Due Diligence: Job Search with Rationally Inattentive Workers

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Abstract

We develop a model of late in life job search that accounts for end of life labor force exit and re-entry. Our key assumptions are that job offers consist of both wage and complex non-wage characteristics and that older workers care more about the non-wage characteristics of a job. In equilibrium, young workers choose jobs with high wages, but poor non-wage characteristics, while older workers are willing to trade off lower wages for better non-wage characteristics. However, due to rational inattention, older workers may ex-post regrettably accept low wage jobs with poor non-wage characteristics. Such mistakes produce welfare losses and generate employment patterns in the model consistent with the empirical patterns of older US workers.

1 Introduction

Accepting a job offer, an offer of college admission, or a marriage proposal entails personal, social, and legal commitments. As a result, individuals invest time and effort in understanding the quality of any such opportunities. Despite such due diligence, they may fail to notice warning signals, suffering consequences that can be profoundly costly. However, increasing the amount of due diligence is costly too, so regrettable actions may be understandable and possibly even inevitable.

With a focus on the essential role of uncertainty and information in decision making, we develop a stylized search model of the post-career labor market in which workers perform costly due diligence about job offers. This information friction can account for features of data from the US labor market, including volatile job spells of older workers utilizing bridge jobs in their transition out of the labor force. There are two key assumptions in our model that deliver these features. The first is that job offers are complex

and multidimensional, so it is difficult for workers with a job offer to know exactly all utility relevant characteristics of that offer. While wages are easily observable, non-wage characteristics can be a large component of job satisfaction and are more difficult to assess. Second, we assume that worker preferences towards wage and non-wage aspects of jobs shift with age, with older workers valuing non-wage characteristics more than younger workers. Because we do not specify the specific non-wage features, this shift in preferences could capture non-convexities in leisure preferences (**?**, **?**), changing preferences toward flexibility and aged-mixed environments (**?**), or increased preference towards a low physical or mentally stressful job **?**. This is consistent with evidence that nonmonetary aspects of a work environment are especially important for explaining the labor force behavior of older workers.

In our model, the degree of due diligence is determined endogenously and is modeled as in **?**. When a worker receives an offer of employment, the utility that would result from acceptance is initially uncertain. This uncertainty can be reduced before accepting a job via costly attention, and once on the job, experience quickly resolves all remaining uncertainty about the characteristics of a job. Employee-employer mismatch can lead to rapid quits that impact efficiency in the labor market, which has welfare consequences for both employers and employees.

The amount and type of attention responds rationally to the economic environment, delivering equilibrium job acceptance and quit rates. Furthermore, information acquisition interacts with the finite horizon and changes in preferences to affect job search, job acceptance, and job turnover as an individual ages. Because search also has a fixed cost that can be larger than the value of search, retirement can emerge endogenously among older workers .

Apart from the addition of attention costs, the search setting is standard. There is a fixed cost of search and there is discounting of future payoffs. Time is discrete, there is at most one offer per period, and there is no recall of past offers. We allow for both finite and infinite horizons as well as for voluntary and involuntary separations. In all cases, we show the optimal strategy to be of a "reservation utility" type, albeit with a twist: the indifference condition between acceptance and rejection that pins down reservation utility does not characterize observed behavior. Rather, this behavior is characterized by two distinct posterior distributions over job types, associated respectively with acceptance and rejection of the offer. There is a clear payoff difference between these. The posterior associated with acceptance yields strictly higher expected utility than does that associated with rejection.

We solve for optimal search and attention using the posterior-based approach of **?** (hereafter referred

to as CDL).[1] They show that "invariant likelihood ratio" (ILR) conditions characterize rational inattention based on Shannon entropy cost. Inserting these conditions into the corresponding Bellman equation reveals the optimal strategy to have simple structure. We examine comparative static properties in relation to changes in prior uncertainty, costs of attention, search costs, the time horizon, and preferences. These model parameters may vary systematically over the life cycle of employment, reflecting the changing importance of the due diligence phase over the course of a career.

To close the model, we consider market in which there are overlapping generations of workers and a continuum of firms who post the wage and non-wage characteristics of vacancies they wish to fill. We provide two examples of this equilibrium, one in which older workers "retire" before the end of their working lives, and one in which older workers rapidly quit jobs with poor non-wage characteristics. In the latter, firms are indifferent between offering high wage jobs with poor non-wage characteristics, low wage jobs with poor non-wage characteristics, and low wage jobs with excellent non-wage characteristics. Young workers choose jobs with high wages, but poor non-wage characteristics, while older workers seek out jobs with low wages, but excellent non-wage characteristics. However, due to rational inattention to job offers, older workers may mistakenly accept low wage jobs with poor non-wage characteristics, which they quickly exit.

By way of application, we consider dynamic patterns of employment and search in the Health and Retirement Study (HRS). We confirm and extend prior findings concerning the high rates of job separation in the later stages of life after both voluntary and involuntary exit from the "career job". **?** and **?** analyze employment patterns of workers of ages 50 and above after involuntary job loss. While the majority of such workers who search for employment do re-enter the labor force within 2 years, the resulting employment spells are short-lived and volatile. Similarly, following a voluntary job exit many workers engage in a transitory "bridge job"period that is characterized by short, frequently part time, employment spells prior to permanent labor force exit **??**. We identify similar patterns: in our model this is best explained by the reduced time horizon of older workers, changing preferences over wage and non-wage elements of a job, and the difficulty in knowing the utility delivered by non-wage elements of a job. Rather than assessing relatively simple pecuniary characteristics, older workers may assign greater importance to non-pecuniary job characteristics, which may be harder to assess. We examine comparative statics that allow us to address many of the leading theories about late in life attachment to the labor force.

In section 2 we present several stylized facts regarding late in life labor patterns that motivate our model. In section 3 we describe the optimal attentional strategy for a class of decision problems. In section

[1] This is closely related to findings in **?**.

4 we present an infinite horizon model of search with rational inattention and characterize optimal search strategies. In sections 5 and 6 we develop the finite horizon labor search model, with section 4 focusing on older workers who value non-wage job characteristics and section 5 focusing on younger workers. In section 6 we present the firm's problem and define a competitive search equilibrium.

2 Late in Life Labor Patterns

The model in this paper presents a theory capable of capturing two stylized facts regarding labor supply patterns late in life. Furthermore, any successful model of late in life labor decisions must generate these two facts. Specifically, many individuals exit a career job late in life and engage in a bridge job phase prior to permanent labor force exit. As others in the literature have done, we define a career job as a job that an individual holds for at least ten years, and a bridge job as any job that occurs post-career but pre-retirement. Second, during this bridge job phase individuals are likely to engage in a series of jobs as they search for a satisfactory employment arrangement, causing a sequence short job spells and labor market churn. In this section, we establish these facts using the HRS.

The HRS is a bi-annual representative panel of Americans over age 50 that was started in 1992 to collect information on the health and labor dynamics of older Americans. Since the original survey, individuals in the sample have been re-surveyed every 2 years to track their health and labor market transitions. This sample is the most commonly used resource to study the labor market transitions of older Americans. Others previously have used the HRS to conduct studies on bridge job transitions (see**?** for a detailed analysis), and the work in this section is consistent with their results. However, as shown below, previous work that has relied on the RAND HRS likely understates the prevalence of bridge jobs due to its ignoring short term job spells that occur between waves. In this section we correct for this, and present tabulations of post-career labor patterns.

For our analysis, we restrict our sample to the original HRS cohort and omit later cohorts for which post-career waves are unlikely to have occured. Furthermore, we omit individuals missing all labor market records, providing a base sample of 9388 of original HRS respondents. Amongst this original HRS sample, 62.1%, or 5830 individuals, are observed in a career job at some point between 1992 and 2010. Given that we are primarily interested in labor force patterns after career job exit, these individuals will be the primary sub-sample we analyze. As shown in table 1, individuals leave their career jobs in several different ways. Fortunately, 71.4% of these individuals are observed to exit their career and transition either to other work

	Observed Exit	Exit to Death	Attrition	No observed exit
Percent	71.4	3.5	10.3	14.9

Table 1: How left career

or retirement in at least one wave after career job exit. For the 3.5% that die while working at their career job, 10.3% that exit the HRS due to non-response, and 14.9/

For the individuals that are observed post-career job, row 1 of table **??** calculates the proportion that are observed in new employment in subsequent waves. Here, we see that 42.0% of the relevant sample engage in some sort of bridge job transition, a number near identical to that in **?**. In addition, column 1 of table 3 presents the self-reported labor force status in the wave following the career job exit. Here we see that while 63.7% of individuals report being retired the wave after exiting their career job, there is significant heterogeneity in the subsequent labor force transitions.

While these tabulations demonstrate the importance of bridge jobs in late in life labor market transitions and are consistent with previous studies, we claim that in fact they are under-measured. Specifically, if a bridge job is held for less than 2 years and does not overlap with interview date, it is not recorded in standard HRS analysis. Fortunately, the HRS also collects information on employment spells between waves that is omitted in the more commonly used RAND HRS. To account for these employment spells, we supplement the RAND HRS with these survey responses and repeat our bridge job tabulations. The results are presented in the second row of table 2. Here we observe that after accounting for between wave employment spells an individual is just as likely to work after a career job as retire, with 49.8% of individuals being found to engage in a bridge job of some kind. In column 2, of table 3, we present the proportion of individuals in each post-career labor state that reported having a job spell between waves. We observe that the transition to each state is complex, with typically 6-23% of individuals in each state having a richer transition path than employment taken at time of survey would suggest.

The presence of jobs between waves presented in table 3 not only suggests that more individuals take bridge jobs, but also that individuals take more bridge jobs. To examine this, in table 4 we present measures of the number of jobs held for each measurement approach. This table provides two key insights in to the nature of bridge job transtions. First, we see that amongst those that engage in a bridge job, 64.9% have more than one employment spell before exiting the labor force. Furthermore, we see that 20.7% of individuals that hold a bridge job are observed to have at least 4 post-careert employment work spells. Given

	Permanent Retirement	Bridge Job
Only at Waves	58.0	42.0
Including b/w Waves	50.2	49.8

Table 2: Bridge Job

Status wave after career exit	Percent	Percent with between non-career wave employment
Full time	11.8	19.5
Part time	2.8	22.6
Unemployed	2.5	17.5
Partly Retired	12.9	14.2
Fully Retired	63.7	9.0
Disabled	2.7	6.3
Not in Labor force	3.7	9.1

Table 3: Labor force status wave after career exit

that everyone in this sample had previously worked in the same position at least 10 years uninterrupted, this table suggests that the bridge job phase is marked by significant labor transition and churn.

In conclusion, we observe evidence of significant labor market participation and bridge jobs following exit from a career late in life. Furthermore, we show that the importance of this bridge job state has been understated in previous studies due to their exclusion of between wave job spells. After accounting for these, we present evidence that the bridge job period is characterized by multiple short, transitory job spells for a large proportion of the population. The model we present in the coming sections is capable of replicating these facts.

3 The Infinite Horizon Model

Here we present our baseline model, a search equilibrium in which the agents face costly identification of jobs and engage in rationally inattentive search when deciding whether to accept or reject a job offer.

Number of jobs post-career	Measurement at Wave	Measurement between Waves
0	58.0	50.2
1	17.3	17.5
2	14.0	14.4
3	5.8	7.6
4	3.0	4.2
5	1.3	2.7
6+	.6	3.4

Table 4: Number of jobs held after career exit

3.1 Environment

We first define the environment of our model.

Job Set There exist a finite number of job types $\omega \in \Omega = \{1, ..., M, \emptyset\}$ with the null state corresponding to unemployment. Associated with each job type $\omega \in \Omega$ is a period utility (including utility from wages) u_ω. Jobs are ordered according to the flow utility they provide so that $u_1 < u_2 < ... < u_M$.

Workers There exist a unit mass of workers that search for jobs. Each period an agent that is employed chooses either to remain employed or to quit and immediately become unemployed. An unemployed person can remain unemployed or search for a job, by paying search cost C. A searching worker matches with a firm and receives a job offer with probability α_w. With a job offer in hand, a worker engages in costly learning to try and identify the offer's job type.

Firms There exist an continuum of firms in the market. Each period a firm that is not matched with a worker decides whether to create a job or not, and enters the labor market if it chooses to do so. There is a probability α_f that a firm with an open vacancy will match with a worker. A firm that is matched with a worker continues with this match until it is broken either through an agents decision or an exogenous separation (occurs with probability δ). Associated with each job type $\omega \in \Omega$ is a cost of vacancy posting,

F, and a period profit function for a firm matched with a worker π_ω. The period profit corresponds to the utility the job provides, so that $\pi_\omega > \pi_{\omega'}$ iff $u_\omega < u_{\omega'}$. This captures the fact that the only meaningful trade-off is on non-wage elements of a job that are desired by workers and costly to provide for firms.

Market Each period unemployed agents and vacant firms enter a matching market with an imperfect matching technology that regulates the probability firms and workers meet. Specifically, we assume a match technology of the following form

$$m(U,V) = \theta U^a V^{1-a} \ , \ a \in (0,1).$$

We denote the mass of vacant firms as

$$V = \sum_{\omega=1}^{M} V_\omega,$$

where V_ω denotes a firm offering job type ω. This allows us to write the match probability of workers and firms as a function of the ratio of vacancies to unemployed workers:

$$\alpha_f \equiv \frac{m(U,V)}{V} = \min\left\{\theta\left(\frac{V}{U}\right)^{-a}, 1\right\}$$

and

$$\alpha_w \equiv \frac{m(U,V)}{U} = \min\left\{\theta\left(\frac{V}{U}\right)^{1-a}, 1\right\}.$$

3.2 Worker Problem

At the start of each period a worker is either employed in a job of type ω or unemployed. If the worker is employed, then the worker decides whether to remain on their current job or quit to unemployment. If the worker decides to stay, then they start the next period in the same job with probability δ and start the next period unemployed with probability $1-\delta$, where δ is an exogenous match separation probability.

If the agent is unemployed, then the agent must decide whether to remain unemployed or search for a job. If the agent decides to search for a job, the agent must first pay a fixed cost C to enter the matching market where an offer is received with probability α_w. Since the agents belief about the ratio $\frac{V}{U}$ in a market with posted wage W is fixed in equilibrium, when appropriate we suppress notation separately tracking beliefs and equilibrium match probabilities. If the match is unsuccessful the worker enters the start of next period

unemployed. If the match is successful, the worker receives a job offer of type ω, with ω unknown to the recipient. Although the job offer is unknown to the worker, the worker is able to engage in costly learning prior to the acceptance of the job to resolve some of the uncertainty. This attentional effort results in a posterior distribution of job type that an individual uses to decide whether to accept or to reject the offer in hand. If accepted, the job gives rise to flow utility during the period, revealing its type for the following period. If rejected, the only option for the period is to remain unemployed and to receive the corresponding utility $u_\emptyset$.

Before displaying and solving the full worker's problem, we first describe in greater detail the decision problem that an agent must solve prior to acceptance. Upon successful job search, an individual holds a job offer of unknown ω but is endowed with a prior

$$\mu : \Omega \to [0, 1]$$

over the job type. The worker is then able to inspect the job and update their beliefs over the job type to form a posterior belief distribution

$$\gamma : \Omega \to [0, 1]$$

over the job distribution prior to acceptance. An agent is constrained in his ability to resolve uncertainty due to attentional costs. These attentional costs are designed to capture all of the costly information acquisition that can be performed to learn about a job before accepting the job, from non-wage monetary job components like expected overtime pay and education/training reimbursement to non-wage non-monetary job components like schedule flexibility, physical demands of the job, and workplace culture. Expected attentional costs are represented by the Shannon cost function

$$S(\gamma, P) \equiv \lambda \left[H(\mu) - \sum_{f \in B} P^f H(\gamma^f)) \right],$$

where λ is the cost parameter, H is the Shannon entropy function

$$H(\gamma^f) \equiv - \left[\sum_{\omega=1}^{M} \gamma_\omega^f \ln \gamma_\omega^f \right] > 0,$$

P^f is the unconditional probability of taking action f from the feasible set of acts B, and γ_ω^f is the posterior probability a job is type ω when taking action f. An attentional strategy for a problem of this type can be represented as $\left(\left\{ P^f, \gamma^f \right\}_{f=1}^{B} \right) \in \Lambda^{\mu, B}$. Given the nature of our problem, the action set consists of the acceptance or rejection of the job in hand, so $B = \{A, R\}$, and so for the remainder of the paper we denote

$P^A = P$, $P^R = 1 - P$, and $\Lambda^{\mu,B} = \Lambda$. Furthermore, **?** show that there exists a unique optimal posterior for a given action given the specified cost function, so that the optimal attentional strategy can be represented as $(P, \gamma^A, \gamma^R) \in \Lambda$ with expected attentional costs

$$S(\gamma, P) \equiv \lambda \left[H(\mu) - (PH(\gamma^A) + (1-P)H(\gamma^R)) \right].$$

Given this specification of attentional strategies and costs, we are now able to fully represent the value of unemployment $V(\emptyset)$, the value of search $N(\gamma, P)$ for a given attentional strategy (γ, P) , and the value of employment $V(\omega)$ for a given a job type ω. First an individual who is unemployed must decide whether to remain unemployed or search for a job. If an individual chooses to remain unemployed they enter next period with the same decision, and if they choose to search for a job they must choose an optimal attentional strategy from the feasible set Λ, and follow the chosen (γ, P) strategy should they match with a firm.[2]

$$V(\emptyset) = \max \left[u_\emptyset + \beta V(\emptyset), \sup_{(\gamma,P)\in\Lambda} N(\gamma, P) \right]. \tag{1}$$

For a given attentional strategy, the worker receives in expected value the following search payoff

$$\begin{aligned} N(\gamma, P) &= -C - \alpha_w S(\gamma, P) + \alpha_w P \left[\sum_{\omega\in\Omega} \gamma^A_\omega (u_\omega + \beta[(1-\delta)V(\omega) + \delta V(\emptyset)]) \right] + (1 - \alpha_w P) \sum_{\omega\in\Omega} \gamma^R_\omega \left[U_\emptyset + \beta V(\emptyset) \right] \\ &= -C - \alpha_w S(\gamma, P) + \alpha_w P \left[\sum_{\omega\in\Omega} \gamma^A_\omega (u_\omega + \beta[(1-\delta)V(\omega) + \delta V(\emptyset)]) \right] + (1 - \alpha_w P)[U_\emptyset + \beta V(\emptyset)] \end{aligned} \tag{2}$$

Here, the $-C$ represents fixed costs of search, $\alpha_w S(\gamma, P)$ denote expected attentional costs conditional on matching, $\left[\sum_{\omega\in\Omega} \gamma_\omega \left[u_\omega + \beta(1-\delta)V(\omega) + \beta\delta V(\emptyset)\right]\right]$ represents the expected value of a job conditional on matching and accepting, and $[U_\emptyset + \beta V(\emptyset)]$ denotes the expected value of a job conditional on matching and rejecting. The value of being employed in a job type ω is denoted as the max of remaining employed in job type ω or quitting to unemployment, and is denoted as

$$V(\omega) = \max\{u_\omega + \beta[(1-\delta)V(\omega) + \delta V(\emptyset)], V(\emptyset)\} \tag{3}$$

We denote the probability of searching σ according to the following rule

$$\sigma \begin{cases} = 1 \text{ if } u_\emptyset + \beta V(\emptyset) < \sup_\Lambda N(\gamma_H, \gamma_L, P) \\ \in [0,1] \text{ if } u_\emptyset + \beta V(\emptyset) = \sup_\Lambda N(\gamma_H, \gamma_L, P) \\ = 0 \text{ if } u_\emptyset + \beta V(\emptyset) > \sup_\Lambda N(\gamma_H, \gamma_L, P). \end{cases}$$

[2]In an awful abuse of notation, we have labeled the value of a worker as V and the level of vacancies as V. Notation will change in the future, but hopefully throughout the paper the V to which we are referring is clear from context.

We denote the quit strategy for a job ω as

$$\eta_\omega \begin{cases} = 1 \text{ if } [u_\omega + \beta(1-\delta)V(\omega)] + \beta\delta V(\emptyset) < V(\emptyset) \\ \in [0,1] \text{ if } [u_\omega + \beta(1-\delta)V(\omega)] + \beta\delta V(\emptyset) = V(\emptyset) \\ = 0 \text{ if } [u_\omega + \beta(1-\delta)V(\omega)] + \beta\delta V(\emptyset) > V(\emptyset). \end{cases}$$

We now will characterize the optimal policy functions of a searching individual. To do so we first introduce a simplified two period decision problem parametrized by a value E and prior μ that is used to characterize the attentional strategy of a searching individual.

3.2.1 Two Period Decision Problems

Consider a class of simple accept/reject decision problems $D(E,\mu)$ with payoffs parametrized by $E \in [\min\{u_\emptyset, u_1\}, u_M]$. In particular, define

$$D(E,\mu) = \sup_{(\gamma,P)\in\Lambda} P\left[\sum_{\omega\in\Omega} \gamma_\omega W^A_\omega(E)\right] + (1-P)W^R(E) - S(\gamma,P), \tag{4}$$

where we define payoffs $W^A_\omega(E)$

$$\begin{aligned} W^A_\omega(E) &= \begin{cases} \frac{E}{1-\beta} + \frac{(u_\omega - E)}{1-\beta(1-\delta)} \text{if } u_\omega > E; \\ \frac{E}{1-\beta} - (E - u_\omega) \text{ if } u_\omega \leq E. \end{cases} \\ W^R(E) &= \frac{E}{1-\beta} - (E - u_\emptyset). \end{aligned}$$

This specification corresponds to expected utility in a two period model with the property that a decision maker who has an offer in hand yet rejects gets $u_\emptyset$ in period 1 and earns second period payoff $\left[\frac{\beta E}{(1-\beta)}\right]$. The alternative is to accept the job. In this case, if the state is ω, the DM receives u_ω in period 1, and if the job survives (probability $(1-\delta)$), thereafter chooses either to hold the job and receive $\frac{(1-\beta)u_\omega + \beta\delta E}{(1-\beta)[1-\beta(1-\delta)]}$, or to take second period payoff $\left[\frac{\beta E}{(1-\beta)}\right]$. If the job does not survive the second period payoff is $\left[\frac{\beta E}{(1-\beta)}\right]$.

To solve this model we apply a result of CDL specialized to an accept/reject decision. Rejection provides utility W^R, while acceptance provides W^A_ω. CDL provide a direct characterization of the solution based on

$$z_\omega^A = \exp\left\{\frac{\frac{E}{1-\beta} + \frac{(u_\omega - E)}{1-\beta(1-\delta)}}{\lambda}\right\} \text{ if } u_\omega > E; \tag{5}$$

$$z_\omega^A = \exp\left\{\frac{\frac{E}{1-\beta} - (E - u_\omega)}{\lambda}\right\} \text{ if } u_\omega < E; \tag{6}$$

$$z^R = \exp\left\{\frac{\frac{E}{1-\beta} - (E - U_\emptyset)}{\lambda}\right\} \tag{7}$$

Lemma 1 *If* $\min\left[\sum_{\omega=1}^M \mu_\omega \left(\frac{z_\omega^A}{z^R}\right), \sum_{\omega=1}^M \mu_\omega \left(\frac{z^R}{z_\omega^A}\right)\right] > 1$, *then the optimal acceptance probability* $P \in (0,1)$ *is the unique solution to,*

$$\sum_{\omega=1}^M \frac{\mu_\omega (z_\omega^A - z^R)}{P z_\omega^A + (1-P) z^R} = 0.$$

Furthermore, the following conditional posteriors hold:

$$\gamma_\omega^A := \Pr(\omega|A) = \frac{\mu_\omega z_\omega^A}{P z_\omega^A + (1-P) z^R} \tag{8}$$

$$\gamma_\omega^R := \Pr(\omega|R) = \frac{\mu_\omega z^R}{P z_\omega^A + (1-P) z^R} \tag{9}$$

$$p_\omega^A := \Pr(A|\omega) = \frac{P z_\omega^A}{P z_\omega^A + (1-P) z^R} \tag{10}$$

$$p_\omega^R := \Pr(R|\omega) = \frac{(1-P) z^R}{P z_\omega^A + (1-P) z^R} \tag{11}$$

If $\sum_{\omega=1}^M \mu_\omega \left(\frac{z_\omega^A}{z^R}\right) \leq 1$, *then the optimal acceptance probability is* $P = 0$. *If* $\sum_{\omega=1}^M \mu_\omega \left(\frac{z^R}{z_\omega^A}\right) \leq 1$, *then the optimal acceptance probability is* $P = 1$.

By way of elucidation, note that Bayes' rule works,

$$P\gamma_\omega^A + (1-P)\gamma_\omega^R = \mu_\omega \left[\frac{P z_\omega^A + (1-P) z^R}{P z_\omega^A + (1-P) z^R}\right] = \mu_\omega,$$

so that,

$$P \sum_{\omega=1}^M \gamma_\omega^A + (1-P) \sum_{\omega=1}^M \gamma_\omega^R = \sum_{\omega=1}^M \mu_\omega = 1.$$

Hence, by the defining property of P we know that both sums are 1,

$$\sum_{\omega=1}^M \gamma_\omega^A = \sum_{\omega=1}^M \frac{\mu_\omega z_\omega^A}{P z_\omega^A + (1-P) z^R} = \sum_{\omega=1}^M \frac{\mu_\omega z^R}{P z_\omega^A + (1-P) z^R} = \sum_{\omega=1}^M \gamma_\omega^R = 1$$

3.2.2 Reservation Expected Utility

We define $\Delta(E)$ as the difference between the overall value of searching and the value of taking reservation $\frac{E}{1-\beta}$ instead of searching. The overall value of searching is comprised of the cost of searching, plus if matched the payoff from the optimized decision problem (over attention strategies) $D^*(E,\mu)$ or if not matched the value of not matching.

$$\begin{aligned}\Delta(E,\mu) &:= \left[\alpha_w D^*(E,\mu) + (1-\alpha_w)\left(u_\phi - E + \frac{E}{1-\beta}\right) - C\right] - \frac{E}{1-\beta} \\ &= \alpha_w \left[D^*(E,\mu) - \frac{E}{1-\beta}\right] + (1-\alpha_w)(u_\phi - E) - C. \end{aligned} \tag{12}$$

Theorem 1 reveals the key role of this function.

Theorem 1 *Define $\hat{u} \in (u_\emptyset, u_M)$ by $\Delta(\hat{u}) = 0$ if $\Delta(u_\emptyset) > 0$, with $\hat{u} = u_\emptyset$ otherwise. Then the value function satisfies,*

$$\begin{aligned} V(\omega) &= \begin{cases} \frac{\hat{u}}{1-\beta} + \frac{(u_\omega - \hat{u})}{1-\beta(1-\delta)} & \text{if } u_\omega > \hat{u}; \\ \frac{\hat{u}}{1-\beta} & \text{if } u_\omega \leq \hat{u}. \end{cases} \\ V(\emptyset) &= \frac{\hat{u}}{1-\beta}. \end{aligned}$$

It is optimal to search and apply rational inattention strategy $\left(\hat{\gamma}, \hat{P}\right)$ corresponding to accept/reject decision problem $D(\hat{u},\mu)$ iff $\Delta(u_\emptyset) > 0$. It is optimal to stay in identified job $\omega \in \Omega$ if and only if $u_\omega \geq \hat{u}$. If $\Delta(u_\emptyset) \leq 0$ it is optimal not to search.

Proof. *See Appendix A.* ■

We call $\hat{u}$ **reservation expected utility (REU)**. Its definition is intuitive. Note that $\Delta(\hat{u}) = 0$ specifies,

$$C = \alpha_w \left[D^*(\hat{u},\mu) - \frac{\hat{u}}{1-\beta}\right] + (1-\alpha_w)(u_\phi - \hat{u});$$

where,

$$D^*(\hat{u},\mu) - \frac{\hat{u}}{1-\beta} = \hat{P}\left(\sum_\omega \hat{\gamma}_\omega (u_\omega - \hat{u}) 1_{\{u_\omega \leq \hat{u}\}} + \sum_\omega \hat{\gamma}_\omega \left[\frac{u_\omega - \hat{u}}{1-\beta(1-\delta)}\right] 1_{\{u_\omega > \hat{u}\}}\right) + (1-\hat{P})(u_\emptyset - \hat{u}) - S(\hat{\gamma},\hat{P}).$$

Substitution yields,

$$C + \alpha_w S(\hat{\gamma},\hat{P}) + (1-\alpha_w\hat{P})(\hat{u} - u_\emptyset) + \alpha_w \hat{P} \sum_\omega \hat{\gamma}_\omega (\hat{u} - u_\omega) 1_{\{u_\omega \leq \hat{u}\}} = \alpha_w \hat{P} \sum_\omega \hat{\gamma}_\omega \left[\frac{u_\omega - \hat{u}}{1-\beta(1-\delta)}\right] 1_{\{u_\omega > \hat{u}\}}. \tag{13}$$

3.3 The Firm Problem

We suppose that there exist a continuum of firms that decide whether to offer a job of type ω each period. A firm can post at most 1 vacancy, and if a firm decides to post a vacancy it must pay a fixed cost of posting: F. If a firm pays the fixed cost they match with a worker with probability α_f, which is determined in equilibrium as a function of $\frac{V}{U}$.[3] If it is matched with a worker, the worker accepts the offer with probability p_ω^A and reject the offer with probability $1 - p_\omega^A$. We denote the beliefs over these job rejection probabilities as Ξ^ω. Should the offer be filled, the firm receives a per period profit of

$$\pi_\omega = R^\omega - W > 0 \text{ for all } \omega$$

where R^ω is the revenue a match for job type ω and W is the posted wage in the market. We assume that a firm dies and is replaced if either the match is terminated or the firm fails to find a match in one period. Note that because the per period profit is positive for all job types the firm never has reason to end the job, and thus a matched job persists until either the match is exogenously terminated or an agent quits with probability η_ω. This simplification allows us to write the value function of a matched firm offering offering a job of type ω as the discounted flow of profits as long as the match remains open,

$$\begin{aligned} J^M(\omega) &= \pi_\omega + \beta(1-\delta)(1-\eta_\omega) J^M(\omega) \\ &= \frac{\pi_\omega}{1 - \beta(1-\delta)(1-\eta_\omega)}. \end{aligned} \tag{14}$$

Using the above value function for a matched firm, we denote the value function of a firm that decides to post a job of type ω as

$$\begin{aligned} J^S(\omega) &= -F + \alpha_f \times p_\omega^A J^M(\omega) \\ &= -F + \alpha_f \times p_\omega^A \frac{\pi_\omega}{1 - \beta(1-\delta)(1-\eta_\omega)}. \end{aligned} \tag{15}$$

Our assumption of a large number of firms ensures that $J^S(\omega) = 0$. More specifically, if $J^S(\omega) > 0$ then a positive mass of non-searching firms would enter the market until α_f is sufficiently low that there is no incentive for any other firms to enter. Finally, we assume that firms employ a mixed strategy ξ_ω upon entry.

[3] We are again suppressing the distinction between firm beliefs and equilibrium matching probabilities for notational simplicity.

Recall the total mass of vacancies is V. This assumption means that each firm posting a vacancy chooses job offer ω with probability ξ_ω, so that the mass of vacancies of job type ω, denoted V_ω, is

$$V_\omega = V\xi_\omega$$

and that the V_ω ensure

$$F = \alpha_f \times p_\omega^A J^M(\omega).$$

Finally, it will be required that a worker's prior beliefs satisfy $\mu_\omega = \xi_\omega$ in equilibrium.

3.4 Matching Market and Flows

As specified in the environment, unemployed workers workers of mass U match with vacancies according to the following match technology,

$$m(U,V) = \theta(V)^{1-a}(U)^a$$

thus yielding the following firm and worker specific match probabilities

$$\begin{aligned}
\alpha_f &\equiv \frac{m(U,V)}{V} = \min\left\{\theta\left(\frac{V}{U}\right)^{-a}, 1\right\} \\
\alpha_w &\equiv \frac{m(U,V)}{U} = \min\left\{\theta\left(\frac{V}{U}\right)^{1-a}, 1\right\}.
\end{aligned}$$

To characterize the equilibrium number of vacancies and unemployment, we use the unemployment and vacancy transition rules defined by the search and quit strategies fo firms and workers. We denote the total number of firms of type ω in the market as M_ω, thus implying that there are $M_\omega - V_\omega$ of the firms are matched with a worker.

We begin by deriving the transition rule for unemployment, remembering that the total number of workers is held fixed at 1. A worker that is currently employed in a job of type ω can enter unemployment next period either by quitting, which occurs with probability η_ω or through an exogenous separation shock. Thus, matches of type ω at time t will contribute $\delta + (1-\delta)\eta_\omega$ new unemployed individuals at time $t+1$. A worker that is currently unemployed will remain unemployed if either they fail to find a match (probability $(1-\alpha_w)$), they match with a firm of type ω and reject the offer (probability $\alpha_w\mu_\omega(1-p_\omega^A)$), they match with a firm of type ω and accept the offer but either quit or exogenously separated after one period (probability

$\alpha_w \mu_\omega p^A_\omega((1-\delta)\eta_\omega + \delta))$. Combining these cases yields the following unemployment transition

$$U(t+1) = \sum_{\omega=1}^{M}(\delta + (1-\delta)\eta_\omega)(M_\omega(t) - V_\omega(t)) + U(t)\left[(1-\alpha_w) + \alpha_w \sum_{\omega=1}^{M} \mu_\omega \left[(1-p^A_\omega) + p^A_\omega(\delta + (1-\delta)\eta_\omega)\right]\right] \tag{16}$$

We now turn our attention to the transition rules for vacancies of type ω under the added assumption that the mass of firms is unchanging over time (i.e., $M_\omega(t) = M_\omega(t+1)$). At the most general level, this implies that the new mass of vacancies of type ω, V_ω, is the product of all firms that died the previous period times the probability a new firm decides to offer a job of type ω, ξ_ω. Matches can terminate in several ways. First, a match terminates exogenously or by decision of the worker with probability $((1-\delta)\eta_\omega + \delta)$ resulting in $((1-\delta)\eta_\omega + \delta)(M_\omega(t) - V_\omega(t))$ new vacancies at time t. Next, a vacancy at time t dies with probability $(1-\alpha_f)$ if it doesn't find a match, dies with probability $\alpha_f\left[(1-p^A_\omega)\right]$ if a match is made but the offer is rejected, and dies with probability $p^A_\omega((1-\delta)\eta_\omega + \delta)$ if the match is made and offer accepted but terminated after one period. Adding these up and multiplying by the number of vacancies of type ω the previous period yields $V_\omega(t)\left[(1-\alpha_f) + \alpha_f\left[(1-p^A_\omega) + p^A_\omega((1-\delta)\eta_\omega + \delta)\right]\right]$ new t+1 vacancies. Thus, we can denote the new vacancies at time $t+1$ as

$$V_\omega(t+1) = \xi_\omega\left[\sum_{m=1}^{M}((1-\delta)\eta_m + \delta)(M_m(t) - V_m(t)) + V_m(t)\left[(1-\alpha_f) + \alpha_f\left[(1-p^A_m) + p^A_m((1-\delta)\eta_m + \delta)\right]\right]\right].$$

While correct, this expression is not as useful as another slightly less general law of motion. If we assume furthermore that the the mass of vacancies of each type is fixed ($V_\omega(t) = V_\omega(t+1)$), then for each death of a firm offering job type ω at time t a new vacancy of type ω is created at time $t+1$. This yields the following simpler law of motion

$$V_\omega(t+1) = ((1-\delta)\eta_\omega + \delta)(M_\omega(t) - V_\omega(t)) + V_\omega(t)\left[(1-\alpha_f) + \alpha_f\left[(1-p^A_\omega) + p^A_\omega((1-\delta)\eta_\omega + \delta)\right]\right].$$

Given that all analysis in this paper will be done under a maintained steady state assumption, we utilize the simpler expression above and solve for the steady state vacancy and unemployment masses as

$$(M_\omega - V_\omega) = \frac{V_\omega\left(1 - \left[(1-\alpha_f) + \alpha_f\left[(1-p^A_\omega) + p^A_\omega((1-\delta)\eta_\omega + \delta)\right]\right]\right)}{((1-\delta)\eta_\omega + \delta)} \tag{17}$$

$$U = \frac{\sum_{\omega=1}^{M}(\delta + (1-\delta\eta_\omega)(M_\omega - V_\omega)}{1 - \left[(1-\alpha_w) + \alpha_w \sum_{\omega=1}^{M} \mu_\omega\left[(1-p^A_\omega) + p^A_\omega((1-\delta)\eta_\omega + \delta)\right]\right]} \tag{18}$$

Finally, we know that the total mass of workers in the market is 1, and summing over all matched and unemployed workers gives the following identity.

$$U + \sum_{\omega=1}^{M} (M_\omega - V_\omega) = 1.$$

3.5 Equilibrium

We now define an equilibrium for our infinite horizon model.

Definition 1 *A competitive search Perfect Bayesian Equilibrium is:*

1. *Worker reservation wage strategy* $\hat{u}$
2. *Worker decisions to search* σ
3. *Worker attentional strategies:*(γ, P)
4. *Worker quit strategies:* η_ω *for* $\omega \in \Omega$
5. *Worker beliefs about the market tightness:* Y_w
6. *Firm mixed strategies of job characteristics:* ξ_ω *;*
7. *Firm beliefs about the market tightness:* Y_f *;*
8. *Firm beliefs about the rejection rates for an offer* ω*:* Ξ^ω
9. *Firm maximization and zero profit condition:* $E\left[J^S(\omega)\right] \leq 0$ *for all* $\omega \in \Omega$
10. *Worker optimization with the associated value function* $V(\omega)$ *for all* $\omega \in \Omega$.
11. *Bayes' Rule and Rational Expectations:* $\frac{V}{U} = Y_w = Y_f$ *,* $\mu_\omega = \xi_\omega$ *, and*

$$\Xi^\omega = \frac{\gamma_\omega P}{\mu_\omega}$$

    $w \in W^P$*, and* $z \in Z^w$.

12. *Market clearing*

$$U + \sum_{\omega \in \Omega} (M_\omega - V_\omega) = 1.$$

3.6 Equilibrium Characterization

To demonstrate an equilibrium of this type, we turn our attention to an equilibrium in which $\Omega = \{H, L, \emptyset\}$ with $u_L < u_\emptyset < u_H$.

Proposition 1 *Two-state equilibrium.*

Suppose that $\Omega = \{H, L, \emptyset\}$ with $u_L < u_\emptyset < u_H$. Define the following payoffs

$$\begin{aligned} \tilde{W}_H^A(\hat{u}) &= (u_H - \hat{u}) + \frac{\beta(1-\delta)(u_H - \hat{u})}{1 - \beta(1-\delta)} > 0; \\ \tilde{W}_L^A &= u_L - \hat{u} < 0; \\ \tilde{W}^R &= 0. \end{aligned}$$

and

$$y := \exp\left\{\frac{\tilde{W}_L^A}{\lambda}\right\} < 1 = \exp\left\{\frac{\tilde{W}^R}{\lambda}\right\} < \exp\left\{\frac{\tilde{W}_H^A(\hat{u})}{\lambda}\right\} =: z.$$

Finally, define $\bar{\pi} \equiv \frac{\pi_L}{\pi_H}$. Then the following policy functions and aggregate variables constitute an equilibrium

$$\sigma = 1$$

$$\eta_\omega = \begin{cases} 1 \text{ if } \omega = L \\ 0 \text{ if } \omega = H \end{cases}$$

$$P(z^\star, \mu) = \begin{cases} 1 \text{ if } \mu \geq \frac{z^\star(1-y)}{z^\star - y}; \\ \frac{\mu(z-1) - (1-\mu)(1-y)}{(z-1)(1-y)} \text{ if } \mu \in \left(\frac{1-y}{z^\star - y}, \frac{z^\star(1-y)}{z-y}\right); \\ 0 \text{ if } \mu \leq \frac{1-y}{z^\star - y}. \end{cases}$$

$$\begin{aligned} (\gamma_H^A, \gamma_L^A) &= (p_H^A(z^\star, \mu), p_L^A(z^\star, \mu)) = \left(\frac{z^\star(1-y)}{z^\star - y}, \frac{y(z^\star - 1)}{z^\star - y}\right); \\ (\gamma_H^R, \gamma_L^R) &= (p_H^R(z^\star, \mu), p_L^R(z^\star, \mu)) = \left(\frac{1-y}{z^\star - y}, \frac{z^\star - 1}{z^\star - y}\right). \end{aligned}$$

$$\mu(z^\star) = \frac{z^\star(1-y)}{z^\star(1 + (\bar{\pi} - 1)y) - \bar{\pi}y},$$

$$U^* = \frac{\delta}{\delta + (1-\delta)\alpha_w(z^*)p_H^A(z^*)\mu(z^*)}.$$

$$V^* = U^* r_f(z^*).$$

$$\begin{aligned} V_H^* &= \mu(z^*)V^*; \\ V_L^* &= V^* - V_H^*. \end{aligned}$$

$$\begin{aligned} M_L^* &= V_L^*; \\ M_H^* &= V_H^* + \frac{\alpha_f(z^*) p_H^A(z^*) V_H^*}{\delta}. \end{aligned}$$

$$p_L^A(z) = \frac{\pi_H \left(z^* - \bar{\pi} y\right)}{\pi_L (z^* - y)}.$$

$$\alpha_f(z^*) = \frac{F}{\pi_L p_L^A(z^*)} = \left(\frac{F}{\pi_H}\right)\left[\frac{z-y}{z-\bar{\pi}y}\right] = \left[\frac{F(z^*-y)}{z^*\pi_H - y\pi_L}\right].$$

$$\alpha_w(z^*) = \frac{C + \hat{u}(z^*) - u_\emptyset}{P(z^*)\left[p_H^A(z^*)\left[\frac{u_H - \hat{u}(z^*)}{1-\beta(1-\delta)}\right] + \hat{u}(z^*) - u_\emptyset - (1 - p_H^A(z))(\hat{u}(z^*) - u_L)\right] - \tilde{S}(z^*)}$$

$$\hat{u}(z^*) = u_H - \frac{\left[u_\emptyset + \lambda \ln z^* - u_H\right]\left[1 - \beta(1-\delta)\right]}{\beta(1-\delta)}.$$

if z^* solves the following equation

$$\left[\frac{\alpha_f(z)}{\theta}\right]^{-\frac{1}{a}} = \left[\frac{\alpha_w(z)}{\theta}\right]^{\frac{1}{1-a}}$$

Proof. See appendix. ■

Appendix

A Proof of Theorem 1: Reservation Utility

Theorem 1 Let $\underline{\omega} := \min \omega \in \Omega$ and $\bar{\omega} := \max \omega \in \Omega$. Define $\hat{u} \in [u_{\underline{\omega}}, u_{\bar{\omega}}]$ by $\Delta(\hat{u}) = 0$ if $\Delta(u_{\underline{\omega}}) > 0$, with $\hat{u} = u_\emptyset$ otherwise. Then the value function satisfies,

$$\begin{aligned} V(\omega) &= \begin{cases} \frac{\hat{u}}{1-\beta} + \frac{(u_\omega - \hat{u})}{1-\beta(1-\delta)} \text{ if } u_\omega > \hat{u}; \\ \frac{\hat{u}}{1-\beta} \text{ if } u_\omega \leq \hat{u}. \end{cases} \\ V(\emptyset) &= \frac{\hat{u}}{1-\beta}. \end{aligned}$$

It is optimal to search and apply rational inattention strategy $\left(\hat{\gamma}, \hat{P}\right)$ corresponding to accept/reject decision problem $D(\hat{u}, \mu)$ iff $\Delta(u_\emptyset) > 0$. It is optimal to stay in identified job $\omega \in \Omega$ if and only if $u_\omega \geq \hat{u}$. If $\Delta(u_\emptyset) \leq 0$ it is optimal not to search.

A.1 $\Delta(E)$ is strictly monotonically decreasing in E

Proof. We first establish that $\Delta(E)$ is strictly monotonically decreasing in E. That is, we want to show, for all $u_H, u_L \in [u_{\underline{\omega}}, u_{\bar{\omega}}]$ with $u_H > u_L$, that $\Delta(u_H) - \Delta(u_L) < 0$.

Recall the definition of $\Delta(E)$:

$$\Delta(E) := \alpha_w \left[D^* \left(E, \mu\right) - \frac{E}{1-\beta} \right] + (1 - \alpha_w)(u_\phi - E) - C.$$

First, we can establish a lower bound on $\Delta(u_L)$. Since the optimal search and attention strategy associated with the decision problem parameterized by u_H, namely $(\gamma(u_H), P(u_H))$, is also feasible in the decision problem with payoff parameter u_L, the $\Delta(u_L)$ associated with optimal attention strategies $(\gamma(u_L), P(u_L))$ must be at least as good as those associated with attention strategies $(\gamma(u_H), P(u_H))$. Thus,

$$\Delta(u_L) \geq \alpha_w \left[P(u_H) \sum_\Omega \gamma_\omega(u_H) W^A_\omega(u_L) + (1 - P(u_H)) W^R(u_L) - [C + S(u_H)] - \frac{u_L}{1-\beta} \right] + (1 - \alpha_w)(u_\emptyset - u_L).$$

Using this weak lower bound, we can derive a weak upper bound for $\Delta(u_H) - \Delta(u_L)$.

$$\begin{aligned} \Delta(u_H) - \Delta(u_L) \leq \alpha_w \Bigg[& P(u_H) \sum_\Omega \gamma_\omega(u_H) \left[W^A_\omega(u_H) - W^A_\omega(u_L) \right] + (1 - P(u_H)) \left[W^R(u_H) - W^R(u_L) \right] - \frac{u_H - u_L}{1-\beta} \Bigg] \\ & + (1 - \alpha_w)(u_L - u_H) \end{aligned} \tag{A.1}$$

Note that if

$$\max\left\{\max_{\omega\in\Omega}\left[W^A_\omega(u_H)-W^A_\omega(u_L)\right],\left[W^R(u_H)-W^R(u_L)\right]\right\}<\frac{u_H-u_L}{1-\beta} \tag{A.2}$$

then, equation A.1 provides a strict upper bound on the difference:

$$\begin{aligned}\Delta(u_H)-\Delta(u_L)&<\alpha_w\left[\frac{u_H-u_L}{1-\beta}\left(P(u_H)\sum_\omega\gamma_\omega(u_H)+(1-P(u_H))\right)-\frac{u_H-u_L}{1-\beta}\right]+(1-\alpha_w)(u_L-u_H)\\&=(1-\alpha_w)(u_L-u_H)<0.\end{aligned} \tag{A.3}$$

Thus, to prove $\Delta(E)$ is strictly monotonically decreasing in E it is sufficient to establish the inequality in equation A.2.

Note first directly from the definition that,

$$W^R(u_H)-W^R(u_L)=\frac{\beta(u_H-u_L)}{1-\beta}.$$

Hence equation A.2 follows if,

$$W^A_\omega(u_H)-W^A_\omega(u_L)\leq\frac{\beta(u_H-u_L)}{1-\beta}, \tag{A.4}$$

To show this holds, first note by definition that

$$W^A_\omega(u_H)-W^A_\omega(u_L)=\frac{\beta\delta(u_H-u_L)}{1-\beta}+\frac{\beta(1-\delta)}{1-\beta}\left[\max\left[\frac{(1-\beta)u_\omega+\delta\beta u_H}{1-\beta(1-\delta)},u_H\right]-\max\left[\frac{(1-\beta)u_\omega+\delta\beta u_L}{1-\beta(1-\delta)},u_L\right]\right].$$

Hence it suffices to establish,

$$\max\left[\frac{(1-\beta)u_\omega+\delta\beta u_H}{1-\beta(1-\delta)},u_H\right]-\max\left[\frac{(1-\beta)u_\omega+\delta\beta u_L}{1-\beta(1-\delta)},u_L\right]\leq u_H-u_L. \tag{A.5}$$

If $u_\omega\leq u_L$, this holds as an equality.

If $u_\omega\geq u_H$, since $\beta\in(0,1)$, inequality A.5 becomes

$$\frac{\delta\beta(u_H-u_L)}{1-\beta+\delta\beta}<u_H-u_L,.$$

Finally, if $u_\omega\in(u_L,u_H)$, inequality A.5 becomes

$$u_H-\frac{(1-\beta)u_\omega+\delta\beta u_L}{1-\beta+\delta\beta}<u_H-u_L.$$

This establishes the inequality in equation A.2 and completes the proof that $\Delta(E)$ is strictly diminishing in E. Given non-degeneracy, we know that $\mu_{\bar{\omega}}<1$ and that $\Delta(u_{\bar{\omega}})<0$. Hence if $\Delta(u_\emptyset)>0$, there exists a unique $\hat{u}\in(u_{\underline{\omega}},u_{\bar{\omega}})$ such that $\Delta(\hat{u})=0$. ■

A.2 V is a fixed point of the Bellman operator: $TV = V$

Proof. In this case we show that the function above is a fixed point of the Bellman operator $TV = V$.[4]

To confirm the fixed point property for $m \in \Omega$, consider first a state with $U_m > \hat{U}$. In this case,

$$\begin{aligned} TV(m) &= \max[U_m + \frac{\beta(1-\delta)\left[(1-\beta)U_m + \beta\delta\hat{U}\right]}{(1-\beta)\left[1-\beta(1-\delta)\right]} + \frac{\beta\delta\hat{U}}{1-\beta}, \frac{\hat{U}}{1-\beta}] \\ &= \frac{U_m\left((1-\beta)1 - \beta(1-\delta) + \beta(1-\delta)\right) + \hat{U}\left(\beta\delta\left[\beta(1-\delta) + 1 - \beta(1-\delta)\right]\right)}{(1-\beta)\left[1-\beta(1-\delta)\right]} \\ &= \frac{(1 - \beta U_m + \beta\delta\hat{U}}{(1-\beta)\left[1-\beta(1-\delta)\right]} = V(m) \end{aligned}$$

To confirm for a state $m \in \Omega$ with $U_m \leq \hat{U}$, note that in this case

$$U_m + \frac{\beta(1-\delta)\left[(1-\beta)U_m + \beta\delta\hat{U}\right]}{(1-\beta)\left[1-\beta(1-\delta)\right]} \leq \frac{\hat{U}}{1-\beta},$$

directly confirming $TV(m) = \frac{\hat{U}}{1-\beta} = V(m)$.

To establish the fixed point property for state $\emptyset$, note first that,

$$TV(\emptyset) = \max\left[U_\emptyset + \frac{\beta\hat{U}}{1-\beta}, \sup_\Lambda \hat{T}N(\gamma, P)\right]; \tag{A.6}$$

where,

$$\hat{T}N(\gamma, P) = P\left[\sum_m \gamma_m\left[U_m + \beta(1-\delta)V(m)\right] + \beta\delta V(\emptyset)\right] + (1-P)\left[U_\emptyset + \beta V(\emptyset)\right] - S(\gamma, P) - C. \tag{A.7}$$

To prove the fixed point property for this state when $\Delta(U_\emptyset) > 0$ therefore requires us to show that,

$$\sup_\Lambda \hat{T}N(\gamma, P) = \frac{\hat{U}}{1-\beta}.$$

The proof follows from substitution. Note first that,

$$\begin{aligned} U_m + \beta(1-\delta)V(m) + \beta\delta V(\emptyset) &= U_m + \beta(1-\delta)\max\left[\frac{(1-\beta)U_m + \beta\delta\hat{U}}{(1-\beta)\left[1-\beta(1-\delta)\right]}, \frac{\hat{U}}{1-\beta}\right] + \frac{\beta\delta\hat{U}}{1-\beta} \\ &= W_m^A(\hat{U}). \end{aligned}$$

Note in addition that

$$U_\emptyset + \beta V(\emptyset) = U_\emptyset + \frac{\beta\hat{U}}{1-\beta} = W^R(\hat{U}),$$

[4] As written, this part of the proof is for the case when $\alpha_w = 1$. To be updated for the $\alpha_w \in (0, 1)$ case. The update is straight forward and follows exact same approach.

and that $\sum_m \gamma_m = 1$. Substitution in () then implies,

$$\hat{T}N(\gamma, P) = P\sum_m \gamma_m W_m^A(\hat{U}) + (1-\hat{P})W^R(\hat{U}) - S(\gamma, P) - C.$$

With this, we show that $\hat{T}N(\gamma, P) \geq \frac{\hat{U}}{1-\beta}$ by evaluating $\hat{T}N(\gamma, P)$ at the strategy $(\hat{\gamma}, \hat{P}) = (\gamma(\hat{U}), P(\hat{U}))$ that achieves the optimum in decision problem $D(\hat{U}, \mu)$,

$$\hat{T}N(\hat{\gamma}, \hat{P}) = \hat{P}\sum_m \hat{\gamma}_m W_m^A(\hat{U}) + (1-\hat{P})W^R(\hat{U}) - S(\hat{\gamma}, \hat{P}) - C = D^*(\hat{U}, \mu) - C = \frac{\hat{U}}{1-\beta},$$

since $\Delta(\hat{U}) = 0$. To show that $\hat{T}N(\gamma, P) \leq \frac{\hat{U}}{1-\beta}$, we suppose to the contrary that there exists $\left(\dot{\gamma}, \dot{P}\right) \in \Lambda$ such that,

$$\hat{T}N\left(\dot{\gamma}, \dot{P}\right) = \dot{P}\sum_m \dot{\gamma} W_m^A(\hat{U}) + (1-\dot{P})W^R(\hat{U}) - S\left(\dot{\gamma}, \dot{P}\right) - C > \frac{\hat{U}}{1-\beta}.$$

This directly contradicts optimality of $(\hat{\gamma}, \hat{P})$ in problem $D(\hat{U}, \mu)$,

$$D^*(\hat{U}, \mu) = C + \frac{\hat{U}}{1-\beta} \geq \dot{P}\sum_m \dot{\gamma} W_m^A(\hat{U}) + (1-\dot{P})W^R(\hat{U}) - S\left(\dot{\gamma}, \dot{P}\right).$$

To prove theorem 1 for $\Delta(U_\emptyset) \leq 0$, we confirm that in such cases $V(m) = \max\left[\frac{(1-\beta)U_m + \beta\delta U_\emptyset}{(1-\beta)[1-\beta(1-\delta)]}, \frac{U_\emptyset}{1-\beta}\right]$ and $V(\emptyset) = \frac{U_\emptyset}{1-\beta}$ is a fixed point of the Bellman operator. Consider first $m \in \Omega$ with $U_m > U_\emptyset$. In this case,

$$\begin{aligned} TV(m) &= \max[U_m + \frac{\beta(1-\delta)\left[(1-\beta)U_m + \beta\delta U_\emptyset\right]}{(1-\beta)\left[1-\beta(1-\delta)\right]} + \frac{\beta\delta U_\emptyset}{1-\beta}, \frac{U_\emptyset}{1-\beta}] \\ &= \frac{(1-\beta)U_m + \beta\delta U_\emptyset}{(1-\beta)\left[1-\beta(1-\delta)\right]} = V(m) \end{aligned}$$

Given state $m \in \Omega$ with $U_m \leq U_\emptyset$,

$$U_m + \frac{\beta(1-\delta)\left[(1-\beta)U_m + \beta\delta U_\emptyset\right]}{(1-\beta)\left[1-\beta(1-\delta)\right]} \leq \frac{U_\emptyset}{1-\beta},$$

so that again $TV(m) = \frac{U_\emptyset}{1-\beta} = V(m)$. Finally, consider the Bellman operator for state $\emptyset$,

$$TV(\emptyset) = \max\left[\frac{U_\emptyset}{1-\beta}, \sup_\Lambda T_\emptyset N(\gamma, P)\right];$$

where,

$$T_\emptyset N(\gamma, P) = P\left[\sum_m \gamma_m \left[U_m + \beta(1-\delta)\max\left[\frac{(1-\beta)U_m + \beta\delta U_\emptyset}{(1-\beta)\left[1-\beta(1-\delta)\right]}, \frac{U_\emptyset}{1-\beta}\right] + \frac{\beta\delta U_\emptyset}{1-\beta}\right]\right] + (1-P)\left[\frac{U_\emptyset}{1-\beta}\right] - S(\gamma, P) - C.$$

Substitution as above reveals,

$$T_\emptyset N(\gamma, P)) = P\sum_m \gamma_m W_m^A(U_\emptyset) + (1-\hat{P})W^R(U_\emptyset) - S(\gamma, P) - C.$$

To complete the proof of theorem 1 for $\Delta(U_\emptyset) \leq 0$ requires us to show that,

$$\sup_\Lambda T_\emptyset N(\gamma, P) \leq \frac{U_\emptyset}{1-\beta}.$$

Suppose to the contrary that there exists $\left(\dot{\gamma}, \dot{P}\right) \in \Lambda$ such that,

$$\dot{P}\sum_m \dot{\gamma}_m W_m^A(U_\emptyset) + (1-\dot{P})W^R(U_\emptyset) - S(\dot{\gamma}, \dot{P}) - C - \frac{U_\emptyset}{1-\beta} > 0.$$

This directly contradicts the fact that,

$$\Delta(U_\emptyset) = \arg\max_\Lambda P\sum_m \gamma_m W_m^A(U_\emptyset) + (1-P)W^R(U_\emptyset) - S(\gamma, P) - C - \frac{U_\emptyset}{1-\beta} \leq 0,$$

completing the proof of Theorem 1. ■

B Equilibrium Characterization: Two Types of Jobs

1. We will be working to identify an appropriate value of $\hat{u}$ and all other equilibrium conditions for the corresponding two period model

$$\begin{aligned} W_\omega^A(\hat{u}) &= \begin{cases} \frac{\hat{u}}{1-\beta} + \frac{(u_\omega - \hat{u})}{1-\beta(1-\delta)} \text{ if } u_\omega > \hat{u}; \\ \frac{\hat{u}}{1-\beta} - (\hat{u} - u_\omega) \text{ if } u_\omega \leq \hat{u}; \end{cases} \\ W^R(\hat{u}) &= \frac{\hat{u}}{1-\beta} - (\hat{u} - u_\emptyset). \end{aligned}$$

   Note that for the most part we will be concerned only with properties of the solution, not the resulting value. For such purposes note that the problem is unchanged if we subtract $W^R(\hat{u})$ from all payoffs. An interesting solution involves rejecting low wage and accepting high wage offers yielding payoffs.[5] Thus we will examine equilibria with the following payoff structure:

$$\begin{aligned} \tilde{W}_H^A(\hat{u}) &= (u_H - u_\emptyset) + \frac{\beta(1-\delta)(u_H - \hat{u})}{1-\beta(1-\delta)} > 0; \\ \tilde{W}_L^A &= u_L - u_\emptyset < 0; \\ \tilde{W}^R &= 0. \end{aligned}$$

[5] Which can be translated into conditions on model primitives.

2. We now focus precisely on the rational inattention model fixing rejection to having payoff $w^R = 0$, with $w_L < 0$ fixed (because it is a function only of parameters) and therefore with only two unknown variables, w_H the payoff in the high state and $\mu \in [0,1]$ the proportion of high type firms. The model is easiest solved in terms of exponentiated returns,

$$y := \exp\left\{\frac{\tilde{W}_L^A}{\lambda}\right\} < 1 = \exp\left\{\frac{\tilde{W}^R}{\lambda}\right\} < \exp\left\{\frac{\tilde{W}_H^A(\hat{u})}{\lambda}\right\} =: z.$$

The return to the high type job is a function of $\hat{u}$, which will be suppressed until we complete the equations that enable us to identify equilibria. Thus, we have a decision problem parameterized by unknowns (z, μ).

3. Define $P(z,\mu)$ as the corresponding optimal unconditional acceptance probability. Directly from CDL we can express this in closed form as:

$$P(z,\mu) = \begin{cases} 1 & \text{if } \mu \geq \frac{z(1-y)}{z-y} \\ \frac{\mu(z-1)-(1-\mu)(1-y)}{(z-1)(1-y)} & \text{if } \mu \in \left(\frac{1-y}{z-y}, \frac{z(1-y)}{z-y}\right) \\ 0 & \text{if } \mu \leq \frac{1-y}{z-y} \end{cases}$$

Note that for $\mu \in \left(\frac{1-y}{z-y}, \frac{z(1-y)}{z-y}\right)$ the probability satisfies $P(z,\mu) \in (0,1)$:

$$\begin{aligned} \mu(z-1)-(1-\mu)(1-y) &= \mu(z-y)-(1-y) > 0 \Longleftrightarrow \mu > \frac{1-y}{z-y}; \\ \mu(z-1)-(1-\mu)(1-y) &< (z-1)(1-y) \Longleftrightarrow \mu(z-y)-(1-y) < z(1-y)-(1-y) \Longleftrightarrow \mu < \frac{z(1-y)}{z-y} \end{aligned}$$

   (a) If $P(z,\mu) = 0$ then there is no acceptance hence no search in the first place and no one is employed. Hence we do not study equilibria of this form.

4. Given $\mu \in \left(\frac{1-y}{z-y}, \frac{z(1-y)}{z-y}\right)$ so that $P(z,\mu) \in (0,1)$ CDL establish the invariant likelihood ratio condition holds.

$$\begin{aligned} \frac{p_H^A(z,\mu)}{p_H^R(z,\mu)} &= \frac{\exp\left\{\frac{W_H^A}{\lambda}\right\}}{\exp\left\{\frac{W^R}{\lambda}\right\}} = z; \\ \frac{p_L^A(z,\mu)}{p_L^R(z,\mu)} &= \frac{\exp\left\{\frac{W_L^A}{\lambda}\right\}}{\exp\left\{\frac{W^R}{\lambda}\right\}} = y \end{aligned}$$

This enables us to compute posterior beliefs given acceptance and rejection:

$$\begin{aligned} \left(p_H^A(z,\mu), p_L^A(z,\mu)\right) &= \left(\frac{z(1-y)}{z-y}, \frac{y(z-1)}{z-y}\right); \\ \left(p_H^R(z,\mu), p_L^R(z,\mu)\right) &= \left(\frac{1-y}{z-y}, \frac{z-1}{z-y}\right). \end{aligned}$$

Note that these are always qualifying posteriors with all terms are strictly below unity and conditionals appropriately adding to 1. Note also that they are independent of μ provided it is in the appropriate range. Qualitatively, acceptance is associated with an increased probability of the good state above the prior belief, rejection with a lower such probability.

5. The zero profit conditions for firms making high and low utility offers are,

$$\alpha_f p_H^A(z,\mu) J^M(H) = F = \alpha_f p_L^A(z,\mu) J^M(L),$$

where F is the cost of posting either type of job, and $J^M(H)$ and $J^M(L)$ are the present discounted value of profits from filling a high and low type job respectively. Hence for both types of firm to be in operation we need a specific ratio condition to be satisfied,

$$\frac{p_H^A(z,\mu)}{p_L^A(z,\mu)} = \frac{J^M(L)}{J^M(H)} =: \bar{\pi},$$

where we have defined $\bar{\pi}$ to be the ratio of the value of a low-type matched firm to a high-type matched firm. Applying Bayes' rule we identify a necessary restriction on μ as a function of z for this ratio condition to be satisfied

$$\frac{p_H^A(z,\mu(z))}{p_L^A(z,\mu(z))} = \frac{(1-\mu(z))p_H^A(z)}{\mu(z)p_L^A(z)} = \frac{(1-\mu(z))z(1-y)}{\mu(z)y(z-1)} = \bar{\pi}.$$

Hence,

$$\mu(z) = \frac{z(1-y)}{z(1+(\bar{\pi}-1)y) - \bar{\pi}y} \Longrightarrow 1-\mu(z) = \frac{\bar{\pi}y(z-1)}{z(1+(\bar{\pi}-1)y) - \bar{\pi}y}.$$

Note $\mu(z) \in (0,1)$ always since,

$$0 < z(1-y) < z(1-y) + \bar{\pi}y(z-1) = z(1+(\bar{\pi}-1)y) - \bar{\pi}y.$$

Note that we can sign the derivative $\mu'(z)$,

$$\mu'(z) \sim (1-y)\left[z(1+(\bar{\pi}-1)y) - \bar{\pi}y\right] - z(1-y)\left[1+(\bar{\pi}-1)y\right] = -\bar{\pi}y(1-y) < 0.$$

6. It is important to note the implied restriction on the data for the profit ratio condition to hold. Specifically it requires that,

$$\bar{\pi} > 1.$$

To see this, note that, since $\mu(z) < \frac{z(1-y)}{z-y}$,

$$\bar{\pi} = \frac{(1-\mu(z))z(1-y)}{\mu(z)y(z-1)} > \frac{(\frac{y(z-1)}{z-y})z(1-y)}{(\frac{z(1-y)}{z-y})y(z-1)} = 1.$$

We assume this from now on.

(a) Note that if $\bar{\pi} leq 1$ then there are only high type firms in the equilibrium, which implies that all offers are accepted and the model is uninteresting.

7. We now substitute for $\mu(z)$ in the RI solution to define $P(z) \equiv P(z, \mu(z))$,

$$\begin{aligned} P(z) &= \frac{\mu(z)(z-1) - (1-\mu(z))(1-y)}{(z-1)(1-y)} = \\ &= \frac{z(1-y)(z-y) - (1-y)\left[z(1+(\bar{\pi}-1)y) - \bar{\pi}y\right]}{(z-1)(1-y)\left[z(1+(\bar{\pi}-1)y) - \bar{\pi}y\right]} \\ &= \frac{z(z-y) - \left[z(1+(\bar{\pi}-1)y) - \bar{\pi}y\right]}{(z-1)\left[z(1+(\bar{\pi}-1)y) - \bar{\pi}y\right]} = \frac{z(z-1) - \bar{\pi}y(z-1)}{(z-1)\left[z(1+(\bar{\pi}-1)y) - \bar{\pi}y\right]} \\ &= \frac{z - \bar{\pi}y}{z(1+(\bar{\pi}-1)y) - \bar{\pi}y}. \end{aligned}$$

Again we can sign the derivative $P'(z)$,

$$\begin{aligned} P'(z) &\sim \left[z(1+(\bar{\pi}-1)y) - \bar{\pi}y\right] - (z - \bar{\pi}y)\left[1+(\bar{\pi}-1)y\right] \\ &= \bar{\pi}y\left[1+(\bar{\pi}-1)y\right] > \bar{\pi}y\left[1-y\right] > 0. \end{aligned}$$

8. Note that,

$$\begin{aligned} p_L^A(z) &= \frac{P(z)p_L^A(z,\mu(z))}{1-\mu(z)} = \frac{P(z)y(z-1)}{(z-y)(1-\mu(z))} \\ &= \frac{\left[z-\bar{\pi}y\right]y(z-1)}{(z-y)\bar{\pi}y(z-1)} = \frac{z-\bar{\pi}y}{\bar{\pi}(z-y)} = \frac{\pi_H(z-\bar{\pi}y)}{\pi_L(z-y)}. \end{aligned}$$

9. We define $\alpha_f(z)$ to reflect the zero profit condition for low type firms,

$$\alpha_f(z) = \frac{F}{J^M(L)p_L^A(z)} = \left(\frac{F}{J^M(H)}\right)\left[\frac{z-y}{z-\bar{\pi}y}\right] = \left[\frac{F(z-y)}{zJ^M(H) - yJ^M(L)}\right].$$

We want to understand when $\alpha_f(z) \in [0,1]$ as this is necessary it to be a meaningful match probability. We can identify the sign of $\alpha_f'(z)$ in terms of the parameters by signing the derivative of the second part of the product,

$$\alpha_f'(z) \sim (z - \bar{\pi}y) - (z-y) = (1-\bar{\pi})\,y \sim 1 - \bar{\pi} < 0.$$

(a) Given strict monotonicity, the key is exploring the value of this function at the lower boundary of $z = 1$ and in the limit as z increases to infinity,

$$\lim_{z\to\infty} \alpha_f(z) = \left(\frac{F}{J^M(H)}\right) > 0,$$

so that,

$$F < J^M(H),$$

ensures meaningfulness as z increases. Note that this is also a necessary condition for an interior solution, given the zero profit condition in which F is paid always and that not all offers are accepted, even those of high type.

(b) To complete the solution note that the function has a discontinuity at $z = \bar{\pi}y$ going from negative to positive infinity. When,

$$y > \frac{1}{\bar{\pi}},$$

we know that,

$$\alpha_f(1) = \frac{F(1-y)}{J^M(H) - yJ^M(L)} < 0.$$

This means for sure that we are in the case in which not all values of $z > 1$ produce meaningful solutions, so that we can identify the value $\bar{z}$ at which $\alpha_f(\bar{z}) = 1$ as,

$$\bar{z} = \frac{y(\pi_L - F)}{(\pi_H - F)} > 1,$$

so that $z \geq \bar{z}$ is consistent with equilibrium.

(c) When $y < \frac{J^M(H)}{J^M(L)}$ we know that $\alpha_f(1) > 0$. In this case that $\alpha_f(1) \leq 1$ holds if and only if,

$$F(1-y) \leq J^M(H) - yJ^M(L) \Longleftrightarrow y \leq \frac{J^M(H) - F}{J^M(L) - F},$$

a more stringent condition. If so, then there is always a meaningful solution. If not, we again identify the value at which $\alpha_f(\bar{z}) = 1$ so that $z \geq \bar{z}$ is consistent with equilibrium as,

$$\bar{z} = \frac{y(J^M(L) - F)}{(J^M(H) - F)}.$$

10. We now analyze the reservation equation which requires transforming returns back to the actual level (the level of reservation utility $\hat{u}$ now is to be solved for). Note first that

$$y \equiv \exp\left\{\frac{u_L - u_\emptyset}{\lambda}\right\} \Longrightarrow u_L = u_\emptyset + \lambda \ln y.$$

The key though is to define $\hat{u}(z)$:

$$z = \exp\left\{\frac{(u_H - u_\emptyset) + \frac{\beta(1-\delta)(u_H - \hat{u})}{1-\beta(1-\delta)}}{\lambda}\right\} \Longrightarrow \lambda \ln z = (u_H - u_\emptyset) + \frac{\beta(1-\delta)(u_H - \hat{u}(z))}{1-\beta(1-\delta)};$$

hence,

$$\hat{u}(z) = u_H - \frac{[u_\emptyset + \lambda \ln z - u_H]\,[1-\beta(1-\delta)]}{\beta(1-\delta)}.$$

11. Define $\alpha_w(z)$ to satisfy the corresponding reservation search equation with low types quit and high type jobs accepted,

$$\begin{aligned} & C + \alpha_w(z)\tilde{S}(z) + (1 - \alpha_w P(z))(\hat{u}(z) - u_\emptyset) + \alpha_w P(z)(1 - p_H^A(z))(\hat{u}(z) - u_L) \\ &= \alpha_w(z)P(z)p_H^A(z)\left[\frac{u_H - \hat{u}(z)}{1 - \beta(1-\delta)}\right] \\ &= \alpha_w(z)P(z)p_H^A(z)\left[\frac{u_\emptyset + \lambda \ln z - u_H}{\beta(1-\delta)}\right]. \end{aligned}$$

where $\tilde{S}(z)$ is the optimized Shannon cost associated with RI problem with parameters $(z, \mu(z))$ hence utility,

$$\tilde{W}_H^A(z) = (u_H - u_\emptyset) + \frac{\beta(1-\delta)(u_H - \hat{u}(z))}{1 - \beta(1-\delta)} = (u_H - u_\emptyset) + u_\emptyset + \lambda \ln z - u_H = \lambda \ln z,$$

and,

$$\tilde{W}_L^A = u_L - u_\emptyset = \lambda \ln y,$$

with $\tilde{W}^R = 0$. Rearrangement yields,

$$\alpha_w(z) = \frac{C + \hat{u}(z) - u_\emptyset}{P(z)\left[p_H^A(z)\left[\frac{u_H - \hat{u}(z)}{1-\beta(1-\delta)}\right] + \hat{u}(z) - u_\emptyset - (1 - p_H^A(z))(\hat{u}(z) - u_L)\right] - \tilde{S}(z)}.$$

12. Note that the numerator has derivative,

$$\frac{d\hat{u}(z)}{dz} = -\frac{\lambda[1 - \beta(1-\delta)]}{\beta(1-\delta)z} < 0.$$

Hence to sign this derivative needs us to study the denominator $D(z)$,

$$D(z) \equiv P(z)\left[p_H^A(z)\left[\frac{u_H - \hat{u}(z)}{1 - \beta(1-\delta)}\right] + \hat{u}(z) - u_\emptyset - (1 - p_H^A(z))(\hat{u}(z) - u_L)\right] - \tilde{S}(z).$$

There is reason to believe that an increase in z, which raises the reward to employment in the high state, will raise the corresponding surplus as computed above, so that $D'(z) > 0$ and $\alpha_w'(z) < 0$ as a result.

13. Provided $\alpha_f(z) < 1$, we uniquely define the corresponding vacancy-to-unemployment ratio as,

$$r_f(z) = \left[\frac{\alpha_f(z)}{\theta}\right]^{-\frac{1}{a}}.$$

If $\alpha_f(z) = 1$, we can place only a bound on the vacancy to unemployment rate: higher unemployment is fine hence a lower such ratio which still generates all firms receiving offers for sure,

$$r_f(z) \leq \left[\frac{1}{\theta}\right]^{-\frac{1}{a}}.$$

14. Provided $\alpha_w(z) < 1$, we define the corresponding vacancy to unemployed ratio as,

$$r_w(z) = \left[\frac{\alpha_w(z)}{\theta}\right]^{\frac{1}{1-a}}.$$

15. If $\alpha_w(z) = 1$, we can place only a bound on the vacancy to unemployment rate: higher vacancies are fine hence a higher such ratio which still generates all firms receiving offers for sure,

$$r_w(z) \geq \left[\frac{1}{\theta}\right]^{\frac{1}{1-a}}$$

16. An interior equilibrium z^* with has the property that the two implied ratios are equal,

$$r_f(z^*) = r_w(z^*).$$

    There are also corner equilibria with either $\alpha_w(z^*) = 1$ so that all workers receive offers and $\alpha_f(z^*) = 1$ so that all firms get matched.

17. The existence of a z^* that equates $r_f(z^*) = r_w(z^*)$ yields existence of an equilibrium. Since the first derivative of $\alpha_f(z)$ is negative, if $\alpha_w(z)$ is monotonic and we can sign the derivative, then the equilibrium is unique. Still to do.

18. From this point on, we can use the market clearing condition to solve for quantity variables, U, V_H, and V_L

    (a) Solve for unemployment rate from market clearing,

$$U^* = \frac{\delta}{\delta + (1-\delta)\alpha_w(z^*)p_H^A(z^*)\mu(z^*)}.$$

    (b) Solve for vacancies from vacancy to unemployment ratio,

$$V^* = U^* r_f(z^*).$$

    (c) Solve for mass of high/low type vacancies from prior μ^*.

$$\begin{aligned} V_H^* &= \mu(z^*)V^*; \\ V_L^* &= V^* - V_H^*. \end{aligned}$$

    (d) Solve for masses of high and low type firms from steady state equations,

$$\begin{aligned} M_L^* &= V_L^*; \\ M_H^* &= V_H^* + \frac{\alpha_f(z^*)p_H^A(z^*)V_H^*}{\delta}. \end{aligned}$$