0 + \lambda(1-p_0)}$	-

Symbols used only in appendices		
ζ	probability that a low-cost dealer quotes an offer above the reservation price of slow traders	An additional subscript 'c' indicates that the quantity depends on c (in the benchmark case); the superscript * denotes the quantity in equilibrium.
X_Δ	$G(v-\Delta)\mathbb{E}\left(v-c-\Delta \middle\vert c \leq v-\Delta\right)$	Expected gain from trade when only high-cost dealers are present
$\vartheta(\zeta)$	$\frac{(1-\gamma)^{N-1}}{p_0(1-\gamma\zeta)^{N-1}+(1-p_0)\frac{1-(1-\gamma\zeta)^N}{N\gamma\zeta}}$	$\vartheta(1)=\kappa$
$\tilde{\alpha}(\zeta)$	$\int_0^1 \left(1+\frac{1-(1-\gamma\zeta)^{N-1}}{(1-\gamma)^{N-1}}p_0\vartheta(\zeta)\Phi(z;\,\zeta)\right)^{-1} dz$	$\tilde{\alpha}(1)=\alpha(1,\,1)$
$\Phi(z;\,\zeta)$	$\frac{\sum_{k=1}^{N-1}\binom{N-1}{k}z^k(\gamma\zeta)^k(1-\gamma\zeta)^{N-1-k}}{1-(1-\gamma\zeta)^{N-1}}$	strictly increasing polynomial, $\Phi(0;\,\zeta)=0$, $\Phi(1;\,\zeta)=1$
$\alpha_h(\lambda)$	$\int_0^1 \left(1+\frac{q(\lambda,0)(1-\gamma)^{N-1}}{1-q(\lambda,0)}z^{N-1}\right)^{-1} dz$	$\alpha_h(0)=0$
$\phi(\lambda)$	$\frac{1-q(\lambda,0)}{1-(1-(1-\gamma)^{N-1})q(\lambda,0)}$	$\phi(0)=0$