

Optimal sovereign debt policy with private trading: Explaining allocation puzzle

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Abstract

This paper studies optimal sovereign debt policy of the government with limited commitment and compare the optimal policies in economies with and without government's capital control. The comparison of optimal sovereign debt policies can rationalize why more financially open market economies show more severe allocation puzzle — more negative relationship between growth and public capital flows, which is observed in the data.

Keywords: allocation puzzle, sovereign debt, capital control

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1 Introduction

The textbook neoclassical growth model predicts that countries with faster productivity growth should invest more and attract more foreign capital. Recently, however, Gourinchas and Jeanne (2013) show that the allocation of capital flows across developing countries is the opposite of this prediction: the correlation between growth and capital flows across developing countries is negative, showing that capital does not flow more to countries that invest and grow more. They call this feature “allocation puzzle.”

More detailed analysis of this allocation puzzle by Gourinchas and Jeanne (2013) shows that the allocation puzzle is mostly a feature of public flows and the allocation puzzle applies more strongly to financially more open economies. That is, the negative relationship between growth and capital flows is driven by public capital flows and the financial openness strengthens the negative relationship between growth and public capital flows. In this paper, we analyze the stronger allocation puzzle in more open economies and explain this feature as the result of optimal sovereign debt policy when there is a limited commitment of the government with and without the government’s private capital control.

Aguiar and Amador (2011) develop a model with limited commitment of the government on sovereign debt and foreign capital investment and they show that optimal sovereign debt policy with this limited commitment can explain the negative relationship between growth and public capital flows. In the optimal sovereign debt contract with limited commitment, there is an incentive to backload consumption because increasing future consumption reduces the default incentive of today as well as that of future. This backloaded consumption is associated with the decrease of the sovereign debt over time because when it is due payments of more consumption to the domestic government, the foreign bond holders’ continuation payoff is reduced, decreasing government debts. Moreover, foreign capitalists under-invest in earlier periods in order to reduce the incentive for the government to deviate, and as consumption promise to the domestic increases over time, more foreign capital can be invested into domestic firms and the domestic economy grows. Thus, along the growth path of a developing economy, there are opposite movements between sovereign debt and foreign capital investment, which explains the negative correlation between net public capital inflows and growth.

The key assumption of Aguiar and Amador (2011), however, is that the private domestic agents cannot access to the international financial market implying that the government can perfectly control private capital flows. So, they cannot say anything about how market

openness affects the allocation puzzle. Aguiar and Amador (2011) indeed focus more on how the political friction (the preference on current consumption of politicians) slows down the growth (slowing down a convergence to the steady state). In this paper, however, we focus on how the market openness can amplify the mechanism of generating negative relationship between growth and public capital flows in Aguiar and Amador (2011).

When private agents can access to the international market and trade bonds to smooth their consumption, the government's ability of "consumption backloading" is limited because private agents will borrow from the international market. Thus, it is more costly for the foreigners to prevent domestic government from deviating, which requires more decrease in the value to the foreign bond holders for the same growth of promise utility to the domestic government. This implies more decrease of sovereign debt for the same growth in more open economies without capital control. That is, the mechanism of Aguiar and Amador (2011) in financially open economies, generating even stronger negative relationship between the foreign capital investment and government's external debt.

There has been a growing literature that tries to explain the allocation puzzle, after this feature was pointed out by Gourinchas and Jeanne (2007). We do not intend to provide full list of the literature here, but we should remark that most studies try to explain this allocation puzzle feature focusing on private agent's saving and investment behavior. For example, low domestic financial development of the growing economies increases domestic savings and reduces the response of the investment to growth, which lead to larger current account surpluses (Caballero et al. (2008), Song et al. (2011)). Also, the rise in idiosyncratic risk that is associated with growth of economy can explain the negative relationship between growth and capital if precautionary saving is encouraged due to low domestic financial development (Carroll and Jeanne (2009), Sandri (2010)).¹ These studies, however, did not seriously consider that the allocation is mostly a feature of public capital flows, not private capital flows. Aguiar and Amador (2011) is the first paper that develops a model to explain the allocation puzzle, focusing on the role of the government and public capital flows. This paper builds on the model of Aguiar and Amador (2011) to explain why the allocation puzzle gets stronger for more open economies, which cannot be explained by Aguiar and Amador (2011) because of their assumption that government can perfectly control private capital flows.

More generally, this paper is related to the literature on puzzling observations of saving, growth and investment. Positive correlation between savings and growth is one puzzling fact

¹Gourinchas and Jeanne (2013) (section 6) discuss more literature on allocation puzzle.

established in the literature (Carroll and Summers (1991) and Carroll and Weil (1994)). The literature has also established related puzzling fact — strongly positive correlation between savings and investment (Feldstein and Horioka (1980)). As Gourinchas and Jeanne (2007) pointed out, the allocation puzzle is stronger than these two puzzles because it shows that the difference between saving and investment (capital outflows) is positively correlated with growth.

Decentralization of the constrained efficient allocation as a competitive equilibrium in an economy with government's limited commitment on sovereign debt has been studied by Kehoe and Perri (2004) and Aguiar et al. (2009). As in these studies, we also decentralize the constrained efficient allocation using taxes on the returns to capital investment, but we allow private agents trade in the international market in comparison with the perfect capital control of the government in their studies. Decentralization of the optimal sovereign debt in an economy where private agents can trade in the international financial market has been studied in Jeske (2006) and Wright (2006), but they study risk sharing contract in an endowment economy where every country can default while this paper studies the risk sharing contract with one-sided limited commitment — international lenders can commit while borrowing countries (government) cannot commit — in a production economy.

Methodologically, this paper is also related to the literature on optimal allocation with hidden private trading. In the literature on mechanism design with moral hazard or private information, there are several papers that study optimal contract with private unobservable trading (Cole and Kocherlakota (2001), Werning (2002), Golosov and Tsyvinski (2007), Ábrahám and Pavoni (2008)). On the other hand, we incorporate private trading to the dynamic optimal insurance contract with limited commitment, but the way we exploit the Euler equation as an additional constraint in the optimal contracting problem resembles the methodology in those papers. In particular, we adopt the simplifying method of Werning (2002) to simplify the recursive problem, which enables us to obtain some analytical characterization.

The remainder of the paper is organized as follows. In section 2, we show empirical facts — allocation puzzle. In section 3, we present the economic environment. Section 3 analyzes the benchmark optimal allocation in an economy with government's capital control and section 4 analyzes the optimal allocation without capital control. Section 5 concludes.

Figure 1: Feature of the Allocation puzzle (Table 6 of Gourinchas and Jeanne (2013))

Variable: $\Delta D^i/Y_0$	(1)	(2)	(3)	(4)	(5)	(6)
	Public flows		Private flows		PPG debt	Reserves
	(Std. Err.)	(Std. Err.)	(Std. Err.)	(Std. Err.)	(Std. Err.)	(Std. Err.)
Productivity catch-up (π)	−0.843*** (0.185)	−1.182*** (0.219)	0.319** (0.158)	0.428** (0.177)	−0.037 (0.097)	−1.145*** (0.149)
Initial capital abundance (k_0/y_0)	−0.177* (0.103)	−0.112 (0.093)	0.080 (0.088)	0.059 (0.089)	−0.085* (0.049)	−0.027 (0.075)
Initial debt (d_0/y_0)	−0.001 (0.003)	−0.002 (0.003)	0.002 (0.003)	0.003 (0.003)	0.001 (0.002)	−0.002 (0.002)
Population growth (n)	−0.208** (0.087)	−0.148* (0.078)	0.091 (0.074)	0.072 (0.075)	0.070* (0.041)	−0.218*** (0.063)
Openness (Chinn–Ito)	−0.155** (0.060)	−0.131** (0.054)	0.006 (0.051)	−0.002 (0.051)	0.030 (0.028)	−0.161*** (0.043)
Openness $\times \pi$		−0.693*** (0.174)		0.222 (0.166)	−0.025 (0.091)	−0.691*** (0.140)
Intercept	0.668 (0.270)	0.504 (0.244)	−0.040 (0.231)	0.119 (0.230)	0.208 (0.124)	0.298 (0.196)
Number of observations	62	62	62	62	62	62
Adjusted- R^2	0.369	0.501	0.056	0.069	0.107	0.592

*, **, and *** significant at 10, 5, and 1% respectively.

2 Empirical facts Gourinchas and Jeanne (2013)

During last decades, there have been many researches that try to answer “where” and “why” capital flows have been. Gourinchas and Jeanne (2007) look at the net capital inflows for non-OECD countries over the period 1980-2000 and find that countries that have grown at a higher rate have not imported more capital. In fact, they found that the cross-country correlation between growth and net capital inflows is negative. Gourinchas and Jeanne (2007) name this feature *allocation puzzle*, because through the lenses of the neoclassical growth model, this feature is against the prediction from the theory which predicts that capital flows to high return places. Aguiar and Amador (2011), however, pointed out that this negative relationship between growth and capital flows is a mostly feature of public capital flows, not of private capital flows. Alfaro et al. (2011) also show that this negative relationship is driven mainly by sovereign-to-sovereign transactions and argue that this feature is not a *puzzle*, because neoclassical growth model pertains to private market behavior.

After these observations, Gourinchas and Jeanne (2013) confirm that the negative relationship between growth and capital flows is a feature of public capital flows and provide more detailed analysis by further decomposing public capital flows and carrying out regressions that controls important determinants of the capital flows. Figure 1 is the regression result of Gourinchas and Jeanne (2013). The first and second columns of the table show that the

public capital flows are negatively correlated with growth. We also notice that the interaction term between financial openness and growth on total capital inflows is negative and significant. This implies that the negative relationship between growth and public capital flows gets severe for more financially open economies which is the main focus of this paper.

Before going to the environment, we briefly discuss the measure of openness Gourinchas and Jeanne (2013). They use Chinn-Ito index for the openness measure. This measure was created by Chinn and Ito (2008) and measures the extent of openness in capital account transactions. This index is constructed based on the binary dummy variables that codify the tabulation of restrictions on cross-border financial transactions reported in IMF's Annual Report on Exchange Arrangements and Exchange Restrictions.

3 Economic environment

In this section, we describe the model environment which is based on Aguiar et al. (2009). Time is discrete, and runs from 0 to infinity. There is a small open economy with production. The economy is populated by domestic agents (workers), foreign capitalists, and a government. Domestic workers provide labor and foreign capitalists own capital and operate domestic firms which produce a single good by hiring domestic labor and using capital.

The economy receives a productivity shock z every period. For simplicity, we assume that the shock z follows an i.i.d. process, but this can be relaxed for most of results we get below. The realization of shock lies in a finite set $Z = \{z_1, \dots, z_N\} \in R$, where $z_i < z_{i+1}$ for all $i = 1, \dots, N-1$, and we denote by $z_1 = \underline{z}$ and $z_N = \bar{z}$. The probability of state z is denoted by $\pi(z)$. Let $z^t = \{z_0, z_1, \dots, z_t\}$ be a history of shocks up to time t and the probability of the history z^t is denoted by $\pi(z^t)$.

There is an international financial market that buys and sells risk-free bonds with a constant return denoted by $R = 1 + r$. We assume a small open economy where interest rate r is exogenously given. We want to remark that this non-contingent risk-free bonds market assumption does not intend to model incomplete market. We will assume state-contingent capital taxes below and then the combination of state-contingent capital taxes and riskless bonds is sufficient transfer resources across histories, implying equivalence to the complete markets.²

²That is, the equilibrium allocation of this economy will be equivalent to the equilibrium allocation of the economy with international market where there is a full set of state-contingent whose price is $Q(z^t) = \frac{\pi(z^t)}{R^t}$. See Judd (1989), Zhu (1992), and Chari et al. (1994) for more detailed account.

Firms The competitive foreign capitalists own domestic firms and use domestic labor and their own capital to produce a single good. The production function of the domestic firms is

$$y = zf(k, l),$$

where f is constant returns to scale with $f_k > 0$, $f_{kk} < 0$, and satisfying Inada conditions. Labor is hired by the firms in a competitive domestic labor market at an equilibrium wage $w(z^t)$.

The capitalists have access to the international financial markets. Capital is fully mobile internationally at the beginning of every period, but after it is installed (before the shock and tax rate are realized) it cannot be moved until the end of the period. The depreciation rate of capital is denoted by δ . Per capita profits of the firm before taxes and depreciation are denoted by

$$\Pi(z^t) = z_t f(k(z^{t-1}), l) - w(z^t)l.$$

The government taxes the firm profits at a state-contingent rate $\tau(z^t)$, thus firms maximize discounted value of after-tax profits net of depreciation

$$E_0 \left[\sum_t \left(\frac{1}{1+r} \right)^t \left((1 - \tau(z^t))\Pi(z^t) - k(z^t) + (1 - \delta)k(z^{t-1}) \right) \right].$$

Profit maximization implies the following conditions:

$$w(z^t) = A(z^t) f_l(k(z^{t-1}), l(z^t)) \tag{1}$$

$$r + d = \sum_{z_t} \pi(z_t) (1 - \tau(z^t)) z_t f_k(k(z^{t-1}), l(z^t)). \tag{2}$$

In economies without taxes, the optimal capital is constant given the i.i.d. shock assumption, and we denote by k^* the first best level of capital that satisfies

$$\sum_{z \in Z} \pi(z) z f_k(k^*, l) = r + \delta.$$

Domestic workers and the government Domestic workers supply l units of labor inelastically every period for wage $w(z^t)$. Then the consumption of domestic workers assigned by the government is given by :

$$x(z^t) = w(z^t)l + T(z^t), \tag{3}$$

where $T(z^t)$ are lumpsum transfers received from the government at history z^t

We denote by the actual consumption of the domestic workers $c(z^t)$. The actual consumption is either equal to or different from consumption assignment by the government $x(z^t)$, depending on the domestic worker's access to the international market. If the government has perfect private capital control, then workers do not have access to the international financial markets and thus domestic workers consume exactly the amount assigned by the government: $c(z^t) = x(z^t)$. On the other hand, when the government does not control private capital, domestic workers can access to the international financial markets and trade risk-free bonds for themselves. In this case, the actual consumption of the workers will be the solution of the following problem:

$$(WP) \quad \hat{V}(\{x(z^t)\}) = \max_{c,s} \sum_t \sum_{z^t} \beta^t \pi(z^t) U(c(z^t))$$

$$s.t. \quad c(z^t) + s(z^t) = x(z^t) + (1+r)s(z^{t-1})$$

$$s(z_0) = 0$$

The representative domestic worker values flows of consumption according to period utility function $U(c)$, with $U' > 0, U'' \leq 0$. Let $\beta \in (0, \frac{1}{R}]$ denote the discount factor of the domestic workers. The preferences of the domestic workers are represented by following the present discounted utility:

$$\sum_{t=0}^{\infty} \sum_{z^t} \beta^t \pi(z^t) U(c(z^t)). \quad (4)$$

We assume benevolent government whose objective function is the present discounted utility of representative domestic worker and assume that the capitalists do not enter the government objective function, which can be relaxed for main results.

We assume that the government receives an endowment income each period $g(z) > 0$, just to reflect that income is not zero absent foreign investment, as in Aguiar et al. (2009). The government also taxes capital profits at a state-contingent linear rate $\tau(z^t)$ and give transfers $T(z^t)$ to the domestic workers. It also trades a non-contingent bond in the international financial markets. Thus, government's budget constraint is:

$$g(z^t) + \tau(z^t)\Pi(z^t) + b(z^t) = T(z^t) + (1+r)b(z^{t-1}), \quad (5)$$

where $b(z^t)$ is the outstanding debt of the government at history z^t .

Following Aguiar et al. (2009), we define the total output of the economy as $F(z, k, l)$:

$$F(z, k, l) \equiv zf(k, l) + g(z)$$

The government and workers combined resource constrained is expressed by combining equations (1), (3), (5):

$$F(z_t, k(z^{t-1}), l) - (1 - \tau(z^t))F_k(z_t, k(z^{t-1}), l)k(z^{t-1}) + b(z^t) = x(z^t) + (1 + r)b(z^{t-1}). \quad (6)$$

Limited Commitment At any point in time, the government can decide not to repay its debt or expropriate foreign capital invested domestically by taxing returns to capital as much as possible and redistributing the proceeds to the workers. That is, tax policies and debt payments promised are subject to limited commitment of the government.

We consider self-enforcing equilibria that are supported by a punishment equilibrium. Let $\underline{V}(z^t, k(z^{t-1}))$ denote the value of deviation to the government at history z^t with installed capital $k(z^{t-1})$. The punishment value depends on the degree of punishment, more specifically, the degree of accessibility of the government to the international financial market and the investment of the foreign capitalists after default. The result of this paper, however, does not depend on specific assumption on this punishment. It just requires that the punishment utility is independent of outstanding debt at the time of deviation, and depends on the existing capital stock installed in domestic firm. More specifically, we require that $\underline{V}(z^t, k(z^{t-1}))$ is weakly increasing in $k(z^{t-1})$. For example, the most standard assumption on the punishment in the sovereign debt literature is that expropriated capital cannot be operated by the government on deviation and the foreign creditors and capitalists can commit to punish default with complete exclusion, so that the country has to live in autarky after the government deviates. With this harshest penalty the value of deviation is:

$$\underline{V}(z^t, k(z^{t-1})) = U(F(z_t, k(z^{t-1}))) + \beta V_{aut}, \quad \text{where} \quad V_{aut} = \sum_{z \in Z} \pi(z) \frac{U(F(z, 0))}{1 - \beta}.$$

Then, the following participation constraint should be satisfied to enforce the the taxes and debt payments of the government.

$$\sum_{s=t}^{\infty} \sum_{z^s} \beta^{s-t} \pi(z^s | z^t) U(c(z^s)) \geq \underline{V}(z^t, k(z^{t-1})), \quad \forall z^t \quad (7)$$

We now define a self-enforcing equilibrium for both *with capital control* and *without capital control*.

Definition 1. A self enforcing equilibrium ***with capital control*** is a sequence of functions $c(z^t), x(z^t), b(z^t), \tau(z^t), w(z^t)$, and $k(z^t)$ such that

$$(i) \quad c(z^t) = x(z^t), \quad \forall z^t$$

- (ii) firms maximize profits given taxes and wages
- (iii) the labor market clears
- (iv) the resource constraint (6) and associated no Ponzi condition hold given some initial debt $b(-1)$
- (v) the participation constraint holds give deviation payoffs $\underline{V}(z^t, k(z^{t-1}))$:

Definition 2. A self enforcing equilibrium **without capital control** is a sequence of functions $c(z^t)$, $x(z^t)$, $b(z^t)$, $\tau(z^t)$, $w(z^t)$, and $k(z^t)$ such that

- (i) domestic workers solve (WP)

and conditions (ii),(iii),(iv),(v) of the self enforcing equilibrium with capital control hold.

4 Optimal allocation with capital control - benchmark

In this section, we first analyze the optimal allocation of the benchmark model — an economy with capital control, where domestic workers cannot access to the international financial market and the government directly controls the consumption of the workers. The government solves a version of Ramsey problem to find optimal sequence of capital taxes and government debts which implements the optimal allocation. As in a standard Ramsey problem, the benevolent government maximizes lifetime utility of the agents domestic workers by choosing the allocation in the set of allocations that are supported as a self enforcing equilibrium. Such a set of allocations is characterized by an implementability constraint and participation constraints, where the implementability constraint is the combination of (1), (2) and (6):

$$\sum_{t=0}^{\infty} \sum_{z^t} \frac{\pi(z^t)}{(1+r)^{t+1}} \left(z_t f(k(z^{t-1}), l) + g(z^t) - (r+d)k(z^{t-1}) - c(z^t) \right) \geq b(-1). \quad (8)$$

Then, the Ramsey problem solves for the allocation $c(z^t)$, $k(z^t)$ that maximizes (4) subject to (8), (7) for given an initial debt $b(-1)$.

We now write down this Ramsey problem in a (inverse) recursive formulation as in Aguiar et al. (2009). Let us denote by v the maximal amount of utility in an optimal allocation, given an initial debt $b \in \mathbf{b}$, where $\mathbf{b}$ is the set of possible debt levels for which the constraint set of the Ramsey problem is non-empty. We assume that $\mathbf{b}$ is bounded below.

Let us denote by B the function such that $(1+r)b = B(v)$, for any $b \in \mathbf{b}$. Then, function B can be represented as a value function of promise utility v , for all v in some closed interval $[V_{aut}, V_{max}]$, where V_{max} is assumed to satisfy $V_{max} \geq U(F(\bar{z}, k^*)) + \beta V_{aut}$. After we define $B(v)$ on $[V_{aut}, V_{max}]$, we then characterize the subset of $[V_{aut}, V_{max}]$ for which $B(v) \in \mathbf{b}$.

Aguiar et al. (2009) show that optimal allocations solve the following Bellman equation where the state variable is promise utility v and the choice variables are capital stock, state-contingent flow utility $u(z)$ and state-contingent promised utility $\omega(z)$:

$$\begin{aligned} B(v) = & \max_{u(z), \omega(z), k} \sum_z \pi(z) \left[z f(k) + g(z) - c(u(z)) - (r+d)k + \frac{1}{1+r} B(\omega(z)) \right] \\ \text{s.t.} \quad & v \leq \sum_z \pi(z) [u(z) + \beta \omega(z)] \\ & W^{aut}(z, k) \leq u(z) + \beta \omega(z). \end{aligned}$$

We know that $(1+r)b(-1) = B(v)$. Moreover, if we solve the problem by iterating the recursive problem ($v_t = \omega(z^{t-1})$) for each shock history, then we get the sequence of optimal government debt by $(1+r)b(z^{t-1}) = B(v_t)$. We want to remark that this inverse representation is much more convenient for us to analyze the relationship between debt and growth which is the key to explain the allocation puzzle. In the remaining of this section, we restate the result of Aguiar and Amador (2011), using this inverse recursive formulation.³ We, however, abstract from political economy friction and focus on the allocation puzzle explained by the limited commitment of the government so that we can analyze the pure effect of market openness (removal of the government capital control) on the allocation puzzle in the next section.

To characterize optimal allocation with capital control, we first define expected net output

$$h \equiv E z f(k) - (r+d)k,$$

and assume the following for the convexity of the problem.

Assumption 3. $W^{aut}(z, K(h))$ is convex in h , where $K(h)$ is the inverse function of h .

This is exactly the assumption of Aguiar et al. (2009). Heuristically, this assumption is satisfied if the utility function is less concave than the production function. Then, the

³Aguiar and Amador (2011) analyzes the relationship between debt and growth with sequential Ramsey formulation for a deterministic economy.

problem can be rewritten as a convex problem:

$$\begin{aligned}
(\mathbf{P1}) \quad B(v) = & \max_{u(z), \omega(z), h} \sum_z \pi(z) \left[h + g(z) - c(u(z)) + \frac{1}{1+r} B(\omega(z)) \right] \\
(\gamma) \quad & s.t. \quad v \leq \sum_z \pi(z) [u(z) + \beta \omega(z)] \\
(\mu(z)) \quad & W^{aut}(z, K(h)) \leq u(z) + \beta \omega(z).
\end{aligned}$$

We now present the properties of the value function and characterize the optimal allocation and then briefly discuss how the model can generate negative relationship between growth and public capital flows. Following three propositions are presented without proofs because the proofs can be found in Aguiar et al. (2009). We start with the properties of the foreigner's value function $B(v)$.

Proposition 4. *Under the stated assumptions,*

- (i) *the value function $B(v)$ is concave and differentiable on $[V_{aut}, V_{max}]$*
- (ii) *there exists $V_{min} > V_{aut}$ such that $B'(v) = 0$ for all $v \in [V_{aut}, V_{min}]$, promise keeping holds with strict equality for $v \geq V_{min}$, and $\mathbf{b} = [B(V_{min}), B(V_{max})]$*
- (iii) *$B(v)$ is strictly decreasing for $v \in (V_{min}, V_{max}]$ and strictly concave for $v \in [V_{min}, V_{max}]$*
- (iv) *for each $v \in [V_{aut}, V_{max}]$, there exists an optimal $(k, u(z), \omega(z))$ with $k > 0$ and non-negative multipliers $(\gamma, \lambda(z))$ that satisfy:*

$$c'(u(z)) = \gamma + \frac{\lambda(z)}{\pi(z)} \quad (9)$$

$$B'(\omega(z)) = -\beta(1+r) \left(\gamma + \frac{\lambda(z)}{\pi(z)} \right) \quad (10)$$

$$\sum_z \pi(z) F_k(z, k) - (r + \delta) = \sum_z \lambda(z) U'(F(z, k)) F_k(z, k) \quad (11)$$

We now present the properties of optimal allocation. Let $g^{(i)}(v)$ denote the policies in an optimal plan for $i = u(z), \omega(z)$, and k at state v . Define $V^* = U(F(\bar{z}, k^*)) + \beta V_{aut}$

Proposition 5. *In an optimal allocation,*

- (i) *$g^i(v)$ is single valued and continuous for $i = u(z), \omega(z)$, and k for all $v \in [V_{min}, V_{max}]$*
- (ii) *for all $v \in [V_{aut}, V_{max}]$, $g^k(v) \leq k^*$*

(iii) $g^k(v)$ is non-decreasing in v , strictly increasing for all $v \in [V_{min}, V^*]$ and $g^k(v) = k^*$ for any $v \geq V^*$

(iv) $g^{\omega(z)}(v)$ and $g^{u(z)}(v)$ are strictly increasing in v for all $v \in [V_{min}, V_{max}]$

(v) $g^{\omega(z_1)}(v) \geq g^{\omega(z_0)}(v)$ if $z_1 > z_0$ and $g^{\omega(\bar{z})}(v) > g^{\omega(\underline{z})}$.

Define $V^* = U(F(\bar{z}, k^*)) + \beta V_{aut}$. Note that for any $v \geq V^*$, $g^k(v) = k^*$. The following proposition characterizes the transition path and steady state of the optimal allocation.

Proposition 6. *In an optimal allocation,*

(i) *if $\beta(1+r) = 1$, $v \leq g^{\omega(z)}(v)$ for all $v \in [V_{min}, V_{max}]$ and for all $z \in Z$, and v and k converge to V^* and k^* respectively*

(ii) *if $\beta(1+r) < 1$, v and k converge to unique, non-degenerate ergodic distribution with respective supports that lie strictly below V^* and k^**

Let us first discuss the case when discount factor of the domestic workers is equal to that of the foreign lender ($\beta(1+r) = 1$). With such discount factor, by combining (10) and envelope theorem, we get

$$-B'(\omega(z)) = -B'(v) + \frac{\lambda(z)}{\pi(z)},$$

which implies that whenever the participation constraint binds in the transition path, $\omega(z) \geq v$ and $\omega(z) = v$ when the participation constraint is not binding. Thus, the proposition states that the promise utility v monotonically increases over time in the transition path and achieves the first best level of promise utility V^* in the steady state. This growth in the transition path is typical properties of the optimal allocation of the one-sided limited commitment problem. Because of the forward looking participation constraint, if the government backloads consumption of the domestic workers by increasing future promise utility, then it reduces not only the incentive to deviate in the future but also the incentive to deviate along the history from today to the future. We can also notice that a growth of the economy is also driven by the productivity shock because (v) of proposition 5 shows that future promise utility is nondecreasing in productivity shock z .

On the other hand, if the government (or equivalently domestic workers) is impatient ($\beta(1+r) < 1$), then the promise utility in the transition path does not monotonically increase and even in the long-run, the first-best level of promise utility is not achieved

and the volatility generated by the limited commitment becomes a permanent feature, as analyzed in Aguiar et al. (2009).

In this paper, to simplify the analysis, we focus on the case when $\beta(1 + r) = 1$. But for most of the important results that we get in this paper, this is not crucial assumption and we will discuss this in more detail in the next section. From now on, we assume that $\beta(1 + r) = 1$. Thus, along the transition, the economy grows with monotonically increasing k and v .

We finish this section with the corollary which presents that the optimal contract can generate the allocation puzzle — negative relationship between growth and public capital flows.

Corollary 7. *The stock of the economy's outstanding sovereign debt decreases (increases) monotonically if the sequence k_t is increasing (decreasing).*

We can easily show this corollary by noticing that that in the optimal allocation, $g^k(v)$ is non-decreasing in v (strictly increasing for all $v \in [V_{min}, V^*]$) and $B'(v) \leq 0$ (with strict inequality for all $v \in [V_{min}, V^*]$). Along the transition, promise utility increases whenever the participation constraint is binding. When promise utility increases, foreign capital investment increases but the debt of the government ($B(v)$) decreases, which is consistent with the empirical observation — the allocation puzzle.

We want to remark that the analysis of the allocation puzzle in this section was the restatement of the analysis Aguiar and Amador (2011) with some modification — we abstract from the political economy assumption but add productivity shock, and we analyzed the relationship between growth and capital flows using the inverse recursive formulation of the Ramsey problem, while Aguiar and Amador (2011) analyzed this using standard infinite horizon Ramsey problem. We use inverse recursive formulation because it will make the comparison of optimal allocations with and without capital control more convenient.

5 Optimal allocation without capital control : stronger allocation puzzle

In this section, we relax the assumption of the government's perfect capital control and analyze the optimal allocation when private agents can also access to the international financial market. We show that in this financially open economy which allows private agents to trade

in the international financial market, the sovereign debt is even more negatively correlated with growth, implying more severe allocation puzzle in open economies.

Due to domestic worker's access to the financial market, the set of allocation that can be implemented as a self-enforcing equilibrium without capital control are now characterized by additional condition:

$$\{c(z^t)\} \text{ solves } (WP) \quad (12)$$

as well as resource constraint (6) and implementability constraint (8). Thus, the Ramsey problem in this economy with capital control solves for the allocation $c(z^t), k(z^t)$ that maximizes (4) subject to (8), (7), and (12) for given initial debt $b(-1)$.

Following lemma rewrites the Ramsey problem.

Lemma 8. *An optimal allocation without capital control $\{c(z^t), k(z^t)\}$ is a solution to the problem that maximizes (4) subject to (8), (7), and*

$$u'(c_t) = \beta(1+r) \sum_{z^{t+1}} \pi(z^{t+1}) u'(c_{t+1}). \quad (13)$$

Compared to the Ramsey problem in an economy with capital control, the government in this economy without capital control has additional Euler equation constraint (13). And this additional constraint is binding when the participation constraint is binding, because the optimal allocation without additional Euler equation exhibits backloading of consumption compared to the allocation that satisfies the Euler equation. This is because future consumption can relax all participation constraints along the history. Moreover, the equality constraint (13) can be replaced by the inequality constraint

$$u'(c_t) \leq \beta(1+r) \sum_{z^{t+1}} \pi(z^{t+1}) u'(c_{t+1}), \quad (14)$$

because the other direction of inequality never binds. In this economy with limited commitment, the government wants to backload consumption so that it can reduce the incentive of the government to default, and workers try to borrow to smooth consumption.⁴

As a side remark, we can notice that if we relax the assumption that the discount factor of the government and that of the domestic workers are the same, then the Euler equation

⁴Notice that the direction of the inequality constraint is the opposite from that of Werning (2002). In an economy with moral hazard, the government wants to frontload consumption (or tax capital) so that it can provide more incentive for workers to make effort in the future, and workers try to save to smooth consumption.

can be binding in either direction. If the government is less patient than domestic workers due to political economy friction, then despite the backloading incentive of the government, it is not clear who wants to save more between domestic workers and the government. Then, the Euler equation inequality can be binding in either direction depending on the size of the discount factor. On the other hand, if the government is more patient than domestic workers due to myopic workers, then the inequality (14) will be binding only.

5.1 Recursive formulation and simplification

In order to characterize the optimal allocation without capital control and compare it with the one with capital control, we first rewrite the Ramsey problem recursively. Adapting Werning (2002)'s methodology to our environment⁵, we can state the problem recursively using additional state variable, u , which is flow utility of previous period. Since there is one-to-one relationship between flow utility u and marginal utility, we denote the function which maps from flow utility by $\tilde{u}_c(\cdot)$. That is, $\tilde{u}_c(u) \equiv U_c(c(u))$. Now, we can write the recursive problem :

$$\begin{aligned}
(\mathbf{P2}) \quad \tilde{B}(v, u) = & \max_{u(z), \omega(z), h} \sum_z \pi(z) \left[h + g(z) - c(u(z)) + \frac{1}{1+r} \tilde{B}(\omega(z), u(z)) \right] \\
(PK) \quad s.t. \quad & v \leq \sum_z \pi(z) [u(z) + \beta \omega(z)] \\
(PC) \quad & W^{aut}(z, K(h)) \leq u(z) + \beta \omega(z) \\
(EE) \quad & \tilde{u}_c(u) \leq \beta(1+r) \sum_z \pi(z) \tilde{u}_c(u(z)) \\
(DR) \quad & (u(z), \omega(z)) \in \Delta(z)
\end{aligned}$$

We need domain restriction (DR) condition because there are combinations of $(u(z), \omega(z))$ that are not consistent with Euler equation of the future. Notice that the condition (EE) only guarantees the Euler equation of the current period, not the Euler equation in the future. Thus, for well defined problem, we need (DR).

To proceed further, we need additional assumption on the functional form of the utility.

Assumption 9. $U(c)$ is either CARA ($U(c) = -\frac{1}{\psi} \exp(-\psi c)$) or CRRA ($U(c) = \frac{1}{1-\sigma} c^{1-\sigma}$).

⁵The environment of Werning (2002) has moral hazard problem when there is unobservable private saving, but the technique of simplifying recursive problem in his papaer can be applied to the environemtn with limited commitment with some modification.

With this assumption, we can simplify the problem (P2) a lot, as in Werning (2002). We will show that with Assumption 9, we can express (DR) as a simple inequality constraint ($\omega(z) \leq \kappa u(z)$), for some constant κ) and we will show that we can drop (EE) constraint. Then, we don't need to keep additional state variable u for recursive problem. We will show that for each utility.

We first consider CARA utility: $U(c) = -\frac{1}{\psi} \exp(-\psi c)$. Using Euler equation and law of iterated expectation, we get $U_{c,t} \leq E_t U_{c,t+k}$, $k \geq 0$. Since $U(c) = -\frac{1}{\psi} U_c(c)$, we get following inequalities:

$$\begin{aligned} V_{t+1}(z^t) &= E_t \sum_{k=0}^{\infty} \beta^k U_{t+1+k}(z^{t+1+k}) = \sum_{k=0}^{\infty} \beta^k \left(-\frac{1}{\psi} E_t U_{c,t+1+k} \right) \\ &\leq \sum_{k=0}^{\infty} \beta^k \left(-\frac{1}{\psi} U_{c,t}(z^t) \right) = \frac{1}{1-\beta} U_t(z^t). \end{aligned}$$

Thus, domain restriction can be written as following inequality constraints:

$$\omega(z) \leq \phi(u(z)), \quad \text{where} \quad \phi(u) = \frac{1}{1-\beta} u$$

We now show that we can drop the (EE) constraint by showing that the solution to the relaxed problem satisfies (EE). We start with the first claim that the Domain restriction of the relaxed problem (problem without (EE) constraint) is binding. This claim can be proved by suppose not: $\tilde{\omega}(z) < \frac{1}{1-\beta} \tilde{u}(z)$, which implies $c'(\tilde{u}(z)) < -\tilde{B}'(\tilde{\omega}(z))$. Then, it is easy to show that ϵ -perturbation $(\tilde{u}(z) - \epsilon, \tilde{\omega}(z) + \frac{\epsilon}{\beta})$ reduces the objective function which is against to the optimality of the allocation.

Now, we claim that the solution of the relaxed problem automatically satisfies (EE). this claim can be proved by the following inequalities,

$$-u \leq -(1-\beta)v = -(1-\beta) \sum_z \pi(z) \left(u(z) + \frac{\beta}{1-\beta} u(z) \right) = - \sum_z \pi(z) u(z),$$

where the first inequality is directly from (DR) and the second equality holds because we know that the solution of the relaxed problem satisfies (DR) with equality from the previous claim. Since $U(c) = -\frac{1}{\psi} U_c(c)$ for the CARA utility, above inequality implies that (EE) is automatically satisfied. Thus, we can drop (EE) constraint.

Second, we consider CRRA utility: $U(c) = \frac{1}{1-\sigma} c^{1-\sigma}$. Notice that $\tilde{u}_c(u) = [(1-\sigma)u]^{\frac{-\sigma}{1-\sigma}}$. The domain $\Delta(z)$ for $(u(z), \omega(z))$ can be defined in terms of frontier of function $\phi(u)$:

$$\Delta(z) = \{(u(z), \omega(z)) | \omega(z) \leq \phi(u(z))\},$$

where ϕ is the fixed point of the Bellman operator,

$$\begin{aligned} T[\phi](u) &= \max_{u(z), \omega(z)} \sum_z \pi(z) \{u(z) + \beta \omega(z)\} \\ \text{s.t.} \quad & [(1 - \sigma)u]^{-\frac{\sigma}{1-\sigma}} \leq \sum_z \pi(z) [(1 - \sigma)u(z)]^{-\frac{\sigma}{1-\sigma}} \\ & \omega(z) \leq \phi(u(z)). \end{aligned} \quad (15)$$

$$(16)$$

Since (EE) of this Bellman equation (15) is binding, (16) is also binding. We now guess that $\phi(u)$ has the functional form of $\phi(u) = \kappa u$ and we can verify that it is indeed the functional form of the fixed point. Exploiting binding (DR) and guess function, we rewrite the Bellman equation:

$$\begin{aligned} \phi(u) &= \max_{u(z)} (1 + \beta \kappa) \sum_z \pi(z) u(z) \\ \text{s.t.} \quad & [(1 - \sigma)u]^{-\frac{\sigma}{1-\sigma}} = \sum_z \pi(z) [(1 - \sigma)u(z)]^{-\frac{\sigma}{1-\sigma}} \end{aligned}$$

Since $(u, (u(z))_z)$ are homogeneous of degree 1, we know that $\sum_z \pi(z) u(z) = Au$ for some constant A . Thus, κ should satisfy $\kappa = (1 + \beta \kappa)A$, resulting in $\kappa = \frac{A}{1 - \beta A}$. We then have verified that the fixed point of the Bellman operator ϕ has the form $\phi(u) = \kappa u$ for some constant κ . In sum, the domain restriction of (P2) can be written $\omega(z) \leq \phi(u(z))$, where $\phi(u) = \kappa u$ for some constant κ . Exploiting the binding (EE) of the Bellman equation and $\sum_z \pi(z) u(z) = Au$, we also get the following equality which will be used below.

$$A^{\frac{\sigma}{1-\sigma}} \left[(1 - \sigma) \sum_z \pi(z) u(z) \right]^{\frac{-\sigma}{1-\sigma}} = \sum_z \pi(z) [(1 - \sigma)u(z)]^{\frac{-\sigma}{1-\sigma}} \quad (17)$$

Going back to the Bellman equation of the problem (P2), we can show that the (DR) of the relaxed problem of (P2) (without EE) is binding using the same argument we used for the CARA utility.

We then claim that the solution of the relaxed problem of (P2) automatically satisfies (EE) constraint, which is shown from the following.

$$\begin{aligned} [(1 - \sigma)u]^{\frac{\sigma}{1-\sigma}} &\leq \kappa^{\frac{\sigma}{1-\sigma}} [(1 - \sigma)v]^{\frac{-\sigma}{1-\sigma}} \\ &= \kappa^{\frac{\sigma}{1-\sigma}} (1 + \beta \kappa)^{\frac{-\sigma}{1-\sigma}} [(1 - \sigma) \sum_z \pi(z) u(z)]^{\frac{-\sigma}{1-\sigma}} \\ &= A^{\frac{\sigma}{1-\sigma}} \left[(1 - \sigma) \sum_z \pi(z) u(z) \right]^{\frac{-\sigma}{1-\sigma}} \\ &= \sum_z \pi(z) [(1 - \sigma)u(z)]^{\frac{-\sigma}{1-\sigma}}, \end{aligned}$$

where the second equality holds because the solution of the relaxed problem satisfies (DR) with equality and the last equality is from (17). This proves that (EE) is automatically satisfied.⁶

Thus, under assumption 9, (DR) constraints can be represented as the simple inequality $\omega(z) \leq \kappa u(z)$ for some constant κ , and we can drop (EE) constraint. The problem (P2) is simplified as follows.

Abusing the notation, we denote by $\tilde{B}(v) = \tilde{B}(v, \phi^{-1}(v))$.

$$\begin{aligned}
(\mathbf{P2}') \quad \tilde{B}(v) = & \max_{u(z), \omega(z), h} \sum_z \pi(z) \left[h + g(z) - c(u(z)) + \frac{1}{1+r} \tilde{B}(\omega(z)) \right] \\
(\tilde{\gamma}) \quad & s.t. \quad v \leq \sum_z \pi(z) [u(z) + \beta \omega(z)] \\
(\tilde{\mu}(z)) \quad & W^{aut}(z, K(h)) \leq u(z) + \beta \omega(z) \\
(\tilde{\xi}(z)) \quad & \omega(z) \leq \kappa u(z)
\end{aligned}$$

Notice that $-\tilde{B}'(v) = \tilde{\gamma}$ by the envelope theorem, where $\tilde{\gamma}$ is the shadow price of increasing v .

Compared to the problem with capital control (**P1**), the problem without capital control (**P2'**) has additional constraint. If this additional constraint raises the shadow cost ($\tilde{\gamma}$) of increasing v , then by the envelope theorem, we know that the value function $\tilde{B}(v)$ decreases faster than $B(v)$ for the same amount of increase in v , implying that the government debt in open economy (without capital control) decreases more for the same growth of promise utility v compared to the case with capital control.

We first provide a simple example which allows closed form solution to discuss this intuition and then provide more general results in the next subsection.

⁶We can get further information on the magnitude of κ . By the binding Euler equation and the convexity of $\tilde{u}_c(u) = [(1-\sigma)u]^{\frac{\sigma}{1-\sigma}}$, we get the following inequalities.

$$\tilde{u}_c(u) = \sum_z \pi(z) \tilde{u}_c(u(z)) \geq \tilde{u}_c\left(\sum_z \pi(z) u(z)\right)$$

Since $\tilde{u}_c(\cdot)$ is decreasing function, $u \leq \sum_z \pi(z) u(z) = Au$. Thus, $A \leq 1$ (since $u < 0$), implying $\kappa = \frac{A}{1-\beta A} \leq \frac{1}{1-\beta}$. Then, $\omega(z) = \kappa u(z) \geq \frac{1}{1-\beta} u(z)$, which implies that $v \leq \sum_z \pi(z) \omega(z)$ (which shows that in average, promised utility increases over time).

5.2 Simple example and discussion

In this subsection, we consider simple deterministic example so that we can discuss the intuition of stronger allocation puzzle with analytical solution for (P1) and (P2'). Utility of the domestic worker has the CRRA functional form, and there is no shock in this example. Thus, $h = F(k) - (r - \delta)k$. In order to get analytical solution we also assume that $W^{aut}(K(h)) = U(h)$. That is, agents in autarky cannot access to the international market in this example. With this simple set up, we can get the analytical solution for both problems.

First, for capital control case, we can guess the functional form of value function $B(v)$: $B(v) = -A[(1 - \sigma)v]^{\frac{1}{1-\sigma}}$, where A is some constant, and verify this functional form by evaluating value function using solutions for given this functional form. We assume that the promise utility v lies in the range where the participation constraint is binding. Then, using the first order conditions, the solutions for given $B(v) = -A[(1 - \sigma)v]^{\frac{1}{1-\sigma}}$ can be obtained as follows.

$$u = \frac{A^{\frac{1-\sigma}{\sigma}}}{A^{\frac{1-\sigma}{\sigma}} + \beta}, \quad \omega = \frac{1}{A^{\frac{1-\sigma}{\sigma}} + \beta}, \quad h = [(1 - \sigma)v]^{\frac{1}{1-\sigma}}$$

Then, by plugging in this solutions into the objective function of the value function, we can get the condition that A should satisfy: $\frac{1}{A} + 1 = [A^{\frac{1-\sigma}{\sigma}} + \beta]^{\frac{-\sigma}{1-\sigma}}$. Here, we consider $\sigma = 2$ to get the analytical solution for A , then we get $A = \frac{4\beta^2}{(1-\beta^2)^2}$.

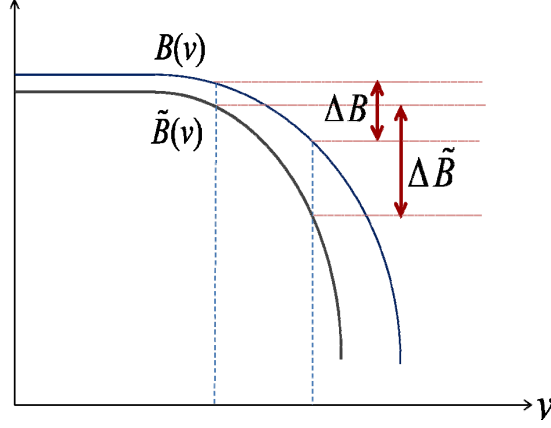
Next, for no capital control case, there is additional binding (DR) constraint $\omega \leq \kappa u$. We can also notice that since this example is deterministic, $\kappa = \frac{1}{1-\beta}$ from the above analysis. Then, we can also guess the same functional form of the value function: $\tilde{B}(v) = -\alpha[(1 - \sigma)v]^{\frac{1}{1-\sigma}}$, for some constant α , and we can verify this by plugging in solutions to the objective function. Again, we assume that the promise utility v is such that the participation constraint is binding. Using the binding (PK), (PC), and (DR), we get the following solutions for given functional form of the value function.

$$\tilde{u} = (1 - \beta)v, \quad \tilde{\omega} = v, \quad \tilde{h} = [(1 - \sigma)v]^{\frac{1}{1-\sigma}}$$

Then, by plugging this into the objective function, we get $\alpha = \frac{(1-\beta)^{\frac{1}{1-\sigma}} - 1}{1-\beta}$. Thus, for $\sigma = 2$, $\alpha = \frac{\beta}{(1-\beta)^2} > \frac{4\beta^2}{(1-\beta^2)^2} = A$.

Then, $-B'(v) = A[(1 - \sigma)v]^{\frac{\sigma}{1-\sigma}} < -\alpha[(1 - \sigma)v]^{\frac{\sigma}{1-\sigma}} = -\tilde{B}'(v)$, which implies $0 < -B'(v) < -\tilde{B}'(v)$. Thus, if promise utility increases, for same increase of promise utility, the government debt will decrease more when there is no capital control. Figure 2 shows this graphically. In addition, since $h(v) = \tilde{h}(v) = [(1 - \sigma)v]^{\frac{1}{1-\sigma}}$, for the same increase in h

Figure 2: decrease of sovereign debt with and without capital control



(equivalently, for the same increase in k), the government debt will decrease more in case of no capital control.

From this simple example, we show that the higher concavity of the value function in the economy with no capital control than that in the economy with capital control is the key to explain the severe allocation puzzle for more open economies. Why does the concavity of the value function matter? Notice that the outstanding sovereign debt which is the value function is the maximum present discount utility to the foreign creditors for given promise utility to the domestic government (or to the workers). Thus, decreasing and concave value function shows that the marginal cost of increasing marginal promise utility gets higher as the promise utility increases, which leads to higher amount of decrease in the utility to the foreign creditors. Moreover, the degree of concavity of this value function determines the speed of decrease in sovereign debt. If marginal cost of increasing promise utility is raised because of eliminating capital control, then the utility to the foreign creditor decreases even more in the absence of capital control, which is represented as more concave value function, and this implies more decrease of sovereign debt.

In the next subsection, we analyze this more formally. That is, we show that additional Euler equation constraint in open economies makes the value function more concave which implies severe allocation puzzle for more open economies.

Before proceeding to the formal analysis, we want to remark two things of the model environment. First, in this model with limited commitment, it is true that *for given shock process*, growth of the economy is slowed down when capital control is removed, because household's consumption smoothing through international financial market disrupts the natural consumption backloading incentive of the government, which leads to the slow down (slow down

of the increase of promise utility) of the economy.⁷ However, this model generates that *for given sequence of increasing promise utility v* (or for given sequence of increasing output) not for the given history of productivity shock, the sovereign debt decreases more in open economies (without capital control), and this relationship between growth of output and sovereign debt is the allocation puzzle we see in the data.

That is, this model then implicitly assumes that if two economies — one with capital control and the other without capital control — show the same realization growth in promise utility or output, then the open economy without capital control is the economy that was hit by higher productivity shock compared to the economy with capital control. The productivity shock in the model can be interpreted as total factor productivity shock which measures the economy's technological change, and we often observe that more financially open economies tend to adopt more technological innovation. The model's implicit assumption that there is a positive correlation between the financial openness and TFP shock is consistent with these observations.

Second, the degree of slow down and the degree of pay down of the sovereign debt depend on the assumption on the domestic worker's financial trading. We assume that the domestic workers can fully commit to the their debt payment in the international market, but if this assumption is relaxed, then domestic workers will also have incentive to backload consumption. But in the real world we often observe that private agents tend to borrow excessively and the government wants to reduce excessive private borrowing, which shows that the government's incentive to backload consumption is stronger than that of the private. As long as this is the case, the mechanism of this paper will remain.

5.3 More general results

In the previous subsection, we explained how the limited commitment as well as capital control can generate stronger allocation puzzle using simple example and discussed that the concavity of the value function is the key for this analysis. In this subsection, we analyze the allocation puzzle in a general set up.

⁷Deterministic example of this subsection is the extreme example where there is no growth in the economy with no capital control: $\omega = v$.

5.3.1 Characterization of the optimal allocation without capital control

(i), (ii), (iii) of Proposition 4 of the benchmark economy (with capital control) also holds in an economy without capital control. That is, the value function $\tilde{B}(v)$ is concave and differentiable on $[V_{aut}, V_{max}]$, and there exists $\tilde{V}_{min} > V_{aut}$ such that $\tilde{B}'(v) = 0$ for all $v \in [V_{aut}, \tilde{V}_{min}]$ and $\tilde{B}$ is strictly decreasing for $v \in (\tilde{V}_{min}, V_{max}]$ and strictly concave for $v \in [\tilde{V}_{min}, V_{max}]$. In addition, for each $v \in [V_{aut}, V_{max}]$, there exists an optimal $(\tilde{k}, \tilde{u}(z), \tilde{\omega}(z))$ and non-negative multipliers $(\tilde{\gamma}, \tilde{\lambda}(z), \tilde{\xi}(z))$ that satisfy:

$$c'(\tilde{u}(z)) = \tilde{\gamma} + \frac{\tilde{\lambda}(z)}{\pi(z)} + \frac{\kappa \tilde{\xi}(z)}{\pi(z)} \quad (18)$$

$$-\tilde{B}'(\tilde{\omega}(z)) = \beta(1+r) \left(\tilde{\gamma} + \frac{\tilde{\lambda}(z)}{\pi(z)} \right) - \frac{\tilde{\xi}(z)}{\beta \pi(z)} \quad (19)$$

$$\sum_z \pi(z) F_k(z, \tilde{k}) - (r + \delta) = \sum_z \tilde{\lambda}(z) U'(F(z, \tilde{k})) F_k(z, \tilde{k}) \quad (20)$$

Properties of the policy functions (Proposition 5) in the benchmark economy also holds in this economy and the characterization of the steady state (Proposition 6) does not change. That is regardless of the capital control, when the domestic government is patient as much as the foreign creditors ($\beta(1+r) = 1$) — the assumption that we take, v and k converge to V^* and k^* .⁸ In the transition path, however, the promise utility does not monotonically grow in this economy without capital control, which can be seen by combining the envelope condition and (19):

$$-\tilde{B}'(\tilde{\omega}(z)) = -\tilde{B}'(v) + \frac{\tilde{\mu}(z)}{\pi(z)} - \frac{(1+r)\tilde{\xi}(z)}{\pi(z)}.$$

Since optimal consumption backloading of the economy with capital control is disturbed by the private consumption frontloading incentive, which is captured by the additional term $\frac{(1+r)\tilde{\xi}(z)}{\pi(z)}$, the promise utility does not necessarily increases when the participation constraint is binding. Thus, it is true that eliminating capital control slows down for the same sequence of productivity shocks. As we discussed in the previous subsection, however, not only optimal contract with limited commitment but also productivity shock drive growth of the economy, and we are interested in analyzing how eliminating capital control changes the optimal sovereign debt for given growth (more precisely, for given increase of promised utility).

⁸If the domestic government is less patient than foreigners ($\beta(1+r) < 1$), then v and k converge to non-degenerate ergodic distribution whose supports strictly lie below V^* and k^* . The distribution, however, is different from that of the economy with capital control.

5.3.2 Stronger allocation puzzle in more open economies

Proposition 10. *Let us denote the subset of the promise utility by $\mathbf{V} \subset [V_{aut}, V_{max}]$ such that $-B'(v) < -\tilde{B}'(v)$ for all $v \in \mathbf{V}$. Then, the stock of the country's outstanding sovereign debt decreases (increases) more in the country without capital control than in the country with capital control, if the promise utility of both countries is increasing (decreasing) from v_t to v_{t+1} and $[v_t, v_{t+1}] \subset \mathbf{V}$ ($[v_{t+1}, v_t] \subset \mathbf{V}$).*

This proposition shows that for the range of promise utility where the shadow cost of marginally increasing promise utility is higher in the country without capital control than in the country with capital control, the country without capital control shows stronger allocation puzzle — stronger negative relationship between growth and public capital flows, which is consistent with the feature of the data. We then need to investigate the range of the promise utility where the inequality $-B'(v) < -\tilde{B}'(v)$ is satisfied or under which condition the inequality is satisfied.

Since it is not easy to find global conditions that guarantees more concave value function in the country without capital control ($-B'(v) < -\tilde{B}'(v)$), we only investigate local analysis. Since the allocation puzzle is the feature across developing countries, we are specifically interested in investigating the relationship between growth and sovereign debt at lower levels of promise utilities that are relevant to developing countries. Thus, we want to show that at lower levels of promise utilities, the inequality $-B'(v) < -\tilde{B}'(v)$ holds. We can show this if we make additional assumption on the minimum level of consumption: there is a subsistence level of consumption $\underline{c}$ that the contract should guarantee. Since there is one to one relationship between utility and consumption, we can present the assumption in terms of utility:

$$u(z) \geq \underline{u} \text{ for all } z \in Z, \text{ for some constant } \underline{u}. \quad (21)$$

Abusing notations slightly, we denote by $B(v)$ and $\tilde{B}(v)$ the foreigner's value functions with additional constraint (21) in countries with capital control and without capital control respectively and denote by V_{min} and $\tilde{V}_{min}$ associated maximum level of promise utility such that $B(V_{min}) = B(V_{aut})$ and $\tilde{B}(\tilde{V}_{min}) = \tilde{B}(V_{aut})$. We also denote by $\underline{U}$ the set of subsistence levels of utility such that $B(V^{aut}) = \tilde{B}(V^{aut})$. There is nonempty $\underline{U}$, because if the subsistence level of utility is high enough, then there are range of promise utilities at which the promise keeping constraint, participation constraint, and domain constraint do not bind and additional minimum consumption constraints (21) bind in optimal contract problems of both countries (with and without capital control).

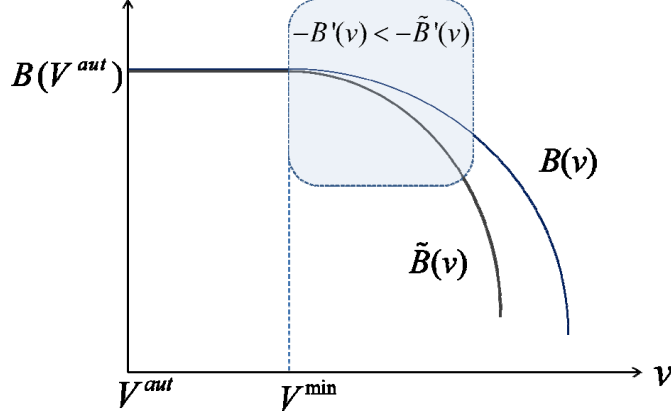


Figure 3: Value functions with subsistence level of consumption

Proposition 11. *If $\underline{u} \in \underline{U}$, then there exists $V^0 > V_{min}$ such that for $v \in (V^{min}, V^0)$, $-B'(v) < -\tilde{B}'(v)$, where $V^{min} = \min\{V_{min}, \tilde{V}_{min}\}$.*

Proof By the definition of $\underline{U}$, $B(V^{aut}) = \tilde{B}(V^{aut})$. This implies that $B(V^{min}) = \tilde{B}(V^{min})$. Because of additional domain restriction constraint, it is obvious that $B(v) > \tilde{B}(v)$ for all $v \in (V^{min}, V^{DB})$ where $V^{DB} \leq V^*$ is the maximum value at which domain restriction binds. Since the value functions are concave and differentiable, there exists $V^0 \leq V^{DB}$ such that $-B'(v) < -\tilde{B}'(v)$ for all $v \in (V^{min}, V^0)$. ■

Figure 3 shows this proposition graphically. For two concave functions which are decreasing starting from the same value, the function which decreases faster is more concave. Thus, developing countries whose promise utilities lies in this range satisfy stronger allocation puzzle for more open economies.

6 Conclusion

In this paper, we investigated how financial openness changes the relationship between growth and sovereign debt. Optimal sovereign debt contract with limited commitment generates negative relationship between growth and public capital flows and this negative relationship gets stronger for more open economies because private agent's ability to smooth consumption disrupts government's ability to optimally backload consumption, amplifying the mechanism generated by limited commitment.

Quantitative analysis to investigate quantitative effects of financial openness will be the future work. Analyzing the optimal contract further using the model that can distinguish sovereign-to-sovereign transactions and sovereign-to-private transactions would be also interesting because recent empirical evidences from Alfaro et al. (2011) show that the allocation puzzle only applies to sovereign-to-sovereign transactions.

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