Optimal Time-Consistent Macroprudential Policy*

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September, 2014

Abstract

The collateral constraints driving the amplification mechanism of a large class of models of financial crises feature a pecuniary externality: Agents making current borrowing decisions do not internalize a future collapse of collateral prices in the event of a crisis. As a result, agents in a competitive equilibrium borrow "too much" during credit expansions compared with a financial regulator who internalizes this externality. Under commitment, however, the credibility of the regulator is called into question because of a time-inconsistency problem: It promises low future consumption to prop up current asset prices when collateral constraints bind, but this is not optimal ex post. Instead, we study the optimal, time-consistent policy of a regulator who cannot commit to future policies. Quantitative analysis shows that this policy reduces the incidence and magnitude of crises, removes fat tails from the distribution of returns and reduces risk premia. The main element of this policy is a state-contingent macro-prudential debt tax, of about 1 percent on average, which is levied in normal times when a financial crisis has positive probability next period. Constant debt taxes also reduce the frequency of crises but are less effective at reducing their severity and reduce welfare when credit constraints bind.

KEYWORDS: Financial crises, macroprudential policy, systemic risk, collateral constraints
JEL CLASSIFICATION CODES: D62, E32, E44, F32, F41

*We thank Fernando Alvarez, Gianluca Benigno, John Cochrane, Lars Hansen, and Charles Engel for useful comments and discussions. We are also grateful for the comments by conference and seminar participants at the Bank of Korea, Chicago Booth, Cornell, Duke, Federal Reserve Bank of Chicago, Queen's University, Riksbank, University of Wisconsin, Yale, the May 2013 Meeting of the Macro Financial Modeling group, and the '2nd Rome Junior Conference on Macroeconomics' of the Einaudi Institute. Mendoza acknowledges gratefully the hospitality and support of the Bank for International Settlements while working on this project under a 2014 Research Fellowship. The support of the National Science Foundation under awards 1325122 (Mendoza) and 1324395 (Bianchi) is also gratefully acknowledged. Some material included in this paper circulated earlier under the title "Overborrowing, Financial Crises and Macroprudential Policy", NBER WP 16091, June 2010.

1 Introduction

The cross-country analysis of credit boom episodes by Mendoza and Terrones (2008) shows that credit booms in advanced and emerging economies are relatively rare, occurring at a frequency of 2.8 percent in a sample of 61 countries spanning the period 1960-2010. They also found, however, that when they occur they display a clear cyclical pattern of economic expansion in the upswing followed by a steep contraction in the downswing. Strikingly, 1/3rd of these credit booms are followed by full blown financial crises, and this frequency is about the same in advanced and emerging economies. Similarly, Reinhart and Rogoff (2009) found that banking crises are preceded by boom-bust credit cycles and a marked upswing in private credit in historical cross-country data going back two centuries. From this perspective, and with all dimensions properly taken, what happened in 2008 in the United States is a recurrent event.

The realization that credit booms are rare but perilous events that often end in financial crises and deep recessions has resulted in a strong push for implementing a new "macroprudential" form of financial regulation. As described in the early work by Borio (2003) or a recent exposition by Bernanke (2010), the objective of this macroprudential approach to regulation is to take a macroeconomic perspective of credit dynamics, with a view to defusing credit booms in their early stages as a prudential measure to prevent them from turning into crises. The efforts to move financial regulation in this direction, however, have moved faster and further ahead than our understanding of how financial policies influence the transmission mechanism driving financial crises, particularly in the context of quantitative macroeconomic models that can be used to design and evaluate the performance of these policies.

This paper aims to fill this gap by answering three key questions: First, can credit frictions affecting individual borrowers turn into a significant macroeconomic problem, in terms of both producing financial crises with quantitative features similar to those we see in the data and influencing ordinary business cycles? Second, what is the optimal design of macroprudential policy when financial regulators lack the ability to commit to future policies (i.e. when the policy needs to avoid the classic time-inconsistency problem that emerges if we assume that the regulator could act under commitment)? Third, how powerful is this policy for affecting the incentives of private credit market participants in a prudential manner, and for reducing the magnitude and incidence of financial crises?

This paper proposes answers to these questions based on the quantitative predictions of a dynamic stochastic general equilibrium model of asset prices and business cycles with credit frictions. We start by developing a simple normative theory for the design of macroprudential policy. Then we extend this theory to a richer model with production and working capital financing, and show that, in the absence of macroprudential policy, this model's financial amplification mechanism produces financial crises with realistic quantitative features. Then we characterize and solve for the optimal, time-consistent macroprudential policy of a financial regulator who lacks the ability to commit to future policies, and quantify a state-contingent schedule of debt taxes that supports the allocations of this policy in a decentralized equilibrium.

A central feature of the framework examined in this paper is a pecuniary externality in a similar vein of those used in the related literature on credit booms and macroprudential policy (e.g. Lorenzoni, 2008; Korinek, 2009; Bianchi, 2011; Stein, 2012): Individual agents facing a collateral constraint do not internalize how their borrowing decisions in "good times" affect the market price of collateral, and hence the aggregate borrowing capacity, in "bad times" in which the collateral constraint binds. This creates a market failure that results in equilibria that can be improved upon by a financial regulator who faces the same credit frictions but internalizes the externality.

The collateral constraint is modeled as an occasionally-binding limit on the total amount of debt (one-period debt and within-period working capital loans) as a fraction of the market value of physical assets that can be posted as collateral, which are in fixed aggregate supply. This constraint is the engine of the mechanism by which the model can produce financial crises with realistic features as an equilibrium outcome. This is because, when the constraint binds, Irving Fisher's classic debt-deflation financial amplification mechanism is set in motion. The result is a financial crisis driven by a nonlinear feedback loop between asset fire sales and borrowing ability.

In this setup, the pecuniary externality of the credit constraint appears as a wedge between the marginal costs and benefits of borrowing considered by individual agents and those faced by the regulator. Private agents fail to internalize when making their borrowing plans taking collateral prices as given that, if the collateral constraint binds in the future, fire sales of assets will cause a Fisherian debt-deflation spiral, which will cause asset prices to decline sharply and the economy's overall borrowing ability to shrink.[1] Moreover, in the model we propose, when the constraint binds production plans are also affected, because working capital financing is needed in order to pay for a fraction of factor costs, and working capital loans are also subject to the collateral constraint. This results in a sudden increase in effective factor costs and a fall in output when the credit constraint binds. In turn, this affects expected dividend streams and therefore equilibrium asset prices, and introduces an additional vehicle for the pecuniary externality to operate, because private agents do not internalize the supply-side effects of their borrowing decisions either.

We study the optimal policy problem of a financial regulator that chooses the level of credit to maximize the private agents' utility subject to resource and credit constraints with two key features: First, the regulator internalizes the pecuniary externality. Second, the regulator cannot commit to future policies. The first feature leads the regulator to impute a higher social marginal cost to choosing higher debt and leverage in good times, because the regulator takes into account

[1]For this reason, the literature also refers to this externality as a systemic risk externality, because individual agents contribute to the risk that a small shock can lead to large macroeconomic effects, or as a fire-sale externality, because as collateral prices drop, agents fire-sale the goods or assets that serve as collateral to meet their financial obligations.

that higher leverage can cause a Fisherian asset price deflation in bad times. The second feature implies that the regulator's optimal policy is time-consistent, in contrast with the time-inconsistent policy chosen by a regulator acting under commitment. Under commitment, we show that if the collateral constraint binds, it is optimal for the regulator to make promises of lower future consumption with the aim to prop up current asset prices, but reneging is optimal ex post. Hence, in the absence of effective commitment devices, this policy strategy is not credible. Instead, we explicitly model the regulator's inability to commit to future policies, and solve for optimal time-consistent macroprudential policy as part of a Markov perfect equilibrium in which the effect of current optimal plans of the regulator on future plans is taken into account.

The paper develops some theoretical results and conducts a quantitative analysis in a version of the model calibrated to data for industrial economies. The theoretical analysis keeps the model tractable by abstracting from production and working capital, assuming that borrowing ability depends on the aggregate supply of assets, instead of individual asset holdings, and modeling exogenous dividend shocks as the only underlying shocks hitting the economy. These three assumptions are relaxed in the quantitative analysis.

The quantitative results show that financial crises in the competitive equilibrium are significantly more frequent and more severe than in the equilibrium attained by the regulator. The incidence of financial crises is about three times larger. Asset prices drop about 30 percent in a typical crisis in the decentralized equilibrium, versus 5 percent in the regulator's equilibrium. The size of the output drop is about 20 percent larger, because the fall in asset prices reduces access to working capital financing. The more severe asset price collapses also generate an endogenous "fat tail" in the distribution of asset returns in the decentralized equilibrium, which causes the price of risk to rise 1.5 times and the equity premium to rise by 5 times, in both tranquil times and crisis times.

We also show that the regulator can replicate its equilibrium allocations as a decentralized equilibrium with an optimal state-contingent schedule of taxes on debt. A key element of this schedule is a macroprudential debt tax levied in good times when the probability of a financial crisis the following period is positive (i.e. when collateral constraints do not bind at date t but can bind with positive probability at $t+1$). Analytical results show that this macroprudential debt tax is always positive, and in the calibrated model we find it to be about 1 percent on average and positively correlated with leverage. The optimal tax schedule when the constraint binds also includes two other components, which can be positive or negative: One that captures the regulator's "ex post" incentives to influence asset prices to prop up credit when collateral constraints are already binding, and one that captures its incentives to influence the optimal plans of future regulators due to the inability to commit to future policies.

This paper contributes to the growing literature in the intersection of Macroeconomics and Finance by developing a non-linear quantitative framework suitable for the normative analysis of macroprudential policy. The non-linear global methods are necessary in order to quantify accurately the macro implications of occasionally binding collateral constraints in models with incomplete asset markets and subject to aggregate shocks. This is important for determining whether the model provides a reasonable approximation to the non-linear macroeconomic features of actual financial crises, and thus whether it is a useful laboratory for policy analysis, and also for capturing the prudential aspect of macroprudential policy, which works by altering the incentives of economic agents to engage in precautionary behavior in "good times," when credit and leverage are building up. Moreover, using non-linear global methods is also key for solving the Markov perfect equilibrium that characterizes optimal time-consistent macroprudential policy.

Most of the recent Macro-Finance literature, including this article, follows in the vein of the research program on fire sales and financial accelerators initiated by Bernanke and Gertler (1989), Kiyotaki and Moore (1997).[2] In particular, we follow Mendoza (2010) in the analysis of non-linear dynamics caused by credit constraints. He conducted only a positive analysis to show how an occasionally binding collateral constraint generates financial crises with realistic features that are nested within regular business cycles as a result of shocks of standard magnitudes. We focus instead on normative analysis, and develop a framework for designing optimal, time-consistent macroprudential regulation that can reduce the risk of financial crises and improve welfare.

As noted earlier, the pecuniary externality at work in our model is related to those examined in the theoretical work of Caballero and Krishnamurthy (2001), Lorenzoni (2008) , and Korinek (2009), which arises because private agents do not internalize the amplification effects caused by financial constraints that depend on market prices.[3] There are also studies of this externality with a quantitative focus similar to ours. In particular, Bianchi (2011) makes a quantitative assessment of a prudential tax on borrowing, but in a setting in which the borrowing capacity is linked to the relative price of nontradable goods to tradable goods. Benigno et al. (2013) show that there can also be a role for ex-post policies to reallocate labor from the non-tradables sector to the tradables sector and show how this reduces the level of precautionary savings.

This paper differs from the above quantitative studies in several important aspects. First, it focuses on asset prices as a key factor driving debt dynamics and the pecuniary externality, instead of the relative price of nontradable goods. This is important because private debt contracts, particularly mortgage loans like those that drove the high household leverage ratios of many industrial countries in the years leading to the 2008 crisis, use assets as collateral. Second, from a theoretical standpoint, a collateral constraint linked to asset prices introduces forward-looking effects that are absent with a credit constraint linked to goods prices. In particular, expectations of a future financial crisis affect the discount rates applied to future dividends and distort asset prices even in periods of financial tranquility. This also leads to the time consistency issues that

[2] More recent papers include Jermann and Quadrini (2012), Perri and Quadrini (2011), Khan and Thomas (2013), Bigio (2011), Boissay et al. (2012), Arellano et al. (2012).

[3] For a generic result on constrained inefficiency in incomplete markets see e.g. Geneakoplos and Polemarchakis (1986).

we tackle in this study and that were absent from previous work. Finally, our model differs because it introduces working capital financing subject to the collateral constraint, which leads the externality to affect adversely production, factor allocations and dividend rates, and thus again asset prices. In contrast, Bianchi (2011) studies an endowment economy and in Benigno et al. (2013) firms are not affected by credit constraints.

This paper is also related to Jeanne and Korinek (2010) who study the quantitative effects of macroprudential policy in a model in which assets serve as collateral. In their model, however, output follows an exogenous Markov-switching process and individual credit is limited to the sum of a fraction of aggregate, rather than individual, asset holdings plus a constant term. Since in their calibration this second term dwarfs the first, and the probability of crises matches the exogenous probability of a low-output regime, the debt tax they examine has no effect on the frequency of crises and has small effects on their magnitude. In contrast, in our model both the probability of crises and output dynamics are endogenous, and macroprudential policy reduces sharply the incidence and magnitude of crises. Our approach also differs from Jeanne and Korinek in that they impose restrictions on the ability of the planner to distort asset prices when the collateral constraint binds, which bypasses the time consistency problem. In particular, they assume that the planner takes as given an asset pricing function consistent with the Euler equation of asset holdings of the decentralized equilibrium.[4] In contrast, we study a Markov perfect equilibrium taking into account explicitly the inability of the planner to commit to future policies without restricting otherwise its ability to influence prices. This approach also allows us to provide a clearer analytical characterization of the pecuniary externality.[5]

Our analysis is also related to other recent studies exploring alternative theories of inefficient borrowing and their policy implications. For instance, Schmitt-Grohé and Uribe (2012) and Farhi and Werning (2012) examine the use of prudential capital controls as a tool for smoothing aggregate demand in the presence of nominal rigidities and a fixed exchange rate regime. In earlier work, Uribe (2006) examined an economy with an aggregate borrowing limit and compared the borrowing decisions with an economy where the borrowing limit applies to each individual agent. He provided an exact "no overborrowing" result in a canonical endowment economy model and also showed that borrowing decisions are almost identical in a version of the model where the borrowing capacity depends on asset prices and this exact equivalence does not hold. Our analysis differs in that we conduct a normative analysis where the social planner takes borrowing decisions internalizing price effects from borrowing decisions.

[4] We followed a similar approach in Bianchi and Mendoza (2010) by setting up the optimal policy problem of the planner in recursive form using the asset pricing function of the unregulated decentralized equilibrium to value collateral.

[5] In particular, we show that the optimal macroprudential tax is positive in states in which the collateral constraint is not binding, which rationalizes the lean-against-the-wind argument of macroprudential policy. By contrast, Jeanne and Korinek provide an expression for the tax that depends on equilibrium objects with a potentially ambiguous sign.

The literature on participation constraints in credit markets initiated by Kehoe and Levine (1993) is also related to our work, because it examines the role of inefficiencies that result from endogenous borrowing limits. In particular, Jeske (2006) showed that if there is discrimination against foreign creditors, private agents have a stronger incentive to default than a planner who internalizes the effects of borrowing decisions on the domestic interest rate, which affects the tightness of the participation constraint. Wright (2006) then showed that as a consequence of this externality, subsidies on capital flows restore constrained efficiency.

From a methodological standpoint, this paper is related to the literature on the use of Markov perfect equilibria to solve optimal time consistent policy. In particular, our paper is related to the work of Klein et al. (2008) on government expenditures and Klein, Paul and Quadrini, Vincenzo and Rios-Rull (2005) on international taxation. Our paper extends these methods to solve for models with an occasionally binding endogenous constraint, in our case a collateral constraint.

The rest of the paper is organized as follows: Section 2 presents the simple version of the model used for the analytical work and characterizes the unregulated competitive equilibrium. Section 3 conducts the normative analysis of the simple model. Section 4 extends the model for the quantitative analysis by endogenizing production, introducing working capital financing, allowing borrowing capacity to depend on individual asset holdings, and adding interest-rate shocks and financial shocks. Section 5 calibrates the model and discusses the quantitative findings. Section 6 provides conclusions.

2 A Simple Fisherian Model of Financial Crises

This Section characterizes the decentralized competitive equilibrium of a simple model of financial crises driven by a collateral constraint. We use this model to develop the normative analysis of the pecuniary externality and optimal time-consistent macroprudential policy in a tractable way. The main features of this analysis will be preserved in the more general model that we use for the quantitative analysis later in the paper.

2.1 Economic Environment

The economy is inhabited by a continuum of identical, infinitely lived agents with preferences given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

In this expression, $\mathbb{E}(\cdot)$ is the expectations operator, β is the subjective discount factor. The utility function $u(\cdot)$ is a standard concave, twice-continuously differentiable function that satisfies the Inada condition.

In each period, agents hold one-period non-state contingent bonds b_t and an asset k_t that pays a random dividend z_t each period, where z_t is an aggregate shock that follows a first-order Markov process. The asset is in fixed unit supply, so that the market clearing condition in the asset market is simply $k_t = 1$. We denote by q_t the market price of this asset. Hence, the budget constraint is:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R} = k_t(z_t + q_t) + b_t \tag{2}$$

where R is an exogenous gross real interest rate. This last assumption can be interpreted as implying that the economy is a price taker in world financial markets.[6] This is a reasonable assumption for most of the advanced economies considered in the quantitative experiments of Section 5. Moreover, assuming a representative borrower allows us to focus on efficiency gains from policy, setting aside distributional effects that would arise if borrowers are heterogeneous.

Agents also subject to a credit constraint by which it cannot borrow more than a fraction κ of the market value of the economy's aggregate quantity of assets:

$$\frac{b_{t+1}}{R} \geq -\kappa q_t \tag{3}$$

The assumption that borrowing ability depends on the aggregate market value of assets simplifies the analytical expressions that characterize the planner's problem of the next Section, but is not necessary in general. Hence, in Section 4 we extend the model for the quantitative analysis by assuming the more realistic scenario in which individual asset holdings determine borrowing capacity.

The agent chooses consumption, asset holdings and bond holdings to maximize (1) subject to the budget constraint (2) and the collateral constraint (3). This maximization problem yields the following first-order conditions for c_t, b_{t+1} and k_{t+1} respectively:

$$\lambda_t = u'(c_t) \tag{4}$$

$$\lambda_t = \beta R \mathbb{E}_t \lambda_{t+1} + \mu_t \tag{5}$$

$$q_t \lambda_t = \beta \mathbb{E}_t \left[\lambda_{t+1} \left(z_{t+1} + q_{t+1}\right)\right] \tag{6}$$

where $\lambda_t > 0$ and $\mu_t \geq 0$ are the Lagrange multipliers on the budget constraint and collateral constraint respectively. Condition (4) is standard. Condition (5) is the Euler equation for bonds. When the collateral constraint binds, this condition implies that the effective marginal cost of borrowing for additional consumption today exceeds the expected marginal utility cost of repaying R units of goods tomorrow by an amount equal to the shadow value of the credit constraint (i.e. the household faces an effective real interest rate higher than R). Condition (6) is the Euler equation for assets, which equates the marginal cost and benefit of holding them. Since the

[6] An alternative assumption that yields an equivalent formulation is to assume deep-pockets, risk-neutral lenders that discount future utility at the rate $\beta^* = 1/R$. These agents would remain indifferent to the policies we consider because the return they obtain on their savings remains the same.

collateral constraint depends on the aggregate quantity of assets, this condition is not affected by μ_t.

The interaction between the collateral constraint and asset prices at work in this simple model can be illustrated by studying how standard asset pricing conditions are altered by the constraint. In particular, combining (5), (6) and the definition of asset returns ($R^q_{t+1} \equiv \frac{z_{t+1}+q_{t+1}}{q_t}$), it follows that the expected excess return on assets relative to bonds (i.e. the equity premium, $R^{ep}_t \equiv \mathbb{E}_t(R^q_{t+1} - R)$), satisfies the following condition:

$$R^{ep}_t = \frac{\mu_t}{u'(c_t)\mathbb{E}_t m_{t+1}} - \frac{cov_t(m_{t+1}, R^q_{t+1})}{\mathbb{E}_t m_{t+1}} \tag{7}$$

where $m_{t,t+j} \equiv \frac{\beta^j u'(c_{t+j})}{u'(c_t)}$ is the stochastic discount factor, and for simplicity when $j = 1$ we denote it just as m_{t+1}.

Following Mendoza and Smith (2006), we can denote the first term in the right-hand-side of (7) as a direct (first-order) effect of the collateral constraint, which reflects the fact that a binding collateral constraint exerts pressure to fire-sell assets, depressing the current price and increasing excess returns.[7] There is also an indirect (second-order) effect of the collateral constraint given by the fact that $cov_t(m_{t+1}, R^q_{t+1})$ is likely to become more negative, because the collateral constraint makes it harder for agents to smooth consumption.

Condition (6) yields a forward looking solution for asset prices:

$$q_t = \mathbb{E}_t \sum_{j=1}^{\infty} m_{t,t+j} z_{t+j} \tag{8}$$

Using the definition of asset returns we can rewrite this pricing condition as follows:

$$q_t = \mathbb{E}_t \sum_{j=0}^{\infty} \left(\prod_{i=0}^{j} \mathbb{E}_{t+i} R^q_{t+1+i} \right)^{-1} z_{t+j+1}, \tag{9}$$

As in Mendoza and Smith (2006), it follows that a binding collateral constraint at date t increases expected excess returns and lowers asset prices at t. This mechanism is at the core of the pecuniary externality in our model: larger levels of debt lead to more frequent fire sales, driving excess returns up and depressing asset prices, which in turn reduce the borrowing capacity of the economy as a whole. Moreover, because expected returns rise whenever the collateral constraint is expected to bind at any future date, condition (9) also implies that asset prices at t are affected by collateral constraints not just when the constraints binds at t, but whenever it is expected to bind at any future date along the equilibrium path. Hence, expectations about future excess returns and risk

[7] When we extend the model in Section 4 to assume that individual asset holdings at the beginning of the period are posted as collateral, this direct effect is weakened by an additional effect due to the fact that the agent also attaches additional value to holding assets as collateral.

premia feed back into current asset prices, and this interaction will be important for the analysis of macroprudential policy, as shown in the next Section.

2.2 Recursive Competitive Equilibrium

We now characterize the competitive equilibrium in recursive form. Since agents are atomistic and take all prices as given, the recursive formulation separates individual bond holdings b that are under the control of the agent at date t from the economy's aggregate bond position B on which all prices depend. Hence, the state variables for the agent's problem are the individual states (b, k) and the aggregate states(B, z). Aggregate capital is not carried as a state variable because it is in fixed supply. In order to be able to form expectations of future prices, the agent also needs to consider the law of motion governing the evolution of the economy's bond position $B' = \Gamma(B, z)$.

For given $B' = \Gamma(B, z)$ and $q(B, z)$, the agent's recursive optimization problem is:

$$V(b, k, B, z) = \max_{b',k',c} u(c) + \beta \mathbb{E}_{z'|z} V(b', k', B', z') \tag{10}$$

$$\begin{aligned} s.t. \quad & q(B, z)k' + c + \frac{b'}{R} = k\left(q(B, z) + z\right) + b \\ & -\frac{b'}{R} \leq \kappa q(B, z) \end{aligned}$$

The solution to this problem is characterized by the decision rules $\hat{b}(b, k, B, z)$, $\hat{k}(b, k, B, z)$, and $\hat{c}(b, k, B, z)$. The decision rule for bond holdings induces an actual law of motion for aggregate bonds, which is given by $\hat{b}(B, 1, B, z)$. In a recursive rational expectations equilibrium, as defined below, the actual and perceived laws of motion must coincide.

Definition (Recursive Competitive Equilibrium). *A recursive competitive equilibrium is defined by an asset pricing function $q(B, z)$,a perceived law of motion for aggregate bond holdings $\Gamma(B, z)$, and decision rules $\hat{b}'(b, k, B, z), \hat{k}'(b, k, B, z), \hat{c}(b, k, B, z)$ with associated value function $V(b, k, B, z)$ such that:*

1. *$\left\{\hat{b}(b, k, B, z), \hat{k}(b, k, B, z), \hat{c}(b, k, B, z)\right\}$ and $V(b, k, B, z)$ solve the agent's recursive optimization problem, taking as given $q(B, z)$ and $\Gamma(B, z)$.*

2. *The perceived law of motion for aggregate bonds is consistent with the actual law of motion: $\Gamma(B, z) = \hat{b}(B, 1, B, z)$.*

3. *Market clearing: Asset markets clear $\hat{k}(B, 1, B, z) = 1$ and the resource constraint holds $\frac{\hat{b}'(B,1,B,z)}{R} + \hat{c}(B, 1, B, z) = z + B$*

3 Normative Analysis

In this Section, we conduct a normative analysis of the model we just laid out. First we make a brief comparison of the competitive equilibrium with an efficient equilibrium in which there is no collateral constraint. Then we study a constrained-efficient social planner's (or financial regulator's) problem in which the regulator chooses the bond position for the private agent while lacking the ability to commit to future policies. Finally, we show that the allocations of this planner's problem can be decentralized with state-contingent taxes on borrowing.

3.1 Equilibrium without collateral constraint

In the absence of the collateral constraint (3), the competitive equilibrium allocations can be represented as the solution to the following standard planning problem:

$$H(B, z) = \max_{B',c} u(c) + \beta \mathbb{E}_{z'|z} H(B', z')$$

$$s.t. \qquad c + \frac{B'}{R} = z + B$$

and subject also to either this problem's *natural debt limit*, which is defined by $B' \geq -min(z)/(R-1)$, or a tighter ad-hoc time- and state-invariant debt limit.

The common strategy followed in quantitative studies of the macro effects of collateral constraints (e.g. Mendoza, 2010) is to compare the allocations of the competitive equilibrium with the collateral constraint with those corresponding to the above problem without collateral constraint. Private agents borrow *less* in the former because the collateral constraint limits the amount they can borrow, and also because they build precautionary savings to self-insure against the risk of the occasionally binding credit constraint. Compared with the constrained-efficient allocations we examine next, however, we will show that the competitive equilibrium with collateral constraints displays overborrowing when the collateral constraint does not bind. Hence, the competitive equilibrium of the economy with the collateral constraint features *underborrowing* relative to the equilibrium without collateral constraints but *overborrowing* relative to the constrained-efficient equilibrium with the collateral constraint.

3.2 A Constrained Efficient, Time-Consistent Planner

Consider now a constrained-efficient social planner who makes the choice of debt for the representative agent subject to the same collateral constraint and lacking the ability to commit to future policies. A key assumption in defining this planner's problem relates to how the equilibrium price of collateral is determined, because this determines the economy's borrowing capacity and plays a central role in the time-consistency issues discussed later in this Section. We assume that this price is determined in a competitive market, and hence the social planner cannot control it directly (i.e. agents retain access to asset markets). The planner does, however, internalize how its borrowing decisions affect asset prices, and makes optimal use of its debt policy to influence them.[8]

3.2.1 Private Agents Optimization Problem

Since the government chooses bond holdings, the optimization problem faced by private agents reduces to choosing consumption and asset holdings taking as given a government transfer T_t, which corresponds to the resources added or subtracted by the planner's debt choices:

Problem 1 (Agent's Problem in Constrained-Efficient Equilibrium)

$$\begin{aligned} \max_{\{c_t, k_{t+1}\}_{t\geq 0}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \\ s.t. \quad c_t + q_t k_{t+1} &= k_t(q_t + z_t) + T_t \end{aligned}$$

The first-order condition of this problem with respect to assets is standard:

$$q_t u'(c_t) = \beta \mathbb{E}_t u'(c_{t+1}) \left(z_{t+1} + q_{t+1}\right) \tag{11}$$

This condition enters as an implementability constraint in the planner's problem. This is a key constraint, because as we explain in the analysis below, it links the planner's policy rules with the market price of assets. In particular, it drives the mechanism by which these rules influence the relationship between expectations about future consumption and asset prices and today's asset prices. As we explain below, this mechanism causes a planner assumed to be committed to future policies to display a time-inconsistency problem, which motivates our interest in formulating optimal macroprudential policy as a time-consistent problem of a planner that lacks the ability to commit.[9]

[8]Since we focus on macroprudential policy, which by definition aims to prevent crises by altering behavior in pre-crises times, this notion of constrained efficiency leaves out policies that may relax directly the credit constraint and make crisis less severe ex-post. These policies are examined by Benigno et al. (2013) in the context of a model in which the collateral constraint depends on goods prices. Nevertheless, as we show later in this Section, our planner's optimal policy does respond to incentives to prop up asset prices and borrowing capacity when credit constraints are binding

[9]The time inconsistency problem does not arise in Lorenzoni (2008)'s classic model of fire sales because the asset price is determined by a static condition linking relative productivity of households and entrepreneurs, rather than expectations about future marginal utility as in our setup. Similarly, in Bianchi (2011), borrowing capacity is determined by a static price of non-tradable goods. Bianchi and Mendoza (2010) and Jeanne and Korinek (2010) impose time-consistency by construction in models with asset prices by imposing pricing conditions as explained in the Introduction.

3.2.2 Social Planner's Optimization Problem

As in Klein et al. (2005), we focus on Markov stationary policy rules, which set the values of bond holdings, consumption and asset prices as functions of the payoff-relevant state variables (b, z). Since the planner is unable to commit to future policy rules, it chooses its policy rules at any given period taking as given the policy rules that represent future planners' decisions, and a Markov perfect equilibrium is characterized by a fixed point in these policy rules. At this fixed point, the policy rules of future planners that the current planner takes as given to solve its optimization problem match the policy rules that the current planner finds optimal to choose. Hence, the planner does not have the incentive to deviate from other planner's policy rules, thereby making these rules time-consistent.

Let $\mathcal{B}(b, z)$ be the policy rule for bond holdings of future planners that the planner takes as given, and $\mathcal{C}(b, z)$ and $\mathcal{Q}(b, z)$ the associated recursive functions that return the private consumption allocations and the market price of assets under that policy rule. Given these functions, we can use the fact that the first-order condition of the households' problem is an implementability constraint in the planner's problem to illustrate how by choosing b' the planner affects the stochastic discount factor that determines current asset prices. In particular, the implementability constraint (11) can be rewritten by replacing private consumption using the budget constraint of private agents evaluated at equilibrium together with the planner's budget constraint ($T_t = b_t - \frac{b_{t+1}}{R}$). The resulting expression indicates that the equilibrium asset price must satisfy:

$$\mathcal{Q}(b_t, z_t) = \frac{\beta \mathbb{E}_t u'\left(b_{t+1} + z_{t+1} - \frac{\mathcal{B}(b_{t+1}, z_{t+1})}{R}\right)(z_{t+1} + \mathcal{Q}(b_{t+1}, z_{t+1}))}{u'\left(b_t + z_t - \frac{b_{t+1}}{R}\right)} \tag{12}$$

while $\mathcal{C}(b_t, z_t) = b_t + z_t - \frac{\mathcal{B}(b_t, z_t)}{R}$.The right-hand-side of this expression shows that the debt choice of the planner affects asset prices directly, by inducing agents to reallocate consumption between t and $t+1$ which affects the stochastic discount factor, and indirectly by affecting the bond holdings chosen by future governments, which also affects c_{t+1}. These effects will be reflected in the optimality conditions that characterize the social planner's equilibrium. This equilibrium can be defined in recursive form as follows.

Problem 2 (Recursive Representation of the Planner's Problem) *Given the policy rule of future planners $\mathcal{B}(b, z)$, and the associated consumption allocations $\mathcal{C}(b, z)$ and asset prices $\mathcal{Q}(b, z)$ the* *planner's problem is characterized by the following Bellman equation:*

$$\begin{aligned}
\mathcal{V}(b,z) &= \max_{c,b',q} u(c) + \beta \mathbb{E}_{z'|z} \mathcal{V}(b',z') \\
c + \frac{b'}{R} &= b + z \\
\frac{b'}{R} &\geq -\kappa q \\
u'(c)q &= \beta \mathbb{E}_{z'|z} u'\left(b' + z' - \frac{\mathcal{B}(b',z')}{R}\right)\left(\mathcal{Q}(b',z') + z'\right)
\end{aligned} \tag{13}$$

In the above problem, the planner chooses $b'(b,z)$ optimally to maximize the household's utility subject to three constraints: First, the economy's resource constraint (with Lagrange multiplier λ), which states that the consumption plan must be consistent with what private agents choose optimally given their budget constraint, market clearing in the asset market, and the planner's transfer. Second, the collateral constraint (with Lagrange multiplier μ), which the planner faces just like private agents. Third, the implementability constraint (with Lagrange multiplier ξ), which requires that the asset price be consistent with the optimality condition that holds in the private asset market.

Assuming that the equilibrium policy functions and the value function are differentiable, we can apply the standard Envelope theorem results to the first-order conditions of the planner's problem in order to recover the corresponding optimality conditions for c_t, b_{t+1} and q_t in sequential form. These optimality conditions are:

$$c_t :: \qquad \lambda_t = u'(c_t) - \xi_t u''(c_t) q_t \tag{14}$$

$$\begin{aligned}
b_{t+1} :: \qquad u'(c_t) &= \beta R \mathbb{E}_t \left\{ u'(\mathcal{C}(b_{t+1},z_{t+1})) - \xi_{t+1} u''(\mathcal{C}(b_{t+1},z_{t+1}))\mathcal{Q}(b_{t+1},z_{t+1}) + \xi_t \Omega_{t+1} \right\} \\
&\quad + \xi_t u''(c_t) q_t + \mu_t
\end{aligned} \tag{15}$$

$$q_t :: \qquad \xi_t = \frac{\kappa \mu_t}{u'(c_t)} \tag{16}$$

with $\Omega_{t+1} \equiv u''(\mathcal{C}(b_{t+1},z_{t+1}))\mathcal{C}_b(b_{t+1},z_{t+1})(\mathcal{Q}(b_{t+1},z_{t+1}) + z_{t+1}) + \mathcal{Q}_b(b_{t+1},z_{t+1})u'(\mathcal{C}(b_{t+1},z_{t+1}))$.

The key differences between the unregulated competitive equilibrium and the financial regulator's equilibrium can be described intuitively by comparing the above optimality conditions with those of the decentralized competitive equilibrium. Compare first condition (14) with the analogous condition in the decentralized equilibrium, equation (4). Condition (4) states that for private agents the shadow value of wealth is equal to the marginal utility of consumption, but (14) shows that for the regulator it equals the marginal utility of consumption plus the effect by which an increase in consumption relaxes the implementability constraint.[10] Moreover, condition (16) shows that the planner sees a social benefit from relaxing the implementability constraint if and only if the collateral constraint is currently binding, i.e., $sign(\mu_t) = sign(\xi_t)$. Hence, when the collateral constraint binds, having an additional unit of wealth has a social benefit derived from how an increase in consumption raises equilibrium asset prices, which in turn relaxes the collateral constraint. This is clearer if we use (16) to rewrite the additional shadow value of wealth for the planner in (14) as $-u''(c_t)q_t\frac{\kappa\mu_t}{u'(c_t)}$. If the collateral constraint does not bind, $\mu_t = \xi_t = 0$ and the shadow values of wealth of the regulator and private agents in the decentralized equilibrium coincide.

Compare next the planner's Generalized Euler equation for bonds (15) with the analogous Euler equation in the competitive equilibrium (5). These equations differ in two key respects: First, condition (15) reflects the fact that the differences identified above in the valuation of bond holdings of the regulator and the private agents "ex post," when the collateral constraint binds, also result in valuation differences "ex ante," when the constraint is not binding, which arise because both the regulator and the agents are forward looking. In particular, if $\mu_t = 0$, the marginal cost of increasing debt at date t for private agents in the competitive equilibrium is simply $\beta R\mathbb{E}_t u'(c_{t+1})$. In contrast, the second term in the right-hand-side of (15) shows that the regulator attaches a higher social marginal cost to borrowing, because it internalizes the effect by which the larger debt at t reduces tomorrow's borrowing ability if the credit constraint binds then.[11] In other words, because the planner values more consumption when the constraint binds ex-post compared to private agents, it borrows less ex-ante. Moreover, this mechanism captures the standard pecuniary externality of the related literature on macroprudential regulation, because it reflects the response of the regulator who takes into account how equilibrium asset prices tomorrow respond to the debt choice of today if the constraint becomes binding tomorrow. Since asset prices are determined in private markets, the equilibrium response is captured by the changes in the pricing kernel reflected in $u''(c_{t+1})$.

The second difference between the two Euler equations for bonds is in that condition (15) includes additional dynamic effects from current borrowing choices resulting from the forward looking nature of asset prices. Because of its inability to commit, the regulator aims to influence future marginal utilities by changing the endogenous state variable of the next-period's regulator, as reflected in the partial derivatives of the future policy rule and pricing function with respect to b included in the term Ω_{t+1} in the right-hand-side of (15). These incentives are only relevant, however, if the borrowing constraint is binding at t, because otherwise they vanish when $\xi_t = 0$.

We can now define the constrained-efficient equilibrium formally:

Definition. *The recursive constrained-efficient equilibrium is defined by the policy rule* $b'(b, z)$

[10]Note that $-\xi_t u''(c_t)q_t > 0$ because $u''(c_t) < 0$ and $\xi_t > 0$, as condition (16) implies. Hence, $\lambda_t > u'(c_t)$.

[11]We can use again (16) to make this more evident mathematically by rewriting the second term in the right-hand-side of (15) as $-u''(c_{t+1})q_{t+1}\frac{\kappa\mu_{t+1}}{u'(c_{t+1})}$, which is positive for $\mu_{t+1} > 0$.

with associated consumption plan $c(b, z)$, pricing function $q(b, z)$ and value function $\mathcal{V}(b, z)$, and the conjectured functions characterizing the policy rule of future planners $\mathcal{B}(b, z)$ and its associated consumption allocations $\mathcal{C}(b, z)$ and asset prices $\mathcal{Q}(b, z)$, such that the following conditions hold:

1. *Planner's optimization: $\mathcal{V}(b, z)$,$b'(b, z)$,$c(b, z)$ and $q(b, z)$ solve the Bellman equation defined in Problem (2) given $\mathcal{B}(b, z)$,$\mathcal{C}(b, z)$, $\mathcal{Q}(b, z)$.*

2. *Time consistency (Markov stationarity): The conjectured policy rule, consumption allocations, and pricing function that represent optimal choices of future planners match the corresponding recursive functions that represent optimal plans of the current regulator: $b'(b, z) = \mathcal{B}(b, z)$, $c(b, z) = \mathcal{C}(b, z)$, $q(b, z) = \mathcal{Q}(b, z)$.*

Note that the requirements that the consumption allocation must be an optimal choice for households according to (1) and that the holdings of assets by households satisfy $k_t = 1$ are redundant, because the former is implied by the planner's resource constraint and the latter is the planner's implementability constraint. The transfer T_t is also implicit from the planner choices for bonds.

3.3 Decentralization

We show now that a state-contingent tax on debt can decentralize the constrained-efficient, time-consistent allocations.[12] With a tax τ_t on borrowing, the budget constraint of private agents in the regulated competitive equilibrium becomes:[13]

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R(1+\tau_t)} = k_t(z_t + q_t) + b_t + T_t \tag{17}$$

where T_t represents lump-sum transfers by which the government rebates all its tax revenue. The agents' Euler equation for bonds becomes:

$$u'(c_t) = \beta R(1+\tau_t)\mathbb{E}_t u'(c_{t+1}) + \mu_t \tag{18}$$

Analyzing the optimality conditions of the planner's problem together with those of the regulated and unregulated decentralized equilibria leads to the following proposition:

Proposition 1 (Decentralization) *The constrained-efficient equilibrium can be decentralized using a state contingent tax on debt with its revenues rebated as a lump-sum transfer. The tax on debt*

[12] Following Bianchi (2011), it is also possible to decentralize the planner's problem using measures targeted directly to financial intermediaries; in particular using capital requirements, reserve requirements or loan-to-value ratios.

[13] The tax can also be expressed as a tax on the price of bonds (i.e. on the income generated by borrowing), so that the post-tax price would be $(1-\tau^R)(1/R)$. The two treatments are equivalent if we set $\tau^R = \tau/(1+\tau)$.

is given by:

$$\tau_t = \frac{R\mathbb{E}_t\left\{-\xi_{t+1}u''(\mathcal{C}(b_{t+1},z_{t+1}))\mathcal{Q}(b_{t+1},z_{t+1}) + \xi_t\Omega_{t+1} + \xi_t u''(c_t)q_t\right\}}{\mathbb{E}_t u'(\mathcal{C}(b_{t+1},z_{t+1}))}$$

Proof: See Appendix A.1

The above optimal state-contingent tax schedule can be broken down into three components. The first is the *macroprudential debt tax*, τ^{MP}, which is defined as the one levied when the collateral constraint is not binding at t but may bind with positive probability at $t+1$. This is in line with the operational definition of macroprudential policies, which are defined as those aimed to tackle credit growth in "good times" to lower the risk of financial instability. Using (16), the macroprudential debt tax reduces to:

$$\tau_t^{MP} = \frac{-R\mathbb{E}_t\frac{\kappa\mu_{t+1}}{u'(\mathcal{C}(b_{t+1},z_{t+1}))}u''(\mathcal{C}(b_{t+1},z_{t+1}))\mathcal{Q}(b_{t+1},z_{t+1})}{\mathbb{E}_t u'(\mathcal{C}(b_{t+1},z_{t+1}))}$$

This tax is strictly positive, since $u' > 0, u'' < 0$ and $\xi \geq 0$. In particular, the tax is strictly positive whenever there is a positive probability that the collateral constraint (or equivalently the implementability constraint, given condition (16)) can become binding at $t+1$.

The other components of the optimal schedule of debt taxes are the terms that include the multiplier ξ_t in eq.(1). These components can be positive or negative, and they are present only when the collateral constraint is binding at t, since $sign(\mu_t) = sign(\xi_t)$. These components reflect the fact that, when the constraint binds at t, the optimal tax schedule must incorporate two effects. First, the regulator internalizes that one more unit of current consumption raises current asset prices, which leads to a subsidy on debt when the collateral constraint binds (i.e. the regulator has the incentive to subsidize debt when the collateral constraint binds in order to prop up asset prices and provide more borrowing capacity).[14] Second, the regulator has incentives to influence the behavior of future planners by altering the bond holdings they receive, which are captured by the terms that include $\mathcal{C}_b(t+1)$ and $\mathcal{Q}_b(t+1)$ in $\Omega(b_{t+1},z_{t+1})$.

It is worth noting also that in quantitative applications of this simple version of the model it is possible to set $\tau = 0$ without affecting equilibrium allocations and prices when $\mu_t > 0$. This is because private agents borrow the maximum amount, which is independent of the tax. As shown in the Appendix A.1, the role of the tax when $\mu_t > 0$ is only to implement the planner's shadow value from relaxing the collateral constraint, and since this shadow value in the decentralization is affected by τ even if the constraint is binding, the analytical expression for the tax can be positive or negative. This result, however, does not extend to the more general model we solve in Section

[14] This is captured in the terms $\mu_t + \xi_t u''(c_t)q_t$ of eq.(1) which form an expression with ambiguous sign, because the first term is positive but the second negative, with the latter capturing the incentive to prop up asset prices at t.

5, for which the value and sign of all the components of the state-contingent tax schedule are uniquely determined even if $\mu_t > 0$.

3.4 Time Inconsistency under Commitment

We close this Section with some remarks illustrating how a time-inconsistency problem emerges if instead of studying the constrained-efficient, time-consistent social planner's problem we set up the analogous problem of a regulator assumed to be able to commit to future policies. This is useful because, as we mentioned earlier, our interest in studying time-consistent macroprudential policy is motivated in part by the fact that under commitment the planner's optimal policies are time-inconsistent.[15]

Under commitment, the planner chooses at time 0 its policy rules in a once-and-for-all fashion. The first-order conditions of the planner's problem in sequential form are ($\forall t > 0$):

$$\lambda_t = u'(c_t) - \xi_t q_t u''(c_t) + u''(c_t)\xi_{t-1}(q_t + z_t) \tag{19}$$

$$\lambda_t = \beta R \mathbb{E}_t \lambda_{t+1} + \mu_t \tag{20}$$

$$\xi_t = \xi_{t-1} + \frac{\mu_t \kappa}{u'(c_t)} \tag{21}$$

The time inconsistency problem is evident from the presence of the lagged multipliers in these optimality conditions.[16] According to (19), the planner internalizes how an increase in consumption at time t helps relax the borrowing constraint at time t and makes it tighter at $t-1$. As (21) shows, this implies that the Lagrange multiplier on the implementability constraint ξ_t follows a positive, non-decreasing sequence, which increases every time the constraint binds. Intuitively, when the constraint binds at t, this planner likes to promise lower future consumption so as to prop up asset prices and borrowing capacity at t, but ex post when $t+1$ arrives it would be sub-optimal to keep this promise. We can show that under commitment, a state contingent tax on borrowing is also sufficient to implement the constrained-efficient solution, except that again it would be a non-credible policy because of the planner's incentives to deviate from announced policy rules ex post.

[15] As noted earlier, in Bianchi and Mendoza (2010) we followed an ad-hoc approach to construct a time-consistent macroprudential policy, by proposing a conditionally-efficient planner restricted to value collateral using the same pricing function of the unregulated competitive equilibrium. Decentralizing this planner's allocations requires, in addition to the debt tax, a state-contingent tax on dividends.

[16] It should be understood that time $t-1$ variables include the history up to time $t-1$ and time t variables represent the history up to time $t-1$ in addition to a time t exogenous disturbance.

4 Model for Quantitative Analysis

The remainder of the paper focuses on studying the model's quantitative predictions. Before proceeding, however, we introduce three modifications that are important for improving the model's ability to produce financial crises with features more in line with actual crises, so that the model can be viewed as a sound benchmark to conduct quantitative policy assessments. First, we introduce production and factor demands using a working capital channel which creates a link between financial amplification and the supply-side of the economy. Second, we modify the collateral constraint so that borrowing capacity is limited by individual asset holdings, instead of the aggregate supply of assets. Third, we introduce shocks to the interest rate and to the collateral constraint to incorporate additional exogenous driving forces of business cycles and financial crises. In the preceding analytical sections we abstracted from these features to keep the model tractable, and while these changes introduce effects that obviously interact with the pecuniary externality, the main features of macroprudential regulation highlighted in the normative analysis are still present.

4.1 Firm-Households Optimization Problem

We follow Mendoza (2010) to add production into the model by replacing the representative agent of the simple model with a representative firm-household, which we also refer to as an agent. This agent makes both production plans and consumption-savings choices.

The agent's preferences are given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t - G(h_t)) \tag{22}$$

where h_t is the agent's labor supply. The argument of $u(\cdot)$ is the composite commodity $c_t - G(h_t)$ defined by Greenwood et al. (1988). $G(h)$ is a convex, strictly increasing and continuously differentiable function that measures the disutility of working. This formulation of preferences removes the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor depend on labor only. This assumption ensures that the model does not deliver a counterfactual increase in labor supply during crises.

The representative firm-household combines physical assets, imported intermediate goods (ν_t), and domestic labor services (h_t) to produce final goods using a production technology such that $y = z_t F(k_t, h_t, v_t)$, where F is a twice-continuously differentiable, concave production function. Hence, z_t is now a standard productivity shock instead of an exogenous dividends process. This shock has compact support and follows a finite-state, stationary Markov process. Imported inputs are purchased in competitive world markets at a constant exogenous price p_ν in terms of the domestically produced goods (i.e. p_ν can be interpreted as the terms of trade).

The profits of the agent are given by $z_t F(k_t, h_t, v_t) - p_v v_t$, and the agent's budget constraint can be written as:

$$q_t k_{t+1} + c_t + \frac{b_{t+1}}{R_t} = q_t k_t + b_t + [z_t F(k_t, h_t, v_t) - p_v v_t] \tag{23}$$

where R_t is the real interest rate, which we continue to treat as exogenous. Like the productivity shocks, interest rate shocks also follow a finite-state, stationary Markov process with compact support. The two shocks can be modeled as correlated or independent processes.

As noted earlier, the assumption that the interest rate is exogenous is equivalent to assuming that the economy is a price-taker in world credit markets, as in other studies of the U.S. financial crisis like those of Boz and Mendoza (2010), Corbae and Quintin (2009) and Howitt (2011), or alternatively it implies that the model can be interpreted as a partial-equilibrium model of the household sector. This assumption is adopted for simplicity, but is also in line with evidence indicating that the observed decline in the U.S. risk-free rate in the era of financial globalization has been driven largely by outside factors, such as the surge in reserves in emerging economies and the persistent collapse of investment rates in South East Asia after 1998. Warnock and Warnock (2009) provide econometric evidence of the significant downward pressure exerted by foreign capital inflows on U.S. T-bill rates since the mid 1980s. Mendoza and Quadrini (2009) document that about 1/2 of the surge in net credit in the U.S. economy since then was financed by foreign capital inflows, and more than half of the stock of U.S. treasury bills is now owned by foreign agents. From this perspective, assuming a fluctuating R_t around a constant mean is actually conservative, as in reality the pre-crisis boom years were characterized by a *falling* real interest rate, which would strengthen our results. Still, we study later in the sensitivity analysis how our quantitative results vary if we relax this assumption and consider instead an exogenous inverse supply-of-funds curve, which allows the real interest rate to increase as debt rises.

The agent also faces a working capital constraint which requires it to pay for a fraction of the cost of input purchases in advance of production using foreign financing. In particular, a foreign working capital loan is used to pay for a fraction θ of the cost of imported inputs $p_v v_t$ at the beginning of the period and repaid at the end of the period. In the conventional working capital setup, a cash-in-advance-like motive for holding funds to pay for inputs implies that the effective marginal cost of inputs carries a financing cost determined by R_t. In contrast, here we simply assume that working capital funds are within-period loans so that the interest rate on working capital is effectively zero. We follow this approach so as to show that the effects of working capital in our analysis hinge only on the need to provide collateral for working capital, as explained below, and not on the effect of interest rate fluctuations on effective factor costs, which is the standard mechanism in business cycle models with working capital (e.g. Uribe and Yue, 2006).[17] Moreover, we consider only imported inputs in the working capital constraint because this constraint relates

[17] We could also change to the standard setup, but in our calibration, $\theta = 0.5$ and the average interest rate is $R = 1.028$, and hence working capital loans would only add 1.4 percent to the cost of imported inputs.

to external financing, which is thus natural to connect to inputs acquired abroad (i.e. we view working capital as akin to trade credit from foreign suppliers of intermediate goods). Labor can be viewed as using working capital loans from domestic creditors that do not require collateral without altering our setup.

The agent faces a collateral constraint that limits total debt, including both intertemporal debt and within period working capital loans, not to exceed a possibly stochastic fraction κ_t of the market value of beginning-of-period asset holdings (i.e. κ_t imposes a ceiling on the leverage ratio):

$$-\frac{b_{t+1}}{R_t} + \theta p_v v_t \leq \kappa_t q_t k_t \tag{24}$$

We interpret shocks to κ_t as financial shocks that lead creditors to adjust collateral requirements on borrowers. It is important to note, however, that neither the nature of the amplification mechanism nor the normative arguments stated in the previous Section rely on κ_t being stochastic, and that even with κ constant models in this class can produce crises dynamics with realistic features (see Mendoza (2010) and Bianchi and Mendoza (2010)). Fluctuations in κ_t are useful for improving the model's ability to match the co-movements linking financial flows and business cycles (see Jermann and Quadrini (2012)) and to generate sharp credit expansions in pre-crisis periods (see (Boz and Mendoza, 2010)).

Collateral constraints similar to the one proposed above are often defined using k_{t+1} as collateral (e.g. Kiyotaki and Moore, 1997 and Aiyagari and Gertler, 1999) instead of k_t. This is immaterial for the equilibrium amount of assets that can be pledged as collateral in this model, because assets remain in fixed unit aggregate supply. As we show below, the assumption does affect the timing of the shadow values of binding collateral constraints in the agent's Euler equation for assets, but what is critical is that the collateral constraint depends on asset prices. As long as this is the case, the intuition for the role that the collateral constraint plays and the incentives of the planner to manage the pecuniary externality are qualitatively the same. We used k_t as collateral for tractability and because it facilitates providing a contractual foundation for the existence of the constraint. In particular, we show in Appendix A.3, the collateral constraint (24) can be obtained as an implication of incentive compatibility constraints on the part of borrowers in an environment in which limited enforcement prevents lenders from collecting more than a fraction κ_t of the value of the k_t owned by a defaulting debtor.

4.2 Unregulated Decentralized Equilibrium

In the unregulated decentralized competitive equilibrium (DE), agents maximize (22) subject to (23) and (24) taking asset and factor prices as given. Also, the markets of goods, assets and factors of production clear.

The maximization problem of the agent yields the following sequence of optimality conditions for each date t:

$$z_t F_h(k_t, h_t, v_t) = G'(h_t) \tag{25}$$

$$z_t F_v(k_t, h_t, v_t) = p_v(1 + \theta\mu_t/u'(t)) \tag{26}$$

$$u'(t) = \beta R_t \mathbb{E}_t u'(t+1) + \mu_t \tag{27}$$

$$q_t u'(t) = \beta \mathbb{E}_t \left[u'(t+1)\left(z_{t+1}F_k(k_{t+1}, h_{t+1}, v_{t+1}) + q_{t+1}\right) + \kappa_{t+1}\mu_{t+1}q_{t+1}\right] \tag{28}$$

where $\mu_t \geq 0$ is the Lagrange multiplier on the collateral constraint and $u'(t)$ denotes $u'(c_t - G(h_t))$.

Condition (25) is the labor market optimality condition equating the marginal disutility of labor supply with the marginal productivity of labor demand, which implicitly determines the wage rate. Condition (26) is a similar condition setting the demand for imported inputs by equating their marginal productivity with their marginal cost. Note, however, that there is key difference in the latter, because the marginal cost of imported inputs includes the extra financing cost $\theta\mu_t/u'(t)$ which is incurred in states of nature in which the collateral constraint binds.

The last two conditions are the Euler equations for bonds and assets respectively, and they yield similar implications for the effects of binding collateral constraints as in the simple model. When the collateral constraint binds, condition (27) implies that the marginal utility of reallocating consumption to the present exceeds the expected marginal utility cost of borrowing in the bond market by an amount equal to the shadow price of relaxing the credit constraint. Condition (28) equates the marginal cost of an extra unit of assets with its marginal gain. The fact that assets serve as collateral increases the benefits of holding the assets by $\beta\mathbb{E}_t\kappa_{t+1}\mu_{t+1}q_{t+1}$.

Proceeding again as we did with the simple model, we can combine the Euler equations for bonds and assets to derive this model's expression for the equity premium:

$$R_t^{ep} = \frac{\mu_t}{u'(t)\mathbb{E}_t m_{t+1}} - \frac{\mathbb{E}_t\left(\phi_{t+1}m_{t+1}\right)}{\mathbb{E}_t m_{t+1}} - \frac{cov_t(m_{t+1}, R_{t+1}^q)}{\mathbb{E}_t\left[m_{t+1}\right]} \tag{29}$$

where $\phi_{t+1} \equiv \kappa_{t+1}\frac{\mu_{t+1}}{u'(c_t)}\frac{q_{t+1}}{q_t}$ represents a *collateral effect* that contributes to reduce excess returns because an extra unit of assets improves the ability to borrow.[18]

The collateral effect can in turn be decomposed into a first-order effect and a risk (or second-order) term:

$$\frac{\mathbb{E}_t\left(\phi_{t+1}m_{t+1}\right)}{\mathbb{E}_t m_{t+1}} = \frac{cov_t\left(\phi_{t+1}, m_{t+1}\right)}{\mathbb{E}_t m_{t+1}} - \mathbb{E}_t\phi_{t+1}$$

Notice that $cov_t(m_{t+1}, R_{t+1}^q)$ and $cov_t\left(\phi_{t+1}, m_{t+1}\right)$ have opposite signs as collateral is most valued when the household is more constrained, which coincides with high marginal utility.

Given the definitions of Sharpe ratio ($SR_t \equiv R_t^{ep}/\sigma_t(R_{t+1}^q)$) and price of risk ($\sigma_t(m_{t+1})/\mathbb{E}_t m_{t+1}$),

[18] A similar effect is present when k_{t+1} serves as collateral instead of k_t, but its timing also changes. In this case, the marginal benefit of holding more assets as collateral shows up as the term $-\mu_t\kappa_t$ in the equity premium expression (see Mendoza and Smith, 2006 and Bianchi and Mendoza, 2010)

we can rewrite the expected excess return and the Sharpe ratio as:

$$R_t^{ep} = S_t \sigma_t(R_{t+1}^q), \qquad S_t = \frac{\mu_t - \mathbb{E}_t\left(\phi_{t+1} m_{t+1}\right)}{u'(t)\mathbb{E}_t m_{t+1} \sigma_t(R_{t+1}^q)} - \rho_t(R_{t+1}^q, m_{t+1}) \frac{\sigma_t(m_{t+1})}{\mathbb{E}_t m_{t+1}} \tag{30}$$

where $\sigma_t(R_{t+1}^q)$ is the date-t conditional standard deviation of asset returns and $\rho_t(R_{t+1}^q, m_{t+1})$ is the conditional correlation between R_{t+1}^q and m_{t+1}. Thus, the collateral constraint has direct and indirect effects on the Sharpe ratio analogous to those it has on the equity premium. The indirect effect reduces to the usual expression in terms of the product of the price of risk and the correlation between asset returns and the stochastic discount factor. The direct effect is normalized by the variance of returns. These relationships will be used later to quantify the effects of the credit friction and the macroprudential policy on asset pricing conditions.

The solution method that we implement in the next Section works using the recursive representation of the competitive equilibrium. We use s to denote the triplet of shocks $s = \{z_t, \kappa_t, R_t\}$. The agent's recursive optimization problem is:

$$V(b,k,s) = \max_{b',k',c,h,\nu} u(c - G(h)) + \beta \mathbb{E}_{s'|s} V(b',k',B',s') \tag{31}$$

$$\begin{aligned} s.t. \quad q(B,s)k' + c + \frac{b'}{R} &= q(B,s)k + b + [zF(k,h,\nu) - p_v v] \\ -\frac{b'}{R} + \theta p_v v &\leq \kappa q(B,s)k \end{aligned}$$

The solution to this problem is characterized by the decision rules $\hat{b}(b,k,s)$, $\hat{k}(b,k,s)$, $\hat{c}(b,k,s)$, $\hat{\nu}(b,k,s)$ and $\hat{h}(b,k,s)$. The decision rule for bond holdings induces an actual law of motion for aggregate bonds, which is given by $\hat{b}(B,1,B,s)$.

Definition (Recursive Competitive Equilibrium)**.** *A recursive competitive equilibrium is defined by an asset pricing function $q(B,s)$, a perceived law of motion for aggregate bond holdings $\Gamma(B,s)$, and decision rules $\hat{b}'(b,k,s), \hat{k}'(b,k,s), \hat{c}(b,k,s), \hat{h}(b,k,s), \hat{v}(b,k,s)$ with associated value function $V(b,k,s)$ such that:*

1. *$\left\{\hat{b}(b,k,s), \hat{k}(b,k,s), \hat{c}(b,k,s), \hat{h}(b,k,s), \hat{v}(b,k,s), \hat{\mu}(b,k,s)\right\}$ and $V(b,k,s)$ solve the agents' recursive optimization problem, taking as given $q(B,s)$ and $\Gamma(B,s)$.*

2. *The market for assets clear and the resource constraint holds $\frac{\hat{b}'(B,1,B,s)}{R} + \hat{c}(B,1,B,s) = zF(1,\hat{n}(B,1,B,s),\hat{v}(B',1,B,s')) + B - p_v\hat{v}(b,1,B,s)$, and $\hat{k}(B,1,B,s) = 1$*

3. *The perceived law of motion for aggregate bonds is consistent with the actual law of motion: $\Gamma(B,z) = \hat{b}(B,1,B,z)$.*

4.3 Planner's Problem and Macroprudential Policy

As in the simple model, we continue to assume that a constrained-efficient, time-consistent social planner (SP) chooses directly the amount of debt, while consumption, asset holdings, and now production, labor, and imported inputs are chosen competitively by private agents. As shown in the Appendix A.2, taking as given a policy rule for bond holdings of future regulators $\mathcal{B}$ and the associated recursive functions governing labor $\mathcal{H}$, consumption $\mathcal{C}$, imported inputs ν, and asset prices $\mathcal{Q}$, the current regulator's optimization problem can be written as the following Bellman equation:

$$
\begin{aligned}
\mathcal{V}(b,s) &= \max_{c,b',q,\mu,h,\nu} u(c-G(h)) + \beta \mathbb{E}_{s'|s}\mathcal{V}(b',s') \qquad (32)\\
c+\frac{b'}{R} &\leq b + zF(1,h,\nu) - p_v v\\
zF_h(1,h,\nu) &= G'(h)\\
zF_v(1,h,\nu) &= p_v\left(1+\frac{\theta\mu}{u'(c-G(h))}\right)\\
\frac{b'}{R}-\theta p_v v &\geq -\kappa q\\
\mu\left(\frac{b'}{R}-\theta p_v v+\kappa q\right) &= 0\\
qu'(c-G(h)) &= \beta\mathbb{E}_{s'|s}\left\{u'(\mathcal{C}(b',s')-G'(\mathcal{H}(b',s')))(\mathcal{Q}(b',s')+z'F_k(1,\mathcal{H}(b',s'),\nu(b',s')))\right.\\
&\quad \left.+\kappa'\mu(b',s')\mathcal{Q}(b',s')\right\}
\end{aligned}
$$

Finally, following Proposition 1, the constrained efficient allocations can again be decentralized with an optimal, state-contingent schedule of taxes on debt (see Appendix A.2). As before, this schedule has a macroprudential component that is levied when the constraint does not bind at t but can bind with positive probability at $t+1$ and two other components that apply when the constraint binds at t, one reflecting the incentive to prop up asset prices to improve borrowing capacity and the other reflecting the aim to influence the behavior of future regulators given the inability to commit of the financial regulator.

We also show in the Appendix A.2 that the optimal macroprudential debt tax is given by:

$$
\tau_t^{MP} = \frac{1}{\mathbb{E}_t u'(t+1)} R\mathbb{E}_t\left\{\frac{\zeta_{t+1}^v p_v\theta\mu(t+1)u''(t+1)}{u'(t)^2} - \xi_{t+1}u''(t+1)\mathcal{Q}(b_{t+1},z_{t+1})\right\} \qquad (33)
$$

where ζ_t^v is the shadow value of relaxing the implementability constraint associated with the private agents' limited access to working capital financing for purchasing imported inputs. As with the simple model, the tax is zero if there is zero probability of a binding collateral constraint tomorrow. On the other hand, when there is a positive probability of hitting the constraint at $t+1$ the optimal tax can no longer be unambiguously signed because there is an additional term that captures the incentive compatibility constraint associated with the choice of imported inputs. This is the first term in the numerator in the right-hand-side of (33), which has an ambiguous sign because ζ_t^v has an ambiguous sign (since the implementability constraint must hold with equality). Quantitatively, however, the tax is largely dominated by the second term in that numerator, which is positive and identical to the expression for the macroprudential tax in the simple model, and hence yields strictly positive values for τ^{MP} in all the numerical exercises we conducted.

5 Quantitative Analysis

5.1 Calibration

We calibrate the model to annual frequency using data from advanced economies. For some variables (e.g. value of housing wealth, utilization-adjusted TFP, Frisch elasticity of labor supply), we used only U.S. data because of data availability limitations, but we examine the implications of parameter variations in the sensitivity analysis.

The functional forms for preferences and technology are the following:

$$u(c - G(h)) = \frac{\left(c - \chi \frac{h^{1+\omega}}{1+\omega}\right)^{1-\sigma} - 1}{1-\sigma} \quad \omega > 0, \sigma > 1$$
$$F(k, h, \nu) = z k^{\alpha_k} \nu^{\alpha_\nu} h^{\alpha_h}, \qquad \alpha_k, \alpha_\nu, \alpha_h \geq 0 \quad \alpha_k + \alpha_\nu + \alpha_h \leq 1$$

We set $\sigma = 1.5$, which is in the range of commonly used values in open-economy DSGE models. The Frisch elasticity of labor supply $(1/\omega)$ is set equal to 1, in line with evidence for the United States provided by Kimball and Shapiro (2008). The parameter χ is inessential and is set so that mean hours are equal to 1, which requires $\chi = \alpha_h$ (with α_h calibrated as described below).

The production function of gross output is Cobb-Douglas. To calibrate the share of imported inputs, we use data reported by Goldberg and Campa (2010) on the average ratio of imported to domestic intermediate goods for 16 advanced economies. The average ratio across them is 25 percent. At standard average ratios of total intermediate goods to gross output of 45 percent, the implied share of imported inputs in gross output is $\alpha_\nu = 0.124$. The factor share of labor is then set so that in terms of value added we obtain the standard share of 0.64, which is similar across industrial countries (see Stockman and Tesar, 1995). This implies $\alpha_h = 0.64^*(1 - \alpha_\nu) = 0.56$. Since capital in the model is in fixed supply, we do not set the capital share to the standard 1/3rd of GDP, because this factor share measures capital income accrued to the entire capital stock. Instead, we set α_k so that the model matches an estimate of the ratio of capital in fixed supply to GDP based on the value of the housing stock. Consistent data across countries on this component of household wealth are not available, so we measured the ratio for the United States using data from the *Flow of Funds* database of the Federal Reserve. In particular, we used the ratio as of 2007, which was about 1.3, because it corresponds to the last year before the start of the 2008 financial crisis. The model matches this ratio, given the other parameter values, if we set $\alpha_k = 0.05$.[19] Notice that this implies that production effectively has decreasing returns to scale, but this is not critical for the results because of the unit supply of capital and because profits return to private agents as income.

We follow Schmitt-Grohe and Uribe (2007) in taking M1 money balances owned by firms as a proxy for working capital. Based on the observations that in the United States about two-thirds of M1 are held by firms (Mulligan, 1997) and that M1 was 10 percent of GDP in 2007, we calibrate the working capital-GDP ratio to (2/3)*0.1 = 0.066. Given $\alpha_\nu = 0.124$ and that the ratio of GDP to gross output is $1 - \alpha_\nu$, and assuming also that the collateral constraint does not bind, the value of θ is solved for as $\theta = 0.066[(1 - \alpha_\nu)/\alpha_\nu]$, which is about 0.5.

The value of β is set to the standard value in DSGE models of 0.96, but in addition in this calibration it supports an average household debt-income ratio in a range that is in line with U.S. data from the *Flow of Funds* database. Before the mid-1990s this ratio was stable at about 30 percent. Since then and until 2007, it rose steadily to a peak of almost 70 percent. By comparison, the average debt-income ratio in the stochastic steady-state of the model with the baseline calibration is 38 percent. A mean debt ratio of 38 percent is sensible because 70 percent was an extreme at the peak of a credit boom and 30 percent is an average from a period before the substantial financial innovation of recent years.

The shocks to the interest rate and productivity are set to fit a discrete approximation to estimates of the following VAR process:

$$\begin{pmatrix} z_t \\ R_t \end{pmatrix} = \boldsymbol{\rho} \begin{pmatrix} z_{t-1} \\ R_{t-1} \end{pmatrix} + \begin{pmatrix} \epsilon_{z,t} \\ \epsilon_{R,t} \end{pmatrix},$$

where $(\epsilon_{z,t}, \epsilon_{R,t})$ follow a bivariate normal distribution with zero mean and contemporaneous variance-covariance matrix **Cov**. The interest rate is measured as the ex-post real interest rate on U.S. three-month T-bills, which is the standard measure of the exogenous world real interest rate in International Macro models. TFP is measured using the TFP estimates adjusted for changes in utilization and relative input prices constructed by Fernald (2009). The VAR estimation yields the following results:

$$\boldsymbol{\rho} = \begin{bmatrix} 0.755972 & -0.030037 \\ -0.074327 & 0.743032 \end{bmatrix}, \qquad \mathbf{Cov} = \begin{bmatrix} 0.0000580 & -0.0000107 \\ -0.0000107 & 0.0001439 \end{bmatrix}.$$

The discrete approximation to this VAR process is constructed using the Tauchen-Hussey quadrature method, set to produce a Markov chain with three realizations of each shock. Their

[19] Estimates of the value of capital in fixed supply vary depending on whether they include land used for residential or commercial purposes, or owned by government at different levels. We used an estimate based on residential property because it is a closer match to the structure of the model (for example, the discount factor was set to match the household debt ratio).

Table 1: Calibration

Description	*Value*	*Source / target*
Risk aversion	$\sigma = 1.5$	Standard value
Discount factor	$\beta = 0.96$	Standard value
Share of imported inputs	$\alpha_v = 0.124$	Cross-country data from Goldberg and Campa (2010)
Share of labor	$\alpha_n = 0.56$	0.64 cross-country data from Stockman and Tesar (1995)
Share of assets	$\alpha_k = 0.05$	2007 U.S. housing stock/GDP =1.3
Labor disutility coefficient	$\chi = 0.56$	Normalizatition for average $h = 1$
Frisch elasticity parameter	$\omega = 1$	Kimball and Shapiro (2008)
Working capital coefficient	$\theta = 0.5$	2007 U.S. firms' $M1/GDP = 0.1$
Financial shock		
	$\kappa^L = 0.3$	Average leverage = 30%
	$P_{HH} = 0.9$	10-year mean duration of κ^H (crisis prob.=0.03)
	$P_{LL} = 0.1$	Prob. of κ^L=0.1 (3-year crisis duration)
TFP and Interest Rate Shocks	see text	

algorithm uses as inputs $\boldsymbol{\rho}$ and $\mathbf{Cov}$, and yields the Markov realization and transition probability matrices, which are available from the authors on request.

The regime-switching process of κ is assumed to be independent from the bivariate process of (z_t, R_t). In particular, κ_t follows a standard two-state, regime-switching Markov process with regime values given by $\{\kappa^L, \kappa^H\}$ and transition matrix

$$P = \left[\begin{array}{cc} P_{L,L} & 1 - P_{L,L} \\ 1 - P_{H,H} & P_{H,H} \end{array} \right].$$

The values of κ^H and κ^L are difficult to calibrate directly from the data because of the model's high level of aggregation and the wide dispersion in loan-to-value restrictions and ability to leverage across households and firms of various characteristics in the data. We set κ^H high enough so that the collateral constraint never binds with strictly positive probability when the economy is in that regime, so that effectively the actual value of κ^H is immaterial. The value of κ^L is set to 0.30, which in the unregulated competitive equilibrium yields an average leverage ratio of 30 percent. This is consistent with measures of household and corporate leverage, which were 0.2 and 0.45 respectively at the onset of the 2008 financial crisis.[20] The continuation probability $P_{H,H}$ is set so as to produce a mean duration of the κ^H regime of 10 years, which yields $P_{H,H} = 0.9$.[21] The

[20]These leverage ratios were computed using *Flow of Funds* data (total assets and credit market debt outstanding of households and nonprofit organizations, and total assets and debt outstanding of the domestic nonfinancial business sector). The resulting leverage ratios are lower than maximum loan-to-value ratios in home mortgages, which reached around 95 percent in the sub-prime market at the peak of the housing boom, but the lower ratio in the *Flow of Funds* data suggests that this is not a representative figure for the broader housing sector.

[21]The κ^L and κ^H regimes have mean durations of $1/(1 - P_{L,L})$ and $1/(1 - P_{H,H})$ respectively.

value of $P_{L,L}$ is then set so that, given $P_{H,H}$, the long-run probability of the κ^L regime is only 10 percent. This long-run probability can be expressed as $(1-P_{H,H})/(2-P_{H,H}-P_{L,L})$, which yields $P_{L,L}=0.105$. Under this parameterization, the DE produces financial crises events, defined in a similar way as in the empirical literature, with a frequency of 3 percent and a mean duration of 3 years, both of which are consistent with empirical estimates (see Reinhart and Rogoff, 2009).[22]

The model is solved using a global, nonlinear solution method that takes into account the occasionally binding, stochastic credit constraint. In the SP's problem, the algorithm also takes into account the need to solve for the Markov stationary policy rules that support time-consistency. Further details are provided in Appendix B.

5.2 Frequency and Severity of Financial Crises

The first important result of the quantitative analysis is that the time-consistent macroprudential policy reduces significantly the frequency and severity of financial crises. With regard to the former, recall that the calibrated process for κ implies that the DE has financial crises with the same 3-percent frequency observed in the data. Under the same baseline calibration, the frequency of crises in the SP economy is only 0.2 percent. Thus, the absence of regulation results in financial crises that are 15 times more likely.[23]

The reduced severity of financial crises in the SP economy is illustrated by conducting an event analysis of crisis episodes using simulated data for the DE and SP economies. The results are presented in Figure 1, which shows nine-year event windows for total credit (bonds plus working capital), asset prices, output, and consumption, all expressed as deviations from long-run averages in the DE equilibrium, as well as two windows that show the evolution of the exogenous shocks.[24]

We construct comparable event windows for the two economies following this procedure: First, we simulate the DE for 100,000 periods and identify financial crisis events using the event-study methodology we borrowed from the empirical literature as described in the calibration (see footnote 22). Second, we construct nine-year event windows centered in the crisis year by computing the averages of all endogenous variables across the cross section of crisis events at each date. This produces the DE dynamics plotted as the red, continuous lines in Figure 1. For TFP and interest rate shocks we also show the cross sectional average in each year, but for the financial shock we show the fraction of the time the κ^L regime was observed in the cross section of each year. We

[22] Following Forbes and Warnock (2012) methodology to identify the timing and duration of sharp changes in international capital flows, we defined a financial crisis as an event in which credit falls by at least two standard deviations, with a starting (ending) date set at the year within the previous (following) two years in which credit first fell below (rose above) one standard deviation. In addition, we require that the collateral constraint be binding.

[23] We identify financial crises for the SP using the credit thresholds of the DE in levels. Re-computing the thresholds using the standard deviation of credit in the SP equilibrium, which is smaller, the frequency of crises for the regulator rises to 0.6 percent, which is still much lower than in the DE. Our results are also largely robust to alternative definitions of crises.

[24] We also produced event windows expressing the data as deviations from the long-run averages of each economy, instead of both in percent of DE means, but this did not result in significant differences.

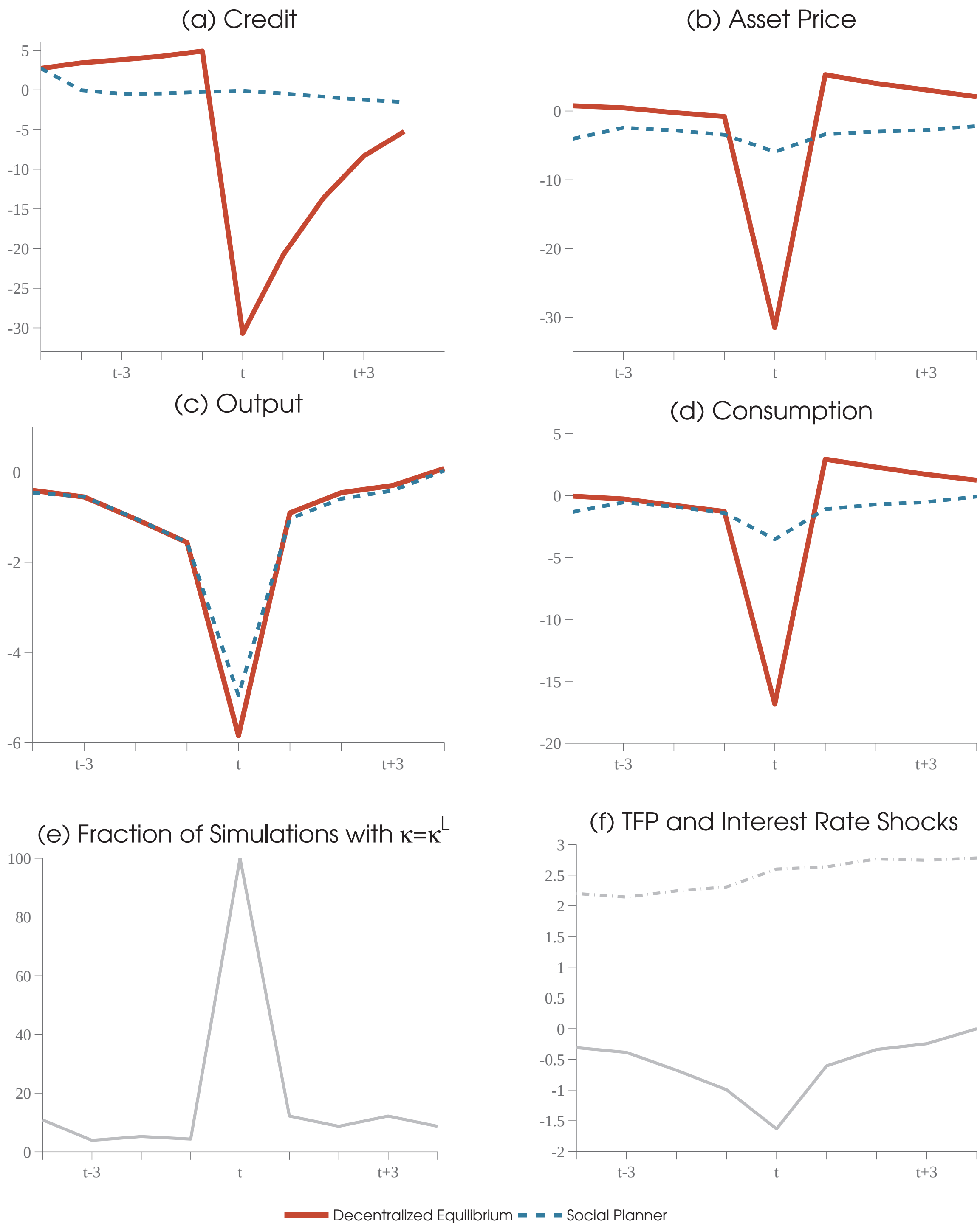

Figure 1: Financial Crises Event Analysis.

Note: all panels except panel (e) plotted as % differences relative to DE unconditional averages do this because, as mentioned earlier, the value of κ^H was set high enough so that it does not bind with positive probability, and hence any value that satisfies this criterion yields the same equilibrium dynamics. Third, we take the initial bond position at $t-5$ of the DE and the sequences of shocks the DE went through in the nine years of each crisis event, and then pass them through the policy functions of the SP. Finally, we average in each date the cross-sectional sample of the SP equilibrium to generate the averages shown as the blue, dashed line in Figure 1.

Panel (a) of Figure 1 shows that the pecuniary externality results in higher borrowing in the DE in the periods preceding the crisis. While credit in the SP remains very close to the DE long-run average throughout, credit in the DE is above its long-run average by 2.5 to 5 percentage points from $t-4$ to $t-1$. The cumulative difference relative to the SP is close to 15 percent. As a result of this overborrowing, the DE builds up higher levels of leverage and experiences a larger collapse in credit when the financial crisis hits. Credit falls about 30 percentage points more, and although it rises at a fast pace after the crisis, four years later it remains 5 percentage points below both its long-run average and the SP level.[25]

Asset prices (panel b), output (panel c) and consumption (panel d) also fall more sharply in the DE than the SP. The declines in consumption and asset prices are particularly larger (-17 v. -4 percent for consumption and -31 v. -6 percent for asset prices). The asset price collapse plays an important role in explaining the more pronounced decline in credit in the DE, because it reflects the impact of the Fisherian deflation mechanism. Output falls 1 percentage point more in the DE than in the SP, because of the higher shadow price of imported inputs due to the effect of the tighter binding credit constraint on access to working capital. This may seem a small difference, but consider that it reflects the impact of the higher financing cost of inputs that have a factor share of only 12.4 percent.

Panel (e) shows that in the DE simulation the economy spends close to 90 percent of the time in the κ^H regime, in line with the long-run probability of κ^H, but in the period in which the crisis hits it always switches to the κ^L regime. Panel (f) shows that TFP is declining on average before the financial crisis, and reaches a through of about -1.5 percent below the mean when the crisis hits, and after that it recovers at a fast pace. The real interest rate is stable at just above 2 percent in the years before the crisis, and when the crisis hits it rises about 50 basis points and then remains stable near 2.5 percent in the years that follow. Thus, financial crises in the DE are associated *on average* with adverse TFP, interest rate and financial shocks. Note, however, that the model also generates crises with positive TFP shocks when leverage is sufficiently high and an adverse financial shock hits.

It is important to note that the DE simulation does a reasonable job at matching key features of actual financial crises (i.e. crises episodes are characterized by unusually large recessions and asset price declines). Moreover, these financial crises are infrequent events nested within regular business

[25] The model produces very large drops in credit partly because all intertemporal credit is in the form of one-period bonds, whereas loans in the data have on average a longer maturity.

cycles that have a standard deviation of GDP of about 2.5 percent, close to what is observed in industrial countries (e.g. the standard deviation of HP- filtered U.S. GDP is 2.1 percent in 1947-2007 data).[26] These are important first steps for making the normative implications of the model relevant.

Interestingly, the implications of the SP equilibrium for the frequency and magnitude of financial crises are not very different from those we obtained in Bianchi and Mendoza (2010) imposing time-consistency by assuming that the planner takes as given the pricing function of the unregulated DE. This suggests that the DE asset pricing function may not differ markedly from the one that the time-consistent planner supports, which in turn suggests that, under our baseline calibration, the Markov stationary debt policy rule of the planner unable to commit restricts the ability to affect the equilibrium asset prices when the collateral constraint binds.

5.3 Borrowing Decisions and Amplification

The financial amplification mechanism and the pecuniary externality at work in the DE simulation, and the credit effects of the SP's policy, can be illustrated further by studying the differences in their borrowing decisions. These borrowing decisions are reflected in the policy functions for bonds of each economy ($\mathcal{B}^{DE}(B,s)$ and $\mathcal{B}^{SP}(B,s)$) shown in Figure 2 as the red-continuous and blue-dashed curves respectively.[27] These policy functions indicate each economy's choice of bonds for $t+1$ in the vertical axis (B') for each value of bond holdings at t in the horizontal axis (B), keeping the values of exogenous shocks at κ^L, average R and a negative TFP shock equal to 1.5 standard deviations below the mean. The Figure also shows the limit on intertemporal borrowing that each economy faces ($\overline{\mathcal{B}}^{DE}(B,s)$ and $\overline{\mathcal{B}}^{SP}(B,s)$), computed using the asset pricing functions of each economy valued at the corresponding (B,s) pairs.[28]

An important result illustrated in this Figure is that the Fisherian deflation mechanism generates V-shaped bond policy functions, instead of the typical monotonically-increasing ones of both standard Bewley-style incomplete markets models of heterogeneous agents and RBC models of the small open economy. The point at which the policy functions switch slope corresponds to the value of B at which the collateral constraint is marginally binding in each economy (i.e. it holds with equality but the choice of debt is exactly the same debt allowed by the credit constraint). To the right of this point, the collateral constraint does not bind and the policy functions are upward sloping. To the left of this point, the policy functions are *decreasing* in B, because a reduction in B results in a sharp reduction in asset prices, which tightens the borrowing constraint, thus increasing B'.[29] In line with these results, the policy functions lie above their corresponding bor-

[26] Mendoza (2010) shows that these features of this class of models hold even in richer variants with capital accumulation.

[27] Note that $\mathcal{B}^{DE}(B,s) \equiv \Gamma(B,z)$ from the definition of the recursive equilibrium.

[28] The borrowing limits are defined as $-\kappa q^{DE}(B,s) + p_\nu \hat{\nu}^{DE}(B,1,B,s)$ and $-\kappa q^{SP}(B,s) + p_\nu \hat{\nu}^{SP}(B,s)$

[29] In Bianchi (2011), there is also a non-monotonicity in the bonds decision rule, but arising from how consump-

rowing limits to the right of the values of B at which the constraint becomes binding in the DE and SP, and to the left of those critical values the decision rules must equal to their corresponding borrowing limits.

The bonds policy functions can be divided into three regions: a "constrained credit region," a "positive crisis probability region" and a "stable credit region." The constrained credit region is defined by the values of B with sufficiently high initial debt (low B) such that the collateral constraint binds for the SP. This is the range with $B \leq -0.33$. In this region, the collateral constraint must bind also for private agents in the DE, because the pecuniary externality affecting agents in this economy implies that the constraint starts binding at higher values of B than in the SP, as we show below.

The total debt (working capital plus debt in bonds) in the constrained credit region is very similar in the DE and SP outcomes. It is not identical because asset prices differ, as the SP is optimally using its policy function to influence asset prices and thus manage borrowing capacity, but within the limits of what its inability to commit allows. To see why asset prices must differ, suppose by way of contradiction that total debt were the same and hence asset prices were the same with the constraint binding. However, because future marginal utilities differ under the DE and SP, this would imply different asset prices. In addition, the bond choices are also similar (as shown in Figure 2), because the choices of imported inputs are also similar in the two economies.[30]

The positive crisis probability region is located to the right of the constrained region, and it includes the interval $-0.33 < B < -0.24$. Here, the regulator chooses uniformly higher B' (lower debt) than private agents, because of the effect that internalizing the externality has on the SP's decisions when the constrained region is near. In fact, private agents hit the credit constraint at $B = -0.32$, while at this B the regulator still retains some borrowing capacity. Moreover, this region is characterized by "financial instability," in the sense that the values of B' chosen in the DE are high enough so that some combinations of shocks at $t+1$ can lead to a binding credit constraint and a financial crisis. The financial amplification dynamics that can occur when the economy is in this region were discussed in the previous section. As shown in the theoretical analysis, this is the region of the state space in which the regulator uses the macroprudential debt tax.

The stable credit region is the interval for which $B \geq -0.24$. In this region, the probability of a binding constraint next period is zero for both DE and SP. The bond choices of the two still differ, however, because expected marginal utilities differ under the two equilibria. But here the regulator does not use the macroprudential debt tax, because even negative shocks cannot lead to a binding credit constraint at $t+1$.

tion affects the marginal rate of substitution between tradables and non-tradables and thus the relative price of nontradables, rather than through the effects on as in this model.

[30]Notice that the planner still treats asset prices as endogenous in these states, which could in principle lead it to consume more to raise asset prices and relax the constraint. However, doing this is generally not feasible because the increase in debt to sustain higher consumption violates the collateral constraint.

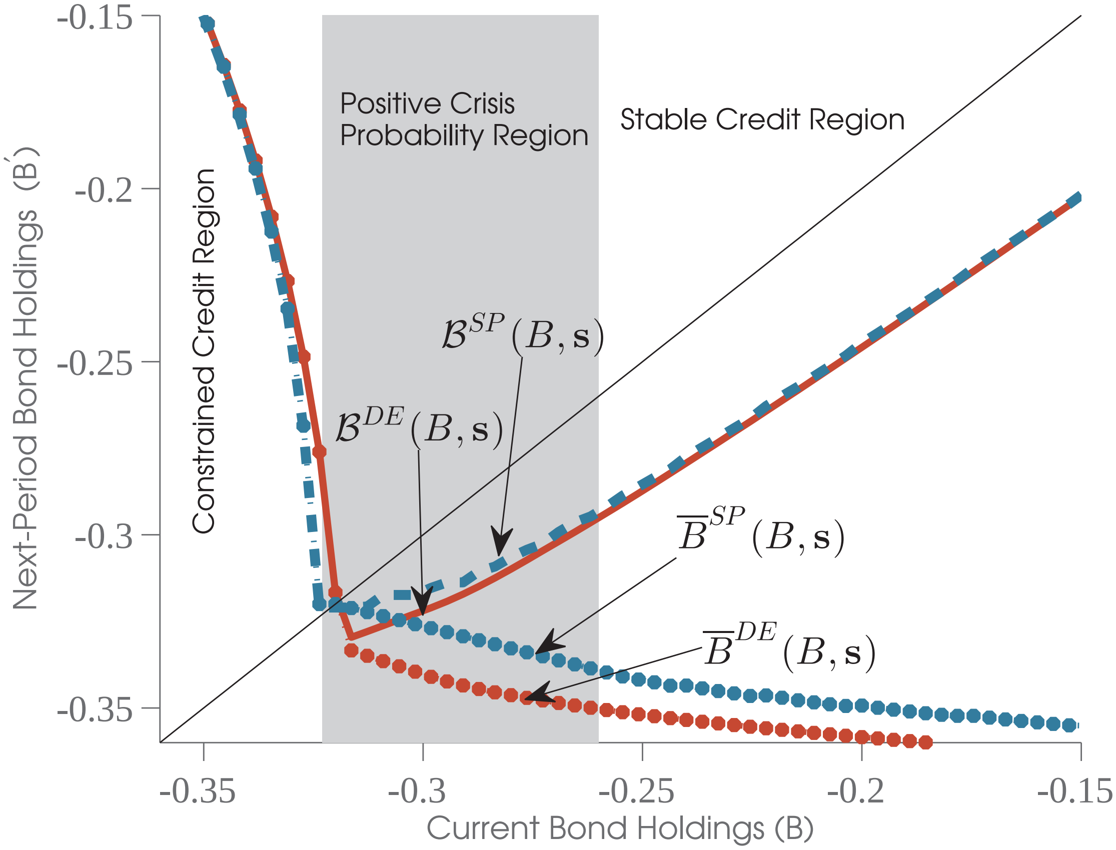

Figure 2: Bond policy functions and borrowing limits.

The larger debt (i.e. lower B') choices of private agents relative to the regulator in the positive crisis probability region provide a measure of the overborrowing effect at work in the DE. The regulator accumulates extra precautionary savings above and beyond what private individuals consider optimal in order to self-insure against the risk of financial crises. This effect may seem small in terms of the difference between the two bond choices, but as we showed in Figure 1, it still leads to large differences in financial amplification. In fact, Figure 2 also reflects the large differences in macroeconomic outcomes when the constraint binds despite the small differences in bond choices when it does not.

The long-run probabilities with which the DE (SP) visit the three regions of the policy functions of bonds in Figure 2, which are conditional on the particular state s used in the Figure, are: 3 (2) percent for the constrained credit region, 86 (88) percent for the positive crisis probability region, and 11 (10) percent for the stable credit region. Hence, both economies spend close to 90 percent of the time in the second region, but the prudential actions of the regulator reduce the probability of entering in the constrained region. This is consistent with the finding mentioned earlier that financial crises are much less frequent in the SP equilibrium than in the unregulated DE.

The significantly nonlinear debt dynamics that result from the financial amplification mechanism, and the SP's ability to weaken them, are illustrated in Figure 3. This Figure shows the policy functions for bonds of the DE and SP over the interval [-0.35,-0.2] of their domain for two different triples of s. One is labeled *positive shock*, which is a state with higher TFP and lower R, and the second is labeled *negative shock*, which has lower TFP and higher R.[31] The financial regime is kept at κ^L in both cases. Hence, the plot illustrates amplification dynamics in response to shocks of standard magnitudes to TFP and R *without* changes in κ. The ray from the origin is the stationary choice (45 degree) line, where $B' = B$. We use a narrower range of B than in Figure 2 to highlight the differences in policy functions in the region that is relevant for financial amplification.

The amplification dynamics to which the DE is exposed can be illustrated as follows. Assume the economy starts in a hypothetical first period with bonds at point D, which is the intersection of DE's policy function with positive shocks with the 45 degree line. At this point, the choice of B' in the DE is identical to the initial B, hence the DE ends the period with the same amount of bonds it started with. Assume then that the second period arrives and the realizations of TFP and R shocks shift to the negative-shock state. The DE starts at point D but now the collateral constraint becomes binding, and the Fisherian deflation dynamics force a sharp, nonlinear upward adjustment of the bond position such that B' increases to point D', reducing debt from -0.34 to -0.22.

Compare the above financial amplification dynamics with what the same experiment produces under the SP equilibrium. The planner starts also at point D, but its policy function for that initial condition moves it to point P, so the planner chooses a value of B' that reduces its debt slightly below what the DE chose (the planner's B' is just about 50 basis points higher than what the DE's chooses at point D). This occurs because the regulator builds precautionary savings and borrows less, since it faces the higher marginal cost of borrowing that results from internalizing the pecuniary externality due the positive probability that the credit constraint may bind in the second period. The second period arrives with the same shift to the negative-shock-state as in the DE economy. The planner starts from point P because of its lower borrowing choice in the first period, and its policy function for the negative-shock state indicates that its bond choice will increase to point P'. Hence, the slight difference in initial debt of the SP v. the DE in the second period results in a sharply smaller upward adjustment in B' for the planner (about 2 v. 12 percent of GDP). This illustrates the dynamics of the mechanism that results in the significantly smaller magnitude of the crisis episodes in the SP v. the DE shown in Figure 1.

The differences in borrowing decisions are also reflected in differences in the long-run distributions of leverage of the DE and SP. Figure 4 shows the cumulative ergodic distributions of leverage ratios (measured as $\frac{-b_{t+1}+\theta p_\nu \nu_t}{q_t}$) in the two economies. The stronger precautionary savings motive of the SP results in an ergodic distribution of leverage that concentrates less probability at higher

[31] In the positive-shock state, TFP is above and R below their means by about 1.3 times their standard deviations, and in the negative-shock state R reverts to its mean and TFP falls below its mean also by about 1.3 times its standard deviation.

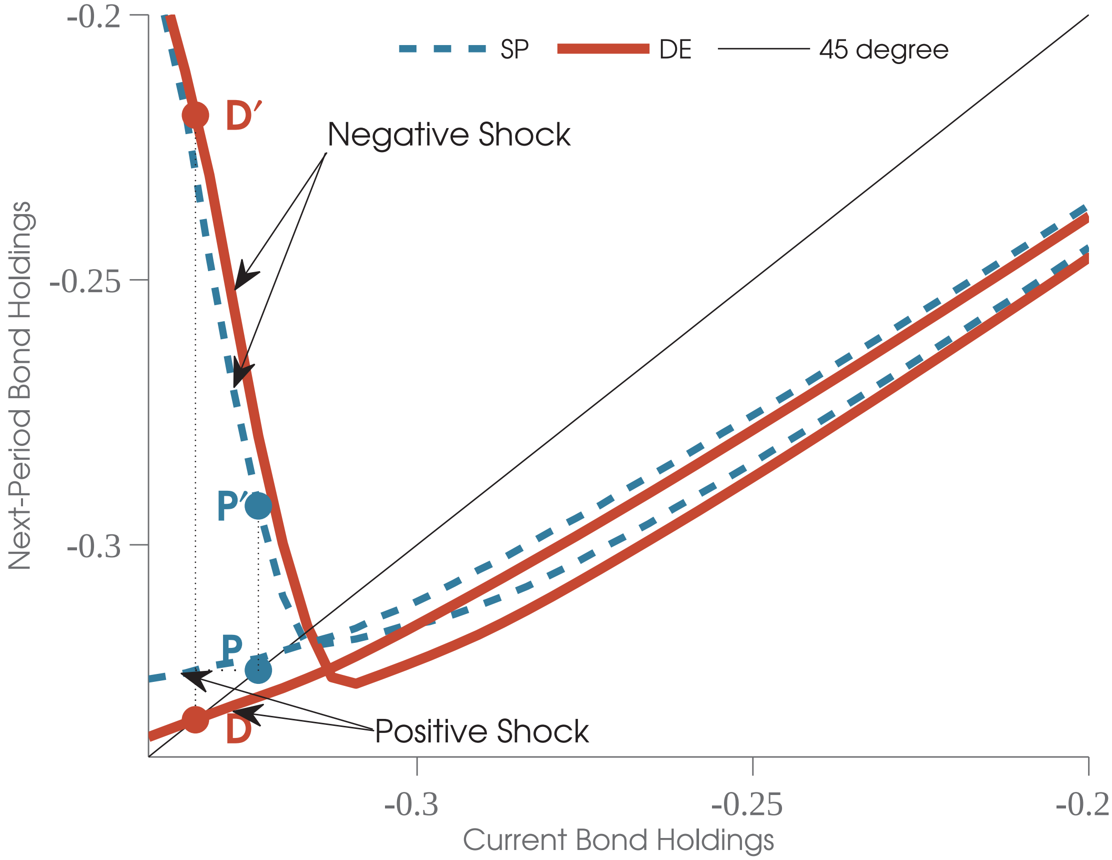

Figure 3: Amplification dynamics in response to adverse shocks.

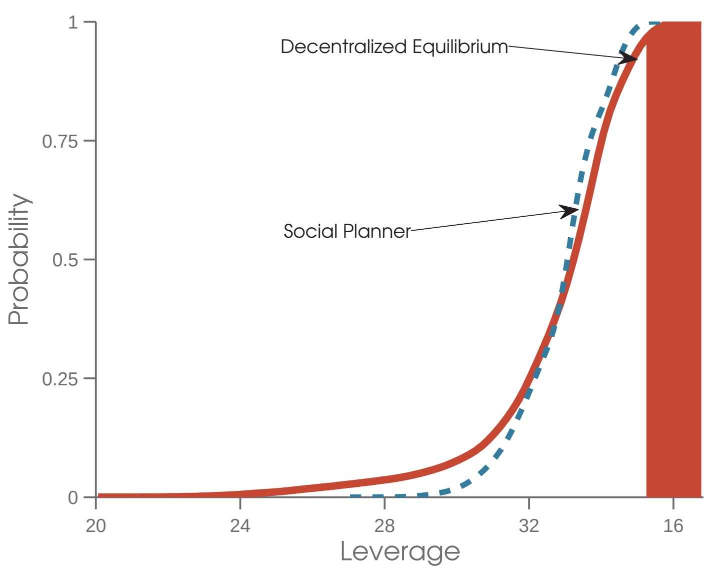

Figure 4: Ergodic Cumulative Distribution of Leverage $\left(\frac{-b_{t+1}+\theta p_\nu \nu_t}{q_t}\right)$

leverage ratios than in the DE. In fact, the leverage ratio in the DE exceeds the maximum of observable under the SP close to 4 percent of the time. Comparing averages across these ergodic distributions, however, mean leverage ratios differ by less than 1 percent. Hence, overborrowing may seem a relatively minor problem when comparing unconditional long-run averages of leverage ratios. Behind this apparently negligible difference hide the large differences in crises probabilities and magnitudes that result from the strongly nonlinear dynamics of financial amplification in the DE. Note also that these same features result in a longer left tail in the DE's distribution due to the deleveraging that occurs following a crisis. In the SP equilibrium the leverage ratio is never below 28 percent with positive probability, whereas in the DE leverage ratios near 25 percent are still positive-probability states.

5.4 Asset Pricing

The forward-looking nature of asset prices and their determinants can be illustrated by looking at their recursive functions in "good" states of nature. Figure 5 shows plots of six key asset pricing variables as functions of B in the DE and SP economies when TFP and the interest rate take their average values and $\kappa = \kappa^H$ (this is in contrast with Figure 2, which plotted policy functions for a "bad" state with low TFP and κ^L). The variables plotted are the expected return on assets, the price of assets, the Sharpe ratio, the volatility of returns, the risk premium, and the price of risk.

In this Figure, for sufficiently high initial values of B that pertain to the stable credit region (with average TFP and R, and $\kappa = \kappa^H$), all the variables plotted except asset prices (panel (b)) are almost the same for the DE and SP economies. This is because from those initial states at date t the collateral constraint is not expected to bind at $t+1$. In contrast, for lower initial B that pertains to the region with a positive probability of a crisis at $t+1$, expected returns are higher in the SP than the DE (panel (a)), and the opposite is true for Sharpe ratios, return volatilities, risk premia and the price of risk (panels (c)-(f)). The higher expected return is due to the fact that in this region the SP is actively using the macroprudential debt tax (which is used only when $E[\mu_{t+1} > 0]$), and by arbitrage of returns between assets and bonds this tax increases the expected return on assets. The higher risk premia, return volatilities, risk prices and Sharpe ratios, which reflect higher order moments, are due to the fact that the DE is significantly more risky than the SP economy. Thus, there is a clear over-compensation for risk-taking in the former relative to the latter.

The higher expected returns in the positive crisis probability region of panel (a) contribute to explain the uniformly lower asset prices of the SP relative to the DE *for all* the domain of B in panel (b). This occurs because even in states where B is in the stable credit region as of an initial date t, there are sequences of shocks and bond decisions that can move the economy to the region in which expected returns are higher for the SP at some future $t+j$, and as explained in Section 2 higher expected returns for those future dates imply heavier discounting of future dividends and

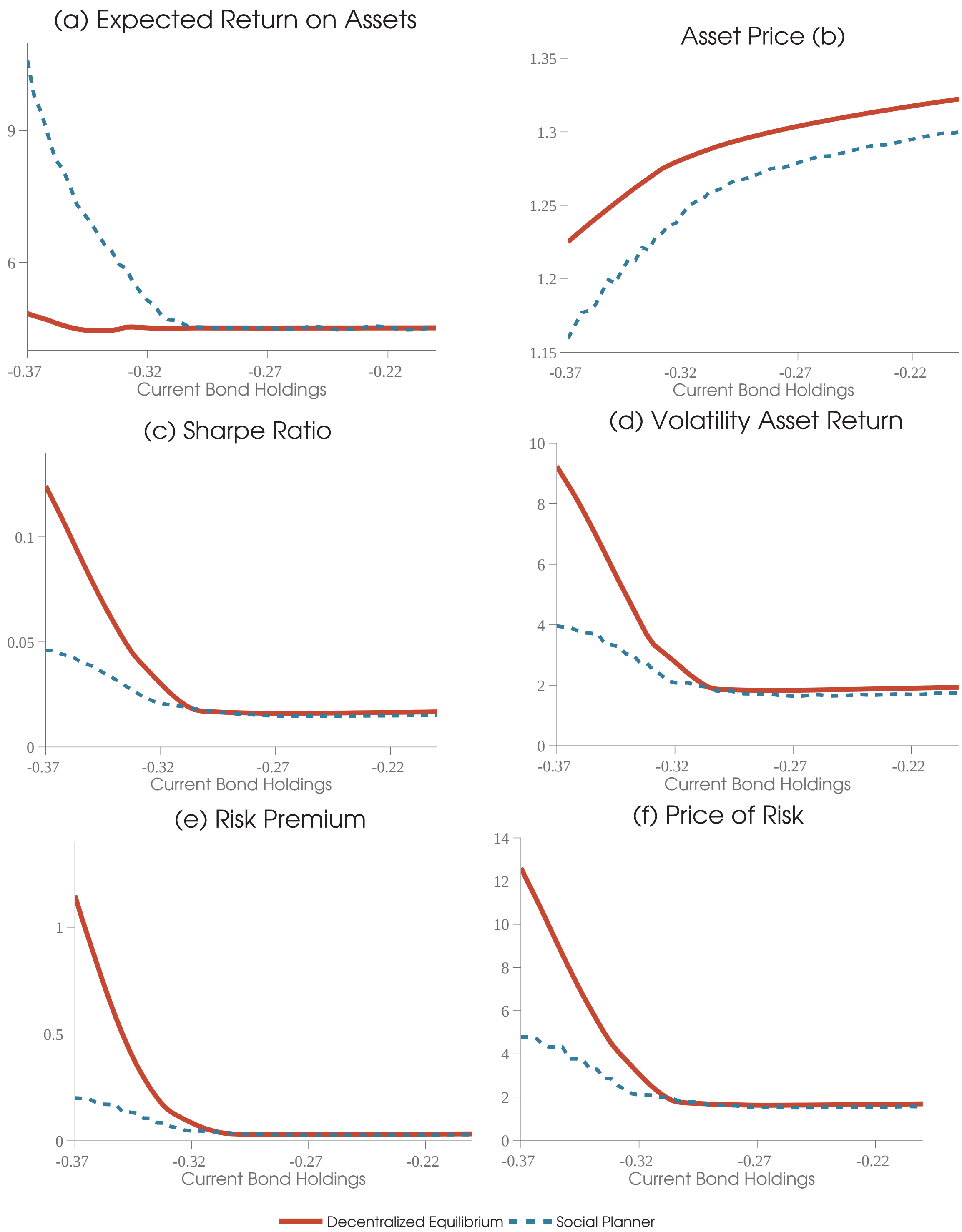

Figure 5: Asset Pricing Variables in “Good” States of Nature

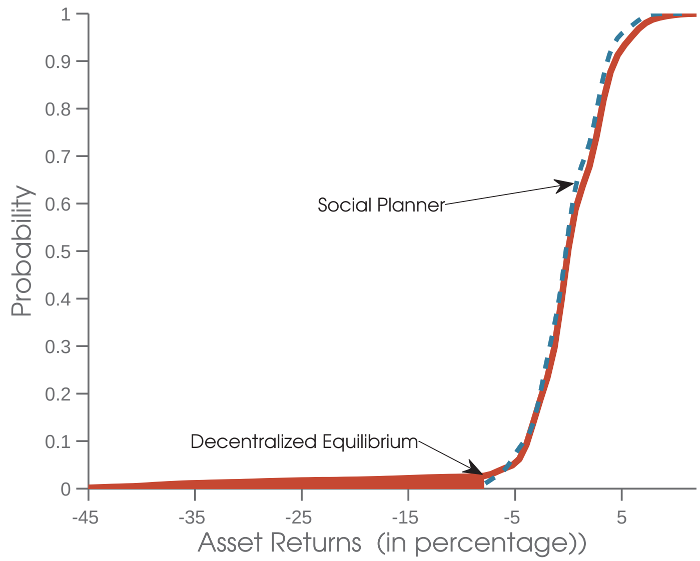

Figure 6: Ergodic Cumulative Distribution of Realized Asset Returns

thus lower asset prices at t.

The higher risk premia, Sharpe ratios and return volatilities in the DE than the SP should in principle push asset prices down by reducing excess returns. At equilibrium, however, this effect is more than offset by the first-order effect of the SP's debt tax, which as explained above increases expected returns and moves prices lower in the SP than the DE.

It is also important to note in Figure 5 the significant nonlinearities in the asset pricing variables within the DE itself (and keeping in mind we are looking at these variables for realizations of shocks in a "good" state as of date t). In particular, in the region with a positive probability of a crisis at $t+1$, the Sharpe ratio, return volatility, risk premium and price of risk are steep decreasing functions of B, while they are virtually flat in the stable credit region. Similarly, asset prices are a steeper function of B in the positive crisis probability region than in the stable credit region.

Figure 6 shows the long-run distributions of realized asset returns for the DE and SP. The distribution for the former displays a sizable fat tail at the low end that is not present in the latter. The 99th percentile of returns is about -30.5 percent in the DE v. -3.6 percent for the regulator. The fat tail in the DE case corresponds to states in which negative shocks hit when agents have a relatively high level of debt. Intuitively, the standard effect of negative shocks reducing expected dividends and putting downward pressure on asset returns is amplified by the effect of asset fire-sales that occur if the collateral constraint becomes binding.

The fat tail of the distribution of asset returns in the DE, and its substantial effects on the risk premium due to the associated time-varying risk of financial crises, are important results because they are an endogenous equilibrium outcome resulting from the non-linear asset pricing dynamics in the DE when the debt-deflation mechanism is at work. Fat tails in asset returns are also highlighted in the recent literature on asset pricing and "rare disasters" but this literature generally treats financial disasters as resulting from exogenous stochastic processes specified to feature fat tails and time-varying volatility.[32]

Table 2 reports statistics that characterize the main properties of asset pricing behavior in the DE and SP. The Table shows expected excess returns ($\mathbb{E}_t[R^q_{t+1}]$) in column (1) and its two components, namely the after-tax risk free rate and the equity premium (R^{ep}_t) in columns (2) and (3) respectively. Using eq. (29), R^{ep}_t is decomposed into the two components that result from the effect of collateral constraints that bind at t (column (4)) or are expected to bind at $t+1$ (column (5)), and the standard risk premium component given by the covariance between the stochastic discount factor and asset returns (column (6)). The equity premium in column (3) is equal to the sum of these three components. In addition, the Table reports the market price of risk (column (7)), the log standard deviation of returns (column (8)) and the Sharpe ratio (column (9)). All of these statistics are reported for the unconditional long-run distributions of each economy as well as for distributions conditional on the collateral constraint being binding and not binding.

Consider first unconditional expected returns. These are larger in the DE than in the SP by about 40 basis points (4.3 v. 3.9 percent). The optimal tax schedule makes the risk-free rate inclusive of tax higher in the SP than the DE (3.4 v. 2.8 percent) but the equity premium is three times larger in the DE than the SP. Since the difference in equity premium is larger than in the risk-free rates, the expected returns in the DE are higher.

Conditional on unconstrained states (i.e. $\mu_t = 0$), the mean expected return of the SP is higher because of the macroprudential debt tax that is levied in states in which $\mathbb{E}_t[\mu_{t+1}] > 0$. In contrast, conditional on constrained states ($\mu_t > 0$) the equity premium surges to nearly 47 percent in the DE and 27 percent in the SP. In the former this translates into large expected returns with a mean of nearly 50 percent, while in the latter the optimal tax schedule calls for a sizable subsidy that reduces the risk free rate inclusive of tax to -22 percent and lowers mean excess returns to 4.5 percent. This is in line with the SP's incentives to prop up asset prices to help sustain borrowing capacity when the collateral constraint is binding. The economy spends roughly 97 and 99.5 percent of the time in the unconstrained region the DE and SP economies respectively, so the unconditional moments weigh significantly more the unconstrained states than the constrained states. Moreover, while the optimal policy schedule calls for a substantial debt subsidy when $\mu_t > 0$, states in which this policy is actually implemented have near zero probability, so the debt subsidy is almost never used.

Next we examine the equity premium and its components. The unconditional average of R^{ep}_t is 1.5 percent in the DE v. 0.5 percent in the SP. The equity premium in the DE is relatively large, about half as large as the average of R (which is 2.8 percent), but much lower than empirical estimates which are in the 6-9 percent range (see Mehra and Prescott, 2003). Still, the sizable

[32]The literature on disasters typically uses Epstein-Zin preferences in order to match the large observed equity premia. Here, we use standard CRRA preferences with a risk aversion coefficient of 1.5, and as we show below, we can obtain much larger risk premia than in the typical CRRA setup without credit frictions.

Table 2: Asset Pricing Moments

	(1) Expected Return	(2) Risk-free Plus Tax	(3) Equity Premium	(4) Collateral Current	(5) Collateral Expected	(6) Risk Premium	(7) Price of Risk	(8) $\sigma_t(R^q_{t+1})$	(9) SR_t
			Decentralized Equilibrium						
Unconditional	4.3	2.8	1.5	1.3	-0.2	0.4	5.8	4.7	0.3
Constrained	49.7	2.8	46.9	46.9	-0.007	0.06	2.0	3.1	15.0
Unconstrained	2.9	2.8	0.1	0.0	-0.25	0.37	5.9	4.7	0.03
			Social Planner						
Unconditional	3.9	3.4	0.5	0.6	-0.17	0.07	2.6	2.6	0.2
Constrained	4.5	-22.1	26.6	27.4	-0.99	0.24	4.8	4.3	6.2
Unconstrained	3.9	4.0	-0.09	0.0	-0.15	0.07	2.6	2.6	-0.03

Note: The Sharpe ratio for each row is computed as the average of all corresponding equilibrium realizations of excess returns divided by the standard deviation of excess returns. All figures except the Sharpe ratios are in percent.

equity premium in the DE contrasts sharply with existing findings (e.g. Heaton and Lucas, 1996) showing that credit frictions without the Fisherian deflation mechanism do not produce large premia.[33] In both the DE and the SP, the unconditional equity premium is dominated by the component that reflects the effect of contemporaneously binding collateral constraints, while the effects of expected binding collateral constraints and the standard risk premium tend to offset each other. Note, however, that the risk premium term at 0.4 percent still plays an important role in the DE, and is significantly larger than in the SP.

Conditional on constrained states, the equity premium is driven by the large current collateral effects present when $\mu_t > 0$, which dwarf the other two components of the equity premium in both the DE and the SP. Conditional on unconstrained states, the current collateral effects are absent by construction, because the unconstrained states are those with $\mu_t = 0$. In these states, while the risk premium is still near 0.4 percent in the DE, the effect of future binding constraints reduces the equity premium to 0.1. For SP, this effect turns the equity premium into marginally negative.

With regard to the decomposition of the equity premium in terms of the Sharpe ratio and the volatility of returns in eq. (30), the main result from Table 2 is the higher Sharpe ratios in the DE v. the SP. The unconditional Sharpe ratio of the DE is about 0.3 v. 0.2 in the SP, and conditional on being credit constrained the DE Sharpe ratio is 15 v. 6.2 in the SP. Hence, risk-taking is "overcompensated" in the competitive equilibrium relative to the compensation it receives under the optimal policy. Interestingly, the unconditional Sharpe ratio in the DE is similar to U.S. data

[33] He and Krishnamurthy (2013) also found that financial frictions can produce large equity premia, close to 4 percent, in a model where specialists face an equity constraint. However, their result uses an exogenous income process with a standard deviation of 9 percent, whereas in our model TFP shocks have a standard deviation of 1.4 percent and GDP fluctuates 2.5 percent. The effect of credit constraints on asset pricing is also emphasized by Alvarez and Jermann (2001). Both of these studies, however, do not investigate the effects of credit frictions on asset pricing from a constrained-efficient standpoint.

estimates of around 0.3 (Campbell, 2003). Notice that the unconditional equity premium is larger in the DE, as noted above, but returns are also more variable. Still, the unconditional Sharpe ratio is higher in the DE than the SP because the equity premium rises proportionally more than the variability of returns.

The unconditional average of the price of risk and the average conditional on being unconstrained are about twice as large in the DE than in the SP. This reflects the fact that consumption, and therefore the pricing kernel, fluctuate significantly more in the former than in the latter. Conditional on being constrained, the opposite occurs and the price of risk is about twice as large in the SP than in the DE.

5.5 Macroprudential Debt Tax

We now study the characteristics of the macroprudential debt tax. As defined in Section 2, this tax is levied in "good times" when the collateral constraint is not binding but debt and leverage are sufficiently high for it to become binding the following period with positive probability.

Figure 7 shows two plots that illustrate the main features of the macroprudential debt tax. Panel (a) shows the tax schedule as a function of B for the same "good" states of TFP, R and κ used in Figure 5. Panel (b) shows the simulated event analysis evolution of the tax around financial crises episodes.

Panel (a) shows that when B is high enough to be in the stable credit region, the tax is zero because the probability of hitting the collateral constraint at $t+1$ is also zero. For $B < -0.3$, the constraint is still not binding at t, given the "good" states of TFP, R and κ, but the economy enters into the positive-crisis-probability region, where the constraint could bind at $t+1$ in some states of nature. In this region, the tax is higher at lower B (i.e, it is increasing in the level of current debt), because this makes it more likely that the constraint will become binding at $t+1$, and that if it does it will be more binding than at higher values of B. The tax can be as high as 5.5 percent when debt is about 37 percent.

Panel (b) shows that the tax is positive and rising rapidly in the four years prior to the financial crisis. Recall from Figure 1 that these are also the years in which credit and leverage would be rising in the unregulated DE. Hence, the policy is taxing debt to reduce the overborrowing that would otherwise occur when private agents fail to internalize the pecuniary externality, so as to mitigate the magnitude of a financial crisis if it occurs. The tax peaks at about 4.5 percent in the year just before the crisis hits. When the crisis starts the tax is zero, because at this point $\mu_t > 0$ and hence the prudential aspect of the policy vanishes, and remains close to zero one year after the crisis. In the second year after the crisis and beyond the tax rises gradually to about 1 percent at $t+4$.

The long-run average of the macroprudential debt tax is about 1.1 percent. It has a standard deviation equal to about 2/3rds of the standard deviation of GDP, and a positive correlation

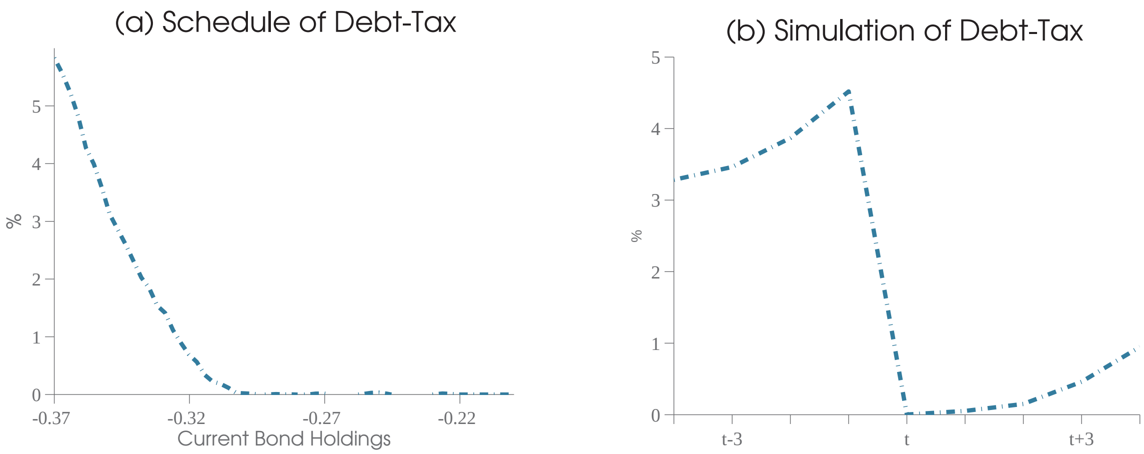

Figure 7: Macroprudential Debt Tax: Schedule in "Good States" and Event Analysis

of 0.8 with the leverage ratio. This is consistent with the prudential rationale behind the tax: The tax is high when leverage is building up and low when the economy is deleveraging. Note, however, that since leverage itself is negatively correlated with GDP, the tax also has a negative GDP correlation. Finally, the tax also has a positive correlation with credit conditions measured by κ_t, again in line with arguments often used to favor macroprudential policy.

It is important to note that Jeanne and Korinek (2010) also computed a schedule of macroprudential taxes on debt to correct a similar pecuniary externality due to a collateral constraint that depends on asset prices. Their findings, however, are quite different. In particular, their results show that the debt taxes lessen the effects of financial crises much less than in our setup and have no effect on the probability of crises. These different results are due to key differences in the structure of the borrowing constraints, the behavior of output, and the design of the quantitative experiments. In their setup, the credit constraint is determined by the aggregate level of assets $\bar{K}$ and by a linear state- and time-invariant term ψ (i.e. their constraint is defined as $\frac{b_{t+1}}{R} \geq -\kappa q_t \bar{K} - \psi$). The fact that this constraint depends on aggregate rather than individual asset holdings, as in our model, matters because it implies that agents do not value additional asset holdings as a mechanism to manage their borrowing ability. But more importantly, in their quantitative analysis they calibrate parameters to $\kappa = 0.046$, $\psi = 3.07$ and $q_t K = 4.8$, which imply that the effects of the credit constraint are driven mainly by ψ, and only 7 percent of the borrowing ability depends on the value of assets (0.07=0.046*4.8/(0.046*4.8+3.07)). As a result, the Fisherian deflation effect and the pecuniary externality are weak, and thus macroprudential policy cannot be very effective at containing financial crises. The asset price drop is reduced from 12.3 to 10.3 percent, and the consumption drop is reduced from 6.2 to 5.2 percent (compared with declines from 30 to 6 and 16 to 4 percent respectively in our model). Moreover, since they model output (or dividends) as an exogenous, regime-switching Markov process such that the probability of a crisis (i.e. binding credit constraint) coincides with the probability of a bad output realization, macroprudential policy cannot affect the probability of crises.

5.6 Welfare Effects

We study the welfare implications of the optimal policy by calculating welfare gains as compensating consumption variations for each initial state (B, s) that equalize expected utility under the allocations of the DE and those attained with the SP's optimal policy (both conditional on that (B, s)). Formally, for a given initial state (B, s) at date 0, the welfare gain of the optimal policy is computed as the value of γ_0 that satisfies this condition:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{DE}(1+\gamma_0) - G(h_t^{DE})) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^{SP} - G(h_t^{SP})) \tag{34}$$

The welfare gains of the optimal policy arise from two sources. The first source is the reduced variability of consumption in the SP v. the DE, due to the fact that the credit constraint binds more often in the DE, and when it binds it induces a larger adjustment in asset prices and consumption. The second is the efficiency loss in production that occurs in the DE due to the effect of the credit friction on working capital. Without the binding collateral constraint limiting access to working capital, the marginal productivity of imported inputs equals their constant price and hence the allocation of these inputs is efficient. With the working capital constraint, however, the shadow marginal cost of these inputs rises when the constraint binds, and this drives a wedge between their marginal product and their market price. Again, since the collateral constraint binds more often in the DE than in the SP, this implies a larger efficiency loss in the former.

The welfare gains of the optimal policy are illustrated in Figure 8. Panel (a) shows the state-contingent schedule of welfare gains as a function of B for the "good" state of nature with average TFP, average R, and κ^H. Panel (b) shows the event-analysis dynamics of the welfare gains around financial crises events. In looking at these results, keep in mind that overall the values of γ are small because the model is in the class of stationary-consumption, representative-agent models with CRRA preferences that produce small welfare effects due to consumption variability (see Lucas, 1987). Moreover, although the efficiency loss in production at work when the collateral constraint binds can be relatively large and add to the welfare effects, this is only a low probability event because of the low probability of hitting the constraint.

The schedule of welfare gains as a function of B in panel (a) is bell-shaped in the region with a positive probability of crisis at $t+1$. It rises sharply as B rises from -0.36, peaking at about a 0.1 welfare gain when $B = -0.35$, and then falls gradually. The welfare gains continue to fall gradually, and almost linearly, as B moves into the stable credit region, and fall to about 0.07 percent when $B = -0.3$. This pattern is due to the differences in the optimal plans of the regulator vis-a-vis private agents in the DE. In the region where a crisis is possible at $t+1$, the SP's allocations projected as of date t for the future differ sharply from those of the DE, because of the latter's higher magnitude and frequency of crises, and this generally enlarges the welfare gains of the optimal policy (or the losses due to the pecuniary externality). Notice that, since the

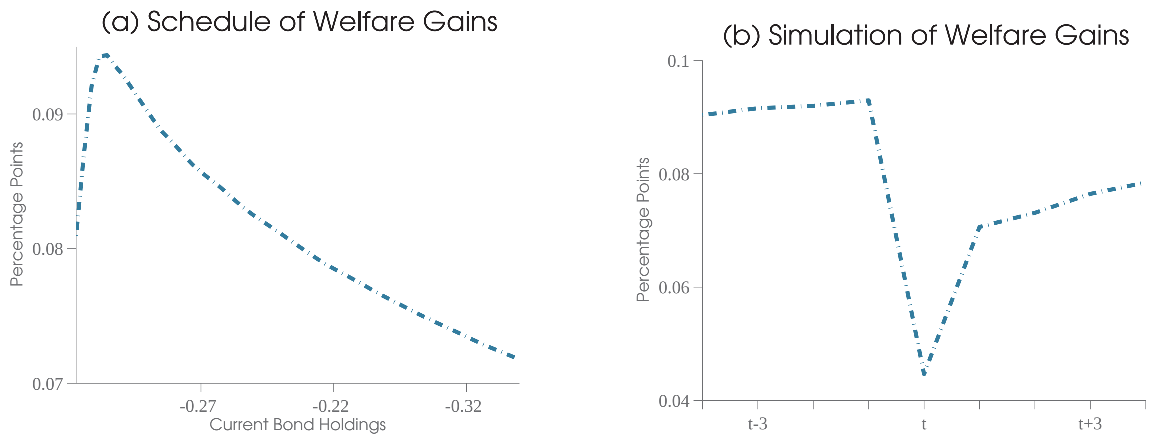

Figure 8: Welfare Gains: Schedule in "Good States" and Event Analysis Dynamics

regulator's allocations involve more savings and less current consumption, there are welfare losses in terms of current utility for the regulator, but these are far outweighed by less vulnerability to sharp decreases in future consumption during financial crises. As the level of debt falls and the economy enters the stable credit region, financial crises are unlikely at $t + 1$, or are likely much further into the future, and thus the welfare gains of the policy (or the costs of the externality) decrease.

Panel (b) shows that the welfare gains of the optimal policy are fairly stable in the four years before the crisis, increasing only slightly from about 0.09 percent at $t - 4$. When the crisis hits the welfare gain drops to close to 0.04 percent, because by then the crisis has arrived and the prudential aspect of the optimal policy is less valuable, but right after the crisis the welfare gain increases sharply and continues to increase 4 years late. The unconditional average welfare gain computed using the DE's ergodic distribution of bonds and exogenous shocks is equal to a 0.09 percent compensating variation in consumption.

5.7 Simple Policies

The state-contingent nature of the optimal debt tax policy raises a familiar criticism posed in the context of Ramsey optimal policy analysis: State-contingent policy schedules are impractical because of the evident limited flexibility of policy-making institutions to adhere to complex, predetermined, time-varying rules for adjusting policy instruments.[34] Moreover, there is also a broader concern over the ability of regulators to keep close track of financial conditions and fine-tune macroprudential policy as needed in a timely fashion (see Cochrane, 2013). For instance, in the case of our optimal macroprudential debt tax, as we documented earlier it varies widely with a standard deviation of 2.2 percent, up to a maximum around 5 percent, and is positively correlated

[34] A related criticism is that Ramsey optimal policies are generally time-inconsistent, but this does not apply to our findings because we solved for the optimal time-consistent policy of a financial regulator who is unable to commit to future policies.

with the leverage ratio.

In light of the above concerns, we examine here the performance of policy rules that are simpler than the optimal policy. In particular, we study the effects of a fixed debt tax constant across time and states of nature. While the optimal policy calls for *increasing* τ^{MP} when leverage is building up, this simpler policy restricts the government to keep the debt tax fixed.

The results show that while fixed debt taxes still reduce the probability and severity of financial crises, they have markedly different welfare implications across states of nature. In particular, fixed taxes still produce welfare gains when computed conditional on initial states for a date t in which the constraint is not binding, but can produce welfare losses if the constraint is binding at t. This is due to the negative short-run effects of debt taxes on asset prices and the tightness of the collateral constraint: Debt taxes have a negative effect on asset prices because the increased cost of borrowing shifts demand from assets to bonds. There is also a positive effect due to a reduction in the riskiness of assets, but this effect is dominated by the first-order effect of taxes on the relative demand for bonds.

Figure 9 shows two plots that illustrate the effects of fixed debt taxes ranging from 0 to 1 percent on the long-run probability of financial crises (panel(a)) and social welfare (panel (b)). With regard to the former, the constant taxes prove very effective for sharply reducing the likelihood of crises from 3 to 1 percent. This is natural because as debt is taxed more, agents build less leverage and are less vulnerable to crises, but this does not mean that they are necessarily better off.

Panel (b) shows welfare gains for constant taxes in the 0 to 1 percent range measured relative to the DE allocations. The plot shows the average, maximum and minimum welfare gain under each tax, and for comparison it also shows the average obtained under the SP (with the optimal state-contingent tax schedule). This plot illustrates two main results. First, fixed taxes are always inferior to the optimal policy. This is reflected in the fact that the maximum (average) welfare gain of fixed taxes peaks at about 0.03 (0.02) percent with a tax of 0.6 percent, which are gains about 1/3rd the size of the SP's average welfare gain.[35] Second, fixed taxes can produce welfare losses (negative gains). Fixed taxes always induce welfare losses in a subset of the state space, as illustrated by the fact that the minimum welfare gain is always negative for all debt taxes in panel (b), and this subset grows as the fixed tax rate rises (the minimum welfare gain drops to -0.25 percent as the tax approaches 1 percent). When the subset of the state space with welfare losses is large enough, the average welfare gain also turns negative. This occurs for debt taxes above 0.82 percent.

Fixed taxes can produce welfare losses because they deliver lower asset prices, which make crises worse, when the collateral constraint binds. This suggests that a regulator constrained to consider only taxes of this form would want to trade off the prudential benefit of the taxes

[35] The fact that fixed taxes can deliver welfare gains is related to Bianchi (2011), but the trade-off we highlight here is not present in his setup because there are no adverse effects from current or future taxes on borrowing capacity.

in restraining credit growth in good times against their adverse effects in financial crises events. This tradeoff is clearly reflected in panel (b), which indicates that the optimal fixed tax would be about 0.6 percent. This fixed tax is roughly 50 basis points smaller than the average of the optimal macroprudential tax, and it achieves about 30 percent of the welfare gains obtained with the optimal policy.

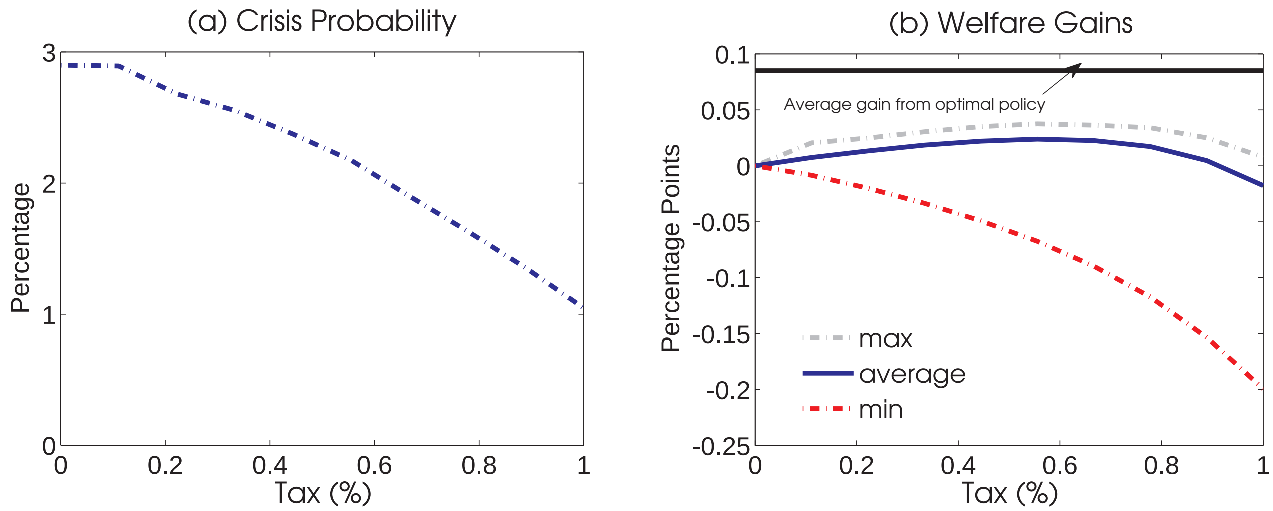

Figure 9: Effects of Fixed Debt Taxes on Probability of Crises and Welfare

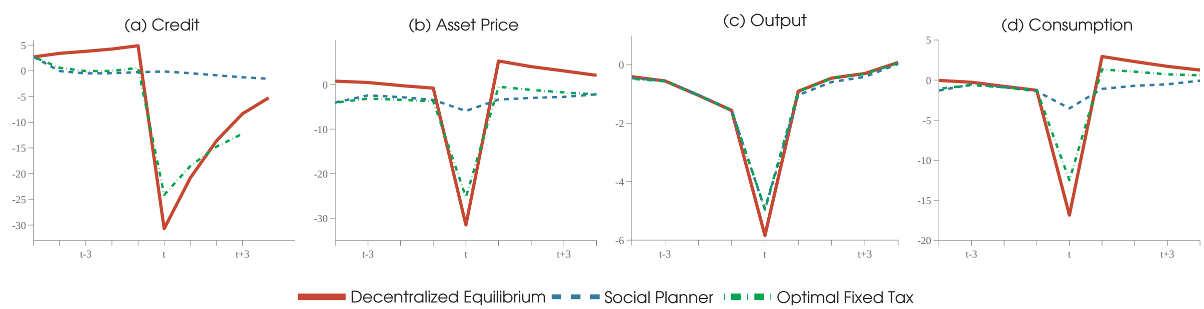

Figure 10: Event Analysis: Decentralized Equilibrium, Optimal Policy and Fixed Tax

Performing event analysis of financial crises events for the welfare-maximizing, on average, fixed debt tax (0.6 percent) we found that the stabilizing effects of taxing debt are still present, although weaker than under the optimal policy (compare Figure 10 v. Figure 1). With the fixed tax set at 0.6 percent, the probability of financial crises drops from 3 percent in the DE to about 2 percent, and the declines in credit, asset prices, and consumption are still smaller than in the DE (e.g. asset prices fall 24 percent with the fixed tax v. 31 percent in the DE).

Overall, the above results show that our findings highlight both the potential benefits of macroprudential policies as well as their dangers: If institutional constraints limit the ability of regulators to optimally fine-tune macroprudential policy instruments, it is possible to end up with environments in which the policy is welfare-reducing. In particular, maintaining time-invariant debt taxes or other credit management tools at play in crisis times can be welfare reducing due to their adverse effects on asset prices.

5.8 Sensitivity

Table 3 reports the results of a sensitivity analysis in which we show how the main results change as we vary the coefficient of relative risk aversion, the value of κ^L, and the working capital coefficient θ, and also as a result of modifying the model to consider an environment in which the real interest rate varies with the level of debt (as an approximation to a setup with an endogenous interest rate). The table shows the unconditional averages of the macroprudential debt tax and welfare gains implied by the implementation of the SP's optimal policy, the crises probabilities under the DE and SP, the average size of the asset price collapses in simulated financial crises events of both economies, and the unconditional risk premia. In this Table, differences across the DE and SP columns reflect how the parameter changes affect the effects of the externality and the Fisherian amplification, while differences across rows, in a given DE or SP column, reflect how the parameter changes affect each equilibrium.

The results of this sensitivity analysis, particularly those for the macroprudential debt tax, can be understood more easily by referring to the expression that defines τ^{MP} (eq. (33)). For given R, τ^{MP} is given by the expected sum of two terms. These terms capture the effect of the pecuniary externality on borrowing capacity for working capital (the first term) and intertemporal debt (the second term). The first term is quantitatively small because the choice of intermediate inputs does not differ significantly between the SP and DE. The second term, which can be expressed as $-\kappa_{t+1}\mu_{t+1}u''(t+1)q_{t+1}/u'(t+1)$, is the main quantitative determinant of the debt tax, and as explained earlier this term is the product of the shadow value of relaxing the credit constraint multiplied by the effect on the value of collateral (i.e. asset prices) resulting from changes in consumption allocations.

The results in Table 3 show that the coefficient of relative risk aversion σ is a key parameter, because altering the curvature of the utility function has first- and higher-order effects that affect how asset prices change when consumption changes, which as explained above are key for the size of the optimal debt tax. Given a sequence of future prices and consumption, a drop in current consumption leads to a larger drop in asset prices the higher is σ. A high σ also makes the stochastic discount factor more sensitive to changes in consumption, and therefore makes asset prices react more to changes in bond holdings. Accordingly, rising σ from 1.5 in the baseline to 2 rises the welfare cost of the credit externality by about 40 percent, and widens the differences in risk premia across the SP and DE. Stronger precautionary savings reduce the probability of crises in the DE, and financial crises become a zero-probability event in the SP. In line with these changes, the average of τ^{MP} increases slightly to 1.2 percent. Conversely, reducing σ to 1 weakens the externality, which results in smaller gaps in the crisis impact effect on asset prices and risk

Table 3: Sensitivity Analysis

	Macroprudential Debt Tax	Welfare Gains	Probability of Crises DE	SP	Asset Price Drop DE	SP	Risk Premium DE	SP
Baseline	1.1	0.09	2.9	0.05	-28.6	-5.6	0.37	0.07
$\sigma = 2$	1.2	0.148	1.3	0.000	-58.9	-6.9	0.64	0.12
$\sigma = 1$	0.8	0.034	4.6	0.008	-11.9	-5.0	0.17	0.05
$\kappa^L = 0.32$	1.1	0.12	1.9	0.01	-32.5	-5.0	0.44	0.07
$\kappa^L = 0.28$	1.1	0.07	3.0	0.05	-19.7	-5.6	0.30	0.07
$\theta = 0.45$	1.1	0.08	2.6	0.01	-25.9	-4.8	0.34	0.06
Endogenous R	1.3	0.13	2.3	0.00	-17.6	-7.3	0.36	0.05

premia across DE and SP, and a smaller welfare cost of the externality. The probability of crises rises, since the incentives for precautionary savings are weaker, but the crises that occur are much less severe. The result of these changes is that the optimal policy of the SP can be decentralized with a lower average debt tax of 0.8 percent.

The value of κ^L also plays an important role because it alters the amount by which asset price changes affect borrowing ability (since κ is a key determinant of the pecuniary externality on intertemporal debt noted above). Increasing κ^L for a given price response when the constraint is binding implies a larger change in borrowing ability, which makes the externality stronger. On the other hand, a higher κ^L makes the constraint less likely to bind, which weakens the externality (since it depends on the expectation that $\mu_{t+1} > 0$). The effects of changes in κ^L are clearly non-monotonic. If $\kappa^L = 0$, there is no effect of prices on borrowing ability. At the same time, for high enough values of κ^L, the constraint never binds (for sure as κ^L approaches κ^H since the latter never binds by construction). In both cases, there is no Fisherian deflation and the externality does not play any role.

Quantitatively, Table 3 shows that small changes in κ^L around the baseline value are positively associated with the size of the externality. The average tax does not change noticeably, but the average welfare gain is higher with the higher κ^L. The probability of crisis is lower in the DE relative to the baseline, but the size of the asset price drop during a crisis and the average risk premium are higher. These results suggest that while increasing credit access by rising κ^L may increase welfare relative to a more financially constrained environment, rising κ^L also makes macroprudential policy more desirable.

To examine the effects of varying θ, we report results for a case reducing θ from 0.5 in the baseline to 0.45, but keeping the two experiments comparable by adjusting κ so as to obtain the same average borrowing ability in terms of one-period bonds (this requires a slight reduction in κ^L to 0.3 to 0.295). The effects of this small reduction in θ have a negligible impact on the results relative to the baseline. It is also obvious, however, that a large reduction in this coefficient would not be as neutral. In the limit, with $\theta = 0$ the model would not have a mechanism linking the supply-side of the economy with the financial transmission mechanism. When a financial crisis hits, output and factor allocations would remain the same as in a frictionless RBC model, and also the stream of dividends entering in the asset pricing function would become independent of whether the collateral constraint binds or not.

In the last sensitivity experiment we modify the model to relax the assumption of a perfectly elastic supply of funds at a constant rate R. We introduce an exogenous function making the real interest rate vary with aggregate bond holdings. In particular, we assume that the net interest rate is now given by $r(B') = r_t - \varrho(e^{-(B'-\bar{B})} - 1)$, where $\bar{B}$ denotes the average value of bond holdings.[36] With $\varrho > 0$, the interest rate increases with the debt of the economy. In principle, this could work to attenuate the Fisherian deflation and the pecuniary externality, because of the endogenous self-correcting mechanism increasing the cost of borrowing as debt increases, but we found that quantitatively this did not result in large changes relative to the baseline.[37] This is shown in the last row of Table 3 for a value of $\varrho = 0.05$. With this value of ϱ, the real interest rate reaches a minimum of -1.5 percent in the simulations which is around the minimum value observed in the data between 1980 and 2012.

6 Conclusions

This paper examined the positive and normative effects of a pecuniary externality that arises in a dynamic stochastic general equilibrium model because a collateral constraint limits access to debt and working capital to a fraction of the market value of an asset in fixed supply. We compared the allocations, prices, and welfare attained by private agents in a competitive equilibrium in which agents face this constraint taking prices as given, with those attained by a constrained-efficient financial regulator who lacks the ability to commit to future policies. This regulator faces the same borrowing limits as private agents but takes into account how its current borrowing choices affect asset prices and the borrowing decisions of the regulators in future periods.

The regulator's inability to commit is important because it implies that the optimal macro-

[36] This specification is proposed by Schmitt-Grohe and Uribe (2003) to avoid the problem with the unit root in net foreign assets that arises when using perturbation methods to solve small open economy models. In our model, its purpose is only to approximate what would happen if the interest rate could respond to debt choices in a richer general equilibrium model.

[37] The average macroprudential debt tax and welfare gain are higher because now the planner also internalizes how borrowing affects the interest rate, which is taken as exogenous by individual agents. The welfare gains of moving from the constrained efficient equilibrium with an exogenous interest rate to the decentralized equilibrium with the endogenous interest rate are about the same as in the baseline, which again suggests that the exogeneity of the interest rate does not have significant effects on the quantitative results.

prudential policy we study is endogenously time-consistent. Under commitment, the regulator would be tempted to promise lower future consumption to prop up current asset prices when the constraint binds, and then renege when the future arrives. Up to now, time-inconsistency had been set aside in the literature on macroprudential policy by setting up problems without the forward-looking elements that drive time inconsistency under commitment (which emerge naturally in asset pricing models), or imposed exogenously by assuming constraints on the ability of so-called conditionally efficient financial regulators to alter asset pricing functions of unregulated equilibria (e.g. Bianchi and Mendoza, 2010 and Jeanne and Korinek, 2010).

The financial regulator internalizes the debt-deflation process that drives macroeconomic dynamics during financial crises, and hence borrows less in periods in which the collateral constraint does not bind, so as to weaken the debt-deflation process in the states in which the constraint becomes binding. Conversely, private agents overborrow in periods in which the constraint does not bind, and hence are exposed to the stronger adverse effects of the debt-deflation mechanism when a financial crisis occurs. Moreover, the regulator responds to the incentives implied by its inability to commit, thus internalizing how the debt choices of future regulators, and the associated future consumption allocations and asset prices, respond to current borrowing decisions.

We also show that the allocations of the optimal, time-consistent financial regulator can be decentralized by means of a state-contingent schedule of debt taxes. A key component of this tax schedule is a macroprudential debt tax that is positive in states in which the collateral constraint does not bind but may become binding in the following period. As Bianchi (2011) showed, however, policies like capital requirements or loan to value ratios can be used with equivalent results as macroprudential debt taxes.

Our analysis quantifies the effects of the debt deflation process and the pecuniary externality in a setup in which the credit friction has effects on both aggregate demand and supply. On the demand side, consumption drops as access to debt becomes constrained, and this induces an endogenous increase in excess returns that leads to a decline in asset prices. Because collateral is valued at market prices, the drop in asset prices tightens the collateral constraint further and leads to fire-sales of assets and a spiraling decline in asset prices, consumption and debt. On the supply side, production and demand for imported inputs are affected by the collateral constraint, because firms buy the latter using working capital loans that are limited by the collateral constraint. Hence, when the constraint binds the effective cost of imported inputs rises, so the demand for these inputs and output drops. This affects dividend rates and hence feeds back into asset prices. Previous studies in the Macro/Finance literature have shown how these mechanisms can produce financial crises with features similar to actual financial crises, but the literature had not conducted a quantitative analysis comparing regulated v. competitive equilibria in an equilibrium model of business cycles and asset prices.

We conducted a quantitative analysis in a version of the model calibrated to data for industrial countries. This analysis showed that, while the externality results in only slightly larger average ratios of debt and leverage to output compared with the regulator's allocations, the externality does produce financial crises that are significantly more severe and more frequent than in the regulated equilibrium. There are also important asset pricing implications. In particular, the externality and its associated higher macroeconomic volatility in the competitive equilibrium produce equity premia, Sharpe ratios, and market price of risk that are much larger than in the regulated economy.

In terms of the macroprudential debt tax, our results show that the optimal, time-consistent taxes can fully neutralize the externality and increase social welfare. The tax is about 1 percent on average, and positively correlated with leverage. Hence, the tax is higher when the economy is building up leverage and becoming vulnerable to a financial crisis, but before a crisis actually occurs, so as to induce private agents to value more the accumulation of precautionary savings than they do in the competitive equilibrium without regulation.

We also examine the implications of policies simpler than the state-contingent optimal tax, in the form of time- and state-invariant taxes. These fixed debt taxes can also reduce the severity and frequency of financial crises, although not as much as the optimal policy. Moreover, fixed debt taxes that remain in place in crises times need to tradeoff the prudential benefits of weakening the pecuniary externality in good times against the costs of making crises worse in bad times. Hence, the welfare-maximizing fixed tax is lower than the average optimal tax, and it produces an average welfare gain that is roughly 1/3rd the gain under the optimal policy.

We recognize that despite the positive findings of this paper about the potential benefits of macroprudential policy in terms of reducing the frequency and severity of financial crises, there are also important hurdles. One has to do with the complexity of actual financial markets vis-a-vis the simple structure of the model. In reality, there is a large set of financial constraints affecting borrowers in credit markets at the level of households, nonfinancial firms and financial intermediaries, and the regulator aiming for the optimal policy would thus be faced with severe informational requirements in terms of both coverage and timeliness of leverage and debt positions. A second important hurdle relates to incomplete information and financial innovation as the optimal macroprudential response could be different depending on the factors driving the credit cycle In Bianchi et al. (2012) we examined the limitations of macroprudential regulation in an environment in which financial innovation occurs but agents, including the regulator, are imperfectly informed about it.

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A Proofs

A.1 Proof of Proposition 1

Define the tax as:

$$1+\tau_t = \frac{\beta R \mathbb{E}_t \left\{ u'(\mathcal{C}(b_{t+1}, z_{t+1})) - \xi_{t+1} u''(\mathcal{C}(b_{t+1}, z_{t+1}))\mathcal{Q}(b_{t+1}, z_{t+1}) + \xi_t \Omega_{t+1} \right\} + \xi_t u''(c_t) q_t}{\beta R \mathbb{E}_t u'(\mathcal{C}(b_{t+1}, z_{t+1}))} \quad (35)$$

The constrained efficient equilibrium can be characterized by sequences $\{c_t, k_{t+1}, b_{t+1}, q_t, \lambda_t, \mu_t\}_{t=0}^{\infty}$ that satisfy (2), (14), (15), (16), $k_t = 1$ together with complementary slackness condition.

The regulated decentralized equilibrium is characterized by a sequence $\{c_t, k_{t+1}, b_{t+1}, q_t, \lambda_t, \mu_t\}_{t=0}^{\infty}$ that satisfy (2), (3), (4), (5),

$$u'(c_t) = \beta R (1+\tau_t) \mathbb{E}_t u'(c_{t+1}) + \mu_t, \quad (36)$$

together with complementary slackness conditions. Using the expression for the tax (35) and (36), yields condition (15) and identical conditions characterizing the two equilibria.

A.2 Quantitative Model: Construction of the Planner's Problem and Proof of Decentralization

Households in the constrained efficient allocations solve

$$\begin{aligned} & \max_{\{c_t, h_t, \nu_t, k_{t+1}\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t - G(h_t) \\ s.t. \quad c_t + q_t k_{t+1} &= k_t q_t + z_t F(k_t, h_t, v_t) - p_v v_t + T_t \\ \frac{b_{t+1}}{R} - \theta p_\nu \nu_t &\geq -\kappa_t q_t k_t \end{aligned}$$

This yields conditions

$$z_t F_h(k_t, h_t, \nu_t) = G'(h_t) \quad (37)$$

$$z_t F_\nu(k_t, h_t, \nu_t) = p_\nu(1 + \theta \mu_t / u'(t)) \quad (38)$$

$$q_t u'(t) = \beta \mathbb{E}_t \left\{ u'(t+1) \left(z_{t+1} F_k(k_{t+1}, h_{t+1}, \nu_{t+1}) + q_{t+1} \right) + \kappa q_{t+1} \mu_{t+1} \right\} \quad (39)$$

These equations together with market clearing in asset markets ($k_t = 1$) are the implementability constraints in the planner's problem 32.

Let υ_t be the multiplier on the complementary slackness condition and μ^* the multiplier on the collateral constraint and ζ_t^ν, ζ_t^h be the multiplier on the first order conditions with respect to

ν_t and h_t.[38] First-order conditions with respect to $c, b, h,\ q$ and μ and envelope condition are:

$$c_t: \quad \lambda_t = u'(t) + \frac{\zeta_t^{\nu} p_\nu \theta \mu_t u''(t)}{u'(t)^2} - \xi_t u''(t) \mathcal{Q}_t$$

$$b_{t+1}: \quad \lambda_t = \mu_t^* + \beta R \mathbb{E}_t (V_b(b_{t+1}, s_{t+1}) + \xi_t \hat{\Omega}_{t+1}) + \mu_t \upsilon_t$$

where $\hat{\Omega}_{t+1} \equiv u''(t+1)\mathcal{C}_b(b_{t+1}, s_{t+1})(Q_{t+1} + z_{t+1}F_k(t+1)) + u'(t+1)Q_b(b_{t+1}, s_{t+1}) + z_{t+1}F_h(t+1)\mathcal{H}_b(b_{t+1}, s_{t+1}) + F_\nu(t+1)\nu_b(b_{t+1}, s_{t+1}) + \mu_b(b_{t+1}, s_{t+1})$.

$$q_t: \quad \xi_t u'(t) = \kappa(\mu_t \upsilon_t + \mu_t^*)$$

$$\begin{aligned} h_t: \quad & u'(t)G'(h_t) = \lambda_t z_t F_h(t) + \zeta_t^h [G''(h) - z_t F_{hh}(t)] + \zeta_t^{\nu} \left[\frac{p_\nu \theta \mu_t u''(t)}{u'(t)^2} G'(h_t) \right. \\ & \left. - z_t F_\nu(_t, h_t, \nu_t) \right] + Q_t u''(t) G'(h) \xi_t \\ \nu_t: \quad & \lambda_t z_t F_\nu(t) = p_\nu[\lambda_t + \mu_t^* + \mu_t \nu_t] + \zeta_t^h z_t F_{h\nu}(t) + z_t F_{\nu\nu}(t) \\ \mu_t: \quad & \upsilon_t \left(\frac{b'}{R} - p_\nu \nu_t + \kappa \mathcal{Q}_t \right) = p_\nu \frac{\theta \mu_t}{u'(c_t - G(h_t))} \zeta_t^{\nu} \\ & V'(b_t) = \lambda_t \end{aligned}$$

The Euler condition for bonds is the following:

$$\begin{aligned} u'(t) \quad &= \quad \beta R \mathbb{E}_t \left[u'(t+1) + \frac{\zeta_{t+1} p_\nu \theta \mu_{t+1} u''(t+1)}{u'(t)^2} - \xi_{t+1} u''(t+1) + \xi_t \hat{\Omega}_{t+1} \right] \\ & \quad + \mu_t \upsilon_t - \frac{\zeta_t p_\nu \theta \mu_t u''(t)}{u'(t)^2} + \xi_t u''(t) q_t + \mu_t^* \end{aligned}$$

Following the proof in section A.1, using (40) we can show that the tax that decentralizes the constrained-efficient allocations is given by:

$$\begin{aligned} 1 + \tau_t \quad &= \quad \frac{1}{\beta \mathbb{E}_t u'(t+1)} \beta R \mathbb{E}_t \left[\frac{\zeta_{t+1} p_\nu \theta \mu_{t+1} u''(t+1)}{u'(t)^2} - \xi_{t+1} u''(t+1) \mathcal{Q}_{t+1} + \xi_t \hat{\Omega}_{t+1} \right] \\ & \quad + \frac{1}{\beta \mathbb{E}_t u'(t+1)} \left[\mu_t \upsilon_t - \frac{\zeta_t p_\nu \theta \mu_t u''(t)}{u'(t)^2} + \xi_t u''(t) q_t \right] \end{aligned}$$

When the collateral constraint is not binding, we obtain the following macroprudential debt tax.

$$1 + \tau_t = \frac{\beta R \mathbb{E}_t \left(\frac{\zeta_{t+1}^{\nu} p_\nu \theta \mu_{t+1} u''(t+1)}{u'(t)^2} - \xi_{t+1} u''(t+1) \mathcal{Q}_{t+1} \right)}{\beta R \mathbb{E}_t u'(t+1)}$$

[38]Of course, μ_t is different from μ_t^* as the private and social values from relaxing the collateral constraint are different. Note that the relevant one for the asset price is μ_t.

A.3 Derivation of Collateral Constraint

We provide a derivation of the collateral constraint (24) as an incentive compatibility constraint resulting from a limited enforcement problem. Debt contracts are signed with creditors in a competitive environment. Financial contracts are not exclusive, i.e., agents can always switch to another creditor at any point in time. Households borrow at the beginning of the period, before the asset market open. Within period, households can divert future revenues and avoid any costs from defaulting next period when debt becomes due. At the end of the period, there are no more opportunities for households to divert revenues and repayment of previous bonds is enforced. Financial intermediaries can costlessly monitor diversion activities at time t. If creditors detect the diversion scheme, they can seize a fraction κ_t of the household assets. After defaulting, a household regains access to credit markets instantaneously and a repurchases the assets that investors sell in open markets. Given this environment, a household that borrows $\tilde{d}_{t+1}$ and engage in diversion activities gains $\tilde{d}_{t+1}$ and loses a $\kappa_t q_t k_t$.

Formally, let V^R and V^d be the value of repayment and default respectively, and V be the continuation value. If a household borrows $\tilde{d}$ at the beginning of the period and defaults, it gets

$$V^d(\tilde{d}, b, k, X) = \max_{b',k',c,v} u(c - G(h)) + \beta \mathbb{E}_{s'|s} V(b', k', B', s') \tag{40}$$

$$s.t. \quad q(B,s)k' + c + \frac{b'}{R} = \tilde{d} + q(B,s)k(1-\kappa) + b + zF(k,h,\nu) - p_\nu \nu$$

$$-\frac{b'}{R} + \theta p_\nu \nu \leq \kappa q(B,s)k$$

where the budget constraint reflects the fact that the household regain access to credit markets and can borrow b' and the collateral constraint reflects that agents buy back the assets from investors. If the household does not default, it gets the utility from current consumption plus the continuation value of starting next period with debt b' as stated in 31.

$$V^r(b, k, X) = \max_{b',k',c,h} u(c - G(n)) + \beta \mathbb{E}_{s'|s} V(b', k', B', s') \tag{41}$$

$$s.t. \quad q(B,s)k' + c + \frac{b'}{R} = q(B,s)k + b + zF(k,h,\nu) - p_\nu \nu$$

$$-\frac{b'}{R} + \theta p_\nu \nu \leq \kappa q(B,s)k$$

A simple inspection at the budget constraints implies that households repay if and only if $\tilde{d}_{t+1} \leq \kappa_t q_t k_t$

Notice that for the constrained-efficient equilibrium, the derivation of the feasible credit positions is analogous to the case in the decentralized equilibrium. If the planner engages in diversion, creditors can seize a fraction κ_t of assets in the economy. Moreover, households can buy back the assets at the market price q_t. This implies that the same collateral constraint applies in the constrained-efficient equilibrium in this environment.

B Computational Algorithm

B.1 Numerical Solution Method for Decentralized Equilibrium

Following Bianchi (2011), we use Coleman (1990)'s time iteration algorithm, modified to address the occasionally binding endogenous constraint, that operates directly on the first-order conditions. Formally, the computation of the competitive equilibrium requires solving for functions $\{\mathcal{B}(b,s), \mathcal{Q}(b,s), \mathcal{C}(b,s), \nu(b,s), \mathcal{H}(b,s), \mu(b,s)\}$ such that:

$$\mathcal{C}(b,s) + \frac{\mathcal{B}(b,s)}{R} = zF(1, \mathcal{H}(b,s), \nu(b,s)) + b - p_v \nu(b,s) \tag{42}$$

$$-\frac{\mathcal{B}(b,s)}{R} + \theta p_\nu \nu(b,s) \leq \kappa \mathcal{Q}(b,s) \tag{43}$$

$$u'(\mathcal{C}(b,s) - G'(\mathcal{H}(b,s))) = \beta R \mathbb{E}_{s'|s}\left[u'(\mathcal{C}(\mathcal{B}(b,s),s') - G'(\mathcal{H}(\mathcal{B}(b,s),s))\right] + \mu(b,s) \tag{44}$$

$$zF_n(1, \mathcal{H}(b,s), \nu(b,s)) \quad = \quad G'(\mathcal{H}(b,s)) \tag{45}$$

$$zF_\nu(1, \mathcal{H}(b,s), \nu(b,s)) \quad = \quad p_\nu(1 + \theta\mu(b,s)/u'(\mathcal{C}(b,s))) \tag{46}$$

$$\begin{aligned} qu'(c - G(h)) \quad = \quad & \beta \mathbb{E}_{s'|s}\{u'(\mathcal{C}(b',s') - G'(\mathcal{H}(b,s)))(\mathcal{Q}(b',s') + z'F_k(1, \mathcal{H}(b',s'), \nu(b',s'))) \\ & +\kappa' \mu(b',s')\mathcal{Q}(b',s')\} \end{aligned} \tag{47}$$

The algorithm follow these steps:

1. Generate a discrete grid for the economy's bond position $G_b = \{b_{1,} b_2, ... b_M\}$ and the shock state space $G_s = \{s_{1,} s_2, ... s_N\}$ and choose an interpolation scheme for evaluating the functions outside the grid of bonds. We use 300 points in the grid for bonds and interpolate the functions using a piecewise linear approximation.

2. Conjecture $\mathcal{B}_k(b,s), \mathcal{Q}_k(b,s), \mathcal{C}_k(b,s), \mathcal{H}_k(b,s), \nu_k(b,s), \mu_k(b,s)$ at time $K, \forall b \in G_b$ and $\forall s \in G_s$

3. Set $j = 1$

4. Solve for the values of $\mathcal{B}_{k-j}(b,s), \mathcal{Q}_{k-j}(b,s), \mathcal{C}_{k-j}(b,s), (b,s), \mu_{k-j}(b,s)$ at time $k-j$ using (42)-(48) and $\mathcal{B}_{k-j+1}(b,s), \mathcal{Q}_{k-j+1}(b,s), \mathcal{C}_{k-j+1}(b,s)$ $\mathcal{H}_{k-j+1}(b,s), \mu_{k-j+1}(b,s) \forall\ b \in G_b$ and $\forall\ s \in G_s$:

   (a) Assume collateral constraint (43) is not binding. Set $\mu_{k-j}(b,s) = 0$ and solve for $\mathcal{H}_{k-j}(b,s)$ and ν using (45) and (46). Solve for $\mathcal{B}_{k-j}(b,s)$ and $\mathcal{C}_{k-j}(b,s)$ using (42) and(44) and a root finding algorithm.

   (b) Check whether $-\frac{\mathcal{B}_{k-j}(b,s)}{R} + \theta p_\nu \nu_{k-j}(b,s) \leq \kappa \mathcal{Q}_{k-j+1}(b,s)$ holds.

(c) If constraint is satisfied, move to next grid point.

(d) Otherwise, solve for $\mu(b,s), \nu_{k-j}(b,s), \mathcal{H}_{k-j}(b,s), \mathcal{B}_{k-j}(b,s)$ using (43, (44) and(45) with equality.

(e) Solve for $\mathcal{Q}_{k-j}(b,s)$ using (48)

5. Evaluate convergence. If $sup_{B,s}\|x_{k-j}(b,s) - x_{k-j+1}(b,s)\| < \epsilon$ for $x = \mathcal{B}, \mathcal{C}, \mathcal{Q}, \mu, \mathcal{H}$ we have found the competitive equilibrium. Otherwise, set $x_{k-j}(b,s) = x_{k-j+1}(b,s)$ and $j \rightsquigarrow j+1$ and go to step 4.

B.2 Numerical Solution Method for Constrained-Efficient Equilibrium

The algorithm uses a nested fixed point algorithm.

1. Generate a discrete grid for the economy's bond position $G_b = \{b_1, b_2, ...b_M\}$ and the shock state space $G_s = \{s_1, s_2, ...s_N\}$ and choose an interpolation scheme for evaluating the functions outside the grid of bonds. We use 300 points in the grid for bonds and interpolate the functions using a piecewise linear approximation.

2. Guess policy functions $\mathcal{B}, \mathcal{Q}, \mathcal{C}, \nu, \mu$ at time K $\forall$ $b \in G_b$ and $\forall$ $z \in G_z$. We use as initial policies the policies of the decentralized equilibrium, and we check that we obtain the same equilibrium when starting from alternative policies.

3. For given $\mathcal{C}, \mathcal{Q}, \mathcal{H}, \nu, \mu$ solve for the value function and policy functions :

$$
\begin{aligned}
V(b', s') &= \max_{c,b',\mu,h,\nu} u(c - G(h)) + \beta \mathbb{E}_{s'|s} V(b', s') && (48)\\
c + \frac{b'}{R} &= b + zF(k, h, \nu) - p_\nu \nu && (49)\\
zF_h(k, h, \nu) &= G'(h) && (50)\\
zF_\nu(k, h, \nu) &= p_\nu \left(1 + \frac{\theta \mu}{u'(c - G(h))}\right) && (51)\\
\mu \left(\frac{b'}{R} - \theta p_\nu \nu + \kappa q\right) &= 0 && (52)\\
\frac{b'}{R} - \theta p_\nu \nu &\geq -\kappa q && (53)
\end{aligned}
$$

$$
\begin{aligned}
qu'(c - G(h)) &= \beta \mathbb{E}_{s'|s} \{u'(\mathcal{C}(b', s') - G'(\mathcal{H}(b, s)))(\mathcal{Q}(b', s') + z' F_k(1, \mathcal{H}(b', s'), \nu(b', s'))\\
&\quad + \kappa' \mu(b', s') \mathcal{Q}(b', s')\} && (54)
\end{aligned}
$$

This recursive problem is solved using value function iteration. The value functions and policy functions are approximated using linear interpolation whenever the bond position is not in the grid. To solve the optimal choices in each state, we first assume the collateral constraint is not binding. We search over a grid of debt and then using the solution to this problem as an initial guess in a non-linear optimization routine. If the collateral constraint is binding, we solve for every b', the combinations of c, h, ν, q, μ that satisfy these 6 conditions (49)-(54), with (54) holding with equality.

4. Denote by $\sigma^i, i = c, q, h, \nu, \mu$ the policy functions that solve the recursive problem in step (3) Compute the sup distance between $\mathcal{B}, \mathcal{Q}, \mathcal{C}, \nu, \mu$and $\sigma^i, i = c, q, h, \nu, \mu$. If the sup distance is higher than 1.0e-6, update $\mathcal{B}, \mathcal{Q}, \mathcal{C}, \nu, \mu$ and solve again the recursive problem.