

Endogenous Uncertainty and Credit Crunches*

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Preliminary and Incomplete

Abstract

This paper examines the relationship between uncertainty and financial crises. In particular, we present a model where the amount of funding a firm receives depends on how financial markets assess the firm's business conditions. Financial agents endogenously learn about a firm's business conditions from local business indicators, but financial constraints impair the usefulness of this information when a firm is short of funds. This is because financially constrained firms respond less to their private information, reducing the informativeness of business indicators. As a result, a temporary aggregate shock to the economy's financial capacity causes a persistent cycle of uncertainty and financial constraints, where financial markets grow increasingly uncertain about firms without funding. While this feedback loop bears little effect on firms with access to funds, the severe effects it has on constrained firms generate a deep and long-lasting aggregate recession, characterized by an increased misallocation of credit, an increased cross-sectional dispersion of output across firms, and highly volatile and pessimistic financial markets.

Keywords: Credit crises, endogenous uncertainty, financial frictions, learning, misallocation, output and belief dispersion, persistence of pessimism.

JEL Classification: D83, E32, E44, G01.

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1 Introduction

Financial crises often entail long-lasting recessions. Commentators who have attempted to explain such recession episodes have frequently pointed to uncertainty as a key factor. For example, Olivier Blanchard (2009) speculated at the height of the recent financial crisis that “the crisis would largely go away” if it were not for uncertainty.¹ However, relatively little theoretical work has analyzed the reasons why uncertainty increases in times of financial distress and how this affects the transmission of financial shocks throughout the economy. This paper explores the joint propagation of uncertainty and financial distress, looking at a model where adverse shocks to the financial sector endogenously generate uncertainty about firm-specific fundamentals.

The model is based on two ingredients. First, firms’ access to finance depends on how financial markets assess their business conditions. When financial markets are optimistic and uncertainty is low, funding will be easier than when markets are pessimistic and uncertainty is high. Second, although financial markets may have good information on the aggregate state of the economy, they have only limited information about local business conditions. Their evaluation of a firm’s business conditions therefore depends on noisy business indicators, such as firm- or sector-specific employment and production decisions. This opens the door to “belief traps”, where a temporary shortage of funding can develop into a long-lasting funding problem, independently of a firm’s true business fundamentals.

In particular, a shortage of funds will reduce the information that can be gained from looking at a firm’s real business activity. This is because subject to a shortage of funds, a firm’s production decisions become less sensitive to business fundamentals and are instead more dictated by financial constraints. A temporary shortage of funds therefore reduces the arrival of new information, reinforcing any prevailing uncertainty and/or pessimism. With uncertainty and pessimism feeding back into financial markets, this perpetuates financial distress and creates a persistent cycle of uncertainty and financial constraints, in which a firm’s production decisions are virtually decoupled from its business fundamentals.

In this paper, we explore how this two-way interaction between beliefs and financial constraints affects the allocation of credit to firms. In particular, we study the implications of an aggregate shock to the financial sector, which increases the likelihood of any given firm to be short of funding.

The analysis is based on a standard neoclassical economy where the economy is split into multiple islands with heterogeneous productivities. Each island accommodates a competitive banking sector, supplying firms with credit. Banks operate subject to a bank leverage requirement and are exposed to local firms through equity holdings on their balance sheet. Balance sheets are priced mark-to-market

¹Similarly, Bloom et al. (2014) reported how uncertainty was repeatedly recognized by the Federal Open Market Committee as a key driver of both, the recession that followed the dot-com bubble in 2001, and the recent Great Recession. In an empirical study, Stein and Stone (2013) find that uncertainty, proxied by options-implied volatilities, approximately doubled in the 2007–2009 crisis, accounting for one third of the decline in U.S. capital investments and hirings during that period. More generally, Reinhart and Rogoff (2009), Hall (2014), and Ball (2014) document the long-lasting nature of financial crises, and Stock and Watson (2012), Caldara et al. (2013), and Gilchrist, Sim and Zakrajšek (2014) present evidence regarding the importance of uncertainty during financial crisis.

by traders in a financial market, thus linking the local credit supply to traders’ beliefs, which reflect information from a combination of market prices, noisy signals about the local productivity and recent production decisions of local firms. In addition to being exposed to traders’ beliefs, bank balance sheets are also subject to both idiosyncratic and aggregate financial shocks. There is no uncertainty about the aggregate state of the economy.

In this environment, the degree to which an island’s production sector will be financially constrained depends on three factors. First, it is driven by optimism and pessimism, defined as the wedge between an island’s true productivity and traders’ belief about it. Conditional on this wedge, a commonly known increase in an island’s productivity (or an increase in aggregate demand) does not change the degree to which firms are *effectively* constrained, since the increasing demand for credit will be exactly offset by an expansion of credit limits due to the concurrent rise in bank value. Second, due to risk aversion, an island is more likely to be constrained if traders become more uncertain about an island’s local productivity. Lastly, idiosyncratic and aggregate financial shocks move financial constraints without affecting credit demand and therefore also affect the likelihood of being constrained. In equilibrium, the measure of islands that will be constrained at any given point in time is therefore determined by the cross-sectional distribution of optimism/pessimism, uncertainty, and idiosyncratic financial shocks, as well as the aggregate financial state.

An adverse aggregate shock to the financial sector influences the measure of constrained islands in two ways. On the one hand, for a given cross-sectional distribution, it increases the measure of states, in which an island turns out to be constrained, because islands will be less resilient to idiosyncratic financial distress. On the other hand, it also skews the cross-sectional distribution of states into the direction of islands being more constrained. In particular, an adverse aggregate shock inhibits learning as described above, causing more islands to be uncertain and pessimistic. To see what causes the increase in pessimism, note that in the cross-section, pessimistic islands are more likely to be constrained, implying that learning about pessimistic islands is on average less effective. In consequence, pessimism becomes “sticky” when financial constraints are tight. An aggregate financial shock therefore increases pessimism across the economy—even though fundamentals remain unchanged and signals are on average unbiased. In that sense, markets are naturally pessimistic during financial crises.

In terms of real economic activity, the change in the cross-sectional distribution of beliefs translates into a divergence in the paths of marginally constrained and marginally unconstrained firms. While adverse financial shocks affect the latter only via general equilibrium effects, the former are subjected to the aforementioned spiral of uncertainty, pessimism, and financial constraints. Endogenous uncertainty is therefore able to generate a positive co-movement between financial distress and the cross-sectional dispersion of economic activity or beliefs.² In particular, the model

²More precisely, the positive co-movement requires that the uncertainty-driven divergence is strong enough to overturn the natural tendency for firms to converge when their production choices become more constrained. The latter tendency implies that firms always converge during financial crises, when shutting down the endogenous uncertainty in our model.

implies a left-skewed distribution for log output (or log sales) during times of financial distress and a close to symmetric distribution in normal times, consistent with the evidence reported by [Bloom et al. \(2014\)](#).

On an aggregate level, an increase in the measure of constrained islands translates into a loss in employment and output through an increase in the economy’s labor and efficiency wedges, reflecting a misallocation of credit.³ These responses are characterized by a high degree of internal persistence that is tightly linked to the endogenous nature of uncertainty.

We use a simple numerical experiment to explore the quantitative potential of the model. In particular, we explore how the model propagates a temporary financial shock with a half-life of four quarters and compare it with counterfactual responses, where we exogenously fix the informativeness of all signals as if the economy were unconstrained. While a four quarter shock to the counterfactual economy produces a short-lasting recession with a half-life of 2 quarters, the same shock produces a long-lasting recession with a half-life of 11 quarters in the endogenous uncertainty economy. Summarizing the above discussion, the responses are characterized by an increased misallocation of credit across firms, an increased cross-sectional dispersion of firm-specific business activity, and high levels of uncertainty and pessimism in financial markets. In addition, high levels of uncertainty naturally imply high levels of risk-premia as well as an increasing disagreement among traders and highly volatile asset prices. To see the mechanism behind the last two effects, notice that traders place more weight on private information—which is inherently diverse—when less can be learned from the production sector. At the same time, traders also try to extract more information from asset price movements, amplifying any noise.

Related literature This paper relates to a recent literature following [Bloom \(2009\)](#) that puts forth the idea of uncertainty-driven business cycles resulting from *exogenous* “uncertainty” shocks (e.g., [Sim, 2008](#); [Bachmann and Bayer, 2009](#); [Bloom et al., 2014](#); [Arellano, Bai and Kehoe, 2012](#); [Backus, Ferriere and Zin, 2014](#)). Complementing these works, we provide a theory that microfounds fluctuations in uncertainty. Specifically, our theory explains why uncertainty is high during times of financial distress, when empirical measures of uncertainty are highest. A distinct feature of our microfoundation is that it unleashes a feedback loop between uncertainty and financial constraints, implying that—unlike the quick recoveries typical for the exogenous uncertainty literature—high uncertainty goes along with deep and long-lasting recessions.

On the other side, the exogenous uncertainty literature complements this paper in that it highlights a number of additional channels through which uncertainty is likely to adversely affect the economy. Among the channels explored by the exogenous uncertainty literature, our modeling approach is most similar to [Christiano, Motto and Rostagno \(2009\)](#) and [Gilchrist, Sim and Zakrajšek \(2014\)](#) who share with us the propagation of uncertainty via the financial sector. In support of a financial transmission channel, [Caldara et al. \(2013\)](#) and [Gilchrist, Sim and Zakrajšek \(2014\)](#) present

³For simplicity, the model abstracts from physical capital and instead works with a working-capital requirement. In a more quantitative environment, financial constraints would manifest themselves as investment wedges as well.

evidence that uncertainty strongly affects investments *via* increasing credit spreads, but has virtually no impact on investments when controlling for credit spreads.

At a methodological level, this paper relates most closely to a recently developing area of macroeconomics that incorporates information frictions into standard business cycle models (Lorenzoni, 2009; Angeletos and La’O, 2010, 2013; Maćkowiak and Wiederholt, 2012; Hassan and Mertens, 2014a,b; Hellwig and Venkateswaran, 2011; Acharya, 2013; Atolia and Chahrour, 2013; Angeletos, Collard and Dellas, 2014). Specifically, La’O (2010) shares with us the combination of learning with financial frictions, but rules out any effect on uncertainty by resolving all dispersion of information by the time agents learn from financially constrained firms; and Angeletos, Lorenzoni and Pavan (2010) also study how learning from the real sector affects financial markets, focusing on a feedback channel that encourages over-investing. However, all these paper have in common that uncertainty is constant along the equilibrium path.

Formally, uncertainty is state-dependent in this paper, because learning from financially constrained firms gives rise to a *concave* signal structure where the slope is decreasing in the “constrainedness” of an island (i.e., the higher business conditions are relative to credit limits). In a related contribution (Straub and Ulbricht, 2014), we show that signal nonlinearities generally imply higher posterior uncertainty when the signal realizes in flatter regions.⁴ One technical challenge in analyzing the dynamic properties of our model is that nonlinear Gaussian signal structures do not pair with any (reasonable) conjugate prior distribution. In this paper, we address this issue by developing a simple approximation approach, which captures the key features of the nonlinear learning problem while preserving tractability.⁵

Our finding that financial frictions destroy information also relates to van Nieuwerburgh and Veldkamp (2006) who explore an alternative mechanism through which uncertainty may fluctuate across the business cycle. In particular, Van Nieuwerburgh and Veldkamp explore the idea that learning about total factor productivity is slow in recessions when total business activity is low (see also Veldkamp, 2005; Ordoñez, 2012). The reason is that if output (in levels) is perturbed by an additive noise term, then this noise term contributes relatively more to output when output is low, leading to higher uncertainty in recessions. Chamley and Gale (1994) and Fajgelbaum, Schaal and Taschereau-Dumouchel (2014) have combined this idea of business activity-driven uncertainty with a version of the “wait-and-see” feedback mechanism propagated in Bloom (2009). The mechanism studied in this paper differs from the mechanism in these papers in that business activity does not *generate* signals, but *is* a signal. Accordingly, as discussed above, the efficiency of learning is governed by the degree to which the economy is constrained rather than by the level of output. In particular, in contrast to the aforementioned papers which predict that uncertainty is unconditionally low in all crisis, the mechanism explored here predicts that uncertainty is in particular caused by

⁴Mertens (2011) and Hassan and Mertens (2014b) also encounter nonlinear signal structures. However, in their papers, the information structure is such that nonlinearities are transformed away before adding noise, so that agents actually update according to a linear signal structure, avoiding endogenous uncertainty.

⁵See Straub and Ulbricht (2012) for an alternative approach where we construct noise terms in such a way that the nonlinear signals behave *as if* they were linear Gaussian signals.

nonfundamental crisis. This approach is shared with two related contributions by [Yuan \(2005\)](#) and [Albagli \(2011\)](#). Both papers focus, however, on one-shot financial market settings, preventing them from analyzing the transmission of financial shocks through the economy, which is at the core of our contribution.⁶

At a quantitative level, our approach to modeling endogenous uncertainty also differs from the previous literature in that we note that business cycle variations in aggregate business activity (or aggregate constraints) are likely too small to induce quantitatively significant variations in uncertainty. Specifically, it seems hard to justify why even during a crisis as severe as the great recession where up to 20 percent of firms were affected, markets would be unable to extract any aggregate information from the remaining fraction of firms. In this paper, we therefore take the extreme point of view that aggregates are fully observable, and instead focus on learning about local business conditions. In our model this implies that very small fluctuations in the financial constraints (with almost no impact on business activity by themselves) can have severe and long-lasting effects on *some* firms. Because aggregation is linear (up to general equilibrium effects), this approach of modeling uncertainty at the local level can easily explain why small fluctuations in (average) uncertainty can have severe effects on the aggregate economy.

Our paper also relates to a growing area of macroeconomics that explores the links between the real economy and the financial sector building upon financial frictions. Our modeling approach to the financial sector is relatively stylized, mainly aimed at capturing the idea that less optimistic views about a firm limit that firm's access to finance (or, equivalent for our purposes, increase that firm's cost of financing). Here we adopt a version of the bank balance sheet channel (e.g., [Bernanke and Gertler, 1989](#); [Bernanke, Gertler and Gilchrist, 1999](#); [Brunnermeier and Sannikov, 2012](#)) based on collateral à la [Kiyotaki and Moore \(1997\)](#), but notice that any other financial friction that links adverse beliefs to tightened credit supply would integrate similarly in our model. In the context of the balance sheet channel literature, an interesting observation is that tighter constraints *reduce* a firms responsiveness to business conditions despite the simultaneous amplification of fundamental shocks. This is due to that the amplification is via the market valuation of bank assets, which by assumption is public information. Hence, while fluctuations in fundamentals are amplified via the adjustments in credit constraints, the sensitivity of firms to fundamentals *conditional* on constraints is decreasing whenever the constraints are tight. Moreover, since market evaluations are driven by *expected* changes in fundamentals, the feedback channel will be less strong in a model of imperfect information, in particular when information is scarce.

Finally, our results on misallocation also relate to the literature on misallocation and endogenous productivities (e.g., [Moll, 2014](#), and references therein). In this context, we provide a particular simple environment in which financial frictions generate aggregate efficiency wedges and labor wedges, and show how they in combination with information frictions can be the source of a severe and long-lasting misallocation after short-lived financial perturbations. By linking misallocation to

⁶See also [Nimark \(2014\)](#) for another mechanism where uncertainty increases endogenously after unusual events.

information frictions, we relate closely to [David, Hopenhayn and Venkateswaran \(2014\)](#) who explore the amount of misallocation that can be attributed to information frictions in the absence of financial frictions.

Outline The plan for the rest of the paper is as follows. The next section introduces the model economy. Section 3 characterizes the equilibrium. Section 4 explores the core mechanism of how to learn from financially constrained firms. Section 5 studies the working of belief traps for a single island in isolation. Section 6 analyzes the cross-sectional dynamics of the model economy. Section 7 contains our simulation of a financial crisis, and Section 8 concludes.

2 Model

The economy consists of three sectors: a representative household, a production sector, and a financial sector. The production and financial sectors are active on a continuum of “islands”. Each island accommodates a continuum of firms and banks, which are organized into firm-bank pairs. Firms produce subject to a working capital requirement that is financed by credits from their bank counterparts. Banks are in turn funded through household deposits that are in perfectly elastic supply. The key friction is a maximum leverage restriction on banks, which effectively imposes a credit limit on firms. Through equity positions on their balance sheets, banks are exposed to local profits, linking the local credit supply to market beliefs about local business conditions.

Time is discrete and indexed by $t \in \{0, 1, 2, \dots\}$. Islands are indexed by $i \in I$. Firms and banks are indexed by $(i, j) \in I \times J = [0, 1]^2$.

Households The preferences of the representative household are given by

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t U(C_t, N_t),$$

with separable isoelastic preferences over consumption C_t and labor supply N_t ,

$$U(C_t, N_t) = \log C_t - \frac{1}{1+\epsilon} N_t^{1+\epsilon},$$

where $\epsilon \geq 0$ and $\beta \in (0, 1)$. Here, C_t is a composite consumption good given by:

$$C_t = \left[\int_{I \times J} C_{ij,t}^{\frac{\xi-1}{\xi}} dj \right]^{\frac{\xi}{\xi-1}},$$

where $C_{ij,t}$ is the consumption of good (i, j) at time t , and $\xi > 1$.

The representative household owns the equity of all banks $(i, j) \in I \times J$ and provides them with liquidity L_t^d in an economy-wide deposit market. For simplicity, bank deposits are assumed to be

made within periods, implying zero opportunity costs to providing liquidity. The budget constraint of the household is

$$\int_{I \times J} P_{ij,t} C_{ij,t} d(i, j) \leq W_t N_t + \int_{I \times J} \Pi_{ij,t}^b d(i, j) + (R_t^d - 1) L_t^d, \quad (1)$$

where $P_{ij,t}$ is the price of good (i, j) , W_t is the nominal wage rate, R_t^d is the deposit rate, and $\Pi_{ij,t}^b$ are profits of bank (i, j) .

In equilibrium households choose consumption, deposits, and hours worked to maximize expected utility subject to their budget constraint. From the household's optimization problem it follows that demand for good (i, j) is

$$C_{ij,t} = \left(\frac{P_{ij,t}}{P_t} \right)^{-\xi} C_t,$$

where

$$P_t = \left[\int_{I \times J} P_{ij,t}^{1-\xi} d(i, j) \right]^{1/(1-\xi)}$$

is the ideal price index. Moreover, because there are no opportunity costs to depositing liquidity, the optimal credit supply is infinitely elastic, yielding

$$R_t^d = 1.$$

Firms Each good $(i, j) \in I \times J$ is produced by a monopolistically competitive firm which has access to a linear production technology

$$Y_{ij,t} = A_{ij,t} N_{ij,t},$$

where $A_{ij,t}$ is the firm's productivity, and $N_{ij,t}$ is the firm's employment. Wages must be paid before production takes place and are financed by (within-period) working capital loans $L_{ij,t}^f = W_t N_{ij,t}$ that are provided by bank (i, j) . Profits are given by

$$\Pi_{ij,t}^f = P_{ij,t} Y_{ij,t} - R_{ij,t}^f L_{ij,t}^f,$$

where $R_{ij,t}^f$ is the gross rate at which firm (i, j) lends from its bank counterpart (i, j) .

Banks Banks intermediate the household's financing of working capital. For simplicity, each bank is associated with exactly one firm, so that the funds available to bank (i, j) define the credit limit of firm (i, j) .⁷ To fund their lending, banks use the liquidity deposits made by the household. Being mainly interested in banks as a transmission channel of financial shocks, we exogenously fix their non-loan asset positions (described below) and normalize them such that the liquidity $L_{ij,t}^d$ deposited

⁷While the adopted firm-bank structure is mainly chosen for its tractability, recent evidence suggests that a typical firm indeed only approach a single bank for credit, even if its credit demand is constrained by that bank (Jiménez et al., 2012).

Table 1: Bank balance sheets

Assets		Liabilities	
Dividends	$\int_J \Pi_{ij,t}^f dj$	Deposits	$R_t^d L_{ij,t}^d$
Loans	$\int_J R_{ij,t}^f L_{ij,t}^f dj$		
Other assets	$\Upsilon_{i,t}$	Equity	
Total assets	$\int_J P_{ij,t} Y_{ij,t} dj + \Upsilon_{i,t} \equiv X_{i,t}$	Bank equity	$X_{i,t} - R_t^d L_{ij,t}^d$

in bank (i, j) flows exclusively into corporate bank lending; i.e., $L_{ij,t}^f = L_{ij,t}^d \equiv L_{ij,t}$.⁸ The key friction in our model is a leverage restriction on bank lending that is meant to reflect a combination of institutional requirements and agency (or “skin in the game”) constraints. In particular, banks are required to comply with a maximum debt-to-asset ratio of $\chi \in (0, 1)$, implying

$$L_{ij,t} \leq \chi Q_{ij,t} \equiv \bar{L}_{ij,t}, \quad (2)$$

where $Q_{ij,t}$ is the mark-to-market value of bank (i, j) ’s assets.

Asset pricing To determine the market value $Q_{ij,t}$ of bank (i, j) ’s assets, we first have to specify the bank’s asset positions. Our choice of these positions complements the overall design of the financial sector in our model, which is devised to capture the idea that favorable market beliefs about local business conditions translate into less restrictive credit lines for the corresponding firms. The following assumptions are aimed at achieving this in the most tractable way.

First, each bank on island i holds a representative portfolio claiming the current profits of island- i firms: $\int_J \Pi_{ij,t}^f dj$. Second, each bank on island i is exposed to an asset with value $\Upsilon_{i,t}$. The purpose of this asset is to introduce some exogenous variations in the bank’s net worth. In particular, we will later use correlated movements in $\Upsilon_{i,t}$ across islands as the source of aggregate financial shocks. Finally, we assume that banks hedge their lending returns within islands, meaning that all banks on island i earn $\int_J R_{ij,t}^f L_{ij,t}^f dj$ on their lending activities. All three assumptions are rigged to ensure that the market value of bank assets is the same within islands ($Q_{ij,t} = Q_{i,t}$ for all $j \in J$), implying that credit limits are identical within islands ($\bar{L}_{ij,t} = \bar{L}_{i,t}$ for all $j \in J$). Table 1 summarizes the balance sheet positions of banks in island i .

Let $X_{i,t}$ denote the combined asset claims of banks in island i :

$$X_{i,t} = \int_J \left(\Pi_{ij,t}^f + R_{ij,t}^f L_{ij,t} \right) dj + \Upsilon_{i,t} = \int_J P_{ij,t} Y_{ij,t} dj + \Upsilon_{i,t}.$$

In order to obtain the aforementioned effect of beliefs on lending, we assume there exists a collection of competitive traders who trade derivatives with the same payoff as $X_{i,t}$. This effectively generates a valuation $Q_{i,t}$ of bank assets without the need of modeling a market for bank shares. A trader

⁸Non-loan related usages of debt can be easily accounted for by adjusting the permissible leverage ratio. See Footnote 28 for details.

$k \in K \equiv [0, 1]$ is island-specific, one-period lived and endowed with an information set $\mathcal{I}_{ik,t}$. Traders value portfolios according to the following mean-variance expected utility function:

$$\mathbb{E}\{\log X_{i,t} - \log Q_{i,t} | \mathcal{I}_{ik,t}\} \theta_{ik,t} - \frac{\text{Var}\{\log X_{i,t} | \mathcal{I}_{ik,t}\}}{2\gamma^{-1}} \theta_{ik,t}^2,$$

where $\theta_{ik,t}$ is trader (i, k) 's demand for derivatives, and $\gamma > 0$ captures his risk-aversion. Notice that these types of preferences naturally emerge in the case where traders have CRRA preferences and $X_{i,t}$ is lognormally distributed, which will be the assumption for the most part of the paper.

Given this specification, and normalizing the financial market to have supply $1 - \eta_{i,t}$ of derivatives, where $\eta_{i,t}$ captures some random (idiosyncratic) financial shock, market clearing ($\int \theta_{ik,t} dk = 1 - \eta_{i,t}$) will imply

$$\log Q_{i,t} = \bar{\mathbb{E}}_{i,t}\{\log X_{i,t}\} - (1 - \eta_{i,t})\gamma \text{Var}\{\log X_{i,t} | \mathcal{I}_{ik,t}\}, \quad (3)$$

where $\bar{\mathbb{E}}_{i,t}\{\cdot\}$ denotes the average expectation of traders, $\bar{\mathbb{E}}_{i,t}\{\cdot\} = \int_K \mathbb{E}\{\cdot | \mathcal{I}_{ik,t}\} dk$.⁹ Substituting (3) in (2) illustrates how firms' credit limits depend on market expectations.

Fundamentals and information The source of uncertainty are the productivities of firms. Productivities have an island-specific component $Z_{i,t}$ and a firm-specific component. There is no uncertainty about aggregate productivities.

Conditional on $Z_{i,t}$, within-island productivities are i.i.d. and log-normally distributed:

$$\text{Prob}(\log A_{ij,t} \leq a) = F_a(a | Z_{i,t}),$$

with

$$F_a(a | Z_{i,t}) = \Phi\left(\frac{a - \log Z_{i,t}}{\sigma_a}\right),$$

where Φ denotes the cumulative distribution function of the standard normal distribution and σ_a measures the within-island dispersion of productivities. As becomes clear below, this firm-specific dispersion is introduced for technical reasons and should be thought of being small.

The fundamentals of interest are the island-specific components of firms' productivities, which define the local business conditions. Having in mind the mere technical motivation behind the within-island dispersion, our preferred interpretation is that one island corresponds to one firm in the data. The island-specific component follows an i.i.d. Markov process, denoted by:

$$\text{Prob}(\log Z_{i,t} \leq z | Z_{i,t-1}) = F_z(z | Z_{i,t-1}).$$

In this environment, correlated shocks to $\Upsilon_{i,t}$ constitute the only source of aggregate volatility. Given that any correlation in $\Upsilon_{i,t}$ can be deduced from the cross-section of islands, even given

⁹More accurately, $\log Q_{i,t}$ will be given by the above expression whenever posterior uncertainty $\text{Var}\{\log X_{i,t} | \mathcal{I}_{ik,t}\}$ is the same across traders in a given island i . Given the informational assumptions underlying our analysis of the credit market in Section 5, this will indeed be the case.

reasonable deviations from full information, it seems hard to conceive that any uncertainty about the aggregate economy would play a significant role in the propagation of economic shocks. To highlight the independence of our results from any aggregate uncertainty, we assume full information about the aggregate state of the economy: Υ_t is common knowledge among all agents of the economy, where Υ_t defines a sufficient aggregate statistic, such that $\Upsilon_{i,t} | (\Upsilon_{i',t}, \Upsilon_t) = \Upsilon_{i,t} | \Upsilon_t$ for all $i, i' \in I$.¹⁰

Imposing full information on the aggregate state of the economy is also convenient, since it allows us to solve the household problem in a completely standard way, without the need to make any distributional assumptions on $\Upsilon_{i,t}$. To focus on the propagation of financial shocks, we further assume that $A_{ij,t}$ is known by firm (i, j) at date t , so that—like the household—firms follow the same decision rules as they would under full information.

We are left to specify traders' information sets $\{\mathcal{I}_{ik,t}\}$, which will determine the market expectations about local business conditions and, hence, the credit supply in a given island. Since the asset positions of banks are fully hedged within islands, all relevant learning ensues with respect to $Z_{i,t}$, and there is no need to specify traders' information regarding the firm-specific productivity components.

Each trader has access to three signals that help assessing the local business conditions $Z_{i,t}$. First, each trader on island i observes the history of working capital $L_{i,t}$ invested in island i up to date $t - 1$, perturbed by some noise. This is meant as a stand-in for observing news about investments and other activity-related signals, which are often thought to be valuable indicators about local business conditions. We use $\omega_{i,t}^l$ to denote this signal and Ω_l to denote the conditional distribution of $\omega_{i,t}^l$, such that

$$\text{Prob}(\omega_{i,t}^l \leq w | L_{i,t-1}) = \Omega_l(w | L_{i,t-1}).$$

Second, traders have access to a private signal $\omega_{ik,t}^p$ that directly communicates $Z_{i,t}$ perturbed by some noise, and has the conditional distribution

$$\text{Prob}(\omega_{ik,t}^p \leq w | Z_{i,t}) = \Omega_p(w | Z_{i,t}).$$

As mentioned before, traders are short-lived and so their private information does not persist from one period to the next, preventing us from having to deal with Townsend's (1983) infinite regress problem.

Finally, $\mathcal{I}_{ik,t}$ includes the history of asset prices up to date t , reflecting the possibility of traders to condition their demand on current prices. Accordingly, the information set of trader k , active in island i , is given by

$$\mathcal{I}_{ik,t} = \{\omega_{ik,t}^p\} \cup \{\omega_{i,s}^l, Q_{i,s}, \Upsilon_s\}_{s=0}^t.$$

We do not provide traders active in island i with information from other islands. This is without loss of generality, given that economy-wide aggregates are fully observable and island-specific shocks

¹⁰See [Straub and Ulbricht \(2012\)](#) for an earlier version of this paper where learning was with respect to aggregate business conditions.

are mutually orthogonal, so that there is nothing to learn from observing information about other islands.

Completing the description of the model, we assume that island-specific financial noise $\eta_{i,t}$ follows an autoregressive process, such that

$$\eta_{i,t} \sim \mathcal{N}(\rho_\eta \eta_{i,t-1}, \sigma_\eta^2),$$

with $0 \leq \rho_\eta < 1$.

Labor, product and credit markets Deposit, labor, and product markets operate on an economy-wide scale. The clearing conditions read

$$L_t^d = \int_{I \times J} L_{ij,t} d(i, j), \quad (4)$$

$$N_t = \int_{I \times J} N_{ij,t} d(i, j), \quad (5)$$

and

$$C_{ij,t} = Y_{ij,t} \quad \text{for all } (i, j) \in I \times J. \quad (6)$$

For simplicity, we assume that both banks and firms are price takers in the credit markets.¹¹ Substituting for bank credit supply, market clearing then requires

$$L_{ij,t} = \begin{cases} 0 & \text{if } R_{ij,t}^f < R_t^d \\ [0, \bar{L}_{i,t}] & \text{if } R_{ij,t}^f = R_t^d \\ \bar{L}_{i,t} & \text{if } R_{ij,t}^f > R_t^d \end{cases} \quad \text{for all } (i, j) \in I \times J. \quad (7)$$

Timing The timing within periods can be summarized in two stages: In stage 1, all available information $\{\mathcal{I}_{ik,t}\}$ realizes and the financial markets operate, pinning down the credit limits $\{\bar{L}_{i,t}\}$. In stage 2, production takes place and credit, labor, and product markets operate.

2.1 Notational conventions

Before proceeding to characterizing the equilibrium of the model economy, let us define some notational shortcuts. In particular, let $a_{ij,t} = \log A_{ij,t}$, $\bar{a}_{i,t} \equiv \log \bar{A}_{i,t}$, $z_{i,t} \equiv \log Z_{i,t}$, $l_{i,t} = \log L_{i,t}$, $v_t \equiv \log \Upsilon_t$, and $z_t = \log Z_t$. Moreover, we use $\hat{\sigma}_{i,t}^2 \equiv \text{Var}\{z_{i,t}|\mathcal{I}_{ik,t}\}$ whenever $\text{Var}\{z_{i,t}|\mathcal{I}_{ik,t}\}$ is the same across traders in a given island.

¹¹At the expense of additional structure, price-taking behavior by firms and banks in the credit market could be microfounded by replacing a bank (i, j) by a continuum of identical banks, and replacing a firm (i, j) by a continuum of firms, with equal productivities but producing separate varieties in the goods market.

3 Equilibrium

3.1 Definition

The equilibrium in our economy is a standard competitive equilibrium with three modifications: The firm sector is subject to a working capital requirement; the price for working capital potentially exceeds the deposit rate due to capital requirements in the banking sector; and whether capital requirements bind or not depends on the market values of banks' balance sheets, determined by a competitive asset market (with imperfect information). Formally the equilibrium is defined as follows.

Definition. Given a stochastic process for $\{Z_{i,t}, \Upsilon_{i,t}, \omega_{i,t}^l, \eta_{i,t}\}_{i \in I}$, $\{A_{ij,t}\}_{(i,j) \in I \times J}$ and $\{\omega_{ik,t}^p\}_{(i,k) \in I \times K}$, a *competitive equilibrium* consists of a stochastic process for C_t, N_t, Y_t, L_t^d , $\{C_{ij,t}, N_{ij,t}, Y_{ij,t}, L_{ij,t}\}_{(i,j) \in I \times J}$ and $\{\theta_{ik,t}\}_{(i,k) \in I \times K}$ together with the associated price processes W_t, R_t^d , $\{Q_{i,t}\}_{i \in I}$ and $\{P_{ij,t}, R_{ij,t}^f\}_{(i,j) \in I \times J}$, such that:

- (i) The process for N_t, L_t^d and $\{C_{ij,t}\}_{(i,j) \in I \times J}$ maximizes the utility of the representative household subject to its budget constraint, and given W_t, R_t^d and $\{P_{ij,t}\}_{(i,j) \in I \times J}$.
- (ii) The process for $\{P_{ij,t}, N_{ij,t}, L_{ij,t}\}_{(i,j) \in I \times R}$ maximizes the profits of firms subject to their individual demand schedules, production technologies, working capital requirements, and given W_t and $\{R_{ij,t}^f\}$.
- (iii) The process for $\{L_{ij,t}\}_{(i,j) \in I \times J}$ maximizes the profits of banks subject to their capital requirements, and given R_t^d and $\{R_{ij,t}^f\}_{(i,j) \in I \times J}$.
- (iv) The process for $\{\theta_{ik,t}\}_{(i,k) \in I \times K}$ maximizes the expected utility of traders conditional on their information sets, and given $\{Q_{i,t}\}_{i \in I}$.
- (v) All product, labor, deposit, loan, and asset markets clear.

This section characterizes the competitive equilibrium of this economy. Throughout, the composite consumption basket C_t is chosen as the numeraire, yielding $P_t = 1$. Recalling that in equilibrium it holds that $R_t^d = 1$, we henceforth drop the superscript from $R_{ij,t}^f$ and use $R_{ij,t}$ to refer to the bank-to-firm lending rate.

3.2 Characterization

Product and labor markets Recall that firms and households effectively optimize under full information. Then, following standard steps, the solutions to their problems can be used to express the behavior of the economy as a function of credit market distortions $\{R_{ij,t}\}$. The following lemma summarizes the equilibrium allocations; details can be found in the appendix.

Lemma 1. *It holds that*

$$\begin{aligned} N_t &= (1 - \tau_t^N)^{1/(1+\epsilon)} \\ Y_t &= (1 - \tau_t^A) A^{eff} N_t, \end{aligned}$$

where

$$A^{eff} = \left[\int_{I \times J} A_{ij,t}^{\xi-1} d(i,j) \right]^{1/(\xi-1)}$$

is the efficient aggregate productivity level that would obtain if marginal (labor) productivities were equalized across all firms, and τ_t^N and τ_t^A are the labor and efficiency wedges, defined by

$$\begin{aligned} 1 - \tau_t^A &= \frac{MPN_t}{A^{eff}} = \frac{1}{A^{eff}} \frac{\left(\int_I \phi_{i,t}^1 di \right)^{\xi/(\xi-1)}}{\int_I \phi_{i,t}^2 di} \\ 1 - \tau_t^N &= \frac{MRS_t}{MPN_t} = \frac{\xi - 1}{\xi} \frac{\int_I \phi_{i,t}^2 di}{\int_I \phi_{i,t}^1 di}, \end{aligned}$$

where

$$\begin{aligned} \phi_{i,t}^1 &\equiv \int_J A_{ij,t}^{\xi-1} R_{ij,t}^{1-\xi} dj \\ \phi_{i,t}^2 &\equiv \int_J A_{ij,t}^{\xi-1} R_{ij,t}^{-\xi} dj. \end{aligned}$$

In this environment, all aggregate fluctuations can be written as a function of a labor wedge τ_t^N and an efficiency wedge τ_t^A as in [Chari, Kehoe and McGrattan \(2007\)](#). These wedges summarize all credit market distortions $R_{ij,t}$, which in turn summarize the propagation of shocks through the financial sector.

A positive labor wedge reflects an inefficiently low labor demand by firms whose working capital constraint would be binding if they could access credit at the deposit rate $R_{ij,t} = R_t^d = 1$. This is the case whenever the credit demand for $R_{ij,t} = 1$ exceeds bank (i,j) 's credit limit $\bar{L}_{ij,t}$, causing a distortion between the household's marginal rate of substitution and the firm's marginal labor productivity. The efficiency wedge in turn reflects that in the presence of credit constraints marginal productivities are not equalized across firms, decreasing the effective (Solow) productivity in the economy through credit misallocation.

It follows immediately that credit inefficiencies strictly reduce aggregate output, whenever there is a positive measure of “constrained” firms.

Proposition 1. *Suppose F_a has full support on $\mathbb{R}$. Then $(1 - \tau_t^A) < 1$ and $(1 - \tau_t^N) < (\xi - 1)/\xi < 1$.*

Notice that owing to our assumption that $R_{ij,t}^f$ is priced competitively, firms are not actually constrained in the sense that they desire more credit than they are allocated in the market. “Binding”

credit constraints are instead fully reflected in the credit spread $R_{ij,t}^f - 1$. In the following we use the phrase “being constrained” loosely, referring to a firm that faces a strictly positive credit spread.

Credit markets The following lemma characterizes equilibrium lending rates $R_{ij,t}$ conditional on the credit limit.

Lemma 2. *Credit market clearing implies*

$$R_{ij,t} = \max \left\{ 1, \left(\frac{A_{ij,t}}{\bar{A}_{ij,t}} \right)^{(\xi-1)/\xi} \right\},$$

where the total credit to firm (i, j) is given by

$$L_{ij,t} = \min \{ A_{ij,t}, \bar{A}_{i,t} \}^{\xi-1} Z_t,$$

where

$$Z_t \equiv \left(\frac{\xi - 1}{\xi} \right)^\xi \frac{C_t}{W_t^{\xi-1}}$$

summarizes the general equilibrium effects on the local credit markets, and where

$$\bar{A}_{i,t} = \left(\frac{\bar{L}_{i,t}}{Z_t} \right)^{1/(\xi-1)}$$

is the credit limit $\bar{L}_{i,t}$ on island i denoted in productivity-units.

Lemma 2 characterizes the credit rates $R_{ij,t}$ and credits $L_{ij,t}$ as a function of firm-specific and aggregate fundamentals, $A_{ij,t}$ and Z_t , and a transformation of the island-specific credit limit $\bar{A}_{i,t}$. In particular, firms are constrained when their (stochastic) productivity realization $A_{ij,t}$ exceeds the credit limit $\bar{A}_{i,t}$, in which case firm (i, j) ’s demand for working capital exceeds the bank’s lending constraint.

Asset markets Finally, to complete the characterization of equilibrium, we examine the credit limit $\bar{A}_{i,t}$ that is consistent with asset market clearing. In particular, it must hold that given market beliefs about island productivity $Z_{i,t}$, the credit limit solves a version of the balance sheet fixed point (Kiyotaki and Moore, 1997).

By design, bank balance sheets are log-linear up to the financial asset $\Upsilon_{i,t}$. To simplify the analysis, and to have the usual interpretation of shocks entering in a log-linear way, we assume that the idiosyncratic component of $\Upsilon_{i,t}$ scales with the bank’s total asset positions, such that in a

log-linear representation only an aggregate component Υ_t of $\Upsilon_{i,t}$ enters $X_{i,t}$.¹²

$$\log X_{i,t} = \log \left(\int_J P_{ij,t} Y_{ij,t} dj \right) + \log \Upsilon_t.$$

For convenience, we define a pricing operator $\mathcal{P}_{i,t}$, so that for any positive random variable X

$$\log \mathcal{P}_{i,t}(X) = \bar{\mathbb{E}}_{i,t}\{\log X\} - (1 - \eta_{i,t})\gamma \text{Var}\{\log X | \mathcal{I}_{ik,t}\}, \quad (8)$$

so that in equilibrium, optimizing behavior of traders implies that $Q_{i,t} = \mathcal{P}_{i,t}(X_{i,t})$ as in equation (3).¹³ With this definition at hand, the following lemma characterizes the asset market equilibrium.

Lemma 3. *Suppose it holds that $Q_{i,t} = \mathcal{P}_{i,t}(X_{i,t})$. Then Asset markets clear if and only if*

$$\log \bar{A}_{i,t}^{\xi-1} = \log c_a + \log \mathcal{P}_{i,t} \left(\int_J \left[A_{ij,t} \min \{ A_{ij,t}, \bar{A}_{i,t} \}^{\xi-1} \right]^{1-1/\xi} dj \right) + \log \Upsilon_t \quad (9)$$

where $c_a \equiv \chi\xi/(\xi - 1)$.

Equation (9) resembles the model's version of the balance-sheet channel: A relaxation of the credit limit $\bar{A}_{i,t}$ increases expected returns (inside the argument of $\mathcal{P}_{i,t}$), which in turn increases the market valuation captured by $\mathcal{P}_{i,t}$, which then in turn relaxes the credit limit. Any shock to the right hand side (either in the form of Υ_t or $\eta_{i,t}$, or in the form of beliefs entering through the definition of $\mathcal{P}_{i,t}$) is amplified by this fixed point mechanism.

3.3 Outlook

In this section, we provided a compact representation of the competitive equilibrium. To fully solve for the equilibrium, it is however necessary to explicitly solve the asset market fixed point (9) for each island i . The next two sections deal with this problem: In a first step, Section 4 explicitly characterizes the belief formation when agents learn from financially constrained firms (which is underlying the $\mathcal{P}$ -operator). Then, Section 5 addresses the balance sheet fixed point in Lemma 3 and discusses how the combination of endogenous belief formation and the balance sheet fixed point gives rise to “belief traps”.

¹²For $\Upsilon_{i,t}$ this requires that $\log(\Upsilon_{i,t}) = \log(\int_J P_{ij,t} Y_{ij,t} dj) + \log(\Upsilon_t - 1)$. What is key here is the log-linear form, which greatly simplifies the ensuing analysis. Intuitively, this assumption on $\Upsilon_{i,t}$ means that there is a single aggregate financial shock to bank balance sheets, which has a larger impact on banks with larger bank balance sheets. Even if we allowed for idiosyncratic financial shocks on top of this aggregate shock, all results would remain virtually unchanged as long as the idiosyncratic component enters $\Upsilon_{i,t}$ log-linearly, as such a component can be absorbed into $\eta_{i,t}$.

¹³Again, the simple representation of the pricing operator $\mathcal{P}_{i,t}(\cdot)$ requires posterior uncertainty $\text{Var}\{\log X | \mathcal{I}_{ik,t}\}$ to be constant across traders within island i . To streamline the exposition, we anticipate that this will indeed be the case given the information structure introduced in Section 4.2 and state Lemma 3 below keeping in mind these qualifications.

4 Core effect: Learning from financially constrained firms

The information sets $\{\mathcal{I}_{ik,t}\}$ underlying the economy's belief formation contain two endogenous signal types: $\{\omega_{i,t}^l\}$ and $\{Q_{i,t}\}$. The asset pricing signal $Q_{i,t}$ and the full learning problem are analyzed in the next section. In this section, we focus on the working capital signal $\omega_{i,t}^l$, showing how this signal is inherently *nonlinear* in the presence of financial constraints. This leads the tightness of financial constraints to naturally change the informativeness of the working capital signal, creating the possibility for dangerous feedback loops between financial constraints, uncertainty, and pessimistic beliefs—*belief traps*.

The outline of this section is as follows. In the first subsection, it is established that in our model, tighter constraints inhibit learning. To stress that the effect is far more general than our particular application, we do not make parametric assumptions on the information structure. Rather we build on a general theorem about the interaction of signal nonlinearity and learning proved in [Straub and Ulbricht \(2014\)](#). In the second subsection, we then present a simple linearization method which allows us to capture all the main effects induced by the nonlinearity of the working capital signal while preserving tractability.

4.1 Nonlinearity of lending and information failures

The information loss caused by financial constraints is due to the fact that, for each island, working capital responds nonlinearly to changes in credit limits. The basic intuition for this is simple and generalizes to other types of constraints: Whenever firms are financially constrained, their choices will be guided less by their information about fundamentals, and are instead dictated by the financial constraint. Therefore, markets aggregate less information.¹⁴

Fix an island i . Following Lemma 2, total working capital on that island is given by,

$$L_{i,t} = Z_t \int_J \min\{A_{ij,t}, \bar{A}_{i,t}\}^{\xi-1} dj. \quad (10)$$

The following proposition characterizes island i 's (log) working capital $l_{i,t}$ as a function of $z_{i,t}$ and $\bar{a}_{i,t}$.

Proposition 2. *Working capital $l_{i,t}$ on island i takes the form*

$$l_{i,t} = z_t + (\xi - 1)\bar{a}_{i,t} + \mathcal{L}(z_{i,t} - \bar{a}_{i,t}), \quad (11)$$

where $\mathcal{L} : \mathbb{R} \rightarrow \mathbb{R}_-$ is a smooth, strictly concave, and increasing function, with $\lim_{x \rightarrow -\infty} \mathcal{L}(x) = -\infty$, $\lim_{x \rightarrow \infty} \mathcal{L}(x) = 0$, $\lim_{x \rightarrow -\infty} \mathcal{L}'(x) = (\xi - 1)$, $\lim_{x \rightarrow \infty} \mathcal{L}'(x) = 0$. In particular,

¹⁴As discussed above, credit limits impact firm's choices in the present setup via rising credit rate spreads, but results would be the same if firms were quantity constrained instead. More generally, tightened credit conditions will reduce the responsiveness of firms to fundamentals regardless of their origin and regardless whether they manifest themselves through quantity constraints or increased credit spreads.

- (a) *in the absence of credit constraints, $\bar{a}_{i,t} \rightarrow \infty$, the equilibrium sensitivity of working capital to fundamentals, $\partial l_{i,t}/\partial z_{i,t}$, is constant in credit supply $\bar{a}_{i,t}$ and island fundamentals $z_{i,t}$.*
- (b) *in the presence of credit constraints, $\bar{a}_{i,t} < \infty$, the equilibrium sensitivity of working capital to fundamentals, $\partial l_{i,t}/\partial z_{i,t}$, is increasing in credit supply $\bar{a}_{i,t}$ and decreasing in island fundamentals $z_{i,t}$.*

Equation (11) follows as a special case from Lemma 8 in the appendix. Lemma 8 also shows how $\mathcal{L}$ can be solved for in closed form. The decomposition in (11) demonstrates that working capital depends on three terms: First, it depends positively on economy-wide business conditions z_t (i.e., reflecting aggregate demand and the level of wages). Second, it depends on the credit limit $\bar{a}_{i,t}$ imposed on firms, since loose credit limits naturally translate into higher business activity. Finally, and crucially, working capital depends on island i 's "relative demand" ($z_{i,t} - \bar{a}_{i,t}$) for credit: The island-specific fundamental $z_{i,t}$ drives island i 's demand for credit, while $\bar{a}_{i,t}$ measures the credit supply (in productivity units). If the relative demand for credit is low, there are sufficient funds for most firms to operate without being financially constrained. In this case, the equilibrium is governed by demand $z_{i,t}$ and working capital is sensitive to fluctuations in demand. If, however, there is a substantial "excess demand" for funds, a significant fraction of firms is financially constrained. Then island i 's working capital is mostly determined by the credit constraint $\bar{a}_{i,t}$ and hence almost insensitive to fluctuations in fundamentals $z_{i,t}$.

The sensitivity of working capital $\partial l_{i,t}/\partial z_{i,t}$ is key in our model as it determines the information content of the working capital signal $\omega_{i,t}^l$. Indeed, when the sensitivity is small, the magnitude of the noise induced by the conditional distribution Ω_l of the working capital signal $\omega_{i,t}^l$ given a certain amount of working capital $l_{i,t-1}$ will become large in relative terms, and vice-versa if the sensitivity is large. The goal of the following paragraphs is to formalize this intuition.

To this end, we impose a small set of assumptions on the economy's information structure. First, we assume that the distribution Ω_l of $\omega_{i,t}^l$ gives rise to a "stochastically monotone" relationship of $\omega_{i,t}^l$ and $l_{i,t-1}$, in the sense that conditional on a signal realization $\omega_{i,t}^l = \omega$, the posterior over $l_{i,t-1}$ is increasing in ω with respect to the monotone likelihood ratio property (MLRP). This ensures that the signal structure Ω_l associates large signals with large values for working capital. Our second assumption is that the signal structure Ω_l does not become more accurate for larger signal realizations, so that the decrease in the sensitivity of working capital does not become mechanically overturned by an exogenous reduction in noise. Formally, we require that the variance of the posterior of $l_{i,t-1}$ conditional on signal $\omega_{i,t}^l = \omega$ is nondecreasing in ω . In particular, the variance can be constant.

Based on these two assumptions, we use the results of [Straub and Ulbricht \(2014\)](#) to prove the following result.

Proposition 3. *Suppose the distribution $l_{i,t-1} | (\omega_{i,t}^l = \omega)$ is (i) increasing in ω in the sense of the MLRP and (ii) its variance is nondecreasing in ω . Then,*

- (a) conditional on a level of the constraint $\bar{a}_{i,t-1}$, posterior uncertainty $\text{Var}\{z_{i,t-1}|\omega_{i,t}^l = \omega\}$ is increasing in ω , and
- (b) conditional on a realization of the working capital signal, $\omega_{i,t}^l = \omega$, posterior uncertainty $\text{Var}\{z_{i,t-1}|\omega_{i,t}^l = \omega\}$ is decreasing in $\bar{a}_{i,t-1}$.

Proposition 3 has two parts, which relate to the two determinants of relative demand ($z_{i,t} - \bar{a}_{i,t}$): The first part formalizes the above intuition. By assumption (i) in Proposition 3, a large realization of the working capital signal $\omega_{i,t}^l$ corresponds (stochastically) to a large working capital $l_{i,t-1}$, which—keeping $\bar{a}_{i,t-1}$ constant—corresponds to a large underlying value of $z_{i,t-1}$. In Proposition 2(a) we showed that the sensitivity of working capital was decreasing in $z_{i,t-1}$, which causes a reduction in the informational content contained in $\omega_{i,t}^l$, as measured by an increase of the posterior variance $\text{Var}\{z_{i,t-1}|\omega_{i,t}^l = \omega\}$. In sum, higher credit demand, as signaled through high $\omega_{i,t}^l$ relative to a fixed $\bar{a}_{i,t}$, decreases the informativeness of the working capital signal.

The second part of Proposition 3 focuses in turn on changes in the credit supply of island i , given by changes in $\bar{a}_{i,t-1}$. Again, we can let Proposition 2 guide us through the intuition: Suppose that $\bar{a}_{i,t-1}$ increases. By Proposition 2(b) this heightens the sensitivity of working capital $l_{i,t-1}$ to changes in fundamentals $z_{i,t-1}$, causing an increase in the informational content contained in $\omega_{i,t}^l$, as measured by a decrease in the posterior variance $\text{Var}\{z_{i,t-1}|\omega_{i,t}^l = \omega\}$. In sum, higher credit supply, as given by an increase in $\bar{a}_{i,t-1}$, increases the informativeness of the working capital signal.

4.2 Approximate Gaussian updating

For the remainder of the paper, we focus on a standard Gaussian information structure, with an AR(1) process for $z_{i,t}$,

$$F_z(z|z_{i,t-1}) = \Phi\left(\frac{z - \rho_z z_{i,t-1}}{\sigma_z}\right), \quad (12)$$

where $\rho_z \in (0, 1)$, and

$$\Omega_l(\omega|l_{i,t-1}) = \Phi\left(\frac{\omega - l_{i,t-1}}{\sigma_\psi}\right), \quad (13)$$

$$\Omega_p(\omega|z_{i,t}) = \Phi\left(\frac{\omega - z_{i,t}}{\sigma_p}\right). \quad (14)$$

This is an important simplification, especially because of the conjugate prior property of normal distributions. In order to deal with the endogenous signal precision of the working capital signal in a tractable way, we introduce here a way of approximating the true nonlinear updating problem while preserving all the key properties derived above.

Given the Gaussian structure, the working capital signal can be written as:

$$\omega_{i,t}^l = \underbrace{\mathcal{L}(z_{i,t-1} - \bar{a}_{i,t-1})}_{\text{fundamental}} + \underbrace{\psi_{i,t}}_{\text{noise}}, \quad (15)$$

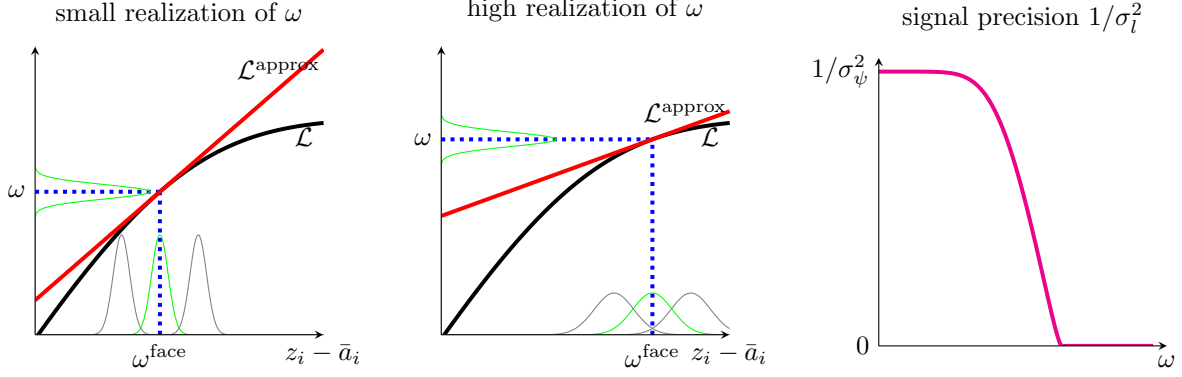


Figure 1: Approximate Gaussian updating. The nonlinear signal structure gives rise to signal-dependent uncertainty that is preserved using the linearization (red line). The right panel displays the resulting signal precision as a function of $\omega = \mathcal{L}(z_{i,t-1} - \bar{a}_{i,t-1}) + \psi_{i,t}$.

up to the constant $(z_{t-1} + (\xi - 1)\bar{a}_{i,t-1})|\mathcal{I}_{ik,t}$, which can be ignored w.l.o.g., and where $\psi_{i,t} \sim \mathcal{N}(0, \sigma_\psi^2)$. Here the first term reflects fundamental variations in the signal that are driven by the relative demand; whereas $\psi_{i,t}$ reflects the noise in the signal structure.

Based on this decomposition, the “approximate” updating rule that we use to quantify the model can be described as follows. After observing a realization of the working capital signal $\omega_{i,t} = \omega$, agents linearize the function $\mathcal{L}$ to do standard Gaussian updating. We let the linearization point, however, depend on the realization ω : Large signal realizations should correspond to larger values of the fundamental $z_{i,t-1}$, and hence to *flatter* regions of the concave function $\mathcal{L}$. To reflect this in the linearization, the linearization point is taken to be the “face value” of $\omega_{i,t}^l$: the implied relative demand if there was no noise in the signal structure, $\omega_{i,t}^{\text{face}} = \mathcal{L}^{-1}(\omega_{i,t})$.¹⁵ Notice that the expression for the face value of $\omega_{i,t}$ exactly becomes the agent’s belief about the true relative credit demand $z_{i,t-1} - \bar{a}_{i,t-1}$ in the limit of the signal $\omega_{i,t}^l$ becoming perfectly informative. Taking linearization and updating together, this procedure implies the following posterior beliefs.

Lemma 4. *Suppose agents linearize $\mathcal{L}$ around the signal’s “face-value” $\omega_{i,t}^{\text{face}}$ to assess the likelihood of observing $\omega_{i,t}^l | z_{i,t-1}$. Then, agents update as if $\omega_{i,t}^{\text{face}}$ was the realization of a “fictitious” Gaussian signal $\omega_{i,t}^{\text{face}} \sim \mathcal{N}(\mu_l, \sigma_l^2)$ with*

$$\begin{aligned} \mu_l &= z_{i,t-1} - \bar{a}_{i,t-1} \\ \sigma_l &= \frac{\sigma_\psi}{\mathcal{L}'(\omega_{i,t}^{\text{face}})}, \end{aligned}$$

and where agents update as if σ_l were exogenous.

The updating behavior described in Lemma 4 is graphically depicted in Figure 1. It is evident how larger signal realizations—driven by increases in relative demand $(z_{i,t-1} - \bar{a}_{i,t-1})$ and by noise $\psi_{i,t}$ —lead the agent to suspect the actual fundamental state in regions where $\mathcal{L}$ is flatter, rendering

¹⁵In the case where $\omega_{i,t} > \mathcal{L}(\infty)$, we let $\omega_{i,t}^{\text{face}} = \infty$.

the agent more uncertain about the state's position. Notice that this approximate Gaussian updating requires the function $\mathcal{L}$ to be differentiable, which is the reason for having a positive within-island dispersion of technologies in the model.

From the definition of $\omega_{i,t}^{\text{face}}$, the following result immediately follows.

Proposition 4. *The signal variance of the approximate Gaussian working capital signal is given by $\sigma_l(\bar{a}_{i,t-1} - z_{i,t-1}, \psi_{i,t})$, where σ_l is increasing in its first and decreasing in its second argument. The median signal precisions (i.e. $\psi_{i,t} = 0$) satisfy*

$$\lim_{\bar{a} \rightarrow +\infty} \sigma_l(\bar{a}, 0) = \sigma_\psi / (\xi - 1)$$

and

$$\lim_{\bar{a} \rightarrow -\infty} \sigma_l(\bar{a}, 0) = \infty.$$

Proposition 4 summarizes the working of the approximate Gaussian updating procedure. As explained in the general case above, the information content in the working capital signal depends on the relative demand ($z_{i,t-1} - \bar{a}_{i,t-1}$), reflecting the degree to which firms are constrained by a particular credit limit $\bar{a}_{i,t-1}$, and on the exogenous noise in the signal structure $\psi_{i,t}$. Here it is useful to think about the relative demand as the systematic determinant of an island's ability to learn from the business sector. The next section explores the endogenous nature of this systematic component, ($\bar{a}_{i,t-1} - z_{i,t-1}$), which we also refer to as an island's "information capacity".

The right panel of Figure 1 illustrates the resulting signal precision $1/\sigma_l^2$ plotted against $\omega = \mathcal{L}(z_{i,t-1} - \bar{a}_{i,t-1}) + \psi_{i,t}$. In the limit where firms on island i are essentially unconstrained, the information content in the signal is equivalent to the exogenous noise, $\sigma_l = \sigma_\psi / (\xi - 1)$. In the opposing case where all firms are suspected to be constrained, observing the working capital contains no information, $\sigma_l = \infty$.

5 Belief traps

We now turn to the feedback loop between financial constraints, uncertainty, and pessimistic beliefs at the core of this contribution. Much of the insights can be gained at the island-level, which we explore in this section. We first characterize the equilibrium credit limits (Section 5.1) and learning dynamics (Section 5.2). Equipped with these results, we then turn to the interaction between the two and illustrate the feedback loop (Section 5.3).

5.1 Equilibrium credit limits

To ensure log-normality of the price signal, we expand $\log X_{i,t}$ (the argument of $\mathcal{P}_{i,t}$) in (9) around $z_{i,t} = \mathbb{E}_{i,t}\{z_{i,t}\}$ (recall that $a_{ij,t}$ is distributed around $z_{i,t}$ to see why $\mathcal{P}_{i,t}$ depends on $z_{i,t}$). Working with this approximation, we derive the following result.

Lemma 5. *The fixed point to (9) exists and has a unique solution, given by*

$$\bar{a}_{i,t} = \bar{\mathbb{E}}_{i,t}\{z_{i,t}\} + f(v_t, (1 - \eta_{i,t}) \hat{\sigma}_{i,t}^2), \quad (16)$$

where (for all $\eta_{i,t} \leq 1$) f is increasing in its first and decreasing in its second argument.

The properties of the unique solution to the balance-sheet fixed point summarizes the working of the banking sector and asset market. As we have discussed above, it links the credit limit available to firm (i, j) to market beliefs and certain orthogonal financial conditions. Specifically, the credit limit $\bar{a}_{i,t}$ increases in the average market expectation about business conditions $\bar{\mathbb{E}}_{i,t}\{z_{i,t}\}$, it decreases in uncertainty $\hat{\sigma}_{i,t}^2$, and it decreases in adverse financial shocks, $-\eta_{i,t}$ and $-v_t$.¹⁶

With our model structure, as an island becomes more constrained, the strength of the balance-sheet feedback is increasing in a way that exactly offsets that working capital becomes less responsive with respect to $z_{i,t}$. This explains the linearity of $\bar{a}_{i,t}$ in $\bar{\mathbb{E}}_{i,t}\{z_{i,t}\}$ here. The link between $\bar{a}_{i,t}$ and $\eta_{i,t}$, v_t , and $\hat{\sigma}_{i,t}^2$, on the other hand, is generally nonlinear and cannot be solved for in closed form. To keep solving the model computationally feasible, we henceforth work with a linear approximation of f in its two arguments (see Appendix C for details):

$$f(u_1, u_2) \simeq \pi_0 + \pi_v u_1 - \pi_\sigma u_2, \quad \pi_v, \pi_\sigma > 0 \quad (17)$$

Recall from Proposition 4 that information content in the working capital signal is increasing in $(\bar{a}_{i,t} - z_{i,t})$. Substituting for $\bar{a}_{i,t}$, we can derive the determinants of the economy's information capacity:

$$\underbrace{\bar{a}_{i,t} - z_{i,t}}_{\text{information capacity}} = \underbrace{\bar{\mathbb{E}}_{i,t}\{z_{i,t}\} - z_{i,t}}_{\text{optimism/pessimism}} - \underbrace{\pi_\sigma \hat{\sigma}_{i,t}^2}_{\text{uncertainty}} + \underbrace{\pi_v v_t + \pi_\sigma \hat{\sigma}_{i,t}^2 \eta_{i,t}}_{\text{financial conditions}} + \pi_0, \quad (18)$$

The following proposition summarizes the result.

Proposition 5. *Let $\eta_{i,t} \leq 1$. Then the noise in the working capital signal, $\sigma_l(\bar{a}_{i,t} - z_{i,t}, \psi_{i,t+1})$, is increasing in pessimism $-(\bar{\mathbb{E}}_{i,t}\{z_{i,t}\} - z_{i,t})$, uncertainty $\hat{\sigma}_{i,t}^2$, and adverse financial shocks $(-\eta_{i,t})$ and $(-v_t)$.*

The information capacity depends on three factors, reflecting the three factors that determine the degree to which an island is constrained. First, it is driven by optimism and pessimism, defined as the wedge between an island's true productivity and traders' average belief about it. Conditional on this belief wedge, changes in an island's productivity have no impact on the degree an island is constrained and therefore on the information capacity. This is because a commonly known increase in $z_{i,t}$ increases credit supply in the same way as it increases demand for credit. Second, the information

¹⁶ For the case where net asset supply becomes sufficiently negative, the derivatives of f may actually switch sign, reflecting that traders become *de facto* risk-seeking when they are short-selling.

capacity is decreasing in uncertainty.¹⁷ Third, the information capacity is driven by the financial shocks $\eta_{i,t}$ and v_t . While financial states are exogenous, beliefs are not, opening the door for a feedback loop between uncertainty, pessimism, and financial constraints.

5.2 Sticky beliefs

We now characterize the evolution of beliefs in equilibrium. To do so, we need to explicitly characterize the information content in the asset prices $\{Q_{i,t}\}$. Given the (approximate) log-normal structure of $X_{i,t}$, this can be done using techniques similar to those used when solving a standard CARA-Normal asset pricing equilibrium (e.g., Hellwig, 1980). We approach this slightly different than the previous literature, noting that rationality implies that the equilibrium structure is common knowledge and, hence, also the impact of $Q_{i,t}$ on beliefs is common knowledge. By factoring out all the common components to the belief formation, we can then explicitly characterize the information content in asset prices without the need of solving a fixed point (see the proof to Lemma 6 for details).

With the solution to the asset pricing problem at hand, we can then use the Kalman filter to recursively filter through all public information up to period $t - 1$, and then use the filter one last time where we also take into account the private information available in period t .

In the following, some additional notation will be useful. Letting $\mathcal{I}_{i,t}^p \equiv \mathcal{I}_{ik,t} \setminus \{\omega_{ik,t}^p\} = \{\omega_{i,s}^l, Q_{i,s}, \Upsilon_s\}_{s=0}^t$ denote the publicly observable history of signals, we define $b_{i,t}^z \equiv \mathbb{E}\{z_{i,t} | \mathcal{I}_{i,t}^p\}$, $b_{i,t}^\eta \equiv \mathbb{E}\{\eta_{i,t} | \mathcal{I}_{i,t}^p\}$ and $\varsigma_{i,t}^2 \equiv \text{Var}\{z_{i,t} | \mathcal{I}_{i,t}^p\}$ as “public” posteriors. With this at hand, the following lemma summarizes the result. Since the lemma is rather cumbersome to read, the reader should feel free to skip the details of the statement and instead read the summary below.

Lemma 6. *In equilibrium, average beliefs are given by*

$$\begin{aligned} \bar{\mathbb{E}}_{i,t}\{z_{i,t}\} &= \left(\kappa^p z_{i,t} + \kappa_{i,t}^l \rho_z \tilde{\omega}_{i,t}^{face} + \kappa_{i,t}^q \omega_{i,t}^q + \kappa_{i,t}^0 \rho_z b_{i,t-1}^z \right) \hat{\sigma}_{i,t}^2 \\ \hat{\sigma}_{i,t}^2 &= \left(\kappa^p + \kappa_{i,t}^l + \kappa_{i,t}^q + \kappa_{i,t}^0 \right)^{-1}, \end{aligned}$$

with public beliefs given by

$$\begin{aligned} b_{i,t}^z &= \left(\kappa_{i,t}^l \rho_z \tilde{\omega}_{i,t}^{face} + \kappa_{i,t}^q \omega_{i,t}^q + \kappa_{i,t}^0 \rho_z b_{i,t-1}^z \right) \varsigma_{i,t}^2 \\ b_{i,t}^\eta &= c_q \left(\omega_{i,t}^q - b_{i,t}^z \right) + \rho_\eta b_{i,t-1}^\eta \\ \varsigma_{i,t}^2 &= \left(\kappa_{i,t}^l + \kappa_{i,t}^q + \kappa_{i,t}^0 \right)^{-1}, \end{aligned}$$

¹⁷As mentioned above, the average trader is short-selling when $\eta_{i,t} \geq 1$, implying that the financial market becomes *de facto* risk-seeking and the sign in front of uncertainty flips.

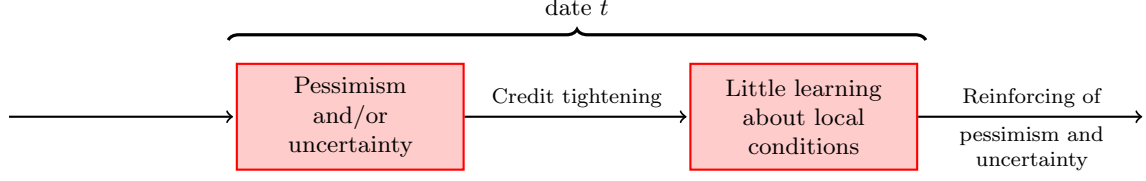


Figure 2: Schematic illustration of belief trap on a given island

where observing $Q_{i,t}$ is equivalent to observing $\omega_{i,t}^q \sim \mathcal{N}(z_{i,t}, 1/\kappa_{i,t}^q)$:

$$\begin{aligned}\omega_{i,t}^q &= z_{i,t} + c_q^{-1} \left(\eta_{i,t} - \rho_\eta b_{i,t-1}^\eta \right) \\ \kappa_{i,t}^q &= \left(c_q^2 \rho_\eta^2 \varsigma_{i,t-1}^2 + \sigma_\eta^2 \right)^{-1} c_q^2,\end{aligned}$$

and where $\tilde{\omega}_{i,t}^{face} = \omega_{i,t}^{face} + \bar{a}_{i,t-1}$, $\kappa^p = \sigma_p^{-2}$, $\kappa_{i,t}^l = \Delta_{i,t} \sigma_{l,i,t}^{-2}$, $\kappa_{i,t}^0 = \Delta_{i,t} \varsigma_{i,t-1}^{-2}$, $c_q = \kappa^p / \pi_\sigma$, and

$$\Delta_{i,t}^{-1} = \rho_z^2 + \left(\sigma_{l,i,t}^{-2} + \varsigma_{i,t-1}^{-2} \right) \sigma_z^2.$$

The Gaussian signal structure implies that expectations are a convex combination of the available signals with weights given by the signal precisions, κ^p , $\kappa_{i,t}^l$, $\kappa_{i,t}^q$, and $\kappa_{i,t}^0$, relative to the posterior precision. When the working capital signal becomes less precise (a fall in $\kappa_{i,t}^l$), traders increase the weight on the other signals. As we will discuss below this implies an increase in the within-island dispersion of beliefs across traders (as traders place more weight on their private signals) as well as an increase in the volatility of asset prices (as traders place more weight on price signals, so that markets become more exposed to financial noise).

Moreover, a fall in $\kappa_{i,t}^l$ leads traders to also increase the weight on prior information, creating a “stickiness” in expectations, so that prior expectations $b_{i,t-r}^z$ have a larger impact on future beliefs $b_{i,t+s}^z$ (and hence on $\bar{\mathbb{E}}_{i,t+s}\{z_{i,t+s}\}$) for all $r, s > 0$.¹⁸ At the same time, also high levels of uncertainty are inherited via the prior, so that a decrease in signal precision equally implies a reinforcement of high uncertainty. In combination with Propositions 5 this defines what we refer to as belief traps: A combination of pessimism and uncertainty combined with tight financial constraints, such that learning is inhibited, which in turn reinforces pessimism and uncertainty and financial constraints. Figure 2 provides a schematic overview of this mechanism. As we illustrate in the next section, this feedback loop can temporarily cause output to become virtually decoupled from fundamentals $z_{i,t}$.

5.3 Impact of idiosyncratic financial conditions: Illustration of belief traps

To illustrate the two-way interaction between beliefs and credit constraints, consider a single island that is hit by a $-1.5\sigma_\eta$ shock to $\eta_{i,t}$, while the aggregate financial state v_t is set constant and the economy is in its stochastic steady state (i.e, the cross-sectional distribution of productivities and

¹⁸See the previous working paper version [Straub and Ulbricht \(2012\)](#) for a formal definition of belief stickiness, an extensive discussion of the result, and a general proof applying to all Gaussian information structures.

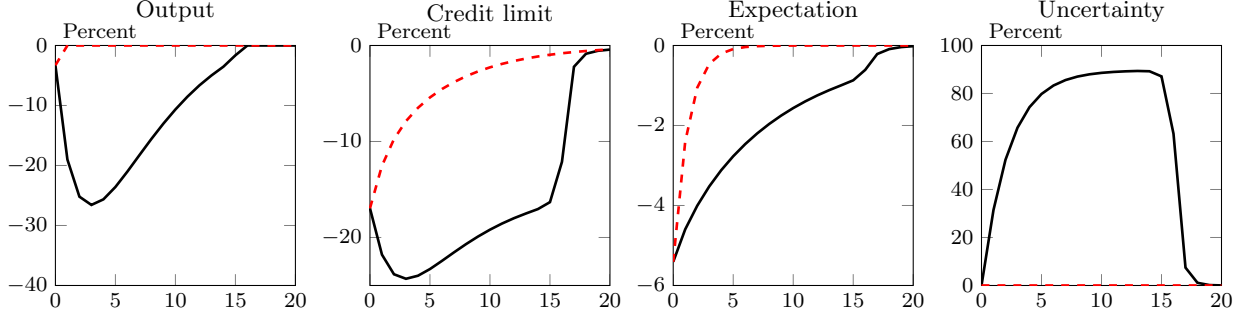


Figure 3: Comparative dynamics to island-specific financial shock. Solid black lines are impulse responses to a $-1.5\sigma_\eta$ shock to $\eta_{i,t}$. Dashed red lines are counterfactual responses to the same shock where the information precision is exogenously fixed at its unconstrained level. All responses are in percentage deviations.

beliefs are in their invariant states).¹⁹ It is instructive to compare the (island-specific) dynamics of the model to such a shock with the hypothetical dynamics that would emerge if the same shock hit the same island, but where the signal precision $\sigma_{l,i,t}^{-2}$ is fixed at the level it would have taken when the island remained to be unconstrained.²⁰ Figure 3 displays the model dynamics (solid black lines) alongside the counterfactuals (dashed red lines).

It can be seen that upon impact the financial shock affects the model in the same way as the counterfactual response. This is because with traders observing lagged investments, the increase in uncertainty will only affect the dynamics with a delay of at least 1 period. On impact, the financial shock decreases the credit limit $\bar{a}_{i,t}$ and leads to more pessimistic expectations $\mathbb{E}_t\{z_{i,t}\}$ since traders cannot distinguish the idiosyncratic financial shock from variations in the asset price that are driven by the fact that other traders might be better informed.

The difference between the model and the hypothetical counterfactual is that the decrease in the credit limit and the resulting increase in the constrainedness lead to a reduction in the information capacity starting one period later. On the one hand this increases uncertainty. On the other hand, in the absence of the possibility to learn from the real economy, traders lack an important signal source and hence their pessimistic expectations recover more slowly compared to the counterfactual (this is the “stickiness” of pessimism). The combination of uncertainty and pessimism then reinforces financial constraints, which lead to a continued collapse in information, and so on. In fact, uncertainty accumulates over time, given that in the absence of new reliable information the initial prior becomes increasingly discounted, so that the loss in output increases over the first few periods after impact.

Given the mean-reverting process of both fundamentals (and, hence, beliefs) and financial noise, expectations and credit limits will eventually (mechanically) recover (i.e., for purely exogenous reasons). Once they hit the point where the island is sufficiently unconstrained so that a sufficient amount of new information is aggregated, there is a sharp drop in uncertainty. At this point, the island has essentially left the “belief trap”, and recovery proceeds quickly.

¹⁹All numerical illustrations use the parametrization of our baseline economy in Section 7.

²⁰The situation is set up, so that virtually all firms in the considered island are unconstrained in the absence of the shock.

Note that throughout the whole impulse response path, the fundamentals remained unchanged. All the adverse effects were due to the combination of beliefs and financial constraints. While in the counterfactual that has lead only to a temporary slump, the same shock has led to a virtual decoupling of business activity from business conditions in the model with endogenous uncertainty. This decoupling is what we refer to as belief traps.

6 Cross-sectional dynamics

We now move to the aggregate economy. An important step in understanding the model dynamics is understanding which islands are constrained in equilibrium. To highlight the mechanics at play, we begin by keeping the aggregate financial state v_t constant to illustrate the cross-sectional distribution. Section 6.2 then looks at how aggregate financial conditions v_t affect the cross-sectional distribution of economic activity.

There are three exogenous sources of idiosyncratic behavior in our model: the cross-sectional distribution of productivities $\{z_{i,t}\}$, the cross-sectional distribution of noise in the signal-extraction $\{\psi_{i,t}\}$, and the cross-sectional distribution of financial noise $\{\eta_{i,t}\}$. Given the history of these idiosyncratic realizations, certain islands will be sufficiently constrained to exhibit the “belief trap” dynamics illustrated in the previous section. Importantly, while in the previous section those dynamics were triggered by an exogenous shock to $\eta_{i,t}$, they in more general will be the result of any combination of idiosyncratic shocks.

A sufficient statistics for the history of idiosyncratic shocks $\{z_{i,s}, \psi_{i,s}, \eta_{i,s}\}_{s=0}^t$ is the quadruple $(z_{i,t}, \bar{\mathbb{E}}_{i,t}\{z_{i,t}\}, \hat{\sigma}_{i,t}, \eta_{i,t})$.²¹ Let $\mathcal{S}_{i,t} \equiv (z_{i,t}, \bar{\mathbb{E}}_{i,t}\{z_{i,t}\}, \hat{\sigma}_{i,t})$. Then conditional on $\mathcal{S}_{i,t}$ and v_t , the degree of constrainedness of island i can be written as a threshold $\bar{\eta}_p(\mathcal{S}_{i,t}, v_t)$, indicating that for all $\eta_{i,t} \leq \bar{\eta}_p(\mathcal{S}_{i,t}, v_t)$, at least fraction p of firms in islands i are constrained, where²²

$$\bar{\eta}_p(\mathcal{S}_{i,t}, v_t) = \frac{1}{\pi_\sigma \hat{\sigma}_{i,t}^2} (\sigma_a \Phi^{-1}(1-p) + z_{i,t} - \bar{\mathbb{E}}_{i,t}\{z_{i,t}\} - \pi_v v_t - \pi_0) + 1. \quad (19)$$

6.1 Divergence around the marginally constrained island

How does the degree to which an island is constrained impact that island’s economic trajectory? In the previous section, we discussed how constrained islands can fall into belief traps, where a powerful feedback mechanism causes a pronounced and prolonged economics crisis. As we show now, the emergence of a belief trap depends crucially on the degree to which an island is constrained. In particular, our model implies a divergence between islands that are “marginally” constrained and

²¹To see this, note that the idiosyncratic output can be written in terms of $\bar{A}_{i,t}$ and $Z_{i,t}$ which by Lemma 5 are functions of $\bar{\mathbb{E}}_{i,t}\{z_{i,t}\}$, $\hat{\sigma}_{i,t}$, $Z_{i,t}$, $\eta_{i,t}$, and the aggregate state Υ_t only.

²²To see this note that the fraction of constrained firms on island i is $p = 1 - \Phi\left(\frac{\bar{a}_{i,t} - z_{i,t}}{\sigma_a}\right)$, hence $(\bar{a}_{i,t} - z_{i,t})|_p = \sigma_a \Phi^{-1}(1-p)$.

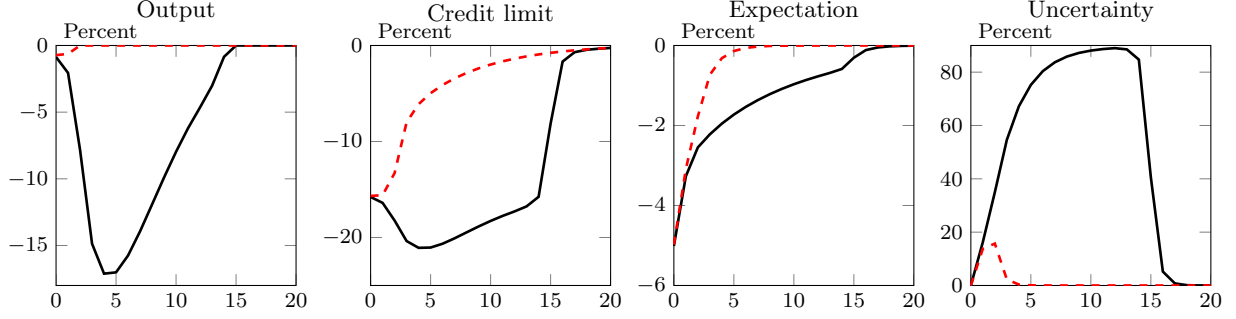


Figure 4: Comparative dynamics to island-specific financial shock so that island is “marginally constrained”. Solid black lines and dashed red lines are impulse responses to $-1.39\sigma_\eta$ and $-1.40\sigma_\eta$ shocks to $\eta_{i,t}$, respectively. All responses are in percentage deviations.

those that are “marginally” unconstrained.²³

To illustrate this, consider Figure 4. Here we plot impulse response functions to a negative financial shock to $\eta_{i,t}$, as we did in Figure 3, only that this time, we choose the initial impact to be “marginal”: A slightly larger impact leads the island to enter a belief trap, whereas a slightly smaller impact avoids it. Specifically, the solid black lines are impulse responses to an initial innovation in $\eta_{i,t}$ set to $\frac{1}{100}$ standard deviations larger than the ones of the dashed red lines (set to $-1.40\sigma_\eta$, the critical value in this case).

It is evident that the solid black lines closely resemble the impulse responses of the model plotted in Figure 3, whereas the dashed red lines resemble the ones of the counterfactual.²⁴ In fact, the marginally unconstrained island’s economy barely suffers any breakdown in the information capacity.

In sum, there is *de facto* a discontinuity in the dynamics of the marginally constrained island and the marginally unconstrained island. While output of the marginally unconstrained island is virtually unaffected by the reduction in the credit limit, a slightly more constrained island remains constrained for a considerable amount of time, with severe consequences for resource misallocation and output (see first panel in figure 4).

6.2 Impact of aggregate financial conditions

Aggregate financial conditions, as represented by v_t , affect the aggregate economy through their effect on the thresholds $\bar{\eta}_p(\mathcal{S}_{i,t}, v_t)$ and thus on the likelihood of an island to fall into a belief trap. The following proposition states that, indeed, a negative aggregate financial shock—a decrease in v_t —causes the thresholds $\bar{\eta}_p(\mathcal{S}_{i,t}, v_t)$ to decrease for every fraction p of constrained firms.

²³As we have discussed in Section 4.2, we have designed our model so that there is a continuous measure of constrainedness. Numerically, however, we keep the within-island dispersion small, so that islands are practically either unconstrained (that is, less than one percent of firms within the island are constrained), or are largely constrained (more than 99 percent of firms are constrained). In our calibration, the average likelihood of an $\eta_{i,t}$ realization, such that $\bar{\eta}_{0.99}(\mathcal{S}_{i,t}, v_t) < \eta_{i,t} < \bar{\eta}_{0.01}(\mathcal{S}_{i,t}, v_t)$ is 0.4 percent, so that it makes sense to think about an island as being either constrained or unconstrained. With this in mind, we define an island to be marginally constrained (unconstrained) if $\eta_{i,t}$ is marginally below (above) $\bar{\eta}_{0.99}(\mathcal{S}_{i,t}, v_t)$.

²⁴The difference between the model responses here and there is that the impact here is marginal (i.e., a $-1.4\sigma_\eta$ shock compared to a $-1.5\sigma_\eta$ shock), making the belief trap slightly less pronounced.

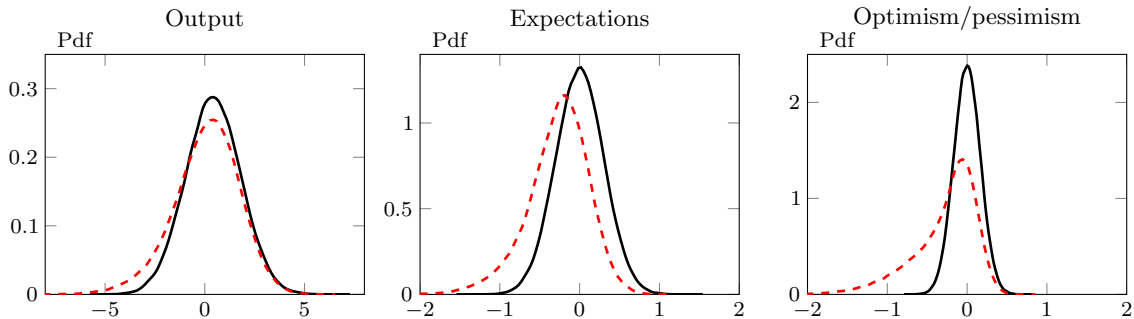


Figure 5: Invariant cross-sectional distributions of output $\log Y_{i,t}$, expectations $\mathbb{E}_t\{z_{i,t}\}$, and optimism/pessimism $\mathbb{E}_t\{z_{i,t}\} - z_{i,t}$. Solid black lines are distributions for $v_t = \bar{v}'$ where less than 1 percent of all firms are constrained. Dashed red lines are distributions for $v_t = \bar{v}'' < \bar{v}'$ where about 50 percent of all firms are constrained.

Proposition 6. *The threshold $\bar{\eta}_p(\mathcal{S}_{i,t}, v_t)$ is decreasing in v_t for all p and $\mathcal{S}_{i,t}$.*

According to the previous subsection, islands in belief traps experience significant economic crises, whereas all other islands are considerably less affected. Because v_t mainly affects the fraction of islands which are in belief traps, a decrease in v_t increases the mass of the lower tail in the cross-sectional distributions of island beliefs and island outputs. Figure 5 illustrates this effect. There, we plot the invariant distributions of log output, expectations and relative optimism or pessimism (as defined in (18)) in two steady states. The black lines show the invariant distributions for a relatively healthy economy with v_t constant at a level $\bar{v}'$, at which less than 1 percent of all firms are constrained in the steady state. There, the three distributions are highly symmetric, due to the underlying Gaussian distribution of $z_{i,t}$. In contrast, the dashed red lines plot the respective invariant distributions for a v_t constant—and for illustrative purposes, extreme—level $\bar{v}'' < \bar{v}'$ where about 50 percent of all firms are constrained. It can be seen that this leads to fatter tails in the cross-sectional distributions.

On an aggregate level, this suggests that islands follow more diverse trajectories during a financial crisis. Naturally, however, the fact that more islands find themselves in a belief trap, with similar consequences, generates comovement across islands. This second effect opposes the first, which is why the response of cross-sectional dispersion to aggregate financial shocks is theoretically ambiguous. And indeed, as we will discuss in Section 7.4, both effects can dominate, depending on the precise parametrization of the information-related parameters.

6.3 Endogenous pessimism

An interesting implication of our result on belief stickiness is that the average pessimism in the economy endogenously increases as v_t falls, even though v_t is common knowledge and has no direct effect on beliefs. The reason for this is an endogenous selection effect: On the one hand, islands with pessimistic beliefs are more likely to fall into a belief trap (see Proposition 5) and expectations in such islands are endogenously persistent due to the collapse in the islands' information aggregation

capacities. More loosely speaking, this causes pessimism to be a (temporarily) absorbing state. On the other hand, on optimistic islands, learning about fundamentals is not impeded by financial constraints, and so islands quickly notice if their optimism was unjustified. This creates an asymmetry between optimism and pessimism that causes the average expectation in the overall economy to fall.

The second and third panels in Figure 5 illustrate these effects. Specifically, in the third panel it is visible that pessimism is strongly skewed to the left due to the described persistence. The second panel shows how this increase in pessimism maps to a more moderate view of business conditions that is strongest for islands that are perceived to be least productive, but extends to more productively perceived islands as well.²⁵

7 Simulation of financial crisis

In this section, we use numerical simulations to explore how a temporary aggregate financial disturbance propagates through the model economy. While the model is too stylized for a full quantitative exploration, we do think this exercise underscores the sizable effects belief traps can have on the local and aggregate economy.

7.1 Parametrization

For the numerical experiment, we interpret one period as a quarter. Parameter values are chosen to be broadly consistent with the existing business cycle literature (see Table 2 for an overview). In particular, the inverse Frisch elasticity of labor supply ϵ is set to 0.5, the elasticity of substitution between consumption varieties ξ is set to 4, and the risk aversion parameter γ is set to 2. The parameters for the productivity processes are chosen bearing in mind our preferred interpretation of an island as corresponding to a single firm. Accordingly, we set $\rho_z = 0.9$ and $\sigma_z = 0.15$, consistent with the existing empirical estimates based on firm-level data.²⁶ The productivity dispersion within islands σ_z is set to a small 0.01, reflecting the mere technical nature of the within island dispersion in the model.

Our choice of the learning parameters is guided by the following two considerations. First, it seems likely that, at a quarterly frequency, information about current innovations to fundamentals is limited. Second, we conjecture that financial markets have access to relatively precise local business indicators (corresponding to the working capital signal in the model) that play an important role for shaping the financial markets' expectations. Based on these considerations, we set the noise in the working capital signal to be small relative to the noise in the private signal of traders and the noise in financial markets, and shift the levels to imply a moderate steady state uncertainty that averages to about one quarter of the unconditional uncertainty ($\sigma_\psi = 0.3$, $\sigma_p = \sigma_\eta = 0.7$; see Appendix D for alternative specifications of the information parameters).

²⁵This is because being productive does not systematically prevent an island from becoming constrained and thereby being subject to a belief trap, as becomes evident from equation (18).

²⁶See, for example, Gilchrist, Sim and Zakrajšek (2014, Appendix 4).

Table 2: Parameter values used in numerical example

Parameter		Value
inverse elasticity of labor supply	ϵ	0.50
elasticity of substitution among goods	ξ	4.00
risk aversion financial market	γ	2.00
maximal debt-to-asset ratio	χ	0.13
persistence of financial noise	ρ_η	0.84
persistence of firm-level productivity	ρ_z	0.90
dispersion of productivity innovations across islands	σ_z	0.15
dispersion of productivities within islands	σ_a	0.01
standard deviation of firm signal	σ_ψ	0.30
standard deviation of private signal	σ_p	0.70
standard deviation of financial noise	σ_η	0.70

It remains to specify the maximum debt-to-asset ratio χ and the persistence of financial noise ρ_η . With the above parametrization and the way we specify the process for v_t (see below), it turns out that χ only scales v_t but has no impact on any of the results below.²⁷ With this in mind we set χ to 0.13, corresponding to a maximal leverage ratio of 20, when adjusting for bank exposure to financial assets not explicitly modeled.²⁸ Finally, we set $\rho_\eta = 0.84$, reflecting a four quarter half-life of the financial noise shocks.

7.2 Computation

The dynamics of the model are fully characterized by Lemmas 1–6. We refer to Appendix C for details on the numerical algorithm.

7.3 Simulation

For our simulation, we let the initial aggregate financial state be $v_0 = \dots = v_{t-1} = \bar{v}$, such that 2.5 percent of firms are constrained in the steady state. We simulate the economy’s impulse response path to a temporary shock at date t with a half-life of 4 quarters:

$$v_{t+s} = \bar{v} - (0.5)^{s/4} \Delta.$$

The size of the initial impact Δ is chosen, so that on average 20 percent of firms are constrained within the first year of impact, consistent with the number of firms that reported to be “very affected”

²⁷More precisely, it turns out that χ has no measurable impact on π_σ and π_v , and only shifts π_0 , which is absorbed by our below calibration strategy for v_t .

²⁸In the model, bank-to-firm lending is the only use of bank deposits, precluding any other source of bank leverage. Our choice of χ accounts for potential other sources of leverage by scaling bank liabilities with the ratio they are used for commercial and industrial loans in June 2007. Based on seasonally adjusted data for commercial banks in the US (released by the Federal reserve), the ratio between bank liabilities and loans was approximately 7.11, yielding $\chi = 0.95/7.11 \approx 0.13$.

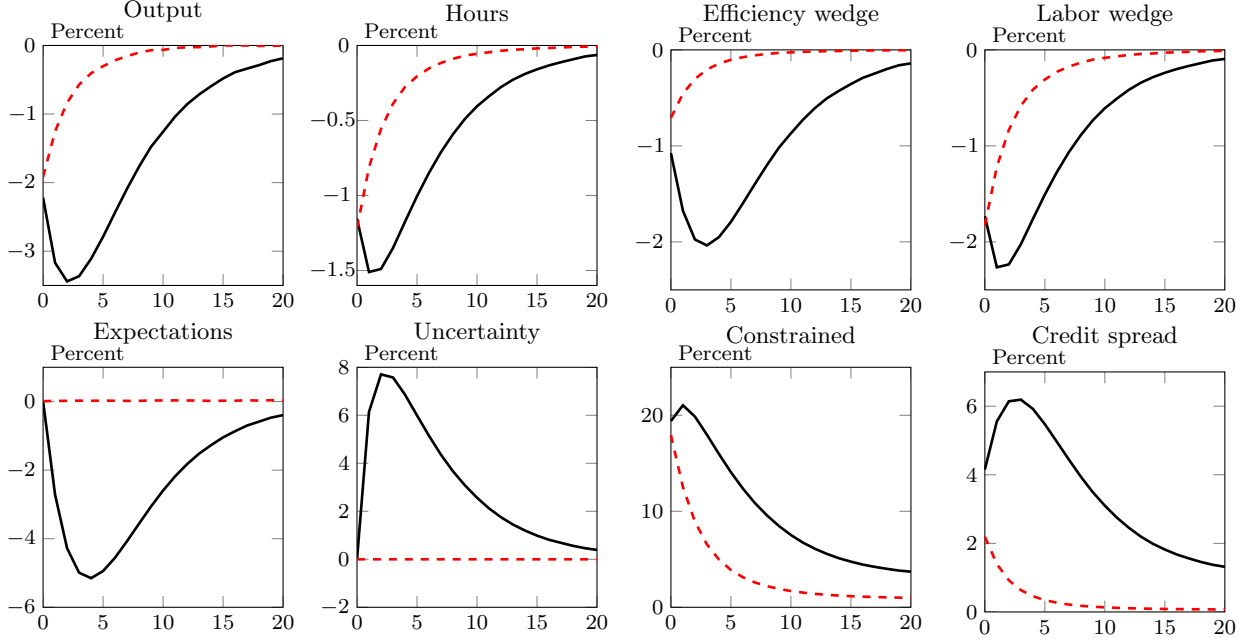


Figure 6: Comparative dynamics to aggregate financial shock (calibrated to 20 percent of firms being constrained upon impact). Solid black lines are impulse responses of the aggregated (or averaged) model economy; dashed red lines are counterfactual responses where the signal precision is exogenously fixed at its unconstrained level. The number of constrained firms and the credit spread are in percentage points, all other responses are in percentage deviations.

by difficulties in accessing the credit market during the recent financial crisis (Campello, Graham and Harvey, 2009).²⁹

Figure 6 depicts the responses of aggregate output, employment, the efficiency wedge, the labor wedge, the average expectations, average uncertainty, the fraction of constrained firms, and the average credit spread.³⁰ All responses are measured in percentage deviations from the steady state, except for the fraction of constrained firms and the credit spread which are measured in percentage points.

The solid black lines show the responses in the model economy. To illustrate the effects of the belief trap mechanism, we also plot counterfactual responses where we exogenously fix $\sigma_{l,i,t}$ at their unconstrained level, shutting down any amplification and persistence stemming from belief traps. The counterfactual responses reflect the impact of the credit tightening on the model economy on its own right. Any difference between the counterfactual and the model economy is due to the breakdown in learning and the belief trap mechanism it causes.

By construction, the simulated shock increases the fraction of constrained firms to 20 percent upon impact, which is further reflected in an increase in the average credit spread. Tighter constraints

²⁹In Section 7.5 we show that the model scales approximately linear in the percent of firms that are constrained upon impact. Accordingly, the response paths to other values of Δ look very similar (see Appendix D for responses to a shock, where 10 percent of firms are constrained upon impact).

³⁰I.e., Y_t , N_t , $(1 - \tau_t^A)$, $(1 - \tau_t^N)$, $\int_I \mathbb{E}_{i,t}\{z_{i,t}\} di$, $\int_I \hat{\sigma}_{i,t}^2 di$, $\int_I \Phi((\bar{a}_{i,t} - z_{i,t})/\sigma_a) di$, and $\int_{I \times J} (R_{ij,t} - 1) d(i, j)$, respectively.

Table 3: Half-life of output and employment to an aggregate financial shock with a half-life of 4 quarters. The size of the shock is calibrated, so that 10 or 20 percent of firms are constrained within the first 4 quarters of impact.

	10% constrained		20% constrained	
	model	counterfactual	model	counterfactual
financial shock	4	4	4	4
output	12	2	11	2
hours	10	2	9	2

then lead to credit and resource misallocation, illustrated by increases in the efficiency and labor wedges, and further causing aggregate output and employment to fall (see first row of Figure 6).

Notice that upon impact, there is no conceptional difference between the counterfactual responses and the model economy’s—all visible differences are due to variations in the steady state distributions between the two economies. Starting with the first period after the initial impact, however, the responses between the model and the counterfactual diverge as learning in the model economy is inhibited for islands facing tighter financial constraints. As discussed above, the increased uncertainty then reinforces any financial difficulties caused by the fundamental shock to v_t , aggravating economic conditions in the model responses.³¹ This is further exacerbated by the spreading of pessimism due to its stickiness, which manifests itself in an overall decline in average expectations (see Section 6.3).

Together, these effects cause a persistent downturn of the aggregate economy. An interesting observation is that, abstracting from general equilibrium effects, the typical firm is almost unaffected by this downturn. Virtually all the downturn originates from the severe and long-lasting effects financial shocks have on firms that turn out to be in lack of financing. Without the disproportionate effects that belief traps induce on those firms, the economy recovers relatively quickly, as seen in the counterfactual responses.

On an aggregate level, the disproportionately long-lasting contraction of firms in belief traps shows as a discrepancy between the underlying aggregate financial shock, which was set to an half-life of 4 quarters, and the endogenous responses of the economy. In particular, Table 3 lists the half-lives of output and employment in both the model economy and for the counterfactual. It is evident that the endogenous uncertainty model has a high degree of internal persistence, implying that the half-life of output (11 quarters) and employment (9 quarters) significantly outlasts the financial disturbance that caused the crisis. The small internal persistence of the same shocks in the exogenous uncertainty economy (2 quarters) illustrates the crucial role of belief traps underlying this result. (See Appendix C for a simulation where only 10 percent of firms are constrained upon impact; in summary, the economy scales approximately proportionally in the fraction of firms being constrained upon impact.)

³¹Specifically, there are two reasons for the progressive unfolding of belief traps in the model economy. First, for islands that are constrained, past information about business conditions becomes increasingly useless due to continued innovations. In the absence of reliable information about recent innovations, this causes uncertainty to increase over time, intensifying existing belief traps. Second, in the cross-section of islands, an increasing number of islands will be affected by belief traps, due to their “absorbing” nature. Taken together, these two effects explain the hump-shaped response of the model economy. In the absence of belief traps, the counterfactual responses by contrast peak on impact.

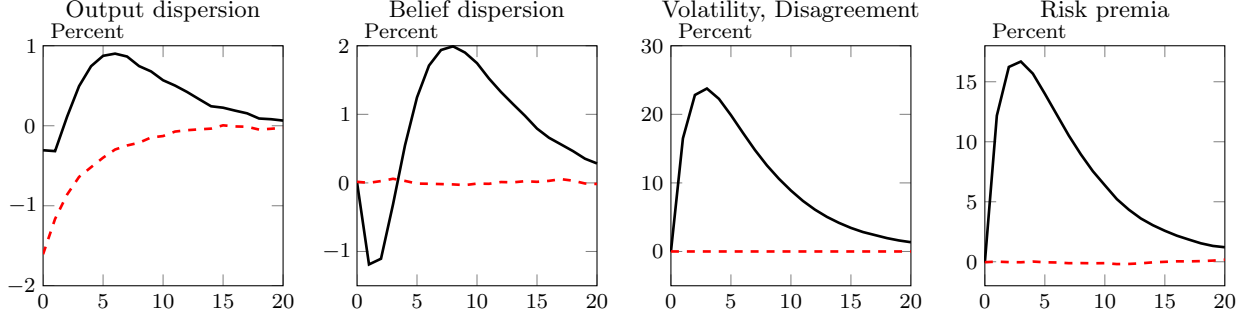


Figure 7: Further comparative dynamics to aggregate financial shock (calibrated to 20 percent of firms being constrained upon impact). Solid black lines are impulse responses of the aggregated (or averaged) model economy; dashed red lines are counterfactual responses where the signal precision is exogenously fixed at its unconstrained level. All responses are in percentage deviations.

7.4 Risk premia, volatility, disagreement, and cross-sectional dispersion

In Figure 7, we report impulse responses for the cross-sectional dispersion of output and expectations, the average volatility of asset prices, the average disagreement among traders (which is numerically equivalent to the volatility response), and the average risk premium for the simulated financial shock. Again, the solid black lines correspond to the model responses and the dashed red lines correspond to counterfactual responses where the signal precision is exogenously fixed at its unconstrained level.

These measures are often used to empirically proxy for uncertainty in macroeconomic and financial contexts. At first sight, the figure shows that the model is able to generate increases in the risk premia, asset price volatility, and disagreement, whereas the counterfactual is not.

Clearly, the average risk premium (defined in terms of $\log Q_{i,t}$) is closely linked to the *perceived* risk $\hat{\sigma}_{i,t}^2$, explaining the positive co-movement of uncertainty and risk-premia in the model.³²

Regarding the volatility, we define asset price volatility as the variance of $\log Q_{i,t}$ conditional on the complete history up to date $t - 1$ and all aggregate information including Z_t and v_t . To see why our measure of volatility is equally increasing throughout the crisis, notice that movements in $\log Q_{i,t}$, which is an affine transformation of $\bar{a}_{i,t}$, are driven by variations in $\bar{\mathbb{E}}_{i,t}\{z_{i,t}\}$ and the financial noise term $\pi_\sigma \hat{\sigma}_{i,t}^2 \eta_{i,t}$. Clearly the latter term becomes more volatile as $\hat{\sigma}_{i,t}$ increases, reflecting that rational traders decrease their asset positions as the risk increases, so that financial noise has a larger impact on the price. Similar, expectations become more volatile when traders become more uncertain, since they then attribute a larger weight to new information (see Lemma 6), contributing further to volatility.

Finally, disagreement among traders, defined as dispersion of beliefs among traders regarding the same $z_{i,t}$ (i.e., $\text{Var}\{\mathbb{E}\{z_{i,t}|\mathcal{I}_{ik,t}\}|i, t\}$), increase on average, since trader's attribute more weight to

³²In particular, the risk-premium is proportional to $(1 - \eta_{i,t})\hat{\sigma}_{i,t}^2$; i.e., the risk weighted by the average asset position of a rational trader. Aggregating across islands, the average risk premium is proportional to $\int_I \hat{\sigma}_{i,t}^2 di - \text{Cov}\{\eta_{i,t}, \hat{\sigma}_{i,t}^2\}$. The covariance term is approximately zero whenever islands are (almost) unconstrained. When islands are constrained, the covariance term becomes negative, reflecting that information is increasing in $\eta_{i,t-1}$ (see Proposition 5). The correlation pattern hence reinforces the link between uncertainty and risk-premia.

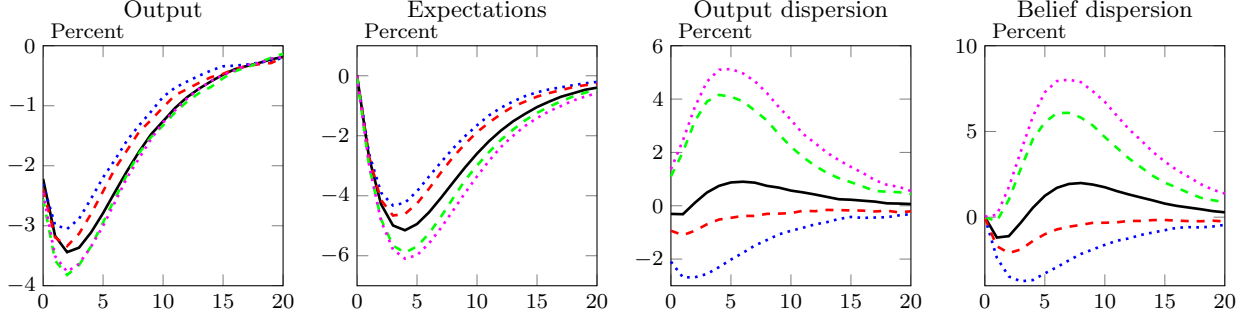


Figure 8: Impulse responses for output, expectations, and the cross-sectional dispersion under alternative parametrizations. The solid black lines correspond to the baseline parametrization ($\sigma_p = \sigma_\eta = 0.7$). The dashed and dotted lines correspond to variations of the information parameters to $\sigma_p = \sigma_\eta = 0.5$, $\sigma_p = 0.5$ and $\sigma_\eta = 0.7$, $\sigma_p = 0.7$ and $\sigma_\eta = 0.9$ and $\sigma_p = \sigma_\eta = 0.9$. The small variation in the responses for output and expectations are representative for all other responses displayed in Figure 6; only the dispersion measures are significantly affected by variations in the information parameters (see Appendix D for a full set of responses). All responses are in percentage deviations.

private information, when less can be learned from business indicators. Since uncertainty is constant in the counterfactual, so are the counterfactual risk premia, volatility and disagreement.

We now turn to the cross-sectional dispersion of output and beliefs.³³ Above we found the effect of an aggregate financial shock on cross-sectional dispersion to be theoretically ambiguous. In our main model calibration, cross-sectional dispersion decreases initially, due to the co-movement of the firms mainly affected by the financial shock. As argued before, this force for co-movement can be overturned by the diverging trajectories of firms in belief traps compared to all other firms. In the chosen parametrization, this is the case beginning with the third quarter after impact.

For a comparison, Figure 8 shows the responses for output, expectations, and their cross-sectional dispersions under four alternative specifications of the information parameters. While the information parameters have little impact on the responses of output, expectations, and all other responses shown in Figure 6 (see Appendix D for the full set of responses under the alternative specifications), the responses for the cross-sectional dispersion vary significantly under the various parametrization. Specifically, as discussed above, the dispersions may both increase or decrease during times of financial distress.

A similar logic applies to the cross-sectional dispersion of expectations: In the first three quarters after impact, expectations on islands in belief traps become homogeneously more pessimistic, putting less weight on the working capital signal. As the crisis continues and selection leads to an increase in average pessimism among constrained islands, the increasing divergence of these islands from the unconstrained fraction of the population eventually dominates and leads to an increased cross-sectional dispersion of beliefs.

Importantly, the increase in the cross-sectional dispersion of output and beliefs crucially depends on the endogenous uncertainty mechanism. Whereas there is no effect on the dispersion of beliefs in the the fixed uncertainty counterfactual, the absence of belief traps always leads to a fall in the

³³In contrast to the disagreement of traders, which is driven by the within-island dispersion of beliefs due to a shift in updating weights, the dispersion of beliefs across islands is entirely driven by the stickiness of beliefs.

dispersion of output in the counterfactual.

The model is thus able to explain the increase in risk premia, volatility, disagreement, and the cross-sectional dispersion of output and beliefs, as well as their co-movement with uncertainty. Related to the co-movement it also offers a simple explanation to the empirical observation that risk premia and volatility are positively correlated with disagreement (Carlin, Longstaff and Matoba, 2013), based on their positive mutual co-movement with uncertainty.

7.5 Scaling of shocks

The following two aspects of the cross-sectional structure of our model are important to understand the impact size of a financial shock: First, the cross-sectional structure is important for the model to generate any quantitatively significant results. This is because with the relevant uncertainty being about local business conditions, all amplification through belief traps is occurring at the local level. Aggregation then implies that the model scales approximately linearly in the fraction of firms that are constrained (see also the left panel of figure 9).³⁴ That is, impulse response for a shock which initially causes 20% of all firms to be constrained is simply a scaled up version of the impulse responses causing 10% of firms to be constrained initially (see Appendix D). This proportionality is crucial for the model to generate quantitative effects.

To see this, suppose that learning were instead about an aggregate fundamental. Then, in order for any belief or uncertainty trap to operate, learning about economy aggregates would have to break down significantly. This, however, requires that a unrealistically high proportion of firms must be financially constrained. For a realistic proportion of affected firms, a mechanism built on aggregate learning therefore fails to generate quantitatively significant effects. In contrast, with learning about local business conditions, it is sufficient that learning about a few firms breaks down significantly.

The other important implication is that in contrast to the linear scaling in the fraction of constrained firms, the model economy responds in a convex fashion to aggregate financial distress (see the right panel of figure 9). Intuitively, for small shocks to v only firms which are hit by extreme adverse idiosyncratic shocks are affected, which under reasonable distributional assumptions is limited to a few. For larger financial shocks, however, the marginally affected firm moves closer to the median firm and thus disproportionately many firms become affected by the shock, causing the convex response.

This nonlinearity generates a discrepancy between high-frequency day-to-day fluctuations in financial markets, which seem to have little impact on real economy, and rare tail events causing pronounced real economy responses.

³⁴Scaling is not exactly linear since (i) a larger fraction of constrained islands also implies that each island is *more* constrained, and (ii) the general equilibrium term z_t further effects on all islands.

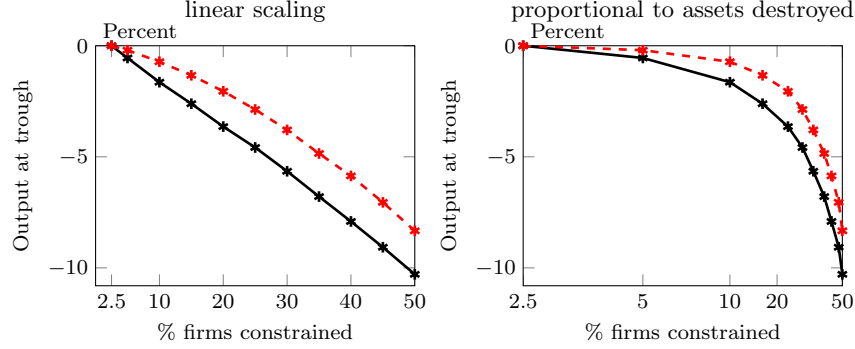


Figure 9: Output deviation in percent at the trough of financial crisis as a function of impact. The left panel scales the x-axis by the average fraction of firms being constrained during the first four quarters after impact (including 2.5 percent of firms being constrained in the steady state); the right panel scales the axis proportional to the assets being destroyed upon impact.

8 Concluding remarks

This study explores the effects of financial crises on the real economy by causing uncertainty about the economy’s fundamentals. When firms see their marginal cost of funds increase during times of financial distress, they respond by cutting hiring—and more generally investment—for projects they deem profitable according to their own private information. This makes information about firm fundamentals *endogenously scarce*, generating uncertainty. We mention a number of consequences uncertainty has within our model—e.g. dispersion of output and beliefs, pessimism, and asset market volatility—but the most important is the feedback effect through the financial sector’s belief about firm profitability: Financial actors see higher credit risks associated with lending to (or more generally investing into) distressed firms, worsening the shortage of funds even further. We illustrate that after aggregate shocks to the financial sector this vicious circle entails not only significant losses for distressed firms, but also for the whole economy.

There are two key novel externalities in our model. Both pertain to the way in which agents fail to internalize the effects of their actions on information generation and hence future financial constraints. First, in our model, constrained firms find it optimal to use up their whole credit limit—with the consequence that their actions become flat in their private information about productivity. In the aggregate, this implies a greater loss of information than if agents were pursuing actions that vary according to their private information. Of course, given credit limits, this alternative might entail a short-run reduction of an island’s productivity but would greatly help to reduce the amplification and persistence of financial shocks.

Second, traders do not internalize their effect on information generation. Given that information plays the role of a public good in this economy, agents have inefficiently low demand for assets: Higher demands would cause higher prices for assets and hence relax firms’ financial constraints. This externality is particularly strong when the marginal effect of prices on information aggregation is large. Traders underinvest *precisely* when it would be needed most to support the economy.

The core mechanism in our model is the effect of financial constraints on the information contained in firms' decisions. There is a number of ways to model this channel. We opted to have publicly observable constraints and exogenous “noise” in signals about firm statistics. An essentially equivalent alternative would have been to assume financial constraints to be private knowledge, endogenously introducing noise into firm statistics. Irrespective of the specific way of modeling it, however, we believe there to be various interesting extensions and other applications of this mechanism. We see for example applications to financial constraints on households, rather than firms, and to borrowing constraints for sovereigns as two especially promising avenues for future research.

A Mathematical appendix

A.1 Proof of Lemma 1

Firms' optimality implies that

$$\begin{aligned} P_{ij,t} &= \frac{\xi}{\xi-1} \frac{W_t R_{ij,t}}{A_{ij,t}} \\ Y_{ij,t} &= P_{ij,t}^{-\xi} C = \left(\frac{\xi-1}{\xi} \right)^\xi \left(\frac{A_{ij,t}}{W_t R_{ij,t}} \right)^\xi C_t \\ L_{ij,t} &= W_t N_{ij,t} = \left(\frac{\xi-1}{\xi} \right)^\xi \left(\frac{A_{ij,t}}{W_t} \right)^{\xi-1} R_{ij,t}^{-\xi} C_t. \end{aligned} \quad (20)$$

Aggregating over labor and output, yields

$$N_t = \left[(1 - \tau_t^A) A^{\text{eff}} \right]^{\xi-1} \left[(1 - \tau_t^N) W_t^{-1} \right]^\xi C_t \quad (21)$$

$$Y_t = \left[(1 - \tau_t^A) A^{\text{eff}} \right] N_t, \quad (22)$$

where

$$A^{\text{eff}} = \left[\int_{I \times J} A_{ij,t}^{\xi-1} d(i,j) \right]^{1/(\xi-1)}$$

is the economy-wide efficient productivity, and where

$$1 - \tau_t^A = \frac{\text{MPN}_t}{\text{MPN}_t^{\text{opt}}} = \frac{Y_t/N_t}{A^{\text{eff}}} = \frac{1}{A^{\text{eff}}} \frac{\left(\int_I \phi_{i,t}^1 di \right)^{\xi/(\xi-1)}}{\int_I \phi_{i,t}^2 di}$$

and

$$1 - \tau_t^N = \frac{\text{MRS}_t}{\text{MPN}_t} = \frac{W_t}{P_t(1 - \tau_t^A) A^{\text{eff}}} = \frac{\xi-1}{\xi} \frac{\int_I \phi_{i,t}^2 di}{\int_I \phi_{i,t}^1 di},$$

define the aggregate efficiency and labor wedge, with $\phi_{i,t}^1$ and $\phi_{i,t}^2$ corresponding to the island-specific distortions defined in the statement of the lemma.

The household's optimization yields

$$C_t N_t^\epsilon = W_t. \quad (23)$$

Collecting equations and setting $C_t = Y_t$, aggregate employment, output, and the wage rate are pinned down by the solution to (21), (22) and (23), yielding

$$\begin{aligned} W_t &= (1 - \tau_t^N)(1 - \tau_t^A) A^{\text{eff}} \\ N_t &= (1 - \tau_t^N)^{1/(1+\epsilon)} \\ Y_t &= (1 - \tau_t^A) A^{\text{eff}} N_t. \end{aligned}$$

A.2 Proof of Proposition 1

Clearly, $R_{ij,t} \geq 0$ for all $(i, j) \in I \times J$. Moreover, for F_a having full support on $\mathbb{R}_+$, there necessarily exists a positive measure of firms (in each island), who's credit demand exceeds $\bar{L}_{i,t}$ for $R_{ij,t} = 1$. Hence, $R_{ij,t} > 0$ for some $(i, j) \in I \times J$. It follows trivially that $(1 - \tau_t^N) < (\xi - 1)/\xi$. To see that $(1 - \tau_t^A) < 1$, rearrange to obtain

$$\left(\int_I x_i^{\frac{\xi-1}{\xi}} y_i^{\frac{(\xi-1)^2}{\xi}} di \right)^\xi < \left(\int_I x_i^{\xi-1} di \right) \left(\int_I y_i^{\xi-1} di \right)^{\xi-1}.$$

To show that this is true, note that $(\cdot)^\xi$ is a convex transformation, so that

$$LHS < \int_I x_i^{\xi-1} y_i^{(\xi-1)^2} di,$$

where the inequality is strict, since the dispersion across islands is strict by assumption. Hence, it suffices to show that

$$\int_I x_i^{\xi-1} y_i^{(\xi-1)^2} di \leq \left(\int_I x_i^{\xi-1} di \right) \left(\int_I y_i^{\xi-1} di \right)^{\xi-1},$$

or

$$\int_I \tilde{x}_i \tilde{y}_i di \leq \left(\int_I \tilde{x}^p di \right)^{1/p} \left(\int_I \tilde{y}^q di \right)^{1/q},$$

where

$$q = 1/(\xi - 1)$$

$$p = 1.$$

Note that $\frac{1}{p} + \frac{1}{q} = 1$. Applying Hölder's inequality (Rogers, 1888; Hölder, 1889), the last inequality holds, establishing $(1 - \tau_t^A) < 1$.

A.3 Proof of Lemma 2

Rearranging (20), we obtain

$$R_{ij,t}^\xi = \frac{A_{ij,t}^{\xi-1}}{L_{ij,t}} Z_t,$$

where

$$Z_t \equiv \left(\frac{\xi - 1}{\xi} \right)^\xi \frac{C_t}{W_t^{\xi-1}}.$$

From (7), it follows that

$$R_{ij,t} = \max \left\{ 1, \frac{A_{ij,t}^{\xi-1}}{\bar{L}_{i,t}} Z_t \right\}^{1/\xi} = \max \left\{ 1, \left(\frac{A_{ij,t}}{\bar{A}_{i,t}} \right)^{(\xi-1)/\xi} \right\}$$

and

$$L_{ij,t} = \min \{ A_{ij,t}, \bar{A}_{i,t} \}^{\xi-1} Z_t$$

for

$$\bar{A}_{i,t} \equiv \left(\frac{\bar{L}_{i,t}}{Z_t} \right)^{1/(\xi-1)}.$$

A.4 Proof of Lemma 3

From firms' optimization, optimal revenues are given by

$$P_{ij,t} Y_{ij,t} = \frac{\xi}{\xi-1} \left(\frac{A_{ij,t}}{R_{ij,t}} \right)^{\xi-1} Z_t.$$

Using Lemma 2, this simplifies to

$$P_{ij,t} Y_{ij,t} = \frac{\xi}{\xi-1} \left(A_{ij,t} \min \{ A_{ij,t}, \bar{A}_{i,t} \}^{\xi-1} \right)^{1-1/\xi} Z_t.$$

Aggregating across firms and substituting into (2), we get that

$$\bar{L}_{ij,t} = \chi \mathcal{P}_{i,t}(X_{i,t}) = \chi \mathcal{P}_{i,t} \left(\frac{\xi}{\xi-1} Z_t \int_J \left[A_{ij,t} \min \{ A_{ij,t}, \bar{A}_{i,t} \}^{\xi-1} \right]^{1-1/\xi} dj \times \Upsilon_t \right),$$

or

$$\bar{A}_{i,t}^{\xi-1} Z_t = \chi \frac{\xi}{\xi-1} Z_t \Upsilon_t \mathcal{P}_{i,t} \left(\int_J \left[A_{ij,t} \min \{ A_{ij,t}, \bar{A}_{i,t} \}^{\xi-1} \right]^{1-1/\xi} dj \right),$$

since $\bar{A}_{i,t} \equiv (\bar{L}_{i,t}/Z_t)^{1/(\xi-1)}$ and $\mathcal{P}_{i,t}(cX) = c\mathcal{P}_{i,t}(X)$ for all constants c (proved in Lemma 7 in Appendix B), where $Z_t \Upsilon_t | \mathcal{I}_{ik,t}$ is constant given our assumption of no aggregate uncertainty. Rearranging yields the result.

A.5 Proof of Proposition 2

Note that the integral in (10) implicitly uses measure $F_A(a_{ij,t}|z_{i,t}) = \Phi \left(\frac{a_{ij,t} - z_{i,t}}{\sigma_a} \right)$. So, another way to express $L_{i,t}$ is,

$$L_{i,t} = Z_t \int_{-\infty}^{\infty} e^{(\xi-1) \min \{ u, \bar{a}_{i,t} \}} d\Phi \left(\frac{u - z_{i,t}}{\sigma_a} \right). \quad (24)$$

In Appendix B, we provide a general Lemma 8 that characterizes how to aggregate arbitrary truncated log-normal variables. Applying Lemma 8 to (24) and taking logs, we obtain

$$l_{i,t} = z_t + (\xi - 1)\bar{a}_{i,t} + \mathcal{L}(z_{i,t} - \bar{a}_{i,t})$$

where

$$\begin{aligned} \mathcal{L}(x) &\equiv \tilde{h}((\xi - 1)x, (\xi - 1)\sigma_a) \\ &= \log \left[e^{(\xi-1)x + \frac{1}{2}(\xi-1)^2\sigma_a^2} \Phi\left(-\frac{x}{\sigma_a} - (\xi - 1)\sigma_a\right) + \Phi\left(\frac{x}{\sigma_a}\right) \right], \end{aligned}$$

where, by Lemma 8, $\tilde{h}$ is strictly increasing and strictly concave in its first argument, and has the described limit properties.

A.6 Proof of Proposition 3

Both proofs rely on Straub and Ulbricht (2014). Before we prove the two parts, notice that equation (11) can be solved for $z_{i,t}$ as function of $l_{i,t}$ and $\bar{a}_{i,t}$,

$$z_{i,t} = g(l_{i,t}, \bar{a}_{i,t}),$$

where, due to the properties of $\mathcal{L}$, g is increasing and strictly convex in $l_{i,t}$, and decreasing in $\bar{a}_{i,t}$. Moreover, g as a function of $l_{i,t}$ becomes “less convex” as $\bar{a}_{i,t}$ increases, in the following sense: The function $G(z) = g(g(\cdot, \bar{a}_1)^{-1}(z), \bar{a}_2)$ is concave and increasing for $\bar{a}_1 \leq \bar{a}_2$. Also note that G is differentiable with derivative between 0 and 1 since $\lim_{z \rightarrow -\infty} G'(z) = 1$.

Now consider part (a). Define two random variables X and Y as having distributions equal to $l_{i,t} | (\omega_{i,t}^l = \omega_1)$ and $l_{i,t} | (\omega_{i,t}^l = \omega_2)$ with $\omega_1 < \omega_2$.³⁵ As we know that $\text{Var}\{l_{i,t} | \omega_{i,t}^l = \omega\}$ is nondecreasing in ω but $l_{i,t} | (\omega_{i,t}^l = \omega)$ is MLRP-increasing in ω , Assumption 1 in Straub and Ulbricht (2014) is satisfied. Therefore, we can apply the strict version of Theorem 1 using g as strictly convex function of $l_{i,t}$ to yield $\text{Var}\{z_{i,t} | \omega_{i,t}^l = \omega_1\} < \text{Var}\{z_{i,t} | \omega_{i,t}^l = \omega_2\}$, which proves the result.

For part (b), let $\bar{a}_1 < \bar{a}_2$ and define G as above. Define random variable Z_j by its distribution $z_{i,t} | (\omega_{i,t}^l = \omega, \bar{a}_{i,t} = \bar{a}_j)$ for $j = 1, 2$. Notice that by definition, $G(Z_1) = Z_2$. This, together with the fact that $0 \leq G'(z) \leq 1$ lets us apply Lemma 2 in Appendix B in Straub and Ulbricht (2014), concluding that $\text{Var}\{Z_2\} < \text{Var}\{Z_1\}$, or in other words, that $\text{Var}\{z_{i,t} | (\omega_{i,t}^l = \omega, \bar{a}_{i,t})$ is decreasing in $\bar{a}_{i,t}$ for any given fixed ω .

³⁵Since their distribution is all that matters for this result, the joint distribution of X and Y is allowed to be anything.

A.7 Proof of Lemma 4

Suppose the working capital signal $\omega_{i,t}^l$ realizes at some ω . If agents linearize the function $\mathcal{L}$ around the face value $\omega^{\text{face}} = \mathcal{L}^{-1}(\omega)$, that means they replace $\mathcal{L}$ by the following linearized function in their information updating problem,

$$\mathcal{L}_{\text{linear}}(x) = \mathcal{L}(\omega^{\text{face}}) + \mathcal{L}'(\omega^{\text{face}})(x - \omega^{\text{face}}).$$

This then implies that agents perceive the nonlinear signal $\omega_{i,t}^l$ as if it came from the corresponding “fictitious” linearized signal,

$$\mathcal{L}_{\text{linear}}(z_{i,t-1} - \bar{a}_{i,t-1}) + \psi_{i,t},$$

or informationally equivalent to this, they update as if they saw the signal

$$\begin{aligned} \omega_{i,t}^{\text{face}} &= (\mathcal{L}_{\text{linear}})^{-1}(\mathcal{L}_{\text{linear}}(z_{i,t-1} - \bar{a}_{i,t-1}) + \psi_{i,t}) \\ &= z_{i,t-1} - \bar{a}_{i,t-1} + \frac{1}{\mathcal{L}'(\omega^{\text{face}})} \psi_{i,t}, \end{aligned}$$

realizing at $\omega_{i,t}^{\text{face}} = \omega^{\text{face}}$. This proves the lemma.

A.8 Proof of Proposition 4

We define

$$\sigma_l(x, \psi) = (\mathcal{L}'(\mathcal{L}^{-1}(\mathcal{L}(-x) + \psi)))^{-1} \sigma_\psi,$$

so that $\sigma_l = \sigma_l(\bar{a}_{i,t-1} - z_{i,t-1}, \psi_{i,t})$ with σ_l from Lemma 4. Obviously, because $\mathcal{L}$ is increasing and $\mathcal{L}'$ is decreasing, σ_l is increasing in its first and decreasing in its second argument.

Now, setting $\psi = 0$, we find that

$$\sigma_l(x, 0) = (\mathcal{L}'(-x))^{-1} \sigma_\psi,$$

giving rise to $\lim_{x \rightarrow \infty} \sigma_l(x, 0) = \sigma_\psi / (\xi - 1)$ and $\lim_{x \rightarrow -\infty} \sigma_l(x, 0) = \infty$ using Proposition 2.

A.9 Proof of Lemma 5

From Lemma 3, $\bar{A}_{i,t}$ is given by the fixed point to

$$\log \bar{A}_{i,t}^{\xi-1} = \log c_a + \log \mathcal{P}_{i,t}(cX_{ij,t}) + v_t,$$

where

$$cX_{ij,t} = \int_J \left[A_{ij,t} \min \{ A_{ij,t}, \bar{A}_{i,t} \}^{\xi-1} \right]^{1-1/\xi} dj$$

for $c = \frac{\xi}{\xi-1} Z_t \Upsilon_t | \mathcal{I}_{ik,t} = \text{const.}$ Clearly, taking a first-order approximation to $\log X_{ij,t}$ is equivalent to taking a first-order approximation to $\log(cX_{ij,t}) = \log c + \log X_{ij,t}$. Accordingly, let

$$H(z_{i,t}, \bar{a}_{i,t}) \equiv \int_J \left[A_{ij,t} \min \{A_{ij,t}, \exp\{\bar{a}_{i,t}\}\}^{\xi-1} \right]^{1-1/\xi} dj,$$

as defined by Lemma 8 in Appendix B, and let $h(z_{i,t}, \bar{a}_{i,t}) \equiv \log H(z_{i,t}, \bar{a}_{i,t})$. Expanding h around $z_{i,t} = \bar{\mathbb{E}}_{i,t}\{z_{i,t}\}$ yields

$$h(z_{i,t}, \bar{a}_{i,t}) \simeq h(\bar{\mathbb{E}}_{i,t}\{z_{i,t}\}, \bar{a}_{i,t}) + \frac{\partial h}{\partial z_{i,t}}(\bar{\mathbb{E}}_{i,t}\{z_{i,t}\}, \bar{a}_{i,t}) \times (z_{i,t} - \bar{\mathbb{E}}_{i,t}\{z_{i,t}\}).$$

Substituting into $\mathcal{P}_{i,t}$, we have

$$\log \mathcal{P}_{i,t}(\exp\{h(z_{i,t}, \bar{a}_{i,t})\}) = h(\bar{\mathbb{E}}\{z_{i,t}\}, \bar{a}_{i,t}) - (1 - \eta_{i,t})\gamma \left[\frac{\partial h}{\partial z}(\bar{\mathbb{E}}\{z_{i,t}\}, \bar{a}_{i,t}) \right]^2 \hat{\sigma}_{i,t}^2$$

Substituting into (9), we get

$$\Gamma(\bar{a}, \mu, u_1, u_2) \equiv (\xi - 1)\bar{a} - u_1 - h(\mu, \bar{a}) + \left[\frac{\partial h(\mu, \bar{a})}{\partial \mu} \right]^2 \gamma u_2 = 0, \quad (25)$$

with

$$\begin{aligned} \bar{a} &\equiv \bar{a}_{i,t} \\ \mu &\equiv \bar{\mathbb{E}}_{i,t}\{z_{i,t}\} \\ u_1 &\equiv \log c_a + v_t \\ u_2 &\equiv (1 - \eta_{i,t})\hat{\sigma}_{i,t}^2. \end{aligned}$$

Equation 25 defines the solution $\bar{a}$ to the balance sheet fixed point Γ as a function of μ , u_1 , and u_2 . Using Lemma 8 (defined in Appendix B), we have that

$$h(\mu, \bar{a}) = \frac{(\xi - 1)^2}{\xi} \bar{a} + \frac{\xi - 1}{\xi} \mu + \frac{(\xi - 1)^2}{2\xi^2} \sigma_a^2 + \tilde{h}(\mu - \bar{a}) \quad (26)$$

for some continuously differentiable $\tilde{h} : \mathbb{R} \rightarrow \mathbb{R}$ with $\tilde{h}' > 0$ and $\tilde{h}'' < 0$.³⁶ It follows that

$$\frac{\partial h}{\partial \mu} = \frac{\xi - 1}{\xi} + \tilde{h}'(\mu - \bar{a}). \quad (27)$$

³⁶In comparison to Lemma 8, the implicit definition of $\tilde{h}$ is slightly different here. Using the definition of $\tilde{h}$ used in Lemma 8, the present one follows after setting $z = \frac{(\xi-1)^2}{\xi}(\mu - \bar{a}) + \frac{(\xi-1)^3}{\xi^2}\sigma_a^2$ and then redefining $\tilde{h}$ accordingly to make it a function of $(\mu - \bar{a})$. Clearly, the transformation sustains the increasing and concave property of $\tilde{h}$.

Linearity in μ We first show that $\bar{a}$ is linear in μ . To see this, substitute (26) and (27) into Γ and rearrange to obtain

$$\Gamma(\hat{a}, u_1, u_2) = \frac{\xi - 1}{\xi} \hat{a} - u_1 - \frac{(\xi - 1)^2}{2\xi^2} \sigma_a^2 - \tilde{h}(-\hat{a}) + \left[\frac{\xi - 1}{\xi} + \tilde{h}'(-\hat{a}) \right]^2 \gamma u_2, \quad (28)$$

where $\hat{a} \equiv \bar{a} - \mu$. We see that Γ is an equation in $\hat{a}$ only. Differentiating on Γ , it therefore trivially holds that $\partial \bar{a} / \partial \mu = 1$.

Comparative statics to u_1 and u_2 Given linearity in μ , write

$$\hat{a} = f(u_1, u_2)$$

for some function f . Differentiating on (28), we use (27) to obtain

$$\frac{\partial f}{\partial u_1} = \frac{1}{\frac{\partial h}{\partial \mu} (1 - 2\gamma \tilde{h}'' u_2)} > 0$$

and

$$\frac{\partial f}{\partial u_2} = \frac{-\frac{\partial h}{\partial \mu} \gamma}{1 - 2\gamma \tilde{h}'' u_2} < 0,$$

since $\partial h / \partial \mu > 0$, $\tilde{h}'' < 0$, and $u_2 \geq 0$ (given $\eta_{i,t} \leq 1$).

Uniqueness Finally, consider uniqueness of f . Fix u_1 and u_2 and suppose that $\Gamma(\hat{a}, u_1, u_2) = 0$ had two solutions, $\hat{a}_1$ and $\hat{a}_2$. Without loss of generality, take $\hat{a}_1 > \hat{a}_2$. Subtracting $\Gamma(\hat{a}_2, u_1, u_2)$ from $\Gamma(\hat{a}_1, u_1, u_2)$, and denoting $\Delta \hat{a} \equiv \hat{a}_1 - \hat{a}_2 > 0$ and $\tilde{h}_i \equiv \tilde{h}(-\hat{a}_i)$, we get

$$\frac{\xi - 1}{\xi} \Delta \hat{a} + \tilde{h}_2 - \tilde{h}_1 + (\tilde{h}'_1 - \tilde{h}'_2) \left[\tilde{h}'_1 + \tilde{h}'_2 + \frac{\xi - 1}{\xi/2} \right] \gamma u_2 = 0,$$

or

$$\frac{\xi - 1}{\xi} - \frac{\tilde{h}_1 - \tilde{h}_2}{\Delta \hat{a}} + \frac{\tilde{h}'_1 - \tilde{h}'_2}{\Delta \hat{a}} \left[\tilde{h}'_1 + \tilde{h}'_2 + \frac{\xi - 1}{\xi/2} \right] \gamma u_2 = 0. \quad (29)$$

Given that $\tilde{h}' > 0$ and $\tilde{h}'' < 0$, we have that

$$\frac{\tilde{h}_1 - \tilde{h}_2}{\Delta \hat{a}} = \frac{\tilde{h}(-\hat{a}_1) - \tilde{h}(-\hat{a}_2)}{\Delta \hat{a}} < 0$$

and

$$\frac{\tilde{h}'_1 - \tilde{h}'_2}{\Delta \hat{a}} = \frac{\tilde{h}'(-\hat{a}_1) - \tilde{h}'(-\hat{a}_2)}{\Delta \hat{a}} > 0,$$

so that the left-hand side of (29) is strictly positive, a contradiction.

A.10 Proof of Proposition 5

From Proposition 4 we have that $\sigma_l(\bar{a}_{i,t} - z_{i,t}, \psi_{i,t+1})$ is increasing in its first argument and decreasing in the second. Since $\psi_{i,t+1}$ is orthogonal noise, any systematic variation in σ_l is caused by changes in $(\bar{a}_{i,t} - z_{i,t})$. By Lemma 5,

$$\bar{a}_{i,t} - z_{i,t} = \bar{\mathbb{E}}\{z_{i,t}\} - z_{i,t} + f(v_t, (1 - \eta_{i,t}) \hat{\sigma}_{i,t}^2),$$

where f is increasing in its first and decreasing in its second argument, yielding the result.

A.11 Proof of Lemma 6

Consider the information set of trader (i, k) at time t , $\mathcal{I}_{ik,t} = \{\omega_{ik,t}^p\} \cup \{\omega_{i,s}^l, Q_{i,s}, \Upsilon_s\}_{s=0}^t$. By assumption, Υ_t is orthogonal to $z_{i,t}$ and can thus be ignored for the purpose of learning about $z_{i,t}$. Given our approximation approach, the remaining elements of $\mathcal{I}_{ik,t}$ are Gaussian signals so that we can characterize $\mathbb{E}\{z_{i,t} | \mathcal{I}_{ik,t}\}$ using a standard Kalman filter. In particular, since $\mathcal{I}_{i,t}^p = \{\omega_{i,s}^l, Q_{i,s}, \Upsilon_s\}_{s=0}^t$ is common knowledge, we can characterize beliefs recursively by first filtering through the publicly observable history $\mathcal{I}_{i,t}^p$, and then applying the filter one last time to process the information contained in $\{\omega_{ik,t}^p, \omega_{i,t}^l, Q_{i,t}\}$.

From Lemma 4, $\omega_{i,t}^l$ is informational equivalent to observing

$$\omega_{i,t}^{\text{face}} \sim \mathcal{N}(z_{i,t-1} - \bar{a}_{i,t-1}, \sigma_{l,i,t}^2),$$

or

$$\tilde{\omega}_{i,t}^{\text{face}} \equiv \omega_{i,t}^{\text{face}} + \bar{a}_{i,t-1} \sim \mathcal{N}(z_{i,t-1}, \sigma_{l,i,t}^2),$$

since $\bar{a}_{i,t-1}$ is known. Asset prices $\{Q_{i,t}\}$ are thus the only endogenous signals that remain to be characterized. Substituting (2) in $\bar{A}_{i,t}$ (as defined in Lemma 2), we have that $Q_{i,t}$ is a positive monotonic transformation of $\bar{a}_{i,t}$. Further substituting (17) in (16), and subtracting all common knowledge terms, then implies that

$$\bar{\mathbb{E}}_{i,t}\{z_{i,t}\} + \pi_\sigma \hat{\sigma}_{i,t}^2 \eta_{i,t} \tag{30}$$

is a positive monotonic (and, hence, informationally equivalent) transformation of $Q_{i,t}$. Suppose for now that observing (30) is informationally equivalent to observing a signal

$$\omega_{i,t}^q \sim \mathcal{N}(z_{i,t}, 1/\kappa_{i,t}^q)$$

for some yet to be determined precision $\kappa_{i,t}^q$.

Letting $b_{i,t-1}^z$ and $\varsigma_{i,t-1}^2$ denote the (public) prior mean and variance given $\mathcal{I}_{i,t-1}^p$, we are now ready to update beliefs given $\tilde{\omega}_{i,t}^{\text{face}}$, $\omega_{i,t}^q$, and $\omega_{ik,t}^p$. Since $\tilde{\omega}_{i,t}^{\text{face}}$ is a signal about $z_{i,t-1}$, we split the updating into two steps, first forming expectations about $z_{i,t-1}$ using only $\tilde{\omega}_{i,t}^{\text{face}}$ and the prior.

Standard Bayesian updating yields

$$\mathbb{E}\{z_{i,t-1}|\mathcal{I}_{i,t-1}^p, \tilde{\omega}_{i,t}^{\text{face}}\} = \frac{\varsigma_{i,t-1}^{-2} b_{i,t-1}^z + \sigma_{i,t,l}^{-2} \tilde{\omega}_{i,t}^{\text{face}}}{\varsigma_{i,t-1}^{-2} + \sigma_{i,t,l}^{-2}},$$

$$\text{Var}\{z_{i,t-1}|\mathcal{I}_{i,t-1}^p, \tilde{\omega}_{i,t}^{\text{face}}\}^{-1} = \varsigma_{i,t-1}^{-2} + \sigma_{i,t,l}^{-2}$$

Projecting forward, we get

$$\mathbb{E}\{z_{i,t}|\mathcal{I}_{i,t-1}^p, \tilde{\omega}_{i,t}^{\text{face}}\} = \rho_z \frac{\varsigma_{i,t-1}^{-2} b_{i,t-1}^z + \sigma_{i,t,l}^{-2} \tilde{\omega}_{i,t}^{\text{face}}}{\varsigma_{i,t-1}^{-2} + \sigma_{i,t,l}^{-2}}$$

$$\text{Var}\{z_{i,t}|\mathcal{I}_{i,t-1}^p, \tilde{\omega}_{i,t}^{\text{face}}\}^{-1} = \left(\varsigma_{i,t-1}^{-2} + \sigma_{i,t,l}^{-2}\right) \Delta_{i,t},$$

where

$$\Delta_{i,t}^{-1} = \rho_z^2 + \left(\varsigma_{i,t-1}^{-2} + \sigma_{i,t,l}^{-2}\right) \sigma_z^2.$$

Now treating $\mathbb{E}\{z_{i,t}|\mathcal{I}_{i,t-1}^p, \tilde{\omega}_{i,t}^{\text{face}}\}$ and $\text{Var}\{z_{i,t}|\mathcal{I}_{i,t-1}^p, \tilde{\omega}_{i,t}^{\text{face}}\}^{-1}$ as prior, updating with respect to $\omega_{i,t}^q$ and $\omega_{ik,t}^p$ yields

$$\mathbb{E}\{z_{i,t}|\mathcal{I}_{i,t-1}^p, \tilde{\omega}_{i,t}^{\text{face}}, \omega_{i,t}^q, \omega_{ik,t}^p\} = \left(\kappa^p \omega_{i,k,t}^p + \rho_z \kappa_{i,t}^l \tilde{\omega}_{i,t}^l + \kappa_{i,t}^q \omega_{i,t}^q + \rho_z \kappa_{i,t}^0 b_{i,t-1}^z\right) \hat{\sigma}_{i,t}^2 \quad (31)$$

and

$$\hat{\sigma}_{i,t}^{-2} = \kappa^p + \kappa_{i,t}^l + \kappa_{i,t}^q + \kappa_{i,t}^0, \quad (32)$$

where $\kappa^p \equiv \sigma_p^{-2}$, $\kappa_{i,t}^l = \Delta_{i,t} \sigma_{i,t,l}^{-2}$, and $\kappa_{i,t}^0 = \Delta_{i,t} \varsigma_{i,t-1}^{-2}$. Aggregating across agents, we have that

$$\bar{\mathbb{E}}_{i,t}\{z_{i,t}\} = \left(\kappa^p z_{i,t} + \rho_z \kappa_{i,t}^l \tilde{\omega}_{i,t}^l + \kappa_{i,t}^q \omega_{i,t}^q + \rho_z \kappa_{i,t}^0 b_{i,t-1}^z\right) \hat{\sigma}_{i,t}^2. \quad (33)$$

To complete the characterization, we still have to find the precision $\kappa_{i,t}^q$ and verify that observing $Q_{i,t}$ is indeed a Gaussian signal. For this, substitute (33) back into (30). Note, however, that the last three terms (including $\hat{\sigma}_{i,t}$) in (33) are all common knowledge, so that (30) is informationally equivalent to observing

$$\kappa^p \hat{\sigma}_{i,t}^2 z_{i,t} + \pi_\sigma \hat{\sigma}_{i,t}^2 \eta_{i,t}$$

or

$$z_{i,t} + \underbrace{\frac{\pi_\sigma}{\kappa^p}}_{\equiv c_q^{-1}} \eta_{i,t},$$

after dividing by $\kappa^p \hat{\sigma}_{i,t}^2$, or

$$\omega_{i,t}^q \equiv z_{i,t} + c_q^{-1} \left(\eta_{i,t} - \rho_\eta b_{i,t-1}^\eta\right), \quad (34)$$

after adding another common knowledge term

$$\begin{aligned} b_{i,t-1}^\eta &\equiv \mathbb{E}\{\eta_{i,t-1} | \mathcal{I}_{i,t-1}^p\} \\ &= c_q \left(\omega_{i,t-1}^q - b_{i,t-1}^z \right) + \rho_\eta b_{i,t-2}^\eta. \end{aligned} \quad (35)$$

The term (34) is the Gaussian transformation of the price signal that we are looking for. To see this, note that

$$\left(\eta_{i,t} - \rho_\eta b_{i,t-1}^\eta \right) \Big| \mathcal{I}_{i,t-1}^p \sim \mathcal{N}(0, 1/\kappa_{i,t}^\eta)$$

where

$$\kappa_{i,t}^\eta = \left[\rho_\eta^2 \text{Var}\{\eta_{i,t-1} | \mathcal{I}_{i,t-1}^p\} + \sigma_\eta^2 \right]^{-1},$$

and

$$\text{Var}\{\eta_{i,t-1} | \mathcal{I}_{i,t-1}^p\} = \text{Var} \left\{ \frac{\omega_{i,t-1}^q - z_{i,t-1}}{c_q^{-1}} \Big| \mathcal{I}_{i,t-1}^p \right\} = c_q^2 \varsigma_{i,t-1}^2.$$

Hence,

$$\omega_{i,t}^q \sim \mathcal{N}(z_{i,t}, 1/\kappa_{i,t}^q),$$

where

$$\kappa_{i,t}^q = c_q^2 \kappa_{i,t}^\eta.$$

We hence have characterized $\bar{\mathbb{E}}\{z_{i,t}\}$ and $\hat{\sigma}_{i,t}$ as a function of exogenous terms and the public priors $b_{i,t-1}^z$, $b_{i,t-1}^\eta$ and $\varsigma_{i,t-1}$ only. Equation (35) already gives a recursive representation for $b_{i,t-1}^\eta$. As discussed in the beginning of the proof, $b_{i,t}^z$ and $\varsigma_{i,t}$ can be characterized by filtering through the public history $\mathcal{I}_{i,t}^p$. Following the same updating steps that lead to $\mathbb{E}\{z_{i,t} | \mathcal{I}_{ik,t}^p\}$ and $\hat{\sigma}_{i,t}^{-2}$, but ignoring $\omega_{ik,t}^p$ (or setting $\kappa^p = 0$ in (31) and (32)), we get

$$b_{i,t}^z = \left(\rho_z \kappa_{i,t}^l \tilde{\omega}_{i,t}^l + \kappa_{i,t}^q \omega_{i,t}^q + \rho_z \kappa_{i,t}^0 b_{i,t-1}^z \right) \varsigma_{i,t}^2$$

and

$$\varsigma_{i,t}^{-2} = \kappa_{i,t}^l + \kappa_{i,t}^q + \kappa_{i,t}^0.$$

Collecting terms, this completes the proof.

A.12 Proof of Proposition 6

The claim is an immediately consequence of (19) and the fact that $\pi_\sigma > 0$ (which follows from Lemma 5).

B Auxiliary Lemmas and Proofs

B.1 Associative property of the $\mathcal{P}$ -operator

Lemma 7. *Let X be some random variable, and let c be some constant. Then it holds that $\mathcal{P}_{i,t}(cX) = c\mathcal{P}_{i,t}(X)$.*

Proof Let $\mathcal{P}_{i,t}$ be defined as in the text:

$$\mathcal{P}_{i,t}(X) = \exp \left[\bar{\mathbb{E}}_{i,t}\{\log X\} - (1 - \eta_{i,t})\gamma \text{Var}\{\log X | \mathcal{I}_{ik,t}\} \right].$$

Notice that

$$\bar{\mathbb{E}}_{i,t}\{\log(cX)\} = \log c + \bar{\mathbb{E}}_{i,t}\{\log X\}.$$

At the same time,

$$\text{Var}\{\log(cX) | \mathcal{I}_{ik,t}\} = \text{Var}\{\log X | \mathcal{I}_{ik,t}\},$$

since c is constant. Hence,

$$\mathcal{P}(cX) = \exp \left[\log c + \bar{\mathbb{E}}_{i,t}\{\log X\} - (1 - \eta_{i,t})\gamma \text{Var}\{\log X | \mathcal{I}_{ik,t}\} \right] = c\mathcal{P}(X).$$

B.2 Aggregation of truncated log-normally distributed variables

Lemma 8. *Let $\log X \sim \mathcal{N}(\mu, \sigma)$. Then for any $s_1 > s_2$ it holds that*

$$H \equiv \int_{-\infty}^{\infty} \min\{e^{c_1} X^{s_1}, e^{c_2} X^{s_2}\} d\Phi \left(\frac{\log X - \mu}{\sigma} \right)$$

satisfies

$$\log H = c_2 + s_2 \mu + \frac{1}{2} s_2^2 \sigma^2 + \tilde{h}(z, t),$$

where

$$\tilde{h}(z, t) \equiv \log \left[e^{z + \frac{1}{2} t^2} \Phi \left(-\frac{z}{t} - t \right) + \Phi \left(\frac{z}{t} \right) \right]$$

and

$$\Delta_c \equiv c_1 - c_2$$

$$\Delta_s \equiv s_1 - s_2$$

$$z \equiv \Delta_c + \Delta_s \mu + \Delta_s s_2 \sigma^2$$

$$t \equiv \Delta_s \sigma.$$

Further, for all $(z, t) \in \mathbb{R} \times \mathbb{R}_+$, $\tilde{h}$ is strictly increasing, strictly concave in z , with $\lim_{z \rightarrow -\infty} \tilde{h}(z, t) = -\infty$, $\lim_{z \rightarrow +\infty} \tilde{h}(z) = 0$, $\lim_{z \rightarrow -\infty} \tilde{h}_z(z, t) = 1$, and $\lim_{z \rightarrow +\infty} \tilde{h}_z(z, t) = 0$.

Proof We want to examine

$$H = \int_{-\infty}^{\infty} e^{\min\{c_1+s_1x, c_2+s_2x\}} d\Phi\left(\frac{x-\mu}{\sigma}\right), \quad s_1 > s_2 \geq 0.$$

Clearly, the first term in the integrand is binding for all

$$x \leq \frac{c_2 - c_1}{s_1 - s_2} \equiv x_0.$$

Hence

$$H = \int_{-\infty}^{x_0} e^{c_1+s_1x} d\Phi\left(\frac{x-\mu}{\sigma}\right) + \int_{x_0}^{\infty} e^{c_2+s_2x} d\Phi\left(\frac{x-\mu}{\sigma}\right),$$

or

$$H = \int_{-\infty}^{c_1+s_1x_0} e^u d\Phi\left(\frac{u-(c_1+s_1\mu)}{s_1\sigma}\right) + \int_{c_2+s_2x_0}^{\infty} e^u d\Phi\left(\frac{u-(c_2+s_2\mu)}{s_2\sigma}\right),$$

after a change in variables, or

$$H = e^{c_1+s_1\mu+\frac{1}{2}s_1^2\sigma^2} - \int_{c_1+s_1x_0}^{\infty} e^u d\Phi\left(\frac{u-(c_1+s_1\mu)}{s_1\sigma}\right) + \int_{c_2+s_2x_0}^{\infty} e^u d\Phi\left(\frac{u-(c_2+s_2\mu)}{s_2\sigma}\right).$$

Computing the partial expectations, we have

$$H = e^{c_1+s_1\mu+\frac{1}{2}s_1^2\sigma^2} \Phi\left(-\frac{\mu-x_0}{\sigma} - s_1\sigma\right) + e^{c_2+s_2\mu+\frac{1}{2}s_2^2\sigma^2} \Phi\left(\frac{\mu-x_0}{\sigma} + s_2\sigma\right),$$

or, after taking logs,

$$\begin{aligned} \log H &= c_2 + s_2\mu + \frac{1}{2}s_2^2\sigma^2 + \log \left[e^{\Delta_c + \Delta_s\mu + \frac{1}{2}(s_1^2 - s_2^2)\sigma^2} \Phi\left(-\frac{\mu-x_0}{\sigma} - s_1\sigma\right) + \Phi\left(\frac{\mu-x_0}{\sigma} + s_2\sigma\right) \right] \\ &= c_2 + s_2\mu + \frac{1}{2}s_2^2\sigma^2 + \underbrace{\log \left[e^{z+\frac{1}{2}t^2} \Phi\left(-\frac{z}{t} - t\right) + \Phi\left(\frac{z}{t}\right) \right]}_{\equiv \tilde{h}(z,t)} \end{aligned}$$

where Δ_c , Δ_s , z , and t are defined as above.

Properties of $\tilde{h}$ We show first that $\tilde{h}$ is increasing and concave. Differentiating, we get

$$\frac{\partial \tilde{h}(z,t)}{\partial z} = \frac{e^{z+\frac{1}{2}t^2} \Phi\left(-\frac{z}{t} - t\right) - \frac{1}{t} e^{z+\frac{1}{2}t^2} \phi\left(-\frac{z}{t} - t\right) + \frac{1}{t} \phi\left(\frac{z}{t}\right)}{\exp\{\tilde{h}(z,t)\}}. \quad (36)$$

The denominator of (36) is always positive. Consider the numerator:

$$\begin{aligned}
& e^{z+\frac{1}{2}t^2} \Phi\left(-\frac{z}{t}-t\right) - \frac{1}{t} e^{z+\frac{1}{2}t^2} \phi\left(-\frac{z}{t}-t\right) + \frac{1}{t} \phi\left(\frac{z}{t}\right) \\
&= e^{z+\frac{1}{2}t^2} \Phi\left(-\frac{z}{t}-t\right) + \frac{1}{t} \phi\left(\frac{z}{t}\right) \left[1 - e^{z+\frac{1}{2}t^2} \phi(t)\right] \\
&= e^{z+\frac{1}{2}t^2} \Phi\left(-\frac{z}{t}-t\right) + \frac{1}{t} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2t^2}} \left[1 - \frac{1}{\sqrt{2\pi}} e^z\right], \tag{37}
\end{aligned}$$

where we used $\phi(x+y) = \phi(x)\phi(y)$, $\phi(-x) = \phi(x)$, and that $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$.

Before showing that (37) is positive, we explore concavity. Differentiate the above expression once more to obtain:³⁷

$$\begin{aligned}
& e^{z+\frac{1}{2}t^2} \left[\Phi\left(-\frac{z}{t}-t\right) - \frac{1}{t} \phi\left(-\frac{z}{t}-t\right) \right] - \frac{z}{t^3} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2t^2}} \left[1 - \frac{1}{\sqrt{2\pi}} e^z \right] - \frac{1}{2\pi t} e^{z-\frac{z^2}{2t^2}} \\
&= e^{z+\frac{1}{2}t^2} \Phi\left(-\frac{z}{t}-t\right) - \frac{1}{t} e^{z+\frac{1}{2}t^2-\frac{z^2}{2t^2}} \phi(t) - \frac{z}{t^3} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2t^2}} - \frac{1}{2\pi t} e^{z-\frac{z^2}{2t^2}} \left[1 - \frac{z}{t^2} \right] \\
&= e^{z+\frac{1}{2}t^2} \left[\Phi\left(-\frac{z}{t}-t\right) - \frac{1}{t} \phi\left(\frac{z}{t}\right) \phi(t) \right] - \frac{z}{t^3} \phi\left(\frac{z}{t}\right) - \frac{1}{t} \frac{1}{\sqrt{2\pi}} \phi\left(\frac{z}{t}\right) e^z \left[1 - \frac{z}{t^2} \right] < 0.
\end{aligned}$$

This establishes concavity. Hence, to show that the first derivative is positive everywhere, it is sufficient to show that it is positive for $z \rightarrow +\infty$. From (37),

$$\lim_{z \rightarrow \infty} e^{-\frac{z^2}{2t^2}} \left[1 - \frac{1}{\sqrt{2\pi}} e^z \right] = \lim_{z \rightarrow \infty} \frac{\sqrt{2\pi} - e^z}{\sqrt{2\pi} e^{\frac{z^2}{2t^2}}} = -\frac{1}{\sqrt{2\pi}} e^{\lim_{z \rightarrow \infty} z - \frac{z^2}{2t^2}} = 0.$$

Given strict concavity, the entire expression remains strictly positive for all $z < \infty$.

Finally, consider the limit properties of $\tilde{h}$. Taking limits, we get

$$\begin{aligned}
\lim_{z \rightarrow -\infty} \tilde{h}(z, t) &= \lim_{z \rightarrow -\infty} \log \left[\underbrace{e^{z+\frac{1}{2}t^2} \Phi\left(-\frac{z}{t}-t\right)}_{\rightarrow 1} + \underbrace{\Phi\left(\frac{z}{t}\right)}_{\rightarrow 0} \right] \\
&= \lim_{z \rightarrow -\infty} \left\{ z + \frac{1}{2}t^2 \right\} = -\infty
\end{aligned}$$

and, hence,

$$\lim_{z \rightarrow -\infty} \frac{\partial \tilde{h}(z, t)}{\partial z} = 1.$$

³⁷It is sufficient to differentiate the numerator, as this gives us the behavior of the expression inside the logarithm. Since $\log(\cdot)$ is itself a concave function, the result follows.

Similarly,

$$\begin{aligned}\lim_{z \rightarrow +\infty} \tilde{h}(z, t) &= \lim_{z \rightarrow +\infty} \log \left[e^{z + \frac{1}{2}t^2} \Phi \left(-\frac{z}{t} - t \right) + \Phi \left(\frac{z}{t} \right) \right] \\ &= \lim_{z \rightarrow +\infty} \left\{ z + \frac{1}{2}t^2 + \underbrace{\log \left[\Phi \left(-\frac{z}{t} - t \right) \right]}_{\rightarrow 0} + e^{-z - \frac{1}{2}t^2} \underbrace{\Phi \left(\frac{z}{t} \right)}_{\rightarrow 1} \right\} = 0,\end{aligned}$$

and where we have already established above that

$$\lim_{z \rightarrow +\infty} \frac{\partial \tilde{h}(z, t)}{\partial z} = 0$$

C Numerical solution method

As described in the main text, our numerical solution involves three approximations. First, we use the approximate Gaussian updating approach outlined in Section 4.2. This keeps information inference from the nonlinear working capital signal tractable. The other two approximations are used to solve for the balance sheet fixed point. First, we expand $\log X_{i,t}$ in the argument of $\mathcal{P}_{i,t}$ around $z_{i,t} = \bar{\mathbb{E}}_{i,t}\{z_{i,t}\}$, ensuring that (the approximated) $\log X_{i,t}$ is log-normally distributed. Given its log-normal distribution, Lemma 5 establishes a quasi-linear relation between $\bar{a}_{i,t}$, equilibrium beliefs and exogenous shocks that solves the balance-sheet fixed point. Together with Lemma 6, this then pins down equilibrium beliefs as a function of signal realizations, which in turn determines the equilibrium credit line $\bar{a}_{i,t}$ for a given island and a given history of exogenous shocks. Our third approximation allows us to explicitly solve the last two steps in a computational feasible way. Specifically, it expands f (defined in Lemma 5) in its two terms.

In particular, let

$$\bar{a}_0 \equiv \mu_0 + f(v_0, \hat{\sigma}_0^2)$$

denote the solution to (25) as characterized by Lemma 5 for the approximation point

$$(\bar{\mathbb{E}}\{z_{i,t}\}, \hat{\sigma}_{i,t}^2, v_t, \eta_{i,t}) = (\mu_0, \hat{\sigma}_0^2, v_0, 0),$$

where μ_0 , $\hat{\sigma}_0^2$, and v_0 are set to their unconditional means.³⁸ Then expanding f around $(u_1, u_2) = (v_0, \hat{\sigma}_0^2)$ yields

$$f(u_1, u_2) \simeq \pi_0 + \pi_v u_1 - \pi_\sigma u_2,$$

where

$$\begin{aligned}f_0 &= \underbrace{\bar{a}_0 - \mu_0}_{=f(v_0, \hat{\sigma}_0^2)} - \pi_v v_0 + \pi_\sigma \hat{\sigma}_0^2\end{aligned}$$

³⁸Note that the unconditional mean of $\eta_{i,t}$ equals zero.

and

$$\pi_v = \frac{\left(\frac{\partial h}{\partial \mu_0}\right)^{-1}}{1 - 2\gamma \tilde{h}'' \hat{\sigma}_0^2} > 0 \quad \pi_\sigma = \frac{\frac{\partial h}{\partial \mu_0} \gamma}{1 - 2\gamma \tilde{h}'' \hat{\sigma}_0^2} > 0.$$

For details, see the proof to Lemma 5.

Using these simplifications, the equilibrium path of the economy can be computed recursively, by setting I to a large number of islands (1×10^6 in our simulations). At date t , the state variables are $S_t = \{z_{i,t}, \eta_{i,t}, \psi_{i,t}, b_{i,t-1}^z, b_{i,t-1}^\eta, \varsigma_{i,t-1}, l_{i,t-1}, \bar{a}_{i,t-1}\}_{i \in I} \cup v_t$. For a given state, the recursion substeps can be summarized by

1. compute the distribution of fictitious lending signals $\{\omega_{i,t}^l, \kappa_{i,t}^l\}_{i \in I}$ via the approximate Gaussian updating approach outlined in Lemma 4
2. compute the equilibrium beliefs as given by Lemma 6
3. compute $\{\bar{a}_{i,t}\}_{i \in I}$ as given by Lemma 5
4. compute $\{\phi_{i,t}^1\}_{i \in I}$ and $\{\phi_{i,t}^2\}_{i \in I}$ and aggregate to get N_t and Y_t as given by Lemma 1
5. simulate $\{z_{i,t+1}, \eta_{i,t+1}, \psi_{i,t+1}\}_{i \in I}$ given their exogenous processes, and proceed to period $t + 1$.

In the simulation we initialize the economy using an arbitrary approximation to the invariant distribution over $\{z_{i,t}, \eta_{i,t}, \psi_{i,t}, b_{i,t-1}^z, b_{i,t-1}^\eta, \varsigma_{i,t-1}, l_{i,t-1}, \bar{a}_{i,t-1}\}_{i \in I}$ and then let the economy run for 100 periods to converge to its true invariant distribution, before shocking the economy.

D Additional simulations

This appendix contains additional simulation experiments discussed in the main text.

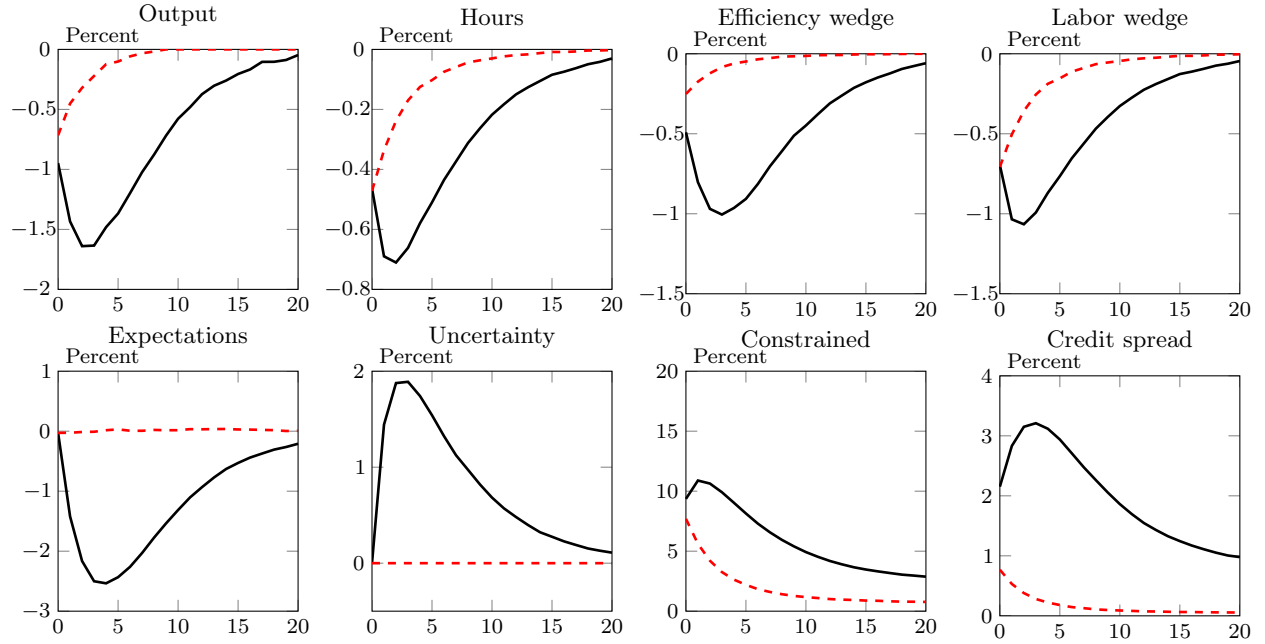


Figure 10: Comparative dynamics to aggregate financial shock (calibrated to 10 percent of firms being constrained upon impact). Solid black lines are impulse responses of the aggregated (or averaged) model economy; dashed red lines are counterfactual responses where the signal precision is exogenously fixed at its unconstrained level. The number of constrained firms and the credit spread are in percentage points, all other responses are in percentage deviations.

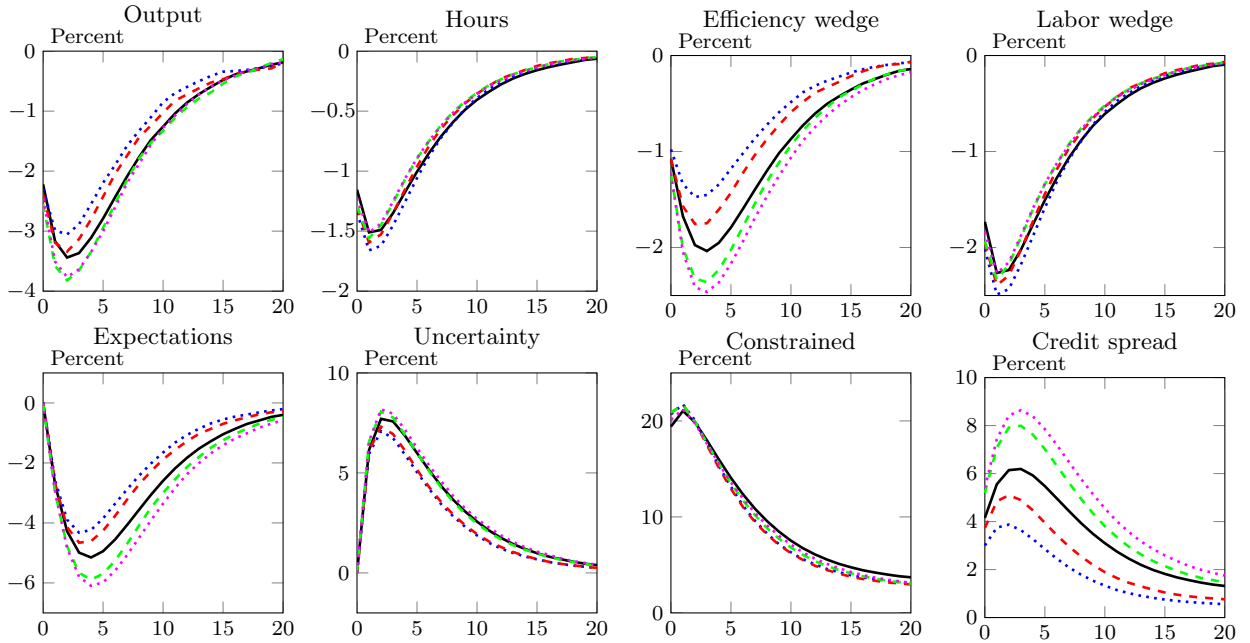


Figure 11: Impulse responses for output, expectations, and the cross-sectional dispersion under alternative parametrizations. The solid black lines correspond to the baseline parametrization. The dashed and dotted lines correspond to variations of the information parameters to $\sigma_p = \sigma_\eta = 0.5$, $\sigma_p = 0.5$ and $\sigma_\eta = 0.7$, $\sigma_p = 0.7$ and $\sigma_\eta = 0.9$ and $\sigma_p = \sigma_\eta = 0.9$. All responses are in percentage deviations.

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