

Shades of Darkness: A Pecking Order of Trading Venues¹

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First version: October 16, 2014

This version: November 15, 2014

¹ For helpful comments and suggestions, we thank Matthijs Breugem, Adrian Buss, Hui Chen, John Core, Bernard Dumas, Frank Hatheway, Pekka Hietala, Erik Loualiche, Jun Pan, Jonathan Parker, Joël Peress, Tālis Putniņš, Astrid Schornick, Pedro Saffi, Jeff Smith, Adrien Verdelhan, Jiang Wang, as well as seminar/conference participants at VU University, MIT, and INSEAD. Menkveld gratefully acknowledges NWO for a VIDI grant. We are grateful to NASDAQ for providing data. We also thank Sunny Li and Michael Abrahams for excellent research assistance.

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Abstract

Investors trade in various types of venues. When demanding immediacy they face a basic tradeoff between price impact and execution uncertainty. Venues can be sorted accordingly along a “pecking order,” with mid-point dark pools and lit markets at the top and bottom, and non-midpoint pools in the middle. A simple model formalizes this pecking order hypothesis. We test it using a unique dataset that disaggregates U.S. dark trading into various categories. A higher VIX or larger earnings surprise tilts trading volumes from the top of the pecking order to the bottom, confirming the hypothesis.

Keywords: dark pool, pecking order, fragmentation, high-frequency trading

JEL Classifications: G12, G14, G18, D47

1 Introduction

A salient trend in global equity markets over the last decade is the explosion of off-exchange, or “dark” trading venues. In the United States, dark venues now account for about 30% of equity trading volume (see Figure 1(a) for an illustration of Dow-Jones stocks). In Europe, dark venues execute about 40% of trading volume in leading equity indices (see Figure 1(b)).

Equally salient is the wide fragmentation of trading volume across dark venues. The United States has more than 30 “dark pools” and more than 200 broker-dealers that execute trades away from exchanges (see SEC 2010). Dark pools, which are automated traded systems that allow investor-to-investor trades without dealer intermediation, have grown fast in market shares and now account for about 15% of equity trading volume in the U.S., according to industry estimates.¹ In Europe, dark venues also face a high degree of fragmentation, with at least 10 multilateral dark venues operating actively.²

The fragmentation of trading—between exchanges and dark venues, as well as across dark venues—is a double-edged sword. It creates a conflict between the efficient interaction among investors and investors’ demand for a diverse set of trading mechanisms. The SEC (2010, p. 11-12) highlights this tradeoff in its Concept Release on Equity Market Structure:

“Fragmentation can inhibit the interaction of investor orders and thereby impair certain efficiencies and the best execution of investors’ orders. . . . On the other hand, mandating the consolidation of order flow in a single venue would create a monopoly and thereby lose the important benefits of competition among markets. The benefits of such competition include incentives for trading centers to create new products, provide high quality trading services that meet the needs of investors, and keep trading fees low.”

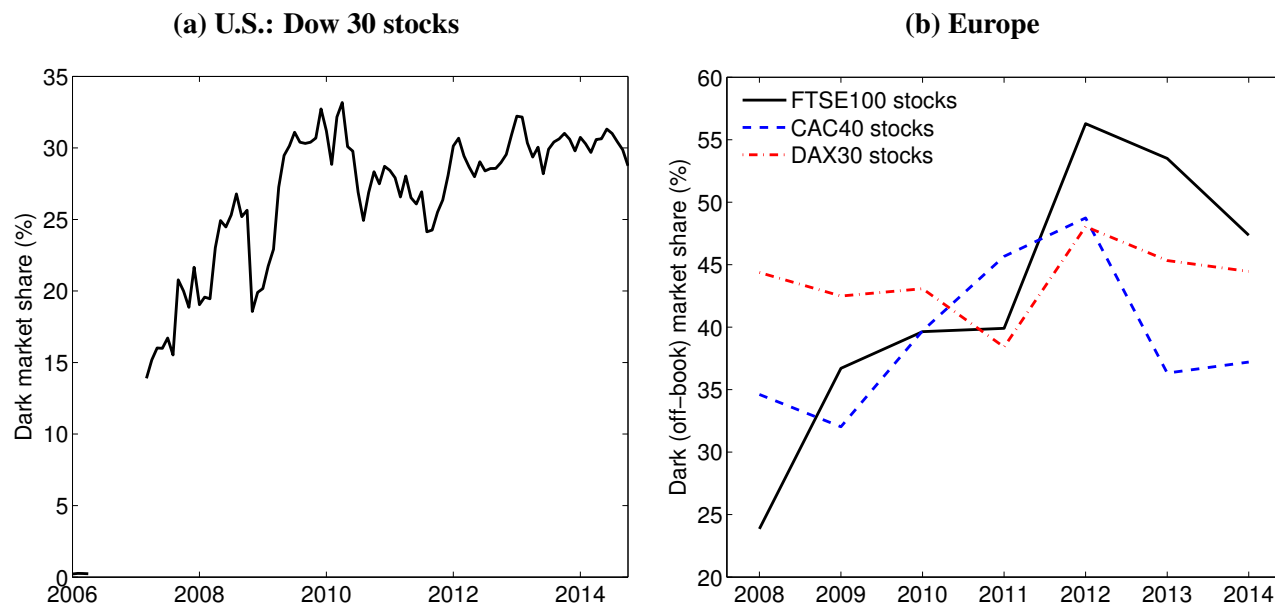
These two views of fragmentation neatly correspond to two complementary strands of theories on fragmented markets. Early theories involving multiple trading venues show that investors are attracted to venues where other investors are trading (see, for example, Pagano 1989 and Chowdhry and Nanda 1991). This “liquidity-begets-liquidity” insight suggests that fragmentation reduces the efficiency of trading among investors. Applied to today’s equity markets, this argument further implies that fragmentation of dark venues, which provide little or no pre-trade transparency, causes a particular concern because investors cannot observe the presence of counterparties *ex ante* and must engage in costly search in multiple dark venues.

¹ Industry estimates are provided by Tabb Group, a consultancy firm, and Rosenblatt Securities, a broker. On June 2, 2014, FINRA started publishing weekly statistics of transaction volumes in alternative trading systems (ATS), with a two-week lag. Many dark pools are registered as ATS. For more details, see <https://www.finra.org/Newsroom/NewsReleases/2014/P519139>.

² See “European Dark Trading Analysis,” Fidessa, October 2013.

Figure 1: Dark market share in United States and Europe

This figure shows the market shares of dark trading in the U.S. and in Europe. Panel (a) plots the monthly average dark shares of the 30 stocks in the Dow Jones Industrial Average from 2006 to October 2014. We use the same stocks that are currently (November 2014) in the Dow index. Volume data are obtained from Bloomberg and TAQ. Dark trades are defined by those reported to FINRA (code “D” in TAQ definition). Between May 2006 to February 2007 the estimates are missing because (i) Bloomberg data do not cover this period, and (ii) the TAQ data mix trades reported to FINRA and some trades on NASDAQ. Panel (b) plots the averages of dark shares of FTSE100, CAC40, and DAX30 index stocks. These estimates are directly obtained from Fidessa.



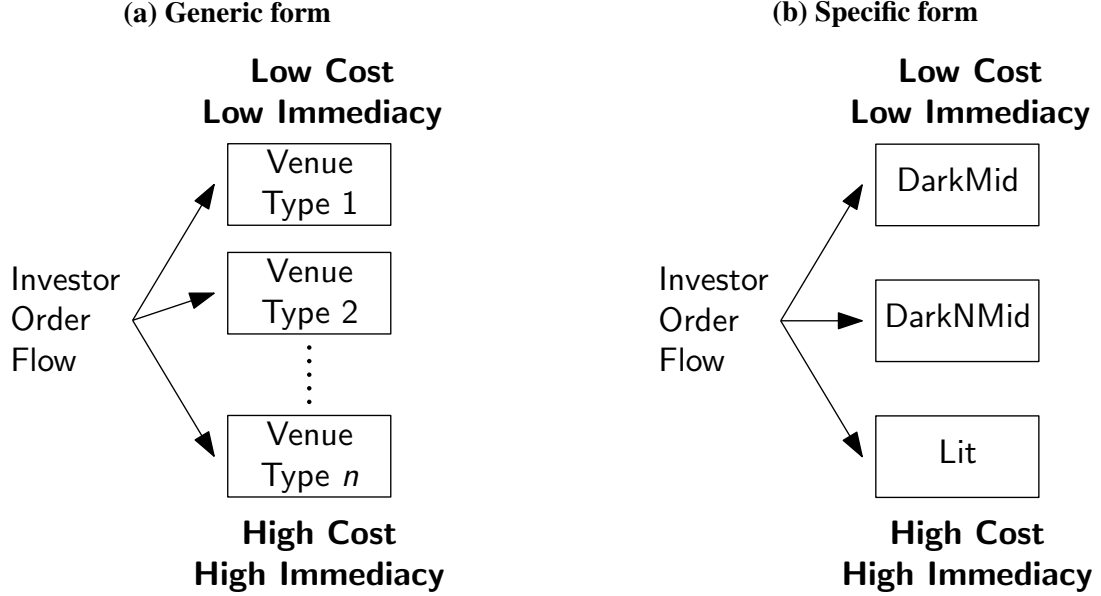
In contrast, recent theories of dark pools show that precisely because of their pre-trade opacity and the associated execution uncertainty, dark venues attract a different type of investors from those on the exchanges (Hendershott and Mendelson 2000, Degryse, Van Achter, and Wuyts 2009, Buti, Rindi, and Werner 2011a, Ye 2011, Zhu 2014, Brolley 2014). Under this “venue heterogeneity” or “separating equilibrium” view, fragmentation can be an equilibrium response to the heterogeneity of investors and time-varying market conditions.

Pecking order hypothesis

This paper characterizes the dynamic fragmentation of U.S. equity markets. We propose and test a “pecking order hypothesis” for the fragmentation of trading volumes, in line with the second view in the SEC remark. We hypothesize that investors “sort” dark and lit venues by the associated costs

Figure 2: Pecking order hypothesis

This figure depicts the pecking order hypothesis. Panel (a) shows the generic form. Panel (b) shows the specific form. On the top is midpoint crossing dark pools, DarkMid. In the middle is midpoint non-crossing dark pools, DarkNMid. Lit market sits in the bottom. Detailed descriptions of various dark pool types are collated in Section 3.1.



(price impact, information leakage) and immediacy, in the form of a “pecking order.”³ The top of the pecking order consists of venues with the lowest cost and lowest immediacy, and the bottom of the pecking order consists of venues with the highest cost and highest immediacy. The pecking order hypothesis predicts that, on average, investors prefer searching for liquidity in low-cost, low-immediacy venues but will move towards higher-cost, higher-immediacy venues if their trading needs become more urgent. This intuitive sorting is illustrated in Figure 2(a).

More concretely, recent theories of dark pools mentioned above predict that dark venues are at the top of the pecking order, whereas lit venues are at the bottom. In addition, through a simple stylized model, we rank two important categories of dark pools according to investors’ cost and immediacy considerations.

The specific ordering of the three types of venues is illustrated in Figure 2(b). This sorting combines—in an intuitive description of the underlying theory—“exchanges are liquidity of last resort” and “not all dark pools are created equal.”

We test the pecking order hypothesis by exploiting a unique dataset on dark trading in U.S. eq-

³ We choose “immediacy” here as we believe it is the appropriate interpretation of liquidity in the context of securities trading (Grossman and Miller, 1988).

uity markets. Our dataset disaggregates dark transactions into five categories by trading mechanism, including the two types of dark pools shown in Figure 2(b). The other three categories of dark transactions are internalized trades with retail brokers, average-price trades, and other (mostly institutional) trades. (The detailed descriptions are provided in Section 3.) To the best of our knowledge, this dataset provides the most comprehensive and granular view of U.S. dark trading that is accessible by academics.

We estimate a panel vector-autoregressive model with exogenous variables (VARX) to characterize the dynamic interrelation among dark volumes, high-frequency trading activity in lit venues, and various market condition measures. The focus of our empirical strategy is two exogenous variables, VIX and earnings surprise, which we use as proxies for shocks to investors' demand for immediacy, due to aggregate volatility and firm-specific news. The pecking order hypothesis predicts that, following an upward shock to VIX or earnings surprise, trading volumes should migrate from low-cost low-immediacy venues to high-cost high-immediacy venues; therefore, the change in volume share of a venue should become progressively larger the further out it is in the pecking order.

The data support the pecking order hypothesis. Following an upward shock to VIX, dark pools that cross orders at the midpoint lose a substantial market share, dark pools that allow some price flexibility lose a moderate market share, and lit venues gain market share. The same pattern is observed following an earnings-surprise shock.

We make a few cautionary remarks in interpreting our results. First, the pecking order hypothesis does not make a welfare statement. In markets with heterogeneous agents, welfare conclusions are difficult to draw.⁴ Second, while our results suggest that the fragmentation among dark venue types can be an equilibrium response to investor heterogeneity and time-varying market conditions, our analysis is silent on the fragmentation within each dark venue type. The latter question requires more detailed data on venue identities, not only venue types. Third, the pecking order hypothesis implicitly assumes that at least some investors make rational venue choices based on correct information of how these venues operate. This last point is important in light of recent cases about alleged fraud by a couple of dark pools for misrepresenting information to investors.⁵

Our primary contribution to the literature is to characterize the *dynamic* fragmentation of dark and lit venues, and provide a pecking order representation of trading volumes in U.S. equity markets.

⁴ For instance, while dark venues can reduce trading costs of some investors, the migration of those investors into dark venues can worsen liquidity in lit venues and increase the transaction costs of investors there. The welfare implication then depends on the weights assigned by the social planner to different agents, which is outside the model.

⁵ For example, on June 25, 2014, Eric Schneiderman, Attorney General of the State of New York, alleges that Barclays has falsified marketing material and misrepresented information to clients about the presence of high-frequency traders in its dark pool. In October 2011, SEC finds that Pipeline, a dark-pool operator that claimed to only allow buy-side firms to participate, had filled the majority of customer orders through its own trading affiliate (see www.sec.gov/news/press/2011/2011-220.htm).

Prior empirical studies of fragmentation and dark venues rely on either the aggregated off-exchange trading volumes or transactions in a selected few dark pools. Studies taking the former approach include [O’Hara and Ye \(2011\)](#) and [Hatheway, Kwan, and Zheng \(2013\)](#) (in the United States), [De-gryse, de Jong, and van Kervel \(2014\)](#) (in the Netherlands), and [Comerton-Forde and Putniņš \(2013\)](#) (in Australia), among others. Studies taking the latter approach include [Hendershott and Jones \(2005\)](#) (Island ECN), [Ready \(2014\)](#) (Liquidnet and POSIT), [Buti, Rindi, and Werner \(2011b\)](#) (11 anonymous dark pools), [Boni, Brown, and Leach \(2012\)](#) (Liquidnet), [Nimalendran and Ray \(2014\)](#) (one anonymous dark pool), [Foley, Malinova, and Park \(2013\)](#) (dark order types on Toronto Stock Exchange), and [Foley and Putniņš \(2014\)](#) (four dark venues in Canada), among others. These papers mostly focus on the relation between dark trading (or fragmentation) and market quality.

[Kwan, Masulis, and McInish \(2014\)](#) use a similar dataset from NASDAQ. They focus on how the minimum tick size affects the competitiveness of exchanges relative to dark venues, which is a very different research question from ours. For a comprehensive review of the recent empirical literature on fragmentation, see [SEC \(2013\)](#).

Venue pecking order and high-frequency trading

One commonly cited reason for the increase in dark volume is that investors “fear” high-frequency traders (HFTs) in lit markets.⁶ HFTs are proprietary traders who use “extraordinarily high-speed and sophisticated computer programs for generating, routing, and executing orders” ([SEC 2010](#)). Some investors and regulators are concerned that HFT may engage in “front-running” or “predatory trading.” In the context of the pecking order hypothesis, the perceived risk of interacting with HFTs is one form of cost for trading in lit venues.

Motivated by these concerns, we are interested in the extent to which HFT activity affects investors’ order flow along the venue pecking order. Without an exogenous shock to HFT activity, we attempt a preliminary, partial answer by studying the responses of HFT activity to VIX and earnings shocks. Specifically, if HFT activity declines after the VIX and earnings shocks, this decline would be at least “consistent with” the conjecture that investors trade more in lit venues because they face a lower risk of interacting with HFTs. If the data show the opposite, then we can reject this conjecture.

Empirically, we find that HFT quoting and trading activities both increase significantly in large stocks following an upward shock to VIX. The changes in HFT activity after earnings shocks are statistically insignificant. This result suggests that investor order flow migrates to lit venues under a higher aggregate volatility *not because* HFTs scale back in activity. Instead, it suggests that investors’ fear for interacting with HFT, if any, is dominated by their demand for immediacy in high volatility situations.

⁶ See, for example, “As market heats up, trading slips into shadows,” New York Times, March 31, 2013.

Our finding that a high HFT activity is associated with a high *exogenous* volatility contributes to an emerging literature on HFT and volatility. [Brogaard, Hendershott, and Riordan \(2014\)](#) document that HFTs participate more on high volatility days and generally trade in the direction of public news (e.g., macro news, market returns, or order book imbalances). [Jovanovic and Menkveld \(2011\)](#) show that HFTs participate more on days when a stock’s volatility largely reflects market-wide volatility (i.e., low idiosyncratic volatility). Our evidence is consistent with theirs. Existing evidence on the relation between HFT activity and the size of endogenous (short-term) pricing errors is mixed. For example, [Boehmer, Fong, and Wu \(2014\)](#), [Egginton, Ness, and Ness \(2013\)](#), and [Hasbrouck \(2013\)](#) suggest a positive relation, whereas [Brogaard, Hendershott, and Riordan \(2014\)](#) and [Hagströmer and Nordén \(2013\)](#) suggest the opposite.

2 A Pecking Order Hypothesis of Trading Venues

In this section we further motivate the pecking order hypothesis in its specific form. It is this specific-form hypothesis that will be the main hypothesis discussed and tested throughout the remainder of the paper. First, we discuss how various dark pool papers could be interpreted as suggestive of the predicted ordering in venues. Second, we develop an empirical strategy to test the hypothesis.

2.1 The pecking order hypothesis in its specific form

Panel (b) of Figure 2 illustrates the specific form of the pecking order hypothesis: dark pools that cross orders at the midpoint of NBBO (labeled “DarkMid”) are on the top of the pecking order, dark pools that allow some price flexibility (labeled “DarkNMid”) are in the middle, and transparent venues (labeled “Lit”) are at the bottom.

DarkNMid is a “lighter shade of dark,” sitting between DarkMid and Lit. In DarkNMid an order imbalance can move the price so that the dark pool operator can create more matched volume and the trade-through restriction guarantees that investors still get a price improvement relative to Lit. In other words, in DarkNMid execution is more likely than in DarkMid, but earning the full half-spread is less likely. If an investor moves from DarkNMid to Lit, execution becomes guaranteed but he loses the ability to earn any of the spread at all. In Section 6 we propose a simple candidate model that characterizes the competition among the three types of venues: Dark, DarkMid and Lit. The model and its analysis formalize the intuition underpinning the pecking order hypothesis of trading venues.

The specific form of the pecking order hypothesis is richer than existing theories of dark pools. For example, models of [Hendershott and Mendelson \(2000\)](#), [Degryse, Van Achter, and Wuyts \(2009\)](#), [Buti, Rindi, and Werner \(2011a\)](#), [Ye \(2011\)](#), and [Zhu \(2014\)](#) all have the feature that midpoint dark

pools provides potential price improvement relative to exchange prices, but they do not guarantee the execution of investors' orders. In all these models, investors have access to all venues and actively choose the best type of venue that serves their speculative or hedging needs.⁷ While these models predict that dark venues are near the top of the pecking order and lit venues near the bottom, they do not distinguish different types of dark pools.

There is an alternative motivation for the pecking order hypothesis, based on an agency conflict between investors and their brokers. Its starting point is that brokers decide where to route investors' orders, and investors monitor brokers insufficiently. If brokers earn more profits by routing investors' orders first to their own dark pools, then orders would first flow into broker-operated dark pools and then to other dark and lit venues. Only when investors tell brokers to execute quickly will they have no choice but to send it to lit venues. This alternative motivation is less rich as it does not suggest a particular ranking of dark venue types. It simply states that dark venues take priority, whatever type of dark venue the broker is running, except when investors emphasize quick execution.

2.2 Empirical strategy for testing the pecking order hypothesis

Because venues are sorted by the tradeoff between immediacy and trading cost, an upward shock to investors' urgency to trade should predict a loss of market share by venues at the top of the pecking order and a gain of market share by venues at the bottom. Testing the pecking order hypothesis requires (i) disaggregated data on dark volumes and (ii) an exogenous shock to demands for immediacy and an econometric model.

Section 3 discusses the high-frequency disaggregated dark volume data available to us. Our empirical strategy is to use the high-frequency nature of the data to identify the dynamic interrelation among dark volumes in the various categories. We use two types of exogenous shocks as instruments for urgency: shocks to VIX and shocks to earnings.

We first study trade response to a market-wide volatility shock implemented through a doubling of VIX. Because VIX is calculated from prices of short-dated (within one month maturity) S&P 500 index options, we are reasonably confident that fragmentation of order flows is unlikely to change VIX. Rather, VIX is driven mostly by systematic risks and the time-varying risk premium in the aggregate U.S. equity market, both of which can increase the effective cost of holding an undesirable inventory.

Next, we study trade response to an idiosyncratic shock to firms' fundamental values, measured by

⁷ Yet another motivation for trading in dark venues is suggested by [Boulatov and George \(2013\)](#). They show that if liquidity providers are informed, displaying their orders reduces their information rents and discourages informed traders from providing liquidity. As a result, an opaque, or dark, market can have a narrower bid-ask spread and more efficient prices than a transparent one. Different from the dark pool studies, however, the model of [Boulatov and George \(2013\)](#) has only one market at a time and does not make predictions regarding the fragmentation of dark and lit venues.

earnings surprises on announcement dates relative to pre-announcement expectations. Since earnings announcements are scheduled far in advance and are made outside trading hours, they are not affected by trading activities. Trades naturally happen given the new information from the announcements. For example, investors may wish to liquidate speculative or hedging positions established before earnings announcements. Trades can also be generated when investors interpret the same earnings news differently (Kim and Verrecchia, 1994). The larger the earnings surprise, the more immediacy we expect investors to demand to adjust their positions in the stock.

The next two sections discuss the data and the econometric approach in full detail. Section 5 discusses the test results.

3 Data

Our data sample covers 117 stocks in October 2010 (21 business days). In addition to TAQ trading volumes and National Best Bid and Offer, we use two main proprietary data sources: (1) transactions in five types of dark venues, (2) HFT activity in the NASDAQ main market. We complement these trading data with intraday VIX and earnings announcements by firms in our sample. These data sources are described below in detail.

3.1 Dark volumes

In the United States, off-exchange transactions in all dark venues are reported to trade-reporting facilities (TRFs). The exact venue in which the dark trade takes place is not reported in public data. (Recently, FINRA started to publish weekly transaction volumes in alternative trading systems, but these volumes are aggregated and not on a trade-by-trade basis.⁸) The NASDAQ TRF is the largest TRF, accounting for about 92% of all off-exchange volumes in our sample. Our first data source is the dark transactions reported to NASDAQ TRF. These trades are executed with limited pre-trade transparency.⁹ A salient feature of our data is that the dark transactions are disaggregated into five categories by trading mechanism. The exact method of such disaggregation is proprietary to NASDAQ, but we know their generic features. The five categories include:

1. **DarkMid.** These trades are done in dark pools that use midpoint crossing as much as possible. Midpoint crossing means the buyer and the seller in the dark venue transact at the midpoint

⁸ See <https://www.finra.org/Newsroom/NewsReleases/2014/P519139>.

⁹ Our dark transaction data do not include trades on electronic communication networks (ECNs). ECNs are transparent venues that register as alternative trading systems (ATS), but they are not exchanges for regulatory purposes. In our sample, ECNs account for a very small fraction of total transaction volume. BATS and DirectEdge, two major exchanges that recently merged, used to be ECNs, but they have converted to full exchange status in November 2008 and July 2010, respectively.

of the National Best Bid and Offer (NBBO). “Agency-only” dark pools (i.e. those with no proprietary order flows) typically operate this way.¹⁰

2. **DarkNMid.** These trades are done in dark pools that allow flexibility in execution prices (not necessarily midpoint). By this features, we infer that dark pools operated by major investment banks belong to this category.
3. **DarkRetail.** These trades are internalized volume by broker-dealers. Retail brokers often route order flows submitted by retail investors to major broker-dealers, who then fill these orders as principal or agent. These transactions would be classified as DarkRetail.
4. **DarkPrintB.** These trades are “average-price” trades. A typical example is that an institutional investor agrees to buy 20,000 shares from a broker, at a volume-weighted average price plus a spread. This trade of 20,000 shares between the investor and the broker would be classified as a “print back” trade, abbreviated as “PrintB.”
5. **DarkOther.** These are other dark trades not covered by the categories above. A typical example in this category would be a negotiated trade between two institutions on the phone (i.e., not done on any electronic platform).

We emphasize that each category is not a single trading venue, but a collection of venues that are qualitatively similar in terms of their trading mechanisms. In the interest of brevity, however, we will use the terms “venue” and “type of venue” interchangeably.

Figure 3 shows the market shares of the five types of dark venues as a fraction of total trading volume (obtained from TAQ) in our sample. We label the complement of these five dark venues as the “lit” venues. The “lit” label is an approximation, but it is a good one.¹¹ We observe that dark venues account for 27.2% of total transaction volume in the 117 stocks in October 2010. Ranked by market shares, the five dark categories are DarkRetail (10.8%), DarkNMid (7.7%), DarkOther (5.8%), DarkMid (2.1%), and DarkPrintB (0.9%).

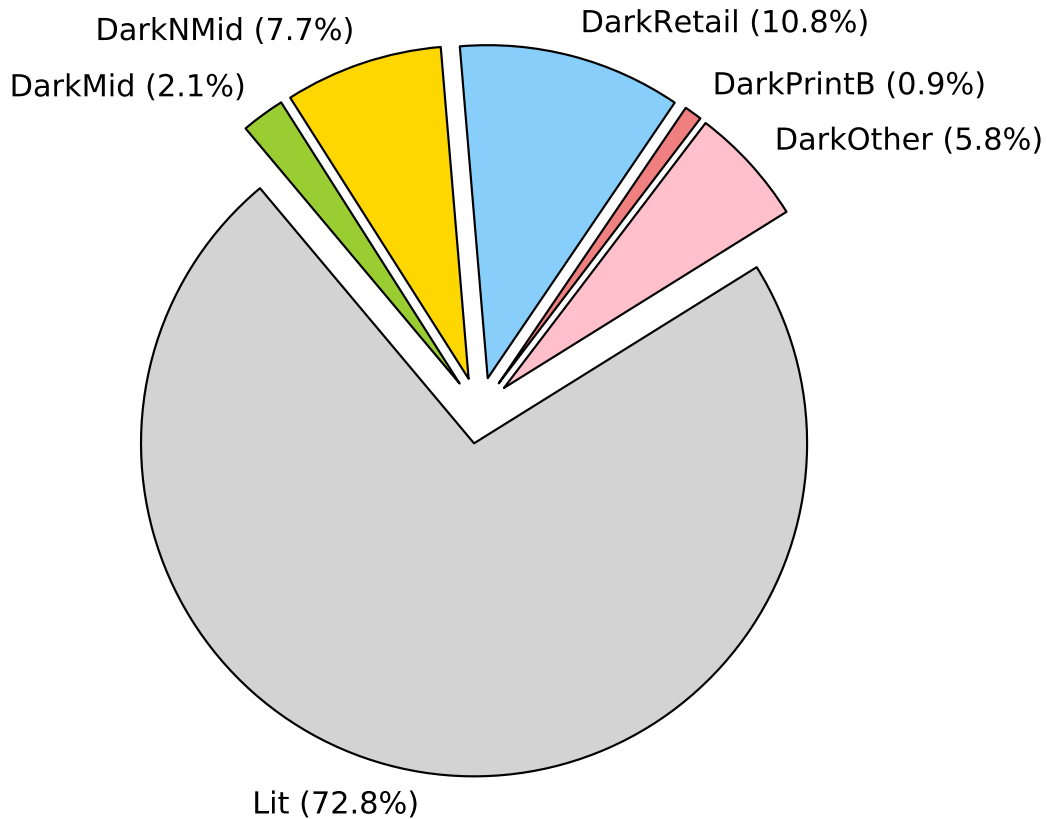
It is informative to compare our five-way categorization of dark trading venues to that of the SEC. SEC (2010) classify opaque trading centers into dark pools and broker-dealer internalization. If an approximate correspondence is to be made, our DarkMid and DarkNMid types roughly fall into the SEC’s dark pool category, and our DarkRetail, DarkPrintB, and DarkOther types roughly fall into the SEC’s broker-dealer internalization category.

¹⁰ The NASDAQ classification puts a couple of midpoint-crossing dark pools that have large transaction sizes into “DarkOther” rather than “DarkMid,” in order to guarantee that one cannot reverse-engineer the identities of these venues.

¹¹ Since NASDAQ TRF accounts for 92% of all off-exchange trading volumes in our sample, the “lit” category also contains the remaining 8% of off-exchange volumes, or about 2.4% of total volumes. For our purpose of characterizing the qualitative nature of venue types, this measurement noise is likely inconsequential.

Figure 3: Volume shares of dark venues and aggregate lit venue

This chart illustrates the average volume share of the various types of dark pools in the data sample.



Using one week of FINRA audit trail data in 2012, [Tuttle \(2014\)](#) reports that about 12.0% of trading volumes in U.S. equities are executed in off-exchange alternative trading systems (ATS), the majority of which are dark pools; and that about 18.8% of U.S. equity volume are executed off exchanges without involving ATS, which can be viewed as a proxy for trades intermediated by broker-dealers. The dark-pool volumes and internalized volumes implied by our dataset are comparable to those reported by [Tuttle \(2014\)](#).

3.2 High-frequency trading and quoting activity in NASDAQ market

Our second data source contains the trading and quoting activities of high-frequency trading firms in the NASDAQ market. These firms are known to employ high-speed computerized trading, but their identities and their strategies are unknown to us. This dataset has two parts.

First, for each transaction on NASDAQ, we observe the stock ticker, transaction price, number of shares traded, an indicator of whether the buyer or the seller is the active side, and an indicator on

whether either side of the trade is an HFT firm. We refer to a trade as an HFT trade if at least one side of the trade is an HFT firm. All transactions are time-stamped to milliseconds. Second, we observe the minute-by-minute snapshot of the NASDAQ limit order book. For each limit order, we observe the ticker, quantity, price, direction (buy or sell), a flag on whether the order is displayed or hidden, and a flag on whether or not the order is submitted by a high-frequency trading firm. For additional details about the NASDAQ HFT dataset, see [Brogaard, Hendershott, and Riordan \(2014\)](#).

3.3 VIX and earnings announcements

The last components of our data include two measures of shocks: VIX and earnings announcements. The Chicago Board of Exchange (CBOE) disseminates VIX every 15 seconds. Minute-by-minute VIX data are obtained from [pittrading.com](#). Among the 117 firms in our sample, 68 firms announced earnings in October 2010. For each of these earnings announcements, we download the announcement dates, time stamps, announced earnings per share, and expected earnings per share from Bloomberg. (We are able to collect EPS forecast from Bloomberg for 67 of the firms, and hence can construct the *EpsSurprise* variable defined below.)

Figure 4 shows the time series of the volume shares of the five types of dark venues and VIX. They are aggregated at the daily level and averaged across the 117 stocks. One interesting observation is that in the time series, a higher VIX is associated with a lower dark market share. In the next few sections we will further explore VIX shocks at a higher frequency to test the pecking order hypothesis.

4 A VARX Model of Dark Volumes and High-Frequency Trading

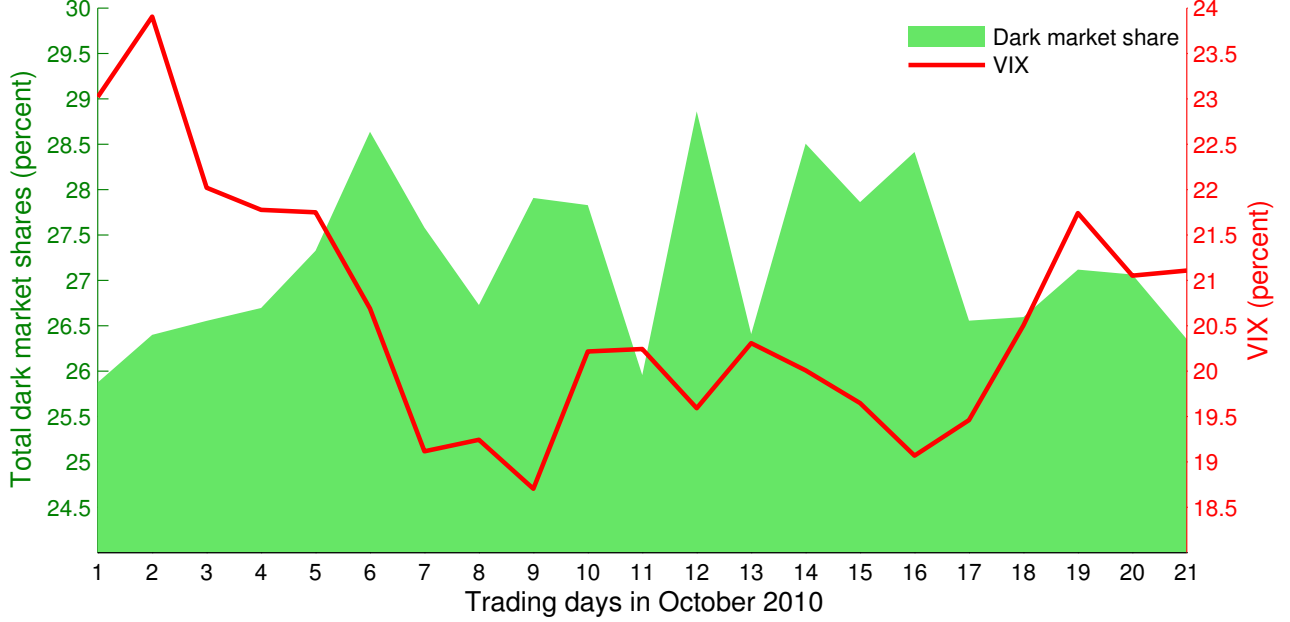
In this section, we characterize the dynamic interrelation among dark volumes, HFT participation, and various standard trade variables through a panel vector autoregressive model with exogenous variables (a panel VARX).

4.1 Model components

From the raw data we calculate three broad types of variables for our analysis: (1) transaction volumes in five types of dark venues, (2) characteristics of the NASDAQ lit market, and (3) overall market conditions. We describe these components below. For ease of reference, these variables and their descriptions are tabulated in Table 1. All variables are constructed from the raw data at the minute frequency during the normal trading hours (9:30am to 4:00pm). This gives us a stock-minute panel data with $117 \times 21 \times 390 = 958,230$ observations.

Figure 4: Time series of dark volume share, HFT participation, and VIX

This chart illustrates the average volume share of the five types of dark venues combined (shaded area, left axis) and VIX (solid line, right axis). All series are aggregated at the daily level and averaged across all stocks.



Dark volumes. We separately calculate trading volumes across the five types of dark venues: *VDarkMid*, *VDarkNMid*, *VDarkRetail*, *VDarkPrintB*, and *VDarkOther*. Volume is measured in thousands of shares, which we prefer over dollar value as it ensures that quantity variables are not contaminated by price effects. For example, two consecutive 1000 share transactions are considered to be of equal size, even though they may be executed at slightly different prices.

Characteristics of the NASDAQ lit market. We use two commonly-used measures of liquidity: spread and depth. *BASpread* is the relative bid-ask spread of the NASDAQ lit market, measured in basis points. *TopDepth* is the sum of visible liquidity supply on the bid side and the ask side of the NASDAQ limit order book, measured in thousands of shares. The motivation for using the number of shares (rather than market value) to measure depth is the same as that for transaction volumes, as discussed earlier.

In addition, we use two measures of HFT activity: *HFTinVolume* and *HFTinTopDepth*. *HFTinVolume* is the number of shares traded by HFT (on at least one side) on the NASDAQ market divided by the number of shares traded on the NASDAQ market.¹² *HFTinTopDepth* is the number of shares

¹² The stock-day-minute average of *HFTinVolume* is around 40% in our sample period (see table 2). If HFT volumes

Table 1: Variable descriptions

This table lists and describes all variables used in this study. All variables are generated for one-minute intervals. Variables that enter the econometric model (Section 4) are underscored. The subscript j indexes stocks; t indexes minutes. Type “Y” and “Z” are described in the panel VARX model.

Type	Variable Name	Description
<i>Panel A: Dark venue trading volumes</i>		
Y	<u>VDarkMid_{jt}</u>	Volume of midpoint-cross dark pools
	<u>VDarkNMid_{jt}</u>	Volume of non-midpoint dark pools
	<u>VDarkRetail_{jt}</u>	Volume of retail flow internalization
	<u>VDarkPrintB_{jt}</u>	Volume of average-price trades (“print back”)
	<u>VDarkOther_{jt}</u>	Volume of other dark venues
	<u>VLit_{jt}</u>	Total volume minus all dark volume
<i>Panel B: NASDAQ lit market characterization</i>		
	<u>BASpread_{jt}</u>	NASDAQ lit market bid-ask spread divided by the NBBO midpoint
	<u>TopDepth_{jt}</u>	Sum of NASDAQ visible best bid depth and best ask depth
	<u>HFTinTopDepth_{jt}</u>	<u>Depth_{jt}</u> based on only HFT limit orders divided by <u>Depth_{jt}</u>
	<u>HFTinVolume_{jt}</u>	NASDAQ lit volume in which HFT participates divided by total NASDAQ lit volume
<i>Panel C: Overall market conditions</i>		
	<u>TAQVolume_{jt}</u>	TAQ volume
	<u>RealVar_{jt}</u>	Realized variance, i.e., sum of one-second squared NBBO midquote returns
	<u>VarRat10S_{jt}</u>	Variance ratio, i.e., ratio of realized variance based on ten-second returns relative to realized variance based on one-second returns (defined to be one for a minute with only one-second returns that equal zero)
Z	<u>VIX_t</u>	One-month volatility of S&P500 index (in annualized percentage points)
	<u>EpsSurprise_{jt}</u>	Surprises in announced EPS, calculated as the absolute difference in announced EPS and the forecast EPS, scaled in share price: announced EPS – forecast EPS /(closing price on the day before). This variable is zero if a stock-day pair does not have a earnings announcement.

posted (quoted) by HFT on the top of the NASDAQ limit order book divided by the total depth on the top of the NASDAQ limit order book.

Overall market conditions. We add endogenous and exogenous variables describing the overall market conditions. For each stock and each minute, the endogenous variables are total transaction volume *TAQVolume*, realized return variance within the minute *RealVar*, and the variance ratio within

are first aggregated across days and stocks, then the total HFT volume accounts for about 45% of the total NASDAQ volume in our sample.

the minute *VarRat10S*. The first variable is based on TAQ data, whereas the other two are based on millisecond-level NBBO data provided by NASDAQ.

Two exogenous variables are included: the one-month volatility on the S&P 500 index, *VIX*, and earnings surprise, *EpsSurprise*. *VIX* is based on option market prices and therefore is a forward-looking volatility measure. It is sometimes referred to as the “fear index.”

There are 68 earnings announcements, 67 of which can be matched with a forecast, in our sample. Consistent with the accounting literature (e.g. [Kinney, Burgstahler, and Martin, 2002](#)), the earnings surprise is calculated as the absolute difference between announced EPS and pre-announcement expected EPS, then divided by the closing price on the business day immediately before the announcement. If stock j announced its earnings on a particular day, we set the variable $EpsSurprise_{jt}$ to this surprise measure for stock j and all minutes t in the business day immediately following the earnings announcement.¹³ Otherwise, $EpsSurprise_{jt}$ is set to zero. This variable captures the presence of material news for the firm’s fundamental as well as the magnitude of the surprise. Earnings announcements are typically scheduled months or years ahead by company management; therefore, they are exogenous.

4.2 Data preparation and summary statistics

We convert all variables into logs, except the earnings surprise. A log-linear model has a couple of advantages over a linear model. First, a log-linear model comes with a natural interpretation that estimated coefficients are elasticities. Second, all endogenous variables (e.g., volume, realized variance, and depth) are guaranteed to remain nonnegative. In other words, the error term does not need to be bounded from below, which would be the case for a linear model.

In order to take the log, data need to be winsorized to eliminate the zeros. We use the following procedure. If a particular dark venue has a zero transaction volume for stock j and minute t , its volume for that stock-minute is reset to one share. If a particular stock j does not trade in period t on the NASDAQ market, the *HFTinVolume* variable is undefined. In this case, to not lose data, *HFTinVolume* is forward filled from the start of the day. The motivation is that market participants may learn HFT activity on NASDAQ by carefully parsing market conditions. If there is no update in a particular time interval, they might rely on the last observed value they learned about. Zero entries for all other variables are left-winsorized at the 0.01% level.

Table 2 reports the summary statistics of the model variables, before taking the log. One important observation is that the data preparation procedure discussed above has a minimum effect on the raw data. Total trading volume per stock-minute is about 12,300 shares on average. The market shares

¹³ That is, if the earnings announcement is made before the market opens (9:30am), the immediate following business day is the same business day. If the earnings announcement is made after the market closes (4pm), the immediate following business day is the next business day.

of the five types of dark venues are shown in Figure 3. (For completeness, we also include $VLit$, defined as $TAQVolume$ less the sum of the five dark volumes, although $VLit$ will not be part of the panel VARX model.) On average, HFT participation in quoting at the best NASDAQ prices, $HFTinTopDepth$, is about 35%, whereas HFT participation in transactions on NASDAQ, $HFTinVolume$, is about 40%. The average depth and relative spread on NASDAQ limit order book are about 4440 shares and 16.8 basis points, respectively. VIX in our sample has an average of 20.6%. The earnings surprises measure, $EpsSurprise$, has an average of about 47 basis points across the 68 firms that made earnings announcements in our sample.¹⁴

4.3 Panel VARX model

The panel VARX model used for the main empirical analysis has the following form:

$$y_{jt} = \alpha_j + \Phi_1 y_{j,t-1} + \cdots + \Phi_p y_{j,t-p} + \Psi_1 z_{j,t-1} + \cdots + \Psi_r z_{j,t-r} + \varepsilon_{jt}, \quad (1)$$

where, for stock j and period t :

y_{jt} is a vector of log-transformed endogenous variables. Our baseline model includes the following variables, with detailed explained in Table 1:

- Volumes in the five types of dark venues;
- Other endogenous variables that characterize trade conditions (e.g., volatility, the bid-ask spread, etc.).

z_{jt} is a vector of arguably exogenous variables, including the log transformations of VIX_{jt} and the raw (not logged) earnings surprise measure $EpsSurprise_{jt}$.

A stock fixed effect α_j ensures that only time variation is captured, not cross-sectional variation, as our focus is on dynamic interrelations among variables. The number of lags (in minutes) is determined based on the Bayesian Information Criterion (BIC): $p = 2$ and $r = 1$. Further details on the implementation of the estimation are provided in Appendix A.

4.4 Estimation results

Table 3 reports the estimation results of the VARX model. Panel (a) reports the estimated coefficients $\{\Phi_1, \Phi_2, \Psi_1\}$, which should be interpreted as elasticities, and Panel (b) reports the correlations of the

¹⁴ The statistics of $EpsSurprise$ reported in Table 2 are across all $117 \times 21 \times 360$ stock-day-minutes. It should be reminded that there are only 67 $EpsSurprises$ during our sample period. The sample average based on these 67 observations is about 0.15% and the sample standard deviation is 0.89%. Based on these numbers, we give a 1% $EpsSurprise$ shock in the pecking order analysis below (see Figure 6).

Table 2: Summary statistics

This table reports summary statistics of the variables used throughout the paper. These statistics are calculated for the raw data as well as the prepared data, before taking the logarithm. The sample frequency is minute. The units of each series is in the square brackets.

	mean		std. dev.		skewness		min		max		zeros		count	
	raw	prep/d	raw	prep/d	raw	prep/d	raw	prep/d	raw	prep/d	raw	prep/d	raw	prep/d
<i>VDarkMid</i> [1k shares]	0.259	0.259	3.161	3.161	377.994	378.016	0	0.001	2032.340	2032.340	737273	0	954892	954892
<i>VDarkNMid</i> [1k shares]	0.941	0.941	5.437	5.437	54.600	54.602	0	0.001	1040.352	1040.352	574616	0	954892	954892
<i>VDarkRetail</i> [1k shares]	1.329	1.330	6.197	6.196	34.609	34.611	0	0.001	1641.229	1641.229	592060	0	954892	954892
<i>VDarkPrintB</i> [1k shares]	0.106	0.107	8.040	8.040	269.541	269.542	0	0.001	3524.920	3524.920	924829	0	954892	954892
<i>VDarkOther</i> [1k shares]	0.709	0.709	10.798	10.798	104.222	104.223	0	0.001	3018.263	3018.263	690408	0	954892	954892
<i>VDark</i> [1k shares]	3.437	3.438	41.120	41.120	414.272	414.273	0	0.001	23616.586	23616.586	440491	0	958230	958230
<i>VLit</i> [1k shares]	8.932	8.932	45.397	45.397	60.567	60.568	0	0.001	11892.866	11892.866	233599	0	958230	958230
<i>RelSpread</i> [bps]	16.776	16.776	28.466	28.466	37.114	37.114	0.162	0.334	6681.449	6681.449	0	0	957769	957769
<i>TopDepth</i> [1k shares]	4.443	4.443	13.909	13.909	6.058	6.058	0	0.004	502.515	502.515	9	0	958230	958230
<i>HFTinTopDepth</i> [percent]	35.199	35.225	29.512	29.480	0.384	0.386	0	0.105	100	100	244386	0	958221	958230
<i>TAQVolume</i> [1k shares]	12.264	12.268	55.722	55.722	40.816	40.817	0	0.001	12196.768	12196.769	208687	0	958230	958230
<i>HFTinVlm</i> [percent]	39.985	37.157	23.939	26.038	0.053	0.166	0	0.125	100	100	72830	0	583561	955059
<i>RealVar</i> [bps]	6.139	6.188	9.112	9.079	14.518	14.668	0	0.172	1367.197	1367.197	274466	0	958218	958218
<i>VarRat10S</i> [percent]	99.772	99.778	44.438	44.426	1.142	1.144	0	0.317	853.407	853.407	16561	0	958218	958218
<i>VIX</i> [percent]	20.626	20.626	1.369	1.369	0.656	0.656	17.940	17.940	24.330	24.330	0	0	954603	954603
<i>EpsSurprise</i> [percent]	0.013	0.013	0.150	0.150	18.871	18.871	0	0	4.190	4.190	931710	931710	957840	957840

residuals. We make a few observations. First, a higher VIX or a higher earnings surprise forecasts higher volumes in dark venues and total TAQ volume, but the elasticity of TAQ volume is higher than that of dark volumes. This suggests that the market shares of various venue types are likely to have heterogeneous reactions to VIX and earnings shocks. Second, a higher HFT participation in this minute tends to forecast a mildly lower volumes in all venues in the next minute. Third, dark volumes in different types of venues have heterogeneous dynamic correlations with spreads, depths, and variance ratios, sometimes with opposite signs (see the first five lines of Panel (a)).

In addition to the parameter estimates, we also calculate and plot standard impulse-response functions (IRFs) of endogenous variables to shocks. With few exceptions, we set the magnitudes of the initial shocks such that the shocked variable doubles in its level (i.e. shock the logged variable by $\log(2)$). For a generic upward shock to some variable in y or z , the IRF essentially identifies two types of elasticities:

- an immediate one-period response of some variable in y , and
- a cumulative long-term response of some variable in y .

IRFs make the dynamic interrelations among variables transparent. Besides its intuitive appeal, another advantage of IRFs is that they shows the duration of the responses. Thus, we will mostly rely on IRFs for exposition in the remaining of the paper. As the IRF is a non-linear function of parameter estimates, we calculate the 95% confidence bounds of the IRF through simulations. In each iteration a parameter value is drawn from a normal distribution with a mean equal to the point estimate and a covariance matrix equal to the estimated parameter covariance matrix. Details of this simulation method are provided in Appendix A. In the next two sections, we present the IRFs that are most directly related to our main findings.

5 Results: Pecking Order

In this section we test the pecking order hypothesis laid out in Section 2.

5.1 Testing the pecking order hypothesis

We focus on testing the specific form of the pecking order hypothesis (Figure 2(b)) because it makes stronger (i.e. more specific) predictions. Evidence supporting the specific form would also support the generic form. For completeness, after presenting results for DarkMid, DarkNMid, and Lit, we also discuss the other three dark venue types (DarkRetail, DarkPrintB, and DarkOther).

Table 3: VARX Estimation

This table reports the estimation result of the VARX model at the minute frequency. All variables are log-transformed except the earnings announcement surprise *EpsSurprise*. Panel (a) reports the estimated coefficients. Panel (b) reports the correlations of the estimated residuals. In Panel (b) we also included *VDark*, defined as the sum of the volumes in five types of dark venues, and *VLit*, defined as *TAQVolume* less *VDark*.

(a) Estimated coefficients

	<i>VDarkMid</i>	<i>VDarkNMid</i>	<i>VDarkRetail</i>	<i>VDarkPrintB</i>	<i>VDarkOther</i>	<i>RelSpread</i>	<i>TopDepth</i>	<i>HFTinTopDepth</i>	<i>TAQVolume</i>	<i>HFTinVlm</i>	<i>RealVar</i>	<i>VarRat10S</i>
Endogenous variables: 1 minute lag												
<i>VDarkMid</i> (-1)	0.213**	0.037**	0.030**	0.005**	0.041**	-0.001*	0.002**	-0.000	0.019**	0.001	0.005**	-0.001*
<i>VDarkNMid</i> (-1)	0.033**	0.196**	0.053**	0.004**	0.049**	-0.000	0.005**	0.001	0.042**	0.000	0.003*	-0.002**
<i>VDarkRetail</i> (-1)	0.020**	0.035**	0.112**	0.002**	0.040**	0.001*	-0.001	-0.001	0.020**	0.003**	0.007**	0.002**
<i>VDarkPrintB</i> (-1)	0.009*	0.007	0.009*	0.054**	0.018**	-0.000	0.001	0.002	0.003	0.000	0.003	0.000
<i>VDarkOther</i> (-1)	0.021**	0.036**	0.038**	0.002**	0.146**	0.000	-0.001**	-0.000	0.019**	-0.002*	0.006**	-0.001
<i>RelSpread</i> (-1)	-0.072**	-0.089**	-0.038**	-0.003	-0.061**	0.409**	-0.066**	-0.100**	-0.368**	-0.116**	0.233**	-0.039**
<i>TopDepth</i> (-1)	0.104**	0.170**	0.077**	0.006	0.082**	-0.040**	0.391**	-0.028*	0.158**	-0.011*	-0.134**	0.014**
<i>HFTinTopDepth</i> (-1)	-0.001	-0.001	-0.003*	-0.000	-0.004**	-0.004**	-0.002*	0.336**	-0.002	0.044**	0.001	0.002**
<i>TAQVolume</i> (-1)	0.015**	0.051**	0.047**	0.001*	0.032**	-0.003**	0.004**	-0.005	0.213**	-0.006*	0.033**	0.002
<i>HFTinVlm</i> (-1)	-0.003*	-0.005**	-0.002	-0.000	-0.007**	0.000	-0.000	0.004*	-0.004**	0.546**	0.001	0.001
<i>RealVar</i> (-1)	-0.025**	-0.046**	-0.013**	-0.000	-0.033**	0.010**	-0.016**	0.005	-0.037**	0.010**	0.234**	-0.022**
<i>VarRat10S</i> (-1)	0.004	0.001	0.018**	0.002	0.005	0.004**	-0.002**	0.001	0.013**	0.007**	0.023**	0.048**
Endogenous variables: 2 minutes lag												
<i>VDarkMid</i> (-2)	0.179**	0.024**	0.018**	0.003**	0.029**	-0.000	0.003**	0.001	0.011**	-0.001	0.004**	0.000
<i>VDarkNMid</i> (-2)	0.021**	0.143**	0.037**	0.002*	0.033**	0.001**	0.005**	0.001	0.026**	-0.001	0.002	-0.000
<i>VDarkRetail</i> (-2)	0.015**	0.026**	0.092**	0.001	0.031**	0.002**	-0.001*	-0.001	0.018**	0.002*	0.012**	0.002**
<i>VDarkPrintB</i> (-2)	0.009*	0.005	0.010*	0.031**	0.006	-0.000	0.001	-0.000	-0.001	-0.001	-0.001	-0.000
<i>VDarkOther</i> (-2)	0.010**	0.022**	0.031**	0.000	0.113**	0.000	-0.001**	-0.002	0.010**	-0.000	0.003**	0.000
<i>RelSpread</i> (-2)	0.021**	0.065**	0.066**	-0.006*	0.075**	0.223**	-0.035**	-0.101**	0.073**	0.010	0.113**	0.014**
<i>TopDepth</i> (-2)	0.009	0.039**	-0.011	0.005	-0.043**	-0.018**	0.235**	-0.001	-0.010	-0.014**	-0.102**	0.003
<i>HFTinTopDepth</i> (-2)	-0.001	-0.002	-0.004**	0.000	-0.004**	-0.005**	-0.000	0.171**	-0.008**	0.007**	-0.004**	-0.000
<i>TAQVolume</i> (-2)	0.001	0.030**	0.029**	0.003**	0.004*	-0.010**	0.010**	-0.007**	0.144**	-0.003	-0.009**	0.002*
<i>HFTinVlm</i> (-2)	0.002	0.002	-0.001	-0.000	0.002	-0.001**	0.000	0.018**	0.001	0.116**	0.005**	0.001
<i>RealVar</i> (-2)	-0.003	-0.014**	0.005*	-0.002*	-0.001	0.009**	-0.011**	0.000	0.002	0.005*	0.151**	-0.010**
<i>VarRat10S</i> (-2)	-0.001	-0.007*	0.004	0.001	-0.003	0.002**	0.001	0.002	-0.006*	0.005*	-0.001	0.030**

Table 3 continued...

(a) Estimated coefficients (continued)

	<i>VDarkMid</i>	<i>VDarkNMid</i>	<i>VDarkRetail</i>	<i>VDarkPrintB</i>	<i>VDarkOther</i>	<i>RelSpread</i>	<i>TopDepth</i>	<i>HFTinTopDepth</i>	<i>TAQVolume</i>	<i>HFTinVlm</i>	<i>RealVar</i>	<i>VarRat10S</i>
Exogenous variables												
<i>VIX</i> (-1)	-0.142	0.268*	-0.401**	0.155**	0.353**	0.046**	0.009	-0.107	0.785**	0.176**	0.813**	-0.001
<i>EpsSurprise</i>	0.077*	0.184*	0.221**	0.086*	0.115*	0.005	-0.000	0.071	0.293**	0.035	0.079**	0.006
R^2	0.130	0.138	0.070	0.005	0.087	0.366	0.353	0.190	0.143	0.398	0.162	0.008
# obs.	937674	937674	937674	937674	937674	937674	937674	937674	937674	937674	937674	937674

*, ** Significant, respectively, at 5%, and 1%. All tests are two sided.

(b) Residual correlations

	<i>VDarkNMid</i>	<i>VDarkRetail</i>	<i>VDarkPrintB</i>	<i>VDarkOther</i>	<i>VDark</i>	<i>VLit</i>	<i>RelSpread</i>	<i>TopDepth</i>	<i>HFTinTopDepth</i>	<i>TAQVolume</i>	<i>HFTinVlm</i>	<i>RealVar</i>	<i>VarRat10S</i>
<i>VDarkMid</i>	0.185**	0.123**	0.010**	0.253**	0.361**	0.143**	0.004**	0.009**	0.010**	0.199**	0.022**	0.124**	-0.015**
<i>VDarkNMid</i>		0.212**	0.011**	0.226**	0.675**	0.227**	0.016**	0.014**	0.014**	0.335**	0.029**	0.171**	-0.014**
<i>VDarkRetail</i>			0.006**	0.161**	0.813**	0.182**	0.015**	0.004**	0.004**	0.318**	0.013**	0.129**	-0.002*
<i>VDarkPrintB</i>				0.009**	0.027**	0.033**	0.001	0.003*	-0.001	0.035**	0.000	0.006**	0.001
<i>VDarkOther</i>					0.460**	0.199**	0.019**	-0.008**	0.007**	0.270**	0.028**	0.176**	-0.011**
<i>VDark</i>						0.288**	0.022**	0.009**	0.011**	0.451**	0.031**	0.220**	-0.012**
<i>VLit</i>							0.059**	-0.027**	0.015**	0.985**	0.065**	0.495**	-0.005**
<i>RelSpread</i>								0.104**	0.150**	0.059**	0.040**	0.125**	-0.004**
<i>TopDepth</i>									0.067**	-0.024**	-0.018**	-0.095**	-0.003**
<i>HFTinTopDepth</i>										0.016**	0.010**	0.025**	0.001
<i>TAQVolume</i>											0.066**	0.501**	-0.007**
<i>HFTinVlm</i>												0.108**	0.001
<i>RealVar</i>													-0.106**

*, ** Significant, respectively, at 5%, and 1%. All tests are two sided.

A VIX shock

Starting with our estimated VARX model and the steady state, we shock VIX by 100% upward (i.e. shock $\ln(VIX)$ by $\ln(2)$) and examine the resulting changes in volume shares of DarkMid, DarkNMid, and Lit in the following minutes.¹⁵ Their volume shares are denoted as $SDarkMid$, $SDarkNMid$, and $SLit$, respectively. The pecking order hypothesis stated in Section 2 predicts that the elasticities of Lit, DarkNMid, and DarkMid market shares to VIX are positive, mildly negative, and most negative, respectively. These findings remain unchanged if one were to study generalized impulse response functions that account for contemporaneous correlation by setting other variables at their expected value given the variable that is shocked (see Koop, Pesaran, and Potter 1996; Pesaran and Shin 1998). The reason is that in the data the contemporaneous correlation of any of the system's innovations with the VIX innovation is negligibly small.

Figure 5 depicts the findings. The ordering of these three venue types conforms to the pecking order hypothesis. In the minute immediately following the VIX shock, $SDarkMid$ shows the most negative reaction, falling from the steady state of 2.3% to 1.2%, a 50% reduction. $SDarkNMid$ also falls from the steady state of 7.1% to 5%, but its loss of market share is a smaller fraction (about 30%) of its steady state counterpart. By sharp contrast, $SLit$ increases from 77.5% to nearly 90%. In all three venue types, the effects on market shares last for about 5 minutes before dying out.

The intuitive sorting of the three venue types shown in Figure 5 is supported by formal econometric tests (a theory that motivates these tests is found in Section 6):

Null 1: The elasticities of $SDarkMid$ and $SDarkNMid$ to VIX shocks are the same;

Null 2: The elasticities of $SDarkNMid$ and $SLit$ to VIX shocks are the same.

The test results are reported in Table 4. Both nulls are rejected by the data. The tests show that after the upward VIX shock, the percentage change of $SDarkMid$ is more negative than that of $SDarkNMid$ at the 1% significance level over the next 5 minutes. The percentage change of $SDarkNMid$ is more negative than that of $SLit$ at the 1% significance level over the next 4 minutes.

Overall, the evidence from VIX shocks supports the pecking order hypothesis.

¹⁵ While the VARX model is written in terms of volumes, the calculation of market shares is straightforward from the estimated coefficients. Specifically, following the VIX shock at the steady state, the VARX model we already estimated spells out the future paths of all transaction volumes in the next 1, 2, 3, ... minutes. From these volumes we calculate the market shares and associated confidence bound by simulation. The steady state levels of the market shares are defined as the average of all the minute-day-stock observations. The steady-state market shares calculated this way are slightly different from those shown in Figure 3, in which the market share of each type is computed as the sum of volume in that venue type divided by the sum of TAQ volume during the entire sample period.

Figure 5: Pecking order following a shock in VIX

This figure plots the impulse response functions of the market shares of DarkMid, DarkNMid, and Lit following an 100% upward shock to VIX. The impulse responses are plotted for a 15-minute horizon, toward the end of which the market shares of the venues revert to their steady state levels. At each minute, the two-standard-deviation confidence bounds are constructed by simulation. The sequence of the venues is motivated by the specific form of the pecking order hypothesis (see Figure 2(b)).

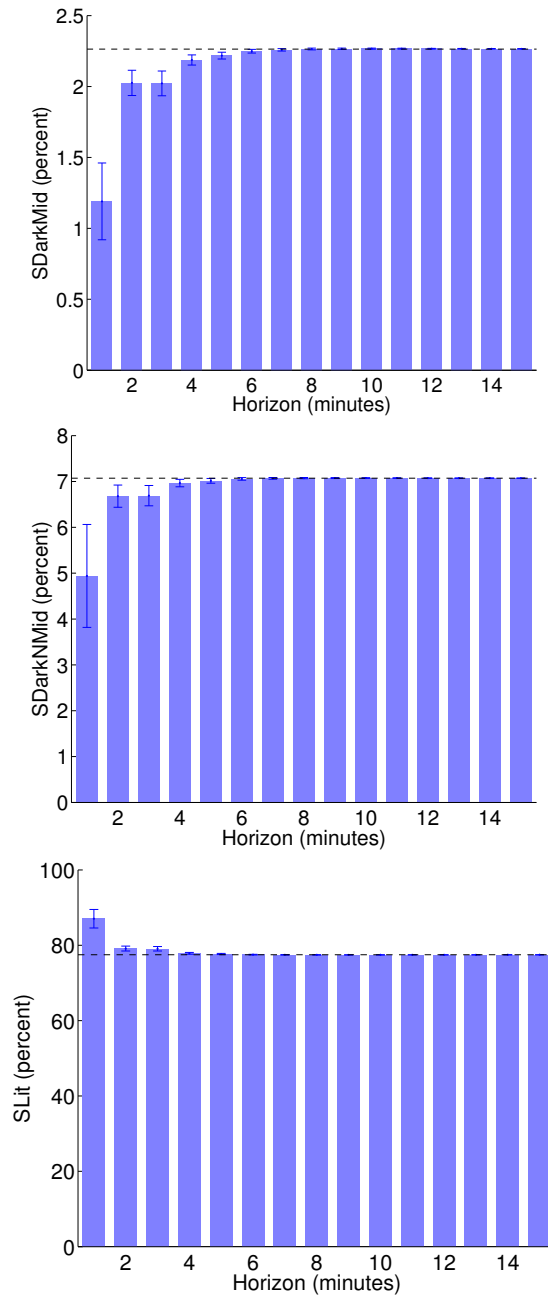


Table 4: Testing the pecking order of trade venues

This table shows the statistical significance of the difference in volume share elasticities (with respect to a VIX shock). Confidence bounds at 99% are constructed, based on 10,000 simulations, for the differences in the volume share changes for the 10 lags (10 minutes) after the VIX shock. If both the upper and the lower bounds are of the same sign, the difference of the elasticities is significantly different from zero. These significant numbers are shaded.

	1 min.	2 min.	3 min.	4 min.	5 min.	6 min.	7 min.	8 min.	9 min.	10 min.
$(\Delta SDMid/SDMid) - (\Delta SDNMid/SDNMid)$										
upper bound (99.5%)	-1.1	-0.7	-1.2	-0.2	-0.1	0.3	0.4	0.4	0.4	0.4
lower bound (0.5%)	-45.7	-13.2	-13.5	-5.2	-3.5	-1.5	-0.9	-0.4	-0.2	-0.1
$(\Delta SDNMid/SDNMid) - (\Delta SLit/SLit)$										
upper bound (99.5%)	-18.7	-2.8	-3.3	-0.2	0.2	0.6	0.7	0.7	0.6	0.5
lower bound (0.5%)	-79.8	-18.1	-17.2	-5.5	-3.2	-1.2	-0.6	-0.3	-0.1	-0.1

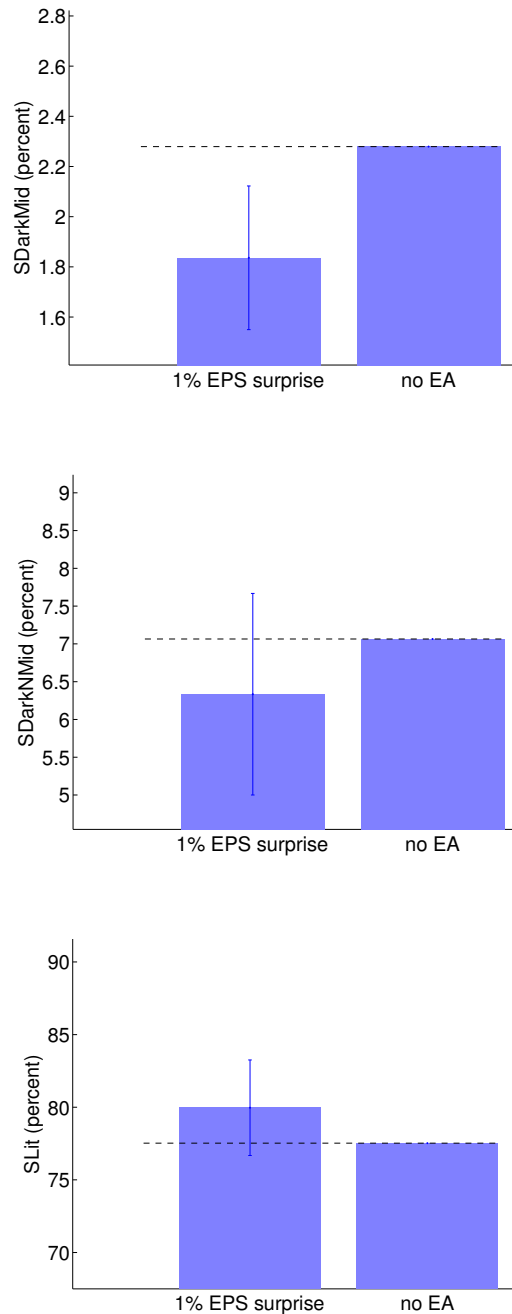
An earnings announcement shock

The second shock we exploit is the earnings surprises of individual firms. Starting with the estimated VARX model and the steady state, we impose a 1% earnings surprise and calculate the new steady-state market shares of DarkMid, DarkNMid, and Lit. For each venue type, this level is compared with the steady-state market share on days with no earnings announcements. Again, the pecking order hypothesis predicts that the percentage changes in the market shares should have the following ordering: DarkMid, DarkNMid, and Lit, from the most negative to the most positive.

Figure 6 plots the results, where the venue types are arranged the same way as in Figure 5. A 1% higher earnings surprise significantly reduces $SD_{DarkMid}$ by about 50%. While $SD_{DarkNMid}$ and $SLit$ do not show statistically significant changes, the point estimates point to the same direction as in Figure 5. The limited statistical significance here is likely due to the small number of firm-day observations that have earnings announcements (67 out of 117×21). Overall, the results for earnings announcement still support the pecking order hypothesis, although the statistical significance only shows at the DarkMid-DarkNMid step.

Figure 6: Pecking order following a shock in earnings surprises

This figure plots the steady-state values of venue shares for days with no earnings announcements (EA) and the corresponding venue shares following a 1% EPS surprise on an earnings day. The two-standard-deviation confidence bounds are constructed on the shocked market shares by simulation.



5.2 Reactions of DarkRetail, DarkPrintB, and DarkOther to VIX and earnings shocks

The specific form of pecking order hypothesis does not make predictions on the remaining three types of dark venues: DarkRetail, DarkPrintB, and DarkOther. This is because the trading mechanisms in these three types of venues often involve substantial amount of intermediation by broker-dealers as principal. First, institutional investors always have the options of trading directly with broker-dealers if they fail to execute everything on their own or if they do not have the execution capacity to start with. DarkPrintB represents one such option. Second, retail order flows are predominantly routed to large broker-dealers for execution, in the form of payment for order flow. Third, the mechanism of DarkOther is not fully transparent, and we do not have enough details to preclude the possibility of substantial dealer involvement in this type of dark venue. To be prudent, we have excluded all three venue types from the specific form of pecking order hypothesis.

That said, it is still interesting study the responses of *SDarkRetail*, *SDarkPrintB*, and *SDarkOther* to shocks in VIX and earnings surprises. Figure 7 shows the results. Two salient patterns stand out. On the one hand, market shares in all three types of venues decrease following a positive shock to VIX. This can reflect broker-dealers' reluctance in providing liquidity (at attractive enough prices) under a high aggregate volatility. This interpretation is consistent with the finding by Nagel (2012) that the return from liquidity provision in equity markets is highly predicted by VIX. On the other hand, following an earnings shock, the two institution-oriented dark venue types, DarkPrintB and DarkOther, significantly lose market shares, but the retail-oriented venue type, DarkRetail, does not. This difference may reflect the concern of adverse selection by broker-dealers when providing liquidity to institutional investors after firm-specific news, but this concern seems mild or nonexistence when broker-dealers trade against retail investors.

5.3 Venue pecking order and high-frequency trading

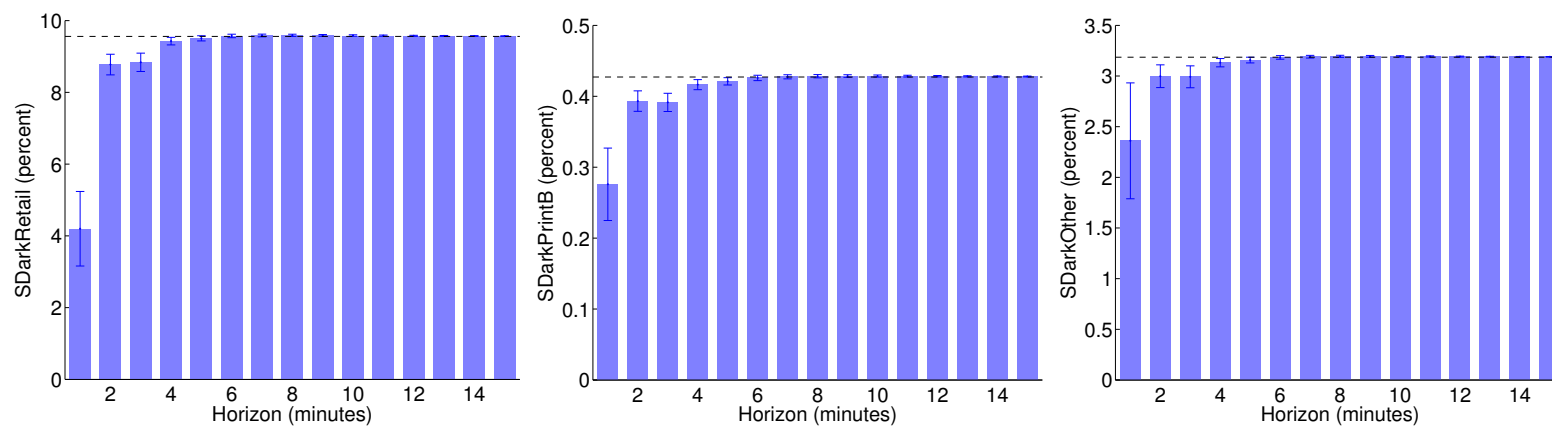
We close our empirical analysis by zooming in on high-frequency trading (HFT) and studying how it relates to the pecking order hypothesis. The prevalence of HFTs is commonly cited as one reason why investors are increasingly using dark venues. In the context of the pecking order hypothesis, investors' concern for interacting with HFTs is one form of cost for trading in lit venues.

If concerns regarding HFTs encourage investors to execute orders in the dark, a natural question arises: How does HFT activity affect investors' order flow down the pecking order? A definitive answer requires some exogenous shock that affects HFT activity but not volumes in dark venues. We do not (yet) have such exogenous variation. Nonetheless, we attempt to provide a partial answer by examining the responses of HFT participation in lit venues to VIX and earnings shocks. More

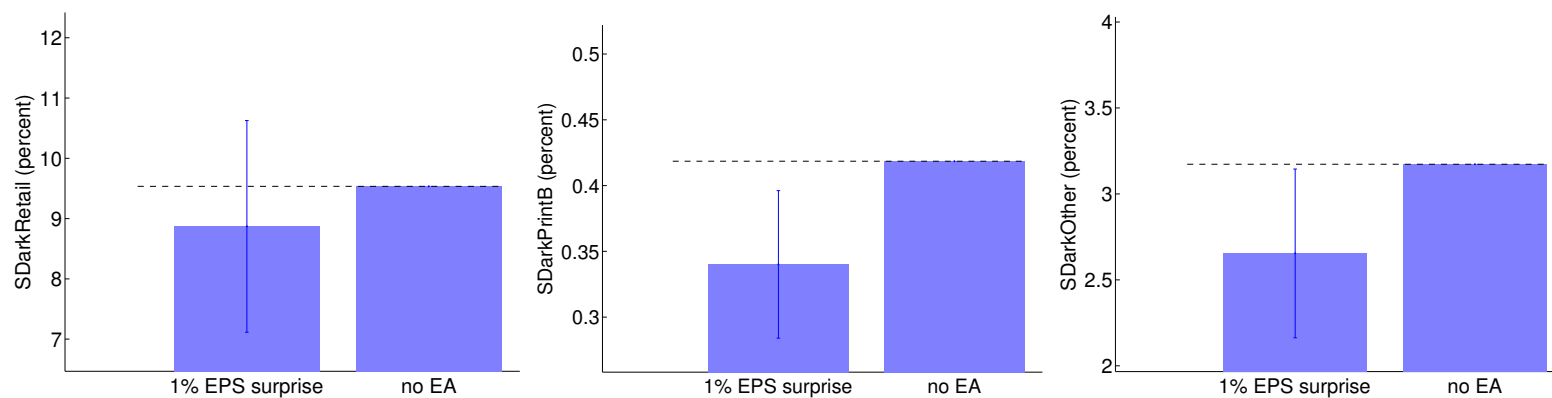
Figure 7: Pecking order of remaining venue types

This figure plots the responses of the market shares of the remaining three dark venue types—DarkRetail, DarkPrintB, and DarkOther—to a VIX shock (Panel (a)) and a earnings surprise shock (Panel (b)).

(a) Responses to a VIX shock



(b) Responses to an earnings surprise shock



specifically, if HFT participation declines after the VIX and earnings shocks, this pattern would be at least “consistent with” the conjecture that investors trade more in lit venues because they face a lower risk of interacting with HFTs. If the data show the opposite, then we can reject this conjecture. In this exercise we only focus on large-cap stocks (top tercile of our sample), as HFTs are generally most active for actively traded stocks.

Panel (a) of Figure 8 depicts how HFT participation responds to a 100% upward shock to VIX. In the Nasdaq market, HFTs participation in both trading and quoting increase by about 20% after a 100% VIX shock. *HFTinTopDepth* reverts to the steady states rather quickly, in about two minutes, but *HFTinVolume* reverts back to the steady state after about 10 minutes. To the best of our knowledge, the result that HFTs participation increases in VIX is new to the HFT literature.

Panel (b) of Figure 8 depicts the steady-state HFT participation measures, *HFTinTopDepth* and *HFTinVolume*, on days with no earnings announcement and after a one-percentage-point shock to *EpsSurprise*. Again, probably because of a small sample of firm-days with earnings announcements, none of these tests show statistical significance.

The evidence reveals that HFTs become more active in trading and quoting under a higher aggregate volatility. If investors fear HFT for the risk of “front-running” or “predatory trading,” this evidence suggests that they still prefer immediate transactions to avoiding HFT when volatility is high. In the context of the pecking order hypothesis, the evidence here is inconsistent with the conjecture that investors use lit venues more under a higher volatility *because* HFTs become less active in these situations.

6 Formalizing Pecking Order Hypothesis: A Candidate Model

This section proposes a simple model that characterizes investors’ choices among three venue types: DarkMid, DarkNMid, and Lit. As discussed above, relative to existing theories of dark pools, our simple model distinguishes different types of dark venues. The model and its analysis formalize the intuition that led to the pecking order hypothesis.

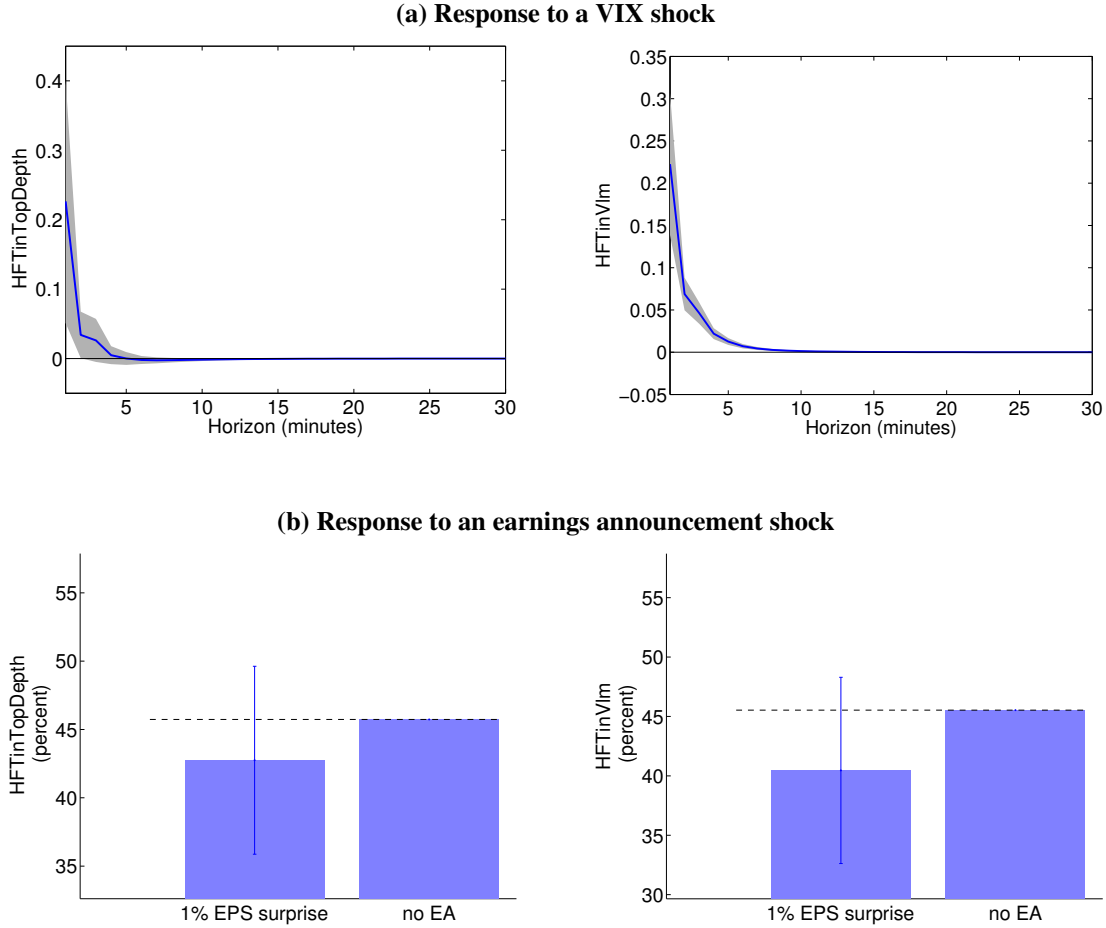
6.1 Model setup

Asset. There is one traded asset. Its fundamental (common) value is normalized to be zero. All players in this model have symmetric information about the asset and value it at zero. To formalize a pecking order hypothesis based on the urgency of trades, a symmetric-information setting suffices.

Venues. There are three trading venues: Lit, DarkNMid, and DarkMid.

Figure 8: HFT responses to shocks in VIX and earnings

This figure shows the responses of the two HFT activity measures after a shock to VIX (Panel (a)) and earnings (Panel (b)). In each panel, the left-hand graph shows the IRF of the participation fraction of HFT in the top depth, and the right-hand graph shows the IRF of the HFT trading volume fraction. The two-standard-deviation confidence bounds are constructed by simulation.



- Lit is populated by infinitely many competitive and infinitesimal liquidity providers who have the same marginal cost β (> 0) for taking on one unit of the asset per capita, either in long or short. The cost can be an operation cost or a margin cost. Together, these liquidity providers supply infinite depth at prices β and $-\beta$. This construct is similar to the “trading crowd” assumption in, for example, [Seppe \(1997\)](#) and [Parlour \(1998\)](#).
- DarkNMid is run by a single competitive liquidity provider who starts with inventory zero, but incurs an inventory cost of $-\eta x^2/2$ for taking a (long or short) position of x , where $\eta > 0$. The

liquidity provider posts an ask price schedule,

$$p^+ = \delta q^+,$$

where $q^+ > 0$ denotes the quantity to buy, and a bid price schedule,

$$p^- = -\delta q^-,$$

where $q^- > 0$ denotes the quantity to sell. The constant δ will be determined in equilibrium such that the liquidity provider breaks even in expectation. (We restrict to linear schedules for simplicity.) The average transaction price for sending in a buy (a sell) order of size q is $\delta q/2$ ($-\delta q/2$). The liquidity provider can be viewed as the broker-dealer who operates this dark venue.

- DarkMid crosses buy and sell orders at the midpoint price, i.e., at 0. If unbalanced, only the matched part of the order flow gets executed. For example, if there are buy orders for 100 units in total and sell orders for 40 units then only 40 units are matched and executed.

Investors. There are two large “representative” investors, a buyer and a seller. Having only two investors, rather than many, simplifies the exposition greatly at little cost of economics. The large buyer and seller receive independent and identically distributed liquidity shocks. The buyer starts with a short position $-Z^+ < 0$ and wishes to buy quantity Z^+ . The seller starts with a long position $Z^- > 0$ and wishes to sell Z^- . Each of Z^+ and Z^- is high (equal to h) with probability ϕ and low (equal to l) with probability $(1 - \phi)$, where $\phi \in (0, 1)$ and $0 \leq l < h$.

If a large investor is left with a non-zero inventory after the trading round (i.e., the difference between the desired trade amount and the actual traded amount), say x , he incurs a quadratic cost of $(\gamma/2)x^2$, with $\gamma > 0$. Here, γ could be an inventory cost, a proxy for risk-aversion, or the cost of a missed opportunity to trade on a short-lived private signal.

The parameter γ is the key parameter of the model. We interpret it as investors’ urgency to trade: The higher is γ , the larger is the cost of holding a non-zero net position, and hence investors are more eager to trade. After deriving the equilibrium, we focus on the comparative statics with respect to γ .^{16,17}

¹⁶ It can be argued that a higher demand for immediacy also changes other parameters, such as the Lit liquidity providers’ cost β and the DarkNMid liquidity provider’s inventory cost η . As is made clear shortly, the equilibrium strategies and outcomes are fully defined by the ratios γ/β and γ/η . We should interpret a higher γ as a higher γ/β and a higher γ/η .

¹⁷ One may also consider how an upward shock to urgency changes desired trade size (h, l) . We caution, however, that portfolio managers of buy-side firms are unlikely to make investment decisions at the minute-by-minute frequency. What

Timing. There is one trading round. Before trading, the large buyer and seller observe, privately, their shock sizes. They then decide simultaneously on the order sizes they send to the three venues. Venues execute trades simultaneously according to their specific trading protocols.

6.2 Equilibrium

We use “ x ” to denote the quantities sent to the trading venues. The superscripts “+” and “−” denote variables associated with the buyer and the seller, respectively. The subscripts “ L ”, “ N ”, and “ M ” denote Lit, DarkNMid, and DarkMid, respectively. We focus on a symmetric-strategy equilibrium, i.e., the buyer and the seller choose the same order flow sizes (but different signs): $x_i^+(Z) = x_i^-(Z)$ for all $(i, Z) \in \{L, N, M\} \times \{l, h\}$.

Optimal order sizes. Fix the seller’s strategy $x_i^-(Z^-)$ and consider the buyer’s choice of $x_i^+(Z^+)$ for all $(i, Z^+, Z^-) \in \{L, N, M\} \times \{l, h\} \times \{l, h\}$. Suppose that the buyer receives a shock of size z . Note that from the buyer’s perspective, the seller’s order size Z^- is a Bernoulli random variable. To emphasize this randomness, we use capital letters to denote X_i^- , for $i \in \{L, N, M\}$, and let $V_M^+ := \min\{x_M^+, X_M^-\}$. Then, the buyer’s expected profit is

$$\begin{aligned} \pi^+(z) = & \underbrace{-\beta \cdot x_L^+(z)}_{\text{price to pay in Lit}} \underbrace{-\frac{\delta}{2} x_N^+(z)^2}_{\substack{\text{price to pay} \\ \text{in DarkNMid}}} \underbrace{-\mathbb{E}[0 \cdot V_M^+(z)]}_{\text{price to pay in DarkMid}} \\ & + \underbrace{\mathbb{E}\left[0 \cdot (z - x_L^+(z) - x_N^+(z) - V_M^+(z))\right]}_{\text{liquidation value of remaining position}} \underbrace{-\frac{\gamma}{2} \mathbb{E}(z - x_L^+(z) - x_N^+(z) - V_M^+(z))^2}_{\text{quadratic cost of urgency to trade}}, \end{aligned}$$

which can be simplified to

$$\pi^+(z) = -\beta x_L^+(z) - \frac{\delta}{2} x_N^+(z)^2 - \frac{\gamma}{2} \mathbb{E}(z - x_L^+(z) - x_N^+(z) - V_M^+(z))^2. \quad (2)$$

The buyer thus chooses six parameters to maximize his expected profit (2): $x_L^+(l)$, $x_N^+(l)$, $x_M^+(l)$, $x_L^+(h)$, $x_N^+(h)$, and $x_M^+(h)$.

Because we look for a symmetric equilibrium, from this point on we suppress the superscript $\pm$ unless we need to explicitly distinguish a buyer from a seller.

is likely to matter more at this frequency is the response of trading desks or executing brokers to urgency shocks, holding fixed the size of orders that they get from portfolio managers. For this reason, we believe the comparative statics with respect to h and l are less relevant than that with respect to γ .

Proposition 1 (Equilibrium order flows). *If*

$$h - l \leq \Delta \equiv \beta \left(\frac{1}{(1 - \phi)\gamma} + \frac{1}{\delta} \right), \quad (3)$$

then there exists an equilibrium with the following strategies:

$$x_M(l) = l; \quad x_M(h) = l + \frac{\delta}{\delta + (1 - \phi)\gamma} (h - l), \quad (4)$$

$$x_N(l) = 0; \quad x_N(h) = \frac{(1 - \phi)\gamma}{\delta + (1 - \phi)\gamma} (h - l), \quad (5)$$

$$x_L(l) = 0; \quad x_L(h) = 0. \quad (6)$$

If $h - l > \Delta$, then there exists an equilibrium with the following strategies:

$$x_M(l) = l; \quad x_M(h) = l + \frac{\beta}{(1 - \phi)\gamma}, \quad (7)$$

$$x_N(l) = 0; \quad x_N(h) = \frac{\beta}{\delta}, \quad (8)$$

$$x_L(l) = 0; \quad x_L(h) = h - l - \Delta. \quad (9)$$

In both cases, the DarkNMid liquidity provider sets the slope of price schedules $\delta = (1 - \phi)\eta$.

The intuition of this equilibrium is as follows. Investors start with DarkMid because it has the best transaction price (saving half the spread). Naturally, a small investor sends all the order to DarkMid because his order size of l can be matched for sure. Because a large investor's excess order size $h - l$ cannot be executed in DarkMid at zero cost, he must strike the balance among nonexecution risk in DarkMid, the price impact in DarkNMid, and the spread in Lit.

The following heuristic argument illustrates this tradeoff. A large buyer's expected marginal cost of sending an additional infinitesimal quantity to DarkMid is $(1 - \phi)(x_M(h) - l)\gamma$, where we use the fact that the order size $x_M(h) - l$ is not matched if the seller is small, and that the marginal inventory cost of holding unexecuted quantity q is γq . The marginal cost of sending an additional infinitesimal quantity to DarkNMid is $\delta x_N(h)$. In equilibrium, these two marginal costs must be equal, for otherwise the large buyer would want to take a marginal unit from one dark venue to the other. Thus, we have

$$(1 - \phi)(x_M(h) - l)\gamma = \delta x_N(h). \quad (10)$$

The above equation says that $x_M(h) - l$ and $x_N(h)$ must increase simultaneously in a fixed proportion as h increases.

If $h - l \leq \Delta$, we can show that the marginal cost of using DarkNMid $\delta x_N(h)$ is no larger than the

Lit spread β . Thus, Lit is not used. The equilibrium order sizes can be solved by observing that in equilibrium $x_M(h) = h - x_N(h)$, that is, the desired quantity in DarkMid is equal to the residual. If $h - l > \Delta$, equation (10) hits a corner at

$$(1 - \phi)(x_M(h) - l)\gamma = \delta x_N(h) = \beta, \quad (11)$$

from which we can solve $x_M(h)$ and $x_N(h)$. Lit order size is given by $x_L(h) = h - x_M(h) - x_N(h)$.

Note that the equilibrium of Proposition 1 is not unique. There exists a trivial equilibrium in which no one uses DarkMid.

Interestingly, the DarkNMid liquidity provider is more aggressive (i.e. sets prices that respond less to additional quantity) if large orders are more likely. This is because he makes more profits from large investors, even though it also increases his inventory cost.

6.3 Urgency elasticity of venue market shares

To formalize the pecking order hypothesis, we are interested in how the market share of each of the three venues responds to a change in investor urgency. From this point on we focus on the second case of Proposition 1 because it implies a positive market share for all venues, as in reality.

To do so, we first compute the equilibrium expected market shares and then study their elasticities with respect to the urgency parameter γ . Using market shares, rather than raw volume, is consistent with empirical studies of dark pools and fragmentation (see, for example, O'Hara and Ye 2011 and Buti, Rindi, and Werner 2011b) and our subsequent empirical tests.

The expected volumes in the three venues and the total volume are given by

$$\bar{v}_M = \phi^2(2x_M(h)) + (1 - \phi)^2(2x_M(l)) + 2\phi(1 - \phi)(2x_M(l)) = 2l + \frac{2\phi^2\beta}{(1 - \phi)\gamma}, \quad (12)$$

$$\bar{v}_N = \phi^2(2x_N(h)) + (1 - \phi)^2(2x_N(l)) + 2\phi(1 - \phi)(x_N(h) + x_N(l)) = 2\frac{\phi\beta}{\delta}, \quad (13)$$

$$\bar{v}_L = \phi^2(2x_L(h)) + 2\phi(1 - \phi)x_L(h) = 2\phi\left(h - l - \frac{\beta}{(1 - \phi)\gamma} - \frac{\beta}{\delta}\right), \quad (14)$$

$$\bar{v} = \bar{v}_M + \bar{v}_N + \bar{v}_L = 2l + 2\phi(h - l) - \frac{2\phi\beta}{\gamma}. \quad (15)$$

Note that in the above calculation we double-count volume in DarkMid, but our results are not affected if DarkMid volume is single-counted.

The volume shares of different venues are defined as,

$$s_i := \frac{\bar{v}_i}{\bar{v}}, \quad \text{for } i \in \{M, N, L\}.$$

Signing partial derivatives of volume shares with respect to γ yields the following proposition.

Proposition 2 (Venue share and urgency). *As investor urgency increases, Lit volume share increases and the dark volume share decreases. Furthermore, DarkMid is more sensitive to urgency than DarkNMid:*

$$\frac{\partial s_M/s_M}{\partial \gamma/\gamma} < \frac{\partial s_N/s_N}{\partial \gamma/\gamma} < 0 < \frac{\partial s_L/s_L}{\partial \gamma/\gamma}.$$

Proposition 2 formalizes this paper’s pecking order hypothesis, shown in Panel (b) of Figure 2. The empirical tests of Section 5.1 correspond closely to this proposition. We believe that an increase in the urgency parameter γ is a reasonable proxy for a positive shock to VIX or an earnings surprise. Intuitively, a higher VIX shock implies a higher market-wide volatility or risk aversion, which tends to make it more costly to hold undesirable, yet unexecuted, positions. Investors and broker-dealers would have a higher urgency to trade. Similarly, the larger is the magnitude of an earnings surprise (positive or negative), the more losses are accumulated to investors who “made the wrong bet,” and the more urgently they want to close out these losing positions (due to margin constraint).

The candidate model of this section is meant to be simple but yet able to demonstrate that the pecking order hypothesis can emerge endogenously as an equilibrium outcome. There are a couple of potential extensions to enrich the model. One direction is to directly model the shocks (h, l) of investors from first principle. Another interesting but technically difficult direction is to consider a fully dynamic model in which investors can route orders across the three venue types. We leave these potential extensions for future research.

7 Conclusion

In this paper, we propose and test a pecking order hypothesis for the dynamic fragmentation of U.S. equity markets. The hypothesis posits that investors disperse their orders into a series of venue types, sorted along a pecking order. The position of venue types on the pecking order depends on the tradeoff between cost (price impact) and immediacy (execution certainty). On the top of the pecking order are low-cost, low-immediacy venues such as midpoint dark pools, whereas in the bottom are high-cost, high-immediacy venues such as lit exchanges; in the middle of the pecking order are non-midpoint dark pools. A positive shock to investors’ urgency to trade tilts their order flows from the top of the pecking to the bottom; therefore, the elasticities of venue market shares to urgency shocks are progressively less negative and more positive further down the pecking order. We propose a simple theoretical model that formalizes this intuition.

We test the pecking order hypothesis using a unique dataset that identifies the minute-by-minute trading volumes in five different types of dark venues in the U.S. equity markets. A panel VARX model characterizes the dynamic interrelation between dark trading volumes, HFT participation in lit venues, and market conditions, as well as two exogenous shocks to investors' demand for immediacy: VIX and earnings surprises.

Consistent with the pecking order hypothesis, we find that an upward shock to VIX substantially reduces the market share of midpoint dark pools, moderately reduces the shares of non-midpoint dark pools, but increases the share of lit venues. After shocks to earnings surprises, the share of midpoint pools also declines significantly. Taken together, our results offer a pecking order representation for the dynamic fragmentation of U.S. equity markets.

Appendix

A Details on the implementation of the panel VARX model

In this appendix we discuss the details of the panel VARX model.

The estimation is implemented via OLS by stacking the observations associated with different stocks into a single vector. The stock fixed effect is accounted for by adding dummy variables to the set of regressors. Lags of the variables are only constructed intraday.

The optimal numbers of lags p and r are chosen according to Bayesian Information Criterion (BIC). Specifically, for each of the 117 stocks, the VARX model is estimated for all pairs of $(p, r) \in \{1, 2, \dots, 10\} \times \{1, 2, \dots, 5\}$. Then the best (according to BIC) model is chosen as (p_j, r_j) for stock j . That is, we confine the search of the optimal lags within 10 and 5 lags, respectively, for endogenous and exogenous variables. The above procedure generates 117 pairs of optimal lags of $\{p_j\}_{j=1}^{117}$ and $\{r_j\}_{j=1}^{117}$. There are 16 p_j that are found to be 1, 93 to be 2, and the other 8 to be 3. All r_j are 1. We hence choose $p = 2$ and $r = 1$ for parsimony.

The standard errors for panel data estimators should account for potential correlation through time and across stocks. One standard way to account for these issues is to do “double-clustering” (Petersen, 2009). The laborious (but most flexible) way of implementing such clustering is by calculating

$$\text{cov}(\hat{\beta}_i, \hat{\beta}_j) = (X'X)^{-1} V (X'X)^{-1} \text{ with } v_{ij} = \sum_{kt,ls} x_{ikt} \hat{\varepsilon}_{ikt} \hat{\varepsilon}_{jls} x_{jls} \times 1_A(ktls), \quad (16)$$

where $i, j \in \{1, \dots, N\}$ where N is the number of regressors, $k, l \in \{1, \dots, J\}$ where J is the number of stocks, and $s, t \in \{1, \dots, T\}$ where T is the number of time periods. $1_A(ktls)$ is the indicator function where the subset A of the index value space identifies which auto- or cross-correlations a researcher worries about. If error terms are independent and identically distributed, then the indicator function equals one if $k = l$ and $t = s$. The subset A for an indicator function in a standard double-clustering is such that:

$$1_A(ktls) = \begin{cases} 1 & \text{if } k = l \text{ or } s = t, \\ 0 & \text{otherwise.} \end{cases} \quad (17)$$

A researcher can easily be more conservative and also account for non-zero cross-autocorrelations by also including changing the $s = t$ condition by, say, $|s - t| \leq 5$.

The cumulative impulse response function is most easily calculated by stacking the estimated Φ matrices, as any VAR can always be written as a first-order VAR. Consider, for example, a VAR with two lags. This VAR can be written as

$$\begin{bmatrix} y_t \\ y_{t-1} \end{bmatrix} = \begin{bmatrix} \Phi_1 & \Phi_2 \\ I & 0 \end{bmatrix}' \begin{bmatrix} y_{t-1} \\ y_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix}. \quad (18)$$

The t -period cumulative impulse response of the j^{th} variable to a unit impulse in the i^{th} variable is the j^{th} element of the vector

$$\begin{bmatrix} \Phi_1 & \Phi_2 \\ I & 0 \end{bmatrix}' e_k, \quad (19)$$

where I is the identity matrix and e_k is the unit vector where the k^{th} element is one and all other elements are zeros.

Confidence intervals on the impulse response function (IRF) are obtained through simulation. The IRF is a non-linear transformations of the VARX coefficient estimates denoted by $\hat{\theta}$. Each simulation involves a draw from the multivariate normal distribution $N(\hat{\theta}, \Sigma_{\hat{\theta}})$, where $\Sigma_{\hat{\theta}}$ is the estimated double-clustered covariance matrix of the coefficients. Note that this distribution is asymptotically true given the assumption that the VARX model is correctly specified with normal residual terms. We perform 10,000 independent draws of the coefficients $\hat{\theta}$ and for each draw compute the IRFs at all lags. Thus, we obtain, for each IRF, an i.i.d. sample of size 10,000. The confidence bounds are then chosen at the 2.5 and the 97.5 (or 0.5 and 99.5) percentiles of the simulated IRFs. The significance levels shown in table 3 are based on whether the estimates exceed the confidence bounds found above.

B Transformation between logarithms and levels of market share variables

In the VARX model implementation, the variables are log-transformed (except *EpsSurprise*). Log-transformation has several advantages. For example, the strictly positive variables (e.g. volume, spread, depth, etc.) are converted to a possibly negative support; the concavity in logarithm discourages the abnormal effects of outliers; the estimation coefficients can be readily interpreted as elasticity. The key variables of our focus are the (logged) trading volumes in the five dark venues and the lit venue, denoted by $\log v_1, \dots, \log v_5$, and $\log v_6$, where the first five are for the five dark venues and the last v_6 is for the trading volume in the lit. (Each of these variables has stock-day-minute granularity.) For the pecking order hypothesis, it is however useful to think in terms of market shares, defined as

$$s_j = \frac{v_j}{\sum_j v_j}$$

for $j \in \{1, \dots, 6\}$.

The purpose of this appendix is to derive the closed-form, exact transformation formula from a shock in trading volume in one venue to the response of all market shares. Specifically, given a shock of $\Delta \log v_i$, we want to know the immediate response Δs_j , for *all* $j \in \{1, \dots, 6\}$. Reverse directions from Δs_i to $\Delta \log v_j$ will also be dealt with. These formulas are used in generating the impulse responses in testing the pecking order hypothesis.

In the derivation below, we shall use the following additional notations. Let v be the total volume: $v = \sum_j v_j$. We shall use a superscript of “+” to denote the variables after a shock; for example, $\log v_j^+ = \log v_j + \Delta \log v_j$. Similarly, while s_j denotes the market share of venue j , s_j^+ denotes the market share after the shock. In the IRF exercise, the pre-shock values will be chosen as the stock-day-minute average across all raw sample observations. Consider the following cases.

From $\Delta \log v_i$ to Δs_j . By construction, $\log v_i^+ = \log v_i + \Delta \log v_i$. Taking the exponential on both sides gives the *level* of the post-shock trading volume: $v_i^+ = v_i \exp \Delta \log v_i$. The post-shock market share by construction is

$$s_i^+ = \frac{v_i^+}{v^+} = \frac{v_i^+}{v + (v_i^+ - v_i)}.$$

Substituting with the expression of v_i^+ and then subtracting $s_i = v_i/v$ yields

$$\Delta s_i = s_i^+ - s_i = \dots = s_i \cdot (e^{\Delta \log v_i - \Delta \log v} - 1) \quad (20)$$

where $\Delta \log v = \log v^+ - \log v = \log(\sum_{j \neq i} v_j + v_i^+) - \log v$. The above formula actually applies to both the venue i whose volume is shocked and any other venue $j \neq i$ whose volume is not shocked. The only difference is, as can be seen after substituting the index i with a different j , that $\Delta \log v_j = 0$ for $j \neq i$. Finally, we can immediately derive the dark volume share change as the complement of the change in the lit share: $\sum_{j \leq 5} \Delta s_j = -\Delta s_6$, simply because the identity of $s_6 = 1 - \sum_{j \leq 5} s_j$.

From $\Delta \log v$ to Δs_i , assuming proportionally scaling across all venues. Now we shock the total volume such that $\log v^+ = \log v + \Delta \log v$ and make the assumption that the increase in volume is proportionally scaled across all venues. That is, for each venue i , $v_i^+ - v_i = s_i \cdot (v^+ - v)$, or

$$v_i^+ = v_i + s_i \Delta v = s_i v + s_i \Delta v = s_i \cdot (v + \Delta v) = s_i v^+.$$

Taking logarithm on both sides gives $\log v_i^+ = \log s_i + \log v^+$. Substitute $\log s_i$ with $\log s_i = \log(v_i/v) = \log v_i - \log v$ and then

$$\log v_i^+ = \log v_i - \log v + \log v^+ \implies \Delta \log v_i = \Delta \log v. \quad (21)$$

Applying equation (21) to (20) immediately gives $\Delta s_i = 0$. Clearly, this holds for all $i \in \{1, \dots, 6\}$.

From Δs_i to $\Delta \log v_j$ by shocking $\log v_i$ and proportionally offsetting in other venues, without changing total volume v . Finally we do the reverse. Suppose we shock s_i by Δs_i . Such a change in market share must be driven by some change(s) in trading volume(s). Here a particular change is considered: Let v_i change in the same direction as s_i but all other $v_{j \neq i}$ move in the other direction so that the total volume does not change, i.e. $v = v^+$. We want to know, given the size of Δs_i , what are the sizes of $\Delta \log v_j$ for all $j \in \{1, \dots, 6\}$.

First, consider $j = i$. By construction, $\Delta s_i = v_i^+/v^+ - s_i$. Because the total volume is assumed to be unchanged, we have $v_i^+ = (s_i + \Delta s_i)v$. This enables the second equality below:

$$\begin{aligned} \Delta \log v_i &= \log v_i^+ - \log v_i \\ &= \log(s_i + \Delta s_i) + \log v - \log v_i = \log(s_i + \Delta s_i) + \log \frac{v}{v_i} = \log(s_i + \Delta s_i) - \log s_i = \log \left(1 + \frac{\Delta s_i}{s_i} \right). \end{aligned} \quad (22)$$

Consider next $j \neq i$. To offset Δv_i , summing over all $j \neq i$ gives $\sum_{j \neq i} \Delta v_j = -\Delta v_i$. Because the changes are proportional according to s_j , we have

$$v_j^+ = v_j - \frac{s_j}{\sum_{h \neq i} s_h} \Delta v_i = s_j \cdot \left(v - \frac{\Delta v_i}{\sum_{h \neq i} s_h} \right).$$

Take logarithm on both sides and expand $\log s_j = \log v_j - \log v$ to get

$$\log v_j^+ - \log v_j = \Delta \log v_j = -\log v + \log \left(v - \frac{\Delta v_i}{\sum_{h \neq i} s_h} \right) = \log \left(1 - \frac{\Delta v_i/v}{\sum_{h \neq i} s_h} \right) = \log \left(1 - \frac{\Delta s_i}{\sum_{h \neq i} s_h} \right), \quad (23)$$

where the last equality follows because the total volume is assumed to be unchanged: $\Delta v_i/v = v_i^+/v - v_i/v = v_i^+/v - v_i/v = s_i^+ - s_i = \Delta s_i$.

C Proofs

C.1 Proof of Proposition 1

We directly verify the strategies. Without loss of generality, we consider the strategy of a buyer, who might be small ($z^+ = l$) or large ($z^+ = h$).

The first case, $h - l \leq \Delta$

Clearly, under these strategies, a small buyer executes all his orders at the price of zero. He has no incentive to deviate to any other venue because the transaction price in Lit or DarkNMid would be positive.

The large buyer's cost is

$$-\pi^+(h) = \beta x_L(h) + \frac{\delta}{2} x_N(h)^2 + \frac{\gamma}{2} \mathbb{E}[(h - x_L(h) - x_N(h) - V_M(h))^2]. \quad (24)$$

Verify $x_N(h)$. The first order condition with respect to $x_N(h)$ yields

$$0 = \delta x_N(h) - \gamma(h - x_L(h) - x_N(h) - \mathbb{E}[V_M(h)]), \quad (25)$$

which is easily verified by substituting the conjectured strategies.

Verify $x_L(h) = 0$. The marginal cost of buying the last unit in DarkNMid is

$$\delta x_N(h) = (h - l) \frac{\delta(1 - \phi)\gamma}{\delta + (1 - \phi)\gamma} < \beta. \quad (26)$$

So the large buyer has no incentive to deviate to Lit.

Verify $x_M(h)$. Suppose the large buyer chooses some other order size, y , in DarkMid. Note that $x_M(h) = h - x_N(h)$. Clearly, the large buyer has no incentive to deviate to $y > x_M(h)$; this deviation does not change this payoff regardless of the size of the seller. Deviating to $y \in [l, x_M(h))$ is also suboptimal: if the seller is large, this deviation reduces matched volume; if the seller is small, this deviation does not change the payoff. Lastly, the buyer does not want to deviate to $y < l$. If he did, we know that $V_M = y$. Taking the derivative of $-\pi^+$ with respect to y gives

$$-\gamma(h - x_L(h) - x_N(h) - y) = -\gamma(x_M(h) - y) < 0, \quad (27)$$

which means that the buyer should increase order size to l . But choosing order size of l is dominated by choosing order size of $x_M(h)$.

The second case, $h - l > \Delta$

The optimality of the small investor's strategy is verified as before.

In this case, the conjecture is that $x_L(h) > 0$. This means the first order condition of $-\pi^+$ with respect to $x_L(h)$ and $x_N(h)$ must both hold with equality. That is,

$$\delta x_N(h) - \gamma(h - x_L(h) - x_N(h) - \mathbb{E}[V_M(h)]) = 0, \quad (28)$$

$$\beta - \gamma(h - x_L(h) - x_N(h) - \mathbb{E}[V_M(h)]) = 0. \quad (29)$$

It is easy to verify that these two first order conditions hold at the conjectured strategies. To verify the optimality of $x_M(h)$, we note $x_M(h) = h - x_L(h) - x_N(h)$ and use the same argument as before.

Calculating equilibrium δ

The last step is to calculate the equilibrium δ , such that the DarkNMid liquidity provider makes zero expected profit, net of inventory costs. Thus, the expected profit is equal to the expected inventory cost, i.e.,

$$\phi^2 \delta x_N(h)^2 + (1 - \phi)^2 \delta x_N(l)^2 + 2\phi(1 - \phi) \left(\frac{\delta}{2} x_N(h)^2 + \frac{\delta}{2} x_N(l)^2 \right) = 2\phi(1 - \phi) \cdot \frac{\eta}{2} (x_N(h) - x_N(l))^2. \quad (30)$$

From this we solve

$$\delta = \eta \frac{\phi(1 - \phi)(x_N(h) - x_N(l))^2}{\phi x_N(h)^2 + (1 - \phi)x_N(l)^2} = \eta(1 - \phi), \quad (31)$$

where the last equality follows from the equilibrium strategy $x_N(l) = 0$.

C.2 Proof of Proposition 2

We first calculate the partial derivatives of expected volumes with respect to γ . For each of the venue:

$$\begin{aligned} \frac{\partial \bar{v}_M}{\partial \gamma} &= -\frac{2\phi^2\beta}{(1 - \phi)\gamma^2} < 0 \\ \frac{\partial \bar{v}_N}{\partial \gamma} &= 0 \\ \frac{\partial \bar{v}_L}{\partial \gamma} &= \frac{2\phi\beta}{(1 - \phi)\gamma^2} > 0 \end{aligned}$$

and for the total volume:

$$\frac{\partial \bar{v}}{\partial \gamma} = \frac{2\phi\beta}{\gamma^2} > 0.$$

Recall the definition of the market shares: $s_i := \bar{v}_i/\bar{v}$ for $i \in \{L, N, M\}$. Following the above calculation, we have 1) $\partial s_N/\partial \gamma < 0$ and 2) $\partial s_M/\partial \gamma < 0$, because while the denominator $\bar{v}$ increases in γ , the denominators (weakly) decrease. Then, by the accounting identity of $s_L + s_N + s_M = 1$ (hence $\partial(s_L + s_N + s_M)/\partial \gamma = 0$), we have 3) $\partial s_L/\partial \gamma > 0$. That is, both dark venue shares reduces and the lit share grows, as urgency γ increases.

It remains to rank the volume share sensitivity to urgency. Define the sensitivity as the elasticity of market share to urgency: $(\partial s_i/\partial \gamma)/(s_i/\gamma)$. This definition matches our empirical approach. Comparing DarkNMid and DarkMid gives

$$\begin{aligned} \frac{\partial s_N}{\partial \gamma} \frac{\gamma}{s_N} - \frac{\partial s_M}{\partial \gamma} \frac{\gamma}{s_M} &= \frac{\gamma}{\bar{v}^2} \left(\overbrace{\frac{\partial \bar{v}_N}{\partial \gamma} \frac{\bar{v}}{s_N} - \frac{\partial \bar{v}}{\partial \gamma} \frac{\bar{v}_N}{s_N}}^{=0} \right) - \frac{\gamma}{\bar{v}^2} \left(\frac{\partial s_M}{\partial \gamma} \frac{\bar{v}}{s_M} - \frac{\partial \bar{v}}{\partial \gamma} \frac{\bar{v}_M}{s_M} \right) \\ &= \frac{\gamma}{\bar{v}^2} \left[\underbrace{\frac{\partial \bar{v}}{\partial \gamma} \left(\frac{\bar{v}_M}{s_M} - \frac{\bar{v}_N}{s_N} \right)}_{=\bar{v}-\bar{v}=0} - \frac{\partial s_M}{\partial \gamma} \frac{\bar{v}}{s_M} \right] > 0. \end{aligned}$$

Hence, the sensitivities of volume shares to urgency can be ranked as stated in proposition 2.

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