Debt Covenants and Macroeconomic Dynamics*

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Abstract

Debt covenants are an important non-price mechanism through which credit is allocated to nonfinancial firms, and are strongly countercyclical. Macroeconomic models however abstract from this margin and focus on price measures such as credit spreads. We propose a simple extension of the canonical Bernanke-Gertler-Gilchrist (1999; BGG) model that gives a role for covenants and that allows to study both their determination, and their macroeconomic impact. In the model, covenants allocate control rights of investment across states of natures, and are determined by a trade-off between the risk of letting the entrepreneur invest excessively, and the cost of letting the lender reduce investment excessively. We demonstrate that covenants alter impulse response functions, relative to BGG, and that the pre-determined convenant tightness is an important state variable for the economy. Finally, we show that the demand for savings and expectations of future productivity are important determinants of covenant tightness.

Keywords: debt contract, financial accelerator, covenants, non-price terms, credit supply, cyclicality.

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1 Introduction

The majority of external funds raised by nonfinancial corporations takes the form of debt contracts, which are characterized by the interest rate charged, the maturity and collateral, but also importantly by various covenants. These covenants restrict the debtor's freedom to engage in certain actions that would reduce the likelihood that the creditor is paid back in full. For instance, the debtor may be restricted from issuing more debt of equal seniority unless the net worth of the firm increases significantly. Covenants also often take the form of a threshold for a financial ratio (such as debt to EBITDA), that, if exceeded, allow the creditor to ask for accelerated repayment. Since accelerated repayment is typically very costly, in practice this gives the creditor significant bargaining power to renegotiate the loan terms, and affect the lender's investment and hiring decisions. The empirical corporate finance literature documents that covenants are often triggered, and that violations of covenants have significant effects on investment and other real decisions (see for instance Chava and Roberts, 2008, Roberts and Sufi 2009, and Falato and Liang 2012). Overall, covenants appear an important mechanism through which agency costs of overindebtness are limited. Moreover, the tightness of covenants introduced in debt contracts appears to vary over the business cycle, and in particular so-called covenant-lite lending was prevalent in the mid 2000s and since 2012.

There is by-now a large literature in macroeconomics that studies the effect of financial frictions on corporate investment and macroeconomic dynamics more generally (Bernanke and Gertler (1989), Bernanke Gertler and Gilchrist (1999) and Kiyotaki and Moore (1997)). This literature however focuses on stylized debt contracts, in environments which give no role for covenants (e.g. because debt is short-term). As a result, the literature has focused on price measures such as credit-spreads, rather than the effect of non-price terms such as covenants.

Our goal in this paper is to propose a first approach to study the effect of covenant tighteness on macroeconomic dynamics. To illustrate in the clearest possible way how covenants alter the standard financial accelerator, we start from the canonical model of Bernanke, Gertler and Gilchrist (1999, BGG) and modify it in a limited way to give a role for covenants. Specifically, we assume that, after the debt contract has been signed, a signal about future productivity is revealed; debt contracts can be contingent on this signal. Two main dimensions can be modified: the interest rate charged on the loan, and the investment decision. We prove that the entrepreneur will always want to invest more than the lender. This reflects the convex payoff of entrepreneurs (concave for lenders), and this gives a role for debt covenants as allocation of control rights. When the signal is low, the lender is in control, and will restrict investment; and when the signal is high, the entrepreneur is in control, and will increase investment. The

optimal contract (within a limited class) trades-off the cost of excess investment in some states, with the cost of too low investment in other states.

This simple model has several interesting implications. First, we show how the financial accelerator is mitigated relative to BGG. This is because the presence of covenants allows to make lending less cyclical. There is less need to restrict credit ex-ante, because covenants allow to control the use of funds ex-post.

Second, the level of predetermined covenants affects the response of the economy to macroeconomic shocks. When many borrowers have tight covenants, a shock will lead lenders to be in control, which will lead to a sharp cutback of investment. Inversely, if covenants are loose, the economy may be less affected by a macroeconomic shock due to entrepreneurs being more often in control.

Third, we start exploring the macroeconomic trends that affect covenant tightness. Clearly, when the economy is expected to do well, it is optimal to use loose covenants since curtailing investment opportunities is not optimal. Hence, expected growth is an important determinant of covenant lending. Second, the overall demand for savings affects negatively the interest rate and credit spreads, but also affects non-price terms such as covenant tightness. Hence, both demand and supply factors can affect covenant tightness.

Overall, while our model remains stylized and abstracts from several important real-life considerations, we think the simple modification brings important new insights. The current version does not demonstrate yet the quantitative importance of these results, but this is work in progress.

The rest of this paper is organized as follows. We first document the cyclicality of the tightness of covenants (Section 2). We then set up our model (Section 3) and study it first in a steady-state to demontrate how covenants are determined. We finally demonstrate numerically the effect of introducing covenants on macroeconomic dynamics (Section 5). [More to be added.] Section 6 concludes.

2 Facts

The goal of this section (which is still work in progress) is to document some basic facts regarding the cyclicality of covenant tightness both for the United States and for the Euro area. The key conclusions are that covenants for new loans become notably tighter in recessions.

There are a number of empirical challenges when trying to measure covenant tightness.

Most importantly, there is no data covering the universe of debt contracts.[1] Moreover, because debt contracts are often tailor-made to the specific situation, it is difficult to summarize the contract in a single statistic. For these reasons, we use as a first illustration the surveys of banks conducted respectively by the Federal Reserve and the European Central Bank.[2]

We start with the United States. The Senior Loan Officer Survey (SLOOS) has included since 1990Q2 a question asking domestic banks if their covenants had become tighter over the past quarter, separately for large-middle-market borrowers and for small borrowers.[3] Figure 1 presents the net change in covenant tightness;[4] yellow bars indicate NBER recessions. Clearly, covenants are tightened during economic recessions, and then become looser during expansions. The effects are somewhat more pronounced for small firms.

Because the question reports *changes* in standards, Figure 2 presents an index of the overall level of covenant tightness, which is simply the cumulated sum of the net of changes. This index clearly shows that covenants are loose during booms and tighter during recessions. This conforms with the numerous reports of "covenant-lite" lending in 2004-2006 and in 2013-2014.[5]

Turning to the Euro Area, the ECB started in 2003 a survey very similar to the US SLOOS. Figure 3 shows the response to the question regarding covenant changes; yellow bars indicate business cycles peak and troughs as measured by the Euro Area business cycle dating committee.[6] While the sample is much shorter, it is clear that, in Europe as well as the U.S., covenant tightness was affected by the credit cycle, with looser terms in 2004-2006 and a strong tightening of covenants during the financial crisis and following the sovereign debt crisis of Summer 2011.[7] Figure 4 shows the breakdown between small and medium-sized firms, on one hand, and large firms, on the other, which is available since 2008 only. Here too, we see that covenants are tightened more for smaller firms.

We conclude from this section, consistent with earlier literature, that covenant tightness is strongly countercyclical and volatile.

[1] Information about loans is dispersed depending notably on the loan originator (a bank, a finance company, a syndicate of lenders, an insurer or other large stand-alone lender, etc.). For the subset of publicly listed corporations, some researchers have performed text searches in financial statements since these corporations are required by law to disclose material changes to their debt contracts.

[2] In future work, we may use measures of covenant tightness that are built up from the loan-level as z-scores, using measures of volatility of financial ratios.

[3] Small is defined as sales less than 5 million dollars.

[4] The response to the question ranges from 1 (markedly tighter) to 5 (markedly looser) and the data available are for the net change (the sum of responses to 1 and 2 minus the responses to 4 and 5).

[5] The trend in this index likely reflects measurement issues given our coarse measure, which we can improve somewhat.

[6] The committee has not yet called the trough of the recession that started in 2011Q3.

[7] These patterns are stronger in the periphery countries.

3 Model

The economy consists of households, entrepreneurs, firms and lenders. The model is real and consumption is the numeraire.

3.1 Households

There is a continuum of homogeneous households that maximize expected utility over consumption (c_t) and hours worked (h_t).

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) \tag{1}$$

Households build and own the physical stock of capital. Capital accumulates according to

$$K_t^s = (1-\delta)K_{t-1}^s + i_t^n, \tag{2}$$

with gross investment i_t^g, and net investment i_t^n,

$$i_t^n = \left(1 - \frac{\phi}{2}\left(\frac{i_t^g}{i_{t-1}^g} - 1\right)^2\right) i_t^g. \tag{3}$$

The parameter ϕ controls a quadratic investment adjustment cost such that the price of capital is not one. Every period households sell the capital K_t^s to the entrepreneurs at price Q_t. Simultaneously, and at the same price, households repurchase the undepreciated capital sold last period to the entrepreneurs, $(1-\delta)K_{t-1}^s$.

Households can save via bank deposits $\left(D_t^H > 0\right)$ that return the rate R_t^D. The budget constraint for households is:

$$c_t + i_t^g + D_t^H + Q_t(1-\delta)K_{t-1}^s = W_t h_t + R_{t-1}^D D_{t-1}^H + Q_t K_t^s. \tag{4}$$

Where W_t is the real wage. The representative household chooses c_t, h_t, K_t^s, i_t^g and D_t^H to maximize (1) subject to (2), (3) and (4).

3.2 Firms

Firms are price takers in the input markets. They rent labor from households, h_t, and entrepreneurs, h_t^E. They also rent capital K_{t-1}^s from entrepreneurs, paying a rental price r_t at the beginning of each period, to return it to entrepreneurs at the end of each period. The production function is:

$$y_t = e^{z_t} \left(K_{t-1}^s\right)^{\Lambda_K} H_t^{1-\Lambda_K}, \tag{5}$$

with total labor (H_t) being the composite

$$H_t = h_t^{1-\Lambda_H} \left(h_t^E\right)^{\Lambda_H}. \tag{6}$$

Λ_H is the share of the entrepreneurs' labor input. The aggregate productivity shock z_t follows the standard lognormal $AR(1)$ process:

$$z_t = \rho_z z_{t-1} + \xi_t, \tag{7}$$

where ξ_t are Normal i.i.d. shocks.

3.3 Entrepreneurs and Debt Covenants

There is a continuum of risk-neutral entrepreneurs who start the period with some inherited wealth, N_t, and can borrow L_t. They invest in two ways: a) they can purchase capital from the households at price Q_t. Then, next period, entrepreneurs rent the capital to the firms at rental rate r_{t+1}, and resell it at price Q_{t+1}. The expected return per unit of capital is:

$$R_{t+1}^K = \frac{r_{t+1} + Q_{t+1}(1-\delta)}{Q_t}. \tag{8}$$

b) Entrepreneurs can save via bank deposits, $D_t^E \geq 0$, that return R_t^D. Thus, the assets, A_t, of each entrepreneur are

$$A_t = D_t^E + Q_t K_t, \tag{9}$$

where $Q_t K_t$ is the value of the capital holdings, and D_t^E the bank deposits. Entrepreneur's liabilities are

$$A_t = N_t + L_t. \tag{10}$$

We denote by α_t the optimal portfolio mix allocated to capital:

$$Q_t K_t = \alpha_t A_t, \tag{11}$$

$$D_t^E = (1 - \alpha_t) A_t. \tag{12}$$

The timing of the model is as follows:

Stage 1) At the start of the period the entrepreneurs meet with the lenders and negotiate a financial contract characterized by three objects: a) State contingent interest rates, $R_{i,t+1}^L$, to insure lenders' break even in any state of the world. b) Loan amount L_t. Or, equivalenty, a leverage ratio $\kappa_t = \frac{A_t}{N_t}$. c) Covenant threshold $\bar{\omega}_t$, such that all entrepreneurs with idiosyncratic productivity shock $\omega < \bar{\omega}_t$ transfer the control rights about their investment decision α_t to the lenders. As it will be explained below, ω affects the return of entrepreneur's capital holdings. Thus, a low ω means low value of the assets and we can interpret $\bar{\omega}_t$ as a covenant on leverage. Perfect competition among lenders determines the three objects of the contract to maximize the entrepreneur's value subject to lenders' zero profit condition.

Stage 2) Once the contract has been signed, each entrepreneur receives a partially observable idiosyncratic shock ω that affects its ability to converts K_t units of capital into ωK_t efficiency units. Thus, an entrepreneur who draws a shock ω has an effective rate of return per unit of capital equal to ωR_{t+1}^K. These shocks are a stand-in for more complicated processes such as changes in demand or the stochastic quality of projects. These shocks are i.i.d. across time and across entrepreneurs, they come from a log-normal distribution $F(\omega)$ where μ_t and σ_t are, respectively, the mean and standard deviation of the natural logarithm of ω, and $\mathbb{E}[\omega] = 1$. There are no idiosyncratic shocks affecting the return from bank deposits. The timing assumption insures that all entrepreneurs borrow the same amount and with the same covenant level since the ω shocks are not forecastable in Stage 1. The ω shocks are partially observable in Stage 2. The agents receive a signal $I(\omega, \bar{\omega}_t)$,

$$I(\omega, \bar{\omega}_t) = \left\{ \begin{array}{l} 1 \text{ if } \omega \geq \bar{\omega}_t \\ \\ 0 \text{ if } \omega < \bar{\omega}_t \end{array} \right\}, \tag{13}$$

such that the signal determines if the debt covenant is violated or not. In other words, in Stage 2 the agents know if the entrepreneur is above or below the covenant threshold.

If $I(\omega, \bar{\omega}_t) = 1$ then the entrepreneur can decide her portfolio mix, that we denote as α_t^E since all entrepreneurs with control rights receive the same signal and thus make the same

choice. However, if $I(\omega, \bar{\omega}_t) = 0$ then it is the lender who decides the portfolio mix, we denote her choice by α_t^L. Therefore,

$$\alpha_t(\omega, \bar{\omega}_t) = \left\{ \begin{array}{l} \alpha_t^E \text{ if } I(\omega, \bar{\omega}_t) = 1 \\ \\ \alpha_t^L \text{ if } I(\omega, \bar{\omega}_t) = 0 \end{array} \right\}. \tag{14}$$

Stage 3) At the start of next period the aggregate shock z_{t+1} is realized and the return R_{t+1}^K is determined. The earnings of an entrepreneur are

$$\max\left\{0, \left(\omega \alpha_t^i R_{t+1}^K + \left(1 - \alpha_t^i\right) R_t^D\right) A_t - R_{i,t+1}^L L_t\right\}, \ i = E, L. \tag{15}$$

The max operator captures limited liability, that is, entrepreneur's losses are bounded below to never exceed her wealth. Thus, those entrepreneurs unable to pay $R_{i,t+1}^L L_t$ default. There are two default thresholds at $t+1$ because the portfolio mix, α_t^i, is different depending on whether the lender or the entrepreneur was in control at t, that is,

$$\hat{\omega}_{t+1}^i(\omega, \bar{\omega}_t) = \left\{ \begin{array}{l} \hat{\omega}_{t+1}^E \text{ if } I(\omega, \bar{\omega}_t) = 1 \\ \\ \hat{\omega}_{t+1}^L \text{ if } I(\omega, \bar{\omega}_t) = 0 \end{array} \right\}, \tag{16}$$

To appropriate entrepreneur's assets, lenders suffer a bankruptcy cost ($0 < \mu < 1$) proportional to the assets.

The state contingent lending rates are:

$$R_{i,t+1}^L = \left\{ \begin{array}{c} R_{E,t+1}^L L_t = \left[\hat{\omega}_{t+1}^E \alpha_t^E R_{t+1}^K + \left(1 - \alpha_t^E\right) R_t^D\right] A_t \text{ if } i = E \\ \\ R_{L,t+1}^L L_t \text{ if } i = L \end{array} \right\} \tag{17}$$

Like in BGG, rates $R_{i,t+1}^L$ are a function of the aggregate shock, R_{t+1}^K changes with z_t, to ensure lender's zero profit condition in every state of nature. Specification (17) allows us to show that our model maps into the BGG when there are no covenants.

The aggregate demand for capital from the entrepreneurs is the sum of the capital demand by the entrepreneurs under lender's control plus the demand from those entrepreneurs who control their own decisions. By market clearing the aggregate holdings of capital by the entrepreneurs

must equal the value of the total amount of capital in the economy, K_t^s.

$$Q_t K_t^s = \left[F\left(\bar{\omega}_t\right)\alpha_t^L + \left[1 - F\left(\bar{\omega}_t\right)\right]\alpha_t^E\right] A_t, \tag{18}$$

where $F\left(\bar{\omega}_t\right)$ is the share of entrepreneurs for which the covenant at t is binding.

3.4 The Financial Contract

The Financial Contract is decided in Stage 1, although it depends on the expected decisions in Stage 2. In Stage 1, the value function of an entrepreneur with net wealth N_t and financial contract $\left(R_{i,t+1}^L, L_t, \bar{\omega}_t\right)$ is

$$V\left(N_t, R_{i,t+1}^L, L_t, \bar{\omega}_t\right) = F\left(\bar{\omega}_t\right) V^L\left(N_t, R_{L,t+1}^L, L_t\right) + \left[1 - F\left(\bar{\omega}_t\right)\right] V^E\left(N_t, R_{E,t+1}^L, L_t\right), \tag{19}$$

where $F\left(\bar{\omega}_t\right)$ is the probability with which the entrepreneur will be controlled by the lender. $V^L\left(N_t, L_t\right)$ is the value function in Stage 2 for entrepreneurs under lender's control. In that case the lender makes the portfolio mix decision, α_t^L, to maximize lender's expected revenue:

$$\begin{aligned} &\max_{\alpha_t^L} \mathbb{E}_t \left[\begin{array}{c} \int_{\hat{\omega}_{t+1}^L}^{\bar{\omega}_t} R_{L,t+1}^L L_t dF\left(\omega|\omega < \bar{\omega}_t\right) + \\ + \int_0^{\hat{\omega}_{t+1}^L} (1-\mu)\left[\alpha_t^L \omega R_{t+1}^K + \left(1-\alpha_t^L\right) R_t^D\right] A_t dF\left(\omega|\omega < \bar{\omega}_t\right) \end{array} \right] \\ &s.t.\ (10)\ \text{and}\ (17). \end{aligned} \tag{20}$$

$F\left(\omega|\omega < \bar{\omega}_t\right)$ is the conditional expectation based on the information contained in the signal. Those entrepreneurs with idiosyncratic shocks between $\hat{\omega}_{t+1}^L$ and $\bar{\omega}_t$ are under lender's control and can repay. Those below $\hat{\omega}_{t+1}^L$ default and lose their assets. Then, the lender receives the value of the assets, net of bankruptcy costs. When the lender is in control the entrepreneur receives:

$$V^L\left(N_t, R_{L,t+1}^L, L_t\right) = \mathbb{E}_t \left[\int_{\hat{\omega}_{t+1}^L}^{\bar{\omega}_t} \left[\left[\alpha_t^L \omega R_{t+1}^K + \left(1-\alpha_t^L\right) R_t^D\right] A_t - R_{L,t+1}^L L_t\right] dF\left(\omega|\omega < \bar{\omega}_t\right) \right]. \tag{21}$$

$V^E\left(N_t, R_{E,t+1}^L, L_t\right)$ is the value function for entrepreneurs when the entrepreneur is in control. In this case she decides the portfolio mix:

$$V^E\left(N_t, R_{E,t+1}^L, L_t\right) = \max_{\alpha_t^E} \mathbb{E}_t \left[\int_{\hat{\omega}_{t+1}^E}^{\infty} \left[\left[\alpha_t^E \omega R_{t+1}^K + \left(1-\alpha_t^E\right) R_t^D\right] A_t - R_{E,t+1}^L L_t\right] dF\left(\omega|\omega \geq \bar{\omega}_t\right) \right] \tag{22}$$

$s.t.$ (10) and (17) .

For lenders, the revenue from lending L_t to entrepreneurs with net worth N_t, under covenant $\bar{\omega}_t$, with rates $R^L_{i,t+1}$, is:

$$W\left(N_t, R^L_{i,t+1}, L_t, \bar{\omega}_t\right) = F\left(\bar{\omega}_t\right) W^L\left(N_t, R^L_{L,t+1}, L_t\right) + \left[1 - F\left(\bar{\omega}_t\right)\right] W^E\left(N_t, R^L_{E,t+1}, L_t\right), \quad (23)$$

where W^L is the revenue from the loan when the lender is in control. That is,

$$W^L\left(N_t, R^L_{L,t+1}, L_t\right) = \begin{array}{c} \int_{\hat{\omega}^L_{t+1}}^{\bar{\omega}_t} R^L_{L,t+1} L_t dF\left(\omega|\omega < \bar{\omega}_t\right) + \\ + \int_0^{\hat{\omega}^L_{t+1}} (1-\mu)\left[\alpha^L_t \omega R^K_{t+1} + \left(1-\alpha^L_t\right) R^D_t\right] A_t dF\left(\omega|\omega < \bar{\omega}_t\right). \end{array} \quad (24)$$

Similarly, W^E is the revenue from the loan when the entrepreneur is in control:

$$W^E\left(N_t, R^L_{E,t+1}, L_t\right) = \begin{array}{c} \int_{\hat{\omega}^E_{t+1}}^{\infty} R^L_{E,t+1} L_t dF\left(\omega|\omega \geq \bar{\omega}_t\right) + \\ + \int_{\bar{\omega}_t}^{\hat{\omega}^E_{t+1}} (1-\mu)\left[\alpha^E_t \omega R^K_{t+1} + \left(1-\alpha^E_t\right) R^D_t\right] A_t dF\left(\omega|\omega \geq \bar{\omega}_t\right). \end{array} \quad (25)$$

Perfect competition among lenders implies that the covenant $\bar{\omega}_t$, interest rates $R^L_{i,t+1}$, and new loan amounts, L_t, maximize entrepreneurs' utility

$$\max_{L_t, R^L_{i,t+1}, \bar{\omega}_t} \mathbb{E}_t\left[V\left(N_t, R^L_{i,t+1}, L_t, \bar{\omega}_t\right)\right] \quad (26)$$

$s.t.$

$$W\left(N_t, R^L_{i,t+1}, L_t, \bar{\omega}_t\right) = R^D_t L_t. \quad (27)$$

That is, the lender makes zero profits for any realization of R^K_{t+1}. That implies to receive enough repayments to cover lender's cost of funds.

3.5 Dynamics of Net Worth and Market Clearing

Entrepreneurs inelastically supply one unit of labor $\left(h^E_t = 1\right)$ to final goods producers and receive from them labor income $W_t h^E_t$. This labor income, along with the earnings from buying, renting and reselling capital, determines entrepreneurs' net worth N_{t+1}. To ensure that this wealth does not become so large that entrepreneurs do not need to borrow, there is a deterministic probability $(1 - \gamma_E)$ that an entrepreneur dies at the end of period t. The entrepreneurs that die consume a fraction γ_E of their end of period earnings. Those entrepreneurs that remain in the market accumulate a fraction γ_E of these earnings.

The net worth of the entrepreneurs at the start of period N_{t+1} is:

$$N_{t+1} = \gamma_E \left[\begin{array}{c} \int_{\hat{\omega}^L_{t+1}}^{\bar{\omega}_t} \left[\left[\alpha^L_t \omega R^K_{t+1} + \left(1-\alpha^L_t\right) R^D_t \right] A_t - R^L_{L,t+1} L_t \right] dF(\omega) + \\ + \int_{\max\left\{\bar{\omega}_t, \hat{\omega}^E_{t+1}\right\}}^{\infty} \left[\left[\alpha^E_t \omega R^K_{t+1} + \left(1-\alpha^E_t\right) R^D_t \right] A_t - R^L_{E,t+1} L_t \right] dF(\omega) \end{array} \right] + W^E_t h^E_t. \tag{28}$$

We do not use conditional densities because we aggregate over types. The $\max\left\{\bar{\omega}_t, \hat{\omega}^E_{t+1}\right\}$ allows for the possibility that no entrepreneur in control defaults. For entrepreneurs controlled by lenders we do not need the $\max\left\{0, \hat{\omega}^L_{t+1}\right\}$ operator because we know that $F(\omega) = 0$ if $\hat{\omega}^L_{t+1} \leq 0$.

The consumption of the entrepreneurs that exit from the market, c^E_t is:

$$c^E_{t+1} = (1-\gamma_E) \left[\begin{array}{c} \int_{\hat{\omega}^L_{t+1}}^{\bar{\omega}_t} \left[\left[\alpha^L_t \omega R^K_{t+1} + \left(1-\alpha^L_t\right) R^D_t \right] A_t - R^L_{L,t+1} L_t \right] dF(\omega) + \\ + \int_{\max\left\{\bar{\omega}_t, \hat{\omega}^E_{t+1}\right\}}^{\infty} \left[\left[\alpha^E_t \omega R^K_{t+1} + \left(1-\alpha^E_t\right) R^D_t \right] A_t - R^L_{E,t+1} L_t \right] dF(\omega) \end{array} \right] \tag{29}$$

The labor income is not in the consumption function.

Market clearing in the market for capital is (18) . Market clearing in credit markets implies that loans equal deposits,

$$L_t = D^H_t + \left[F\left(\bar{\omega}_t\right) \left(1-\alpha^L_t\right) + \left[1 - F\left(\bar{\omega}_t\right)\right] \left(1-\alpha^E_t\right) \right] A_t. \tag{30}$$

Finally, the market clearing in goods markets is that total consumption ($\tilde{c}_t = c_t + c^E_t$), gross investment and default cost are financed with the total production of the economy.

$$y_t = c_t + c^E_t + i^g_t + \mu \left(F\left(\hat{\omega}^L_t\right) + \left[\max\left\{0, F\left(\hat{\omega}^E_t\right) - F\left(\bar{\omega}_{t-1}\right)\right\}\right] \right). \tag{31}$$

The $\max\left\{0, F\left(\hat{\omega}^E_t\right) - F\left(\bar{\omega}_{t-1}\right)\right\}$ captures the area of default by entrepreneurs in control.

4 What Determines the Covenant Tightness

In this Section first we show analytically that $\alpha^E_t \geq \alpha^L_t$. That is, control rights affect the level of investment in capital because the entrepreneur wants to invest more than the lender. Then we show what determines the level of covenant tightness.

4.1 Investment and Control Rights

First, let's characterize the choice of α_t^E when the entrepreneurs are in control. Substituting (17) into (22)

$$V^E\left(N_t, R_{E,t+1}^L, L_t\right) = \max_{0\leq\alpha_t^E\leq 1} \mathbb{E}_t\left[\int_{\hat{\omega}_{t+1}^E}^{\infty}\left(\omega - \hat{\omega}_{t+1}^E\right)\alpha_t^E R_{t+1}^K A_t dF\left(\omega|\omega\geq\bar{\omega}_t\right)\right] \tag{32}$$

Since R_{t+1}^K and A_t are both strictly positive and since the integration is over all ω types such that $\omega \geq \hat{\omega}_{t+1}^E$, the expectation is strictly positive. Hence, it is optimal for the entrepreneur in control to always choose

$$\alpha_t^E = 1. \tag{33}$$

This is the risk shifting problem in debt contracts, the entrepreneur protected with limited liability invests in a riskier way that if financed with equity since the rewards from risk are for her, while the losses are bounded. The payoff function for a lender is different, it is concave and not convex on the risky investment. Substituting (17) into (24) we obtain

$$\max_{0\leq\alpha_t^L\leq 1} \mathbb{E}_t\left[\alpha_t^L R_{t+1}^K A_t\left[\begin{array}{c}\hat{\omega}_{t+1}^L\left[1-F\left(\hat{\omega}_{t+1}^L|\omega<\bar{\omega}_t\right)\right]+\\ +\int_0^{\hat{\omega}_{t+1}^L}\left(1-\mu\right)\omega dF\left(\omega|\omega<\bar{\omega}_t\right)\end{array}\right]+\left(1-\alpha_t^L\right)L_t R_t^D\left[1-\mu F\left(\hat{\omega}_{t+1}^L|\omega<\bar{\omega}_t\right)\right]\right] \tag{34}$$

The term that multiplies α_t^L is the expected return that the lender will receive if she invests in capital purchases, it is the sum of the earnings if the entrepreneur repays and those if she defaults. The term multiplying $(1-\alpha_t^L)$ is expected return from investing in riskless bank deposits. The optimal α_t^L will thus depend on how the two returns compare to each other.

(33) and (34) imply that $\alpha_t^E \geq \alpha_t^L$. That is, investment in capital will be different depending on the covenant level since lenders and borrowers invest differently. We prove that the entrepreneur will always want to invest more than the lender. This reflects the convex payoff of entrepreneurs (concave for lenders), and this gives a role for debt covenants as allocation of control rights. The result is consistent with the empirical corporate finance literature that documents that violations of covenants has significant effects on investment (Chava and Roberts, 2008, Roberts and Sufi 2009).

The interesting case is $\alpha_t^E > \alpha_t^L$. Otherwise both borrower and lender pick the same investment level. We focus on $\alpha_t^L = 0$ such that no entrepreneur controlled by a lender defaults.

4.2 Lending Rates, Leverage and Covenant Tightness

Using $\alpha_t^E = 1$ and $\alpha_t^L = 0$ we can write entrepreneur's expected revenue in Stage 1 as

$$V\left(N_t, \hat{\omega}_{t+1}^E, L_t, \bar{\omega}_t\right) = N_t \mathbb{E}_t \left[\int_{\hat{\omega}_{t+1}^E}^{\infty} \left(\omega - \hat{\omega}_{t+1}^E\right) R_{t+1}^K \kappa_t dF\left(\omega\right)\right] + F(\bar{\omega}_t) R_t^D N_t. \tag{35}$$

That is, entrepreneur's expected revenue is the weighted sum of the gains of investing all assets in capital and not defaulting, and of earning the risk free rate in her equity. The weight is the probability of the covenant being binding or not. Following the notation in BGG, we can write (35) as

$$N_t \left[\kappa_t \mathbb{E}_t \left[R_{t+1}^K (1 - \Gamma(\hat{\omega}_{t+1}^E)\right] + F(\bar{\omega}_t) R_t^D\right]. \tag{36}$$

Where

$$\Gamma(\hat{\omega}_{t+1}^E) \equiv \hat{\omega}_{t+1}^E [1 - F(\hat{\omega}_{t+1}^E)] + \int_0^{\hat{\omega}_{t+1}^E} \omega dF\left(\omega\right). \tag{37}$$

The lender's zero profit condition is

$$\mathbb{E}_t \left[\begin{array}{c} \left[1 - F(\hat{\omega}_{t+1}^E)\right] \left[R_{t+1}^K \hat{\omega}_{t+1}^E A_t\right] + \int_{\bar{\omega}_t}^{\hat{\omega}_{t+1}^E} \omega R_{t+1}^K \left(1 - \mu\right) A_t dF\left(\omega\right) + \\ + F(\bar{\omega}_t) L_t R_t^D \end{array}\right] = R_t^D L_t. \tag{38}$$

In other words, in every state of nature the lenders face 3 potential types of borrowers: 1) Borrowers above $\hat{\omega}_{t+1}^E$, with them the lender gains $R_{t+1}^K \hat{\omega}_{t+1}^E \left(N_t + L_t\right) - R_t^D L_t$. 2) Borrowers in the area $\left[\bar{\omega}_t, \hat{\omega}_{t+1}^E\right]$, the lenders loses $\omega R_{t+1}^K (1 - \mu) \left(N_t + L_t\right) - R_t^D L_t$ on each of them. 3) For borrowers in area $\left[0, \bar{\omega}_t\right]$, the lender covers her cost of funds. Using the BGG expressions the lender's zero profit condition is

$$\kappa_t R_{t+1}^K [\Gamma(\hat{\omega}_{t+1}^E) - \mu G(\hat{\omega}_{t+1}^E) - (1 - \mu) H(\bar{\omega}_t)] + (\kappa_t - 1) R_t^D F(\bar{\omega}_t) = R_t^D (\kappa_t - 1). \tag{39}$$

where

$$\mu G(\hat{\omega}_{t+1}^E) \equiv \mu \int_0^{\hat{\omega}_{t+1}^E} \omega dF\left(\omega\right), \tag{40}$$

$$H(\bar{\omega}_t) \equiv \int_0^{\bar{\omega}_t} \omega dF\left(\omega\right). \tag{41}$$

The optimal contract maximizes (36) relative to the covenant level $(\bar{\omega}_t)$, default threshold

$\left(\hat{\omega}_{t+1}^{E}\right)$, and leverage ratio (κ_t) subject to (39). From the FOCs we obtain:

$$\bar{\omega}_t = \frac{R_t^D + \mathbb{E}_t\left[\lambda_{t+1}(\kappa_t - 1)R_t^D\right]}{\mathbb{E}_t\left[\lambda_{t+1}(1-\mu)\kappa_t R_{t+1}^K\right]} = \frac{R_t^D(1 + \mathbb{E}_t\left[\lambda_{t+1}\right](\kappa_t - 1))}{\mathbb{E}_t\left[\lambda_{t+1}(1-\mu)\kappa_t R_{t+1}^K\right]}, \tag{42}$$

$$\kappa_t = \frac{R_t^D(1 - F(\bar{\omega}_t))\mathbb{E}_t\lambda_{t+1}}{\mathbb{E}_t\left[R_{t+1}^K(1 - \Gamma(\hat{\omega}_{t+1}^E))\right]}, \tag{43}$$

$$\lambda_{t+1} = \frac{\Gamma'(\hat{\omega}_{t+1}^E)}{\Gamma'(\hat{\omega}_{t+1}^E) - \mu G'(\hat{\omega}_{t+1}^E)} \tag{44}$$

Where λ_{t+1} is the Lagrange multiplier in (39). This system of three equations together with (39) characterize the financial contract $\left(\hat{\omega}_{t+1}^E, \kappa_t, \bar{\omega}_t, \lambda_{t+1}\right)$ in partial equilibrium.

Figure 5 illustrates the tradeoffs for a lender. It plots how the lending rates change as a function of the covenant threshold. That is, it maps (39) for two levels of leverage. Higher covenants minimize default risk for the lender, and thus lending rates are lower. Higher leverage leads to higher defaults, and thus higher lending rates.

Figure 6 shows the effect of a change in the cost of funds, for example due to monetary policy or capital flows. Higher cost of funds leads to tighter covenants and lower leverage. Lower leverage leads to lower default thresholds. The same dynamics are obtained with a decrease in the expected return from capital. Thus, debt covenants are countercyclical.

5 Covenant Tightness and Macroeconomic Dynamics

Our model collapses into the standard BGG framework when there are no covenants $(\bar{\omega}_t = 0)$. This can be shown for the case with no aggregate uncertanty like in Section 3 of the BGG handbook chapter. The external finance premium is $s_t = \frac{R_{t+1}^K}{R_t^D}$. And from the FOC for κ_t we obtain

$$s_t = \frac{\lambda_{t+1}(1 - F(\bar{\omega}_t))}{(1 - \Gamma(\hat{\omega}_{t+1}^E)) + \lambda_{t+1}[\Gamma(\hat{\omega}_{t+1}^E) - \mu G(\hat{\omega}_{t+1}^E) - (1-\mu)H(\bar{\omega}_t)]}. \tag{45}$$

For $\bar{\omega}_t = 0$ we have exactly the same premium as the BGG (equation A.1 in the handbook chapter). Similarly, we can write the leverage ratio as

$$\kappa_t = 1 + \frac{\lambda_{t+1}[\Gamma(\hat{\omega}_{t+1}^E) - \mu G(\hat{\omega}_{t+1}^E)]}{(1 - \Gamma(\hat{\omega}_{t+1}^E))} - \frac{\lambda_{t+1}(1-\mu)H(\bar{\omega}_t)}{(1 - \Gamma(\hat{\omega}_{t+1}^E))}.$$

Which is the BGG leverage ratio except for $\frac{\lambda_{t+1}(1-\mu)H(\bar{\omega}_t)}{(1-\Gamma(\hat{\omega}^E_{t+1}))}$, (equation A.2 in the BGG handbook chapter).

Figure 7 illustrates how covenants alter the reaction of the economy to shocks relative to the BGG. When covenants are tight then more entrepreneurs are under control of the lenders, they invest less in capital, hence for any bad shock losses are smaller.

6 Conclusions

TBA

Summary of Model Equations

27 Unknowns: $D^H, h, i^g, i^n, R^D, K^s, y, H, r, Q_k, h^E, W^E, W, R^K, D^E, A, \alpha^E, \alpha^L, \hat{\omega}^L, \hat{\omega}^E, \bar{\omega}, \kappa, L, \lambda, c^E, N, c$

27 Equations:

$$u_c(c_t, h_t) = \beta \mathbb{E}_t \left[u_c(c_{t+1}, h_{t+1}) R_t^D \right] \tag{1}$$

$$W_t = -\frac{u_h(c_t, h_t)}{u_c(c_t, h_t)} \tag{2}$$

$$1 + Q_{k,t} \left(\begin{array}{c} \phi \left(\frac{i_t^g}{i_{t-1}^g} - 1 \right) \frac{i_t^g}{i_{t-1}^g} + \\ - \left(1 - \frac{\phi}{2} \left(\frac{i_t^g}{i_{t-1}^g} - 1 \right)^2 \right) \end{array} \right) = \beta \mathbb{E}_t \left[\frac{u_c(c_{t+1}, h_{t+1})}{u_c(c_t, h_t)} Q_{k,t+1} \phi \left(\frac{i_{t+1}^g}{i_t^g} - 1 \right) \left(\frac{i_{t+1}^g}{i_t^g} \right)^2 \right] \tag{3}$$

$$K_t^s = (1 - \delta) K_{t-1}^s + i_t^n \tag{4}$$

$$i_t^n = \left(1 - \frac{\phi}{2} \left(\frac{i_t^g}{i_{t-1}^g} - 1 \right)^2 \right) i_t^g \tag{5}$$

$$y_t = e^{z_t} \left(K_{t-1}^s \right)^{\Lambda_K} H_t^{1-\Lambda_K} \tag{6}$$

$$H_t = h_t^{1-\Lambda_H} \left(h_t^E \right)^{\Lambda_H} \tag{7}$$

$$r_t = z_t \Lambda_K \left(\frac{h_t^{1-\Lambda_H} \left(h_t^E \right)^{\Lambda_H}}{K_{t-1}^s} \right)^{1-\Lambda_K} \tag{8}$$

$$h_t^E = 1 \tag{9}$$

$$W_t = z_t \left(K_{t-1}^s \right)^{\Lambda_K} (1 - \Lambda_H) (1 - \Lambda_K) \left(\frac{h_t^E}{h_t} \right)^{\Lambda_H (1-\Lambda_K)} h_t^{-\Lambda_K} \tag{10}$$

$$W_t^E = z_t \left(K_{t-1}^s \right)^{\Lambda_K} \Lambda_H (1 - \Lambda_K) \left(\frac{h_t}{h_t^E} \right)^{(1-\Lambda_H)(1-\Lambda_K)} \left(h_t^E \right)^{-\Lambda_K} \tag{11}$$

$$R_{t+1}^K = \frac{r_{t+1} + Q_{t+1}(1 - \delta)}{Q_t} \tag{12}$$

$$A_t = D_t^E + Q_{k,t} K_t \tag{13}$$

$$A_t = N_t + L_t \tag{14}$$

$$Q_t K_t^s = \left[F\left(\bar{\omega}_t\right)\alpha_t^L + \left[1 - F\left(\bar{\omega}_t\right)\right]\alpha_t^E\right] A_t \tag{15}$$

$$\kappa_t = \frac{A_t}{N_t} \tag{16}$$

$$\alpha_t^E = 1 \tag{17}$$

$$\alpha_t^L = 0 \tag{18}$$

$$\hat{\omega}_{t+1}^L = 0 \tag{19}$$

$$\bar{\omega}_t = \frac{R_t^D + \mathbb{E}_t\left[\lambda_{t+1}(\kappa_t - 1)R_t^D\right]}{\mathbb{E}_t\left[\lambda_{t+1}(1-\mu)\kappa_t R_{t+1}^K\right]} = \frac{R_t^D(1 + \mathbb{E}_t\left[\lambda_{t+1}\right](\kappa_t - 1))}{\mathbb{E}_t\left[\lambda_{t+1}(1-\mu)\kappa_t R_{t+1}^K\right]} \tag{20}$$

$$\kappa_t = \frac{R_t^D(1 - F(\bar{\omega}_t))\mathbb{E}_t\lambda_{t+1}}{\mathbb{E}_t\left[R_{t+1}^K(1 - \Gamma(\hat{\omega}_{t+1}^E))\right]} \tag{21}$$

$$\lambda_{t+1} = \frac{\Gamma'(\hat{\omega}_{t+1}^E)}{\Gamma'(\hat{\omega}_{t+1}^E) - \mu G'(\hat{\omega}_{t+1}^E)} \tag{22}$$

$$\kappa_t R_{t+1}^K[\Gamma(\hat{\omega}_{t+1}^E) - \mu G(\hat{\omega}_{t+1}^E) - (1-\mu)H(\bar{\omega}_t)] + (\kappa_t - 1)R_t^D F(\bar{\omega}_t) = R_t^D(\kappa_t - 1) \tag{23}$$

$$N_{t+1} = \gamma_E \left[\begin{array}{c} \int_{\hat{\omega}_{t+1}^L}^{\bar{\omega}_t} \left[\left[\alpha_t^L \omega R_{t+1}^K + \left(1 - \alpha_t^L\right) R_t^D\right] A_t - R_{L,t+1}^L L_t\right] dF(\omega) + \\ + \int_{\max\left\{\bar{\omega}_t, \hat{\omega}_{t+1}^E\right\}}^{\infty} \left[\left[\alpha_t^E \omega R_{t+1}^K + \left(1 - \alpha_t^E\right) R_t^D\right] A_t - R_{E,t+1}^L L_t\right] dF(\omega) \end{array} \right] + W_t^E h_t^E \tag{24}$$

$$c_{t+1}^E = (1 - \gamma_E) \left[\begin{array}{c} \int_{\hat{\omega}_{t+1}^L}^{\bar{\omega}_t} \left[\left[\alpha_t^L \omega R_{t+1}^K + \left(1 - \alpha_t^L\right) R_t^D\right] A_t - R_{L,t+1}^L L_t\right] dF(\omega) + \\ + \int_{\max\left\{\bar{\omega}_t, \hat{\omega}_{t+1}^E\right\}}^{\infty} \left[\left[\alpha_t^E \omega R_{t+1}^K + \left(1 - \alpha_t^E\right) R_t^D\right] A_t - R_{E,t+1}^L L_t\right] dF(\omega) \end{array} \right] \tag{25}$$

$$L_t = D_t^H + \left[F\left(\bar{\omega}_t\right)\left(1 - \alpha_t^L\right) + \left[1 - F\left(\bar{\omega}_t\right)\right]\left(1 - \alpha_t^E\right)\right] A_t \tag{26}$$

$$y_t = c_t + c_t^E + i_t^g + \mu\left(F\left(\hat{\omega}_t^L\right) + \left[\max\left\{0, F\left(\hat{\omega}_t^E\right) - F\left(\bar{\omega}_{t-1}\right)\right\}\right]\right) \tag{27}$$

Figures

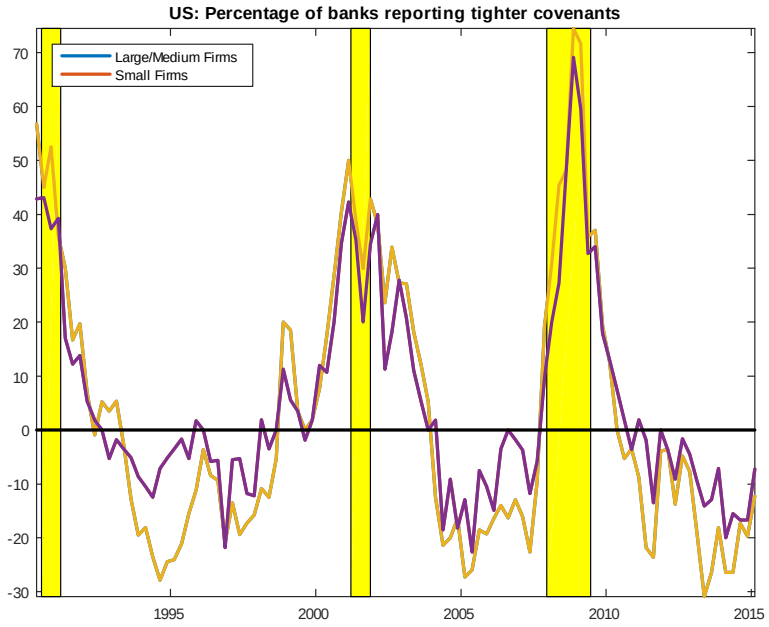

Figure 1.

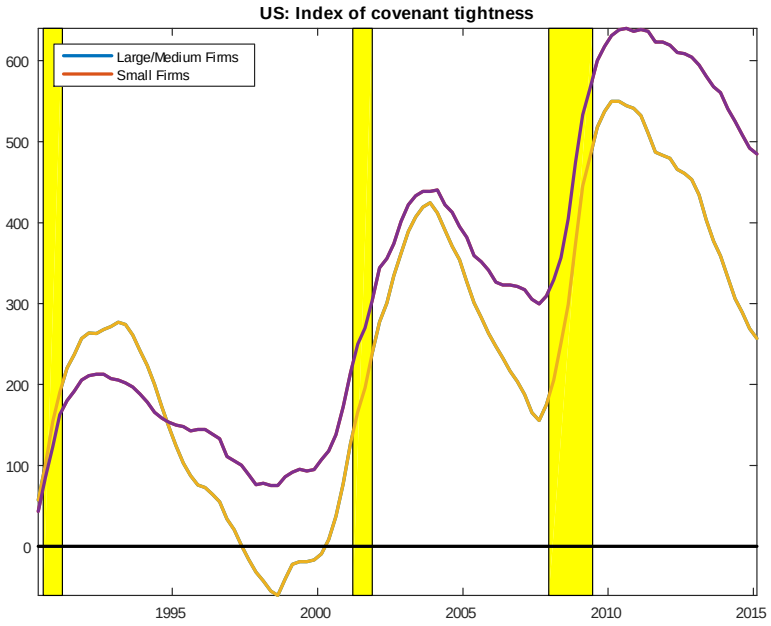

Figure 2.

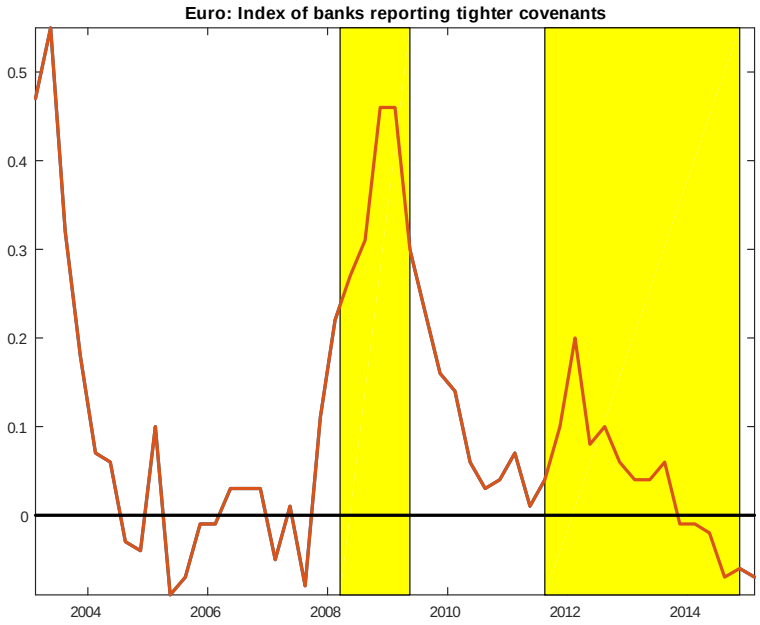

Figure 3.

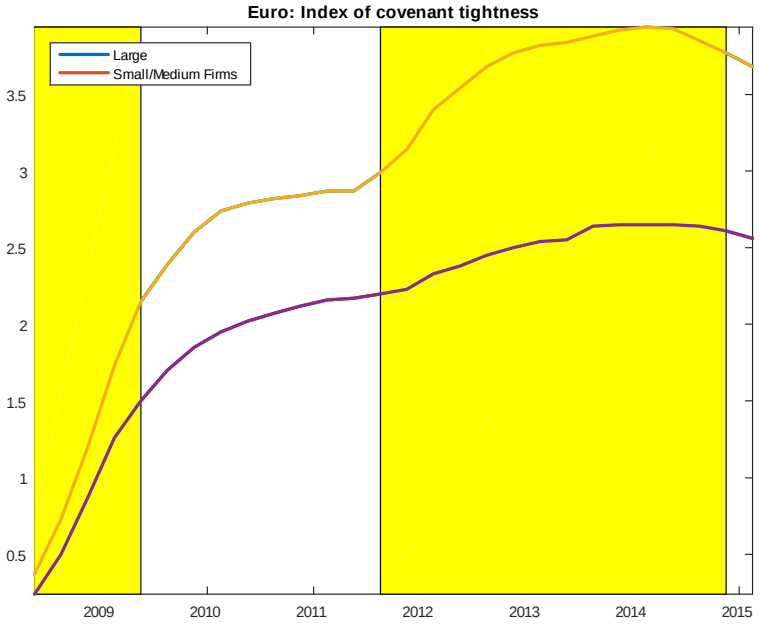

Figure 4

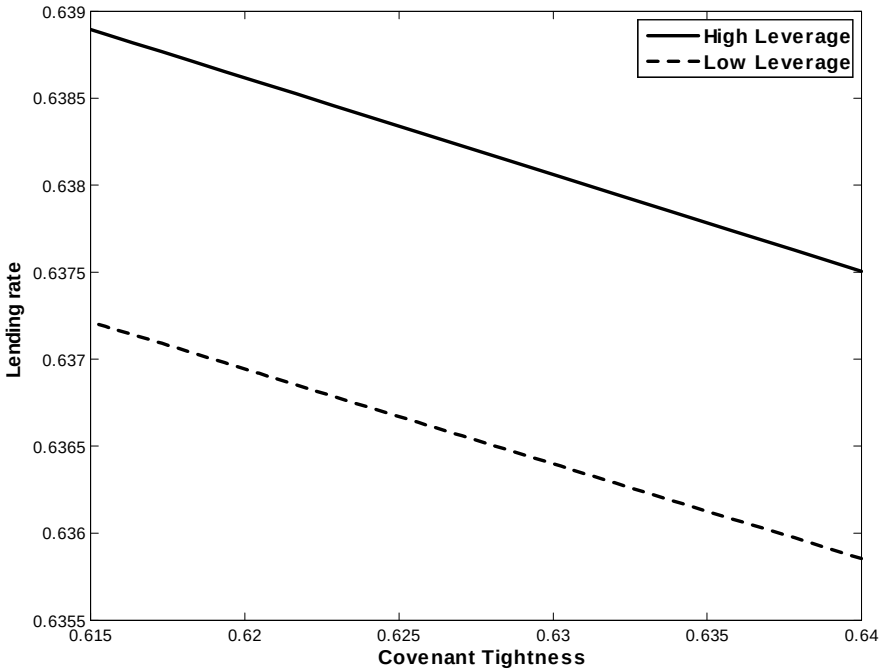
High Leverage
Low Leverage
Lending rate
Covenant Tightness
0.639
0.6385
0.638
0.6375
0.637
0.6365
0.636
0.6355
0.615
0.62
0.625
0.63
0.635
0.64

Figure 5

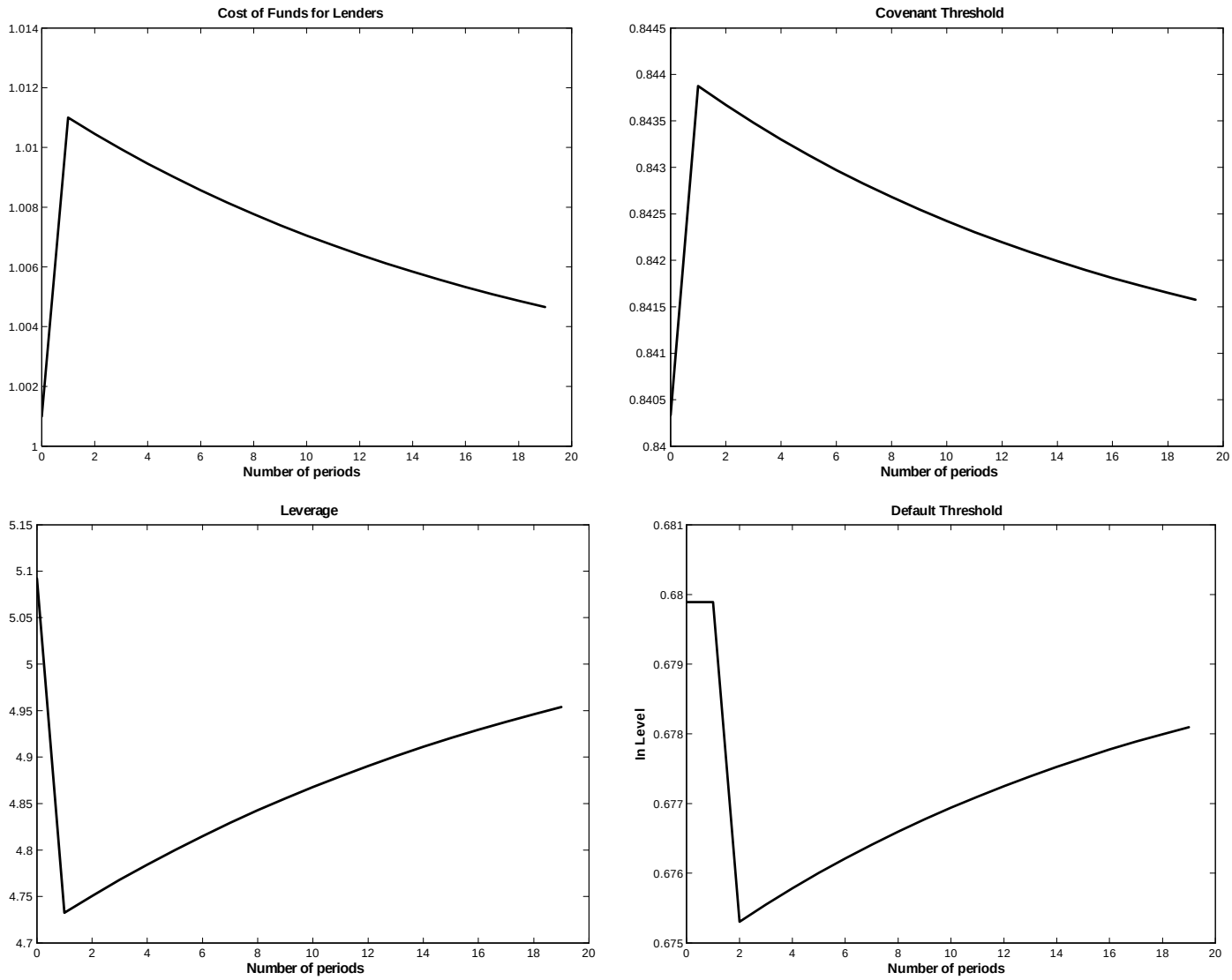
Cost of Funds for Lenders
1.014
1.012
1.01
1.008
1.006
1.004
1.002
1
0 2 4 6 8 10 12 14 16 18 20
Number of periods
Covenant Threshold
0.8445
0.844
0.8435
0.843
0.8425
0.842
0.8415
0.841
0.8405
0.84
0 2 4 6 8 10 12 14 16 18 20
Number of periods
Leverage
5.15
5.1
5.05
5
4.95
4.9
4.85
4.8
4.75
4.7
0 2 4 6 8 10 12 14 16 18 20
Number of periods
Default Threshold
0.681
0.68
0.679
0.678
0.677
0.676
0.675
ln Level
0 2 4 6 8 10 12 14 16 18 20
Number of periods

Figure 6

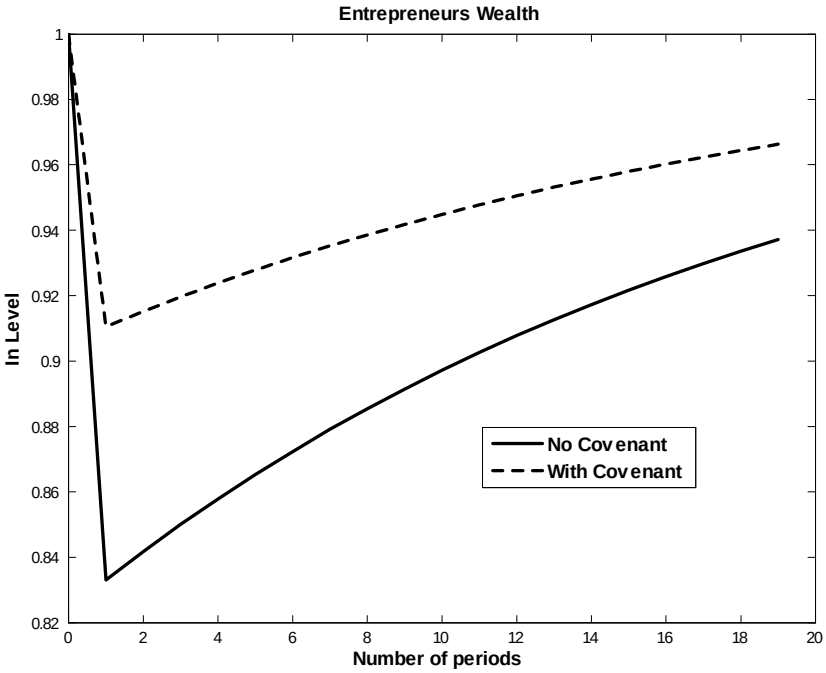
Entrepreneurs Wealth
In Level
Number of periods
No Covenant
With Covenant
1
0.98
0.96
0.94
0.92
0.9
0.88
0.86
0.84
0.82
0
2
4
6
8
10
12
14
16
18
20

Figure 7