

A Search Theory of Retail Market Structure

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Abstract

This paper studies how consumer multiproduct shopping and search frictions jointly affect retail market structure. Single-product shops with different products can merge to form a multiproduct retailer. By doing this, they provide one-stop shopping convenience and thus attract more consumers. However, the conglomerate merger also changes the market structure and affects price competition. We find that an asymmetric market structure with one multiproduct retailer and smaller single-product competitors often leads to the weakest price competition. We solve for the equilibrium market structure, and show that it is asymmetric whenever the search friction is not too high. Conglomerate merger which leads to such an asymmetric market structure often raises market prices and harms consumers. Due to the endogeneity of the market structure, reducing the search friction does not necessarily induce lower market prices and higher consumer welfare.

Keywords: consumer search, conglomerate merger, multiproduct pricing, retail market structure

JEL classification: D11, D43, D83, L13

1 Introduction

Retailing is an important sector of the economy. For example, in 2013 in the US, the sales revenue generated by department stores was \$199.4 billion, and that generated by supermarkets and other grocery stores amounted to some \$546.6 billion (which is greater than that of the auto industry). Retailers are increasingly offering consumers more choice, by branching out into product lines which they traditionally would not have stocked. For example drugstores like Walgreens and Rite Aid are attempting to sell more fresh food and grocery items, while supermarkets like Wal-Mart and Publix have been offering pharmacies and clinical services. According to the Food Marketing Institute, between 1975 and 2008, the number of products in the average supermarket increased from an average of 8,948 to almost 47,000.

One important feature of the retail market is that consumers often need buy a basket of products in their shopping trips. But in the same time consumers are often constrained in terms of how much time they can spend searching around for the products they want. For this reason they value the convenience of “one-stop shopping” provided by large retailers. For example, Messinger and Narasimhan (1997) provide empirical evidence that time-saving convenience is the most important driver of the growth in supermarket store size. However, large multiproduct retailers still often coexist with smaller competitors such as specialist shops which have much narrower product ranges.

This paper intends to study how consumer multiproduct shopping and search frictions together might shape the retail market structure. The investigation will also help us understand questions like: What is the nature of competition between large and small retailers? Why might some firms choose to remain small? Why might retail market structure be different on- and off-line? (The online market structure appears more asymmetric among competitors. For example, in 2012 Amazon sells more online than its 12 top competitors combined.) How do conglomerate mergers between firms with different products, affect market performance?

Our benchmark model has two products (or product categories), each of which is sold by two single-product retailers. Consumers wish to buy one unit of each product, but differ with respect to their shopping costs. Some consumers (“shoppers”) are able to visit all retailers without incurring any cost, while other consumers (“non-shoppers”) are time-constrained and are only able to visit one retailer. Single-product firms can choose whether or not to merge with another one that sells a different product. The

resulted market structure is observed by all participants, and firms then engage in price competition. Shoppers learn all prices and so buy each product at the lowest price available. Non-shoppers do not observe prices, but visit a store which they believe will offer them the highest (expected) surplus. The fraction of non-shoppers is interpreted as a measure of the search friction in the market.

The paper first solves for equilibrium pricing in each of the three possible market structures: i.e., four single-product firms, two multi-product firms, and an asymmetric market with one multiproduct firm and two single-product firms. Prices are drawn using mixed strategies, because firms face the usual dilemma of pricing low to attract shoppers, or pricing high to exploit non-shoppers. As we explain below, the existing related literature focuses on settings where firms have identical product ranges. As such, our analysis of the pricing equilibrium for the case of an asymmetric market structure is of independent interest. Here we show that the multiproduct firm charges higher prices than the single-product firms (in the sense of first order stochastic dominance), and yet still attracts a disproportionate share of non-shoppers. Intuitively this is because the multiproduct firm offers one-stop shopping convenience, which more than compensates for the higher prices that it charges. Moreover, we also demonstrate that this asymmetric market structure often leads to the weakest price competition. This is because the multiproduct retailer focuses more on exploiting its one-stop shopping convenience through higher prices, which softens competition with the two single-product retailers.

Using the previous insight, the paper then proceeds to characterize equilibrium market structure. A conglomerate merger is shown to have two distinct effects. Firstly, when two single-product firms merge they provide one-stop shopping convenience, and so enjoy an increase in demand (a *“search order effect”*). Secondly though, the merger changes market structure - and either softens or strengthens price competition, depending upon whether the new market structure is symmetric or not (a *“price competition effect”*). Consequently there is no equilibrium with four single-product firms: if two of them engage in a conglomerate merger, they enjoy higher demand and also soften price competition. More interestingly, the size of the search friction then determines whether or not a second conglomerate merger occurs. In particular when the search friction is relatively low, the price competition effect dominates, such that the two remaining single-product firms prefer not to merge. Thus the model is able to generate both symmetric and asymmetric market structures, depending upon parameters. We also analyze the welfare consequences of conglomerate mergers, and show amongst other

results that a conglomerate merger which leads to an asymmetric market structure are detrimental for consumers, due to the way in which they relax price competition.

Compared to the existing literature, a distinctive feature of our paper is the endogeneity of market structure. Whilst several recent papers consider consumer search when firms sell multiple products, all these papers assume symmetry, such that competing firms sell exactly the same (exogenously given) products. For example Zhou (2014) shows that when all firms sell more products, prices decline due to a joint search effect. Rhodes (2014) considers the relationship between a multiproduct retailer’s advertised and unadvertised prices, and how the former can signal a low price image. See also Lal and Matutes (1994). McAfee (1995) and Shelegia (2012) examine when and how multiproduct firms might correlate their prices. However none of these models consider the case of an asymmetric market, where firms sell different (numbers of) products. In a single-product search environment, Goldmanis, Hortacsu, Syverson, and Emre (2010) show that lower search costs have generated more asymmetric market structures, by pushing business towards a small number of very large retailers (in terms of market shares, instead of the product range).

The rest of the paper proceeds as follows. Section 2 outlines a benchmark model, characterizes price distributions, and solves for the equilibrium market structure. Section 3 then considers several extensions, including a variation with differentiated products which allows for explicit consumer search. Section 4 then concludes with a discussion of future avenues for research. All omitted proofs are available in the appendix.

2 A Basic Model

A unit mass of consumers is interested in buying two products 1 and 2. Each consumer has unit demand, and is willing to pay up to v for each product. Initially there are four single-product firms in the market: two of them, denoted by 1_A and 1_B , sell a homogenous product 1, and the other two, denoted by 2_A and 2_B , sell a homogenous product 2. Supplying each product involves a constant marginal cost which is normalized to zero. Consumers are divided into two groups. A fraction $\alpha \in (0, 1)$ of consumers are shoppers, who can visit firms costlessly and multi-stop shop. If they find two firms offering the same price for a product, they purchase one randomly. The remaining fraction $1 - \alpha$ of consumers are non-shoppers, who can visit only one firm (but can do

so costlessly).¹

We assume that conglomerate merger between two firms supplying different products is possible and costless. We also assume that horizontal merger between two firms selling the same product is *not* permitted (or is too costly), for instance due to antitrust policy. We consider a two-stage game. In the first stage, each pair of firms $(1_k, 2_k)$, $k = A, B$ has the opportunity to merge. Their merger decisions can be simultaneous or sequential. In the second stage, after observing the market structure firms choose their prices simultaneously. Shoppers buy each product at the lowest price. Non-shoppers form (rational) expectations about each firm's price distribution, and choose to visit the firm which they believe will deliver them highest consumer surplus. In turn, each firm sets its prices to maximize expected profits, given consumer search strategies and the other firms' pricing strategies. We assume that multiproduct firms charge separate prices for each product, and therefore do not use bundling.

Notice that $1 - \alpha$ can be regarded as a measure of search frictions in the market. Our primary aim in this paper is to understand how these search frictions affect equilibrium market structure. In order to do this, we need to solve for equilibrium pricing in all three potential market structures: (i) if no merger occurs, a market with four independent single-product firms, (ii) if only one pair of firms merges, an asymmetric market structure with one multiproduct firm and two single-product firms, and (iii) if both pairs of firm choose to merge, a symmetric market with two multiproduct firms. Indeed we believe that the pricing outcome for the asymmetric case is of independent interest. This is because the search literature rarely investigates the case where multiproduct firms compete with single-product firms, despite its obvious relevance.

Finally, a few comments on our modelling assumptions. When firms sell a homogenous product, the standard way to avoid the Diamond (1971) paradox is to make consumers differentially informed. One approach - which we use here - is to assume that some consumers observe one firm's price, whilst other consumers observe at least two firms' prices. This induces firms to adopt a mixed pricing strategy, which creates price dispersion. (See, for example, Varian, 1980; Burdett and Judd, 1983.) An alternative approach would be to assume that non-shoppers search sequentially, and incur a cost each time they search a firm (no matter whether it is a single-product or multiproduct firm). (See, for example, Stahl, 1989.) However this is complicated to analyze in a mul-

¹For example, non-shoppers do not visit another firm because the cost of doing so is very high.

tiproduct context, because typically there are multiple mixed-strategy pricing equilibria which are not outcome equivalent (see McAfee, 1995).² Finally an alternative way to relax the Diamond paradox is to introduce product differentiation. In section 3.3, we provide an extension with product differentiation, and show that the main results are qualitatively similar to those of our baseline model.

A recurring component in our analysis below is a pricing game where two (single-product) firms which are visited by potentially different numbers of non-shoppers choose their prices simultaneously. The following lemma reports the equilibrium in this pricing game.

Lemma 1 *Consider a simultaneous pricing game between two firms A and B which supply a homogenous product at zero cost. Let N_k , $k = A, B$, be the number of non-shoppers who visit firm k only. Suppose $N_A \geq N_B \geq 0$ with at least one strict inequality. Let $S > 0$ be the number of shoppers.*

- (i) *There is no pure-strategy Nash equilibrium.*
- (ii) *If $N_A = N_B = N > 0$, the unique equilibrium is that each firm charges a random price according to the atomless price distribution*

$$F(p) = 1 - \frac{N}{S} \left(\frac{v}{p} - 1 \right) \quad (1)$$

which has a support $[\underline{p}, v]$ with

$$\underline{p} = \frac{N}{N + S} v . \quad (2)$$

- (iii) *If $N_A > N_B \geq 0$, the unique equilibrium is that firm A charges a random price according to the price distribution*

$$F_A(p) = 1 + \frac{N_B}{S} - \left(\frac{N_B}{S} + \lambda \right) \frac{v}{p} \quad (3)$$

which has support $[\underline{p}, v]$ with

$$\underline{p} = \frac{N_A}{N_A + S} v \quad (4)$$

and has a mass point on v with size

$$\lambda = \frac{N_A - N_B}{N_A + S} , \quad (5)$$

²In the most interesting class of equilibria, a firm's prices are negatively correlated across its products. However the characterization of this equilibrium is complex.

and firm B charges a random price according to the atomless price distribution

$$F_B(p) = 1 - \frac{N_A}{S} \left(\frac{v}{p} - 1 \right) \quad (6)$$

which has a support $[\underline{p}, v)$.

Proof. (i) The proof is standard. It is impossible that both firms charge the same price $p > 0$. Otherwise, each firm would have an incentive to undercut given $S > 0$. It is also impossible that both firms charge $p = 0$ given the existence of non-shoppers for at least one firm. The last possible pure-strategy equilibrium is that the two firms charge different prices, say, $p_A > p_B$. Then firm B will have an incentive to raise its price by $\varepsilon < p_A - p_B$, since no consumers will leave firm B because of this price adjustment.

(ii) We first verify the configuration is an equilibrium. Given the other firm is using the equilibrium strategy F , a firm's profit at $p < v$ is $p[N + S(1 - F(p))]$. (N non-shoppers will buy for sure, and shoppers will also buy if the rival is charging a price above p .) Its profit at $p = v$ is Nv since only non-shoppers will buy in that case. The expression for F in (1) equalizes these two profits such that each firm is indifferent among all prices in $[\underline{p}, v]$, where the lower bound $\underline{p}$ is given in (2) and is derived from $F(\underline{p}) = 0$. It is also clear that each firm has no unilateral incentive to charge a price below $\underline{p}$. The uniqueness of the equilibrium has been shown in Varian (1980) (which proves no other symmetric equilibria) and Baye, Kovenock, and de Vries (1992) (which proves no asymmetric equilibria in the duopoly version of Varian, 1980).

(iii) Again we first verify the proposed configuration is an equilibrium. Consider firm A first. Given firm B is using the equilibrium strategy F_B , its profit at $p < v$ is $p[N_A + S(1 - F_B(p))]$, and its profit at $p = v$ is $N_A v$. The expression for F_B in (6) just equalizes these two profits. The lower bound of support $\underline{p}$ in (4) is derived from $F_B(\underline{p}) = 0$. Firm A is then indifferent among all prices between $\underline{p}$ and v , and it also has no incentive to charge a price below $\underline{p}$.

Now consider firm B . Given firm A is using the equilibrium strategy F_A , its profit at $p < v$ is $p[N_B + S(1 - F_A(p))]$. When p converges to v from below, the profit converges to $v[N_B + S\lambda]$ given F_A has a mass λ at $p = v$. The expression for F_A in (3) equalizes these two profits. Given the mass point of F_A , firm B never wants to charge a price exactly at $p = v$ because it is dominated by a price slightly below v . That is why the support of F_B is open at v . λ in (5) is derived from $F_A(\underline{p}) = 0$. Then firm B has no incentive to charge a price below $\underline{p}$ either. The uniqueness of the equilibrium has been

shown in Baye, Kovenock, and de Vries (1992). (Our case is a special case of their section V.) ■

Two useful observations about the case with $N_A > N_B$ follow from this lemma: (i) From F_A in (3) and F_B in (6), we can see that the corresponding density functions f_A and f_B must have the form of a constant times v/p^2 . Using (5), one can verify that

$$f_A(p) = (1 - \lambda)f_B(p) \quad (7)$$

for $p \in [\underline{p}, v]$.³ This indicates that firm A charges higher prices than firm B in the sense of first-order stochastic dominance (FOSD).

We now proceed by studying equilibrium pricing in each of the three possible market structures outlined above, starting with the simplest case of four independent single-product firms. (In the welfare analysis, total welfare is defined as the sum of industry profit and consumer surplus.)

Lemma 2 *With four independent single-product firms, non-shoppers randomly visit one firm and each firm uses a mixed pricing strategy as characterized in (ii) in Lemma 1 with $N = \frac{1}{4}(1 - \alpha)$ and $S = \alpha$. Each firm earns profit $\frac{1}{4}(1 - \alpha)v$ and industry profit is $(1 - \alpha)v$. Total welfare is $(1 + \alpha)v$ and consumer surplus is $2\alpha v$.*

Proof. We first show that in equilibrium non-shoppers must randomly visit one firm such that each firm has $\frac{1}{4}(1 - \alpha)$ non-shoppers. Suppose, in contrast, in at least one product market more non-shoppers visit firm A than firm B . Then Lemma 1 implies that firm A will charge a higher price than firm B in the sense of FOSD. This is inconsistent with non-shoppers' search behavior. Another possibility is that in each product market non-shoppers visit a firm randomly, but more of them visit one market, say, the market for product 1, than the other. Then firms 1_A and 1_B will charge a higher price than firms 2_A and 2_B . Again, this contradicts non-shoppers' search behavior.

Therefore, in each product market we must have $N_A = N_B = \frac{1}{4}(1 - \alpha)$ in equilibrium. The equilibrium characterization then just follows from Lemma 1. When a firm charges the monopoly price v , only $\frac{1}{4}(1 - \alpha)$ non-shoppers buy from it. So each firm's equilibrium profit is $\frac{1}{4}(1 - \alpha)v$ and industry profit is $(1 - \alpha)v$. Since non-shoppers are only able

³This is also clear from the facts $\int_{\underline{p}}^v f_A(p)dp = 1 - \lambda$ and $\int_{\underline{p}}^v f_B(p)dp = 1$.

to buy one product but shoppers buy both products, total welfare is $(1 - \alpha)v + \alpha 2v = (1 + \alpha)v$. Consumer surplus is therefore $2\alpha v$. ■

Another simple case is when both pairs of firms choose to merge and then the market has two multiproduct firms. In terms of equilibrium outcome, it is without loss of generality to focus on an equilibrium where each firm charges independent prices across the two products.⁴

Lemma 3 *With two multiproduct firms, non-shoppers randomly visit one firm, and each firm uses a mixed pricing strategy for each product according to a price distribution as characterized in (ii) in Lemma 1 with $N = \frac{1}{2}(1 - \alpha)$ and $S = \alpha$. Each firm earns profit $\frac{1}{2}(1 - \alpha)v$ from each product and industry profit is $2(1 - \alpha)v$. Total welfare is $2v$ and consumer surplus is $2\alpha v$.*

Proof. The argument for that non-shoppers must randomly visit one firm in equilibrium is similar as in Lemma 2. Then we must have $N_A = N_B = \frac{1}{2}(1 - \alpha)$, and the equilibrium characterization simply follows from Lemma 1. The welfare calculation is straightforward since now all consumers buy both products. ■

The last possibility is the case where firms 1_A and 2_A merge and form a multiproduct firm A , while firms 1_B and 2_B remain as single-product firms. Intuitively, non-shoppers tend to visit the multiproduct firm A since they can find both products there while in a single-product firm they can only buy one product. However, firm A can exploit this advantage and charge higher prices than its single-product rivals. This can in turn discourages non-shoppers from visiting firm A . Therefore, it is *ex ante* not clear whether all non-shoppers visit the multiproduct firm A in equilibrium. As we will see below, it actually depends on the relative proportion of non-shoppers in the population. We will show that when the proportion of non-shoppers is high, only some non-shoppers visit firm A in equilibrium. This prevents the multiproduct firm from charging too high prices, and thus rationalizes non-shoppers' search behavior.

⁴Given the rival firm's pricing strategy F (which can be a joint price distribution with correlation) and non-shoppers' shopping behavior, only the two marginal distributions of F matter for a firm's profit calculation. Therefore, for any equilibrium where each firm uses a joint price distribution, we can construct an alternative equilibrium with the same outcome by having each firm charging prices independently across their products according to the corresponding marginal distributions.

Suppose N_A non-shoppers visit firm A and N_B non-shoppers visit firm B in equilibrium. Then the pricing equilibrium is described as in Lemma 1 with $S = \alpha$. To check the optimality of non-shoppers' search decision, we need to compare their expected surplus from visiting firm A

$$2 \int_{\underline{p}}^v (v - p) f_A(p) dp$$

with that from visiting a single-product firm B

$$\int_{\underline{p}}^v (v - p) f_B(p) dp .$$

By using (5) and (7), the former is weakly higher if and only if

$$2(1 - \lambda) \geq 1 \Leftrightarrow \frac{N_B + S}{N_A + S} \geq \frac{1}{2} . \quad (8)$$

The following lemma reports two possible equilibria in the asymmetric market.

Lemma 4 *Consider the asymmetric market structure with one multiproduct firm A and two single-product firms 1_B and 2_B .*

(i) *If there are more shoppers than non-shoppers in the market (i.e., if $\alpha \geq \frac{1}{2}$), then all non-shoppers visit the multiproduct firm A . Firm A uses a pricing strategy F_A for each of its products, and firms 1_B and 2_B use a pricing strategy F_B , both characterized as in (iii) in Lemma 1 with $N_A = 1 - \alpha$, $N_B = 0$ and $S = \alpha$. Firm A 's per product profit is $(1 - \alpha)v$, each single-product firm's profit is $\alpha(1 - \alpha)v$, and industry profit is $2(1 - \alpha^2)v$. Total welfare is $2v$, and consumer surplus is $2\alpha^2v$.*

(ii) *If there are fewer shoppers than non-shoppers in the market (i.e., if $\alpha < \frac{1}{2}$), then a fraction $\frac{1}{2(1-\alpha)}$ of non-shoppers visit the multiproduct firm A and the remaining non-shoppers randomly visit a single-product firm. Firm A uses a pricing strategy F_A for each of its products, and firms 1_B and 2_B use a pricing strategy F_B , both as characterized in (iii) in Lemma 1 with $N_A = \frac{1}{2}$, $N_B = \frac{1}{4}(1 - 2\alpha)$ and $S = \alpha$. Firm A 's per product profit is $\frac{1}{2}v$, each single-product firm's profit is $\frac{1}{4}v$, and industry profit is $\frac{3}{2}v$. Total welfare is $(\frac{3}{2} + \alpha)v$ and consumer surplus is αv .*

Proof. Notice that it is impossible that in equilibrium more non-shoppers visit one single-product firm than the other. We then must have $N_B = \frac{1}{2}(1 - \alpha - N_A)$ in equilibrium. It is also impossible to have an equilibrium where no non-shoppers visit firm A . Otherwise, the single-product firms would charge higher prices than firm A , and then all non-shoppers should visit firm A .

(i) An equilibrium with all non-shoppers visiting firm A exists if (8) holds with $N_A = 1 - \alpha$, $N_B = 0$ and $S = \alpha$. This yields the condition $\alpha \geq \frac{1}{2}$. The equilibrium price distributions follows from (iii) in Lemma 1. From (5) we have $\lambda = 1 - \alpha$. In equilibrium, firm A 's profit from each product is $N_A v = (1 - \alpha)v$, and each single-product firm's profit is $(N_B + S\lambda)v = \alpha(1 - \alpha)v$. Hence, industry profit is $2(1 - \alpha^2)v$. Total welfare is $2v$ since even non-shoppers buy both products by visiting the multiproduct firm. Consumer surplus is then $2\alpha^2v$.

(ii) Now consider the case with $\alpha < \frac{1}{2}$, and the only possible equilibrium is that a fraction $X \in (0, 1)$ of the non-shoppers visit the multiproduct firm A such that $N_A = (1 - \alpha)X$ and $N_B = \frac{1}{2}(1 - \alpha)(1 - X)$. According to (8), non-shoppers will be indifferent between visiting a multiproduct firm and a single-product firm only if $\lambda = \frac{1}{2}$, or equivalently

$$\frac{N_B + S}{N_A + S} = \frac{1}{2} \Leftrightarrow X = \frac{1}{2(1 - \alpha)}.$$

(Given $0 < \alpha < \frac{1}{2}$, we have $X \in (\frac{1}{2}, 1)$.) Then $N_A = \frac{1}{2}$ and $N_B = \frac{1}{4}(1 - 2\alpha)$. In equilibrium, firm A 's profit from each product is $N_A v = \frac{1}{2}v$ and each single-product firm's profit is $(N_B + S\lambda)v = \frac{1}{4}v$. Industry profit is then $\frac{3}{2}v$. Since only a fraction X of the non-shoppers now buy both products, the total number of consumers who buy both products is $\alpha + (1 - \alpha)X = \frac{1}{2} + \alpha$. So total welfare is $(\frac{1}{2} + \alpha)2v + (\frac{1}{2} - \alpha)v = (\frac{3}{2} + \alpha)v$. Then consumer surplus is αv . ■

For convenience, we summarize all relevant welfare variables in the following table:

	4 sp firms	2 mp firms	asymmetric ($\alpha \geq 1/2$)	asymmetric ($\alpha < 1/2$)
Per product profit	$\frac{1}{4}(1 - \alpha)v$	$\frac{1}{2}(1 - \alpha)v$	$A : (1 - \alpha)v$ $B : \alpha(1 - \alpha)v$	$A : \frac{1}{2}v$ $B : \frac{1}{4}v$
Industry profit	$(1 - \alpha)v$	$2(1 - \alpha)v$	$2(1 - \alpha^2)v$	$\frac{3}{2}v$
Consumer surplus	$2\alpha v$	$2\alpha v$	$2\alpha^2v$	αv
Total welfare	$(1 + \alpha)v$	$2v$	$2v$	$(\frac{3}{2} + \alpha)v$

Table 1: Profit and welfare comparison across market structures

We can now examine the equilibrium market structure when both pairs of firms $(1_A, 2_A)$ and $(1_B, 2_B)$ have the opportunity to merge before engaging in price competition. The basic trade-off in the merger decision is as follows. If a pair of firms

choose to merge, they become more attractive to non-shoppers because of the resulting one-stop shopping convenience. This induces more non-shoppers to visit the merged multiproduct firm (*“the search order effect”*). Meanwhile, the merger also affects price competition in the market. When the other pair of firms do not merge, the merger creates an asymmetric market structure. This softens price competition (*“the price competition effect”*): multiproduct firm A can charge higher prices because it has more non-shoppers to exploit, and this further relaxes price competition among firms. As we can see from Table 1, all firms benefit in the asymmetric market structure compared to the initial situation with four single-product firms. As a result, the initial situation with four single-product firms cannot be an equilibrium outcome, because both effects push in the same direction.⁵

If the other pair of firms also choose to merge, they can win some non-shoppers back since they are now also providing one-stop shopping convenience. However, the restored symmetric market structure may intensify price competition. Therefore, whether or not the second pair of firms chooses to merge will depend on the relative strengths of the search order effect and the price competition effect. As we can see from Table 1, when $\alpha \geq \frac{1}{2}$ (i.e., when there are more shoppers than non-shoppers in the market), the price competition effect is stronger and so the second pair of firms have no incentive to merge. But when $\alpha < \frac{1}{2}$, the search order effect dominates and the second pair of firms will merge. (Notice that in the case with $\alpha \geq \frac{1}{2}$, if the two pairs of firms make their merger decisions simultaneously, there are two asymmetric pure-strategy equilibria and one mixed-strategy equilibrium.)

The following result reports the equilibrium market structure and welfare outcomes (the welfare outcomes are straightforward from Table 1) :

Proposition 1 (i) *When there are more shoppers than non-shoppers in the market (i.e., when $\alpha \geq \frac{1}{2}$), the unique (pure-strategy) equilibrium outcome is that the market has a multiproduct firm and two single-product firms. Compared to the initial situation, this leads to higher industry profit and total welfare but lower consumer surplus.*
(ii) *When there are more non-shoppers than shoppers in the market (i.e., when $\alpha < \frac{1}{2}$), the unique equilibrium outcome is that the market has two multiproduct firms. Compared to the initial situation, this leads to higher industry profit and total welfare, and*

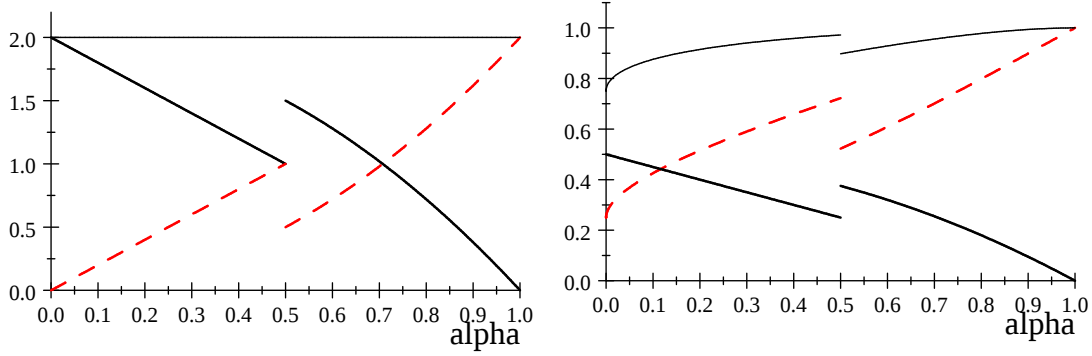
⁵Of course, if conglomerate merger involves a sufficiently high fixed cost, then the initial situation can remain as an equilibrium outcome.

consumer surplus remains unchanged.

In either case, the market outcome with conglomerate merger improves total welfare relative to the initial situation with four single-product firms. This is simply because of a positive market coverage effect from conglomerate merger—now non-shoppers are able to buy both products by visiting a multiproduct firm. (Notice that in the current setting with inelastic demand, consumer payment is a pure transfer and so the price competition effect of conglomerate merger does not affect total welfare.) In either case, the market outcome also improves industry profit relative to the initial situation. This is because price competition is relaxed either because of the resulting asymmetric market structure (when $\alpha \geq \frac{1}{2}$) or because more non-shoppers visit each multiproduct firm (when $\alpha < \frac{1}{2}$).

In either case, the market outcome does not improve consumer welfare relative to the initial situation. In the case with an asymmetric market outcome, consumers suffer from the conglomerate merger because the negative price competition effect dominates the positive market coverage effect. In the case with two multiproduct firms, these two effects happen to cancel out each other.

The fraction of non-shoppers (i.e., $1 - \alpha$) is a measure of search frictions in this baseline model. Proposition 1 implies that when the search friction increases, the equilibrium market structure moves from an asymmetric one to a symmetric one with two multiproduct firms. Intuitively, this is because the search order effect becomes more important as the search friction becomes larger. Since the asymmetric market structure is often associated with weaker price competition, this also suggests that a higher search friction does not always harm consumers due to the endogeneity of market structure. Figure 1(a) below describes how total welfare (the top horizontal line), industry profit (the thick solid lines), consumer surplus (the dashed lines) vary with α when $v = 1$. Total welfare is independent of α because of the unit-demand feature in the model. (This is no longer true if we consider elastic demand as indicated in Figure 1(b). The case with elastic demand will be analyzed in section 3.1.) Consumer surplus does not always increase in α (or decrease in $1 - \alpha$), and it has a jump-down at $\alpha = 0.5$. Similarly, industry profit does not always decline in α , and it has a jump-up at $\alpha = 0.5$.



(a): unit demand

(b): linear demand

Figure 1: Welfare and the number of shoppers

3 Extensions and an Alternative Model

3.1 Elastic demand

The analysis can be extended to the case with elastic demand without changing the main result. Suppose now consumers are willing to buy $Q(p)$ units of a product when its price is p . Here $Q(p)$ is a downward sloping demand function. Let $v(p)$ be the associated indirect utility function with $v'(p) = -Q(p)$, and let $\pi(p) \equiv pQ(p)$ be the profit function.

Assumption 1 *Suppose the demand function $Q(p)$ is such that $\pi(p)$ has a unique maximum π_M at the monopoly price p_M , and suppose $\pi(p)$ increases in $p < p_M$.*

The results in Lemma 1 remain unchanged as long as we replace the profit with price p by $\pi(p)$ (in particular, the monopoly profit is now π_M) and the upper bound v of the price support by p_M . For instance, (1) becomes

$$F(p) = 1 - \frac{N}{S} \left(\frac{\pi_M}{\pi(p)} - 1 \right) ,$$

and the boundary condition (2) becomes

$$\pi(\underline{p}) = \frac{N}{N+S} \pi_M .$$

In particular, the mass point condition (5) remains the same. The analysis of the two symmetric market structures is essentially the same. In the case with four single-product firms, each firm has $\frac{1}{4}(1 - \alpha)$ non-shoppers, so each firm's profit is $\frac{1}{4}(1 - \alpha)\pi_M$. In the case with two multiproduct firms, each has $\frac{1}{2}(1 - \alpha)$ non-shoppers, so each firm's per product profit is $\frac{1}{2}(1 - \alpha)\pi_M$.

The following lemma reports the main results in the asymmetric case. (All omitted proofs are provided in the appendix.)

Lemma 5 *Consider the asymmetric market structure with one multiproduct firm A and two single-product firms 1_B and 2_B . Suppose the demand function $Q(p)$ satisfies Assumption 1.*

(i) *If*

$$2(1 - \alpha)v(p_M) + (2\alpha - 1) \int_{\underline{p}}^{p_M} v(p)f_B(p)dp \geq 0 \quad (9)$$

where $f_B(p) = \frac{(1-\alpha)\pi_M}{\alpha} \frac{\pi'(p)}{\pi(p)^2}$ and $\underline{p}$ solves $\pi(\underline{p}) = (1 - \alpha)\pi_M$, there is an equilibrium where all non-shoppers visit firm A. In this equilibrium, firm A's per product profit is $(1 - \alpha)\pi_M$ and each single-product firm's profit is $\alpha(1 - \alpha)\pi_M$.

(ii) *If (9) does not hold, there is an equilibrium where $X \in (\frac{1}{2}, 1)$ fraction of non-shoppers visit firm A. In this equilibrium, firm A's per product profit is $(1 - \alpha)X\pi_M$, and each single-product firm's profit is $[\frac{1}{2}(1 - \alpha)(1 - X) + \alpha\lambda]\pi_M$, where $\lambda = \frac{(1-\alpha)(3X-1)/2}{\alpha+(1-\alpha)X}$ is the size of the mass point on p_M in firm A's price distribution.*

In the unit demand case, $v(p_M) = 0$ and so condition (9) degenerates to $\alpha \geq \frac{1}{2}$. This is consistent with Lemma 4. With an elastic demand, $v(p_M) > 0$ and so a sufficient condition for (9) is $\alpha \geq \frac{1}{2}$. In particular, in the case with a linear demand $Q(p) = 1 - p$, we can show that (9) holds for any α . But of course this cannot be true for any elastic demand function given our result in the unit demand case.

Although the analysis of the asymmetric market structure is more complicated, the result about the equilibrium market structure is the same as in the baseline model.

Proposition 2 *For any elastic demand function satisfying Assumption 1, if $\alpha \geq \frac{1}{2}$ the unique (pure-strategy) equilibrium outcome is that the market has a multiproduct firm and two single-product firms, and if $\alpha < \frac{1}{2}$ the unique equilibrium outcome is that the market has two multiproduct firms.*

For a general demand function $Q(p)$, it is hard to compare welfare. Therefore we consider an example with a linear demand curve $Q(p) = 1 - p$. Then $v(p) = \frac{1}{2}(1 - p)^2$, $p_M = \frac{1}{2}$ and $\pi_M = \frac{1}{4}$. In this case, as we already claimed all non-shoppers visit the multiproduct firm in the asymmetric market structure. Industry profit is maximized in the asymmetric market structure and minimized with four single-product firms. Figure 1(b) above depicts how industry profit (the thick solid lines), consumer surplus (the dashed lines), and total welfare (the thin solid lines) vary with α . This is similar to the unit demand case, except that now total welfare is no longer constant and it also has a jump-down at $\alpha = 0.5$.

3.2 Coexistence of single-product and multiproduct consumers

This section extends the basic model by considering both single-product and multiproduct consumers. Suppose each product is needed by a consumer with probability $\gamma > 0$. The purchase interest is independent across products, and also independent of whether the consumer is a shopper or non-shopper. Then a fraction γ^2 of consumers want to buy both products, a fraction $\gamma(1 - \gamma)$ of them need product 1 only, a fraction $\gamma(1 - \gamma)$ need product 2 only, and the rest of consumers buy nothing. The baseline model in section 2 corresponds to the case with $\gamma = 1$.

The logic of how to analyze this extended model is similar. The main results are reported here, and all the details are relegated to the appendix. When γ is sufficiently high, the results are qualitatively similar to those in the baseline model with $\gamma = 1$.

Proposition 3 *Suppose each product is needed by a consumer with probability $\gamma > 0$.*

(i) *When $\gamma > \frac{1}{2}$ and $\alpha \geq \frac{\gamma}{1+\gamma}$, the unique (pure-strategy) equilibrium outcome is that the market has a multiproduct firm and two single-product firms. All non-shoppers who want both products visit the multiproduct firm while all non-shoppers who only need one product visit a relevant single-product firm. Compared to the initial situation, this leads to higher industry profit and total welfare but lower consumer surplus.*

(ii) *When $\gamma > \frac{1}{2}$ and $\alpha < \frac{\gamma}{1+\gamma}$, the unique equilibrium outcome is that the market has two multiproduct firms. All non-shoppers randomize their choice of where to shop. Compared to the initial situation, this leads to higher industry profit and total welfare, and consumer surplus remains unchanged.*

(iii) *When $\gamma \leq \frac{1}{2}$, any market structure except for the case with four single-product firms can emerge as an equilibrium outcome, and they lead to the same welfare outcome.*

(The asymmetric market structure is the only outcome if a conglomerate merger involves any cost.) All non-shoppers who need both products visit the multiproduct firm, but a fraction of the non-shoppers who need one product only also visit the multiproduct firm. Compared to the initial situation, this leads to higher industry profit and total welfare, and consumer surplus remains unchanged.

3.3 A model with differentiated products

In the baseline model with homogenous products, for tractability we have assumed two extreme types of consumer search behavior: one type of consumers can visit all firms costlessly, while the other can visit one firm only. In this section, we provide an alternative model where consumers can choose how many firms to visit but visiting each firm is costly. As we pointed out earlier, however, this creates some complications when firms sell homogeneous products and there are two multiproduct firms in the market. In order to have a relatively tractable model, we therefore introduce horizontal product differentiation. We argue that the basic insight we learned in the baseline model about how the size of search friction affects the market structure will carry over in this alternative model.

Consider a two-stage game as before, starting with four independent single-product firms. Suppose now firms i_A and i_B , $i = 1, 2$, supply two horizontally differentiated product i . We use the random utility framework to model product differentiation. The match utility of each product i is a random draw from a common distribution $F_i(u)$ with support $[\underline{u}, \bar{u}]$ and a density $f_i(u)$. The realization of the match utility is i.i.d. across consumers and firms. (This reflects for example consumers' idiosyncratic tastes.) As in the baseline model, we assume the two products are symmetric (i.e., $F_1 = F_2 = F$). Initially consumers have imperfect information about each product's match utility and price, though they know the match utility distribution $F(u)$ and also hold rational expectations about each firm's pricing strategy. To reach a firm and learn its product match utility and price, a consumer has to incur a search cost s . The search process is sequential. To capture the idea of one-stop shopping convenience, we assume the search cost is the same no matter whether a consumer visits a single-product or a multiproduct firm. To have an active search market in each possible scenario, the search cost cannot

be too high. More precisely, this requires

$$s < \int_{\underline{u}}^{\bar{u}} (u - \underline{u}) dF(u) . \quad (10)$$

(This condition guarantees active search even in the single-product case.) For simplicity, we make two more assumptions which are often adopted in the literature: (i) Consumers have free recall. That is, they can return to a previously visited firm without paying any extra cost. (ii) The market is fully covered. That is, all consumers buy both products in equilibrium. With product differentiation there is pure-strategy pricing equilibrium in each possible market structure. In the following, we report the main results but relegate the details to the appendix.

With four single-product firms, a consumer's search process is separate across the two product markets. In each market, we have a duopoly version of the sequential search model in Anderson and Renault (1999). With two multiproduct firms, we have a multiproduct search model as analyzed in Zhou (2014). In either case, we can derive a symmetric equilibrium where both firms adopt the same pricing strategy and consumers search in a random order. Let p_0 be each product's price in the case with four single-product firms, and let p_2 be each product's price in the case with two multiproduct firms. There is a qualitative difference compared to the basic model with homogenous products. In the basic model, each product's price is higher in the case with two multiproduct firms than in the case with four single-product firms. (This is because in the former case each multiproduct firm has half of non-shoppers while in the latter case each single-product firm has only one quarter of non-shoppers. This is due to the assumption that a non-shopper can visit one firm only even if they are all single-product firms.) In the current model the opposite is true. That is, each product's price is lower in the case with two multiproduct firms (i.e., $p_0 > p_2$). This is due to a joint-search effect in a multiproduct search model: when a firm reduces the price of one product, more consumers who visit it first will stop searching and buy both products. That is, reducing one product's price can increase the demand for the other product as well. This makes the two products in each firm like complements even if they are physically independent. This joint-search effect intensifies price competition. In the baseline model, no consumers have a real search decision after they arrive a firm, so this joint-search effect does not appear.

In the asymmetric case with a multiproduct firm A and two single-product firms 1_B and 2_B , we show that there is an equilibrium where the multiproduct firm sets

lower prices for each product than its single-product rivals, and all consumers visit the multiproduct firm first. The consumer search order is optimal not only because of the convenience of one-stop shopping but also because of the lower prices offered by the multiproduct firm. Let p_A be the multiproduct firm's price and p_B be each single-product firm's price. The prediction that $p_A < p_B$ is also opposite to what we observed in the baseline model with homogenous products. But both p_A and p_B tend to be higher than p_0 and p_2 as we will see below. This is similar to the baseline model.

For a general match utility distribution, it is hard to compare the outcome across regimes and to study the equilibrium market structure. For this reason, we rely on the uniform distribution case with $F(u) = u$. In that case, the search cost condition (10) requires $s < \frac{1}{2}$. Figure 2(a) below compares prices across three possible market structures, where the dashed curve is p_0 , the top two solid curves are p_B and p_A respectively, and the lowest solid curve is p_2 . As in the baseline model, the asymmetric market structure generates the highest market prices.

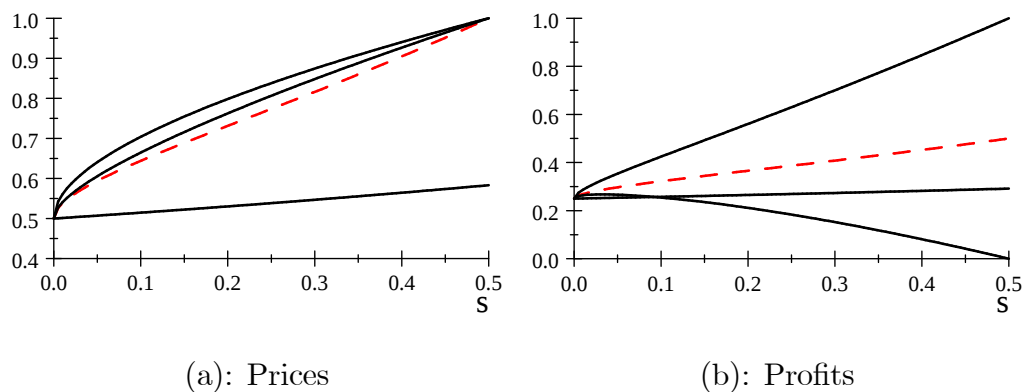


Figure 2: Price and profit comparison with differentiated products

Figure 2(b) compares per product profit across regimes. The dashed curve is π_0 (each firm's profit in the case with four single-product firms), the middle (almost horizontal) solid curve is π_2 (each firm's per product profit in the case with two multiproduct firms), and the other high and low curves are respectively π_A and π_B (the multiproduct firm's per product profit and each single-product firm's profit in the asymmetric case). A few observations follow: (i) $\pi_A > \pi_0$, so starting from the initial situation with four single-product firms, each pair of firms have a unilateral incentive to merge. (ii) $\pi_B > \pi_2$ if and only if s is less than about 0.092. We can then deduce that if s is less

than 0.092, the (pure-strategy) equilibrium outcome is an asymmetric market with a multiproduct firm and two single-product firms. (Numerical simulations also show that industry profit is higher in this case than in the initial situation with four single-product firms.) While if s is greater than 0.092, the equilibrium outcome is two multiproduct firms. In this case, industry profit is lower than in the initial situation and so firms end up in a prisoner's dilemma. Therefore, in this uniform distribution example we have a qualitatively similar result concerning how the size of search friction affects market structure as in the baseline model with homogeneous products. It again reflects the trade-off between the search order effect and the price competition effect. When there is already a multiproduct retailer in the market, remaining as two single-product firms suffers from consumers' non-random search order but it can sustain relatively high prices; in contrast, merger can restore consumers' random search order but it also causes a fiercer price competition.

In this uniform distribution example, one can also investigate how the magnitude of the search cost affects consumer surplus and total welfare. The outcome is qualitatively similar as in Figure 1(b): due to the endogeneity of the market structure, reducing the search cost does not always improve consumers surplus and total welfare.

4 Conclusion

The paper has analyzed equilibrium market structure, within a model of consumer search where firms are able to engage in conglomerate mergers. The model permits a simple and tractable analysis of multiproduct competition, which we believe will be useful in other applications. Amongst other results, we have shown that the size of the search friction plays a key role in determining whether the equilibrium market structure is symmetric or not. In particular when search frictions are relatively low, some single-product firms choose to remain unmerged, in order to weaken the amount of price competition in the market. In equilibrium, larger retailers earn more profit per-product, and thus benefit from their greater size. This is due to their ability to offer one-stop shopping convenience, which expands their pool of potential customers. Of course the model neglects an important practical consideration - namely that conglomerate mergers may be costly to propose, but could also generate synergies and therefore long-term cost savings. Nevertheless introducing this into the model would not change the main qualitative insights.

Our main results extend straightforwardly to richer environments. We demonstrated this for the case where consumers have arbitrary elastic demands, and where products are differentiated and consumers engage in explicit sequential search. We are also able to solve the model for the case where there are initially an arbitrary number of single-product firms. Although the pricing solution is now more complicated, the search order and price competition effects still play a prominent role. Finally, there are two other promising avenues that the model could address. The first is to think about market structure not as the result of conglomerate mergers, but as the result of firms optimally choosing the size and contents of their product line. The second avenue would be to consider other effects of conglomerate mergers, especially their ability to deter entry. We intend to think more about these possibilities in future work.

Appendix: Proofs and Omitted Details

Proof of Lemma 5: (i) We first consider the possible equilibrium where all non-shoppers visit the multiproduct firm A (i.e., $N_A = 1 - \alpha$ and $N_B = 0$) and F_A has a mass point at p_M with size λ . The two density functions f_A and f_B satisfies the same relationship

$$f_A(p) = (1 - \lambda)f_B(p)$$

as in (7) holds, where

$$f_B(p) = \frac{(1 - \alpha)\pi_M}{\alpha} \frac{\pi'(p)}{\pi(p)^2} .$$

Notice that

$$\lambda = \frac{N_A - N_B}{N_A + S} = 1 - \alpha .$$

A non-shopper's expected surplus from visiting firm A is

$$2\lambda v(p_M) + 2 \int_{\underline{p}}^{p_M} v(p) f_A(p) dp ,$$

where the first term is because F_A has a mass point on p_M . (In the baseline model with unit demand, $v(p_M) = 0$, so the first term vanishes.) While a non-shopper's expected surplus from visiting a single-product firm is

$$\int_{\underline{p}}^{p_M} v(p) f_B(p) dp .$$

The equilibrium with all non-shoppers visiting firm A exists if the former surplus is greater, or if

$$2(1 - \alpha)v(p_M) + (2\alpha - 1) \int_{\underline{p}}^{p_M} v(p)f_B(p)dp \geq 0 .$$

This is condition (9). It is clearly the case if $\alpha \geq \frac{1}{2}$, and given $v(p_M) > 0$ it also holds for some $\alpha < \frac{1}{2}$.⁶ In this equilibrium, firm A 's per product profit is $N_A\pi_M = (1 - \alpha)\pi_M$ and each single-product firm's profit is $(N_B + S\lambda)\pi_M = \alpha(1 - \alpha)\pi_M$.

(ii) If (9) does not hold, we show that there exists an equilibrium where $X \in (0, 1)$ fraction of non-shoppers visit the multiproduct firm A in equilibrium (such that $N_A = (1 - \alpha)X$ and $N_B = \frac{1}{2}(1 - \alpha)(1 - X)$). This type of equilibrium can exist only if firm A charges higher prices, i.e., if $N_A > N_B$ or equivalently $X > \frac{1}{3}$. Using $f_A = (1 - \lambda)f_B$, we know that a non-shopper is indifferent between visiting firm A and a single-product firm if

$$2\lambda v(p_M) + (1 - 2\lambda) \int_{\underline{p}}^{p_M} v(p)f_B(p)dp = 0 , \quad (11)$$

where

$$\lambda = \frac{N_A - N_B}{N_A + S} = \frac{(1 - \alpha)(3X - 1)/2}{\alpha + (1 - \alpha)X} \quad (12)$$

and

$$f_B(p) = \frac{(1 - \alpha)X\pi_M}{\alpha} \frac{\pi'(p)}{\pi(p)^2} .$$

(When $\alpha \geq \frac{1}{2}$, one can check that the left-hand side of (11) must be positive, and so there is no equilibrium where only a fraction of non-shoppers visit firm A . So this type of equilibrium can exist only if $\alpha < \frac{1}{2}$.) When $X = 1$, $\lambda = 1 - \alpha$ and the left-hand side of (11) equals the left-hand side of (9), so it must be negative when (9) does not hold. On the other hand, when $X = \frac{1}{3}$, $\lambda = 0$ and the left-hand side of (11) must be positive. Hence, when (9) does not hold, (11) must have a solution $X \in (\frac{1}{3}, 1)$. This proves the existence.

In this type of equilibrium, given $v(p_M) > 0$, (11) implies $\lambda > \frac{1}{2}$, or equivalently

$$X > \frac{1}{2(1 - \alpha)} > \frac{1}{2} . \quad (13)$$

⁶Notice also that (9) holds for α close to zero. Notice $\underline{p} \rightarrow p_M$ as $\alpha \rightarrow 0$. So in the limiting case all firms charge the monopoly price p_M . Then it is optimal for a non-shopper to visit the multiproduct firm A . (In the unit demand case, the surplus is zero at the monopoly price, so this is trivially the case.)

Firm A 's per product profit is $N_A \pi_M = (1 - \alpha)X \pi_M$, and each single-product firm's profit is $(N_B + S\lambda) \pi_M = [\frac{1}{2}(1 - \alpha)(1 - X) + \alpha\lambda] \pi_M$.

Proof of Proposition 2: When $\alpha \geq \frac{1}{2}$, all non-shoppers visit the multiproduct firm A in the asymmetric case. Then the payoff matrix in the first stage is:

	Merge	Not
Merge	$\frac{1}{2}(1 - \alpha)\pi_M, \frac{1}{2}(1 - \alpha)\pi_M$	$(1 - \alpha)\pi_M, \alpha(1 - \alpha)\pi_M$
Not	$\alpha(1 - \alpha)\pi_M, (1 - \alpha)\pi_M$	$\frac{1}{4}(1 - \alpha)\pi_M, \frac{1}{4}(1 - \alpha)\pi_M$

The only (pure-strategy) equilibrium outcome is that one pair of firms merge but the other do not.

When $\alpha < \frac{1}{2}$, there are two possibilities. One possibility is that the same payoff matrix is relevant (e.g., in the linear demand case), in which case the only equilibrium is that both pairs of firms merge. Another possibility is that only part of non-shoppers visit the multiproduct firm A in the asymmetric market. In that case, the relevant payoff matrix in the first stage is:

	Merge	Not
Merge	$\frac{1}{2}(1 - \alpha)\pi_M, \frac{1}{2}(1 - \alpha)\pi_M$	$(1 - \alpha)X\pi_M, [\frac{1}{2}(1 - \alpha)(1 - X) + \alpha\lambda]\pi_M$
Not	$[\frac{1}{2}(1 - \alpha)(1 - X) + \alpha\lambda]\pi_M, (1 - \alpha)X\pi_M$	$\frac{1}{4}(1 - \alpha)\pi_M, \frac{1}{4}(1 - \alpha)\pi_M$

Here $X > \frac{1}{2}$ as shown in (13), and λ is given in (12). Using $\alpha < \frac{1}{2}$ and (12) one can show that⁷

$$\frac{1}{2}(1 - \alpha) > \frac{1}{2}(1 - \alpha)(1 - X) + \alpha\lambda.$$

In the same time $(1 - \alpha)X > \frac{1}{4}(1 - \alpha)$ given $X > \frac{1}{2}$. Therefore, the only equilibrium is that both pairs of firms merge and two multiproduct firms prevail in the market.

The details of the model with both single-product and multiproduct consumers. In this extended model, the number of non-shoppers who want both products is

$$N_b = (1 - \alpha)\gamma^2,$$

⁷This is equivalent to

$$\frac{1}{2}(1 - \alpha)X > \alpha\lambda \Leftrightarrow \frac{1 - \alpha}{\alpha}X^2 - 2X + 1 > 0.$$

This must be the case given $\frac{1 - \alpha}{\alpha} > 1$ and $X \neq 1$.

and the number of non-shoppers who want only product i is

$$N_i = (1 - \alpha)(1 - \gamma)\gamma .$$

(In the following, we also call the two types of non-shoppers as N_b and N_i , respectively.)

Both the case with four single-product firms and the case with two multiproduct firms are straightforward to analyze. We omit the details of the price distributions but only give the relevant welfare variables. In the former case, N_b visit a firm randomly and N_i visit a relevant firm randomly. So for each firm, at the product level the number of non-shoppers it has is

$$\frac{1}{4}N_b + \frac{1}{2}N_i = \frac{1}{4}(1 - \alpha)\gamma(2 - \gamma) ,$$

and the number of relevant shoppers is $\alpha\gamma$. Then each firm's profit is $\frac{1}{4}(1 - \alpha)(2 - \gamma)\gamma v$, and industry profit is $(1 - \alpha)(2 - \gamma)\gamma v$. Each shopper who wants both products generates surplus $2v$, and each of the other active consumers generates surplus v . So total welfare is $(2 - (1 - \alpha)\gamma)\gamma v$, and consumer surplus is $2\alpha\gamma v$.

In the case with two multiproduct firms, each non-shopper visits a firm randomly. So for each firm, at the product level the number of non-shoppers is

$$\frac{1}{2}N_b + \frac{1}{2}N_i = \frac{1}{2}(1 - \alpha)\gamma ,$$

and the number of relevant shoppers is $\alpha\gamma$. Then each multiproduct firm's per product profit is $\frac{1}{2}(1 - \alpha)\gamma v$, and so industry profit is $2(1 - \alpha)\gamma v$. Both non-shoppers and shoppers can buy the product(s) they want, so total welfare is $\gamma^2 2v + 2\gamma(1 - \gamma)v = 2\gamma v$, and consumer surplus is $2\alpha\gamma v$.

Now consider the asymmetric case with a multiproduct firm A and two single-product firms 1_B and 2_B . There are three cases to discuss.

Case (i): all N_b visit firm A and all N_i visit firm i_B , $i = 1, 2$. Let F_A be the price distribution of firm A 's each product, and let F_B be the price distribution of each single-product firm. We look for an equilibrium where F_A has a mass point at v with size λ , while F_B has no mass point.⁸ The result (iii) in Lemma 1 applies with $N_A = N_b$, $N_B = N_i$, and $S = \alpha\gamma$. From (5) we have

$$\lambda = \frac{N_A - N_B}{N_A + S} = \frac{(1 - \alpha)(2\gamma - 1)}{\alpha + (1 - \alpha)\gamma} .$$

⁸It is impossible that both F_A and F_B have a mass point at v , and it can also be shown that there is no equilibrium where F_B has a mass point at v but F_A does not.

This is positive only if $\gamma > \frac{1}{2}$.⁹ The relationship between the two density functions is as usual: $f_A = (1 - \lambda)f_B$. N_i 's search behavior is obviously justified given firm A are charging higher prices. According to (8), N_b 's search behavior is justified if

$$\lambda \leq \frac{1}{2} \Leftrightarrow \alpha \geq \frac{3\gamma - 2}{3\gamma - 1}.$$

Therefore, we can conclude that the type of equilibrium we are looking for exists if $\gamma > \frac{1}{2}$ and $\alpha \geq \frac{3\gamma - 2}{3\gamma - 1}$. (In particular, when $\gamma \in (\frac{1}{2}, \frac{2}{3}]$ the condition for α is always satisfied.) In this equilibrium, firm A 's per product profit is $N_A v = (1 - \alpha)\gamma^2 v$ and each single-product firm's profit is $(N_B + S\lambda)v = (1 - \alpha)\frac{1 - (1 - \alpha)\gamma}{\alpha + (1 - \alpha)\gamma}\gamma^2 v$. So industry profit is $\frac{2(1 - \alpha^2)}{\alpha + (1 - \alpha)\gamma}\gamma^2 v$. Total welfare is $\gamma^2 2v + 2\gamma(1 - \gamma)v = 2\gamma v$, and so consumer surplus is $2\alpha\frac{1 - (1 - \alpha)\gamma}{\alpha + (1 - \alpha)\gamma}\gamma v$.

Case (ii): some N_b visit firm A and all N_i visit firm i_B . We look for an equilibrium where a fraction $X \in (0, 1)$ of N_b visit the multiproduct firm A , a fraction $\frac{1}{2}(1 - X)$ of N_b go to each single-product firm, and all N_i go to firm i_B . Suppose F_A has a mass point at v with mass λ .¹⁰ The result (iii) in Lemma 1 applies again with $N_A = XN_b$, $N_B = N_i + \frac{1}{2}(1 - X)N_b$, and $S = \alpha\gamma$. From (5) we have

$$\lambda = \frac{N_A - N_B}{N_A + S} = \frac{(1 - \alpha)\left(\frac{1 + 3X}{2}\gamma - 1\right)}{\alpha + (1 - \alpha)\gamma X}.$$

According to (8), the existence of this equilibrium requires

$$\lambda = \frac{1}{2} \Leftrightarrow X = \frac{1 + (1 - \alpha)(1 - \gamma)}{2(1 - \alpha)\gamma}.$$

To have $X < 1$, we need $\alpha < \frac{3\gamma - 2}{3\gamma - 1}$. This condition is non-empty only if $\gamma > \frac{2}{3}$, which is consistent with the condition derived in case (i). Given $\lambda = \frac{1}{2}$, we have $f_A = \frac{1}{2}f_B$, so N_i 's search behavior is justified.

Therefore, we can conclude that the type of equilibrium we are looking for exists if $\gamma > \frac{2}{3}$ and $\alpha < \frac{3\gamma - 2}{3\gamma - 1}$. In this equilibrium, firm A 's per product profit is $N_A v = \frac{1}{2}[1 + (1 - \alpha)(1 - \gamma)]\gamma v$, and each single-product firm's profit is $(N_B + S\lambda)v = \frac{1}{4}[1 + (1 - \alpha)(1 - \gamma)]\gamma v$. Then industry profit is $\frac{3}{2}[1 + (1 - \alpha)(1 - \gamma)]\gamma v$. One can check that total welfare is $\frac{1}{2}[5 + (1 - \alpha)(1 - 3\gamma)]\gamma v$, and then consumer surplus is $\alpha\gamma v$.

⁹This is intuitive. Considering any one product, the multiproduct firm A attracts γ^2 non-shoppers whilst the independent store attracts $\gamma(1 - \gamma)$ of them. In order for firm A to have a mass point, it must attract more non-shoppers, which happens if and only if $\gamma \geq 1/2$.

¹⁰It can be shown that there is no such an equilibrium where both F_A and F_B have a mass point on v , or only F_B has a mass point on v .

Case (iii): all N_b visit firm A and some N_i also visit firm A . The remaining case is $\gamma \leq \frac{1}{2}$. For N_i to randomize their choices, we need

$$\int_{\underline{p}}^v (v - p)f_A(p)dp = \int_{\underline{p}}^v (v - p)f_B(p)dp.$$

If F_A has a mass point at v with mass λ , then $f_A = (1 - \lambda)f_B$ and the equality can never hold. Therefore we look for an equilibrium where $F_A = F_B$ and both are atomless. Let Y be the probability that an N_i visits firm A . The result (iii) in Lemma 1 applies with $N_A = N_b + YN_i$, $N_B = (1 - Y)N_i$, and $S = \alpha\gamma$. The requirement $\lambda = 0$ and (5) imply

$$N_A = N_B \Leftrightarrow Y = \frac{1 - 2\gamma}{2 - 2\gamma}.$$

Y is positive only if $\gamma \leq \frac{1}{2}$. Given $F_A = F_B$, all non-shoppers' search behavior can be justified. In this equilibrium, firm A 's per product profit is $N_A v = \frac{1}{2}(1 - \alpha)\gamma v$, and each single-product firm's profit is the same. Industry profit is then $2(1 - \alpha)\gamma v$. Total welfare is $\gamma^2 2v + 2\gamma(1 - \gamma)v = 2\gamma v$, and consumer surplus is $2\alpha\gamma v$. The outcome is exactly the same as in the case with two multiproduct firms.

Given all the welfare variables derived above, it is straightforward to verify the results stated in Proposition 3.

The details of the model with product differentiation. Consider first the simplest case with four single-product firms. A consumer's search process is then totally separate across the two product markets. In each market, we have a duopoly version of the sequential search model in Anderson and Renault (1999). Consider the market for product i . We aim to derive a symmetric equilibrium where both firms charge the same price p_0 and consumers search in a random order. In the symmetric equilibrium, the optimal stopping rule is characterized by a reservation utility level a which solves

$$\int_a^{\bar{u}} (u - a)dF(u) = s. \quad (14)$$

(The left-hand side is the expected benefit from sampling the second firm when the first firm offers match utility a .) This equation has a unique solution $a \in (0, \bar{u})$ given the search cost condition (10). A consumer will buy immediately at the first visited firm if and only if its match utility is no less than a .

To derive the symmetric equilibrium price, suppose now firm i_A unilaterally deviates and charges a price p'_0 . For the consumers who visit it first, they will stop searching

and buy immediately if $u_A - p'_0 \geq a - p_0$. This generates demand $\frac{1}{2}[1 - F(a - p_0 + p'_0)]$, where $\frac{1}{2}$ is because half of the consumers visit firm i_A first. For those who continue to visit firm i_B , they will return and buy from firm i_A if $u_A - p'_0 > u_B - p_0$. This generates demand

$$\frac{1}{2} \Pr[u_B - p_0 < u_A - p'_0 < a - p_0] = \frac{1}{2} \int_{\underline{u}}^{a - p_0 + p'_0} F(u_A - p'_0 + p_0) dF(u_A) .$$

Another demand source is the consumers who visit firm i_B first. Since those consumers hold equilibrium belief about firm i_A 's price, they will come to visit if $u_B < a$. They will then buy at firm i_A if $u_A - p'_0 > u_B - p_0$. This generates demand

$$\frac{1}{2} \int_{\underline{u}}^a [1 - F(u_B - p_0 + p'_0)] dF(u_B) .$$

In equilibrium, each firm has half of the consumers due to symmetry, and from the above three demand components one can also check that the equilibrium demand slope is the negative of $\frac{1}{2}f(a)[1 - F(a)] + \int_{\underline{u}}^a f(u)^2 du$. Then the first-order condition for p_0 to be the equilibrium price is¹¹

$$\frac{1}{p_0} = f(a)[1 - F(a)] + 2 \int_{\underline{u}}^a f(u)^2 du .$$

In the uniform distribution example with $F(u) = u$, we have $a = 1 - \sqrt{2s}$ and the search cost condition (10) requires $s < \frac{1}{2}$. The first-order condition implies

$$p_0 = \frac{1}{2 - \sqrt{2s}} . \quad (15)$$

In equilibrium, firms share the market equally, so each firm's profit is $\pi_0 = \frac{1}{2}p_0$.

With two multiproduct firms A and B , we have a multiproduct search model as analyzed in Zhou (2014). After a consumer visits the first firm, she faces the following options: stop searching and buy both products, or buy one product and keep searching for the other, or keep searching for both products. However, given the search cost occurs at the firm level and the consumer has free recall, the second option is always dominated by the third one. If she chooses to visit the second firm, she can thereafter freely mix and match among the two firms. Consider a consumer who visits firm A

¹¹In this type of search model with product differentiation, it is straightforward to investigate when the first-order condition is also sufficient for defining the equilibrium price. In this case with single-product firms, a sufficient condition is $p[1 - F(p)]$ being concave.

first. In a symmetric equilibrium where both firms charge the same prices, she will stop searching and buy both products at firm A if the match utilities (u_{1A}, u_{2A}) satisfy

$$\int_{u_{1A}}^{\bar{u}} (u_{1B} - u_{1A}) dF(u_{1B}) + \int_{u_{2A}}^{\bar{u}} (u_{2B} - u_{2A}) dF(u_{2B}) \leq s .$$

(The left-hand side is the expected benefit from sampling the second firm B .) The equality of this condition defines a reservation frontier $u_{2A} = \phi(u_{1A})$ where $\phi(\cdot)$ is a decreasing and convex function. If the match utilities (u_{1A}, u_{2A}) at firm A are such that $u_{2A} \geq \phi(u_{1A})$, the consumer will stop searching. Otherwise, she will continue to visit firm B . The analysis of how to derive the symmetric equilibrium prices is more involved than in the single-product search model. We refer the details to Zhou (2014). For a general distribution, the equilibrium price p_2 is given by

$$\frac{1}{p_2} = 2 \int_{u_2 \leq \phi(u_1)} f(u_1)^2 f(u_2) d\mathbf{u} + \int_a^{\bar{u}} [1 - F(\phi(u))] f(\phi(u)) dF(u) + \int_a^{\bar{u}} [1 - F(u)] f(\phi(u)) dF(u) ,$$

where a is the reservation utility defined in (14). In general, due to the joint-search effect (i.e., reducing one product's price also increases the demand for the other product), p_2 is lower than p_0 , the price in the case with four single-product firms. In the uniform distribution example, we have

$$p_2 = \frac{1}{2 - (\frac{1}{2}\pi - 1)s} , \quad (16)$$

where $\pi \approx 3.14$ is the mathematical constant. In equilibrium, firms share the market equally and each firm's per product profit is $\pi_2 = \frac{1}{2}p_2$.

Finally, consider the asymmetric market with a multiproduct firm A and two single-product firms 1_B and 2_B . We show below that there is an equilibrium where the multiproduct firm charges p_A for each product, the two single-product firms charge price $p_B > p_A$, and all consumers visit the multiproduct store first. Notice that the cost of visiting each single-product firm is separable. Then a consumer's search decision when she is at the multiproduct firm is also separable between the two products: she will continue to visit the single-product firm i_B only if the multiproduct firm's product i has a surplus less than $a - p_B$, where a is defined in (14). Hence, the multiproduct firm is competing with its smaller rivals in two separate single-product markets where consumers search non-randomly. (In this case, there is actually no the joint-search effect. The non-random search in each market is similar as in Armstrong, Vickers, and

Zhou, 2009.¹²)

Consider the market for product i . The demand for the multiproduct firm's product, if it charges p'_A while its rival sets the equilibrium price p_B , is

$$[1 - F(a - p_B + p'_A)] + \int_{\underline{u}}^{a - p_B + p'_A} F(u - p'_A + p_B) dF(u) . \quad (17)$$

(All consumers visit firm A first. For those who find $u_A - p'_A \geq a - p_B$, they will buy immediately at firm A . This generates the first term. For those who find $u_A - p'_A < a - p_B$, they will continue to sample firm i_B but will return to buy from firm A if $u_B - p_B < u_A - p'_A$. This generates the second term.) The demand for firm i_B 's product, if it charges price p'_B while firm A sticks to its equilibrium price p_A , is

$$\int_{\underline{u}}^{a - p_B + p_A} [1 - F(u - p_A + p'_B)] dF(u) . \quad (18)$$

(All consumers visit firm A first. They hold equilibrium belief about firm i_B 's price. So if they find $u_A - p_A < a - p_B$, they will come to visit firm i_B . They will then buy from it if $u_B - p'_B > u_A - p_A$.)

Define $\Delta \equiv p_B - p_A$, and

$$Q(\Delta) \equiv 1 - \int_{\underline{u}}^{a - \Delta} [1 - F(u + \Delta)] dF(u) .$$

Then $Q(\Delta)$ is the equilibrium demand for firm A (i.e., (17) evaluated at $p'_A = p_A$), and $1 - Q(\Delta)$ is the equilibrium demand for firm i_B (i.e., (18) evaluated at $p'_B = p_B$). They only depend on the price difference Δ , given the assumption of full market coverage. One can then check that the first-order conditions for the equilibrium prices (p_A, p_B) are¹³

$$p_A = \frac{Q(\Delta)}{Q'(\Delta)} \quad (19)$$

and

$$p_B = \frac{1 - Q(\Delta)}{Q'(\Delta) - [1 - F(a)]f(a - \Delta)} . \quad (20)$$

Notice that the analysis so far is predicated on that all consumers visit firm A first and a technical condition $a - \Delta > \underline{u}$. The following claim gives a sufficient condition for the

¹²In Armstrong, Vickers, and Zhou (2009), the assumption of full market coverage is not imposed, but they focus on the uniform distribution case.

¹³If $p[1 - F(p)]$ is concave, the first-order conditions are also sufficient for defining the equilibrium prices.

system of (19) and (20) to have a solution $\Delta \in (0, a - \underline{u})$. (With $\Delta > 0$, the consumer search order is then justified.)

Claim 1 *Suppose $1 - F$ is strictly logconcave and the search cost condition (10) holds. Then the system of (19) and (20) has a solution $\Delta \in (0, a - \underline{u})$.*

Proof. From (19) and (20), we have

$$\Delta = \frac{1 - Q(\Delta)}{Q'(\Delta) - f(a - \Delta)[1 - F(a)]} - \frac{Q(\Delta)}{Q'(\Delta)} \equiv \Phi(\Delta) .$$

From the expression for $Q(\Delta)$, we have

$$Q'(\Delta) = [1 - F(a)]f(a - \Delta) + \int_{\underline{u}}^{a - \Delta} f(u + \Delta)dF(u) .$$

Then it is ready to check that $\Phi(0) > 0$ if

$$\frac{1}{F(a) - \frac{1}{2}F(a)^2} - \frac{f(a)[1 - F(a)]}{\int_{\underline{u}}^a f(u)dF(u)} < 2 .$$

This condition holds because

$$\int_{\underline{u}}^a f(u)dF(u) = \int_{\underline{u}}^a \frac{f(u)}{1 - F(u)}[1 - F(u)]dF(u) < \frac{f(a)}{1 - F(a)}[F(a) - \frac{1}{2}F(a)^2] .$$

(The inequality used the logconcavity of $1 - F$.)

On the other hand, using L'Hôpital's rule one can check that

$$\Phi(a - \underline{u}) = \frac{1 - F(a)}{f(a)} - \frac{1}{f(\underline{u})[1 - F(a)]} < \frac{1}{f(\underline{u})}\left[1 - \frac{1}{1 - F(a)}\right] < 0 < a - \underline{u} .$$

(The first inequality used the logconcavity of $1 - F$.) Therefore, $\Phi(\Delta) = \Delta$ has a solution between 0 and $a - \underline{u}$. ■

A consumer will visit a single-product firm only if she is unsatisfied with the multi-product firm's product. (In other words, her search order reveals information about her preferences.) This gives the single-product firm an extra monopoly power and induces it to charge a higher price.¹⁴ In the uniform distribution example, the system of (19) and (20) simplifies to

$$p_A = \frac{1}{1 - \Delta}[1 - a + \Delta + \frac{1}{2}(a^2 - \Delta^2)], \quad p_B = 1 - \frac{1}{2}(a - \Delta) ,$$

¹⁴Following the information revelation argument, one may conjecture another possible equilibrium in which consumers visit the two single-product firms first and they charge lower prices than the larger multiproduct firm. (The price difference should be large enough to compensate the extra search cost involved in visiting single-product firms first.) It is analytically difficult to exclude this possibility in general, but in the uniform distribution example this can be ruled out.

where $a = 1 - \sqrt{2s}$. The system has a unique solution:

$$p_A = \frac{3}{16}\sqrt{17a^2 - 30a + 49} - \frac{5}{16}(1 + a) ,$$

and

$$p_B = \frac{1}{16}\sqrt{17a^2 - 30a + 49} + \frac{1}{16}(9 - 7a) .$$

(It can be verified that $p_A < p_B$ for $a \in (0, 1)$. When $a = 0$ or 1 (i.e., when $s = \frac{1}{2}$ or 0), the two prices coincide with each other.) Given the prices, we can calculate each firm's (per product) profit $\pi_A = p_A Q(\Delta)$ and $\pi_B = p_B(1 - Q(\Delta))$.

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