Diffusing Coordination Risk*

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January, 2015
Preliminary and Incomplete

Abstract

This paper designs an optimal mechanism to correct coordination failure. A planner wants her agents to coordinate on a cooperative action. Agents gather noisy private information regarding the underlying fundamental and decide whether to cooperate or not. The global game literature uniquely identifies the chance of coordination failure when the coordination risk is concentrated at one point in time. We analyze the case when the planner diffuses the coordination risk over time. The planner approaches the agents sequentially - only a proportion of agents at a time and advancing further only when the coordination failure has been averted so far. The public information of survival works as a coordination device and helps in mitigating the coordination risk. We show that if the planner can diffuse the coordination risk enough, then she can achieve the first best as the unique equilibrium outcome. However, if the planner has only limited power to diffuse the coordination risk, multiple equilibria can arise. A max-min planner should diffuse the coordination risk as much as possible. We also show that if some groups are more reluctant to cooperate than others, a max-min planner should approach the more reluctant groups first. Our mechanism is robust to various generalizations and can be applied to a wide range of coordination games.

1 Introduction

Coordination failure leads to economic turmoil and recessions. Pessimistic investors worry about the non-participation of other potential investors and decide to walk away from a new investment opportunity. Countries trying to attract investment or financial institutions trying to convince their creditors to rollover their money are often faced with such challenges. Is there a way to lower the risk of coordination failure? To what extent, can this risk can be lowered?

Coordination failure can be thought of as a bad equilibrium arising from a binary action game where agents can either take a *cooperative* action or a *non-cooperative* action and payoffs satisfy the properties of strategic complementarity. In such strategic situations, there can be a good equilibrium

*We are extremely grateful to Douglas Gale and David Pearce for their substantial guidance and support throughout this project. We thank Viral Acharya, Jess Benhabib, Itay Goldstein, Zhiguo He, Laurent Mathevet, Cecilia Parlatore, Ennio Stacchetti, Laura Veldkamp, Neil Wallace for helpful comments and suggestions.

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outcome where every agent believes that the other agents will take the cooperative action inducing them to coordinate on the cooperative action. There can be a bad equilibrium as well where every agent believes that the other agents will not take the cooperative action inducing them to take the non-cooperative action. This is what we call the coordination failure. If there is a planner e.g. a country or a financial institution who wants the agents to coordinate on the cooperative action, her objective would be to minimize the chance of coordination failure. To define the chance of coordination failure we will use the equilibrium selection following Carlsson and Van Damme (1993).[1] This equilibrium selection is commonly referred to as global games: there is a continuum of agents who gather noisy private information regarding an underlying fundamental strength. Global game uniquely identifies the chance of coordination failure which is the probability that the fundamental value is below some threshold. We will refer to this chance of coordination failure as *coordination risk*.

The question that we will ask then is: can the planner design a policy to reduce this fundamental threshold? We will consider a particular type of policy. We call it diffusing the coordination risk. Instead of approaching all the agents at the same time, the planner approaches them sequentially in groups. She advances further only when coordination failure has been avoided so far. Thus the coordination risk that was concentrated at one point in time, gets diffused over time. For example, consider a borrower who wants her creditors to rollover their lendings for some project. Suppose the project's return will be realized in one year. Instead of letting creditors make their rollover decisions at the end of the sixth month, the borrower can design the debt contract which asks 10% share of creditors to make their decisions at the end of each month from the first month to the tenth month. Since agents will care about the aggregate risk that has been diffused over time, diffusion does not necessarily mean that the coordination risk gets reduced. Although agents are facing less strategic uncertainty from agents who have been approached at the same time, they have a dynamic concern about what agents will do in future. Choi et al. (2014) show that corporate bond issuers are usuing similar policies through diversification of debt rollovers across maturity dates. Hedge fund managers also set redemption gate, which limits the amount of withdrawals from the fund during a redemption period. [2]

The underlying fundamental strength may be so bad that even if all agents coordinate on the cooperative action the project will still fail. We call such projects *doomed*. This provides a lower bound to how much the fundamental threshold can be reduced. The first best outcome is when agents coordinate on the cooperative action whenever the project is not doomed. So, the first best is achieved when the planner can reduce the threshold fundamental to it's lower bound. Our first main result is to show that the planner can achieve the first best as the unique equilibrium outcome if she can diffuse the coordination risk enough.

The planner approaches the remaining groups of agents only when the project is not yet doomed i.e. the fundamental strength is such that if all the remaining agents coordinate on taking the cooperative action, the project may still be successful. So the agents, when approached, publicly know

[1] We can talk about Big Push theory (Ray (2000)) without going into equilibrium selection. However, for practicality, we assume the planner does not have enough money to give a big push. In fact we will assume that planner has no money.

[2] The example of hedge fund is slightly different from our model. The group of creditors (or investors) who make their decisions during a certain redemption time is endogenous. We talk about this problem in a companion paper.

that the project has survived so far. This breaks down the uniqueness result we get from global game. In particular, there can be an equilibrium when agents ignore their private information and completely rely on this public information of survival. Thus, the first best outcome is achievable. However, there is no certainty that agents will indeed do so. We can characterize a worst equilibrium where the chance of coordination failure is maximum. We will show that even in the worst equilibrium the diffused coordination risk will be lower than the concentrated coordination risk.

Our optimal policy design is based on the interplay of two policy forces: (1) diffusing coordination risk and (2) the public information of survival. We will show that diffusion without any public information of survival does not affect the coordination risk at all, while the public information of survival will always reduce the coordination risk even without diffusion. When we combine the two forces, diffusion increases the impact of public information of survival, in the sense that the coordination risk is less in the worst equilibrium. This brings us to our second main result: if the planner cannot diffuse the coordination risk as much as she wants then a max-min or *cautious* planner who designs her policy anticipating the worst, should diffuse the coordination risk as much as possible.

This, however, does not guarantee that there is no limit to how much diffusion can help reducing coordination risk. It is possible that ther are projects which are not doomed and yet can not be made successful by diffusing the coordination risk. Our main result shows otherwise i.e. the first best outcome can be achieved uniquely through diffusing coordination risk. What if achieving this requires the planner to infinitely diffuse the coordination risk? Our main contribution is to show that this is not the case. There exists a uniform finite upper bound on how much diffusion is required to design a policy to achieve the first best. This upper bound depends on the parameters of the model. The planner can achieve the first best with less diffusion if agents have higher payoff that makes them less reluctant to take the cooperative action or if agents have less precise private information.

Consider a project that requires 60% of the agents to coordinate on the cooperative action for being successful i.e. it can withstand 40% non-cooperation. Supppose this 0.4 reperesents the fundamental strength of the project. Now, suppose the planner has already approached 50% of the agents and only 50% of them have cooperated so far. So the residual strength now is $0.4 - 0.5 * 0.5 = 0.15$ i.e. the project can still withstand 15% non-cooperation. When the planner approaches the agents, they gather private information about this current strength. We will call it *residual strength*.[3] The last group of agents do not have any dynamic concern. If the mass of creditors in the last period is small enough, we will show that the public information of survival overcomes all the coordination risk. The unique rationalizable action for agents is to ignore their private information and take the cooperative action. The second last group of agents can rationally anticipate the strategy of the last group. By backward induction, we show that diffusing the coordination risk enough unravels the coordination risk from the end and thus enables the planner to completely avoid the coordination failure. The unraveling starts because diffusion makes the private information noisy when agents try to form their belief about the per capita residual strength. However, if agents gather more

[3] We will assume the agents gather information right before they are going to make their decision. This seems more natural when the time of move is fixed by the planner. If agents can choose the timing then we expect the agents to gather information repeatedly.

precise private information when the planner diffuses the coordination risk, this unraveling may not work. We derive a sufficient condition under which this unraveling mechanism will work. We also construct an example when diffusion may increase the coordination risk. We can think of the planner as an information designer (as defined in Kamenica and Gentzkow (2011) or Bergemann and Morris (2013a)) but with limited means to manipulate the agents' beliefs. By diffusing the coordination risk enough, the planner manipulates the agents' beliefs in a way that her favorite outcome is the unique outcome.

What if the planner lacks the power to diffuse the coordination risk? Suppose, there are two heterogeneous groups of agents and the only policy the planner can take is to choose the order in which she will approach them. Which group should she approach first? We show that the *max-min* planner should first approach the group which is more reluctant to take the cooperative action. Approaching the less reluctant agents first is equivalent to giving an incentive right away[4], while approaching the more reluctant agents first is equivalent to providing an incentive later. A late incentive will reduce the coordination risk later and early agents will anticipate that. We show that this combined effect of late incentive is better than the early incentive.

Morris and Shin (1998, 2003, 2007) have shown that the global game perturbation works for a broad class of games that satisfy some payoff assumptions. We show that our results are robust to such payoff generality. The optimal policy we have designed is simple and can be applied to many real life problems like bank runs or credit freezing or adoption of a new industry standard. Financing the long-term investment by short-term borrowing or the maturity mismatch problem is at the center of current financial crisis (Brunnermeier, 2009). The coordination failure among creditors impairs the stability of financial system by inducing fire sale and drying up the market liquidity. This paper provides a feasible way to minimize this coordination risk among creditors. Our comparative statics results are in line with the recent studies (Gorton and Ordoñez, 2014) about the factors causing of financial crisis. In economic booms, the return rate of debt is high and creditors tend to ignore their private information of the firm's liquidity. In order to achieve the first best outcome without coordination failure, the borrower does not need to diffuse the debt structure much.[5] The high interest rate will make creditors more likely to roll over and their decision will be based more on the public information of survival. However, creditors start to acquire information about borrower's liquidity and the expected rate of return is lower before recessions. At that time, the desired debt structure to achieve the first best outcome is much more dispersed and that is why there will be ample coordination risk and panic-based runs given the debt structure design more suitable for good times.

Related Literature

We begin with a coordination problem faced by a mass of agent. When all agents take their decision simultaneously, the game typically has multiple equilibria. Carlsson and Van Damme (1993) consider the refinement idea of relaxing the common knowledge of payoffs and obtain a unique equi-

[4] Think of two groups that are similar to to start with. So, it does not matter whom the planner approaches first. The planner giving an incentive early is equivalent to a policy when the planner approaches the less reluctant agents first and vice versa.

[5] Suppose diffusing debt structure has some minor costs, we will discuss this in more detail in Section 6.

librium prediction. Morris and Shin (1998, 2003, 2007) (henceforth MS) developed the idea further. This strand of literature is commonly referred to as Global Games. Agents privately gather information regarding an underlying the fundamental strength and this leads to a unique equilibrium which plays out in threshold strategy. Agents take the cooperative action if and only if they get a good enough signal. The risk of coordination is concentrated at one point in time. This is the basic problem we will start with.

When the planner diffuses this coordination risk over time, the early agents have concerns regarding actions of future agents. There have been several works which focus on specific features of this dynamic concern. The paper which comes closest to ours is Dasgupta (2007). The fundamental of the project is chosen ex-ante and remains fixed (unlike Chassang (2010)). Agents gather relevant information privately before they make their decisions. Dasgupta et al. (2012) and Mathevet and Steiner (2013) also talk about a similar private learning environment. In addition to private information agents get a public linformation of survival. We see similar public information in Angeletos et al. (2007) (henceforth AHP). However, we have two major difference with AHP . First, unlike AHP, in our model, agents do not get to choose the timing of their action. Agents move at an exogenously specified point in time. Second, in AHP agents only care about whether enough agents will coordinate on the coperative action today, but in our model agnts not only care about what their fellow agents will do today but also what agents will do in future .

Our research question is very different from the works we have mentioned above. The planner wants to design a policy to minimize the chance of coordination failure. In terms of the research question, our work has close relation with the work of Bergemann and Morris (2013b) (henceforth BM) and Kamenica and Gentzkow (2011). The palnner can be thought of as an information designer but with limited means to manipulate the agents beliefs. Like BM we ask the question: can the palnner achieve her favorite outcome as the unique equilibrium outcome? We showed the answer is yes and we prove it by construction. Sakovics and Steiner (2012) also asked the same question: how to solve coordination failure. They designed the optimal subsidy or deposit insurance when agents are heterogeneous in terms of their willingness to invest or roll over. They showed that the planner should subsidize the more reluctant agents first. We show that a max-min planner should approach the more reluctant agents first.

It is a common practice for firms to spread creditors' rollover decisions over time to reduce the liquidity risk of having to roll over large quantities of debt at the same time (He and Xiong, 2012). In this paper, we justify the diffused debt structure without liquidity shocks to the borrower. Diffused debt structure is similar to the asynchronous debt structure in finance literature (Leland and Toft, 1996; He and Konstantin, 2014). Stationary debt structure requires the borrower to roll over a fixed fraction of their outstanding debt at every instant of time. Instead of taking the stationary debt structure as given, this paper rationalizes it from minimizing the coordination risk between creditors.

Unlike Diamond-Dybivg debt run model, creditors in our model have no preference shocks and the allocation of asset is absent. We focus only on the information aspect of the coordination problem. Hence, we consider a dynamic coordination problem as in the global game literature[6]. Green and Lin (2003) designed a direct mechanism to achieve the first best (no bank run and ex-ante

[6]See Goldstein and Pauzner (2005) for a global game refinement of the Diamond-Dybivg model

efficient allocation) under sequential service constraint in Diamond-Dybivg model. The basic idea of their mechanism is to make payoff dependent on the position and reported type of creditors. The optimal mechanism in our paper is the design of debt structure, which gives same payoff to each creditor.

This paper is also related to the work of Gale (1995). In Gale (1995) a finite number of players facing dynamic coordination problem with endogenous delays. Efficiency can be achieved when agents move sequentially because of complete information. But in our model creditors have incomplete information. They collect information privately. The only publicly available information is that project is not yet hopeless.

Outline

The paper is organized as follows: In section 2 we will describe the benchmark where all the risk of coordination is concentrated at one point in time. Readers familiar with the global game literature can skip this section. In section 3 we will characterize monotone equilibria recursively when the planner diffuses the coordination risk. Then we will construct the optimal policy. In section 4 we will talk about the interplay of the two policy forces namely, public information of survival and diffusion of coordination risk. In section 5 we will consider the case when agents are divided in two heterogeneous groups and the planner chooses in which order she will approach the groups. In section 6 we check the robustness of our results and discusses more general setting. In section 7 we talk about some application where we can use our optimal policy and section 8 concludes. The proofs omitted in the main paper are in the appendix.

2 Concentrated Coordination Risk

In this section, we consider a model where all players move simultaneously to make decision of coordinating or not. The coordination risk will be characterized in the equilibrium. There are a continuum of players in the model, $i \in [0, 1]$. The strength of fundamental θ is defined as the maximum share of non-cooperation the game can sustain for a success. For example, if the success of a coordination game requires at least 60% of cooperation, the fundamental θ is 40%. Coordination failure can be avoided only when the non-cooperation is less than 40%.

Players are risk neutral and there is no discounting. Player i's payoff u depends on her binary action a_i ($a_i = 1$ for cooperative action, $a_i = 0$ for non-cooperative action), the aggregate non-cooperation $w \equiv \int_i 1(a_i = 0)di$ and the strength of the fundamental θ

$$u(1, w, \theta) = \begin{cases} r & \text{if } w \leq \theta \\ q & \text{if } w > \theta \end{cases}, \; u(0, w, \theta) = \begin{cases} b & \text{if } w \leq \theta \\ c & \text{if } w > \theta \end{cases}$$

in which $q < c,\ b < r$. The payoff structure captures strategic complementarity. The game succeeds iff the aggregate noncooperation w is not higher than θ. The constant payoffs only depend on whether the coordination game is successful or not, i.e. whether $\theta \geq w$. This assumes away the possibility that payoffs depend continuously on the fundamental θ and/or the aggregate non-cooperation w. We will extend the model to include that in the next subsection.

If the fundamental θ is commonly known, there are multiple equilibria: all players cooperating is an equilibrium and all players defecting is another equilibrium. It is natural to expect that the project with a higher θ is more likely to succeed. Following Carlsson and Van Damme (1993), we relax the common knowledge assumption. Suppose the nature picks the state of the world θ from the commonly known prior $U[\underline{\theta}, \bar{\theta}]$. Player i gets private signal about θ [7].

$$s_i = \theta + \sigma\epsilon_i \text{ where } \epsilon_i \sim F(\epsilon)$$

Assume that $supp(F) = [-0.5, 0.5]$. The error ϵ_i is independent of the fundamental and i.i.d with zero mean, i.e. $\int \epsilon dF(\epsilon) = 0$. The standard deviation of the private signal σ is idential accross players. Assume that $\underline{\theta} \leq -\sigma$ and $\bar{\theta} \geq 1 + \sigma$. [8]Let $\tau = \frac{1}{\sigma^2}$ represent the precision of the signal.

From private information s_i, players can learn the fundamental and other player's belief (others' beliefs about fundamental, others' beliefs about others' beliefs and so on). Players receive higher signals believe that the project can withstand more noncooperative actions. They will be more optimistic towards the other player's belief as well. Therefore, players with higher signals are more likely to cooperate since the coordination failure is less likely to happen. We will look into the monotone equilibrium where player cooperates if and only if her signal is higher than some threshold $s^\star$. In equilibrium, the higher the fundamental θ is, the larger share of players will receive sufficiently high information ($s > s^\star$). Thus, there exists $\theta^\star$, such that the project succeeds if and only if θ is greater than $\theta^\star$. The monotone equilibrium can be summarized by $(\theta^\star, s^\star)$. Morris and Shin (2003) show that the monotone equilibrium is indeed the unique equilibrium for this standard global game.

Proposition 1 *There is a unique equilibrium where player cooperates if and only if she gets a signal $s \geq s^\star$. The project succeeds iff $\theta \geq \theta^\star = \frac{1}{1+r'}$.*

Proof. See appendix. ■

The unique equilibrium is $(\theta^\star, s^\star)$,where $\theta^\star = \frac{1}{1+r'} = \frac{1}{1+\frac{r-b}{c-q}}$, $s^\star = \theta^\star + \frac{1}{\sqrt{\tau}} F^{-1}(\theta^\star)$. The ex ante probability of coordination failure is the prior probability that $\theta < \theta^\star$, so $\theta^\star$ is the sufficient statistics for the coordination risk. The designer or planner's[9] goal of minimizing coordination risk is equivalent to minimizing $\theta^\star$. The best scenario the planner can achieve is to make $\theta^\star$to be 0. There is no improvement the planner can make if the realization of θ is negative. In those states, the project fails even if all players coordinate.

Notice that, if we restrict the prior to non-negative, i.e. $\underline{\theta} \geq 0$, the lower dominance region is absent by iterated elimination. If players believe all the other players will cooperate, given the non-negative prior, they will cooperate independent of their private information . This equilibrium with $\theta^\star = 0$ cannot be ruled out. However, coordination risk still exists since the equilibrium of $\theta^\star = 0$ is not the only equilibrium can be played. We will discuss the non-negative prior in more detail in the discussion part.

[7]Although there aresome technical concerns regarding the existence of continuum of independent random variables, Judd (1985) shows this assumption is still appropriate to work with

[8]This assumption guarantees the existence of the dominance regions (of private noisy signal). As will be discussed later, this assumption helps us to get unique equilibrium in this benchmark model but it is not essential to our main result. For example, we could have $\underline{\theta} = 0$.

[9]"Designer" is someone who can design and commit to a certain rule of game, diffusion in our case. "Designer" is equivalent to "Planner" who is a social welfare maximizer favoring the equilibrium with lowest possible coordination risk, but cannot design contracts providing payoff incentive to the agents.

General Payoff Structure

We can relax the assumption of constant payoff to allow the payoffs further depend on the value of w and θ. As long as the general payoff satisfies the following assumptions, we can construct the unique equilibrium of the coordination game, which characterizes the coordination risk.

Assumption 1 *Suppose the player i's payoff u depends on the her action a_i, the aggregate state w and the fundamental strength θ.*

$$u(a_i = 1, \theta, w) = \begin{cases} r(\theta, w) & \text{if } \theta \geq w \\ q(\theta, w) & \text{if } \theta < w \end{cases}$$

$$u(a_i = 0, \theta, w) = \begin{cases} b(\theta, w) & \text{if } \theta \geq w \\ c(\theta, w) & \text{if } \theta < w \end{cases}$$

Assumption 1.1

$$r(\theta, w) - b(\theta, w) = \bar{u}(\theta, w) > 0$$

$$q(\theta, w) - c(\theta, w) = \underline{u}(\theta, w) < 0$$

$\bar{u}(\theta, w)$ and $\underline{u}(\theta, w)$ are non-decreasing in θ and non-increasing in w.

Assumption 1.2

$$0 < \underline{n} < \bar{u} < \bar{n}$$

$$0 < \underline{m} < -\underline{u} < \bar{m} < \underline{n}$$

in which $\underline{m}, \bar{m}, \underline{n}, \bar{n}$ are constants and $\underline{m} < \bar{m} < \underline{n} < \bar{n}$.

Assumption 1.2 guarantees the motive to coordinate. The incentive to cooperate is non-decreasing in w, which the incentive to take non-cooperative action is non-increasing in w. Based on Assumption 1.3, the payoff differences are bounded. The previous assumed payoff structure is a special example of this more general one.

Proposition 2 *[Morris and Shin('03)] Under the general payoff structure, if Assumption 1 holds true, there is a unique equilibrium where agent will coordinate on th cooperative action if and only if she gets a signal $s \geq s^\star$. The project succeeds iff $\theta \geq \theta^\star$.*

Proof.
See Appendix.
■

3 Diffusing Coordination Risk

Section 1 characterizes the coordination risk in the concentrated coordination model in the previous section. In this section, we first present a model of bifurcated diffusion, which is helpful in understanding the timing and information structure of the dynamic coordination game. Then, we will discuss how diffusing coordination could reduce coordination risk and achieve the first best case.

3.1 Bifurcated Diffusion

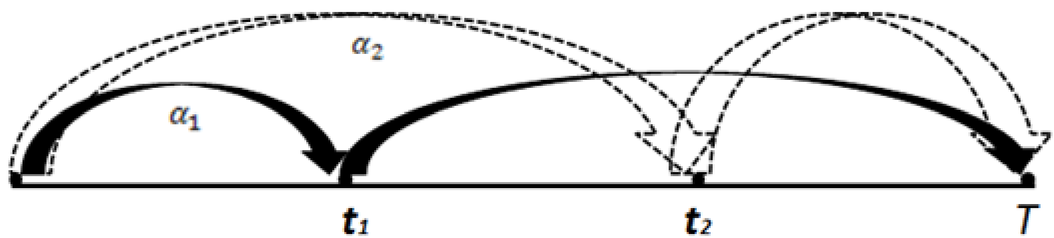

Figure 1: Bifurcated Term Structure: α_1 proportion of agents take their decision at t_1, α_2 proportion of agents take their action at t_2

Consider the two period case. All agents are moving at a pre-specified time and they are making decisions only once. Once the decisions have been made, agents cannot revise their choices. The designer approaches α_1 share of agents for their decisions at $t = 1$ and α_2 ($\alpha_2 = 1 - \alpha_1$) share of agents at $t = 2$. As in the benchmark model, the early agents make their decisions based private noisy information $s_{1i} = \theta_1 + \sigma\epsilon_{1i}$[10]. The proportion of non-cooperation among α_1 share of early group is w_1. Thus, the strength of fundamental left for the late agents is $\theta_2 \equiv \theta_1 - \alpha_1 w_1$.[11] Agents are instantaneous information gatherers. The late group cannot learn θ_1 directly but have private noisy information regarding the residual fundamental θ_2, i.e. $s_{2i} = \theta_2 + \sigma\epsilon_{2i}$. [12] The errors ϵ_{1i}, ϵ_{2i} are uncorrelated, independent of the fundamental and distributed according to F.

The designer advances to approach the late agents only if she can withstand the non-cooperation among the early group, i.e. if the project is not *hopeless*. A project is *hopeless* if the underlying fundamental strength is so bad that even if all agents coordinate on the cooperative action the project will still fail. Once the designer commits to this algorithm, the late agents get the public information that the designer has already sustained the non-cooperation in the first period. In other words, they learn that the residual strength $\theta_2 = \theta - \alpha_1 w_1 \geq 0$. This is what we refer to as

[10] $\theta_1 = \theta$ is the fundamental strength for group 1.

[11] Taking w as a percentage, the linear relation between the orignal fundamental and the residual fundamental is very natural. We focus on the the linear transformation of fundamental dynamics in the main part of the paper while a discussion about other functional forms will be provided in the robustness part.

[12] The assumption of instantaneous information gathering is not essential here. We can allow the prior of θ to be informative and the later agents have noisy private information about the aggregate non-cooperation among early agents, w_1. This complication will not change the main result of the paper according to Dasgupta (2007). What matters for the late agents is still the current fundamental, θ_2. If the information about non-cooperation is available, they will have to aggregate two pieces of noisy information to learn θ_2.

the public information of survival. It is very natural to have this public information in a dynamic setting, since if the strength is insufficient to sustain the non-cooperation of early agents, then the designer has no reason to approach the late agents because the project will fail independent of the later agents' decisions. Based on both the private signal and the public information of survival, the late agents decide whether to cooperate or not. The project succeeds if the residual strength can withstand the non-cooperation in second period, or $\theta_1 \geq \alpha_1 w_1 + \alpha_2 w_2$. The payoffs are the same as in the benchmark model and will be realized after the late agents make their decisions.

The public information of survival is in AHP's dynamic coordination game as well, where the agents see the central bank has survived past currency attacks. There are two substantial differences between this model and AHP. First, agents in this model move at a pre-specified time while in AHP agents decide when to attack the currency. Second, in this model, the early agents have dynamic concerns about what agents will do later, while in AHP agents attacking the currency are only worried about what other agents will do today.

Definition 1 $(\theta_t^\star, s_t^\star)_{t=1}^T$ *is said to be a monotone Bayesian Nash Equilibrium if agents in period* t *cooperate iff* $s_{ti} \geq s_t^\star$ *and consequently if the project reaches peiod* t*, it will not fail iff* $\theta_t \geq \theta_t^\star$*.*

The following proposition describes the existence of equilibrium and equilbrium conditions.

Proposition 3 *There exists monotone equilibria* (s_t^*) *such that agents cooperate in period* t *iff* $s_t > s_t^*$, $t = 1, 2$*. Consequently, the project will not fail from* t *to* $T = 2$ *iff* $\theta_t \geq \theta_t^*$*, where*

$$\theta_1^* = \frac{\alpha_1}{1+r'} + \theta_2^*$$

$$\frac{\frac{\theta_2^\star}{\alpha_2}}{F(\sqrt{\tau}\theta_2^\star + F^{-1}(\frac{\theta_2^\star}{\alpha_2}))} = \frac{1}{1+r'} \, or \ \theta_2^* = 0$$

Proof. This is a special case of proposition 4. ■

As can be seen in Figure 2, when there is no public information of survival, the threshold agent believes that the project will succeed with probability $\frac{\theta_2^\star}{\alpha_2}$, which is an increasing function of $\theta_2^\star$. This gives the unique solution (point b in Figure 2) of $\frac{\theta_2^\star}{\alpha_2} = \frac{1}{1+r'}$. However, when there is truncated information, the belief of the threshold agent is not monotonic (G in Figure 2), which gives us multiple solutions for $\theta_2^\star$.In addition to the (potential) multiple monotone equilibria in the model, there may exist non-monotonic equilibria. Observe that $\theta_2^* = 0$ is always an equilibrium. To see this suppose agents always coordinate irrespective of whatever signal they get in period 2. If time 2 agent believes that, she would cooperate only if she believes that $\theta_2 \geq 0$ with probability greater than $\frac{1}{1+r'}$. If it is already publicly known that $\theta_2 \geq 0$, we can not eliminate any never best responses. So, we can never rule out the strategy that agents always cooperate irrespective of their signal as a never best response. Therefore, conditional on reaching period 2, there is no chance of coordination failure if $\theta_2^\star = 0$. Consequently, given the bifurcated policy (α_1, α_2), the equilibrium with least chance of coordination failure is the monotonic equilibrium with fundamental thresholds $(\theta_1^\star = \frac{\alpha_1}{1+r'},\ \theta_2^\star = 0)$.

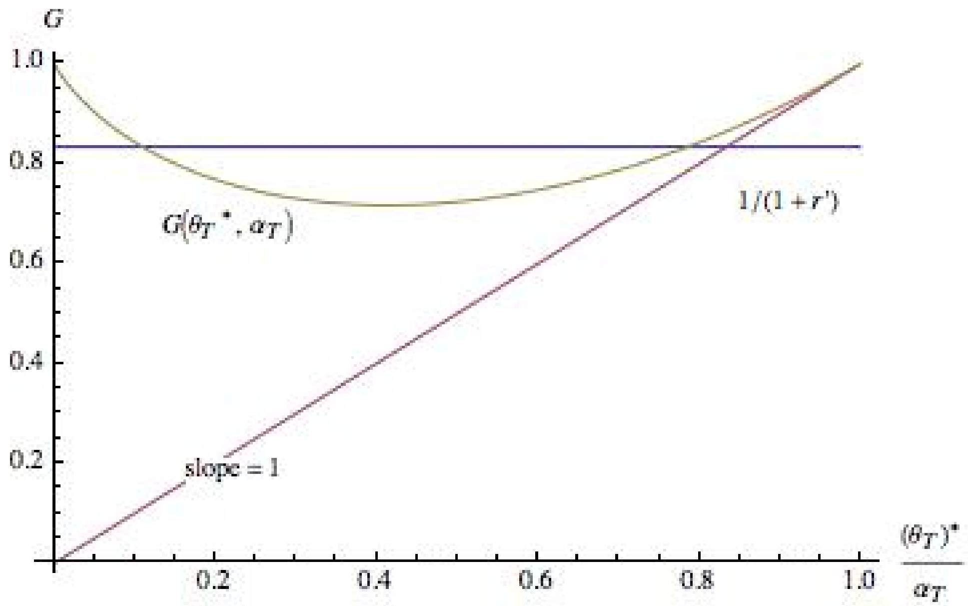

Figure 2: Effect of Truncated Information, $T = 2$

3.2 General Diffusion

Suppose the designer is able to separate the agents into T groups and approach them sequentially. At any time t $(1 \leq t < T)$, the designer will ask mass α_t agents to cooperate. If the designer can sustain non-cooperation then the designer asks the next group of mass α_{t+1} agents to make their decisions and keeps doing so until period T, where she exhausts the whole set of agents, i.e. $\sum_{t=1}^{T} \alpha_t = 1$. The end period agents is exactly the same as in the bifurcated case. Given the equilibrium $\theta_T^\star$, at $T-1$, the agents believe that the project succeeds if $\theta_{T-1} - \theta_T^\star \geq \alpha_{T-1} w_{T-1}$. If $\theta_{T-1}^\star$ is the equilibrium threshold, then there is a threshold for private signal, $s_{T-1}^\star$, such that $s_{T-1}^\star = \theta_{T-1}^\star - \theta_T^\star + \frac{1}{\sqrt{\tau}} F^{-1}(\frac{\theta_{T-1}^\star - \theta_T^\star}{\alpha_T})$. The belief of the threshold agent that the project will succeed is $P(\theta_{T-1} > \theta_{T-1}^\star | s_{T-1}^\star, \theta_{T-1} \geq 0)$. In equilibrium, the threshold agent will be indifferent between the cooperative action and the non-cooperative action.

$$P(\theta_{T-1} > \theta_{T-1}^\star | s_{T-1}^\star, \theta_{T-1} \geq 0) = \frac{1}{1+r'}$$

This solves for $\theta_{T-1}^\star$ and go backwards we can solve for the sequence of $\{\theta_t^\star\}_{t=1}^{T}$. In the equilibrium, the agent with the threshold private signal must believe the probability of success is $\frac{1}{1+r}$. Similar to the argument we made in 2 period model, the resulting $\theta_1^\star$ will be smaller than the case without truncated information.

Proposition 4 *There exists monotone equilibria $(s_t^*), (t = 1, 2 \ldots, T)$ such that agents cooperate in period t iff $s_t \geq s_t^*$. The project will not fail from time t to time T iff $\theta_t \geq \theta_t^\star$. Consequently, the project succeeds iff $\theta > \theta_1^*$, where $\theta_{T+1}^\star = 0$,*

$$\theta_1^* = \frac{\alpha_1}{1+r'} + \theta_2^*$$

$$\forall t \geq 2, \ \frac{\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t}}{F(\sqrt{\tau}\theta_t^\star + F^{-1}(\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t}))} = \frac{1}{1+r'} \ or \ \theta_t^* = 0 \tag{1}$$

Proof. See appendix. ■

The above recursive relation does not have unique solution similar to the bifurcated policy case. $\theta_t^\star = 0$ for all $t = 2, 3, \ldots, T$ is always a solution. Therefore, there is an equilibrium where conditional on reaching period 2, there is no chance of coordination failure. If agents in period 1 believes that this will happen from period 2 onwards, then the equilibrium thresholds are $(\theta_1^\star = \frac{\alpha_1}{1+r}, \ \theta_2^\star = 0 \ldots, \theta_T^\star = 0)$. This monotone equilibrium has the least chance of coordination failure among all possible equilibria.

3.3 Optimal Policy

We will see in Proposition 6 that when the designer approaches the agents sequentially and there is public information of survival, the probability of coordination failure is always lower (for any equilibrium) than the case where there is no truncated information. Thus diffusion of coordination risk helps in reducing the coordination risk if there is truncated information. We would like to know how the designer can design a policy $(T, (\alpha))$ where $T \in \mathbb{N}$ and $(\alpha) \equiv (\alpha_1, \alpha_2, \ldots \alpha_T) \in \Delta^{T-1}$,[13] such that the probability of having coordination failure is minimized. Let $P(T, (\alpha))$ be the coordination risk given policy $(T, (\alpha))$. The designer chooses $(T, (\alpha))$ to minimize $P(T, (\alpha))$. If each policy induces a unique equilibrium then the objective function is straight forward. However, as we have seen there can be multiple equilibria when there is truncated information. So we first need to define the objective function when there are multiple equilibria corresponding to any policy $(T, (\alpha))$.

Let us first rank all the equilibria in order of coordination risk[14] Following Milgrom and Roberts (1990) (Henceforth MR) we can say the best and worst equilibrium are in monotone strategy. We have already shown that given any policy $(T, (\alpha))$, the *best* equilibrium is the monotone equilibrium with threshold fundamental $(\theta_1^{\star l} = \frac{\alpha_1}{1+r}, \ \theta_2^{\star l} = 0 \ldots, \theta_T^{\star l} = 0)$. In this equilibrium, agents always cooperate from $t = 2$ onwards irrespective of their private signals. The *worst* equilibrium is the monotone equilibrium corresponding to the maximum solution to equation 1. Let $\{\theta_t^{*h}(T, (\alpha))\}_{t=2}^T$ be the maximum solution to 1. Then the *worst* equilibrium is a monotone equilibrium with threshold fundamental $(\theta_1^{\star h} = \frac{\alpha_1}{1+r} + \theta_2^{\star h}, \ \theta_2^{\star h} \ldots, \theta_T^{\star h})$. Let us define $\underline{P}(T, (\alpha)) := P(\theta < \theta_1^{\star l}(T, (\alpha)))$ be the coordination risk to the *best* equlibrium and $\bar{P}(T, (\alpha)) := P(\theta < \theta_1^{\star h}(T, (\alpha)))$ for the *worst* equlibrium. So, given any policy $(T, (\alpha))$,

$$\bar{P}(T, (\alpha)) \geq P(T, (\alpha)) \geq \underline{P}(T, (\alpha))$$

P is the prior belief of the designer. It is possible that the designer has better information than the agents and so P may be different from the prior belief of the agents. If $(T, (\alpha))$ is designed after the designer has acquired this information then in equilibrium the agents would have learned more about the fundamental after the designer announces the policy. We are assuming that either the designer announces $(T, (\alpha))$ before she acquires more information or she shares the same prior as the agents. Thus the agents do not learn anything from the chosen policy. So, if there is a

[13] where $\Delta^{T-1} := \{(\alpha_1, \alpha_2, \ldots, \alpha_T) \in \mathbb{R}^T | \ \alpha_t \geq 0 \ \forall t = 1, 2 \ldots T, \ \sum_{t=1}^T \alpha_t = 1\}$ is the standard $T - 1$ simplex

[14] We do not need a complete order, we only need a lower bound and an upper bound on $P(T, (\alpha))$ for any $(T, (\alpha))$.

threshold value of fundamental such that the project succeeds whenever θ is beyond this threshold, the designer would like to reduce the threshold.

Given the potential multiplicity of equilibria, we can say that for any policy $(T, (\alpha))$, there is $\theta_1^{*l}(T, (\alpha))$ and $\theta_1^{*h}(T, (\alpha))$ such that : (1) If $\theta \geq \theta_1^{*h}(T, (\alpha))$ the project succeeds irrespective of whatever equilibrium is played. (2) If $\theta < \theta_1^{*l}(T, (\alpha))$ the project fails irrespective of whatever equilibrium is played and (3) if $\theta \in [\theta_1^{*l}(T, (\alpha)), \theta_1^{*h}(T, (\alpha)))$, there exists some equilibrium such that the project may fail.

Let us define

$$\underline{\theta}_1^* := \inf_{(T,(\alpha))} \theta_1^{*l}(T, (\alpha)) \text{ and } \bar{\theta}_1^* := \inf_{(T,(\alpha))} \theta_1^{*h}(T, (\alpha)).$$

We know $\theta_1^{*l}(T, (\alpha)) = \frac{\alpha_1}{1+r}$ for all $(T, (\alpha))$. Therefore, $\underline{\theta}_1^* = 0$. One way to define the designer's objective will be to minimize $\theta_1^{*h}(T, (\alpha))$. We will call such a designer a *Cautious* designer or a *max-min* designer. The designer wants to minimize the coordination risk anticipatng the worst can happen (see Gilboa and Schmeidler (1989)). Of course this is not the only reasonable objective of the designer when there are multiple equilibria. For example, we can think of a designer who is *optimistic* i.e. always anticipates the best possible equilibrium will be played. The following theorem represents the main result of this paper. The validity of this theorem is not limited to the specific assumption regarding the designer's preference. Theorem 1 says that when the designer can diffuse the policy enough, she can place small enough α_t agents in every round and she can make sure that the project succeeds for all $\theta > 0$.

Theorem 1 *When the designer approaches the agents sequentially and there is public information of survival,*

1. $\bar{\theta}_1^* = \underline{\theta}_1^* = 0$.
2. *Given (r', τ) there exists $T^* < \infty$, such that for any $\eta > 0$ (however small), the designer can design a policy $(T, (\alpha))$ such that the project succeeds for all $\theta \geq \eta$, if the designer can diffuse the policy enough i.e. $T \geq T^*$.*

Proof.
Let us define

$$G(x, \alpha) := \frac{x}{F(\sqrt{\tau}\alpha x + F^{-1}(x))}, \ \alpha \in [0, 1], \ x \in [0, 1] \tag{2}$$

$$x(\alpha) := \arg\min_{x\in[0,1]} G(x, \alpha) \tag{3}$$

$$y(\alpha) := G(x(\alpha), \alpha) \tag{4}$$

By theorem of maximum we know $y(\alpha)$ is well defined and it is continuous on α. $\forall \alpha_1 < \alpha_2$, $y(\alpha_1) = G(x(\alpha_1), \alpha_1) > G(x(\alpha_1), \alpha_2) > G(x(\alpha_2), \alpha_2) = y(\alpha_2)$. So, $y(\alpha)$ is decreasing with α. Hence, $y(0) = \lim_{\alpha\to 0} y(\alpha) = 1 > \frac{1}{1+r'}$. If $y(1) < \frac{1}{1+r'}$ then $\exists \alpha^* > 0$ such that $\forall \alpha < \alpha^\star, \forall x \in [0, 1], G(x, \alpha) > \frac{1}{1+r'}$. If $y(1) > \frac{1}{1+r'}$ then $\alpha^* = 1$. Consider period T, from proposition 4 we know in equilibrium either $G(\frac{\theta_T^*}{\alpha_T}, \alpha_T) = \frac{1}{1+r'}$or $\theta_T^* = 0$. Therefore, if $\alpha_T < \alpha^*$ then $\min_{\theta_T^*} G((\frac{\theta_T^*}{,}\alpha_T) > \frac{1}{1+r'}$. Hence $\theta_T^* = 0$ is the unique equilibrium. Also note that if $\alpha_T \geq \alpha^*$ then there is always equilibrium with $\theta_T^* > 0$. Therefore if less than α^* agents move in period T, then the all agents will take cooperative action if it reaches period T.

Suppose $\alpha_T < \alpha^*$, then agents in period $T-1$ know that the project will survive the next period if it survives the current period. So they are essentially facing static coordination risk while it is publicly known that $\theta_{T-1} \geq 0$. Therefore, if $\alpha_{T-1} < \alpha^*$ then $\theta_{T-1}^* = 0$ is the unique equilibrium. Proceeding the same way if $\alpha_t < \alpha^*$ for all $t \geq 2$ then $\theta_2^* = 0$.

Let $T^* = \frac{1}{\alpha^*} + 1 < \infty$. If $T > T^*$ then the planner can design a term structure such that it assigns ϵ (however small) proportion of agents in period 1 and rest $(1-\epsilon)$ proportion of agents in the rest $T-1$ periods such that $\alpha_t < \alpha^*$. Then $\theta_1^* = \frac{\epsilon}{1+r}$. Take $\epsilon \to 0$ and then $\theta_1^* \to 0$. Therefore, $\bar{\theta}_1^* = 0$.

■

It follows from definition of $\bar{\theta}_1^*$ that for any $\eta > 0$ the designer can design a policy $(T, (\alpha))$ such that the project will always succeed for any $\theta \geq \eta$. The theorem claims more than that. It says for any $\eta > 0$ (however small), there is a uniform bound on how much the designer needs to diffuse the policy to make sure the project succeeds whenever $\theta \geq \eta$. In practice, if the designer can diffuse the concentration of coordination risk without cost, then she can almost costless make sure the project succeeds for any $\theta > 0$.

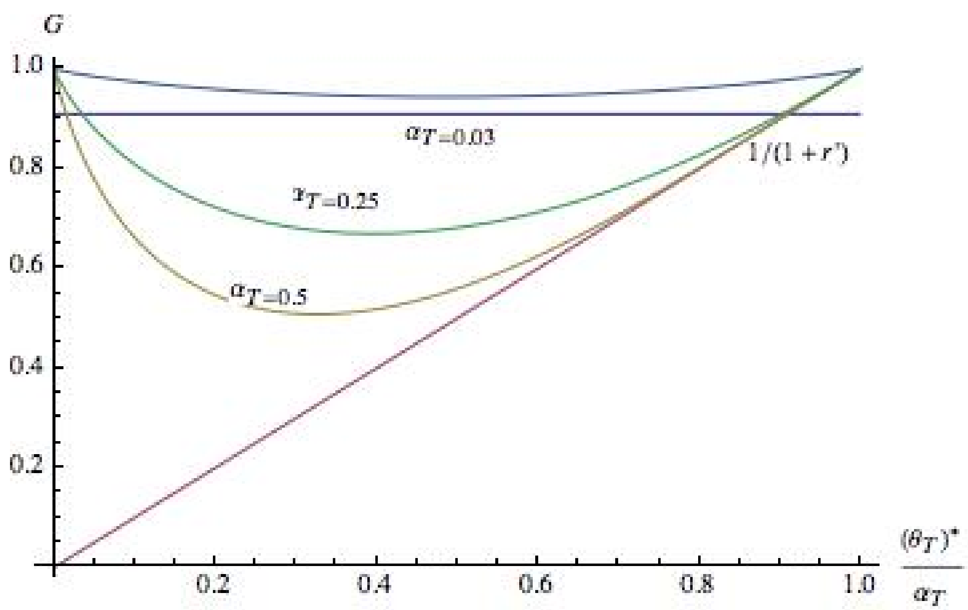

Figure 3: Effect of α_T on θ_T^*

The proof is constructive. When the designer separates the agents into several groups but there is no public information, the last group of agents problem is just a scaled down version of the static benchmark problem. We can think of the problem with effective fundamental per capita $\frac{\theta_T}{\alpha_T}$. Thus in equilibrium $\frac{\theta_T^*}{\alpha_T} = \frac{1}{1+r'}$. Now they also have the public information $\theta_T \geq 0$ whenever they need to make decisions on coordination. We are assuming that agents gather information regarding the residual strength with constant precision. So, when α_T becomes small, i.e. only a small fraction of agents are left, agents private information becomes very noisy for learning per capita residual strength. However, α_T cannot affect the effectiveness of public information at time T. Agents understand the per capita fundamental is non-negative, i.e. $\frac{\theta_T}{\alpha_T} \geq 0$. There exists a threshold level of mass α^* such that if $\alpha_T < \alpha^*$, then the private information become so vague that the positive effects from public information completely outweighs the effect of private information. No matter what private information agents receive, the public information of survival make agent believe the probability of success is higher than $\frac{1}{1+r'}$. Thus, $a_{iT} = 1$ becomes dominant strategy for agents at T independent of their private information. If the designer designs the policy such that $\alpha_T < \alpha^*$ then conditional on reaching the last group of agent there is no chance of coordination failure, i.e. $\theta_T^* = 0$.

Now consider the last but one group. In absence of public information and given any θ_T^*, their problem is similar to the static problem with effective fundamental per capita $\frac{\theta_{T-1}-\theta_T^*}{\alpha_{T-1}}$. Thus when there is no truncated information $\frac{\theta_{T-1}^*-\theta_T^*}{\alpha_{T-1}} = \frac{1}{1+r'}$. So if the designer has designed a policy such that $\alpha_T < \alpha^*$, then the $T-1$ group of agents know $\theta_T^* = 0$, i.e. all agents at T will cooperate if the designer can withstand the non-cooperation in period $T-1$. Thus their problem is exactly the same as the group T agents. Therefore if the designer chooses $\alpha_{T-1} < \alpha^*$, then $\theta_{T-1}^* = 0$ is the only equilibrium.

Continuing this way until group 1, if the designer design a policy that places $\alpha_t < \alpha^*$ for all $t = 2, 3 \ldots T$, then $\theta_2^* = 0$ i.e. group 1 agents know that if the designer can withstand non-cooperation at time 1, then there is no chance of failure. So they are playing a static game. Let $T^* := [\frac{1}{\alpha^*} + 1]$. Consider the term structure $(T^*, (\alpha))$ such that $\alpha_1 = \epsilon$ and $\alpha_t < \alpha^*$ for all $t = 2, 3 \ldots, T$. Then, there is unique equilibrium with fundamental threshold $\left(\theta_1^* = \frac{\epsilon}{1+r'},\ \theta_2^\star = 0, \ldots,\ \theta_T^\star = 0\right)$. Now take $\epsilon \to 0$. So the projects succeeds for all $\theta > 0$.

4 Interplay of Diffusion and Survival Information

In the optimal policy we designed the planner approaches the agents sequentially group by group and advances further only when the coordination failure has not occurred yet. This policy has two components: (1) diffusion and (2) public information of survival. Although this public information of survival is generated quite naturally when the planner diffuses the coordination risk, we would like to understand what these two policy components do on their own. This will give us a better understanding of how these two policy components interact with each other.

Let us first consider diffusion as an independent policy. Suppose the planner diffuses the coordination risk but advance further anyway, independent of whether there is any hope left or not. This may not sound a practical policy, but we want to understand the effect of only diffusion without any survival information. The following proposition says that diffusion on its own is not an effective policy.

Proposition 5 *When there is no survival information, diffusion does not change θ_1^* from $\frac{1}{1+r'}$.*

Proof. See appendix ■

It is no surprise that the public information of survival helps in reducing the coordination risk. If there is some way the designer can provide the public information of survival without diffusion, it would have helped even then. For example, suppose the planner is a borrower and the fundamental strength is some outside funding she can use, when her investors (agents) withdraws their money. So, it will be natural that agents know $\theta \geq 0$. There will be multiple equilibria but in any equilibrium the chance of default will be less as shown in proposition 6.

Proposition 6 *Suppose $\theta \geq 0$ is publicly known, then in any equilibrium the threshold θ_1^* will be below $\frac{1}{1+r'}$ irrespective of whether the coordination risk is concentrated or diffused.*

Proof. See appendix ■

However, there is a limitation to how much this public information can help in reducing the chance of coordination failure. If agents believe that all other agents will ignore their respective private information and will coordinate on the cooperative action when they know $\theta \geq 0$, they will do the same. Thus the first best is an equilibrium outcome. However, there is no certainty that agents will indeed ignore their private information and rely on this public information that $\theta \geq 0$.

So the driving force behind our result is the interplay of these two policies. When the planner diffuse the coordination risk, she naturally provides the survival news repeatedly. We design a way to repeat this survival news. We showed that given the parameters of the model, there is a threshold number of repetitions such that if the planner can repeat beyond this threshold then agents will always ignore their private information and coordinate on the cooperative action upon receiving the survival news. However, if the planner cannot repeat the survival information enough then we have multiplicity of equilibria. We will consider a max-min planner who wants to do the best anticipating the worst equilibrium will be played. The following proposition establishes that a max-min planner thinks she can improve the impact of the survival information by diffusing the coordination risk.

Theorem 2 *For a max-min designer diffusion enlarges the positive effect of public information of survival i.e. the more diffusion, the lower the coordination risk. In case the designer cannot make $\alpha_t < \alpha^*$, she will diffuse coordination as much as possible i.e. a max-min designer will always prefer a policy $((\alpha_1, \alpha_2, ..., \alpha_t, \alpha_{t+1}, \alpha_{t+2} \ldots, \alpha_T), T)$ over $((\alpha_1, \alpha_2, ..., \alpha_t + \alpha_{t+1}, \alpha_{t+2}, ..., \alpha_T), T-1)$.*

Proof.

Denote $\theta_c^\star$ as the maximum solution to the model of concentrated coordination risk with public information of survival. Suppose now the planner diffuse coordination into T groups, the group sizes are $\{\alpha_1, \alpha_2, ..., \alpha_T\}$. Denote the worst possible equilibrium is $\theta_d^\star$. We want to show that $\theta_d^\star < \theta_c^\star$. Define $G(\theta, \alpha) := \dfrac{\frac{\theta}{\alpha}}{F(\sqrt{\tau}\theta + F^{-1}(\frac{\theta}{\alpha}))}$. We know that at the maximum solution $\theta^\star$ of $G(\theta^*, \alpha) = \frac{1}{1+r'}$, $\frac{\partial G}{\partial \theta}|_{\theta=\theta^\star} > 0$.

The equilibrium condition for $\theta_c^\star$ is

$$G(\theta_c^\star, 1) = \frac{\theta_c^\star}{F(\sqrt{\tau}\theta_c^\star + F^{-1}(\theta_c^\star))} = \frac{1}{1+r'}$$

In case of diffusion, at $t = T$, we have

$$G(\theta_T^\star, \alpha_T) = \frac{\frac{\theta_T^\star}{\alpha_T}}{F(\sqrt{\tau}\theta_T^\star + F^{-1}(\frac{\theta_T^\star}{\alpha_T}))} = \frac{1}{1+r'}$$

So we have $\frac{\theta_T^\star}{\alpha_T} < \theta_c^\star$ since $G(\theta_c^\star, 1) = G(\theta_T^\star, \alpha_T) > G(\frac{\theta_T^\star}{\alpha_T}, 1)$. At any $t = T-1$, we have the equilibrium condition

$$\frac{\frac{\theta_{T-1}^\star - \theta_T^\star}{\alpha_T}}{F(\sqrt{\tau}\theta_{T-1}^\star + F^{-1}(\frac{\theta_{T-1}^\star - \theta_T^\star}{\alpha_{T-1}}))} = \frac{\frac{\theta_T^\star}{\alpha_T}}{F(\sqrt{\tau}\theta_T^\star + F^{-1}(\frac{\theta_T^\star}{\alpha_T}))} = \frac{1}{1+r'}$$

Based on the monotonicity of $G(\theta, \alpha)$, we have $\frac{\theta_{T-1}^\star - \theta_T^\star}{\alpha_{T-1}} < \frac{\theta_T^\star}{\alpha_T}$ and thus $\theta_{T-1}^\star < (\alpha_{T-1} + \alpha_T)\theta_c^\star$. By induction, we can show that $\theta_{dT}^\star < \theta_c^\star$.

Further, think about two policies $((\alpha_1, \alpha_2, \ldots, \alpha_t, \alpha_{t+1}, \alpha_{t+2} ..., \alpha_T), T)$ and $((\alpha_1, \alpha_2, \ldots, \alpha_t + \alpha_{t+1}, \alpha_{t+2}, ..., \alpha_T), T-1)$. The former one is a more diffused structure than the later one. Notice that the former one has T groups while the later one has $T-1$ groups. Denote the thresholds for the former one $\theta_{F,t}^\star$ and the later one $\theta_{L,t}^\star$. We want to show that $\theta_{F,1}^\star < \theta_{L,1}^\star$. It is easy to see that $\theta_{F,t+2}^\star = \theta_{L,t+1}^\star$. Define $G'(x, y, \alpha) = \frac{\frac{x-y}{\alpha}}{F(\sqrt{\tau}x + F^{-1}(\frac{x-y}{\alpha}))}$. From the equilibrium condition, we have

$$G'(\theta_{L,t}^\star, \theta_{L,t+1}^\star, \alpha_t + \alpha_{t+1}) = G'(\theta_{F,t+1}^\star, \theta_{F,t+2}^\star, \alpha_{t+1}) = G'(\theta_{F,t}^\star, \theta_{F,t}^\star, \alpha_t)$$

Notice that G' is increasing in x, decreasing in y. Based on the monotonicity of G,

$$G'(\frac{\alpha_{t+1}\theta_{L,t}^\star + \alpha_t\theta_{L,t+1}^\star}{\alpha_t + \alpha_{t+1}}, \theta_{F,t+2}^\star, \alpha_{t+1}) > G'(\theta_{L,t}^\star, \theta_{L,t+1}^\star, \alpha_t + \alpha_{t+1}) = G'(\theta_{F,t+1}^\star, \theta_{F,t+2}^\star, \alpha_{t+1})$$

Thus

$$\theta_{F,t+1}^\star < \frac{\alpha_{t+1}\theta_{L,t}^\star + \alpha_t\theta_{L,t+1}^\star}{\alpha_t + \alpha_{t+1}}.$$

Similarly, we have

$$\begin{aligned} G'(\theta_{L,t}^\star, \frac{\alpha_{t+1}\theta_{L,t}^\star + \alpha_t\theta_{L,t+1}^\star}{\alpha_t + \alpha_{t+1}}, \alpha_t) &= G'(\theta_{L,t}^\star, \theta_{L,t+1}^\star, \alpha_t + \alpha_{t+1}) \\ = G'(\theta_{F,t}^\star, \theta_{F,t}^\star, \alpha_t) &> G'(\theta_{F,t}^\star, \frac{\alpha_{t+1}\theta_{L,t}^\star + \alpha_t\theta_{L,t+1}^\star}{\alpha_t + \alpha_{t+1}}, \alpha_t) \end{aligned}$$

So, we proved that $\theta_{F,t}^\star < \theta_{L,t}^\star$. Obviously, $\theta_{F,1}^\star < \theta_{L,1}^\star$.

■

We constructed the optimal policy based on the result that for small α_T, θ_T^* will be 0. Our optimal policy design is feasible only when the designer can choose small enough groups. What happens when such policies are not feasible? The above proposition says that a max-min planner will always diffuse the coordination risk as much as she can. In the next section we will address a slightly different problem. We will consider a situation when the groups are predefined and the planner does not have any negotiation power to make the groups any finer. However the groups may be heterogeneous. We will investigate which group should a max-min planner approach first.

If the designer advances further only when the aggregate non-cooperation so far has not exhausted all her strength, survival information arises quite naturally when the designer adopts a diffused design. Agents privately receive information about the residual strength and publicly observes that the residual strength is positive. Depending on the precision of their private signal agents then put some appropriate weights to the public information. Since the designer cannot increase the noise of the private information agents gather, she cannot make the agents put more weights on this public information as compared to the weights they put on their private information. One can think of diffusing the coordination risk as a way to make this private information noisy as compared to the public truncated information. To see this consider the last period problem. Agents gather information about the residual fundamental strength θ_T. But only α_T proportion of agents are left. So, in terms of per capita residual fundamental strength $\frac{\theta_T}{\alpha_T}$, the private information is quite noisy as α_T becomes small. So agents will rely more on the public truncated information that the designer has sustained all non-cooperative action so far.

Thus we saw that diffusion as an independent policy does not help, while survival information as an independent policy helps in reducing the chance of default. When these two policies are combined together, diffusion improves the impact of survival information. More importantly (1) there is no upperbound to how much diffusion can help improving the impact of survival information and (2) given the parameters of the model, there is an uniform bound on how much the planner has to diffuse. These two features together gives our main result that for any $\eta > 0$, however small, there is an uniform T such that the planner can design a policy (α, T) with $T \geq T^*$ for which there is no coordination failure whenever $\theta \geq \eta$. Thus the designer can achieve the first best outcome uniquely.

Endogenous Information Gathering

Suppose agents choose the precision of their private information. What if a diffused policy design makes the agents gather more precise private information? As we have understood, the optimal policy design exploits the fact that agents will rely less on their private information when the designer adopts a diffused policy. Therefore diffusion may not help reducing the chance of default if it makes agents to gather more precise information regarding the residual strength. The following theorem provides a necessary and sufficient condition for the validity of our optimal policy. Suppose, $\tau_t(T, (\alpha))$ be the endogenously chosen precision of group t agents, given the policy $(T, (\alpha))$. Let us call a policy a policy $(T', (\alpha'))$ more diffused than $(T, (\alpha))$ if $T' > T$ and $\alpha'_t \geq \alpha_{t+T'-T}$ for all $t = 1, 2, \ldots T'$. Surely it is not a complete order, but we can say always design a more diffused policy from a given policy by splitting a group into two groups (and putting one of them in the beginning).

Theorem 3 *If (1) $\alpha_t\sqrt{\tau_t(T,(\alpha))}$ decreases as the designer reduces α_t by adopting a more diffused policy and (2) $\lim_{\alpha_t \to 0} \alpha_t\sqrt{\tau_t(T,(\alpha))} = 0$, then the designer can achieve the first best by adopting the diffused enough policy. However, if $\alpha_t\sqrt{\tau_t(T,(\alpha))}$ is a constant, then diffusion increases the chance of coordination failure.*

Proof. Following the proof of Theorem 1, $x = \frac{\theta^\star_T}{\alpha_T}$ is the solution to the following equation

$$G(x,\alpha) = \frac{x}{F(\alpha\sqrt{\tau}x + F^{-1}(x))} = \frac{1}{1+r'}$$

The condition that $\alpha\sqrt{\tau(\alpha)}$ is an increasing function of α guarantees $y(\alpha)$ decreases with α, and the condition $\lim_{\alpha\to 0} \alpha\sqrt{\tau(\alpha)} = 0$ guarantees $\lim_{\alpha\to 0} y(\alpha) = 1$. Since $y(\alpha)$ is well defined and continuous in α, we can find $\alpha^\star(r')$. $\forall \alpha \leq \alpha^\star$, $\forall x \in (0,1)$, $G(x,\alpha) > \frac{1}{1+r'}$. Hence, $\alpha\sqrt{\tau(\alpha)}$ is increasing in α

and $\lim_{\alpha\to 0}\alpha\sqrt{\tau(\alpha)} = 0$ are the sufficient conditions to this result. They are necessary conditions as well since otherwise $y(\alpha)$ is not decreasing in α or $\lim_{\alpha\to 0} y(\alpha) < \frac{1}{1+r'}$.

In case $\alpha\sqrt{\tau(\alpha)}$ is constant in α, diffusion actually increases the coordination risk. We will show this by comparing the worst possible equilibrium in concentrated coordination model (with public information of survival) to that for the model of diffusing coordination risk. Suppose $\alpha\sqrt{\tau(\alpha)} = z$. When coordination risk is concentrated at one time with public information of survival, the threshold fundamental in worst possible equilibrium $\theta^\star$is the maximal solution to the following equation.

$$\frac{\theta^\star}{F(z + F(\theta^\star))} = \frac{1}{1+r'}$$

When diffusing coordination to two groups $\{\alpha_1, \alpha_2\}$, the worst possible equilibrium $\{\theta_1^\star, \theta_2^\star\}$ solves the following two equations

$$\frac{\frac{\theta_2^\star}{\alpha_2}}{F(z\frac{\theta_2^\star}{\alpha_2} + F(\frac{\theta_2^\star}{\alpha_2}))} = \frac{1}{1+r'}, \quad \frac{\frac{\theta_1^\star - \theta_2^\star}{\alpha_1}}{F(z\frac{\theta_1^\star}{\alpha_1} + F(\frac{\theta_1^\star - \theta_2^\star}{\alpha_1}))} = \frac{1}{1+r'}$$

By comparing these equations, it is clear that $\frac{\theta_2^\star}{\alpha_2} = \theta^\star$ and $\frac{\theta_1^\star - \theta_2^\star}{\alpha_1} > \theta^\star$. Thus, $\theta_1^\star > \theta^\star$. ■

5 Heterogeneity

In the previous section, we consider a borrower who has full flexibility in separating creditors into different groups. We have seen that when the borrower can diffuse the term structure enough, she can make sure the project succeeds for any positive fundamental. In this section, we will consider the case when the groups areexogenously defined and the planner cannot separate them and make any finer groups. We already know from Theorem 2 that a max-min planner will diffuse the coordination risk as much as he can. In this section we posit a different question. What if the groups are heterogeneous in nature? In which order should the planner approach the groups? The information structure is as before (with truncated information). From our analysis before, multiple equilibria may arise because of the public information of survival. So we need an objective criteria for the planner. The planner is assumed to be *cautious* or *max-min* economic agent, i.e. she wants to minimize the chance of default anticipating the *worst* equilibrium can be played.

Example Consider two equally informed groups of mass $\frac{1}{2}$. Suppose, $r_1' < r_2'$. Since agents in group 1 will take the cooperative action only when they believe that the project will succeed with probability greater than $\frac{1}{1+r_1'} > \frac{1}{1+r_2'}$, we will say group 1 agents are more reluctant to take the cooperative action. The planner has to decide whether she will go to the more reluctant group first or the less reluctant group first. Alternatively, we can think of the problem as that of a homogenous groups but the planner is choosing whether to give a payoff incentive to the early group or the late group. An early incentive reduces θ_1^* directly while a late incentive reduces θ_2^* directly. We will see that under some mild condition, the direct impact of payoff incentive on θ_2^* is higher than the direct impact of payoff incentive on θ_1^*. Since, once the game has reached the late agents they are facing less coordination risk than their predecessors. Finally, when the planner provides a late incentive the early agents anticipate that θ_2^* will decrease, this in turn reduces θ_1^*. If there is no public information of survival in the first period then $\frac{\partial\theta_1^*}{\partial\theta_2^*} = 1$. If there is public information of survival in the first period as well then this anticipation effect is even higher. Thus, a late payoff incentive reduces θ_1^* more. Equivalently, we can say that it is better to approach the more reluctant agents first.

Suppose there are n heterogeneous groups. Groups may be hererogeneous in terms of payoff, informativeness or group sizes. We will consider one type of heterogeneity at a time keeping the groups same in all other respects. Let us denote $\mathcal{T}_n$ as the set of all possible permutation of $(1, 2, \ldots n)$, i.e. $\mathcal{T}_n := \{(t(1), t(2), \ldots t(n)) | t : \{1, 2, \ldots n\} \to \{1, 2, \ldots n\}$ is a permutation$\}$. The borrower's problem is to choose $(t(1), t(2), \ldots t(n)) \in \mathcal{T}_n$ i.e. a period for every group to minimize the highest possible threshold θ_1^{*h}.

Let us define (abusing notation)

$$G(x, y, m) := \frac{x}{F(mx + F^{-1}(x) + my)}, \ x, y \in [0, 1] \tag{5}$$

Let $\psi(k, y, m)$ be such that

$$G(\psi(k, y, m), y, m) \equiv k \tag{6}$$

One can easily check that: $\psi_k = \frac{1}{G_x} > 0$ (since we are considering θ^{*h})), $\psi_y = -\frac{G_y}{G_x} > 0$ (since $G_y < 0$) and $\psi_m = -\frac{G_m}{G_x} > 0$ (since $G_m < 0$).

In equalibrium at any period $t = 1, 2 \ldots T$,

$$G(\frac{(\theta_t^* - \theta_{t+1}^*)}{\alpha_t}, \frac{\theta_{t+1}^*}{\alpha_t}, \alpha_t \sqrt{\tau_t}) = \frac{1}{1 + r'_{(t)}} \tag{7}$$

Therefore,

$$\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t} = \psi(\frac{1}{1 + r'_t}, \frac{\theta_{t+1}^*}{\alpha_t}, \alpha_t \sqrt{\tau_t}) \tag{8}$$

This gives us the following recursive relation in θ_t^*:

$$\theta_{t-1}^* = \alpha_{t-1} \psi(\frac{1}{1 + r'_{t-1}}, \frac{\theta_t^*}{\alpha_{t-1}}, \alpha_{t-1} \sqrt{\tau_{t-1}}) + \theta_t^* \tag{9}$$

$$= \alpha_{t-1} \psi(\frac{1}{1 + r'_{t-1}}, \frac{\theta_t^*}{\alpha_{t-1}}, \alpha_{t-1} \sqrt{\tau_{t-1}}) + \alpha_t \psi(\frac{1}{1 + r'_t}, \frac{\theta_{t+1}^*}{\alpha_t}, \alpha_t \sqrt{\tau_t}) + \theta_{t+1}^* \tag{10}$$

However if there is no public information of survival in period 1, then

$$\theta_1^* = \alpha_1 \frac{1}{1 + r'_1} + \alpha_2 \psi(\frac{1}{1 + r'_2}, \frac{\theta_3^*}{\alpha_2}, \alpha_2 \sqrt{\tau_2}) + \theta_3^* \tag{11}$$

Suppose there are two groups. So the planner can either choose the permutation $(1, 2)$ or $(2, 1)$. Since we are considering only the worst equilibrium θ^{*h}, we will drop the superscript h from further notation. The planner optimally chooses permutation $(1, 2)$ if $\theta_1^*(1, 2) < \theta_1^*(2, 1)$ and vice versa.

Information Heterogeneity Suppose group 1 is more informative than group 2 i.e. $\tau_1 > \tau_2$. Suppose groups are of equal mass ie. $\alpha_1 = \alpha_2 = \frac{1}{2}$ and the payoff are same i.e. $\frac{1}{1+r'_1} = \frac{1}{1+r'_2} = k$ (say). Then from equation 11

$$\theta_1^*(1,2) - \theta_1^*(2,1) = \frac{k}{2} + \frac{1}{2}\psi(k, 0, \frac{\sqrt{\tau_2}}{2}) - \frac{k}{2} - \frac{1}{2}\psi(k, 0, \frac{\sqrt{\tau_1}}{2}) < 0$$

The last inequality holds since $\psi_m > 0$ and $\tau_1 > \tau_2$. Thus we see that a planner should approach the more informed group first. This because the less informed group will be more affected by the survival news and early agents will anticipate that.

Heterogeneity in Group Size Suppose group 1 has greater mass than group 2, i.e. $\alpha_1 > \alpha_2$. Suppose the agents are equally informative i.e. $\tau_1 = \tau_2 = \tau$ (say) and they have same payoff i.e. $\frac{1}{1+r'_1} = \frac{1}{1+r'_2} = k$ (say). We can see from efuation 11

$$\begin{aligned}
\theta_1^*(1,2) - \theta_1^*(2,1) &= \alpha_1 k + \alpha_2 \psi(k, 0, \alpha_2\sqrt{\tau}) - \alpha_2 k + \alpha_1 \psi(k, 0, \alpha_1\sqrt{\tau}) \\
&< \alpha_1 k + \alpha_2 \psi(k, 0, \alpha_1\sqrt{\tau}) - \alpha_2 k + \alpha_1 \psi(k, 0, \alpha_1\sqrt{\tau}) \text{ (since } \alpha_1 > \alpha_2 \text{ and } \psi_m > 0) \\
&= (\alpha_1 - \alpha_2)(k - \psi(k, 0, \alpha_1\sqrt{\tau})) \\
&< 0 \text{ (since } \frac{\psi}{F} = G = k \text{ we have } \psi < k)
\end{aligned}$$

The larger group will anticipate that the survival news will affect the smaller group later more than what smaller groups believes what survival news will do to larger group later. Thus we see that a planner should approach the larger group first.

Payoff Heterogeneity Suppose $r'_1 < r'_2$ ie.e. group 1 is more reluctant to take the cooperative action than group 2 i.e. $k_1 = \frac{1}{1+r'_1} > \frac{1}{1+r'_2} = k_2$. Suppose the group sizes are same i.e. $\alpha_1 = \alpha_2 = \frac{1}{2}$ and the two groups are equally informative i.e. $\tau_1 = \tau_2 = \tau$. From equation 11 we see

$$\begin{aligned}
\theta_1^*(1,2) - \theta_1^*(2,1) &= \frac{1}{2}[\{k_1 + \psi(k_2, 0, \frac{1}{2}\sqrt{\tau})\} - \{k_2 + \psi(k_1, 0, \frac{1}{2}\sqrt{\tau})\}] \\
&= \frac{1}{2}[\{k_1 - \psi(k_1, 0, \frac{1}{2}\sqrt{\tau})\} - \{k_2 - \psi(k_2, 0, \frac{1}{2}\sqrt{\tau}\})]
\end{aligned}$$

Define $F(k) := \frac{1}{2}[k - \psi(k, 0, \frac{1}{2}\sqrt{\tau})]$. We have $F'(k) = 1 - \psi_k$. Note that $\psi_k = \frac{1}{G_x}$ captures how θ_2^* changes following a payoff incentive. When there is no public information of survival this effect is 1. Under some regularity assumption we can say that with public information of survival the effect will be larger i.e. $\psi_k = \frac{1}{G_x} > 1$. Hence $F'(k) < 0$. Hence, $\theta_1^*(1,2) - \theta_1^*(2,1) < 0$. The less reluctant agents are more affected by the survival news. So if the planner goes to the more reluctant agents first they will anticipate this and consequently θ_1^* will be lower.

6 Robustness and Discussion

6.1 General Payoff

Our analysis of diffusing coordination so far is based on the assumption that player's payoff only depends on the binary action of cooperation and the success or failure of the project. In this subsection, we will show our results are robust to more general payoff structure. This extension will enlarge the potential applicability of our theory, e.g. the self-fulfilling runs in the application part. In addition to the assumption 1 on the static payoffs, the general payoff for time t player satisfies the following assumption 2.

Assumption 2 *At time t ($t \in [1,T]$)of the diffused coordination risk model, suppose the player i's payoff u depends on the her action a_{it}, the current non-coordination $\alpha_t w_t$, the future aggregate non-coordination $w^t \equiv \sum_{u=t+1}^{T} \alpha_u w_u$ and the current fundamental strength θ_t.*

$$u(a_{it}=1,\theta_t,\alpha_t w_t,w^t)=\begin{cases} r(\theta_t,\alpha_t w_t,w^t) & \text{if } \theta_t \geq \alpha_t w_t + w^t \\ q(\theta_t,\alpha_t w_t,w^t) & \text{if } \theta_t < \alpha_t w_t + w^t \end{cases}$$

$$u(a_{it}=0,\theta_t,\alpha_t w_t,w^t)=\begin{cases} b(\theta_t,\alpha_t w_t,w^t) & \text{if } \theta_t \geq \alpha_t w_t + w^t \\ c(\theta_t,\alpha_t w_t,w^t) & \text{if } \ \theta_t < \alpha_t w_t + w^t \end{cases}$$

in which $w_{T+1} = w^T = 0$. Notice that, in case the game ends before time t, or if there exists any $v < t$ such that $\theta_v < \alpha_v w_v + w^v$, time t players have no chance to make a decision and the payoff is irrelavent.

Assumption 2.1

$$\bar{u}(\theta_t,\alpha_t w_t,w^t) = r(\theta_t,\alpha_t w_t,w^t) - b(\theta_t,\alpha_t w_t,w^t) > 0,$$

$$\underline{u}(\theta_t,\alpha_t w_t,w^t) = q(\theta_t,\alpha_t w_t,w^t) - q(\theta_t,\alpha_t w_t,w^t) < 0$$

for any given value of $(\theta_t,\alpha_t w_t,w^t)$. $r(\theta_t,\alpha_t w_t,w^t), b(\theta_t,\alpha_t w_t,w^t), c(\theta_t,\alpha_t w_t,w^t), q(\theta_t,\alpha_t w_t,w^t)$ are all non-decreasing in θ_t and non-increasing in $\alpha_t w_t$ and w^t.

Assumption 2.2

$$0 < \underline{n} < \bar{u} < \bar{n}$$

$$0 < \underline{m} < -\underline{u} < \bar{m} < \underline{n}$$

in which $\underline{m}, \bar{m}, \underline{n}, \bar{n}$ are constants and $\underline{m} < \bar{m} < \underline{n} < \bar{n}$.

Proposition 7 *Under general payoff structure satisfying assumption 2, our mechanism of diffusing coordination could work to completely avoid the coordination risk.*

Proof.
In the appendix.
■

Corollary 1 *As long as $\bar{u}$ and $-\underline{u}$ are non-negative and bounded, our mechanism of diffusing coordination could work to completely avoid the coordination risk.*

Proof.
In the appendix.
■

6.2 General Dynamics of Residual Fundamental

In this subsection, we show that our mechanism works with more general law of motion of fundamentals, which can have other reasonable function form than the linear one.

Assumption 3 *The law of motion of fundamental is in form of*

$$\theta_{t+1} = h(\theta_t, \alpha_t w_t)$$

h has the following properties

$$\frac{\partial h}{\partial \theta} > 0, \ \frac{\partial h}{\partial w} < 0 \text{ and } h(0,0) = 0$$

Proposition 8 *Under general dynamics of fundamental satisfying assumption 3, our mechanism of diffusing coordination could work to completely avoid the coordination risk.*

Proof.
In the appendix.
■

6.3 General Equilibrium Concern

In Theorem 1, we assume that the interest rate paid to creditors is fixed at r, which does not depend on the probability of default in equilibrium. It is worthwhile to consider the incentive of creditors in a general equilibrium framework. In order to make the diffused debt contract feasible, creditors would ask a higher interest rate if the coordination failure is more likely to happen. Consider the debt contract $(T, (\alpha))$, where $T \in \mathbb{N}$ and $\alpha \equiv (\alpha_1, \alpha_2, ..., \alpha_T) \in \Delta^{T-1}$. As defined in Section 4, the probability of default corresponding to the worst equilibrium is $\bar{P}(T,(\alpha)) := P(\theta < \theta_1^{*h}(T,(\alpha)))$. The participation constraint for any creditors is

$$\left[1 - \bar{P}(T,(\alpha))\right] \times [1 + r(T,(\alpha))] \geq 1 + R$$

in which R is the creditors outside option and $r(T,(\alpha))$ is the interest rate paid to creditor given the project succeeds. In equilibrium, borrower will choose the optimal debt contract to maximize the expected profit by taking the creditors' participation constraint into consideration. Here, we assume that the borrower cannot set the interest rate high or low to subsidize all creditors or a group of creditors. The interest rate is uniform to each creditor. Borrower will set the interest rate at $r(T,(\alpha)) = \frac{1+R}{1-\bar{P}(T,(\alpha))} - 1$ to make creditors breakeven. Given this interest rate, creditors act exactly the same as in the partial equilibrium model discussed before.

Hence, our mechanism could work when making the price (interest rate) endegeneous in the model. The first best outcome is still implementable in the general equilibrium model. Borrower's objective is still to minimize the probability of coordination failure as the interest rate $r(T,(\alpha))$ is increasing with the coordination risk. When the $\alpha_t < \alpha^\star$ for $t = 2, 3, ...T$, and $\alpha_1 \to 0$, the unique equilibrium gives the first best outcome $\bar{P}(T,(\alpha)) = \underline{P}(T,(\alpha)) = P(T,(\alpha)) \to 0$ and the interest rate would be the outside option of creditors R. Deviating from this debt structure will increase the probability of default and thus increase the cost of borrowing, so the best debt structure is exactly the same.

6.4 No lower Dominance Region

For some coordination game we may naturally have $\theta \geq 0$. For example, think of a borrower who wants the creditors to rollover their debts. If θ is some outside source of funding the brrower uses to pay back the creditors, then it is natural to assume that the borrower is known to have non-negative outside funding. In this case, we lose the lower dominance region, i.e. the project cannot fail if all agents coordinate on the cooperative action. The public information of survival raises the threshold agent's belief about the probability of success. With this modification in the benchmark model, multiple equilibria arise but the probabililty of failure will be lower no matter which equilibrium is selected. The best equilibrium threshold is $\theta_1^{\star l} = 0$ and worst equilibrium threshold is $\theta_1^{\star h}$, which is the maximum solution to equation 1 with $T = 1,\ \alpha_T = 1$.

Consider the diffused structure $(T, (\alpha))$ with public information of survival such that $\alpha_t < \alpha^\star$for all $t = 1, \ldots, T$. If agents in the first period know $\theta \geq 0$, then $\theta_1^\star = 0$. Compare this with $\theta_1^\star = \frac{\alpha_1}{1+r}$ in case when the information of $\theta \geq 0$ was not available. The designer can achieve the best case scenario with zero coordination risk without restricting α_1 to be tending to zero. Therefore, it is easier to construct the diffusion for the validity of Theorem 1.

Consider the concentrated coordination risk model and suppose $\theta \geq 0$ information was available. There are multiple equilibria outcome including the first best. If the planner can control the precision of private signals then she can achieve the first best outcome as the unique equilibrium outcome by choosing a small enough precision so that the good news overcome the coordination risk. But the planner is not likely to have such unlimited manipulation power. The diffusion of coordination risk serves the same purpose. Although the planner can not control the precision of private signal, she can control the mass of agents taking their decision at any point in time. There is a critical mass such that the good news effect overcomes the coordination risk. Using this result we design a mechanism that the coordination risk unravel from the end.

6.5 Information Design

An information designer, as defined in Kamenica and Gentzkow (2011), can commit to the information process that the agents have access to. But she cannot influence the realization of the signal. Our planner is essentially an information designer but unlike the grand information designer, the planner may not have the power to implement all Bayes-plausible distribution of posterior belief of the agents. So, the planner has only limited power of manipulation. Instead of looking into all possible beliefs the planner can possibly induce, we look into a policy that can be easily implemented. We focus on the posterior belief corresponding to this simple policy.

The first best outcome is - whenever $\theta \geq 0$, all agents coordinate on the cooperative action. We can easily construct an information structure when this can be an equilibrium outcome e.g. suppose agents know θ. So, if the planner can reveal all information about θ the first best outcome can occur as a Bayes Nash Equilibrium (BNE). Bergemann and Morris (2013b) shows that a Bayes Correlated Equilibrium (BCE)[15] is a BNE with some extra information. So, agents coordinating on

[15]See Bergemann and Morris (2013b) for formal definition of Bayes Nash Equilibrium and Bayes Correlated Equilibrium.

the cooperative action for all $\theta \geq 0$ is a BCE. We know that if agents know θ then the first best is one possible outcome but so is all agents taking the non-cooperative action for all $\theta < 1$. *Can the information designer design the information structure so that the resulting outcome is always the first best outcome?*

Suppose, the planner does not intervene at all. Then the standard global game argument tells us there is a unique Bayes Nash Equilibrium (BNE). The project succeeds if the fundamental strength is beyond a threshold level, otherwise not. But the threshold is not zero as the planner wants. Now think of what way the iplanner can possibly influence the outcome? Suppose the planner can commit to publicly disclosure of information of the following form: whenever the project is not hopeless it sends a good signal, otherwise a bad signal. Although the first best is a BNE, multiple equilibria arise. But the planner does not have unlimited manipulation power. So let us qualify our earlier question: *Can the planner design an information structure with her limited manipulation power such that her favorite BCE is the unique BNE?*

The answer is yes and we have proven it by construction. The design of the optimal policy is based on the idea of diffusing coordination risk coupled with public information of survival. Note that diffusion of coordination risk alone does not reduce coordination risk. However, if the planner approaches the agents sequentially and advancing further only when the project is not yet hopeless, the public information of survival is naturally generated. This public information of survival changes the creditors' beliefs. Consequently the coordination risk is less even for the worst possible equilibrium. So the next question we ask:

Is there a limit to how much the planner can improve by diffusing the coordination risk?

We showed that borrower can achieve the first best as the unique BNE outcome if she can diffuse the coordination risk enough. The basic argument is that the effect of public news of survival can completely overcome the coordination risk if the risk comes from a small enough mass of agents. Then the coordination risk unravels from back. Thus, whenever the planner has full freedom to make the groups and diffuse the coordination risk she can achieve the the first best as the unique BNE.

7 Application

Self-Fulfilling Runs

Self-fulfilling bank runs is nothing but a coordination failure between depositors. Depositors can withdraw (noncooperative action) or roll over (cooperative action) their deposit in the bank. Banks have comparative advantage in pooling depositor's savings together and making long-term profitable investment. However, the demand deposit contracts are prone to runs (see Diamond and Dybvig (1983); Allen and Gale (1998)). Banks cannot afford to pay all earlier withdrawals because of their illiquid investments. Depositors expecting other depositors withdraw will also withdraw, which will give rise to a bank run even if the bank is financially solvent. Similarly, if financial institutions finances its long-term investment by issuing short-term debts, the coordination failure between their creditors could give rise to self-fulfilling runs. This coordination risk in maturity mismatch problem is one of the main causes of the recent financial crisis (Brunnermeier, 2009). Coordinating on rolling over debts could avoid the costly liquidation of long-term investment and contribute to the stability of financial system. The question to be answered here is bow can banks or financial institutions design the debt contracts to reduce the chance of self-fulfilling runs? This section applies our theory to show that by diffusing coordination, the borrower can completely avoid self-fulfilling bank runs.

Suppose a borrower has a continuum of debt holders with a profitable long-term investment, which produces a riskless return R . Before the return from long-term investment realizes, each debt holder has a chance to withdraw their investment. The long-term investment cannot be reversed and thus the borrower has limited liquidity θ to rollover the withdrawals. θ is the value of liquid assets hold by the borrower and the collateral value of any pledgable legacy asset, including the liquidation value of thelong-term investment. [16] If the total withdrawal w is not higher than the liquidity of borrower θ, the project will succeed and who roll over their debt will have a return of $1+r$ $(1+r<R)$. The withdrawers will receive their principal back $b=1$. If the total withdrawal w is higher than θ, the borrower will have to liquidate the long-term investment at a fire sale price and use the liquidation value to pay earlier withdrawers. Creditor who didn't withdraw will get nothing left. The details of payoff structure are as followings.

$$u(a=1,\theta,w)=\begin{cases}1+r & w\leq\theta\\ 0 & w>\theta\end{cases}$$

$$u(a=0,\theta,w)=\begin{cases}1 & w\leq\theta\\ \frac{\theta}{w} & w>\theta\end{cases}$$

The payoff structure is slightly different to the standard coordination game. The strategic complementarity is lowered in the withdrawal side. If more withdrawals happened, the expected payoff from withdrawing is lower, which give less incentive to withdraw (see Goldstein and Pauzner (2005)). Creditors don't have complete information about θ but only receive a noisy private information about θ. They will make rollover or withdrawal decision based on their private information. Goldstein and Pauzner (2005) show that there exists unique equilibrium to this game and the ex-ante probability of bank runs is non-negative. Assume that, instead of asking creditors to make their decisions at once, the borrower could approach them sequentially group by group (group sizes are $\alpha_1,\alpha_2,...,\alpha_T$). The borrower can commit that she will only approach the next group of creditors if the withdrawals so far are less than the liquid asset holdings. The dynamics of residual fundamental is $\theta_{t+1}=\theta_t-\alpha_t w_t$. Therefore, whenever creditors need to make rollover decisions, they understand that the borrower still have some liquid asset to roll over the future withdrawals, i.e. $\theta_t\geq 0$. Since the payoff structure gives bounded payoff difference, applying corollary 1, as long as the borrower can divide creditors into sufficiently many groups, the borrower can completely avoid self-fulfilling runs. This mechanism is welfare improving since it avoids the costly liquidation of profitable long-term project.

The self-fulfilling feature of bank runs comes from the strategic complementarity between creditors. There are a few papers, e.g. Green and Lin (2003); Andolfatto et al. (2014), build on the idea

[16] In the Diamond and Dybvig model, θ can be taken as the excessive liquid asset the bank holds for the later (patient) creditors, assuming there is no aggregate uncertainty about the share of (im)patient creditors. In this sense, our model also addresses the question: how much excessive liquid asset should the bank holds to avoid self-fulfilling bank runs. In order to have the lower dominance region, we implicitly assume that the realization of θ can be slightly negative. But this assumption is not the key to our result, as shown in the discussion part.

of lining up finite number of creditors to reduce the uncertainties between them. However, they rely on the assumption that the later creditors are able to observe exactly what the earlier ones' type or their actions and thus make the debt contract contingent on that information. Different from theirs, we focus on the information part to persuade coordination among a continuum of creditors but not giving any monetary incentive to them. In our model, the noisy private information tells the borrower's ability to roll over withdrawals and tells how likely other creditors will withdraw. The higher the private noisy information means the borrower are more able to sustain withdrawals and the creditors are less likely to withdraw. Thus, the creditor's decision is (weakly) monotone in their private information. The will reduce the multiple equilibrium to a unique equilibrium as Goldstein and Pauzner (2005).

When the borrower approaches creditors sequentially, the strategic complementarities have dynamic features. Since the creditors are making their decisions only once, whenever they decide to roll over or withdraw, they think about what other creditors will do in the future. The dynamic debt run problem has been investigated by He and Xiong (2012). While they focus on a time varying fundamental problem with complete information, our focus is the incomplete information case with fixed fundamental. When the borrower divides creditors into groups and approach them sequentially, it is very natural for the appearance of the public information that the borrower survives the earlier withdrawals. In the dynamic runs problem, creditors are forward-looking. Based on their private information about the current fundamental, they form beliefs about the current aggregate withdrawals, the fundamental left for future creditors and the equilibrium will be played next period based on the residual fundamental. This piece of public information will make creditors more positive towards the borrower's ability in sustaining the current and future withdrawals and the belief towards other creditors' cooperative actions. As we have showed, dividing creditors into smaller groups can make the private information noisier. This will force creditors put a higher weight on the public information of survival, which makes the creditors even more optimistic. Hence, if the borrower diffuses the coordination enough, the only rationalizable action is to roll over.

Our mechanism is related to the idea of preventing bank runs by suspension of convertibility (see Diamond and Dybvig (1983))[17]. Instead of completely suspending withdrawals, our mechanism allows creditors to withdraw. But the option is restrictive in the sense that it is open at a certain point of time. We argue that this mechanism is feasible to avoid self-fulfilling runs because there is empirical evidence the granularity of corporate debts has a lot of variation across firms and across time, see Choi et al. (2014). It tells that financial firms could issue debt with different maturities for their long-term investment. However, the optimal mechanism proposed here is still fragile to some exogenous shocks outside the model [18]Suppose due to some exogenous reason, the creditors start to actively gather more accurate private information about the fundamental. More precise private information will require more diffusion ($\frac{\partial \alpha^\star}{\partial \tau} < 0$). Thus, the pre-designed diffusion structure may not be sufficient to avoid runs. This is consistent with the explanation for financial cirisis by Gorton and Ordoñez (2014). They argue that the trigger of financial crisis is that some shocks make creditors more skeptical and thus they try to gather more precise information. In their case, the crisis takes place because some projects with positive Net Present Value cannot be financed

[17]Ennis and Keister (2009) shows that complete suspension of convertibility is not ex post efficient and thus not an ex ante credible commitment. It may not necessarily reduce the chance of runs.

[18]Suppose there are some negligible costs of diffusion, the borrower will only divide the group size into $\alpha^\star$to exactly avoid any bank runs. For instance, there might be some issuance cost for each type of debt. One can interpret the cost of diffusing debt structure as the illiquidity discount, since more debt issues with smaller sizes will have a less liquid secondary market than few debt issues with larger sizes (Choi et al., 2014).

because of the cost of acquiring new information. While in our case, more precise information forces creditors put a lower weight on the positive public information. Based on our theory, if the borrower can adjust the debt structure, she would adopt a more diffused debt structure in facing this shock.Choi et al. (2014) provides empirical evidence to support this idea. They found that right before the financial crisis financial institutions adopted more diffused term structures.

Credit Freezes

Bebchuk and Goldstein (2011) models credit market freezes as a coordination failure among banks. Firms rely on bank's loans to finance profitable investment opportunities. Banks are able to distinguish good firms from bad firms, while the government cannot. For good firms, the investment costs \$1 with return $1 + R$. Banks can choose to make the loans (coordinate) or withdraw. The project will succeed only if the macroeconomic fundamental θ is strong enough and/or there are sufficient number of banks to make loans, i.e $\theta \geq b - a + aw$ (or $h(\theta, w) = \theta + a - b - aw \geq 0$). a and b are some constants to capture the level of complementary and how tough the condition for success is. w is the aggregate non-cooperation. If the bank makes the loan, the payoff is $1 + r$ if the project succeeds and -1 if it fails. The payoff is 0 if bank chooses not to make loans. Banks don't observe θ directly but receive a noisy information about it. The strategic complementarity comes from the fact that more banks make loans will incentivize each bank to make loans. The credit freezes could be self-fulfilling because if each bank believes no other banks will make loans, they will not lend to non-financial firms. As in our benchmark model where the coordination risk is concentrated at a point, when banks make their decision simultaneously, they will make loans iff the noisy information is higher than some threshold $s^\star$. If the fundamental θ is lower than the threshold $\theta^\star$, the project will fail and banks who make loans will have a loss.

Our theory proposes a feasible way to correct the coordination failure. Instead of obtaining financing from all banks at one time, the (good) firms could act as a group approaching banks sequentially. Based on earlier banks' decisions on extending the loan, only if there are still hope that the project can be successful, they approach the next bank for loans and share the information of attracting loans so far. Based on Theorem 1, we show that as long as the macroeconomic fundamental is not too bad, or the realization of θ is bigger than $b - a$, [19], this mechanism will help to complete avoid self-fulfilling credit freezes.

The mechanism of diffusing coordination could work only if banks are separable in term of their decision making and their knowledge about the economy. Suppose there is a large bank holding company who controls several banks as their subsidies, all subsidies are individual banks could make their decisions individually. However, the appearance of large banking holding companiesl makes diffusing coordination among banks more difficult to be applied. Therefore, from the perspective of avoiding self-fulfilling credit freeze, we provide some rationale for breaking up the large banks[20]. Large banks in our model are not as good as small banks because they prevents all banks to coordinate on making loans to productive firms. With large banks, avoiding the inefficient credit market freezes and making the real economy to grow or recover becomes more difficult.

[19] Note that if $\theta < b - a$, the project cannot be successful even if all banks lend to good firms

[20] Notice that the reason that big banks are inferior to small banks has nothing to do with "to big to fail", or the implicit government guarantess or moral hazard problem. Rather, according to our model, big banks will make the real economy more likely to fail.

Network Technology

Suppose there is a 'standardization board' (planner) who wants the firms to adopt a new better standard (see Farrell and Saloner (1985)). If she just advises them to adopt it, they may think others will not adopt it and thus coordination failure may arise. We can say if the new standard is good enough ($\theta > \theta^*$) then only the industry switch to the new standard. Thus many better standards which are below the threshold will never get adopted. We are suggesting that if the board approach these firms sequentially and proceeding further only when there is still hope that the new standard may do good (i.e. things may still work out if all left over firms adopt it), then the board can make sure (by diffusing enough) that any better standard will get adopted. Also, notice that the planner can influence the nature of truncated information by taking a 'posture' regarding when she gives up. We may want to look into this as well.

8 Conclusion

We design a policy that can achieve the first best as the unique equilibrium outcome in a coordination problem. The design is based on the idea of diffusing the coordination risk. The planner approaches the agents sequentially and advancing further only when the coordination failure has been averted so far. This policy is simple to implement. However, if the planner cannot diffuse the coordination risk as much as she wants, then we show a cautious planner should diffuse as much as possible. Also, we show that if the groups are heterogeneous and the planner can only choose which group to approach first, then a cautious planner should first approach the more reluctant group of agents. This is not the only way the planner can diffuse the coordination risk. The planner can diffuse the mass over time by restricting the choice of agents, e.g a financial institute allows the agents to withdraw only 10% of their money at a time. We expect such diffusion to have similar effects. We will pursue this in our companion paper.

Appendix

Proof of Proposition 1 This is a special case of the Proposition 2. □

Proof of Proposition 2 Given the private information s, the play will choose to cooperate if she believes the probability of success is sufficiently high, i.e.

$$P(\theta \geq w|s) \geq \frac{c(\theta,w)-q(\theta,w)}{r(\theta,w)-b(\theta,w)+c(\theta,w)-q(\theta,w)} = \frac{1}{1+\frac{\bar{u}(\theta,w)}{-\underline{\mathrm{u}}(\theta,w)}}$$

Given the monotonicity of $\bar{u}$ and $\underline{\mathrm{u}}$, the maximum value of $\frac{1}{1+\frac{\bar{u}(\theta,w)}{-\underline{\mathrm{u}}(\theta,w)}}$ is obtained when $w=1$ and the minimum is obtained when $w=0$. Define $\bar{p}=\max(\frac{-\underline{\mathrm{u}}}{\bar{u}-\underline{\mathrm{u}}})=\frac{\bar{m}}{\underline{\mathrm{n}}+\underline{\mathrm{m}}}$ and $\underline{\mathrm{p}}=\min(\frac{-\underline{\mathrm{u}}}{\bar{u}-\underline{\mathrm{u}}})=\frac{\underline{\mathrm{m}}}{\bar{n}+\bar{m}}$.

Upper Dominance Region Suppose the player believes that all the other players will take the non-cooperative action irrespective of their private information $i.e. w=1$. Under this worst possible scenario, she would still cooperate if

$$P(\theta \geq 1|s) \geq \bar{p}$$

the player will choose to cooperate for sure. Thus, if the private signal $s \geq \bar{s}_1 = \sigma F^{-1}(\bar{p}) + 1$, the player will cooperate irrespective of other players' action.

Lower Dominance Region Similarly, consider the best possible case where$w = 0$. Player with private information $s < \underline{s}_1 = \sigma F^{-1}(\underline{p})$ would take the non-cooperative action irrespective of other players' action, since

$$P(\theta \geq 0|s) \leq \underline{p}$$

Iterated Elimination of Never Best Response For the upper dominance region, suppose now (after iterated elimination of never best response) the upper dominance region for s is $\bar{s}_n$. In other words, players with private signal higher than $\bar{s}_n$will cooperate irrespective of the other player's actions. Given all agents hold this belief about other agents' strategy, we can derive the next threshold $\bar{s}_{n+1}$ as a function of $\bar{s}_n$. If the fundamental is higher than $\theta^\star(\bar{s}_n)$, the measure of non-cooperative actions will be smaller than the fundamental and thus no coordination failure could occur. $\theta^\star(\bar{s}_n)$ solves the following equation

$$P(s < \bar{s}_n|\theta^\star) = \theta^\star \Longrightarrow \sigma F^{-1}(\theta^\star(\bar{s}_n)) + \theta^\star(\bar{s}_n) = \bar{s}_n$$

Consider the worst possible case with $\bar{p}$[21] and given $\theta^\star(\bar{s}_n)$, $\bar{s}_{n+1}$ solves the following equation

$$P(\theta \geq \theta^\star(\bar{s}_n)|\bar{s}_{n+1}) = \bar{p} \Longrightarrow \sigma F^{-1}(\bar{p}) + \theta^\star(\bar{s}_n) = \bar{s}_{n+1}$$

Notice that $\theta^\star(\bar{s}_0) \equiv 1$, we have two equations for $\bar{s}_1$:

$$\sigma F^{-1}(\theta^\star(\bar{s}_1)) + \theta^\star(\bar{s}_1) = \bar{s}_1, \ \sigma F^{-1}(\bar{p}) + 1 = \bar{s}_1$$

So, obviously $\theta^\star(\bar{s}_1) < \theta^\star(\bar{s}_0) = 1$. Recursively,

$$\sigma F^{-1}(\bar{p}) + \theta^\star(\bar{s}_n) = \bar{s}_{n+1}$$

if $\theta^\star(\bar{s}_n) < \theta^\star(\bar{s}_{n-1})$, it is easy to see that $\bar{s}_{n+1} < \bar{s}_n$. Given the decreasing sequence $(\bar{s}_j)_{j=1}^\infty$ is in the compact set $[\underline{\theta} - \frac{\sigma}{2}, \bar{\theta} + \frac{\sigma}{2}]$, we know that there exist a limit for this sequence, $\bar{s} = \lim_{j\to\infty} \bar{s}_j$. Similarly, there is a upbound for the lower dominance region, $\underline{s} = \lim_{j\to\infty} \underline{s}_j$.

Unique Equilibrium The previous section shows that there exist $\bar{s} \equiv \lim_{n\to\infty}(\bar{s}_n)$, and $\underline{s} \equiv \lim_{n\to\infty}(\underline{s}_n)$ such that the only rationalizable action for any player is to cooperate when $s \geq \bar{s}$ and to take non-cooperative action when $s < \underline{s}$. We don't have much knowledge in the region of $(\underline{s},\ \bar{s})$. Is it possible that $\underline{s} = \bar{s}$? We construct the upper and lower dominance region based on the highest (lowest) possible payoff difference parameter, or $\bar{p}$ ($\underline{p}$). However, when the profile of strategies changes, e.g. $\bar{s}_n$ changes, the proportion of players who take the cooperative action and incooperative action will adjust. $\bar{p}$ or $\underline{p}$ cannot be supported as the best or worst possible case. We will provide a proof to show the iterated elimination of dominated strategies will generate a unique equilibrium to this coordination game.

[21] Notice that this condition used in iterated elimination is sufficient but not necessary. Holding the belief that other player's strategy based on $\bar{s}_n$, the maximal non-cooperative action is $P(\epsilon_i - \epsilon_J < \bar{s}_n - \bar{s}_{n+1}) < 1$. We use the $\bar{p}$ here to illustrate the idea of iterated elimination of never best response. It will be shown later that the iterated elimination will produce a unique equilibrium.

Equilibrium Definition Given $s^\star$, for any fundamental value θ, the criteria for success is that

$$w \lesseqqgtr \theta \Longleftrightarrow F(\frac{s^\star - \theta}{\sigma}) \lesseqqgtr \theta$$

The criteria can be written as$\theta \lesseqqgtr \theta^\star$, in which $\theta^\star$satisfies $\sigma F^{-1}(\theta^\star) + \theta^\star = s^\star$.We know that for the thresold player, who has received the private information $s^\star$,she is indifferent between the cooperative action and the non-cooperative action. Define the function $H(s, s^\star)$ as the expected payoff difference from the cooperative action and from the non-cooperative action for player with private noisy signal s given $s^\star$.

$$H(s, s^\star) \equiv \int_{\theta \geq \theta^\star} \bar{u}(\theta, w(\theta)) dF(\theta|s) + \int_{\theta < \theta^\star} \underline{u}(\theta, w(\theta)) dF(\theta|s)$$

For the threshold player with private information $s^\star$, $H(s^\star, s^\star) = 0$. The equilibrium is defined as $(\theta^\star, s^\star)$ such that players will cooperate if and only if $s \geq s^\star$, and the project will be successful if and only if $\theta \geq \theta^\star$. In equilibrium, the following two conditions hold.

$$H(s^\star, s^\star) = 0$$

$$\sigma F^{-1}(\theta^\star) + \theta^\star = s^\star$$

We will show that there is a unique threshold equilibrium$(\theta^\star, s^\star)$ such that players cooperate only when receiving private information $s \geq s^\star$, and the project succeeds only when the fundamental $\theta \geq \theta^\star$.

It is equivalent to show that there exists a unique solution $s^\star$to $H(s^\star, s^\star) = 0$. Since $w(\theta, s^\star) = F(\frac{s^\star - \theta}{\sigma})$, we can write θ as $\theta \equiv v(w, s^\star) \equiv s^\star - \sigma F^{-1}(w)$. w is uniformly distributed for the threshold player as

$$P(w \leq W) = P(F(\frac{s^\star - \theta}{\sigma}) \leq W) = P(\theta \geq s^\star - \sigma F^{-1}(W)) = W$$

Transforming the integral from θ to w,

$$\begin{aligned} H(s^\star, s^\star) &= \int_{\theta \geq \theta^\star(s^\star)} \bar{u}(\theta, w(\theta)) dF(\theta|s^\star) + \int_{\theta < \theta^\star(s^\star)} \underline{u}(\theta, w(\theta)) dF(\theta|s^\star) \\ &= \int_0^{w(\theta^\star, s^\star)} \bar{u}(v(w, s^\star), w) dw + \int_{w(\theta^\star, s^\star)}^1 \underline{u}(v(w, s^\star), w) dw \end{aligned}$$

If there are two different solutions to $H(s^\star, s^\star) = 0$, i.e. $H(s_1^\star, s_1^\star) = H(s_2^\star, s_2^\star) = 0$. W.L.O.G, assume that $s_1^\star > s_2^\star$. For any given w, $v(w, s_1^\star) > v(w, s_2^\star)$, thus

$$\bar{u}(v(w, s_1^\star), w) > \bar{u}(v(w, s_2^\star), w) > 0 > \underline{u}(v(w, s_1^\star), w) > \underline{u}(v(w, s_2^\star), w)$$

We know that in equilibrium, $w(\theta^\star, s^\star) = F(\frac{s^\star - \theta^\star}{\sigma}) = \theta^\star(s^\star)$. Therefore, $w(\theta^\star, s_1^\star) > w(\theta^\star, s_2^\star)$,

$$\begin{aligned} H(s_1^\star, s_1^\star) - H(s_2^\star, s_2^\star) &= \int_0^{w(\theta^\star, s_2^\star)} \left(\bar{u}(v(w, s_1^\star), w) - \bar{u}(v(w, s_2^\star), w)\right) dw \\ &\quad + \int_{w(\theta^\star, s_1^\star)}^1 \left(\underline{u}(v(w, s_1^\star), w) - \underline{u}(v(w, s_2^\star), w)\right) dw \\ &\quad + \int_{w(\theta^\star, s_2^\star)}^{w(\theta^\star, s_1^\star)} \left(\bar{u}(v(w, s_1^\star), w) - \underline{u}((v(w, s_2^\star), w))\right) dw > 0 \end{aligned}$$

which contradicts the assumption that $H(s_1^\star, s_1^\star) = H(s_2^\star, s_2^\star) = 0$ and $s_1^\star > s_2^\star$. From the proof, it is easy to see that actually $H(s^\star, s^\star)$ is increasing in $s^\star$ and $H(\frac{1}{2}\sigma, \frac{1}{2}\sigma) > 0 > H(-\frac{1}{2}\sigma, -\frac{1}{2}\sigma)$. Hence, there is only one threshold equilibrium for this coordination game. Following Milgrom and Roberts (1990), the unique threshold equilibrium is the unique equilibrium for the coordination game.

The general payoff structure nests the case all payoffs are constant only depending on $w \lesseqgtr \theta$. So, for proposition 1.1, we have

$$P(s \leq s^\star|\theta^\star) = \theta^\star,\ P(\theta \geq \theta^\star|s^\star) = \frac{1}{1+r'} = \frac{1}{1+\frac{r-b}{c-q}}$$

So, the unique equilibrium is $\theta^\star = \frac{1}{1+r'}$, $s^\star = \theta^\star + \sigma F^{-1}(\theta^\star)$ □

Proof of Proposition 3 This is a special case of proposition 4 □

Proof of Proposition 4 Let $(\theta_t^*, s_t^*)_{t=1}^T$ be the equilibrium threshold. At $t = 1$ we have

$$\theta_1^* = (\frac{\alpha_1}{1+r}) + \theta_2^* \tag{12}$$

For any $t \geq 2$ let us define the net pay off from cooperating when an agent gets a signal s_t and it is commonly known that $\theta_t \geq 0$ as follows:

$$y(s_t, \theta_t^*) = \begin{cases} (r-b+c-q)\frac{F(\sqrt{\tau}(s_t-\theta_t^*))}{F(\sqrt{\tau}s_t)} - (c-q) & \text{if } \theta_t^* > 0 \\ r-b & \text{if } \theta_t^* = 0 \end{cases} \tag{13}$$

since $P(\theta_t \geq \theta_t^\star|s_t, \theta_t \geq 0) = \frac{P(\theta_t \geq \theta_t^*|s_t)}{P(\theta_t \geq 0|s_t)} = \frac{F(\sqrt{\tau}(s_t-\theta_t^*))}{F(\sqrt{\tau}s_t)}$. Let $s_t^*(\theta_t^*)$ be the threshold signal such that if player will cooperate iff $s_t \geq s_t^*(\theta^*)$. $\theta_t \geq \alpha_t w(\theta_t) + \theta_{t+1}^*$ iff $\theta_t \geq \theta_t^*$. Therefore,

$$s_t^*(\theta_t^*) = \theta_t^* + \frac{1}{\sqrt{\tau}}F^{-1}(\frac{\theta_t^* - \theta_{t+1}^*}{\alpha_t}) \tag{14}$$

Finally define the net payoff from cooperating for the threshold agent who gets the signal $s_t^*(\theta^*)$ and it is publicly known that $\theta_t \geq 0$ as follows:

$$Y(s_t, \theta_t^*) = \begin{cases} \lim_{s_t \to -\infty} y(s_t, \theta_t^*) & \text{if } \theta_t^* = 0 \\ y(s_t^*(\theta^*), \theta_t^*) & \text{if } \theta_t^* \in (0,1) \\ \lim_{s_t \to \infty} y(s_t, \theta_t^*) & \text{if } \theta_t^* = 1 \end{cases} \tag{15}$$

In equilibrium the threshold agent is indifferent. So $Y(s_t, \theta^*) = 0$. Also, $\theta_t^* = 0$ is always a solution but $\theta_t^* = 1$ is never a solution. This gives us the recursive relation 1. □

Proof of Proposition 5 Consider a T period model. At the last period $t = T$, in equilibrium, $(\theta^\star_T, s^\star_T)$ satisfies

$$P(\theta_T \geq \theta^*_T | s^*_T) = \frac{1}{1+r'}$$

$$\alpha_T P(s_T \leq s^*_T | \theta^*_T) = \theta^*_T$$

This is a scale down version of the benchmark model. Hence, the unique equilibrium is

$$\theta^*_T = \frac{\alpha_T}{1+r'},\ s^*_T = \frac{\alpha_T}{1+r'} + \frac{1}{\sqrt{\tau}} F^{-1}(\frac{1}{1+r'})$$

When $t \leq T$, we have the recursive condition for equilibrium $(\theta^\star_t, s^\star_t)$,

$$P(\theta_t \geq \theta^*_t | s^*_t) = \frac{1}{1+r'}$$

$$\alpha_t P(s_t \leq s^*_t | \theta^*_t) = \theta^*_t - \theta^\star_{t+1}$$

Hence, we have

$$\theta^*_t = \frac{\alpha_t}{1+r'} + \theta^\star_{t+1},\ s^*_t = \theta^\star_t + \frac{1}{\sqrt{\tau}} F^{-1}(\frac{1}{1+r'})$$

By induction, we have

$$\theta^*_1 = \frac{1}{1+r'},\ s^*_1 = \frac{1}{1+r'} + \frac{1}{\sqrt{\tau}} F^{-1}(\frac{1}{1+r'})$$

Thus we see that there is a unique monotone equilibrium. Extending the MR argument [22] we can then say that it is indeed the unique equilibrium in general. So, probability of default given any α is $P(D|\alpha) = P(\theta \leq \theta^*_1)$ is independent of α. □

Proof of Proposition 6

Case of concentrated coordination $(T = 1)$ Suppose there is public information that the fundamental is non-negative in the model of concentrated coordination risk, then the monotone equilibrium $(\theta^\star, s^\star)$ statisfies the following two conditions

$$P(\theta \geq \theta^\star | s^\star, \theta \geq 0) = \frac{1}{1+r'}$$
$$P(s < s^\star | \theta^\star) = \theta^\star$$

Combining these two conditions, we have

$$\frac{\theta^\star}{F(\sqrt{\tau}\theta^\star + F^{-1}(\theta^\star))} = \frac{1}{1+r'}$$

Since $F(\sqrt{\tau}\theta^\star + F^{-1}(\theta^\star)) \leq 1$, we proved that $\theta^\star \leq \frac{1}{1+r'}$. We know that in the model of concentrated coordination risk without public information of survival, the unique equilibrium gives the threshold of fundamental $\frac{1}{1+r'}$. Notice that there could be multiple equilibria for this coordination game and thus multiple solution for $(\theta^\star, s^\star)$. However, no matter which equilibrium is selected, we proved that public information of survival could reduce coordination risk when coordination is concentrated.

[22] It is a very straight forward extension where we just need to treat the agents in period 1 and 2 diffrently. So we omit the details.

Case of Diffused Coordination Risk $(T > 1)$ Start from the last period T, it is easy to see that $\frac{\theta_T^\star}{\alpha_T} \leq \frac{1}{1+r'}$, since the equilibrium condition can be summarized as

$$\frac{\frac{\theta_T^\star}{\alpha_T}}{F(\sqrt{\tau}\theta_T^\star + F^{-1}(\frac{\theta_T^\star}{\alpha_T}))} = \frac{1}{1+r'}$$

When $t \leq T$, we can summarize the equilibrium condition as

$$\frac{\frac{\theta_t^\star - \theta_{t-1}^\star}{\alpha_t}}{F(\sqrt{\tau}\theta_t^\star + F^{-1}(\frac{\theta_t^\star - \theta_{t-1}^\star}{\alpha_t}))} = \frac{1}{1+r'}$$

Thus, we have $\frac{\theta_t^\star - \theta_{t+1}^\star}{\alpha_t} \leq \frac{1}{1+r'}$. Notice that there could be multiple equilibrium in every subgame starting from t. We showed that the inequality is true for any possible equilibrium. Recursively, we have $\theta_t^\star \leq \frac{1}{1+r'} \sum_{u \geq t} \alpha_t$. Hence, we proved that $\theta_1^\star \leq \frac{1}{1+r'}$, no matter which equilibrium is selected.

Proof of Proposition 6 with general Payoff From the earlier proof, there exists at least one monotone equilibrium for the diffused coordination with public information of survival. Now consider the alternative problem without public information for the time T players. Denote the unique equilibrium of the subgame at time t without public information as $(\tilde{s}_t^\star, \tilde{\theta}_t^\star)$, while the equilibrium with public truncated information is $(s_t^\star, \theta_t^\star)$. Note that $(\tilde{s}_t^\star, \tilde{\theta}_t^\star)$ is unique but depending on the equilibrium played in later periods $u (u \geq t)$, while $(s_t^\star, \theta_t^\star)$ may not be unique but also depends on the later solutions. The proof will compare $\tilde{s}_t^\star$ to the largest possible $s_t^\star$given the same expectation of equilibria played after t. In the last period T, the unique equilibrium$(\tilde{s}_T^\star, \tilde{\theta}_T^\star)$ satisfies

$$\int_0^{w_T(\theta_T^\star, s_T^\star)/\alpha_T} \bar{u}(v(w_T, \tilde{s}_T^\star), \alpha_T w_T, w^T = 0) dw_T + \int_{w_T(\theta_T^\star, s_T^\star)/\alpha_T}^1 \underline{u}(v(w_T, \tilde{s}_T^\star), \alpha_T w_T, w^T = 0) dw_T = 0$$

For any possible solution for the model with truncated public informaiton $s_T^\star$, if $s_T^\star \geq \tilde{s}_T^\star$, we have$w_T(\theta_T^\star, s_T^\star) \geq w_T(\theta_T^\star, \tilde{s}_T^\star)$ and $v_T(\theta_T^\star, s_T^\star) \geq v_T(\theta_T^\star, \tilde{s}_T^\star)$. It is easy to see that

$$\begin{aligned}
\int_0^{w_T(\theta_T^\star, s_T^\star)/\alpha_T} \bar{u}(v(w_T, s_T^\star), \alpha_T w_T, w^T = 0) dw_T + \int_{w_T(\theta_T^\star, s_T^\star)/\alpha_T}^{F(\frac{s_T^\star}{\sigma})/\alpha_T} \underline{u}(v(w_T, s_T^\star), \alpha_T w_T, w^T = 0) dw_T &> \\
\int_0^{w_T(\theta_T^\star, s_T^\star)/\alpha_T} \bar{u}(v(w_T, s_T^\star), \alpha_T w_T, w^T = 0) dw_T + \int_{w_T(\theta_T^\star, s_T^\star)/\alpha_T}^{1/\alpha_T} \underline{u}(v(w_T, s_T^\star), \alpha_T w_T, w^T = 0) dw_T &> \\
\int_0^{w_T(\theta_T^\star, s_T^\star)/\alpha_T} \bar{u}(v(w_T, \tilde{s}_T^\star), \alpha_T w_T, w^T = 0) dw_T + \int_{w_T(\theta_T^\star, s_T^\star)/\alpha_T}^{1/\alpha_T} \underline{u}(v(w_T, \tilde{s}_T^\star), \alpha_T w_T, w^T = 0) dw_T &= 0
\end{aligned}$$

This violates the indifference condition for threshold agent with $s_T^\star$. So, we proved that for any possible equilibrium $s_T^\star < \tilde{s}_T^\star$ and thus $\theta_t^\star < \tilde{\theta}_T^\star$ at $t = T$. In the second last period, $t = T - 1$, for any given equilibrium $s_T^\star$, suppose the solution for the case without public information of survival is $\tilde{s}_{T-1}^\star(s_T^\star)$, we have

$$\begin{aligned}
&\int_0^{w_{T-1}(\theta_{T-1}^\star, \tilde{s}_{T-1}^\star)/\alpha_{T-1}} \bar{u}(v(w_{T-1}, \tilde{s}_{T-1}^\star), \alpha_{T-1} w_{T-1}, \alpha_T w_T) dw_{T-1} \\
&+ \int_{w_{T-1}(\theta_{T-1}^\star, \tilde{s}_{T-1}^\star)/\alpha_{T-1}}^{1/\alpha_{T-1}} \underline{u}(v(w_{T-1} \tilde{s}_{T-1}^\star), \alpha_{T-1} w_{T-1}, \alpha_T w_T) dw_{T-1} = 0 \qquad (16)
\end{aligned}$$

in which

$$v(w_{T-1}, \tilde{s}^\star_{T-1}) = \tilde{s}^\star_{T-1} - \sigma F^{-1}(w_{T-1}),$$

$$w_T = F(\frac{s^\star_T - \theta_T}{\sigma}) = F(\frac{s^\star_T - \tilde{s}^\star_{T-1} + \sigma F^{-1}(w_{T-1}) + \alpha_{T-1}w_{T-1}}{\sigma})$$

It is clear that the LHS of the above equation is increasing in $\tilde{s}^\star_{T-1}$ but decreasing in $s^\star_T$. Thus, the unique equilibrium $\tilde{s}^\star_{T-1}(s^\star_T)$ it is (locally) increasing in $s^\star_T$. We proved that the public information of survival lowered the threshold $s^\star_T$ in the last period. It in turn lowers the threshold in the second last period $\tilde{s}_{T-1}$, even there is no public information at $T-1$. Applying the similar argument as in the last period, it is easy to show that, for any possible equilibrium with truncated public information, $s^\star_{T-1}(s^\star_T)$ is lower than $\tilde{s}^\star_{T-1}(s^\star_T)$. Similarly, we can show that for any $u \in [1, T-1]$, the unique equilibrium (without public information of survival) $\tilde{s}^\star_u(s^\star_{u+1}, .., s^\star_T)$ is increasing in every component and thus any possible monotone equilibrium (with public information of survival) $s^\star_u(s^\star_{u+1}, .., s^\star_T)$ is lower than $\tilde{s}^\star_u(s^\star_{u+1}, .., s^\star_T)$. So, we proved by induction that any possible $s^\star_1$ is lower than the threshold without public information of survival. Since $\theta^\star_1 + \sigma F^{-1}(\theta^\star_1) = s^\star_1$, $\theta^\star_1$ is increasing in $s^\star_1$, which completes the proof.

□

Proof of Proposition 7

Step 1: There are multiple equilibria for diffusing coordination to$T$ groups. There is an equilibrium with lowest chance of failure, $\theta^\star_2 = \theta^\star_3 = ... = \theta^\star_T = 0$, $s^\star_2 = s^\star_3 = ... = s^\star_T = -\frac{\sigma}{2}$. $\theta^\star_1$ and $s^\star_1$ satisfy

$$\alpha_1 F(\frac{s^\star_1 - \theta^\star_1}{\sigma}) = \theta^\star_1$$

$$\int_0^{F(\frac{s^\star_1-\theta^\star_1}{\sigma})} \bar{u}(v(w_1, s^\star_1), \alpha_1 w_1, w^1 = 0)dw + \int_{F(\frac{s^\star_1-\theta^\star_1}{\sigma})}^1 \underline{u}(v(w_1, s^\star_1), \alpha_1 w_1, w^1 = 0)dw = 0.$$

The equilibrium is defined by threshold of private signals $(s^\star_1, s^\star_2, ..., s^\star_T)$, and thresholds for the fundamental $\{\theta^\star_1, \theta^\star_2, ..., \theta^\star_T\}$. The player will take cooperative action only if $s_t \geq s^\star_t$ and there will be no failure in any period $u \geq t$ iff the fundamental $\theta_t \geq \theta^\star_t,\ t = 1, 2, ..., T$. At any time t, the aggregate non-cooperative action is defined as following:

$$w_t(\theta_t, s^\star_t) = F(\frac{s^\star_t - \theta_t}{\sigma})$$

For the time $t(2 \leq t \leq T)$ players, they take $(s^\star_u)_{u \geq t}$as given when they make their decisions. The equilibrium conditions for $\{\theta^\star_t, s^\star_t\}$ can be characterized as following:

$$H(s^\star_t, s^\star_t) = \int_{\theta_t \geq \theta^\star_t} \bar{u}(\theta_t, \alpha_t w_t, w^t)dF(\theta_t | s^\star_t, \theta_t \geq 0) + \int_{\theta_t < \theta^\star_t} \underline{u}(\theta_t, \alpha_t w_t, w^t)dF(\theta_t | s^\star_t, \theta_t \geq 0) = 0$$

$$\alpha_t P(s_t < s^\star_t | \theta^\star_t) = \theta^\star_t - \theta^\star_{t+1}$$

in which $\theta^\star_{T+1} = w^T = 0$. The first period problem is the same except the public information of survival:

$$H(s_1^\star, s_1^\star) = \int_{\theta_1 \geq \theta_1^\star} \bar{u}(\theta_1, \alpha_1 w_1, w^1) dF(\theta_1 | s_1^\star) + \int_{\theta_1 < \theta_1^\star} \underline{\mathrm{u}}(\theta_1, \alpha_1 w_1, w^1) dF(\theta_1 | s_1^\star) = 0$$

$$\alpha_1 P(s_1 < s_1^\star | \theta_1^\star) = \theta_1^\star - \theta_2^\star$$

It is easy to see how the multiplicity of equilibria could arise in the last period $t = T$. The distribution of w_T is shifted by the truncated information in the following way

$$\begin{aligned} P(w_T &\leq W_T | s_T^\star, \theta_T \geq 0) = P(F(\frac{s_T^\star - \theta_T}{\sigma}) \leq W_T | s_T^\star, \theta_T \geq 0) \\ &= \frac{P(\theta_T \geq s_T^\star - \sigma F^{-1}(W_T))}{P(\theta_T \geq 0)} = \frac{W_T}{F(\frac{s_T^\star}{\sigma})} \end{aligned}$$

Rewrite the integral in term of w, we have

$$H(s_T^\star, s_T^\star) = \frac{1}{F(\frac{s_T^\star}{\sigma})} \left(\int_0^{w(\theta_T^\star, s_T^\star)} \bar{u}(v(w_T, s_T^\star), \alpha_T w_T, w^T = 0) dw_T + \int_{w(\theta_T^\star, s_T^\star)}^{F(\frac{s_T^\star}{\sigma})} \underline{\mathrm{u}}(v(w_T, s_T^\star), \alpha_T w_T, w^T = 0) dw_T \right)$$

In equilibrium, $H(s_T^\star, s_T^\star) = 0$. Since $w(\theta_T^\star, s_T^\star) = F(\frac{s_T^\star - \theta_T^\star}{\sigma})$ and $v(w_T, s_T^\star) \equiv s_T^\star - \sigma F^{-1}(w_T)$, $w(\theta_T^\star, s_T^\star)$ and $v(w_T, s_T^\star)$ are both increasing in $s_T^\star$. Since $F(\frac{s_T^\star}{\sigma})$ increases with $s_T^\star$, increasing $s_T^\star$ might enlarge the range $\left(F(\frac{s_T^\star}{\sigma}) - F(\frac{s_T^\star - \theta_T^\star}{\sigma}) \right)$ of admitting negative payoff difference $\underline{\mathrm{u}}(v(w_T, s_T^\star)$. Thus, $H(s_T^\star, s_T^\star)$ might be not monotone in $s_T^\star$ depending on the distribution of F. That is why we could have multiple equilibria in the subgame of $t = T$ for any realization of θ_T. Similar argument can be applied to $t < T$. Hence, we could have multiple monotone equilibria for this dynamic game. If $\theta_T^\star = 0$, $s_T^\star = -\frac{\sigma}{2}$, $F(\frac{s_T^\star}{\sigma}) = \theta_T^\star$. The payoff difference for any $s_T \geq s_T^\star$ is always non-negative and the dominant strategy is to coordinate. This constitutes an equilibrium for the subgame of $t = T$ for any realization of θ_T. If the players in$t = T - 1$ expects this equilibrium will be played in the last period, the problem for $T - 1$ players face exactly the same problem as in the last period. The backward induction stops at $t = 2$, since there is no truncated information at $t = 1$. Given $\theta_2^\star = \theta_3^\star = ... = \theta_T^\star = 0$, $s_2^\star = s_3^\star = ... = s_T^\star = -\frac{\sigma}{2}$, the first period problem is as following: $\theta_1^\star$ and $s_1^\star$ satisfy

$$\alpha_1 F(\frac{s_1^\star - \theta_1^\star}{\sigma}) = \theta_1^\star$$

$$\int_0^{F(\frac{s_1^\star - \theta_1^\star}{\sigma})/\alpha_1} \bar{u}(v(w_1, s_1^\star), \alpha_1 w_1, w^1 = 0) dw + \int_{F(\frac{s_1^\star - \theta_1^\star}{\sigma})/\alpha_1}^{1} \underline{\mathrm{u}}(v(w_1, s_1^\star), \alpha_1 w_1, w^1 = 0) dw = 0.$$

There is a unique solution of $\theta_1^\star$and $s_1^\star$ as in the proof of concentrated coordination risk with general payoff structure. Thus, we proved that this is an equilibrium for the model of diffused coordination risk with public information of survival.

Step 2: We begin our proof from the last period and then complete it by backward induction. We need to show that if α_T in the last period is small enough, then the only rationalizable action for the player is to cooperate. In other words, for any possible $s_T^\star$ and $\theta_T^\star$ satisfying

$$\alpha_T P(s_T < s_T^\star | \theta_T^\star) = \theta_T^\star$$

the following expected payoff difference is always non-negative

$$H(s_T^\star, s_T^\star) \equiv \int_{\theta_T \geq \theta_T^\star} \bar{u}(\theta_T, \alpha_T w_T, w^T = 0) dF(\theta_T | s_T^\star, \theta_T \geq 0) + \int_{\theta_T < \theta_T^\star} \underline{u}(\theta_T, \alpha_T w_T, w^T = 0) dF(\theta_T | s_T^\star, \theta_T \geq 0)$$

First of all, the expected payoff difference can be written as an integral of w_T

$$H(s_T^\star, s_T^\star) = \frac{1}{F(\frac{s_T^\star}{\sigma})} \left(\int_0^{F(\frac{s_T^\star - \theta_T^\star}{\sigma})} \bar{u}(v(w_T, s_T^\star), \alpha_T w_T) dw_T + \int_{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}^{F(\frac{s_T^\star}{\sigma})} \underline{u}(v(w_T, s_T^\star), \alpha_T w_T) dw_T \right)$$

It is sufficient to show that

$$\int_0^{F(\frac{s_T^\star - \theta_T^\star}{\sigma})} \bar{u}(v(w_T, s_T^\star), \alpha_T w_T) dw_T + \int_{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}^{F(\frac{s_T^\star}{\sigma})} \underline{u}(v(w_T, s_T^\star), \alpha_T w_T) dw_T \geq 0$$

for any $s_T^\star$ and $\theta_T^\star$ satisfying $\alpha_T P(s_T < s_T^\star | \theta_T^\star) = \theta_T^\star$. We know that

$$\begin{aligned}
& \int_0^{F(\frac{s_T^\star - \theta_T^\star}{\sigma})} \bar{u}(v(w_T, s_T^\star), \alpha_T w_T) dw_T + \int_{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}^{F(\frac{s_T^\star}{\sigma})} \underline{u}(v(w_T, s_T^\star), \alpha_T w_T) dw_T \\
> \quad & F(\tfrac{s_T^\star - \theta_T^\star}{\sigma}) \times \underline{n} + \left(F(\tfrac{s_T^\star}{\sigma}) - F(\tfrac{s_T^\star - \theta_T^\star}{\sigma}) \right) \times (-\bar{m}) \\
= \quad & F(\tfrac{s_T^\star - \theta_T^\star}{\sigma}) \times (\underline{n} + \bar{m}) - F(\tfrac{s_T^\star}{\sigma}) \times \bar{m}
\end{aligned}$$

So, it is sufficient to show that there exist $\alpha^\star \in (0,1)$ such that for any $\alpha_T \leq \alpha^\star$, the following inequality always hold for any $\theta_T^\star \in [0, \alpha_T]$.

$$\frac{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}{F(\frac{s_T^\star}{\sigma})} = \frac{\frac{\theta_T^\star}{\alpha_T}}{F(\frac{1}{\sigma}\theta_T^\star + F^{-1}(\frac{\theta_T^\star}{\alpha_T}))} \geq \frac{\bar{m}}{\underline{n} + \bar{m}}$$

Equivalently, we need to show that $\forall x \in [0,1]$

$$G(\alpha_T, x) = \frac{x}{F(\frac{1}{\sigma}\alpha_T x + F^{-1}(x))} \geq \frac{\bar{m}}{\underline{n} + \bar{m}}$$

Notice that $G(0,x) = 1$ and $\frac{\partial G}{\partial \alpha_T} < 0$. $\forall \alpha_T \in [0,1]$, $G(\alpha_T, x)$ is continuous in x. So, $H(\alpha_T) \equiv \min_{x \in [0,1]} G(\alpha_T, x)$ is well defined and it is continuous by the theorem of maximum. Obivously, $H(\alpha_T)$ is decreasing in α_T. $\max_{\alpha_T} H(\alpha_T) = \lim_{\alpha_T \to 0} H(\alpha_T) = 1$. Thus, for $\frac{\bar{m}}{\underline{n} + \bar{m}} < 1$, there always exists $\alpha^\star$ such that $\forall \alpha_T \leq \alpha^\star$

$$H(\alpha_T) \geq \frac{\bar{m}}{\underline{n} + \bar{m}}.$$

Thus, if $\alpha_T \leq \alpha^\star$, $\theta_T^\star = 0$ and $s_T^\star = -\frac{\sigma}{2}$. This means all player in the last period will cooperate independent of their private information. In other words, the aggregate noncooperation is 0, or $w_T = 0$. Given this unique solution in the last period, the coordination game at $T-1$ is exactly the same as time T . The argument will go backwards to the second period. For the first period without public information of survival, we know that there is a unique equilibrium with $\theta_1^\star \leq \alpha_1$.Thus, given the parameters in the model with general payoff, or $\bar{m}, \bar{n}, \underline{m}, \underline{n}$ and σ, there exists $\alpha^\star$ and $T^\star = [\frac{1}{\alpha^\star} + 1]$, for any positive fundamental $\eta > 0$ (however small). The planner can design a policy $(T, (\alpha))$, or $\forall \alpha_1 \in (0, \eta)$, $\alpha_2 = \alpha_3 = ... = \alpha_{T^\star} = \alpha_{T^\star+1} = \frac{1-\alpha_1}{T^\star}$ such that the project succeeds for all $\theta > \eta$. $\square$

Proof of Corollary 1 The proof is the same as the proof of Proposition 7. In that proof, we don't need the monotonicity of $\bar{u}$ or $\underline{u}$ on θ or w_t or w^t. As long as $\bar{u}$ is bounded downwards and $\underline{u}$ is bounded upwards, that proof is valid.

$\square$

Proof of Proposition 8 Suppose the policy is $(T, (\alpha))$. The definition of the (monotone) equilibrium is $\{\theta_1^\star, \theta_2^\star, ...\theta_T^\star; s_1^\star, s_2^\star, ..., s_T^\star\}$ such that agents will coordinate iff $s_t \geq s_t^\star$, the project will not fail after time t iff $\theta_t \geq \theta_t^\star$. For $t \geq 2$

$$\theta_{t+1}^\star = h\left(\theta_t^\star, \alpha_t P(s_t < s_t^\star | \theta_t^\star)\right) \text{ and } \theta_{T+1}^\star = 0 \tag{17}$$

$$P(\theta_t \geq \theta_t^\star | \theta_t \geq 0, s_t^\star) = \frac{1}{1+r'} \tag{18}$$

For $t = 1$,

$$P(\theta_1 \geq \theta_1^\star | s_1^\star) = \frac{1}{1+r'},\ \theta_2^\star = h\left(\theta_1^\star, \alpha_1 P(s_1 < s_1^\star | \theta_1^\star)\right)$$

,,At time T, from equation (2), we have

$$F\left(\frac{s_T^\star - \theta_T^\star}{\sigma}\right) = \frac{1}{1+r'} F\left(\frac{s_T^\star}{\sigma}\right) \tag{19}$$

Take $s_T^\star$ as a function of $\theta_T^\star$, we have

$$\frac{ds_T^\star}{d\theta_T^\star} = \frac{f(\frac{s_T^\star - \theta_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma}) - \frac{1}{1+r'} f(\frac{s_T^\star}{\sigma})} \tag{20}$$

It is easy to show that$s_T^\star$ increases with $\theta_T^\star$ and $(s_T^\star - \theta_T^\star)$ decreases with $\theta_T^\star$ since

$$\frac{ds_T^\star}{d\theta_T^\star} = \frac{f(\frac{s_T^\star - \theta_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma}) - \frac{1}{1+r'} f(\frac{s_T^\star}{\sigma})} = \frac{f(\frac{s_T^\star - \theta_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma}) - \frac{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}{F(\frac{s_T^\star}{\sigma})} f(\frac{s_T^\star}{\sigma})} > 0$$

$$\frac{d\left(s_T^\star - \theta_T^\star\right)}{d\theta_T^\star} = \frac{-\frac{1}{1+r'} f(\frac{s_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma}) - \frac{1}{1+r'} f(\frac{s_T^\star}{\sigma})} = \frac{-\frac{1}{1+r'} f(\frac{s_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma}) - \frac{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}{F(\frac{s_T^\star}{\sigma})} f(\frac{s_T^\star}{\sigma})} < 0$$

Plug (3) into (1), we have

$$h\left(\theta_T^\star, \alpha_T F(\frac{s_T^\star - \theta_T^\star}{\sigma})\right) = h(\theta_T^\star, \frac{\alpha_T}{1+r'} F(\frac{s_T^\star}{\sigma})) = 0$$

Differetiate $h(\theta_T^\star, \frac{\alpha_T}{1+r'} F(\frac{s_T^\star}{\sigma}))$ with respect to $\theta_T^\star$, we have

$$\begin{aligned}
\frac{dh\left(\theta_T^\star, \alpha_T P(s_T < s_T^\star | \theta_T^\star)\right)}{d\theta_T^\star} &= \frac{\partial h}{\partial \theta} - (-\frac{\partial h}{\partial w}) \times \frac{\alpha_T}{1+r'} \times \frac{1}{\sigma} \times f(\frac{s_T^\star}{\sigma}) \times \frac{ds_T^\star}{d\theta_T^\star} \\
&= \frac{\partial h}{\partial \theta} - (-\frac{\partial h}{\partial w}) \times \frac{\alpha_T}{1+r'} \frac{1}{\sigma} \times \frac{f(\frac{s_T^\star - \theta_T^\star}{\sigma}) f(\frac{s_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma}) - \frac{1}{1+r'} f(\frac{s_T^\star}{\sigma})} \\
&= \frac{\partial h}{\partial \theta} - (-\frac{\partial h}{\partial w}) \times \frac{\alpha_T}{1+r'} \frac{1}{\sigma} \times f(\frac{s_T^\star}{\sigma}) \frac{1}{1 - \frac{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma})} / \frac{F(\frac{s_T^\star}{\sigma})}{f(\frac{s_T^\star}{\sigma})}}
\end{aligned}$$

Define $L(\theta_T^\star)$ as

$$L(\theta_T^\star) \equiv \frac{1}{1 - \frac{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma})} / \frac{F(\frac{s_T^\star}{\sigma})}{f(\frac{s_T^\star}{\sigma})}}$$

From the log-concavity of F, and the monotonicity of $s_T^\star$ and $s_T^\star - \theta_T^\star$ on $\theta_T^\star$, we know $L(\theta_T^\star)$ is decreasing in $\theta_T^\star$. So we know that $\forall \theta_T^\star > 0,\ L(\theta_T^\star) < \lim_{\theta_T^\star \to 0} L(\theta_T^\star)$. Note that corner solution $(\theta_T^\star = 0,\ s_T^\star = -\frac{1}{2})$ always exists since $P(\theta_T \geq 0 | \theta_T \geq 0, s_T^\star) = 1 > \frac{1}{1+r'}$. We have $\lim_{\theta_T^\star \to 0} L(\theta_T^\star) = \lim_{\theta_T^\star \to 0} \frac{f(\frac{s_T^\star}{\sigma})}{f(\frac{s_T^\star}{\sigma}) - \frac{1}{1+r'} f(\frac{s_T^\star}{\sigma})} = \frac{1+r'}{r'}$. $\forall \theta_T^\star \geq 0$, we have

$$\begin{aligned}
\frac{dh\left(\theta_T^\star, \alpha_T P(s_T < s_T^\star | \theta_T^\star)\right)}{d\theta_T^\star}\Big|_{\theta_T^\star \geq 0} &= \frac{\partial h}{\partial \theta}\Big|_{\theta > 0} - \left(-\frac{\partial h}{\partial w}\right)\Big|_{w > 0} \times \frac{\alpha_T}{1+r'} \frac{1}{\sigma} \times f\left(\frac{s_T^\star}{\sigma}\right) \frac{1}{1 - \frac{F(\frac{s_T^\star - \theta_T^\star}{\sigma})}{f(\frac{s_T^\star - \theta_T^\star}{\sigma})} / \frac{F(\frac{s_T^\star}{\sigma})}{f(\frac{s_T^\star}{\sigma})}} \\
&\geq \underline{\mathrm{k}}_1 - \bar{k}_2 \times \frac{\alpha_T}{1+r'} \frac{1}{\sigma} \times f\left(\frac{s_T^\star}{\sigma}\right) \lim_{\theta_T^\star \to 0} L(\theta_T^\star) \\
&\geq \underline{\mathrm{k}}_1 - \bar{k}_2 \times \frac{\alpha_T}{1+r'} \frac{1}{\sigma} \times \max_{\theta_T^\star} f\left(\frac{s_T^\star(\theta_T^\star)}{\sigma}\right) \frac{1+r'}{r'}
\end{aligned}$$

We know that F is continuously differentiable, so the PDF f is bounded. There exists $\bar{f} > 0$ such that $\max_{x \in [-\frac{1}{2}, \frac{1}{2}]} f(x) \leq \bar{f}$. Thus, if $\alpha_T \leq \alpha^\star = \frac{\underline{\mathrm{k}}_1 \sigma r'}{\bar{k}_2 \bar{f}}$, $\frac{dh\left(\theta_T^\star, \alpha_T P(s_T < s_T^\star | \theta_T^\star)\right)}{d\theta_T^\star} \geq 0$ holds for all $\theta_T^\star > 0$. Since $h\left(\theta_T^\star = 0, \alpha_T P(s_T < s_T^\star = -\frac{1}{2} | \theta_T^\star)\right) = 0$, the only solution to equation 1 is $(\theta_T^\star = 0,\ s_T^\star = -\frac{1}{2}\sigma)$. Given the unique Bayesian Nash Equilibrium at T is the $\theta_T^\star = 0,\ s_T^\star = -\frac{1}{2}\sigma$, the problem at time $T-1$ is exactly the same problem as in time T. The argument will go backwards to $t = 1$. This completes this generalization.

□

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