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Too Big to Cheat: Efficiency and Investment in Partnerships*

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Abstract

Many economic activities are organized as partnerships. These ventures are formed with capital contributions by partnership members who obtain a share of ownership in exchange. The design of the partnership dictates how much of the profits is distributed among the members and how much is reinvested. In this paper, we study the optimal design of partnerships under the assumption that partners privately observe shocks to their liquidity needs. When the ownership share of a partner is large enough, his incentives to misreport vanish. This occurs because a fraction of the increase in his payouts after reporting high liquidity needs is financed by disinvesting in the partnership. When his ownership share is not big enough, the ownership structure of the partnership must vary over time to prevent misreporting. The limiting distribution of shares depends on the initial ownership structure. Under certain conditions, if the partnership starts with approximately equally distributed shares, both partners are too big to cheat and the ownership structure remains unchanged forever. Instead, if the initial ownership structure is such that one of the partners is too big to cheat but the other is not, the share of the initially larger partner ends up either reaching 100% (i.e., sole proprietorship forever) or decreasing to the point at which both partners are too big to cheat (i.e., shares are approximately equally distributed forever).

Keywords: Partnerships, Asymmetric Information, Investment, Financing, Firm Size.

JEL Codes: D82, D86, D92, G32

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1 Introduction

Many economic activities are organized as partnerships. They are unincorporated organizations formed by two or more persons that join to carry on a trade or business. Each partner contributes money, property, labor, or skill, and each expects to share in the profits and losses.¹ They do not have access to the stock market and the ownership and control are vested in the partners [Magill and Quinzii, 1996].² In 2010, there were 3,248,481 partnerships that generated 27% of the total net income of all U.S. businesses. Table 1 displays the percent of total net income by type of business and industry. Partnerships accounts for a significant share of all these industries.

Table 1: Net income by type of business and industry

	Sole proprietorships (%)	Partnerships (%)	Corporations (%)
Total	15	27	58
Legal services	23	65	11
Professional services	39	42	19
Accounting services	26	59	15
Agriculture	13	58	29
Mining	3	52	45
Finance and insurance	5	55	40
Consulting services	56	19	25

Note: Net income by type of business and industry, U.S. Internal Revenue Service, 2008.

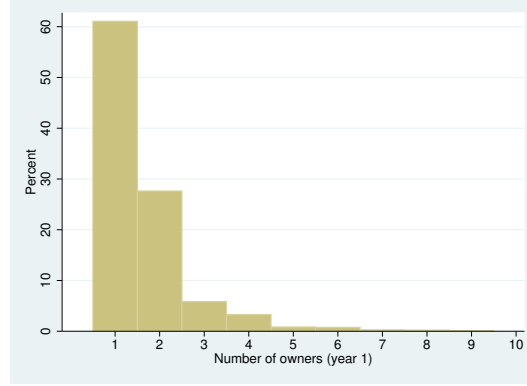
Partnerships can potentially mitigate the effect of financial frictions. Several entrepreneurs may contribute their own capital to form a joint venture and thus minimize the extend of external financing. The formation of partnerships, however, may be limited by informational frictions. The cost of dealing with private information may offset the benefits. In fact, in the U.S. about 90% of start-ups have 2 or fewer owners (Figure 1).³

¹This definition is provided by the U.S. Internal Revenue Service.

²Corporations, on the other hand, represent the other set of firms in which the rest of economic activities are organized. They are distinguished from partnerships along two critical dimensions: (i) the financial sources to which they have access (i.e. they can issue shares in the stock market and can borrow and lend in the credit market) and (ii) the ownership and control structure (i.e. it is owned by shareholders, while it is typically run by a separate group of managers). See Magill and Quinzii [1996], Chapter 6, Section 31, for a thorough discussion.

³Section 6 uses this data to study the dynamics of ownership shares and compare it with the predictions of the model.

Figure 1: Number of owners, distribution



Source: Kauffman Firm Survey.

We model partnerships as follows. Two risk-averse partners contribute some initial capital to form a partnership. The partners face liquidity needs that vary from period to period. The partnership finances itself internally.⁴ The design of the partnership should dictate which fraction of profits is reinvested in capital that will be available for production the next period (*the investment plan*) and how to divide the rest among the partners as payouts for consumption (*the distribution plan*). Ideally, the profits each member receives should depend on his willingness to consume or, more generally, on his liquidity needs. However, if liquidity needs are private information, the design of the partnership must also deal with incentive problems. This paper studies the optimal arrangement in a setting in which liquidity needs are private information and the partners must undertake investment decisions.

Capital accumulation further complicates the analysis in this setting with private information. Solving this problem with the method developed by [Abreu et al. \[1990\]](#), APS hereafter, would imply iterating on utility possibility correspondences instead of functions. However, while this is theoretically feasible, a method based on iterations of correspondences makes the computation very demanding.⁵ We propose an alternative method to overcome these difficulties that characterizes the efficient frontier of a convex utility possibility correspondence by means of weights attached to each partner, which can be interpreted as each partner's ownership shares.⁶ These shares

⁴This assumption is in line with a broad literature, surveyed by [Hubbard \[1998\]](#), that documents credit constraints, especially for small firms.

⁵[Espino \[2005\]](#) adapted APS's method to study a similar framework with capital accumulation. [Abraham and Pavoni \[2008\]](#) used APS's approach to compute the constrained efficient allocation of a model with hidden actions and hidden savings.

⁶The idea of substituting utility levels with ownership shares is borrowed from [Lucas and Stokey \[1984\]](#).

together with capital become endogenous state variables that summarize the history. Our approach complements the methods pioneered by the seminal work of [Spear and Srivastava \[1987\]](#) and [Abreu et al. \[1990\]](#). Indeed, it can be interpreted as a combination of APS and the [Marcet and Marimon \[1994\]](#)'s Lagrangean method.⁷ [Marcet and Marimon \[1994\]](#) sidesteps the requirement that future utilities must lie in the utility correspondence next period by mapping utility levels into welfare weights. In order to do so, they identify the corresponding law of motion for those weights from the sequential formulation of the problem. Our method derives the optimal law of motion of these welfare weights endogenously. The [Marcet and Marimon \[1994\]](#) formulation in general states a saddle point functional equation, which is analogous but not equal to a Bellman equation. In turn, ours postulates a generalized version of a Bellman equation and provides a convergence theorem that allows the analyst to compute the exact solution in convex settings. In particular, since the operator is not necessarily a contraction, we follow the APS strategy to approximate the utility frontier from above.⁸

The environment is chosen so that in the absence of private information, the problem of analyzing the optimal design of the partnership becomes trivial: It simplifies to maximizing the lifetime utility of the owner of a sole proprietorship with a peculiar shock to his liquidity needs. Importantly, efficiency dictates that the ownership structure (i.e. the share of ownership held by each partner) is constant forever. In each period, as the liquidity needs of one partner increase, the efficient arrangement implies that his current payout increases, the current payout of the other partner decreases, and investment in the partnership decreases. Thus, a liquidity shock triggers an increase in a partner's payout that is financed from two sources: *redistribution* (as the payout of the other partner decreases) and *disinvestment* (as investment also decreases).

Under private information, the ownership structure becomes critical. In particular, there is a threshold for the share of the ownership held by each partner such that this partner's incentives to misreport vanish for levels above that threshold. To understand this result, recall that a part of the increase in this partner's payout after a high liquidity shock comes from reducing the other partner's payout (redistribution), and the other part comes from decreasing investment (disinvestment). Interestingly, under full information, the fraction of the increase in the payout financed through disinvestment is increasing in his ownership share. Thus, if this partner chooses

⁷The work of [Mele \[2014\]](#) adapts [Marcet and Marimon \[1994\]](#) to private information problems. See also [Messner et al. \[2012\]](#) for a throughout analysis of the connection between recursive primal and dual approaches.

⁸A related approach is developed by [Beker and Espino \[2013\]](#) to analyze endowment economies with limited commitment and belief heterogeneity.

to misreport, he will consume more this period at the cost of inducing underinvestment. As his share in the partnership increases, eventually the cost of misreporting (underinvestment) is larger than the benefits (a higher current payout). Disinvestment more heavily affects his future payouts and thus provides incentives for truthful revelation.

Our theory also has implications for the dynamics of the ownership structure: it is optimal to make ownership structure fluctuate to alleviate information problems. If one partner's share is below the threshold mentioned above, whenever he reports high liquidity needs, he receives a larger share of current payouts in exchange for a lower future ownership share. Thus, under private information the optimal ownership structure fluctuates over time to provide the right incentives, provided ownership is such that at least one of the partners is not too big to cheat.

The long-run implication of our theory is that the limiting distribution of the ownership structure is such that private information does not matter. To further study this issue we divide the analysis into two cases. In the first case, only one of the partners faces liquidity need shocks (hereafter *the founder* of the partnership). We refer to the partner who does not face shocks to his liquidity needs as *the associate*. This setting is interesting for two reasons. First, it helps us derive results that can be generalized to the case in which both partners face shocks. Second, it directly relates to the literature studying the borrower-lender relationship under private information.⁹ In this case, there are two possible extreme structures. On the one hand, for a sufficiently long sequence of low liquidity shocks, the founder's ownership share becomes sufficiently large to render the incentive problem irrelevant (i.e. incentives to misreport disappear). When that level is reached, the ownership structure remains unchanged forever and private information does not matter. On the other hand, if the founder is hit by a sufficiently long sequence of high shocks, his share in the partnership converges to zero; that is, the ownership structure is concentrated in the associate, and private information does not matter.

Second, we consider a more general case in which both partners face liquidity need shocks. We show that, under certain conditions, in partnerships with equally distributed ownership shares private information is irrelevant. The underlying mechanism resembles the previous case. Here, when ownership shares are approximately equally distributed, both partners are indeed too big to cheat. This implies that for approximately equally distributed shares the ownership structure does not need to change to provide incentives. The limiting distribution of shares depend on the initial ownership structure. On the one hand, if the partnership starts with approximately equally

⁹Below, we discuss how it relates to previous studies in this strand of literature.

distributed shares, the ownership structure remains unchanged forever. On the other hand, if the initial ownership structure is such that one of the partners is too big to cheat but the other is not, the shares of the initially larger partner eventually either reach 100% (i.e. sole proprietorship forever) or decrease to the point at which both partners are too big to cheat (i.e. shares are approximately equally distributed forever).

The model predicts that ownership shares may not need to change to provide incentives if ownership shares are approximately equally distributed. We document that this prediction is in line with empirical evidence. In particular, we split the sample of start-ups with 2 and 3 owners in the Kauffman Firm Survey according to the ownership structure. Among the partnerships with equally distributed shares, we find that 88% do not change the ownership structure between the first and second year of life. In contrast, only 39% do not change the ownership structure among partnerships with unequally distributed shares. Partnerships consisting of few owners with equally distributed shares need to change shares less frequently, as predicted by our model.

1.1 Related literature

The object of study in our paper resembles the concept of a partnership in general equilibrium theory and game theory. In general equilibrium theory, a partnership is a group of agents (the partners) who come together and contribute capital to set up and carry on a business in which limited liability limits the access of a partnership to loanable funds [Magill and Quinzii, 1996]. In game theory, a partnership is the paradigmatic model of many-sided moral hazard [Dutta and Radner, 1994]. The only difference with our case is that we consider privately observed liquidity needs, à la Diamond and Dybvig [1983], instead of hidden effort.

Recent contribution to the study of partnerships focus on the formation of partnerships. Some papers study the choice between partnerships and corporations [Kaya and Vereshchagina, 2014, Levin and Tadelis, 2005, among others]. More related, other papers study how the initial ownership structure affects the provision of incentives [Bolton and von Thadden, 1998, Vereshchagina, 2013]. In particular, Vereshchagina [2013] introduces a static model of team production in which there are four agents, two with low capital and two with high capital. The problem is not studying investment, which is taken exogenously, but characterizing the best way to organize production. If there are decreasing returns and moral hazard, the optimal arrangement shows a positive correlation between the owners' contributions (one firm has the owners with high capital and other the owners with low capital). In addition, Vereshchagina [2013] shows that the data displays

that positive correlation between contribution. The emphasis is very different than in our work, however. We abstracted away from the formation of the partnership and instead study its dynamics and long-run implications.¹⁰ In our model, ownership shares may change without extra capital contributions because they are used to prevent cheating.

Dynamic contracts are considered in another related strand of literature which study the relationship between a lender and a firm. [Marcet and Marimon \[1992\]](#) assume that output is subject to privately observed productivity shocks to show that a *risk-neutral* investor with *unlimited resources* can make a risk-averse entrepreneur follow the first-best investment plan. As the lender is not only risk neutral but also has access to unlimited resources, private information does not affect capital accumulation since the lender can absorb all possible fluctuations at not extra cost.¹¹ [Clementi and Hopenhayn \[2006\]](#) study a setting in which a risk-neutral lender finances a project under limited liability that is run by a risk-neutral entrepreneur who privately observes productivity shocks. Once the level of promised utility of the entrepreneur reaches some threshold, promised utilities can be spread out and the firm attains its efficient size forever. On the other hand, liquidation of the firm is also an absorbing state and the entrepreneur attains its reservation value. While the interaction of two frictions—limited liability and private information—is the key to attain this limiting result, investment plays no role. Our theory does not appeal to limited liability and thus it isolates the interaction of private information and investment.¹² More recently, [Clementi et al. \[2010\]](#) study a venture in which a risk-neutral investor cannot monitor the entrepreneur’s effort. They provide a rationale for a firm’s decline since the incentive provision becomes more costly as the entrepreneur’s wealth increases, which leads to a decreased return on investment. This sort of immiserization differs from our results as their source of the informational friction is different and the investor is assumed to be risk neutral.

Finally, our work is methodologically related to pioneering contributions in the literature on constrained efficient allocations with private information. In pure exchange economy settings, [Green \[1987\]](#), [Spear and Srivastava \[1987\]](#), [Thomas and Worrall \[1990\]](#), and [Atkeson and Lucas \[1992\]](#) show that constrained efficiency dictates extreme levels of “immiserization,” that is, the

¹⁰However, a simple extension of our model in which two partners make capital contributions and bargain over the initial shares would provide a link between capital contributions and ownerships shares even without private information. Since with private information equally distributed ownership shares are desirable, that extension could also rationalize the positive correlation between contributions.

¹¹We further analyze the relationship with our work in Section 4.3.2.

¹²There are two other related papers in the spirit of [Clementi and Hopenhayn \[2006\]](#). In a model with moral hazard, [Quadrini \[2004\]](#) studies the design of renegotiation-proof contracts. In a more recent paper, [Cole et al. \[2012\]](#) study the decision to adopt new technologies in a model with private information and costly monitoring.

utility of (almost) every agent in the economy converges to the lower bound with probability 1. This result is quite robust and independent of the initial distribution. These studies abstracted from capital accumulation as their main goal has been to study wealth distribution. Our long-run analysis shows that this result does not hold if investment opportunities are available and there are only a few agents such that the disinvestment effect plays the critical role described above.

The paper is organized as follows. Section 2 describes the physical environment of the model, defines feasibility and incentive compatibility, and shows how efficient plans can be represented with ownership shares. Section 3 characterizes the optimal design of partnerships under full information. Sections 4 and 5 study the optimal design of partnerships under private information when only one partner has liquidity shocks and when both partners face shocks, respectively. Section 6 presents evidence on the dynamics of ownership shares. Section 7 provides conclusions. Proofs are provided in the appendix, Section 8.

2 Model

Time is discrete and the time horizon is infinite. At date 0, consider a partnership that is formed by two agents, indexed $i = 1, 2$.¹³ They operate a decreasing return to scale technology that delivers a profits function $f(K) = K^\alpha$ with $0 < \alpha < 1$.¹⁴ The partnership starts with capital $K_0 = K_{0,1} + K_{0,2}$, contributed by agents 1 and 2, respectively. Capital depreciates at the rate $\delta \in (0, 1)$. Given technological assumptions, there exists some $\bar{K}$ such that $X = [0, \bar{K}]$ denotes the sustainable levels of capital. Given their initial capital contributions, partners get ownership shares in the partnership.

Partners have preferences à la [Diamond and Dybvig \[1983\]](#) as they face liquidity shocks.¹⁵ At the beginning of date t , partner i privately observes his shock $s_{i,t} \in S_{i,t} = \{s_L, s_H\}$, where $s_H > s_L$ and define $S_t = S_{1,t} \times S_{2,t}$. These shocks are assumed to be i.i.d. across time and partners, where $\pi(s_{i,t}) > 0$ is the probability of $s_{i,t}$. Let $s_t = (s_{1,t}, s_{2,t}) \in S_t$ be the joint shock at date t with probability $\pi(s_t) = \pi(s_{1,t}) \times \pi(s_{2,t})$, while $s^t = (s_0, \dots, s_t) \in S^t = \times_{j=0}^t S_j$ denotes the history of events from date 0 to date t . The probability at date 0 of any particular history s^t is given by $\pi(s^t) = \pi(s_0) \dots \pi(s_t)$.

¹³All the analysis regarding the solution method also applies to the general case in which there is an arbitrary number of partners I .

¹⁴The technology to produce output is represented by $F(K, L) = (K^\omega L^{1-\omega})^\gamma$ for $\gamma, \omega \in (0, 1)$. As the firm hires labor at the wage w , then the profit function reduces to $f(K) = \chi K^{\frac{\omega\gamma}{1-(1-\omega)\gamma}}$ where the constant χ depends on the (γ, ω, w) and it is normalized to 1.

¹⁵This class of preferences is standard in the literature; see [Tirole \[2005\]](#), Chapter 12. Moreover, liquidity shocks are multiplicative as in [Atkeson and Lucas \[1992\]](#).

Given a consumption path $\{c_{i,t}\}_{t=0}^{\infty}$ such that $c_{i,t} : S^t \rightarrow \mathbb{R}_+$, partner i 's preferences are represented by

$$E \left\{ \sum_{t=0}^{\infty} \beta^t s_{i,t} u(c_{i,t}) \right\} = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) s_{i,t} u(c_i(s^t)),$$

where $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is strictly increasing, strictly concave, and twice differentiable; $\lim_{c_t \rightarrow 0} u'(c_t) = +\infty$; and $\beta \in (0, 1)$. A higher value of the partners liquidity needs implies that he is willing to take more resources from the partnership to consume more today compared with the future.

Since the problem of choosing the best investment and distribution plans for the partnership is restricted to solutions that are incentive compatible (i.e. partners have no incentives to misreport liquidity shocks), here we describe the reporting strategies available to the partners.¹⁶

At date t , partner i has privately observed the partial history s_i^t . Let $z_i = \{z_i(s_i^t)\}_{t=0}^{\infty}$ represent his reporting strategy, where $z_{i,t} : \times_{h=0}^t S_{i,h} = S_i^t \rightarrow S_{i,t}$ for all t (i.e. $z_i(s_i^t) \in S_{i,t}$ for all t). Let $z = (z_1, z_2)$ denote the joint reporting strategy. Let z_i^* be the truth-telling reporting strategy for which $z_i^*(s_i^t) = s_{i,t}$ for all t and $s_i^t \in S_i^t$.

Let $K' = \{K_{t+1}\}_{t=0}^{\infty}$ be an *investment plan* that every period allocates next-period capital, given a history of joint reports (i.e. $K_{t+1} : S^t \rightarrow \mathbb{R}_+$ for all t). Similarly, let $C = \{C_t\}_{t=0}^{\infty}$ be a *distribution plan*, where $C_t : S^t \rightarrow \mathbb{R}_+^2$ for all t . To interpret this, consider any joint realization s^t up to date t and any joint reporting strategy z . Partner i 's consumption at date t is given by $C_i(z^t(s^t))$. Similarly, the stock of capital at date $t + 1$ will be given by $K(z^t(s^t)) \geq 0$. Any (C, K') satisfying these properties is called a *sequential plan*.

Definition 1 Given K_0 , a sequential plan (C, K') is feasible for the partnership, if for all joint reporting strategies z , all t , all s^t , and $K_0 = K$,

$$K(z^t(s^t)) + \sum_{i=1}^2 C_i(z^t(s^t)) \leq f(K(z^{t-1}(s^{t-1}))) + (1 - \delta)K(z^{t-1}(s^{t-1})). \quad (1)$$

Intuitively, feasibility at the firm's level means that part of the output is reinvested in the partnership, $K(z^t(s^t)) - (1 - \delta)K(z^{t-1}(s^{t-1}))$, and part is paid out to the partners, $C_1(z^t(s^t)) + C_2(z^t(s^t))$. Importantly, note that there is no external finance available to the partnership.¹⁷ In what follows, we refer to K as the stock of capital as well as the size of the partnership indistinctly.

¹⁶This restriction is without loss of generality since it can be shown that the relevant version of the celebrated Revelation Principle holds.

¹⁷In Section 4.3.2 we study the case in which partners can obtain unlimited resources at a given interest rate r .

Now we introduce the concept of incentive compatibility. Let s^{t-1} be any arbitrary joint partial history reported. Let $\widehat{z}$ be an joint continuation reporting strategy from period t onward. Given a sequential plan (C, K') , partner i 's utility at date t is

$$\begin{aligned} & U_{i,t}(C, K', \widehat{z} || s^{t-1}) \\ &= \sum_{j=0}^{\infty} \beta^j \sum_{s^{j+1}} \pi(s^{j+1}) s_{i,t+j} u(C_i(s^{t-1}, \widehat{z}^j(s^j))), \\ &= \sum_{s_t \in S_t} \pi(s_t) \left(s_{i,t} u(C_i(s^{t-1}, \widehat{z}_0(s_t))) + \beta U_{i,t+1}(C, K', \widehat{z}'(s_t) || (s^{t-1}, \widehat{z}_0(s_t))) \right), \end{aligned}$$

where $\widehat{z}'(s)$ is the continuation reporting strategy from period $t+1$ onward induced by $\widehat{z}$ when the first element s is kept constant. When $t=0$, we directly write $U_i(C, K', z) = U_{i,0}(C, K', z)$ for any z .

The following definition states that a sequential plan is *incentive compatible* if truth-telling is the best response for each partner. Let s_{-i} be a liquidity shock that excludes partner i 's element (e.g., $s_{-1} = s_2$).

Definition 2 Given $K \in X$, a sequential plan (C, K') is incentive compatible if, for each partner i , for all $t \geq 0$, all s^{t-1} , and any continuation z'_i ,

$$\begin{aligned} & \sum_{s_{-i}} \pi(s_{-i}) \left(s_i u(C_i(s^{t-1}, s_i, s_{-i})) + \beta U_{i,t+1}(C, K', z_i^*, z_{-i}^* || (s^{t-1}, s_i, s_{-i})) \right) \\ & \geq \sum_{s_{-i}} \pi(s_{-i}) \left(s_i u(C_i(s^{t-1}, \widetilde{s}_i, s_{-i})) + \beta U_{i,t+1}(C, K', z'_i, z_{-i}^* || (s^{t-1}, \widetilde{s}_i, s_{-i})) \right) \end{aligned} \quad (2)$$

for all $(s_i, \widetilde{s}_i)$.

Thus, incentive compatibility ensures that partners have no incentive to misreport their liquidity shocks. Note that Definition 2 takes into account that partners can choose a continuation reporting strategy every period after they have observed their own history of shocks.¹⁸

2.1 Efficient partnerships

To solve for the efficient arrangement, we consider a fictitious planner who chooses among plans that are incentive compatible and feasible. Let $\Psi^*(K)$ be the utility possibility correspondence—that is, the level of utility of the partners that can be attained given a partnership of size K and

¹⁸This implementation concept can be interpreted as the natural extension of Bayesian implementation for this particular dynamic environment. See Espino [2005].

plans that are incentive compatible and feasible,

$$\Psi^*(K) \equiv \left\{ w \in \mathbb{R}_+^2 : \exists (C, K') \text{ satisfying (1) – (2)} \right. \\ \left. \text{and } w_i \leq U_i(C, K', z^*) \forall i, K_0 = K \right\}.$$

Definition 3 *An incentive-compatible, feasible sequential plan (C^*, K^*) is efficient if there is no alternative incentive-compatible, feasible sequential plan (C, K') such that $U_i(C, K', z^*) > U_i(C^*, K^*, z^*)$ for all i .*

It can be shown that $\Psi^*(\cdot)$ is a continuous, compact-valued, and convex correspondence (see [Espino \[2005\]](#)). The following results are convenient to implement our approach.

Remark 1 *The set of constrained efficient plans can be parameterized by $(\theta_1, \theta_2) \in \mathbb{R}_+^2$. That is, (C^*, K^*) is constrained efficient if and only if (C^*, K^*) is the corresponding plan that solves*

$$h^*(K, \theta) = \max_{w \in \Psi^*(K)} (\theta_1 w_1 + \theta_2 w_2), \quad (3)$$

for some $(\theta_1, \theta_2) \in \mathbb{R}_+^2$.

Remark 2 *$h^*(K, \theta)$ is continuous with respect to (K, θ) and concave with respect to K .¹⁹*

The intuition of the solution strategy proposed below to characterize efficient partnerships is as follows. Imagine that the partnership evaluates alternative plans weighting the utility of partner i using a coefficient θ_i . These weights map into property rights on both current and future payouts to the partners. Although the assignment is indirect, hereafter we refer to these coefficients (θ_1, θ_2) as the *ownership shares* of partners 1 and 2, respectively. So the best distribution and investment plan maximizes the weighted sum of the partners' utility given these ownership shares.²⁰

2.2 Recursive representation

Here we provide a recursive representation of the dynamic problem discussed above. Hereafter we restrict the ownership shares of the partners to add up to 1; that is, $\Delta \equiv \{\theta \in \mathbb{R}_+^2 : \theta_1 + \theta_2 = 1\}$.²¹ In the appendix, we show that h^* solves the following operator:

$$(Th)(k, \theta) = \max_{(c, w', k')} \sum_{i=1}^2 \theta_i \left\{ \sum_s \pi(s) \left[s_i u(c_i(s)) + \beta w'_i(s) \right] \right\}, \quad (4)$$

¹⁹Continuity follows as an application of the Theorem of the Maximum as $\Psi^*(K)$ is a continuous correspondence. Concavity is a direct consequence of the concavity of F and u .

²⁰Our approach to define the objective of the partnership is in the spirit of the general equilibrium approach under incomplete markets; see [Magill and Quinzii \[1996\]](#), Chapter 6, Section 31.

²¹This restriction is innocuous because solutions are homogenous of degree 0 with respect to (θ_1, θ_2) .

subject to

$$k'(s) + \sum_{i=1}^2 c_i(s) = f(k) + (1 - \delta)k \quad (5)$$

$$\begin{aligned} & \sum_{s_{-i}} \pi(s_{-i}) (s_i u(c_i(s_i, s_{-i})) + \beta w'_i(s_i, s_{-i})) \\ & \geq \sum_{s_{-i}} \pi(s_{-i}) (s_i u(c_i(\bar{s}_i, s_{-i})) + \beta w'_i(\bar{s}_i, s_{-i})) \end{aligned} \quad (6)$$

for all $(s_i, \bar{s}_i)$ and

$$c_i(s) \geq 0, \quad w'_i(s) \geq 0 \quad \text{for all } s \text{ and all } i, \quad (7)$$

$$\min_{\theta' \in \Delta} \left[h(k'(s), \theta') - \sum_{i=1}^2 \theta'_i(s) w'_i(s) \right] \geq 0 \text{ for all } s. \quad (8)$$

Notice that the optimization problem takes as given the size of the firm k and the share of ownerships θ , distributes output between current payouts (consumption of the partners) and investment, and assigns continuation utility levels to the partners. The optimization problem defined in condition (8) characterizes the set of continuation utility levels attainable at the future stock of capital k' and ownership shares θ' .²² The values of θ' that attain the minimum in (8) are the next-period ownership shares that are consistent with the entitlement of continuation utilities.

Let $(\widehat{c}, \widehat{k}', \widehat{w}')$ denote the set of policy functions solving (4) - (8), while $\widehat{\theta}'$ denotes the corresponding next-period ownership shares solving (8). Given (k_0, θ_0) , we say that a set of policy functions $(\widehat{c}, \widehat{k}', \widehat{w}')$ generates a sequential plan $(\widehat{C}, \widehat{K}')$ if

$$\begin{aligned} \widehat{C}_i(s^t) &= \widehat{c}_i(\widehat{K}(s^{t-1}), \theta(s^{t-1}))(s_t), \\ \theta(s^{t-1}, s_t) &= \widehat{\theta}'(\widehat{K}(s^{t-1}), \theta(s^{t-1}))(s_t), \\ \widehat{K}(s^{t-1}, s_t) &= \widehat{k}'(\widehat{K}(s^{t-1}), \theta(s^{t-1}))(s_t), \end{aligned} \quad (9)$$

for all i and all (t, s^t) , where K_0 and θ_0 are given.

Proposition 1 h^* is a fixed point of T .

²²In the appendix, we argue that condition (8) holds if and only if

$$\min_{\bar{\theta} \in \Delta^I} \left[h(K, \bar{\theta}) - \sum_{i=1}^I \bar{\theta}_i w_i \right] \geq 0.$$

Importantly, it can be shown that the following version of the Principle of Optimality holds.²³

Remark 3 *A plan (C^*, K^*) is constrained efficient at K_0 if and only if it is generated by the set of policy functions at (9).*

Thus, the value of any plan that can be attained with an incentive-compatible, feasible sequential plan (C, K') can also be attained by splitting output between total current payouts and investment and then delivering current payouts and contingent future ownership shares to each partner.

Now we provide an algorithm capable of finding the value function h^* and its corresponding policy functions. Let h^{**} be the value function solving the recursive problem for which the incentive compatibility constraints are ignored (i.e. the full information case). Evidently, $h^*(k, \theta) \leq h^{**}(k, \theta)$ for all (k, θ) .

Proposition 2 *Let $h_0 = h^{**}$ and denote $h_n = T^n(h^{**})$. Then, $\{h_n\}$ is a monotone decreasing sequence and $\lim_{n \rightarrow \infty} h_n = h^*$.*

Our method complements the traditional APS approach in terms of tractability. It identifies attainable levels of next-period utility by iterating directly on the utility possibility frontier with no need to know the utility possibility correspondence a priori. This greatly simplifies the computational burden. Of course, the APS approach outperforms ours if the utility possibility correspondence is not convex valued. Indeed, our approach (at least as stated herein) is not appropriate to handle such problems.

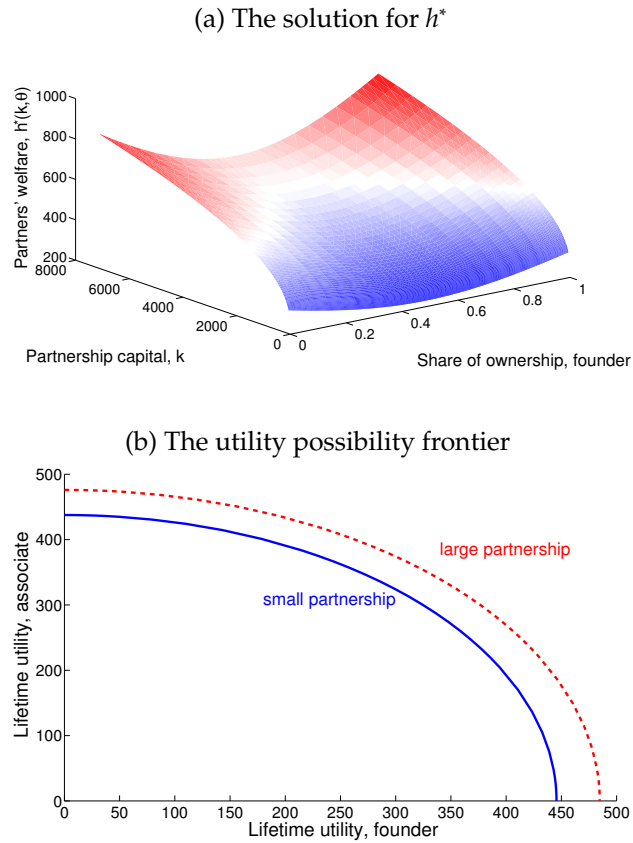
The results of applying the method described above are now illustrated in a numerical example.²⁴ Figure 2a shows the solution for function h^* . It has the properties described in the theoretical characterization. The function h^* is increasing and concave in the size of the partnership. Additionally, this function is also convex in the founder's share of ownership in the partnership, θ . There is an alternative way of displaying this function: utility possibility frontiers. They display the combination of lifetime utility levels that the founder and the associate can achieve given the

²³Proof available upon request.

²⁴For this exercise, we assume that the utility function is $\frac{\epsilon(1-\sigma)}{1-\sigma}$, the profit function is k^α , and the shocks are $s(L) = 1 - \epsilon$ and $s(H) = 1 + \epsilon$. The parameter $\sigma = 0.5$. This value is lower than what it is usual in macroeconomics, but it is in line with other studies of private information such as those by Hopenhayn and Nicolini [1997] and Pavoni [2007]. Setting $\sigma = 1/2$ is useful because $u(0)$ is finite and the results in previous sections assume the utility function is bounded. The parameter $\alpha = 0.7$ is in the range of estimations of Cooper and Ejarque [2003]. We set $\delta = 0.07$ and $\beta = 0.97$ as is standard in the literature and $\epsilon = 0.2$ simply for illustrative proposes.

current size of the partnership. The solid blue line in Figure 2b corresponds to a small partnership, while the dashed red line corresponds to a large partnership. As expected, the frontier is concave and increasing in k .

Figure 2: Results of applying the new recursive approach



3 Design of partnerships under full information

This section provides the main insights about the design of partnerships in the benchmark case in which the observability of liquidity shocks is perfect (i.e. there is full information, FI hereafter). Both agent 1 (hereafter the founder) and agent 2 (the associate) face liquidity shocks.²⁵ In what follows, we refer to (θ_t, k_t) as the ownership structure and the size of the partnership at date t , respectively. Since $\theta_1 = 1 - \theta_2$, to simplify notation hereafter we refer to the founder's ownership share of the partnership directly as θ .

The full information case reduces to the problem of a sole proprietorship with an investor with preferences featuring peculiar shocks to liquidity needs. That is, this shock will generally be a weighted average of the liquidity shocks of both partners.

To see this more clearly assume that the partners have constant relative risk aversion (CRRA, hereafter) preferences with relative risk aversion coefficient σ . The necessary and sufficient first-order conditions of the full information problem imply

$$c_1(s) = \frac{(\theta s_1)^{1/\sigma}}{(\theta s_1)^{1/\sigma} + ((1 - \theta) s_2)^{1/\sigma}} (f(k) + k(1 - \delta) - k'(s)), \quad (10)$$

$$c_2(s) = \frac{((1 - \theta) s_2)^{1/\sigma}}{(\theta s_1)^{1/\sigma} + ((1 - \theta) s_2)^{1/\sigma}} (f(k) + k(1 - \delta) - k'(s)), \quad (11)$$

where $(f(k) + k(1 - \delta) - k'(s))$ is the total amount of resources available to be paid to the partners. Importantly, first-order conditions with respect to w' imply that

$$\theta'(s) = \theta \quad (12)$$

for all $s = (s_1, s_2)$. This means that efficiency dictates that the ownership shares of the partnership are kept constant. Indeed, this result holds more generally for full information plans and, in particular, it does not depend on CRRA preferences. The underlying intuition for keeping these ownership shares optimally constant can be grasped as follows. The planner's problem is an artificial device to characterize a particular set of plans in which ownership shares, $(\theta, 1 - \theta)$, are the ex-ante planner's valuation of delivering one more unit of expected discounted utility to each partner. Thus, the ex-ante relative valuation is $\theta/(1 - \theta)$. Similarly, $(\theta'(s), 1 - \theta'(s))$ denotes the valuation of delivering one more unit of expected discounted utility to the partners next period

²⁵Importantly, the associate is risk averse and has no access to unlimited resources. These two features are critical for the results; see Section 4.3 for a discussion.

if today's shock is s . Consider now the relative valuation of delivering one unit of expected discounted utility next period after a realization of the shock s . In the full information case, this is given by the ratio $\frac{\theta'(s)}{(1-\theta'(s))} = \frac{\theta\beta\pi(s)}{(1-\theta)\beta\pi(s)}$ and thus the relative valuation remains unchanged since agents discount the future equally. On the other hand, when there is private information future expected discounted utility plays an additional role since it provides incentives for truthful reporting. Its relative valuation can vary as time and uncertainty unfold.

Observe that (10)-(12) imply that the problem reduces to choosing total payouts and investment in a sole proprietorship with an "aggregate" investor with preferences at date t ,

$$S(\theta, s_t) \frac{(C_t)^{1-\sigma}}{1-\sigma},$$

where $C_t = c_{1,t} + c_{2,t}$ denotes total payouts and

$$S(\theta, s_t) = \left((\theta s_{1,t})^{1/\sigma} + ((1-\theta) s_{2,t})^{1/\sigma} \right)^\sigma$$

is an "aggregate" liquidity shock. An important observation is that this shock depends on the ownership structure through θ . It follows by standard arguments that investment is decreasing in S_t . So the fraction of the increase in each partner's payout that is financed through disinvestment is increasing in his ownership share. This is a key aspect in the analysis below.

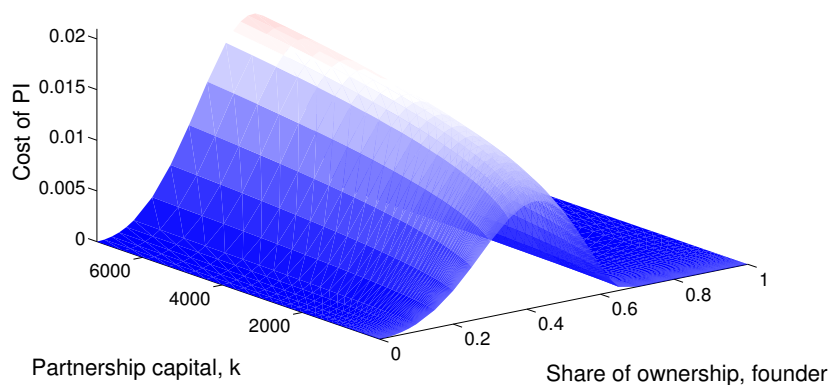
4 Design of partnerships under private information: founder only with liquidity shocks

This section studies the problem of designing an efficient partnership between a risk-averse founder, who has a project and varying liquidity needs, and a risk-averse associate, who makes a capital contribution to start the firm. Many economic insights in this section carry over to the symmetric case in which both partners face privately observed liquidity shocks.

Figure 3 illustrates the cost of private information relative to full information.²⁶ Notice that this cost initially increases with the founder's ownership share until it reaches a maximum and it becomes zero when his share is sufficiently large. This is discussed in more detail below.

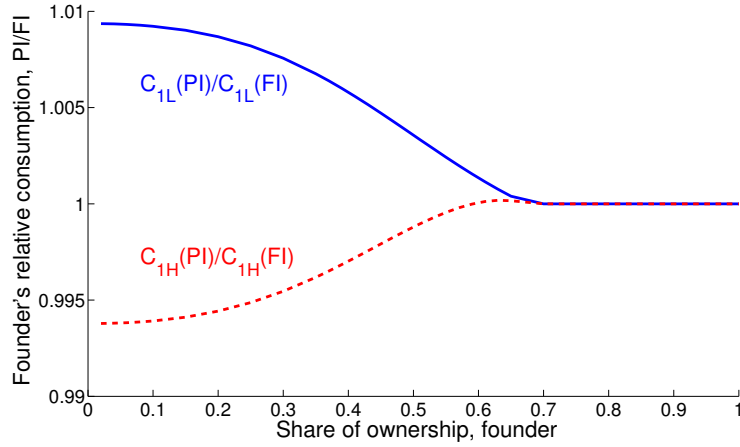
²⁶Cost of PI is defined as $\gamma(k, \theta) = \left[\left(\frac{h^*(k, \theta)}{h^*(k, \theta)} \right)^{\frac{1}{1-\sigma}} - 1 \right] 100$.

Figure 3: The cost of private information



To analyze how private information (PI, hereafter) shapes the payouts, we focus on Figure 4. It plots the founder's payout under private information, relative to his payout under full information, as a function of his ownership share of the partnership. First, consider the solid blue line: the founder's payout when he reports a low liquidity shock. Notice that this line is above 1 for low values of the founder's ownership share of the partnership and is equal to 1 for shares larger than 0.7, when the founder's ownership share reaches a threshold. The fact that the ratio is greater than 1 should be interpreted together with the payout of the founder after he reports a high liquidity shock (dashed red line). This ratio is smaller than 1 for low values of the founder's ownership share of the partnership and equal to 1 for shares larger than 0.6. Together, these patterns show that under private information the founder's payout does not react to liquidity needs as much as under full information. This occurs because, to make his report compatible with incentives, the founder receives a higher payout when his report is low and a lower payout when it is high, compared with the full information case. Hence, under private information, the founder cannot fully insure his liquidity needs.

Figure 4: Relative consumption: PI vs. FI



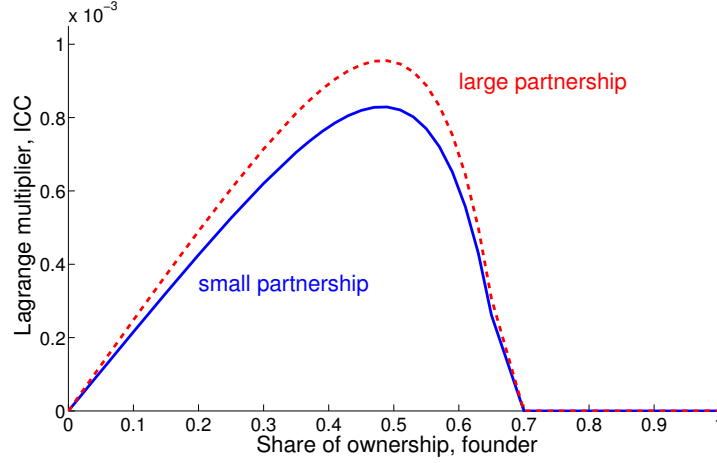
4.1 Too big to cheat

We argue that the best arrangement between partners under full information and under private information coincides under a certain ownership structure. For a given size of firm k , the following result establishes that the founder's incentive to misreport vanishes as his share of the partnership becomes big enough.

Proposition 3 (Too big to cheat) *For each partnership size k , there exists some value of the founder's share of ownership $\bar{\theta}(k) \in (0, 1)$ such that for all (θ, k) with $\theta \in [\bar{\theta}(k), 1]$ the full information plan satisfies the incentive compatibility constraints.*

Figure 5 shows the Lagrange multiplier of the incentive compatibility constraint as a function of the founder's ownership share of the partnership. As his share starts increasing from 0, the Lagrange multiplier of the incentive compatibility constraint also increases, indicating that relaxing this constraint would increase both partners' welfare, h^* . However, when the value of the founder's ownership share is around 0.47, the multiplier peaks and then starts to decrease, reaching 0 when his share is around 0.7. Thereafter, relaxing the incentive compatibility constraint does not increase the partners' welfare: *Private information does not matter*.

Figure 5: Too big to cheat



Note: ICC, incentive compatibility constraint.

The underlying intuition for this result can be grasped as follows. As mentioned previously, as the founder reports high liquidity needs, he receives more funds. These resources come from two sources: (i) decreasing the funds received by the associate and (ii) reducing investment or disinvestment. If the founder's ownership share is large enough, most of the extra funds he receives are financed by disinvestment. Two main reasons explain this finding. First, there is not much left to redistribute from the associate as his payout approaches zero. And second, the associate's allowance is small; therefore, the cost of taking away an extra unit of consumption is large.²⁷ Since disinvestment implies that misreporting can heavily decrease the future utility of the founder himself, as the founder's participation in the partnership passes a threshold, cheating becomes undesirable.

To illustrate this feature, consider the increment of the founder's payout as he reports high liquidity needs compared with the case in which he reports low needs. Define the share of this increment that is financed by means disinvestment as

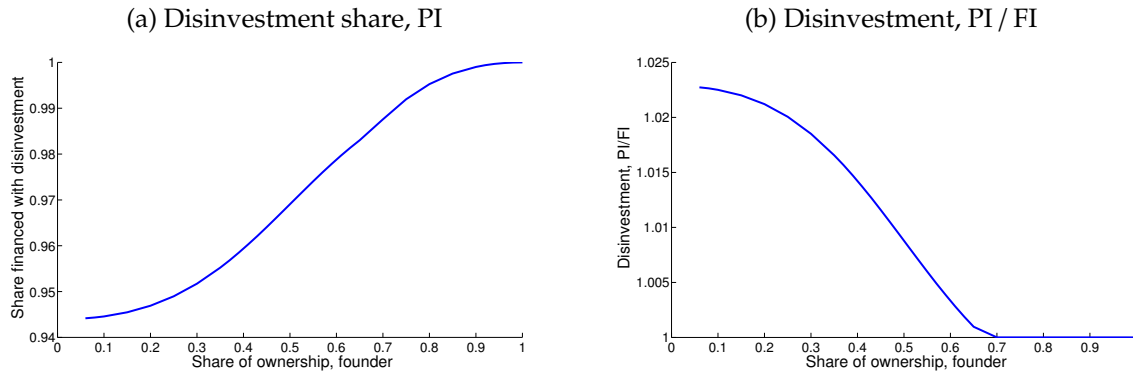
$$\text{Disinvestment share} = \frac{k'_L - k'_H}{c_{1H} - c_{1L}}; \quad (13)$$

²⁷If we assume that the partner's utility function has constant absolute risk aversion (CARA) instead of CRRA, this second mechanism does not operate but the result still hold because the first mechanism operates. In that case, there exists a threshold value of the founder's ownership share such that for larger shares the payout of the partner hits the zero lower bound and all of the extra payout received by the founder is financed with disinvestment. Thus, the role of the first mechanism is exacerbated in this case.

that is, the fraction of the higher payout financed by investing less. Figure 6a shows the disinvestment share defined in (13) and illustrates that it is increasing in the founder's ownership share. Notice that the founder's incentives for misreporting stem from the fact that he can take resources from the associate. Thus, the reduction in the future size of the partnership after a report of a high shock provides an incentive for truthful revelation since this lowers the founder's payouts from tomorrow onward.

Now, if disinvestment is useful to provide incentives, we should see more disinvestment under private information. The share financed with disinvestment under private information is actually higher than under full information (Figure 6b). This implies that investment is distorted, and thus the size of the partnership is altered, by the presence of private information. Every time the founder reports a high shock, investment decreases more under private information than under full information because this helps to provide incentives for truthful revelation.²⁸

Figure 6: The role of disinvestment



Now we move a step forward to argue that private information does not matter under a certain ownership structure of the partnership. That is, we show that one can always find a subset of ownership shares such that the investment and distribution plans under private information actually coincide with the full information plans.

²⁸This is the key difference with the work of [Marcet and Marimon \[1992\]](#), as discussed below.

Consider the full information policy function for capital accumulation $k'_{FI}(\theta, k)(s_L)$ and $k'_{FI}(\theta, k)(s_H)$ and define the bounds of sustainable levels of capital as

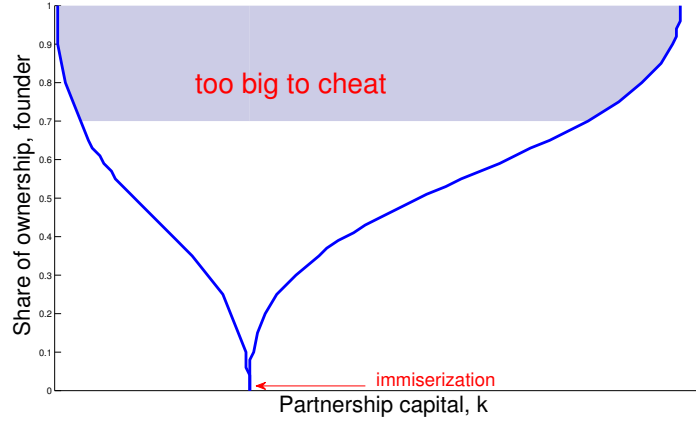
$$\begin{aligned} k'_{FI}(\theta, k_{\min}(\theta))(s_H) &= k_{\min}(\theta), \\ k'_{FI}(\theta, k_{\max}(\theta))(s_L) &= k_{\max}(\theta). \end{aligned}$$

Since policy functions are continuous, the bounds $k_{\min}(\theta)$ and $k_{\max}(\theta)$ are both continuous functions with respect to θ . Let $\Gamma \equiv \{(\theta, k) : \theta \in [0, 1], k \in [k_{\min}(\theta), k_{\max}(\theta)]\}$, and define $\theta^* \equiv \sup\{\bar{\theta}(k) : (\bar{\theta}(k), k) \in \Gamma\} \in (0, 1)$.

Proposition 4 (Too big to cheat, forever) *Suppose that $(\theta_t, k_t) \in [\theta^*, 1] \times [k_{\min}(\theta_t), k_{\max}(\theta_t)]$ at some t . Then the efficient and the full information plans coincide, and $\theta_{t+n} = \theta_t$ for all $n \geq 0$.*

This is our main result, which argues that the ownership structure remains unchanged and private information becomes irrelevant once the ownership structure and the size of the partnership reach a region in which (i) the founder's ownership share becomes big enough and (ii) the size of the partnership converges to an interval with bounds determined in the full information case. Figure 7 characterizes the ergodic set in the space of the state variables: the size of the partnership and the founder's ownership share. Once the state variables take values in this region, they stay there forever. There are actually two regions. The clearest is the shaded region with the founder's ownership share of the partnership between 0.7 and 1: the too-big-to-cheat-region. The founder's share remains constant anywhere within that region but the capital stock that defines the size of the partnership fluctuates between the boundaries. The other region contains exactly one point in the state space: where the founder's ownership share of the partnership is 0. In this second region, the capital stock—and thus the size of the partnership—is constant because only the founder has a liquidity shock but his ownership share is 0.

Figure 7: Ergodic set



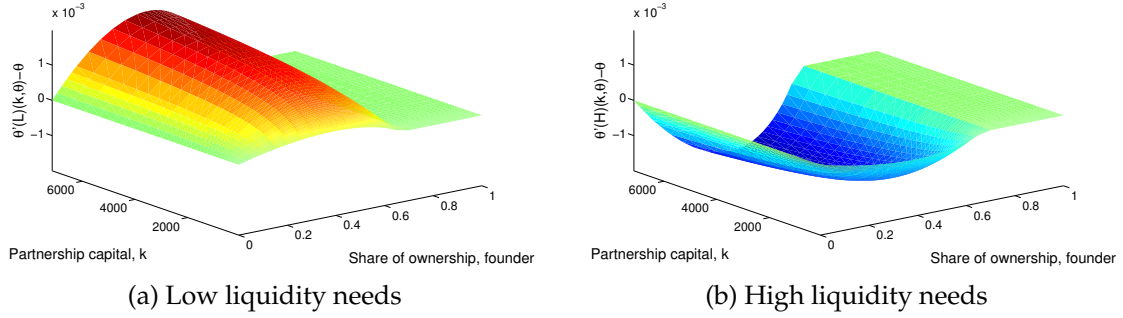
The too-big-to-cheat region (defined in Proposition 4) can be reached either immediately (as the initial ownership structure and the initial size of the partnership starts there) or in the long run (as the ownership structure and the size of the partnership converge as time and uncertainty unfold). We discuss this latter possibility in what follows.

We have assumed that the liquidity shock can take only two values. Before we move to study dynamics and long-run convergence, it is useful to briefly discuss the case with continuous state space for the liquidity need shocks. The striking fact is the impact of allowing for investment. With no investment possibilities, i.e. a partnership with exogenously given output, if the partners are offered the full information plan, any partner would always have incentive to report the highest liquidity shock as consumption is increasing in the report, independently of his share of ownership in the partnership. At the individual level, there are no incentives to reporting something smaller since the increment in payout would always come from trying to grab some consumption from the other partners. In contrast, in the economy with investment, as the founder ownership share increases, his desired report approaches his realized shock because the part of his increment in consumption that is financed with disinvestment is increasing in his size. Actually, for $\theta = 1$, the full information plan is incentive compatible as the incentive compatibility constraint would be slack (with strict inequality) for any two different liquidity shocks as uniqueness of the maximum dictates. The distinction with the discrete case is that the difference between telling the truth and misreporting for any two values of s is not bounded, in the sense that the levels of utility approach to each other as the values of the shocks approach to each other.

4.2 Dynamics and long-run convergence

We now consider the evolution of the ownership structure of the partnership. Figures 8a and 8b display changes in the founder's ownership share as a function of capital and his current ownership share. In particular, the figures show that if the founder's ownership share is not too big, his share next period will be larger (smaller) if he reports low (high) liquidity needs. In the too-big-to-cheat region, ownership shares are constant because the plan coincides with the full information plan.

Figure 8: Evolution of ownership shares



To analyze the long-run convergence of ownership shares, let S^∞ be the set of infinite sequences of liquidity shocks, with typical element $s^\infty = \{s_0, \dots, s_t, \dots\}$, where $\mathcal{B}(S^\infty)$ are the corresponding Borel sets. Let $\{\theta_t\}_{t=0}^\infty$ be the stochastic process for ownership shares generated by the set of policy functions solving (9). That is, $\theta_t : S^\infty \rightarrow [0, 1]$, where $\theta_t(s^\infty)$ denotes a particular realization at date t . It will be shown that the stochastic process of the ratio of ownership shares is a nonnegative martingale.

Proposition 5 (Ownership share dynamics) *The ratio of ownership shares satisfies the following properties:*

1. It is a nonnegative martingale; i.e. for all t and all s^t ,

$$\mathbb{E} \left[\frac{\theta_{t+1}}{(1 - \theta_{t+1})} \parallel s^t \right] = \frac{\theta_t(s^\infty)}{(1 - \theta_t(s^\infty))} \quad s^\infty - a.s.$$

2. It follows by the martingale convergence theorem that

$$\frac{\theta_t(s^\infty)}{(1 - \theta_t(s^\infty))} \rightarrow \frac{\widehat{\theta}(s^\infty)}{(1 - \widehat{\theta}(s^\infty))} \quad s^\infty - a.s.$$

for some random variable $\widehat{\theta}$ on $(S^\infty, \mathcal{B}(S^\infty))$.

The next result shows that private information does not matter in the long run; that is, the limiting allocation is incentive compatible and so coincides with the full information allocation.

Proposition 6 (Long-run convergence to full information) *The partnership's ownership structure and size, (θ_t, k_t) , reach the region in which the constrained efficient and the full information plans coincide with probability 1 in which $\theta_t \rightarrow \{0, [\theta^*, 1]\}$ a.s.*

This proposition shows that incentives to misreport vanish in the long run. This occurs because in the long run either the associate owns the entire partnership or the founder's ownership share is sufficiently large. In the former case, private information does not matter because there is only one active partner. That case actually resembles the immiserization result found in other settings with private information. In the latter case, the founder's share is sufficiently large that the costs of lying that he internalizes, as opposed to the total costs to the partnership, dominate the benefits of misreporting.

The initial distribution of shares is key to determining which of these two possibilities will be the long-run outcome. For instance, if the initial shares of the founder are big enough, implying $\theta_0 \geq \theta^*$, the best arrangement under private information coincides with the one of full information from the beginning of the partnership formation. On the other hand, if the initial shares of the founder are not big enough (i.e. $\theta_0 < \theta^*$), the long-run ownership structure could reach both outcomes with positive probability. Which outcome is realized depends on the realization of shocks and the initial conditions. To characterize the role of these factors, we analyze the convergence of 1,000 simulations for different starting conditions. Table 2 shows that the initial ownership structure matters for the ownership structure in the long run, but the initial size does not. When the founder's initial share of ownership is small ($\theta = 0.2$), the partnership converges to the too-big-to-cheat region 11% of the time if either the initial partnership size is small or large. In contrast, if the founder's initial share is larger ($\theta = 0.4$), the partnership structure converges to the too-big-to-cheat region more frequently (30% of the times) for either small or large initial partnership size.

Table 2: Long-run convergence

	Low θ , low k			High θ , low k			Low θ , high k			High θ , high k		
	Full Info	Private Info TBTC	Private Info IM	Full Info	Private Info TBTC	Private Info IM	Full Info	Private Info TBTC	Private Info IM	Full Info	Private Info TBTC	Private Info IM
Mean(k)*	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Coef. var.(k)	0.00	0.07	0.00	0.03	0.07	0.00	0.00	0.07	0.00	0.03	0.07	0.00
Mean(θ)	0.20	0.69	0.00	0.40	0.69	0.00	0.20	0.69	0.00	0.40	0.69	0.00
Obs, %	100	11	89	100	30	70	100	11	89	100	30	70

Note: TBTC, too big too cheat; IM, immiserization. Simulations from 1,000 paths.

(*) Normalized by the mean(k) in the full information case.

4.3 Discussion of key assumptions

This section discusses the role of the main assumptions in our analysis above.

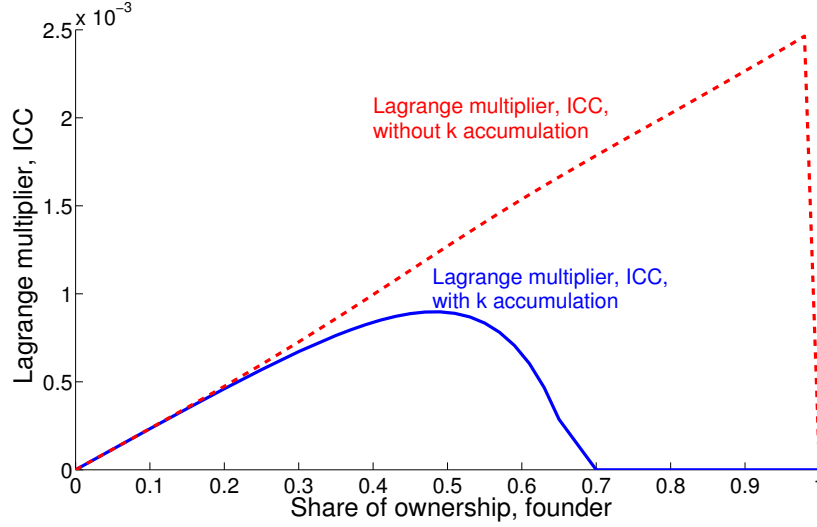
4.3.1 The role of investment

What role do investment and capital accumulation play? To answer this question, we study the same problem as above but assume that the resources available cannot be affected by investment and thus are given every period. Thus, this framework resembles that of [Atkeson and Lucas \[1992\]](#) but with only two agents.²⁹

Figure 9 shows the value of the Lagrange multiplier of the incentive compatibility constraint in both cases. The solid blue line is the Lagrange multiplier in the case with capital accumulation. As shown in Figure 5, the multiplier starts to decrease when the founder's ownership share of the partnership increases and eventually reaches 0. This means that the full information allocation is actually implementable under private information. The dashed red line represents the value of the Lagrange multiplier of the incentive compatibility constraint in the case without capital accumulation. Remarkably, it is increasing in the founder's ownership share of the partnership in the interval $[0, 1)$ and drops discontinuously to 0 as this share is 1. This illustrates our argument that investment plays a key role as an additional instrument to provide incentives.

²⁹To facilitate the comparison, the level of resources fixed in the case without capital accumulation is set at the output produced at the mean of the steady-state level of capital in the case with capital accumulation.

Figure 9: Lagrange multiplier of the incentive compatibility constraint



Note: ICC, incentive compatibility constraint.

4.3.2 Risk neutrality

Here we discuss the role of risk aversion and internal financing, as opposed to the assumption of risk neutrality. This is important to contrast our results with the work of [Marcet and Marimon \[1992\]](#), who find that private information does not distort optimal investment.

Suppose that the associate is risk neutral. Notice that this is the case if the associate has both linear preferences and unlimited access to funds (i.e. deep pockets).³⁰ In this case, it is simple to show that, as a direct consequence of evaluating the investment decision with the intertemporal marginal rate of substitution of the risk-neutral agent, the first-order condition that characterizes optimal investment is

$$1 = \beta (f'(k^*) + (1 - \delta)). \quad (14)$$

This implies that the the size of the partnership, k , defines the size of the partnership will jump directly from k_0 to k^* to stay at that level forever. Consequently, since one partner is risk neutral

³⁰The latter implies that the associate can have negative consumption.

in this setting, the stationary size of the partnership under private information coincides with the size under full information. Therefore, the reports of the partner with private information do not distort the optimal investment plan.

Why has the too-big-to-cheat result disappeared? With a risk-neutral agent with deep pockets, the Euler equation described above implies that the size of the partnership, k , is constant. Therefore, this case resembles the case without investment in which output is fixed at $f(k^*) - \delta k^*$. All the reasonings mentioned for that case also apply here.

Now, why is capital accumulation not used to provide incentives? As in the full information case with a risk-neutral associate, output is independent of liquidity shocks and thus it is optimal to finance all changes in the founder's payout by redistribution from the risk-neutral associate. With private information, the best arrangement also needs to provide incentives to report truthfully. This is optimally achieved only by redistribution since manipulating the associate's payout does not imply extra costs as a consequence of risk neutrality.

5 Design of partnerships under private information: founder and associate with liquidity shocks

Here we extend the analysis to the case in which both partners face liquidity shocks. In order to do that, we assume that the value of the shocks are $s_L = 1 - \epsilon$ and $s_H = 1 + \epsilon$. As before, these values occur at any t with probabilities $\pi(s_L)$ and $\pi(s_H)$, respectively.

The physical characteristics of a partnership considered above depend on several parameters. The following proposition analyzes the role of one of those parameters. In particular, it characterizes the size of the liquidity needs shocks that make it possible carry over the results in the previous section.³¹ We say that a plan is *strictly incentive compatible* if it satisfies the corresponding incentive compatibility constraints with strict inequality.

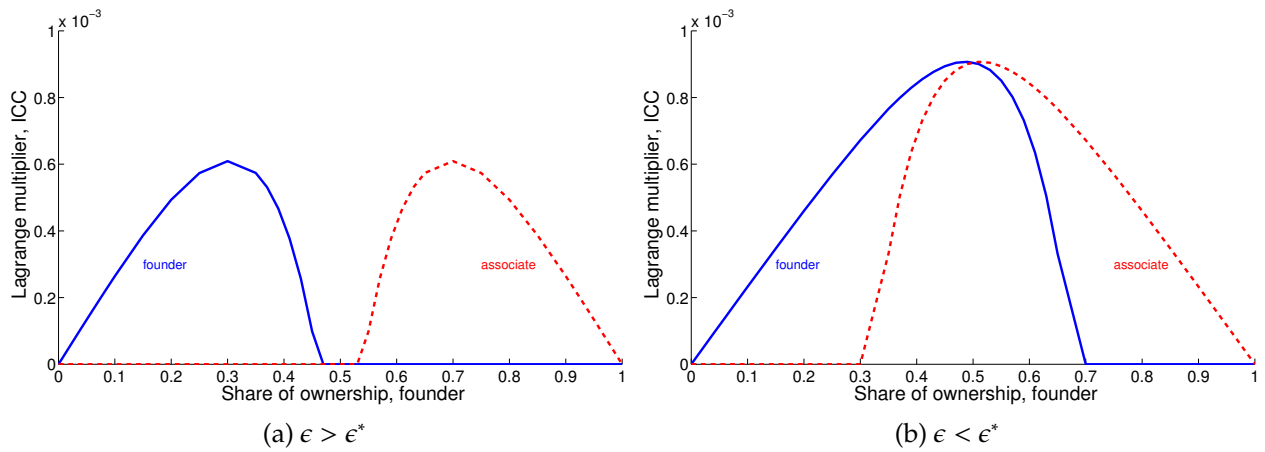
Proposition 7 *For any partnership, there exists some $\epsilon^* \in (0, 1)$ such that for all $\epsilon \in (\epsilon^*, 1)$ the full information plan is strictly incentive compatible for both agents at $\theta = 1/2$ for all k .*

The two cases identified by Proposition 7 are illustrated in Figures 10a and 10b. They show the value of the Lagrange multipliers associated with the incentive compatibility constraint of both

³¹Of course, a similar analysis could be performed with any of the other parameters that define the characteristics of the partnership.

agents as a function of the founder's ownership share (for a given k).³² In both cases, the Lagrange multipliers for both partners are initially increasing in their own shares, reach a peak, and then they decrease until they reach zero. By symmetry, they mirror each other and so they equalize when ownership shares are equally distributed. The key difference between these two cases is that in Figure 10a there exists a region around $\theta = 1/2$ in which they are both equal to zero.

Figure 10: Too big to cheat



Note: ICC, incentive compatibility constraint.

In Figure 10b, at least one incentive compatibility constraint is binding for all $\theta \in (0, 1)$. As a consequence, ownership shares fluctuate permanently and investment is distorted to provide incentives. In the long run, the partnership will be owned by one of the partners. Notice that this case did not exist in the previous section, in which only the founder had liquidity shocks. The key for its existence is that as more partners (with privately observed shocks) are added to the partnership, it is more difficult that all of them have ownership shares sufficiently large to internalize the impact of misreporting. This is one key message of our analysis: the costs imposed by private information are increasing in the number of members.

Hereafter, we focus on the case described in Figure 10a, for which the best arrangements between partners under full information and under private information coincide under a certain ownership structure. The following result establishes that if both partners (strictly) prefer truthful

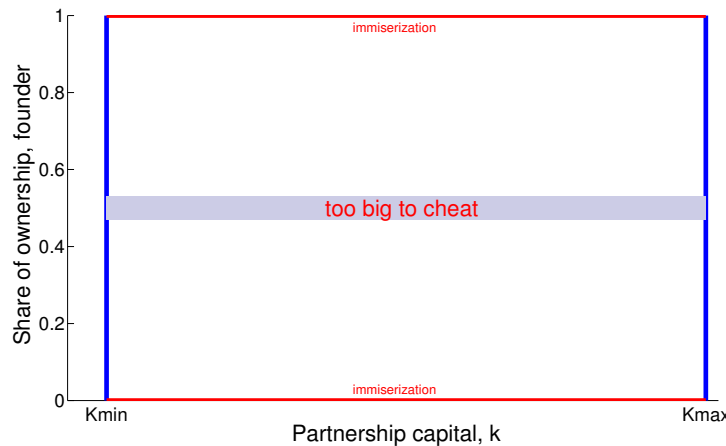
³²For expositional purposes we chose $\epsilon = 0.6 > \epsilon^*$ in Figure 10a and $\epsilon = 0.2 < \epsilon^*$ in Figure 10b. The value $\epsilon = 0.6$ is also used in 11 and 12.

reporting when the ownership structure is equally distributed, then there exists a region around $\theta = 1/2$ in which private information does not matter. In addition, if that region is ever reached, the partnership structure does not change.

Corollary 1 *Suppose that the full information plan is strictly incentive compatible at $\theta = 1/2$ for all k . Then, there exists $\theta^* \in (0, 1/2)$ such that if $(\theta_t, k_t) \in [\theta^*, 1 - \theta^*] \times [k_{\min}(\theta_t), k_{\max}(\theta_t)]$ at some t , then the efficient and the full information plans coincide and $\theta_{t+n} = \theta_t$ for all $n \geq 0$.*

Figure 11 displays the ergodic set. It includes the extreme cases of sole proprietorship, $\theta = 0$ and $\theta = 1$, and a region around equally distributed ownership shares described in the previous corollary.

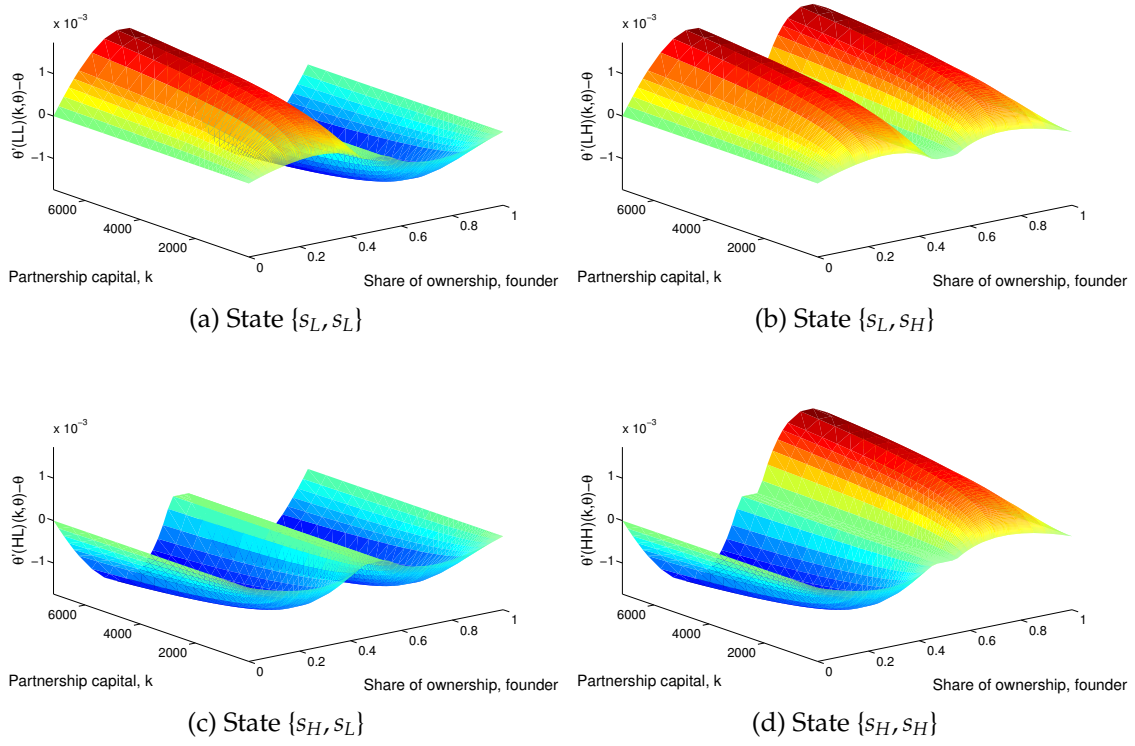
Figure 11: Ergodic set



Now, we consider the evolution of the ownership structure of the partnership. Figures 12a-12d show how the founder's ownership share changes as a function of capital and his current ownership share. Each panel refers to different combinations of liquidity shocks for both partners. For instance, Figure 12a illustrates how the founder's ownership share varies when both he and his partner report low liquidity needs. When the founder's participation in the partnership is relatively small, his share increases to induce him to report truthfully. On the other hand, as the founder's share become sufficiently large, efficiency dictates that the ownership structure must vary to provide incentives to the partner. This accounts for the decline in the founder's share (i.e. his partner's share increases). In the intermediate case, when the ownership structure is close to

equally distributed, both partners' incentives to misreport vanish, so their ownership shares do not vary.

Figure 12: Evolution of ownership shares



The next corollary extends Proposition 6 to the case in which both partners face liquidity shocks. It argues that under certain conditions private information does not matter in the long run. In the long run, the partnership converges to either a sole proprietorship or a 2-owner partnership in which the ownership structure in which shares are sufficiently equally distributed.

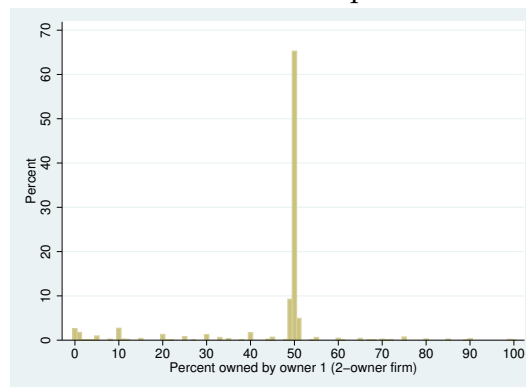
Corollary 2 Suppose that the full information plan is strictly incentive compatible at $\theta = 1/2$ for all k and let θ^* be defined as in Corollary 1. Then, the partnership's ownership structure and size, (θ_t, k_t) , reach the region in which the efficient plan and the full information plan coincide with probability 1, where $\theta_t \rightarrow \{0, [\theta^*, 1 - \theta^*], 1\}$ a.s.

6 Evidence on the dynamics of ownership shares

In this section, we provide evidence on the dynamics of ownership shares from the Kauffman Firm Survey.³³ This survey collects nation-wide data on start-ups and follow them during the next four years. The 4,928 businesses included in the Kauffman Firm Survey are nationally representative of start-ups from 2004. Importantly, this panel contains information about the ownership share of each partner for each year.

Figures 13 and 14 display the distribution of ownership shares in partnerships with 2 and 3 owners, respectively. Notice that more than 60% of the 2-owner partnerships and more than 30% of the 3-owner partnerships have equally distributed ownership shares.³⁴

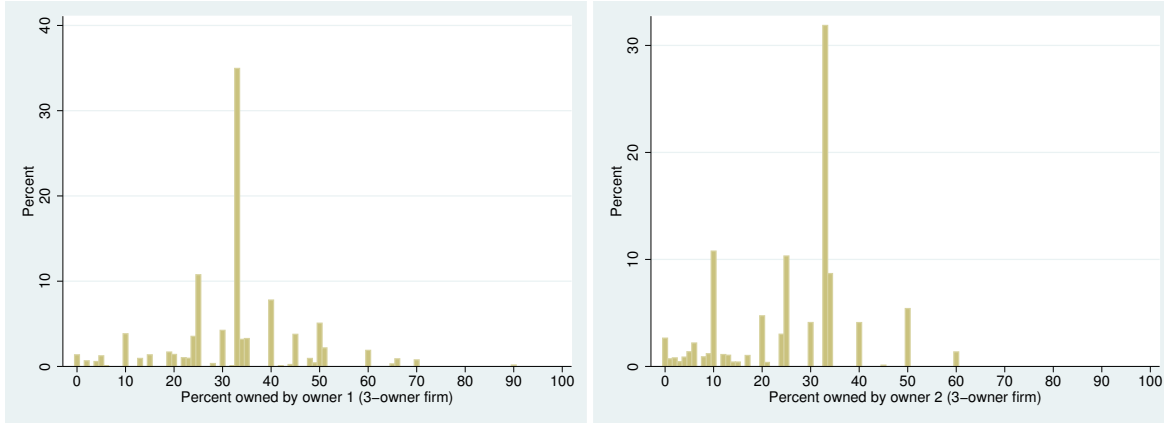
Figure 13: Distribution of ownership shares, 2-owner firms



³³Similar results were obtained using the Panel Study of Entrepreneurial Dynamics (PSED).

³⁴A similar pattern is documented by Vereshchagina [2013].

Figure 14: Distribution of ownership shares, 3-owner firms

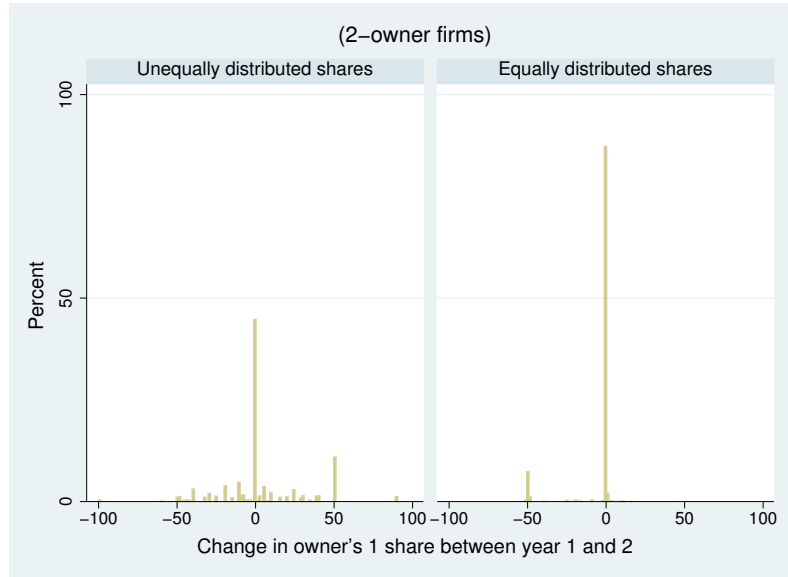


This pattern is interesting because our theory predicts that the incidence of private information tends to be minimized with those ownership structures. If that is actually the reason for the high occurrence of equally distributed ownership structures, we should observe, according to our theory, different ownership dynamics for those partnerships. Recall that private information induces fluctuations in ownership shares to discourage partners from misreporting liquidity needs. But the impact of private information depends on the ownership structure itself. In particular, our theory predicts that fluctuations in ownership shares should be observed more frequently in partnerships with ownership structures that are more unequally distributed. Since the observed fluctuations in ownership shares may also occur for reasons not included in our model, our emphasis is not on changes per se but on the difference between changes in ownership shares among partnerships with different ownership structure.

We computed the changes in shares of one of the owners of the partnerships between the first and second years. Figure 15 shows the histogram of those changes for firms with unequally and equally distributed ownership structures.³⁵ The key finding is that the fraction of partnerships that do not change their ownership structure when shares are equally distributed is almost twice as large as the fraction of partnerships that do not change their ownership structure when shares are unequally distributed (43 vs. 82%). This finding is line with one of the main predictions of the model.

³⁵ A I -owner partnership has an ownership structure that is *equally distributed* if the share of each of its members is between $0.95 \times (1/I)$ and $1.05 \times (1/I)$. The findings, however, are robust to tighter or looser bounds.

Figure 15: Change in ownership shares



One possibility is that there are observable differences between these two groups of partnerships that may account for the dissimilarity in the dynamics of the ownership structure. In order to study the robustness of this finding, we compute the percent of partnerships with unchanged ownership structure between years t and $t + 1$ conditional on different characteristics of the partnership. The results are shown in Table 3. The rows show the alternative groups of partnerships. The columns vary by the age of the partnership after the change in shares. The robustness of the difference between the dynamics of ownership among partnerships with equally and unequally distributed ownership structure is striking. In particular, the findings are robust across partnerships with 1, 2, and 3 years old, 2 and 3 owners, different legal status, profitable and non-profitable, growing and shrinking in terms of employment, and with more and less than 5 workers.

Table 3: Percent of partnerships with unchanged ownership structure

	Age 2		Age 3		Age 4	
	Equal	Unequal	Equal	Unequal	Equal	Unequal
All	87.9	38.8	90.0	45.1	94.7	53.3
2 owners	89.2	43.7	92.5	53.2	96.6	56.3
3 owners	96.1	62.7	95.6	65.3	92.5	70.8
General or limited partnership	88.6	20.5	91.7	36.2	96.7	49.9
Other legal status	87.8	44.8	89.7	52.8	94.4	60.1
Profitable	86.7	46.4	90.5	57.1	97.1	60.2
Nonprofitable	89.2	40.3	89.8	46.5	91.1	59.1
Increased in employment	87.7	48.6	92.6	46.9	96.6	59.9
Decreased employment	88.1	34.2	89.8	45.3	94.5	51.1
No. of employees $\leq$ 5	87.0	39.1	90.1	47.4	94.6	60.0
No. of employees $>$ 5	91.7	38.3	89.6	41.8	94.9	42.5

To summarize, we provide evidence that the current ownership structure is an important determinant of the frequency of changes in ownership structure. Our theory provides a rationale for this finding as one of its main predictions is that if all owners have a sufficiently large share in the partnership, which is more likely if shares are approximately equally distributed, fluctuations in ownerships shares are not required to discourage cheating.

7 Conclusion

This paper studies a venture with two partners who share the stock of capital and must allocate resources to payouts for each partner and investment. Efficiency dictates that the ownership structure and its dynamics are fundamental to lessen the difficulties of private information.

We show that if the ownership share of a partner becomes large enough, his incentives to cheat vanish. This is a consequence of the fact that a fraction of the increase in his payout after reporting a high liquidity shock is financed by disinvesting in the partnership. Thus, allowing for capital accumulation is ultimately crucial. When at least one of the partners' ownership share is not big enough, the occasionally binding incentive compatibility constraint makes that partner's payout increase less under private information than under full information after a high liquidity shock. More importantly, the history dependence caused by private information makes the ownership structure of the partnership vary over time.

Overall, the analysis suggests that capital accumulation, the number of partners, and the ownership structure in a venture are important for determining the extent to which private information matters. When there are fewer partners with large ownership shares in the partnership, they partially internalize the cost of misreporting liquidity needs in terms of distorting capital accumulation, which helps in overcoming the problem of private information. This mechanism tends to reduce the number of members in partnerships. Actually, if the number of initial members is $n > 2$, it would be unlikely that all members are too big to cheat just because $1/n$ may be too small. Although we did not analyze this case formally, we believe that over time those partnerships will converge to either a sole proprietorship or a partnership with 2 or at most 3 partners with approximately equally distributed ownership shares.

Our analysis can be applied to different settings. For instance, the partnership could be reinterpreted as an economic union among several countries. Then, the size of the countries (in terms of how much wealth they have relative to the union) would be important to determine the extent to which misreporting must be prevented by the union's structure and regulations. In an economic union between a large and a small country, our results suggest that the small country would have incentives to misreport if the union regulations are not carefully designed. Moreover, our theory predicts that adding more countries to the union—and thereby reducing the share of the union of each member—would exacerbate information problems.

8 Appendix

This appendix provides all the proofs of our results in the main text.

8.1 Recursive formulation

In this section we provide the proofs for Propositions 1 and 2 in Section 2.2. Our analysis here generalizes to I partners with privately observed shocks to liquidity needs since our alternative recursive approach does not depend on our 2-partner assumption.

Let $\Delta^I \equiv \{\theta \in \mathbb{R}_+^I : \sum_{i=1}^I \theta_i = 1\}$ and $\|h\| = \sup_{(k,\theta)} \{ |h(k, \theta)| : \theta \in \Delta^I \}$ and define

$$F \equiv \{h : X \times \mathbb{R}_+^I \rightarrow \mathbb{R}_+ : h \text{ is continuous and } \|h\| < \infty\},$$

as the set of continuous and bounded functions mapping $X \times \mathbb{R}_+^I$ into $\mathbb{R}_+$, while denoting the subset of functions

$$\bar{F} \equiv \{h \in F : f \text{ is HOD 1 and concave in } k\}.$$

Consider the metric induced by $\|\cdot\|$ and observe that $(\bar{F}, \|\cdot\|)$ is a closed subset of the Banach space $(F, \|\cdot\|)$ and thus a Banach space itself.

Consider the operator T defined on $\bar{F}$ defined as follows:

$$(Th)(k, \theta) = \sup_{(c, w', k')} \sum_{i=1}^I \theta_i \left\{ \sum_s \pi(s) [s_i u(c_i(s)) + \beta w'_i(s)] \right\}, \quad (15)$$

subject to

$$k'(s) + \sum_{i=1}^I c_i(s) = f(k) + (1 - \delta)k \quad (16)$$

$$\begin{aligned} & \sum_{s-i} \pi(s_{-i}) (s_i u(c_i(s_i, s_{-i})) + \beta w'_i(s_i, s_{-i})) \\ & \geq \sum_{s-i} \pi(s_{-i}) (s_i u(c_i(\tilde{s}_i, s_{-i})) + \beta w'_i(\tilde{s}_i, s_{-i})) \end{aligned} \quad (17)$$

for all $(s_i, \tilde{s}_i)$ and

$$c_i(s) \geq 0, \quad w'_i(s) \geq 0 \quad \text{for all } s \text{ and all } i, \quad (18)$$

$$h(k'(s), \theta') \geq \sum_{i=1}^I \theta'_i(s) w'_i(s) \quad \text{for all } \theta' \text{ and } s. \quad (19)$$

Now define

$$\begin{aligned} \Psi(k)(h) & \equiv \{w \in \mathbb{R}_+^I : \exists (c, k', w') \text{ such that (16)-(19) are satisfied} \\ & \text{and } w_i = \sum_s \pi(s) [s_i u(c_i(s)) + \beta w'_i(s)]\}. \end{aligned}$$

Given $h \in \bar{F}$, let $\mathcal{W}(k)(h)$ denote the constraint correspondence defined by (16)-(19) at $k \in X$. Any $(c, w', k') \in \mathcal{W}(k)(h)$ will be referred to as a *feasible, incentive-compatible recursive plan with respect to h* . We say

that $h \in \bar{F}$ is preserved under T if $h(k, \theta) \leq (Th)(k, \theta)$ for all (k, θ) . Importantly, notice that it is straightforward to check that

$$(Th)(k, \theta) = \sup_{w \in \Psi(k)(h)} \sum_{i=1}^I \theta_i w_i$$

The following result establishes that the correspondence $\Psi(\cdot)(h)$ is well behaved.³⁶

Lemma 1 $\Psi(\cdot)(h)$ is a continuous compact-valued correspondence for all $h \in \bar{F}$.

It follows that the sup in the operator T is attained. Furthermore, a straightforward application of the Theorem of the Maximum implies that $T : \bar{F} \rightarrow \bar{F}$. In the next lemma, we establish the convexity of $\Psi(k)(h)$, a property that is key to our approach.

Lemma 2 $\Psi(k)(h)$ is convex for all $k \in X$ and all $h \in \bar{F}$.

Proof. Let w and $\tilde{w} \in \Psi(k)(h)$ as $(c, w', k'), (\tilde{c}, \tilde{w}', \tilde{k}') \in \mathcal{W}(k)(h)$ are the corresponding feasible, incentive-compatible recursive plans with respect to h .

We need to show that $w^\lambda = \lambda w + (1 - \lambda)\tilde{w} \in \Psi(K)(h)$ for any $\lambda \in [0, 1]$. In order to do that, define for each i and all s

$$\begin{aligned} u(c_i^\lambda(s)) &= \lambda u(c_i(s)) + (1 - \lambda)u(\tilde{c}_i(s)), \\ k'^\lambda(s) &= \lambda k'(s) + (1 - \lambda)\tilde{k}'(s) \\ w'^\lambda(s) &= \lambda w'(s) + (1 - \lambda)\tilde{w}'(s) \end{aligned}$$

Notice that the strict concavity of u implies that $c_i^\lambda(s) \leq \lambda c_i(s) + (1 - \lambda)\tilde{c}_i(s)$ for all i , all s .

Step 1. Notice that by construction, it follows that

$$w_i^\lambda = \lambda w + (1 - \lambda)\tilde{w} = \sum_s \pi(s)[s_i u(c_i^\lambda(s)) + \beta w_i'^\lambda(s)]$$

for all i .

Step 2. Feasibility. As (c, w', k') and $(\tilde{c}, \tilde{w}', \tilde{k}')$ are both feasible and $c_i^\lambda(s) \leq \lambda c_i(s) + (1 - \lambda)\tilde{c}_i(s)$ for all i , all s as mentioned, it follows immediately that

$$k'^\lambda(s) + \sum_{i=1}^I c_i^\lambda(s) \leq f(k) + (1 - \delta)k$$

for all s and so $(c^\lambda, w'^\lambda, k'^\lambda)$ is also feasible.

Step 3. Incentive Compatibility. As the liquidity shocks are multiplicative, it follows by the linear construction that $(c^\lambda(s), w'^\lambda(s), k'^\lambda(s))$ satisfy 17.

Step 4. Take any $\theta' \in \Delta^I$ and notice that since $h \in \bar{F}$ is concave in k , it follows that

$$\begin{aligned} h(k'^\lambda, \theta') &\geq \lambda h(k', \theta') + (1 - \lambda)h(\tilde{k}', \theta') \\ &\geq \lambda \sum_{i=1}^I \theta'_i(s) w'_i(s) + (1 - \lambda) \sum_{i=1}^I \theta'_i(s) \tilde{w}'_i(s) = \sum_{i=1}^I \theta'_i(s) w_i'^\lambda(s). \end{aligned}$$

³⁶The proof is omitted as it follows by standard arguments. Details are available upon request.

Since $\theta' \in \Delta^I$ is arbitrary, condition (19) is satisfied. Therefore, we can conclude that $(c^\lambda, w'^\lambda, k'^\lambda)$ is a feasible incentive-compatible recursive plan with respect to h . ■

The next result is one of the key steps to make our alternative approach computationally simpler and is based on [Lucas and Stokey \[1984\]](#) and [Abreu et al. \[1990\]](#).

Lemma 3 *For any $h \in \bar{F}$, $w \in \Psi(k)(h)$ if and only if $w \geq 0$*

$$Th(k, \theta) \geq \sum_{i=1}^I \theta_i w_i \quad \text{for all } \theta \in \Delta^I. \quad (20)$$

Proof. See [Rockafellar \[1970\]](#), Theorem 13.1. ■

Remark. For computational purposes, it is convenient to recall that Condition (20) holds if and only if

$$\min_{\theta \in \Delta^I} \left[Th(K, \theta) - \sum_{i=1}^I \theta_i w_i \right] \geq 0.$$

The next result below is similar in spirit to APS's celebrated self-generation.

Lemma 4 (Self-generating) *If $h \in \bar{F}$ is preserved under T , then*

$$(Th)(k, \theta) \leq h^*(k, \theta)$$

for all (k, θ) .

Proof. Take any arbitrary $(\widehat{c}_0(s_0), \widehat{w}'_0(s_0), \widehat{k}'_1(s_0))_{s_0 \in S} \in \mathcal{W}(k_0)(h)$ and notice that this implies, in particular, that

$$\sum_{i=1}^I \theta'_i w'_{i,0}(s_0) \leq h(\widehat{k}'_1(s_0), \theta') \quad (21)$$

for all $\theta' \in \Delta^I$. On the other hand, as h is preserved under T , this last condition implies that given $\widehat{k}'_1(s_0)$

$$h(\widehat{k}'_1(s_0), \theta') \leq (Th)(\widehat{k}'_1(s_0), \theta') \quad (22)$$

for all $\theta' \in \Delta^I$. Hence, as we couple conditions (21) and (22), we conclude that

$$\theta' \widehat{w}'_0(s_0) \leq (Th)(\widehat{k}'_1(s_0), \theta')$$

for all $\theta' \in \Delta^I$ and therefore $\widehat{w}'_0(s_0) \in \Psi(k'(s_0))(h)$ as a direct implication of Lemma 3. Importantly, this implies that there exists some $(\widehat{c}_1(s_0, s_1), \widehat{w}'_1(s_0, s_1), \widehat{k}'_2(s_0, s_1))_{s_1 \in S} \in \mathcal{W}(\widehat{k}'_1(s_0))(h)$ such that

$$\widehat{w}'_0(s_0) = \sum_{s_1} \pi(s_1) \left[s_i u(\widehat{c}_{i,1}(s_0, s_1)) + \beta \widehat{w}'_{i,1}(s_0, s_1) \right]$$

for each $s_0 \in S$.

As we repeat this strategy T times, we can conclude that for any arbitrary $\theta_0 \in \Delta^I$

$$\begin{aligned}
& \sum_{i=1}^I \theta_{i,0} \left\{ \sum_{s_0} \pi(s_0) [s_i u(\widehat{c}_{i,0}(s_0)) + \beta \widehat{w}'_{i,0}(s_0)] \right\} \\
&= \sum_{i=1}^I \theta_{i,0} \left\{ \sum_{s_0} \pi(s_0) s_i u(\widehat{c}_{i,0}(s_0)) \right. \\
&\quad \left. + \beta \sum_{s_0} \pi(s_0) \sum_{s_1} \pi(s_1) [s_i u(\widehat{c}_{i,1}(s_0, s_1)) + \beta \widehat{w}'_{i,1}(s_0, s_1)] \right\} \\
&= \sum_{i=1}^I \theta_{i,0} E \left(\sum_{t=0}^T \beta^t s_{i,t} u(\widehat{c}_{i,t}) \right) + \beta^{T+1} \sum_{i=1}^I \theta_{i,0} E(\widehat{w}'_{i,T+1})
\end{aligned}$$

Condition (19) implies that

$$\sup \sum_{i=1}^I \theta_{i,0} E(\widehat{w}'_{i,T+1}) \leq \|h\|,$$

and so taking limits on both sides as $T \rightarrow \infty$, it follows from the Dominated Convergence Theorem that

$$\begin{aligned}
& \sum_{i=1}^I \theta_{i,0} \left\{ \sum_{s_0} \pi(s_0) [s_i u(\widehat{c}_{i,0}(s_0)) + \beta \widehat{w}'_{i,0}(s_0)] \right\} \\
& \leq \sum_{i=1}^I \theta_{i,0} E \left(\sum_{t=0}^{\infty} \beta^t s_{i,t} u(\widehat{c}_{i,t}) \right)
\end{aligned} \tag{23}$$

as $\beta \in (0, 1)$.

Consider the sequential plan $(\widehat{c}, \widehat{k}')$ stemming from above. It is immediate that this plan is sequentially feasible by construction. Now we argue that it is incentive compatible as well. To see this, denote recursively $W_{i,t}(s^t) = \widehat{w}'_{i,t}(s_0, \dots, s_t)$ and observe that by construction

$$\begin{aligned}
& \left| U_{i,t}(\widehat{c}, \widehat{k}')(s^t) - W_{i,t}(s^t) \right| \\
&= \beta \left| \sum_{s_{t+1}} \pi(s_{t+1}) (U_{i,t}(\widehat{c}, \widehat{k}')(s^t, s_{t+1}) - W_{i,t+1}(s^t, s_{t+1})) \right| \\
&\leq \beta \sup_{s_{t+1}} \left| U_{i,t}(\widehat{c}, \widehat{k}')(s^t, s_{t+1}) - W_{i,t+1}(s^t, s_{t+1}) \right| \\
&\leq \beta^k \sup_{(s_{t+1}, \dots, s_{t+k})} \left| U_{i,t}(\widehat{c}, \widehat{k}')(s^t, s_{t+1}, \dots, s_{t+k}) - W_{i,t+k}(s^t, s_{t+1}, \dots, s_{t+k}) \right|.
\end{aligned}$$

Observe that $0 \leq W_{i,t}(s^t) \leq \|h\| < \infty$ for all i and all s^t while $\widehat{c}$ is uniformly bounded by construction. Taking the lim sup as $k \rightarrow \infty$ for this last expression, we can conclude that $U_{i,t}(\widehat{c}, \widehat{k}')(s^t) = W_{i,t}(s^t)$ for all i and all s^t and so sequential incentive compatibility follows immediately.

Since both $(\widehat{c}_0(s_0), \widehat{w}'_0(s_0), \widehat{k}'_1(s_0))_{s_0} \in \mathcal{W}(k_0)(h)$ and the corresponding sequential plan $(\widehat{c}, \widehat{k})$ are arbitrary, we take the sup on both sides of (23) to conclude that

$$\begin{aligned}
Th(k, \theta) &= \sup_{(\widehat{c}, \widehat{w}', \widehat{k}') \in \mathcal{W}(k_0)(h)} \sum_{i=1}^I \theta_{i,0} \left\{ \sum_{s_0} \pi(s_0) [s_i u(\widehat{c}_i) + \beta \widehat{w}'_i] \right\} \\
&\leq \sup \sum_{i=1}^I \theta_i E \left(\sum_{t=0}^{\infty} \beta^t s_{i,t} u(\widehat{c}_{i,t}) \right) \\
&= h^*(k, \theta).
\end{aligned}$$

and this completes the proof. ■

Now we are prepared to prove our two main results in Section 2.2.

Proof of Proposition 1. Given (K, θ) , take any $w \in \Psi^*(K)$ for which (C, K') denotes the corresponding feasible incentive-compatible plan. Observe that

$$\sum_{i=1}^I \theta_i U_i(C, K') = \sum_{i=1}^I \theta_i \sum_{s_0 \in S_0} \pi(s_0) [s_{i,0} u(C_i(s_0)) + \beta U_{i,1}(C, K' || (s_0))]$$

Notice that $(U_{i,1}(C, K' || (s_0)))_{i=1}^I \in \Psi^*(K(s_0))$ for all s_0 . It follows by definition of h^* (see Remark 1) that

$$h^*(K'(s_0), \theta') \geq \sum_{i=1}^I \theta'_i U_{i,1}(C, K' || (s_0))$$

for all $\theta' \in \Delta^I$ and all s_0 . Therefore, $(C_i, U_{i,1}(C, K'), K')_{i=1}^I \in \mathcal{W}(K_0)(h^*)$ and then

$$\sum_{i=1}^I \theta_i U_i(C, K') \leq (Th^*)(K, \theta)$$

Since weak inequalities are preserved in the limit, we can conclude that

$$h^*(K, \theta) = \sup_{(C, K')} \sum_{i=1}^I \theta_i U_i(C, K') \leq (Th^*)(K, \theta),$$

for all (K, θ) (i.e. h^* is preserved under T). Thus, Lemma 4 implies that $h^*(K, \theta) = (Th^*)(K, \theta)$ for all (K, θ) . ■

Proof of Proposition 2. Let $\widehat{T}$ be the operator solving the recursive problem for which the incentive compatibility constraints (17) are ignored (i.e. the full information case). It is a routine exercise to show that T is a monotone operator (i.e. if $f \geq g$, then $Tf \geq Tg$). Also, observe that $h^* = Th^* \leq Th^{**} \leq \widehat{T}h^{**} = h^{**}$ by Proposition 1 and monotonicity. Let $h_n = T^n h^{**}$ and then monotonicity implies that $h_n \geq h_{n+1} \geq h^*$ for all n . So $\{h_n\}$ is a monotone decreasing sequence of uniformly bounded functions, and therefore there exists a function $h_\infty \geq h^*$ such that $\lim_{n \rightarrow \infty} h_n = h_\infty$. It remains to show that $h_\infty \leq h^*$, for which it is sufficient that h_∞ is preserved under T due to Lemma 4.

Given (k, θ) , $h_\infty(k, \theta) \leq h_n(k, \theta)$ implies that, for all n , there exists $(c^n, w'^n, k'^n) \in \mathcal{W}(k)(h_n)$ that attains $h_\infty(k, \theta)$. Observe that (c^n, w'^n, k'^n) lies in a compact set and, thus, it has a convergent subsequence with limit point $(\widehat{c}, \widehat{w}', \widehat{k}')$. Suppose for notational simplicity that the convergent subsequence is the sequence itself and

notice that incentive compatibility is preserved in the limit. Also, we have that for all n and all s

$$h_n(k^n(s), \theta') \geq \sum_{i=1}^I \theta'_i w_i^n(s), \text{ for all } \theta'$$

and then

$$h_\infty(\widehat{k}(s), \theta') \geq \sum_{i=1}^I \theta'_i \widehat{w}_i(s),$$

Therefore, $(\widehat{c}, \widehat{w}', \widehat{k}') \in \mathcal{W}(k)(h_\infty)$ and

$$h_\infty(k, \theta) = \sum_{i=1}^I \theta_i \sum_{s_0} \pi(s_0) [u(\widehat{c}_i(s_0)) + \beta \widehat{w}'_i(s_0)].$$

Finally, since the recursive allocation plan $(\widehat{c}, \widehat{w}', \widehat{k}') \in \mathcal{W}(k)(h_\infty)$ is arbitrary, we can conclude that

$$\begin{aligned} (Th_\infty)(k, \theta) &\geq \sum_{i=1}^I \theta_i \sum_{s_0} \pi(s_0) [u(\widehat{v}_i(s_0)) + \beta \widehat{w}'_i(s_0)] \\ &= h_\infty(k, \theta), \end{aligned}$$

for all (k, θ) and thus h_∞ is preserved under T by definition. ■

8.2 Design of partnerships under private information

In order to provide the proofs for Section 4 and 5, it is convenient to state the necessary and sufficient first-order conditions that characterize the solutions. We abuse notation in this Section and use both $(\theta_1, \theta_2) \in \Delta^2$ and $\theta = \theta_1$. Let μ_i be the Lagrange multiplier of (17) for partner i , $\lambda(s)$ be Lagrange multiplier of (16) and $\eta(s)$ be the Lagrange multiplier of (19). The policy functions and Lagrange multipliers must satisfy the following conditions. First-order conditions with respect to consumption and promised utility render

$$\lambda(k, \theta_1, \theta_2)(s_L, s_{-i}) = s_L u'(c_i(k, \theta_1, \theta_2)(s_L, s_{-i}))(\theta_i + \mu_i(k, \theta_1, \theta_2))/\pi(s_L), \quad (24)$$

$$\lambda(k, \theta_1, \theta_2)(s_H, s_{-i}) = s_H u'(c_i(k, \theta_1, \theta_2)(s_H, s_{-i}))(\theta_i - \mu_i(k, \theta_1, \theta_2))/\pi(s_H), \quad (25)$$

$$\eta \theta'_i(k, \theta_1, \theta_2)(s_L, s_{-i}) = \theta_i + \mu_i(k, \theta_1, \theta_2)/\pi(s_L), \quad (26)$$

$$\eta \theta'_i(k, \theta_1, \theta_2)(s_H, s_{-i}) = \theta_i - \mu_i(k, \theta_1, \theta_2)/\pi(s_H), \quad (27)$$

for both partners $i = 1, 2$ and all s_{-i} . The optimal choice of next-period capital implies

$$\lambda(k, \theta_1, \theta_2)(s) = \beta \eta (f'((k, \theta_1, \theta_2)(s)) + (1 - \delta)) \sum_{s'} \pi(s') \lambda((k, \theta'_1(k, \theta_1, \theta_2)(s), \theta'_2(k, \theta_1, \theta_2)(s))(s'), \quad (28)$$

for all s .³⁷ The law of motion of welfare weights and incentive compatibility constraints dictate that

$$h_{\theta i}^* = w'_i(k, \theta_1, \theta_2)(s); \quad (29)$$

³⁷The Euler equation (28) uses the corresponding envelope condition to determine h_k^* .

$$\mu_i(k, \theta_1, \theta_2) \left[\sum_{s_{-i}} \pi(s_{-i}) \left[\left(s_L u(c_i(k, \theta_1, \theta_2)(s_L, s_{-i})) + \beta w'_i(k, \theta_1, \theta_2)(s_L, s_{-i}) \right) - \left(s_L u(c_i(k, \theta_1, \theta_2)(s_H, s_{-i})) + \beta w'_i(k, \theta_1, \theta_2)(s_H, s_{-i}) \right) \right] \right] = 0; \quad (30)$$

for both partners $i = 1, 2$. And finally,

$$k'(k, \theta_1, \theta_2)(s) + \sum_{i=1}^I c_i(k, \theta_1, \theta_2)(s) = f(k) + (1 - \delta)k; \quad (31)$$

$$h(k'(k, \theta_1, \theta_2)(s), \theta'(k, \theta_1, \theta_2)(s)) = \sum_{i=1}^I \theta'_i(k, \theta_1, \theta_2)(s) w'_i(k, \theta_1, \theta_2)(s). \quad (32)$$

Notice that the case of full information is the one for which we basically set $\mu_i = 0$ for $i = 1, 2$. That implies (11), (10), and (12) in the main text. We consider two other special cases below. The first is the case in which only the founder faces liquidity shocks and in that case $\mu_2 = 0$ and, moreover, since shocks to the partner's liquidity needs play no essential role, we put $s_{2,t} = 1$ for all t in some examples. In the second case, both partners face liquidity shocks.

8.2.1 Founder only with liquidity shocks

In this section, we provide the proofs of our main results in Section 4 in which only the founder, agent 1, faces shocks to liquidity needs that are private information.

Proof of Proposition 3. First, we show that the full information plan is incentive compatible as $\theta = 1$.

Consider the recursive problem (15)-(19) for the case in which the incentive compatibility constraints are absent.

Let $(c(\theta, k)(s), c_2(\theta, k)(s), k'(\theta, k)(s), \theta'(\theta, k)(s), w'_1(\theta, k)(s), w'_2(\theta, k)(s))$ be the set of continuous policy functions solving (24) - (32) such that $\mu_i = 0$ for $i = 1, 2$.

Notice that in this case, (26) and (27) for both agents and the fact that $\theta_1 + \theta_2 = 1$ imply that the law of motion of ownership shares satisfies

$$\theta'(\theta, k)(s) = \theta$$

for all s and all (θ, k) .

Since $\theta'(\theta, k)(s) = \theta = 1$, then $h(k'(s), 1) = w'_1(s)$ for all (s, θ, k) and the value function reduces to

$$h(k, 1) = \pi(s_L) \left[s_L u(c_1(1, k)(s_L, s_2)) + \beta w'_1(1, k)(s_L, s_2) \right] + \pi(s_H) \left[s_H u(c_1(1, k)(s_H, s_2)) + \beta w'_1(1, k)(s_H, s_2) \right]$$

for all s_2 . Notice that as $\theta'(\theta, k)(s) = \theta = 1$, then s_2 plays no allocative role. Therefore, consumption, future promised utilities, and capital accumulation are independent of s_2 .

Suppose that the corresponding full information plan is not incentive compatible; i.e.

$$s_L u(c_1(k, 1)(s_L)) + \beta h(k'(k, 1)(s_L), 1) < s_L u(c_1(k, 1)(s_H)) + \beta h(k'(1, k)(s_H), 1)$$

while

$$c_1(k, 1)(s_L) + k'(k, 1)(s_L) = f(k) + (1 - \delta)k$$

$$c_1(k, 1)(s_H) + k'(k, 1)(s_H) = f(k) + (1 - \delta)k$$

This implies that $(c_1(k, 1)(s_H), k'(k, 1)(s_H))$ is feasible at $s_1 = s_L$ and

$$w'_1(k, 1) = h(k'(k, 1), 1)(s_H).$$

This contradicts that $(c_1(k, 1)(s_L), k'(k, 1)(s_L), w'_1(k, 1)(s_L))$ is part of the unique solution at $(k, 1)$.

To complete the proof we need to argue that, conditional upon k , we can find some $\bar{\theta}(k) < 1$ such that the incentive compatibility constraint is also satisfied. Since the solution is unique, the incentive compatibility constraint must hold with strict inequality when $\theta = 1$. Now, since the policy functions in the full information case are continuous, it must be the case that there exists some $\bar{\theta}(k) < 1$ such that the incentive compatibility constraint is also satisfied for $\theta \in [\bar{\theta}(k), 1]$. ■

Proof of Proposition 4. First notice that Γ is a compact set since $k_{\min}(\theta)$ and $k_{\max}(\theta)$ are both continuous functions. Then, for all $\theta \in [\theta^*, 1]$ and $k \in [k_{\min}(\theta), k_{\max}(\theta)]$ the incentive compatibility constraints are not binding. Hence, $\theta'(s)(\theta, k) = \theta$ for all (s, θ, k) and $k' \in [k_{\min}(\theta), k_{\max}(\theta)] = [k_{\min}(\theta'), k_{\max}(\theta')]$. The size of the partnership, k , remains in this ergodic set and, more in general, the corresponding plan coincides with the full information plan. ■

Proof of Proposition 5. Conditions (26) and (27) for both partners and the fact that $\mu_2 = 0$ imply that

$$\frac{\theta'(k, \theta, 1 - \theta)(s_L, s_2)}{(1 - \theta'(k, \theta, 1 - \theta)(s_L, s_2))} = \frac{\theta + \mu_1(k, \theta, 1 - \theta)/\pi(s_L)}{(1 - \theta)} = \frac{\theta}{(1 - \theta)} + \frac{\mu_1(k, \theta, 1 - \theta)}{(1 - \theta) \pi(L)},$$

$$\frac{\theta'(k, \theta, 1 - \theta)(s_H, s_2)}{(1 - \theta'(k, \theta, 1 - \theta)(s_H, s_2))} = \frac{\theta + \mu_1(k, \theta, 1 - \theta)/\pi(s_H)}{(1 - \theta)} = \frac{\theta}{(1 - \theta)} + \frac{\mu_1(k, \theta, 1 - \theta)}{(1 - \theta) \pi(H)},$$

for all s_2 . This implies that

$$\mathbf{E} \left[\frac{\theta'(k, \theta, 1 - \theta)(s_H, s_2)}{(1 - \theta'(k, \theta, 1 - \theta)(s))} \right] = \frac{\theta}{(1 - \theta)}.$$

Let $\{\theta_t\}_{t=0}^\infty$ be the stochastic process for ownership shares generated by the set of policy functions as in (9). That is, $\theta_t : S^\infty \rightarrow [0, 1]$, where $\theta_t(s^\infty)$ denotes a particular realization at date t . Hence the last expression can be rewritten as

$$\mathbf{E} \left[\frac{\theta_{t+1}}{(1 - \theta_{t+1})} \parallel s^t \right] = \frac{\theta_t(s^\infty)}{(1 - \theta_t(s^\infty))} \quad s^\infty - a.s.$$

It follows by the martingale convergence theorem that

$$\frac{\theta_t(s^\infty)}{(1 - \theta_t(s^\infty))} \rightarrow \frac{\widehat{\theta}(s^\infty)}{(1 - \widehat{\theta}(s^\infty))} \quad s^\infty - a.s.$$

for some random variable $\widehat{\theta}$ on $(S^\infty, \mathcal{B}(S^\infty))$. ■

Proof of Proposition 6. Let $\Omega = \{s^\infty \in S^\infty : \theta_t(s^\infty) \rightarrow \widehat{\theta}(s^\infty)\}$ and take any $s^\infty \in \Omega$. If $\widehat{\theta}(s^\infty) = 0$ the limiting plan is trivially first best.

Now if $\widehat{\theta}(s^\infty) \geq \theta^*$, it follows by Proposition (4) that the limiting plan is first best.

Finally, we need to show that $\widehat{\theta}(s^\infty) \notin (0, \theta^*)$; i.e. the limiting plan can converge to a plan where the ICC is binding only for zero-probability sequences.

Case 1: $\bar{\theta}(k) = \theta^*$ for all k

Suppose that the state s_L occurs infinitely often and consider that infinite subsequence $\{s_{t_n}\}_{n=0}^\infty$ in which $s_{t_n} = s_L$ for all n . Since $\{k_{t_n}\}_{n=0}^\infty$ is a sequence in a compact set, it must have a convergent subsequence with limit $\widehat{k}(s^\infty) \in \times [k_{\min}(\widehat{\theta}(s^\infty)), k_{\max}(\widehat{\theta}(s^\infty))]$; to simplify notation, suppose that it is the sequence $\{k_{t_n}\}_{n=0}^\infty$ itself. Since $\theta_{t_n+1} = \theta'(\theta_{t_n+1}, k_{t_n})(s_L)$, it follows by continuity that taking the limit

$$\widehat{\theta}(s^\infty) = \theta'(\widehat{\theta}(s^\infty), \widehat{k}(s^\infty))(s_L);$$

i.e. the ICC does not bind. But this contradicts that $\widehat{\theta}(s^\infty) \notin (0, \theta^*)$ and, consequently, as in [Thomas and Worral \[1990\]](#), $\{\theta_t\}_{t=0}^\infty$ can converge to some number in the interval $(0, \theta^*)$ only for sequences where s_L occurs only finitely often. Those events occur with zero probability.

Case 2: $\bar{\theta}(k)$ can vary with k .

First, if $\widehat{\theta}(s^\infty) < \bar{\theta}(k)$ for all k , then the argument follows as in Case 1.

So, suppose that $\theta^* > \widehat{\theta}(s^\infty) > \bar{\theta}(k)$ for some k and let $k(s^\infty)$ be defined such that $\widehat{\theta}(s^\infty) = \bar{\theta}(k(s^\infty))$. Notice that $k(s^\infty) \in (k_{\min}(\widehat{\theta}(s^\infty)), k_{\max}(\widehat{\theta}(s^\infty)))$ and assume, without loss of generality, that $k(s^\infty) > k$. As long as $\widehat{\theta}(s^\infty) \geq \bar{\theta}(k_i(s^\infty))$, $\theta'_{s_i}(\widehat{\theta}(s^\infty), k_i(s^\infty)) = \widehat{\theta}(s^\infty)$ for all s_i .

Since s^∞ belongs to a set with positive probability, there exists some finite T such that $k_T(s^\infty) \geq k(s^\infty)$.

If $\widehat{\theta}(s^\infty) < \bar{\theta}(k_T(s^\infty))$, the full information plan does not satisfy the incentive compatibility constraint at $(\widehat{\theta}(s^\infty), k_T(s^\infty))$ and so the argument follows as in Case 1.

If $\widehat{\theta}(s^\infty) = \bar{\theta}(k_T(s^\infty))$, the full information plan satisfies the incentive compatibility constraint at T , θ' remains unchanged, and k' moves up and with s . Then, the full information constraint does not satisfy the incentive compatibility constraint at $(\widehat{\theta}(s^\infty), k'_L(\widehat{\theta}(s^\infty), k_T(s^\infty)))$ since $k'_L(\widehat{\theta}(s^\infty), k_T(s^\infty)) > k_T(s^\infty)$. Again, the argument follows as in Case 1. ■

8.2.2 Founder and associate with liquidity shocks

In this section, we provide the proofs of our results in [Section 5](#), in which the setting extended so that both the founder and associate face liquidity shocks.

Proof of Proposition 7. Consider the solution to the full information problem evaluated at $\theta = 1/2$ and let $s_L = 1 - \epsilon$ and $s_H = 1 + \epsilon$. Notice that symmetry implies that

$$\begin{aligned} c_1(1 - \epsilon, 1 - \epsilon) &= c_2(1 - \epsilon, 1 - \epsilon) \\ c_1(1 + \epsilon, 1 + \epsilon) &= c_2(1 + \epsilon, 1 + \epsilon) \\ w'_1(1 - \epsilon, 1 - \epsilon) &= w'_2(1 - \epsilon, 1 - \epsilon) \\ w'_1(1 + \epsilon, 1 + \epsilon) &= w'_2(1 + \epsilon, 1 + \epsilon) \\ k'(1 - \epsilon, 1 - \epsilon) &> k'(1 + \epsilon, 1 + \epsilon) \end{aligned}$$

and also

$$\begin{aligned} c_1(1 - \epsilon, 1 + \epsilon) &= c_2(1 + \epsilon, 1 - \epsilon) \\ c_1(1 + \epsilon, 1 - \epsilon) &= c_2(1 - \epsilon, 1 + \epsilon) \\ w'_1(1 - \epsilon, 1 + \epsilon) &= w'_2(1 + \epsilon, 1 - \epsilon) \\ w'_1(1 + \epsilon, 1 - \epsilon) &= w'_2(1 - \epsilon, 1 + \epsilon) \\ k'(1 - \epsilon, 1 + \epsilon) &= k'(1 + \epsilon, 1 - \epsilon) \end{aligned}$$

Notice that this implies that

$$h'(k(1 - \epsilon, 1 + \epsilon), 1/2, 1/2) = h'(k(1 + \epsilon, 1 - \epsilon), 1/2, 1/2).$$

So in this case, (26) and (27) coupled with the fact that $\theta_1 + \theta_2 = 1$ imply that the law of motion of ownership shares satisfies

$$\theta'(k', 1/2, 1/2)(s_1, s_2) = 1/2,$$

for all (s_1, s_2) and all k' . Therefore, it follows by definition of h that

$$w'_1(1 - \epsilon, 1 + \epsilon) = w'_1(1 + \epsilon, 1 - \epsilon)$$

Given ϵ , consider the incentive compatibility constraint of partner 1 that needs to be satisfied

$$\begin{aligned} & \sum_{s_2} \pi(s_2) \left((1 - \epsilon)u(c_1(1 - \epsilon, s_2)) + \beta w'_1(1 - \epsilon, s_2) \right) \\ & \geq \sum_{s_2} \pi(s_2) \left((1 - \epsilon)u(c_i(1 + \epsilon, s_2)) + \beta w'_i(1 + \epsilon, s_2) \right). \end{aligned} \quad (33)$$

Notice that as h is strictly increasing in k , it follows that $w'_1(1 - \epsilon, s_2) > w'_i(1 + \epsilon, s_2)$ for all s_2 . By the Theory of Maximum, policy functions can be parameterized continuously with respect to ϵ . For each k , since (34) holds with strict inequality as ϵ goes to 1, we can conclude that there exists some $\epsilon^*(k) \in (0, 1)$ such that (34) holds with equality. Importantly, the full information plan is strictly incentive compatible for partner 1 for all $\epsilon \in (\epsilon^*(k), 1)$ at k . Since it follows by standard arguments that $\epsilon^*(k)$ varies continuously, we define $\epsilon^* \equiv \max \{\epsilon^*(k) : k \in [k_{\min}(1/2), k_{\max}(1/2)]\} \in (0, 1)$. Therefore, the full information plan is strictly incentive compatible for partner 1 for all $\epsilon \in (\epsilon^*, 1)$ for all k . It follows by symmetry it is also strictly incentive compatible for partner 2. ■

Proof of Corollary 1. Suppose that the full information plan is strictly incentive compatible at $\theta = 1/2$ for all k . Notice that it follows by symmetry that

$$\begin{aligned} c_1(k, \theta, 1 - \theta)(s) &= c_2(k, 1 - \theta, \theta)(s), \\ k'(k, \theta, 1 - \theta)(s) &= k'(k, 1 - \theta, \theta)(s) \end{aligned}$$

for all s , for all k and for all $\theta \in [0, 1]$. The same arguments developed in the proof of Proposition 4 and the aforementioned symmetry let us conclude that: there exists $\theta^* \in (0, 1/2)$ such that if $(\theta_t, k_t) \in [\theta^*, 1 - \theta^*] \times [k_{\min}(\theta_t), k_{\max}(\theta_t)]$ at some t , then the efficient and the full information plans coincide and $\theta_{t+n} = \theta_t$ for all $n \geq 0$. ■

Proof of Corollary 2. Suppose that the full information plan is strictly incentive compatible at $\theta = 1/2$ for all k and let θ^* be defined as in Corollary 1. Notice that in this case the incentive compatibility constraint for partner 1 never binds for all $\theta \in [0, 1/2]$. Importantly, as $\theta_0 \in [0, \theta^*]$, then the same reasoning behind Proposition 6 makes it possible to conclude that (θ_t, k_t) reach the region in which the efficient plan and the full information plan coincide and $\theta_t \rightarrow \{0, \theta^*\}$ a.s. On the other hand, symmetry implies that the incentive compatibility constraint for partner 2 never binds for all $\theta \in [1/2, 1]$. Therefore, as $\theta_0 \in [1 - \theta^*, 1]$, (θ_t, k_t) reach the region in which the efficient plan and the full information plan coincide and $\theta_t \rightarrow \{1 - \theta^*, 1\}$ a.s. Finally, the case where $\theta_0 \in [\theta^*, 1 - \theta^*]$ was analyzed in Corollary 1. ■

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