

The Sufficient Statistic Approach: Predicting the Top of the Laffer Curve*

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Abstract

We provide a formula that relates the tax rate at the top of the Laffer curve to three elasticities. The formula applies to static models and to steady states of dynamic models. We also explore whether existing methods for estimating elasticities, common in the elasticity of taxable income literature, accurately estimate the theoretically-relevant elasticity. Existing methods work poorly in models with endogenous human capital accumulation but better in models with exogenous human capital.

Keywords: Sufficient Statistic, Laffer Curve, Marginal Tax Rate, Elasticity

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1 Introduction

Imagine that an important public policy issue, involving the functioning of the entire economy, could be settled convincingly by a simple formula together with only a few inputs estimated from data. Imagine further that the simple formula connecting the public policy variable to empirical inputs is general in that it holds within a wide class of theoretical models widely viewed as relevant to the issue at hand. It would seem to be clear that such work, satisfying the trinity of theoretical, empirical and policy relevance, would be celebrated.

The scenario just described is what the sufficient statistic approach hopes to achieve. Chetty (2009) reviews applications of the sufficient statistic approach. Chetty (2009) states “*The central concept of the sufficient statistic approach ... is to derive formulas for the welfare [revenue] consequences of policies that are functions of high-level elasticities rather than deep primitives.*”

One important and strikingly-simple application of the sufficient statistic approach is to predict the tax rate at the top the Laffer curve (i.e. the revenue maximizing tax rate). Thus, the public policy variable under consideration is then a tax rate on some specified component of income or expenditure. This could be the tax rate on labor income, capital income or something more specific such as the top federal tax rate on ordinary income as defined by the Internal Revenue Service in the US.

One may want to determine the tax rate at the top of the Laffer curve for several reasons. First, it may be widely agreed that it is counter productive to push a tax rate beyond the top of the Laffer curve. Thus, part of the public policy dialogue may be centered around first not doing anything foolish. Second, one may argue, following Diamond and Saez (2011), that the revenue maximizing tax rate on top earners closely approximates the welfare maximizing top tax rate for some welfare criteria.

From a quick look at the literature, one might conclude that the work of the theorist on this issue is complete. There already is a widely-used formula $\tau^* = 1/(1 + a\epsilon)$ that characterizes the revenue maximizing marginal tax rate τ^* that applies beyond a threshold y . Moreover, there is also a closely related formula $\tau^* = (1 - g)/(1 - g + a\epsilon)$ for the welfare maximizing marginal tax rate that is stated in terms of the same two empirical inputs (a, ϵ) and a social welfare weight $g \geq 0$ put on top earners marginal consumption. Diamond and Saez (2011) and Piketty and Saez (2013) discuss these results. These formulas are incorporated into the Mirrlees Review - an important public policy document that presents a synthesis of current thinking about what constitutes good tax policy (see Mirrlees et al. (2010)).

A better reading of the literature is that the widely-used formula does not actually apply to a wide class of models relevant to the issue at hand. The widely-used formula $\tau^* = 1/(1 + a\epsilon)$ is not valid in dynamic models. Specifically, it does not apply to either the infinitely-lived agent or the overlapping generations versions of the neoclassical growth model. These are the two workhorse models of modern macroeconomics. The taxation of consumption, labor income and capital income are commonly analyzed within such models. The formula in Theorem 1 of this paper applies to both of these workhorse models and to static models.

In addition, the widely-used formula does not apply to the class of heterogeneous-agent models that are currently the dominant positive models of the distribution of earnings, income,

consumption and wealth.¹ Within this class of models, Badel and Huggett (2014), Guner, Lopez-Daneri and Ventura (2014) and Kindermann and Krueger (2014) analyze the Laffer curve produced by increasing the marginal tax rate on top earners. The formula in Theorem 1 of this paper applies to all three model economies. Thus, the formula builds a bridge between the vast body of quantitative work that analyzes steady states of growth models and the application of a sufficient statistic approach to an important issue in macroeconomics and public economics.

This paper makes two contributions. First, Theorem 1 provides a tax rate formula that can be applied to both static models and to steady states of dynamic models. Thus, the formula in Theorem 1 should replace the widely-used formula in future work. Second, the paper explores whether standard methods to estimate elasticities, from the large literature on the elasticity of taxable income, closely approximate one of the key elasticities that enters the formula in Theorem 1. We show that standard methods substantially underestimate the key elasticity in a state-of-the-art model with endogenous human capital but work better in a model without endogenous skill accumulation.

The remainder of the paper is organized as follows. Section 2 presents the tax rate formula. Section 3 shows that the formula applies in a straightforward way to several classic static and dynamic models. Section 4 applies the formula to a state-of-the-art, quantitative human capital model and examines whether existing methods for estimating a key elasticity accurately approximate one of the three theoretically-relevant elasticities in Theorem 1. Section 5 concludes.

2 Tax Rate Formula

Our tax rate formula is based on three basic model elements: (i) a distribution of agent types $(X, \mathcal{X}, P)$, (ii) an income choice $y(x, \tau)$ that maps an agent type $x \in X$ and a parameter τ of the tax system into an income choice and (iii) a class of tax functions $T(y; \tau)$ mapping income choice and a tax system parameter τ into the total tax paid. Our approach does not rely on specifying an explicit dynamic or static equilibrium model up front. Instead, our tax rate formula can be applied in a straightforward way by mapping equilibrium allocations of specific non-parametric models into the three basic model elements.

2.1 Assumptions

Assumption A1 says that the distribution of agent types is represented by a probability space - a space of types X , a σ -field $\mathcal{X}$ on X and a probability measure P defined over sets in $\mathcal{X}$. Assumption A2 places structure on the class of tax functions. The tax functions differ in a single parameter τ , where τ is interpreted as the linear tax rate that applies to income that exceeds a threshold $\underline{y}$. Below this threshold the tax function can be nonlinear but all tax functions in the class are the same below the threshold. Assumption A3 says that key aggregates are differentiable in τ . The aggregates are based on integrals over the sets $X_1 = \{x \in X : y(x, \tau^*) > \underline{y}\}$ and $X_2 = X - X_1$ in $\mathcal{X}$.

¹Heathcote, Storesletten and Violante (2009) review this literature.

A1. $(X, \mathcal{X}, P)$ is a probability space.

A2. There is $\underline{y} \geq 0$ such that

(i) $T(y; \tau) - T(\underline{y}; \tau) = \tau[y - \underline{y}]$, $\forall y > \underline{y}, \forall \tau \in (0, 1)$ and

(ii) $T(y; \tau) = T(y; \tau')$, $\forall y \leq \underline{y}, \forall \tau, \tau' \in (0, 1)$.

A3. $\int_{X_2} T(y(x, \tau); \tau) dP$ and $\int_{X_1} y(x, \tau) dP$ are strictly positive and are differentiable in τ .

We also consider a generalization. The three basic elements of the generalized model are (i) a distribution of agent types $(X, \mathcal{X}, P)$, (ii) an $n \geq 2$ dimensional income choice $(y_1(x, \tau), \dots, y_n(x, \tau))$ and (iii) a class of tax functions $T(y_1, \dots, y_n; \tau)$ mapping the vector of choices and a tax system parameter τ into the total tax paid.

Assumptions A1' – A3' state assumptions when the tax system depends on $n \geq 2$ components of income. A2' assumes that the tax system is additively separable in that the first component of income y_1 determines a portion of the tax liability of an agent separate from the remaining components. For example, this structure captures a situation where labor income y_1 and capital income y_2 are taxed using separate tax schedules or where labor income y_1 and consumption y_2 are taxed separately. Alternatively, one might view y_1 as being ordinary income and y_2 as being the sum of long-term capital gains and qualified dividends as defined by the Internal Revenue Service in the US. All of these possibilities are consistent with the additive separability assumption. Assumption A2' states that the first component of income is taxed at a fixed tax rate τ beyond a threshold $\underline{y}$.

A1'. $(X, \mathcal{X}, P)$ is a probability space.

A2'. T is separable in that $T(y_1, \dots, y_n; \tau) = T_1(y_1; \tau) + T_2(y_2, \dots, y_n)$, $\forall (y_1, \dots, y_n, \tau)$. Moreover, there is $\underline{y} \geq 0$ such that

(i) $T_1(y_1; \tau) - T_1(\underline{y}; \tau) = \tau[y_1 - \underline{y}]$, $\forall y_1 > \underline{y}, \forall \tau \in (0, 1)$ and

(ii) $T_1(y_1; \tau) = T_1(y_1; \tau')$, $\forall y_1 \leq \underline{y}, \forall \tau, \tau' \in (0, 1)$.

A3'. $\int_{X_2} T(y_1, \dots, y_n; \tau) dP$, $\int_{X_1} y_1 dP$ and $\int_{X_1} T_2(y_2, \dots, y_n) dP$ are strictly positive and are differentiable in τ .

2.2 Theorem

To state Theorem 1, it is useful to write total tax revenue as the sum of tax revenue from the set of agent types with incomes above a threshold $X_1 = \{x \in X : y(x, \tau^*) > \underline{y}\}$ and from all remaining types $X_2 = X - X_1$. Total tax revenue can be stated in the same manner when the tax system depends on $n \geq 2$ components of income by again defining two sets $X_1 = \{x \in X : y_1(x, \tau^*) > \underline{y}\}$ and $X_2 = X - X_1$.

$$\int_X T(y(x, \tau); \tau) dP = \int_{X_1} T(y(x, \tau); \tau) dP + \int_{X_2} T(y(x, \tau); \tau) dP$$

$$\int_X T(y_1, \dots, y_n; \tau) dP = \int_{X_1} T(y_1, \dots, y_n; \tau) dP + \int_{X_2} T(y_1, \dots, y_n; \tau) dP$$

Theorem 1:

(i) Assume A1 – A3. If $\tau^* \in (0, 1)$ is revenue maximizing, then $\tau^* = \frac{1-a_2\varepsilon_2}{1+a_1\varepsilon_1}$, where

$$(a_1, a_2) = \left(\frac{\int_{X_1} y dP}{\int_{X_1} [y - \underline{y}] dP}, \frac{\int_{X_2} T(y; \tau^*) dP}{\int_{X_1} [y - \underline{y}] dP} \right) \text{ and } (\varepsilon_1, \varepsilon_2) = \left(\frac{d \log \int_{X_1} y dP}{d \log(1 - \tau)}, \frac{d \log \int_{X_2} T(y; \tau^*) dP}{d \log(1 - \tau)} \right).$$

(ii) Assume A1' – A3'. If $\tau^* \in (0, 1)$ is revenue maximizing, then $\tau^* = \frac{1-a_2\varepsilon_2-a_3\varepsilon_3}{1+a_1\varepsilon_1}$, where

$$(a_1, a_2, a_3) = \left(\frac{\int_{X_1} y_1 dP}{\int_{X_1} [y_1 - \underline{y}] dP}, \frac{\int_{X_2} T(y_1, \dots, y_n; \tau^*) dP}{\int_{X_1} [y_1 - \underline{y}] dP}, \frac{\int_{X_1} T_2(y_2, \dots, y_n) dP}{\int_{X_1} [y_1 - \underline{y}] dP} \right) \text{ and}$$

$$(\varepsilon_1, \varepsilon_2, \varepsilon_3) = \left(\frac{d \log \int_{X_1} y_1 dP}{d \log(1 - \tau)}, \frac{d \log \int_{X_2} T(y_1, \dots, y_n; \tau^*) dP}{d \log(1 - \tau)}, \frac{d \log \int_{X_1} T_2(y_2, \dots, y_n) dP}{d \log(1 - \tau)} \right).$$

Proof:

(i) If $\tau^* \in (0, 1)$ maximizes revenue then it also maximizes $\tau \int_{X_1} [y(x; \tau) - \underline{y}] dP + \int_{X_2} T(y(x; \tau), \tau) dP$. This holds by subtracting the constant term $\int_{X_1} T(\underline{y}; \tau) dP$ from total revenue and using A2. The following necessary condition then holds:

$$\int_{X_1} [y(x, \tau^*) - \underline{y}] dP - \tau^* \frac{d \int_{X_1} y(x; \tau^*) dP}{d(1 - \tau)} - \frac{d \int_{X_2} T(y(x, \tau^*); \tau^*) dP}{d(1 - \tau)} = 0$$

Divide the necessary condition by $\int_{X_1} [y(x, \tau^*) - \underline{y}] dP$ and rearrange using the elasticities stated in the Theorem. This implies $1 - \frac{\tau^*}{1-\tau^*} a_1 \varepsilon_1 - \frac{1}{1-\tau^*} a_2 \varepsilon_2 = 0$ which in turn implies $\tau^* = \frac{1-a_2\varepsilon_2}{1+a_1\varepsilon_1}$.

(ii) If $\tau^* \in (0, 1)$ maximizes revenue then by assumption A2' it also maximizes $\tau \int_{X_1} (y_1 - \underline{y}) dP + \int_{X_2} T_2(y_2, \dots, y_n) dP + \int_{X_2} T(y_1, \dots, y_n; \tau) dP$. The following necessary condition then holds:

$$\int_{X_1} [y_1 - \underline{y}] dP - \tau^* \frac{d \int_{X_1} y_1 dP}{d(1 - \tau)} - \frac{d \int_{X_1} T_2(y_2, \dots, y_n) dP}{d(1 - \tau)} - \frac{d \int_{X_2} T(y_1, \dots, y_n; \tau^*) dP}{d(1 - \tau)} = 0$$

Divide all terms in the previous equation by $\int_{X_1} [y_1 - \underline{y}] dP$ and then rearrange using the elasticities stated in the Theorem. This implies $1 - \frac{\tau^*}{1-\tau^*} a_1 \varepsilon_1 - \frac{1}{1-\tau^*} a_2 \varepsilon_2 - \frac{1}{1-\tau^*} a_3 \varepsilon_3 = 0$ which in turn implies $\tau^* = \frac{1-a_2\varepsilon_2-a_3\varepsilon_3}{1+a_1\varepsilon_1}$. ||

Comment:

1. Theorem 1 is appealing from the perspective of the sufficient statistic approach. It is stated in terms of at most three elasticities and yet allows a general treatment of agent heterogeneity (agent types are represented by a probability space) and applies to tax systems where taxes are determined based on many different income or expenditure types. The next section will clarify how to map equilibria in static or dynamic models into the three basic model elements used to state Theorem 1.

2. Grouping agent types into the set of types above and below the threshold (i.e the sets X_1 and X_2) highlights three distinct ways in which the widely-used formula can be invalid: (1) agent types in X_2 are impacted by factor price movements that are triggered by the behavioral response of agents in X_1 as τ changes, (2) agent types in X_2 anticipate being impacted by the tax change later in life when they have income above the threshold and therefore change their choices before they are directly impacted, (3) agents types in X_1 have other components of income or expenditure ($y_2, \dots, y_n$) that are subject to taxation and these components vary as τ changes. Points (1) and (2) above highlight circumstances in which the term $a_2\epsilon_2$ may be non-zero, whereas point (3) highlights how the term $a_3\epsilon_3$ may be non-zero. Section 3 and 4 provide examples that illustrate situations where $a_2\epsilon_2$ or $a_3\epsilon_3$ are non-zero.

3. To state the result in Theorem 1 using elasticities requires that each of the integrals (e.g. $\int_{X_2} T(y; \tau) dP$), over which the elasticity is taken, is non-zero. If any of the integral terms is zero, then the result can still be stated but without using an elasticity for that integral. For example, if the integral $\int_{X_2} T(y; \tau) dP$ is zero (i.e. net taxes on agent types below the threshold are zero) and the integral does not vary on the margin as the tax rate τ varies, then the term $a_2\epsilon_2$ in the formula in Theorem 1 can be replaced with a zero. Examples 1 and 3 in the next section illustrate this point.

3 Examples

We now consider three classic models: the Mirrlees model as well as the overlapping generations and infinitely-lived agent versions of the neoclassical growth model. We map equilibrium allocations in each model into the language of Theorem 1.

3.1 Example 1: Mirrlees Model

Mirrlees (1971) considered a static model in which agents make a consumption and labor decision in the presence of a tax system. A government budget constraint also must hold. The model elements are a utility function $u(c, l)$, agent productivity $x \in X$, a productivity distribution P and a class of tax functions $T(y; \tau)$.

Definition: An equilibrium is $(c(x; \tau), l(x; \tau), Tr(\tau))$ such that given any $\tau \in (0, 1)$

1. optimization: $(c(x; \tau), l(x; \tau)) \in \argmax u(c, l) \text{ s.t. } c \leq wxl - T(wxl; \tau) + Tr(\tau), l \geq 0$
2. market clearing: $\int_X c(x; \tau) dP = w \int_X xl(x; \tau) dP$

3. government budget: $Tr(\tau) = \int_X T(wxl(x; \tau); \tau) dP$

It will be helpful to state equilibria in closed form. To this end, we use the following functional forms and restrictions:

$$u(c, l) = c - \alpha \frac{l^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \text{ and } \alpha, \nu > 0$$

$$X = R_+ \text{ and } (X, \mathcal{X}, P) \text{ implies that } \int_X x^{1+\nu} dP \text{ is finite}$$

$$T(y; \tau) = \tau y \text{ is a tax system financing a lump-sum transfer } Tr(\tau)$$

Equilibrium allocations are straightforward to state:

$$l(x; \tau) = \left[\frac{wx(1-\tau)}{\alpha} \right]^\nu$$

$$c(x; \tau) = wx \left[\frac{wx(1-\tau)}{\alpha} \right]^\nu (1-\tau) + \tau \left[\frac{w(1-\tau)}{\alpha} \right]^\nu \int_X x^{1+\nu} dP$$

$$Tr(\tau) = \tau \left[\frac{w(1-\tau)}{\alpha} \right]^\nu \int_X x^{1+\nu} dP$$

We now map these equilibrium allocations into the elements $((X, \mathcal{X}, P), y(x; \tau), T(y; \tau))$ used to state Theorem 1.

Step 1: Set $(X, \mathcal{X}, P)$ to the probability space used in Example 1.

Step 2: Set $y(x; \tau) = wx \left[\frac{wx(1-\tau)}{\alpha} \right]^\nu$

Step 3: Set $T(y; \tau) = \tau y$.

We now calculate the terms in the formula within Example 1. It is clear that $a_1 = 1$ as the threshold is $\underline{y} = 0$. It is also clear that $\int_{X_2} T(y; \tau) dP = 0$ as only agent types with $x = 0$ are in the set X_2 . These agent types produce no labor income and generate zero tax revenue for all tax rates τ . Thus, the term $a_2 \epsilon_2$ in the formula can be replaced with a zero, consistent with Comment 3 to Theorem 1. Finally, it is clear that $\epsilon_1 = \nu$ by a direct calculation of the elasticity.²

In summary, the revenue maximizing tax rate depends only on the elasticity ϵ_1 . This “high-level” elasticity coincides with the preference parameter ν , given the form of the utility function.

$$\tau^* = \frac{1 - a_2 \epsilon_2}{1 + a_1 \epsilon_1} = \frac{1 - 0}{1 + 1 \times \nu} = \frac{1}{1 + \nu} < 1$$

²Theorem 1 directs one to calculate the elasticities and the related coefficients when the sets (X_1, X_2) are defined at the revenue maximizing tax rate τ^* . We calculate $(a_1, \epsilon_1) = (1, \nu)$ when these sets are specified for any fixed value of $\tau \in (0, 1)$.

3.2 Example 2: Diamond Growth Model

Diamond (1965) analyzes an overlapping generations model with two-period lived agents and with an aggregate production function $F(K, L)$. In our version of this model, age 1 and age 2 agents are equally numerous at any point in time and each age group has a mass of 1. Agents solve problem P1, where they face a proportional labor income tax τ and a proportional consumption tax τ_c . Agents receive a lump-sum transfer $Tr(\tau)$ from the government when young and when old.

$$(P1) \quad \max U(c_1, c_2, l) \text{ s.t.}$$

$$(1 + \tau_c)c_1 + k \leq w(\tau)zl(1 - \tau) + Tr(\tau), (1 + \tau_c)c_2 \leq k(1 + r(\tau)) \text{ and } l \in [0, 1]$$

Implicit in problem P1 is the assumption that age 1 agents can work and that age 2 agents cannot work. Age 1 agents are heterogeneous in labor productivity $z \in Z \subset R_+$. The distribution of labor productivity is given by $(Z, \mathcal{Z}, \hat{P})$. Define two aggregates $K(\tau) = \int_Z k(z; \tau) d\hat{P}$ and $L(\tau) = \int_Z zl(z; \tau) d\hat{P}$.

Definition: A steady-state equilibrium is $(c_1(z; \tau), c_2(z; \tau), l(z; \tau), k(z; \tau))$, a transfer $Tr(\tau)$ and factor prices $(w(\tau), r(\tau))$ such that for any $\tau \in (0, 1)$

1. optimization: $(c_1(z; \tau), c_2(z; \tau), l(z; \tau), k(z; \tau))$ solve P1.
2. factor prices: $w(\tau) = F_2(K(\tau), L(\tau))$ and $1 + r(\tau) = F_1(K(\tau), L(\tau))$
3. market clearing: $\int_Z (c_1(z; \tau) + c_2(z; \tau)) d\hat{P} + K(\tau) = F(K(\tau), L(\tau))$
4. government budget: $Tr(\tau) = \tau_c \int_Z (c_1(z; \tau) + c_2(z; \tau)) d\hat{P} + \tau \int_Z w(\tau) zl(z; \tau) d\hat{P}$

We now map equilibrium allocations into the language used to state Theorem 1.

Step 1: Define the probability space of agent types. An agent type is $x = (z, j)$ consisting of the agent's productivity z when young and the agent's current age j . To apply Theorem 1, an agent type needs to be stated in terms of **exogenous variables**. An agent type does not vary as the tax parameter τ varies.

$$x = (z, j) \in X = Z \times \{1, 2\} \text{ and } P(A) = \int_Z \left[\frac{1}{2} 1_{\{(z,1) \in A\}} + \frac{1}{2} 1_{\{(z,2) \in A\}} \right] d\hat{P}, \forall A \in \mathcal{X}$$

Step 2: Define choices (y_1, y_2) . View y_1 as labor income and y_2 as consumption.

$$y_1(x; \tau) = w(\tau)zl(z; \tau) \text{ if } x = (z, 1), z \in Z \text{ and } y_1(x; \tau) = 0 \text{ otherwise}$$

$$y_2(x; \tau) = c_1(z; \tau) \text{ if } x = (z, 1) \text{ and } z \in Z \text{ and } y_2(x; \tau) = c_2(z; \tau) \text{ otherwise}$$

Step 3: Set $T(y_1, y_2; \tau) = \tau y_1 + \tau_c y_2$. With this choice, aggregate taxes $\int_X T(y_1, y_2; \tau) dP$ are proportional to the right-hand side of equilibrium condition 4.

The coefficients premultiplying each of the elasticities in Theorem 1 are easy to determine. The threshold is $\underline{y} = 0$ as the tax τ is a proportional labor income tax. The coefficients are $(a_1, a_2, a_3) = (1, \frac{\tau_c \int_Z c_2(z; \tau^*) d\hat{P}}{w(\tau^*)L(\tau^*)}, \frac{\tau_c \int_Z c_1(z; \tau^*) d\hat{P}}{w(\tau^*)L(\tau^*)})$. The coefficient a_2 from Theorem 1 is the ratio of the total tax revenue from agent types in X_2 to the total incomes y_1 above the threshold $\underline{y}$ for agent types in X_1 . This equals the ratio of total consumption taxes paid by age 2 agents to total labor income. The coefficient a_3 is the ratio of total tax revenue from agents in X_1 on types of income or expenditure other than type y_1 to total incomes y_1 above the threshold.

$$\tau^* = \frac{1 - a_2\varepsilon_2 - a_3\varepsilon_3}{1 + a_1\varepsilon_1} = \frac{1 - \frac{\tau_c \int_Z c_2(z; \tau^*) d\hat{P}}{w(\tau^*)L(\tau^*)} \times \varepsilon_2 - \frac{\tau_c \int_Z c_1(z; \tau^*) d\hat{P}}{w(\tau^*)L(\tau^*)} \times \varepsilon_3}{1 + 1 \times \varepsilon_1}$$

The take-away point from Example 2 is that the formula in Theorem 1 applies to steady states of dynamic models once an agent type is viewed in the right way.

3.3 Example 3: Trabandt and Uhlig

Trabandt and Uhlig (2011) use the neoclassical growth model with an infinitely-lived agent to analyze Laffer curves corresponding to different average tax rates on aggregate labor income, capital income and consumption. They calibrate some model parameters so that steady states of their model match aggregate features of the US economy and 14 European economies and preset other model parameters. They calculate Laffer curves by varying either the tax rate on labor income, capital income or consumption to determine how the steady-state equilibrium lump-sum transfer responds to the tax rate. We show that our sufficient statistic formula also to Laffer curves in their model. We highlight the Laffer curve related to varying the labor income tax.

The equilibrium concept given below differs from that in Trabandt and Uhlig in that it abstracts from steady-state growth and imports in order to simplify the exposition.

$$(P1) \quad \max \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \quad s.t.$$

$$(1 + \tau_c)c_t + (k_{t+1} - k_t(1 - \delta)) + b_{t+1} \leq (1 - \tau_l)w_t l_t + k_t(1 + r_t(1 - \tau_k)) + b_t R_t^b + T r_t$$

$$l_t \in [0, 1] \text{ and } k_0 \text{ is given}$$

Definition: A steady-state equilibrium is an allocation $\{c_t, l_t, k_t\}_{t=0}^{\infty}$, prices $\{w_t, r_t, R_t^b\}_{t=0}^{\infty}$ fiscal policy $\{g_t, b_t, T r_t\}_{t=0}^{\infty}$ such that

1. optimization $(c_t, l_t, k_t, b_t) = (\bar{c}, \bar{l}, \bar{k}, \bar{b}), \forall t \geq 0$ solves P1.
2. prices: $w_t = \bar{w} = F_2(\bar{k}, \bar{l}), r_t = \bar{r} = F_1(\bar{k}, \bar{l}) - \delta, R_t^b = \bar{R}^b, \forall t \geq 0$
3. government: $(g_t, b_t, T r_t) = (\bar{g}, \bar{b}, \bar{T} r), \forall t \geq 0$ and $\bar{g} + \bar{b}(\bar{R}^b - 1) + \bar{T} r = \tau_c \bar{c} + \tau_k \bar{k} \bar{r} + \tau_l \bar{w} \bar{l}$
4. market clearing: $\bar{c} + \bar{k} \delta + \bar{g} = F(\bar{k}, \bar{l})$

We map equilibrium allocations into the language of Theorem 1 in three steps. We denote the labor income tax rate $\tau_l = \tau$. We let bars over variables denote steady-state quantities so that $\bar{c}(\tau)$ denotes the steady-state equilibrium consumption associated with labor income tax rate $\tau_l = \tau$, fixing the other tax rates (τ_c, τ_k) and fixing $(\bar{g}, \bar{b})$. Thus, transfers $\bar{T}r(\tau)$ adjust to changes in revenue when τ is varied.

Step 1: $(X, \mathcal{X}, \mathcal{P})$ is $X = 1, \mathcal{X} = \{\{1\}, \emptyset\}, P(1) = 1, P(\emptyset) = 0$

Step 2: $y_1(x, \tau) = \bar{w}(\tau)\bar{l}(\tau), y_2(x, \tau) = \bar{c}(\tau)$ and $y_3(x, \tau) = \bar{r}(\tau)\bar{k}(\tau)$

Step 3: $T(y_1, y_2, y_3; \tau) = \tau y_1 + \tau_c y_2 + \tau_k y_3 - \bar{g} - \bar{b}(\bar{R}^b(\tau) - 1)$

For the tax system in step 3 to satisfy assumption $A2'$, we need (i) separability (i.e. $T = T_1 + T_2$), (ii) T_1 is eventually linear in τ and (iii) T_2 depends on τ only indirectly via $(y_2, \dots, y_n)$. Setting $T(y_1, y_2, y_3; \tau) = T_1(y_1; \tau) + T_2(y_2, y_3)$, where $T_1(y_1; \tau) = \tau y_1$ and $T_2(y_2, y_3) = \tau_c y_2 + \tau_k y_3 - \bar{g} - \bar{b}(\bar{R}^b(\tau) - 1)$, is potentially problematic. Without further argument, the tax system does not satisfy property (iii). Specifically, since the government bond return $\bar{R}^b(\tau)$ enters T_2 , property (iii) may be violated. However, $\bar{R}^b(\tau) = 1/\beta, \forall \tau \in [0, 1)$ in steady state follows directly from the Euler equation. Thus, the tax function in Step 3 satisfies $A2'$.

Given this mapping, the coefficients pre-multiplying the elasticities are easy to calculate. The two relevant coefficients are $(a_1, a_3) = (1, \frac{\tau_c \bar{c}(\tau^*) + \tau_k \bar{k}(\tau^*) \bar{r}(\tau^*) - \bar{g} - \bar{b}(\bar{R}^b(\tau^*) - 1)}{\bar{w}(\tau^*) \bar{l}(\tau^*)})$. The representative-agent structure implies that there is just one agent type. Thus, the term $a_2 \epsilon_2$ is zero in this model as there are no agent types in the set X_2 having y_1 at or below the threshold $y = 0$. Thus, the revenue from these types is zero at all tax rates. The coefficient a_3 is non-zero as there are other sources of taxes besides the labor tax on agent types in the set X_1 . When the labor tax moves these other sources of tax revenue can move as well.

$$\tau^* = \frac{1 - a_2 \epsilon_2 - a_3 \epsilon_3}{1 + a_1 \epsilon_1} = \frac{1 - \frac{\tau_c \bar{c}(\tau^*) + \tau_k \bar{k}(\tau^*) \bar{r}(\tau^*) - \bar{g} - \bar{b}(\bar{R}^b(\tau^*) - 1)}{\bar{w}(\tau^*) \bar{l}(\tau^*)} \times \epsilon_3}{1 + 1 \times \epsilon_1}$$

The take-away point is that for each of the 15 quantitative model economies considered by Trabandt and Uhlig (2011), there are two high-level elasticities (ϵ_1, ϵ_3) and one coefficient a_3 that determine the top of the model Laffer curve.

4 A Human Capital Model

We now explore the human capital model developed in Badel and Huggett (2014). They calibrate the model to match a number of features of the US age-earnings distribution. They calculate the Laffer curve that results from increasing the tax rate on top earners. Extra tax revenue is used to fund a lump-sum transfer.

This section has two goals. First, it examines whether the formula in Theorem 1 accurately predicts the top of the model Laffer curve under almost ideal conditions. Specifically, the three coefficients (a_1, a_2, a_3) and corresponding elasticities $(\epsilon_1, \epsilon_2, \epsilon_3)$ are computed using model data at the initial steady state, where the top tax rate does not maximize revenue or transfers. Proponents of the sufficient statistic approach (e.g. Diamond and Saez (2011) and the references

cited in the introduction) apply the widely-used formula $\tau^* = 1/(1 + a\epsilon)$ to predict the top of the Laffer curve. Second, it examines how well standard methods from the elasticity of taxable income literature (see the literature reviewed by Saez, Slemrod and Giertz (2012)) work in estimating the elasticities relevant in Theorem 1.

4.1 Equilibrium

We briefly sketch the Badel-Huggett model. Agents solve decision problem P1. Thus, agents maximize expected utility by making a consumption decision c_j and an asset holding decision k_{j+1} each period. Period resources come from labor market earnings $e_j = wh_jl_j$, which are the product of a wage rate w , skill h_j and work time l_j , and from interest paying assets $k_j(1+r)$ brought into the period. Earnings next period depend on next period's human capital h_{j+1} which depends on current human capital h_j , learning time s_j , learning ability a and an idiosyncratic shock z_{j+1} .

Problem P1: $\max E[\sum_{j=1}^J \beta^{j-1} u(c_j, l_j + s_j)]$ subject to

$$c_j + k_{j+1} \leq e_j + k_j(1+r) - T_j(e_j, rk_j) \text{ and } k_{j+1} \geq 0, \forall j$$

$$e_j = wh_jl_j \text{ for } j < \text{Retire} \text{ and } e_j = 0 \text{ otherwise}$$

$$0 \leq l_j + s_j \leq 1, h_{j+1} = H(h_j, s_j, z_{j+1}, a) \text{ and } k_1 = 0$$

We note that the consumption decision $c = (c_1, \dots, c_J)$ is composed of functions $c_j(h_1, a, z^j)$ that map initial condition (h_1, a) , age j and shock history z^j into a period choice. The notational convention also holds for labor time decisions $l = (l_1, \dots, l_J)$, learning decisions $s = (s_1, \dots, s_J)$, asset decisions $k = (k_1, \dots, k_J)$ and human capital decisions $h = (h_1, \dots, h_J)$. These decisions, population fractions μ_j and the distribution ψ of initial conditions define steady-state aggregates (K, L, C, T) . The population fractions μ_j are determined by a constant population growth rate n .

$$K = \sum_j \mu_j \int E[k_j | h_1, a] d\psi \text{ and } L = \sum_j \mu_j \int E[h_j l_j | h_1, a] d\psi$$

$$C = \sum_j \mu_j \int E[c_j | h_1, a] d\psi \text{ and } T = \sum_j \mu_j \int E[T_j(wh_jl_j, rk_j) | h_1, a] d\psi$$

Definition: A steady-state equilibrium consists of decisions (c, l, s, k, h) , factor prices (w, r) and government spending G such that

1. Decisions: (c, l, s, k, h) solve Problem P1, given (w, r) .
2. Prices: $w = F_2(K, L)$ and $r = F_1(K, L) - \delta$
3. Government Budget: $G = T$

4. Feasibility: $C + K(n + \delta) + G = F(K, L)$

The tax system in the Badel-Huggett model has three components: a progressive tax imposed on labor income, a social security system and a proportional tax on capital income. Figure 1 presents the progressive component of the tax system. The progressive tax captures the increasing pattern of marginal tax rates on earnings in the US due to federal and state income taxes, sales taxes and medicare taxes. The top marginal tax rate is 42.5 percent that equals the top rate calculated by Diamond and Saez (2011). This top rate applies to incomes within the top 1 percent of the model income distribution.

Figure 2 presents the Laffer curve that results from increasing the top tax rate in Figure 1 without changing the marginal tax rate schedule at lower income levels. The top of the Laffer curve in the human capital model occurs at a 52 percent top tax rate. Figure 2 also plots the Laffer curve that is produced by a closely-related model called the exogenous human capital model.³ The exogenous human capital model has a much higher revenue maximizing top tax rate (66 percent) and produces a much larger lump-sum transfer compared to the human capital model. Intuitively, this is because aggregate labor supply is less elastic with respect to a change in the top rate compared to the human capital model. Skills fall in response to an increase in marginal tax rates in the human capital model but not in the exogenous human capital model. The exogenous human capital model is similar to the structure of the models analyzed by Guner et al. (2014) and Kindermann and Krueger (2014) as individual labor productivity in these papers is invariant to a change in the tax system.

4.2 How to Map Model Allocations into Theorem 1

We map steady-state equilibrium decisions in the Badel-Huggett model into the language used in Theorem 1. Agent types have to be defined in terms of exogenous variables to apply Theorem 1. A natural formulation is that a type $x = (h_1, a, j, z^j)$ is determined by initial condition (h_1, a) , current age j and shock history z^j up to the current age. All these components of type are exogenous and completely determine equilibrium decisions.

Step 1: $x = (h_1, a, j, z^j) \in X = H \times A \times \{1, \dots, J\} \times Z$

Step 2:

$$y_1(x, \tau) = w(\tau)h_j(h_1, a, z^j; \tau)l_j(h_1, a, z^j; \tau) \text{ for } j < \textit{Retire} \text{ and } y_1(x, \tau) = 0 \text{ otherwise}$$

$$y_2(x, \tau) = b\bar{e} \text{ for } j \geq \textit{Retire} \text{ and } y_2(x, \tau) = 0 \text{ otherwise.}$$

$$y_3(x, \tau) = r(\tau)k_j(h_1, a, z^j; \tau)$$

³Badel and Huggett (2014) describe this model in detail. This model is observationally equivalent to the benchmark human capital model under the benchmark tax system in that earnings, labor hours, consumption, assets and tax payments are identical. However, when the tax system changes then the two models differ. Human capital decisions change in the human capital model in response to the tax reform via the learning time decision s_j . Human capital decisions do not change in the exogenous human capital model. Human capital evolves in the same way regardless of the size of the reform. All other decisions (consumption, labor hours, asset choice) are made optimally in the exogenous human capital model.

Step 3: $T(y_1, y_2, y_3; \tau) = T_1(y_1; \tau) + T_2(y_2, y_3)$

$$T_1(y_1; \tau) = T^{prog}(y_1; \tau) + \tau_{ss} \min\{y_1, emax\} \text{ and } T_2(y_2, y_3) = T^{prog}(y_2) - y_2 + \tau_k y_3$$

In Step 2, (y_1, y_2, y_3) denote labor income, social security transfer income and capital income respectively. The model tax and transfer system depends on these values. Specifically, Step 3 indicates that labor income y_1 is taxed progressively, where T^{prog} was graphed in Figure 1. The top marginal tax rate on earnings is due entirely to the progressive tax function top tax rate τ . This holds because the maximum taxable earnings level in the U.S. social security system, denoted $emax$, is well below the income level of the top tax bracket in Figure 1. Capital income is taxed at a constant tax rate τ_k .

Based on Steps 1-3, we calculate the coefficients (a_1, a_2, a_3) and the elasticities $(\epsilon_1, \epsilon_2, \epsilon_3)$ in Theorem 1.⁴ Table 1 present the results. The predicted top of the Laffer curve $\tau^* = .52$ equals the actual top of the Laffer curve up to two-digit accuracy in the endogenous human capital model. In the exogenous human capital model the predicted top of the Laffer curve $\tau^* = .64$ differs from the actual top rate by two percentage points. This holds even though the initial steady state top tax rate is quite far from the revenue maximizing top rate. Based on the results in Table 1, using the formula to predict the top of the model Laffer curve does not seem to be especially problematic.

Table 1: Revenue Maximizing Top Tax Rate Formula

Terms	Endogenous Human Capital Model	Exogenous Human Capital Model
$a_1 \times \epsilon_1$	$1.97 \times .396 = .780$	$1.97 \times .240 = .473$
$a_2 \times \epsilon_2$	$3.06 \times .019 = .059$	$3.06 \times .014 = .044$
$a_3 \times \epsilon_3$	$.043 \times .508 = .022$	$.043 \times .459 = .020$
$\tau^* = \frac{1 - a_2\epsilon_2 - a_3\epsilon_3}{1 + a_1\epsilon_1}$	.52	.64
τ at peak of Laffer curve	.52	.66

Table 1 also provides a quantitative illustration of what the widely-used formula $\tau^* = 1/(1 + a\epsilon)$ misses. The two additional terms $(a_2\epsilon_2, a_3\epsilon_3)$ in the numerator in the formula in Theorem 1 are both positive in the economies analyzed in Table 1. Thus, if one used the invalid but widely-used formula to predict the top of the Laffer curve in either of these models and used the correct values for the remaining inputs, then one would predict that the top of the Laffer curve occurs at a higher top tax rate than the true top of either the Laffer curve.

4.3 Comparing Model Elasticities to Empirical Elasticities

Saez, Slemrod and Giertz (2012) review the literature that estimates the income elasticity with respect to the net-of-tax rate. Their review is very clear on several points: (1) the elasticities

⁴To calculate elasticity ϵ_1 , set $X_1 = \{x \in X : y_1(x; \tau) = w(\tau)h_j(h_1, a, z^j; \tau)l_j(h_1, a, z^j; \tau) > \underline{y}\}$ when $\tau = .425$. Then calculate $\int_{X_1} y_1 dP$ for multiple values of the top tax rate to approximate the elasticity with a difference quotient.

that this literature produces are the key input into the two sufficient statistic formulas (i.e. $\tau^* = 1/(1 + a\epsilon)$ and $\tau^* = (1 - g)/(1 - g + a\epsilon)$) discussed in the introduction, (2) the existing literature has primarily focused on short-run responses, (3) it is the long-run response to a tax reform that is of most interest for policy making and (4) long-run elasticity estimates are plagued by extremely difficult issues so that there are no convincing long-run elasticity estimates.

This section applies existing techniques from this literature to estimate, using model data, an earnings elasticity for top earners. This exercise is informative about the degree to which existing methods from the literature approximate the long-run model elasticities that determine the top of the model Laffer curve (see Table 1) and that are, according to Saez et al. (2012), of most interest for policy making. To the best of our knowledge, our work is the first in the literature to carry out such an analysis where the relevant long-run elasticity is known in advance.

Much of the literature applies the regression framework developed by Gruber and Saez (2002). They consider the regression equation below, where ϵ is the empirical proxy for the elasticity parameter in the formula, z_{it} is income of individual i at time t , $\tau_t(z_{it})$ is the marginal tax rate, $f(z_{it}) = \beta \log z_{it}$ is an income control and α_t are time dummy variables.

$$\log \left(\frac{z_{it+1}}{z_{it}} \right) = \epsilon \log \left(\frac{1 - \tau_{t+1}(z_{it+1})}{1 - \tau_t(z_{it})} \right) + f(z_{it}) + \alpha_t + \nu_{it+1}$$

Saez et al. (2012) estimate the elasticity of taxable income using U.S. data from 1991-97 and the regression equation above. Their estimation is based on data from a tax reform that occurred in 1993. They calculate that this reform increased the average marginal tax rate of the top 1 percent but did not significantly change the average marginal tax rate for the rest of the top 10 percent. They examine a panel of the top 10 percent of US taxpayers over 1991-97 and run a battery of regressions based on this equation, utilizing different controls, instruments and subperiods.

Saez et al. (2012, Table 2, Panel B) estimate the regression equation above using three different choices of instruments for log net-of-tax-rate changes and a two-stage-least-squares estimator. Instrument 1 is the indicator function taking the value 1 if individual i is in the top 1 percent in 1992 (i.e. $1_{\{i \in T_{1992}\}}$). Thus, T_{1992} denotes the set of individuals in the top 1 percent in 1992 which is the pre-reform year. Instrument 2 is $1_{\{i \in T_{1992} \text{ and } t=1992\}}$. Instrument 3 is $\log \left(\frac{1 - \tau_{t+1}(z_{it})}{1 - \tau_t(z_{it})} \right)$ and represents the log of the ratio of net-of-tax rates across years if income were not to change across years.

We now use the same regression equation and apply the same techniques (i.e. two-stage-least squares) to a tax reform in the two models analyzed in Table 1. The benchmark tax system applies in period $t = 1$ and $t = 2$ and agents view this system as being permanent. Agents are surprised to learn that the model tax system is modified permanently at $t = 3$. They learn this at the start of period $t = 3$. Thus, the model periods $t = 1, \dots, 7$ loosely correspond to the years 1991 – 1997 for the US economy. The tax system is modified by increasing the top tax rate to $\bar{\tau} = 0.52$, which corresponds to the tax rate at the top of the Laffer curve for the human capital model.

This is a partial equilibrium analysis as factor prices and transfers are assumed to be fixed over

Table 2 - Elasticity Estimates: Endogenous Human Capital Model Data

	(1)	(2)	(3)	(4)	(5)	(6)
Mean Elasticity ϵ	0.0089	0.1744	0.1840	0.2189	0.2129	0.2451
S.D.	(.0220)	(.0231)	(.0237)	(.0240)	(.0228)	(.0230)
Income Control $f(z)$	No	Yes	No	Yes	No	Yes
Time Effects α_t	No	No	Yes	Yes	Yes	Yes
Instrument 1: $1_{\{i \in T_2\}}$	Yes	Yes				
Instrument 2: $1_{\{i \in T_2 \text{ and } t=2\}}$			Yes	Yes		
Instrument 3: $\log(\frac{1-\tau_{t+1}(z_{it})}{1-\tau_t(z_{it})})$					Yes	Yes
Use data for time periods	$t = 2, 3$	$t = 2, 3$	All	All	All	All
Long-run Model Elasticity ϵ_1	0.396	0.396	0.396	0.396	0.396	0.396

Note: (1) We draw 100 balanced panel data sets of 30,000 agents. Each data set mimics the structure of the data set used by Saez et al. (2012, Table 2). The agents in each balanced panel have labor income above the 90th percentile of earnings at $t = 1$. We follow these agents from periods $t = 1$ to $t = 7$. The tax reform occurs at $t = 3$. (2) We report means and standard deviations of the point estimates of ϵ across 100 randomly drawn balanced panels.

Table 3 - Elasticity Estimates: Exogenous Human Capital Model Data

	(1)	(2)	(3)	(4)	(5)	(6)
Mean Elasticity ϵ	0.0072	0.2171	0.1982	0.2349	0.2114	0.2448
S.D.	(.0226)	(.0241)	(.0246)	(.0246)	(.0243)	(.0245)
Income Control $f(z)$	No	Yes	No	Yes	No	Yes
Time Effects α_t	No	No	Yes	Yes	Yes	Yes
Instrument 1: $1_{\{i \in T_2\}}$	Yes	Yes				
Instrument 2: $1_{\{i \in T_2 \text{ and } t=2\}}$			Yes	Yes		
Instrument 3: $\log(\frac{1-\tau_{t+1}(z_{it})}{1-\tau_t(z_{it})})$					Yes	Yes
Use data for time periods	$t = 2, 3$	$t = 2, 3$	All	All	All	All
Long-run Model Elasticity ϵ_1	0.240	0.240	0.240	0.240	0.240	0.240

Note: The construction of the balanced panels follows the structure and methods used in Table 2.

time at initial steady-state values. A general equilibrium analysis of this reform is much more costly in computational resources than the partial equilibrium analysis. In the human capital model the final (general equilibrium) steady state factor prices change very little as a result of the reform because aggregate capital and labor inputs fall by roughly the same percent.

The results of this exercise are presented in Table 2 and 3. Each table presents the mean of the estimated elasticity across 100 balanced panel data sets. The instruments used are model analogs of instruments 1-3 used by Saez et al. (2012). Estimation is based on the same two-stage-least squares regression methods employed by Saez et al. (2012). Table 2 finds that the mean of the estimated elasticity for any of the six regressions in Table 2 is at or below $\epsilon = 0.2451$, whereas the model elasticity that is relevant for determining the top of the model Laffer curve is much larger and is $\epsilon_1 = 0.396$ as reported in Table 1. Thus, in the endogenous human capital model the estimated elasticity averages roughly half of the true, long-run model elasticity.

Table 3 finds that the mean of the estimated model elasticity in the exogenous human capital model is typically not too far from the true, long-run model elasticity. The only exception is the result in column (1) where there are no income controls and where the instrument is a dummy variable taking the value 1 if the agent is in the top 1 percent in the pre-reform year. The literature review by Saez et al. (2012) stresses the potential importance of adding income

controls. Thus, the results in columns (2), (4) and (6) in Table 2 and 3 correspond to the regression results that are typically stressed in the literature.

Why is it the case that standard methods imply that the estimated elasticity is roughly half of the true elasticity in the endogenous human capital model but roughly approximates the true elasticity in the exogenous human capital model? Figure 3 helps to answer this question. It plots the transition path for aggregate capital K and labor L input in both models after the tax reform. The tax reform occurs in model period 3 in Figure 3. This is consistent with the timing of the tax reform that underlies the results in Table 2 and 3.

Figure 3 shows that the model dynamics differ sharply after a reform. In the endogenous human capital model, aggregate labor input drops sharply in the period of the reform and then gradually declines to the new steady state value after roughly 40 model periods - the length of the working lifetime. Less than half of the overall change in labor input occurs in the initial reform period. Thus, regression methods that compare an earnings or income measure before the reform and a year or so after the reform will pick up how hours movements impact earnings but not the impact of skill change on earnings. In the human capital model more than half the change in the aggregate labor input is due to skill change and this skill change is concentrated among agents with high learning ability. In the exogenous human capital model there is also a steep fall in labor input in the initial period of the reform. Most of the change in the steady state aggregate labor input occurs in this model period. In fact, aggregate labor input falls by more in the first period of the reform than the total fall in the steady-state aggregate labor input in the exogenous human capital model.

In both models, the movement in aggregate labor input is determined by changes in behavior of agents with very high learning ability who have the greatest chance of becoming a top earner and being directly impacted by the change in the tax system. In the human capital model, agents with high learning ability decrease their time investments in human capital investment over the lifetime. The fall in aggregate labor input which is due to these lower investments takes a long time to be fully realized as the impact of the tax reform on skill accumulation is greatest in percentage terms at the end of the working lifetime. This accounts for the substantially different dynamics across the two models.

5 Discussion

This paper doggedly makes two main points. Point 1 is that Theorem 1 applies broadly to both static models and to steady states of dynamic models. Thus, the formula in Theorem 1 should replace the widely-used formula in future work. Point 2 is that the formula from Theorem 1 works well in predicting the top of the Laffer curve in a state-of-the-art quantitative human capital model and yet existing state-of-the-art methods for estimating elasticities, drawn from the literature on the elasticity of taxable income, work poorly even on an ideal data set drawn from a tax experiment performed within the model.

We think that these two points warrant changes in how macroeconomists and public economists go about predicting the top of the Laffer curve:

1. The elasticity of taxable income literature needs to retool

Papers are still being written based on a two-part premise. The first part is that $\tau^* = 1/(1 + a\epsilon)$ robustly characterizes the top of the Laffer curve. The second part is that the canonical Gruber and Saez (2002) regression equation from section 4 of this paper shines valuable light on the value of ϵ provided only that one is careful in obtaining and constructing the right data set or in creating even better instruments than the instrument proposed by Gruber and Saez (2002). Both parts of this premise are misguided. While the first part of the premise can be fixed simply by using the formula in Theorem 1, the second part of the premise is not so quickly fixed. The data sets constructed in section 4 are ideal data sets in that they are large, have effectively no measurement error and have no sample attrition. In addition, the reform is a permanent reform that only alters the top tax rate. Agents in these data sets have no advanced information of the tax reform. However, state-of-the-art techniques applied to these ideal data sets produce an elasticity that is roughly half of the true, long-run elasticity. A view from leading economists is that the long-run response to a tax reform is of most interest for policy making. Thus, we think that the elasticity of taxable income literature needs to retool to develop estimation methods that work well in a wide range of plausible dynamic models.

2. Needed: Some methodological flexibility amongst macroeconomists

Until recently, macroeconomists approached the same taxation questions as public economists but with a different tool kit. There appeared to be at best a limited understanding of whether the sufficient statistic approach, championed by economists working in public economics, might apply to their models. For example, Trabandt and Uhlig (2011) calculate the top of the Laffer curve for many advanced economies by choosing many parameters of a specific parametric model to match data targets and choose a number of other parameters with separate considerations. Public economists might be skeptical of the merits of such an approach. They might say, after reading section 3 of this paper, that all effort should be focused on estimating exactly two high-level elasticities. We think that macroeconomists should be open to such an approach.

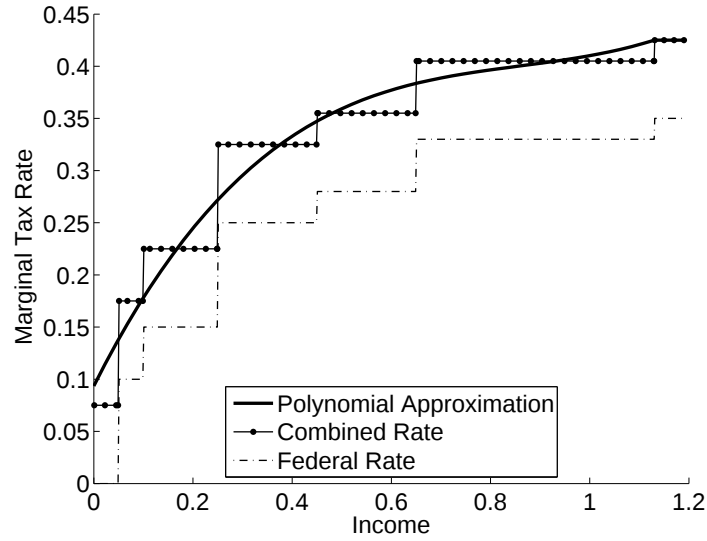
At the same time, macroeconomists should be skeptical of existing attempts in the literature that apply the sufficient statistic approach to predict the top of the Laffer curve. Taking the widely-used formula $\tau^* = 1/(1 + a\epsilon)$ at face value and then inputting values for (a, ϵ) into this formula based on state-of-the-art techniques and an ideal data set leads to badly misleading results. Using $a = 1.97$ from Table 1 and the range of estimates for ϵ from Table 2, the predicted top of the model Laffer curve ranges from a low of $\tau^* = 0.67$ to a high of $\tau^* = 0.98$ when the actual top of the Laffer curve is 52 percent. This paper argues that this occurs due to problems in theory and in methods.

This skepticism should not extend to the sufficient statistic approach in principle. The formula in Theorem 1 applies to a wide range of relevant static and dynamic models. It applies to the dynamic models employed by Badel and Huggett (2014), Guner, Lopez-Daneri and Ventura (2014) and Kindermann and Krueger (2014). It accurately predicts the top of the Laffer curve in the examples considered in section 4. This occurs even when the economy is quite far from the top of the Laffer curve. The remaining problem is one of methods. Existing methods for estimating the elasticities in the formula need to be completely reworked with the structure of dynamic models in mind and successfully applied to data. After this occurs celebrations can begin.

References

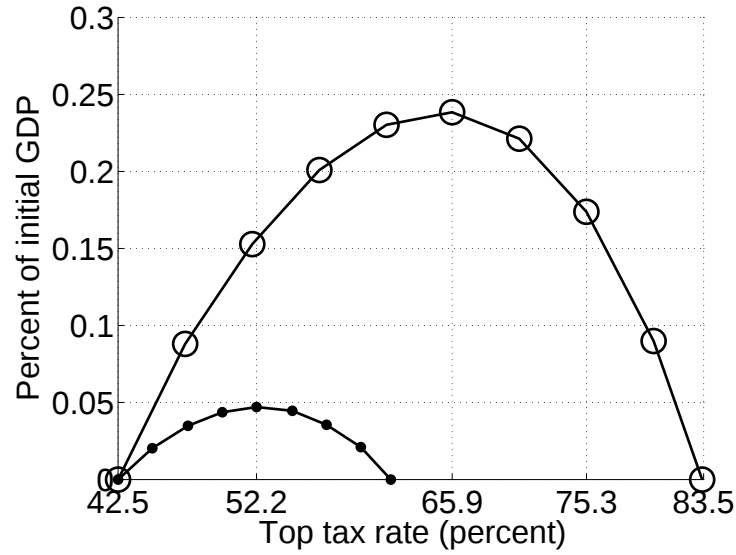
1. Badel, A. and M. Huggett (2014), Taxing Top Earners: A Human Capital Perspective, manuscript.
2. Chetty, R. (2009), Sufficient Statistics for Welfare Analysis: A Bridge Between Structural and Reduced-Form Methods, *Annual Review of Economics*, 1, 451- 88.
3. Diamond, P. (1965), National Debt in a Neoclassical Growth Model, *American Economic Review*, 55, 1126- 50.
4. Diamond, P. and E. Saez (2011), The Case for a Progressive Tax: From Basic Research to Policy Recommendations, *Journal of Economic Perspectives*, 25, 165-90.
5. Goolsbee, A. (2000), What Happens When You Tax the Rich? Evidence from Executive Compensation, *Journal of Political Economy*, 108, 352- 78.
6. Gruber, J. and E. Saez (2002), The Elasticity of Taxable Income: Evidence and Implications, *Journal of Public Economics*, 84, 1-32.
7. Guner, N., Lopez-Daneri, M. and G. Ventura (2014), Heterogeneity and Government Revenues: Higher Taxes at the Top?, manuscript.
8. Heathcote, J., Storesletten, K. and G. Violante (2009), Quantitative Macroeconomics with Heterogeneous Households, *Annual Review of Economics*, 1, 319-54.
9. Kindermann, F. and D. Krueger (2014), The Redistributive Benefits of Progressive Labor and Capital Income Taxation, manuscript.
10. Mirrlees, J. (1971), An Exploration into the Theory of Optimum Income Taxation, *Review of Economic Studies*, 38, 175- 208.
11. Mirrlees, J. and S. Adams, T. Besley, R. Blundell, S. Bond, R. Chote, M. Grammie, P. Johnson, G. Myles and J. Poterba (eds.)(2010), *Dimensions of Tax Design: The Mirrlees Review*, Oxford University Press, Oxford, UK.
12. Piketty, T and E. Saez (2013), Optimal Labor Income Taxation, in *Handbook of Public Economics*, Volume 5, editors A. Auerbach, R. Chetty, M. Feldstein and E. Saez, Elsevier.
13. Saez, E. (2001), Using Elasticities to Derive Optimal Income Tax Rates, *Review of Economic Studies*, 68, 205-29.
14. Saez, E., Slemrod, J. and S. Giertz (2012), The Elasticity of Taxable Income with Respect to Marginal Tax Rates: A Critical Review, *Journal of Economic Literature*, 50:1, 3-50.
15. Trabandt, M. and H. Uhlig (2011), The Laffer Curve Revisited, *Journal of Monetary Economics*, 58, 305- 27.

Figure 1. Progressive Component of Tax System, $T^{prog}(\cdot)$



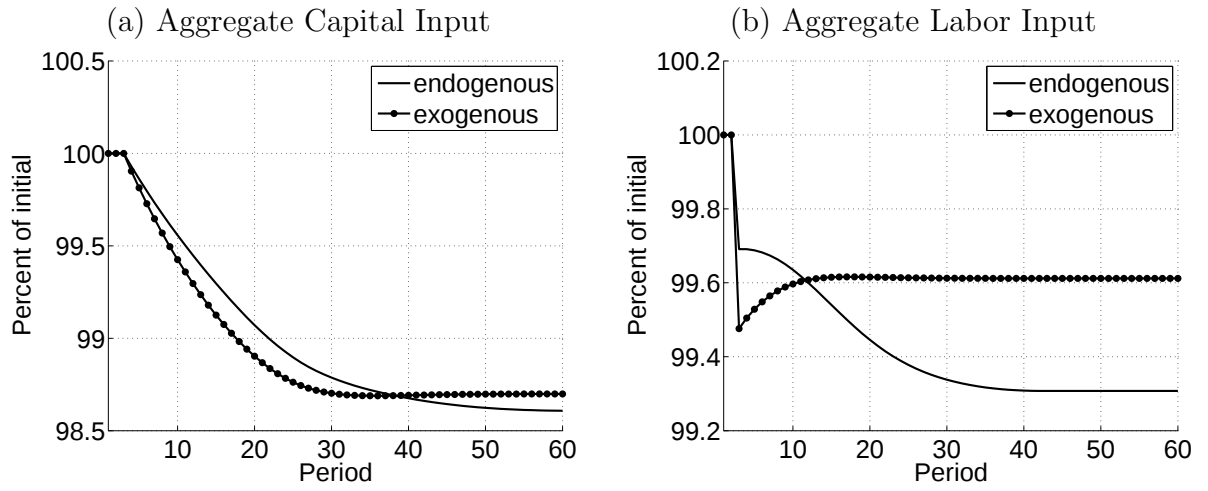
Note: The horizontal axis measures income in multiples of the 99th percentile of income.

Figure 2. Laffer Curves: Endogenous and Exogenous Human Capital



Note: Large circles describe the Laffer curve for exogenous human capital model. Dots describe the Laffer curve for the endogenous human capital model.

Figure 3. Transition Paths



Note: Aggregate capital and labor input are normalized to equal 100 in period 1. The tax reform occurs in model period 3.