Unemployment Cycles*

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Abstract

The labor market by itself can create cyclical outcomes, even in the absence of exogenous shocks. We show that the search behavior of the employed has profound aggregate implications for the unemployed. Active on-the-job search increases vacancy posting by firms: it changes the number of searchers and in the presence of sorting, it improves the quality of the pool of searchers. More vacancy posting in turn makes on-the-job search more attractive, a self-fulfilling belief. The absence of on-the-job search discourages vacancy posting, making costly on-the-job search little attractive. This model of multiple equilibria can account for large fluctuations in vacancies, unemployment, and job-to-job transitions; it provides a rationale for the Jobless Recovery through a novel channel of the employed crowding out the unemployed; and it gives rise to a shift in the Beveridge Curve (the unemployment-vacancy locus). Each of these phenomena are matched in the data.

Keywords. On-the-job search. Jobless Recovery. Unemployment Cycles. Sorting. Mismatch. Job-to-job flows.

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1 Introduction

Business cycles have a wide variety of origins, ranging from financial crises, over oil price shocks, to productivity spurts and slowdowns. Often, of all economic agents, workers are those affected most dramatically, mainly through unemployment. For long, researchers – most notably Diamond (1982) – have asked whether the labor market *by itself* can generate cyclical outcomes, even in the absence of any exogenous shocks or changes in fundamentals. But so far there has been no compelling labor market mechanism that fits the facts. In this paper, we propose a simple theory that generates self-fulfilling unemployment fluctuations and that can account for the key labor market facts: fluctuations in unemployment, vacancies and job-to-job flows. We will argue that our model provides a simple rationale for the Jobless Recovery, the fact that it takes a long time for unemployment to drop even after productivity has recovered, and for the outward shift of the Beveridge Curve.

The main driving force behind unemployment cycles is the *search behavior of those with a job*. Singling out the employed to explain unemployment may seem counterintuitive. But with a share of over ninety percent of the labor force, any minor change in the behavior of the employed, who vie for job openings side by side with the unemployed in the same labor market, has profound aggregate implications for unemployment. Even if they search much less intensively than the unemployed, because of their shear size, on average still about half of the vacancies are filled by workers who were employed already. Most importantly, we document that there is also a *huge variation* in the search behavior of the employed over the business cycle. And it is precisely this variation that has drastic implications for the unemployed during the recession and recovery.

Our model establishes that, based on the interplay between on-the-job search, sorting with mismatch, and job creation, multiple steady states arise naturally. When workers actively search while on-the-job, it is easier for firms to fill vacancies. Not only is the pool of searchers larger, in the presence of sorting, the quality of the match also tends to be higher as the fraction on-the-job searchers to unemployed is higher. It is this *selection externality* (very much in the spirit of Burdett and Coles (1997)) that is at the root of the multiplicity. Higher expected match quality creates incentives for vacancy creation, which in turn creates incentives to actively search on-the-job since it is easier for a worker to find one. Active job search has become self-fulfilling. This high churning outcome corresponds to an economic boom with active on-the-job search, little mismatch, abundant job creation, low unemployment and high aggregate output. But there is also another equilibrium outcome, the recession, where workers do not actively search on-the-job, where the pool of searchers has relatively few on-the-job searchers and is of relative low match quality, where firms have little incentives to post vacancies, resulting in low job turnover and high mismatch, and where aggregate output is low. In the recession, workers experience grim job prospects and hang on to their existing jobs, even if mismatched.

These endogenous fluctuations can account for three features of the labor market and how it evolves between boom and recession. First, even in the absence of any exogenous shocks (such as to productivity and to financial markets) there are big fluctuations in unemployment, vacancies and job-to-job transitions. As self-fulfilling beliefs switch from a high to a low rate of vacancy posting, and the corresponding job search intensity goes from active to inactive, this results in huge fluctuations in unemployment and gross job flows. In particular, and most importantly, this gives rise to large fluctuations in the job-to-job (EE, for employment-to-employment) flows. In a boom, workers switch jobs frequently whereas in a recession they do so much less, as can be seen from Figure 1.A. Before the great recession for example, job-to-job transitions (EE flows) were 50% higher (6% of the labor force in the boom versus 4% in the recession). In our model, when beliefs switch, there is precisely such a sudden drop in job-to-job transitions. Because employed workers constitute the large majority of the labor force, even a minor change in their behavior has an enormous impact on the labor market, in particular on the unemployed.[1]

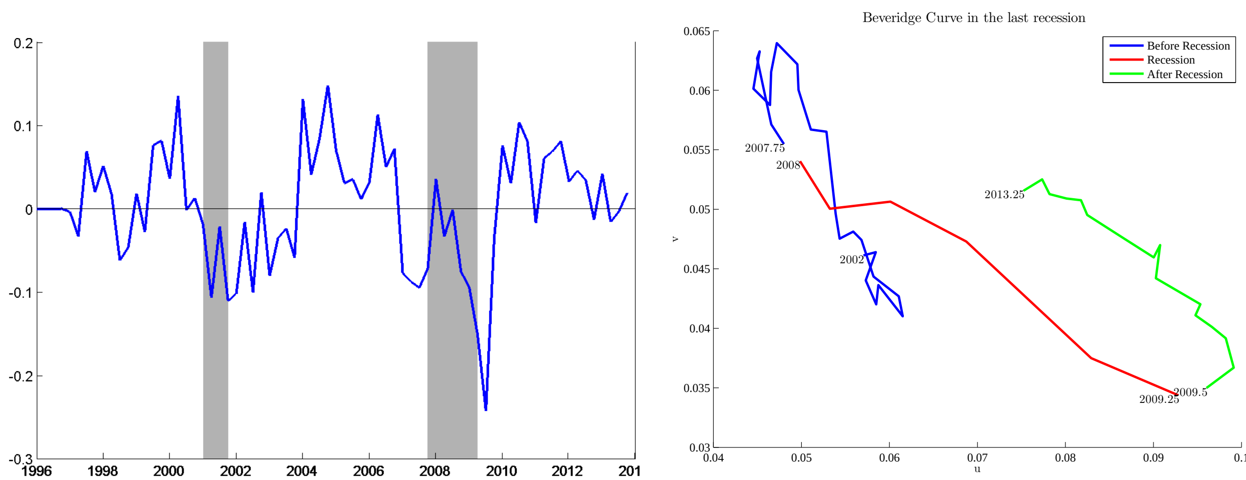

Figure 1: A. Employment-to-Employment flows (detrended). B. Outward Shift of the Beverage Curve.

The second labor market feature the model can account for is the *Jobless Recovery*. Even long after productivity has picked up following a recession, unemployment remains sluggishly high. In late 2014 for example, five years after the end of the great recession, unemployment was still at 6%, more than one percentage point above the rate observed in the preceding boom in 2007. Why does it take so long for unemployment to recover, even after financial markets have turned healthy, productivity has recovered and the economy is growing? Ever since the term was first used in the aftermath of the great depression in 1935, economists have speculated about its possible origins and mechanisms. Popular and largely unfounded explanations aside such as technological change (the Luddite fallacy)[2] or permanent structural change in the labor market, academics have not found a satisfactory explanation for it. Here

[1]In the Appendix, we further document that what drives gross labor market flows is job-to-job movements more so than movements from unemployment. We report both UE and EE flows in Figure 7. Surprisingly, in the recession UE flows increase. This is due to the fact that the number of unemployed increases sharply, even if the individual matching probability decreases. Yet, total hires are down, indicating that the change in EE flows dominate that in UE flows.

[2]During the second industrial revolution, the Luddites, textile workers in England, feared that automated looms would lead to mass unemployment. It is considered a fallacy because the laid off textile workers were eventually hired in new, higher productivity jobs in different industries.

we argue that a jobless recovery is inherent in the cyclical behavior of unemployment due to fluctuations in on-the-job search. We identify a new channel where the employed are *crowding out* the unemployed. At the end of a recession, as beliefs switch to an active on-the-job search regime and firms add many vacancies, the composition of the pool of searchers changes. Almost instantaneously, the on-the-job searchers crowd out the unemployed searchers. Overall, job flows pick up, but with random matching they go disproportionately to the on-the-job searchers who are abundant after the recession, and not to the unemployed. All the renewed activity thus initially translates in better jobs for those with jobs, but no improvement in the prospects for the unemployed. After some time this crowding out fades as the fraction of those with a job who have found a good match has grown, and the number of on-the-job searchers decreases again relative to the searchers who are unemployed.

Third, the endogenous job market fluctuations naturally give rise to a shift in the Beveridge Curve (the locus of vacancies and unemployment). While researchers have pondered over observed shifts of the Beveridge Curve ever since it was introduced, recently there has been a renewed interest because of sizable outward shift following the great recession (see Figure 1.B).[3] An outward shift is often interpreted as a decrease in matching efficiency: to maintain a given level of unemployment, a larger number of vacancies need to be posted. This is puzzling because as the economy recovers, the apparent labor market efficiency deteriorates. One would expect to the contrary that part of the recovery is due to improved matching. We argue that the shift of the Beveridge Curve is not due to matching efficiency but rather due to a shift in the *effective* market tightness. While the Beveridge Curve depicts the ordinary market tightness, i.e., ratio of vacancies to unemployed, the *effective* market tightness is the ratio of vacancies to *all searchers*, including in the denominator the on-the-job searchers as well as the unemployed. As more employed workers search on-the-job, tightness drops. This affects the value of creating a vacancy in multiple ways: negatively, because the job duration on average is shorter due to active on-the-job search; positively, because there are more searchers so it is easy to fill a vacancy; and positively also, because the pool of vacancies has better expected matches. Equilibrium requires that the net effect for the firm is positive, but along the way, for any given level of unemployment and vacancies, with more searchers, the *effective* labor market tightness measured as vacancies over *all* searchers has gone down. Responsible for the shifting Beveridge Curve is therefore an abrupt change in the (endogenous) *argument* of the matching function – i.e., the effective market tightness – and not the exogenous matching technology. This results in lower job finding probabilities for the unemployed, and hence contributes to the Jobless Recovery.

The economic mechanism is based on three components: 1. on-the-job search; 2. sorting (wage ladder) with mismatch and selection; 3. endogenous vacancy creation. Key in our analysis is the role of active on-the-job search. Consider a candidate equilibrium where mismatched workers believe many

[3]The outward shift is substantial. For example, with 4% of vacancy creation, the unemployment rate at the end of 2008 was 7.5% while with the same vacancy creation of 4%, the unemployment rate at the beginning of 2010 was 9.5%. Most people (including the NBER) would argue that 2010 was solid recovery, yet unemployment was higher.

firms out there are posting large numbers of vacancies. Workers will find it optimal to engage in costly on-the-job search due to thick markets: the more vacancies are posted, the more likely it is for a worker to find a job. Now on the firm side, firms that match with a worker coming from an existing job will be more productive due to sorting (only those who are mismatched search on-the-job) than those coming out of unemployment. Therefore, if a lot of workers are searching on-the-job, firms are more likely to find a better match. This increases the value of opening a vacancy, and firms will post large numbers of vacancies. This is self-fulfilling. Likewise, if workers believe firms do not post many vacancies, they will not actively search on-the-job, there will be few high productivity workers in the pool of searchers and the option value of a job opening will be low. Hence firms confirm the workers' belief by not opening many vacancies.

Each of these ingredients is crucial for the self-fulfilling equilibria. Without endogenous on-the-job search, there is no choice on the worker side. Without sorting, say if all jobs are identical and there is no mismatch, there is no efficiency gain from moving from the first job and as a result, yielding a unique equilibrium. In fact, on-the-job search is not only costly to the actively searching worker who incurs a utility cost, it is also costly to the firm because the average job duration is lower: workers actively search and leave as soon as they find a better job. Therefore it is not only necessary that there is sorting, there must be a sufficiently large output gain in order to obtain the multiplicity. Finally, firms must respond to the on-the-job search behavior of the workers, so vacancy creation needs to be endogenous.

We then perform a quantitative exercise. First, we calibrate the model to the US economy and show that our model of multiple equilibria accounts for a significant part of the observed fluctuations in unemployment and vacancies over the business cycle. Then, we analyze a sudden shift in beliefs and show that it can account for the phenomenon of jobless recovery. We also use the model for a counterfactual exercise, feeding in observed labor productivity changes (for a given equilibrium) and compare the implications for unemployment and vacancies with those of the model with multiple equilibria. Compared to movements in labor productivity, our mechanism can match observed fluctuations in labor market outcomes significantly better.

Related Literature

We are intellectually indebted to several earlier contributions, the ideas in which have shaped our thinking on this topic. The most celebrated model of self-fulfilling employment fluctuations is without a doubt Diamond (1982).[4] He proposes a very interesting search mechanism with search in production as well as in exchange in the goods market. Due to thick market externalities, trade in the goods market is faster if the population of traders is larger. This in turn, leads to faster production and lower unemployment due to search in production. In addition to this high production, low unemployment equilibrium, there

[4] Diamond and Fudenberg (1989) further elaborate on the model and analyze the non-stationary rational expectations equilibrium properties.

is also an equilibrium with a small population in the goods market and long unemployment in the production market. The multiplicity is due to the thick market externality: the more people search, the higher the probability they find a job. While the model is interesting conceptually and has broken many barriers of understanding frictional trade, most economists do not find it compelling empirically since it relies on increasing returns to matching, a counterfactual feature of the matching technology (Pissarides and Petrongolo (2001)).

The source of multiplicity in our model is closer in essence to Burdett and Coles (1997). In a simple two-type model with two-sided heterogeneity and non-transferable utility, they show that even with constant returns in the matching technology, there can be multiple equilibria.[5] The driving force is thus not Diamond (1982)'s market size or network externality, but rather a *selection externality*. If high types believe other high types are not selective and accept matches with low types, the equilibrium distribution of searchers will have a low fraction of high types provided the economy-wide share of high types is low enough. This in turn makes the expected value of search low, inducing high types to accept low types. Instead, if the high types believe other high types will be selective and accept matches only with other high types (rejecting all low types), then the steady state fraction of high types is high (provided the economy-wide fraction of high types is below a threshold) and the continuation value of search is high. The best response therefore is to accept high types only. These beliefs are therefore self-fulfilling, driven by the externality from the composition in the stationary distribution of searchers.[6] While the externality of active on-the-job searchers we find here is similar to the selection externality first proposed in Burdett and Coles (1997), the model has quite different predictions. In Burdett and Coles (1997) it is difficult to map the two equilibria into a boom or a recession. In the selective equilibrium, the mismatch is low and output is high (as in a boom), but unemployment is high as well, whereas in the non-selective equilibrium, mismatch is high and output is low (as in the recession), but unemployment is low.

In the spirit of Diamond (1982)'s goods market externality, Kaplan and Menzio (2014) ask the archetypical Keynesian question whether externalities in the goods market can affect production in the labor market: if people do not demand goods, there will be less production, leading to unemployment and hence lower demand.[7] It is well known that with competitive goods markets, prices instantaneously adjust and there is no feedback or propagation from of demand on unemployment. Kaplan and Menzio (2014) model goods demand by means of a frictional Burdett and Judd (1983) goods market where the unemployed search more intensely for low goods prices. As unemployment in the labor market

[5]Their work together with Smith (2006) analyzes matching patterns in the presence of endogenous type distributions. Other work, including Morgan (1996), Bloch and Ryder (1999), Eeckhout (1999), McNamara and Collins (1990), had not noticed the multiplicity because its setup had an exogenous distribution of searchers, known as the cloning assumption. Shimer and Smith (2000) analyze two-sided matching with search frictions under transferable utility and with endogenous distributions of searchers.

[6]In an intriguing piece, Shimer and Smith (2001) study the planner's allocation who cares only about aggregate surplus. They find that the planner's allocation may involve a non-stationary policy, including limit cycles.

[7]In an early contribution, Howitt and McAfee (1992) address a similar question.

is higher, markups in the goods market are lower which in turn leads to lower production and more unemployment. They show that this shopping externality can thus generate multiple self-fulfilling steady state equilibria, which they, as we do here, interpret as business cycles. In Kaplan and Menzio (2014) of course, the business cycle is due to a demand externality, and not and externality in the labor market itself. It can therefore also not produce a shift of the Beveridge curve, nor does their mechanism speak to the jobless recovery.

Two interesting mechanisms in a very different setting are related to ours. In the marriage market, Burdett, Imai, and Wright (2004) unearth a source of multiplicity from the strategic interaction *within a match* that stems from the fact that both partners in the match search (in addition to the economy wide, aggregate feedback of beliefs that drives the multiplicity in our model). If my partner searches "on the marriage", my match will dissolve with higher probability than if she does not search, and therefore my match value is lower. This in turn increases the incentives to search myself, a self-fulfilling belief. In the housing market, Moen and Nenov (2014) derive multiple equilibria where homeowners who switch house decide to sell their current house before or after they buy the new house. While this has small implications for the value of housing for a given individual (the transition of a few months is very small compared to the average duration of ownership, about eight years in the US), it has big aggregate implications which are reflected in housing prices. Using micro level data from housing in Denmark, they find huge changes in the fraction of "on-the-home" buyers versus those who go through a period of being without a home (which is comparable to unemployment in the labor market).

Finally, in his seminal paper, Shimer (2005) argues that in the standard Mortensen-Pissarides model of unemployment, productivity fluctuations cannot account for the fluctuations in unemployment and vacancies observed in the data. Hall (2005) (wage stickiness) and Hagedorn and Manovskii (2008) (the high value of unemployment) have offered explanations to counter Shimer's finding, and can indeed create labor market volatility from small productivity shocks. We do not aim to contribute to this debate because our objective is to explain the observed fluctuations in job-to-job flows in the absence of productivity shocks. We do however need take a stance on the parameter values we use in our calibration. In the light of this debate and of the result of additional recent work,[8] we will use an intermediate value for unemployment. In any event, our mechanism can generate multiple steady states for *any* value of unemployment benefits and we show in our counterfactual exercises that on-the-job search behavior is more important in generating labor market fluctuations than productivity movements, even when unemployment benefits are set relatively high.

The paper is organized as follows. In the next section (2) we set out the environment. In Section 3 we characterize the equilibrium allocation and show that multiple steady states arise. In Section 4 we

[8] Chodorow-Reich and Karabarbounis (2013) *directly* estimate from micro data the value of being unemployed. They find a value of unemployment relative to productivity of 0.75 that is higher than Shimer (2005)'s (0.4), and lower than Hagedorn and Manovskii (2008) (0.95). In addition, they find the contribution of unemployment benefits to the value of unemployment is very small (16%) and that the estimated value of unemployment is cyclical, indicating that any variation due to productivity is offset by the variation in the value of unemployment.

analyze the transition dynamics and the Jobless Recovery. We then calibrate the model in Section 5 and address some extensions in Section 6. Finally, in Section 7 we offer some concluding remarks.

2 Environment

We lay out the model economy, which is a basic version of the Postel-Vinay and Robin (2002) model of on-the-job search in combination with a highly stylized form of sorting, to capture the main forces in the search models with sorting (à la Shimer and Smith (2000)) that incorporate on-the-job search. It is stylized because we do not explicitly model the sorting process based on two-sided heterogeneity. The reason is that we cannot solve that model analytically. In fact, the full blown model of sorting with on-the-job search is analyzed computationally in Robin and Lise (2013) and Lamadon, Lise, Meghir, and Robin (2013).

Because our objective is to derive analytical results, instead we use a key feature of search models with sorting and on-the-job search, namely the implicit job ladder.[9] A worker who already has a job, will only move to a new job if the new job is more productive. Therefore, the types of jobs out of unemployment are on average less productive than those that are accepted when moving from an existing job. We will capture this very crudely by means of two job types: jobs out of unemployment have low productivity, jobs out of an existing job have high productivity.[10]

Agents and Technology. Time is continuous, $t \in \mathbb{R}_+$. There is a measure one of risk neutral workers in the economy. A worker is unemployed searching for a job, or employed, in which case you can choose to search on-the-job . The value of unemployment is b and the flow value of employment is equal to the wage. The search cost when unemployed is normalized to zero and the search cost for *active* search when employed is k.[11]

There is a large measure M of potential jobs (firms). Firms can open a job paying a flow cost c. If they stay inactive their payoff is zero. Denote the measure of job openings by v. All jobs are ex ante identical, but ex post heterogeneous in their job productivity y, with $y \in \{\underline{y}, \overline{y}\}$.[12] When a job is filled by an unemployed worker, the productivity is $\underline{y}$ and when it is filled by a formerly employed worker the productivity is $\overline{y}$, with $\underline{y} \leq \overline{y}$.[13] Firms are risk neutral and maximize the discounted sum of profits.

[9] See also Moscarini and Postel-Vinay (2012) for a job ladder model without explicit sorting.

[10] Observe that our interoperation here is sorting and not human capital accumulation on-the-job, which could be an additional reason why matches out of unemployment are less productive.

[11] For tractability and without loss of generality, we assume that only those in low productivity jobs search. Those in high productivity jobs have an incentive to search merely to increase their wages. We rationalize this by implicitly assuming that the cost of on-the-job search does not compensate for the wage increase.

[12] Below we allow for aggregate productivity to vary, in which case we denote output by py where p is a measure of aggregate productivity. We assume that job-specific productivity y, the cost of on-the-job search k and unemployment benefits b are proportional to p. This is consistent with the findings of Chodorow-Reich and Karabarbounis (2013) that the value of unemployment is pro cyclical.

[13] In order to keep the model tractable, we assume firms commit to hire any worker type, whether she is hired out of unemployment or from an on-the-job move.

Denote the measure of the unemployed by u; the measure of the employed in a low productivity job by γ; the measure of the employed in high productivity jobs after passive search by ξ_0, and after active search by ξ_1. Since the measure of workers is equal to one, feasibility requires that $u + \gamma + \xi_0 + \xi_1 = 1$.

Market Frictions and Wage Setting. Meetings between jobs and workers are stochastic, and are modeled by means of a standard matching function $m(\theta)$, where θ is the ratio of vacancies to job searchers, and m is assumed increasing and concave. The matching probability for a worker is $m(\theta)$ and that of a firm is $q(\theta) = \frac{m(\theta)}{\theta}$. Job separation is exogenous and occurs at rate δ.[14]

We assume all unemployed workers are searching at zero cost. The employed always engage in passive search at no cost (job opportunities arrive without search effort) which leads to a match with probability $\lambda_0 m(\theta)$. They can also engage in active search by incurring the search cost k.[15] In return they get a $(\lambda_0 + \lambda_1)m(\theta)$ chance of a match. Therefore the measure of workers searching for a job is given by $u + \lambda_0\gamma$ if no employed worker actively searches on-the-job and by $u + (\lambda_0 + \lambda_1)\gamma$ if all employed worker actively search on-the-job . λ_0 and λ_1 (with $\lambda_0 + \lambda_1 \leq 1$) denote the efficiency of the on-the-job matching technology. The resulting market tightness is then a function of the total measure of searchers expressed in "efficiency units", denoted by the *effective* number of searchers s, equal to $u+\lambda_0\gamma$ or $u+(\lambda_0+\lambda_1)\gamma$. For example, when all workers actively search on-the-job, the market tightness is given by $\theta = \frac{v}{u+(\lambda_0+\lambda_1)\gamma}$ and when none search actively on-the-job it is given by $\theta = \frac{v}{u+\lambda_0\gamma}$. Notice that we distinguish the effective market tightness $\theta = \frac{v}{s}$ that takes into account *all* the effective job searchers from the conventional market tightness in the Mortensen-Pissarides search model denoted by $\Theta = \frac{v}{u}$, which takes into account only the unemployed job searchers.

Wages are determined in a sequential auction framework as in Postel-Vinay and Robin (2002) (see also Dey and Flinn (2005)). As workers receive outside offers, wages are renegotiated. Current and outside employers Bertrand-compete over the worker. The worker goes to the match that has higher total value and receives a wage such that his continuation value equals the match value with the least productive of the two competing firms. If no outside offer arrives, wages remain unchanged. If the worker is hired out of unemployment, she receives the option value of unemployment.

Individual Decision Problems and Bellman Equations. We denote the value of a vacant job by V, of a filled job by J, of an unemployed worker by U, and of an employed worker by E. When we denote value of a filled vacancy or of an employed worker, we use the notation $\underline{E}$ ($\overline{E}$) to indicate that a worker is employed in a low (high) productivity job. Likewise, $\underline{J}$ ($\overline{J}$) denotes the value of a low (high) productivity job that is filled with a worker coming out of unemployment (a low productivity job). Denote by $\Omega \in \{0, 1\}$ the decision by the individual worker whether to actively search on-the-job

[14] Even though endogenous separations in the presence of stochastic match surplus is an important determinant of labor market dynamics (see amongst others Mortensen and Pissarides (1994) and Bils, Chang, and Kim (2011)), we aim to make the point in this paper under exogenous separations.

[15] An alternative way of modeling this would be through continuous search intensity on-the-job, where workers choose an interior non-zero search intensity. This would then potentially give rise to a high and a low intensity on-the-job search outcome that is determined endogenously. Unfortunately we cannot solve that case analytically.

and which equal to one when the worker searches actively on-the-job and is zero otherwise. And denote by $\mathbf{\Omega} \in \{0, 1\}$ the behavior in a symmetric strategy equilibrium of all other workers in the economy.[16] Throughout we assume that individual search behavior Ω is private information, so firms cannot make wages contingent on search effort. For the remainder, we also use the notation $\lambda(\Omega) = \lambda_0 + \Omega\lambda_1$ for the individual search intensity and $\lambda(\mathbf{\Omega}) = \lambda_0 + \mathbf{\Omega}\lambda_1$ for the aggregate search intensity.

Workers. We can write the values of workers as follows:

$$
\begin{aligned}
rU &= pb + m(\theta(\mathbf{\Omega}))(\underline{E} - U) + \dot{U} && (1)\\
r\underline{E} &= \underline{w}(\mathbf{\Omega}) - \Omega pk + \lambda(\Omega)m(\theta(\mathbf{\Omega}))(\overline{E} - \underline{E}) - \delta(\underline{E} - U) + \dot{\underline{E}} && (2)\\
r\overline{E} &= \overline{w}(\mathbf{\Omega}) - \delta(\overline{E} - U) + \dot{\overline{E}} && (3)
\end{aligned}
$$

where $\dot{U}$ is the time derivative of U (likewise for all other values) and where

$$\theta(\mathbf{\Omega}) = \frac{v}{s(\mathbf{\Omega})} = \frac{v}{u + \lambda(\mathbf{\Omega})\gamma}. \tag{4}$$

It is key to observe here that individual decisions Ω affect only the value of the employed in low productivity jobs through the cost of of job search k and the probability of finding a job λ_1.[17] Aggregate outcomes from the population behavior at large enter into values through two channels: 1. market tightness $\theta(\mathbf{\Omega})$ and 2. wages. The market tightness induces a search externality that affects the job finding probabilities of all job searches: the value of the employed in a low productivity job as well as the value of the unemployed. But also wages change depending on the belief whether workers search on-the-job or not. For much of what follows and whenever it is clear from the context whether $\mathbf{\Omega}$ is zero or one, we will drop the dependence of the market tightness on $\mathbf{\Omega}$ in the notation.

Firms. The value to a firm deciding to open a vacancy is given by:

$$rV = -c + q(\theta(\mathbf{\Omega})) \left[\frac{u}{u + \lambda(\mathbf{\Omega})\gamma}\underline{J} + \frac{\lambda(\mathbf{\Omega})\gamma}{u + \lambda(\mathbf{\Omega})\gamma}\overline{J} - V \right] + \dot{V}. \tag{5}$$

There are $u+\gamma$ searching workers, but the effective measure of searchers expressed in efficiency units is $u + \lambda(\mathbf{\Omega})\gamma$: all the unemployed search, and of all γ employed in low productivity jobs, a effective measure $\lambda_0\gamma$ are searching at no cost and $\mathbf{\Omega}\lambda_1\gamma$ are actively searching on-the-job . Because there is a large measure M of potentially entering firms, the value of a vacancy is driven to zero and V is equal to zero. In equilibrium, equation (5) is evaluated at $V = 0$. Observe that the measure of vacancies adjusts

[16]We focus here on strategies where all agents either do not search or all agents do search, hence $\mathbf{\Omega} \in \{0, 1\}$. But one can imagine a mixed strategy like outcome (or purification) where the fraction of active on-the-job searchers includes interior outcomes $\mathbf{\Omega} \in [0, 1]$.

[17]We could have assumed that on-the-job search is costly also if $\Omega = 0$ but we normalize this cost to zero. Our assumptions imply variable search costs, i.e. $k = 0$ if $\Omega = 0$ and $k > 0$ if $\Omega = 1$.

instantaneously.[18] Whenever there is a positive value for profits, vacancies are created frictionlessly to set the profits back to zero.[19]

The value of a filled job, high or low productivity, is given by:

$$r\overline{J} = p\overline{y} - \overline{w}(\mathbf{\Omega}) - \delta(\overline{J} - V) + \dot{\overline{J}} \quad (6)$$

$$r\underline{J} = p\underline{y} - \underline{w}(\mathbf{\Omega}) - [\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})) + \delta](\underline{J} - V) + \dot{\underline{J}}. \quad (7)$$

The flow value of a high type job is output net of wages. Once it is filled, the job lasts until there is exogenous separation with probability δ. The low type job similarly generates a flow value of $p\underline{y} - \underline{w}$ and separates exogenously at rate δ, but in addition faces separation from on-the-job search.

Labor Market Dynamics. At any point in time, the laws of motion satisfy:

$$1 = u + \gamma + \xi_0 + \mathbf{\Omega}\xi_1 \quad (8)$$

$$\dot{\gamma} = um(\theta(\mathbf{\Omega})) - \gamma[\delta + \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))] \quad (9)$$

$$\dot{\xi}_0 = \gamma\lambda_0 m(\theta(\mathbf{\Omega})) - \xi_0\delta \quad (10)$$

$$\dot{\xi}_1 = \gamma\mathbf{\Omega}\lambda_1 m(\theta(\mathbf{\Omega})) - \mathbf{\Omega}\xi_1\delta. \quad (11)$$

Equation (8) is simply feasibility. The measure of workers consisting of unemployed u, on-the-job searchers γ, high productivity workers who obtained the job through passive on-the-job search ξ_0 and those who get the job through active on-the-job search ξ_1 is equal to the measure of the entire worker population. The flow into low productivity jobs and out of unemployment (the right hand side) is equal to the flow out of low productivity job (the right hand side), which consists of separation δ and the outflow due finding a better job.

Definition of equilibrium. We can now define equilibrium.

Definition 1 *A Perfect Foresight Equilibrium is a sequence* $\{U, \underline{E}, \overline{E}, V, \underline{J}, \overline{J}, \theta, u, \gamma, \xi_0, \xi_1, \Omega, \mathbf{\Omega}\}$ *such that*

1. $U, \underline{E}, \overline{E}, V, \underline{J}, \overline{J}$ *satisfy the Bellman equations above;*

2. Ω *maximizes* $\underline{E}$*;*

3. *There is free entry:* $V = 0$*;*

4. u, γ, ξ_0, ξ_1 *satisfy the laws of motion;*

[18] We assume that there is commitment on the part of the firms. That is, even if the surplus of a low match value is negative, the firms will commit to take the loss, knowing that in expectations they make zero profits. This may be the case if the cost of opening a vacancy c is not too high. Observe that if we did not assume commitment, there cannot be a pure strategy equilibrium if that surplus is negative since all firms would reject low productivity matches. But this in turn implies that no unemployed workers ever get matched and therefore there are no high productivity jobs since those exist only after on-the-job search.

[19] Below in Section 4 we investigate the role of sluggish adjustment process in vacancy creation.

5. $\lim_{t\to\infty} J_t, V_t, U_t, E_t$ *are finite and initial conditions for given* $u_{-1}, \gamma_{-1}, \xi_{0_{-1}}, \xi_{1_{-1}}$.

Aggregate Output. We can evaluate the aggregate impact of different equilibria. To do so, we calculate gross output (where we do not include b since it is interpreted as a preference term, not home production)

$$Y_g = \gamma \underline{y} + (1 - u - \gamma)\overline{y},$$

and net output

$$Y = Y_g - \gamma \boldsymbol{\Omega} k - vc.$$

Calculating welfare here is trivial given free entry of firms and the wage setting. The value to firms is zero, and given that firms extract all the surplus from the unemployed worker who finds a job, the value of the worker is constant.[20]

3 Equilibrium

We first solve for the equilibrium system of equations. Wages are determined following the sequential auction wage setting procedure (Postel-Vinay and Robin (2002)) and are pinned down by the worker's outside option. A firm that hires an unemployed worker will offer her a wage $\underline{w}$ that makes her indifferent between accepting the job and remaining unemployed, i.e. $\underline{E} = U$. Likewise, the firm offers a wage $\overline{w}(\boldsymbol{\Omega})$ that makes the worker with an on-the-job offer indifferent between joining the new, high productivity firm and and staying at the existing low productivity firm, i.e., $\underline{J} = V$. Some algebra (See Appendix) immediately reveals that

$$\begin{aligned} \underline{w}(\boldsymbol{\Omega}) &= pb\left(\frac{r + \lambda(\boldsymbol{\Omega})m(\theta(\boldsymbol{\Omega})) + \delta}{r+\delta}\right) - \frac{\lambda(\boldsymbol{\Omega})m(\theta(\boldsymbol{\Omega}))}{r+\delta} p\underline{y} + \boldsymbol{\Omega} pk \qquad (12) \\ \overline{w}(\boldsymbol{\Omega}) &= p\underline{y}. \qquad (13) \end{aligned}$$

Observe that we have used the population-wide average of active searchers $\boldsymbol{\Omega}$ and not the individual level Ω since the wage reflects the firm's belief about the worker's action whether to search actively or not. Note that the wage of a given worker is constant even out of steady state. Over time, wages can of course vary across workers hired at different points in time, which is captured through the change in the effective market tightness $\theta(\boldsymbol{\Omega})$.

[20]This however is a feature of these two assumptions, and we could easily alter the framework to allow for some intermediate division of the surplus as in Cahuc, Postel-Vinay, and Robin (2006). This would only generalize the setup, and give rise values of the unemployed that depend on the active search intensity. Unfortunately, we cannot explicitly solve for this model.

3.1 Multiple Steady States

We now set the time derivatives to zero and analyze steady state multiplicity. We construct the two candidate equilibria in which either no worker searches actively on the job, $\mathbf{\Omega} = 0$, or all search actively on the job $\mathbf{\Omega} = 1$. To ascertain whether these candidate equilibria are indeed equilibria, we have to check whether there is no deviation by any individual worker.

One-shot deviations. It is sufficient to check one-shot deviations from the workers strategies. To evaluate deviations by an individual worker, we introduce the following notation. We define $\underline{E}(\Omega|\mathbf{\Omega})$ as the worker's payoff in the low productivity job, while taking action Ω *for an instant dt*, and then to revert to the equilibrium action $\mathbf{\Omega}$. This captures the notion of a one shot deviation, or equivalently Bellman optimality.

Given the wage setting, the value of unemployment is equal to that of employment in the low productivity job. As a result, the value of unemployment is independent of the equilibrium search intensity $\mathbf{\Omega}$. Likewise, the worker value in high productivity jobs is independent of the equilibrium value since those workers do not search and exogenous separation leads to unemployment. This implies that U and $\overline{E}$ are independent of $(\mathbf{\Omega})$, and we can directly verify the deviations of those employed in a low productivity job $\underline{E}$. Because on-the-job search behavior is private information, wages are dependent only on the beliefs about aggregate behavior $\mathbf{\Omega}$.

We now verify two possible deviations and under which conditions they are not individually rational: 1. when no one is actively searching on-the-job, there is no deviation of a single agent to search; 2. when all are actively searching on-the-job, there is no deviation by a single agent not to actively search. These two no-deviation conditions give rise to the following result.

Lemma 1 *There are multiple steady state equilibria if and only if*

$$\theta(\mathbf{0}) < m^{-1}\left(\frac{k(r+\delta)}{\lambda_1(\underline{y}-b))}\right) < \theta(\mathbf{1}).$$

Proof. In Appendix. ∎

Of course, $\theta(\mathbf{\Omega})$ is an endogenous object. Unfortunately, we cannot in general calculate conditions under which this is satisfied. We do find a necessary and sufficient condition in terms of the primitives of the model when $\lim \delta \to 0$ and under a particular matching function, the telegraph matching function:

$$m(\theta) = \phi\frac{\alpha\theta}{\alpha\theta+1}, \tag{14}$$

where ϕ is the overall matching efficiency of the matching technology and α is parameter that regulates curvature.[21] In much of what follows, we will use the telegraph for three reasons. First, it has all

[21] This is also the matching function used in the money and search literature, where ϕ and α are set to one. There it is

the desirable features of a matching function, being a special case of the CES matching function. Second, with the level parameter ϕ and the shape parameter α, we can approximate precisely the matching functions used in the literature (Shimer (2005), Hagedorn and Manovskii (2008)). Finally, when implementing the calibration, we need to solve explicitly for vacancies and tightness. This is the only matching function that allows us to do so computationally.

Under these assumptions, we can then establish the following result:

Proposition 1 *Let* $\lim \delta \to 0$*, and* $m(\theta) = \phi \frac{\alpha\theta}{\alpha\theta+1}$*. Then there are multiple steady states if and only if condition* **M** *(in Appendix) is satisfied. Equivalently, only if* $p \in [p^l, p^u]$ *then there are multiple equilibria, where:*

$$\begin{aligned} p^l &= \frac{2c\lambda_1 r(\underline{y}-b)[k(\lambda_0+\lambda_1)+\lambda_1(\underline{y}-b)]}{\alpha[\lambda_1\phi(\underline{y}-b)-kr][b^2\lambda_1+k(\lambda_0+\lambda_1)\overline{y}+\lambda_1(\overline{y}-k)\underline{y}-b(k\lambda_0+\lambda_1\overline{y}+\lambda_1\underline{y})]} \\ p^u &= \frac{2c\lambda_1 r(\underline{y}-b)}{\alpha(\overline{y}-b)[\lambda_1\phi(\underline{y}-b)-kr]}, \end{aligned}$$

and the set $[p^l, p^u]$ *is non-empty for a robust set of parameter values.*

Proof. In Appendix. ■

In other words, this result rewrites the necessary and sufficient condition for multiple equilibria in Lemma 1 as a condition of exogenous model parameters. This in turn is then interpreted in particular as an interval of values for the labor productivity p: values for productivity in that interval are a necessary condition for multiplicity. Moreover, that interval is non-empty for a robust set of parameters.

While analytically we derive this result for the limit case where $\delta \to 0$, we can compute the outcome for any δ. Using the calibration parameters (see Section 5 below), in Figure 2.A we report the equilibrium outcomes v and Θ for different values of p. Both parts of the figure also show the range of multiple equilibria between $p^l = 0.94$ and $p^u = 1.04$. Observe that vacancies and conventional tightness are always increasing in productivity, both with and without OJS. And while vacancies (and tightness) are always higher with OJS for large enough p, for low p vacancies with OJS may actually drop below those without OJS.

The next result states that the existence of multiple equilibria is closely related to the gains from sorting. For low gains from sorting, there is a unique equilibrium with no OJS. OJS has two costs: 1. the direct search cost k incurred by the worker; and 2. the indirect search cost incurred by the firm. The latter is an externality that derives from the fact that active on the job search renders the expected duration of a job shorter. As a result, everything else equal, the opportunity cost of opening a job to the firm is higher. This indirect cost then explains why there cannot be active OJS in equilibrium

interpreted as a matching process where buyers (money holders) and sellers are one population, and hence under uniform random matching, the likelihood of meeting a buyer is proportional to the number of buyers in the total population of buyers and sellers: $m(\theta) = \frac{\theta}{1+\theta} = \frac{b}{b+s}$ where $\theta = \frac{b}{s}$ is the ratio of buyers to sellers.

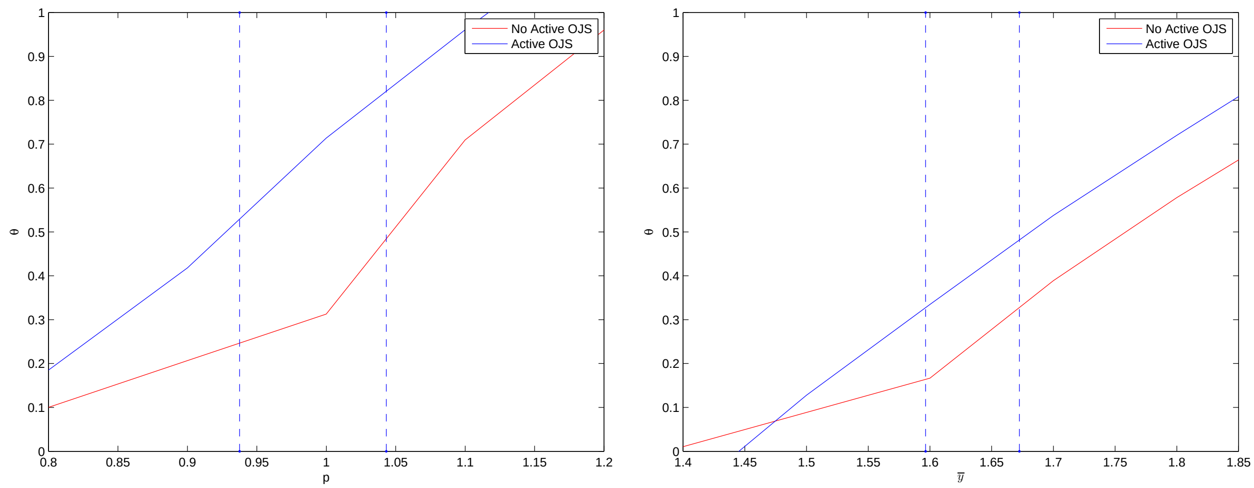

Figure 2: Effective market tightness $\theta = \frac{v}{s}$ with multiplicity range: A. As a function of aggregate productivity p; B. As a function of high job productivity $\overline{y}$. (Parameters for the calibrated economy below).

when the gains from OJS are arbitrarily small. Those gains are measured by the extent of sorting, i.e., the difference between $\overline{y} - \underline{y}$. There is hardly any output gain when finding a job through OJS, yet the opportunity cost of OJS has increased discretely via the job finding probability λ_1. As a result, it is a dominant strategy not to search. It is important to observe here that the gains of a match are distributed between worker and firms through the wage.

At the other extreme, when the output gain is arbitrarily large for given search cost k and vacancy cost c, the gains from OJS swamp the direct and indirect cost, irrespective of the behavior of other workers. It is then a dominant strategy to always search.

Proposition 2 *(Sufficient Sorting needed for active OJS). Let* $\lim \delta \to 0$, *and* $m(\theta) = \phi \frac{\alpha\theta}{\alpha\theta+1}$.

1. *When the gains from sorting are small ($\overline{y} - \underline{y} < \epsilon$) then there is a unique steady state with no on-the-job search;*

2. *When the gains from sorting are arbitrarily high, there is there is a unique steady state with on-the job search.*

3. *For intermediate gains from sorting $\overline{y} \in [\overline{y}^l, \overline{y}^u]$ (for given $\underline{y}$) there are multiple equilibria.*

Proof. In Appendix. ■

This result is represented graphically in Figure 2.B for the model with the telegraph matching technology and for the calibrated values of the economy, including an unbounded $\delta = 0.06$. The graph shows that the limit result also holds more generally. For low values of $\overline{y}$ there is a unique equilibrium, as there is for very high values. If the value of sorting is low (low $\overline{y}$) there is little gain from OJS and the unique equilibrium is one with no active OJS. Instead, if $\overline{y}$ is high, it is a dominant strategy to search on the job and the unique equilibrium is the one with active OJS. In the intermediate range of $\overline{y}\ in[1.59, 1.67]$, there are multiple equilibria. For intermediate values of sorting, there is multiplicity.

Note finally that there can be multiplicity from other sources. In particular, the combination of the composition as in Burdett-Coles and concavity of the matching function (with a linear matching function there is no multiplicity). The source of the composition multiplicity stems from the composition effect on the worker side in conjunction with the concavity of the matching function, thus generating feedback for the firms.

3.2 Steady State Equilibrium Properties

We now depict the equilibrium properties in the canonical Beveridge Curve diagram, i.e., the unemployment-vacancy space. Since this is the convention, we continue to do so in the (u, v) space with corresponding market tightness $\Theta = \frac{v}{u}$. Since matching probabilities are a function of the effective market tightness $\theta = \frac{v}{u+\lambda(\mathbf{\Omega})\gamma}$, we then also report the *effective* Beveridge curve, in the $(u + \lambda(\mathbf{\Omega})\gamma, v)$ space.

We derive the explicit expressions for the Beveridge curve from the flow equations (8)-(11) evaluated in steady state we obtain:

$$u = \frac{\delta}{\delta + m(\theta(\mathbf{\Omega}))} \tag{15}$$

$$\gamma = \frac{\delta m(\theta(\mathbf{\Omega}))}{[\delta + m(\theta(\mathbf{\Omega}))][\delta + \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))]}. \tag{16}$$

With telegraph matching technology, defined in (14), we can solve explicitly for v as a function of u, v as a function of all searchers $s \equiv u + \lambda((\mathbf{\Omega}))\gamma$ as well as γ as a function of u, where $\theta = \frac{v}{s}$:

$$v = \frac{\delta u(1-u)[2\lambda(\mathbf{\Omega})(1-u)+u]}{\alpha[u(\delta+\phi)-\delta][\lambda(\mathbf{\Omega})(1-u)+u]} \tag{17}$$

$$v = -\frac{(\delta s(2\delta(-1+s) + \phi(\lambda(-2+s) + s - \sqrt{\lambda^2(-2+s)^2 + s^2 - 2\lambda s^2}))}{-2\alpha\delta(\delta+2\lambda\phi) + 2\alpha(\delta+\phi)(\delta+\lambda\phi)s} \tag{18}$$

$$\gamma = \frac{u(1-u)}{\lambda(\mathbf{\Omega})(1-u)+u} \tag{19}$$

where the first equation is the explicit expression for the Beveridge Curve in (u, v) space, which depends on the equilibrium $\mathbf{\Omega}$, denoted by $BC(\mathbf{\Omega})$. The second equation is the effecitve Beveridge Curve in (s, v) space, taking *all* searchers into account, denoted by $BC^s(\mathbf{\Omega})$. The effective BC has the same properties as the BC in the standard Pissarides framework (it is downward sloping and convex). For our purpose, of interest are the comparative properties across the multiple equilibria, which are gathered in the Proposition below.

Proposition 3 *Let there be multiple steady states. Then:*

1. *unemployment is lower with active OJS:* $u(\mathbf{1}) < u(\mathbf{0})$*;*

2. *EE flows are higher with active OJS:* $EE(\mathbf{1}) > EE(\mathbf{0})$*;*

and under the telegraph matching function:

3. $BC(\mathbf{1})$ *is shifted outward relative to* $BC(\mathbf{0})$

4. $BC^s(\mathbf{1})$ *is shifted outward relative to* $BC^s(\mathbf{0})$

5. *vacancies are higher with active OJS:* $v(\mathbf{1}) > v(\mathbf{0})$*;*

6. *market tightness is higher with active OJS:* $\Theta(\mathbf{1}) > \Theta(\mathbf{0})$*;*

7. *the effective number of on-the-job searchers is higher with active OJS but, for sufficiently small* δ*, the number of overall searchers is lower:* $\lambda(\mathbf{1})\gamma(\mathbf{1}) > \lambda(\mathbf{0})\gamma(\mathbf{0})$ *and* $s(\mathbf{1}) < s(\mathbf{0})$*.*

Proof. In Appendix. ■

This result describes in greater detail what exactly is the difference between the two equilibria whenever they coexist. Many of these features can be observed in Figure 3. Part A. plots the conventional Beveridge Curve that relates vacancies v to unemployment u with the standard market tightness $\Theta = \frac{v}{u}$; part B. plots the *effective* Beveridge Curve relating v to the *effective* number of job searchers s and corresponding effective market tightness $\theta = \frac{v}{s}$. In line with Lemma 1, if market tightness under active on the job search is high enough (intersecting with the bold part of the red Beveridge curve), then this equilibrium exists. In turn, if market tightness under non-active on the job search is low enough (intersecting with the bold part of the blue Beveridge curve), then the equilibrium with low search intensity exists.

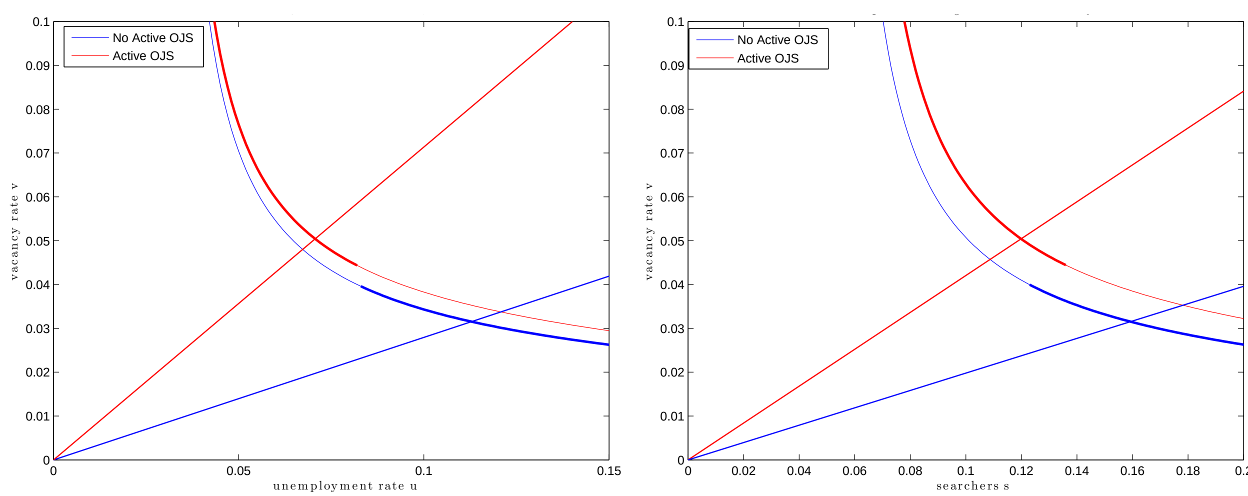

Figure 3: Multiplicity and The Beveridge Curve: A. The conventional BC, in (u, v) with $\Theta = \frac{v}{u}$; B. The effective BC, in (u, s) with $\theta = \frac{v}{s}$. (Parameters taken from the calibrated economy below.)

First, the BC shifts out under active OJS, both in the (u, v) and (u, s)-space (3. and 4.), and hence corresponds to lower match efficiency. There are many more jobs, increasing the matching probability, but the on-the-job searchers crowd out the unemployed. Hence the match efficiency per unemployed worker is lower. Vacancies are higher (4.): there are more job searchers, hence firms match faster and they have more incentives to open vacancies. But there is also a cost since the expected duration of a job goes down. Those workers hired out of unemployment will stay in the job for a shorter period. Despite the lower match efficiency, unemployment is lower under active OJS (1.). This follows immediately from the flow equation for unemployment and the fact that under multiplicity $\theta(\mathbf{1}) > \theta(\mathbf{0})$ (from Lemma 1): the matching probability increases while job separation is unchanged. Not surprisingly, the EE flow is higher under active OJS (2.). Across the two equilibria, the stock of overall searchers, s, is smaller under high search intensity (7.). But the quality of the pool of searchers is higher since there are more effective on-the job-searcher, inducing firms to post more vacancies. Finally, market tightness $\Theta = \frac{v}{u}$ is higher under active OJS (6.), which is immediate from the fact that vacancies are higher and unemployment is lower.

4 Dynamics: the Jobless Recovery

Vacancy Posting with Frictions

In order to analyze the transition dynamics in a more realistic manner, we modify the model slightly. The standard Mortensen-Pissarides, like our benchmark model above, has the feature that in the presence of any change in the environment (productivity, technology, beliefs,...), there is an instantaneous adjustment of the vacancies. The model is not immediately in steady state due to search frictions, and the fact that unemployment only gradually adjust, but vacancies always adjust instantaneously. Recently Coles and Kelishomi (2011) have pointed out that job creation itself is a slow process with frictions. While posting a vacancy by itself is instantaneous, it is the conception of the productive idea that takes time. We adopt a very stylized version of such job creation process with frictions.

To that end, we introduce a incubation state Z during which a firm or entrepreneur enters the market to develop an idea, which involves a flow cost c. The idea transits from non-productive incubation to being productive at the exogenous rate β, in which case the firm can immediately open a vacancy. We assume that there is no additional cost of opening the vacancy since those, without loss, are captured by the cost of developing the idea.[22] Then the value functions are:

$$\begin{aligned} rZ &= -c + \beta(V - Z) + \dot{Z} \\ rV &= q(\theta(\mathbf{\Omega}))\left[\frac{u}{u + \lambda(\mathbf{\Omega})\gamma}\underline{J} + \frac{\lambda(\mathbf{\Omega})\gamma}{u + \lambda(\mathbf{\Omega})\gamma}\overline{J} - Z\right] + \dot{V}, \end{aligned}$$

and all other value functions and specifications are as before. With free entry, in equilibrium Z will be driven to zero. As a result, $Z = \dot{Z} = 0$ and $V = \frac{c}{\beta}$. We can now derive exactly as before the equilibrium value functions. They are identical to those for the benchmark model without frictional

[22] Here for simplicity and without loss, we assume that the idea disappears as the job is terminated.

vacancy creation (equations (22)-(27) in the Appendix), except for the value of a vacancy (25) which now is:

$$V = q(\theta(\mathbf{\Omega}))\left[\frac{u}{u+\lambda(\mathbf{\Omega})\gamma}\left(\frac{p(\underline{y}-b)}{r+\delta} - \frac{pk\mathbf{\Omega}-\dot{J}}{r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))}\right) + \frac{\lambda(\mathbf{\Omega})\gamma}{u+\lambda(\mathbf{\Omega})\gamma}\frac{p(\overline{y}-\underline{y})}{r+\delta}\right] + \dot{V} \neq 0,$$

and together with the fact that $V = \frac{c}{\beta}$, the equilibrium condition (25) now becomes:

$$\dot{V} = \frac{rc}{\beta} - q(\theta(\mathbf{\Omega}))\left[\frac{u}{u+\lambda(\mathbf{\Omega})\gamma}\left(\frac{p(\underline{y}-b)}{r+\delta} - \frac{pk\mathbf{\Omega}-\dot{J}}{r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))}\right) + \frac{\lambda(\mathbf{\Omega})\gamma}{u+\lambda(\mathbf{\Omega})\gamma}\frac{p(\overline{y}-\underline{y})}{r+\delta}\right].$$

There are three differences: 1. in steady state, we can interpret the vacancy creation cost now as $\frac{rc}{\beta}$ instead of c, which we can think of this as a rescaling; 2. the value of the job does not adjust instantaneously, and as a result, $\dot{V} \neq 0$ out of steady state; 3. the value of a vacancy is not zero.

The flow equations are exactly as before with one addition. Now there is also a flow equation for the stock of vacancies that adjust gradually:

$$\dot{v} = \beta z - q(\theta(\mathbf{\Omega}))v,$$

where the stock of ideas in incubation stage is denoted by z, and where z adjusts instantaneously.

In steady state, there is no change compared to the benchmark model above with instantaneous vacancy posting. In fact, the necessary and sufficient conditions for multiple steady states in Lemma 1 are identical (the only change in steady state is to substitute c with $\frac{rc}{\beta}$, but the multiplicity condition does not depend on c). The bounds of the interval $[p^l, p^u]$ are those in the benchmark multiplied with $\frac{r}{\beta}$.

The main difference is in the dynamics. Our objective here is not to just match the steady states, but to use the transition from recession via recovery to boom to estimate value of β and to match as closely as possible all quarterly observations in the transition.

5 Quantitative Exercise

We now perform a quantitative exercise. First we calibrate the model to the US economy. Then, we analyze the implications of a sudden shift in beliefs to capture the jobless recovery. We also perform a counterfactual exercise, feeding observed labor productivity changes into our model (for a given equilibrium) and compare the implications for unemployment and vacancies with those of the model with multiple equilibria.

5.1 Data and Calibration of Steady State Equilibria

We calibrate our model to the US economy. Calibrating our framework requires data on vacancies, unemployment, as well as labor market transitions (employment-unemployment, employment-employment as well as employment-unemployment). Our main data source are the quarterly series from the Current Population Survey (CPS). For vacancies, we use the JOLTS data from the Bureau of Labor Statistics, supplemented by data from the Conference Board on online help-wanted ads.[23] Using this supplements for vacancies, our data spans the period 1996-2013.

We set the parameters r, b, δ, p as well as $\underline{y}, \overline{y}$ outside the model and report the values in Table 1. Our model features a constant separation rate δ across boom and recession, which is not supported by the data. Moreover, our model of multiple equilibria also requires a constant productivity in boom and recession (which we normalize to one). Both constant separations and constant productivity makes it difficult to obtain an exact fit between model and business cycle data.

Table 1: Exogenously Set Parameters

Variable	Value		Notes
r	0.012	discount factor	standard
$\underline{y}$	1	productivity first job	normalization
$\overline{y}$	1.65	productivity second job	to match wage inequality between UE and EE workers
b	0.95	unemployment value	95% of $\underline{y}$; 58% of $\overline{y}$
δ	0.06	job separation rate	average separation rate
p	1	productivity	detrended data; range [.95,1.05]

We determine the remaining parameters endogenously, including those that relate to the search technology, active and passive OJS intensity (λ_0, λ_1), the parameters of the matching function (α, ϕ) and the cost of vacancies c and of OJS k. To pin down these parameters, we target business cycle moments from the Great Recession. Central to our strategy is to target EE transitions in both boom and recession, since we would like to explain business cycle fluctuations through differences in OJS. Moreover, we target matching probabilities in boom and recession. Last we aim to choose parameters such that the model matches the empirical vacancy and unemployment levels in the *boom*. Notice that we *do not* target unemployment and vacancy levels in the recession. Instead, we want to assess whether our model of multiple equilibria is able to quantitatively match the observed fluctuations in vacancies and unemployment despite constant labor productivity.

To get sizable fluctuations in EE flows, we focus on the two data points from the last business cycle where the difference between EE was most pronounced. We denote the third quarter of 2006 as *boom* $(\Omega = 1)$ and the third quarter of 2009 as *recession* $(\Omega = 0)$. We use General Method of Moments to estimate our model, with equal number of moments and parameters. Targeted moments (data and model) as well as estimated parameter values are in tables 2 and 3.

We match the EE flows in boom and recession fairly well, most importantly we match the difference between the two. We match the matching probabilities nearly exactly. The model moments of unemployment and vacancies in the boom are slightly higher than the data moments. The main reason

[23]We thank Regis Barnichon for making this data available to us.

Table 2: Targeted Moments

	Data	Model
$EE(\mathbf{1})$	0.066	0.042
$EE(\mathbf{0})$	0.036	0.024
$u(\mathbf{1})$	0.047	0.071
$v(\mathbf{1})$	0.029	0.050
$m(\theta(\mathbf{1}))$	0.852	0.855
$m(\theta(\mathbf{0}))$	0.511	0.511

Table 3: Estimated Parameters

	Estimate	Parameter Description
λ_0	0.089	passive OJS intensity
λ_1	0.086	active OJS intensity
α	1.593	curvature matching function
ϕ	2.132	overall matching efficiency
c	7.560	vacancy posting cost
k	0.045	job separation rate

for the gap between model and data are constant separation rates and labor productivity across boom and recession.

The parameter estimates suggest that search intensity of on the job searchers is considerably lower than search intensity of unemployed workers, normalized to 1 (i.e. $\lambda_0 + \lambda_1 = 0.17$). Moreover, on the job searchers are about twice as actively searching in boom compared to recession. The curvature parameter of the matching technology α is estimated to be 1.6 and matching efficiency ϕ is estimated to be about 2.1. Notice that the matching efficiency is estimated to be higher than what is suggested by most estimates in the literature, which stems from using a different tightness measure v/s (which is smaller than the conventional v/u since it takes all searchers into account). The costs of OJS are estimated to be a small fraction of the first job's output (about 5%). Finally, the estimated costs of posting a vacancy are comparably high, $c = 7.5$, corresponding to a hiring cost of a little more than a year's output of a high productivity match. The estimated costs are large but in line with a growing literature that argues hiring costs are substantial and, depending on the worker type, can take up more than an annual wage.[24]

Using this calibration, we are interested in the unemployment and vacancy fluctuations that the model predicts only based on multiplicity and without any movements of labor productivity. Table 4 suggests that our model of multiple equilibria performs well in matching the *non-targeted* moments of unemployment and vacancies in the recession. Comparing vacancy and unemployment levels across boom and recession (Table 2 and 4), it generates sizable fluctuations: Our model explains about 61% of the observed increase in unemployment and 74% of the drop in vacancies during the Great Recession. It is important to note that these fluctuations are obtained through multiple equilibria alone and without alluding to any decline in labor productivity, which is held fixed in our exercise. This suggests that the difference in the intensity at which workers search on the job in boom and recession has a profound impact on labor market fluctuations.

Using the estimates from the calibrated economy, we can now compare the output net of search costs in boom and recession. We find that net output is larger in the boom than the recession: $Y(\mathbf{1}) = 0.9502, Y(\mathbf{0}) = 0.8824$. In other words, a difference of $\left(\frac{Y(\mathbf{1})}{Y(\mathbf{0})} - 1\right) * 100 = 7.1\%$. For labor

[24]For evidence on substantial hiring costs, see, for instance, Blatter, Muehlemann, and Schenker (2012) as well as Dube, Freeman, and Reich (2010) and the references therein.

Table 4: Non-Targeted Moments

	Data	**Model**
$u(\mathbf{0})$	0.096	0.113
$v(\mathbf{0})$	0.016	0.032

productivity fluctuations of more than ±10%, we find that net output is always higher in the boom equilibrium where search intensity is high. In this calibrated version of the model, this implies that we can clearly rank the two equilibria in terms of welfare.

5.2 Analysis

Market Tightness and the Decomposition of Flows

The key premise underlying our model is that workers search more actively on the job during boom, inducing firms to post more vacancies compared to the recession. In this section, we provide evidence for this mechanism.

We decompose employment-to-employment flows, given by $EE = \gamma\lambda m(\theta)$, into its three determinants: the stock of on the job searchers γ, the search intensity λ, and matching probability $m(\theta)$. To separate the cyclical component from the trend (which our paper has little to say about), we plot the de-trended variables. We measure the matching probability in the data from $m(\theta) = UE/u$, γ from the steady state flow equation[25]

$$\gamma = \frac{UE - EE}{\delta}$$

and obtain search intensity from $\lambda = (EE/\gamma)(u/UE)$.

EE flows are pro-cyclical in the data (Figure 4.A). This is not driven by the number of on the job searchers γ, which is counter-cyclical (Figure 4.C): The *number* of on-the-job searchers actually increases in the recession, even though they are not actively searching. The reason is that towards the recession, many workers are stuck in low productivity jobs due to low matching rates. With fewer job-to-job transitions, the fraction of those in low productivity jobs is increasing. In the boom, all search actively, hence they leave the pool at a higher rate, implying that the stock in steady state is smaller. Instead, pro-cyclicality of EE flows stem from two sources: First, we know that from boom to recession, v sharply drops, and u sharply increases, yielding pro-cyclical matching probabilities (Figure 4.B). But also search intensity is pro-cyclical (Figure 4.D), being high in the boom and falling sharply during recession. EE flows then go down during recession, because $EE = \lambda\gamma m(\theta)$, and the decrease in both θ (and therefore in $m(\theta)$) and search intensity λ dominate the increase in γ.

This decomposition exercise shows that our mechanism that relies on cyclical fluctuations in search

[25]This flow equation only holds in steady state and thus not over the whole cycle. We take this as a first approximation for γ and address this issue below by measuring γ directly from the data.

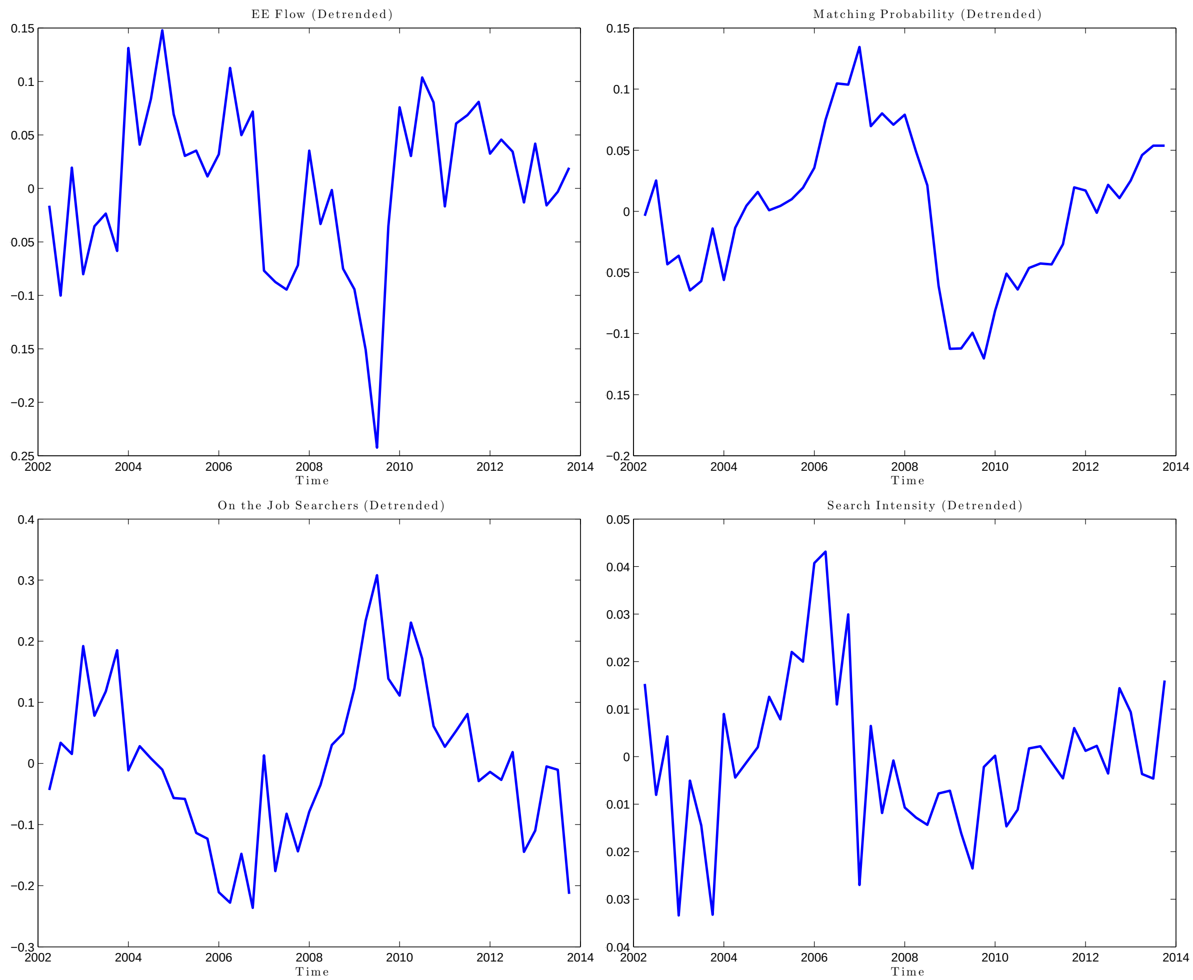

Figure 4: Decomposition of EE Flows (de-trended). A. EE Flows. B. Matching Probability. C. Stock of On the Job Searchers. D. Search Intensity.

intensity and firms responding endogenously to this behavior, has bite. Higher search intensity of workers is rewarded by higher matching rates during booms compared to recessions.[26] Fluctuations in the labor market flows are not merely driven by fluctuations in market tightness and thus matching probabilities. This is not only the case for EE flows but also for UE flows (see Figure 7, Appendix), which *go up* during downturns, even though matching probabilities sharply fall.

The Composition of Jobs

While in the last section, we measure the stock of on the job searchers in the data from a steady state relation, here we aim to take a more direct approach. To do so, we use the panel structure from the CPS Micro-data to assess whether employed individuals transited to their jobs from unemployment, which

[26] This is in line with the literature that documents that search intensity correlates with real outcomes. In fact, it predicts how likely it is to find a job. See, e.g., Krueger and Mueller (2011) who shows that the amount of time devoted to job search helps predict early exits from Unemployment Insurance.

we count into the stock of on the job searchers γ, or from another job, indicating an EE move (which we then count into the stock ξ). The CPS interviews individuals for 4 consecutive months, then gives them a break for 8 consecutive months, and finally interviews them again for 4 consecutive months. We aggregate the monthly data up to the quarterly level. Figure 5 shows the results of this exercise.

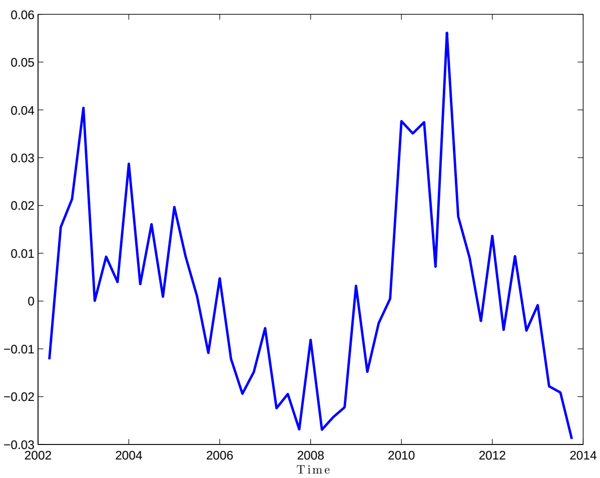

Figure 5: A. Stock of workers in low productivity jobs γ; B. Stock of workers in high productivity jobs ξ. Both de-trended.

In line with what the calibrated version of our model predicts, the stock of on the job searchers is countercyclical (Figure 5.A), indicating that during downturns many workers are stuck at the bottom of the job ladder. But few workers are in high productivity jobs at the top, reflected by a low stock ξ (Figure 5.B). This shows that the recessions negatively affects the composition of jobs, with a considerable bias towards low-productivity jobs. This resembles the result by Moscarini and Postel-Vinay (2012), who state that the job ladder has failed during the Great Recession.

Jobless Recovery and Crowding Out

Below we will analyze the full dynamics of the model. For now, consider a very simple exercise. Suppose we are at the bottom of the recession, and we investigate the impact of a change in beliefs, even if the economy is in the recession equilibrium: all workers in a low productivity job start actively searching for a job and firms instantaneously adjust by posting vacancies so that profits are driven to zero.

The immediate implication is an increase in the active searchers from $s(\mathbf{0}) = \lambda_0\gamma(\mathbf{0})$ to $s^R = (\lambda_0+\lambda_1)\gamma(\mathbf{0})$, where the superscript R stands for Recovery. This also leads to *crowding out.* Conditional on forming a match, the probability that it is with an unemployed worker is now lower since more are searching on the job. Denote by κ the fraction of hires with the worker coming from unemployment. Then:

$$\kappa(\mathbf{0}) = \frac{u(\mathbf{0})}{u(\mathbf{0}) + \lambda_0\gamma(\mathbf{0})} \quad \text{and} \quad \kappa^R = \frac{u(\mathbf{0})}{u(\mathbf{0}) + (\lambda_0+\lambda_1)\gamma(\mathbf{0})}$$

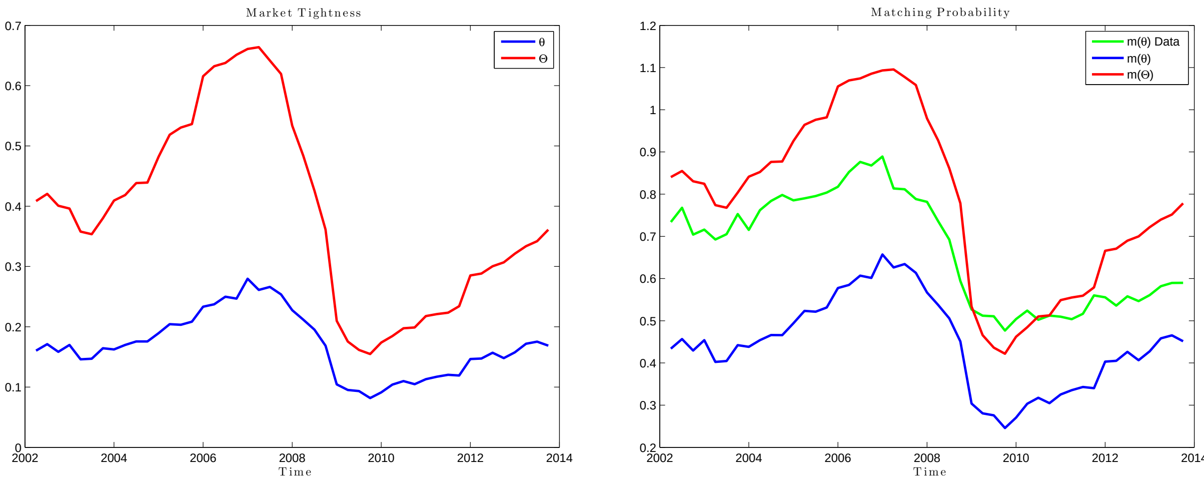

Figure 6: A. Market tightness $\Theta = \frac{v}{u}$ and effective market tightness $\theta = \Theta \frac{UE}{UE+EE}$ in the data; B. Matching probabilities in data and with telegraph matching (using Θ and θ from the data and calibrated parameters).

The stocks $u(\mathbf{0})$ and $\gamma(\mathbf{0})$ have not changed yet, but there is already an immediate response in the search activity. As a result it follows that $\kappa^R < \kappa(\mathbf{0})$. According to the estimates of our calibration, this implies that conditional on the formation of a match, the probability that it is with an unemployed worker goes from $\kappa(\mathbf{0}) = 0.71$ to $\kappa^R = 0.33$. As soon as the recovery starts, the likelihood, conditional on a match formation, that an unemployed workers is selected over a worker with a job drops by a factor of more than 2. This is what we refer to as *crowding out* in the recovery.

Certainly, what matters is not just the conditional likelihood of being drawn. It is also important how fast the matching is. Under the belief that employed workers actively search for a job, the matching rate for firms goes up, and in response, new vacancies are created until profits are driven down to zero again. As a result, the market tightness changes as does the matching probability $m(\theta)$. Therefore, the unconditional matching probability for an unemployed worker is $\kappa m(\theta)$, which drops from $\kappa(\mathbf{0})m(\theta(\mathbf{0})) = 0.36$ in the recession to $\kappa^R m(\theta^R) = 0.33$ in the recovery. The implication of the lower matching rates is that the unemployment rate initially edges up marginally: $\dot{u} > 0$ since the separation rate δ is unchanged. In turn, the matching rate of the employed workers who search doubles, going from $(1 - \kappa(\mathbf{0}))m(\theta(\mathbf{0})) = 0.14$ in the recession to $(1 - \kappa^R)m(\theta^R) = 0.27$ in the recovery.

Within the framework of our model, we can also highlight the role of the effective market tightness for jobless recovery. Since we observe vacancies and unemployment, we can readily construct the conventional market tightness $\Theta = \frac{v}{u}$. We want to compare this to the effective market tightness $\theta = \frac{v}{s}$, which we obtain from the data as follows:

$$\theta = \frac{v}{u + \lambda\gamma} = \frac{v}{\frac{UE}{m(\theta)} + \frac{EE}{m(\theta)}} = \Theta \frac{UE}{UE + EE},$$

since $EE = m(\theta)\lambda\gamma$ and $UE = m(\theta)u$. Figure 6.A has the plot of both θ and Θ. It is apparent that there is not only less fluctuation in θ than there is in Θ but in particular, after the crisis, the recovery of θ is much flatter than that indicated by Θ. This is due to the fact that the effective market tightness θ reflects the change in the number of on-the-job searchers. The implications for fluctuations in matching probabilities follow immediately (Figure 6.B): While the matching probability according to $m(\Theta)$ (using the telegraph matching function) indicates fast recovery, matching probability $m(\theta)$ recovers more slowly, closely resembling the slow recovery of matching probability in the data and fueling jobless recovery.

All this indicates that the immediate impact of the recovery out of the recession looks even bleaker than the recession itself. Due to crowding out, the unemployed initially match at a slower rate and the unemployment rate goes up. Rather than a jobless recovery, this indicates a more accurate term might be a *job-destructive recovery*.

The Role of Productivity Fluctuations

We now consider the role of labor productivity movements in generating fluctuations of labor market outcomes. Shimer (2005) argues that in the standard Mortensen-Pissarides model of unemployment, productivity fluctuations cannot account for the fluctuations in unemployment and vacancies observed in the data. Several studies have offered explanations to counter Shimer's finding, and can indeed create labor market volatility from small productivity shocks, for instance, sufficiently high value of unemployment b (Hagedorn and Manovskii (2008)). To shed light on the role of productivity changes and to isolate it from our channel of multiplicity, we perform the following counterfactual. We pretend the boom equilibrium with active on the job search is the unique equilibrium and feed the labor productivity drop that was observed during the Great Recession into the model (see Figure 8 for detrended labor productivity movements in the data). Can the observed decline in productivity generate fluctuations of similar magnitudes as our model of multiple equilibria? In response to this productivity change, vacancies under active on the job search drop by 10% while unemployment increases by 11%. In comparison, our model of multiplicity, by switching from boom to recession equilibrium, generates a drop in vacancies of 36% and an increase in unemployment by 59%, coming much closer to the observed fluctuations in vacancy (-45%) and unemployment rates (+104%) (see section 5.1 for details).

This suggests that changes in search behavior on the job are more important in generating labor market fluctuations than productivity movements, even when unemployment benefits are set relatively high.

6 Conclusion

We have argued that the labor market behavior of the employed can have profound implications on the unemployed. Moreover, even in the absence of exogenous shocks, this by itself can create multiple equilibria and hence cyclical outcomes. Active on-the-job search by the employed makes it more attractive for firms to post vacancies, which in turn makes on-the-job search more attractive. Self-fulfilling beliefs can thus give rise to either a high activity on-the-job search equilibrium which we interpret as a boom as well as a low activity equilibrium interpreted as a recession.

These beliefs give rise to large fluctuations in vacancies, unemployment and job-to-job transitions, even without any change in the productivity or other primitives. Moreover, in the transition from a low on-the-job search equilibrium to an equilibrium with active on-the-job search, this model naturally generates a jobless recovery. As the employed start to search, they crowd out the unemployed, making the recovery for the unemployed even worse than the recession. In fact, initially unemployment even rises. We believe that this crowding out channel is new in the literature. The model also gives rise to a shift in the Beveridge Curve (the unemployment-vacancy locus). This shift is not driven by an exogenous change in the matching efficiency or the simple transition dynamics around a unique steady state, but by the transition from one steady state to another. We calibrate the model to the US economy and can match each of these phenomena in the data. We then use our calibrated model for counterfactual exercises to assess the importance of labor productivity movements relative to our mechanism that relies on multiple equilibria: Feeding observed productivity movements into one of our two equilibria generates only small fluctuations in labor market outcomes compared to both the fluctuations observed in the data and those generated by our model of multiplicity.

Finally, the multiplicity that we have singled out in this framework should also draw the attention of those estimating models with on-the-job search. From a methodological viewpoint, the identification strategy must change in the presence of multiplicity.

Appendix

UE, EE, UE+EE flows and Labor Productivity

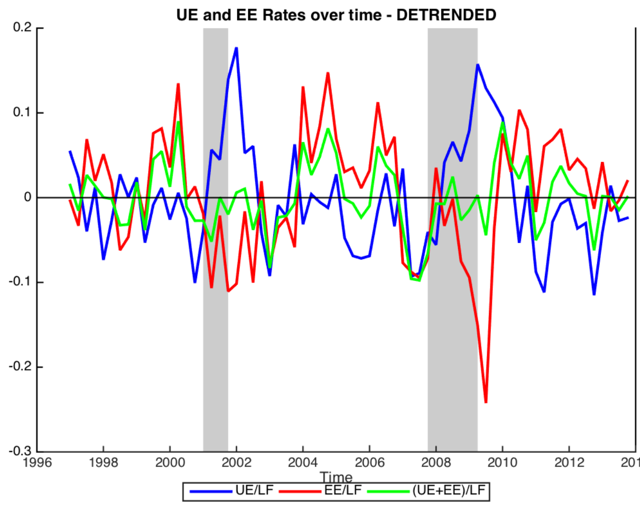

Figure 7: Flows (detrended).

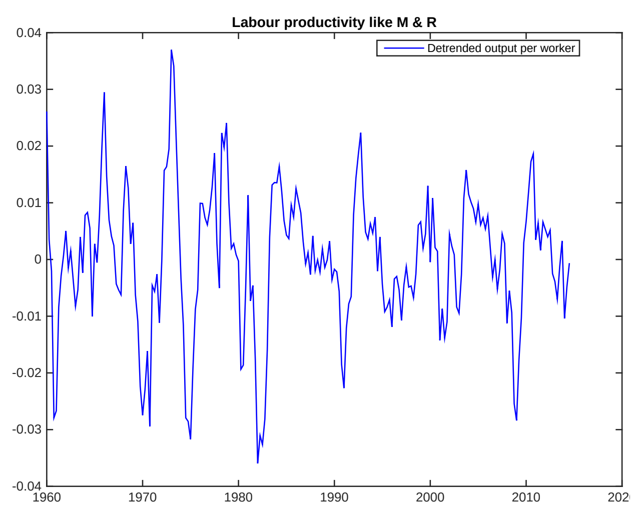

Figure 8: Labor Productivity (Measured as Output per Worker, Detrended).

Equilibrium Value Functions

Firms believe workers take an individual action Ω consistent with the equilibrium belief $\mathbf{\Omega}$, i.e., $\Omega = \mathbf{\Omega}$. Wage setting requires that $\underline{E} = U$, which implies that $\dot{\underline{E}} = \dot{U}$. Solving for U in the Bellman equation (1), using the fact that from wage setting, $\underline{E} = U$ and $\dot{\underline{E}} = \dot{U}$, implies:

$$U = \frac{pb}{r} + \frac{\dot{U}}{r} = \frac{pb}{r}, \tag{20}$$

where the second equality follows from the fact that the first term $\frac{pb}{r}$ is a constant, which immediately implies that $\dot{U} = 0$, and as a result, $\dot{\underline{E}} = 0$. Using this expression for U to solve for $\underline{E}$ in (2) we get:

$$\underline{E} = \frac{\underline{w}(\mathbf{\Omega}) - \mathbf{\Omega} pk + \lambda(\mathbf{\Omega}) m(\theta(\mathbf{\Omega}))\overline{E}}{r + \lambda(\mathbf{\Omega}) m(\theta(\mathbf{\Omega}))}. \tag{21}$$

Solving for (3) implies:

$$\overline{E} = \frac{\overline{w}(\mathbf{\Omega}) + \delta\frac{pb}{r} + \dot{\overline{E}}}{r + \delta}.$$

The equilibrium wage for the high productivity job $\overline{w}$ is pinned down by the sequential auction framework setting $\underline{J} = V = 0$, which applies to any new formed high type match. Since $\dot{V} = 0$, this implies that the wage set in the high productivity job is time invariant. Solving for the wage from $\underline{J} = V = 0$ implies:

$$\overline{w} = p\underline{y},$$

which is independent of the equilibrium $\mathbf{\Omega}$ and time invariant. This implies that

$$\overline{E} = \frac{p\underline{y} + \delta\frac{pb}{r}}{r + \delta},$$

where $\dot{\overline{E}} = 0$ since all other terms are constants.

The equilibrium wage for the low productivity job $\underline{w}$ is pinned down by the sequential auction framework setting $\underline{E} = U$. Using (20) and (21) to solve for $\underline{w}(\mathbf{\Omega})$ implies:

$$\underline{w}(\mathbf{\Omega}) = pb\left(\frac{r + \lambda(\mathbf{\Omega}) m(\theta(\mathbf{\Omega})) + \delta}{r + \delta}\right) - \frac{\lambda(\mathbf{\Omega}) m(\theta(\mathbf{\Omega}))}{r + \delta} p\underline{y} + \mathbf{\Omega} pk,$$

which depends on time only through θ.

From free entry, $V = 0$ and therefore (5), (6) and (7) can be written as:

$$\begin{aligned} 0 &= -c + q(\theta(\mathbf{\Omega}))\left[\frac{u}{u+\lambda(\mathbf{\Omega})\gamma}\underline{J} + \frac{\lambda(\mathbf{\Omega})\gamma}{u+\lambda(\mathbf{\Omega})\gamma}\overline{J}\right] \\ \overline{J} &= \frac{p\overline{y}-\overline{w}}{r+\delta} \\ \underline{J} &= \frac{p\underline{y}-\underline{w}(\mathbf{\Omega})+\dot{\underline{J}}}{r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))}. \end{aligned}$$

Now using the equilibrium wages and substituting all explicit solutions for the values, we obtain the following complete set of equilibrium Bellman equations:

$$U = \frac{pb}{r} \tag{22}$$

$$\underline{E} = \frac{pb}{r} \tag{23}$$

$$\overline{E} = \frac{p\underline{y}+\delta\frac{pb}{r}}{r+\delta} \tag{24}$$

$$V = -c + q(\theta(\mathbf{\Omega}))\left[\frac{u}{u+\lambda(\mathbf{\Omega})\gamma}\left(\frac{p(\underline{y}-b)}{r+\delta} - \frac{pk\mathbf{\Omega}-\dot{\underline{J}}}{r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))}\right) + \frac{\lambda(\mathbf{\Omega})\gamma}{u+\lambda(\mathbf{\Omega})\gamma}\frac{p(\overline{y}-\underline{y})}{r+\delta}\right] = 0 \tag{25}$$

$$\underline{J} = \frac{p(\underline{y}-b)}{r+\delta} - \frac{pk\mathbf{\Omega}-\dot{\underline{J}}}{r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))} \tag{26}$$

$$\overline{J} = \frac{p(\overline{y}-\underline{y})}{r+\delta}. \tag{27}$$

Proof of Lemma 1

Proof. 1. No deviation when no one searches: $\underline{E}(0|\mathbf{0}) > \underline{E}(1|\mathbf{0})$.

In this case, when no one actively searches on-the-job ($\mathbf{\Omega} = \mathbf{0}$), a worker in a low productivity job deviating during an interval dt chooses $\Omega = 1$ and gets a payoff

$$\underline{E}(1|\mathbf{0}) = \frac{1}{1+rdt}\left[dt(\underline{w}(\mathbf{0})-pk) + dt\lambda(1)(1-\delta dt)m(\theta(\mathbf{0}))\overline{E} + (1-\delta dt)(1-dt\lambda(1)m(\theta(\mathbf{0})))\underline{E}(0|\mathbf{0}) + \delta dtU\right]$$

where $\overline{E} = \overline{E}(0|\mathbf{0})$ since that value is the same independent of the argument. There is no deviation provided $\underline{E}(0|\mathbf{0}) > \underline{E}(1|\mathbf{0})$ or:

$$\underline{E}(0|\mathbf{0})(1+rdt) > dt(\underline{w}(\mathbf{0})-pk) + dt\lambda(1)(1-\delta dt)m(\theta(\mathbf{0}))\overline{E} + (1-\delta dt - dt\lambda(1)m(\theta(\mathbf{0})) + dt^2\delta\lambda(1)m(\theta(\mathbf{0})))\underline{E}(0|\mathbf{0}) + \delta dtU.$$

After subtracting $\underline{E}(0|\mathbf{0})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$r\underline{E}(0|\mathbf{0}) > \underline{w}(\mathbf{0}) - pk + \lambda(1)m(\theta(\mathbf{0}))\overline{E} + (-\delta - \lambda(1)m(\theta(\mathbf{0})))\underline{E}(0|\mathbf{0}) + \delta U.$$

Substituting the equilibrium values for $\underline{E}(0|\mathbf{0}), \overline{E}, U$ and $\underline{w}(\mathbf{0})$ we get:

$$(\underline{y}-b)[\lambda(1)-\lambda(0)]m(\theta(\mathbf{0})) - k(r+\delta) < 0. \tag{28}$$

2. No deviation when all search: $\underline{E}(1|\mathbf{1}) > \underline{E}(0|\mathbf{1})$.

In this case, when all actively search on-the-job ($\mathbf{\Omega} = \mathbf{1}$), a worker in a low productivity job deviating for an interval dt chooses $\Omega = 0$ and gets a payoff

$$\underline{E}(0|\mathbf{1}) = \frac{1}{1+rdt}\left[dt\underline{w}(\mathbf{1}) + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{1}))\overline{E} + (1-\delta dt)(1-dt\lambda(0)m(\theta(\mathbf{1})))\underline{E}(1|\mathbf{1}) + \delta dtU\right].$$

There is no deviation provided $\underline{E}(1|\mathbf{1}) > \underline{E}(0|\mathbf{1})$:

$$\underline{E}(1|\mathbf{1})(1+rdt) > dt\underline{w}(\mathbf{1}) + dt\lambda(0)(1-\delta dt)m(\theta(\mathbf{1}))\overline{E} + (1-\delta dt - dt\lambda(0)m(\theta(\mathbf{1})) + dt^2\delta\lambda(0)m(\theta(\mathbf{1})))\underline{E}(1|\mathbf{1}) + \delta dtU.$$

After subtracting $\underline{E}(1|\mathbf{1})$ from both sides and dividing by dt and taking the limit $dt \to 0$, we obtain:

$$r\underline{E}(1|\mathbf{1}) > \underline{w}(\mathbf{1}) + \lambda(0)m(\theta(\mathbf{1}))\overline{E} + (-\delta - \lambda(0)m(\theta(\mathbf{1})))\underline{E}(1|\mathbf{1}) + \delta U.$$

Substituting the equilibrium values for $\underline{E}(1|\mathbf{1}), \overline{E}, U$ and $\underline{w}(\mathbf{1})$ we get:

$$(\underline{y} - b)[\lambda(1) - \lambda(0)]m(\theta(\mathbf{1})) - k(r+\delta) > 0. \tag{29}$$

Putting (28) and (29) together gives the condition in the Lemma. ■

Proof of Proposition 1

Proof. In steady state, the limit $\delta \to 0$ of equation (9) implies $u = \lambda(\mathbf{\Omega})\gamma$. Then $\frac{u}{u+\lambda(\mathbf{\Omega})\gamma} \to \frac{1}{2}$, and equation (25), the free entry condition, is an expression only in terms of the endogenous variable θ, and the parameters. With the functional form of the telegraph matching function, we can actually solve this equation explicitly for θ. For each of the two candidate equilibria we obtain:

$$\begin{aligned}
\theta(\mathbf{1}) &= -\frac{2(\phi(\lambda_0+\lambda_1)+2r)+kr}{4\alpha(\phi(\lambda_0+\lambda_1)+r)} + \overline{y} - \alpha^2 p\phi b \\
&\quad + \frac{\sqrt{\alpha^2(-8cr^2(\phi(\lambda_0+\lambda_1)+r)(2cr-\alpha p\phi(\overline{y}-b-k)) + [cr2(\phi(\lambda_0+\lambda_1)+2r) + \alpha p\phi(kr-(\overline{y}-b)[\phi(\lambda_0+\lambda_1)+r])]^2)}}{4\alpha^2 cr(\phi(\lambda_0+\lambda_1)+r)} \\
\theta(\mathbf{0}) &= -\frac{2(\phi\lambda_0+2r)}{4\alpha(\phi\lambda_0+r)} + \overline{y} - \alpha^2 p\phi b \\
&\quad + \frac{\sqrt{\alpha^2(-8cr^2(\phi\lambda_0+r)(2cr-\alpha p\phi(\overline{y}-b)) + (cr2(\phi\lambda_0+2r)+\alpha p\phi(-(\overline{y}-b)(\phi\lambda_0+r)))^2)}}{4\alpha^2 cr(\phi\lambda_0+r)}
\end{aligned}$$

Then condition **M**, from Lemma 1 is:

$$\begin{aligned}
&-\frac{2(\phi\lambda_0+2r)}{4\alpha(\phi\lambda_0+r)} + \overline{y} - \alpha^2 p\phi b + \frac{\sqrt{\alpha^2(-8cr^2(\phi\lambda_0+r)(2cr-\alpha p\phi(\overline{y}-b)) + (cr2(\phi\lambda_0+2r)+\alpha p\phi(-(\overline{y}-b)(\phi\lambda_0+r)))^2)}}{4\alpha^2 cr(\phi\lambda_0+r)} \\
&\qquad < \frac{kr}{\alpha\left(\phi\lambda_1(\underline{y}-b)-kr\right)} < -\frac{2(\phi(\lambda_0+\lambda_1)+2r)+kr}{4\alpha(\phi(\lambda_0+\lambda_1)+r)} + \overline{y} - \alpha^2 p\phi b \qquad (\mathbf{M}) \\
&+\frac{\sqrt{\alpha^2(-8cr^2(\phi(\lambda_0+\lambda_1)+r)(2cr-\alpha p\phi(\overline{y}-b-k)) + (cr2(\phi(\lambda_0+\lambda_1)+2r)+\alpha p\phi(kr-(\overline{y}-b)(\phi(\lambda_0+\lambda_1)+r)))^2)}}{4\alpha^2 cr(\phi(\lambda_0+\lambda_1)+r)}
\end{aligned}$$

To obtain the p-bounds, we solve the inequalities in ($\mathbf{M}$) with equality and solve for p^l in the first inequality and for p^u in the second one.

$$\begin{aligned} p^l &= \frac{2c\lambda_1 r(\underline{y}-b)[k(\lambda_0+\lambda_1)-\lambda_1(\underline{y}-b)]}{\alpha[\lambda_1\phi(\underline{y}-b)-kr][b^2\lambda_1+k(\lambda_0+\lambda_1)\overline{y}+\lambda_1(\overline{y}-k)\underline{y}-b(k\lambda_0+\lambda_1\overline{y}+\lambda_1\underline{y})]} \\ p^u &= \frac{2c\lambda_1 r(\underline{y}-b)}{\alpha(\overline{y}-b)[\lambda_1\phi(\underline{y}-b)-kr]} \end{aligned}$$

We show that $p^u > p^l$ or

$$\begin{aligned} \frac{2c\lambda_1 r(\underline{y}-b)}{\alpha(\overline{y}-b)[\lambda_1\phi(\underline{y}-b)-kr]} - \frac{2c\lambda_1 r(\underline{y}-b)[k(\lambda_0+\lambda_1)+\lambda_1(\underline{y}-b)]}{\alpha[\lambda_1\phi(\underline{y}-b)-kr][b^2\lambda_1+k(\lambda_0+\lambda_1)\overline{y}-\lambda_1(k-\overline{y})\underline{y}-b((k\lambda_0+\lambda_1\overline{y})+\lambda_1\underline{y})]} &> 0 \\ \frac{1}{(\overline{y}-b)} - \frac{[k(\lambda_0+\lambda_1)+\lambda_1(\underline{y}-b)]}{[b^2\lambda_1+k(\lambda_0+\lambda_1)\overline{y}-\lambda_1(k-\overline{y})\underline{y}-b((k\lambda_0+\lambda_1\overline{y})+\lambda_1\underline{y})]} &> 0 \\ b^2\lambda_1+k(\lambda_0+\lambda_1)\overline{y}-\lambda_1(k-\overline{y})\underline{y}-b((k\lambda_0+\lambda_1\overline{y})+\lambda_1\underline{y})-(\overline{y}-b)[k(\lambda_0+\lambda_1)+\lambda_1(\underline{y}-b)] &> 0 \\ (\overline{y}-\underline{y})(\lambda_0+\lambda_1)k+(\underline{y}-b)\lambda_1(\overline{y}-\underline{y}-k) &> 0. \end{aligned}$$

So, if $(\overline{y}-\underline{y})(\lambda_0+\lambda_1)k+(\underline{y}-b)\lambda_1(\overline{y}-\underline{y}-k) > 0$ and $b^2\lambda_1+k(\lambda_0+\lambda_1)\overline{y}-\lambda_1(k-\overline{y})\underline{y}-b((k\lambda_0+\lambda_1\overline{y})+\lambda_1\underline{y}) > 0$, then $p^u > p^l$. ■

Proof of Proposition 2

Proof. 1. We aim to show that there will be a deviation. An immediate implication of Lemma 1 is that $\theta(\mathbf{0}) < \theta(\mathbf{1})$ and therefore a sufficient condition for a unique equilibrium is that $\theta(\mathbf{0}) > \theta(\mathbf{1})$. To show this, observe that $\underline{J}(\mathbf{1}) = \underline{J}(\mathbf{0}) - K$ where $K = \frac{pk\mathbf{\Omega}}{r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))}$ from equation (26).

From (25), the free entry condition, when $\delta \to 0$ and $\overline{y} \to \underline{y}$, in both equilibria, can be written as:

$$\begin{aligned} \frac{2c}{q(\theta(\mathbf{1}))} &= \underline{J}(\mathbf{0}) - \frac{pk}{r+\delta+\lambda(\mathbf{1})m(\theta(\mathbf{1}))} \\ \frac{2c}{q(\theta(\mathbf{0}))} &= \underline{J}(\mathbf{0}) \end{aligned}$$

Since the LHS is increasing in θ and $\frac{pk}{r+\delta+\lambda(\mathbf{1})m(\theta(\mathbf{1}))} > 0$, it immediately follows that $\theta(\mathbf{1}) < \theta(\mathbf{0})$. By continuity, this holds for $\overline{y} - \underline{y} < \varepsilon$.

2. Fix $\underline{y} < \infty$ and let $\overline{y} \to \infty$. We aim to show that when there is no active on-the-job search by all workers, there is always a profitable deviation by an individual. From Lemma 1 this is equivalent to violating $\theta(\mathbf{0}) < m^{-1}\left(\frac{k(r+\delta)}{\lambda_1(\underline{y}-b)}\right)$. From equation (25) we obtain:

$$\begin{aligned} \frac{2c}{q(\theta(\mathbf{\Omega}))} &= \underline{J}(\mathbf{0}) - \frac{pk\mathbf{\Omega}}{r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))} + \lim_{\overline{y}\to\infty} \frac{\lambda(\mathbf{\Omega})\gamma}{u+\lambda(\mathbf{\Omega})\gamma}\frac{p(\overline{y}-\underline{y})}{r+\delta} \\ &= \underline{J}(\mathbf{0}) - \frac{pk\mathbf{\Omega}}{r+\delta+\lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))} + \infty \end{aligned}$$

which under no active on-the-job search is

$$\frac{2c}{q(\theta(\mathbf{0}))} \quad = \quad \underline{J}(\mathbf{0}) + \infty.$$

This can only be satisfied when $\theta \to \infty$, thus violating the no-deviation condition. By continuity, this holds for finite but large $\overline{y}$.

3. We can compute the bounds for $\overline{y}$ explicitly from a system of two equations (per bound), namely free entry and the no-deviation condition.

$$\begin{aligned}
\overline{y}^l &= b + k + \frac{2cr}{\alpha\phi p} - \frac{2ckr^2}{\alpha b\lambda_1\phi^2 p + \alpha k\phi pr - \alpha\lambda_1\phi^2 p\underline{y}} - \frac{k^2(\lambda_0+\lambda_1)}{k(\lambda_0+\lambda_1)+\lambda_1(-b+\underline{y})} \\
\overline{y}^h &= \frac{\alpha bp(kr+\lambda_1\phi(b-\underline{y})) + 2c\lambda_1 r(b-\underline{y})}{\alpha p(kr+\lambda_1\phi(b-\underline{y}))}
\end{aligned}$$

The set $[\overline{y}^l, \overline{y}^l]$ is non-empty. ■

Proof of Proposition 3

Proof. Each of the items in the proposition hinges on the fact that $\theta(\mathbf{1}) > \theta(\mathbf{0})$, which follows from Lemma 1.

1. From equation (15), $u(\mathbf{1}) < u(\mathbf{0})$ immediately follows from the fact that $\theta(\mathbf{1}) > \theta(\mathbf{0})$ and $m(\theta)$ is increasing.

2. EE flows are defined as:

$$\begin{aligned}
EE(\mathbf{\Omega}) &= \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))\gamma(\mathbf{\Omega}) \\
&= \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega}))\frac{\delta m(\theta(\mathbf{\Omega}))}{(\delta + m(\theta(\mathbf{\Omega})))(\delta + \lambda(\mathbf{\Omega})m(\theta(\mathbf{\Omega})))},
\end{aligned}$$

using (16). Then $EE(\mathbf{1}) > EE((\mathbf{0})$ provided

$$\begin{aligned}
\frac{\delta\lambda(\mathbf{1})m(\theta(\mathbf{1}))^2}{(\delta + m(\theta(\mathbf{1})))(\delta + \lambda(\mathbf{1})m(\theta(\mathbf{1})))} - \frac{\delta\lambda(\mathbf{0})m(\theta(\mathbf{0}))^2}{(\delta + m(\theta(\mathbf{0})))(\delta + \lambda(\mathbf{0})m(\theta(\mathbf{0})))} &> \\
\delta^2\left(\lambda(\mathbf{1})m(\theta(\mathbf{1}))^2 - \lambda(\mathbf{0})m(\theta(\mathbf{0}))^2\right) + \lambda(\mathbf{0})\lambda(\mathbf{1})m(\theta(\mathbf{0}))m(\theta(\mathbf{1}))\left[m(\theta(\mathbf{1})) - m(\theta(\mathbf{0}))\right] & \\
+ m(\theta(\mathbf{0}))m(\theta(\mathbf{1}))\delta\left[\lambda(\mathbf{1})m(\theta(\mathbf{1})) - \lambda(\mathbf{0})m(\theta(\mathbf{0}))\right] &> 0,
\end{aligned}$$

which is holds since $\lambda(\mathbf{1}) > \lambda(\mathbf{0})$ and under multiplicity $m(\theta(\mathbf{1})) > m(\theta(\mathbf{0}))$.

3. Since we need an explicit expression for the matching function to solve explicitly for v, we derive this for the telegraph matching function. For a given unemployment rate u, from (17) we find that $v(\mathbf{1}) > v(\mathbf{0})$ provided

$$\frac{2\lambda(\mathbf{1})(1-u)+u}{\lambda(\mathbf{1})(1-u)+u} > \frac{2\lambda(\mathbf{0})(1-u)+u}{\lambda(\mathbf{0})(1-u)+u}$$

which is satisfied since $\lambda(\mathbf{1}) > \lambda(\mathbf{0})$ and $u \in [0,1]$.

4. TBA. (For given s, v is increasing in λ.)

5. From (17), $v(\mathbf{1}) > v(\mathbf{0})$ provided

$$\frac{\delta u(1-u)[2\lambda(\mathbf{1})(1-u)+u]}{\alpha[u(\delta+\phi)-\delta][\lambda(\mathbf{1})(1-u)+u]} > \frac{\delta u(1-u)[2\lambda(\mathbf{0})(1-u)+u]}{\alpha[u(\delta+\phi)-\delta][\lambda(\mathbf{0})(1-u)+u]}.$$

We know from 1. that $u(\mathbf{1}) < u(\mathbf{0})$, and from 3. that for the same u, v is higher under active OJS. Because the BC is downward sloping, it follows that also for $u(\mathbf{1}) < u(\mathbf{0})$ it must be the case that $v(\mathbf{1}) < v(\mathbf{0})$.

6. This follows from Lemma 1 (i.e. necessary condition for multiplicity $\theta(\mathbf{1}) > \theta(\mathbf{0})$) and

$$\theta(\mathbf{\Omega})\frac{u(\mathbf{\Omega})+\lambda(\Omega)\gamma(\mathbf{\Omega})}{u} = \Theta(\mathbf{\Omega})$$

where $\frac{u+\lambda\gamma}{u} = 2 - \frac{\delta}{\delta+\lambda m(\theta)}$ is larger for $\mathbf{\Omega} = 1$ since $\lambda(1)m(\theta(\mathbf{1}) > \lambda(0)m(\theta(\mathbf{0}))$.

7. $\lambda(\mathbf{1})\gamma(\mathbf{1}) > \lambda(\mathbf{0})\gamma(\mathbf{0})$ follows from (19):

$$\begin{aligned}
\lambda(\mathbf{1})\gamma(\mathbf{1}) &> \lambda(\mathbf{0})\gamma(\mathbf{0}) \\
\lambda(\mathbf{1})u(\mathbf{1})(1-u(\mathbf{1}))(\lambda(\mathbf{0})(1-u(\mathbf{0}))+u(\mathbf{0})) &> \lambda(\mathbf{0})u(\mathbf{0})(1-u(\mathbf{0}))(\lambda(\mathbf{1})(1-u(\mathbf{1}))+u(\mathbf{1})) \\
u(\mathbf{0})u(\mathbf{1})\lambda_0(u(\mathbf{0})-u(\mathbf{1}))+u(\mathbf{0})u(\mathbf{1})\lambda_1(1-u(\mathbf{1})) &> (\lambda_0+\lambda_1)\lambda_0(1-u(\mathbf{1}))(1-u(\mathbf{0}))(u(\mathbf{0})-u(\mathbf{1})) \\
u(\mathbf{0})u(\mathbf{1})\lambda_1(1-u(\mathbf{1}))+\lambda_0(u(\mathbf{0}) &> u(\mathbf{1}))[u(\mathbf{0})u(\mathbf{1})-(\lambda_0+\lambda_1)(1-u(\mathbf{1}))(1-u(\mathbf{0}))] \\
u(\mathbf{0})u(\mathbf{1})\lambda_1(1-u(\mathbf{1}))+\lambda_0(u(\mathbf{0}) &> u(\mathbf{1}))[u(\mathbf{0})u(\mathbf{1})-(1-u(\mathbf{1}))(1-u(\mathbf{0}))] \\
u(\mathbf{0})u(\mathbf{1})\lambda_1(1-u(\mathbf{1}))-u(\mathbf{0})u(\mathbf{1})\lambda_0(1-u(\mathbf{1})-u(\mathbf{0})) &> 0
\end{aligned}$$

for $\lambda_1 \geq \lambda_0$ and $\lambda_0 + \lambda_1 \leq 1$.

$s(\mathbf{1}) < s(\mathbf{0})$ follows from (15) and (16):

$$s \equiv u(\mathbf{\Omega}) + \lambda(\Omega)\gamma(\mathbf{\Omega}) = \frac{\delta^2 + 2\delta\lambda(\Omega)m(\theta(\mathbf{\Omega}))}{(\delta+m(\theta(\mathbf{\Omega})))(\delta+\lambda(\Omega)m(\theta(\mathbf{\Omega})))}$$

We know that both $m(\theta(\mathbf{1})) > m(\theta(\mathbf{0}))$ and $\lambda(1) > \lambda(0)$. Then,

$$\begin{aligned}
\frac{\partial s}{\partial m} &= \frac{\delta(\delta^2(-1+\lambda)-2\delta\lambda m-2\lambda^2 m^2)}{(\delta+m)^2(\delta+\lambda m)^2} < 0 \\
\frac{\partial^2 s}{\partial m \partial \lambda} &= \frac{\delta^2(\delta^2-\delta\lambda m-2\lambda m^2)}{(\delta+m)^2(\delta+\lambda m)^3}
\end{aligned}$$

where the cross-partial is non-positive for δ sufficiently small. ■

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