

Segmented Asset Markets and the Distribution of Wealth

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ABSTRACT

Using the 2004 Survey of Consumer Finances data, we find significant heterogeneity in household portfolio choice across ages and wealth levels. First, 30 percent of the U.S. households hold high-yield assets defined as stocks, bonds, and mutual and hedge funds. Second, the probability of a household participating in high-yield asset markets is rising with age and wealth level. Lastly, wealthy households tend to hold more of their financial assets as high-yield assets.

We study a quantitative overlapping-generations model to explore the effects of segmented assets markets on wealth distribution. At any time, a household is identified by its age, wealth and labour income. There are two asset markets, one associated with a high-yield asset, the other with a low yield asset. At any age, a household gains access to the high-yield asset market through payment of a fixed cost. We estimate earnings processes for households from the Panel Study of Income Dynamics using the minimum method of moments. We choose the distribution of fixed costs associated with access to the high yield asset market, in our model economy, to be consistent with our measure of asset market segmentation from the SCF data.

Solving for stationary equilibrium, we study the effect of different returns on savings on the distribution of wealth. We first find that segmented asset markets lead to a substantial increase in wealth dispersion across households and move the model economy to empirical measures of overall inequality. Specifically, an alternative model without market segmentation, wherein all households earn the average return on assets in our benchmark segmented markets economy, generates a Gini coefficient for wealth that is approximately 7 to 10 percent lower. Moreover the concentration of wealth in the top percentiles is substantially less. Second, we reproduce the empirical findings that households are more likely to hold high-yield assets if they are older and wealthier. Given recent empirical evidence on heterogeneity in households' portfolios, our results suggest asset market segmentation is an important source of wealth inequality.

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1 Introduction

Understanding the large levels of inequality observed in the distribution of wealth has been a primary focus of quantitative macroeconomic research in recent years. Most studies have examined environments in which all households have access to the same assets. However, household data shows that households vary not only in the levels of their wealth, but also in the types of assets they hold. Wealthier households tend to save using assets that offer higher rates of return. Despite this evidence of household portfolio heterogeneity, little effort has been devoted to understanding how the distribution of wealth is shaped by differences in the return on savings across households.

We study a quantitative overlapping-generations model with segmented asset markets. Households differ by age, wealth, income, and the types of assets they may hold. They work in the first stage of their lives, then retire. Each period, households consume and borrow or save. Savings are invested in either of two assets which differ in their rates of return. Access to the higher return asset market requires the one time payment of a fixed cost. Households without current access have a discrete choice of whether or not to pay their current fixed cost to gain access to the high yield asset market, or whether to continue saving using the low yield asset. There are two sources of risk in the model. First, individual income is risky and persistent. Second, we allow for idiosyncratic uncertainty in the costs households face to access the high yield asset market.

Our benchmark model economy is calibrated to be consistent with household earnings from the PSID and data on household portfolio choice. We allow for an earnings process with both a persistent component and a transitory component. The estimated earnings process is very volatile, and earnings risk drives savings as households attempt to smooth consumption against fluctuations in income. We set the rate of return difference between the high and low return assets to be consistent with evidence on the long-run average difference in rates of return to liquid assets and less liquid, higher return assets.

Using the 2004 SCF data, we find 30 percent of households hold more than 1 percent of their total assets in the form of high return assets.¹ Our model economy is consistent

¹Our findings are consistent with that of Kacperczyk, Nosal, and Stevens (2015), who compute a

with this finding. Further, it reproduces the rising participation rate, seen in the data, in both age and age and wealth.² Calibrating our model to 2004 earnings data, we find that asset market segmentation results in a 7 percent increase in the Gini coefficient for wealth as well as a rise in concentration for wealth amongst households in the top percentiles of the distribution. A less variable income process, consistent with the estimated variances for PSID earnings data for 1983, implies a 10 percent increase in the Gini coefficient for wealth. Our results suggest that empirically plausible differences in the return to savings across households are an important source of wealth inequality.

The contribution of asset market segmentation to inequality hinges critically on our realistic quantitative overlapping generations model. In particular, households' savings rates vary not only in response to the real interest rate and income risk, but also over age. Our benchmark economy involves a high level of income risk. This raises the savings rate of households without access to the high return asset market. Furthermore, the share of households using the high return asset rises over age and older households have lower savings rates. A greater proportion of older households having access to the high return asset market implies slower wealth growth among such households, as they have lower savings rates, than would arise if the average age of households in both asset markets was similar. Precautionary savings by low return households increases their wealth. Both effects reduce the impact of market segmentation on wealth inequality. This suggests the importance of a realistic demographic structure, and empirically consistent earnings process, for a quantitative evaluation of market frictions that may affect wealth inequality.

In the U.S., the distribution of wealth is highly concentrated and skewed to the right. For instance, excluding the self-employed, the U.S. wealth Gini coefficient was around 0.77 in 2004, and the fraction of wealth held by the top 1 percent of U.S. households was around 29 percent. A large literature including Aiyagari (1994), Huggett (1996), Krusell and Smith (1998) and Castaneda et al. (2003) has attempted to explain observed earnings and wealth inequality in the U.S. However, most of these models fail to generate both the significant

measure of similar high return asset market segmentation.

²The substantial heterogeneity in the share of household wealth invested in high return assets is consistent with the market participation evidence presented on Vissing-Jorgensen (2002).

concentration of wealth in the top tail and a high fraction of wealth-poor households in the bottom tail. For example, Huggett (1996) produces the right U.S. wealth Gini coefficient, mainly driven by a counterfactually high fraction of households with zero or negative asset holdings. Although Castenada et al. (2003) successfully reproduce both tails of the wealth distribution and the U.S. wealth Gini coefficient in a modified stochastic neoclassical growth model, their results are derived in an environment characterized by a stochastic life-cycle and they choose the earnings of the highest income households. Stochastic aging builds a strong precautionary saving motive, potentially boosting wealth accumulation.

Our work is related to the entrepreneurship with loan market frictions models of Quadrini (1998) and Cagetti and De Nardi (2006). These models focus on explaining wealth inequality through differences in earnings by entrepreneurs relative to workers. For example, Cagetti and De Nardi (2006) assume that entrepreneurs borrow capital in loan markets with limited enforceability of debt contracts. Households with profitable entrepreneurial opportunities accumulate large levels of wealth in order to increase their collateral and thus the level of capital they are able to borrow. Assuming a two-stage life-cycle, with stochastic aging, and allowing for altruism, Cagetti and De Nardi are able to reproduce the empirical Gini coefficient and the concentration of wealth amongst the top percentiles. They identify entrepreneurs in the data with the self-employed. Excluding the self-employed from earnings data, and from our measure of wealth, we focus on the importance of segmented asset markets on wealth inequality.

Our work is also related to Kaplan and Violante's (2014) model where households must pay a fixed cost each time they adjust their holdings of high return assets. In contrast, we assume a one time fixed cost of access to high return assets, and focus on the distribution of wealth.

The remainder of the paper is organized as follows. Section 2 summarizes the U.S. wealth distribution in 1983 and 2004. Section 3 presents the model economy. Section 4 discusses our calibration and earnings process estimation. Section 5 presents results, and Section 6 concludes.

2 Wealth Inequality and Household Portfolio Choice

This section describes the stylized facts of the wealth distribution and household portfolio choice in the U.S using the Survey of Consumer Finances (SCF) data. SCF data is a household triennial data conducted by the Board of Governors of the Federal Reserve System in cooperation with the Statistics of Income Division of the IRS since 1983. The SCF employs a dual frame sample design, one frame is multi-stage national area probability design which provides information on the characteristics of population, and the other is a list sample to provide a disproportionate representation of wealthy households.³

Net worth in the survey is defined as total assets minus total debt. Total assets include financial assets and nonfinancial assets. Financial assets include current values and characteristics of deposits, cash accounts, securities traded on exchanges, mutual funds and hedge funds, annuities, cash-value of life insurance, tax-deferred retirement accounts, and loans made to other people. Nonfinancial assets include current values of principal residences, other real estate not owned by a business, corporate and non-corporate private businesses, and vehicles. Total debt includes the outstanding balances on credit cards, lines of credit and other revolving accounts, mortgages, installment loans for vehicles and education, loans against pensions and insurance policies, and money owed to a business owned at least in part by the family. We select households where the head of households' age is between 25 and 85 years old. Net worth is expressed in 2013 dollars. Full sample weights are used to calculate the wealth inequality measures.

The income process estimated using the PSID data selects households where income is not from self-employment. For consistency, we also measure wealth inequality excluding households who are self-employed. Table 1 summarizes the wealth distribution in the U.S. economy. The wealth distribution of the U.S. is highly concentrated and skewed to the right. Across years, more than 20 percent of total wealth is held by the top 1 percent wealth rich households and approximately 65 percent of wealth in the economy is held by the top 10 percent of households. The wealth Gini coefficient is 0.771 in 2004 which is

³Beginning with the 1989 survey data, the SCF methodology was substantially changed including a multiple-imputation method implemented for missing variables.

higher than the earnings Gini around 0.5.⁴ This is the well-known fact that wealth is more concentrated and unequally distributed than earnings in the U.S.⁵ The share of households with zero or negative asset holdings is around 8 percent in 2004.⁶

Table 1. The Wealth Inequality in the U.S. economy

Year	1%	5%	10%	50%	90%	≤ 0	Gini
1983	36.6	57.6	68.1	96.7	100	8.7	0.789
2004	28.7	50.5	63.5	96.6	100	8.5	0.771

Table 1 shows the share of wealth held by the top 1, 5, 10, 50

and 90 wealthiest households, the wealth Gini coefficient

in the U.S. economy. Source: SCF data.

Among financial assets in SCF survey, we define high-yield assets as stocks, bonds, money market mutual funds, and the share of annuities, trusts, and retirement accounts invested in stocks.⁷ We find that the share of households in 2004 with more than 1 percent of total assets held in high-yield assets is 30 percent. This definition of high-yield assets is comparable to the measure used by Kacperczyk, Nosal, and Stevens (2015). Using the SCF data from 1989 to 2013, they also show that 34 percent of households, on average, invest in stocks, bonds, or mutual funds, or using a brokerage account.

Figure 1 shows the share of households across 5-year age groups, holding more than 1 percent of their total assets as high yield assets. Examining households between 20 and 24 years old, only 10 percent hold high-yield assets. However, participation in high-yield asset markets increases with age until the retirement age of 64. In the 60-64 group, more than 40 percent of households hold their assets in high-yield assets. After retirement, the share

⁴To accurately measure the wealth Gini when some households have negative assets, we follow Chen, Tsaur, and Rhai (1982).

⁵U.S. earnings inequality has risen over the last 30 years. However, there is no clear evidence of corresponding trend in wealth inequality. See Heathcote, Perri, and Violante (2010).

⁶If we include self-employed households, the 2004 wealth distribution is more concentrated. The top 1 percent of households hold around 33 percent of total wealth and the top 10 percent of households hold 69 percent of total wealth. The wealth Gini coefficient is 0.80 and the share of zero or negative asset holdings is 8.5 percent of the sample. However, the inclusion of self-employed households in 1983 data changes the wealth inequality measures little.

⁷For annuity, trust, and retirement accounts, the SCF data provides a variable for the share of amount invested in stocks. However, it does not provide the amount of these accounts invested in other kinds of potentially high-yield assets.

of households with high-yield assets starts to decrease, falling to around 30 percent when households are 80-84 years old.

Figure 2 describes the share of households holding high-yield assets over wealth deciles as well as the share of high-yield assets as a fraction of total financial assets. Less than 10 percent of households in the first wealth decile hold high-yield assets compared to around 80 percent of households in the tenth wealth decile. Participation in high-yield asset markets increases with the level of household wealth. Figure 2 shows wealthy households investing more of their assets in high-yield assets. For example, top 10 percent wealthiest households hold more than 50 percent of their financial assets in high return assets compared to 20 percent for the bottom 10 percent households.⁸

3 Economic Model

3.1 Overview

In the model economy, there are four sets of agents: households, perfectly competitive financial intermediaries, firms, and government. Household demographic structure involves J overlapping generations. Each generation has a fraction of μ_j of the population and the total population is normalized to one. Households value consumption in each period and discount future utility by $\beta \in (0, 1)$. They are endowed with a unit of time in each period. Households enter the labor market at age $j = 1$, retire at age $J_r = 40$, and $J = 60$ is the last age. During their working-life, an agent's efficiency units of labor depends on labor market experience, $l(j)$, and idiosyncratic productivity shocks, ϵ . After retirement, households receive a lump-sum social security payment b .

The economy consists of two segmented financial asset markets, a high-yield asset market and a low-yield asset market. A household may gain permanent access to the high-yield asset market by paying a fixed cost to financial intermediaries. At the start of each period, any household is identified by age, j , assets, a , an idiosyncratic productivity shock, ϵ , and its access to the high-yield asset market. For those without such access, they are also iden-

⁸This is consistent with the finding by Vissing-Jorgensen (2002) that there is substantial heterogeneity in the share of wealth invested in stocks among the stock market participants.

tified by a fixed access cost to financial intermediaries, $\zeta \in (0, \bar{\zeta})$. This cost is denominated in units of output and drawn from a time-invariant distribution $H(\zeta)$ common across to households. Households in low-yield asset market can pay their fixed cost to access the high-yield asset market before consumption and saving decisions. Once a household pays its fixed cost ζ , the return on their asset holdings is r_h . Otherwise, the return on asset remains r_l , where $r_l < r_h$.

Consumption and investment goods are produced by a representative firm with aggregate capital K and labor N , through a strictly concave, constant returns to scale production function $F(K, N)$. An aggregate capital K is a constant elasticity of substitution function, $G(K_1, K_2)$, of two types of capital.

$$G(K_1, K_2) = \left[\lambda K_1^{\frac{\theta-1}{\theta}} + (1 - \lambda) K_2^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

where K_1 and K_2 are distinct capital stocks associated with different sets of households. Specifically, K_1 represents the total capital held by households investing in the high return asset market and K_2 represents the the capital stock held by low return asset market households. The common rate of depreciation for these capital stocks is $\delta \in (0, 1)$. Finally, the government imposes taxes on labor income τ_n . Revenues are used to finance social security payments to retirees.

3.2 A Household in a High-Yield Asset Market

We now describe the behavior of households with existing access to high-yield asset market. Let $V^h(j, a, \epsilon)$ represent the expected discounted value of a household at age j with assets a and an idiosyncratic productivity shock ϵ . Note that households who have already paid the fixed cost gain permanent access to the high-yield asset market. Given a , the household chooses consumption c , saving a' , and labor supply, n . The problem of a working household is

$$V^h(j, a, \epsilon_i) = \max_{c, a', n} \left\{ u(c) + \beta \sum_{l=1}^{N_\epsilon} \pi_{i,l} V^h(j+1, a', \epsilon_l) \right\} \quad (1)$$

subject to

$$c + a' \leq (1 + r_h)a + (1 - \tau_n)wl(j)\epsilon n$$

$$a' \geq \underline{a}, \quad c \geq 0, \quad n \in [0, 1]$$

where $\underline{a}$ is the borrowing limit and $l(j)$ is labor market experience.⁹ Labor income is taxed at rate τ_n .

After retirement, the household does not work $n = 0$ and receives a lump-sum social security benefit payment b . The problem of a retiree is

$$V^h(j, a, \epsilon_i) = \max_{c, a'} \left\{ u(c) + \beta \sum_{l=1}^{N_\epsilon} \pi_{i,l} V^h(j+1, a', \epsilon_l) \right\} \quad (2)$$

subject to

$$c + a' \leq (1 + r_h)a + b$$

$$a' \geq \underline{a}, \quad c \geq 0$$

3.3 A Household in a Low-Yield Asset Market

Households start their life without the access to the high-yield asset market. At the start of each period, a household draws a fixed financial intermediary cost, ζ , and decides its portfolio choice prior to consumption and saving decisions. Let $V^l(j, a, \epsilon)$ represent the value function of a household in the low-yield asset market.

$$V^l(j, a, \epsilon_i) = \int_0^{\bar{\zeta}} \max \{ V_l^l(j, a, \epsilon), V_h^l(j, a, \epsilon, \zeta) \} H(d\zeta) \quad (3)$$

where V_l^l is the value function of a household saving in low return asset and V_h^l is the value function of a household saving in high return asset.

⁹Following Heathcote, Storesletten, and Violante (2010), we assume a return to age as a proxy for labor market experience.

Savings in low-yield asset

Given asset holdings a and idiosyncratic productivity shock ϵ , V_l^l solves a working household's problem

$$V_l^l(j, a, \epsilon_i) = \max_{c, a', n} \left\{ u(c) + \beta \sum_{l=1}^{N_\epsilon} \pi_{i,l} V^l(j+1, a', \epsilon_l) \right\} \quad (4)$$

subject to

$$c + a' \leq (1 + r_l)a + (1 - \tau_n)wl(j)\epsilon n$$

$$a' \geq \underline{a}, \quad c \geq 0, \quad n \in [0, 1]$$

Savings in high-yield asset

Let $V_h^l(j, a, \epsilon, \zeta)$ represent the value function of a household who pays a fixed financial intermediary cost ζ in current period and saves in the high return asset. V_h^l solves a working household's problem

$$V_h^l(j, a, \epsilon_i, \zeta) = \max_{c, a', n} \left\{ u(c) + \beta \sum_{l=1}^{N_\epsilon} \pi_{i,l} V^h(j+1, a', \epsilon_l) \right\} \quad (5)$$

subject to

$$c + a' \leq (1 + r_l)a + (1 - \tau_n)wl(j)\epsilon n - \zeta$$

$$a' \geq \underline{a}, \quad c \geq 0, \quad n \in [0, 1]$$

A retiree's problem, in the low-yield asset market, is the analogue to (2) above.

3.4 Recursive Equilibrium

We define stationary recursive equilibrium. The distribution of households varies over age, wealth, labour productivity and access to the high-yield asset market. Let $\mathbf{J} = \{1, \dots, J\}$ represent the set of indices for household age and $\mathbf{E} = \{\varepsilon_1, \dots, \varepsilon_{N_\varepsilon}\}$ define the support for household shocks to earnings. Households wealth $a \in \mathbf{A} = (\underline{a}, \infty)$ and access to the high-yield asset market is described using $s \in \{0, 1\}$ where $s = 1$ if a household has paid its cost of access and 0 otherwise. Lastly, let $\mathbf{Z} = (0, \infty)$ be the space for fixed costs drawn by households. The product space, $\mathbf{S} = \mathbf{J} \times \mathbf{A} \times \mathbf{E} \times \{0, 1\}$ describe the space for the distribution of households at the start of the period before the realisation of fixed costs. These costs of access to the high-yield asset market are relevant only for households with $s = 0$. Define $\mathcal{S}$ as the Borel algebra generated by the open subsets of $\mathbf{S}$. We define $\psi : \mathcal{S} \rightarrow [0, 1]$ as a probability measure for households.

Households of age 1 all begin with $a_0 \geq 0$ and an initial productivity drawn from $\pi^0 \sim \log N(0, \sigma_\pi^2)$, the invariant distribution for $\{\pi_{i,j}\}_{i,j=1}^{N_\varepsilon}$. We assume all such households have $s = 0$. Let μ_1 be the number of initial households with age $j = 1$, their distribution is described by

$$\psi(1, A, \varepsilon_i, 0) = \pi_i^0 \mu_1 \text{ iff } a_0 \in A. \quad (6)$$

In subsequent period, $j + 1$, $j = 1, \dots, J - 1$, the distribution of households in the high asset market is given by the following. The first term describes low-asset market households that pay their fixed cost draws, draw a labour productivity of ε_j at the start of the next period and save a level of wealth in the measurable set A . The second term involves existing high asset market households of age j ,

$$\begin{aligned} \psi(j+1, A, \varepsilon_j, 1) &= \sum_{i=1}^{N_\varepsilon} \pi_{ij} \int_{\{(a, \varepsilon_i, \zeta) | g^l(j, a, \varepsilon_i, \zeta) \in A \text{ and } \chi(j, a, \varepsilon_i, \zeta) = 1\}} \psi(j, da, \varepsilon_i, 0) H(d\zeta) \\ &+ \sum_{i=1}^{N_\varepsilon} \pi_{ij} \int_{\{(a, \varepsilon_i) | g^h(j, a, \varepsilon_i) \in A\}} \psi(j, da, \varepsilon_i, 1). \end{aligned} \quad (7)$$

The distribution of households in the low asset market of age $j + 1$, $j = 1, \dots, J - 1$, is given by

$$\psi(j + 1, A, \varepsilon_j, 0) = \sum_{i=1}^{N_\varepsilon} \pi_{ij} \int_{\{(a, \varepsilon_i, \zeta) | g^l(j, a, \varepsilon_i, \zeta) \in A \text{ and } \chi(j, a, \varepsilon_i, \zeta) = 0\}} \psi(j, da, \varepsilon_i, 0) H(d\zeta) \quad (8)$$

A *recursive competitive equilibrium* is a set of functions $(V^l, V^h, V_l^l, V_h^l, g^l, g^h, n^l, n^h, c^l, c^h, \chi)$ and prices (r_l, r_h, w) such that:

- i (V^l, V^h, V_l^l, V_h^l) solve (1) - (5), $g^l : \mathbf{J} \times \mathbf{A} \times \mathbf{E} \times \mathbf{Z} \rightarrow \mathbf{A}$ is the associated optimal policy for savings by a household that begins the period in the low asset market, $g^h : \mathbf{J} \times \mathbf{A} \times \mathbf{E} \rightarrow \mathbf{A}$ is the associated optimal policy that attains the maximum in (1) and (2), $n^l : \mathbf{J} \times \mathbf{A} \times \mathbf{E} \times \mathbf{Z} \rightarrow [0, 1]$ is the associated optimal policy for labor supply by a household that begins the period in low asset market, $n^h : \mathbf{J} \times \mathbf{A} \times \mathbf{E} \times \mathbf{Z} \rightarrow [0, 1]$ is the associated optimal policy for labor supply by a household that begins the period in high asset market, $c^l : \mathbf{J} \times \mathbf{A} \times \mathbf{E} \times \mathbf{Z} \rightarrow \mathbf{R}_+$ is the associated optimal policy for consumption by a household that begins the period in low asset market, $c^h : \mathbf{J} \times \mathbf{A} \times \mathbf{E} \times \mathbf{Z} \rightarrow \mathbf{R}_+$ is the associated optimal policy for consumption by a household that begins the period in high asset market, and $\chi^l : \mathbf{J} \times \mathbf{A} \times \mathbf{E} \times \mathbf{Z} \rightarrow \{0, 1\}$ is the decision rule for paying the fixed cost of access to the high-yield asset market.

ii Markets clear

$$K_1 = \sum_{j=1}^J \sum_{i=1}^{N_\varepsilon} \int_{\mathbf{A}} a \psi(j, da, \varepsilon_i, 1)$$

$$K_2 = \sum_{j=1}^J \sum_{i=1}^{N_\varepsilon} \int_{\mathbf{A}} a \psi(j, da, \varepsilon_i, 0)$$

$$N = \sum_{j=1}^{J_r-1} \sum_{i=1}^{N_\varepsilon} \left\{ \int_{\mathbf{A}} l(j) \epsilon_i n^l \psi(j, da, \varepsilon_i, 0) + \int_{\mathbf{A}} l(j) \epsilon_i n^h \psi(j, da, \varepsilon_i, 1) \right\}$$

$$C + \delta K_1 + \delta K_2 = F(K, N) - \sum_{j=1}^J \sum_{i=1}^{N_\varepsilon} \int_{\{(j, a, \varepsilon_i, \zeta) | \chi(j, a, \varepsilon_i, \zeta) = 1\}} \zeta \psi(j, da, \varepsilon_i, 0) H(d\zeta)$$

, where $C = \sum_{j=1}^J \sum_{i=1}^{N_\varepsilon} \left\{ \int_{\mathbf{A}} c^l \psi(j, da, \varepsilon_i, 0) + \int_{\mathbf{A}} c^h \psi(j, da, \varepsilon_i, 1) \right\}$.

iii Government budget is balanced:

$$\tau_n w N = b \sum_{j=J_r}^J \sum_{i=1}^{N_\epsilon} \left\{ \int_{\mathbf{A}} \psi(j, da, \varepsilon_i, 0) + \int_{\mathbf{A}} \psi(j, da, \varepsilon_i, 1) \right\}$$

iv Prices are competitively determined.

$$r_l = D_1 F(K, N) D_2 G(K_1, K_2) - \delta$$

$$r_h = D_1 F(K, N) D_1 G(K_1, K_2) - \delta$$

$$w = D_2 F(K, N)$$

4 Calibration

4.1 Parameters

We set the length of a period to one year, and assume that households begin working at age 25. Given this, we set $J = 60$ and assume that retirement is at age 65, implying $J_r = 40$. The subjective discount factor, $\beta = 0.99$. We assume that period utility function is iso-elastic and set the inverse of the elasticity of intertemporal substitution, $\sigma = 1.5$. In our present analysis, we assume that the distribution of fixed costs required to access the high yield asset market is log-normal. We set the variance of the corresponding Normal distribution to 1, the mean to 3.91. This implies that roughly 1/3 of households save using high return assets. This is consistent with our findings from the SCF, reported in section 2, on the share of households holding more than 1 percent of their total assets as high yield assets. We interpret such households as having access to high yield assets, and we view their return on investment as representing the mean return for any portfolio that includes such assets.

In our present analysis, in lieu of a general equilibrium approach, we follow the approach in Heathcote, Storesletten and Violante (2012) and Kaplan and Violante (2014), and set

the real interest rate exogenously. We assume that the real interest rate paid on low return assets is one percent, broadly consistent with evidence from the Flow of Funds of the real return to liquid assets such as checking and savings accounts and other components of $M1$, $r_l = 0.01$. Next we set the real return on the high yield asset, $r_h = 0.05$, to imply an average real return on savings in our benchmark economy of just above 4 percent a year. The real wage per efficiency unit of labor is normalised as $w = 1$. Future versions of our analysis will solve a general equilibrium model that recovers these returns using two capital stocks as described in section 3. We briefly describe our solution method for this step. We first fix r_l and r_h and solve the equilibrium wage w to clear the goods market. Our CES aggregator function for capital implies that $\frac{r_h + \delta}{r_l + \delta} = \frac{\lambda}{1-\lambda} \left(\frac{K_1}{K_2}\right)^{-\frac{1}{\theta}}$. Given K_1 and K_2 , we set λ to be consistent with our measurement of r_l , r_h , and δ . This determines K . Lastly, we choose aggregate total factor productivity, z , to imply $r_l = zK^{\alpha-1}N^{1-\alpha}(1-\lambda)K^{\frac{1}{\theta}}K_2^{-\frac{1}{\theta}}$.

4.2 Earning Shocks Estimation

We estimate labor market experience and the earning shock process using PSID data between 1968 and 2011. The PSID data is a longitudinal survey of a sample of US individuals and families conducted annually from 1968 to 1997, and biennially since 1997. The original 1968 PSID sample combines the Survey Research Center (SRC) and the Survey of Economic Opportunities (SEO) samples. We use the U.S. population representative SRC sample.

Household earnings are defined as the sum of the annual earnings of the head of household and spouse. Total annual earnings include all income from wages, salaries, bonuses, overtime, commissions, professional practice and the labor part of farm and business income. We select households with no missing values for education and self-employment status where: 1) the head of households' age is between 25 and 59 years old, 2) neither the head or spouse has positive labor income but zero annual hours worked, 3) the hourly wage is not less than half of the minimum wage, 4) income is not from self-employment.¹⁰ Earnings are deflated with the CPI and expressed in 2013 dollars. After the sample se-

¹⁰We also estimated the earnings process using a sample that includes the income of the self-employed. The estimated earnings process changes a little (See below).

lection, there are only 23 top-coded observations out of 42,127 observations.¹¹ Following Autor and Katz (1999), we multiply all top-coded observations by a factor of 1.5 times the top-coded thresholds.

Let $y_{i,j,t}$ be the earnings of household i with head at age j in year t . We run an OLS regression of log earnings on time dummies; interaction term with education, edu_i , and time dummies, labor market experience, θ ; and experience-squared, θ^2 .¹² In the absence of modeling education, the estimated earnings process may understate the earnings dispersion in the economy. See Kim (2015) for an exploration of the role of college wage premium on earnings and wealth inequalities.

$$\log y_{i,j,t} = \beta_{t,0} + \beta_{t,1}edu_i + \beta_3\theta + \beta_4\theta^2 + \hat{r}_{i,j,t}$$

Recall that efficiency units of labor depend on both labor market experience, $l(j) = e^{(\beta_3\theta + \beta_4\theta^2)}$, and idiosyncratic productivity shocks. Figure 3 shows the estimated potential market experience function. Labor market experience increases earnings through the first 27 years when earnings rise 71 percent relative to their initial level. Thereafter, total earnings fall in experience until retirement.

The regression residuals $\hat{r}_{i,j,t}$ are assumed to be the sum of idiosyncratic productivity shocks, $\log \epsilon_{i,j,t}$, and measurement error, $\tilde{v}_{i,j,t}$.¹³ Idiosyncratic productivity shocks consist of both a persistent component, $\log \epsilon^p$, and transitory component, $\log \epsilon^v$. To be specific,

$$\begin{aligned}\log \epsilon_{i,j,t} &= \log \epsilon_{i,j,t}^p + \log \epsilon_{i,j,t}^v \\ \log \epsilon_{i,j,t}^p &= \rho \log \epsilon_{i,j-1,t-1}^p + \eta_{i,j,t}\end{aligned}$$

where $\eta_{i,j,t} \sim N(0, \sigma_{\eta_t}^2)$ and $\log \epsilon_{i,j,t}^v \sim N(0, \sigma_{v_t}^2)$.

Following Heathcote, Storesletten, and Violante (2010), we estimate year-varying shock

¹¹All these top-coded observations are for head of households before 1983. No spouse's earnings is top-coded.

¹²Labor market experience is measured as age minus years of schooling minus 5. In years missing the variable for years of schooling, we proxy years of schooling for the individuals with a college degree as 16 and for the individuals without a college degree as 12.

¹³Given the similarity in the estimates of earnings process to wage process estimated in Kim (2015), we use French (2002)'s estimate of a variance of measurement error in log hourly wages of 0.02.

variances $\{\sigma_{\eta_t}^2, \sigma_{v_t}^2\}$, the persistence of the shock $\{\rho\}$, and the variance of persistent shock for initial age σ_π^2 using minimum distance methods. We use survey data from 1968 to 2011, but only estimate the variances up to 2009 because of the finite sample bias at the end of sample period. As Heathcote et al. (2010) point out, although the variance of the persistent shock for the missing years can be theoretically pinned down by the available information from adjacent years, the resulting estimates for missing years are downward-biased because of insufficient information. Therefore, we follow their approach of Heathcote et al. (2010) for the estimates in the missing years by taking the weighted average of the two adjacent years. We estimate a total of $L = 86$ parameters. The parameter vector is denoted by $\mathcal{P}_{L \times 1}$.

The theoretical moment is defined as

$$m_{t,t+n}^j(\mathcal{P}) = E(r_{i,j,t} r_{i,j+n,t+n})$$

which is the covariance between earnings of individuals at age j in year t and $t+n$.¹⁴ To calculate the empirical moments, we group individuals into 44 year and 26 overlapping age groups. For example, the first age group contains all observations between 25 and 34 years old and second group contains those between 26 and 35 years old. The empirical moment conditions are

$$\hat{m}_{t,t+n}^j - m_{t,t+n}^j(\mathcal{P}) = 0$$

where $\hat{m}_{t,t+n}^j = \frac{1}{I_{j,t,n}} \sum_{i=1}^{I_{j,t,n}} \hat{r}_{i,j,t} \hat{r}_{i,j+n,t+n}$ and $I_{j,t,n}$ is the number of observations of age j at year t existing n periods later.

The minimum distance estimator solves the following problem

$$\min_{\mathcal{P}} [\hat{\mathbf{m}} - \mathbf{m}(\mathcal{P})]' [\hat{\mathbf{m}} - \mathbf{m}(\mathcal{P})]$$

¹⁴The closed form of theoretical moment is

$$E(r_{i,j,t} r_{i,j+n,t+n}) = \rho^n [\rho \min(j-1, t) \sigma_\pi^2 + \sum_{i=0}^{\min(j-1, t)} \rho^{2(i-1)} \sigma_{\eta_{t-i+1}}^2 + \mathbf{1} \sigma_{v_t}^2], \mathbf{1} = \begin{cases} 1 & \text{if } t = t+n \\ 0 & \text{otherwise} \end{cases}$$

where $\hat{\mathbf{m}}$ and $\mathbf{m}$ are vectors of empirical moments and theoretical moments with dimension 8, 342×1 . Note that we use the identity matrix as a weighting matrix.

Figure 4 and Figure 5 display the estimates of the variances of persistent and transitory earning shocks. The estimated persistence of earning shock, ρ , is 0.9825 and the age 1 variance of the persistent earning shock, σ_π^2 , is 0.1546. The benchmark model economy is calibrated to 2004 with the smoothed estimates of the persistent shock variance 0.0574 and transitory shock variance 0.1913.¹⁵

5 Results

In Table 2 we report several moments from the distribution of wealth and the share of households investing in the high-yield asset market, for our benchmark model economy.

Table 2: The Benchmark Economy							
1%	5%	10%	50%	90%	≤ 0	Gini	share
17.5	46.5	64.5	98.3	100	7.3	0.788	0.33

Table 2 shows the share of wealth held by the top 1, 5, 10, 50 and 90 wealthiest households, the wealth Gini coefficient, and the share of households in the high-yield asset market.

Our benchmark parameter set implies that, across all ages, 33 percent of households invest in the high-yield asset market. The higher return on wealth for such households increases inequality in the economy, and, as a result, the Gini coefficient is 0.79, essentially the same as the 2004 empirical counterpart. As another measure of inequality, the model reproduces a substantial fraction of the concentration of wealth seen among the top percentiles of households. The top 10 percent of households hold 65 percent of wealth in the model, compared to 64 percent in the 2004 SCF data. The skewness in the distribution of wealth is less pronounced in the top percentile. The top 1 percent of households hold 29

¹⁵We estimated earnings process with a sample that includes the income from self-employment. The estimated persistence of earnings is 0.9834 and the initial variance of persistent earning shock is 0.1553. The persistent shock variance estimate is 0.056 and transitory shock estimate is 0.1891. The estimates are essentially unchanged by the inclusion of income from self-employment.

percent of wealth in the data, the model counterpart is 18 percent. Despite an absence of strong motives for wealth accumulation among the wealthiest households, as seen in the entrepreneurial borrowing with collateral constraints model of Cagetti and De Nardi (2004) or the income risk model of Castenada et al. (2003), our model economy is able to explain a substantial fraction of the empirical distribution of wealth. This is surprising given the large number of wealthy households in the SCF, which provides the data on wealth, and the scarcity of such households in the PSID, which is the basis of our income process.¹⁶ Part of the reason for this success is our exclusion of the self-employed both in our earnings estimation and in our calculation of moments from the wealth distribution.

Figures 6a and 6b show the distribution of wealth across households saving in the low asset market and those saving in the high asset market. While both sets of households have identical stochastic processes for earnings, the low return on savings (one percent), in the former leads to compression in the distribution of wealth. There is considerably more dispersion in wealth among households in the high asset market where the effect of persistent earnings shocks on wealth are amplified through a higher rate of return on savings (five percent). Both distributions are concentrated in lower levels of earnings, a reflection of the considerable skewness in our log-normal earnings process. Figure 7 illustrates the Lorenz curve for the economy. The very flat initial shape of the Lorenz curve is the result of negative asset holdings by 7 percent of households in our model. This, in turn, implies that the richest 90 percent of households hold roughly 100 percent of wealth.

We now examine asset market segmentation, and its relation to households wealth and age. Figure 8 displays the fraction of households in the high-yield asset market across levels of wealth. Wealthier households tend to invest in the high yield asset market. First, higher levels of wealth make it more likely that a household will choose to pay any fixed cost draw and save in the high-yield asset market. Second, for households already in this market, their higher returns to savings drive more rapid wealth accumulation compared to households saving using the low return asset.¹⁷ Our model captures the monotonic relation,

¹⁶See Heathcote, Perri and Violante (2010) for a discussion of these issues.

¹⁷This is qualitatively consistent with the finding by Vissing-Jorgensen (2002) that participation in high return asset markets is more likely for wealthier households.

between household wealth and holdings of high yield assets, we find in the SCF and report in Figure 2.

As we reported in Figure 1, the SCF for 2004 shows the share of households holding high yield assets to be monotonically increasing with age until 65. In our model, Figure 9 captures this rise and, importantly, the largest share of households over ages, around 0.47, is close to the data. However, our model, in the absence of return risk, does not exhibit the decline in participation over retirement ages seen in the data.

As seen in Figure 9, participation in the high-return market increases with the years a household has been in the labour force. Initially, all households begin in the low asset market. Thereafter, as they accumulate wealth, the probability that they will pay any fixed cost increases and thus their probability of having access to high returns on savings. Moreover, as households repeatedly sample fixed cost draws over time, the probability of a relatively low cost which they will pay rises. Both forces lead to a rise in the fraction of households, as they gain experience, with access to the high return asset. Nonetheless, as households age, their lifetime compounded yield from higher returns decreases. Thus the threshold cost they are willing to pay for access, given any level of wealth, falls. As wealth accumulates slowly in the low return asset market, this leads to an eventual leveling off of the share of households in the high return market. Across ages, 1/3 of households save in the high return asset. This proportion is 0 for the youngest households, and levels off around 48 percent for the oldest.

Asset market segmentation increases wealth inequality in our economy. We establish this using by eliminating segmentation and setting a common return on savings for all households. This return is equal to the wealth-weighted average return, 4.56, in our benchmark model described in Table 2. As seen in Table 3, there is a 7 percent reduction in the wealth Gini which falls to 0.737, and there is less concentration of wealth in the top percentiles of the distribution.

Table 3: An Economy Without Asset Market Segmentation

1%	5%	10%	50%	90%	≤ 0	Gini
15.3	40.9	57.9	96.3	100	1.0	0.737

Table 3 shows the share of wealth held by the top 1, 5, 10, 50 and 90 wealthiest households, the wealth Gini coefficient, and the share of households in the high-yield asset market.

While asset market segmentation, for our benchmark model calibrated to 2004 earnings data, increases inequality, there are several mechanisms in our quantitative overlapping generations model, involving its demographic structure and volatile earnings process, that reduce its impact. We begin our exploration of the determinants of the distribution of wealth with a study of households' savings behaviour over age and across sectors.

Defining savings rates as the change in assets over cash on hand, Figure 10 shows wealth-averaged savings rates over years in the labour force, for households in the low return asset market, the high return asset market and across both markets. The savings rate over working age declines monotonically for households in the high return market. It is 14.15 percent at age 27 falling to 2.17 percent in the last period in the labour force at age 54. In the low return asset market, savings rates over working age are non-monotonic. When age is 27, the savings rate is 7.9 percent, falling to 1.35 percent at age 50 and thereafter rising to 4.23 when 65 years old. Both sets of households dissave in retirement. However, higher wealth accumulation leads to a larger fraction of cash on hand generated by capital income for households using the high yield asset market. As asset holdings are relatively large, compared to total cash on hand, this implies greater dissaving for such households. Consequently, high return households exhibit an average savings rate of -2.29 percent in their first year of retirement, at age 65, which falls to -45.2 percent at age 84. In comparison, savings rates become negative for low asset market households at age 66, when they fall to -6.27 percent. Thereafter they decline to -23.41 percent at age 84. Strong dissaving of financial assets after retirement leads to an overall wealth-weighted savings rate, across all households in the high return market, of -1.27 percent. In the low return market the savings rate is 0.0 percent.

Given positive savings rates before retirement, wealth is accumulated, and inequality generated, during years when of employment. Over working ages, the average savings rate for households in the low asset market is 2.0 percent; households in the high asset market the average savings rate is 6.3 percent. Thus households in the latter market respond to higher rates of return with higher savings rates. Nonetheless, as discussed above, these savings rates fall with age. Further, as seen in figure 7, households in the high asset market tend to be older. As a result, the life-cycle profile of savings rates reduces the accumulation of wealth by households with access to high yields on savings. This is the first mechanism, arising from our model's realistic demographic structure, which dampens the effect of asset market segmentation on wealth inequality. While high asset market households receive a higher return on savings, their wealth grows more slowly than it would in the absence of savings rates that decline with age.

The higher average savings rate, over working ages, of households using the high yield asset is in response to higher rates of return on savings earned in that market. In our alternative model without asset market segmentation, this responsiveness of savings rate results in an average over working ages of 4.8 percent. Compared to the 2.0 percent savings rate in the low return asset market of the segmented markets model, the increased savings in the alternative model increases wealth accumulation over households' working lives, and drives inequality even in the absence of asset market segmentation. As a result, the Gini coefficient without market segmentation remains high.

A second important force reducing the impact of segmented market on wealth inequality involves the relatively high savings rates in the low asset market. The high level of earnings risk in our 2004 calibration implies an average savings rate, over working life of 2.0 percent. This reduces the difference in savings rate between the two sectors, and thus the level of wealth inequality generated by market segmentation. Below, we explore a less volatile income process. The rise in inequality from differences in the rates of return earned by households is amplified when savings rates are less responsive to earnings risk. This is seen in Table 4, the top row reports results for a model where household earnings follow the estimated income process for 1983. This involves different variances of the persistent and transitory shocks to earnings, all other model parameters are unchanged when compared

to those used for Table 1. A less volatile earnings process leads to less inequality, and the Gini coefficient falls to 0.689. Compared to our benchmark model, calibrated to the 2004 estimated income process, there is less concentration of wealth among the wealthiest households.

Table 4: Asset market segmentation with 1983 earnings process

	1%	5%	10%	50%	90%	≤ 0	Gini
segmented markets economy	10.9	33.5	50.3	95.2	100	3.8	0.689
common market economy	8.5	27.4	42.9	91.9	99.9	1.4	0.619

Table 4 shows the share of wealth held by the top 1, 5, 10, 50 and 90 wealthiest households, the wealth Gini coefficient, and the share of households in the high-yield asset market.

Less variable income implies fewer high income households and thus fewer households paying any fixed cost to save using the high return asset. Thus, despite the same returns in the high and low yield asset markets, the share of households, across all levels of wealth and all ages, in the return market falls to 23.5 percent from 1/3 in the benchmark model. As a greater share of total wealth is held in the low return asset market, the average return on savings falls to 3.64 percent. When we eliminate asset market segmentation by providing all households this rate of return, the second row of Table 4 shows the Gini coefficient for wealth falling over 10 percent to 0.619. The larger effect of asset market segmentation arises, because the average savings rate over working life for households using the high return asset is 5.68 percent, 0.62 percentage points less than for such households in our benchmark economy. However, the savings rate falls 2 percentage points to 0 percent for households using the low return asset. This is the result of a reduced insurance motive for savings given less volatile earnings. Consistent with this finding, the savings rate associated with 1983 earnings and no asset market segmentation is now 2.67 percent, 2.14 percentage points lower than the corresponding rate for a 2004 earnings process.

When compared to our 2004 exercise, we see a larger difference in savings rates across sectors in the segmented asset market model calibrated to 1983 earnings data. This increases the effect of asset market segmentation on wealth inequality. Moreover, eliminating segmentation implies a lower savings rate than that observed in the 2004 case. This implies

that segmented asset markets drive a larger percentage increase in wealth inequality.

6 Concluding Remarks

Using the SCF data, we find significant heterogeneity in household portfolios across ages and wealth levels. First, 30 percent of the U.S. households hold high-yield assets. Second, the probability of a household participating in high-yield asset markets is rising with age and wealth level. Lastly, wealthy households tend to hold more of their financial assets as high-yield assets.

To evaluate the effect of asset market segmentation on the distribution of wealth, we develop a rich quantitative overlapping generations model where households face earnings risk and borrowing constraints and asset markets are segmented. We estimate the stochastic process for earnings from the PSID. Importantly, market segmentation is endogenous in our framework. Each period, households face a discrete choice of whether or not to pay a fixed cost and access a high yield asset market. This endogeneity has important implications for the effect of asset market segmentation on wealth inequality. It implies that wealthier, older households tend to invest in the high yield asset, consistent with our empirical findings.

In future work, we will explore a general equilibrium version of our model, that endogenises the return on high and low yield assets using two capital stocks as described using two capital stocks as described in section 4.

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Figure 1. Share of Households holding High-yield Assets

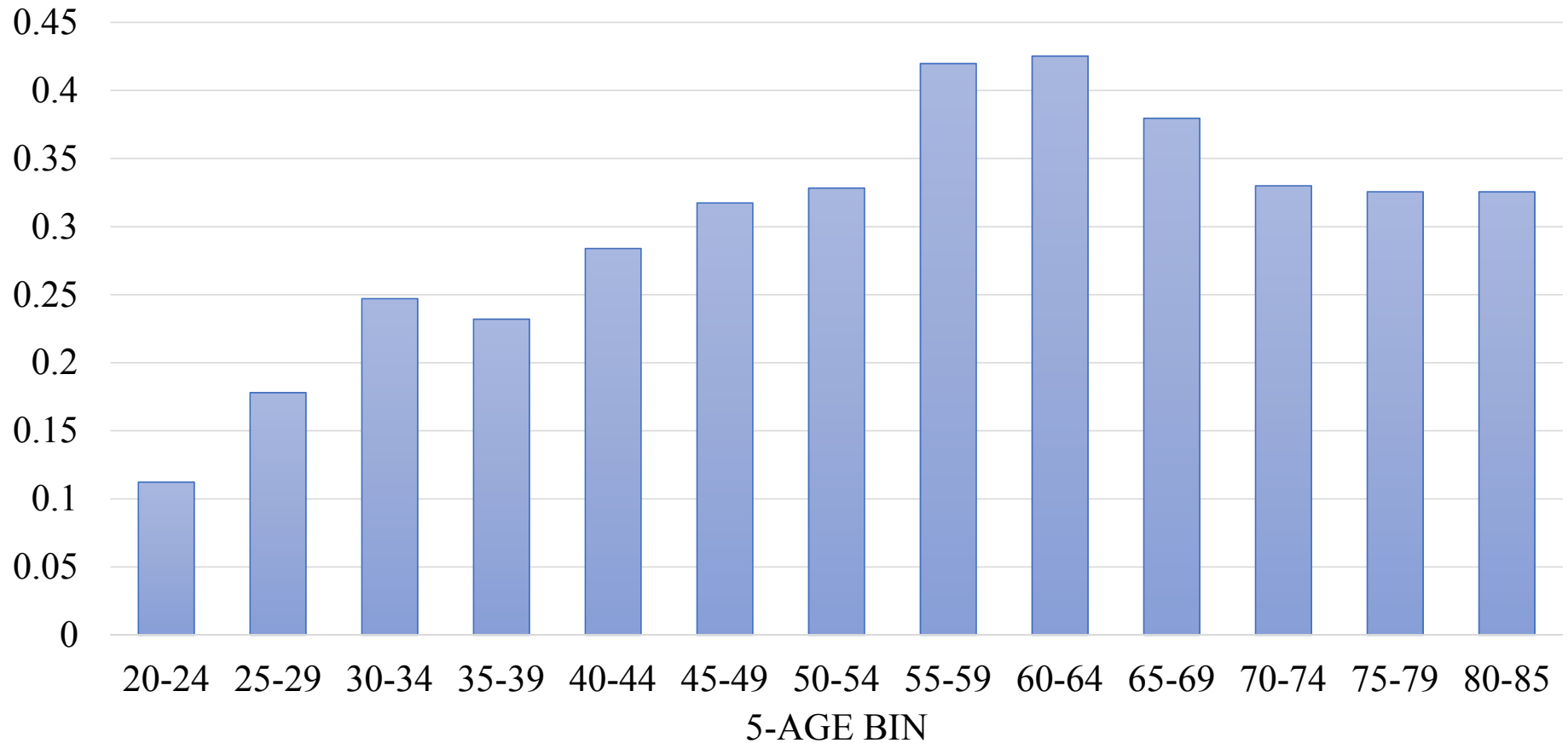


Figure 1. Share of households holding high-yield assets that are more than 1 percent of their total assets across age groups. Source: 2004 SCF data

Figure 2. High-yield Assets over Wealth

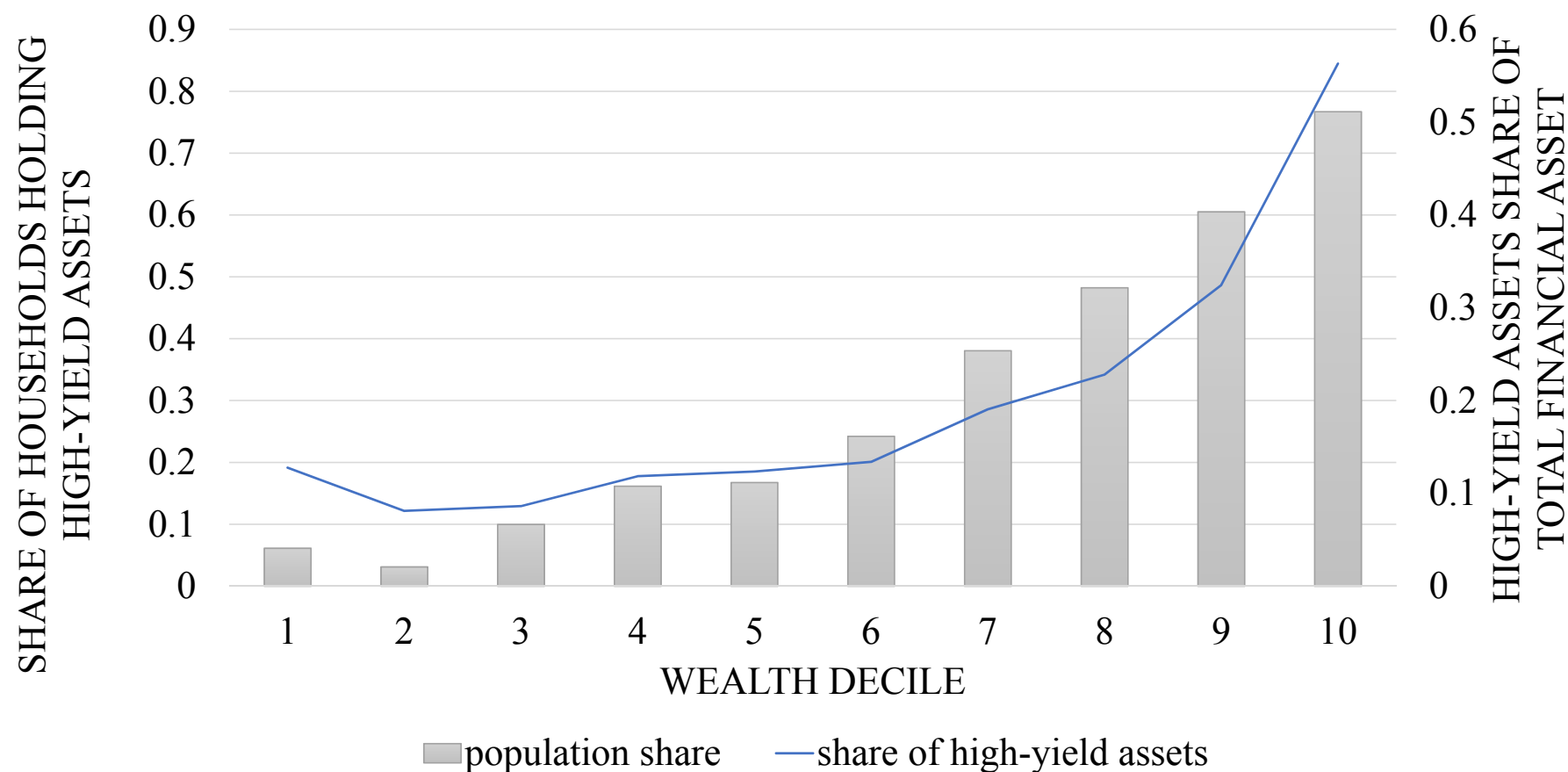


Figure 2. Left axis: Share of households holding high-yield assets exceeding 1 percent of total assets. Right axis: Share of high-yield assets as a proportion of total financial asset. Source: 2004 SCF data

Figure 3. Labor Market Experience

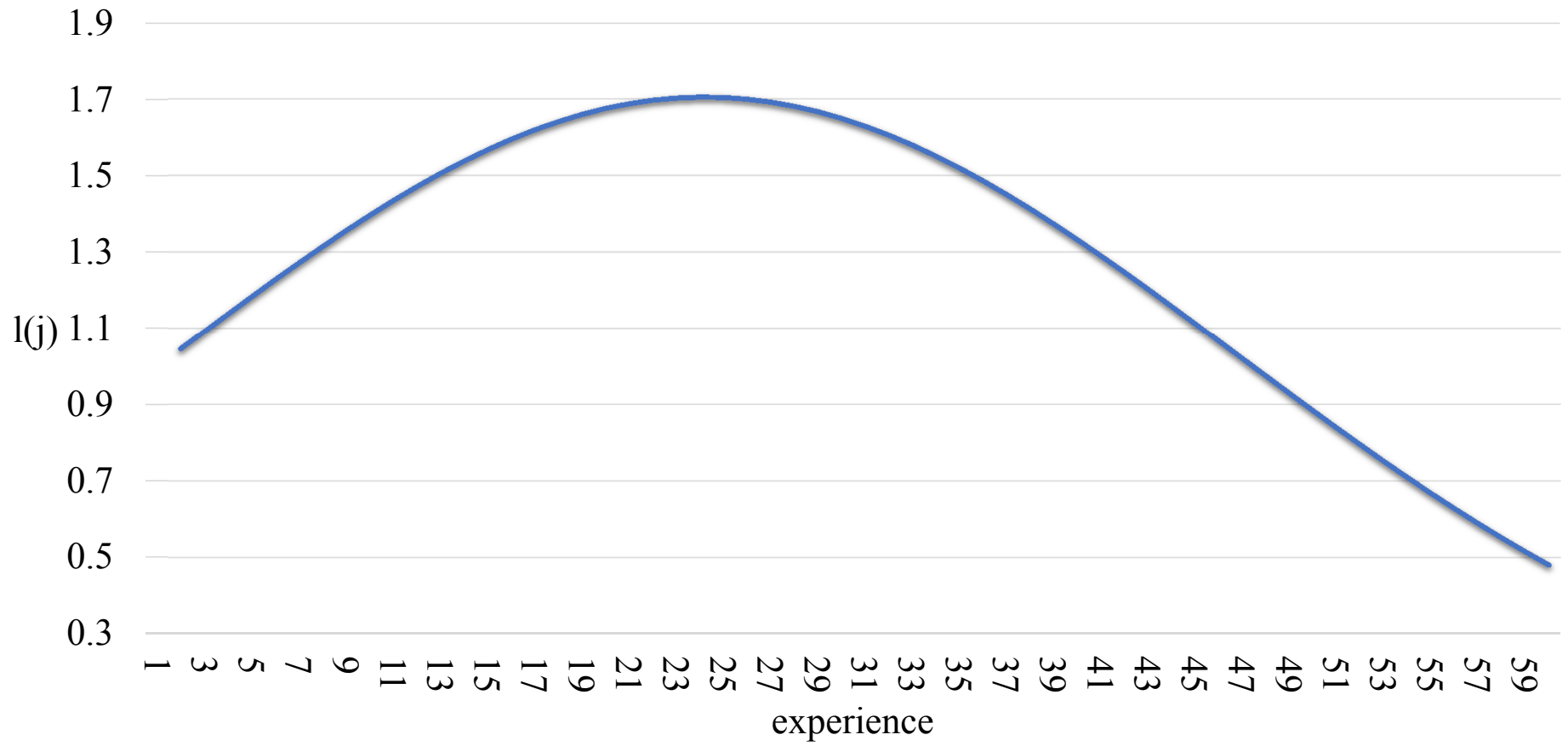


Figure 3. Labor market experience. Source: PSID data (1968-2011)

Figure 4. Variances of Persistent Earning Shocks

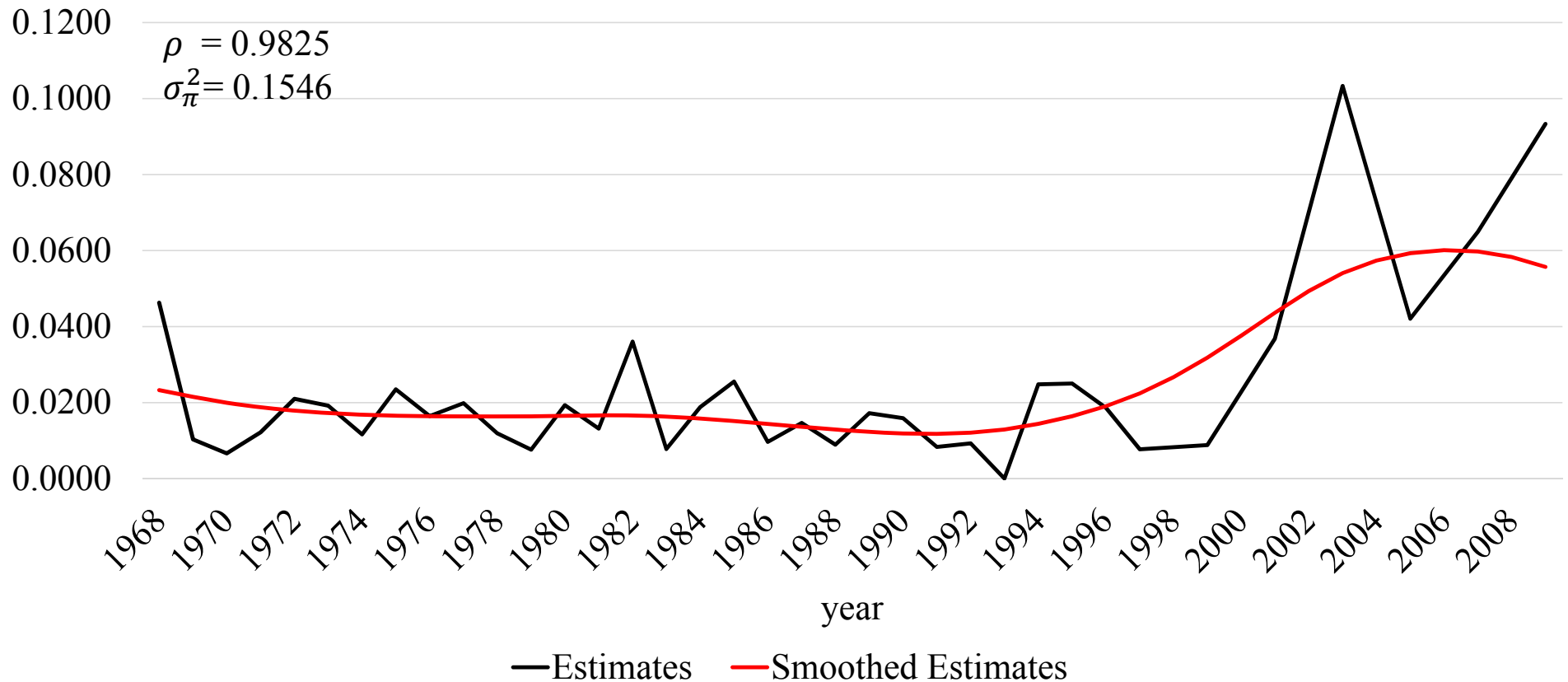


Figure 4. Minimum Distance Estimates of the persistent earning shocks. ρ is the persistence of earning shock. σ_{π}^2 is the initial variance of the persistent shock. Smoothed series are generated using Hodrick-Prescott trend with smoothing parameter of 100.

Figure 5. Variances of Idiosyncratic Earning Shocks

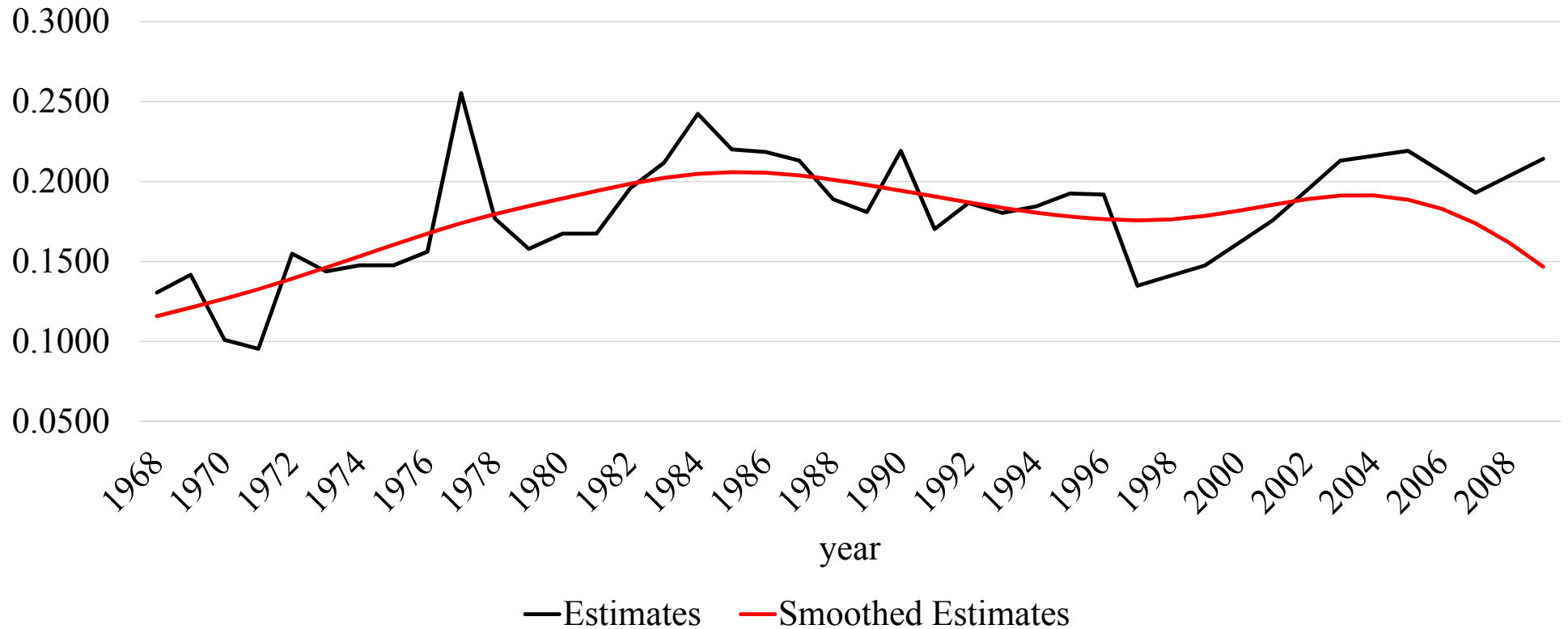


Figure 5. Minimum Distance Estimates of the transitory earning shocks. Smoothed series are generated using Hodrick-Prescott trend with smoothing parameter of 100.

Figure 6a. Distribution of Wealth for Low Yield Asset Market Households

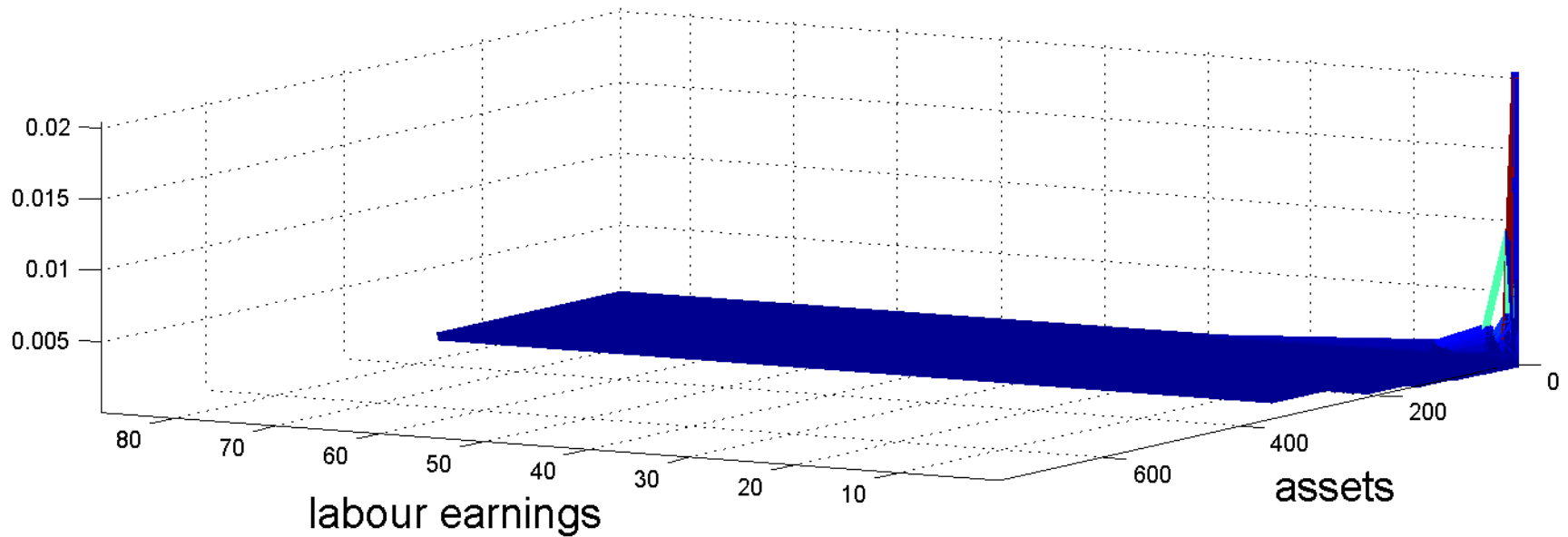


Figure 6b. Distribution of Wealth for High Yield Asset Market Households

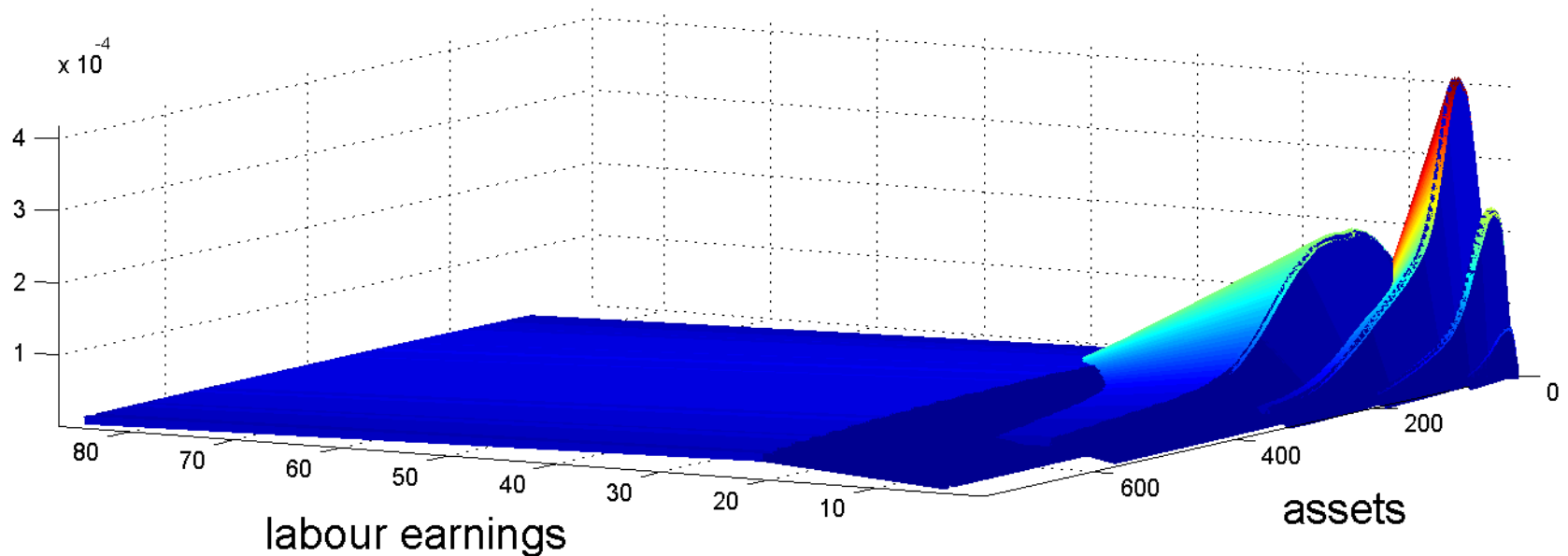


Figure 7. The Lorenz Curve

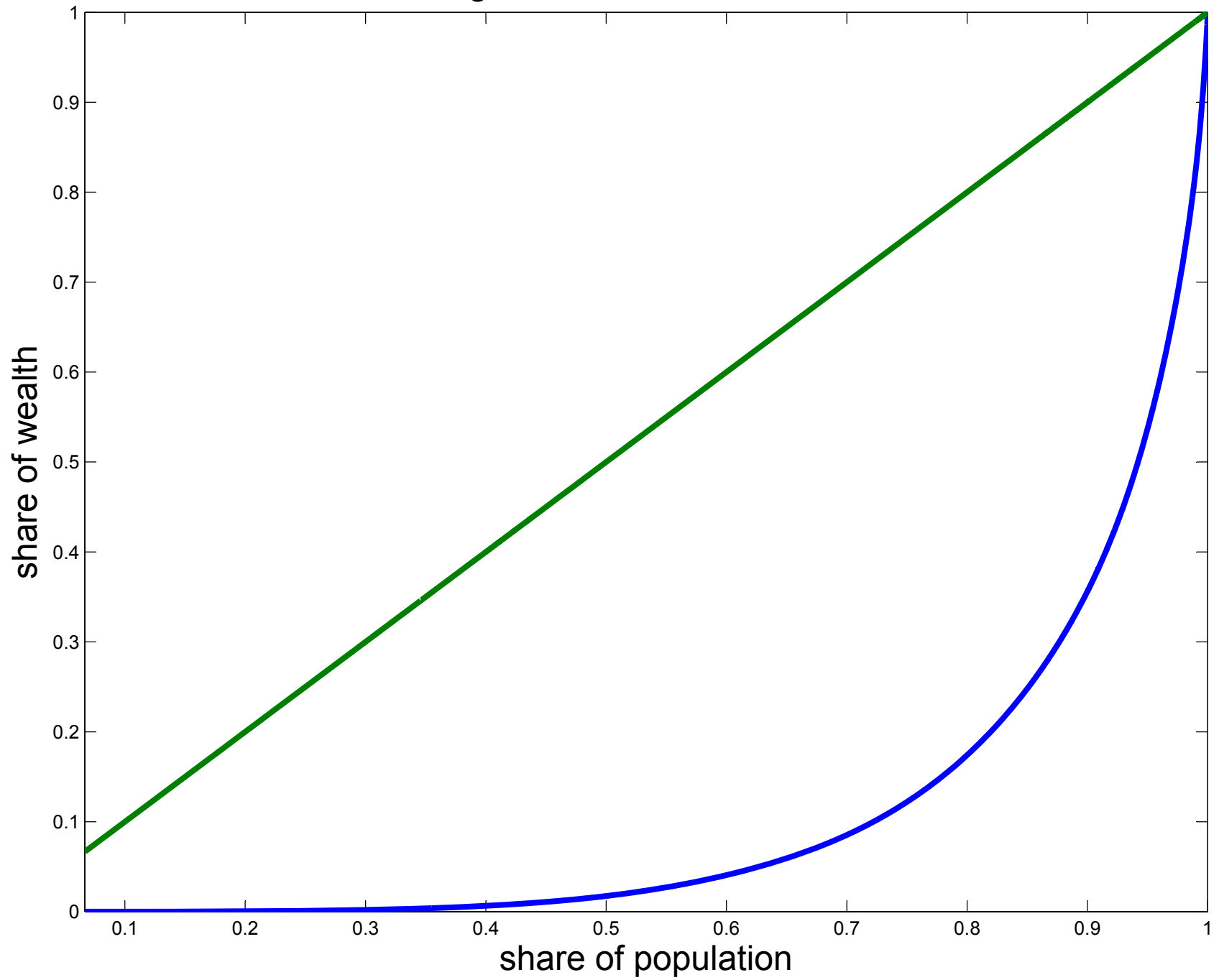


Figure 8. Share of Households in High Asset Market

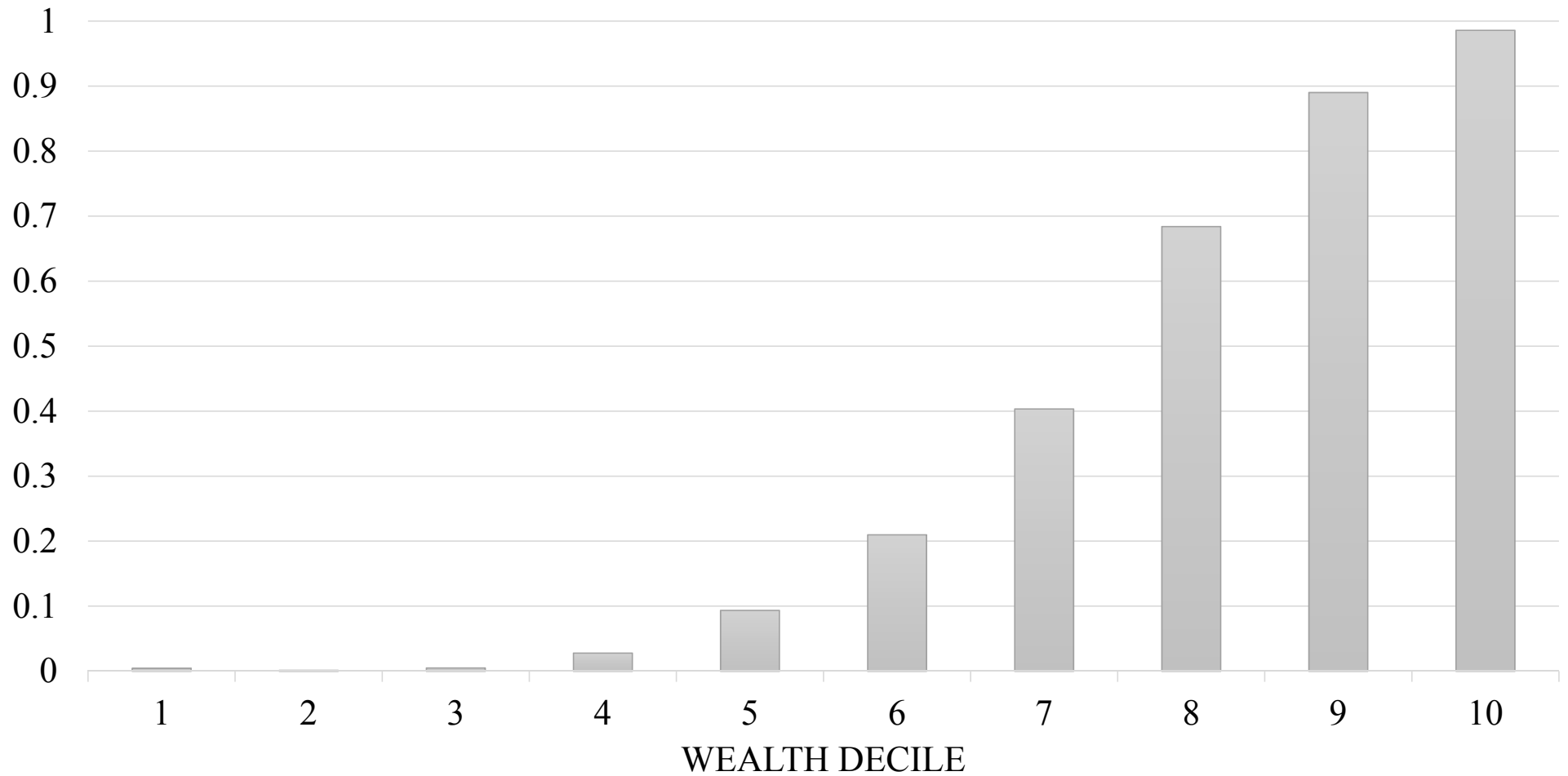


Figure 9. Share of High Asset Market Households over years of work

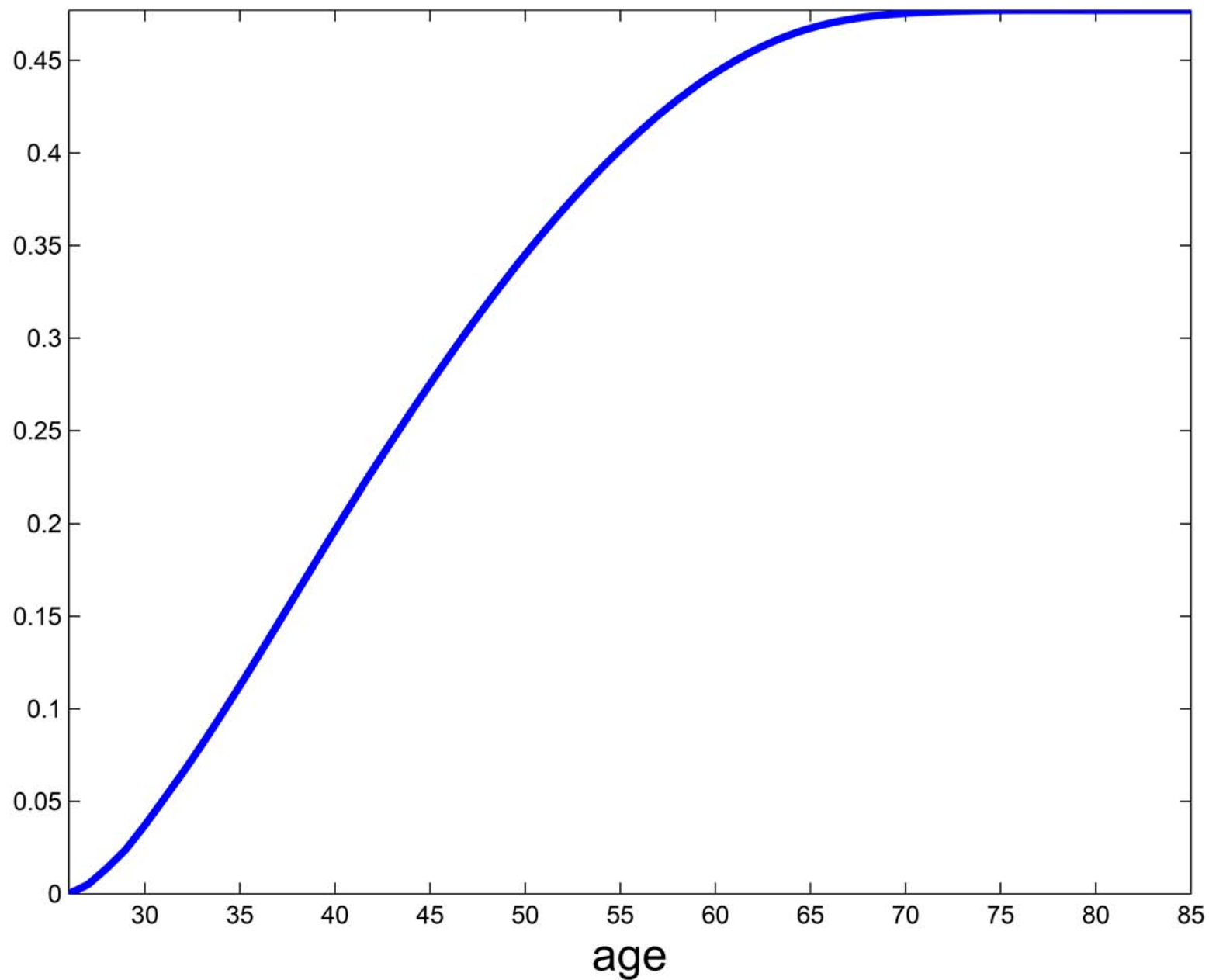


Figure 10. Life-Cycle Savings Rates

