Structural Change with Long-Run Income and Price Effects*

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Abstract

We present a multi-sector model of growth that accommodates long-run real income effects and sectoral price trends. Consistent with post-war data from OECD countries, our model generates constant aggregate growth rates, constant interest rates and non-constant sectoral expenditure shares (non-homothetic Engel curves). Our model is consistent with the decline in agriculture, the hump-shaped evolution of manufacturing and the rise of services both in nominal and real terms. We estimate the demand system derived from the model –which takes a simple log-linear form– using historical data on sectoral shares from 30 different countries and household survey data. We show that our model parsimoniously accounts for the broad patterns of sectoral reallocation observed among rich, miracle and developing economies in the post-war period.

Keywords: Structural Transformation, Non-homothetic preferences,
JEL Classification: E2, O1, O4, O5.

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1 Introduction

Economies undergo large sectoral reallocations of capital and labor as they develop, a phenomenon known as structural transformation (Kuznets, 1973; Maddison, 1980; Vries et al., 2014). These reallocations along the growth process lead to a gradual reduction of the importance of agriculture and an increase in manufacturing. As growth progresses, services eventually emerge as the largest sector in the economy (Herrendorf et al., 2014). Understanding these sweeping changes in sectoral composition along the development path constitutes a core theme of economic growth. Two leading theories of structural transformation focus on mechanisms involving production and demand. Production-side theories explain the reallocation patterns as a result of variations in the rate of technological growth across sectors, as reflected in sectoral prices (Baumol, 1967; Ngai and Pissarides, 2007).[1] Demand-side theories emphasize the role of heterogeneous sectoral income elasticities (non-homothetic demand) in driving structural transformation as an economy grows and becomes wealthier, e.g., Kongsamut et al., 2001.

These two theories have testable predictions for the shape of Engel curves, which can help identify their relevance.[2] If structural transformations are driven by price effects as predicted by supply-side theories, sectoral expenditure shares should be uncorrelated with income (or aggregate consumption) after controlling for relative prices. If instead, structural transformation is driven by wealth effects, sectoral Engel curves should not be flat after controlling for relative prices. In agriculture, the slope should be declining in aggregate consumption, while for services the slope should be increasing. Indeed, since the form of non-homotheticity generally assumed in standard models is transitory, the slope of sectoral Engel curves should vanish over time as the expenditure shares should become eventually constant and the Engel curve, flat. Which of these predictions are borne by the data?

To explore this question, Figure 1 plots the relationship between the (log) expenditure share in agriculture (1a), manufacturing (1b) and services (1c) against (log) consumption in the United States. In all plots, we have controlled for sectoral relative prices. This exercise reveals interesting findings.[3] First, residual expenditure shares and residual consumption are strongly correlated in agricultural goods and services. In the former, the relationship is negative while in the latter it is positive. Second, Engel curves are roughly linear. That is, despite being an advanced country that started its structural transformation almost a century before our sample, the structural transformation in the U.S. has continued at a

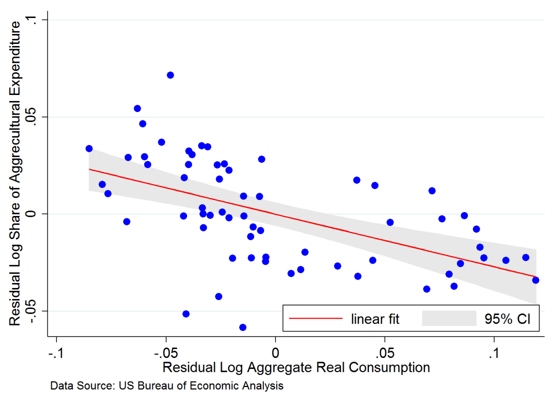
[1] Acemoglu and Guerrieri (2008) provide an alternative view for the role of technology in structural transformation based on capital deepening. In their theory, cross-sector variations in labor shares drives the reallocation of resources in the economy along the growth path. This theory has the same implications for the shape of Engel curves as supply-side tehories.

[2] Engel curves are defined as the relationship between sectoral shares and aggregate real wealth when sectoral prices remain constant.

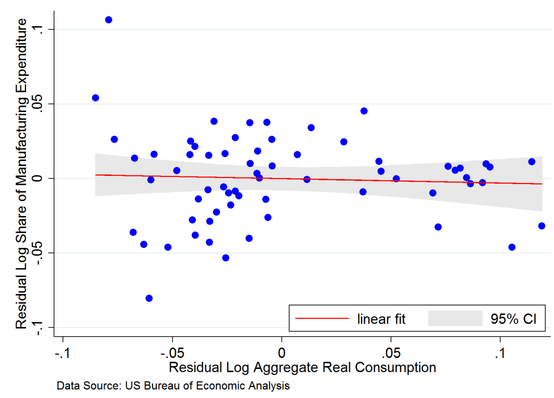
[3] Figure 6 in the Appendix shows that the same results hold when analyzing OECD countries.

Figure 1: Partial Correlations of Expenditure Share and Real Consumption for the U.S. in the postwar period

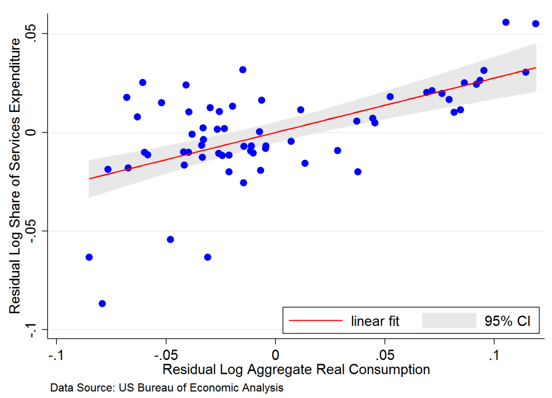
(a) Agriculture



(b) Manufacturing



(c) Services



(d) Regression Table

	Log Expenditure Share in		
	Agriculture	Manufacturing	Services
$\log C$	-0.27	-0.03	0.28
	(0.04)	(0.08)	(0.05)
$\log P_{\text{Agri}}$	0.69	0.26	-0.33
	(0.08)	(0.09)	(0.07)
$\log P_{\text{Manu}}$	0.09	-0.04	0.28
	(0.06)	(0.10)	(0.06)
$\log P_{\text{Serv}}$	-0.84	-0.28	0.04
	(0.06)	(0.11)	(0.07)
R^2	1.00	0.93	0.97

Robust Std. Err. in Parenthesis. BEA data from Herrendorf et al. (2013)

roughly constant pace over the last 60 years.

These findings are inconsistent with existing models of structural transformation. The strong relationship between *residual* (log) expenditure shares and *residual* (log) consumption suggests that forces other than trends in sectoral prices are important to understand structural transformations. The linearity of these relationships speaks against existing models of preference non-homotheticity where structural transformations are transitory phenomena.[4]

In this paper, we provide a simple theory of structural transformation that rationalizes these facts. The model builds on the standard framework used in recent empirical work on structural transformation (e.g., Herrendorf et al., 2014). The key departure from the standard framework is that we introduce a utility function that generates heterogeneous, non-homothetic sectoral demands for all levels of income, including when income grows toward

[4]This has been discussed elsewhere, e.g., Buera and Kaboski (2009) and Matsuyama (2009).

infinity. Moreover, our model also allows for trends in relative prices. Thus, our framework can accommodate asymptotically non-vanishing demand and supply factors affecting the long-run growth rate.

This environment yields the following theoretical results. First, our model is consistent with the presence of structural transformation along a constant growth path, even in the absence of trends in relative prices. In particular, it can account for the log-linear relationships between aggregate consumption and the service and agricultural shares in GDP presneted in Figures (1a) and (1c). Second, unlike standard models with Stone-Geary preferences, our model has a balanced growth path even in the presence of long-run trends in relative prices. Third, in our framework, the long-run growth rate of real aggregate consumption in this economy deviates from the growth rate of GDP and is partly determined by sectoral income elasticities. Intuitively, this occurs because sectoral shares are affected (also asymptotically) by the elasticity of sectoral consumption with respect to aggregate consumption. Finally, under plausible assumptions, we show that our model has an aggregate representation. That is, we can derive expressions for aggregate consumption using simple summary statistics of household expenditures.

To empirically evaluate the model, we use structural equations derived from our theory to estimate the elasticities that characterize our utility function. We use data on cross-country, historical, sectoral shares that differ in the geographies and periods covered and in the measures of economic activity used to capture the structural transformation. A major finding is that the estimates of the elasticity of substitution and the relative slope of the Engel curves in the different sectors are robust to countries, time periods and economic measures of sectoral activity used. This demonstrates that the patterns presented in Figure 1 not only characterize the Engel curves in the U.S. but also apply to broader group of countries. In addition to the high goodness of fit achieved, we take the robustness of the structural parameters as evidence in favor of the model's ability to account for structural transformations.

A key parameter singled out in the literature is the elasticity of substitution between consumption of different goods and services. Our estimate of the elasticity of substitution is around 0.7. Taking advantage of an aggregation result that we derive for our model, we explore whether our estimates are biased by the omission of aggregate drivers of sectoral demand (e.g., sectoral demand shocks). In particular, we reestimate our model using household level data from the Consumer Expenditure Survey (CES). Our estimates from the CES are very similar to the baseline from the aggregate cross-country panel suggesting that they are not biased.

In contrast to Herrendorf et al. (2013), we observe a much smaller change in the elasticity of substitution when we estimate it using data on consumption expenditure rather than value added to measure sectoral economic activity. In particular, our estimate of the elasticity of substitution falls to 0.57 when using gross output for the United States. Herrendorf et al. (2013) and Buera and Kaboski (2009) obtain an estimate of zero using a Stone-Geary specification. Herrendorf et al. (2013) provide the very plausible argument that the different ways to aggregate the consumption in certain substitutable categories in the expenditure versus value added classifications lowers the estimate of the elasticity of susbtitution when using the latter data. However, while we also find that the estimated elasticity of substitution is lower using value added data, we find it reassuring that the our estimate substitution does not drop to zero when using the specification derived from our model.

Both relative prices and income effects turn out to be important contributors to structural transformations. However, we find that income effects are more important than sectoral substitution driven by relative price trends, in contrast to some of the prior empirical findings based on transitory nonhomothetic preferences. The root cause of this finding is that in our framework income effects are not hard-wired to have only transitory effects on the structural transformation. Once we allow for long-run effects of income in sectoral growth, our estimates imply a relatively larger contribution of wealth effects to the realloction patterns observed across the world.

A key feature of structural transformations is that the evolution of nominal and real sectoral measures of activity follow very similar patterns. Standard models of supply-side driven structural transformations cannot account for this basic regularity, as pointed out by Herrendorf et al. (2014). In contrast, our model does account for the high correlation between nominal and real transformations. This is a consequence of the strong role of wealth effects in generating reallocation patterns among different sectors in our framework.

We show in a series of case studies that our model is consistent with the hump-shaped patterns observed in the evolution of manufacturing employment share in some developed and developing economies. We conclude by extending the analysis in two dimensions. First, we consider thinner disaggregations of the data (10 sectors) to show that the framework can be used to understand sectoral reallocation patterns within manufacturing and services –which, as pointed out by Jorgenson and Timmer (2011), is becoming increasingly important to understand structural transformation in advanced economies. Second, we generalize the class of preferences used in our analysis to allow for more flexible relationships between income and sectoral shares. Using this more flexible specification, we employ our estimates from the post-WWII US data to accurately backcast the evolution of the sectoral shares between 1870 and 1945.

Related Literature. Boppart (2013) has recently introduced a sub-class of price-indepenedent-generalized-linear (PIGL) preferences that also yield income effects in the long-run. We note that there are significant differences between our approach and Boppart's. His focus is on accounting for the evolution of consumption of goods relative to services over time and in the cross-section. In consequence, the preferences chosen by Boppart can only have two goods with a non-homothetic demand entering the (indirect) utility function.[5] This is in contrast with the purpose of this paper, in which we are interested in formulating a theory for more than two sectors. Thus, we want to accommodate for potentially more than two sectors with non-homothetic demand.[6]

We note that the nonhomothetic CES preferences that we propose may be a natural choice for the exercise of decomposing structrual reallocations into an income and a price effect. These preferences characterize the income effect with constant sectoral real income elasticities and the price effects with constant elasticities of substitution. In contrast, Stone-Geary preferences or PIGL preferences have embedded a parametric pattern for the evolution of elasticities over time.[7] However, there is no economic rationale *a priori* to assume particular linkages between the income and the substitution effects along the path of development.[8] Nonhomothetic CES preferences structurally decompose the variations in sectoral shares into the effects of prices and real income and therefore enable us to examine the role of each factor separately.

Our paper also relates to the vast and rich empirical and quantitative literature aiming at quantifying the role of non-homotheticities on growth and development (see, among others, Echevarria (1997), Gollin et al., 2002, Duarte and Restuccia, 2010, Alvarez-Cuadrado and Poschke, 2011). Buera and Kaboski (2009) and Dennis and Iscan (2009) have noted the limits of the Stone-Geary utility function to match long time series or cross-sections of countries with different income levels. In recent work, Święcki (2014) uses an indirect utility function that also features non-vanishing income effects. Święcki estimates a static structural model in which he allows for international trade, intersectorial wedges and non-constant elasticities of substitution across goods. He finds that income and price effects account for most of the variation in his panel of countries for 1970-2005.

An alternative formulation that can reconcile demand being asymptotically non-homothetic with balanced growth path is given by hierarchical preferences (e.g., Foellmi and Zweimller, 2008 and Foellmi et al., 2014). This formulation, however, abstracts from different production sectors. Rather, it focuses on a product cycle for products whereby they start as luxury goods, to eventually become a necessity (see, among others, Matsuyama, 2000, 2002 and Saint-Paul, 2006).

The rest of the paper is organized as follows. Section 2 presents the model. Section 3

[5] In fact, the non-homotheticity of demand is governed only by one parameter. Thus, there is a parametric link between the demand systems of the two goods.

[6] One additional advantage of our model is that it generates simple log-linear demand equations that enables structural estimation of the relevant elastiticies using data on cross-country sectoral shares.

[7] In fact, Hanoch (1975) shows that this is generically true for all explicitly additively separable utility functions. The preferences used recently in the international trade literature by Fieler (2011) and Caron et al. (2014) also fall in this category.

[8] Appendix A provides more details on the nature of the differences between these preferences with regard to the evolution of elasticities.

contains the estimation and model evaluation for a panel of 29 countries for the period 1947-2005. Section 4 analyzes aggregate macroeconomic time series and household expenditure data for the United States. Section 5 analyzes in greater detail our cross-country results and extends it in several directions. Section 6 extends the baseline model to allow for heterogeneous capital shares in the production function. Section 7 concludes.

2 Model

This section provides a simple multi-sector growth model where both the rate of technological progress and income elasticities remain asymptotically heterogenous across sectors. The model is identical to Herrendorf et al. (2014), except for the aggregator of sectoral consumptions into aggregate consumption. This single departure from the standard workhorse model delivers the main theoretical results of the paper and the demand system that we use to estimate our data. All proofs are contained in Appendix B.

2.1 Preferences and the Household Problem

A representative households consumes goods and services produced in I different sectors, supplies one unit of labor and owns all capital. This households takes the sequence of wages, rental prices of capital, and prices of goods and services $\left\{w_t, R_t, \{p_{it}\}_{i=1}^{I}\right\}_{t=0}^{\infty}$ as given, and choose a sequence of capital stocks $\{K_t\}_{t=0}^{\infty}$ and aggregate consumption $\{C_t\}_{t=0}^{\infty}$ to maximize its utility

$$\max_{\{C_{it},K_{it}\}} \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\theta} - 1}{1-\theta} \right), \tag{1}$$

subject to the per-period budget constraint

$$K_{t+1} + \sum_{i=1}^{I} p_{it} C_{it} \leq w_t + K_t \left(1 - \delta + R_t\right).$$

Aggregate consumption, C_t, results from the combination of consumption of goods, $\{C_{it}\}_{i=1}^{I}$. Household's preferences to combine goods are given by a nonhomothetic generalization of a Constant Elasticity of Substitution (CES) aggregator, implicitly defined as

$$\sum_{i=1}^{I} \Omega_i \, C_t^{\frac{\epsilon_i - \sigma}{\sigma}} C_{it}^{\frac{\sigma-1}{\sigma}} = 1, \tag{2}$$

where σ is the price elasticity of substitution across goods, and Ω_i are constant weights. Each sector i is identified with a paramter ϵ_i, which is a measure of the income elasticity of demand for good i. The key feature of these preferences is that the importance of consumption of good i changes with the aggregate consumption level C_t at a rate that is controlled by ϵ_i. Note also that the standard CES aggregator is a special case of (2) where the elasticity parameter is the same for all sectors, $\epsilon_i = 1$. In this section, we specialize our analysis to the case that consumption goods are complements, $\sigma \in (0,1)$.[9]

The next lemma characterizes necessary conditions for a sequence of sectoral consumption goods to be a solution to the household problem.

Lemma 1. *Household Behavior. Consider a household with preferences and budget constraint as described by equations (1) and (2). Given a sequence of prices* $\left\{w_t, R_t, \{p_{it}\}_{i=1}^{I}\right\}_{t=0}^{\infty}$, *any interior solution satisfies the following intratemporal and intertemporal conditions.*

1. The intratemporal allocation of consumption goods satisfies

$$C_{it} = \Omega_i \left(\frac{p_{it}}{P_t}\right)^{-\sigma} C_t^{\epsilon_i}, \tag{3}$$

where P_t is the aggregate price index

$$P_t \equiv \frac{E_t}{C_t} = \frac{1}{C_t}\left[\sum_{i=1}^{I} \Omega_i C_t^{\epsilon_i - \sigma} p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{4}$$

and $E_t \equiv \sum_i p_i C_i$ denotes total expenditure.

2. The intertemporal allocation of real aggregate consumption satisfies the Euler equation

$$C_t^{-\theta} = (1 - \delta + R_t)\frac{P_t}{P_{t+1}}\left(\frac{\sum_{i=1}^{I}\omega_{it}\epsilon_i - \sigma}{\sum_{i=1}^{I}\omega_{i,t+1}\epsilon_i - \sigma}\right) C_{t+1}^{-\theta}, \tag{5}$$

where ω_{it} is the nominal expenditure share in sector i, $p_{it}C_{it}/E_t$.[10]

The key insight from Lemma 1 is that the household problem can be decomposed into two sub-problems. First, the consumer solves the *intertemporal* consumption-savings problem, finding the sequence of $\{K_t, C_t\}_t$ to maximize expected utility (1) subject to the constraint

$$K_{t+1} + E\left(C_t, \{p_{it}\}_{i=1}^{I}\right) \leq w_t + K_t\left(1 - \delta + R_t\right),$$

[9] Appendix A discusses some general properties of these preferences, which belong to the class of preferences discussed in Hanoch (1975) and Sato (1975). To avoid a taxonomical analysis, this section focuses on the case $0 < \sigma < 1$, which is the relevant empirical case when studying agriculture, manufacturing and services. However, these preferences can be generalized to $\sigma \geq 1$. Also, different sets of parametric restrictions can be made to ensure that this is a well-defined aggregator. For example, $\Omega_i > 0$ and $\epsilon_i \geq \sigma$ for all consumption sectors $i \in \mathcal{I} \equiv \{1, \ldots, I\}$ are sufficient conditions to ensure that (2) is globally monotonically increasing and quasi-concave.

[10] Furthermore, the sequence of aggregate consumption has to satisfy the No-Ponzi condition $\lim_{t\to\infty} K_t \left(\prod_{t'=1}^{t-1} \frac{1}{1-\delta+R_{t'}}\right) = 0$.

where $E\left(C, \{p_i\}_{i=1}^{I}\right)$ is the expenditure function for the non-homothetic CES preferences, defined in equation (4). As a result of the nonhomotheticity, nominal consumption expenditure does not vary linearly in real aggregate consumption. The price index reflects the changes in the sectoral composition of consumption. In contrast to the Stone-Geary form of nonhomotheticity, the nonlinear relationship between nominal and real aggregate consumption for these preferences exists for all levels of income.[11] The household incorporates this relationship in its Euler equation, (5), where we see a wedge between the marginal cost of real consumption and the aggregate price index. The size of this wedge, given by $(\bar{\epsilon}_t - \sigma)/(1-\sigma)$, depends on the average income elasticities across sectors, $\bar{\epsilon}_t = \sum_{i=1}^{I} \omega_{it}\epsilon_i$, and varies over time. In the case of homothetic CES where $\epsilon_i = 1$, this wedge disappears.

The second part of the household problem involves the *intratemporal* problem of allocating consumption across the I different goods. Equation (3) provides an intuitive and simple expression that determines real consumption in each sector in terms of sectoral price and aggregate consumption. In particular, we can derive the nominal share of sectoral consumption from equation (3) in terms of aggregate consumption and sectoral price or real consumption,

$$\omega_{it} \;=\; \frac{p_{it}C_{it}}{P_t C_t} \;=\; \Omega_i\, C_t^{\frac{\epsilon_i-\sigma}{\sigma}} C_{it}^{\frac{\sigma-1}{\sigma}} \;=\; \Omega_i \left(\frac{p_{it}}{P_t}\right)^{1-\sigma} C_t^{\epsilon_i - 1}. \tag{6}$$

Note that this expression for expenditure shares coincides with each entry of the original nonhomothetic CES aggregator, (2). Thus, we can rewrite (2) as simply $\sum_{i=1}^{I} \omega_{it}{=}1$, and interpret it in terms of expenditure shares.

Equations (3) and (6) together yield the relative real consumption and nominal consumption shares between sectors i and j as a function of relative prices and aggregate consumption,

$$\log\left(\frac{C_{it}}{C_{jt}}\right) \;=\; -\sigma \log\left(\frac{p_{it}}{p_{jt}}\right) + (\epsilon_i - \epsilon_j)\log C_t + \log\left(\frac{\Omega_i}{\Omega_j}\right), \tag{7}$$

$$\log\left(\frac{\omega_{it}}{\omega_{ji}}\right) \;=\; (1-\sigma) \log\left(\frac{p_{it}}{p_{jt}}\right) + (\epsilon_i - \epsilon_j)\log C_t + \log\left(\frac{\Omega_i}{\Omega_j}\right). \tag{8}$$

Equation (7) illustrates the key features of the demand system implied by this nonhomothetic CES preferences. Interpreting C_{it} as the Hicksian demand for good i with aggregate consumption (utility) C_t under prices p_{it}'s, we find a *perfect separation of the price and the income effects.* The first term on the right hand side shows the price effects characterized by constant elasticity of substition σ. More interestingly, the second term on the right hand side shows the change in relative sectoral demand as consumers move across indifference curves.[12] This income effect is characterized by constant sectoral income elasticity parameters ϵ_i's. If $\epsilon_i > \epsilon_j$,

[11] See Appendix A for a precise statement of this relationship.

[12] Nonhomothetic CES preferences inherit this property because they belong to the class of Implicitly Additive preferences (Hanoch, 1975). In contrast, Explicitly Additive preferences such as Stone-Geary preferences do not allow separation of income and price effects (see Appendix A).

demand for good i rises relative to good j as consumers become wealthier. The implied non-homotheticity holds irrespective of prices or the level of wealth. Income elasticities vary across sectors for all levels of aggregate consumption and remain heterogenous asymptotically.

Equations (7) and (8) also show how our model can generate a positive correlation between relative sectoral consumption in nominal and real terms, as it is observed in the data.[13] As in the case with homothetic CES aggregators, trends in relative prices are a force making relative real consumption be negatively correlated with relative expenditures. To see that, that relative real consumption is decreasing in relative prices with an elasticity of $-\sigma$, while relative expenditure is increasing with an elasticity of $1-\sigma$. However, our demand system has an additional force, income effects, which makes both time series co-move in aggregate consumption. Thus, if income effects are sufficiently strong, both time series can be positively correlated. In Section 5.2 we show that this is the case when we estimate our demand system.

In our exposition of the model, we refer to C_t as aggregate consumption, which allows us to connect the log linear specification of real sectoral consumption in expression (8) to our empirical analysis in Section 3. As equations (3) and (6) have shown, the elasticities of real and nominal sectoral consumption of good i with respect the aggregate consumption are ϵ_i and $\epsilon_i - 1$, respectively. This is why we refer to parameter ϵ_i as the sectoral income elasticity parameter. Section 3 and Appendix A provide details on the exact mapping between aggregate real consumption C_t and real consumption as constructed from the data. We can also derive elasticities of sectoral consumption relative to total expenditure,

$$\eta_{it} \equiv \frac{\partial \log C_{it}}{\partial \log E_t} = 1 + \frac{1-\sigma}{\bar{\epsilon}_t - \sigma}\left(\epsilon_i - \bar{\epsilon}_t\right). \tag{9}$$

2.2 Production and Competitive Equilibrium

The production side of the economy and the definition of the competitive equilibrium are as in Herrendorf et al. (2014) and Ngai and Pissarides (2007). There exists a representative firm in each sector producing under perfect competition. They produce investment good, X_t, and consumption goods, Y_{it}, according to

$$\begin{aligned} Y_{it} &= K_{it}^{\alpha}\left(A_{it}L_t\right)^{1-\alpha}, \qquad i \in \mathcal{I}, \\ X_t &= K_{0t}^{\alpha}\left(A_{0t}L_{0t}\right)^{1-\alpha}, \end{aligned}$$

where K_{it} and L_{it} are capital and labor used in the production of sector i at time t (we have identified the sector producing investment good as $i=0$) and $0<\alpha<1$. A_{it} indicates a sector-specific labor-augmenting technology. The aggregate capital stock of the economy, K_t,

[13] With homothetic CES aggregators of complementary goods, i.e., $\sigma < 1$, the correlation between relative sectoral consumptions in nominal and real terms is negative, while it is positive in the data. See (Herrendorf et al., 2014) for further discussion.

accumulates using investment goods and depreciates at the rate δ, $X_t = K_{t+1} - (1-\delta) K_t$.

We study the competitive equilibrium of this economy.[14] Since all sectors produce using Cobb-Douglas production functions with the same capital shares and face the same relative input prices, cost minimization implies identical capital to labor ratios across sectors. This implies that capital labor ratios equal aggregate stock of capital, $K_{it}/L_{it} = K_t$ (as total labor is normalized to one). Sectoral prices are determined based on sectoral productivities

$$p_{it} = \frac{p_{it}}{p_{0t}} = \left(\frac{A_{0t}}{A_{it}}\right)^{1-\alpha}, \tag{10}$$

where we have normalized the price of investment good, $p_{0t} \equiv 1$. Labor and capital demands for goods production are proportional to expenditure shares ω_{it},

$$L_{it} = (1-\alpha)\frac{P_t C_t}{w_t}\omega_{it}, \qquad K_{it} = \alpha\frac{P_t C_t}{R_t}\omega_{it}, \qquad i = 1, \ldots, I. \tag{11}$$

The production side of the economy therefore aggregates to yield $Y_t \equiv w_t + R_t K_t = A_{0t}^{1-\alpha} K_t^{\alpha}$. We can express wages and the rental price of capital as

$$w_t = (1-\alpha) A_{ot}^{1-\alpha} K_t^{\alpha}, \qquad\qquad R_t = \alpha A_{ot}^{1-\alpha} K_t^{\alpha-1}. \tag{12}$$

Goods market clearing implies that supply and demand equalize for all sectoral consumptions, $C_{it} = Y_{it}$ for all $i = 1, \ldots, I$. Henceforth, we denote both by C_{it}. Factor market clearing implies that labor and capital are divided across sectors, $\sum_{i=1}^{I} L_{it} = 1$, $\sum_{i=1}^{I} K_{it} = K_t$.

2.3 Constant Growth Path

In this section, we analyze the dynamic equilibrium allocation when aggregate consumption grows at a constant rate which, following Acemoglu and Guerrieri (2008), we refer to as Constant Growth Path (CGP). There are two notable features of the constant growth path. The first is that despite constant growth of aggregate consumption, growth is non-balanced across sectors. The second, and perhaps more important feature, is that the asymptotic growth rate is pinned down both by technological progress parameters and preference parameters (income elasticities), highlighting the fact that income effects play a role even asymptotically in our

[14] Given initial stock of capital K_0 and a sequence of sectoral productivities $\left\{\{A_{it}\}_{i=1}^{I}\right\}_{t\geq 0}$, a competitive equilibrium is defined as a sequence of allocations $\left\{C_t, K_{t+1}, X_t, \{C_{it}, K_{it}, L_{it}\}_{i=1}^{I}\right\}_{t\geq 0}$, a sequence of prices $\left\{w_t, R_t, \{p_{it}\}_{i=1}^{I}\right\}_{t\geq 0}$, and a sequence of aggregate price indices $\{P_t\}_{t\geq 0}$ such that *(i)* agents maximize the present discounted value of their utility given their budget constraint, equation (1), *(ii)* firms maximize profits and *(iii)* markets clear. This implies that that the sequence of aggregate stocks of capital, real aggregate consumption, real sectoral consumption and investment goods satisfy (3), (4), (5). The the sequence of sectoral prices, wages, and the rental prices satisfy (10) and (12).

framework.

As we are interested in long-run growth, we assume that the rate of sectoral technological progress is constant in what follows. We allow the rates of technological progress to be heterogeneous across sectors, and denote them by

$$\gamma_i = \frac{A_{it+1}}{A_{it}} - 1, \qquad i = 0, 1, \ldots, I. \tag{13}$$

Proposition 2. *There exists a unique CGP in this economy. Let there be at least one sector with $\epsilon_i > \sigma$ and $\mathcal{I} \equiv \{1, \ldots, I\}$ denote the set of consumption sectors, then the asymptotic aggregate consumption growth is*

$$\lim_{t\to\infty} \frac{C_{t+1}}{C_t} = 1 + \gamma^* = \min_{i\in\mathcal{I}:\epsilon_i>\sigma} \left[(1+\gamma_0)^{\alpha}(1+\gamma_i)^{1-\alpha}\right]^{\frac{1-\sigma}{\epsilon_i-\sigma}}. \tag{14}$$

Along the CGP the rental price of capital is constant,

$$R^* = \frac{1+\gamma_0}{\beta\,(1+\gamma^*)^{1-\theta}} - (1-\delta), \tag{15}$$

and nominal expenditure, total nominal output, and the stock of capital all grow at a constant rate $1+\gamma_0$ provided that $R^* > \alpha(\gamma_0+\delta)$. The asymptotic sectoral consumption growth in sector i is

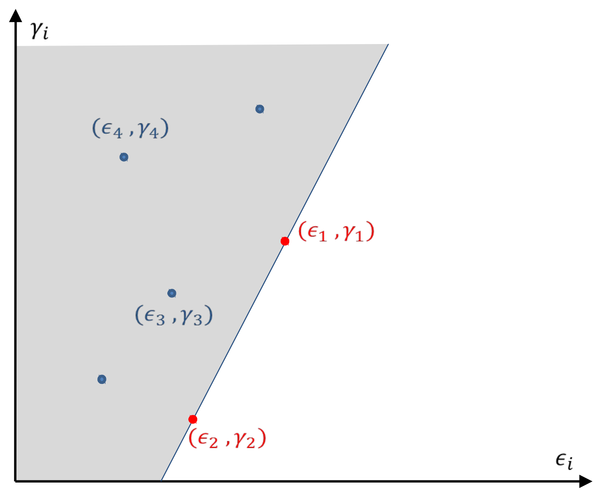
$$\lim_{t\to\infty} \frac{C_{it+1}}{C_{it}} = (1+\gamma_i)^{(1-\alpha)\sigma}(1+\gamma_0)^{\alpha\sigma}(1+\gamma^*)^{\epsilon_i-\sigma}. \tag{16}$$

The shares of nominal consumption expenditure for all sectors asymptotically vanish except for the subset of industries, $\mathcal{I}^* \subseteq \mathcal{I}$, that achieve the minimum in expression (14).

Proposition 2 shows how the long-run growth rate of aggregate consumption γ^* (equation in 14) is affected by income elasticities, in addition to technological progress. This is due to the fact that preferences are asymptotically non-homothetic. Moreover, this implies that the growth rate of real aggregate consumption γ^* diverges from the growth rate of nominal aggregate consumption, which grows at rate γ_0 as total output and investment. Note also that the Kaldor facts hold along the CGP: aggregate variables grow at a constant rate, the interest rate, the capital output ratio and the labor and capital shares are constant. Equation (16) shows how sectoral consumption grows at different growth rates across sectors. The cross-sectional difference in growth rates is again determined by the combination of technological progress and the income elasticity of each sector.

Moreover, only a subset of all consumption sectors, $\mathcal{I}^*$, will asymptotically comprise a nonnegligible share of total consumption expenditure. This subset of sectors is also determined based on the joint sectoral distribution of income elasticities and technological rates

Figure 2: Example of Determination of the Growth Rate of Aggregate Consumption



Notes: Each dot represents a sector (ϵ_i, γ_i) in the economy. The set of industries that asymptotically have nonzero employment shares is $\mathcal{I}^* = \{1, 2\}$.All other sectors, which are located in the grey area, have asymptotically-vanishing expenditure shares.

of growth.[15] This is in contrastwith models with homothetic demands.In this case, $\mathcal{I}^*$ is solely determined based on the growth rate of technological progress across sectors. Suppose that $\epsilon_i = 1$ in all sectors. As in Ngai and Pissarides (2007), we have that asymptotically only industries with the slowest technological growth have nonvanishing expenditure shares, $\gamma_{i^*} = \gamma_{\min}$, because consumption sectors are complements, $\sigma < 1$. However, when we allow nonhomotheticity in preferences. $\epsilon_i \neq 1$, the set of industries that asymptotically constitute a significant share of employment also depends on income elasticities. Intuitively, even if a sector has fast technological progress, it can be nonvanishing if it is sufficiently income elastic. This is illustrated in Figure 2, in which sectors are represented in terms of their income elasticity and their rate of technological progress, (ϵ_i, γ_i). The only sectors with asymptotically non-vanising expenditure shares lie on the blue line and are denoted by red dots. Intuitively, these are the sectors which, for a given level of income elasticity, have a sufficiently low rate of technological progress not to vanish.

[15] The growth rate of nominal consumption expenditure share and employment for sector i is

$$1 + \xi_i \equiv \lim_{t \to \infty} \frac{\omega_{it+1}}{\omega_{it}} = \lim_{t \to \infty} \frac{L_{it+1}}{L_{it}} = \frac{(1 + \gamma^*)^{\epsilon_i}}{\left[(1 + \gamma_0)^{\alpha} (1 + \gamma_i)^{1-\alpha}\right]^{1-\sigma}}. \tag{17}$$

2.4 Structural Transformation in a Four-sector Model

This section shows that our baseline model can generate the stylized facts of observed in the process of long-run development: a decline of agriculture, a rise of the service sector and a hump-shaped evolution of the manufacturing sector. To this end, we consider an economy composed of four sectors: investment x, agriculture a, manufacturing m, and services s.

Proposition 3. *Suppose that $\gamma_a > \gamma_m > \gamma_s$ and $\epsilon_s > \epsilon_m > \epsilon_a > 0$. If the initial capital stock of the economy is below $\underline{K} > 0$, employment and expenditure shares are decreasing in agriculture, increasing in services and hump-shaped (and initially higher than in services) in manufacturing.*

Proposition 3 states that under empirically reasonable assumptions on technological progress and income elasticities, the model can generate the patterns consistent with the broad patterns observed in the process of development.[16] To understand the mechanics of growth, the first condition that the economy has to satisfy is that expenditure shares in services become asymptotically non-negligible and that the shares of agriculture and manufacturing asymptotically vanish, $\mathcal{I}^* = \{s\}$. Rewriting the growth rate of consumption (14) for small rates of technological growth in all sectors $\gamma_i \ll 1$, we have that

$$\gamma^* = \frac{1-\sigma}{\epsilon_s - \sigma} \left(\alpha \gamma_m + (1-\alpha)\, \gamma_s \right). \tag{18}$$

Thus, if $\gamma_a > \gamma_m > \gamma_s$ and $\epsilon_s > \epsilon_m, \epsilon_a$, the growth rate in services implied by the right-hand-side of (18) is the lower than the one we would obtain in manufacturing and agriculture. This implies that services is the only sector satisfying (14) and the only sector non-vanishing asymptotically. To ensure that agriculture that is asymptotically monotonically decreasing, a similar argument shows that the additional sufficient condition needed is that $\epsilon_m > \epsilon_a$.[17]

For manufacturing, the hump-shape comes from the interaction of technological progress and income elasticities. At initial stages, the differences in income elasticities across sectors plays a modest role, and technological progress effects dominate. Employment reallocates from agriculture to both services and manufacturing as the productivity of agriculture is the fastest

[16] The assumption on differential growth rates of technology $\gamma_a > \gamma_m > \gamma_s$ is also made in Ngai and Pissarides (2007). In our cross-country data set, we show that trends in relative prices are consistent with these assumptions and relative prices of services relative to manufacturing have grown while relative prices of agriculture to manufacturing have declined (see Table B.3 in the online appendix). Also, in Section 3 we confirm that the differences in income elasticities are such that $\epsilon_s > \epsilon_m > \epsilon_a > 0$.

[17] Using (17), we find

$$\begin{aligned}
\lim_{t\to\infty} \frac{L_{mt+1}/L_{at+1}}{L_{mt}/L_{at}} = \lim_{t\to\infty} \frac{\omega_{mt+1}/\omega_{at+1}}{\omega_{mt}/\omega_{at}} \quad &= \quad \left(\frac{1+\gamma_a}{1+\gamma_m}\right)^{(1-\alpha)(1-\sigma)} (1+\gamma^*)^{\epsilon_m - \epsilon_a}, \\
&\approx \quad (1-\alpha)\,(1-\sigma)\,(\gamma_a - \gamma_m) + (\epsilon_m - \epsilon_a)\,\gamma^*,
\end{aligned}$$

which is unambiguously positive if $\epsilon_m > \epsilon_a$.

growing. Therefore, the manufacturing share initially increases as it absorbs resources from agriculture. However, as the economy grows, the force of income effects eventually prevails. As services have the highest income elasticity, the loss of resources from manufacturing to services becomes larger than the gain of resources from agriculture. As a result, the manufacturing share eventually begins to diminish.

2.5 Demand Aggregation

So far, we have assumed a representative agent economy. In this section, we analyze whether our demand system for aggregate expenditure can be defined as a result of the aggregation of household demands with nonhomothetic CES preferences.

Consider an economy composed of households differing in their initial level of wealth. Index household types by h, and suppose they are distributed according to the cumulative distribution $F(h)$. The expenditure share in sector i relative to sector j for household h is

$$\omega_{it}^h = \Omega_i^h \left(\frac{p_{it}}{P_t^h}\right)^{1-\sigma} \left(C_t^h\right)^{\varepsilon_i - 1}, \qquad \text{for all } h. \tag{19}$$

Aggregating across households we obtain

$$\omega_{it} \equiv \int \omega_{it}^h dF(h) \quad = \quad \phi_{it} \left(\frac{p_{it}}{P_t}\right)^{1-\sigma} C_t^{\varepsilon_i - 1},$$

with

$$\phi_{it} \quad = \quad \int dF(h) \Omega^h \left(\frac{C_t^h}{C_t}\right)^{\varepsilon_i} \frac{\sum_{j=1}^I C_t^{\varepsilon_j} p_j^{1-\sigma}}{\sum_{j=1}^I \left(C_t^h\right)^{\varepsilon_j} p_j^{1-\sigma}}. \tag{20}$$

Thus, the aggregate demand that we obtain aggregating household preferences consists of the demand system generated by a representative consumer, (8), with the addition of a correction term that captures the importance of consumption inequality across households, ϕ_{it}. The extent to which consumption inequality across households matters for aggregate relative demand is parametrized by ε_i. This implies that consumption inequality specially matters when comparing aggregate consumption of goods with very different income elasticities. Indeed, if i and j have the same income elasticity, the inequality term disappears in the relative demand. Finally, note that (20) is homogeneous of degree 0 in C_t^h. Thus, if we generalize our definition of CGP to the household level, so that $\lim_{t\to\infty} \frac{C_{t+1}^h}{C_t^h} = 1 + \gamma^*$ for all h, the inequality measure ϕ_{it} becomes constant over time.

3 Estimation of the Preference Parameters in a Country-Panel

3.1 Empirical Strategy

Our empirical strategy uses the solution for intratemporal problem and the production decisions of firms to estimate the preference parameters of utility function (2). Taking the logarithms of the sectoral demands (8) and using that the ratio of sectoral expenditures is equal in equilibrium to the ratio of sectoral labor allocations, we obtain that

$$\log\left(\frac{L^c_{a,t}}{L^c_{m,t}}\right) = \alpha^c_{am} + (1-\sigma)\log\left(\frac{p^c_{a,t}}{p^c_{m,t}}\right) + (\varepsilon_a - \varepsilon_m)\log C^c_t + \nu^c_{am,t}, \tag{21}$$

$$\log\left(\frac{L^c_{s,t}}{L^c_{m,t}}\right) = \alpha^c_{sm} + (1-\sigma)\log\left(\frac{p^c_{s,t}}{p^c_{m,t}}\right) + (\varepsilon_s - \varepsilon_m)\log C^c_t + \nu^c_{sm,t}, \tag{22}$$

where a, m and s denote agriculture, manufacturing and services, respectively, and t, time. The superscript c denotes country, and $\nu^c_{am,t}$ and $\nu^c_{sm,t}$ are the error terms. We allow for country-sector dyad fixed effects, α^c_{am} and α^c_{sm}, as there may be systematic differences in measurement across countries. Thus, the identification of the elasticities of interest come from the within-country time-series. Note also the cross-equation restriction in the estimation that the price elasticity σ and the income elasticities, ε_s, are the same across countries.

Finally, we relate sectoral expenditure shares to employment shares using the production side of our economy. We prefer this specification because in order to construct the price indices we use both nominal and real sectoral consumption data. As a result, if we used expenditures in the left-hand-side of our estimation equations, we would obtain biased estimates of the income and price elasticities. The use of employment shares introduces an additional complication, however. In contrast to relative expenditures, it is important to account for the fact that some goods can be imported and exported, thus affecting the sectoral employment composition. Accordingly, we control for sectorial net exports in our regressions

$$\log\left(\frac{X^c_{s,t}}{M^c_{s,t}}\right)$$

where $X^c_{s,t}$ and $M^c_{s,t}$ denote nominal exports from and imports to country c in sector s at time t.[18]

[18] In a robustness check we control for $\log \tilde{X}^c_{s,t}$ and $\log \tilde{M}^c_{s,t}$ separately and obtain similar results. Controlling separately for exports and imports implies that that we have to use price deflators (hence the tilde in the notation). We use the exports and imports price deflators provided in the Penn World Tables database, eighth version (Feenstra et al., 2013), which builds on the work of Feenstra and Romalis (2014) to adjust for different qualities and prices of imports and exports. We prefer the baseline specification as it deals away with this issue by using the ratio of nominal values.

3.2 Identification

As in Herrendorf et al. (2013), our baseline identification strategy relies on the intra-period allocation of consumption that follows from the solution of the intratemporal problem allocation problem (8). That is, conditional on the observed level of aggregate consumption C_t^c and sectorial prices $P_{i,t}^c$, we use our demand system to estimate relative consumption across sectors. Identification relies on the fact that C_t^c and $P_{i,t}^c$ are predetermined in the system of equations (21) - (22).

Aggregate and sectoral supply-shocks contribute to the identification of the demand system. Sectoral productivity shocks, such as an increase in relative productivity in one of the sectors, affect relative prices and introduce variation in the estimating equations, (21) - (22).[19] Also, they affect the consumption-saving decisions, thereby introducing additional variation through total consumption expenditures. Likewise, aggregate supply shocks, such as an increase in labor or capital productivity, affect the consumption-saving decision and sectoral prices.[20]

Aggregate demand shocks, such an increase in the propensity to spend, are captured through the aggregate expenditure term in (21) - (22) and contribute to the identification of our demand system. However, sectoral taste shocks that induce consumers to spend more in one sector for a given level of aggregate expenditure and sectoral prices are not well-captured in this specification. Given that we have already country-sector dyad fixed effects we cannot add an additional time fixed effect that would control for this. To the extent that these shocks are uncorrelated with the other type of shocks, they will enter as classical measurement error on the left-hand side of the estimating equations (21) - (22) and our estimates are still consistent.

One way to avoid the potential biases from sector-specific demand shocks is by using household-level data. With this kind of micro data, we can include sector-year fixed effects that absorb sectoral demand shocks, and hence eliminate the biases from tehir ommission. In Section 4, we implement this strategy by using household panel data for the U.S. from the Consumer Expenditure Survey (CES) to estimate our demand system. Reassuringly, we find that the estimates for the elasticity of substitution and of the income elasticities are very similar to our estimates from aggregate data.

On the identification of Income Elasticities As noted by Hanoch (1975), there is one degree of freedom that is not pinned down by the non-homothetic utility function (2). This implies that only the relative slopes of the Engel curves can be identified. To see that, consider the monotonic transformation our real consumption measure as $\tilde{C}_t = C_t^{\zeta}$ in the definition of

[19] Note that the fact that we use the log of the ratio of sectoral prices in our estimation has the advantage that we can use directly nominal prices and any cross-country systematic differences in the measurement of prices are going to be captured in the fixed effect.

[20] In fact, in our model, relative sectoral prices would remain unaltered in this case.

the utility function, (2). This would not change the real allocations in our economy nor the expenditure elasticities as defined in (9).[21] However, the implied real consumption elasticities would change from ε_i to $\zeta\varepsilon_i$. Thus, the level of the estimates we obtain in our estimation $\varepsilon_i - \varepsilon_j$ depend on the choice of the definition of real consumption. In Appendix A, we show that there exists a particular monotone transformation that corresponds to real income as defined in national accounts, which will correspond to the level of the elasticities we identify. However, this level is irrelevant for fitting (21) - (22). As a result, the economically meaningful measures to be analyzed from the estimation have to be independent of these type of monotone transformations. For example, the ratios of income elasticities or the sign of the difference of elasticities, $\varepsilon_i - \varepsilon_j$, are informative measures as they are invariant to isoelastic transformations of C_t.[22]

On a more practical front, note that in our estimation equations, nominal consumption expenditure is deflated by the ideal price index to obtain the correct measure of real consumption. However, there are no time series constructed using our price index. To overcome this obstacle, we use time series that are deflated using chained Fisher indices. The justification for this choice comes from the following two facts. First, the exact ideal price index for any non-homothetic continuously differentiable utility function is the Törnqvist-Theil Index up to a second order approximation. Second, the Fisher index and the Törnqvist-Theil Index approximate each other up to second order.[23]

3.3 Data Description

In order to implement our estimation strategy, we need time-series data on sectoral employment shares, sectoral price deflators and real consumption.

We perform our baseline estimation resorting to the GGDC 10-Sector Database for sectoral data and the Barro-Ursua data set for real consumption. The GGDC 10-Sector Database provides a long-run internationally comparable dataset on sectoral outcomes for 10 countries Asia, 9 in Europe, 9 in Latin America, 10 in Africa and the US. Variables covered in the data set are annual series of production value added (nominal and real), and employment for 10 broad sectors starting in 1947. While chained indexing has been used for all countries to compute real value added, there is heterogeneity in the implementation country by country

[21] See Appendix A

[22] An alternative possibility to circumvent this under-identification problem, as suggested by Hanoch (1975), is to pin down the elasticity level for one of the sectors and express the rest relative to them. As, we do not have ex-ante information on what one of this values may be we prefer not to pursue this approach.

[23] Thus, the using a Fisher index provides a second order approximation to our ideal price index. Diewert (1978, 2002) provides a formal proof of these statements and quantitative analysis. To assess the accuracy of this second order estimation we have also run Monte-Carlo simulations using the true price index and the Fisher price index and the estimation results are almost identical. For example, when generating data consistent with the U.S., the estimates were identical up to the fifth decimal. Also, using a simple non-chained deflator did not make a big quantitative difference. Appendix A contains a template of the code used for the curious reader.

given the different data sources, as described in Vries et al. (2014). For most developing countries a fixed-based Laspeyres index is used and this base is usually updated every 5 to 10 years. For richer countries, Fisher indices are typically used. As our identification comes from the within country time series, this cross-country heterogeneity should not bias our estimation. Finally, in our baseline exercise we aggregate ten sectors into agriculture, manufacturing and services.[24] In Section 5.3.1 we estimate our model for 10 sectors

For consumption, we use the time series on consumption per capita from the Barro-Ursua consumption series.[25] Their data has the advantage of using the Fisher chained price index, which allows us to have a meaningful measure of real consumption.[26] Barro-Ursua do not include government services into consumption. Thus, we exclude government services from our baseline estimation.[27] The drawback of using Barro-Ursua is that the only African country they cover is South Africa.

3.4 Results

We estimate jointly (21) and (22) imposing the cross-equation restriction that price and income elasticities have to be the same across countries.[28] Table 1 reports the results of estimating the system of equations for the whole sample of 29 countries and for OECD, Latin American and Asian countries separately.

Columns (1) to (3) in Table 1 report our estimates for the entire sample of countries. Column (1) reports the estimates without using country-sector fixed effects (and thus using cross-country variation in levels to identify the parameters). Column (2) reports our estimates using country-sector fixed effects (whereby using only within country variation to identify the elasticities) and column (3) includes trade controls. Our estimates of the price elasticity of substitution across sectors ranges from .66 to .75 and is precisely estimated in all three specifications (standard errors are clustered at the country level). In fact, we cannot reject the null that these three estimates are statistically identical. However, we can reject the null that they are equal to one, implying that these three sectors are complements. These estimates of

[24] The ten sectors are agriculture, mining, manufacturing, construction, public utilities, retail and wholesale trade, transport and communication, finance and business services, other market services and government services. More information on the data set is available online at http://www.rug.nl/research/ggdc/data/10-sector-database.

[25] http://scholar.harvard.edu/barro/publications/barro-ursua-macroeconomic-data

[26] The use of the Fisher price index is justified in our framework because it approximates up to second order the Törnqvist-Theil Index. The Törnqvist-Theil Index For is the local exact price index (up to a second-order approximation) of any continuously differentiable non-homothetic utility function. Consistent with this fact, these two price indices are almost identical when computed with U.S. time series. Diewert (1978, 2002) provides a formal proof of these statements and a quantitative analysis.

[27] In fact, the GGDC 10-sector database also lacks data on government services for Latin-American countries.

[28] We report the results of using Maximum Likelihood Estimation rather than Seemingly Unrelated Regressions, so that we can cluster the standard errors at the country level. The point estimates we obtain are very similar for both cases.

Table 1: Baseline Estimates for the Cross-Country Sample

Dep. Var.:	World			OECD		Asia		Latin America	
Rel. Emp.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
σ	0.66	0.75	0.72	0.69	0.69	0.73	0.77	0.77	0.68
	(0.19)	(0.11)	(0.11)	(0.17)	(0.18)	(0.18)	(0.23)	(0.08)	(0.05)
$\varepsilon_a - \varepsilon_m$	-0.81	-1.09	-1.03	-0.99	-0.94	-1.19	-1.26	-1.20	-0.90
	(0.24)	(0.10)	(0.14)	(0.19)	(0.18)	(0.12)	(0.17)	(0.25)	(0.17)
$\varepsilon_s - \varepsilon_m$	0.32	0.32	0.32	0.40	0.49	0.07	0.09	0.59	0.54
	(0.08)	(0.10)	(0.13)	(0.19)	(0.15)	(0.04)	(0.08)	(0.14)	(0.11)
Obs.	1006	1006	916	436	407	319	297	295	245
$c \cdot sm$ FE	N	Y	Y	Y	Y	Y	Y	Y	Y
Trade Controls	N	N	Y	N	Y	N	Y	N	Y

Note: Standard errors clustered at the country level.

the price elasticity of substitution are in line with the literature (e.g., Herrendorf et al., 2013 and Buera and Kaboski, 2009).

The estimates for the difference of income elasticities yield sensible results that are very stable across the three specifications. We find that the difference in income elasticities between agriculture and manufacturing, $\varepsilon_a - \varepsilon_m$, is negative, while the difference between services and manufacturing, $\varepsilon_s - \varepsilon_m$, is positive. All these estimates are significant at conventional levels.

Columns 4 through 9 estimate the parameters of the utility function separately for different regions of the world. We find that the estimates of the elasticity of substitution are very similar to our baseline estimates. They range from .68 to .77, and we cannot reject that they are statistically identical. We also find that the difference in income elasticities between agriculture and manufacturing, $\varepsilon_a - \varepsilon_m$, is negative, while the difference between services and manufacturing, $\varepsilon_s - \varepsilon_m$, is positive for all specifications. The magnitudes we find for the income elasticity of agriculture relative to manufacturing is stable across specifications around -1 and we cannot reject the null that they are statistically identical. For the income elasticity of services relative to manufacturing, we find very similar values for OECD and Latin American countries, around .5. However, the magnitude is significantly smaller for Asian countries, .09. In fact, we cannot rule out that this elasticity is different from zero at conventional confidence levels.[29] The income elasticity of services in Asian countries notwithstanding, we conclude that, overall, our estimates paint a picture of stable price and income elasticities across different regions and groupings of countries according to their level of development.

Next, we analyze how much each of the two channels, prices and consumption, account for structural transformation in our setting. To this end, we report different information

[29] This result seems to be driven by a few countries in our sample. We are currently investigating whether there are systematic differences in the way their aggregate data are collected and computed.

Table 2: Contribution of Relative Prices and Consumption

Specification	Log-Likelihood	LR Test χ^2	LR Test p-value	AIC	BIC
FE Only	-324.28	–	–	754.56	1010.02
FE + Prices	-270.30	107.96	0.00	648.61	908.89
FE + Consumption	363.34	1375.25	0.00	-616.68	-351.58
Full Specification	412.08	1472.71 97.47	0.00 0.00	-712.15	-442.23

Note: AIC refers to the Akaike Information Criterion, BIC refers to the Bayesian Information Criterion. The first two Likelihood Ratio Tests are done against the model that has only country-(relative)sector fixed effects. The last Likelihood Ratio Test compares the full model against one with fixed effects and consumption.

criteria comparing the fit of models with and without one of these channels in Table 2.[30] The first column reports the raw log-likelihood of estimating our sample with just country-sector fixed effects (first line in Table 2), using fixed effects and the price series only (second line), using fixed effects and the consumption series only (third line), and the full specification, which contains both the price and consumption series. We see that the log-likelihood increases relative to the fixed benchmark when we add either the prices or consumption series. However, the increase is an order of magnitude higher when we add the consumption series. This suggests that the role of non-homotheticity of demand is preponderant. The full specification has the highest log-likelihood, which suggests that the information contained in the relative prices and consumption time series are complementary. The second and third columns of Table 2 test these claims formally. The first three lines report the likelihood ratio tests done relative to the baseline of using only fixed effects. We find that all of them yield very high values of the test-statistic with associated p-values of the order of 0.00. Note moreover that the increase in the χ^2 statistic is biggest when we add the consumption time series. This confirms the idea that income effects have a predominant role. However, prices also play a significant role. This can be seen in the last row of the likelihood ratio test, which compares the full model to one with fixed-effects and consumption time series only. Again, the p-value is of the order of 0.00, which confirms that both channel are quantitatively significant. Finally, we also report the Akaike and Bayesian Information Criterion which convey the same message.

Table 3 reports partial correlations for each of our baseline regressions, equations (21) and (22). The partial correlation between a left-hand side variable and a regressor informs us of what would the correlation between the two be if we keep the rest of the regressors constant. Essentially, it is the correlation between the two variables after partialling out the other

[30] As a first pass, we have that the R^2 for equations (21) and (22) with only fixed effects .91 and .51. Including only price controls, it becomes .93 and .51, including consumption as additional control, .97 and .71.

Table 3: Partial Correlations

Regression Equation	Consumption		Relative Prices	
	Partial Corr.	Partial Corr.^{2}	Partial Corr.	Partial Corr.^{2}
L_a/L_m	-0.85	0.72	0.11	0.01
L_s/L_m	0.46	0.21	0.17	0.03

Note: Country*(relative)sector fixed effects included. Suppose that y is determined by $x_1, x_2, \ldots, x_k$. The partial correlation between y and x_1 is estimates the correlation that would be observed between y and x_1 if the other x's did not vary. The squared correlations estimate the proportion of the variance of y that is explained by each.

regressors on both variables. We find that the correlation of consumption is greater in absolute value than the correlation of prices. Indeed, note that the correlation of consumption with the equation that measures the relative employment share of agriculture to manufacturing is negative, reflecting the fact that the income elasticity of manufacturing is larger than the income elasticity of agriculture. We also report the squared of the partial correlation, which is to be interpreted as the fraction of the variance in the left-hand-side variable that is "exclusively" explained by the regressor of interest. We find that the squared partial correlations are an order of magnitude higher for consumption. Overall, the evidence presented in Tables 2 and 3 suggests that income effects play the lion's share in accounting for structural transformation in our sample. These findings are in line with the results of Herrendorf et al. (2013) for the United States. On the other hand, Boppart (2013) finds that the contribution of price and income effects are roughly of equal sizes. A first difference between our paper and his is that his exercise studies expenditure in services relative to the rest of consumption expenditures. A second difference is that the parametric assumption made in the demand system used in Boppart imposes that the price elasticity of services relative to the rest of consumption is declining as the economy grows. As noted by Buera and Kaboski (2009) among others, as relative services expenditure grows at a faster rate than prices, a declining price elasticity improves the explanatory power of relative prices. In contrast, our proposed specification holds the demand price elasticity constant.

4 Consumption Expenditure: Macro and Micro Estimates

In this section we analyze in more detail the case of the U.S., for which we can obtain more detailed data. First, we estimate our demand system using household data from the Consumption Expenditure Survey. This allows us to control for sectoral preference shocks using time fixed effects, which is not possible in the aggregate panel used above. Second, we estimate the parameters of the utility function using data on consumption aggregate time series. Building on the work of Herrendorf et al. (2013), we measure consumption both as expenditure and value added and analyze the robustness of the estimates to the alternative definitions.

4.1 Micro Data Estimation: Consumer Expenditure Survey

In this section we use household data to estimate our demand system. We estimate our demand system using household quarterly consumption data for the United States for the period 1980-2006. We use Consumption Expenditure Survey data as constructed in Heathcote et al. (2010). We follow Heathcote et al. and focus on a sample of households with a present household head with age between 25 and 60. We also use the same consumption categories, except that we separate food from the rest non-durables consumption.[31] We estimate the demand system using expenditure shares for each household in the left hand side. To control for household fixed characteristics, we estimate the demand system using household fixed effects,

$$\log\left(\frac{\omega^h_{i,t}}{\omega^h_{nd,t}}\right) = (1-\sigma)\log\left(\frac{p_{i,t}}{p_{nd,t}}\right) + (\epsilon_i - \epsilon_{nd})\log C^h_t + \delta^h + \delta_{i,t} + \eta^h_{i,t}. \tag{23}$$

The superscript h denotes a household, and nd denotes non-durables –which we use as reference in our regressions. The price indexes $p_{i,t}$ come from the corresponding sectorial CPI-Us of the BLS. Aggregate consumption expenditure C^h_t is deflated using a household specific CPI, as suggested by our theory.[32] Household fixed effects are denoted by δ^h, while time-sector fixed-effects are denoted by $\delta_{i,t}$.

Since the BLS CPIs vary regionally, we can identify the price elasticity of consumption shares even after including time-sector effects. Variation in the prices is arguably exogenous to households. In an analogous manner to the cross-country panel, the identification comes from within household variation in total consumption expenditure.[33] Finally, following Johnson

[31] Consumption measures are divided by the number of adult equivalents in the household. We use the categorization of Heathcote et al. (2010) for expenditures. The consumption categories in non-durables are: alcoholic beverages, tobacco, personal care, fuels, utilities and public services, public transportation, gasoline and motor oil, apparel, education, reading, health services and miscellaneous. Our data for services comes from entertainment expenditures. These includes among others fees for recreational lessons, TV and music related expenditures, pet-related expenditures, toys, games. Durables comprises vehicles (purchases/services derived from it and car maintenance and repair) and household equipment. Housing comprises the rents or imputed rents (if the dwelling is owned) as well as from "other dwellings" (primarily vacation homes). For each household we have a maximum of 4 observations (one per semester). The consumption data comes from the Family Characteristics and Income files except for years 1982 and 1983 for which the Detailed Expenditures files were used. See Heathcote et al. (2010) and Krueger and Perri (2006) for further discussion on the construction of the data set and its characteristics.

[32] We note that if we deflated consumption expenditures using the aggregate CPI, we obtain similar estimates. When we include durables in our regressions, we use the aggregate CPI, as we lack information on the purchase timing used in the imputation method.

[33] One additional possible concern is the fact that even after controlling for household fixed characteristics

et al. (2006), we instrument total expenditure using the random timing of arrivals of tax rebates for the subsample period 2001-2002. We find very similar elasticities.

Table 4 reports the results of our regressions with and without including durables consumption, and with and without sector-year fixed effects. The main observation is that the estimates are very similar across all four specifications. This suggests that sector-specific demand shocks are not a significant source of bias in our demand system. The estimate of the price elasticity is around .64. As we shall see in Section 4.2, this value is within the range of estimates we obtain from the aggregate U.S. time series.[34] With respect to the income elasticities, food has the lowest elasticity, followed by housing, services and we find the highest income elasticity for durables. As the purchase of durables is lumpy and our most of our consumption for durables is imputed, we take this latter estimate with a grain of salt. Finally, we should keep in mind that consumption expenditure is known to be noisily measured in household survey data, so to the extent that first-differencing does not correct for mis-measurement, we would expected some downward bias in the estimation of income elasticities.[35]

IV Strategy One possible concern with the previous regression is that total consumption expenditure is an endogenous choice of the household that may be correlated with some omitted variable. To address this concern, we use an instrumental variable approach as in Johnson et al. (2006). The instrument is based on the fact that the timing of the 2001 federal income tax rebates was done as a function of the last digit of the recipient's social security number. Thus, it was effectively random.[36] Columns (1) and (2) in Table 5 report the OLS estimated as in our baseline estimation (23) for the period of interest (2001-2002). Note that the estimates are very similar to the whole sample. Columns (3) and (4) report IV estimates excluding and including durables. We instrument consumption expenditure with a dummy for whether or not a household received the rebate at a given point in time. We note that the estimates remain very similar to the baseline estimation. The income elasticity of non-durables decreases somewhat after instrumenting but the ranking of income elasticities

there is an unobserved and persistent shock driving both aggregate consumption expenditure and some particular consumption category, most likely durable goods and housing. Note that we are imputing the flow services obtained from housing and vehicles, which should attenuate these concerns.

[34]One possible reason for why we find a different price elasticity than the macro consumption expenditure is that the consumption categories we have access to are not identical. An alternative explanation can be that perfect competition does not hold and prices and consumption expenditures co-move by a third omitted variable. This can induce an upward bias in the aggregate time series.

[35]As the consumption categories of the household expenditure survey are different from the cross-country estimation, it is not entirely clear whether this is the case.

[36]Because the data requirements to construct detailed household expenditures in Herrendorf et al. (2013) are different than in Johnson et al. (2006), we can merge only a third of the tax rebate data with our baseline data set. The merge is done according to total gross income and nominal food expenditure.

Table 4: Consumer Expenditure Survey, 1980-2006

	(1)	(2)	(3)	(4)
σ	0.64	0.64	0.63	0.63
	(0.12)	(0.11)	(0.14)	(0.14)
Food	-0.46	-0.44	-0.49	-0.48
	(0.13)	(0.13)	(0.16)	(0.16)
Housing	-0.31	-0.31	-0.27	-0.26
	(0.16)	(0.15)	(0.18)	(0.17)
Services	0.57	0.52	0.62	0.57
	(0.24)	(0.25)	(0.33)	(0.33)
Durables			0.94	0.93
			(0.58)	(0.58)
Time FE	N	Y	N	Y
Observations	346631	346631	241470	241470

Note: Std. Errors clustered at the household level. Elasticity estimates are relative to non-durables consumption. Data from Heathcote et al. (2010).

remains the same and the magnitudes are also very similar.[37]

Quartiles Estimation We also estimate the baseline regression (23) by income quartiles in the pooled data, 1980-2006. Columns (1) to (4) of Table 6 report the elasticity estimates for the first to the fourth quartiles. We find that the value of all elasticities is quite stable at different income levels.

4.2 Aggregate Consumption: Value Added and Expenditure

In this section, we estimate our demand system using consumption time series for the United States. We use the data on the three sectors from more the Bureau of Economic Analysis as constructed by Herrendorf et al. (2013). Expenditure data are decomposed in two different ways. Consumption expenditure classifies sectors according to final good expenditure. The data on consumption value added decomposes each dollar of final expenditure of a good to the share of value added attributable to agriculture, manufacturing and services using U.S. input output tables. For example, purchases of food from supermarkets is included in agriculture in the final expenditure computation, while it is broken down into food (agriculture), food processing (manufacturing) and distribution (services) when using the value added formulation. See Herrendorf et al., 2013 for details. Thus, the final expenditure and value added represen-

[37]We also note that using the value of the rebate yields very similar results. However, as the magnitude of the rebate was not random, we prefer to use only the indicator

Table 5: Instrumental Variables Strategy
Consumption Expenditure Survey, 2001-2002

	(1)	(2)	(3)	(4)
σ	0.69	0.64	0.69	0.64
	(0.02)	(0.02)	(0.02)	(0.02)
Food	-0.44	-0.43	-0.45	-0.44
	(0.02)	(0.02)	(0.02)	(0.02)
Housing	-0.17	-0.16	-0.18	-0.17
	(0.03)	(0.03)	(0.03)	(0.03)
Services	0.51	0.52	0.51	0.52
	(0.04)	(0.04)	(0.04)	(0.04)
Non Durables		1.31		0.93
		(0.09)		(0.09)
			First Stage	
Tax Rebate Indicator			0.02	0.02
			(0.01)	(0.01)

Note: Std. Errors clustered at the household level. Elasticity estimates are relative to non-durables consumption. All estimates contain household and time-sector fixed effects. Data from Heathcote et al. (2010) and Johnson et al. (2006).

tation of consumption are two alternative classifications of the same underlying data.[38]

We repeat the exercise done by Herrendorf et al. (2013) of estimating the demand system for sectoral consumption value added and expenditure using our preference specification. Herrendorf et al. (2013) estimate a CES Stone-Geary utility and find that this distinction yields quantitatively very different results. Using value added measures, Herrendorf et al. (2013) find that the elasticity of substitution across sectors is not statistically different from 0. Buera and Kaboski (2009) report a similar finding for the period 1870-2000. This result is quite extreme and some authors have fount it discomforting, e.g. Buera and Kaboski (2009). In terms of the value of the subsistence level relative to total consumption, they find that for agriculture in 1947 it represents 17% and in 2010, a 4%. For services, the figures are 73% and 32%. When estimating the model with final expenditure Herrendorf, et al. find that the elasticity of substitution is .85 and the role of non-homotheticities is less pronounced (but still important). The value of the subsistence level of agriculture relative to total consumption is 8% in 1947 and .4% in 2010. For services, the values are 86% and 32%. Herrendorf et al. (2013) convincingly argue that the elasticity of substitution should be greater when measuring expenditure because expenditure measures even at sectoral level are likely to embed inputs from the three sectors. However they do not provide a good intuition for why a Leontief production function is a reasonable benchmark.

[38] Note also that neither classification maps one-to-one to the classification of our baseline dataset.

Table 6: Estimation by Quartiles
Consumer Expenditure Survey, 1980-2006

	(1)	(2)	(3)	(4)
σ	0.63	0.76	0.75	0.67
	(0.01)	(0.01)	(0.01)	(0.01)
Food	-0.44	-0.31	-0.41	-0.48
	(0.02)	(0.07)	(0.09)	(0.02)
Housing	-0.20	-0.44	-0.37	-0.23
	(0.02)	(0.06)	(0.07)	(0.02)
Services	0.47	0.75	0.77	0.68
	(0.03)	(0.15)	(0.18)	(0.04)

Note: Std. Errors clustered at the household level. Elasticity estimates are relative to non-durables consumption. (1) Corresponds to the first quartile, (2), to the second, etc. All estimates contain household and time-sector fixed effects. Data from Heathcote et al. (2010).

We estimate a demand system analogous to our baseline estimation,

$$\log\left(\frac{\omega_{a,t}}{\omega_{m,t}}\right) = \alpha_{am} + (1-\sigma)\log\left(\frac{p_{a,t}}{p_{m,t}}\right) + (\varepsilon_a - \varepsilon_m)\log C_t + \nu_{am,t}, \tag{24}$$

$$\log\left(\frac{\omega_{st}}{\omega_{mt}}\right) = \alpha_{sm} + (1-\sigma)\log\left(\frac{p_{s,t}}{p_{m,t}}\right) + (\varepsilon_s - \varepsilon_m)\log C_t + \nu_{sm,t}, \tag{25}$$

where $\omega_{i,t}$ denotes consumption expenditure or value added in sector i at time t. As we are using relative consumption shares rather than employment, there is no need to control for international trade in this regression because it is subsumed in the consumption expenditure. Our estimates are reported in table 7. For the elasticity of substitution, we also find that it is larger when estimated for expenditure data. However, we find that for value added measures, the estimated elasticity is .57 with a standard deviation of .1. Clearly, the preferences implied by our estimate differ significantly from the Leontief specification found in the Stone-Geary setting. When estimating our demand system with expenditure data, we find an estimate of .88 which is very similar to the .85 found in Herrendorf et al.. For the income elasticities, we find estimates that are very similar using both data sets. In fact, the point estimates are identical for $\varepsilon_a - \varepsilon_m$, with an elasticity of $-.63$. The estimates for $\varepsilon_s - \varepsilon_m$ are .62 for value added data and .55 for expenditure data. We cannot reject the null that they are statistically identical. Thus, our estimates of the non-homotheticity paint a picture in which the role for non-homotheticities is very similar regardless of whether utility is specified in terms of value added or expenditure.

Table 7: Consumption Expenditure and Value Added for the U.S.

	σ	$\varepsilon_a - \varepsilon_m$	$\varepsilon_s - \varepsilon_m$	Obs.	R^2_{am}	R^2_{sm}
Value Added	0.57	-0.63	0.62	63	0.77	0.93
	(0.10)	(0.12)	(0.06)			
Expenditure	0.88	-0.63	0.55	63	0.98	0.91
	(0.03)	(0.03)	(0.03)			

5 Further Analysis of Cross-Country Data

5.1 Case Studies for Three Sectors

From the estimation based on the relative shares, (21) and (22), we have obtained estimates of $\{\sigma, \varepsilon_a - \varepsilon_m, \varepsilon_s - \varepsilon_m\}$ and the country-sector dyad fixed effects $\{\alpha_{c,am}, \alpha_{c,sm}\}$. Using this information and the fact that employment shares sum up to one, we can recover the predicted evolution of individual shares over time (not only the relative shares).[39]

Figures 3, 4 and 5 show the predicted series of the employment shares and the actual time series. Note that the estimates $\{\sigma, \varepsilon_a - \varepsilon_m, \varepsilon_s - \varepsilon_m\}$ are the same for all countries. Figures C.1, C.2 and C.3 in the online appendix report the fit for all countries in our sample. We see that despite the parsimonious approach of using the same three parameters in the utility function for all countries, the overall fit is good. The model captures well the trends in all sectors for countries at very different stages of development.[40] Figure 3 shows two

[39]To see this, denoting the exponent of the fitted values as

$$\widehat{\frac{L_{s,ct}}{L_{manu,ct}}} = \exp\left(\log\left(\widehat{\frac{L_{agri,ct}}{L_{manu,ct}}}\right)\right), \qquad s = \{agri, serv\}, \tag{26}$$

we have that the predicted shares are

$$\widehat{\frac{L_{manu}}{L}} = \left(1 + \widehat{\frac{L_{agri,ct}}{L_{manu,ct}}} + \widehat{\frac{L_{serv,ct}}{L_{manu,ct}}}\right)^{-1} \tag{27}$$

$$\widehat{\frac{L_{agri}}{L}} = \widehat{\frac{L_{agri}}{L_{manu}}}\,\widehat{\frac{L_{manu}}{L}} \tag{28}$$

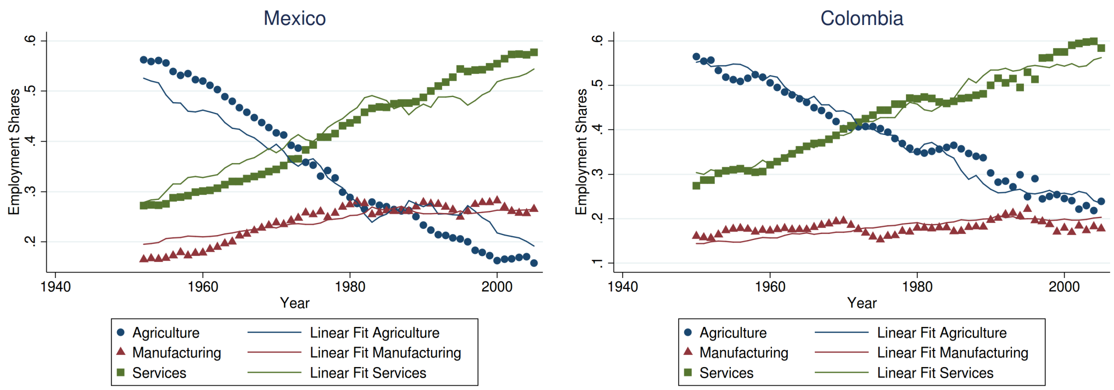
$$\widehat{\frac{L_{serv}}{L}} = \widehat{\frac{L_{serv}}{L_{manu}}}\,\widehat{\frac{L_{manu}}{L}} \tag{29}$$

[40]We have performed an analogous exercise using Stone-Geary preferences for the intra-period problem as in Herrendorf et al. (2013),

$$C_t = \left(\omega_a^{\frac{1}{\sigma}}\left(C_{at} - \overline{C}_a\right)^{\frac{\sigma-1}{\sigma}} + \omega_m^{\frac{1}{\sigma}} C_{mt}^{\frac{\sigma-1}{\sigma}} + \omega_s^{\frac{1}{\sigma}}\left(C_{st} + \overline{C}_s\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{30}$$

where ω_i are positive weights that add up to one, and $\overline{C}_a$, $\overline{C}_s$ are positive reals that capture the "subsistence" levels. We find the fit to be much worse. The fitted time series are reported in the online appendix, Figures D.1, D.2 and D.3. We find that the role for non-homotheticities is much smaller in this case. Broadly speaking, the fit is good for countries in which there trends in relative prices are important, while the fit is much poorer

Figure 3: Examples of Latin American Countries



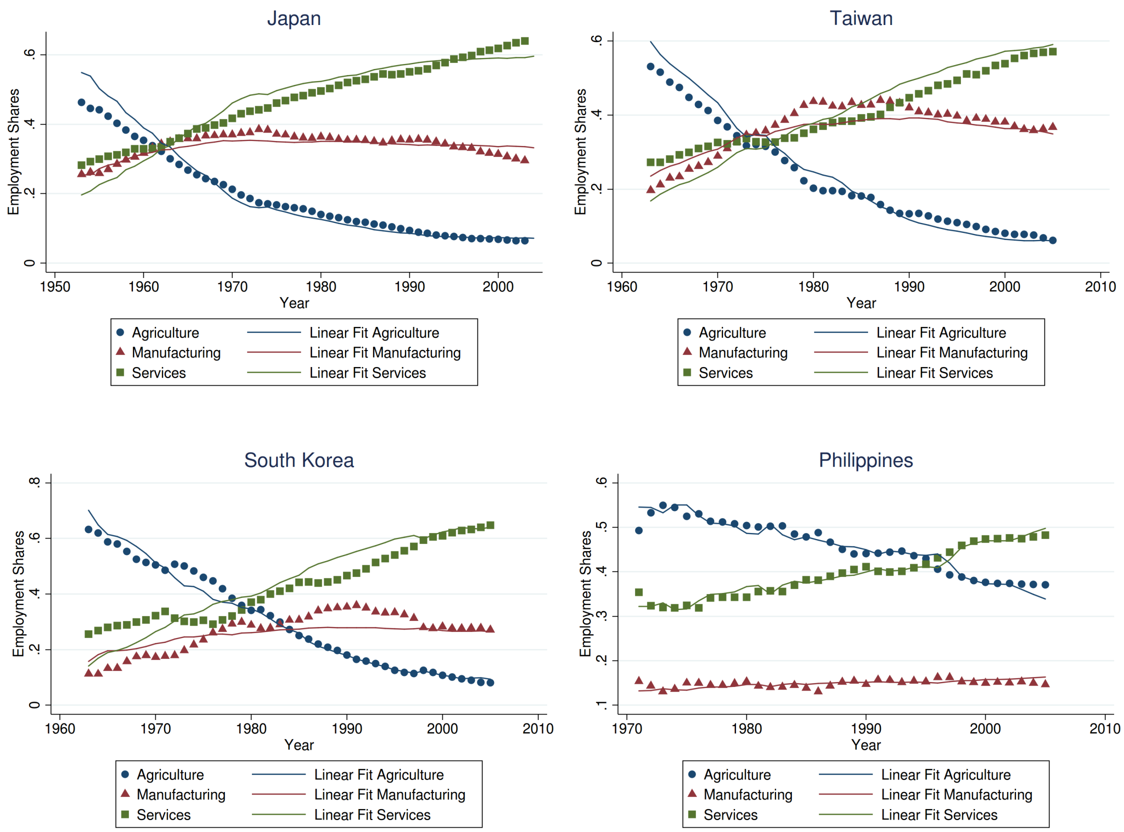
Latin American countries, Mexico and Colombia (Figure C.3 contains the plots for all Latin-American countries). We see that the fit captures well the trends in all sectors. However, for Mexico, the fit appears to overstate the contribution of Services in the first half of the period and understate it for Agriculture, while the trend is reversed in 1980. This is partly due to the fact that value added measures are constructed from different data sources pre-1980 (which uses U.N. national accounts and uses growth rates to link data series backwards) and post-1980, which uses higher quality data from the Central Bank of Mexico. The model captures the trend on average, "compensating" for the change in the design of the consumption basket. For Colombia, which uses the same data source pre-1980 (U.N. national accounts) and ECLAC data (Statistical Year Book for Latin America and the Caribbean) for the 1980-1993 data we do not observe any discontinuity in the value added series for this period.[41]

Figure 4 depicts the fit of our baseline estimation for four Asian economies: Japan, Taiwan, South Korea and the Philippines (Figure C.2 contains the plots for all Latin-American countries). The overall trends are well captured by our model for all countries. For Japan and Taiwan, we see that our model generates a hump-shape for employment in manufacturing of the same order of magnitude as in the data. For Taiwan, the predicted initial level of the employment share in manufacturing is 21%, it goes up to 39% and back to 35% at the end of the period. The observed levels are 20%, 43% and 37%. However, for South Korea, we find that the fitted trend for manufacturing has a hump shape, albeit less pronounced than in the data. Our predicted level of initial employment share is 13%, it goes up to 28% and then declines to 27%, while in the observed data we have that the initial level is 11%, 36% and 27%. This result may appear surprising, as in these three countries the evolution of consumption is similiar. Moreover, the relative price of services to manufacturing grew at a

for countries with no significant price trends.

[41] From 1991 onwards the data source changes to the National Statistics Department and we observe a small bump in the aggregate consumption time series that translates into the predicted time series.

Figure 4: Examples of Asian Countries



much faster rate prior to 1990 in Korea than in Japan and Taiwan, which would exacerbate the increasing part in the hump. The key difference is that prior to 1985, the relative price of agriculture to manufacturing was increasing at 2.5% annually in Korea.[42] This introduces a countervailing force in our model that tends to reduce the increase in employment shares in manufacturing.[43]

Finally, figure 5 reports the fit for two OECD countries, the USA and Spain (Figure C.1 in the online appendix shows the fit for all OECD countries). We see that our model fits the trends well. For the USA, however, we see that the evolution of the employment shares in services and manufacturing are steeper than predicted by our model. This feature is shared

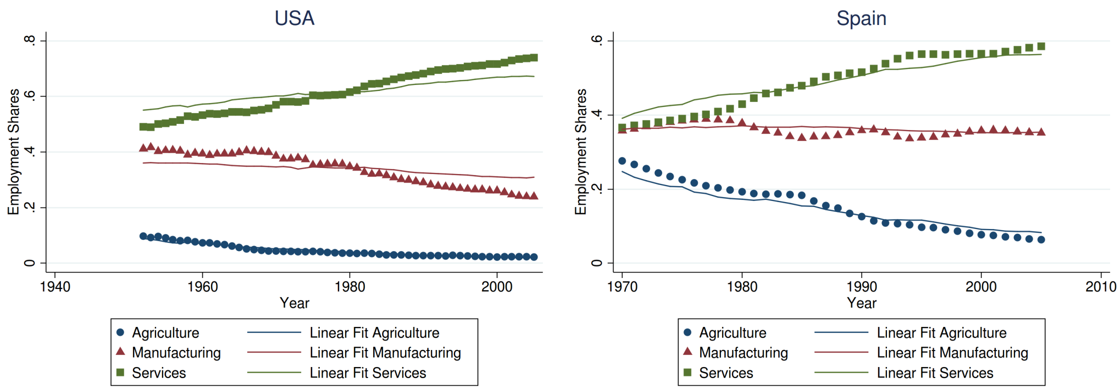
[42] The relative price of agriculture to manufacturing has no trends in Taiwan during this period and it is also increasing for Japan but at a slower rate, 1.7%.

[43] From equation 27. it follows that the growth rate of the employment share in manufacturing, is equal to

$$\gamma_m = -L_m(\gamma_{L_a/L_m} + \gamma_{L_s/L_m}), \tag{31}$$

where γ denotes growth rates. From here it is clear that a high and positive growth rate of the relative price of agriculture to manufacturing increases the growth rate γ_{L_a/L_m} which decreases γ_m.

Figure 5: Examples of OECD Countries



with other OECD countries. This reflects the fact that the income elasticity of services is greater for these set of countries, as column (5) in Table 1 shows. Indeed, if we plot the predicted fit using the estimates $\{\sigma, \varepsilon_s - \varepsilon_m, \varepsilon_a - \varepsilon_m\}$ for only OECD countries this problem goes entirely away, as can be seen in Figure C.4 in the online appendix.

5.2 Real and Nominal Value Added

We next investigate whether our model can quantitatively reproduce the positive correlation between nominal and real sectoral demands observed in the data. Combining the preference parameters $\{\sigma, \varepsilon_a - \varepsilon_m, \varepsilon_s - \varepsilon_m\}$ obtained in our baseline estimation with sectoral demands, equations (7) and (8), we can generate the predicted evolution of nominal and real sectoral demands and compute its correlation.

Table 8 reports the correlation predicted by our estimated model and the empirical correlation. We find that the model is able to generate correlations that are similar to the data, even though in the estimation of the preference parameters we did not target any of these correlations. In particular, we find that our model generates a correlation of .93 between the nominal and real relative demand of agricultural goods to manufactures, while in the data it is .95. For services, the model generates a correlation of .71 while in the data it is .80. Note that this result highlights the importance of using a non-homothetic CES framework. If we had used an homothetic framework as Ngai and Pissarides (2007), in which all structural transformation is generated through trends in relative prices, the correlation generated by the model would have been negative.[44] Our framework also has a price elasticity of substitution that is less than one, which in a homothetic framework would generate a negative correlation.

[44] To see that, note that the relative trend in nominal values $\omega_{i,t}/\omega_{j,t}$ would be proportional to $(p_{i,t}/p_{j,t})^{1-\sigma}$. For real values, $c_{i,t}/c_{j,t}$, would be proportional to $(p_{i,t}/p_{j,t})^{-\sigma}$. As $0 < \sigma < 1$, both trends would move in opposite directions.

Table 8: Correlation of Nominal and Real Value Added

	Correlation	
	Data	Model
Agriculture/Manufacturing	0.95	0.93
Services/Manufacturing	0.80	0.71

The reason this does not happen is that, at the estimated parameter values, the income effects are sufficiently strong to overcome the relative price effects.

5.3 Extensions

5.3.1 Beyond Three Sectors

Jorgenson and Timmer (2011) have pointed out that in order to understand how structural transformation progresses in rich countries, it is important to zoom in the service sector, as it represents the majority of rich economies' consumption shares. Our framework lends itself to this purpose very well, as it can accommodate any arbitrary number of sectors in an analogous way as the homothetic CES framework. In this section we use the richness of the GDDC database to extend our estimation to 10 sectors: (1) agriculture, forestry and fishing, (2) mining and quarrying, (3) manufacturing, (4) public utilities, (5) construction, (6) wholesale and retail trade, hotels and restaurants, (7) transport, storage and communication, (8) finance, insurance, real state, (9) community, social and personal services, (10) government services.

We estimate a demand system analogous to the one used in our baseline estimation, where we use manufacturing as a reference sector (even though in this case it is more narrowly defined than in the baseline estimation)

$$\log\left(\frac{L^c_{i,t}}{L^c_{m,t}}\right) = \alpha^c_{im} + (1-\sigma)\log\left(\frac{p^c_{i,t}}{p^c_{m,t}}\right) + (\varepsilon_i - \varepsilon_m)\log C^c_t + \eta^c_{i,t} \tag{32}$$

with i denoting any of our sectors and c, a country index. Our panel estimates a reported in Table 9. Column (1) shows that we find an elasticity of substitution of .82 which is close to the .72 we found in our baseline, three-sector, estimation. We find that the smallest income elasticities correspond to mining and agriculture, while the highest correspond to service sectors, such as finance, insurance, real state and government services. Columns (2) to (4) show that the ranking of sectors in terms of their income elasticity is consistently estimated when we estimate OECD, Asian and Latin American countries separately.[45]

[45] Table 11 in the Appendix reports the estimation results from a nested CES structure. That is, we estimate separately the demand for each of the sectors separately that belong to services or manufacturing separately.

Table 9: 10-Sector Regression

	World	OECD	Asia	Latin America
Elasticity σ	0.82	0.93	0.85	0.96
	(0.01)	(0.01)	(0.01)	(0.01)
Mining	-1.43	-0.91	-1.55	-0.64
	(0.03)	(0.05)	(0.04)	(0.06)
Agriculture	-0.96	-0.66	-1.07	-1.12
	(0.03)	(0.04)	(0.03)	(0.05)
Public Utilities	-0.02	0.18	-0.11	0.27
	(0.02)	(0.03)	(0.02)	(0.07)
Transp., Storage, Comm.	0.10	0.60	-0.10	0.52
	(0.02)	(0.03)	(0.02)	(0.04)
Construction	0.18	0.47	0.13	0.83
	(0.02)	(0.03)	(0.02)	(0.07)
Wholesale and Retail	0.37	0.95	0.14	1.22
	(0.02)	(0.02)	(0.02)	(0.06)
Community, Social and Personal Serv.	0.44	1.29	0.18	0.37
	(0.03)	(0.03)	(0.04)	(0.05)
Finance, Insurance, Real State	0.94	1.92	0.55	1.22
	(0.03)	(0.04)	(0.03)	(0.09)
Government Services	0.82	0.93	0.85	–
	(0.01)	(0.01)	(0.01)	
Observations	517	342	125	295

Note: All sectoral elasticities computed relative to Manufacturing. Source: 10 Sector database. Robust Standard Errors shown in parenthesis.

5.3.2 Non-constant Elasticity of Consumption

In Section 5.1, when we discussed the fit for OECD countries, we argued that the income elasticity of services relative to manufacturing estimated for the whole sample was too low to fit the steep increases in employment shares in services. Indeed, in column (5) in Table 1 we show that estimating $\{\sigma, \varepsilon_a - \varepsilon_m, \varepsilon_s - \varepsilon_m,\}$ for only OECD countries yields a higher $\varepsilon_s - \varepsilon_m$, which improves the fit of the time series for OECD countries, as shown in Figure C.4. Motivated by this evidence, in this section we explore the possibility of relaxing the assumption of constant income elasticity across sectors.[46] To this end, we allow for a second order term in the income elasticity term, $\log^2 C_{ct}$. This represents a second order approximation to an

This is done at the expense of not having sectoral consumption data. We use aggregate sectoral value added instead. The results are similiar to the ones reported in the main text.

[46]In this case, the existence of an Asymptotic Growth Path is ensured as long as the long-run elasticity is constant.

Table 10: Non-constant elasticity of substitution estimation

Dep. Var.: Rel. Lab. Sh.	σ	$\varepsilon_a - \varepsilon_m$		$\varepsilon_s - \varepsilon_m$		Obs.	R^2_{am}	R^2_{sm}
		$\log C$	$\log^2 C$	$\log C$	$\log^2 C$			
World, 1947-2005	0.80	-1.40	0.45	-3.18	4.61	1005	0.98	0.79
	(0.02)	(0.20)	(0.27)	(0.16)	(0.21)			
OECD	0.92	-1.93	1.36	-4.95	7.21	436	0.96	0.87
	(0.03)	(0.32)	(0.43)	(0.20)	(0.26)			
Latin America	0.83	-1.10	0.08	-0.96	1.98	350	0.96	0.70
	(0.04)	(0.89)	(1.10)	(0.80)	(0.99)			
Asia	0.82	-0.61	-0.73	-1.11	1.60	319	0.98	0.82
	(0.05)	(0.29)	(0.39)	(0.18)	(0.25)			

Note: 10 sector database.

arbitrary function and estimate the system

$$\log\left(\frac{L^c_{a,t}}{L^c_{m,t}}\right) = \alpha^c_{am} + (1-\sigma)\log\left(\frac{p^c_{a,t}}{p^c_{m,t}}\right) + \varepsilon_{1,am}\log C^c_t + \varepsilon_{2,am}\log^2 C^c_t + \nu^c_{am,t} \quad (33)$$

$$\log\left(\frac{L^c_{s,t}}{L^c_{m,t}}\right) = \alpha^c_{sm} + (1-\sigma)\log\left(\frac{p^c_{s,t}}{p^c_{m,t}}\right) + \varepsilon_{1,sm}\log C^c_t + \varepsilon_{2,sm}\log^2 C^c_t + \nu^c_{sm,t} \quad (34)$$

where we also control by sectorial next exports in each regression. Our results in Table 10 show that when estimating the system of equations (33)−(34) in our full panel the quadratic term for the income elasticity of agriculture relative to manufacturing is zero, while the quadratic term for services relative to manufacturing is positive. This indicates that the real income elasticity of services relative to manufacturing (and agriculture) increases as countries become richer. Perhaps interestingly, the Stone-Geary framework imposes the opposite by assumption. This finding is confirmed when analyzing different regions of the world separately.

6 Extended Model

In this section we provide an extended version of the model presented in Section 2. This section has two goals. First, we show that the production side of the economy can be extended to allow for heterogeneous capital shares in the production function of different goods. We show the existence, uniqueness and local saddle-path stability in this case.[47] Moreover, we show that our estimating equations go through in this more general setup. Second, we introduce a generalization of the baseline utility function. We provide precise statements for the problem to be well-defined. This formulation also allows us to have a precise mapping from aggregate

[47] We choose a continuous time formulation because it allows a simpler analysis of the stability properties of the dynamic system.

consumption in the data and the theory.[48]

6.1 Household Preferences and Demand

Consider a unit mass of households with identical preferences over a stream of real consumption per capita $[c(t)]_{t=0}^{\infty}$ according to

$$U \equiv \int_0^{\infty} e^{-(\rho-n)t} u(c(t))\, dt, \tag{35}$$

where the instantaneous utility function u is asymptotically isoelastic, such that $\lim_{c\to\infty} \eta_u \equiv cu'/u = 1-\theta$, with $\theta > 0$ and $n \geq 0$ denotes population growth. We make the standard assumption that $n < \rho$.

Per capita real consumption $c(t)$ aggregates cosumption of a bundle $\{c_i(t)\}_{i=1}^{I}$ of goods according to generalized nonhomothetic CES preferences defined implicitly through

$$\sum_{i=1}^{I} \left[g(c(t))^{-\epsilon_i} c_i(t)\right]^{\frac{\sigma-1}{\sigma}} = 1, \tag{36}$$

where the elasticity of substitution is $0 < \sigma < 1$, and each consumption good is characterized by an income elasticity parameter $\epsilon_i > 0$ and the monotonic function $g(\cdot)$, to be introduced in what follows. next. When consumers face prices $\boldsymbol{p}(t) \equiv \{p_i(t)\}_{i=1}^{I}$, the expenditure function corresponding to the preferences above is be given by (see derivations in Appendix C.1):

$$e(c(t);\boldsymbol{p}(t)) \equiv \left(\sum_{i=1}^{I} [g(c(t))^{\epsilon_i} p_i(t)]^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{37}$$

In the remainder of the exposition, we will often drop the dependence of function e on the prices whenever possible in order to simply notation. Appendix C.1 discusses conditions for this function to always remain strictly monotically increasing and therefore one-to-one.

We define function $g(\cdot)$ in such a way that aggregate consumption per capita is expressed in terms of constant prices $\boldsymbol{q}$, i.e., in real terms. To achieve this, we need to simply let $g \equiv e^{-1}(\cdot;\boldsymbol{q})$, that is, define $g(\cdot)$ implicitly through:

$$c(t) \equiv \left(\sum_{i=1}^{I} [g(c(t))^{\epsilon_i} q_i]^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{38}$$

With this definition, the function (36) aggregates bundles of sectoral consumption goods in such a way that if consumers were facing prices $\boldsymbol{q}$ their optimal bundle costs them exactly

[48] In the theory presented in 2 there was one degree of freedom. We show that this degree of freedom can be eliminated by imposing more structure on the definition of the consumption aggregator.

$c(t)$. The consumption expenditure share of good i is given by

$$\omega_i(t) \equiv \frac{p_i(t)\, c_i(t)}{e(c(t);\boldsymbol{p})} = \left(\frac{g(c(t))^{\epsilon_i}\, p_i(t)}{e(c(t);\boldsymbol{p})}\right)^{1-\sigma}.$$

Proposition 4. *Suppose the elasticity of intertemporal substition is bounded above by $1/\underline{\theta}$, that is, $\eta_{u'}(c) < -\underline{\theta}$ for all c, and that we have that*

$$\underline{\theta} > \frac{\epsilon_{max} - \epsilon_{min}}{\epsilon_{max}} + (1-\sigma)\frac{(\epsilon_{max} - \epsilon_{min})^2}{4\epsilon_{min}^2}. \tag{39}$$

Let $\eta_{e'} \equiv \frac{c\partial^2 e/\partial c^2}{\partial e/\partial c}$ denote the elasticity of the marginal nominal cost of real consumption. Then, the Euler Equation

$$\frac{\dot{c}}{c} = \frac{r(t) - \rho}{\eta_{e'} - \eta_{u'}},$$

along with the transversality condition

$$\lim_{t\to\infty} a(t)\, e^{-\int_0^t [r(\tau)-n]d\tau} \geq 0,$$

characterize the unique path of real consumption $[c(t)]_{t=0}^{\infty}$ maximizing the household intertemporal problem (35) subject to the budget constraint

$$\dot{a}(t) = w(t) + [r(t) - n]\, a(t) - e(c(t);\boldsymbol{p}(t)).$$

The proposition above establishes that

6.2 Production

In this section we provide a rish model of multi-sectoral production that allows both for inter-sector heterogeneity in rates of technological progress, analyzed first by Ngai and Pissarides (2007) as well as inter-sector heterogeneity in factor intensities, studied by Acemoglu and Guerrieri (2008) in a two-sector model. We show that our growth model remains fully tractable even incoporating both these supply side channels. More importantly, we want to emphasize that both these supply side mechanisms are reflected in sectoral prices. Therefore, the empirical strategy that we have introduced in Section 3 remains valid and can isolate the contribution of demand channels to structural change.

Households also inelastically supply labor, $L(t) \equiv L(0)\, e^{nt}$, where $n < \rho$. Capital is accumulated using investment goods produced by sector $i = 0$:

$$\dot{K}(t) = Y_0(t) - \delta K(t),$$

Labor and capital are combined by producers of consumption good sectors i to produce output using a Cobb-Douglas technology

$$Y_i(t) = A_i(t) L_i(t)^{1-\alpha_i} K_i(t)^{\alpha_i}, \qquad \text{for } i \in \{0, \cdots, I\},$$

where production technology in sector i has capital intensity $\alpha_i \in [0,1]$. Thus, sectors differ in their capital intensities. The Hicks-neutral technological progress $A_i(t)$ grows exogenously with the asymptotically constant rate $\lim_{t\to\infty} \dot{A}_i(t)/A_i(t) = \gamma_i > 0$ for all sectors i.

Before continuing with our derivations for the production side of the economy, let us define some auxiliary variables to simplify the exposition in what follows. As before, we normalize the price of the investment sector in each period to unity, $p_0(t) \equiv 1$ and define aggregate nominal output as:

$$\begin{aligned} Y(t) &\equiv \sum_{i=0}^{I} p_i(t) Y_i(t), \\ &= Y_0(t) + L(t) e(t), \end{aligned}$$

where the second equality follows from the definition of $e(t)$ as the per capita consumption expenditure. We define the output weighted average capital share in the consumption sector as

$$\bar{\alpha}(t) \equiv \sum_{i=0}^{I} \omega_i(t) \alpha_i. \tag{40}$$

Since we assume competitive labor and capital markets, the marginal revenue product of labor and capital have to match their prices, that is,

$$w(t) = (1-\alpha_i) \frac{p_i(t) Y_i(t)}{L_i(t)}, \tag{41}$$

$$r(t) = \alpha_i \frac{p_i(t) Y_i(t)}{K_i(t)}. \tag{42}$$

Furthermore, we define sectoral capital-to-labor ratios as:

$$\kappa_i(t) \equiv \frac{K_i(t)}{L_i(t)} = \frac{\alpha_i}{1-\alpha_i} \frac{w(t)}{r(t)}, \qquad \text{for } i \in \{0, \cdots, I\}. \tag{43}$$

We can write total labor and stock of capital employed in the consumption sectors of the economy in terms of aggregate nominal consumption and factor prices as follows:

$$L_C(t) = \sum_{i=1}^{I} L_i(t) = \frac{(1-\bar{\alpha}(t)) e(t) L(t)}{w(t)}, \qquad K_C(t) = \sum_{i=1}^{I} K_i(t) = \frac{\bar{\alpha}(t) e(t) L(t)}{r(t)}. \tag{44}$$

This suggests that the average sectoral capital share $\bar{\alpha}(t)$ corresponds to the capital share in the aggregate consumption sector of the economy. Correspondingly, we can define the aggregate capital to labor ratio in the consumption sectors of the economy,

$$\kappa_C(t) \equiv \frac{K_C(t)}{L_C(t)} = \frac{\bar{\alpha}(t)}{1-\bar{\alpha}(t)} \frac{w(t)}{r(t)}. \tag{45}$$

6.3 Equilibrium and Equilibrium Dynamics

First, we find equilibrium price of sectoral consumption goods in terms of the investment good by equalizing marginal revenue products of capital from Equation (42), finding:

$$p_i(t) \quad = \quad \left(\frac{\alpha_0}{\alpha_i}\right)^{\alpha_i} \left(\frac{1-\alpha_0}{1-\alpha_i}\right)^{1-\alpha_i} \frac{A_0(t)}{A_i(t)} \kappa_0(t)^{\alpha_0-\alpha_i}, \tag{46}$$

where κ_0 is the capital-to-labor ratio in the investment sector. Equation (46) shows that consumption good prices depend only on sectoral TFPs and the capital-to-labor ratio in the investment sector. As expected, goods produced by sectors with lower TFPs are more expensive. The dependence of prices on capital-to-labor ratio in the investment sector in Equation (46) is a proxy for their dependence on the rental price of capital. Goods produced by sectors with higher capital intesities become more expensive as capital-to-labor ratios rise and the rental price of capital falls.

Equation (46) illustrates how both supply-side forces driving structural change appear through sectoral prices. Along any growth path, the technological progress of sector i, $A_i(t)$ (relative to the investment sector) is a force driving prices, as in Ngai and Pissarides (2007). As the economy accumulates capital, the capital intensity of sector i, , $\kappa_0(t)^{\alpha_i-\alpha_0}$ (relative to the investment sector) also affects the relative price of sector i, as in Acemoglu and Guerrieri (2008).

To characterize the dynamics of the aggregate economy, we define standard productivity-adjusted aggregate and investment variables,

$$\tilde{k}(t) \equiv \frac{K(t)}{A_0^{\frac{1}{1-\alpha_0}}(t)\,L(t)}, \qquad \tilde{y}_0(t) \equiv \frac{Y_0(t)}{A_0^{\frac{1}{1-\alpha_0}}(t)\,L(t)}. \qquad \tilde{e}(t) \equiv \frac{e(t)}{A_0^{\frac{1}{1-\alpha_0}}(t)}, \qquad \tilde{\kappa}_0(t) \equiv \frac{\kappa_0(t)}{A_0^{\frac{1}{1-\alpha_0}}(t)}, \tag{47}$$

and denote the share of labor employed in the investment sector be $l_0 \equiv L_0/L$. With this normalization, the rental price of capital and the capital-to-labor ratio in the investment sector have the following one-to-one relationship $r(t) = \alpha_0 \tilde{\kappa}_0^{\alpha_0-1}$ and we can use them interchangably to characterize the path of the aggregate economy.

Productivity-adjusted per capita consumption expenditure $\tilde{e}$ is a function of $(c, \tilde{\kappa}_0)$ but also directly depends on time due to the impact of technological growth on prices:

$$\tilde{e}(t) = \tilde{e}(c(t), \tilde{\kappa}_0(t), t) = \left(\sum_{i=1}^{I} \left[\varphi_i(t) g(c(t))^{\epsilon_i} \tilde{\kappa}_0(t)^{\alpha_0 - \alpha_i} \right]^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \tag{48}$$

where we have substituted from Equations (46) and (47) in definition Equation (37) and have defined (exogenously given) time-dependent function

$$\varphi_i(t) \equiv \left(\frac{\alpha_0}{\alpha_i} \right)^{\alpha_i} \left(\frac{1-\alpha_0}{1-\alpha_i} \right)^{1-\alpha_i} A_0(t)^{-\frac{\alpha_i}{1-\alpha_0}} A_i(t)^{-1}.$$

Similarly, we can write the share of sectosr i in consumption expenditure as a function of $(c, \tilde{\kappa}_0)$ and time as $\omega_i(t) = \omega_i(c(t), \tilde{\kappa}_0(t), t)$. Averages of the income elasticity parameter and capital share in the consumption sector of the economy $\bar{\epsilon}$ and $\bar{\alpha}$ then also become functions of the two state variables and time.

Next, we observe that the output of the investment sector and its share in total labor depend on capital to labor ratio

$$\tilde{y}_0 = \tilde{\kappa}_0^{\alpha_0} - \frac{1-\bar{\alpha}}{1-\alpha_0}\tilde{e}, \tag{49}$$

$$l_0 = 1 - \frac{1-\bar{\alpha}}{1-\alpha_0}\frac{\tilde{e}}{\tilde{\kappa}_0^{\alpha_0}}. \tag{50}$$

To fully describe the dynamics of the aggregate economy in terms of the state variables $(c, \tilde{\kappa}_0)$, we finally need to connect the aggregate stock of capital in the economy to the capital-to-labor ratio in the investment sector. This is established in the following lemma.

Lemma 5. *For a given level of per capita real consumption c and sectoral productivities $\{A_i\}_{i=0}^{I}$, following equation defines a one-to-one mapping between the capital-to-labor ratio in the investment sector $\tilde{\kappa}_0$ and the aggregate capital stock $\tilde{k}$*

$$\tilde{k} = \tilde{\kappa}_0 \left[1 + \frac{\tilde{e}}{\tilde{\kappa}_0^{\alpha_0}} \left(\frac{\bar{\alpha}}{\alpha_0} - \frac{1-\bar{\alpha}}{1-\alpha_0} \right) \right], \tag{51}$$

where we have suppressed the dependence of per capita expenditure $\tilde{e}$ and average capital share in consumption $\bar{\alpha}$ on the capital-to-labor ratio in the investment sector κ_0, through prices as in equation (46).

Proof. See Appendix C.2. ∎

Equation (51) shows that whenever average capital share of the consumption sector exceeds that of the investment sector, the economy-wide capital-to-labor ratio k is greater than the capital-to-labor ratio in the investment sector κ_0. Furthermore, the lemma ensures that the two ratios always move in the same direction along any equilibrium path. This point is critical since it ensures that we can use the investment sector capital-to-labor ratio $\tilde{\kappa}_0$ as the state variable fully characterizing the path of capital accumulation in the economy.

Using Lemma (5) and Equations (49) and (50), we can write the system of dynamic equations describing the equilibrium path of aggregate variables in the economy:

$$\begin{aligned}\dot{\tilde{k}} &= \tilde{\kappa}_0 \left(\tilde{k}\right)^{\alpha_0} - \frac{1-\bar{\alpha}}{1-\alpha_0}\tilde{e} - \left(n + \delta + \frac{\gamma_0}{1-\alpha_0}\right)\tilde{k}, &(52)\\ \frac{\dot{c}}{c} &= \frac{\alpha_0 \tilde{\kappa}_0 \left(\tilde{k}\right)^{\alpha_0 - 1} - \rho}{\eta_{e'} - \eta_{u'}}. &(53)\end{aligned}$$

The first equation simply follows from (49) and the second is found noting that the rental price of capital can be written as $r = \alpha_0 \tilde{\kappa}_0^{\alpha_0}$. The two state variables $(c, \tilde{\kappa}_0)$ at time t are sufficient to fully specify the economy. As we discussed, both $\tilde{e}$ and $\bar{\alpha}$ are functions of c and $\tilde{\kappa}_0$, with the dependence on the latter going through the dependence of functions (37) and (40) on prices, as specified by Equation (46). Knowing the two state variables at any given time t, capital and labor employed in each consumption good sector i may be written as:

$$\begin{aligned}K_i &= \left(\frac{\alpha_i}{\alpha_0}\omega_i\left(c, \tilde{\kappa}_0\right) \cdot \frac{\tilde{e}\left(c, \tilde{\kappa}_0\right)}{\tilde{\kappa}_0^{\alpha_0}}\right) \cdot L A_0{}^{\frac{1}{1-\alpha_0}},\\ L_i &= \left(\frac{1-\alpha_i}{1-\alpha_0}\omega_i\left(c, \tilde{\kappa}_0\right) \cdot \frac{\tilde{e}\left(c, \tilde{\kappa}_0\right)}{\tilde{\kappa}_0^{\alpha_0}}\right) \cdot L,\end{aligned}$$

which completes the characterization of the economy.

6.4 Constant Growth Path

We now study the asymptotic behavior of the economy characterized by the system of dynamic equations (52) and (53), incorporating the assumption of constant technological progress $\dot{A}_i/A_i = \gamma_i$. In particular, substituting the normalized capital-to-labor ratio of the investment sector in Equation (46), we observe that the growth of consumption good prices take the following form

$$\frac{p_i(t)}{p_i(0)} = \left(\frac{\tilde{\kappa}_0(t)}{\tilde{\kappa}_0(0)}\right)^{\alpha_0 - \alpha_i} e^{-\left(\gamma_i - \frac{1-\alpha_i}{1-\alpha_0}\gamma_0\right)t}. \tag{54}$$

Therefore, if the rental price of capital remains constant, price of consumption good i falls at the rate $(\gamma_i - \gamma_0) + \frac{\alpha_i - \alpha_0}{1-\alpha_0}\gamma_0$, where the first terms in the parantheses captures technical growth in sector i and the second term captures the extent to which technical growth in the investment sector differentially impacts growth in sector i through differences in capital intensity.

We start the characterization of the asymptotic behavior of equilibrium paths by introducing the concept of constant growth path.

Definition 6. An equilibrium path is a Constant Growth Path (CGP) if per capita real consumption grows asymptotically at rate γ^*:

$$\lim_{t\to\infty} \frac{\dot{c}(t)}{c(t)} = \gamma^*.$$

From the Euler equation (53), we know that along any CGPs, productivity-adjusted capital-to-labor ratio in the investment sector has to asymptotically converge to a constant $\tilde{\kappa}_0^*$ given by:

$$\frac{\alpha_0 \left(\tilde{\kappa}_0^*\right)^{\alpha_0 - 1} - \rho}{\frac{\bar{\epsilon}^*}{\epsilon_{max}} + \theta} = \gamma^*. \tag{55}$$

It then follows that along a constant growth path share of each sector i in consumption expenditure asymptotically grows at rate (see derivation in Appendix C.3):

$$\lim_{t\to\infty} \frac{\dot{\omega}_i}{\omega} = (1-\sigma)\left[\frac{\epsilon_i - \bar{\epsilon}}{\epsilon_{max}}\gamma^* - \left(\gamma_i + \frac{\alpha_i - \bar{\alpha}}{1-\alpha_0}\gamma_0\right)\right]. \tag{56}$$

Since all shares are bounded above by one, the rate of growth of sectoral shares cannot remain asymptotically positive. Therefore, for the equilibrium to be well defined we need to ensure that these rates of growth are all asymptotically less than or equal zero. This simple rule in fact pins down the rate of growth of real consumption per capita γ^* along any CGP, which in general will be different from the rate of growth of per capita consumption expenditure $\frac{\gamma_0}{1-\alpha_0}$.

Proposition 7. *Under some mild parametric restrictions (specified in Appendix C.3), the equilibrium path of the economy defined above for any initial stock of capital $K(0)$ will converge to a unique constant growth path where the real consumption per capita grows at rate:*

$$\gamma^* = \min_{i\in\mathcal{I}} \left\{\frac{\epsilon_{max}}{\epsilon_i}\left(\gamma_i + \frac{\alpha_i}{1-\alpha_0}\gamma_0\right)\right\}, \tag{57}$$

and the set of sectors i^* that achieve the minimum above, which we denote $\mathcal{I}^*$, includes all consumption good sectors that do not asymptotically vanish.

Proof. See Appendix C.3. ∎

7 Concluding Remarks

This paper presents a tractable model of structural transformation. It features both long-run demand and supply drivers of structural transformation. Our contribution is to propose a utility function that has non-homothetic Engel curves at any level of development. This implies that along the generalized balanced growth path agents' utility does not become asymptotically homothetic. Moreover, we show how this formulation of preferences can accommodate seamlessly supply drivers of the structural transformation that take the form of differential sectorial trends in prices.

Relative to models with Stone-Geary prefrences, our framework has the advantage to deliver a balanced growth path even in the presence of sectorial trends in prices. Moreover, the model can generate non-monotonic patterns like the hump shape in employment shares in manufacturing. Relative to models with differential trends in relative prices and homothetic constant-elasticity preferences, our model has the advantage that can accommodate trends in both real and nominal measures.

We find that our baseline model of constant elasticities fits the data well despite of only using three elasticities. We estimate our model applied to three sectors (agriculture, manufacturing and services) using panel of 29 countries for the postwar period. Our estimates of the price elasticity is around three-quarters. We find that the income elasticities rank as expected. Agriculture has the lowest elasticity, manufacturing has intermediate elasticity and services has the highest elasticity.

The proposed preferences provide a tractable departure from homothetic preferences. We believe that they can be used in many applied general equilibrium settings that currently use homothetic constant elasticity of substitution as their workhorse model, such as international trade. These preferences can be also combined with productions functions without constant shares to study skilled-biased technological change or capital deepening.

References

Acemoglu, D. and V. Guerrieri (2008): "Capital Deepening and Nonbalanced Economic Growth," *Journal of Political Economy*, 116, 467–498.

Alvarez-Cuadrado, F. and M. Poschke (2011): "Structural Change Out of Agriculture: Labor Push versus Labor Pull," *American Economic Journal: Macroeconomics*, 3, 127–58.

Baumol, W. J. (1967): " Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis," *American Economic Review*, 57, 415–426.

Boppart, T. (2013): "Structural change and the Kaldor facts in a growth model with relative price effects and non-Gorman preferences," 2013 Meeting Papers 217, Society for Economic Dynamics.

Buera, F. J. and J. P. Kaboski (2009): "Can Traditional Theories of Structural Change Fit The Data?" *Journal of the European Economic Association*, 7, 469–477.

Caron, J., T. Fally, and J. R. Markusen (2014): "International Trade Puzzles: A Solution Linking Production and Preferences," *Quarterly Journal of Economics*, 129, 1501–1552.

Dennis, B. N. and T. B. Iscan (2009): "Engel versus Baumol: Accounting for structural change using two centuries of U.S. data," *Explorations in Economic History*, 46, 186–202.

Diewert, W. E. (1978): "Superlative Index Numbers and Consistency in Aggregation," *Econometrica*, 46, 883–900.

——— (2002): "The Quadratic Approximation Lemma and Decompositions of Superlative Indexes," *Journal of Economic and Social Measurement*, 28, 63–88.

Duarte, M. and D. Restuccia (2010): "The Role of the Structural Transformation in Aggregate Productivity," *The Quarterly Journal of Economics*, 125, 129–173.

Echevarria, C. (1997): "Changes in Sectoral Composition Associated with Economic Growth," *International Economic Review*, 38, 431–52.

Feenstra, R. C., R. Inklaar, and M. Timmer (2013): "The Next Generation of the Penn World Table," NBER Working Papers 19255, National Bureau of Economic Research, Inc.

Feenstra, R. C. and J. Romalis (2014): "International Prices and Endogenous Quality," *Quarterly Journal of Economics*, 129, 477–527.

Fieler, A. C. (2011): "Nonhomotheticity and Bilateral Trade: Evidence and a Quantitative Explanation," *Econometrica*, 79, 1069–1101.

Foellmi, R., T. Wuergler, and J. Zweimller (2014): "The macroeconomics of Model T," *Journal of Economic Theory*, 153, 617–647.

Foellmi, R. and J. Zweimller (2008): "Structural change, Engel's consumption cycles and Kaldor's facts of economic growth," *Journal of Monetary Economics*, 55, 1317–1328.

Gollin, D., S. Parente, and R. Rogerson (2002): "The Role of Agriculture in Development," *American Economic Review*, 92, 160–164.

Hanoch, G. (1975): "Production and demand models with direct or indirect implicit additivity," *Econometrica*, 43, 395–419.

Heathcote, J., F. Perri, and G. L. Violante (2010): "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States: 1967-2006," *Review of Economic Dynamics*, 13, 15–51.

Herrendorf, B., R. Rogerson, and A. Valentinyi (2013): "Two Perspectives on Preferences and Structural Transformation," *American Economic Review*, 103, 2752–89.

——— (2014): "Growth and Structural Transformation," in *Handbook of Economic Growth*, Elsevier, vol. 2 of *Handbook of Economic Growth*, chap. 6, 855–941.

Johnson, D. S., J. A. Parker, and N. S. Souleles (2006): "Household Expenditure and the Income Tax Rebates of 2001," *American Economic Review*, 96, 1589–1610.

Jorgenson, D. W. and M. P. Timmer (2011): "Structural Change in Advanced Nations: A New Set of Stylised Facts," *Scandinavian Journal of Economics*, 113, 1–29.

Kongsamut, P., S. Rebelo, and D. Xie (2001): "Beyond Balanced Growth," *Review of Economic Studies*, 68, 869–82.

Krueger, D. and F. Perri (2006): "Does Income Inequality Lead to Consumption Inequality? Evidence and Theory," *Review of Economic Studies*, 73, 163–193.

Kuznets, S. (1973): "Modern Economic Growth: Findings and Reflections," *American Economic Review*, 63, 247–58.

Maddison, A. (1980): "Economic Growth and Structural Change in Advanced Countries," in *Western Economies in Transition*, Croom Helm, 41–60.

Matsuyama, K. (2000): "A Ricardian Model with a Continuum of Goods under Nonhomothetic Preferences: Demand Complementarities, Income Distribution, and North-South Trade," *Journal of Political Economy*, 108, 1093–1120.

——— (2002): "The Rise of Mass Consumption Societies," *Journal of Political Economy*, 110, 1035–1070.

——— (2009): "Structural Change in an Interdependent World: A Global View of Manufacturing Decline," *Journal of the European Economic Association*, 7, 478–486.

Ngai, L. R. and C. A. Pissarides (2007): "Structural Change in a Multisector Model of Growth," *American Economic Review*, 97, 429–443.

Saint-Paul, G. (2006): "Distribution and Growth in an Economy with Limited Needs: Variable Markups and 'the End of Work'," *Economic Journal*, 116, 382–407.

Sato, R. (1975): "The Most General Class of CES Functions," *Econometrica*, 43, 999–1003.

Święcki, T. (2014): "Determinants of Structural Change," Mimeo.

Vries, K. d., G. d. Vries, and M. Timmer (2014): "Patterns of Structural Change in Developing Countries," GGDC Research Memorandum GD-149, Groningen Growth and Development Centre, University of Groningen.

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A Nonhomothetic CES Preferences

In this section, we provide an overview of the properties of the general Nonhomothetic CES preferences and their generalizations.

A.1 General Nonhomothetic CES Prefereces

Consider preferences over a bundle $\{C_1, C_2, \cdots, C_I\}$ of goods defined through an implicit utility function:

$$\sum_{i=1}^{I} f_i(U)^{\frac{1}{\sigma}} C_i^{\frac{\sigma-1}{\sigma}} = 1, \tag{A.1}$$

where functions f_i's are differentiable in U and $\sigma \neq 1$ and $\sigma > 0$.[49] Standard CES preferences are a specific example of Equation (A.1) with $f_i(U) = U^{1-\sigma}$ for all i's. The next lemma characterizes the demand for general nonhomothetic CES preferences.

Lemma 8. *Consider any bundle of goods that maximizes the utility function defined in Equation (A.1) subject to the budget constraint $\sum_i p_i C_i \leq E$. For each good i, the real consumption satisfies:*

$$C_i = \left(\frac{E}{p_i}\right)^{\sigma} f_i(U), \tag{A.2}$$

and the share in consumption expenditure satisfies:

$$\frac{p_i C_i}{E} = f_i(U)^{\frac{1}{\sigma}} C_i^{\frac{\sigma-1}{\sigma}} = f_i(U) \left(\frac{p_i}{E}\right)^{1-\sigma}. \tag{A.3}$$

Proof. Let λ and ρ denote the Lagranage multipliers on the budget constraint and constraint (A.1), respectively:

$$\mathcal{L} = U + \rho \left(1 - \sum_i f_i^{\frac{1}{\sigma}} C_i^{\frac{\sigma-1}{\sigma}}\right) + \lambda \left(E - \sum_i p_i C_i\right).$$

The FOCs with respect to C_i yields:

$$\rho \frac{1-\sigma}{\sigma} \frac{\omega_i}{C_i} = \lambda p_i, \tag{A.4}$$

where we have defined

$$\omega_i \equiv f_i(U)^{\frac{1}{\sigma}} C_i^{\frac{\sigma-1}{\sigma}}. \tag{A.5}$$

[49] For the case of $\sigma = 1$, the preferences are simply defined according to $\sum_i f_i(U) \log C_i = 1$.

Equation (A.4) shows that expenditure p_iC_i on good i is proportional to ω_i. Since the latter sums to one from constraint (A.1), it follows that ω_i is the expenditure share of good i, and we have:

$$E = \sum_{i=1}^{I} p_i C_i \quad = \quad \frac{1-\sigma}{\sigma}\frac{\rho}{\lambda}.$$

We can now substitue the definition of ω_i from Equation (A.5) in expression (A.4)and use (A.6) to find (A.2) and (A.3). ∎

Lemma 8 implies the following relationship, implicitly defining the expenditure and indirect utility functions for general Nonhomothetic CES preferences:

$$E^{1-\sigma} = \sum_{i=1}^{I} f_i(U)\, p_i^{1-\sigma}. \tag{A.6}$$

The expenditure function is clearly continuous in prices p_i's and U, and homogenous of degree 1, increasing, and concave in prices. We further need to ensure that the expenditure function is increasing in utility. We compute the elasticity of the expenditure function with respect to utility and find

$$\eta_U^E \equiv \frac{U\partial E}{E\partial U} = \frac{1}{1-\sigma}\sum_i \omega_i\, \eta_U^{f_i} = \frac{1}{1-\sigma}\overline{\eta_U^{f_i}}, \tag{A.7}$$

where $\eta_U^{f_i} \equiv U\partial f_i / f_i \partial U$ is the elasticity of function f_i with respect to U. The next lemma then follows.

Lemma 9. *Equation (A.1) defines a unique, continuous, and monotone utility function $U = F(C_1, \cdots, C_I)$ with $C_i > 0$, if functions f_i's are continuous in U and monotonically increasing (decreasing) when $\sigma < 1$ ($\sigma > 1$).*

Examining sectoral demand from Equation (A.2) along indifference curves, shows the main properties of nonhomothetic CES preferences. As expected, the elasticity of substitution is constant:

$$\eta_{p_i/p_j}^{C_i/C_j} \equiv \frac{\partial \log\left(C_i/C_j\right)}{\partial \log\left(p_i/p_j\right)} = \sigma. \tag{A.8}$$

More interestingly, the elasticity of relative demand with respect to utility is in general different from unity:

$$\eta_U^{C_i/C_j} \equiv \frac{\partial \log\left(C_i/C_j\right)}{\partial \log U} = \frac{\partial \log\left(f_i/f_j\right)}{\partial \log U}. \tag{A.9}$$

Since utility has a monotonic relationship with real income (and hence expenditure), it then follows that the nominal income (consumption expenditure) elasticity of different goods are not the same. More specifically, we can use (A.7) to find the nominal expenditure elasticity of demand:

$$\eta_E^{C_i} \equiv \frac{\partial \log C_i}{\partial \log E} = \sigma + (1-\sigma)\,\eta_U^{f_i}/\overline{\eta_U^{f_i}}. \tag{A.10}$$

The intuition for the normalization in expression (A.10) is that the elasticity $\eta_E^{C_i}$ has to be invariant to all monotonic transformations of utility.

Preferences defined by Equation (A.1) belong to the general class of preferences with *Direct Implicit Addivity*. Hanoch (1975) shows that the latter family of preferences have the nice property that is illustrated by Equations (A.8) and (A.9): the separability of the income and subsitution elasticities of the Hicksian demand. This is in contrast to the stronger requirement of *Explicit Additivity* commonly assumed in nonhomothetic preferences, whereby the utility is explicitly defined as a function $U = F\left(\sum_i f_i(C_i)\right)$. As we will show below in the specific example of generalized Stone-Geary preferences, substitution and income elasticities of Hicksian demand are *not* separable for preferences with explicity additivity.

A.2 Income Isoelastic Nonhomothetic CES Preferences

Now, consider the specific case used in the exposition of our model, of isoelastic functions f_i defined as:

$$f_i(U) = \Omega_i U^{\epsilon_i - \sigma}, \tag{A.11}$$

where $\eta_U^{f_i} = \partial \log f_i / \partial \log U = \epsilon_i - \sigma$, and we retrieve standard CES preferences when $\epsilon_i = 1$ for all i's. For ease of exposition, let us for now identify utility with C, aggregate real income and define a corresponding aggregate price index $P \equiv E/C$. From Equations (A.2) and (A.3), we find demand to be:

$$C_i = \left(\frac{p_i}{P}\right)^{-\sigma} C^{\epsilon_i}, \tag{A.12}$$

$$\omega_i = \frac{p_i C_i}{PC} = \left(\frac{p_i}{P}\right)^{1-\sigma} C^{\epsilon_i - 1}. \tag{A.13}$$

The aggregate price index is

$$P \equiv \frac{E}{C} = \left(\sum_{i=1}^{I} C^{\epsilon_i - 1} p_i^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{A.14}$$

From (A.7), the real income elasticity of the expenditure function is:

$$\eta_C^E \equiv \frac{C}{E}\frac{\partial E}{\partial C} = \frac{\bar{\epsilon} - \sigma}{1 - \sigma}, \tag{A.15}$$

where $\bar{\epsilon} = \sum_i \omega_i \epsilon_i$. Therefore, a sufficient condition for the function $E\left(C; \{p_i\}_{i=1}^I\right)$ to be a one-to-one mapping for all positive prices is that all sectors have an income elasticity larger than the elasticity of substitution $\epsilon_i > \sigma$ if $\sigma < 1$ (and $\epsilon_i < \sigma$ if $\sigma > 1$).[50]

The income elasticities of demand are given by Equations (A.9) and (A.10):

$$\eta_C^{C_i/C_j} = \epsilon_i - \epsilon_j, \tag{A.16}$$

$$\eta_E^{C_i} = \sigma + (1-\sigma)\frac{\epsilon_i - \sigma}{\bar{\epsilon} - \sigma}. \tag{A.17}$$

Each good i is characterized by a parameter $\epsilon_i \in \mathbb{R}$ that is a measure of its real income elasticity.

More generally, the relationship between utility U and real aggregate consumption C in Expression (A.11) can be defined by any monotonic function V such that $U = V(C)$. In particular, let us define $V(\cdot)$ such that C corresponds to nominal consumption expenditure at constant prices $\{q_i\}_i$ such that

$$C^{1-\sigma} = \sum_{i=1}^{I} \Omega_i \, V(C)^{\epsilon_i - 1} q_i^{1-\sigma}. \tag{A.18}$$

Assuming $\sigma \in (0,1)$, if $\epsilon_i > \sigma$ for all i, function $V(\cdot)$ defined through Equation (A.18) is monotonically increasing for all positive C. Therefore, we can approximate the relationship as:

$$\begin{aligned} \log V(C) &\approx \log V(\tilde{C}) + \left.\frac{\partial \log V}{\partial \log C}\right|_{C=\tilde{C}} \cdot \left(\log C - \log \tilde{C}\right), \\ &= \frac{1-\sigma}{\bar{\epsilon} - \sigma} \log C + \text{const.}, \end{aligned} \tag{A.19}$$

where $\bar{\epsilon}$ is the average elasticity parameter at constant price q and real income $\tilde{C}$. For this reason, we simplify our benchmark model by identifying both C and U with real aggregate income.

A.3 Comparison to Generalized Stone-Geary Preferences

For comparison, now consider the following generalization of Stone-Geary preferences:

$$C_t = \left(\sum_{i=1}^{I} (C_{it} - \underline{C}_i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{A.20}$$

where $\underline{C}_i$ are the usual coordinate shifters. In particular, standard 3-sector models of structural transformation generally assume $\underline{C}_a > 0$, $\underline{C}_s < 0$ and $\underline{C}_m = 0$. Define the marginal cost

[50] As Hanoch (1975) shows, the utility function defined by Equation (A.1) is globally valid (monotone and quasi-concave) at all strictly positive consumption bundles either if $0 < \sigma < 1$ and $\epsilon_i > \sigma$, or if $\sigma > 1$ and $\epsilon_i < \sigma$ for all sectors i (p. 403).

of consumption

$$\tilde{P}_t \equiv \frac{\partial E_t}{\partial C_t} = \left(\sum_{i=1}^{I} p_{it}^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \tag{A.21}$$

which is independent of income. We then have:

$$P_t = \frac{E_t}{C_t} = \tilde{P}_t \left(1 + \frac{\sum_i p_{it}\underline{C}_i}{E_t}\right), \tag{A.22}$$

which implies that the dependence of the price index on nominal income E_t vanishes as the latter grows to infinity.

Another important feature of the nonhomothetic CES preferences is the fact that elasticity of substitution σ_{ij} between all goods i and j remains constant σ and remains independent of income elasticities. In contrast, for Stone-Geary preferences we find:

$$\sigma_{ij} = \sigma \cdot \frac{E_t}{E_t - \sum_k p_{kt}\underline{C}_k} \cdot \left(1 - \frac{\underline{C}_i}{C_{it}}\right) \cdot \left(1 - \frac{\underline{C}_j}{C_{jt}}\right), \tag{A.23}$$

$$\eta_i = \frac{E_t}{E_t - \sum_k p_{kt}\underline{C}_k} \cdot \left(1 - \frac{\underline{C}_i}{C_{it}}\right), \tag{A.24}$$

where η_i is the nominal income elasticity of demand in sector i (Hanoch, 1975). It then follows that the elasticities of substitution between goods i and j always satisfies the following equality:

$$\sigma_{ij} = \sigma \eta_i \eta_j, \tag{A.25}$$

creating a direct linkage between elasticities of substitution and income for different sectors. As expected, when E_t goes to infinity we find that $\sigma_{ij} \to \sigma$ and $\eta_i \to 1$ for all sectors.

B Proofs

Proof of Lemma 1. The HH problem can be written as that of finding

$$\max_{\{C_t, K_t, \{C_{it}\}_i\}_t} \sum_t \left\{\beta^t \frac{C_t^{1-\theta} - 1}{1-\theta} + \lambda_t \left(E_t - \sum_{i=1}^{I} p_{it} C_{it}\right) + \rho_t \left(F_t - 1\right)\right\}, \tag{B.1}$$

where $E_t = w_t + K_t(1 - \delta + R_t) - K_{t+1}$ and F_t is defined according to Equation (A.1). It is straightforward to see that the FOCs characterizing the optimal bundle $\{C_{it}\}_i$ for any choice of E_t and C_t are the same as the ones in the static case and are given by:

$$\rho_t \frac{1-\sigma}{\sigma} \frac{\omega_{it}}{C_{it}} = \lambda_t p_{it}, \tag{B.2}$$

resulting in an expression similar to Equation (A.2):

$$C_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\sigma} C_t^{\epsilon_i}. \tag{B.3}$$

Furthermore, aggregate price index P_t and total consumption expenditure E_t could be written according to expressions (A.14) and (A.6), respectively. This shows the first part of the lemma, characterizing the intra-temporal problem.

Now, we can write down the intertemporal problem as:

$$\max_{\{C_t\}_t} \sum_t \left\{\beta^t \frac{C_t^{1-\theta} - 1}{1-\theta} + \lambda_t \left(w_t + K_t(1-\delta+R_t) - K_{t+1} - E(C_t)\right)\right\}, \tag{B.4}$$

where we have defined the expenditure function:

$$E\left(C_t; \{p_{it}\}_{i=1}^I\right) = \left[\sum_{i=1}^I C_t^{\epsilon_i - \sigma} p_i^{1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{B.5}$$

The FOCs with respect to K_t yield:

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{\beta\left(1-\delta+R_t\right)}, \tag{B.6}$$

which will pin down all λ's for any given λ_0.

The discussion above implies that for any initial marginal utility of wealth λ_0, Equation (B.6) gives λ_t, which then allows us to find C_t that satisfies its respective FOC from:

$$C_t^{1-\theta} = \lambda_t E(C_t) \frac{\sum_i \epsilon_i \omega_{it} - \sigma}{1-\sigma}. \tag{B.7}$$

Combining the equation above with Equation (B.6) then yields the Euler equation (5) and completes the proof. ∎

Lemma 10. *Section A.2 describes sufficient conditions for a sequence of consumption allocations to be an optimal solution to problem* (1)*.*

Proof. We showed in Section A that the expenditure function E is monotonically increasing in C_t if $\bar{\epsilon} > \sigma$. In order for problem (1) to have a solution, we should further ensure that the objective function is concave in C_t for all sets of positive prices $\{p_{it}\}_{i=1}^I$. A sufficient condition, therefore, is the convexity of the expenditure function E in C.

Next, we find the set of parameters for which the expenditure function is convex in aggregate real consumption. Let us write the second derivative of expenditure as:

$$\begin{aligned}
\frac{\partial^2 E}{\partial C^2} &= \left(\frac{C}{E}\frac{\partial E}{\partial C}\right)\cdot\frac{\partial}{\partial C}\left(\frac{E}{C}\right)+\left(\frac{E}{C}\right)\cdot\frac{\partial}{\partial C}\left(\frac{C}{E}\frac{\partial E}{\partial C}\right)\\
&= \eta_C^E\frac{\partial}{\partial C}\left(\frac{E}{C}\right)+\left(\frac{E}{C}\right)\cdot\frac{\partial \eta_C^E}{\partial C},
\end{aligned}$$

where in the second equality we have used the definition of real consumption elasticity of expenditure from Equation A.15. We can write the first term, the derivative of average consumption expenditure with respect to real consumption as:

$$\begin{aligned}
\frac{\partial}{\partial C}\left(\frac{E}{C}\right) &= \frac{E}{C^2}\left(\eta_C^E-1\right)\\
&= \frac{E}{C^2}\left(\frac{\bar{\epsilon}-1}{1-\sigma}\right), \qquad \text{(B.8)}
\end{aligned}$$

where again we have used Equation A.15. Some more algebra shows that the derivative of the elasticity η_C^E with respect to aggregate real consumption is given by:

$$\frac{\partial \eta_C^E}{\partial C}=\frac{1}{C(1-\sigma)}\sum_i \omega_i\left(\epsilon_i-\bar{\epsilon}\right)^2, \qquad \text{(B.9)}$$

which is always nonnegative if we have complementarity among goods $\sigma < 1$. It then follows that a sufficient condition for the convexity of the expenditure function is that $\bar{\epsilon} \geq 1$ and $0 < \sigma \leq 1$. ∎

Lemma 11. *Optimal production decisions imply Equations (10) and (12).*

Proof. Capital is allocated across different sectors such that its marginal revenue product equals the rental price of capital:

$$R_t = p_{it}A_{it}^{1-\alpha}\left(\frac{L_{it}}{K_{it}}\right)^{1-\alpha} = p_{it}A_{it}^{1-\alpha}K_t^{\alpha-1}. \qquad \text{(B.10)}$$

Let us now normalize the price of the investment sector to 1, $p_{0t} \equiv 1$. Equation (B.10) then determines sectoral output prices according to (10). Note that the sum of the values of all sectoral outputs has the following aggregate represenation:

$$\begin{aligned}
Y_t \equiv \sum_{i=1}^{I} p_{it}Y_{it}+X_t &= \left(\sum_{i=1}^{I} p_{it}A_{it}^{1-\alpha}L_{sit}+A_{0t}^{1-\alpha}L_{0t}\right)K_t^{\alpha},\\
&= A_{0t}^{1-\alpha}\left(\sum_{i=1}^{I} L_{it}\right)K_t^{\alpha},\\
&= A_{0t}^{1-\alpha}K_t^{\alpha}, \qquad \text{(B.11)}
\end{aligned}$$

where the first and second equalities follow from the equality of capital to labor ratios across sectors and Equation (10), respectively. It then follows that the wage and the rental price of capital are determined at each period by equation (12). ∎

Proof of Proposition 2. We first derive an expression for the asymptotic growth of nominal consumption expenditure shares of different sectors, equation (17),

$$\begin{aligned} 1+\xi_i \equiv \lim_{t\to\infty}\frac{\omega_{it+1}}{\omega_{it}} &= \lim_{t\to\infty}\left(\frac{E_t}{E_{t+1}}\right)^{1-\sigma}\left(\frac{p_{it+1}}{p_{it}}\right)^{1-\sigma}\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i-\sigma}, \\ &= \left(\frac{1}{1+\gamma_0}\right)^{1-\sigma}\left(\frac{1+\gamma_0}{1+\gamma_i}\right)^{(1-\sigma)(1-\alpha)}\left(1+\gamma^*\right)^{\epsilon_i-\sigma}, \\ &= \frac{\left(1+\gamma^*\right)^{\epsilon_i-\sigma}}{\left[\left(1+\gamma_0\right)^{\alpha}\left(1+\gamma_i\right)^{1-\alpha}\right]^{1-\sigma}}. \end{aligned}$$

Since shares have to add up to 1, sectoral growth in shares must be non-positive. Moreover, the growth expression above has to be zero at least for one nonvanishing sector. Now, consider the expression defined in (14) for the growth rate of real consumption. For sectors $i \in \mathcal{I}^*$ that achieve the minimum, the growth of nominal expenditure share becomes zero, and their shares converge to constant values ω_i^*. For sectors $i \notin \mathcal{I}^*$, we find the following expressoin for the growth rate of nominal shares:

$$\xi_i = \left[\frac{1+\gamma^*}{\left[\left(1+\gamma_0\right)^{\alpha}\left(1+\gamma_i\right)^{1-\alpha}\right]^{\frac{1-\sigma}{\epsilon_i-\sigma}}}\right]^{\epsilon_i-\sigma} - 1. \tag{B.12}$$

Assuming $\sigma < 1$ and $\epsilon_i > \sigma$, the expression inside the bracket is less than 1, since we know sector i does not achieve the minimum in (14). If $\epsilon_i < \sigma$, then the expression inside the bracket is greater than 1, since γ^* is by definition positive, whereas the denominator is always less than 1. Therefore, in both cases $\xi_i < 0$ and the nominal shares asymptotically vanish.

Next, we show that if the growth rate of real aggregate consumption given by γ^*, nominal aggregate consumption expenditure asymptotically grows at the same rate γ_0 as total output. From the consumer utility maximization problem, we know that the growth of aggregate consumption expenditure has to satisfies

$$\begin{aligned}
\frac{E_{t+1}}{E_t} &= \left(\frac{\sum_{i=1}^{I}\Omega_i C_{t+1}^{\epsilon_i-\sigma}p_{it+1}^{1-\sigma}}{\sum_{i=1}^{I}\Omega_i C_t^{\epsilon_i-\sigma}p_{it}^{1-\sigma}}\right)^{\frac{1}{1-\sigma}}, \\
&= \left(\frac{\sum_{i=1}^{I}\Omega_i C_t^{\epsilon_i-\sigma}p_{it}^{1-\sigma}\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i-\sigma}\left(\frac{p_{it+1}}{p_{it}}\right)^{1-\sigma}}{\sum_{i=1}^{I}\Omega_i C_t^{\epsilon_i-\sigma}p_{it}^{1-\sigma}}\right)^{\frac{1}{1-\sigma}}, \\
&= \left(\sum_{i=1}^{I}\omega_{it}\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i-\sigma}\left(\frac{p_{it+1}}{p_{it}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \qquad \text{(B.13)}
\end{aligned}$$

Now, using the fact that $\omega_{it} \to 0$ for $i \notin \mathcal{I}^*$, we find that along the CGP:

$$\begin{aligned}
\lim_{t\to\infty}\frac{E_{t+1}}{E_t} &= \lim_{t\to\infty}\left(\sum_{i=1}^{I}\omega_{it}\left(\frac{C_{t+1}}{C_t}\right)^{\epsilon_i-\sigma}\left(\frac{A_{0t+1}/A_{0t}}{A_{it+1}/A_{it}}\right)^{(1-\alpha)(1-\sigma)}\right)^{\frac{1}{1-\sigma}}, \\
&= \left(\sum_{i\in\mathcal{I}^*}\omega_i^*\left(1+\gamma^*\right)^{\epsilon_i-\sigma}\left(\frac{1+\gamma_0}{1+\gamma_i}\right)^{(1-\alpha)(1-\sigma)}\right)^{\frac{1}{1-\sigma}}, \\
&= 1+\gamma_0,
\end{aligned}$$

where in the last equality from the fact that for $i \in \mathcal{I}^*$ we have that $\gamma^* = \left[\left(1+\gamma_0\right)^{\alpha}\left(1+\gamma_i\right)^{1-\alpha}\right]^{\frac{1-\sigma}{\epsilon_i-\sigma}}$.

Note that asymptotically, the average income elasticity of the economy converges to a constant

$$\lim_{t\to\infty}\sum_{i=1}^{I}\epsilon_i\omega_{it} = \sum_{i\in\mathcal{I}^*}\epsilon_i\omega_i^*.$$

Combining (B.7) and (B.6), we find:

$$\begin{aligned}
\lim_{t\to\infty}\left(\frac{C_{t+1}}{C_t}\right)^{1-\theta} &= \lim_{t\to\infty}\left(\frac{\lambda_{t+1}}{\lambda_t}\right)\left(\frac{E_{t+1}}{E_t}\right), \\
\left(1+\gamma^*\right)^{1-\theta} &= \frac{1+\gamma_0}{\beta\left(1-\delta+R^*\right)},
\end{aligned}$$

which implies (15).

To examine the properties of aggregate variables along an equilibrium path, we introduce nominal consumption expenditure, aggregate investment, and stock of capital per efficiency unit of labor as $e_t \equiv E_t/A_{0t}$, $x_t \equiv X_t/A_{0t}$, and $k_t \equiv K_t/A_{0t}$. Substituting for wage and rental price of capital from (12), we can rewrite the economy-wide resource constraint as $x_t + e_t = k_t^{\alpha}$.

Moreover, investment per efficiency unit of labor satisfies

$$x_t \quad = \quad k_t \left[\left(\frac{k_{t+1}}{k_t} \right) (1+\gamma_0) - (1-\delta) \right],$$

Total nominal output and rental price of capital are determined in terms of the aggregate stock of capital

$$y_t = k_t^{\alpha}, \qquad R_t = \alpha k_t^{\alpha-1}.$$

Along an CGP, aggregate variables per (investment) efficiency unit of labor y_t, e_t, x_t, and k_t converge to positive constants y^*, e^*, x^*, and k^*, respectively. The production side of the economy implies the standard results that $k^* = (\alpha/R^*)^{\frac{1}{1-\alpha}}$ and $y^* = (k^*)^{\alpha}$, and $x^* = k^* (\gamma_0 + \delta)$. Therefore, the asymptotic rental price of capital pins down steady state levels of nominal output, investment, and stock of capital. We have:

$$\begin{aligned} x^* &= k^* (\gamma_0 + \delta), \\ k^* &= \left(\frac{\alpha}{R^*} \right)^{\frac{1}{1-\alpha}}, \\ e^* &= (k^*)^{\alpha} \left[1 - (k^*)^{1-\alpha} (\gamma_0 - \delta) \right], \end{aligned}$$

which implies the constraint $R^* > \alpha (\gamma_0 + \delta)$. ∎

Proof of Proposition 3. We aim to prove that there is always a stage along the equilibrium converging to an CGP where the labor share of manufacturing is increasing. Consider an economy starting with initial stock of capital K_0 that consumes areal aggregate consumption C_0, where we further normalize $A_{x0} = 1$. First, we that for any $0 < \kappa < 1$, there always exists K_0 small enough such that the share of services relative to agriculture is smaller than κ, that is

$$\frac{\omega_{s0}}{\omega_{a0}} = \left(\frac{A_{a0}}{A_{s0}} \right)^{(1-\alpha)(1-\sigma)} C_0^{\epsilon_s - \epsilon_m} < \kappa,$$

which implies that initial real aggregate C_0 can always be made small enough to satisfy

$$C_0 < \kappa \left(\frac{A_{s0}}{A_{a0}} \right)^{\frac{(1-\sigma)(1-\alpha)}{\epsilon_s - \epsilon_a}}. \tag{B.14}$$

Note that from the resource constraint:

$$P_0 C_0 \le K_0^{\alpha} + K_0 (1-\delta),$$

for any set of initial prices determined by finite productivities (A_{a0}, A_{m0}, A_{s0}) we can always find $\underline{K}$ such that if $K_0 \le \underline{K}$ condition (B.14) is satisfied.

Next, we use the result above to prove that the share of manufactruing could be made increasing. First, we observe that under conditions (conditions to be specified), any sequence of nominal consumption expenditures has to be nondecreasing, i.e., $E_{t+1}/E_t \geq 1$. From (17), we can write initial growth in the share of manufacturing as:

$$\frac{L_{m1}}{L_{m0}} = \left(\frac{C_1/C_0}{1+\gamma^*}\right)^{\epsilon_m} (1+\xi_m),$$

where ξ_m is the asymptotic (negative) growth rate in the share of manufactruing. Assume to the contrary that the expression above is less than or equal to 1. Note that with the assumptions made in Section 2.4, that is $\epsilon_s > \epsilon_m > \epsilon_a$ and $\gamma_a > \gamma_m > \gamma_s$, we have the following relations among the asymptotic sectoral growth rates of labor shares: $\xi_a < \xi_m < \xi_s = 0$. From (B.13), we can rewrite the growth in nominal aggregate consumption as:

$$\begin{aligned} 1 \leq \frac{E_1}{E_0} &= \omega_{a0}\left(\frac{C_1/C_t}{1+\gamma^*}\right)^{\epsilon_a}(1+\xi_a) + \omega_{m0}\left(\frac{C_1/C_0}{1+\gamma^*}\right)^{\epsilon_m}(1+\xi_m) + \omega_{s0}\left(\frac{C_1/C_0}{1+\gamma^*}\right)^{\epsilon_s}, \\ &= \omega_{m0}\frac{L_{m1}}{L_{m0}} + \omega_{a1}\left(\frac{L_{m1}}{L_{m0}}\right)^{\frac{\epsilon_a}{\epsilon_m}}\left[\frac{1+\xi_a}{(1+\xi_m)^{\epsilon_a/\epsilon_m}} + \left(\frac{\omega_{s0}}{\omega_{a0}}\right)\frac{(L_{m1}/L_{m0})^{\frac{\epsilon_s-\epsilon_a}{\epsilon_m}}}{(1+\xi_m)^{\epsilon_s/\epsilon_m}}\right]. \end{aligned}$$

The first term inside the bracket is always less than 1. From what we proved above, for any L_{m1}/L_{m0}we can always find a $\underline{K}$ such that if the initial level of stock satisfies $K_0 \leq \underline{K}$, then

$$\frac{\omega_{s0}}{\omega_{a0}} < \kappa = \left[1 - \frac{1+\xi_a}{(1+\xi_m)^{\epsilon_a/\epsilon_m}}\right]\frac{(1+\xi_m)^{\epsilon_s/\epsilon_m}}{(L_{m1}/L_{m0})^{\frac{\epsilon_s-\epsilon_a}{\epsilon_m}}}.$$

With such a K_0, we have that

$$\begin{aligned} 1 \leq \frac{E_{t+1}}{E_t} &< \omega_{m0}\frac{L_{m1}}{L_{m0}} + \omega_{a0}\left(\frac{L_{m1}}{L_{m0}}\right)^{\frac{\epsilon_a}{\epsilon_m}}, \\ &< \omega_{m0} + \omega_{a0}, \end{aligned}$$

which yields a contradition, where in the second inequality we have used the assumption that $L_{m1}/L_{m0} \leq 1$ and that $\epsilon_a, \epsilon_m > 0$. This completes our proof that K_0 always exists such that the share of manufacturing is initially growing. ∎

C Proof for the Continuous Time Model

C.1 Demand and the Euler Equation

For a given path of prices $[\boldsymbol{p}(t)]_{t=0}^{\infty}$, the present value Hamiltonian for the consumer problem (35) may be written as:

$$\hat{\mathcal{H}}\left(t, \boldsymbol{c}, c, \lambda, \rho\right) \equiv u\left(c\right) + \lambda\left[w + \left(r-n\right)a - e\left(c; \boldsymbol{p}\left(t\right)\right)\right].$$

The FOCs for the Hamiltonian are as follows:

$$\frac{\partial \hat{\mathcal{H}}}{\partial c} = 0 \quad \Rightarrow u'\left(c\right) - \lambda \frac{\partial e}{\partial c} = 0, \tag{C.1}$$

$$\frac{\partial \hat{\mathcal{H}}}{\partial a} = \left(\rho - n\right)\lambda - \dot{\lambda} \quad \Rightarrow \frac{\dot{\lambda}}{\lambda} = -\left(r-\rho\right). \tag{C.2}$$

This yields the Euler equation:

$$\frac{\dot{c}}{c} = \frac{r-\rho}{\eta_{e'} - \eta_{u'}}.$$

To ensure the sufficiency of the FOCs, first we check that Equation (C.1) is a sufficient condition for c to maximize the Hamiltonian. The second order condition for c is

$$u'' - \lambda \frac{\partial^2 e}{\partial c^2} = \frac{u}{c^2}\eta_u\left(\eta_{u'} - \eta_{e'}\right),$$

which we would like to show always remains negative. The first term on the right hand side, by assumption, is always greater than $\underline{\theta}$. Next, we will compute the elasticities of the expenditure function to evaluate $\eta_{e'}$.

We can rewrite $\eta_{e'}$ as:

$$\begin{aligned} \eta_{e'} &= \frac{\partial \log \eta_e^c}{\partial \log c} + \frac{\partial \log\left(e/c\right)}{\partial \log c}, \\ &= \eta_{\eta_e^c}^c + \eta_e^c - 1. \end{aligned}$$

First, let us compute the consumption elasticity of expenditure:

$$\begin{aligned} \eta_e^c = \frac{\partial \log e}{\partial \log c} &= c\frac{\partial}{\partial c}\log \sum_{i=1}^{I}\left(g^{\epsilon_i}p_i\right)^{1-\sigma}, \\ &= \left(1-\sigma\right)\eta_g \cdot \sum_{i=1}^{I}\epsilon_i\omega_i = \left(1-\sigma\right)\eta_g \cdot \overline{\epsilon\left(c; \boldsymbol{p}\right)}. \end{aligned}$$

Since g is the inverse of the expenditure function at prices $\boldsymbol{q}$, we will have that:

$$\eta_e^c = \frac{\overline{\epsilon\left(c; \boldsymbol{p}\right)}}{\overline{\epsilon\left(c; q\right)}},$$

which is positive if we have $\epsilon_i > 0$ for all sectors. This ensures that the function e is monotonically increasing and one-to-one.

Now, let us compute the second order elasticity of the expenditure function. We have:

$$\eta^c_{\eta^c_e} = \frac{\partial \log \eta^c_e}{\partial \log c} = \frac{\partial \log \overline{\epsilon\left(c;\boldsymbol{p}\right)}}{\partial \log c} + \frac{\partial \log \eta_g}{\partial \log c},$$

where the second term on the right hand side can be written as:

$$\eta_{\eta_g} = 1 + \eta_{g'} - \eta_g.$$

We can rewrite the first term on the right hand side as follows:

$$\begin{aligned} \frac{\partial \log \overline{\epsilon\left(c;\boldsymbol{p}\right)}}{\partial \log c} &= c\frac{\partial}{\partial c} \log \sum_{i=1}^{I} \epsilon_i \left(g^{\epsilon_i} p_i\right)^{1-\sigma} - (1-\sigma)\frac{\partial \log e}{\partial \log c}, \\ &= (1-\sigma)\,\eta_g \frac{\sum_{i=1}^{I} \epsilon_i^2 \omega_i}{\sum_{i=1}^{I} \epsilon_i \omega_i} - (1-\sigma)\,\eta_e, \\ &= (1-\sigma)\,\eta_g \left(\frac{\overline{\epsilon^2\left(c;\boldsymbol{p}\right)}}{\overline{\epsilon\left(c;\boldsymbol{p}\right)}} - \overline{\epsilon\left(c;\boldsymbol{p}\right)}\right), \\ &= (1-\sigma)\,\eta_g \frac{Var\left(\epsilon; c, \boldsymbol{p}\right)}{\overline{\epsilon\left(c;\boldsymbol{p}\right)}} \end{aligned}$$

Substituting for $\log \eta_g = -\log \overline{\epsilon\left(c;\boldsymbol{q}\right)}$, we find that:

$$\eta^c_{\eta^c_e} = (1-\sigma)\,\eta_g \left[\frac{Var\left(\epsilon; c, \boldsymbol{p}\right)}{\overline{\epsilon\left(c;\boldsymbol{p}\right)}} - \frac{Var\left(\epsilon; c, \boldsymbol{q}\right)}{\overline{\epsilon\left(c;\boldsymbol{q}\right)}}\right].$$

Combining all the results above, we find the second order condition to require:

$$\theta + \frac{\overline{\epsilon\left(c;\boldsymbol{p}\right)}}{\overline{\epsilon\left(c;\boldsymbol{q}\right)}} - 1 + (1-\sigma)\left[\frac{\overline{\epsilon\left(c;\boldsymbol{p}\right)}}{\overline{\epsilon\left(c;\boldsymbol{q}\right)}}\left(\frac{Var\left(\epsilon; c, \boldsymbol{p}\right)}{\overline{\epsilon\left(c;\boldsymbol{p}\right)}^2}\right) - \frac{Var\left(\epsilon; c, \boldsymbol{q}\right)}{\overline{\epsilon\left(c;\boldsymbol{q}\right)}^2}\right] > 0,$$

for all c. Remembering that the variance of any distribution on $\{\epsilon_i\}_{i=1}^{I}$ is bounded above by $\frac{1}{4}\left(\epsilon_{max} - \epsilon_{min}\right)^2$, it immediately follows that condition (39) is a sufficient condition for the FOC to always characterize an optimal solution.

C.2 Proof of Equilibrium and Equilibrium Dynamics

First, let us derive Equation (46) for the price of consumption goods. Equalizing marginal revenue products of either capital or labor in the investment sector and any given section i

we find:

$$\begin{aligned} p_i(t) &= \frac{\alpha_0^{\alpha_0}(1-\alpha_0)^{1-\alpha_0}}{\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}} \cdot \frac{A_0(t)}{A_i(t)} \cdot \left(\frac{w(t)}{r(t)}\right)^{\alpha_0-\alpha_i}, \\ &= \left(\frac{\alpha_0}{\alpha_i}\right)^{\alpha_i} \left(\frac{1-\alpha_0}{1-\alpha_i}\right)^{1-\alpha_i} \frac{A_0(t)}{A_i(t)} \kappa_0(t)^{\alpha_0-\alpha_i}, \end{aligned}$$

where in the second equality, we have substituted for relative price of inputs from Equation (43). Using the definition of productivity-adjusted capital-to-labor ratio of the investment sector, and the assumption $A_i(t) = A_i(0)\, e^{\gamma_i t}$, we reach Equation (54).

For Equations (49) and (50), we start with the observation that:

$$\begin{aligned} \tilde{y}_0 &= \frac{1}{A_0^{\frac{1}{1-\alpha_0}} L} A_0 L_0 \kappa_0^{\alpha_0}, \\ &= l_0 \tilde{\kappa}_0^{\alpha_0}, \end{aligned} \tag{C.3}$$

where we have used the definition of the share of total labor in the investment sector as $l_0 = L_0/L$. We can compute this share as follows:

$$\begin{aligned} l_0 \equiv \frac{L_0}{L} &= \frac{1}{1+\frac{L_C}{L_0}}, \\ &= \frac{1}{1+\frac{1-\bar{\alpha}}{1-\alpha_0}\frac{e}{y_0}}, \\ &= \frac{(1-\alpha_0)\,\tilde{y}_0}{(1-\alpha_0)\,\tilde{y}_0 + (1-\bar{\alpha})\,\tilde{e}}. \end{aligned} \tag{C.4}$$

Combining Equations (C.3) and (C.4), we find:

$$\begin{aligned} \tilde{y}_0 &= \tilde{\kappa}_0^{\alpha_0} - \frac{1-\bar{\alpha}}{1-\alpha_0}\tilde{e}, \\ l_0 &= 1 - \frac{1-\bar{\alpha}}{1-\alpha_0}\frac{\tilde{e}}{\tilde{\kappa}_0^{\alpha_0}}. \end{aligned}$$

Lemma. *See Lemma (5).*

Proof. Aggregate capital to labor ratio in the economy may be written as:

$$
\begin{aligned}
k = \frac{K}{L} &= \frac{L_0}{L}\kappa_0 + \frac{L_C}{L}\kappa_C, \\
&= l_0\kappa_0 + (1-l_0)\,\kappa_0\frac{\kappa_C}{\kappa_0}, \\
&= \kappa_0\left[l_0 + (1-l_0)\,\frac{\bar{\alpha}/\,(1-\bar{\alpha})}{\alpha_0/\,(1-\alpha_0)}\right], \\
&= \kappa_0\left[1 + \frac{\tilde{e}}{\tilde{\kappa}_0^{\alpha_0}}\left(\frac{\bar{\alpha}}{\alpha_0} - \frac{1-\bar{\alpha}}{1-\alpha_0}\right)\right],
\end{aligned} \tag{C.5}
$$

where in the third equality, we have used the expressions for capital-to-labor ratios in Equations (43) and (45), and in the last equality, used the expression for the share of labor in the investment sector from Equation (50). We will now show that Equation (C.5) defines a one-to-one monotonically increasing function mapping k to κ_0. Adjusting both sides of the equation above with respect to the productivity in the

Taking the derivative of the expression with respect to $\tilde{\kappa}_0$, we find:

$$
\frac{\partial \tilde{k}}{\partial \tilde{\kappa}_0} = 1 + \left(\frac{\bar{\alpha}}{\alpha_0} - \frac{1-\bar{\alpha}}{1-\alpha_0}\right)\frac{\partial}{\partial \tilde{\kappa}_0}\left(\tilde{\kappa}_0^{1-\alpha_0}\tilde{e}\right) + \left(\tilde{\kappa}_0^{1-\alpha_0}\tilde{e}\right)\frac{\partial}{\partial \tilde{\kappa}_0}\left(\frac{\bar{\alpha}}{\alpha_0} - \frac{1-\bar{\alpha}}{1-\alpha_0}\right).
$$

Lemma (12) below proves $\eta_{\tilde{e}}^{\tilde{\kappa}_0} = \alpha_0 - \bar{\alpha}$ and $\eta_{\bar{\alpha}}^{\tilde{\kappa}_0} = -(1-\sigma)\,Var(\alpha)\,/\bar{\alpha}$, two relations that we use to compute the two derivates in the second and third terms of the right hand side in the expression above:

$$
\begin{aligned}
\frac{\partial}{\partial \tilde{\kappa}_0}\left(\tilde{\kappa}_0^{1-\alpha_0}\tilde{e}\right) &= \tilde{\kappa}_0^{-\alpha_0}\tilde{e}\frac{\partial \log\left(\tilde{\kappa}_0^{1-\alpha_0}\tilde{e}\right)}{\partial \log \tilde{\kappa}_0}, \\
&= \tilde{\kappa}_0^{-\alpha_0}\tilde{e}\left(1-\alpha_0-\eta_{\tilde{e}}^{\tilde{\kappa}_0}\right), \\
&= \tilde{\kappa}_0^{-\alpha_0}\tilde{e}\,(1-\bar{\alpha}),
\end{aligned}
$$

and,

$$
\begin{aligned}
\frac{\partial}{\partial \tilde{\kappa}_0}\left(\frac{\bar{\alpha}}{\alpha_0} - \frac{1-\bar{\alpha}}{1-\alpha_0}\right) &= \frac{\bar{\alpha}}{\alpha_0\,(1-\alpha_0)\,\tilde{\kappa}_0}\frac{\partial \log \bar{\alpha}}{\partial \log \tilde{\kappa}_0}, \\
&= -\frac{(1-\sigma)\,Var(\alpha)}{\alpha_0\,(1-\alpha_0)\,\tilde{\kappa}_0}.
\end{aligned}
$$

Now, we combine these two terms to find:

$$\begin{aligned}
\frac{\partial \tilde{k}}{\partial \tilde{\kappa}_0} &= 1+\tilde{\kappa}_0^{-\alpha_0}\tilde{e}\left[\left(\frac{\bar{\alpha}}{\alpha_0}-\frac{1-\bar{\alpha}}{1-\alpha_0}\right)(1-\bar{\alpha})-\frac{(1-\sigma)\,Var\,(\alpha)}{\alpha_0\,(1-\alpha_0)}\right], \\
&= 1+(1-l_0)\left(\frac{\bar{\alpha}-\alpha_0}{\alpha_0}-\frac{(1-\sigma)\,Var\,(\alpha)}{\alpha_0\,(1-\bar{\alpha})}\right), \\
&= l_0+\frac{1-l_0}{\alpha_0\,(1-\bar{\alpha})}\left[\bar{\alpha}\,(1-\bar{\alpha})-(1-\sigma)\,Var\,(\alpha)\right].
\end{aligned}$$

where in the second equality, we have again used the expression for the share of labor in the investment sector in Equation (50). Finally, note that the expression within the square bracket on the right hand side is always positive. This is because:

$$\begin{aligned}
\bar{\alpha}\,(1-\bar{\alpha})-(1-\sigma)\,Var\,(\alpha) &\geq \bar{\alpha}\,(1-\bar{\alpha})-Var\,(\alpha)\,, \\
&= \bar{\alpha}-\overline{\alpha^2}>0.
\end{aligned}$$

where the inequality in the second line follows from the fact that for all sectors i, we have $0<\alpha_i<1$. This completes the proof that the mapping of $\tilde{k}$ to $\tilde{\kappa}_0$ is monotonic and one-to-one. ∎

Lemma 12. *Elasticity of the average capital share of the consumption sector with respect to the capital-to-labor ratio in the investment sector is always negative and depends on the variance of capital shares in the consumption sector according to:*

$$\eta_{\bar{\alpha}}^{\tilde{\kappa}_0}=-\frac{1-\sigma}{\bar{\alpha}}Var\,(\alpha)\,.$$

Proof. Equation (46) implies that the investment sector capital-to-labor elasticity of sectoral prices are:

$$\eta_{p_i}^{\tilde{\kappa}_0}\equiv\frac{\partial \log p_i}{\partial \log \tilde{\kappa}_0}=\alpha_0-\alpha_i,$$

which allows us to find the elasticity of the expenditure function:

$$\eta_{\tilde{e}}^{\tilde{\kappa}_0}\equiv\frac{\partial \log \tilde{e}}{\partial \log \tilde{\kappa}_0}=\alpha_0-\bar{\alpha},$$

where we have used the fact that $\eta_e^{p_i}=\omega_i$. Next, we compute the elasticity of the sectoral share of sector i with respect to the investment sector capital-to-labor ratio:

$$\begin{aligned}
\eta_{\omega_i}^{\tilde{\kappa}_0} &\equiv \frac{\partial \log \omega_i}{\partial \log \tilde{\kappa}_0}, \\
&= (1-\sigma)\left(\eta_{p_i}^{\tilde{\kappa}_0}-\eta_e^{\tilde{\kappa}_0}\right), \\
&= (1-\sigma)\,(\bar{\alpha}-\alpha_i)\,.
\end{aligned}$$

A bit more algebra now yields the desired result:

$$
\begin{aligned}
\eta_{p_i}^{\tilde{\kappa}_0} &\equiv \frac{\partial \log p_i}{\partial \log \tilde{\kappa}_0}, \\
&= \frac{1}{\bar{\alpha}} \sum_{i=1}^{I} \alpha_i \omega_i \eta_{\omega_i}^{\tilde{\kappa}_0}, \\
&= \frac{1-\sigma}{\bar{\alpha}} \left(\bar{\alpha}^2 - \overline{\alpha^2} \right).
\end{aligned}
$$

∎

C.3 Derivations for Constant Growth Path

We first derive the asymptotic growth rate of share of sector i in consumption expenditure given by Equation (56). Sectoral shares shares evolve along an equilibrium path according to:

$$
\begin{aligned}
\gamma_{\omega_i} \equiv \frac{\dot{\omega}_i}{\omega_i} &= (1-\sigma) \left(\eta_g \epsilon_i \frac{\dot{c}}{c} + \frac{\dot{p}_i}{p_i} - \frac{\dot{e}}{e} \right), \\
&= (1-\sigma) \left[\eta_g \left(\epsilon_i - \bar{\epsilon} \right) \frac{\dot{c}}{c} + \gamma_{p_i} - \overline{\gamma_{p_i}} \right],
\end{aligned}
$$

where $\gamma_{p_i} \equiv \dot{p}_i / p_i$ is the rate of growth of the price of sector i, and in the second equality, we have used the relation:

$$
\frac{\dot{e}}{e} = \eta_e \frac{\dot{c}}{c} + \sum_{i=1}^{I} \omega_i \left(\frac{\dot{p}_i}{p_i} \right).
$$

From the definition of a CGP, we have that $\gamma_c \equiv \dot{c}/c \to \gamma^*$ and from Equation (54), we find that:

$$
\begin{aligned}
\gamma_{p_i} &= \frac{1-\alpha_i}{1-\alpha_0} \gamma_0 - \gamma_i + \left(\alpha_0 - \alpha_i \right) \frac{\dot{\tilde{\kappa}}_0}{\tilde{\kappa}_0}, \\
&\to \frac{1-\alpha_i}{1-\alpha_0} \gamma_0 - \gamma_i.
\end{aligned}
$$

Combining the expressions above and $\eta_g \to 1/\epsilon_{max}$, we find the desired result in Equation (56).

Lemma. *See Proposition 7.*

Proof. First, we will show that a CGP exists in the limit satisfies the dynamic Equations 52 and 53. Because of the uniqueness of the equilibrium, it then follows that the equilibrium path of the economy asymptotically converges the CGP found here.

Assume that real consumption per capita grows asymptotically as $c^* e^{\gamma^* t}$ for some constant c^* and for γ^* specified by Equation (57). Lemma 13 below shows that for such a path, we have that

$$\lim_{t\to\infty} g\left(c\left(t\right)\right) = \left(\frac{c^*}{q_{i_{max}}}\right)^{\frac{1}{\epsilon_{max}}} e^{\frac{\gamma^*}{\epsilon_{max}}t}, \tag{C.6}$$

where i_{max} is the sector with the highest income elasticity parameter ϵ_{max} and $q_{i_{max}}$ is the base price of that sector from Equation (38), in which real consumption is expressed.

Furthermore, assume a path of $\tilde{\kappa}_0\left(t\right)$ that asymptotically converges to the constant capital-to-labor ratio $\tilde{\kappa}_0^*$, as specified by Equation (55). Note that along such a path for $\left(c\left(t\right), \tilde{\kappa}_0\left(t\right)\right)$, from Equation (48), the growth-adjusted per capita consumption expenditure converges to:

$$\begin{aligned}
\lim_{t\to\infty} \tilde{e}\left(c(t), \tilde{\kappa}_0\left(t\right), t\right) &= \lim_{t\to\infty} \left(\sum_{i=1}^{I} \left[\varphi_i\left(0\right) e^{-\gamma_i - \frac{\alpha_i}{1-\alpha_0}\gamma_0} g\left(c\left(t\right)\right)^{\epsilon_i}\right]^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \\
&= \left(\sum_{i\in\mathcal{I}^*} \left[\varphi_i\left(0\right)\left(\frac{c^*}{q_{i_{max}}}\right)^{\frac{\epsilon_i}{\epsilon_{max}}}\right]^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, \\
&= \tilde{e}\left(c^*\right),
\end{aligned}$$

where the second equality follows from the definition of γ^* in Equation (57), and the last time, with a little abuse of notation, emphasizes that the limit is a monotonically increasing function of c^*. With a similar logic, the consumption expenditure shares of different sectors converge to a distribution:

$$\lim_{t\to\infty} \omega_i\left(t\right) = \omega_i^* = \begin{cases} \dfrac{\left[\varphi_i(0)\left(\frac{c^*}{q_{i_{max}}}\right)^{\frac{\epsilon_i}{\epsilon_{max}}}\right]^{1-\sigma}}{\sum_{i'\in\mathcal{I}^*}\left[\varphi_{i'}(0)\left(\frac{c^*}{q_{i_{max}}}\right)^{\frac{\epsilon_{i'}}{\epsilon_{max}}}\right]^{1-\sigma}}, & i \in \mathcal{I}^*, \\ 0, & i \notin \mathcal{I}^*, \end{cases}$$

suggesting further that average capital share of the consumption sector converges to a constant $\lim_{t\to\infty} \bar{\alpha}\left(t\right) = \bar{\alpha}^*$. Note that the

Finally, we can find c^* such that it satisfies Equation (52), as:

$$\tilde{e}\left(c^*\right) = \frac{1-\alpha_0}{1-\bar{\alpha}^*}\left[\left(\tilde{\kappa}_0^*\right)^{\alpha_0} - \left(n + \delta + \frac{\gamma_0}{1-\alpha_0}\right)\tilde{k}\left(\tilde{\kappa}_0^*, \tilde{e}\left(c^*\right)\right)\right],$$

where $\tilde{k}$ is a function of both $\tilde{\kappa}_0$ and $\tilde{e}$ according to Equation (51). ∎

Lemma 13. *For any path of $c\left(t\right)$ such that $\lim_{t\to\infty} c\left(t\right) = c^* e^{\gamma^* t}$, Expression (C.6) holds.*

Proof. From the definition of function $g\left(\cdot\right)$ in Equation (38), we know that asymptotically, the following has to be satisfied:

$$\lim_{t\to\infty} \frac{\left(\sum_{i=1}^{I} \left[g\left(c\left(t\right)\right)^{\epsilon_i} q_i\right]^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}{c^* e^{\gamma^* t}} = 1.$$

Since $g\left(\cdot\right)$ is monotonically increasing, it implies that:

$$\lim_{t\to\infty} \left(\sum_{i=1}^{I} \left[g\left(c\left(t\right)\right) q_i\right]^{1-\sigma}\right)^{\frac{1}{1-\sigma}} = g\left(c\left(t\right)\right)^{\epsilon_{max}} q_{i_{max}},$$

from which the desired result immediately follows. ∎

D Tables and Figures

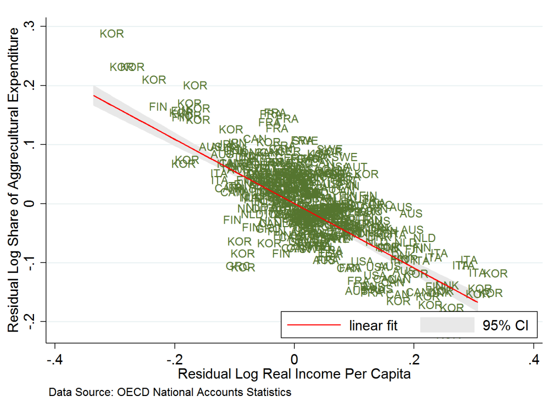
Table 11: Nested CES Regression

Within Manufacturing (ε rel. to Industry)	
Elasticity σ	0.86
	(0.02)
Mining	-0.47
	(0.02)
Construction	0.16
	(0.01)
Observations	1602
Within Services (ε rel. to Public Utilities)	
Elasticity σ	0.88
	(0.01)
Transp., Storage, Comm.	0.03
	(0.02)
Wholesale and Retail	0.24
	(0.02)
Community, Social and Personal Serv.	0.07
	(0.02)
Finance, Insurance, Real State	0.48
	(0.02)
Observations	950

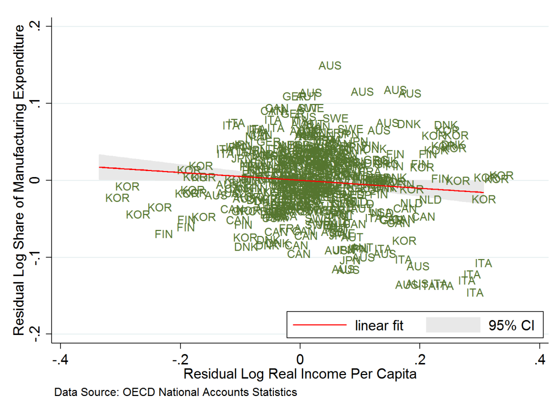
Regressions for the entire sample. Robust standard errors in parentheses.

Figure 6: Partial Correlations of Expenditure Share and Real Consumption for OECD countries

(a) Agriculture



(b) Manufacturing



(c) Services

(d) Services