The Social Value of Information in a Business-Cycle Model*

George-Marios Angeletos
MIT

Luigi Iovino
Bocconi University

Jennifer La'O
Columbia University

February 15, 2015

Abstract

Does welfare improve when firms have more information about the state of the economy and can better coordinate their production and pricing decisions? We address this question in a business-cycle model that highlights how informational frictions can be the source of both nominal and real rigidity. We then elaborate on how the answer to this question depends on each of these rigidities, on the sources of the business cycle, and on the conduct of monetary policy.

JEL codes: C7, D6, D8.
Keywords: Fluctuations, informational frictions, strategic complementarity, beauty contests, public information, central-bank transparency.

*Earlier versions of this paper circulated under the title "Cycles, Gaps, and Social Value of Information." *Email addresses:* angelet@mit.edu, luigi.iovino@unibocconi.it, jenlao@columbia.edu.

1 Introduction

In modern economies, market participants have access to a variety of sources of information about the state of the economy, some of which are private and some of which are public. By informing each agent about the activity of other agents, public information can ease coordination, whereas private information can hinder it. In this paper, we study the welfare consequences of this mechanism within the context of a business-cycle model in which firms make their pricing and production choices under incomplete information about the state of the economy.

Background. We are not the first to study how information affects coordination and welfare. In an influential article, Morris and Shin (2002) used a "beauty contest" game—a linear-quadratic game in which actions were strategic complements—to formalize the coordinating role of public information and to study its welfare implications. In such a game, public signals have a disproportionate effect on equilibrium outcomes, relative to what is warranted on the basis of their informational content regarding the fundamentals, due to the fact that the players use such signals, not only to predict the fundamentals, but also to coordinate their actions. In this regard, public signals play, in part, a role akin to that of sunspots and can contribute to higher volatility and lower welfare.

Because strategic complementarity emerges naturally from the aggregate demand externalities that are embedded in macroeconomic models, Morris and Shin's analysis was used to inform the debate on the pros and cons of central-bank transparency.[1] However, subsequent work raised questions about the validity of applying Morris and Shin's lessons to a macroeconomic context.

On the one hand, using a broader game-theoretic framework, Angeletos and Pavan (2007) highlighted that Morris and Shin's welfare conclusion hinges on the assumption that coordination is socially harmful—an assumption that need not be valid in workhorse macroeconomic models. On the other hand, a line of applied work that includes Baeriswyl and Cornand (2010), Hellwig (2005), Lorenzoni (2010), Roca (2005), and Welsh (2007) found different welfare effects than those suggested by Morris and Shin in incomplete-information variants of the baseline New-Keynesian model.

This line of applied work has pushed the analysis of the question of interest from abstract games to micro-founded business-cycle models—a crucial step, as "anything goes" without the discipline of particular micro-foundations. Yet, this work faces two limitations. First, it treats the informational friction as synonymous to a particular form of nominal rigidity: following Woodford (2002) and Mankiw and Reis (2002), it assumes that firms set their prices on the basis of incomplete information, which seems relevant in reality; but it also allows output to adjust freely to the realized shocks, thus abstracting from the bite that the informational friction can have on employment and production irrespectively of nominal rigidity. Second, this work intertwines the welfare effects of information with those of particular monetary policies, occasionally confounding the role of market distortions with those of policy distortions.

[1] See, e.g., Morris and Shin's original article, as well as the follow-up *AER* articles by Svensson (2006), Morris et al. (2006), and James and Lawler (2011).

Our contribution. Seeking to overcome the aforementioned limitations, in this paper we let the incompleteness of information to be the source of both *real* and *nominal* rigidity: we consider a framework in which firms make, not only their pricing choices, but also certain production choices on the basis of dispersed private information about the underlying aggregate shocks. In addition, we dissect how the welfare effects of such information depend on whether monetary policy coincides with, or deviates from, either the one that replicates flexible prices or the one that maximizes welfare in the absence of policy frictions—two familiar benchmarks that were often suppressed in the aforementioned work. This leads to a certain taxonomy, and a sharper understanding, of how the welfare effects of either private or public information depend on the firms' coordination motives, on the sources of the business cycle, and on the conduct of monetary policy.

Isolating the real rigidity. The first part of the paper (Sections 2 through 4) assumes that firms choose employment on the basis of incomplete information, thus accommodating real rigidity, but lets prices adjust freely to the realized state of nature, thus abstracting from nominal rigidity and shutting down any role for monetary policy. This part therefore concentrates on the polar case than the one studied in prior work and serves as a stepping stone towards the more general analysis that follows in the second part of the paper. It also facilitates a precise mapping between our micro-founded framework and the abstract games studied in Morris and Shin (2002) and Angeletos and Pavan (2007), helping clarify whether the coordination motives that are embedded in workhorse macroeconomic models are aligned with social objectives.

We first show that the available information has ambiguous, and often conflicting, effects on two familiar welfare components: the volatility of the aggregate output gap and the inefficient cross-sectional dispersion in relative prices and quantities. These ambiguities are governed by three factors: (i) preference and technology parameters that relate to the degree of strategic complementarity; (ii) whether the information is private or public; and (iii) the underlying sources of the business cycle. We next show that, despite these ambiguities in the component effects, the overall welfare effect is governed *solely* by the sources of the business cycle, as explained next.

Suppose first that the business cycle is driven by non-distortionary forces such as technology shocks. In this case, more information unambiguously increases welfare, irrespectively of whether the additional information is public or private. This is because more information brings the incomplete-information equilibrium allocation closer to its complete-information counterpart, which itself co-varies perfectly with the first-best allocation along the business cycle. What is more, the beneficial effect of public information is actually *higher* when the strategic complementarity is stronger. This is because the micro-foundations of our model guarantee that the private value that firms assign to the coordination of their production choices coincides with the corresponding social value.

Suppose next the business cycle is driven by distortions in product and labor markets, such as shocks to monopoly markups. Once again, information has ambiguous and possibly conflicting effects on volatility and dispersion. Yet, the combined welfare effect is now unambiguously negative.

This is because of two factors: that more information now facilitates a stronger response to shocks that only cause the economy to fluctuate *away* from the first best; and that, despite this fact, the coordination motives remain otherwise aligned with social objectives.[2]

These findings offer a different policy message than that of Morris and Shin (2002). At face value, they also appear to contradict some of the results and intuitions found in prior work. Hellwig (2005), Welsh (2007) and Baersiwyl and Conrand (2010) document ambiguous welfare effects for certain types of information and attribute these ambiguities to a trade off between volatility and dispersion; relatedly, Angeletos and Pavan (2007) conjecture that the firms' motives to coordinate in baseline macroeconomic models are inefficiently low. By contrast, we find no discrepancy between private and social values of coordination; because of this, we also obtain unambiguous overall welfare effects, despite a potential trade off between volatility and dispersion. The resolution to the apparent inconsistency between the prior work and ours rests on dissecting the role of monetary policy.

Adding nominal rigidity. In the second part of the paper (Sections 5 and 6), we study a more general framework, in which we let the informational friction impede, not only the firms' production choices, but also their price-setting behavior.

The models used in the aforementioned applied work are nested in this framework as special cases that shut down the real rigidity and that assume a particular monetary policy. By contrast, our analysis allows the two rigidities to coexist and seeks to dissect the role of different monetary policies. We thus anchor our taxonomy around the benchmark of a monetary policy that neutralizes the nominal rigidity, in the sense of implementing the same real allocation as the one that would have obtained under flexible prices.

At this benchmark, the welfare properties of the extended model are qualitatively the same as those of the baseline model: welfare increases (respectively, decreases) with either private or public information when the business cycle is driven by technology shocks (respectively, markup shocks). One can thus readily recast our baseline analysis as one that allows for both nominal and real rigidity, but focuses on a familiar policy benchmark.

Away from this benchmark, an additional effect emerges: information can matter, not only via the real rigidity, but also by affecting the firms' ability to forecast the monetary policy's deviations from the aforementioned benchmark. Whether this additional effect contributes to less or more welfare then depends on whether these deviations were desirable to start with or not.

When the business cycle is driven by technology shocks, *any* deviation from flexible prices is welfare-deteriorating. Increasing the information that is available to firms may then improve welfare, not only by alleviating the real rigidity, but also by helping the firms forecast, and undo, the "mistakes" in monetary policy. In this sense, "transparency" is good.

[2] In general, a countervailing effect may also be at work: by reducing the idiosyncratic risk faced by the firms, more information can help raise the mean level of economic activity. We characterize this effect in Proposition 5, but abstract from it thereafter by assuming, as often done in the literature, that a non-contingent subsidy is used to eliminate the mean (or "steady-state") distortion in economic activity.

When, instead, the business cycle is driven by markup shocks or other distortions, an appropriate deviation from flexible prices is desirable, for reasons once gain familiar from the New-Keynesian framework: the optimal policy now seeks to *exploit* the nominal rigidity in order to substitute for a missing tax instrument, namely, the state-contingent subsidy that would have offset the markup shock. More information in the hands of the private sector can then be detrimental for welfare, not only for the reasons highlighted in our baseline model, but also by reducing the effectiveness of monetary policy.[3] In this sense, "opacity" becomes preferable.

All in all, we obtain a taxonomy of how the welfare effects of information depend on the two distinct types of rigidity, on the sources of the business cycle, and on the conduct of monetary policy. This taxonomy contains a particularly sharp answer to the question of interest under certain policy benchmarks; but it also provides a roadmap for understanding the welfare effect of information away from them. As an illustration of the latter point, Section 6 revisits some of the findings of the aforementioned prior work under the lenses of our taxonomy.

2 The baseline model

The baseline model builds on Angeletos and La'O (2009). The economy consists of a "mainland" and a continuum of "islands". Each island is inhabited by a continuum of workers and a continuum of monopolistic firms. Firms employ local workers through a competitive labor market and produce differentiated commodities, which they ultimately sell in a centralized market in the mainland. The latter is inhabited by a continuum of consumers, each of whom is tied to one worker and one firm from *every* island in the economy. Along with the fact that there will be no heterogeneity *within* islands, this guarantees that the economy admits a representative household: we can think of the latter as a "big family" that is comprised of all agents, collects all income, and consumes all output in the economy. Nevertheless, the aforementioned geography introduces an informational friction: we assume that firms and workers observe the fundamentals on their own island, but face incomplete information about the underlying aggregate shocks and the choices that other agents (their "siblings") make on other islands. Finally, islands are indexed by $i \in I = [0, 1]$; firms, workers, and commodities by $(i, j) \in I \times J = [0, 1]^2$; and periods by $t \in \{0, 1, 2, ...\}$.

Fundamentals. The utility of the representative household is given by

$$\mathcal{U} = \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - \int_I \int_J \chi_{it} V(n_{ijt}) dj di \right],$$

where $U(C) = \frac{1}{1-\gamma} C^{1-\gamma}$, $V(n) = \frac{1}{1+\epsilon} n^{1+\epsilon}$, and $\gamma, \epsilon \geq 0$. Here, n_{ijt} is the labor input in firm j of island i (or the effort of the corresponding worker), χ_{it} is an island-specific shock to the disutility of

[3] A similar effect is highlighted in Baeriswyl and Cornand (2010), albeit in a model that rules out the real rigidity we accommodate here and shifts attention to a certain signaling mechanism. See the discussion in Section 6.

labor, and C_t is aggregate consumption. The latter is given by the following nested CES structure:

$$C_t = \left[\int_I (c_{it})^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}}$$

where c_{it} is a composite of the goods produced by island i, itself defined by

$$c_{it} = \left[\int_J (c_{ijt})^{\frac{\eta_{it}-1}{\eta_{it}}} dj \right]^{\frac{\eta_{it}}{\eta_{it}-1}},$$

where c_{ijt} denotes the consumption of commodity j from island i. Note here that ρ and η identify the elasticities of substitution across and within islands. In equilibrium, ρ ends up controlling the strength of aggregate demand externalities, while η controls the degree of monopoly power. We let $\eta \neq \rho$ so as to isolate the distinct roles of these two forces; we then let η_{it} be random so as to accommodate markup, or cost-push, shocks.

Recall that the representative household receives labor income and profits from all islands in the economy. Its budget constraint is thus given by the following:

$$\int_I \int_J p_{ijt} c_{ijt} dj dj + B_{t+1} \leq \int_I \int_J \pi_{ijt} di dj + \int_I (1 - \tau_{it}) w_{it} n_{it} di + R_t B_t + T_t,$$

Here, p_{ijt} is the period-t price of the commodity produced by firm j on island i, π_{ijt} is the period-t nominal profit of that firm, w_{it} is the period-t nominal wage on island i, R_t is the period-t nominal gross rate of return on the riskless bond, and B_t is the amount of bonds held in period t.

The variables τ_{it} and T_t satisfy $T_t = \int_I \tau_{it} w_{it} n_{it} di$. One can thus interpret τ_{it} as an island-specific distortionary tax and T_t as the lump-sum transfers needed to balance the budget. Alternatively, we can consider a variant of our model with monopolistic labor markets as in Blanchard and Kiyotaki (1987), in which case τ_{it} could re-emerge as an island-specific markup between the wage and the marginal revenue product of labor. In line with much of the DSGE literature, we can thus introduce exogenous variation in $1 - \tau_{it}$ and interpret this variation as shocks to the "labor wedge".

Finally, the output of firm j on island i during period t is given by

$$y_{ijt} = A_{it} n_{ijt}$$

where A_{it} is the island-specific TFP, and the firm's realized profit is given by $\pi_{ijt} = p_{ijt} y_{ijt} - w_{it} n_{ijt}$.

Information structure. Different authors have motivated informational frictions on the basis of either market segmentation (Lucas, 1972; Lorenzoni, 2010; Angeletos and La'O, 2013) or some form of inattention (Sims, 2003, Mankiw and Reis, 2002, Woodford, 2002). In either case, the key friction is an agent-specific measurability constraint, reflecting the dispersed private information upon which certain economic decisions are conditioned. In this paper we wish to understand the welfare effects of relaxing this constraint, not its micro-foundations. Furthermore, we seek to isolate the impact of this constraint on *coordination*, or equivalently the role of strategic uncertainty.

With these points in mind, we assume that the firms and workers of any given island know the local fundamentals, but have incomplete information about the aggregate state of the economy. Following Morris and Shin (2002) and Angeletos and Pavan (2007), we then model the available information as a combination of private and public signals. The details, and a justification, are provided in Section 4. For now, we note that the results of Section 3 use only the weaker assumption that the stochastic structure is Gaussian.

3 Equilibrium, Welfare, and Coordination

The equilibrium is defined in a familiar manner: prices clear markets and quantities are optimal given the available information. Following the same steps as in Angeletos and La'O (2009),[4] one can show that equilibrium output is pinned down by the following fixed-point relation:

$$\chi_{it} V' \left(\frac{y_{it}}{A_{it}} \right) = \frac{1}{\mathcal{M}_{it}} \mathbb{E}_{it} \left[U' \left(Y_t \right) \left(\frac{y_{it}}{Y_t} \right)^{-\frac{1}{\rho}} \right] A_{it}, \tag{1}$$

where $\mathcal{M}_{it} \equiv \frac{1}{1-\tau_{it}} \frac{\eta_{it}}{\eta_{it}-1}$ measures the overall wedge due to monopoly power, taxes, and/or labor-market distortions, $\mathbb{E}_{it}$ denotes the expectation conditional on the information that is available to island i, and Y_t denotes aggregate output (with $Y_t = C_t$, since there is no capital).

In the absence of informational friction, condition (1) holds without the expectation operator; in its presence, equilibrium outcomes diverge from their complete-information counterparts insofar as aggregate output, Y_t, is not commonly known. This underscores how the real rigidity we accommodate in this paper inhibits the coordination of production. Building on this observation, the following lemma helps reveal a formal connection between the *positive* properties of our model and those of the class of beauty-contest games studied by Morris and Shin (2002), Angeletos and Pavan (2007), and Bergemman and Morris (2012).

Lemma 1. *The equilibrium level of output is pinned down by the following fixed-point relation:*

$$\log y_{it} \ = \phi_0 + \phi_a a_{it} + \phi_\mu \mu_{it} + \alpha \, \mathbb{E}_{it} \left[\log Y_t \right] \tag{2}$$

where ϕ_0*,* $\phi_a > 0$*, and* $\phi_\mu < 0$ *are scalars,* $a_{it} \equiv \log A_{it} - \frac{1}{1+\epsilon} \log \chi_{it}$ *and* $\mu_{it} \equiv \log \mathcal{M}_{it}$ *capture the local shocks, and*

$$\alpha \equiv \frac{1 - \rho \gamma}{1 + \rho \epsilon} < 1. \tag{3}$$

Condition (2), which is a log-linear transformation of condition (1), is formally identical to the best-response condition that characterizes the aforementioned class of beauty-contest games. Under these lenses, the scalar α identifies the degree of strategic complementarity and encapsulates the *private* value of coordination: it measures how much the players in the game (the firms in our

[4] The characterization of the equilibrium, and a variant of Lemma 1 below, can be found also in Angeletos and La'O (2009). Our contribution starts with the welfare decomposition in Proposition 1.

model) care to align their actions (their production levels). In an abstract game, this value can be a "free variable"; in a particular application, it is dictated by the underlying micro-foundations.

In our setting, α in pinned down by the balance of two forces. On the one hand, an increase in aggregate income raises the demand faced by each firm, which stimulates firm profits, production, and employment; this effect captures the "aggregate demand externality". On the other hand, an increase in aggregate income discourages labor supply and raises real wages, which has the opposite effect on firm profits, production, and employment. In our view, the most plausible scenario is one in which the former effect dominates, so that $\alpha > 0$. To simplify the exposition, the discussion in the main text and the comparative statics of volatility and dispersion in Propositions 2 and 3 focus on this case. However, our key welfare results (Theorems 1, 2, and 3) and the entire analysis in the Appendix dispense with this assumption.

Lemma 1 permits one to characterize the *positive* properties of our baseline model as a direct translation of the positive properties of the aforementioned class of beauty-contest games. For example, one can readily show that, other things equal, a higher α maps to higher sensitivity of equilibrium production to noisy public news, mirroring a similar result from Morris and Shin (2002). Alternatively, following Bergemman and Morris (2012), one could show that the entire set of equilibrium allocations that obtain under arbitrary Gaussian information structures can be spanned with the simpler signal structures we specify in the next section.

None of these facts, however, informs us about the *normative* properties of our model. To understand these properties, we start by developing a certain decomposition of the welfare losses that obtain in equilibrium relative to the first best. Thus let y_{it}^* and Y_t^* denote the first-best levels of, respectively, local and aggregate output. Next, define the local and aggregate output gaps by, respectively, $\log \widetilde{y}_{it} \equiv \log y_{it} - \log y_{it}^*$ and $\log \widetilde{Y}_t \equiv \log Y_t - \log Y_t^*$. Finally, let

$$\Sigma \equiv \text{Var}\left(\log \widetilde{Y}_t\right) \qquad \text{and} \qquad \sigma \equiv \text{Var}\left(\log \widetilde{y}_{it} - \log \widetilde{Y}_t\right).$$

measure, respectively, the volatility of the aggregate output gap and the cross-sectional dispersion in local output gaps. We can the reach the following characterization of equilibrium welfare.

Proposition 1. *There exists a decreasing function w, which is invariant to the information structure, such that equilibrium welfare is given by*

$$\mathcal{W} = \Delta w\left(\Lambda\right)$$

where Δ depends only on the mean value of aggregate output and where

$$\Lambda = \Sigma + \frac{1}{1-\alpha}\sigma, \tag{4}$$

with $\alpha < 1$ defined as in (3).

This result and a set of companion results we provide in Section 5 extend the kind of welfare decompositions that are familiar from the New-Keynesian framework (Woodford, 2003, Gali, 2008) to the incomplete-information economies we are interested in. While these decompositions need not be surprising on their own right, they facilitate two purposes. First, they help identify the different channels through which information can affect welfare. Second, they complete the mapping between the macroeconomic models of interest and the abstract games studied in Morris and Shin (2002), Angeletos and Pavan (2007), and Bergemman and Morris (2012), thus also clarifying whether there is any discrepancy between the private and the social value of coordination in these models.

The first point will become evident as we proceed, especially once we add nominal rigidity. To understand the second point, consider any of the games studied in the aforementioned papers and momentarily let Σ and σ stand for the volatility and the dispersion of the gaps between the equilibrium and the first-best actions in that game. Following Angeletos and Pavan (2007), the combined welfare loss due these gaps can be shown to be proportional to the following sum:

$$\Lambda = \Sigma + \frac{1}{1-\alpha^*}\sigma,$$

where α^* is a scalar that depends on the payoff structure of the game and that encapsulates the *social* value of coordination.[5] In general, this scalar may defer from the one that measures the degree of strategic complementarity, reflecting a divergence between private and social motives to coordinate. In the light of Proposition 1, however, our economy maps to a game in which $\alpha = \alpha^*$, meaning that there is no such divergence.[6]

Fact. *In our setting, the private value of coordination coincides with its social counterpart.*

This fact underscores a crucial difference between our setting and that of Morris and Shin (2002): the latter study a game in which $\alpha > 0$ but $\alpha^* = 0$, meaning that coordination is socially wasteful. As we emphasize in the sequel, the same fact is also key to understanding why the *combined* effect of information on Λ turns out to be unambiguous, even though its component effects on volatility and dispersion are ambiguous and often in conflict to one another.

4 The effects of information on volatility, dispersion, and welfare

In this section, we characterize the comparative statics of the volatility measure Σ, the dispersion measure σ, and overall welfare $\mathcal{W}$ with respect to the information structure. We do so by distinguishing two polar cases. In the first, the underlying fundamental uncertainty is over technology or preferences. In the second, it is over monopoly power or labor wedges. The first case helps us

[5] Formally, α^* is defined as the degree of strategy complementarity in a fictitious game whose equilibrium strategy coincides with the strategy that maximizes welfare in the economy under consideration; it therefore reflects how much agents *should* care to coordinate, as opposed to how much they actually do in equilibrium.

[6] The exact mapping between our model and the framework of Angeletos and Pavan (2007) is complicated by two facts. First, our model is linear-quadratic in *logs* rather than levels. Second, the term Δ may depend on the information structure. The first difference is inconsequential. The second matters only in the case of markup shocks: see Proposition 5 and the discussion surrounding it.

capture the scenario in which the business cycle would have been efficient had information been complete; in this case, Σ and σ are non-zero only due to the incompleteness of information. The second case permits us to study the scenario in which the business cycle originates from distortions in product and labor markets; in this case, Σ and σ reflect the combination of the informational friction with such distortions.

Efficient fluctuations. In this part, we fix $\mathcal{M}_{it} = \bar{\mathcal{M}}$ for all (i,t) and concentrate on the case in which the business cycle is driven by technology shocks; the case of preference shocks is formally identical in terms of welfare properties. The fact that the monopoly (or labor) distortion is fixed guarantees that Δ is also fixed. It follows that the welfare effects of information are captured solely by Λ, the combined effect of volatility and dispersion.

To facilitate sharp comparative statics, we specify the stochastic structure of the economy as follows. First, we let local productivity be $a_{it} \equiv \log A_{it} = \bar{a}_t + \xi_{it}$, where $\bar{a}_t$ is the aggregate productivity shock and ξ_{it} is an idiosyncratic productivity shock. The aggregate shock $\bar{a}_t$ is i.i.d. over time, drawn from $\mathcal{N}(0,\sigma_a^2)$, while the idiosyncratic shock ξ_{it} is i.i.d. across both t and i, independent of $\bar{a}_t$, and drawn from $\mathcal{N}(0,\sigma_\xi^2)$.[7] Next, we summarize all of the private (local) information of island i regarding the underlying aggregate shock $\bar{a}_t$ in an island-specific signal x_{it} given by

$$x_{it} = \bar{a}_t + u_{it}, \tag{5}$$

where the noise term u_{it} is i.i.d. across i and t, orthogonal to $\bar{a}_t$, and drawn from $\mathcal{N}(0,\sigma_x^2)$.[8] Similarly, we summarize all of the public (aggregate) information in a public signal z_t given by

$$z_t = \bar{a}_t + \varepsilon_t, \tag{6}$$

where the noise term ε_t is i.i.d. across t, orthogonal to all other shocks, and drawn from $\mathcal{N}(0,\sigma_z^2)$. Finally, to ease notation, we let $\kappa_a \equiv \sigma_a^{-2}$, $\kappa_\xi \equiv \sigma_\xi^{-2}$, $\kappa_x \equiv \sigma_x^{-2}$, and $\kappa_z \equiv \sigma_z^{-2}$.

The subsequent analysis focuses on the comparative statics of the equilibrium volatility, dispersion, and welfare with respect to the scalars κ_x and κ_z, which measure the precisions of, respectively, the available private and public information. When interpreting our results, however, it is worth keeping in mind the following point. As noted before, the results of Bergemman and Morris (2012) guarantee that the equilibrium allocation obtained by *any* Gaussian information structure can always be replicated with an information structure like the one specified above. This means that the adopted specification is without serious loss of generality and that the scalars κ_x and κ_z represent more generally a convenient parameterization of the overall precision of the available information

[7] Throughout, $\mathcal{N}(\mu,\sigma^2)$ denotes the Normal distribution with mean μ and variance σ^2.

[8] Note that local productivity is itself a private signal of aggregate productivity. The sufficient statistic x_{it} is meant to include this information. More precisely, $x_{it} \equiv (1-\omega)a_{it} + \omega x'_{it}$, where: $x'_{it} = \bar{a} + u'_{it}$ is a signal that captures any private information *other* than the one contained in local productivity; u'_{it} is the noise in that signal, which is i.i.d. across i and t, orthogonal to $\bar{a}_t$ and ξ_{it}, and drawn from $\mathcal{N}(0,\sigma_x'^2)$; $\sigma_x^{-2} \equiv \sigma_\xi^{-2} + \sigma_x'^{-2}$; and $\omega \equiv \sigma_x'^{-2}/\sigma_x^{-2}$.

and of the extent to which this information is commonly shared.[9]

We start by studying the comparative statics of Σ and σ.

Proposition 2. *Suppose $\alpha > 0$. (i) An increase in κ_z necessarily reduces dispersion σ, whereas it reduces volatility Σ iff κ_z is high enough. (ii) Symmetrically, an increase in κ_x necessarily reduces Σ, whereas it reduces σ iff κ_x is high enough.*

To understand part (i), note that an increase in the precision of public information induces firms and workers to reduce their reliance on their private signals, which in turn reduces the contribution of idiosyncratic noise to cross-sectional dispersion. At the same time, because these agents increase their reliance on public signals, the contribution of public information to aggregate output gaps is ambiguous: the reduction in the level of the noise itself tends to reduce Σ, while the increased reaction of the agents tends to raise Σ. Which effect dominates depends on how large the noise is, which explains part (i). The intuition for part (ii) is symmetric.

While the effects of the information structure on the volatility and dispersion components of welfare are ambiguous in general, the *combined* welfare effect is not: as we show in the Appendix, Λ necessarily decreases with either κ_x or κ_z. This is thanks to the coincidence of the social and private values of coordination: if the scalar that governs the relative contribution of volatility and dispersion in Λ were different from the one that governs the strategic complementarity—as, for example, happens in Morris and Shin's (2002) game—then either private or public information could have had a detrimental welfare effect. Along with the fact that Δ is constant, this gives us the following result.

Theorem 1. *Welfare necessarily increases with the precision of either public or private information, no matter the value of α. Moreover, when $\alpha > 0$, the marginal impact of public information on welfare increases with α.*

Under the lenses of the mapping we have developed between our model and the class of games in Angeletos and Pavan (2007), the above result can be seen as a variant of Proposition 6 in that paper: when the level of the monopoly distortion is fixed, Fact 1 implies that our economy maps to a game in the equilibrium use of information is efficient, guaranteeing that welfare improves with either type of information. However, this conclusion could not have been reached on the basis of that paper alone, because that paper does not recognize that the micro-foundation of workhorse macroeconomic models do not imply a divergence between private and social motives to coordinate. In fact, borrowing certain intuitions from Hellwig (2005), Angeletos and Pavan (2007) speculate *incorrectly* that such a divergence obtains generally in these models from the CES specification of preferences, whereas, as we will clarify later on, such divergence is active in the particular exercise conducted in Hellwig (2005) because, and only because, of a particular friction in monetary policy.

[9] Formally, the sum $\kappa_x + \kappa_z$ governs the precision of the firms' forecasts of the underlying technology shock, whereas the ratio κ_z/κ_x governs the correlation of the forecast errors across agents.

Inefficient fluctuations. We now shift focus to the case of inefficient fluctuations, which we capture with shocks to monopoly markups and/or labor-wedges. We thus fix $A_{it} = \chi_{it} = 1$ for all i, t and let the log of the local wedge be given by

$$\mu_{it} \equiv \log \mathcal{M}_{it} = \bar{\mu}_t + \xi_{it},$$

where $\bar{\mu}_t$ is the aggregate markup (or labor-wedge) shock and ξ_{it} is a purely idiosyncratic shock. The former is i.i.d. across t, drawn from $\mathcal{N}(0, \sigma_\mu^2)$; the latter is i.i.d. across both t and i, independent of $\bar{\mu}_t$, and drawn from $\mathcal{N}(0, \sigma_\xi^2)$. Finally, the information structure is modeled in the same way as in the previous section: the available signals are given by (5) and (6), replacing $\bar{a}_t$ with $\bar{\mu}_t$.

In the case studied in the previous section, equilibrium allocations could fluctuate away from the first best *only* because of the incompleteness of information about the state of the economy. Here, by contrast, the *entire* variation in equilibrium allocations represents a deviation from the first best, no matter whether this variation originates in the noise or the fundamentals themselves. The comparative statics of the resulting aggregate and cross-sectional gaps are described below.

Proposition 3. *Suppose $\alpha > 0$. (i) Volatility Σ increases with either κ_x or κ_z. (ii) Dispersion σ decreases with κ_z, and is generally non-monotone in κ_x.*

In spite of these ambiguous and conflicting component effects, the micro-foundations of our setting guarantee that the overall welfare effects are once again unambiguous—but now of the opposite sign than in the case of technology shocks.

Proposition 4. *The combined welfare loss due to volatility and dispersion, Λ, increases with either κ_x or κ_z, no matter α.*

As before, it is useful to relate our findings to those Angeletos and Pavan (2007). Corollary 9 of that paper uses an abstract example in which $\alpha^* = \alpha = 0$ to illustrate the basic insight that information can be detrimental for welfare when it regards shocks that only move the complete-information equilibrium away from the first best. However, by leaving open the question of whether $\alpha^* \neq \alpha$ in workhorse macroeconomic models, that paper also left open the door for information about markup shocks to have ambiguous welfare effects. The sharpness of the welfare effects documented in the above proposition and throughout our paper therefore hinges on the property that, for the economies of interest, the equilibrium and the socially optimal degrees of coordination coincide.

A similar point applies to Beariswyl and Conrand (2010). In one of their benchmarks (Section 4.3), they document that private information about markup shocks has an ambiguous welfare effect and they explain this finding though the conflicting effects that information can have on the volatility and dispersion terms of welfare. In light of our results, that explanation seems unsatisfactory: even though our analysis features similarly ambiguous and conflicting effects on volatility and dispersion, we obtain an unambiguous combined effect. Instead, as we explain in the sequel, the ambiguity stems exclusively from a certain friction in the conduct of monetary policy.[10]

Moving on, we note that welfare depends, not only on the second-order term Λ, which we characterized above, but also on the first-order term Δ, which remains to be studied. In the previous section, this step was redundant because Δ was pinned down by the mean wedge $\bar{\mathcal{M}}$ and was invariant to the information structure. Here, instead, Δ varies with the level of noise. In fact, the effect of information through Δ turns out to be of the exact opposite sign of the combined volatility-and-dispersion effect documented above.

Proposition 5. *The welfare loss due to the mean distortion, which is captured by Δ, decreases with either κ_x or κ_z, no matter α.*

This finding can be explained as follows. The mean level of employment and output decreases with the strategic uncertainty firms and workers face when making their choices. This effect captures a form of risk aversion and is present irrespective of the nature of the underlying aggregate shocks. The welfare consequences of this effect, however, crucially hinge on the nature of the shocks. In the case of technology shocks, the equilibrium use of information is socially optimal. By the same token, the aforementioned effect does not represent a distortion. In the case of markup or labor-wedge shocks, instead, the planner would prefer the agents not to respond to the underlying uncertainty. The aforementioned effect is thus associated with a first-order welfare loss. By reducing the uncertainty that firms and workers face, more information also reduces this kind of distortion, which explains the above result.

The fact that information has conflicting effects through Λ and Δ implies that the overall welfare effect is ambiguous in general. However, one may expect the Λ term to dominate when the mean distortion in the level of economic activity is small enough. Furthermore, because the equilibrium degree of coordination is socially optimal, one may expect the effects of private information to be symmetric to those of public information. The following result verifies these intuitions.

Theorem 2. *Welfare decreases with the precision of either public or private information if and only if the mean wedge $\bar{\mu}$ is low enough.*

The related literature has focused exclusively on the volatility and dispersion effects, thus ignoring the first-order effect we have characterized here. Although we find this effect to be of its own interest, we abstract from it in the rest of the paper and, instead, extend the analysis in the direction of adding nominal rigidity.

5 Nominal rigidity and monetary policy

In the preceding analysis we isolated the role of the informational friction as a source of real rigidity. We now extend the analysis to the more realistic scenario in which the informational friction is also

[10] Having said that, we should clarify that the core contribution of Beariswyl and Conrand (2010) rests on a different matter: a certain signaling mechanism, which is beyond the scope of our paper. See also the discussion in **??**

a source of nominal rigidity: firms set their nominal prices on the basis of the kind of noisy private and public signals that were feature in our preceding analysis.

Setup. As usual, the introduction of nominal rigidity requires that we also introduce a margin of adjustment in quantities: at least one input must be free to adjust to the realized level of demand, or else supply would could fail to meet demand at the posted prices. Accordingly, we allow for two types of labor: one that is chosen on the basis of incomplete information, thus preserving the type of real rigidity that was at the core of our baseline model; and another that adjusts freely to the underlying state of nature, thus preserving market clearing in the presence of the nominal rigidity.

More specifically, we assume that the output of the typical firm in island i is now given by

$$y_{it} = A_{it} n_{it}^{\theta} \ell_{it}^{\eta}$$

where n_{it} is the labor input that is chosen the basis of incomplete information (as in our baseline analysis), ℓ_{it} is the alternative input that adjusts to the realized state (so that markets can clear), and θ, η are positive scalars, with $\theta + \eta \leq 1$. One may think of n_{it} as bodies of employed workers whom the firm hires on the basis of incomplete information and of ℓ_{it} as labor utilization, overtime work, or other margins that adjust to realized demand. The precise interpretation of these inputs, however, is not essential. Rather, the key is that this specification helps parameterize the welfare significance of the real rigidity by θ: a lower θ means that a smaller fraction of output is fixed on the basis of incomplete information. The scalar η, on the other hand, parameterizes the margin of adjustment in quantities in the presence of nominal rigidity.

We next let the per-period utility of the representative household be

$$\tfrac{1}{1-\gamma} C_t^{1-\gamma} - \tfrac{1}{1+\epsilon} \int_I n_{it}^{1+\epsilon} di - \tfrac{1}{1+\zeta} \int_I \ell_{it}^{1+\zeta} di,$$

where ϵ and ζ parameterize the Frisch elasticities of the two types of labor. To simplify the algebra, and without serious loss, we let $\zeta = \epsilon$. We then collect all the exogenous parameters of the model in the vector $\mathcal{E} \equiv (\gamma, \epsilon, \eta, \theta, \kappa_a, \kappa_\mu, \kappa_x, \kappa_z)$.

To close the model, we need to specify the monetary policy. In general, this opens the door to delicate modeling issues. What is the information upon which the monetary authority acts? What is the policy instrument under its immediate control and how does it affect economic activity? What are the objectives, targets, or policy rules that guide its behavior? How one chooses to answer these questions is bound to affect the welfare properties of the model, because of the non-neutrality of monetary policy. To attain a satisfactory understanding of the welfare effects of information, we bypass any specific answer to the aforementioned questions and, instead, develop a taxonomy that spans directly the set of equilibrium allocations that obtain from arbitrary monetary policies.

Lemma 2. *Suppose that monetary policy takes the form of an arbitrary Gaussian process for the nominal interest rate. In equilibrium, nominal GDP, $M_t \equiv P_t Y_t$, satisfies*

$$\log M_t = \lambda \bar{s}_t + \zeta z_t + m_t, \tag{7}$$

where λ and ζ are scalars, and m_t is a random variable, which is orthogonal to both $\bar{s}_t$ and z_t and is Normally distributed with mean zero and variance σ_m^2. Conversely, for any triplet $(\lambda, \zeta, \sigma_m)$, there exists an interest-rate process such that (7) holds in equilibrium.

The first part of the lemma follows from projecting (regressing) the equilibrium nominal GDP on the fundamental and the public signal, and letting (λ, ζ) be the projection coefficients and m_t the residual; the converse follows from reverse-engineering the required interest-rate process. One may wish to interpret M_t as a policy target, λ as a measure of how much the policy "accommodates" the underlying shock, and m_t as a "mistake" caused by imperfect control of M_t. Alternatively, one may assume that policy is conducted according to a Taylor rule of the form

$$\log R_t = r_z z_t + r_y(\log Y_t + \epsilon_t^y) + r_p(\log P_t + \epsilon_t^p) + \tilde{r}_t,$$

where R_t is the (gross) nominal interest rate, (r_z, r_y, r_p) are policy coefficients, ϵ_t^y and ϵ_t^p are measurement errors in the monetary authority's observation of real output and the price level, and $\tilde{r}_t$ is a monetary shock. In this case, the projection coefficients (λ, ζ) are transformation of the aforementioned policy coefficients, and m_t is a combination of the monetary shock and the aforementioned measurement errors. However, no such particular interpretation is required. The above lemma subsumes the specification of monetary policy and, instead, lets the triplet $(\lambda, \zeta, \sigma_m)$ index the entire set of the stochastic processes for nominal GDP that can emerge in a monetary equilibrium.

A familiar benchmark. With the aforementioned representation at hand, the next lemma identifies a familiar benchmark, which serves as a reference point for our subsequent welfare analysis.

Lemma 3. *For every $\mathcal{E}$, there exists a unique λ^* such that a monetary policy replicates flexible prices if and only if (7) holds with $\lambda = \lambda^*$ and $\sigma_m = 0$.*

This lemma is a special case of a more general result in Angeletos and La'O (2013): just as in the New-Keynesian framework there are monetary policies that can undo the nominal rigidity induced by Calvo-like sticky prices, in the class of incomplete-information models studied in this paper and in the related literature there are monetary policies that can undo the nominal rigidity induced by informational frictions. These policies presume that the policy maker can observe perfectly the state of the economy and can also control perfectly aggregate demand. They are therefore note meant to be realistic. They are nevertheless a useful reference point, facilitating sharp normative results.

When these policies are in place, information matters for welfare only through the real rigidity. This suggests a possible connection to our baseline analysis, which we formalize in the next lemma.

Proposition 6. *Let $q_{it} \equiv A_{it} n_{it}^{\theta}$ denote the component of output that is determined on the basis of incomplete information, define the corresponding aggregate as*

$$Q_t \equiv \left[\int_I (q_{it})^{\frac{\hat{\rho}-1}{\hat{\rho}}} di \right]^{\frac{\hat{\rho}}{\hat{\rho}-1}},$$

and finally let

$$\hat{\alpha} \equiv \frac{1-\hat{\rho}\,\hat{\gamma}}{1+\hat{\rho}\,\hat{\epsilon}} < 1. \tag{8}$$

where $\hat{\rho} > 0, \hat{\epsilon} > 0$, and $\hat{\gamma} > 0$ are transformations of the underlying preference and technology parameters (see Appendix for the exact formulas). There exist scalars $\hat{\phi}_a > 0, \hat{\phi}_\mu < 0$, and $\hat{\phi}_{\bar{\mu}}$, and a decreasing function w, such that at the flexible-price allocation the following is true for any information structure:

(i) The equilibrium value of q_{it} is determined by the solution to the following fixed-point relation:

$$\log q_{it} \;=\; \hat{\phi}_a\, a_{it} \,+\, \hat{\phi}_\mu\, \mu_{it} \,+\, \hat{\phi}_{\bar{\mu}}\, \mathbb{E}_{it}[\bar{\mu}_t] \,+\, \hat{\alpha}\, \mathbb{E}_{it}[\log Q_t], \tag{9}$$

(ii) Welfare is given by $w(\Lambda)$, where

$$\Lambda = \Sigma + \frac{1}{1-\hat{\alpha}}\sigma + \omega, \tag{10}$$

where Σ and σ are defined in the same way as in the baseline model, modulo replacing output y with the component q defined above and the scalar α with the scalar $\hat{\alpha}$, and where ω is a scalar that does not depend on either κ_x or κ_z and that vanishes in the absence of markup shocks.

This proposition extends Lemma 1 and Proposition 1 from our baseline model to the flexible-price allocation of the present model. If we compare condition (9) to condition (2), the look-alike condition in our baseline model, we see three differences. First, q_{it} and Q_t have taken the place of, respectively, y_{it} and Y_t. This is because it is only q_{it} (or n_{it}), not y_{it} (or ℓ_{it}), that is restricted to depend on incomplete information. Second, the hatted scalars $\hat{\alpha}$, $\hat{\rho}$, etc have taken the place of the corresponding un-hatted scalars in the baseline model. This reflects the more general specification of preferences and technologies allowed in our extended model. Finally, a new term has emerged: in addition to the firm's own markup, the expected aggregate markup enters the firm's best-response condition. This is because the realized aggregate markup affects the realized aggregate demand for any given Q_t, implying in turn that a firm's optimal choice of q_{it} depends on its expectation of the aggregate markup beyond and above its expectation of Q_t.[11]

A new term shows up also in the definition of Λ. Even if we hold constant the firms' ex-ante input choices (and therefore the allocation of q_{it}), the realized markup distorts the firms' ex-post choices (namely, ℓ_{it}). This explains why Λ contains, not only the terms Σ and σ, but also the term ω in condition (10), which is proportional to the volatility of the markup shock.

Notwithstanding these differences in the micro-foundations and the corresponding interpretations of conditions (9) and (10), the scalar $\hat{\alpha}$ that captures the degree of strategic complementarity in the former condition continues to determine the relative welfare costs of volatility and dispersion in the latter condition. Similarly to our baseline model, this facilitates a direct mapping from the present framework we are studying to the taxonomy of games developed in Angeletos and Pavan (2007) and proves the following point, which has already been anticipated.

[11]Of course, all these differences vanish when $\eta = 0$ and $\theta = 1$, because the extended model then reduces to the baseline one and $q_{it} = y_{it}$.

Fact. *The private and the social value of coordination are perfectly aligned as long as monetary policy replicates flexible prices.*

Since the present framework nests the models studied in Woodford (2002), Hellwig (2005), Welsh (2007) and Bariswyl and Conrand (2010), this also implies that any misalignment of these values in these prior works derives exclusively from a deviation of monetary policy from the benchmark of replicating flexible prices—not from the micro-foundations of preferences and technologies, as instead suggested by Hellwig (2005) and Angeletos and Pavan (2007).

Furthermore, the flexible-price allocation of the present framework leads to the same answer to the question of interest as the baseline model. To understand why, note first that, insofar monetary policy replicates flexible prices, the informational friction has no bite on ℓ_{it} conditional on q_{it}: monetary policy guarantees that the ex-post input choice adjusts to the realized supply and demand conditions *as if* information were complete.[12] The ω term in (10) is therefore invariant to κ_x and κ_z, and can thus be ignored for our purposes. Consider now the case of technology shocks. In this case, q solves *exactly* the same fixed-point relation as y did in the baseline model, guaranteeing that the mapping from the information structure to the equilibrium values of Σ, σ, and Λ is also the same, modulo the replacement of the scalars α and ϕ_a with their hatted counterparts. When instead the business cycle is driven by markup shocks, the presence of the aggregate markup in condition (9) breaks the exact mathematical equivalence between the two models. Nevertheless, because this term represents only an additional wedge from to the first best, its presence only reinforces the adverse welfare effects of information that were obtained in the baseline model. We thus arrive to the following extension of Theorems 1 and 2.

Theorem 3. *Suppose that monetary policy replicates flexible prices. Welfare increases with both private and public information in the case of technology shocks, and decreases with both of them in the case of markup shocks.*

The sign of the overall welfare effect of either type of information is therefore determined solely by the nature of the underlying business-cycle forces—not by preference and technology parameters or any apparent trade-off between volatility and dispersion. As in the baseline model, this sharpness owes to the coincidence of the private and the social value of coordination. It also contrasts with some of the ambiguities reported in prior work, a point which we revisit in the next section.

Away from the benchmark. We now turn to policies, and equilibria, that deviate from the benchmark of replicating flexible prices. Following Lemma 2, any such deviation can contain at most three components: one that is correlated with the underlying fundamentals; one that is correlated with the public signal; and a residual one. The second component has no welfare consequences, reflecting the fact that the response of monetary policy to public information is perfectly, and

[12]This is akin to our earlier point that the informational friction matters in the prior work only because monetary policy deviates from flexible prices.

commonly, anticipated when firms set prices. We thus reach the following decomposition of welfare for *any* monetary policy.

Proposition 7. *There exists a decreasing function w such that, for any monetary policy and any information structure, the equilibrium level of welfare is given by*

$$\mathcal{W} = w(\Lambda + \mathcal{K} + \mathcal{T}),$$

where the following are true:

(i) Λ is the welfare loss at the flexible-price allocation.

(ii) $\mathcal{K}$ is the welfare effect of $\lambda \neq \lambda^*$, which can be expressed as follows:

$$\mathcal{K} = \mathcal{K}(\lambda) \equiv \begin{cases} \Theta(\lambda - \lambda^*)^2\sigma_a^2 & \textit{in the case of technology shocks} \\ -2\Theta_1(\lambda - \lambda^*)\sigma_\mu^2 + \Theta_2(\lambda - \lambda^*)^2\sigma_\mu^2 & \textit{in the case of markup shocks} \end{cases} \tag{11}$$

where Θ, Θ_1, and Θ_2 are scalars, which depend on (κ_x, κ_z) and vanish as $\kappa_x \to \infty$ or $\kappa_z \to \infty$.

(iii) $\mathcal{T}$ is the welfare loss caused by $\sigma_m \neq 0$.

This result complements Proposition 1 from our baseline analysis and formalizes the sense in which the two forms of rigidity map into two channels through which information affects welfare: the role of the real rigidity is captured by Λ; the additional effect of the nominal rigidity is captured by the sum $\mathcal{K} + \mathcal{T}$. This sum is non-zero only insofar as monetary policy deviates from the benchmark of replicating flexible prices. By contrast, Λ is necessarily positive.[13]

As already mentioned, $\mathcal{T}$ captures the welfare consequences of the component of any given deviation that is orthogonal to the underlying fundamental and that can potentially be interpreted as a monetary shock. Whatever its interpretation, the fact that, by construction, m_t is orthogonal to $\bar{s}_t$ implies that $\mathcal{T}$ is necessarily non-negative, as well as that it is independent of the available information about $\bar{s}_t$.By contrast, $\mathcal{K}$ depends on that information precisely because it captures the deviations that are correlated with $\bar{s}_t$. Furthermore, both the sign and the comparative statics of this term with respect to the available information depend on the nature of the underlying business-cycle forces.

From part (ii), we see that the minimum of $\mathcal{K}$ is zero and it is attained at $\lambda = \lambda^*$ when the business cycle is driven by technology shocks, whereas it is positive and it is attained at $\lambda \neq \lambda^*$ when the business cycle is driven by markup shocks. This verifies that the following policy lesson extends from the New-Keynesian setting to the present framework:

Fact. *A monetary policy that replicates flexible prices is optimal in the case of technology shocks, but not in the case of markup shocks.*

Notwithstanding the apparent similarity to the New-Keynesian framework, the following difference is worth mentioning in the case of technology shocks: unless $\theta = 0$, replicating flexible

[13] With the exemption, of course, of the extreme case in which $\theta = 0$ (no real rigidity), in which case $\Lambda = 0$.

prices does *not* implement the first-best allocation, nor is it synonymous to targeting price stability (Angeletos and La'O, 2013). Turning to the case of markup shocks, certain deviations from the flexible-price allocation are welfare-improving because they substitute for the missing tax instrument, namely the state-contingent subsidy that would have offset the markup shock. The only key difference from the New-Keynesian framework then is that, since the nominal rigidity originates in an informational friction rather that Calvo-like sticky prices, the ability of the monetary authority to counter the markup shock hinges on its ability to respond to information that is not available to the firms when the latter set their prices. This suggests that more precise information in the firms' hands may contribute towards lower welfare, not only by reducing the "base" level of welfare that obtains in the flexible-price allocation, but also by limiting the ability of the monetary authority to counteract the markup shock. As it turns out, this intuition is only partially correct.

Lemma 4. *Suppose the business cycle is driven by markup shocks. The optimal policy corresponds to* $\lambda = \lambda^{**}$ *and* $\sigma_m = 0$*, where* $\lambda^{**} \equiv \arg\min_{\lambda} \mathcal{K}(\lambda)$*. Let* $K(\kappa_x, \kappa_z)$ *be this minimum.*

(i) Suppose $\theta = 0$*. Then,* $K(\kappa_x, \kappa_z)$ *is increasing in both* κ_x *and* κ_z

(ii) Suppose $\theta > 0$ *and let* $\bar{K}(\kappa, r)$ *be the function defined by* $K(\kappa_x, \kappa_z)$ *along the locus in which* $\kappa_x + \kappa_z = \kappa$ *and* $\frac{\kappa_x}{\kappa_\mu + \kappa_z} = r$ *Then* $\bar{K}(\kappa, r)$ *increases in* κ*.*

(iii) Suppose $\theta > 0$*. There are values of the preferences and technology parameters for which* $K(\kappa_x, \kappa_z)$ *is non-monotone in either* κ_x *or* κ_z*.*

Part (i) verifies the aforementioned intuition in the limit case that shuts down the real rigidity. Part (ii) shows that the intuition applies more generally if the improvement in the available information comes without a change in its composition (meaning an increase in the overall precision, κ, without a change in the relative precision, r). Part (iii) then qualifies the intuition by showing that an increase in the precision of either private or public information can have a non-monotone effect on the welfare gain that the optimal policy attains relative to the flexible-price allocation.

We do not fully comprehend the economics behind this non-monotonicity. Following previous work, one could attribute it to the underlying trade off between volatility and dispersion. In light of our other results, however, such an explanation would be imprecise. At this point, we only know that this non-monotonicity derives from the interaction of the real rigidity with changes in the relative precision of private and public information: if either the real rigidity is shut down, in the sense that $\theta = 0$, or the composition of information is kept constant, in the sense of part (ii) of the above proposition, more information necessarily reduces the aforementioned welfare gain. Noting then that the ratio r defined in part (ii) of the above proposition captures, more generally, the extent to which information is correlated across firms,[14] we reach the following summary.

Theorem 4. *Suppose that monetary policy is optimal.*

(i) When the business cycle is driven by technology shocks, more information improves welfare by, and only by, improving the efficiency of the underlying flexible-price allocation.

[14] See Angeletos and Pavan (2007); the latter refer to this ratio as the "commonality of information".

(ii) When instead the business cycle is driven by markup shocks, more information contributes to lower welfare both by exacerbating the inefficiency of the underlying flexible-price fluctuations and by reducing the ability of the monetary authority to combat these fluctuations. Nevertheless, an ambiguous effect can obtain from the interaction of real rigidity with changes in the extent to which information is correlated.

We wish to emphasize that the above result refers to the solution of a Ramsey problem where the planner is free to select an arbitrary Gaussian process for the interest rate or, equivalently, to induce any triplet $(\lambda, \zeta, \sigma_m)$ he wishes in condition (7). We now discuss what happens when the monetary policy is suboptimal *relative* to this "unconstrained" optimum. This could be because the monetary authority has imperfect control of aggregate demand; because it observes the underlying shocks and/or the endogenous economic outcomes with noise; because its objectives diverge from the welfare criterion in the model; or because of any other reason that is left outside our model.

Not surprisingly, not much can be said if one puts no structure on the aforementioned deviations from optimality: we can always find a λ that turns upside down the welfare effects we have documented so far. To understand the logic, consider the case of technology shocks. When the nominal rigidity is shut down either by assumption or by the optimal monetary policy, the informational friction represents a real distortion that moves the equilibrium away from the first best. Increasing the precision of the available information necessarily reduces the welfare cost of this distortion. When this is the only distortion, more information is unambiguously welfare-improving. But if an additional distortion is present due to the combination of nominal rigidity and suboptimal policy, a second-best result applies: reducing the available information may increase welfare by having the one distortion offset the other.

The opposite scenario, however, is also possible. In fact, we think that this scenario is relevant for the following reason. To the extent that she is guided by the New-Keynesian framework, the policy maker may fail to incorporate how the informational friction affects the nature of the optimal allocation and the corresponding policy targets. In so doing, the policy maker may inadvertently introduce distortions in addition to those induced by the informational friction. But when the latter vanishes, the policy maker's "mistake" also vanishes. Under this scenario, more information helps increase welfare, not only by attenuating the real rigidity, but also by alleviating the policy suboptimality.

We illustrate this logic in Figure 1.[15] This figure reports a numerical simulation of the welfare effects of public information under three alternative specifications of the monetary policy: the optimal policy (solid blue line), a policy that stabilizes the aggregate output gap (dotted red line), and a policy that stabilizes the price level (dashed green line). The last two policies are suboptimal

[15] Figure 1 is based on the following parameterization of the model: $\theta = \eta = 0.5$, $\epsilon = 1$, $\gamma = 0.25$, $\rho = 0.5$, $\sigma_a = \sigma_x = \sigma_z = .02$, and $\sigma_m = \sigma_\epsilon = 0$. The qualitative effects we report in this figure appear to be robust across various plausible parameterizations.

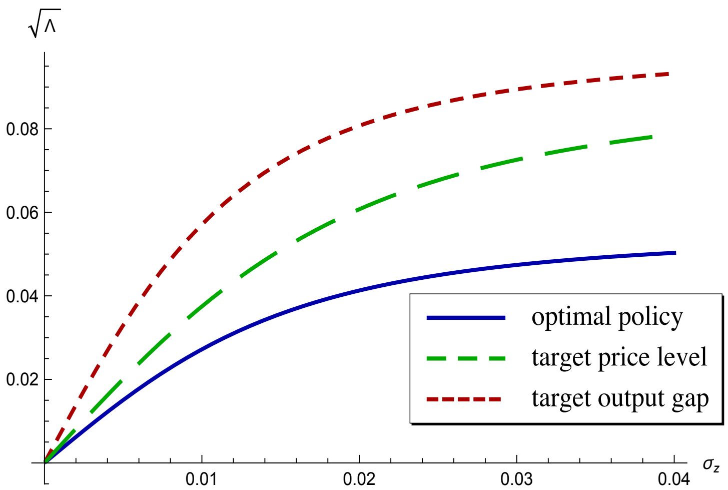

Figure 1: The welfare losses due to incomplete information, under different levels of noise in the public signal and under different policy rules.

in our model due to the presence of the informational friction, but are useful reference points because they are optimal in the prototypical New-Keynesian model and they also seem relevant in practice.[16]

For any level of noise $\sigma_z > 0$, the welfare losses associated with targeting either the price level or the output gap are higher than those associated with the optimal policy. Nevertheless, the welfare gap between these policies and the optimal one decreases with the precision of the available information, and vanishes in the limit as $\sigma_z \to 0$. It follows that, under either of these two policies, more information improves welfare via two effects: by bringing the equilibrium allocation closer to the optimal one; and by raising the welfare attained at the optimal allocation.[17]

Monetary shocks. We conclude this section by studying $\mathcal{T}$, the welfare term corresponding to deviations from the flexible-price benchmark that are orthogonal to the underlying fundamentals. As already noted, these deviations can be interpreted as monetary shocks and the resulting welfare losses are invariant to the available information insofar as the latter regards exclusively the "real" fundamentals, namely preferences, technologies, markups, and wedges. But now suppose that the firms receive signals about the aforementioned kind of monetary shocks. How does this distinct type of information matter for welfare?

The answer to this question is trivial in the case of public information. In equilibrium, firms always coordinate their prices in a manner than undoes completely the real effect of any *commonly* predictable variation in m_t. Furthermore, any residual variation in m_t necessarily contributes to welfare losses, because m_t is, by contraction, orthogonal to the underlying preferences and tech-

[16] For a detailed analysis of how informational frictions impacts the nature of the optimal monetary policy, we refer the reader to Angeletos and La'O (2014).

[17] In line with these numerical findings, one can show that the scalar Θ, which shows up in condition (**??**) and captures the welfare cost of any given deviation between λ and λ^*, vanishes as $\sigma_z \to 0$.

nologies. It follows that more precise public information about m_t necessarily improves welfare (equivalently, it reduces $\mathcal{T}$).

As shown in Hellwig (2005), the answer to the above question is less obvious in the case of private information. Similarly to public information, private information helps dampen the aggregate real effect of any monetary shocks. But unlike public information, private information can also exacerbate the friction in the firms' coordination of their pricing decisions. It follows that more precise private information can have a non-monotone welfare effect. We refer the reader to Hellwig (2005) for a more detailed explanation.[18]

6 Related literature

In this section we discuss in further detail how our paper relates to prior work. In so doing, we illustrate more generally how our analysis can facilitate a useful anatomy of the welfare effects of information in baseline macroeconomic models.

As already noted, prior work such as Hellwig (2005), Welsh (2007), Baeriswyl and Conrand (2010), and Lorenzoni (2010) have ruled out real rigidity, a scenario that is nested in the extended framework of Section 5 by letting $\theta = 0$. In this limit case, a monetary policy that replicates flexible prices implements the complete-information allocation. This is because the absence of real rigidity guarantees that the informational friction ceases to have a bite on real allocations and welfare once the "right" monetary policy is in place. Of course, such a policy may be unrealistic or undesirable. But the point we wish to make, for pedagogical reasons, is the welfare effects reported in prior work hinge on deviations from the benchmark of replicating flexible prices. Some times these deviations are by brute force, others by the delicate workings of other mechanisms.

Consider Hellwig (2005). Here, the deviation is by brute force. First, an exogenous stochastic process is assumed for M_t, the level of nominal GDP. And second, there are not shocks to preferences and technologies, implying that the first-best level of output is constant and that it is attained with a policy that targets a constant M_t. It follows that all the volatility in M_t can be interpreted as a monetary shock of the type described in Lemma 2. As noted in the end of the previous section, the exercise conducted in Hellwig (2005) therefore boils down to studying the comparative statics of $\mathcal{T}$ with respect to the information that firms have about the monetary shock, and the key unexpected finding is that private information has a non-monotone welfare effect. At face value, this may appear to contradict the monotone effects we have documented in this paper under either flexible prices or optimal policy. But there is no contradiction once one recognizes that Hellwig's analysis regards random deviations from these policy benchmarks.

Consider next Welsh (2007). That paper focuses on the question of whether central bank transparency—modeled as the policy maker's revelation of the information she has about the un-

[18] Hellwig (2005) establishes the aforementioned non-monotonicity in a model that shuts down the real rigidity, but the result applies more generally.

derlying state of the economy—is socially desirable. It also allows monetary policy to react systematically to shocks in preferences and markups.[19] A deviation from optimality, however, obtains because of two kinds of policy frictions: measurement error in the monetary authority's observation of the underlying shocks; and the assumption of ad-hoc policy objectives that differ from social welfare. In the light of our analysis, it is then not surprising that Welsh (2007) is unable to obtain sharp theoretical results regarding the question of interest and, instead, resorts on numerical explorations. Perhaps this is the cost of a realistic specification of monetary policy. But if one is willing to abstract from the aforementioned policy frictions, a particularly sharp answer becomes available on the basis of our results: transparency is desirable in the face of benign forces such as technology shocks, opacity is desirable in the face of distortionary forces such as markup shocks.

Consider next Baeriswyl and Cornand (2010). The key contribution of that paper rests on the study of a signaling mechanism, which we describe in the sequel. Before doing that, we wish to highlight a core ingredient of their model: the requirement that nominal GDP, $M_t \equiv P_t Y_t$, satisfy

$$\log M_t = \lambda_a(\bar{a}_t + \epsilon_{a,t}) + \lambda_\mu(\bar{\mu}_t + \epsilon_{\mu,t}), \tag{12}$$

where $\bar{a}_t$ and $\bar{\mu}_t$ are the underlying technology and markup shocks, $\epsilon_{a,t}$ and $\epsilon_{\mu,t}$ are exogenous noises, and λ_a and λ_μ are scalars under the control of the monetary authority.[20] Baeriswyl and Cornand (2010) interpret M_t as the policy instrument, $(\epsilon_{a,t}, \epsilon_{\mu,t})$ as measurement errors in the policy maker's observation of the underlying shocks, and condition (12) as a policy rule. Putting aside any quibbles one may have with this interpretation,[21] condition (12) represents a restriction on the set of implementable allocations that a Ramsey planner can choose from. It therefore drives the optimal policy in that paper away from the unconstrained optimum we have studied in the previous section. Furthermore, condition (12) is nested in the representation we provided in Lemma 2 by letting $m_t = \lambda_a \epsilon_{a,t} + \lambda_\mu \epsilon_{\mu,t}$. That is, the aforementioned kind of measurement errors maps into a particular kind of monetary shock, or "policy mistake", which by itself contributes to welfare losses.

Let us now momentarily abstract from the technology shock. In Section 4.3 of their paper, Baeriswyl and Cornand (2010) report that an increase in the precision of the firms' private information about the markup shock has a *non-monotone* effect on welfare under the optimal policy and attribute this finding to a trade off between volatility. At face value, this finding appears to

[19] Welsh refers to the former type of shock as "demand shocks" and to the latter as "cost-push shocks".

[20] The aforementioned condition can be found in the top of page 686 of Baeriswyl and Cornand (2010). We have only adapted the notation of that condition to make it consistent with the notation we use in the rest of our paper.

[21] We find this interpretation somewhat problematic for two reasons. First, it seems more natural to assume that the policy instrument is the nominal interest rate. But when the latter is set on the basis of noisy signals of the underlying fundamentals, nominal GDP is endogenously determined and need not satisfy condition (12). Second, a realistic specification of monetary policy would probably have to allow the monetary authority to condition the interest rate, not only on exogenous signals of the underlying shocks (which one could interpret as the policy maker's priors of the underlying shocks), but also on signals of endogenous economic outcomes such as GDP and the price level. This would make the information that is available to the monetary authority endogenous to its own choices—an intriguing possibility that has not been addressed.

contradict part (i) of Lemma 4 in our paper. The proposed explanation also feels incomplete, because we obtained an unambiguous overall welfare effects despite a trade-off between volatility and dispersion. The resolution rests on "policy mistakes" induced by the aforementioned measurement error. In its absence, our result guarantees an unambiguously negative welfare effect. But once the measurement error is present, a countervailing effect emerges. Because an increase in the precision of the firms' information reduces the efficacy of countercyclical monetary policy, it induces the monetary authority to respond less to its own noisy signal. As this happens, the volatility of the (endogenous) monetary shock gets smaller, which in turn contributes towards higher welfare.

Let us next turn to the core contribution of Baeriswyl and Cornand (2010), which depends on allowing the firms to observe signals of M_t. Baeriswyl and Cornand (2010) interpret the latter as policy signals (signals of action of the policy maker). In the eyes of a firm, such signals contain three kinds of information: information about the underlying technology shock; information about the underlying markup shock; and information about the policy mistake triggered by measurement error. The first kind of information tends to contribute to higher welfare; the second has a non-monotone effect, for the reason just explained above; the third also has a non-monotone effect, for the reason first explained in Hellwig (2005). It follows that the overall welfare effect of the aforementioned policy signals is ambiguous, precisely because these signals confound different kinds of information. More intringingly, the monetary authority can control the extent of this confounding by its choice of the coefficients λ_a and λ_μ. This introduces a delicate trade off between the stabilization role of the monetary policy and its signaling role. The key contribution of Baeriswyl and Cornand (2010) is to identify this trade off and to study how it affects the design of the optimal monetary policy.

The confounding of different kinds of information appears to explain also the result reported in Lorenzoni (2010) regarding the social value of public information. Consider the following two modifications of our framework, which helps capture two essential aspects of Lorenzoni's model: first, allow for firm-specific demand shocks; second, suppose that a firm's uncertainty about its demand is inversely related to the available information about the aggregate technology shock. In the absence of the latter effect, the welfare that obtains at the optimal policy is invariant to information about the aggregate shocks, because of the absence of real rigidity, but depends on the available information about idiosyncratic demand shocks, because their combination with nominal rigidity induces distortions in relative prices. It follows that, once the two types of information are confounded, more information about the aggregate shocks can improve welfare because, and only because, it reduces the latter kind of distortion.[22]

[22] In Lorenzoni (2010), the aforementioned confounding obtains naturally from a segmentation in consumer markets: each firm is matched with an idiosyncratic set of consumers, whose demand is affected by their own imperfect information about the state of the economy; more precise public information about the aggregate technology shock then maps to less demand uncertainty in the eyes of the firm because it reduces the uncertainty faced by its customers. Our discussion abstracted from these micro-foundations only to facilitate a clearer mapping between that paper and our results.

7 Conclusion

By assuming away incomplete information and strategic uncertainty, standard macroeconomic models presume that firms can perfectly coordinate their production and pricing decisions. By contrast, in this paper we allowed an informational friction to inhibit the coordination of these decisions and proceeded to study how this shapes the social value of information within an elementary business-cycle model. The key lessons we reached can be summarized as follows:

- The welfare effects of information can be decomposed in two channels: the real rigidity that emerges as firms make production choices on the basis of incomplete information; and the nominal rigidity that emerges as firms also set prices on the basis of such information.
- The first channel is present irrespectively of the conduct of monetary policy. It also has sharp comparative statics: more information is welfare-improving through this channel if the business cycle is driven by technology shocks (efficient fluctuations), and welfare-deteriorating if the business cycle is driven by markup shocks (inefficient fluctuations).
- By contrast, the second channel hinges on the conduct of monetary policy. Similarly to the New-Keynesian framework, there is a policy that neutralizes the nominal rigidity. At this benchmark, the welfare effects of information are shaped solely by the real-rigidity channel, as described above. Away from it, they hinge on whether the provision of more information dampens or amplifies the deviation, and on whether the deviation was desirable to start with.
- When the business cycle is driven by technology shock, a monetary policy that replicates flexible prices is optimal. When, instead, the business cycle is driven by markup shocks, a deviation from this benchmark is desirable. More information then tends to decrease welfare, not only because it exacerbates the inefficiency of the underlying flexible-price fluctuations, but also because it curtails the monetary authority's ability to combat these fluctuations.

We view the sharpness of these lessons and their connection to familiar normative properties of RBC and New-Keynesian models as the main strengths of our contribution. This sharpness, however, was possible only because of appropriate abstractions. We narrowed the analysis within the context of an elementary micro-founded business-cycle model, or else "anything goes". We also treated the information structure as exogenous, thus bypassing the question of either how the available information is shaped by markets and other social institutions, or what policy instruments can affect it.[23] Perhaps more crucially for applied purposes, we allowed the informational friction to inhibit the coordination of the production and pricing decisions of firms, but assumed away such a frictions in, say, the consumption and saving choices of households or the trades of financial

[23]See, however, Appendix A for a discussion of how our analysis can be extended to settings that endogenize the information structure, with a focus on the role of macroeconomic indicators.

investors. The bite of incomplete information on the social efficiency of the latter kind of economic decisions is an important research question, which is however beyond the scope of this paper.

Appendix A: Endogenous information

We now discuss how our baseline analysis can be adapted to a setting in which the available public signal regards the level of economic activity rather than the underlying fundamental, as it is the case for macroeconomic indicators. Apart from being more realistic, this scenario endogenizes the available public information: the informational content of macroeconomic indicators is endogenous to the behavior of the agents. In so doing, it also adds an informational externality, which of course affects the welfare properties of the model.

To understand whether this interferes with our key lessons, we augment our model with the following indicator of aggregate output:

$$\omega_t = \log Y_t + \varepsilon_t \tag{13}$$

where $\varepsilon_t \sim N\left(0, \sigma_\omega^2\right)$ represents classical measurement error and $\sigma_\omega > 0$ parameterizes the level of this measurement error. We then proceed to study the welfare effects of $\kappa_\omega \equiv \sigma_\omega^{-2}$, which we can think of as a proxy for the precision of macroeconomic statistics.

Suppose that local output is given by

$$\log y_{it} = \varphi_s s_{it} + \varphi_x x_{it} + h_t,$$

where s_{it} is the local productivity or markup shock, x_{it} is the private signal, φ_s and φ_x are known constants, and h_t is an arbitrary function of the available public information. Aggregate output is then given by $\log Y_t = (\varphi_s + \varphi_x)\bar{s}_t + h_t$, where $\bar{s}_t$ is the aggregate productivity or markup shock. Observing the statistic ω_t is thus equivalent to observing the signal ω_t', where

$$\omega_t' \equiv \frac{\omega_t - h_t}{\varphi_s + \varphi_x} = \bar{s}_t + \varepsilon_t', \quad \text{where} \quad \varepsilon_t' \equiv \frac{1}{\varphi_s + \varphi_x}\varepsilon_t.$$

This is akin to the public signal z_t featured in our earlier analysis, except for one key difference: the magnitude of the noise is now inversely proportional to the sum $\varphi_a + \varphi_x$ and, in this sense, the precision of this signal is endogenous to the behavior of the agents.

Notwithstanding this qualification, the available public information can be summarized in a Gaussian sufficient statistic, given by a weighted average of the exogenous signal z_t and the transformed endogenous signal ω_t'. This means that the analysis of Sections **??** and **??** goes through once we replace κ_z with $\widetilde{\kappa}_z$, where

$$\widetilde{\kappa}_z = \kappa_z + (\varphi_a + \varphi_x)^2 \kappa_\omega, \tag{14}$$

is the precision of the aforementioned sufficient statistic. Finally, and importantly, we can show that the equilibrium value of $\widetilde{\kappa}_z$ increases with κ_ω: policies that reduce the measurement error in macroeconomic statistics map into an improvement in the overall quality of the available public information. It then follows directly from Theorems 1 and 2 that such policies are welfare-improving when the business cycle is driven by technology shocks, but can be detrimental when the business cycle is driven by markup shocks.

Now, consider an alternative setting that endogenizes the collection of private information. In this case, increasing the precision of the available public information may crowd out the collection of private information, in a manner similar to Amador and Weill (2011). The welfare effect of public information can then be decomposed in two terms: a direct effect; and an indirect effect due to the endogenous response of κ_x. The overall effect may therefore be ambiguous in general, but in light of our results one at least knows that the sign of the component effects is pinned down by the nature of the business-cycle forces.

Putting aside the specifics of the scenarios discussed above, the broader lesson is the following: although the precise translation of our results to variants of our framework that endogenize the available information depends on the details of how this is done, Theorems 1 and 2 may continue to characterize the social value of information *irrespectively* of where that information originates from.

Appendix B: Proofs

Derivation of equation (1). Let p_{it} be the price index for the consumption basket of the goods produced in island i. The optimal consumption decision satisfy the following conditions:

$$c_{it} = \left(\frac{p_{it}}{P_t}\right)^{-\rho} C_t,$$

for the aforementioned basket; and

$$c_{ijt} = \left(\frac{p_{ijt}}{p_{it}}\right)^{-\eta_{it}} c_{it},$$

for the particular good produced by firm j in island i. In equilibrium, consumption levels coincide with production levels. It follows that the inverse demand function faced by firm j in island i is given by

$$p_{ijt} = D_{it}\; y_{ijt}^{-\frac{1}{\eta_{it}}}, \tag{15}$$

where

$$D_{it} \equiv p_{it} y_{it}^{\frac{1}{\eta_{it}}} = P_t Y_t^{\frac{1}{\rho}} y_{it}^{\frac{1}{\eta_{it}} - \frac{1}{\rho}}$$

is exogenous to the firm but endogenous to the island.

Consider now the optimal behavior of the aforementioned firm. Given that marginal value of (nominal) income for the representative household is $U'(Y_t)/P_t$, the firm's objective is simply the local expectation of its profit times $U'(Y_t)/P_t$. Using (15), this can be expressed as follows:

$$\mathbb{E}_{it}\left[\frac{U'(Y_t)}{P_t}\left(D_{it} y_{ijt}^{1-\frac{1}{\eta_{it}}} - w_{it} n_{ijt}\right)\right]$$

Using $y_{ijt} = A_i n_{ijt}$ and taking the FOC with respect to n_{ijt} gives

$$\mathbb{E}_{it}\left[\left(1 - \frac{1}{\eta_{it}}\right) A_{it} U'(Y_t)\frac{D_{it} y_{ijt}^{-\frac{1}{\eta_{it}}}}{P_t} - U'(Y_t)\frac{w_{it}}{P_t}\right] = 0.$$

By the fact that all firms within a given island are symmetric, we have that, in equilibrium, $n_{ijt} = n_{it}$, $y_{ijt} = y_{it}$, and $p_{ijt} = p_{it}$. It follows that $D_{it} y_{ijt}^{-\frac{1}{\eta_{it}}} = P_t Y_t^{\frac{1}{\rho}} y_{it}^{-\frac{1}{\rho}}$ and the above condition reduces to

$$\mathbb{E}_{it}\left[U'(Y_t)\frac{w_{it}}{P_t}\right] = \mathbb{E}_{it}\left[\left(1-\frac{1}{\eta_{it}}\right)U'(Y_t)\,Y_t^{\frac{1}{\rho}}y_{it}^{-\frac{1}{\rho}}A_{it}\right]$$

Finally, consider the optimal labor supply in island i. The relevant FOC for the household gives

$$\chi_{it}V'(n_{it}) = (1-\tau_{it})\mathbb{E}_{it}\left[U'(Y_t)\frac{w_{it}}{P_t}\right]$$

Combining the above two conditions and letting $\mathcal{M}_{it} \equiv \frac{1}{1-\tau_{it}}\frac{\eta_{it}}{\eta_{it}-1}$ gives condition (1). ■

Proof of Lemma 1. Taking logs of both sides of (1) and rearranging gives

$$\left(\frac{1}{\rho}+\epsilon\right)\log y_{it} = -\mu_{it} + \log\mathbb{E}_{it}\left[Y_t^{\frac{1}{\rho}-\gamma}\right] + (1+\epsilon)\,a_{it}.$$

Assuming that Y_t is log-normal (verified below) the latter can be rewritten as

$$\left(\frac{1}{\rho}+\epsilon\right)\log y_{it} = -\mu_{it} + \left(\frac{1}{\rho}-\gamma\right)\mathbb{E}_{it}\left[\log Y_t\right] + \frac{1}{2}\left(\frac{1}{\rho}-\gamma\right)^2 Var\left(\log Y_t\right) + (1+\epsilon)\,a_{it}$$

or

$$\log y_{it} = \phi_0 + \phi_a a_{it} + \phi_\mu \mu_{it} + \alpha\mathbb{E}_{it}\left[\log Y_t\right],$$

where

$$\phi_0 \equiv \frac{1}{2}\frac{\rho}{1+\rho\epsilon}\left(\frac{1}{\rho}-\gamma\right)^2 Var\left(\log Y_t\right),\ \phi_a \equiv \frac{\rho\,(1+\epsilon)}{1+\rho\epsilon},\ \phi_\mu \equiv -\frac{\rho}{1+\rho\epsilon},\ \alpha = \frac{1-\rho\gamma}{1+\rho\epsilon}.$$

■

Proof of Proposition 1. Welfare is given by

$$\mathcal{W} = \mathbb{E}\left[\sum \beta^t W_t\right]$$

where

$$W_t \equiv \mathbb{E}\left[\frac{Y_t^{1-\gamma}}{1-\gamma} - \tfrac{1}{1+\epsilon}\int \chi_{it}\left(\frac{y_{it}}{A_{it}}\right)^{1+\epsilon} di\right]$$

measures the period-t unconditional expected utility flow. Clearly, the comparative statics of welfare with respect to the information structure are pinned down by those of W_t. We henceforth focus on the latter. To simplify the notation, we drop the time index t. The log-normality assumption allows us to rewrite the first component of W (the one that corresponds to consumption) as follows:

$$\mathbb{E}\left(Y^{1-\gamma}\right) = \left[\mathbb{E}\left(Y\right)\right]^{1-\gamma}\exp\left\{-\tfrac{1}{2}\gamma\left(1-\gamma\right)Var\left(\log Y\right)\right\}$$

Similarly, letting $b_i \equiv A_i^{1+\epsilon}/\chi_i$, denoting with B the cross-sectional mean of b_i, and noting that $Y \equiv \left(\int y_i^{\frac{\rho-1}{\rho}} di\right)^{\frac{\rho}{\rho-1}}$, we can express the component of W that corresponds to the disutility of leisure as follows:

$$\int \chi_i \left(\frac{y_i}{A_i}\right)^{1+\epsilon} di = \mathbb{E}\left[\int \frac{y_i^{1+\epsilon}}{b_i}\right] = \frac{Y^{1+\epsilon}}{B}\exp(H)$$

where

$$H \equiv \tfrac{1}{2}\left(\epsilon + \tfrac{1}{\rho}\right)(1+\epsilon)Var\left(\log y_i|\Theta\right) + \tfrac{1}{2}Var\left(\log b_i|\Theta\right) - (1+\epsilon)Cov\left(\log y_i, \log b_i|\Theta\right)$$

and where $\Theta \equiv (Y, B)$. Because of the Gaussian specification, the variance and covariance terms that enter in H above are constants (not random variables). It follows that

$$\mathbb{E}\left[\int \chi_i \left(\frac{y_i}{A_i}\right)^{1+\epsilon} di\right] = \mathbb{E}\left[\frac{Y^{1+\epsilon}}{B}\right]\exp(H) = \frac{\mathbb{E}[Y]^{1+\epsilon}}{\mathbb{E}[B]}\exp(G)$$

where

$$G \equiv H + \tfrac{1}{2}\epsilon(1+\epsilon)Var\left(\log Y\right) + Var\left(\log B\right) - (1+\epsilon)Cov\left(\log Y, \log B\right).$$

Following a similar step for the consumption term, we infer that

$$W = \tfrac{1}{1-\gamma}\left[\mathbb{E}\left(Y\right)\right]^{1-\gamma}\exp\left\{-\tfrac{1}{2}\gamma\left(1-\gamma\right)Var\left(\log Y\right)\right\} - \tfrac{1}{1+\epsilon}\frac{\left[\mathbb{E}\left(Y\right)\right]^{1+\epsilon}}{\mathbb{E}\left(B\right)}\exp(G).$$

Next, let us define $\hat{Y}$ as the value of $\mathbb{E}(Y)$ that maximizes the aforementioned expression for W, taking as given B, G, and $Var(\log Y)$. Clearly, this is given by the solution to the following condition:

$$\hat{Y}^{1-\gamma}\exp\left\{-\tfrac{1}{2}\gamma\left(1-\gamma\right)Var\left(\log Y\right)\right\} = \frac{\hat{Y}^{1+\epsilon}}{\mathbb{E}\left(B\right)}\exp(G) \tag{16}$$

We can then restate W as follows:

$$W = \left\{\tfrac{1}{1-\gamma}\left[\frac{\mathbb{E}\left(Y\right)}{\hat{Y}}\right]^{1-\gamma} - \tfrac{1}{1+\epsilon}\left[\frac{\mathbb{E}\left(Y\right)}{\hat{Y}}\right]^{1+\epsilon}\right\}\frac{\hat{Y}^{1+\epsilon}}{\mathbb{E}\left(B\right)}\exp(G)$$

If $\mathbb{E}(Y)$ happens to equal $\hat{Y}$, then $W = \hat{W}$, where

$$\hat{W} \equiv \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}\frac{\hat{Y}^{1+\epsilon}}{\mathbb{E}\left(B\right)}\exp(G). \tag{17}$$

Letting

$$\delta \equiv \frac{\mathbb{E}(Y)}{\hat{Y}} \qquad \text{and} \qquad \Delta \equiv \frac{U(\delta) - V(\delta)}{U(1) - V(1)} = \frac{\tfrac{1}{1-\gamma}\delta^{1-\gamma} - \tfrac{1}{1+\epsilon}\delta^{1+\epsilon}}{\tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}},$$

we conclude that

$$W = \Delta\hat{W}. \tag{18}$$

Δ therefore identifies the wedge between actual welfare and the welfare that would have obtained if a planner had an instrument that permitted him to scale up and down the allocation under consideration by a factor δ and could choose this factor so as to maximize welfare.[24]

Next, from (16) we have that

$$\hat{Y} = [\mathbb{E}(B)]^{\frac{1}{\epsilon+\gamma}} \exp\left\{-\tfrac{1}{\epsilon+\gamma}\left[G + \tfrac{1}{2}\gamma(1-\gamma) Var(\log Y)\right]\right\},$$

which together with (17) gives

$$\hat{W} = \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}[\mathbb{E}(B)]^{\frac{1-\gamma}{\epsilon+\gamma}} \exp\left\{G - \tfrac{1+\epsilon}{\epsilon+\gamma}\left[G + \tfrac{1}{2}\gamma(1-\gamma) Var(\log Y)\right]\right\}$$

Equivalently,

$$\hat{W} = \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}[\mathbb{E}(B)]^{\frac{1-\gamma}{\epsilon+\gamma}} \exp\left\{-\tfrac{1}{2}\tfrac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma}\hat{\Omega})\right\} \tag{19}$$

where

$$\begin{aligned}
\hat{\Omega} \;\equiv\;& \tfrac{2}{1+\epsilon}G + \gamma Var(\log Y) \\
=\;& (\epsilon+\gamma) Var(\log Y) + \tfrac{2}{1+\epsilon} Var(\log B) - 2Cov(\log Y, \log B) \\
&+ \left(\epsilon + \tfrac{1}{\rho}\right) Var(\log y_i|\Theta) + \tfrac{1}{1+\epsilon} Var(\log b_i|\Theta) - 2Cov(\log y_i, \log b_i|\Theta)
\end{aligned}$$

Now, note that the first-best levels of output are given by the fixed-point to the following:

$$\log y_i^* = (1-\alpha)\tfrac{1}{\epsilon+\gamma}\log b_i + \alpha \log Y^*.$$

It follows that, up to some constants that we omit for notational simplicity,

$$\log Y^* = \tfrac{1}{\epsilon+\gamma}\log B \qquad \text{and} \qquad \log y_i^* - \log Y^* = (1-\alpha)\tfrac{1}{\epsilon+\gamma}(\log b_i - \log B)$$

Using this result towards replacing the terms in $\hat{\Omega}$ that involve b_i and B, we get

$$\begin{aligned}
\hat{\Omega} \;=\;& (\epsilon+\gamma) Var(\log Y) + 2\tfrac{(\epsilon+\gamma)^2}{(1+\epsilon)} Var(\log Y^*) - 2(\epsilon+\gamma) Cov(\log Y, \log Y^*) \\
&+ \left(\epsilon + \tfrac{1}{\rho}\right) Var(\log y_i|\Theta) + \tfrac{(\epsilon+\gamma)^2}{(1+\epsilon)(1-\alpha)^2} Var(\log y_i^*|\Theta) - 2\tfrac{\epsilon+\gamma}{1-\alpha} Cov(\log y_i, \log y_i^*|\Theta)
\end{aligned}$$

Furthermore, the first-best level of welfare is given by

$$W^* = \tfrac{\epsilon+\gamma}{(1-\gamma)(1+\epsilon)}[\mathbb{E}(B)]^{\frac{1-\gamma}{\epsilon+\gamma}} \exp\left\{-\tfrac{1}{2}\tfrac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma}\Omega^*)\right\}$$

where Ω^* obtains from $\hat{\Omega}$ once we replace y_i and Y with, respectively, y_i^* and Y^*. We conclude that

$$\hat{W} = W^* \exp\left\{-\tfrac{1}{2}\tfrac{(1-\gamma)(1+\epsilon)}{\epsilon+\gamma}\left(\hat{\Omega} - \Omega^*\right)\right\} \tag{20}$$

[24] To see this more clearly, note that Δ is strictly concave in δ and reaches its maximum at $\delta = 1$ when $\gamma < 1$, whereas it is strictly convex and reaches its minimum at $\delta = 1$ when $\gamma > 1$. Along with the fact that $\hat{W} > 0$ when $\gamma < 1$ but $\hat{W} < 0$ when $\gamma > 1$, this means that $\hat{W}\Delta$ is always strictly concave in δ and $\delta = 1$ is always the maximal point.

Finally, using the definitions of $\hat{\Omega}$ and Ω^* together with the fact that $1-\alpha = \frac{\epsilon+\gamma}{\epsilon+1/\rho}$, we have

$$\begin{aligned}
\frac{\hat{\Omega}-\Omega^*}{\epsilon+\gamma} &= \{Var(\log Y) + Var(\log Y^*) - 2Cov(\log Y, \log Y^*)\} \\
&\quad + \frac{1}{1-\alpha}\{Var(\log y_i|\Theta) + Var(\log y_i^*|\Theta) - 2Cov(\log y_i, \log y_i^*|\Theta)\} \\
&= Var(\log Y - \log Y^*) + \frac{1}{1-\alpha} Var(\log y_i - \log y_i^*|\Theta)
\end{aligned}$$

Note that conditioning on $\Theta \equiv (\log Y, \log B)$ is equivalent to conditioning on $(\log Y, \log Y^*)$. Furthermore, because $\log Y$ and $\log Y^*$ are the cross-sectional means (expectations) of, respectively, $\log y_i$ and $\log y_i^*$, we have that

$$\begin{aligned}
Var\left(\log y_i - \log y_i^* \middle| \log Y, \log Y^*\right) &= Var\left((\log y_i - \log Y) - (\log y_i^* - \log Y^*) \middle| \log Y, \log Y^*\right), \\
&= Var\left(\log \widetilde{y}_i - \log \widetilde{Y}\right)
\end{aligned}$$

Combining the above results with the definitions of Σ, σ and Λ, gives

$$\frac{\hat{\Omega}-\Omega^*}{\epsilon+\gamma} = \Sigma + \frac{1}{1-\alpha}\sigma = \Lambda.$$

Finally, combining the above with (18) and (20), gives

$$W = W^* \Delta \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\} \tag{21}$$

and completes the proof. ■

Equilibrium with productivity shocks. Suppose the equilibrium production strategy takes a log-linear form:

$$\log y_{it} = \varphi_0 + \varphi_a a_{it} + \varphi_\mu \mu_{it} + \varphi_x x_{it} + \varphi_z z_t, \tag{22}$$

for some coefficients $(\varphi_a, \varphi_x, \varphi_z)$. Aggregate output is then given by

$$\log Y_t = \varphi_0 + X + (\varphi_a + \varphi_x)\,\bar{a}_t + \varphi_z z_t$$

where

$$X \equiv \frac{1}{2}\left(\frac{\rho-1}{\rho}\right) Var(\log y_{it}|\Theta) = \frac{1}{2}\left(\frac{\rho-1}{\rho}\right)\left[\frac{\varphi_a^2}{\kappa_\xi} + \frac{\varphi_x^2}{\kappa_x} + 2\frac{\varphi_a\varphi_x}{\kappa_x}\right]$$

adjusts for the curvature in the CES aggregator. It follows that Y_t is log-normal, with

$$\begin{aligned}
\mathbb{E}_{it}[\log Y_t] &= \varphi_0 + X + (\varphi_a + \varphi_x)\,\mathbb{E}_{it}[\bar{a}_t] + \varphi_z z_t & (23)\\
Var_{it}[\log Y_t] &= (\varphi_a + \varphi_x)^2\, Var_{it}[\bar{a}_t] & (24)
\end{aligned}$$

where, by standard Gaussian updating,

$$\begin{aligned}
\mathbb{E}_{it}[\bar{a}_t] &= \frac{\kappa_x}{\kappa_a+\kappa_x+\kappa_z} x_{it} + \frac{\kappa_z}{\kappa_a+\kappa_x+\kappa_z} z_t & (25)\\
Var_{it}[\bar{a}_t] &= \frac{1}{\kappa_a+\kappa_x+\kappa_z} & (26)
\end{aligned}$$

Because of the log-normality of Y_t, the fixed-point condition (1) reduces to following:

$$\log y_{it} = (1-\alpha)(\Psi a_{it} - \Psi' \log \bar{\mathcal{M}}) + \alpha \mathbb{E}_{it}[\log Y_t] + \Gamma \tag{27}$$

where $\Psi \equiv \frac{1+\epsilon}{\epsilon+\gamma} > 0$, $\Psi' \equiv \frac{1}{\epsilon+\gamma} > 0$, $\log \bar{\mathcal{M}} \equiv -\log\left[\left(\frac{\bar{\eta}-1}{\bar{\eta}}\right)(1-\bar{\tau})\right] \approx \bar{\mu} + \bar{\tau} > 0$ is the overall distortion caused by the monopoly markup and the labor wedge (which are both constant because we are herein focusing on the case with only productivity shocks), and

$$\Gamma = \tfrac{1}{2}\alpha\left(\tfrac{1}{\rho} - \gamma\right) Var_{it}\left[\log Y_t\right] = \tfrac{1}{2}\alpha^2\left(\tfrac{1}{\rho} + \epsilon\right) Var_{it}\left[\log Y_t\right] > 0$$

Next, combining (27) with (23) and (25), we obtain

$$\begin{aligned} \log y_{it} &= \Gamma - (1-\alpha)\Psi'\lambda + (1-\alpha)\,\Psi a_{it} + \alpha\left(\varphi_0 + X + \varphi_z z_t\right) \\ &\quad + \alpha\left(\varphi_a + \varphi_x\right)\left(\frac{\kappa_x}{\kappa_0 + \kappa_x + \kappa_z} x_{it} + \frac{\kappa_z}{\kappa_a + \kappa_x + \kappa_z} z_t\right) \end{aligned}$$

For this to coincide with our initial guess in (22) for every event, it is necessary and sufficient that the coefficients $(\varphi_0, \varphi_a, \varphi_x, \varphi_z)$ solve the following system:

$$\begin{aligned} \varphi_0 &= \Gamma - (1-\alpha)\Psi' \log \bar{\mathcal{M}} + \alpha(\varphi_0 + X) \\ \varphi_a &= (1-\alpha)\,\Psi \\ \varphi_x &= \alpha\left(\varphi_a + \varphi_x\right)\frac{\kappa_x}{\kappa_a + \kappa_x + \kappa_z} \\ \varphi_z &= \alpha\varphi_z + \alpha\left(\varphi_a + \varphi_x\right)\frac{\kappa_z}{\kappa_a + \kappa_x + \kappa_z} \end{aligned}$$

The unique solution to this system is given by the following:

$$\varphi_a = (1-\alpha)\,\Psi > 0, \quad \varphi_x = \frac{(1-\alpha)\,\kappa_x}{\kappa_a + (1-\alpha)\,\kappa_x + \kappa_z}\alpha\Psi,$$

$$\varphi_z = \frac{\kappa_z}{\kappa_a + (1-\alpha)\,\kappa_x + \kappa_z}\alpha\Psi, \qquad \text{and} \qquad \varphi_0 = -\Psi'\lambda + \tfrac{1}{1-\alpha}\left(\alpha X + \Gamma\right)$$

Note then that the coefficients φ_x and φ_z, which capture the individual response to expectations of the aggregate state, are positive if and only if $\alpha > 0$. ■

Proof of Proposition 2. Using the characterization of the equilibrium allocation in the preceding proof along with that of the first best, we can calculate the equilibrium values of the aggregate output gaps

$$\log Y_t - \log Y_t^* = \left(\varphi_a + \varphi_x + \varphi_z\right)\bar{a}_t + \varphi_z \varepsilon_t - \Psi \bar{a}_t$$

and the cross-section dispersion of the local output gaps

$$\log y_{it} - \log y_{it}^* = \varphi_x u_{it},$$

which imply that the volatility of $\log Y_t - \log Y_t^*$ is

$$\Sigma = \frac{\varphi_z^2}{\kappa_z} + \frac{(\varphi_a + \varphi_x + \varphi_z - \Psi)^2}{\kappa_a} = \frac{\alpha^2 (\kappa_a + \kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^2} \Psi^2$$

and the cross-sectional dispersion of $\log y_{it} - \log y_{it}^*$ is

$$\sigma = \frac{\varphi_x^2}{\kappa_x} = \frac{\alpha^2 (1-\alpha)^2 \kappa_x}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^2} \Psi^2.$$

Taking the derivative of Σ with respect to the precision of public information gives

$$\frac{\partial \Sigma}{\partial \kappa_z} = \frac{(1-\alpha)\kappa_x - (\kappa_a + \kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^3} \alpha^2 \Psi^2$$

which is negative if and only if $\kappa_z > (1-\alpha)\kappa_x - \kappa_a$, while taking the derivative of σ gives

$$\frac{\partial \sigma}{\partial \kappa_z} = -2 \frac{\alpha^2 (1-\alpha)^2 \kappa_x}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^3} \Psi^2$$

which is necessarily negative.

Similarly, taking the derivatives with respect to the precision of private information, we obtain

$$\frac{\partial \Sigma}{\partial \kappa_x} = -\frac{2(1-\alpha)(\kappa_a + \kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^3} \alpha^2 (\Psi')^2$$

which is necessarily negative and

$$\frac{\partial \sigma}{\partial \kappa_x} = \frac{\kappa_z + \kappa_a - (1-\alpha)\kappa_x}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^3} (1-\alpha)^2 \alpha^2 (\Psi')^2$$

which is negative if and only if $(1-\alpha)\kappa_x > \kappa_z + \kappa_a$. ■

Proof of Theorem ??. From the proof of Proposition 2, we can rewrite Λ as

$$\Lambda = \Sigma + \frac{1}{1-\alpha}\sigma = \frac{\alpha^2}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)} \Psi^2$$

from which it is immediate that second-order losses are decreasing in the precision of either public or private information. (Note that this would be true even if α were negative.) Furthermore,

$$\frac{\partial^2 \Lambda}{\partial \kappa_z \partial \alpha} = -\frac{2\alpha (\kappa_x + \kappa_z + \kappa_a)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_a)^3} \Psi^2$$

which is itself negative if and only if $\alpha > 0$.

The distortion in the mean level of output is given by

$$\delta = \left[\left(\frac{\bar{\eta}-1}{\eta} \right) (1 - \bar{\tau}) \right]^{\frac{1}{\epsilon+\gamma}} < 1$$

where $\frac{\bar{\eta}-1}{\eta}$ is the monopoly wedge (the reciprocal of the markup) and $1-\bar{\tau}$ is the labor wedge. Since δ, and hence also Δ, is invariant to the information structure, the welfare effects of public information are pinned down by comparative statics of Λ alone, which were established before. It follows that welfare necessarily increases with the precision of either public or private information.
■

Equilibrium with markup shocks. This follows very similar steps as the characterization of equilibrium in the case with productivity shocks. Suppose equilibrium output takes a log-linear form:

$$\log y_{it} = \varphi_{\mu} + \varphi_{\mu}\mu_{it} + \varphi_{x}x_{it} + \varphi_{z}z_{t},$$

for some coefficients $(\varphi_{\mu}, \varphi_{x}, \varphi_{z})$. This guarantees that aggregate output is log-normal, which in turn implies that the fixed-point condition (1) now reduces to

$$\log y_{it} = (1-\alpha)(\Psi\bar{a} - \Psi'\mu_{it}) + \alpha\mathbb{E}_{it}[\log Y_t] + \Gamma$$

where Ψ, Ψ', and Γ are defined as in the case with productivity shocks. Following similar steps as in that case, we can then show that the unique equilibrium coefficients are given by the following:

$$\varphi_{\mu} = -(1-\alpha)\Psi' < 0, \quad \varphi_{x} = -\tfrac{(1-\alpha)\kappa_x}{\kappa_{\mu}+(1-\alpha)\kappa_x+\kappa_z}\alpha\Psi',$$

$$\varphi_{z} = -\tfrac{\kappa_z}{\kappa_{\mu}+(1-\alpha)\kappa_x+\kappa_z}\alpha\Psi', \qquad \text{and} \qquad \varphi_0 = \Psi\bar{a} + \tfrac{1}{1-\alpha}(\alpha X + \Gamma)$$

Note that the sign of the coefficients φ_x and φ_z is once again pinned down by the sign of α. ■

Proof of Proposition 3. With only markup shocks, first-best allocations are constant. The volatility of aggregate output gaps and the dispersion of local output gaps are thus given by the following:

$$\Sigma = \frac{\varphi_z^2}{\kappa_z} + \frac{(\varphi_a+\varphi_x+\varphi_z)^2}{\kappa_{\mu}} = \frac{\alpha^2\kappa_{\mu}\kappa_z + ((1-\alpha)\kappa_{\mu} + (1-\alpha)\kappa_x + \kappa_z)^2}{\kappa_{\mu}(\kappa_{\mu} + (1-\alpha)\kappa_x + \kappa_z)^2}(\Psi')^2$$

$$\sigma = \frac{\varphi_{\mu}^2}{\kappa_{\xi}} + \frac{\varphi_x^2}{\kappa_x} + 2\frac{\varphi_{\mu}\varphi_x}{\kappa_x} = \frac{(1-\alpha)^2}{\kappa_{\xi}}(\Psi')^2 + \frac{\alpha(1-\alpha)^2(2\kappa_{\mu} + (2-\alpha)\kappa_x + 2\kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_{\mu})^2}(\Psi')^2$$

Next, taking the derivatives with respect to the precision of public information, we obtain

$$\frac{\partial\Sigma}{\partial\kappa_z} = \frac{(2+\alpha)(1-\alpha)\kappa_x + (\kappa_z+\kappa_{\mu})(2-\alpha)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_{\mu})^3}\alpha(\Psi')^2$$

which is positive if $\alpha > 0$ and

$$\frac{\partial\sigma}{\partial\kappa_z} = -\frac{2(1-\alpha)^2(\kappa_{\mu}+\kappa_x+\kappa_z)}{((1-\alpha)\kappa_x + \kappa_z + \kappa_{\mu})^3}\alpha(\Psi')^2$$

which is negative if (and only if) $\alpha > 0$.

Similarly, taking the derivatives with respect to the precision of private information, we obtain

$$\frac{\partial\Sigma}{\partial\kappa_x} = \frac{2(1-\alpha)^2(\kappa_x+\kappa_z+\kappa_{\mu})}{((1-\alpha)\kappa_x + \kappa_z + \kappa_{\mu})^3}\alpha(\Psi')^2$$

which is positive if (and only if) $\alpha > 0$ and

$$\frac{\partial\sigma}{\partial\kappa_x} = -\frac{(1-\alpha)^2[(2-\alpha)(1-\alpha)\kappa_x + (2-3\alpha)(\kappa_{\mu}+\kappa_z)]}{((1-\alpha)\kappa_x + \kappa_z + \kappa_{\mu})^3}\alpha(\Psi')^2,$$

which is in general ambiguous. ■

Proof of Proposition 4. From the preceding characterization of Σ and σ, we have that

$$\Lambda = \frac{1-\alpha}{\kappa_\xi}\left(\Psi'\right)^2 + \frac{(1-\alpha)\kappa_x + \kappa_z + (1-\alpha^2)\kappa_\mu}{\kappa_\mu\left((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu\right)}\left(\Psi'\right)^2$$

It follows that

$$\frac{\partial\Lambda}{\partial\kappa_z} = \frac{\alpha^2\left(\Psi'\right)^2}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu\right)^2} > 0 \tag{28}$$

$$\frac{\partial\Lambda}{\partial\kappa_x} = \frac{(1-\alpha)\alpha^2\left(\Psi'\right)^2}{\left((1-\alpha)\kappa_x + \kappa_z + \kappa_\mu\right)^2} > 0, \tag{29}$$

which proves that second-order welfare losses necessarily increase with the precision of either public or private information, no matter the value of α. ■

Proof of Proposition 5. To study the impact of information on first-order losses Δ, we first need to compute δ. Using the equilibrium characterization and after some tedious algebra, we can show that the predictable component of aggregate output, $\mathbb{E}[Y_t]$, satisfies the following condition:

$$\left(\mathbb{E}[Y_t]\right)^{1-\gamma}\exp\left(-\tfrac{1}{2}\gamma\left(1-\gamma\right)Var\left(\log Y_t\right)\right) = \exp\left(\chi - \log(1-\bar{\tau}) + \tfrac{1}{2}D\right)\frac{\left(\mathbb{E}[Y_t]\right)^{1+\epsilon}}{\mathbb{E}[B_t]}\exp(\widetilde{G})$$

where

$$\begin{aligned}
\widetilde{G} &\equiv \tfrac{1}{2}\epsilon(1+\epsilon)Var\left(\log Y_t\right) + \tfrac{1}{2}\left(\epsilon + \tfrac{1}{\rho}\right)(1+\epsilon)Var\left(\log y_{it}|\bar{\mu}_t\right) \\
D &\equiv Var\left(\mu_{it}|\bar{\mu}_t\right) + 2\left(1+\epsilon\right)Cov\left(y_{it},\mu_{it}|\bar{\mu}_t\right) \\
&= \frac{1}{\kappa_\xi} + \frac{1}{\kappa_\mu} + 2\left(1+\epsilon\right)\left(\frac{\varphi_\mu}{\kappa_\xi} + \frac{\varphi_x}{\kappa_x} + \frac{\varphi_\mu + \varphi_x + \varphi_z}{\kappa_\mu}\right) \\
&= \frac{1}{\kappa_\xi} + \frac{1}{\kappa_\mu} - 2\left(1+\epsilon\right)\left(\frac{1-\alpha}{\kappa_\xi} + \frac{1}{\kappa_\mu} - \frac{\alpha^2}{\kappa_\mu + \left(1-\alpha\right)\kappa_x + \kappa_z}\right)\Psi'
\end{aligned}$$

From (16), on the other hand, we infer that the optimal $\hat{Y}_t$ solves the following condition:

$$\hat{Y}_t^{1-\gamma}\exp\left(-\tfrac{1}{2}\gamma\left(1-\gamma\right)Var\left(\log Y_t\right)\right) = \frac{\hat{Y}_t^{1+\epsilon}}{\mathbb{E}[B_t]}\exp(\widetilde{G})$$

Combining these results, we infer that

$$\delta \equiv \frac{\mathbb{E}[Y_t]}{\hat{Y}_t} = \exp\left[-\Psi'\left(\chi - \log(1-\bar{\tau}) + \tfrac{1}{2}D\right)\right],$$

and therefore

$$\frac{\partial\delta}{\partial\kappa_z} = -\tfrac{1}{2}\delta\Psi'\frac{\partial D}{\partial\kappa_z} = \frac{\alpha^2\left(\Psi'\right)^2}{\left(\left(1-\alpha\right)\kappa_x + \kappa_z + \kappa_\mu\right)^2}\left(1+\epsilon\right)\delta > 0 \tag{30}$$

$$\frac{\partial\delta}{\partial\kappa_x} = -\tfrac{1}{2}\delta\Psi'\frac{\partial D}{\partial\kappa_x} = \frac{(1-\alpha)\alpha^2\left(\Psi'\right)^2}{\left(\left(1-\alpha\right)\kappa_x + \kappa_z + \kappa_\mu\right)^2}\left(1+\epsilon\right)\delta > 0. \tag{31}$$

It follows that, as long as $\delta < 1$, an increase in the precision of either public or private information reduces the mean distortion—it brings δ closer to 1—and thereby also reduces first-order welfare losses. (Once again, this result holds true irrespectively of whether α is positive or negative.) ■

Proof of Theorem 2. To obtain the overall welfare effect, recall that welfare is given by

$$W_t = W_t^* \Delta \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\}$$

Consider first the case of public information. From the above, we have that

$$\frac{\partial W_t}{\partial \kappa_z} = W_t^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\} \left(\frac{\partial \Delta}{\partial \delta}\frac{\partial \delta}{\partial \kappa_z} - \tfrac{1}{2}\Delta(1+\epsilon)(1-\gamma)\frac{\partial \Lambda}{\partial \kappa_z}\right)$$

From (28) and (30), we have that

$$\frac{\partial \delta}{\partial \kappa_z} = \frac{\partial \Lambda}{\partial \kappa_z}(1+\epsilon)\delta$$

It follows that

$$\frac{\partial W_t}{\partial \kappa_z} = W_t^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\} \frac{\partial \Lambda}{\partial \kappa_z} H$$

where

$$\begin{aligned} H &\equiv \frac{\partial \Delta}{\partial \delta}(1+\epsilon)\delta - \tfrac{1}{2}(1-\gamma)(1+\epsilon)\Delta \\ &= \tfrac{(1-\gamma)(1+\epsilon)}{2(\epsilon+\gamma)}\left[2(1+\epsilon)\left(\delta^{1-\gamma} - \delta^{1+\epsilon}\right) - \left((1+\epsilon)\delta^{1-\gamma} - (1-\gamma)\delta^{1+\epsilon}\right)\right] \\ &= \tfrac{(1-\gamma)(1+\epsilon)}{2(\epsilon+\gamma)}\left[(1+\epsilon)\delta^{1-\gamma} - (1+2\epsilon+\gamma)\delta^{1+\epsilon}\right], \end{aligned}$$

and, therefore,

$$\frac{\partial W_t}{\partial \kappa_z} = \tfrac{(1-\gamma)(1+\epsilon)}{2(\epsilon+\gamma)} W_t^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\} \frac{\partial \Lambda}{\partial \kappa_z}\delta\left[(1+\epsilon) - (1+2\epsilon+\gamma)\delta^{\epsilon+\gamma}\right]$$

Note then that the sign of W_t^* is the same as that of $(1-\gamma)$, which together with the facts that $\frac{\partial \Lambda}{\partial \kappa_z} > 0$ and $\delta > 0$ implies that the sign of $\frac{\partial W_t}{\partial \kappa_z}$ is the same as the sign of $(1+\epsilon) - (1+2\epsilon+\gamma)\delta^{\epsilon+\gamma}$. We conclude that

$$\frac{\partial W_t}{\partial \kappa_z} < 0 \quad \text{iff} \quad \delta > \hat{\delta},$$

where

$$\hat{\delta} \equiv \left(\frac{1+\epsilon}{1+2\epsilon+\gamma}\right)^{\frac{1}{\epsilon+\gamma}} \in (0,1).$$

Consider next the case of private information. From (29) and (31), we have that

$$\frac{\partial \delta}{\partial \kappa_x} = \frac{\partial \Lambda}{\partial \kappa_x}(1+\epsilon)\delta,$$

pretty much as in the case of public information. It follows that

$$\begin{aligned} \frac{\partial W_t}{\partial \kappa_x} &= W_t^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\} \left(\frac{\partial \Delta}{\partial \delta}\frac{\partial \delta}{\partial \kappa_x} - \tfrac{1}{2}\Delta(1+\epsilon)(1-\gamma)\frac{\partial \Lambda}{\partial \kappa_x}\right) \\ &= W_t^* \exp\left\{-\tfrac{1}{2}(1+\epsilon)(1-\gamma)\Lambda\right\} \frac{\partial \Lambda}{\partial \kappa_x} H \end{aligned}$$

where H is defined as before. By direct implication,

$$\frac{\partial W_t}{\partial \kappa_x} < 0 \quad \text{iff} \quad \delta > \hat{\delta},$$

where $\hat{\delta}$ is the same threshold as the one in the case of public information. ■

Proofs of Section 5.

We start with two preliminary lemmas. The first derives some useful benchmarks which we will use throughout. The second extends the welfare decomposition in Section 3 to the more general model considered in Section 5. Finally, the third lemma provides some implementability conditions that equilibrium allocations have to satisfy. In the proofs of this section we will often make use of the following objects which we list here for convenience

$$\begin{aligned}
\hat{\epsilon} &\equiv \frac{1+\epsilon}{\theta} - 1, \quad \hat{\gamma} \equiv 1 - \frac{(1-\gamma)(1+\epsilon)}{1+\epsilon-\eta(1-\gamma)}, \quad \hat{\rho} \equiv \frac{1}{1-\rho\nu+\nu}, \\
\hat{\alpha} &\equiv \frac{1-\hat{\rho}\hat{\gamma}}{1+\hat{\rho}\hat{\epsilon}}, \quad \chi \equiv \frac{1+\epsilon}{1+\epsilon-\eta+\gamma\eta} > 0, \quad \nu \equiv \frac{1+\epsilon}{\rho(1+\epsilon-\eta)+\eta}, \\
\hat{\phi}_a &\equiv \frac{\hat{\rho}(1+\hat{\epsilon})}{1+\hat{\rho}\hat{\epsilon}}, \quad \hat{\phi}_\mu \equiv -\frac{\hat{\rho}+(\hat{\rho}-1)\frac{\eta}{1+\epsilon}}{1+\hat{\rho}\hat{\epsilon}}, \quad \bar{\alpha} \equiv \frac{(1-\rho\gamma)\eta}{\rho(1+\epsilon-\eta)+\eta}.
\end{aligned} \tag{32}$$

Lemma 5. *Denote by $\log q^*_{it}$ and $\log Q^*_t$ the first-best level of output and the corresponding aggregate level, respectively, then*

$$\log q^*_{it} = \hat{\phi}_a a_{it} + \hat{\alpha} \log Q^*_t. \tag{33}$$

Similarly, denote by $\log \bar{y}_{it}$ and $\log \bar{Y}_t$ the first-best level of output conditional *on the first-period equilibrium production decision and the corresponding aggregate level, respectively, then*

$$\log \bar{y}_{it} = \rho v \log q_{it} + \bar{\alpha} \log \bar{Y}_t. \tag{34}$$

Proof. Similarly to the baseline model, once we use the equilibrium conditions for the wages and the prices, the first-best level of output satisfies the following FOCs of the firm's profit with respect to the two inputs:

$$n^{\epsilon}_{it} = U'(Y_t)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\left(\theta\frac{y_{it}}{n_{it}}\right) \tag{35}$$

$$l^{\epsilon}_{it} = U'(Y_t)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\left(\eta\frac{y_{it}}{l_{it}}\right) \tag{36}$$

Note that, by the definition of first-best level of output, the markup and the expectation operator are absent from both conditions. In addition, the first-best level of output *conditional* on the first-period equilibrium production decision satisfies (36) conditional on $\log q_{it}$.

We can rewrite (36) to solve for $\log l_{it}$ as (omitting constants)

$$(1+\epsilon)\log l_{it} = \left(\frac{1}{\rho} - \gamma\right)\log Y_t + \left(1 - \frac{1}{\rho}\right)\log y_{it}. \tag{37}$$

Next, using $\log y_{it} = \log q_{it} + \eta \log l_{it}$ to replace l_{it}, we can restate the above as

$$\log y_{it} = b_q \log q_{it} + \bar{\alpha} \log Y_t,$$

where $b_q \equiv \rho\nu$, which is (34). Using the above result to replace y_{it} into (35), taking logs, and using $\log q_{it} = a_{it} + \theta \log n_{it}$, we finally arrive at

$$\log q^*_{it} = \hat{\phi}_a a_{it} + \frac{\hat{\alpha}}{\chi} \log Y_t,$$

where $\hat{\phi}_a$, $\hat{\alpha}$, χ, and ν are defined in (32).

Finally, integrating (37) across different islands gives

$$\int \log l_{it} di = \frac{1-\gamma}{1+\epsilon} \log Y_t,$$

which can be combined with (51) to get

$$\log Y_t = \chi \log Q_t$$

and, thus,

$$\log q^*_{it} = \hat{\phi}_a a_{it} + \hat{\alpha} \log Q_t.$$

■

Similarly to the baseline model, let Σ_Q and σ_q denote the volatility of the first-period aggregate gap, $\log Q_t - \log Q^*_t$, and the cross-sectional dispersion of the first-period local gaps, $\log q_{it} - \log q^*_{it}$, respectively. In addition, let Σ_Y and σ_y denote the volatility of the aggregate gap conditional on the first-period equilibrium production decision, $\log Y_t - \log \bar{Y}_t$, and the cross-sectional dispersion of the local gap conditional on the first-period equilibrium production decision, $\log y_{it} - \log \bar{y}_{it}$, respectively.

Lemma 6. *Consider the extended model and suppose that the optimal unconditional mean levels of the two types of labor are attained.*[25] *Welfare is given by*

$$\mathcal{W} = \mathcal{W}^* \exp\left(-\frac{1}{2}(1-\hat{\gamma})(1+\hat{\epsilon})\Lambda'\right), \tag{38}$$

where $\mathcal{W}^*$ is the first-best level of welfare and

$$\Lambda' = \left(\Sigma_Q + \frac{1}{1-\hat{\alpha}}\sigma_q\right) + \xi\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right), \tag{39}$$

where $\hat{\gamma}$ and $\hat{\epsilon}$ are given in (32) and ξ is a positive scalar pinned down by γ, ϵ , θ, and η.

[25] As in the baseline mode, obtaining the optimal mean level of economic activity typically requires the use of non-contingent subsidies. In the absence of such subsidies a first-order loss Δ' shows up in (38) similarly to condition (21) in the baseline model.

Proof. To simplify the notation, we drop the time index t. We show that when the optimal subsidies on the first and second-period employment are in place, per-period welfare in the general model

$$W = \mathbb{E}\left[\frac{Y^{1-\gamma}}{1-\gamma} - \frac{1}{1+\epsilon}\int\left(\frac{q_i}{A_i}\right)^{\frac{1+\epsilon}{\theta}} di - \frac{1}{1+\epsilon}\int\left(\frac{y_i}{q_i}\right)^{\frac{1+\epsilon}{\eta}} di\right]$$

can be decomposed as follows:

$$W = W^{*}\exp\left(-\frac{1}{2}\left(1-\hat{\gamma}\right)\left(1+\hat{\epsilon}\right)\Lambda'\right).$$

We first focus on the first and third component of W:

$$\mathbb{E}\left[\frac{Y^{1-\gamma}}{1-\gamma} - \frac{1}{1+\epsilon}\int\left(\frac{y_i}{q_i}\right)^{\frac{1+\epsilon}{\eta}} di\right].$$

This expression resembles the expression for welfare in the baseline model with q_i replacing local productivity a_i. We can, thus, follows the same steps as in the proof of Proposition 1 and rewrite the latter expression as (omitting unimportant constants)

$$\mathbb{E}\left[Q\right]^{\frac{1-\gamma}{\frac{1+\epsilon}{\eta}-1+\gamma}\frac{1+\epsilon}{1-\eta}} \exp\left(-\frac{1}{2}\frac{\frac{1+\epsilon}{\eta}\left(1-\gamma\right)}{\frac{1+\epsilon}{\eta}-1+\gamma}\bar{\Omega}\right)\exp\left(-\frac{1}{2}\frac{1+\epsilon}{\eta}\left(1-\gamma\right)\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right),$$

with

$$\bar{\Omega} \equiv \left(\frac{1+\epsilon}{\eta}\gamma+\gamma-1\right)\frac{\frac{1+\epsilon}{\eta}+\gamma-1}{\left(\frac{1+\epsilon}{\eta}\right)^{2}}Var\left(\log\bar{Y}\right) = \frac{\frac{1+\epsilon}{\eta}\gamma+\gamma-1}{\frac{1+\epsilon}{\eta}+\gamma-1}Var\left(\log Q\right),$$

where the second line uses the fact that (34) implies $\log\bar{Y} = \frac{\rho v}{1-\bar{\alpha}}\log Q$.

We can use these results to rewrite W as follows:

$$W = \mathbb{E}\left[Q\right]^{\frac{1-\gamma}{\frac{1+\epsilon}{\eta}-1+\gamma}\frac{1+\epsilon}{\eta}} \exp\left(-\frac{1}{2}\frac{\frac{1+\epsilon}{\eta}\left(1-\gamma\right)}{\frac{1+\epsilon}{\eta}-1+\gamma}\bar{\Omega} - \frac{1}{2}\frac{1+\epsilon}{\eta}\left(1-\gamma\right)\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right) - \frac{1}{1+\epsilon}\mathbb{E}\left[\int\left(\frac{q_i}{A_i}\right)^{\frac{1+\epsilon}{\theta}} di\right]$$

Note that the definition of $\hat{\rho}$ in (32) implies

$$Q \equiv \left(\int q_i^{(\rho-1)\nu} di\right)^{\frac{1}{(\rho-1)\nu}} = \left(\int q_i^{\frac{\hat{\rho}-1}{\hat{\rho}}} di\right)^{\frac{\hat{\rho}}{\hat{\rho}-1}},$$

Also, using the properties of the log-normal distribution,

$$E\left[Q\right]^{1-\hat{\gamma}} = E\left[Q^{1-\hat{\gamma}}\right]e^{\frac{1}{2}\hat{\gamma}(1-\hat{\gamma})Var(\log Q)}$$

and, thus,

$$W = \frac{1}{1-\hat{\gamma}}\mathbb{E}\left[Q^{1-\hat{\gamma}}\right]\Xi - \frac{1}{1+\hat{\epsilon}}\int\left(\frac{q_i}{A_i}\right)^{1+\hat{\epsilon}} di$$

where

$$\Xi \equiv \left(1-\hat{\gamma}\right)\theta\exp\left(\frac{1}{2}\hat{\gamma}\left(1-\hat{\gamma}\right)Var\left(\log Q\right) - \frac{1}{2}\frac{\frac{1+\epsilon}{\eta}\left(1-\gamma\right)}{\frac{1+\epsilon}{\eta}-1+\gamma}\bar{\Omega} - \frac{1}{2}\frac{1+\epsilon}{\eta}\left(1-\gamma\right)\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right),$$

or, using the definition of $\bar{\Omega}$,

$$\Xi = (1-\hat{\gamma})\,\theta \exp\left(-\frac{1}{2}\frac{1+\epsilon}{\eta}(1-\gamma)\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right).$$

Note that then that the functional that maps the strategy q into the welfare level W is the same as the one in the proof of Proposition 1, provided that we make two changes: we replace ρ, γ, and ϵ with, respectively, $\hat{\rho}$, $\hat{\gamma}$, and $\hat{\epsilon}$; and we accommodate the constant Ξ. Thus, following the same steps as in that proof, we can obtain welfare W as

$$W = W^* \exp\left\{-\tfrac{1}{2}(1+\hat{\epsilon})(1-\hat{\gamma})\left(\Sigma_Q - \frac{1}{1-\alpha}\sigma_q\right)\right\}\Xi^{\frac{1+\hat{\epsilon}}{\hat{\epsilon}+\hat{\gamma}}}.$$

Finally, using definition of Ξ and rearranging gives

$$\begin{aligned} W &= W^* \exp\left\{-\tfrac{1}{2}(1-\hat{\gamma})(1+\hat{\epsilon})\left(\Sigma_Q - \frac{1}{1-\alpha}\sigma_q\right)\right\}\exp\left(-\frac{1}{2}\frac{1+\hat{\epsilon}}{\hat{\epsilon}+\hat{\gamma}}(1-\gamma)\frac{1+\epsilon}{\eta}\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right) \\ &= W^* \exp\left\{-\tfrac{1}{2}(1-\hat{\gamma})(1+\hat{\epsilon})\left[\left(\Sigma_Q - \frac{1}{1-\alpha}\sigma_q\right) + \xi\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right)\right]\right\}, \end{aligned}$$

where

$$\xi \equiv \frac{\frac{1+\epsilon}{\eta} + \gamma - 1}{\hat{\epsilon}+\hat{\gamma}}. \blacksquare$$

Lemma 7. *Consider the model of Section 5 and let s_{it} and $\bar{s}_t$ denote the idiosyncratic and aggregate shocks to fundamentals (technology or markups, depending on the case under consideration).*

(i) A pair of strategies

$$\log q_{it} = \varphi_0 + \varphi_s s_{it} + \varphi_x x_{it} + \varphi_z z_t \qquad \text{and} \qquad \log l_{it} = l_0 + l_{\bar{s}}\bar{s}_t + l_s s_{it} + l_x x_{it} + l_z z_t, \tag{40}$$

can be implemented as an equilibrium with monetary policy of the form

$$\log M_t = \lambda \bar{s}_t \tag{41}$$

if and only if the following conditions are satisfied:[26]

$$\begin{aligned} \varphi_s &= \hat{\phi}_s && (42) \\ \varphi_x &= \Gamma_x + \Gamma_x' l_{\bar{s}} && (43) \\ \varphi_z &= \Gamma_z + \Gamma_z' l_{\bar{s}} && (44) \\ l_s &= \frac{1}{\theta}(\varphi_s - \mathbf{1}_{s=a}) && (45) \\ l_x &= \frac{1}{\theta}\varphi_x - \frac{\kappa_x}{\kappa_s+\kappa_x+\kappa_z} l_{\bar{s}} && (46) \\ l_z &= \frac{1}{\theta}\varphi_z - \frac{\kappa_z}{\kappa_s+\kappa_x+\kappa_z} l_{\bar{s}}, && (47) \end{aligned}$$

[26]There is also a restriction between φ_0 and l_0, which we omit because it is of no interest: φ_0 and l_0 are irrelevant for the business-cycle properties of allocations.

where $\hat{\phi}_s$ is given in (32), $\mathbf{1}_{s=a}$ is an indicator that takes the value 1 in the case of technology shocks ($s = a$) and 0 in the case of markup shocks ($s = \mu$), and Γ_x, Γ'_x, Γ_z, Γ'_z are scalars given in the proof and $l_{\bar{s}}$ is an arbitrary coefficient.

(ii) Furthermore, there exist scalars $\bar{\Pi}$, Π such that the policy that implements the aforementioned allocations is given by $\lambda = \bar{\Pi} + \Pi l_{\bar{s}}$.

(iii) When instead prices are flexible, a pair of strategies as in (40) can be implemented with monetary policy as in (41) if and only if the following condition is satisfied in addition to conditions (42)-(47):

$$l_{\bar{s}} = l^*_{\bar{s}} \equiv \frac{(1-\rho\gamma)\left(1+\frac{\eta}{\theta}\right)\left(\hat{\phi}_s + \Gamma_x\right) - \eta(1-\rho\gamma)\mathbf{1}_{s=a}}{\rho(1+\epsilon-\eta) + \eta - (1-\rho\gamma)\left(\left(1+\frac{\eta}{\theta}\right)\Gamma'_x + \eta\frac{\kappa_\mu+\kappa_z}{\kappa_\mu+\kappa_x+\kappa_z}\right)}. \tag{48}$$

(iv) If monetary policy takes the form of

$$\log M_t = \lambda \bar{s}_t + m_t,$$

where m_t is a random variable, which is orthogonal to $\bar{s}_t$, then conditions (42)-(47) remain the same and only the second-period labor choice will respond to m_t:

$$\log l_{it} = l_0 + l_{\bar{s}}\bar{s}_t + l_s s_{it} + l_x x_{it} + l_z z_t + \frac{1}{\eta} m_t.$$

Proof. *Part (i).* The proof of the necessity of conditions (42)-(47) uses similar conditions to those found in the derivation of equation (1). The only difference is that, when prices are not flexible, the firm will choose q_{it} and p_{it} so as to maximize the expected valuation of its profit. After some manipulation, the relevant FOCs can be re-stated as follows:

$$0 = n_{it}^{1+\epsilon} - \mathbb{E}_{it}\left[\mathcal{M}_{it}U'(C_t)\left(\frac{c_{it}}{C_t}\right)^{-\frac{1}{\rho}}\theta c_{it}\right] \tag{49}$$

$$0 = \mathbb{E}_{it}\left[\left(l_{it}^{1+\epsilon} - \mathcal{M}_{it}U'(C_t)\left(\frac{c_{it}}{C_t}\right)^{-\frac{1}{\rho}}\eta c_{it}\right)\right] \tag{50}$$

We conclude that a set of allocations, prices and policies constitute an equilibrium if and only if the following three properties hold: (i) the allocations satisfy conditions (49) and (50) along with the resource constraint

$$C_t = \left[\int \left(q_{it}l_{it}^{\eta}\right)^{\frac{\rho-1}{\rho}} di\right]^{\frac{\rho}{\rho-1}}; \tag{51}$$

(ii) nominal prices satisfy

$$\frac{p_{it}}{P_t} = \left(\frac{q_{it}l_{it}^{\eta}}{C_t}\right)^{-\frac{1}{\rho}}; \tag{52}$$

and (iii) the monetary policy satisfies

$$M_t = P_t C_t. \tag{53}$$

We now seek to translate these properties in terms of the relevant coefficients that parameterize the allocations, prices, and policy under a log-normal specification. Thus let (omitting constants)

$$\begin{aligned}
\log q_{it} &= \varphi_s s_{it} + \varphi_x x_{it} + \varphi_z z_t \\
\log l_{it} &= l_{\bar{s}} \bar{s}_t + l_s s_{it} + l_x x_{it} + l_z z_t \\
\log C_t &= c_{\bar{s}} \bar{s}_t + c_z z_t \\
\log p_{it} &= \psi_s s_{it} + \psi_x x_{it} + \psi_z z_t \\
\log M_t &= \lambda \bar{s}_t
\end{aligned}$$

for some coefficients $(\varphi_s, \varphi_x, ..., \lambda)$, and with the understanding that s stands either for a or μ, depending on whether we are considering the case of technology or markup shocks. Note that the resource constraint (51) is satisfied if and only if

$$c_{\bar{s}} = (\varphi_s + \varphi_x) + \eta(l_s + l_x + l_{\bar{s}}) \tag{54}$$

$$c_z = \varphi_z + \eta l_z \tag{55}$$

while (53) is satisfied if and only if

$$\lambda = c_{\bar{s}} + (\psi_s + \psi_x) \tag{56}$$

Consider the consumer's demand function, which can be expressed as follows:

$$-\rho\left(\log p_{it} - \log P_t\right) = \left(\log c_{it} - \log C_t\right)$$

Using market clearing, the production function, and the proposed strategies, we can express the consumption of good i as follows:

$$\begin{aligned}
\log c_{it} &= \log y_{it} = \log q_{it} + \eta \log l_{it} \\
&= (\varphi_s + \eta l_s) s_{it} + (\varphi_x + \eta l_x) x_{it} + (\varphi_z + \eta l_z) z_t + \eta l_{\bar{s}} \bar{s}_t,
\end{aligned}$$

By implication, aggregate consumption satisfies

$$\log C_{it} = (\varphi_s + \eta l_s + \varphi_x + \eta l_x)\bar{s}_t + (\varphi_z + \eta l_z) z_t$$

Substituting the above two results in the consumer's demand function, and doing a similar substitution for p_{it} and P_t, we infer that the following must hold for all realizations of the shocks and signals:

$$-\rho(\psi_s s_{it} + \psi_x x_{it} - (\psi_s + \psi_x)\bar{s}_t) = (\varphi_s + \eta l_s) s_{it} + (\varphi_x + \eta l_x) x_{it} - (\varphi_s + \eta l_s + \varphi_x + \eta l_x)\bar{s}_t.$$

This is true if and only if

$$\psi_s = -\frac{1}{\rho}(\varphi_s + \eta l_s) \qquad \text{and} \qquad \psi_x = -\frac{1}{\rho}(\varphi_x + \eta l_x) \tag{57}$$

Finally, taking the logs of conditions (49) and (50) and using the properties of log-normal distributions, we can rewrite these conditions as follows:

$$\mathbb{E}_{it}[\log l_{it}] = \frac{1}{\theta}\left(\log q_{it} - a_{it}\right) \tag{58}$$

$$\log q_{it} = \hat{\phi}_a a_{it} + \hat{\phi}_\mu \mu_{it} + \frac{\hat{\alpha}}{\chi}\mathbb{E}_{it}\left[\log C_t\right]. \tag{59}$$

Clearly, condition (58) holds for all i if and only if

$$l_s = \frac{1}{\theta}\left(\varphi_s - \mathbf{1}_{s=a}\right) \tag{60}$$

$$l_x = \frac{1}{\theta}\varphi_x - l_{\bar{s}}\frac{\kappa_x}{\kappa_s+\kappa_x+\kappa_z} \tag{61}$$

$$l_z = \frac{1}{\theta}\varphi_z - l_{\bar{s}}\frac{\kappa_z}{\kappa_s+\kappa_x+\kappa_z}, \tag{62}$$

while condition (59) holds for all i if and only if

$$\varphi_s = \hat{\phi}_s \tag{63}$$

$$\varphi_x = \frac{\hat{\alpha}}{\chi}c_{\bar{s}}\frac{\kappa_x}{\kappa_s+\kappa_x+\kappa_z} \tag{64}$$

$$\varphi_z = \frac{\hat{\alpha}}{\chi}\left(c_{\bar{s}}\frac{\kappa_z}{\kappa_s+\kappa_x+\kappa_z} + c_z\right). \tag{65}$$

We can now use (54)-(55) to replace $c_{\bar{s}}$ and c_z in (64) and (65) to get

$$\varphi_x = \frac{\hat{\alpha}}{\chi}\left(\varphi_s + \varphi_x + \eta\left(l_s + l_x + l_{\bar{s}}\right)\right)\frac{\kappa_x}{\kappa_s+\kappa_x+\kappa_z}$$

and

$$\varphi_z = \frac{\hat{\alpha}}{\chi}\left(\varphi_s + \varphi_x + \eta\left(l_s + l_x + l_{\bar{s}}\right)\right)\frac{\kappa_z}{\kappa_s+\kappa_x+\kappa_z} + \frac{\hat{\alpha}}{\chi}\left(\varphi_z + \eta l_z\right),$$

with the understanding that s stands for either a or μ, depending on the case under consideration. Using (60), (61), (62), and (63) and rearranging gives

$$\left(\frac{\kappa_s+\kappa_x+\kappa_z}{\kappa_x}\frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta}\right)\varphi_x = \hat{\phi}_s + \frac{\eta}{\theta}\left(\hat{\phi}_s - \mathbf{1}_{s=a}\right) + \eta\frac{\kappa_s+\kappa_z}{\kappa_s+\kappa_x+\kappa_z}l_{\bar{s}}$$

and

$$\left(\frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta}\right)\varphi_z = \frac{\chi}{\hat{\alpha}}\frac{\kappa_z}{\kappa_x}\varphi_x - \eta\frac{\kappa_z}{\kappa_s+\kappa_x+\kappa_z}l_{\bar{s}}.$$

or

$$\varphi_x = \Gamma_x + \Gamma_x' l_{\bar{s}},$$

and

$$\varphi_z = \Gamma_z + \Gamma_z' l_{\bar{s}},$$

where

$$\Gamma_x \equiv \frac{\hat{\phi}_s + \frac{\eta}{\theta}\left(\hat{\phi}_s - \mathbf{1}_{s=a}\right)}{\frac{\kappa_s+\kappa_x+\kappa_z}{\kappa_x}\frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta}}, \quad \Gamma_x' \equiv \frac{\eta\kappa_x}{\kappa_s+\kappa_x+\kappa_z}\frac{\kappa_s+\kappa_z}{\left(\kappa_s+\kappa_x+\kappa_z\right)\frac{\chi}{\hat{\alpha}} - \kappa_x\left(1+\frac{\eta}{\theta}\right)},$$
$$\Gamma_z \equiv \frac{\frac{\chi}{\hat{\alpha}}\frac{\kappa_z}{\kappa_x}}{\frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta}}\Gamma_x, \quad \Gamma_z' \equiv \frac{\frac{\chi}{\hat{\alpha}}\frac{\kappa_z}{\kappa_x}\Gamma_x' - \eta\frac{\kappa_z}{\kappa_s+\kappa_x+\kappa_z}}{\frac{\chi}{\hat{\alpha}} - 1 - \frac{\eta}{\theta}}.$$

This completes the proof of the necessity of conditions (42)-(47) for an allocation to be part of an equilibrium.

We now prove sufficiency. Pick arbitrary $l_{\bar{s}}$ and let $(\varphi_s, \varphi_x, \varphi_z, l_s, l_x, l_z)$ be the unique vector that satisfies conditions (42) through (47) for the given $l_{\bar{s}}$. Next, let $(c_{\bar{s}}, c_z, \psi_s, \psi_x)$ be determined as in (54)-(57) and let $\psi_z = -c_z$. Finally, set λ as in (56). By construction, the allocations, prices and policies defined in this way consitute an equilibrium, which completes the sufficiency argument.

Part (ii). Substituting conditions (42)-(47) into (54) and (57) and using (56) shows that λ is a linear function of $l_{\bar{s}}$.

Part (iii). This proof is the same as that of part (i), except for one key difference: now the marginal costs and returns of second-state employment must be equated state-by-state, not just in expectation. It is this additional restriction that pins down $l_{\bar{s}}$ at the value stated in the lemma. A detailed derivation is available upon request.

Part (iv). From (53), when prices are sticky and, thus, cannot react to the noise in the monetary policy, second-period employment and total consumption will respond to m_t one-to-one. Thus, if we conjecture

$$\begin{aligned}\log l_{it} &= l_{\bar{s}}\bar{s}_t + l_s s_{it} + l_x x_{it} + l_z z_t + l_m m_t \\ \log C_t &= c_{\bar{s}}\bar{s}_t + c_z z_t + c_m m_t\end{aligned}$$

and follow analogous steps to those in the proof of Lemma 7, we obtain $c_m = \eta l_m = 1$. Modulo these changes, it is immediate to see that all the results in part (i) remain true. ■

Proof of Lemma 3. From part (ii) and part (iii) of Lemma 7 we know that a monetary policy as in (41) replicates flexible-price allocations if and only if $\lambda = \lambda^*$, where $\lambda^* \equiv \bar{\Pi} + \Pi l_{\bar{s}}^*$ and $l_{\bar{s}}^*$ is given by (48). ■

Proof of Proposition 6. *Part (i).* Similarly to the baseline model, once we use the equilibrium conditions for the wages and the prices, the FOCs of the firm's profit with respect to the two inputs reduce to the following:

$$n_{it}^{\epsilon} = \mathbb{E}_{it}\left[\mathcal{M}_{it}U'\left(Y_t\right)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\left(\theta\frac{y_{it}}{n_{it}}\right)\right] \tag{66}$$

$$l_{it}^{\epsilon} = \mathcal{M}_{it}U'\left(Y_t\right)\left(\frac{y_{it}}{Y_t}\right)^{-\frac{1}{\rho}}\left(\eta\frac{y_{it}}{l_{it}}\right) \tag{67}$$

Note that the first condition is essentially the same as in the baseline model. The second condition, by contrast, does not feature an expectation operator, because the absence of nominal rigidity means that the stage-2 input adjusts so as to equate marginal cost and marginal revenue state-by-state. We can use (67) to solve for $\log l_{it}$ as (omitting constants)

$$(1+\epsilon)\log l_{it} = -\mu_{it} + \left(\frac{1}{\rho} - \gamma\right)\log Y_t + \left(1 - \frac{1}{\rho}\right)\log y_{it}. \tag{68}$$

Next, using $\log y_{it} = \log q_{it} + \eta \log l_{it}$ to replace l_{it}, we can restate the above as

$$\log y_{it} = b_q \log q_{it} - b_\mu \mu_{it} + \bar{\alpha}\log Y_t$$

where

$$b_q \equiv \rho\nu \quad \text{and} \quad b_\mu \equiv \frac{\eta}{1+\epsilon}\rho\nu.$$

Using the above result to replace y_{it} into (66), taking logs of the latter, and also using $\log q_{it} = a_{it} + \theta \log n_{it}$, we finally arrive at

$$\log q_{it} = \hat{\phi}_a a_{it} + \hat{\phi}_\mu \mu_{it} + \frac{\hat{\alpha}}{\chi}\mathbb{E}_{it}[\log Y_t],$$

where $\hat{\phi}_a$, $\hat{\phi}_\mu$, $\hat{\alpha}$, χ, and ν are defined in (32).

Finally, integrating (68) across different islands gives

$$\int \log l_{it} di = \frac{1-\gamma}{1+\epsilon}\log Y_t - \frac{1}{1+\epsilon}\bar{\mu}_t,$$

which can be combined with (51) to get

$$\log Y_t = \chi\left(\log Q_t - \frac{\eta}{1+\epsilon}\bar{\mu}_t\right).$$

Therefore,

$$\log q_{it} = \hat{\phi}_a a_{it} + \hat{\phi}_\mu \mu_{it} + \hat{\phi}_{\bar{\mu}}\mathbb{E}_{it}\left[\bar{\mu}_t\right] + \hat{\alpha}\mathbb{E}_{it}\left[\log Q_t\right].$$

where $\hat{\phi}_{\bar{\mu}} \equiv -\hat{\alpha}\frac{\eta}{1+\epsilon}$.

Part (ii). From Lemma 6 we have that, irrespectively of whether the nominal rigidity is present or not, welfare is a decreasing function of

$$\Lambda' = \left(\Sigma_Q + \frac{1}{1-\hat{\alpha}}\sigma_q\right) + \xi\left(\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y\right).$$

When the policy of Lemma 5 is in place, the equilibrium quantities satisfy the FOC (67) which, as showed in part (i), can be rearranged as

$$\log y_{it} = b_q \log q_{it} - b_\mu \mu_{it} + \bar{\alpha}\log Y_t$$

On the other hand, Lemma 5 shows that the first-best level of output conditional on the first-period equilibrium production decision satisfies

$$\log \bar{y}_{it} = b_q \log q_{it} + \bar{\alpha} \log \bar{Y}_t.$$

Therefore, the aggregate and local gaps are given by, respectively,

$$\log Y_t - \log \bar{Y}_t = -b_{\bar{\mu}} \bar{\mu}_t$$

where $b_{\bar{\mu}} \equiv \frac{b_\mu}{1-\bar{\alpha}}$ and

$$\log y_{it} - \log \bar{y}_{it} = -b_\mu \mu_{it} - \bar{\alpha} b_{\bar{\mu}} \bar{\mu}_t.$$

Importantly, note that b_μ and $b_{\bar{\mu}}$ are independent of the information structure. It follows that the second term in Λ' reduces to

$$\Sigma_Y + \frac{1}{1-\bar{\alpha}} \sigma_y = b_{\bar{\mu}}^2 \sigma_{\bar{\mu}}^2 + \frac{1}{1-\bar{\alpha}} b_\mu^2 \sigma_\xi^2$$

which is independent of the information structure. Therefore, (39) reduces to

$$\Lambda' = \Sigma_Q + \frac{1}{1-\hat{\alpha}} \sigma_q + \omega$$

where $\omega \equiv \xi \frac{b_\mu^2}{1-\bar{\alpha}} \left(\frac{1}{1-\bar{\alpha}} \sigma_{\bar{\mu}}^2 + \sigma_\xi^2 \right)$, which corresponds to (10) in the main text. ■

Proof of Theorem 3. From Proposition 6, when the policy that replicates flexible prices is in place, welfare is a decreasing function of

$$\Lambda' = \Sigma + \frac{1}{1-\hat{\alpha}} \sigma + \omega.$$

Since ω is independent of the information structure, the effects of the precision of private and public information are determined only by the changes in the volatility and dispersion of the first-period gaps.

Consider first the case with technology shocks. In that case $\hat{\phi}_\mu = \hat{\phi}_{\bar{\mu}} = 0$ and $\omega = 0$ and the welfare effects coincide with those found in Proposition **??**. On the other hand, when the business cycle is driven by markup shocks, $\hat{\phi}_\mu$ is given in (32), $\hat{\phi}_{\bar{\mu}} \equiv -\hat{\alpha} \frac{\eta}{1+\epsilon}$, ω is given in Proposition 6, and

$$\Lambda' = \frac{\left(\hat{\alpha}\eta - (1+\epsilon)\hat{\phi}_\mu\right)^2 ((1-\hat{\alpha})\kappa_x + \kappa_z) - (1-\hat{\alpha})(1+\epsilon)\left(2\alpha\eta - (1+\epsilon)(1+\hat{\alpha})\hat{\phi}_\mu\right)\kappa_\mu \hat{\phi}_\mu}{(1-\hat{\alpha})^2 (1+\epsilon)^2 (\kappa_\mu + (1-\hat{\alpha})\kappa_x + \kappa_z)\kappa_\mu} + \frac{\hat{\phi}_\mu^2}{(1-\hat{\alpha})\kappa_\xi} + \omega.$$

Differentiating Λ' with respect to the precision of public and private information gives

$$\frac{\partial \Lambda'}{\partial \kappa_z} = \frac{\hat{\alpha}^2 \left(\eta - (1+\epsilon)\hat{\phi}_\mu\right)^2}{(1-\hat{\alpha})^2 (1+\epsilon)^2 (\kappa_\mu + (1-\hat{\alpha})\kappa_x + \kappa_z)^2}$$

and

$$\frac{\partial \Lambda'}{\partial \kappa_x} = \frac{\hat{\alpha}^2 \left(\eta - (1+\epsilon)\hat{\phi}_\mu\right)^2}{(1-\hat{\alpha})(1+\epsilon)^2 (\kappa_\mu + (1-\hat{\alpha})\kappa_x + \kappa_z)^2},$$

which are both always positive. ■

Proof of Proposition 7. Consider first the case of technology shocks. Denote by $\log \breve{q}_{it}$ and $\log \breve{Q}_t$ the local and aggregate first-period production with flexible prices, respectively. We can rewrite the aggregate volatility of the first-period aggregate gap as follows

$$\Sigma_Q \equiv Var\left(\log Q_t - \log Q_t^*\right) = Var\left(\log Q_t - \log \breve{Q}_t\right) + 2Cov\left(\log Q_t - \log \breve{Q}_t, \log \breve{Q}_t - \log Q_t^*\right) + \breve{\Sigma}_Q,$$

where $\breve{\Sigma}_Q$ is the aggregate volatility of the first-period aggregate gap when prices are flexible. Similarly, the dispersion of the first-period local gaps can be rewritten as

$$\begin{aligned}
\sigma_q &\equiv Var\left(\left(\log q_{it} - \log Q_t\right) - \left(\log q_{it}^* - \log Q_t^*\right)\right) \\
&= Var\left(\left(\log q_{it} - \log Q_t\right) - \left(\log \breve{q}_{it} - \log \breve{Q}_t\right)\right) \\
&\quad + 2Cov\left(\left(\log q_{it} - \log Q_t\right) - \left(\log \breve{q}_{it} - \log \breve{Q}_t\right), \left(\log \breve{q}_{it} - \log \breve{Q}_t\right) - \left(\log q_{it}^* - \log Q_t^*\right)\right) + \breve{\sigma}_q,
\end{aligned}$$

where $\breve{\sigma}_q$ is the dispersion of the first-period local gaps when prices are flexible.

From part (i) of Lemma 7, first-period production satisfies (40) with φ_x and φ_z given by (43) and (44), respectively. Denote by $\breve{\varphi}_x$ and $\breve{\varphi}_z$ the coefficients φ_x and φ_z when the policy replicating flexible prices is in place. Combining part (i) and part (iii) of Lemma 7, $\breve{\varphi}_x$ and $\breve{\varphi}_z$ satisfy

$$\begin{aligned}
\breve{\varphi}_x &= \Gamma_x + \Gamma_x' l_{\bar{a}}^* = \frac{\phi_a}{\kappa_a + (1-\hat{\alpha})\kappa_x + \kappa_z} \hat{\alpha}\kappa_x, \\
\breve{\varphi}_z &= \Gamma_z + \Gamma_z' l_{\bar{a}}^* = \frac{\phi_a}{\kappa_a + (1-\hat{\alpha})\kappa_x + \kappa_z} \frac{\hat{\alpha}\kappa_z}{1-\hat{\alpha}}.
\end{aligned}$$

Finally, first-best production is given by (33). Thus,

$$\log Q_t - \log \breve{Q}_t = \left(\Gamma_x' + \Gamma_z'\right)\left(l_{\bar{a}} - l_{\bar{a}}^*\right)\bar{a}_t + \Gamma_z'\left(l_{\bar{a}} - l_{\bar{a}}^*\right)\varepsilon_t$$

Similarly,

$$\log \breve{Q}_t - \log Q_t^* = \left(-\frac{\hat{\alpha}}{1-\hat{\alpha}}\hat{\phi}_a + \Gamma_x + \Gamma_x' l_{\bar{a}}^* + \Gamma_z + \Gamma_z' l_{\bar{a}}^*\right)\bar{a}_t + \left(\Gamma_z + \Gamma_z' l_{\bar{a}}^*\right)\varepsilon_t$$

Therefore,

$$\begin{aligned}
&Cov\left(\log Q_t - \log \breve{Q}_t, \log \breve{Q}_t - \log Q_t^*\right) \\
&= \left(\Gamma_x' + \Gamma_z'\right)\left(l_{\bar{a}} - l_{\bar{a}}^*\right)\left(-\frac{\hat{\alpha}}{1-\hat{\alpha}}\hat{\phi}_a + \Gamma_x + \Gamma_x' l_{\bar{a}}^* + \Gamma_z + \Gamma_z' l_{\bar{a}}^*\right)\frac{1}{\kappa_a} + \left(\Gamma_z + \Gamma_z' l_{\bar{a}}^*\right)\Gamma_z'\left(l_{\bar{a}} - l_{\bar{a}}^*\right)\frac{1}{\kappa_z} \\
&= -\frac{\hat{\alpha}\hat{\phi}_a\Gamma_x'}{\left(\kappa_a + (1-\hat{\alpha})\kappa_x + \kappa_z\right)(1-\hat{\alpha})}\left(l_{\bar{a}} - l_{\bar{a}}^*\right)
\end{aligned}$$

and

$$Var\left(\log Q_t - \log \breve{Q}_t\right) = \left[\left(\Gamma_x' + \Gamma_z'\right)^2 \frac{1}{\kappa_a} + \left(\Gamma_z'\right)^2 \frac{1}{\kappa_z}\right]\left(l_{\bar{a}} - l_{\bar{a}}^*\right)^2.$$

Similarly, we have

$$\left(\log q_{it} - \log Q_t\right) - \left(\log \breve{q}_{it} - \log \breve{Q}_t\right) = \Gamma'_x \left(l_{\bar{a}} - l^*_{\bar{a}}\right) u_{it}$$

and

$$\left(\log \breve{q}_{it} - \log \breve{Q}_t\right) - \left(\log q^*_{it} - \log Q^*_t\right) = \left(\Gamma_x + \Gamma'_x l^*_{\bar{a}}\right) u_{it}.$$

Therefore,

$$Cov\left(\left(\log q_{it} - \log Q_t\right) - \left(\log \breve{q}_{it} - \log \breve{Q}_t\right), \left(\log \breve{q}_{it} - \log \breve{Q}_t\right) - \left(\log q^*_{it} - \log Q^*_t\right)\right) = \Gamma'_x \left(\Gamma_x + \Gamma'_x l^*_{\bar{a}}\right) \frac{1}{\kappa_x} \left(l_{\bar{a}} - l^*_{\bar{a}}\right).$$

and

$$Var\left(\left(\log q_{it} - \log Q_t\right) - \left(\log \breve{q}_{it} - \log \breve{Q}_t\right)\right) = \left(\Gamma'_x\right)^2 \left(l_{\bar{a}} - l^*_{\bar{a}}\right)^2 \frac{1}{\kappa_x}.$$

Collecting all the terms together, we find that the second-order welfare losses associated with the first-period gaps, $\Sigma_Q + \frac{1}{1-\hat{\alpha}}\sigma_q$, can be rewritten as

$$\begin{aligned}
&\breve{\Sigma}_Q + \frac{1}{1-\hat{\alpha}}\breve{\sigma}_q + \left[\left(\Gamma'_x + \Gamma'_z\right)^2 \frac{1}{\kappa_a} + \left(\Gamma'_z\right)^2 \frac{1}{\kappa_z} + \frac{1}{1-\hat{\alpha}} \left(\Gamma'_x\right)^2 \frac{1}{\kappa_x}\right] \left(l_{\bar{a}} - l^*_{\bar{a}}\right)^2 \\
&+ \frac{1}{1-\hat{\alpha}} \left(\Gamma'_x \left(\Gamma_x + \Gamma'_x l^*_{\bar{a}}\right) \frac{1}{\kappa_x} - \frac{\hat{\alpha}\hat{\phi}_a \Gamma'_x}{\kappa_a + (1-\hat{\alpha})\kappa_x + \kappa_z}\right) \left(l_{\bar{a}} - l^*_{\bar{a}}\right)
\end{aligned}$$

which simplifies to

$$\breve{\Sigma}_Q + \frac{1}{1-\hat{\alpha}}\breve{\sigma}_q + \left[\left(\Gamma'_x + \Gamma'_z\right)^2 \frac{1}{\kappa_a} + \left(\Gamma'_z\right)^2 \frac{1}{\kappa_z} + \frac{1}{1-\hat{\alpha}} \left(\Gamma'_x\right)^2 \frac{1}{\kappa_x}\right] \left(l_{\bar{a}} - l^*_{\bar{a}}\right)^2. \tag{69}$$

Thus,

$$\Sigma_Q + \frac{1}{1-\hat{\alpha}}\sigma_q = \breve{\Sigma}_Q + \frac{1}{1-\hat{\alpha}}\breve{\sigma}_q + \vartheta_a \left(l_{\bar{a}} - l^*_{\bar{a}}\right)^2,$$

where

$$\vartheta_a \equiv \left(\Gamma'_x + \Gamma'_z\right)^2 \frac{1}{\kappa_a} + \left(\Gamma'_z\right)^2 \frac{1}{\kappa_z} + \frac{1}{1-\hat{\alpha}} \left(\Gamma'_x\right)^2 \frac{1}{\kappa_x}.$$

Let's now look at the second-period gaps. With technology shocks flexible-price allocations, $\log \breve{y}_{it}$, satisfy (67) which can be rearranged to get (34), except for a constant capturing the constant markup. Thus, except for an unimportant constant, flexible-price allocations coincide with first-best allocations conditional on first-period production choices. Using the conditions in Lemma 7, we can then compute the volatility and dispersion of second-period gaps:

$$\begin{aligned}
\Sigma_Y &= \left(\left(1+\frac{\eta}{\theta}\right)\Gamma'_x + \left(1+\frac{\eta}{\theta}\right)\Gamma'_z + \frac{\eta\kappa_a}{\kappa_a + \kappa_x + \kappa_z}\right)^2 \frac{\left(l_{\bar{a}} - l^*_{\bar{a}}\right)^2}{\kappa_a} + \left(\left(1+\frac{\eta}{\theta}\right)\Gamma'_z - \frac{\eta\kappa_z}{\kappa_a + \kappa_x + \kappa_z}\right)^2 \frac{\left(l_{\bar{a}} - l^*_{\bar{a}}\right)^2}{\kappa_z} + \sigma \\
\sigma_y &= \left(\left(1+\frac{\eta}{\theta}\right)\Gamma'_x - \frac{\eta\kappa_x}{\kappa_a + \kappa_x + \kappa_z}\right) \frac{\left(l_{\bar{a}} - l^*_{\bar{a}}\right)^2}{\kappa_x}.
\end{aligned}$$

Collecting all the terms together, we find that the second-order welfare losses associated with the second-period gaps, $\Sigma_Y + \frac{1}{1-\hat{\alpha}}\sigma_y$, can be rewritten as

$$\vartheta'_a \left(l_{\bar{a}} - l^*_{\bar{a}}\right)^2 + \sigma^2_m,$$

where

$$\vartheta_a' \equiv \left(\left(1+\frac{\eta}{\theta}\right)\Gamma_x' + \left(1+\frac{\eta}{\theta}\right)\Gamma_z' + \frac{\eta\kappa_a}{\kappa_a+\kappa_x+\kappa_z}\right)^2 \frac{1}{\kappa_a} + \left(\left(1+\frac{\eta}{\theta}\right)\Gamma_z' - \frac{\eta\kappa_z}{\kappa_a+\kappa_x+\kappa_z}\right)^2 \frac{1}{\kappa_z}$$
$$\frac{1}{1-\bar{\alpha}}\left(\left(1+\frac{\eta}{\theta}\right)\Gamma_x' - \frac{\eta\kappa_x}{\kappa_a+\kappa_x+\kappa_z}\right)\frac{1}{\kappa_x}.$$

Finally, from part (ii) of Lemma 7 we can rewrite $l_{\bar{a}} - l_{\bar{a}}^*$ as $(\lambda-\lambda^*)/\Pi$ and the statement of the proposition follows from using Lemma 6 and letting $\Lambda = \breve{\Sigma}_Q + \frac{1}{1-\bar{\alpha}}\breve{\sigma}_q$, $\mathcal{T} = \sigma_m^2$, and

$$\Theta = \frac{\vartheta_a + \xi\vartheta_a'}{\Pi^2\sigma_a^2}.$$

The proof for the case with markup shocks follows similar steps, so here we simply report variances and covariances required to derive $\mathcal{K}$. Remember that with markup shocks first-best allocations are constant. Using part (i) and (iii) of Lemma 7, the first-period aggregate gaps are

$$\log Q_t - \log \breve{Q}_t = \left(\Gamma_x' + \Gamma_z'\right)\left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right)\bar{\mu}_t + \Gamma_z'\left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right)\varepsilon_t$$

and

$$\log \breve{Q}_t - \log Q_t^* = \log \breve{Q}_t = \left(\hat{\phi}_\mu + \Gamma_x + \Gamma_x' l_{\bar{\mu}}^* + \Gamma_z + \Gamma_z' l_{\bar{\mu}}^*\right)\bar{\mu}_t + \left(\Gamma_z + \Gamma_z' l_{\bar{\mu}}^*\right)\varepsilon_t.$$

Therefore,

$$Var\left(\log Q_t - \log \tilde{Q}_t\right) = \left(\frac{\left(\Gamma_x' + \Gamma_z'\right)^2}{\kappa_\mu} + \frac{\Gamma_z'}{\kappa_z}\right)\left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right)^2$$

and

$$\begin{aligned}
& Cov\left(\log Q_t - \log \breve{Q}_t, \log \breve{Q}_t\right) \\
= \; & \left(\Gamma_x' + \Gamma_z'\right)\left(\phi_\mu + \Gamma_x + \Gamma_x' l_{\bar{\mu}}^* + \Gamma_z + \Gamma_z' l_{\bar{\mu}}^*\right)\frac{1}{\kappa_\mu}\left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right) + \left(\Gamma_z + \Gamma_z' l_{\bar{\mu}}^*\right)\Gamma_z' \frac{1}{\kappa_z}\left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right) \\
= \; & \frac{1}{1-\hat{\alpha}}\left(\frac{\hat{\phi}_\mu + \frac{\eta}{1+\epsilon}}{\kappa_\mu + (1-\hat{\alpha})\kappa_x + \kappa_z}\hat{\alpha}\Gamma_x' - \left(\Gamma_x' + \Gamma_z'\right)\left(\hat{\phi}_\mu + \frac{\eta}{1+\epsilon}\hat{\alpha}\right)\frac{1}{\kappa_\mu}\right)\left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right).
\end{aligned}$$

Similarly, the terms capturing the dispersion of first-period local output are

$$\left(\log q_{it} - \log Q_t\right) - \left(\log \breve{q}_{it} - \log \breve{Q}_t\right) = \Gamma_x'\left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right)u_{it}$$

and

$$\log \breve{q}_{it} - \log \breve{Q}_t = \hat{\phi}_\mu \xi_{it} + \left(\Gamma_x + \Gamma_x' l_{\bar{\mu}}^*\right)u_{it},$$

which imply

$$Var\left(\left(\log q_{it} - \log Q_t\right) - \left(\log \breve{q}_{it} - \log \breve{Q}_t\right)\right) = \frac{\left(\Gamma_x'\right)^2}{\kappa_x}\left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right)^2$$

and

$$Cov\left(\left(\log q_{it}-\log Q_t\right)-\left(\log \breve{q}_{it}-\log \breve{Q}_t\right),\log \breve{q}_{it}-\log \breve{Q}_t\right)=\frac{\Gamma'_x}{\kappa_x}\left(\hat{\phi}_\mu+\Gamma_x+\Gamma'_x l^*_{\bar{\mu}}\right)\left(l_{\bar{\mu}}-l^*_{\bar{\mu}}\right).$$

Collecting all the terms together, we find that the second-order welfare losses associated with the first-period gaps, $\Sigma_Q+\frac{1}{1-\hat{\alpha}}\sigma_q$, can be rewritten as

$$\breve{\Sigma}_Q+\frac{1}{1-\hat{\alpha}}\breve{\sigma}_q+\vartheta_\mu\left(l_{\bar{\mu}}-l^*_{\bar{\mu}}\right)^2-2\tilde{\vartheta}_\mu\left(l_{\bar{\mu}}-l^*_{\bar{\mu}}\right),$$

where

$$\vartheta_\mu\equiv\frac{\left(\Gamma'_x+\Gamma'_z\right)^2}{\kappa_\mu}+\frac{\Gamma'_z}{\kappa_z}+\frac{1}{1-\hat{\alpha}}\frac{\left(\Gamma'_x\right)^2}{\kappa_x}$$

and

$$\tilde{\vartheta}_\mu\equiv-\frac{1}{1-\hat{\alpha}}\left(\frac{\Gamma'_x+\Gamma'_z}{\kappa_\mu}\left(\hat{\phi}_\mu-\frac{\eta}{1+\epsilon}\hat{\alpha}\right)+\frac{\Gamma'_x}{\kappa_x}\hat{\phi}_\mu\right).$$

Let's now look at the second-period gaps. From (34) we know that first-best output condition on first-period production is given by

$$\log \bar{y}_{it}=\rho v\log q_{it}+\bar{\alpha}\log \bar{Y}_t$$

and, aggregating across different islands,

$$\log \bar{Y}_t=\frac{\rho v}{1-\bar{\alpha}}\log Q_t.$$

The latter, together with part (i), (iii), and (iv) of Lemma 7, gives

$$\begin{aligned}\log Y_t-\log \breve{Y}_t \quad = \quad & \left(\left(1+\frac{\eta}{\theta}\right)\left(\varphi_x-\breve{\varphi}_x\right)+\left(1+\frac{\eta}{\theta}\right)\left(\varphi_z-\breve{\varphi}_z\right)+\frac{\eta\kappa_\mu}{\kappa_\mu+\kappa_x+\kappa_z}\right)\left(l_{\bar{\mu}}-l^*_{\bar{\mu}}\right)\bar{\mu}_t\\ &+\left(\left(1+\frac{\eta}{\theta}\right)\left(\varphi_z-\breve{\varphi}_z\right)-\frac{\eta\kappa_z}{\kappa_\mu+\kappa_x+\kappa_z}\right)\left(l_{\bar{\mu}}-l^*_{\bar{\mu}}\right)\varepsilon_t+m_t\end{aligned}$$

and

$$\log \breve{Y}_t-\log \bar{Y}_t=-\frac{\eta}{1+\varepsilon}\frac{\rho\nu}{1-\bar{\alpha}}\bar{\mu}_t.$$

In turn, these imply

$$\begin{aligned}&Var\left(\log Y_t-\log \breve{Y}_t\right)\\ =\;&\left[\left(\left(1+\frac{\eta}{\theta}\right)\left(\Gamma'_x+\Gamma'_z\right)+\frac{\eta\kappa_\mu}{\kappa_\mu+\kappa_x+\kappa_z}\right)^2\frac{1}{\kappa_\mu}+\left(\left(1+\frac{\eta}{\theta}\right)\Gamma'_z-\frac{\eta\kappa_z}{\kappa_\mu+\kappa_x+\kappa_z}\right)^2\frac{1}{\kappa_z}\right]\left(l_{\bar{\mu}}-l^*_{\bar{\mu}}\right)^2+\sigma_m^2\end{aligned}$$

and

$$Cov\left(\log Y_t-\log \breve{Y}_t,\log \breve{Y}_t-\log \bar{Y}_t\right)=-\frac{\eta}{1+\varepsilon}\frac{\rho\nu}{1-\bar{\alpha}}\left(\left(1+\frac{\eta}{\theta}\right)\left(\Gamma'_x+\Gamma'_z\right)+\frac{\eta\kappa_\mu}{\kappa_\mu+\kappa_x+\kappa_z}\right)\frac{1}{\kappa_\mu}\left(l_{\bar{\mu}}-l^*_{\bar{\mu}}\right)$$

Similarly, the terms capturing the dispersion local output are

$$\left(\log y_{it}-\log Y_t\right)-\left(\log \breve{y}_{it}-\log \breve{Y}_t\right)=\left(\left(1+\frac{\eta}{\theta}\right)\left(\varphi_x-\breve{\varphi}_x\right)-\frac{\eta\kappa_x}{\kappa_\mu+\kappa_x+\kappa_z}\right)\left(l_{\bar{\mu}}-l^*_{\bar{\mu}}\right)u_{it}$$

and

$$\left(\log \breve{y}_{it} - \log \breve{Y}_t\right) - \left(\log \bar{y}_{it} - \log \bar{Y}_t\right) = -\frac{\eta}{1+\varepsilon}\rho\nu\xi_{it},$$

which imply

$$Var\left(\left(\log y_{it} - \log Y_t\right) - \left(\log \breve{y}_{it} - \log \breve{Y}_t\right)\right) = \left(\left(1+\frac{\eta}{\theta}\right)\Gamma_x' - \frac{\eta\kappa_x}{\kappa_\mu+\kappa_x+\kappa_z}\right)^2 \frac{1}{\kappa_x}\left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right)^2$$

and

$$\begin{aligned} & Cov\left(\left(\log y_{it} - \log Y_t\right) - \left(\log \breve{y}_{it} - \log \breve{Y}_t\right), \left(\log \breve{y}_{it} - \log \breve{Y}_t\right) - \left(\log \bar{y}_{it} - \log \bar{Y}_t\right)\right) \\ = & -\frac{\eta}{1+\varepsilon}\rho\nu\left(\left(1+\frac{\eta}{\theta}\right)\Gamma_x' - \frac{\eta\kappa_x}{\kappa_\mu+\kappa_x+\kappa_z}\right)\frac{1}{\kappa_x}\left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right) \end{aligned}$$

Collecting all the terms together, we find that the second-order welfare losses associated with the second-period gaps, $\Sigma_Y + \frac{1}{1-\bar{\alpha}}\sigma_y$, can be rewritten as

$$\breve{\Sigma}_Y + \frac{1}{1-\hat{\alpha}}\breve{\sigma}_y + \vartheta_\mu'\left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right)^2 - 2\tilde{\vartheta}_\mu'\left(l_{\bar{\mu}} - l_{\bar{\mu}}^*\right) + \sigma_m^2,$$

where

$$\begin{aligned} \vartheta_\mu' \equiv & \left(\left(1+\frac{\eta}{\theta}\right)\left(\Gamma_x'+\Gamma_z'\right) + \frac{\eta\kappa_\mu}{\kappa_\mu+\kappa_x+\kappa_z}\right)^2\frac{1}{\kappa_\mu} + \left(\left(1+\frac{\eta}{\theta}\right)\Gamma_z' - \frac{\eta\kappa_z}{\kappa_\mu+\kappa_x+\kappa_z}\right)^2\frac{1}{\kappa_z} \\ & + \frac{1}{1-\bar{\alpha}}\left(\left(1+\frac{\eta}{\theta}\right)\Gamma_x' - \frac{\eta\kappa_x}{\kappa_\mu+\kappa_x+\kappa_z}\right)^2\frac{1}{\kappa_x} \end{aligned}$$

and

$$\tilde{\vartheta}_\mu' \equiv \frac{\eta}{1+\varepsilon}\frac{\rho\nu}{1-\bar{\alpha}}\left(1+\frac{\eta}{\theta}\right)\left[\frac{\Gamma_x'+\Gamma_z'}{\kappa_\mu} + \frac{\Gamma_x'}{\kappa_x}\right].$$

Finally, from part (ii) of Lemma 7 we can rewrite $l_{\bar{\mu}} - l_{\bar{\mu}}^*$ as $\left(\lambda - \lambda^*\right)/\Pi$ and the statement of the proposition follows from using Lemma 6 and letting $\Lambda = \breve{\Sigma}_Q + \frac{1}{1-\hat{\alpha}}\breve{\sigma}_q + \xi\left(\breve{\Sigma}_Y + \frac{1}{1-\hat{\alpha}}\breve{\sigma}_y\right)$, $\mathcal{T} = \sigma_m^2$,

$$\Theta_1 = \frac{\tilde{\vartheta}_\mu + \xi\tilde{\vartheta}_\mu'}{\Pi^2\sigma_\mu^2}, \text{ and } \Theta_2 = \frac{\vartheta_\mu + \xi\vartheta_\mu'}{\Pi^2\sigma_\mu^2}.$$

■

Proof of Lemma 4. From Proposition 7, the optimal policy can be found by minimizing the term $\Lambda + \mathcal{K} + \mathcal{T}$. The only terms that can be affected by monetary policy are $\mathcal{K}$ and $\mathcal{T}$. Finally, The minimum value for $\mathcal{T}$ is clearly achieved when $\sigma_m^2 = 0$. From (11), $\lambda^{**} = \arg\min_\lambda \mathcal{K}(\lambda) = \Theta_1/\Theta_2$. The minimum value is $K(\kappa_x, \kappa_z) = -\Theta_1^2/\Theta_2$. Also, note that since $\mathcal{K}(0) = 0$ it has to be the case that $K(\kappa_x, \kappa_z) \leq 0$.

Part (i). When real rigidities are absent ($\theta = 0$),

$$K(\kappa_x, \kappa_z) - \frac{(1-\gamma\rho)^2}{(\gamma+\epsilon)^2(1+\epsilon\rho)\left((1+\epsilon\rho)(\kappa_\mu+\kappa_z) + \rho(\gamma+\epsilon)\kappa_x\right)},$$

which is clearly increasing in both κ_x and κ_z.

Part (ii). First note that both Θ_1 and Θ_2 are linear in $\kappa = \kappa_x + \kappa_z$. Thus, $\bar{K}(\kappa, r)$ is linear in κ and the statement follows from the fact that $-\Theta_1^2/\Theta_2 \leq 0$.

Part (iii). When $\kappa_x = .1$, $\kappa_z = .9$, $\epsilon = 1$, $\gamma = 1$, $\eta = .5$, $\theta = .5$, $\rho = 2$, we have that $K(\kappa_x, \kappa_z)$ is increasing in both κ_x and κ_z. On the contrary, when $\kappa_x = .5$, $\kappa_z = .5$, $\epsilon = 1$, $\gamma = 2$, $\eta = .5$, $\theta = .5$, $\rho = 2$, $K(\kappa_x, \kappa_z)$ is decreasing in κ_x and when $\kappa_x = .1$, $\kappa_z = .9$, $\epsilon = .45$, $\gamma = .1$, $\eta = .03$, $\theta = .36$, $\rho = .09$, $K(\kappa_x, \kappa_z)$ is decreasing in κ_z. ■

Proof of Theorem 4. *Part (i).* This part follows from Proposition 6 along with the fact that the optimal policy with technology shocks replicates flexible prices.

Part (ii). This part follows from Proposition 6, Lemma 4, and the discussion in the main text. ■

References

[1] Adam, Klaus (2007), "Optimal monetary policy with imperfect common knowledge," *Journal of Monetary Economics* 54(2), 267-301.

[2] Angeletos, George-Marios, and Jennifer La'O (2009), "Noisy Business Cycles," *NBER Macroeconomics Annual 2009*, 24, 319-378.

[3] Angeletos, George-Marios, and Jennifer La'O (2011), "Dispersed Information over the Business Cycle: Optimal Fiscal and Monetary Policy," MIT/Chicago Booth mimeo.

[4] Angeletos, George-Marios, and Jennifer La'O (2012), "Sentiments," *Econometrica forthcoming*.

[5] Angeletos, George-Marios, and Alessandro Pavan (2007), "Efficient Use of Information and Social Value of Information," *Econometrica* 75(4), 1103-1142.

[6] Baeriswyl, Romain, and Camille Cornand (2010), "The signaling role of policy actions," *Journal of Monetary Economics* 57(6), 682-695.

[7] Blanchard, Olivier J., and Nobuhiro Kiyotaki (1987), "Monopolistic Competition and the Effects of Aggregate Demand," *American Economic Review* 77(4), 647-666.

[8] Fixler, Dennis J., and Bruce T. Grimm (2005), "Reliability of the NIPA Estimates of U.S. Economic Activity," *Survey of Current Business 85*, 8-19.

[9] Gali, Jordi (2008), *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework,* Princeton University Press.

[10] Hellwig, Christian (2005), "Heterogeneous Information and the Welfare Effects of Public Information Disclosures," UCLA mimeo.

[11] James, Jonathan, and Phillip Lawler (2011), "Optimal Policy Intervention and the Social Value of Public Information,"*American Economic Review* 101(4), 1561-74.

[12] Lorenzoni, Guido (2010), "A Theory of Demand Shocks," *American Economic Review* 99(5), 2050-2084.

[13] Lucas, Robert E., Jr. (1972), "Expectations and the Neutrality of Money," *Journal of Economic Theory* 4, 103-124.

[14] Mackowiak, Bartosz, and Mirko Wiederholt (2009), "Optimal Sticky Prices under Rational Inattention," *American Economic Review*, 99, 769-803.

[15] Mankiw, N. Gregory, and Ricardo Reis (2002), "Sticky information versus sticky prices: a proposal to replace the New Keynesian Phillips curve," *The Quarterly Journal of Economics* 117(4), 1295-1328.

[16] Melosi, Leonardo (2011), "A Likelihood Analysis of Models with Information Frictions," London Business School mimeo.

[17] Morris, Stephen, and Hyun Song Shin (2002), "The Social Value of Public Information," *American Economic Review* 92, 1521-1534.

[18] Morris, Stephen, Hyun Song Shin, and Hui Tong (2006), "Social Value of Public Information: Morris and Shin (2002) Is Actually Pro-Transparency, Not Con: Reply," *American Economic Review*, 96(1), 453-455.

[19] Paciello, Luigi, and Mirko Wiederholt (2012), "Exogenous information, endogenous information and optimal monetary policy," Goethe University/EIEF Mimeo.

[20] Roca, Mauro (2005), "Transparency and Monetary Policy with Imperfect Common Knowledge," Columbia University Mimeo.

[21] Sims, Christopher A. (2003), "Implications of rational inattention," *Journal of Monetary Economics* 50(3), 665-690.

[22] Svensson, Lars E. (2006), "Social Value of Public Information: Comment: Morris and Shin (2002) Is Actually Pro-Transparency, Not Con," *American Economic Review* 96(1), 448-452.

[23] Walsh, Carl E. (2007), "Optimal economic transparency," *International Journal of Central Banking* 3(1), 5-36.

[24] Woodford, Michael (2003a), "Imperfect Common Knowledge and the Effects of Monetary Policy," in P. Aghion, R. Frydman, J. Stiglitz, and M. Woodford, eds., *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, Princeton: Princeton University Press.

[25] Woodford, Michael (2003b), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press.