

Zeros and the Gains from Openness

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Abstract

Despite the enormous growth in global trade and investment, most countries still do not trade or invest with one other. I document that 80% of bilateral trade and FDI relationships are zeros. I construct a model that rationalizes these zeros and allows new bilateral relationships to form (aggregate zero-to-one transitions) following policy reform. Firms incur two types of costs when operating internationally: (1) fixed costs preventing them from operating - identified using variation in zeros, and (2) iceberg costs reducing the amount they sell when they operate - identified using variation in positive flows. The global bilateral fixed costs estimated from the zeros are novel to the literature, which has focused on country- or sector-specific fixed costs. I develop an algorithm that enables me to (1) compute an approximate equilibrium where exact equilibria do not exist, and (2) reduce the computational complexity to one that grows linearly, as opposed to exponentially, in the number of countries, mitigating the curse of dimensionality. Welfare gains in models with no aggregate entry and exit account for only 41% of the average gains obtained in the model where zeros matter, signifying that this aggregate extensive margin matters for understanding what countries gain from openness.

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1 Introduction

Globalization has been one of the most important developments of the past half century. Global merchandise trade as a fraction of total output has risen more than twofold in the last four decades (World Trade Organization database) while global FDI as a fraction of total output has risen more than fivefold in the corresponding period (United Nations Conference on Trade and Development STAT). Yet for all this growth the vast majority of bilateral trade and FDI relationships remain full of zeros, i.e. there are no documented flows from one country to the other. Using data from the recently commissioned Coordinated Investment Survey Database and the Direction of Trade Statistics Database, I document that in the period 2009-2011, more than 80% of bilateral trade-FDI relationships in a global sample of over 100 countries contain at least one zero. This raises two key questions: Why do some countries trade and do FDI with each other, but not with others? And if globalization creates winners and losers because countries only trade or conduct FDI selectively, what do countries really stand to gain from openness?

This paper answers these questions quantitatively by estimating heterogeneous bilateral fixed and variable iceberg costs using variation in zeros and positive flows around the world, and computing the welfare impact of trade and financial liberalization within a quantitative general equilibrium model. Both types of costs prove important in accounting for the heterogeneous flows observed in the data: the iceberg costs account for the variation in positive flows where countries are already trading or investing in each other, while fixed costs account for the variation in zeros as country pairs with prohibitive fixed startup costs will not transact with one another. My estimates for the iceberg costs are in line with those obtained from most gravity regressions, and consistent with that literature, I find that these variable costs are quite successful in generating the variation in positive flows in the data. The main contribution of this paper in terms of estimation is that it produces novel estimates for bilateral fixed costs around the world. This is made possible by using a source of variation the literature has largely ignored until now: zeros at the country level. I find that fixed costs for developing countries are higher on average, consistent with the fact that there are more zeros in the subsample of countries in the developing world. There is strong evidence of zeros turning positive following reform,¹ as countries not only trade or invest with existing partners but also start transacting internationally with countries it did not do business with before reform. This means that when one measures the welfare benefit of trade and financial

¹For example, between 1990 and 2005, over 60 countries started exporting to Kenya. Similarly, 45 countries that exported to Argentina in 2005 were not exporting to Argentina in 1990. There were still, however, more than 40 countries not exporting to Argentina in 2005. Additional evidence can be made available upon request.

liberalization using a model where zeros are ignored, one gets an imprecise measure of the true gains from openness, particularly for countries with lots of zeros. Consistent with this intuition, I find that relative to a model where zeros do not matter, my model generates welfare gains that are up to 41% higher for the average country, with the discrepancy larger for countries in the developing world.

The mechanism through which zeros affect welfare is through the variation they generate in the fixed costs that firms have to face when operating internationally. Zeros create variation in fixed costs because if two countries are trade or MP partners such that there is no zero, the fixed cost must be sufficiently low for their firms to cross their borders; conversely, if two countries do no trade or MP, the fixed cost has to be sufficiently high to prevent foreign firms from operating domestically. Even within the set of zero bilateral ties, if a country A has more productive firms than another country B *ceteris paribus*, then the fixed cost for A to enter B must be higher than the fixed cost for B firms to enter A . Given this inherent asymmetry in bilateral fixed costs, the natural question to ask is whether there is a common component to these fixed costs - a global fixed cost. Here again, zeros contain information on the magnitude of this cost: the global fixed cost cannot be too large as otherwise no countries will want to trade or produce internationally (an autarkic equilibrium is clearly at odds with the data).

To quantify and interpret the size of fixed costs obtained from the zeros, the main metric I use is the fixed cost as a fraction of profits. This unitless measure is appropriate as it is what firms consider when they make their entry decisions: is the fixed cost low enough to justify operating overseas. I find that the global fixed cost can be as high as 4.65% of profit. Note that this is not a point estimate but an upper bound given by the restriction that in the model countries trade and produce internationally as they do in the data. For this case where the global fixed cost is highest, the average trade fixed cost is 19.51% of profit, while the average MP fixed cost is 88.59% of profit. Zeros do not provide a lower bound on the global fixed cost, which can be infinitesimally small. In the case where the global fixed cost is negligible and fixed costs are minimized subject to the restriction that in the model countries do not trade or produce internationally when they do not do so in the data, international fixed costs become purely country-pair specific, and are zero for the pairs that do trade or do MP. At this other end of the spectrum, I find that the average trade fixed cost is 0.80% of profit, while the average MP fixed cost is 85.93% of profit. Comparing these numbers to those obtained earlier illustrates just how many more zeros exist in MP compared to trade, and how big the MP fixed costs have to be to induce firms not to produce overseas. The case with fixed costs minimized is the baseline parameterization as this maximizes the welfare impact of greater openness and makes clear the bilateral asymmetry embedded in the zeros (as there

is no common components to the fixed costs). Asymmetry manifests itself in the average fixed cost by destination being vastly different from the average fixed cost by source country. I find that the ratio between the average fixed cost by destination and the average fixed cost by source is negatively correlated with GDP per worker (and similarly negatively correlated with GDP and TFP). This systematic asymmetry is intuitive: it means that while the foreign fixed costs Greek producers face is similar to the average fixed costs foreign producers face entering Greece, foreigners find it much easier to enter Singapore than Singaporean firms producing internationally, as Singapore is an incredibly open economy. Viewed from a zeros perspective, this outcome seems quite natural: if American multinationals do not have a presence in Niger, the MP fixed cost from the US to Niger must be high as US firms are very productive and would have made high profits had it entered (the fixed cost would have to be higher than these profits).

Accounting for these zeros in a general equilibrium framework requires a huge computational effort for two reasons: computational complexity and equilibrium non-existence. Both issues come about because the programming problem not only involves discrete (0-1) choices, it involves many discrete choices. In particular, computing equilibria in this environment involves considering $2^{N \cdot (N-1) \cdot 2}$ cases, where $N > 100$ is the number of countries (to get a sense of how big this number is, note that $2^{20} > 10^6$). The exponential growth in the number of cases to be considered arises from the integer nature of the problem. For each of the N countries, there are $N - 1$ potential partners and two separate modes of transacting with that country (trade or MP), and it can choose to enter or not enter any of these destinations through trade or MP: an equilibrium requires that all decisions rules are internally consistent with the resultant general equilibrium objects and hence necessitates evaluating all possible permutations of these discrete entry choices in search of a fixed point. I develop an algorithm that gets around this complication by iterating in the space of average decision rules, rather than the space of all decision rules.² This reduces the computational complexity to one that grows linearly, rather than exponentially, in the number of countries. I also construct the algorithm to address the second issue of non-existence. Exact equilibrium where firms enter when it is profitable to do so and not otherwise does not always exist in this environment with severe non-convexities. The intuition is similar to the non-existence of pure-strategy Nash equilibrium. Suppose before the reform that entry is not optimal (i.e. a zero exists). Then given the reform, costs are lower so that with no entry, i.e. at the pre-reform general equilibrium environment, it is now optimal to enter (a profitable deviation

²Krusell and Smith (1998) iterate in the space of average decision rules as well, albeit for a very different reason (aggregate uncertainty). Ruhl (2008) is an example of a trade model solved using the Krusell-Smith(1998) approach. The algorithm here, while similar in spirit, is very different in implementation.

exists). However, entering affects the general equilibrium and if the impact is sufficiently significant, equilibrium profit with entry falls below the fixed cost and entry becomes suboptimal as well. Hence, neither not-enter nor enter are optimal decisions. To get around this, I develop an approximate equilibrium concept wherein (1) countries engage in more than 99% of all profitable bilateral relationships available to them, and (2) of the bilateral relationships they engage in, more than 99% yield positive profits.³ Utilizing the type-specific monotonicity that results from the baseline parameterization, the algorithm computes an approximate equilibrium with the zero-to-one transitions that result from liberalization in trade and multinational production. The gains from openness are obtained by comparing welfare in the equilibrium obtained pre- and post-reform.

The strength of the general equilibrium approach this paper takes is that it allows for policy analysis that measures the impact of trade and MP liberalization. In particular, it is not immediately apparent why aggregate zeros should matter for welfare, but that is what this paper finds.⁴ Arkolakis, Costinot, and Rodriguez-Clare (2012) show that the gains from trade arising from a wide array of models depend only on two statistics: the import penetration ratio and the trade elasticity. To the extent that two different models predict that economic policy has the same effect on these two equilibrium objects, these models are identical in terms of welfare. This model under the baseline parameterization generates different welfare implications from Arkolakis, Costinot, and Rodriguez-Clare (2012) because it does not satisfy one of the three restrictions required to obtain their result. In particular, their third restriction requires that the percentage change in relative imports associated with a change in variable costs be symmetric across countries; this does not hold in my environment where zeros turn positive following reform as then there is a discontinuous jump from zero import demand to positive import demand. To interpret my results, I compare my model with an alternative specification that satisfies the restrictions in Arkolakis, Costinot, and Rodriguez-Clare (2012) and find that my model generates welfare gains that are up to 59% larger for the average country. The gains that I consider also do not only come from trade; there is multinational production in my model as well, and I find that for most

³An alternative solution to the issue of nonexistence of exact equilibrium is to consider mixed-strategy equilibria where firms can enter with positive, non-unitary probability and not enter with the complementary probability. We focus on the approximate equilibria defined earlier because interpretation of mixed strategies in this context is unclear: firms in the data either choose to enter or not enter, and I do not observe time variation in entry and exit that can discipline the probability of entry produced by such mixed strategies.

⁴di-Giovanni and Levchenko (2013) show that for a trade model with firm-level heterogeneity, zeros at the firm-level matter little for welfare if the firm-size distribution follows Zipf's Law. To rationalize these seemingly opposing results, it suffices to note that in their case, policy would also have a big impact if it affected the infra-marginal firms (which it would in my world with aggregate zeros) instead of simply affecting lower-productivity firms who are at the cutoff, i.e. a country-level zero implies a zero for all firms, including the ones that are most productive.

countries, MP liberalization has a stronger impact on welfare than trade liberalization. This is not surprising when one takes into account the fact that there are significantly more zeros in MP than trade. The zeros also show up in welfare through the uneven effects of global trade and MP reform: developing countries stand to gain twice as much as developed countries. Most zero bilateral ties involve at least one developing country - and in some cases, two developing countries - especially for trade. This means that relative to the standard model that does not allow aggregate zeros to turn positive, my model predicts that the welfare impact of reform is not only understated across the board, but particularly so for countries in the developing world.

This paper is related to three strands of literature. The first line of research looks at the zeros in trade. Armenter and Koren (2013) propose a statistical model with balls and bins to account for the large number of zeros in international shipments when they are mapped against product categories. Eaton, Kortum, and Sotelo (2012) show that the standard heterogeneous-firm model can be modified to generate an integer number of firms and as a result account well for the zeros in bilateral trade data. Building on the work of Alessandria, Kaboski, and Midrigan (2010), Hornok and Koren (2012) show that administrative trade costs associated with shipping goods across borders result in lumpiness that reduces welfare as shipments do not necessarily coincide with the preferred timing of agents' consumption. Relative to these papers, my work differs along two dimensions. First, it focuses on the aggregate (i.e. country- rather than sector-, firm-, and product-level) zeros in both trade and multinational production. Second, the zeros in this paper are hard zeros in the sense that they are zero with probability one, in contrast with the statistical approach adopted by the aforementioned papers. The message that zeros matter for welfare remains.⁵ Related to this is a literature that focuses on the econometric issues that arise in the estimation of gravity-type equations given the nonlinearities introduced by zeros. Helpman, Melitz, and Rubinstein (2008) employ an instrumental variables approach that allows them to demonstrate the significance of the inclusion of firm-level heterogeneity in the estimation of a gravity-type model. Santos Silva and Tenreyro (2006) emphasizes the difference between gravity estimates obtained from a poisson pseudo-maximum likelihood estimator and those obtained using ordinary least squares. In contrast, because my goal is to quantify the gains from openness in a world with zeros, I perform my analysis within the context of a general

⁵Kehoe and Ruhl (2013) find that increased trade in the set of least-traded goods accounts for a significant fraction of trade growth following trade liberalization and similar structural breaks. Arkolakis (2010) constructs a model where trade liberalization results in a large increase in the trade of goods with previously low volumes of trade. Evenett and Venables (2002) and Hummels and Klenow (2005) find evidence of the importance of the extensive margin for trade growth. Feenstra (1994) and Broda and Weinstein (2006) find that ignoring the extensive margin from additional varieties results in prices that are too high and welfare that is too low.

equilibrium model so as to be able to conduct policy experiments. The second strand of literature examines trade and multinational production within a unified framework. Ramondo and Rodriguez Clare (2013) build on the seminal work of Eaton and Kortum (2002) and study the gains from openness in a model where countries have a motive to engage in both trade and MP, while abstracting from the zeros in both types of flows. I view my work as complementary to theirs. Earlier work on trade and FDI as well as their interaction is vast: Costinot and Rodriguez-Clare (2013) and Antras and Yeaple (2013) in their surveys detail the evolution of this work.⁶ More recently, Irarrazabal, Moxnes and Opromolla (2012), building on Eaton, Kortum and Kramarz (2011), account for intrafirm trade using detailed data on exporting and multinational firms from Norway. In a similar vein, Tintelnot (2013) studies global export platforms and estimates his model using German firm-level data; his subsequent analysis on the effects of liberalization are confined to a sample of 12 European and North American countries, where aggregate zeros are minimal. These firm- and industry-level studies typically focus on firms from a particular country or industries from the group of developed economies; my study, on the other hand, focuses on the variation in entry and sales patterns across countries, with particular emphasis on the zeros in the bilateral relationships between countries big and small. The third line of related work seeks to measure the fixed and variable costs in trade. Recent contributions to the gravity specification used to estimate iceberg variable costs include Anderson and van-Wincoop (2003), Eaton and Kortum (2002), and Waugh (2008). My work adds to these papers by using zeros to estimate fixed costs as well. Papers that measure fixed costs include Djankov et al. (2002), Barseghyan and DiCeccio (2011), and Bollard, Klenow and Li (2014) who estimate domestic costs, as well as Morales, Sheu and Zahler (2011), Moxnes (2010), McCallum (2013), and Alessandria and Choi (2014) who look at international fixed costs for certain countries and certain industries. My work complements this research on country- and sector-specific fixed costs by measuring bilateral fixed costs globally.

The next section presents the key empirical facts. Section 3 presents the model I construct to account for these facts. The quantitative analysis is discussed in Section 4, and the main results are shown in Section 5. The last section concludes. Table and figures can be found in a separate section following the main references. All proofs are relegated to Appendix A. Appendix B contains a description of the algorithm. A two-country example of an approximate equilibrium is presented in Appendix C.

⁶e.g. Helpman, Melitz, and Yeaple (2004), Ramondo, Rappoport, and Ruhl (2012), Markusen (1984), Brainard (1997), Rob and Vettas (2003), Fillat and Garetto (2010), Keller and Yeaple (2009), Carr, Markusen and Maskus (2001), Grossman, Helpman and Szeidl (2006), McGrattan and Prescott (2010), McGrattan (2012), Alfaro and Charlton (2009), Fajgelbaum, Grossman, and Helpman (2013), Bernard, Jensen and Schott (2009)

2 Zeros in the Data

To construct the database of global bilateral trade and FDI flows, I merge data from two main sources: the Coordinated Investment Survey Database and the Direction of Trade Statistics. The Coordinated Investment Survey Database is a recent initiative by the IMF that was commissioned for the purpose of reconciling differences between the reported bilateral FDI flows by reporter and partner countries; this is my main source for bilateral FDI data, and it runs from 2009-2011. Due to the lag in reporting national data to the international organization performing the survey, the 2009 vintage of the Coordinated Investment Survey Database is most complete and will be the focus of this study. Statistics for 2010 and 2011 are very similar and will not be considered in this paper. The bilateral trade data come from the Direction of Trade Statistics Database. Merging these two datasets, I obtain bilateral trade and FDI flows data for over 100 countries.

Given that bilateral trade and multinational production can either be positive or zero, there are four cases in total. The case where both are positive, the case where both are zero, and the cases where only one is positive and not the other. All four cases are observed in the data. Figure 1 documents that of the bilateral pairs in my sample, 80% contain at least one zero, with only roughly a fifth of all bilateral relationships have both positive trade and FDI flows. Moreover, a quarter of the country pairs do not trade with or do FDI with each other. This shows that despite the remarkable growth in world trade and investment in the past half century, we still live in a world that is nowhere near free trade or investment.

[Figure 1 around here]

One might think that these zeros are simply a result of a group of countries not trading or investing with one another. Figure 2 shows a scatterplot of the number of trading partners on the number of FDI partners by reporting country. Clearly, countries that do FDI with more countries also trade with more countries *ceteris paribus*. While there are countries like China and Italy that receive FDI and imports from nearly every country in the sample, there are also countries like New Zealand and Belgium that receive imports from a much larger set of countries than it does FDI. There are naturally countries like Nepal and Zimbabwe that only receive imports and inward FDI from a small subset of countries. In sum, the figure shows that the prevalence of zeros does not just come about because of a certain group of countries, but every country is involved to some extent.

[Figure 2 around here]

Thus far, the analysis has counted all zeros as equal in the sense that a zero between two big countries is considered the same as a zero between two small countries. In the tables that follow, I weigh the zeros by GDP and consider the sum of the GDPs of the countries that do not trade or do MP with the average reporting country, relative to total world GDP. Reporting countries can be big or small, where big countries have GDP's larger than the sample (world) average, which comes out to roughly 0.5% of total world GDP. In the entries highlighted in red, I show that the zeros are not simply between small-small country pairs, but often involve at least one big country.

[Table 1 around here]

Look at zeros across different destinations, one also finds significant heterogeneity. For each destination country, Figures 3 and 4 plot the fraction of the sample that said country does not import or receive investment from against GDP. These two figures show that the zeros are not simply an artifact of a subset of countries, but apply to all countries in the sample. For example, less than half the countries in the world invest in the US, even though the US does trade with all the countries in my sample. The number of zeros is negatively correlated with GDP, implying that small countries are less likely recipients of trade and multinational production. Figures 5 and 6 also count the number of zeros by destination by weigh these zeros by GDP. In comparing Figures 3 and 5, we see that while 70% of the world's countries do not export to Tonga or Samoa, these countries only make up about 15% of total world GDP. Figures 4 and 6 paint a different picture. Here, we see that even for countries like Sweden, Egypt or New Zealand, weighing the zeros by GDP does not drastically alter the finding that big countries do not produce there as the zeros for each of these destinations have GDP's that sum up to roughly half of the world's total GDP. In conclusion, zeros are not just between small countries, but often involve big countries as well.

[Figures 3-6 around here]

3 Model

The model is a monopolistic competition with homogeneous goods setup. There are N countries, and two sets of firms producing differentiated products in each country: a set of firms that produces domestically and exports, and another set that produces domestically and does multinational production. The measure of firms that engage in monopolistic competition is exogenous, and there exists a numeraire good sector as in Chaney (2008). Country i produces w_i units of the freely-traded numeraire good with one unit of labor, and as is

standard in this class of models, I only consider equilibria where this good is produced in all countries, in effect pinning down the wage w_i in country i . Both trade and MP are subject to fixed entry and variable iceberg costs that differ both across country pairs and between trade and MP. There is no free entry condition; firms can choose not to enter countries where the fixed costs exceed expected profits. Profits are aggregated into a global fund and distributed proportionally across households. Goods can be produced or traded internationally, and are produced using labor and capital which are mobile within but not across countries.⁷

3.1 Consumers

There are $i = 1, 2, \dots, N$ countries and there is a measure L_i of consumers in each country i . Consumers maximize utility obtained from consuming goods in three sectors. Utility from the first sector comes from consumption of the numeraire good. Each of the other two sectors consists of consumption of differentiated goods: goods consumed in one sector can be imported from another country, while the goods consumed in the other sector can be produced by a foreign firm through multinational production. An exogenous fraction $1 - 2\mu$ of income is spent on the numeraire good, leaving a fraction μ to be spent on goods from each of the two differentiated sectors. Preferences are CES over varieties of the differentiated goods with constant elasticity of substitution $\sigma > 1$. The problem for the representative consumer in country i can then be written as

$$\begin{aligned} \max_{c_i^0, c_{ij}^M(\omega), c_{ij}^T(\omega)} \quad & (1 - 2\mu) \log c_i^0 + \mu \log c_i^T + \mu \log c_i^M \\ c_i^T = \quad & \left(\sum_{j=1}^N \int_{\Omega_i^T} c_{ij}^T(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \\ c_i^M = \quad & \left(\sum_{j=1}^N \int_{\Omega_i^M} c_{ij}^M(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \\ \sum_{j=1}^N \int_{\Omega_i^T} p_{ij}^T(\omega) c_{ij}^T(\omega) d\omega + \sum_{j=1}^N \int_{\Omega_i^M} p_{ij}^M(\omega) c_{ij}^M(\omega) d\omega + p_0 c_i^0 = \quad & w_i L_i + r_i K_i + 2w_i L_i \pi \end{aligned}$$

⁷My model builds on the seminal work of Krugman (1979), which Arkolakis, Demidova, Klenow and Rodriguez Clare (2008) extend by allowing for endogenous entry; here I adopt exogenous entry where the measure of potential firms is fixed a la Chaney (2008). In contrast to Chaney (2008), however, I parameterize the fixed costs so that zero-to-one transitions following policy reform are observed in equilibrium.

Denote the price indices for trade and MP by P_i^T and P_i^M . These are given by

$$P_i^T = \left[\sum_{j=1}^N \int_{\Omega_i^T} p_{ij}^T(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \quad (1)$$

$$P_i^M = \left[\sum_{j=1}^N \int_{\Omega_i^M} p_{ij}^M(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \quad (2)$$

Consumer optimization generates demand functions of the form (with capital-labor ratio $k_i = K_i/L_i$ and capital share α)

$$c_{ij}^M(\omega) = \mu \frac{p_{ij}^M(\omega)^{-\sigma}}{P_i^{M^{1-\sigma}}} Y_i = \mu \frac{p_{ij}^M(\omega)^{-\sigma}}{P_i^{M^{1-\sigma}}} (w_i L_i + r_i K_i + \pi_i) = \mu \frac{p_{ij}^M(\omega)^{-\sigma}}{P_i^{M^{1-\sigma}}} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) \quad (3)$$

$$c_{ij}^T(\omega) = \mu \frac{p_{ij}^T(\omega)^{-\sigma}}{P_i^{T^{1-\sigma}}} Y_i = \mu \frac{p_{ij}^T(\omega)^{-\sigma}}{P_i^{T^{1-\sigma}}} (w_i L_i + r_i K_i + \pi_i) = \mu \frac{p_{ij}^T(\omega)^{-\sigma}}{P_i^{M^{1-\sigma}}} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) \quad (4)$$

$$c_i^0 = (1 - 2\mu) \frac{Y_i}{p_0} = (1 - 2\mu) \frac{w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right)}{p_0} \quad (5)$$

These demand functions are taken as given by individual suppliers whose technology I discuss next.

3.2 Technology and Barriers to Trade and MP

Consider the two sectors with differentiated goods. All firms in country j operate a technology with productivity ϕ_j .⁸ A firm in the trade sector in country j can access the foreign market i by incurring fixed cost f_{ij}^T ; similarly, a firm belonging to the MP sector in country j can gain access to country i by incurring the fixed cost f_{ij}^M . Exports from j to i are subject to additional variable costs τ_{ij}^T that are of iceberg form. Similarly, there are efficiency losses τ_{ij}^M associated with multinational production in i for firms from j . Labor and capital are required to produce each differentiated good, with Cobb-Douglas production function and capital share α . Firms in the MP sector from any country j then solve N problems, one for

⁸The reason for making this assumption is twofold. First, firm-level data for the fraction of firms that export or do MP are only available for a few countries, and are not available for the majority of countries in my sample. Further, even within the sample of countries for which firm-level data are available, the fraction of exporters (or those doing MP) differs considerably (e.g. 40% of firms are exporters in Norway vs. 13% in the US) so one cannot take any one country as being representative of all the countries in the world. Excluding countries for which data are not available is not advisable in this case as aggregate zeros, the source of variation of primary interest, relies on having a sample with sufficiently many heterogeneous countries. The results for the version of the model with firm-level heterogeneity are qualitatively similar and available upon request.

each destination i , where they maximize profits given by

$$\begin{aligned}
\pi_{ij}^M &= \max_{p_{ij}^M} \left\{ p_{ij}^M c_{ij}^M - \frac{c_{ij}^M}{\phi_j} \frac{\tau_{ij}^M w_i^{1-\alpha} r_i^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha} - f_{ij}^M, 0 \right\} \\
&= \max_{p_{ij}^M} \left\{ p_{ij}^M c_{ij}^M - c_{ij}^M \frac{\tau_{ij}^M w_i}{\phi_j} \left(\frac{1}{k_i} \right)^\alpha \left(\frac{\alpha}{1-\alpha} \right)^\alpha \frac{1}{(1-\alpha)^{1-\alpha} \alpha^\alpha} - f_{ij}^M, 0 \right\} \\
&= \max_{p_{ij}^M} \left\{ p_{ij}^M c_{ij}^M - c_{ij}^M \frac{\tau_{ij}^M w_i}{\phi_j} \left(\frac{1}{k_i} \right)^\alpha \frac{1}{1-\alpha} - f_{ij}^M, 0 \right\} \\
&= \max_{p_{ij}^M} \left\{ \mu \frac{p_{ij}^{M^{1-\sigma}}}{P_i^{M^{1-\sigma}}} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) - \frac{p_{ij}^{M^{-\sigma}}}{P_i^{M^{1-\sigma}}} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) \frac{\tau_{ij}^M w_i}{\phi_j} \left(\frac{1}{k_i} \right)^\alpha \frac{1}{1-\alpha} - f_{ij}^M, 0 \right\}
\end{aligned}$$

Similarly, firms in the trade sector from any country j then solve N problems, one for each destination i , where they maximize profits given by

$$\begin{aligned}
\pi_{ij}^T &= \max_{p_{ij}^T} \left\{ p_{ij}^T c_{ij}^T - \frac{c_{ij}^T}{\phi_j} \frac{\tau_{ij}^T w_j^{1-\alpha} r_j^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha} - f_{ij}^T, 0 \right\} \\
&= \max_{p_{ij}^T} \left\{ p_{ij}^T c_{ij}^T - c_{ij}^T \frac{\tau_{ij}^T w_j}{\phi_j} \left(\frac{1}{k_j} \right)^\alpha \left(\frac{\alpha}{1-\alpha} \right)^\alpha \frac{1}{(1-\alpha)^{1-\alpha} \alpha^\alpha} - f_{ij}^T, 0 \right\} \\
&= \max_{p_{ij}^T} \left\{ p_{ij}^T c_{ij}^T - c_{ij}^T \frac{\tau_{ij}^T w_j}{\phi_j} \left(\frac{1}{k_j} \right)^\alpha \frac{1}{1-\alpha} - f_{ij}^T, 0 \right\} \\
&= \max_{p_{ij}^T} \left\{ \mu \frac{p_{ij}^{T^{1-\sigma}}}{P_i^{T^{1-\sigma}}} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) - \frac{p_{ij}^{T^{-\sigma}}}{P_i^{T^{1-\sigma}}} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) \frac{\tau_{ij}^T w_j}{\phi_j} \left(\frac{1}{k_j} \right)^\alpha \frac{1}{1-\alpha} - f_{ij}^T, 0 \right\}
\end{aligned}$$

Optimality in such a monopolistic competition setup requires that firms charge the Dixit-Stiglitz markup if it enters. On the other hand, if it does not enter, prices have to tend to infinity to be consistent with zero demand. This yields pricing equations

$$p_{ij}^M(\phi_j) = \begin{cases} \tau_{ij}^M \frac{w_i}{\phi_j} \frac{\sigma}{\sigma-1} \left(\frac{1}{k_i} \right)^\alpha \left(\frac{\alpha}{1-\alpha} \right)^\alpha & \text{if } \pi_{ij}^M > 0 \\ \infty & \text{otherwise} \end{cases} \quad (6)$$

$$p_{ij}^T(\phi_j) = \begin{cases} \tau_{ij}^T \frac{w_j}{\phi_j} \frac{\sigma}{\sigma-1} \left(\frac{1}{k_j} \right)^\alpha \left(\frac{\alpha}{1-\alpha} \right)^\alpha & \text{if } \pi_{ij}^T > 0 \\ \infty & \text{otherwise} \end{cases} \quad (7)$$

These pricing rules imply that gross profits are proportional to expenditure, and hence the

equivalence relations

$$\begin{aligned}\pi_{ij}^T > 0 &\Leftrightarrow \mu \frac{1}{\sigma} \left(\frac{p_{ij}^T}{P_i^T} \right)^{1-\sigma} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) > f_{ij}^T \\ \pi_{ij}^M > 0 &\Leftrightarrow \mu \frac{1}{\sigma} \left(\frac{p_{ij}^M}{P_i^M} \right)^{1-\sigma} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) > f_{ij}^M\end{aligned}$$

Firms from j enter country i if profits net of fixed costs are positive. Denote entry by a firm from j in the MP sector in country i by e_{ij}^M and similarly entry into the trade sector by e_{ij}^T . Hence we have the optimal decision rules

$$e_{ij}^M(\phi_j) = \begin{cases} 1 & \text{if } \mu \frac{1}{\sigma} \left(\frac{p_{ij}^M}{P_i^M} \right)^{1-\sigma} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) > f_{ij}^M \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

$$e_{ij}^T(\phi_j) = \begin{cases} 1 & \text{if } \mu \frac{1}{\sigma} \left(\frac{p_{ij}^T}{P_i^T} \right)^{1-\sigma} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) > f_{ij}^T \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

Note the difference between the wage terms that enter into these pricing relations: the labor cost incurred by the foreign multinational is that of the destination country, while the labor cost incurred by the exporter is that of the source country. This is the primary conceptual difference between trade and FDI in this model.

3.3 Profits

As in Chaney, profits made by firms worldwide are pooled into a global mutual fund and redistributed proportionally across households, with the representative household in country i owning $2w_i L_i$ shares (as there is a measure $\theta_i^M = w_i L_i$ firms in country i 's MP sector and similarly a measure $\theta_i^T = w_i L_i$ in its trade sector). Hence profits or dividends per share is given by

$$\pi = \frac{\sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^s}{P_i^s} \right)^{1-\sigma} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) - f_{ij}^s \right] e_{ij}^s}{2 \sum_{i=1}^N w_i L_i} \quad (10)$$

3.4 Equilibrium

An equilibrium consists of consumption plans c_i^0 , c_{ij}^T for trade and c_{ij}^M for MP, production plans y_i^0 , y_{ij}^T for trade and y_{ij}^M for MP, labor allocations l_i^0 , l_{ij}^T for trade and l_{ij}^M for MP, capital allocations k_{ij}^T for trade and k_{ij}^M for MP, entry decisions e_{ij}^T for trade and e_{ij}^M for MP, pricing decisions p_{ij}^T for trade and p_{ij}^M for MP, price indices P_i^T for trade and P_i^M for MP and profits per share π such that the following conditions hold:

- (i) Consumption plans are optimal, and solve the household problem, satisfying (3)-(5).
- (ii) Pricing decisions are optimal, and firm charge the Dixit-Stiglitz markup with entry, and prices tend to infinity otherwise: (6)-(7).
- (iii) Entry decisions are optimal, and firms only enter markets where profits exceed the fixed entry costs: (8)-(9).
- (iv) Production plans are optimal, where output, labor, and capital allocations satisfy

$$y_{ij}^T = \phi_j e_{ij}^T k_{ij}^{T\alpha} l_{ij}^{T1\alpha} \quad (11)$$

$$y_{ij}^M = \phi_j e_{ij}^M k_{ij}^{M\alpha} l_{ij}^{M1\alpha} \quad (12)$$

$$\frac{r_i k_{ij}^T}{w_i l_{ij}^T} = \frac{r_i k_{ij}^M}{w_i l_{ij}^M} = \frac{\alpha}{1 - \alpha} \quad (13)$$

$$y_i^0 = A_i l_i^0 = w_i l_i^0 \quad (14)$$

- (v) Price indices are consistent with the entry and pricing decisions of firms

$$P_i^M = \left[\sum_{j=1}^N \int_{\Omega_i^M} p_{ij}^M(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} = \left[\sum_{j=1}^N w_j L_j p_{ij}^{M1-\sigma} e_{ij}^M \right]^{\frac{1}{1-\sigma}} \quad (15)$$

$$P_i^T = \left[\sum_{j=1}^N \int_{\Omega_i^T} p_{ij}^T(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} = \left[\sum_{j=1}^N w_j L_j p_{ij}^{T1-\sigma} e_{ij}^T \right]^{\frac{1}{1-\sigma}} \quad (16)$$

- (vi) Profits or dividend per share are consistent with entry and pricing decisions of firms: (10).
- (vii) All markets clear.

4 Quantitative Analysis

In this section, I operationalize the model and use it to illustrate the importance of zeros for the gains from openness that arise from two specific policies: double taxation treaties

and regional trade agreements. First, I discuss the baseline calibration of the model and show that it can match the main features of the data discussed earlier. Next, I describe the policy experiments that I perform to gauge the importance of the aggregate extensive margin. Subsequent to this, I describe the properties of the algorithm that I use to compute the equilibrium following the policy reforms. I close this section with a discussion of the main results.

4.1 Benchmark Calibration: With Extensive Margin

I categorize the list of parameters into three groups. The first group consists of parameters common across all countries, as well as the list of country-specific parameters. The second group consists of the country pair-specific iceberg costs, both for trade and MP. Finally, the third group consists of the country pair-specific fixed costs for both trade and MP.

4.1.1 Country-Specific and Common Parameters

A country in the model is defined by a wage, a labor endowment, a capital endowment, and a level of productivity that applies to all its firms. For wages, I extrapolate wage data from the Occupational Wages around the World (OWW) Database constructed by Freeman and Oostendorp (2012). Details for the construction of the wage series can be found in the Appendix. For wages and the other exogenous country-specific parameters given below, values for the US are normalized to 1. Labor endowments are taken from the Penn World Tables (PWT). Capital stocks are constructed using investment and GDP data from the PWT and the perpetual inventory method. Total factor productivity as measured by the Solow residual is taken to be firm-level productivity. The elasticity of substitution σ across differentiated goods in both trade and MP sectors is set to 4. The expenditure share of each of the two differentiated sectors μ is set to 0.25. The capital share α is set to 0.33.

4.1.2 Country Pair-Specific Iceberg Costs

Following the gravity literature, I regress positive trade or MP flows on the standard gravity variables of distance, language and contiguity as well as other control variables. The t-statistics are shown below the regression coefficients. D1 to D6 are distance dummies that represent different intervals, with D1 being the shortest, and D6 the farthest. As expected, these dummies have negative coefficients, and are strongly statistically significant. Contiguity and common language also have the right signs, in that they are both positive, meaning that sharing a common border and language does increase the volume of trade or multinational production across countries. Finally I also present the estimates for two policy

variables: regional trade agreements and double taxation treaties. I find that these variables are statistically significant even at the 1% level and have the right signs. These coefficients can be transformed as in Waugh (2008), and I obtain trade iceberg costs for the OECD that are similar to standard gravity estimates. The iceberg costs for MP are roughly twice as high as those in trade on average, unsurprisingly higher as MP shares are typically smaller than trade shares in the data.

[Table 2 around here]

4.1.3 Country Pair-Specific Fixed Costs

Denote the matrix of trade entry patterns in the data by E^T and similarly the matrix of empirical MP entry patterns by E^M . Given an element e_{ij}^T in E^T , $e_{ij}^T = 1$ means that country i imports from country j in the data and $e_{ij}^T = 0$ means it does not. Given the vector of wages, TFPs, labor endowments, and capital-to-labor ratios, the matrix of iceberg costs and values for the common parameters (σ, α, μ) , we can then construct the prices that firms would charge if they entered each of the N locations according to the Dixit-Stiglitz formula. This yields price matrices P^T and P^M . Given these price and entry matrices, we can construct two matrices of price indices denoted by $\mathbf{P}^T$ and $\mathbf{P}^M$ that would be the price indices firms faced in an equilibrium where entry patterns were exactly as they were in the data. Given the price matrices (P^T, P^M) and matrices of price indices $(\mathbf{P}^T, \mathbf{P}^M)$, we can then write the matrices for bilateral gross profits π^T and π^M with representative elements π_{ij}^T and π_{ij}^M as functions of the global dividend per share π . π_{ij}^T and π_{ij}^M are the profits gross of fixed costs that country j firms in the trade and MP sector can make if it operates in country i . The final general equilibrium object π is then determined given all the aforementioned variables and the fixed cost parameterization specified below.

4.1.3.1 Global Fixed Costs

I parameterize the fixed costs to be functions of the profit and entry matrices. In particular, for a given positive constant ϵ , I set

$$\begin{aligned} f_{ij}^T &= \pi_{ij}^T + \epsilon, & \text{if } e_{ij}^T &= 0 \\ f_{ij}^T &= \epsilon, & \text{if } e_{ij}^T &= 1 \\ f_{ij}^M &= \pi_{ij}^M + \epsilon, & \text{if } e_{ij}^M &= 0 \\ f_{ij}^M &= \epsilon, & \text{if } e_{ij}^M &= 1 \end{aligned} \tag{17}$$

Given that the entry patterns are as they are in the data, the fixed costs are parameterized to guarantee entry where necessary and to suppress it otherwise. With this parameterization of the fixed costs, we can write global dividend per share as a function of ϵ according to (10). Denote this by $\pi(\epsilon)$, to make clear its dependence on the constant ϵ . Then we know that bilateral profits that are functions of the dividend per share, can also be written as functions of ϵ , i.e. $\pi_{ij}^T = \pi_{ij}^T(\pi) = \pi_{ij}^T(\epsilon)$ and $\pi_{ij}^M = \pi_{ij}^M(\pi) = \pi_{ij}^M(\epsilon)$. Then to have an equilibrium where $\pi_{ij}^T(\epsilon) \geq f_{ij}^T = \epsilon$, when $e_{ij}^T = 1$ and $\pi_{ij}^M(\epsilon) \geq f_{ij}^M = \epsilon$, when $e_{ij}^M = 1$, the scalar $\epsilon > 0$ must satisfy the following restrictions:

$$\epsilon \leq \pi_{ij}^T \quad \forall (i, j) \in \Omega^T \tag{18}$$

$$\epsilon \leq \pi_{ij}^M \quad \forall (i, j) \in \Omega^M \tag{19}$$

where the sets Ω^T and Ω^M contain all the country pairs for which there is entry in trade and MP in the data, i.e.

$$\Omega^T = \{(i, j) : e_{ij}^T = 1\} \quad \Omega^M = \{(i, j) : e_{ij}^M = 1\} \tag{20}$$

Define the functions $g_{ij}^T(\epsilon) = \pi_{ij}^T(\epsilon) - \epsilon$ and $g_{ij}^M(\epsilon) = \pi_{ij}^M(\epsilon) - \epsilon$. It is not hard to show that $g_{ij}^T(\epsilon)$ and $g_{ij}^M(\epsilon)$ are strictly decreasing in ϵ , hence there exists $\bar{\epsilon}_{ij}^T > 0$ that solves $g_{ij}^T(\epsilon) = 0$ and similarly $\bar{\epsilon}_{ij}^M > 0$ that solves $g_{ij}^M(\epsilon) = 0$ for each (i, j) pair. Then for any given pair (i, j) such that we need $g_{ij}^T(\epsilon) \geq 0$, we know that as long as $\epsilon \leq \bar{\epsilon}_{ij}^T$, we will have $g_{ij}^T(\epsilon) \geq 0$. Thus there exists ϵ_H such that $\forall \epsilon < \epsilon_H$ we have that both $g_{ij}^T(\epsilon) \geq 0$ and $g_{ij}^M(\epsilon) \geq 0$ for all country pairs (i, j) . We also know that as $\epsilon \rightarrow 0$, the fixed costs are minimized. The next two figures illustrate the average fixed cost by destination for these two values of ϵ , which we can think of as a global fixed cost, in that it is the component of the bilateral fixed costs that is not country- or country-pair specific.

[Figure 7 around here]

How zeros affect welfare is through fixed costs, because the size of fixed costs determines how much countries stand to gain from reform. When fixed costs are really high, there are few zero-to-one transitions, and the effect of having this aggregate extensive margin is minimal. On the other hand, if fixed costs are relatively low, there are more zero-to-one transitions, and the aggregate extensive margin is more important. These figures show the average fixed cost as a fraction of profits (a ratio, hence unitless measure) by destination country for both trade and MP. To understand the size of the fixed costs, I normalize these costs by profits, because this is what matters to firms: how much of its profits have to go towards paying for the initial fixed cost of establishing the trade or multinational relationship. The first figure shows the average fixed costs in the parameterization that maximizes welfare and hence minimizes fixed costs. The x-axis plots the average fixed cost for trade as a fraction of profit by destination, while the y-axis plots the average fixed cost for multinational production as a fraction of total multinational production profits. Three things can be taken away from this picture. First, the two types of fixed costs are positively correlated: countries with high trade fixed costs tend to have high MP fixed costs as well. This is not surprising as fixed costs are identified by zeros, and the trade and MP zeros by destination are also positively correlated. Second, though they are positively correlated, the fixed costs for MP are generally higher than the fixed costs for trade. This again relates back to the zeros: there are more MP zeros than trade zeros. Finally, the average fixed cost as a fraction of profits for developing countries are generally bigger than the average fixed costs for developed countries. This reflects the fact that there are still more zeros in developing countries than there are in developed countries.

[Figure 8 around here]

The next figure once again plots average fixed costs as a fraction of profits, but in this case for the highest ϵ possible. Two things can be seen when comparing these two figures. First, we see that because the global fixed cost ϵ starts to dominate the profits component π_{ij} of the fixed costs in the second figure, the ratio of fixed costs to profits adheres more closely to a diagonal line. When the global fixed cost is low, bilateral fixed costs are more country-pair specific because the π_{ij} terms are country-pair specific (owing to differences in technologies and variable costs, among other things). When the global fixed cost is high, average international fixed costs as a fraction of profits become less country-pair specific, though they remain country-specific. The reason they remain country-specific is because profits (the denominator in the ratio) are country-specific, and given the profit distribution rule (recall that shares are distributed proportionally to country size) and fixed costs that in the extreme (when $\epsilon \gg \max_{i,j} \pi_{ij}$) become constant $\approx \epsilon$, we then have the average fixed

costs as a fraction of profits becoming inversely proportional to size, and countries should lie on the diagonal (with slope equal to π_i^T/π_i^M where $\pi_i^T = \sum_j \pi_{ij}^T$ and $\pi_i^M = \sum_j \pi_{ij}^M$), with bigger countries on the left and smaller countries on the right. Second, we see that in the first figure, the most developed countries lie to the left of the graph and have much lower fixed cost to profit ratios than the rest of the world while this is no longer true in the second figure. This comes about because of both asymmetric fixed costs and profits. In the first figure, for countries with lots of zeros, its average fixed costs will be dominated by the bilateral profits term π_{ij} (as the effect of the global fixed cost is negligible). Only countries with few zeros will have fixed costs that are small; incidentally, such countries tend to be richer (and hence bigger) as well, thereby making more profit, hence the outcome that the average fixed cost-to-profit ratio is particularly small for rich countries. This is no longer true in the second figure because fixed costs are predominantly global in nature and hence even for countries with few zeros, the average fixed cost-to-profit ratio becomes large. The impact of zeros in the second country can be most strongly seen in the countries with lots of zeros, as in these cases, even if the global fixed cost is high, the country-pair specific component π_{ij} of fixed costs f_{ij} when bilateral ties are zero are still significant enough cumulatively to make these countries (e.g. VEN, NGA) deviate from the diagonal. These two figures illustrate the underlying tension when inferring fixed costs from zeros. On the one hand, zeros are informative for inferring fixed costs to both developed and developing countries because a highly productive country not investing in a less productive country implies a higher fixed cost than the opposite direction (because the highly productive country would have made higher profits). But on the other hand, to the extent that the global fixed cost is high (i.e. there are significant costs to doing international trade or multinational production everywhere) zeros become less informative for big, developed countries with few zero ties as their fixed costs are well-approximated by the global fixed cost. There is a silver lining in all this though. The global fixed cost cannot be too high in that a global fixed cost that is too high would imply zeros everywhere (i.e. autarky), an environment that is clearly rejected by the data (especially in trade where there are a lot of countries trading with each other). Hence, while high global fixed costs reduce the bilateral asymmetry in fixed costs that can be inferred from zeros, zeros place an upper bound on size of the global fixed cost and hence remain informative for understanding the size of international fixed costs. I find that the global fixed cost can be as high as 4.65% of profit. Note that this is not a point estimate but an upper bound given by the restriction that in the model countries trade and produce internationally as they do in the data. In the case where the global fixed cost is highest, the average trade fixed cost is 19.51% of profit, while the average MP fixed cost is 88.59% of profit. At the other end of the spectrum where the global fixed cost is negligible and fixed

costs are minimized subject to the restriction that in the model countries do not trade or produce internationally when they do not do so in the data, international fixed costs become purely country-pair specific (and are zero for the pairs that do trade or do MP), I find that the average trade fixed cost is 0.80% of profit, while the average MP fixed cost is 85.93% of profit. Comparing these numbers to those obtained earlier illustrates just how many more zeros exist in MP compared to trade, and how big the MP fixed costs have to be to induce firms not to produce overseas. In sum, regardless of whether one believes that global fixed costs are small or large, zeros hold significant information regarding fixed costs: in the case when the global cost is high, zeros discipline the size of this global cost by placing an upper bound on its range; in the case when the global cost is low, variation in the pattern of zeros can be used to estimate bilaterally asymmetric fixed costs. This asymmetry in fixed costs is what I turn to next.

4.1.3.2 Asymmetric International Fixed Costs

Given that I am interested in understanding the potential of zeros for greater welfare gains, henceforth I will consider the parameterization that minimizes fixed costs and maximizes welfare (so the global fixed cost is set to a positive constant virtually indistinguishable from zero). The earlier discussion alluded to average fixed costs by destination country. A relevant question given the bilateral asymmetry in fixed costs is whether systemic patterns exist between fixed costs that are averaged by source country vis-a-vis those averaged by destination. In particular, consider the ratio between average MP fixed cost by destination and average MP fixed cost by source for any country s

$$\frac{\sum_j f_{sj}^M}{\sum_i f_{is}^M}$$

The next figure plots this ratio against GDP per worker. Similar plots obtain when we plot this ratio against GDP and TFP (which controls for capital differences). It also obtains when the ratio is computed for trade instead of MP. The correlation between the ratio and GDP per worker (and similarly GDP and TFP) is highly negative. To understand this, first consider the ratio. The ratio is around 1 for Greece. This means that the average international fixed cost faced by Greek producers is roughly the same size as the fixed cost international producers face when attempting to enter Greece. The ratio is significantly less than 1 for Singapore. This means that Singapore is relatively open: foreign firms find it to enter the Singapore relative to Singaporean firms trying to break into international markets. The converse is true for many developing or unproductive countries that are relatively closed. Viewed from a zeros perspective, this outcome seems quite natural: if American multina-

tionals do not have a presence in Niger, the MP fixed cost from the US to Niger must be high as US firms are very productive and would have made high profits had it entered (the fixed cost would have to be higher than these profits).

[Figure 9 around here]

4.1.3.3 International Fixed Costs vs. Variable Costs

One can go one step further and link the fixed costs obtained in this section with the iceberg costs estimated earlier. To make the connection between iceberg costs and fixed costs, note that they are linked through profits: to the extent that fixed costs have to be higher than profits to suppress entry and profits are negatively correlated with iceberg costs, for the zero pairs, fixed costs and iceberg costs are negatively correlated as the next figure (estimates for Netherlands) shows.

[Figure 10 around here]

4.2 Alternative Parameterization: No Extensive Margin

To make clear the role of the extensive margin, I consider an alternative parameterization of my model where zero-to-one (aggregate entry) and one-to-zero (aggregate exit) transitions can never be observed following policy reform that lower the iceberg costs to trade and MP. In order for this to be true, fixed costs have to be sufficiently close to zero for country pairs where positive trade and MP are observed and sufficiently close to infinity for country pairs where zero trade and MP are observed before the reform, as the lemmata and the proposition below show.

Lemma 1. Given $f_{ij}^s = \infty$ for all (i, j) such that $e_{ij}^s = 0$, $s \in \{T, M\}$, there is no aggregate entry following policy reform.

Lemma 2. Given $f_{ij}^s = 0$ for all (i, j) such that $e_{ij}^s = 1$, $s \in \{T, M\}$, then there is no aggregate exit post-reform.

Lemma 3. If there is no aggregate entry and exit following policy reform, dividend per share stays unchanged ($\pi' = \pi$).

Lemmas 1 and 2 suggest that the following alternative parameterization is sufficient to

prevent aggregate entry and exit:

$$\begin{aligned}
f_{ij}^T &= \infty, & \text{if } e_{ij}^T &= 0 \\
f_{ij}^T &= 0, & \text{if } e_{ij}^T &= 1 \\
f_{ij}^M &= \infty, & \text{if } e_{ij}^M &= 0 \\
f_{ij}^M &= 0, & \text{if } e_{ij}^M &= 1
\end{aligned} \tag{21}$$

This parameterization is not unique. If fixed costs are sufficiently high for pairs that do not trade or do MP initially and sufficiently low for pairs that do, we also obtain the result that zeros before the reform stay zero after the reform and likewise for those that entered before the reform. The following proposition shows that because of Lemma 3, the gains from openness obtained from all these other parameterizations with no aggregate entry and exit coincide with that obtained in the limiting case just presented, so we can compare our benchmark results against this limiting parameterization without loss of generality.

Proposition 1. The welfare gains computed in the limiting parameterization given by $f_{ij}^s = 0$ for all (i, j) such that $e_{ij}^s = 1$, $s \in \{T, M\}$ and $f_{ij}^s = \infty$ for all (i, j) such that $e_{ij}^s = 0$, $s \in \{T, M\}$ coincide with the welfare gains in an alternative parameterization of fixed costs where $f_{ij}^s = \underline{f}$ for all (i, j) such that $e_{ij}^s = 1$, $s \in \{T, M\}$ and $f_{ij}^s = \bar{f}$ for all (i, j) such that $e_{ij}^s = 0$, $s \in \{T, M\}$ where $\underline{f}$ is sufficiently small and $\bar{f}$ is sufficiently large to ensure that there is no aggregate entry or exit post-reform.

4.3 Approximate Equilibrium

In what follows, I will consider policy reforms that lower the iceberg costs to trade and MP for a large group of country pairs, holding all other parameters constant. In the alternative parameterization with no room for aggregate entry or exit, computation of the equilibrium is simple as by construction the post-reform entry patterns coincide with the pre-reform entry patterns - if there were positive trade or MP before, there will be positive flows after the reform, and similarly for the pairs where there none. In the baseline parameterization, however, this is not the case; pairs where there were initially no flows need not remain zero as the fixed cost is not infinitely large, but rather only epsilon larger than what profits would have been in the pre-reform equilibrium, and lower iceberg costs increase profits ceteris paribus. Further, if there is aggregate entry, there then can also be aggregate exit, as entry of low cost producers lowers the prevailing price index and reduces demand for existing products. This means we need to compute the entry decision for each firm and each potential destination, giving rise to $N \times N$ ($N = 107$ countries) decisions in the traded sector and another $N \times N$

decisions in the MP sector. For an exact equilibrium to exist, these $2 \times N \times N$ decisions then need to be consistent in the sense that the resulting price indices and dividend per share that results from such decisions are exactly the same set of price indices and dividend per share that firms took as given when making their decisions. Only in certain special regions in the parameter space do such exact equilibria exist. Appendix C presents an example of the different regions in the parameter space in a simple two-country world with trade.

In the general case where an exact equilibrium does not exist, the standard approach of iterating on the general equilibrium objects given by the price indices and the dividend per share fails as cycles result and the algorithm does not converge. To get around this problem, I develop an approximate equilibrium concept and an algorithm that computes such approximate equilibria. In such an equilibrium, the entry decisions of agents need not be perfectly consistent with the price indices and dividend per share that such decisions engender. Positive equilibrium profits net of fixed costs do not automatically imply entry and vice versa. The goal then is to compute an equilibrium that is approximately exact in the sense that false positives and false negatives are kept to a minimum. Below I formally define the approximate equilibrium concept.

Definition 1. (Approximate Equilibrium)

An *approximate equilibrium that is $x\%$ accurate* is an equilibrium wherein (1) countries engage in more than $x\%$ of all profitable bilateral relationships available to them, and (2) of the bilateral relationships they engage in, more than $x\%$ yield positive profits.

Given that this study focuses on the effect of aggregate entry and exit on welfare, the accuracy defined in the aforementioned equilibrium concept provides a measure of how consistent aggregate entry and exit decisions are with profits net of fixed costs after reform in the event that an exact equilibrium (equivalently, an approximate equilibrium that is 100% accurate) fails to exist. Rather than iterating in the space of general equilibrium objects, I iterate in decision-rule space. And as I iterate in the space of decision rules, it is important that firms have accurate expectations over the decision rules for the other firms in the world economy. This perception depends on the specification of decision rules that firms take as given; this specification needs to be structured in a way that reflects how different firms behave when faced with the same policy change. Given the parameterization of the fixed costs, the proposition below shows that it is the low cost firms that enter and the high cost firms that exit following policy reform, after controlling for country-pair type. A type is a set of country pairs that are impacted the same way by general equilibrium forces; there are

$4N$ for trade and $4N$ for MP, where N is the number of countries. Given this type-specific monotonicity in entry and exit following reform, I specify cutoff rules for the entry decisions and find that the approximate equilibrium computed is accurate 99% of the time. A more detailed description of the algorithm can be found in Appendix B.

Proposition 2. (Type-Specific Cutoffs)

(i) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} = \tau_{ij}^T$ and $e_{ij}^T = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i1} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i1}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i1}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

(ii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} = \tau_{ij}^T$ and $e_{ij}^T = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i2} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i2}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i2}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

(iii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} < \tau_{ij}^T$ and $e_{ij}^T = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i3} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i3}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i3}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

(iv) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} < \tau_{ij}^T$ and $e_{ij}^T = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i4} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i4}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i4}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

(v) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} = \tau_{ij}^M$ and $e_{ij}^M = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i5} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i5}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i5}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

(vi) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} = \tau_{ij}^M$ and $e_{ij}^M = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i6} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i6}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i6}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

(vii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} < \tau_{ij}^M$ and $e_{ij}^M = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i7} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i7}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i7}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

(viii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} < \tau_{ij}^M$ and $e_{ij}^M = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i8} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i8}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i8}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

Corollary 1. (Type-Specific Monotonicity)

(i) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{T'} = \tau_{ij}^T$, $\tau_{ik}^{T'} = \tau_{ik}^T$, $e_{ij}^T = 0$, and $e_{ik}^T = 0$. If $c_{ik}^T < c_{ij}^T$, then $e_{ik}^{T'} \geq e_{ij}^{T'}$.

(ii) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{T'} = \tau_{ij}^T$, $\tau_{ik}^{T'} = \tau_{ik}^T$, $e_{ij}^T = 1$, and $e_{ik}^T = 1$. If $c_{ik}^T < c_{ij}^T$, then $e_{ik}^{T'} \geq e_{ij}^{T'}$.

(iii) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{T'} < \tau_{ij}^T$, $\tau_{ik}^{T'} < \tau_{ik}^T$, $e_{ij}^T = 1$, and $e_{ik}^T = 1$. If $c_{ik}^T < c_{ij}^T$, then $e_{ik}^{T'} \geq e_{ij}^{T'}$.

(iv) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{T'} < \tau_{ij}^T$, $\tau_{ik}^{T'} < \tau_{ik}^T$, $e_{ij}^T = 0$, and $e_{ik}^T = 0$. If $c_{ik}^T < c_{ij}^T$, then $e_{ik}^{T'} \geq e_{ij}^{T'}$.

(v) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{M'} = \tau_{ij}^M$, $\tau_{ik}^{M'} = \tau_{ik}^M$, $e_{ij}^M = 0$, and $e_{ik}^M = 0$. If $c_{ik}^M < c_{ij}^M$, then $e_{ik}^{M'} \geq e_{ij}^{M'}$.

(vi) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{M'} = \tau_{ij}^M$, $\tau_{ik}^{M'} = \tau_{ik}^M$, $e_{ij}^M = 1$, and $e_{ik}^M = 1$. If $c_{ik}^M < c_{ij}^M$, then $e_{ik}^{M'} \geq e_{ij}^{M'}$.

(vii) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{M'} < \tau_{ij}^M$, $\tau_{ik}^{M'} < \tau_{ik}^M$, $e_{ij}^M = 1$, and $e_{ik}^M = 1$. If $c_{ik}^M < c_{ij}^M$, then $e_{ik}^{M'} \geq e_{ij}^{M'}$.

(viii) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{M'} < \tau_{ij}^M$, $\tau_{ik}^{M'} < \tau_{ik}^M$, $e_{ij}^M = 0$, and $e_{ik}^M = 0$. If $c_{ik}^M < c_{ij}^M$, then $e_{ik}^{M'} \geq e_{ij}^{M'}$.

4.4 Policy Experiments

I consider the effect of two policy reforms: the formation of regional trade agreements as well as the signing of double taxation treaties globally. Of the 11449 bilateral country pairs in my sample, roughly 20% have double taxation treaties, and similarly, about 20% have regional trade agreements, though the subsamples that have these policies in place do not completely coincide. From my gravity estimation, I find that having a regional trade agreement lowers iceberg trade costs by 32%. By the same token, I find that having a double taxation treaty lowers iceberg MP costs by 38%.

I consider the gains from openness that result from a combination of trade and financial liberalization. For trade liberalization, I consider the formation of regional trade agreements worldwide: country pairs that are initially not part of a regional trade agreement form a regional trade agreement after the reform. For financial liberalization, I consider the establishment of double taxation treaties: country pairs with no double taxation treaties sign double taxation treaties after reform. I then ask the following questions. What do countries stand to gain from such policies? Are the gains symmetric across countries? Do these gains come primarily through trade or MP? And finally, how do the gains from the model with an extensive margin that allows aggregate entry and exit differ from the gains obtained from the model with no extensive margin?

4.4.1 Trade vs. MP

The total gains from openness are given by

$$\frac{W'_i}{W_i} = \log \frac{\left(\frac{1}{1-\alpha} + 2\pi'\right) P_i^{M\mu} P_i^{T\mu}}{\left(\frac{1}{1-\alpha} + 2\pi\right) P_i^{M'\mu} P_i^{T'\mu}}$$

These gains can come through greater consumption of the varieties in the traded goods sector, the MP sector or the numeraire good sector. The change in welfare due to trade is given by

$$\frac{W_i^{T'}}{W_i^T} = \log \frac{\left(\frac{1}{1-\alpha} + 2\pi'\right)^\mu P_i^{T\mu}}{\left(\frac{1}{1-\alpha} + 2\pi\right)^\mu P_i^{T'\mu}}$$

Similarly, the change in welfare due to MP is given by

$$\frac{W_i^{M'}}{W_i^M} = \log \frac{\left(\frac{1}{1-\alpha} + 2\pi'\right)^\mu P_i^{M\mu}}{\left(\frac{1}{1-\alpha} + 2\pi\right)^\mu P_i^{M'\mu}}$$

Table 3 shows the total gains to trade and financial liberalization decomposed into the trade and MP channels. The countries are ranked in terms of income, and the gains are averaged across the countries belonging to each quartile. Countries in the bottom quartile, for example, gain 10.9% in real income terms on average, with more than half these gains coming through MP 5.6%, the rest coming through trade 4.6%. In the baseline parameterization, reforms are welfare-improving on average, but do not affect all countries equally, with more gains accruing to low-income countries. This is not surprising as poor countries gain more from consuming new goods obtained from rich countries than rich countries do from consuming new goods obtained from poor countries. Further, notice the asymmetry in the decomposition of the gains into the trade and MP channels across the different income groups. While the bottom half of the income distribution benefit more from MP, welfare gains in the top half are either evenly split between trade and MP or skew toward trade. These results are all given the benchmark parameterization that allows for zero-to-one transitions. To isolate the role of the extensive margin, I now proceed to compare this benchmark case against the model with no aggregate extensive margin.

[Table 3 around here]

4.4.2 Benchmark vs. Model with No Extensive Margin

In the parameterization section, I discuss two versions of the model: one with a role for the aggregate extensive margin, and one without. Table 4 shows the results of running the same reforms on these two different environments. The model with aggregate entry and exit estimates greater gains from openness than the model with no zero-to-one transitions on average: 9.7% vs. 4.0% following trade and financial liberalization. The welfare gains from trade and financial liberalization are understated across the board: average gains in the pure intensive margin model are between 39 – 43% of the gains obtained in the benchmark model, though the table masks the differences for individual countries. It does, however, suggest that the gains from openness for all kinds of countries, not just the smallest or least developed countries, are significantly impacted by the inclusion of zeros. Apart from overestimating the gains, there is another distinction between the two versions: reforms that lower iceberg costs worldwide unambiguously result in welfare gains in the model with no extensive margin, in contrast to the possible losses following reform in the benchmark model. In the model with no extensive margin, no additional fixed costs are incurred following reform as there is no aggregate entry or exit, and income rises and price indices fall, so welfare has to rise. By contrast, the benchmark model allows for aggregate entry and exit, with potentially higher price indices and lower income given the resources lost in paying for the additional

fixed costs. To the extent that previously consumed goods are highly valued, and the fixed costs incurred in the formation of new bilateral relationships are substantial, reforms can result in lower welfare. This does not occur in this particular experiment, but does occur in other experiments (available upon request) where the model with no extensive margin not only overestimates the gains following reform, it predicts positive gains when entry and exit would imply losses. For both its effects on the absolute value of the gains as well as its sign, the aggregate extensive margin is quantitatively important for measuring the gains from openness.

[Table 4 around here]

5 Conclusion

In this paper, I document the prevalence of zeros in trade and multinational production in the data and study the welfare impact of incorporating the extensive margin that arises from these zeros within the context of a general equilibrium framework. Three contributions come out of the analysis. The first contribution relates to the estimation of global bilateral fixed costs identified using variation in the pattern of zeros across countries. Zeros are a rich source of information for quantifying both the common (global) and asymmetric (country pair-specific) components of international fixed costs. I find that the global fixed cost can be as high as 4.65% of profit, and systemic asymmetry manifests itself in the ratio between the average fixed cost by destination and the average fixed cost by source being strongly negatively correlated with GDP per worker (and similarly negatively correlated with GDP and TFP). The second main contribution lies in the algorithm developed to overcome two issues that arise from accounting for the many zeros in general equilibrium: computational complexity and equilibrium non-existence. The algorithm computes an approximate equilibrium where exact equilibrium does not exist and reduces the computational complexity to one that grows linearly, rather than exponentially, in the number of countries. Finally, I show that aggregate zeros matter for welfare. I find that models with no aggregate entry or exit can account for only 41% of the average gains obtained in the model where zeros matter, signifying that the aggregate extensive margin zeros create is important for understanding what countries gain from greater openness.

One can organize potential extensions along the three themes mentioned above. First, as richer data along both spatial and temporal dimensions become available, fixed cost estimates that vary across countries can be refined and extended to costs that vary not only across sectors but also over time (work exists but are typically limited to selected countries

or sectors). There has likewise been growing interest in modeling zeros at the firm- and goods-level; it would be interesting to see how the welfare impact of zeros at these different levels of aggregation differ and how these differences can be reconciled within a unifying framework. By the same token, there has been research that focuses on more complex interactions between trade and multinational production (e.g. intra-firm imports, export-platform multinational production), albeit focusing on firms from a particular country or a smaller group of countries where zeros do not occur. It would be instructive to see what gains result in the context of a model that has both aggregate zero-to-one transitions and such complicated interdependencies between trade and multinational production. Solving such models would require an algorithm that not only reduces computational complexity along the dimension of countries but rather sequences of countries as such complex interactions require one to keep track of the order in which international activity flows. I leave all these extensions for future research.

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6 Tables and Figures

Table 1: GDP-Weighted Zeros in Trade and MP

Reporter Type	Big Trade Zeros	Small Trade Zeros	Total Nonzeros
Larger than Average	0.1	0.3	99
Smaller than Average	2.8	1.8	95
Reporter Type	Big MP Zeros	Small MP Zeros	Total Nonzeros
Larger than Average	18	4.6	77
Smaller than Average	52	6.3	42

Table 2: Gravity Estimates for Estimating Iceberg Costs to Trade and MP

	MP	Trade
d1		0.912** (3.29)
d2	-0.338 (-0.93)	
d3	-1.423*** (-4.23)	-0.997*** (-5.47)
d4	-2.727*** (-7.89)	-2.140*** (-12.13)
d5	-2.967*** (-8.81)	-2.481*** (-14.24)
d6	-3.758*** (-10.79)	-3.128*** (-17.74)
contig	0.389 (1.26)	0.590** (2.59)
comlang	0.749*** (4.71)	0.198* (2.52)
dtc	1.436*** (11.91)	
rta		1.036*** (13.12)
cons	-7.923*** (-23.32)	-4.885*** (-28.27)

Table 3: Percentage Gain in Real GDP After Reform: Trade vs MP

Percentiles	Overall Gains	Trade Gains	MP Gains
00-25	10.9	4.6	5.6
26-50	11.9	5.0	6.1
51-75	10.0	4.8	4.5
76-100	6.2	4.3	1.4
Total	9.7	4.7	4.4

Table 4: Percentage Gain in Real GDP After Reform - The Role of Zeros

Percentiles	Pure Intensive Margin Model	Benchmark Model
00-25	4.7	10.9
26-50	4.8	11.9
51-75	3.9	10.0
76-100	2.6	6.2
Total	4.0	9.7

Figure 1: The Four Cases

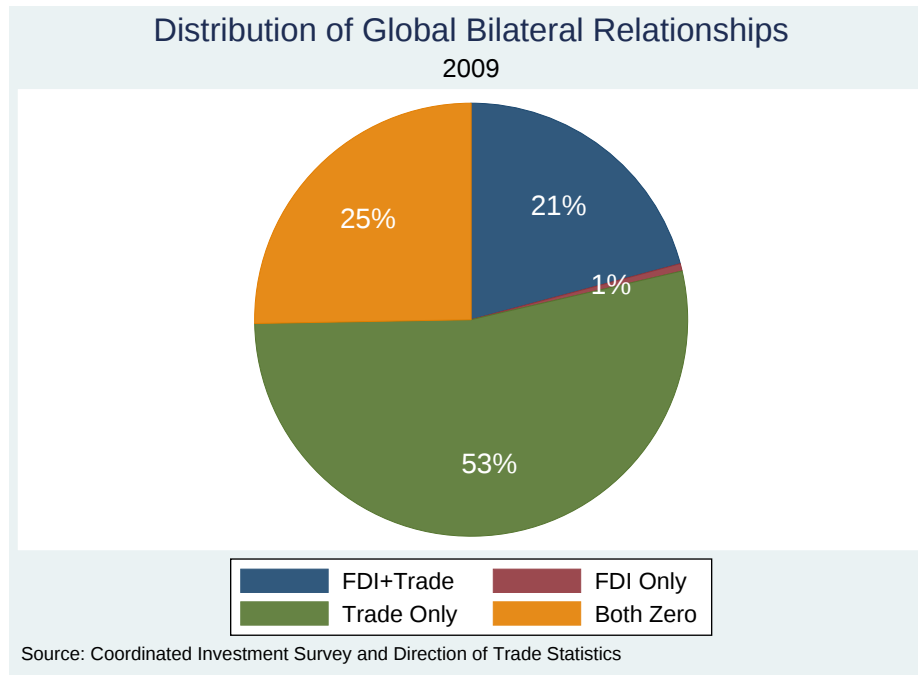


Figure 2: Zeros in Trade vs. Zeros in MP

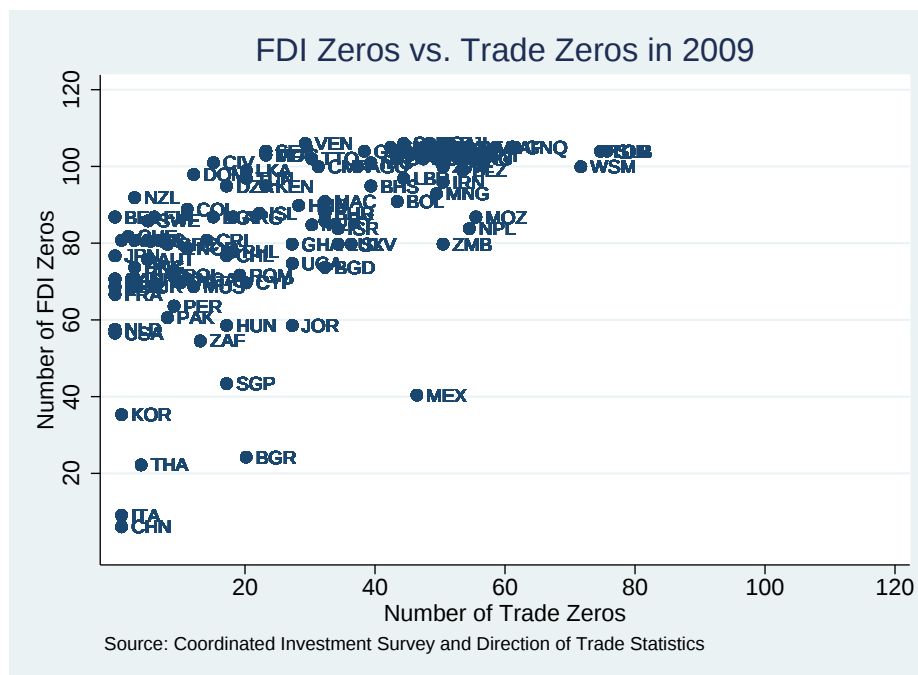


Figure 3: Zeros by Destination Country (Trade)

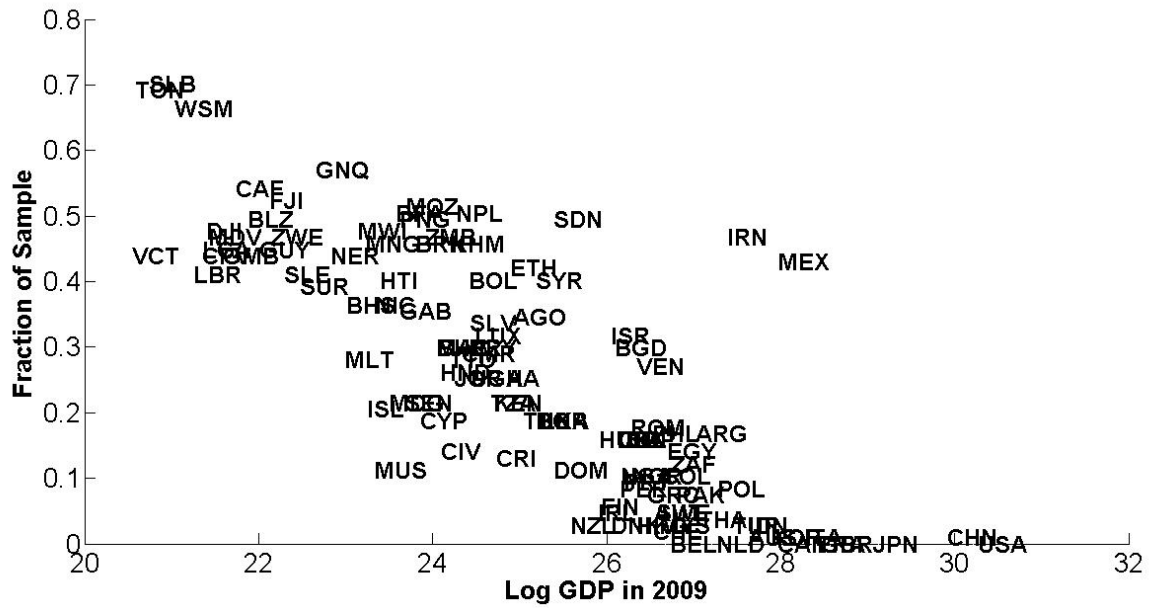


Figure 4: Zeros by Destination Country (MP)

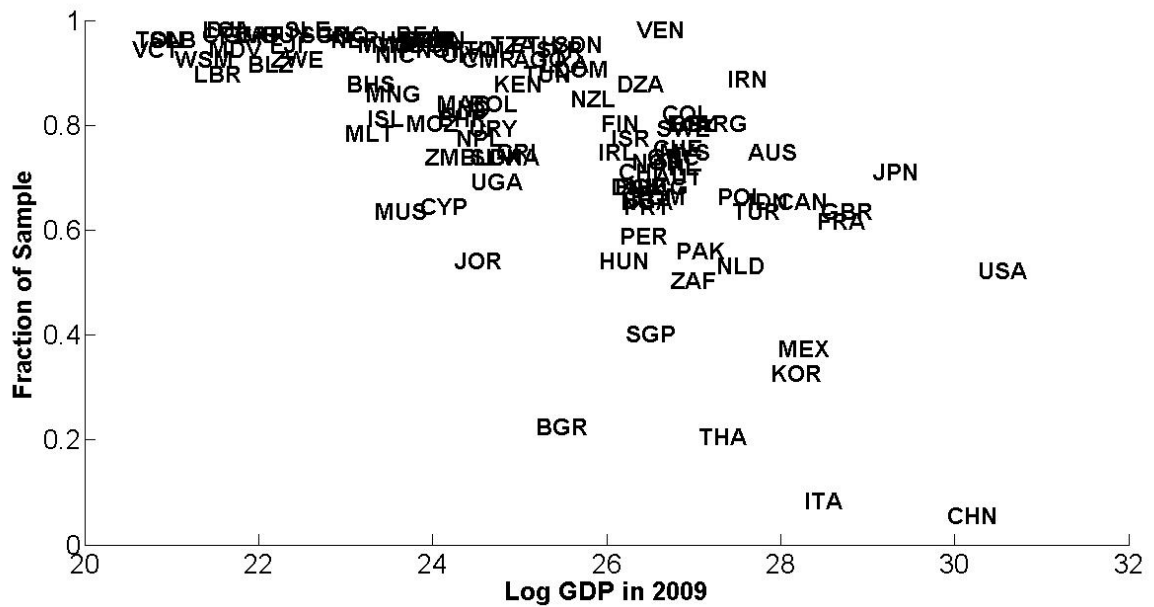


Figure 5: GDP-Weighted Zeros by Destination Country (Trade)

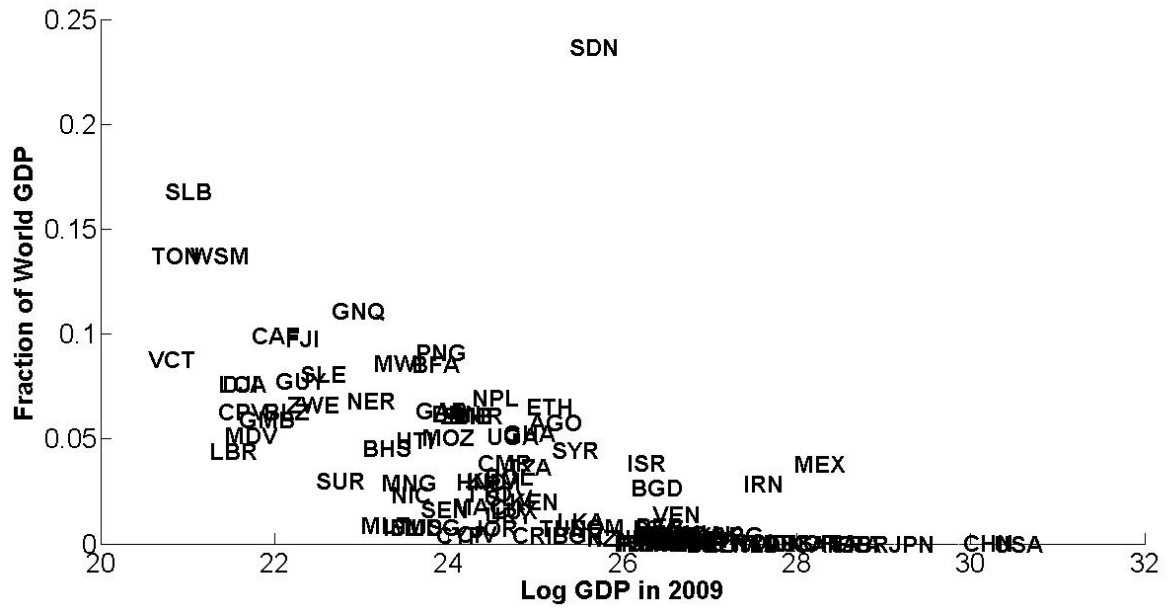


Figure 6: GDP-Weighted Zeros by Destination Country (MP)

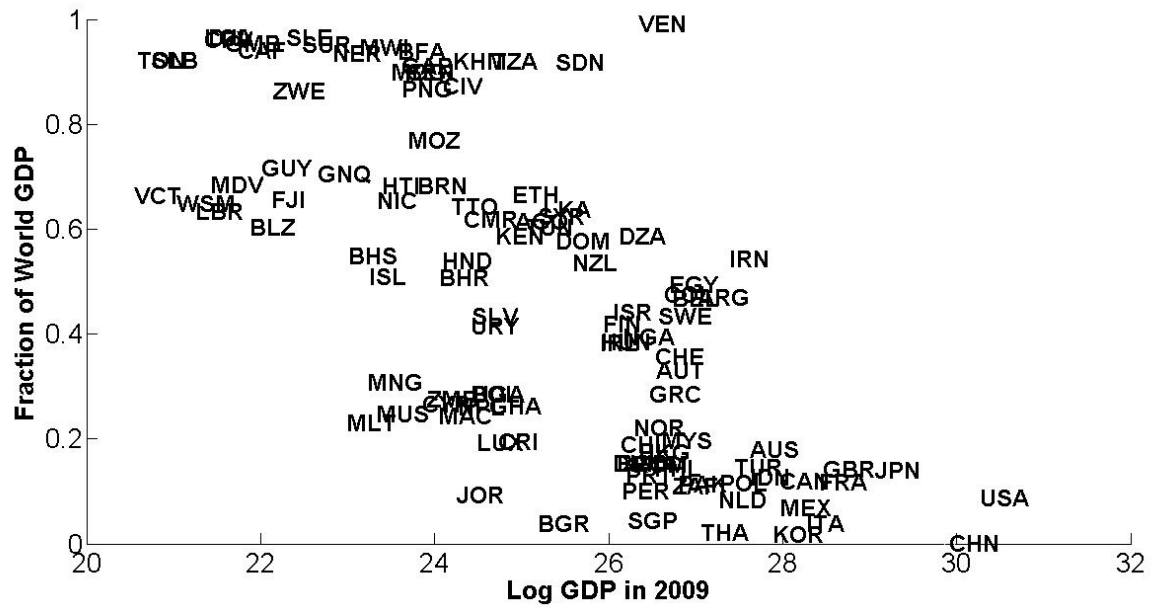


Figure 7: Average Fixed Costs: Identified by Zero Trade and MP

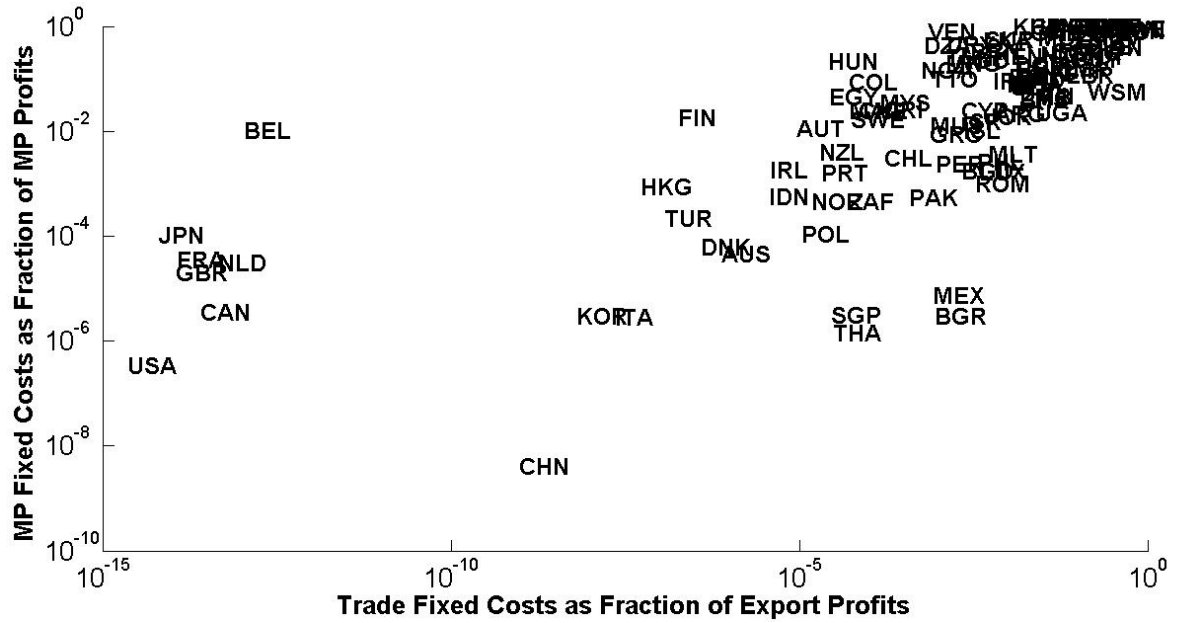


Figure 8: Average Fixed Costs: Maximum Global Fixed Costs

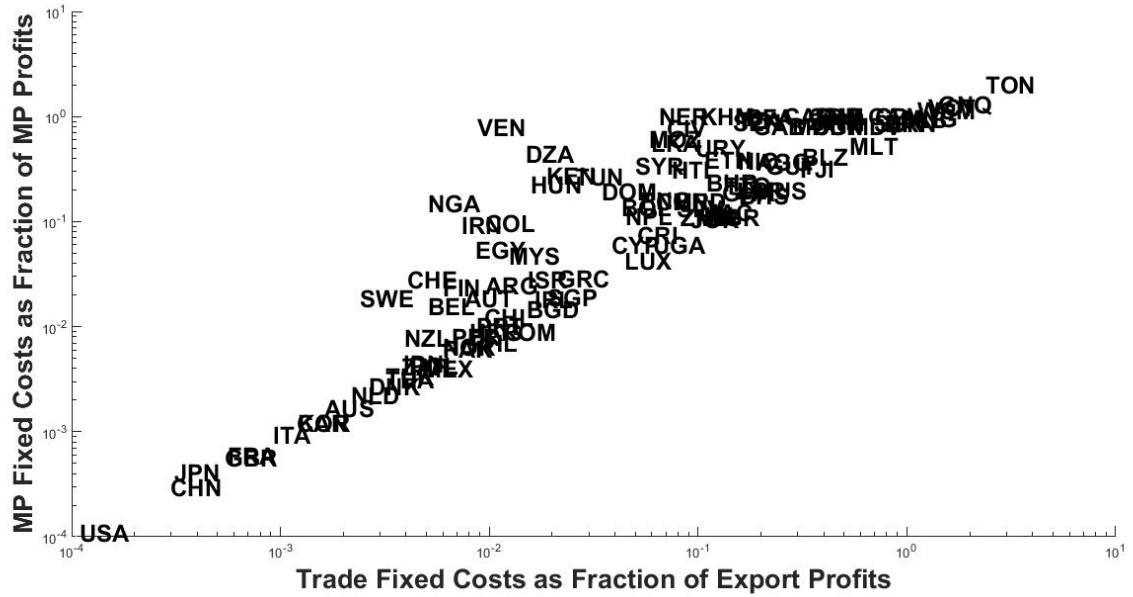


Figure 9: Asymmetry in Fixed Costs

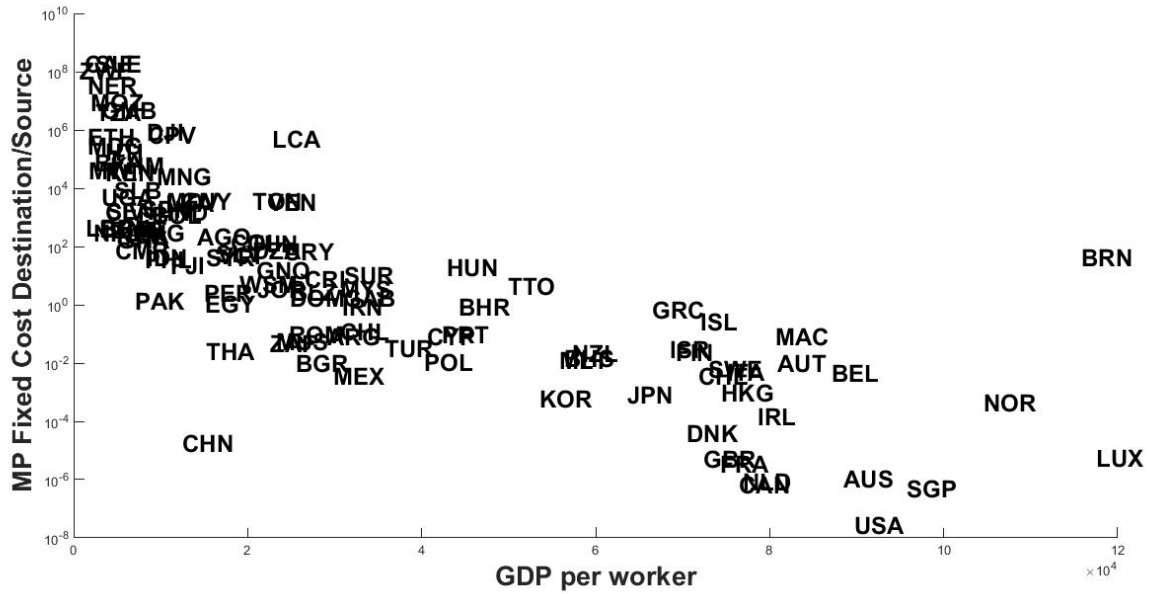
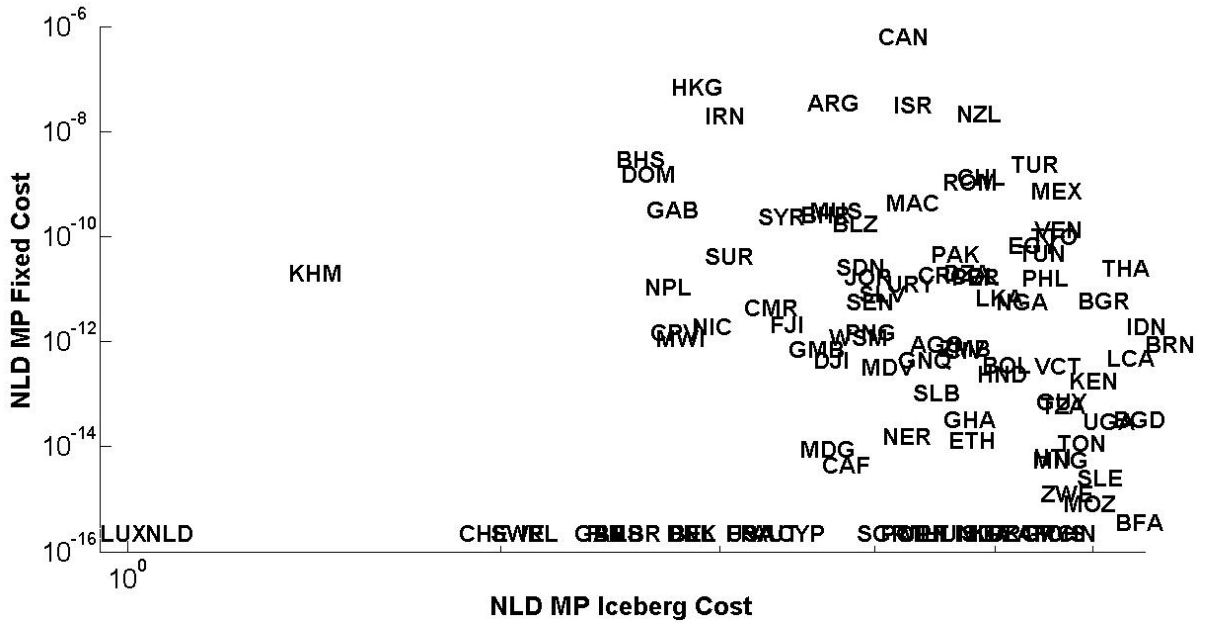


Figure 10: Iceberg Variable Costs & Fixed Costs



7 Appendix A: Proofs

Lemma 1. Given $f_{ij}^s = \infty$. for all (i, j) such that $e_{ij}^s = 0$, $s \in \{T, M\}$, there is no aggregate entry following policy reform.

Proof. Entry following reform requires that there exists (i, j) such that $e_{ij}^s = 0$ and $e_{ij}^{s'} = 1$ for $s \in \{T, M\}$. But this means that $\pi_{ij}^s < f_{ij}^s$ and $\pi_{ij}^{s'} \geq f_{ij}^s$. This cannot be true given that $f_{ij}^s = \infty$ and $\pi_{ij}^{s'} < \infty$, $\pi_{ij}^s < \infty$.

Lemma 2. Given $f_{ij}^s = 0$ for all (i, j) such that $e_{ij}^s = 1$, $s \in \{T, M\}$, then there is no aggregate exit post-reform.

Proof. Exit following reform requires that there exists (i, j) such that $e_{ij}^s = 1$ and $e_{ij}^{s'} = 0$ for $s \in \{T, M\}$. But this means that $\pi_{ij}^s \geq f_{ij}^s$ and $\pi_{ij}^{s'} < f_{ij}^s$. This cannot be true given that $f_{ij}^s = 0$ and $\pi_{ij}^{s'} > 0$, $\pi_{ij}^s > 0$.

Lemma 3. If there is no aggregate entry and exit following policy reform, dividend per share stays unchanged ($\pi' = \pi$).

Proof. Recall the expression for dividend per share

$$\pi = \frac{\sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^S}{P_i^S} \right)^{1-\sigma} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) - f_{ij}^S \right] e_{ij}^S}{2 \sum_{i=1}^N w_i L_i}$$

Rearranging to isolate the terms that are functions of π to yield

$$\begin{aligned} & 2 \left\{ \sum_{i=1}^N w_i L_i - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^S}{P_i^S} \right)^{1-\sigma} w_i L_i \right] e_{ij}^S \right\} \pi \\ &= \frac{1}{1-\alpha} \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^S}{P_i^S} \right)^{1-\sigma} w_i L_i \right] e_{ij}^S - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^S \end{aligned}$$

Similarly, post-reform we have

$$\begin{aligned}
& 2 \left\{ \sum_{i=1}^N w_i L_i - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^{S'}}{P_i^{S'}} \right)^{1-\sigma} w_i L_i \right] e_{ij}^{S'} \right\} \pi' \\
&= \frac{1}{1-\alpha} \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^{S'}}{P_i^{S'}} \right)^{1-\sigma} w_i L_i \right] e_{ij}^{S'} - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^{S'}
\end{aligned}$$

To see that $\pi = \pi'$, it suffices to note that for any $S \in \{T, M\}, i = 1, 2, \dots, N$ we have

$$\begin{aligned}
\sum_{j=1}^N w_j L_j \left[p_{ij}^{S'1-\sigma} e_{ij}^{S'} \right] &= P_i^{S'1-\sigma} \\
\sum_{j=1}^N w_j L_j \left[p_{ij}^{S1-\sigma} e_{ij}^S \right] &= P_i^{S1-\sigma}
\end{aligned}$$

and because there is no aggregate entry or exit, $e_{ij}^{S'} = e_{ij}^S, \forall (S, i, j)$, so that

$$\sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^{S'} = \sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^S$$

Hence

$$\begin{aligned}
\pi &= \frac{\frac{1}{1-\alpha} \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^S}{P_i^S} \right)^{1-\sigma} w_i L_i \right] e_{ij}^S - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^S}{2 \left\{ \sum_{i=1}^N w_i L_i - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^S}{P_i^S} \right)^{1-\sigma} w_i L_i \right] e_{ij}^S \right\}} \\
&= \frac{\frac{1}{1-\alpha} \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^{S'}}{P_i^{S'}} \right)^{1-\sigma} w_i L_i \right] e_{ij}^{S'} - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^{S'}}{2 \left\{ \sum_{i=1}^N w_i L_i - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^{S'}}{P_i^{S'}} \right)^{1-\sigma} w_i L_i \right] e_{ij}^{S'} \right\}} = \pi'
\end{aligned}$$

Proposition 1. The welfare gains computed in the limiting parameterization given by $f_{ij}^s = 0$ for all (i, j) such that $e_{ij}^s = 1, s \in \{T, M\}$ and $f_{ij}^s = \infty$ for all (i, j) such that $e_{ij}^s = 0, s \in \{T, M\}$ coincide with the welfare gains in an alternative parameterization of fixed costs where $f_{ij}^s = \underline{f}$ for all (i, j) such that $e_{ij}^s = 1, s \in \{T, M\}$ and $f_{ij}^s = \bar{f}$ for all (i, j) such that $e_{ij}^s = 0, s \in \{T, M\}$ where $\underline{f}$ is sufficiently small and $\bar{f}$ is sufficiently large to ensure that there is no aggregate entry or exit post-reform.

Proof. Denote the equilibrium objects that arise from the alternative parameterization with hats, e.g. $\hat{P}_i, \hat{W}$. We know given that there is no aggregate entry or exit post-reform,

Lemma 3 implies that $\hat{\pi}' = \hat{\pi}$ (even as $\hat{\pi} \neq \pi$). Next note that because $\hat{e}_{ij}^{TS} = e_{ij}^{TS} = e_{ij}^S$ for all $i, j = 1, 2, \dots, N$ and $S \in \{T, M\}$ we have $P_i^{S'} = \hat{P}_i^{S'}$ for all $i = 1, 2, \dots, N$ and $S \in \{T, M\}$. The normalization stipulates that $P_0 = P'_0 = 1$. Then given that welfare is given by

$$W_i = \Gamma + \log \left\{ w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) \right\} - (1-2\mu) \log P_0 - \mu \log P_i^M - \mu \log P_i^T = \Gamma \log \left(\frac{w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right)}{P_0^{1-2\mu} P_i^{M\mu} P_i^{T\mu}} \right)$$

We have that the gains from openness are given by

$$\frac{W'_i}{W_i} = \log \frac{\left(\frac{1}{1-\alpha} + 2\pi' \right) P_i^{M\mu} P_i^{T\mu}}{\left(\frac{1}{1-\alpha} + 2\pi \right) P_i^{M'\mu} P_i^{T'\mu}}$$

Hence

$$\frac{\hat{W}'_i}{\hat{W}_i} = \log \frac{\left(\frac{1}{1-\alpha} + 2\hat{\pi}' \right) \hat{P}_i^{M\mu} \hat{P}_i^{T\mu}}{\left(\frac{1}{1-\alpha} + 2\hat{\pi} \right) \hat{P}_i^{M'\mu} \hat{P}_i^{T'\mu}} = \log \frac{\hat{P}_i^{M\mu} \hat{P}_i^{T\mu}}{\hat{P}_i^{M'\mu} \hat{P}_i^{T'\mu}} = \log \frac{P_i^{M\mu} P_i^{T\mu}}{P_i^{M'\mu} P_i^{T'\mu}} = \log \frac{\left(\frac{1}{1-\alpha} + 2\pi' \right) P_i^{M\mu} P_i^{T\mu}}{\left(\frac{1}{1-\alpha} + 2\pi \right) P_i^{M'\mu} P_i^{T'\mu}} = \frac{W'_i}{W_i}$$

as desired. Q.E.D.

Proposition 2. (i) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} = \tau_{ij}^T$ and $e_{ij}^T = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i1} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i1}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i1}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$ but $c_{ij}^T \geq x_{i1}, c_{ik}^T < x_{i1}$. This pair of inequalities imply that $\pi_{ik}^T > \pi_{ij}^T$. We also know from $e_{ij}^T = 0, e_{ik}^T = 0$ that

$$\begin{aligned} \pi_{ij}^T &< f_{ij}^T = \pi_{ij}^T + \epsilon \\ \pi_{ik}^T &< f_{ik}^T = \pi_{ik}^T + \epsilon \end{aligned}$$

where $\epsilon > 0$ is a small positive number. Note that

$$\begin{aligned} \frac{\pi_{ik}^{T'}}{\pi_{ik}^T} &= \left(\frac{P_i^{T'}}{P_i^T} \right)^{\sigma-1} \frac{Y'_i}{Y_i} \\ \frac{\pi_{ij}^{T'}}{\pi_{ij}^T} &= \left(\frac{P_i^{T'}}{P_i^T} \right)^{\sigma-1} \frac{Y'_i}{Y_i} \end{aligned}$$

So that the change in bilateral gross profits (through the price index and income from dividends) due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{P_i^{T'}}{P_i^T} \right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$, we also know that $\pi_{ij}^{T'} \geq f_{ij}^T$ and $\pi_{ik}^{T'} < f_{ik}^T$. Take the first of these two inequalities. We have

$$\pi_{ij}^{T'} \geq f_{ij}^T \Rightarrow \pi_{ij}^{T'} \geq \pi_{ij}^T + \epsilon \Rightarrow (\delta - 1)\pi_{ij}^T \geq \epsilon \Rightarrow \delta - 1 \geq \frac{\epsilon}{\pi_{ij}^T}$$

Then the last inequality implies

$$(\delta - 1)\pi_{ik}^T \geq \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \Rightarrow \pi_{ik}^{T'} - \pi_{ik}^T \geq \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \Rightarrow \pi_{ik}^{T'} \geq \pi_{ik}^T + \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T}$$

However, combining this with $\pi_{ik}^{T'} < f_{ik}^T = \pi_{ik}^T + \epsilon$ yields

$$\pi_{ik}^T + \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \leq \pi_{ik}^{T'} < f_{ik}^T = \pi_{ik}^T + \epsilon$$

which is a contradiction as $\pi_{ik}^T > \pi_{ij}^T$.

(ii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} = \tau_{ij}^T$ and $e_{ij}^T = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i2} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i2}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i2}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$ but $c_{ij}^T \geq x_{i2}, c_{ik}^T < x_{i2}$. This pair of inequalities imply that $\pi_{ik}^T > \pi_{ij}^T$. We also know from $e_{ij}^T = 1, e_{ik}^T = 1$ that

$$\begin{aligned} \pi_{ij}^T &> f_{ij}^T = \epsilon \\ \pi_{ik}^T &> f_{ik}^T = \epsilon \end{aligned}$$

where $\epsilon > 0$ is a small positive number. Note that

$$\frac{\pi_{ik}^{T'}}{\pi_{ik}^T} = \left(\frac{P_i^{T'}}{P_i^T} \right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

$$\frac{\pi_{ij}^{T'}}{\pi_{ij}^T} = \left(\frac{P_i^{T'}}{P_i^T} \right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

So that the change in bilateral gross profits (through the price index and income from dividends) due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{P_i^{T'}}{P_i^T} \right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$, we also know that $\pi_{ij}^{T'} \geq f_{ij}^T$ and $\pi_{ik}^{T'} < f_{ik}^T$. Take the second of these two inequalities. We have

$$\pi_{ik}^{T'} < f_{ik}^T \Rightarrow \pi_{ik}^{T'} < \epsilon \Rightarrow \delta \pi_{ik}^T < \epsilon \Rightarrow \delta < \frac{\epsilon}{\pi_{ik}^T}$$

Then the last inequality implies

$$\delta \pi_{ij}^T < \frac{\epsilon \pi_{ij}^T}{\pi_{ik}^T} \Rightarrow \pi_{ij}^{T'} < \frac{\epsilon \pi_{ij}^T}{\pi_{ik}^T} \Rightarrow \pi_{ij}^{T'} < \epsilon$$

where the last implication comes from $\pi_{ik}^T > \pi_{ij}^T$. This is a contradiction as $\pi_{ij}^{T'} \geq f_{ij}^T = \epsilon$.

(iii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} < \tau_{ij}^T$ and $e_{ij}^T = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i3} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i3}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i3}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$ but $c_{ij}^T \geq x_{i3}, c_{ik}^T < x_{i3}$. This pair of inequalities imply that $\pi_{ik}^T > \pi_{ij}^T$. We also know from $e_{ij}^T = 1, e_{ik}^T = 1$ that

$$\pi_{ij}^T > f_{ij}^T = \epsilon$$

$$\pi_{ik}^T > f_{ik}^T = \epsilon$$

where $\epsilon > 0$ is a small positive number. Note that

$$\begin{aligned}\frac{\pi_{ik}^{T'}}{\pi_{ik}^T} &= \left(\frac{\tau_{ik}^{T'}}{\tau_{ik}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i} \\ \frac{\pi_{ij}^{T'}}{\pi_{ij}^T} &= \left(\frac{\tau_{ij}^{T'}}{\tau_{ij}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}\end{aligned}$$

Because $\tau_{ik}^{T'}/\tau_{ik}^T = \tau_{ij}^{T'}/\tau_{ij}^T$ the change in bilateral gross profits due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{\tau_{ij}^{T'}}{\tau_{ij}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$, we also know that $\pi_{ij}^{T'} \geq f_{ij}^T$ and $\pi_{ik}^{T'} < f_{ik}^T$. Take the first of these two inequalities. We have

$$\pi_{ik}^{T'} < f_{ik}^T \Rightarrow \pi_{ik}^{T'} < \epsilon \Rightarrow \delta \pi_{ik}^T < \epsilon \Rightarrow \delta < \frac{\epsilon}{\pi_{ik}^T}$$

Then the last inequality implies

$$\delta \pi_{ij}^T < \frac{\epsilon \pi_{ij}^T}{\pi_{ik}^T} \Rightarrow \pi_{ij}^{T'} < \frac{\epsilon \pi_{ij}^T}{\pi_{ik}^T} \Rightarrow \pi_{ij}^{T'} < \epsilon$$

where the last implication comes from $\pi_{ik}^T > \pi_{ij}^T$. This is a contradiction as $\pi_{ij}^{T'} \geq f_{ij}^T = \epsilon$.

(iv) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} < \tau_{ij}^T$ and $e_{ij}^T = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i4} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i4}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i4}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$ but $c_{ij}^T \geq x_{i4}, c_{ik}^T < x_{i4}$. This pair of inequalities imply that $\pi_{ik}^T > \pi_{ij}^T$. We also know from $e_{ij}^T = 0, e_{ik}^T = 0$ that

$$\begin{aligned}\pi_{ij}^T &< f_{ij}^T = \pi_{ij}^T + \epsilon \\ \pi_{ik}^T &< f_{ik}^T = \pi_{ik}^T + \epsilon\end{aligned}$$

where $\epsilon > 0$ is a small positive number. Note that

$$\begin{aligned}\frac{\pi_{ik}^{T'}}{\pi_{ik}^T} &= \left(\frac{\tau_{ik}^{T'}}{\tau_{ik}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i} \\ \frac{\pi_{ij}^{T'}}{\pi_{ij}^T} &= \left(\frac{\tau_{ij}^{T'}}{\tau_{ij}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}\end{aligned}$$

Because $\tau_{ik}^{T'}/\tau_{ik}^T = \tau_{ij}^{T'}/\tau_{ij}^T$ the change in bilateral gross profits due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{\tau_{ij}^{T'}}{\tau_{ij}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$, we also know that $\pi_{ij}^{T'} \geq f_{ij}^T$ and $\pi_{ik}^{T'} < f_{ik}^T$. Take the first of these two inequalities. We have

$$\pi_{ij}^{T'} \geq f_{ij}^T \Rightarrow \pi_{ij}^{T'} \geq \pi_{ij}^T + \epsilon \Rightarrow (\delta - 1)\pi_{ij}^T \geq \epsilon \Rightarrow \delta - 1 \geq \frac{\epsilon}{\pi_{ij}^T}$$

Then the last inequality implies

$$(\delta - 1)\pi_{ik}^T \geq \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \Rightarrow \pi_{ik}^{T'} - \pi_{ik}^T \geq \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \Rightarrow \pi_{ik}^{T'} \geq \pi_{ik}^T + \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T}$$

However, combining this with $\pi_{ik}^{T'} < f_{ik}^T = \pi_{ik}^T + \epsilon$ yields

$$\pi_{ik}^T + \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \leq \pi_{ik}^{T'} < f_{ik}^T = \pi_{ik}^T + \epsilon$$

which is a contradiction as $\pi_{ik}^T > \pi_{ij}^T$.

(v) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} = \tau_{ij}^M$ and $e_{ij}^M = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i5} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i5}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i5}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$ but $c_{ij}^M \geq x_{i5}, c_{ik}^M < x_{i5}$.

This pair of inequalities imply that $\pi_{ik}^M > \pi_{ij}^M$. We also know from $e_{ij}^M = 0, e_{ik}^M = 0$ that

$$\begin{aligned}\pi_{ij}^M &< f_{ij}^M = \pi_{ij}^M + \epsilon \\ \pi_{ik}^M &< f_{ik}^M = \pi_{ik}^M + \epsilon\end{aligned}$$

where $\epsilon > 0$ is a small positive number. Note that

$$\begin{aligned}\frac{\pi_{ik}^{M'}}{\pi_{ik}^M} &= \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i} \\ \frac{\pi_{ij}^{M'}}{\pi_{ij}^M} &= \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}\end{aligned}$$

So that the change in bilateral gross profits (through the price index and income from dividends) due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$, we also know that $\pi_{ij}^{M'} \geq f_{ij}^M$ and $\pi_{ik}^{M'} < f_{ik}^M$. Take the first of these two inequalities. We have

$$\pi_{ij}^{M'} \geq f_{ij}^M \quad \Rightarrow \quad \pi_{ij}^{M'} \geq \pi_{ij}^M + \epsilon \quad \Rightarrow \quad (\delta - 1)\pi_{ij}^M \geq \epsilon \quad \Rightarrow \quad \delta - 1 \geq \frac{\epsilon}{\pi_{ij}^M}$$

Then the last inequality implies

$$(\delta - 1)\pi_{ik}^M \geq \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M} \quad \Rightarrow \quad \pi_{ik}^{M'} - \pi_{ik}^M \geq \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M} \quad \Rightarrow \quad \pi_{ik}^{M'} \geq \pi_{ik}^M + \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M}$$

However, combining this with $\pi_{ik}^{M'} < f_{ik}^M = \pi_{ik}^M + \epsilon$ yields

$$\pi_{ik}^M + \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M} \leq \pi_{ik}^{M'} < f_{ik}^M = \pi_{ik}^M + \epsilon$$

which is a contradiction as $\pi_{ik}^M > \pi_{ij}^M$.

(vi) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} = \tau_{ij}^M$ and $e_{ij}^M = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i6} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i6}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost

$c_{ik}^M < x_{i6}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$ but $c_{ij}^M \geq x_{i6}, c_{ik}^M < x_{i6}$. This pair of inequalities imply that $\pi_{ik}^M > \pi_{ij}^M$. We also know from $e_{ij}^M = 1, e_{ik}^M = 1$ that

$$\begin{aligned}\pi_{ij}^M &> f_{ij}^M = \epsilon \\ \pi_{ik}^M &> f_{ik}^M = \epsilon\end{aligned}$$

where $\epsilon > 0$ is a small positive number. Note that

$$\begin{aligned}\frac{\pi_{ik}^{M'}}{\pi_{ik}^M} &= \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i} \\ \frac{\pi_{ij}^{M'}}{\pi_{ij}^M} &= \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}\end{aligned}$$

So that the change in bilateral gross profits (through the price index and income from dividends) due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$, we also know that $\pi_{ij}^{M'} \geq f_{ij}^M$ and $\pi_{ik}^{M'} < f_{ik}^M$. Take the second of these two inequalities. We have

$$\pi_{ik}^{M'} < f_{ik}^M \Rightarrow \pi_{ik}^{M'} < \epsilon \Rightarrow \delta \pi_{ik}^M < \epsilon \Rightarrow \delta < \frac{\epsilon}{\pi_{ik}^M}$$

Then the last inequality implies

$$\delta \pi_{ij}^M < \frac{\epsilon \pi_{ij}^M}{\pi_{ik}^M} \Rightarrow \pi_{ij}^{M'} < \frac{\epsilon \pi_{ij}^M}{\pi_{ik}^M} \Rightarrow \pi_{ij}^{M'} < \epsilon$$

where the last implication comes from $\pi_{ik}^M > \pi_{ij}^M$. This is a contradiction as $\pi_{ij}^{M'} \geq f_{ij}^M = \epsilon$.

(vii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} < \tau_{ij}^M$ and $e_{ij}^M = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i7} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i7}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i7}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$ but $c_{ij}^M \geq x_{i7}, c_{ik}^M < x_{i7}$. This pair of inequalities imply that $\pi_{ik}^M > \pi_{ij}^M$. We also know from $e_{ij}^M = 1, e_{ik}^M = 1$ that

$$\begin{aligned}\pi_{ij}^M &> f_{ij}^M = \epsilon \\ \pi_{ik}^M &> f_{ik}^M = \epsilon\end{aligned}$$

where $\epsilon > 0$ is a small positive number. Note that

$$\begin{aligned}\frac{\pi_{ik}^{M'}}{\pi_{ik}^M} &= \left(\frac{\tau_{ik}^{M'}}{\tau_{ik}^M}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i} \\ \frac{\pi_{ij}^{M'}}{\pi_{ij}^M} &= \left(\frac{\tau_{ij}^{M'}}{\tau_{ij}^M}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}\end{aligned}$$

Because $\tau_{ik}^{M'}/\tau_{ik}^M = \tau_{ij}^{M'}/\tau_{ij}^M$ the change in bilateral gross profits due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{\tau_{ij}^{M'}}{\tau_{ij}^M}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$, we also know that $\pi_{ij}^{M'} \geq f_{ij}^M$ and $\pi_{ik}^{M'} < f_{ik}^M$. Take the first of these two inequalities. We have

$$\pi_{ik}^{M'} < f_{ik}^M \Rightarrow \pi_{ik}^{M'} < \epsilon \Rightarrow \delta \pi_{ik}^M < \epsilon \Rightarrow \delta < \frac{\epsilon}{\pi_{ik}^M}$$

Then the last inequality implies

$$\delta \pi_{ij}^M < \frac{\epsilon \pi_{ij}^M}{\pi_{ik}^M} \Rightarrow \pi_{ij}^{M'} < \frac{\epsilon \pi_{ij}^M}{\pi_{ik}^M} \Rightarrow \pi_{ij}^{M'} < \epsilon$$

where the last implication comes from $\pi_{ik}^M > \pi_{ij}^M$. This is a contradiction as $\pi_{ij}^{M'} \geq f_{ij}^M = \epsilon$.

(viii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} < \tau_{ij}^M$ and $e_{ij}^M = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i8} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i8}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i8}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$ but $c_{ij}^M \geq x_{i8}, c_{ik}^M < x_{i8}$. This pair of inequalities imply that $\pi_{ik}^M > \pi_{ij}^M$. We also know from $e_{ij}^M = 0, e_{ik}^M = 0$ that

$$\begin{aligned}\pi_{ij}^M &< f_{ij}^M = \pi_{ij}^M + \epsilon \\ \pi_{ik}^M &< f_{ik}^M = \pi_{ik}^M + \epsilon\end{aligned}$$

where $\epsilon > 0$ is a small positive number. Note that

$$\begin{aligned}\frac{\pi_{ik}^{M'}}{\pi_{ik}^M} &= \left(\frac{\tau_{ik}^{M'}}{\tau_{ik}^M}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i} \\ \frac{\pi_{ij}^{M'}}{\pi_{ij}^M} &= \left(\frac{\tau_{ij}^{M'}}{\tau_{ij}^M}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}\end{aligned}$$

Because $\tau_{ik}^{M'}/\tau_{ik}^M = \tau_{ij}^{M'}/\tau_{ij}^M$ the change in bilateral gross profits due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{\tau_{ij}^{M'}}{\tau_{ij}^M}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$, we also know that $\pi_{ij}^{M'} \geq f_{ij}^M$ and $\pi_{ik}^{M'} < f_{ik}^M$. Take the first of these two inequalities. We have

$$\pi_{ij}^{M'} \geq f_{ij}^M \Rightarrow \pi_{ij}^{M'} \geq \pi_{ij}^M + \epsilon \Rightarrow (\delta - 1)\pi_{ij}^M \geq \epsilon \Rightarrow \delta - 1 \geq \frac{\epsilon}{\pi_{ij}^M}$$

Then the last inequality implies

$$(\delta - 1)\pi_{ik}^M \geq \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M} \Rightarrow \pi_{ik}^{M'} - \pi_{ik}^M \geq \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M} \Rightarrow \pi_{ik}^{M'} \geq \pi_{ik}^M + \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M}$$

However, combining this with $\pi_{ik}^{M'} < f_{ik}^M = \pi_{ik}^M + \epsilon$ yields

$$\pi_{ik}^M + \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M} \leq \pi_{ik}^{M'} < f_{ik}^M = \pi_{ik}^M + \epsilon$$

which is a contradiction as $\pi_{ik}^M > \pi_{ij}^M$.

8 Appendix B: Computational Algorithm

Step 1: Parameterize the distribution of decision rules e'_{ij} for all i, j as follows:

$$e_{ij}^{T'} = \begin{cases} 0 & \text{if } \tau_{ij}^{T'} = \tau_{ij}^T, e_{ij}^T = 0, c_{ij}^T \geq x'_{i1} \\ 1 & \text{if } \tau_{ij}^{T'} = \tau_{ij}^T, e_{ij}^T = 0, c_{ij}^T < x'_{i1} \\ 0 & \text{if } \tau_{ij}^{T'} = \tau_{ij}^T, e_{ij}^T = 1, c_{ij}^T \geq x'_{i2} \\ 1 & \text{if } \tau_{ij}^{T'} = \tau_{ij}^T, e_{ij}^T = 1, c_{ij}^T < x'_{i2} \\ 0 & \text{if } \tau_{ij}^{T'} < \tau_{ij}^T, e_{ij}^T = 1, c_{ij}^T \geq x'_{i3} \\ 1 & \text{if } \tau_{ij}^{T'} < \tau_{ij}^T, e_{ij}^T = 1, c_{ij}^T < x'_{i3} \\ 0 & \text{if } \tau_{ij}^{T'} < \tau_{ij}^T, e_{ij}^T = 0, c_{ij}^T \geq x'_{i4} \\ 1 & \text{if } \tau_{ij}^{T'} < \tau_{ij}^T, e_{ij}^T = 0, c_{ij}^T < x'_{i4} \end{cases}$$

$$e_{ij}^{M'} = \begin{cases} 0 & \text{if } \tau_{ij}^{M'} = \tau_{ij}^M, e_{ij}^M = 0, c_{ij}^M \geq x'_{i5} \\ 1 & \text{if } \tau_{ij}^{M'} = \tau_{ij}^M, e_{ij}^M = 0, c_{ij}^M < x'_{i5} \\ 0 & \text{if } \tau_{ij}^{M'} = \tau_{ij}^M, e_{ij}^M = 1, c_{ij}^M \geq x'_{i6} \\ 1 & \text{if } \tau_{ij}^{M'} = \tau_{ij}^M, e_{ij}^M = 1, c_{ij}^M < x'_{i6} \\ 0 & \text{if } \tau_{ij}^{M'} < \tau_{ij}^M, e_{ij}^M = 1, c_{ij}^M \geq x'_{i7} \\ 1 & \text{if } \tau_{ij}^{M'} < \tau_{ij}^M, e_{ij}^M = 1, c_{ij}^M < x'_{i7} \\ 0 & \text{if } \tau_{ij}^{M'} < \tau_{ij}^M, e_{ij}^M = 0, c_{ij}^M \geq x'_{i8} \\ 1 & \text{if } \tau_{ij}^{M'} < \tau_{ij}^M, e_{ij}^M = 0, c_{ij}^M < x'_{i8} \end{cases}$$

Note the similarity between these decision rules and the lemmata proved earlier.

Step 2: Guess an initial vector of cutoffs $\{(x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5}, x_{i6}, x_{i7}, x_{i8})\}_{i=1,2,\dots,N}$. Do this with percentiles of the elements in the $8N$ sets $\{\Omega_{i1}, \Omega_{i2}, \Omega_{i3}, \Omega_{i4}, \Omega_{i5}, \Omega_{i6}, \Omega_{i7}, \Omega_{i8}\}_{i=1,2,\dots,N}$,

defined by

$$\begin{aligned}
\Omega_{i1} &= \{j : \tau_{ij}^{T'} = \tau_{ij}^T, e_{ij}^T = 0\} \\
\Omega_{i2} &= \{j : \tau_{ij}^{T'} = \tau_{ij}^T, e_{ij}^T = 1\} \\
\Omega_{i3} &= \{j : \tau_{ij}^{T'} < \tau_{ij}^T, e_{ij}^T = 1\} \\
\Omega_{i4} &= \{j : \tau_{ij}^{T'} < \tau_{ij}^T, e_{ij}^T = 0\} \\
\Omega_{i5} &= \{j : \tau_{ij}^{M'} = \tau_{ij}^M, e_{ij}^M = 0\} \\
\Omega_{i6} &= \{j : \tau_{ij}^{M'} = \tau_{ij}^M, e_{ij}^M = 1\} \\
\Omega_{i7} &= \{j : \tau_{ij}^{M'} < \tau_{ij}^M, e_{ij}^M = 1\} \\
\Omega_{i8} &= \{j : \tau_{ij}^{M'} < \tau_{ij}^M, e_{ij}^M = 0\}
\end{aligned}$$

A cutoff x_{ih} is then associated with the percentile $k_{ih} = 100|\hat{\Omega}_{ih}|/|\Omega_{ih}|$ for $h = 1, 2, 3, \dots, 8$, where $\hat{\Omega}_{ih} = \{j : j \in \Omega_{ih}, c_{ij} < x_{ih}\}$.

Step 3: Given the parameterization of the decision rules and guess for the cutoffs, we can then compute what the prices indices P_i^t are for all i as well as the dividend per share π^t . Here the superscript t denotes the t -th iteration.

Step 4: Given the price indices P_i^t and dividend per share π^t , we can compute the decision rule e'_{ij} for firms from each source country j and destination country i . Denote this matrix (i.e. distribution) of decision rules by E . These decision rules then yield price indices P_i^n and dividend per share π^n .

Step 5: Given the price indices P_i^n and dividend per share π^n , we can construct the profits π_{ij}^n that firms from j can make by exporting to i and compare this against the corresponding fixed cost f_{ij} to determine the entry pattern e'_{ij}^n . Denote the matrix of such entry patterns by E^n . If $E = E^n$ then we are done.

Step 6: If $E \neq E^n$, check if the cutoff rules implied by the decision rules just obtained k_{ih}^n coincide with the guess for the cutoff rules k_{ih}^t . If yes, then terminate; in this case we obtain an approximate equilibrium as no further updating can be done and $E \neq E^n$. If no, then proceed to update the cutoffs according to $k_{ih}^{t+1} = \lambda k_{ih}^n + (1 - \lambda)k_{ih}^t$ where the step size is $\lambda \in (0, 1)$. The cutoff rules implied by the decision rules e'_{ij} are constructed as follows:

$k_{ih}^n = 100|\hat{\Omega}_{ih}^n|/|\Omega_{ih}|$ where

$$\begin{aligned}\hat{\Omega}_{i1}^n &= \{j : j \in \Omega_{i1}, e_{ij}^{T'} = 1\} \\ \hat{\Omega}_{i2}^n &= \{j : j \in \Omega_{i2}, e_{ij}^{T'} = 1\} \\ \hat{\Omega}_{i3}^n &= \{j : j \in \Omega_{i3}, e_{ij}^{T'} = 1\} \\ \hat{\Omega}_{i4}^n &= \{j : j \in \Omega_{i4}, e_{ij}^{T'} = 1\} \\ \hat{\Omega}_{i5}^n &= \{j : j \in \Omega_{i5}, e_{ij}^{M'} = 1\} \\ \hat{\Omega}_{i6}^n &= \{j : j \in \Omega_{i6}, e_{ij}^{M'} = 1\} \\ \hat{\Omega}_{i7}^n &= \{j : j \in \Omega_{i7}, e_{ij}^{M'} = 1\} \\ \hat{\Omega}_{i8}^n &= \{j : j \in \Omega_{i8}, e_{ij}^{M'} = 1\}\end{aligned}$$

With the updated cutoff rules, return to Step 3 and iterate until convergence.

9 Appendix C: Two-Country Example

9.1 Introduction

There are two countries $i = 1, 2$. Each has a measure $\theta_i = w_i L_i$ of firms that are considering which markets to enter. For simplicity consider only an environment with trade (i.e. no multinational production). International trade is subject to both iceberg and fixed costs, which can be asymmetric across country pairs. Consider a policy reform that lowers the iceberg costs for firms from 2 to export to 1 from $\tau_{12} = 2$ to $\tau'_{12} = 1$. Let $\tau_{21} = \tau'_{21} = 1$. The trade elasticity is $\sigma = 2$ and the expenditure share for the differentiated goods sector is $\mu = 1/2$. The other exogenous parameters are given below

$$w_1 = 1, \quad w_2 = 1, \quad L_1 = 1, \quad L_2 = 1, \quad \phi_1 = 2, \quad \phi_2 = 1$$

This implies

$$p_{11} = p'_{11} = 1, \quad p_{22} = p'_{22} = 2, \quad p_{12} = \frac{\sigma}{\sigma - 1} \frac{w_2 \tau_{12}}{\phi_2} = 4, \quad p_{21} = \frac{\sigma}{\sigma - 1} \frac{w_1 \tau_{21}}{\phi_1} = 1, \quad p'_{12} = \frac{\sigma}{\sigma - 1} \frac{w_2 \tau'_{12}}{\phi_2} = 2$$

Now we will look at the equilibria that can arise given different values for (f_{12}, f_{21}) , i.e. we will partition the $f_{12} - f_{21}$ space into the regions where the four cases $(e_{12}, e_{21}) \in \{(0, 0), (0, 1), (1, 0), (1, 1)\}$ apply. We will do this both pre-reform and post-reform. Note: In general, one should consider not only four cases, but sixteen cases, as entry decisions e_{11}, e_{22} are also binary choice variables that can be 0 or 1. To ease exposition, I assume throughout

this illustration that these decisions are $e_{11} = 1$ and $e_{22} = 1$.

Note that given our assumptions on the values of the parameters, the profit for a firm from j exporting to i denoted by π_{ij} is simply given by

$$\pi_{ij} = \mu \frac{1}{\sigma} \left(\frac{p_{ij}}{P_i} \right)^{1-\sigma} w_i L_i (1 + \pi) = \frac{1}{2} \frac{1}{2} \left(\frac{p_{ij}}{P_i} \right)^{-1} (1 + \pi) = \frac{1}{4} \frac{P_i}{p_{ij}} (1 + \pi)$$

This will be useful for the characterization that follows.

9.2 Pre-Reform

9.2.1 Case 1: $(e_{12}, e_{21}) = (0, 0)$

In this equilibrium, firms only operate domestically and there is no entry into international markets either way. Hence we have

$$P_1 = p_{11} = 1, \quad P_2 = p_{22} = 2$$

In this case we also have $\pi = \frac{1}{3}$, and

$$\begin{aligned} \pi_{11} &= \frac{1}{4} (1 + \pi) \\ \pi_{12} &= \frac{1}{4} \frac{P_1}{p_{12}} (1 + \pi) = \frac{1}{16} (1 + \pi) = \frac{1}{16} \frac{4}{3} = \frac{1}{12} \leq f_{12} \\ \pi_{21} &= \frac{1}{4} \frac{P_2}{p_{21}} (1 + \pi) = \frac{1}{2} (1 + \pi) = \frac{2}{3} \leq f_{21} \\ \pi_{22} &= \frac{1}{4} \frac{P_2}{p_{22}} (1 + \pi) = \frac{1}{4} (1 + \pi) = \frac{1}{3} \end{aligned}$$

9.2.2 Case 2: $(e_{12}, e_{21}) = (0, 1)$

In this equilibrium, country 2 firms enter (i.e. exports to) country 1 but not the other way around. Hence we have

$$P_1 = p_{11} = 1, \quad P_2 = [p_{21}^{-1} + p_{22}^{-1}]^{-1} = \frac{2}{3}$$

In this case we have

$$\begin{aligned}\pi_{11} &= \frac{1}{4}(1 + \pi) \\ \pi_{12} &= \frac{1}{16}(1 + \pi) \\ \pi_{21} &= \frac{1}{4} \frac{2/3}{1}(1 + \pi) = \frac{1}{6}(1 + \pi) \\ \pi_{22} &= \frac{1}{4} \frac{2/3}{2} = \frac{1}{12}(1 + \pi)\end{aligned}$$

Hence

$$\pi = \frac{\frac{3+2+1}{12}(1 + \pi) - f_{21}}{2} = \frac{1}{3} - \frac{2}{3}f_{21}$$

It is instructive to note that

$$\begin{aligned}\pi_{\max} &= \frac{1}{3}, \quad f_{21} = 0 \\ \pi_{\min} &= \frac{1}{6}(1 + \pi_{\min}) \Rightarrow \pi_{\min} = \frac{1}{5}, \quad f_{21} = \frac{1}{5}\end{aligned}$$

In general

$$\pi_{21} = \frac{1}{16} \left(\frac{4}{3} - \frac{2}{3}f_{21} \right) = \frac{1}{12} - \frac{1}{24}f_{21} \leq f_{12}, \text{ for } f_{21} \in (0, \frac{1}{5})$$

9.2.3 Case 3: $(e_{12}, e_{21}) = (1, 0)$

In this equilibrium, country 1 firms enter (i.e. exports to) country 2 but not the other way around. Hence we have

$$P_1 = [p_{11}^{-1} + p_{12}^{-1}]^{-1} = \frac{4}{5}, \quad P_2 = p_{22} = 2$$

In this case we have

$$\begin{aligned}\pi_{11} &= \frac{1}{4} \frac{P_1}{p_{11}}(1 + \pi) = \frac{1}{4} \frac{4}{5}(1 + \pi) = \frac{1}{5}(1 + \pi) \\ \pi_{12} &= \frac{1}{4} \frac{P_1}{p_{12}}(1 + \pi) = \frac{1}{4} \frac{4/5}{4}(1 + \pi) = \frac{1}{20}(1 + \pi) \\ \pi_{21} &= \frac{1}{4} \frac{P_2}{p_{21}}(1 + \pi) = \frac{1}{4} \frac{2}{1}(1 + \pi) = \frac{1}{2}(1 + \pi) \\ \pi_{22} &= \frac{1}{4} \frac{P_2}{p_{22}}(1 + \pi) = \frac{1}{4}(1 + \pi)\end{aligned}$$

Hence

$$\pi = \frac{\frac{5+4+1}{20}(1+\pi) - f_{12}}{2} = \frac{1}{3} - \frac{2}{3}f_{12}$$

It is instructive to note that

$$\begin{aligned}\pi_{\max} &= \frac{1}{3}, \quad f_{12} = 0 \\ \pi_{\min} &= \frac{\frac{5+4}{20}(1+\pi)}{2} = \frac{9}{40}(1+\pi_{\min}) \Rightarrow \pi_{\min} = \frac{9}{31}, \quad f_{12} = \frac{2}{31}\end{aligned}$$

In general

$$\pi_{21} = \frac{1}{2}\left(\frac{4}{3} - \frac{2}{3}f_{12}\right) = \frac{2}{3} - \frac{1}{3}f_{12} \leq f_{21}, \text{ for } f_{12} \in (0, \frac{2}{13})$$

9.2.4 Case 4: $(e_{12}, e_{21}) = (1, 1)$

In this equilibrium, country 1 firms enter (i.e. exports to) country 2 and vice versa. Hence we have

$$P_1 = [p_{11}^{-1} + p_{12}^{-1}]^{-1} = \frac{4}{5}, \quad P_2 = [p_{21}^{-1} + p_{22}^{-1}]^{-1} = \frac{2}{3}$$

In this case we have

$$\begin{aligned}\pi_{11} &= \frac{1}{5}(1+\pi) \\ \pi_{12} &= \frac{1}{20}(1+\pi) \\ \pi_{21} &= \frac{1}{6}(1+\pi) \\ \pi_{22} &= \frac{1}{12}(1+\pi)\end{aligned}$$

Hence

$$\pi = \frac{1}{3} - \frac{2}{3}(f_{12} + f_{21})$$

$$\pi_{\max} = \frac{1}{3}, \quad f_{12} = 0, \quad f_{21} = 0$$

$$\pi_{\min} = \frac{\frac{5+12}{60}(1+\pi)}{2} = \frac{17}{120}(1+\pi_{\min}) \Rightarrow \pi_{\min} = \frac{17}{103}, \quad f_{21} = \frac{1}{20} \frac{120}{103} = \frac{6}{103}, \quad f_{12} = \frac{1}{6} \frac{120}{103} = \frac{20}{103}$$

In general

$$\begin{aligned}\pi_{12} &= \frac{1}{20}\left(\frac{4}{3} - \frac{2}{3}f_{12} - \frac{2}{3}f_{21}\right) = \frac{2}{30} - \frac{1}{30}f_{12} - \frac{1}{30}f_{21} \leq f_{12} \\ \pi_{21} &= \frac{1}{6}\left(\frac{4}{3} - \frac{2}{3}f_{12} - \frac{2}{3}f_{21}\right) = \frac{2}{9} - \frac{1}{9}f_{12} - \frac{1}{9}f_{21} \leq f_{21}\end{aligned}$$

This implies

$$\begin{aligned} f_{12} &\leq \frac{30}{31} \left(\frac{2}{30} - \frac{1}{30} f_{21} \right) \\ f_{21} &\leq \frac{9}{10} \left(\frac{2}{9} - \frac{1}{9} f_{12} \right) \end{aligned}$$

Hence the endpoints are $\{(\frac{2}{31}, 0), (0, \frac{2}{10})\}$.

9.3 Post-Reform

9.3.1 Case 1: $(e'_{12}, e'_{21}) = (0, 0)$

In this equilibrium, firms only operate domestically and there is no entry into international markets either way. Hence we have

$$P'_1 = p'_{11} = 1, \quad P'_2 = p'_{22} = 2$$

In this case we also have $\pi' = \frac{1}{3}$, and

$$\begin{aligned} \pi'_{11} &= \frac{1}{4}(1 + \pi') \\ \pi'_{12} &= \frac{1}{4} \frac{P'_1}{p'_{12}}(1 + \pi') = \frac{1}{8}(1 + \pi') = \frac{1}{8} \frac{4}{3} = \frac{1}{6} \leq f_{12} \\ \pi'_{21} &= \frac{1}{4} \frac{P'_2}{p'_{21}}(1 + \pi') = \frac{1}{2}(1 + \pi') = \frac{2}{3} \leq f_{21} \\ \pi'_{22} &= \frac{1}{4} \frac{P'_2}{p'_{22}} = \frac{1}{4}(1 + \pi') = \frac{1}{3} \end{aligned}$$

9.3.2 Case 2: $(e'_{12}, e'_{21}) = (0, 1)$

In this equilibrium, country 2 firms enter (i.e. exports to) country 1 but not the other way around. Hence we have

$$P'_1 = p'_{11} = 1, \quad P'_2 = [p'^{-1}_{21} + p'^{-1}_{22}]^{-1} = \frac{2}{3}$$

In this case we have

$$\begin{aligned}\pi'_{11} &= \frac{1}{4}(1 + \pi') \\ \pi'_{12} &= \frac{1}{8}(1 + \pi') \\ \pi'_{21} &= \frac{1}{6}(1 + \pi') \\ \pi'_{22} &= \frac{1}{12}(1 + \pi')\end{aligned}$$

Hence

$$\pi' = \frac{1}{3} - \frac{2}{3}f_{21}$$

It is instructive to note that

$$\begin{aligned}\pi'_{\max} &= \frac{1}{3}, & f_{21} &= 0 \\ \pi'_{\min} &= \frac{1}{5}, & f_{21} &= \frac{1}{5}\end{aligned}$$

In general

$$\pi'_{21} = \frac{1}{8}\left(\frac{4}{3} - \frac{2}{3}f_{21}\right) = \frac{1}{6} - \frac{1}{12}f_{21} \leq f_{12}, \text{ for } f_{21} \in (0, \frac{1}{5})$$

9.3.3 Case 3: $(e'_{12}, e'_{21}) = (1, 0)$

In this equilibrium, country 1 firms enter (i.e. exports to) country 2 but not the other way around. Hence we have

$$P'_1 = [p'^{-1}_{11} + p'^{-1}_{12}]^{-1} = \frac{2}{3}, \quad P'_2 = p'_{22} = 2$$

In this case we have

$$\begin{aligned}\pi'_{11} &= \frac{1}{6}(1 + \pi') \\ \pi'_{12} &= \frac{1}{12}(1 + \pi') \\ \pi'_{21} &= \frac{1}{2}(1 + \pi') \\ \pi'_{22} &= \frac{1}{4}(1 + \pi')\end{aligned}$$

Hence

$$\pi' = \frac{\frac{2+1+13}{12}(1 + \pi') - f_{12}}{2} = \frac{1}{3} - \frac{2}{3}f_{12}$$

It is instructive to note that

$$\begin{aligned}\pi'_{\max} &= \frac{1}{3}, \quad f_{12} = 0 \\ \pi'_{\min} &= \frac{5}{24}(1 + \pi'_{\min}) \Rightarrow \pi'_{\min} = \frac{5}{19}, \quad f_{12} = \frac{2}{19}\end{aligned}$$

In general

$$\pi'_{21} = \frac{1}{2} \left(\frac{4}{3} - \frac{2}{3} f_{12} \right) = \frac{2}{3} - \frac{1}{3} f_{12} \leq f_{21}, \text{ for } f_{12} \in (0, \frac{2}{19})$$

9.3.4 Case 4: $(e_{12}, e_{21}) = (1, 1)$

In this equilibrium, country 1 firms enter (i.e. exports to) country 2 and vice versa. Hence we have

$$P'_1 = [p'^{-1}_{11} + p'^{-1}_{12}]^{-1} = \frac{2}{3}, \quad P'_2 = [p'^{-1}_{21} + p'^{-1}_{22}]^{-1} = \frac{2}{3}$$

In this case we have

$$\begin{aligned}\pi'_{11} &= \frac{1}{6}(1 + \pi') \\ \pi'_{12} &= \frac{1}{12}(1 + \pi') \\ \pi'_{21} &= \frac{1}{6}(1 + \pi') \\ \pi'_{22} &= \frac{1}{12}(1 + \pi')\end{aligned}$$

Hence

$$\pi' = \frac{1}{3} - \frac{2}{3}(f_{12} + f_{21})$$

$$\begin{aligned}\pi'_{\max} &= \frac{1}{3}, \quad f_{12} = 0, \quad f_{21} = 0 \\ \pi'_{\min} &= \frac{1}{8}(1 + \pi'_{\min}) \Rightarrow \pi'_{\min} = \frac{1}{7}, \quad f_{21} = \frac{1}{6} \frac{8}{7} = \frac{4}{21}, \quad f_{12} = \frac{1}{12} \frac{8}{7} = \frac{2}{21}\end{aligned}$$

In general

$$\begin{aligned}\pi'_{12} &= \frac{1}{12} \left(\frac{4}{3} - \frac{2}{3} f_{12} - \frac{2}{3} f_{21} \right) = \frac{1}{9} - \frac{1}{18} f_{12} - \frac{1}{18} f_{21} \leq f_{12} \\ \pi'_{21} &= \frac{1}{6} \left(\frac{4}{3} - \frac{2}{3} f_{12} - \frac{2}{3} f_{21} \right) = \frac{2}{9} - \frac{1}{9} f_{12} - \frac{1}{9} f_{21} \leq f_{21}\end{aligned}$$

This implies

$$\begin{aligned} f_{12} &\leq \frac{18}{19} \left(\frac{1}{9} - \frac{1}{18} f_{21} \right) \\ f_{21} &\leq \frac{9}{10} \left(\frac{2}{9} - \frac{1}{9} f_{12} \right) \end{aligned}$$

Hence the endpoints are $\{(\frac{2}{19}, 0), (0, \frac{2}{10})\}$.

9.4 Equilibria Pre- and Post-Reform

Given the characterization of the two previous sections, we can partition the $f_{12} - f_{21}$ space into the following regions.

Pre-reform, we have the five regions P_i , $i = 0, 1, \dots, 4$ where P_i corresponds to the region that applies to the i -th case described earlier (e.g. P_2 corresponds to $(e_{12}, e_{21}) = (0, 1)$), and the 0-th case is the region in the parameter space where no equilibria exist.

$$\begin{aligned} P_1 &= \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 1/12, f_{21} \geq 2/3\} \\ P_2 &= \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 1/12 - 1/24 f_{21}, f_{21} \leq 1/5\} \\ P_3 &= \{(f_{12}, f_{21}) \in R_+^2 : f_{21} \geq 2/3 - 1/3 f_{12}, f_{12} \leq 2/31\} \\ P_4 &= \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 30/31(2/30 - 1/30 f_{21}), f_{21} \geq 9/10(2/9 - 1/9 f_{12})\} \\ P_0 &= R_+^2 \setminus \{P_1 \cup P_2 \cup P_3 \cup P_4\} \end{aligned}$$

Post-reform, the fixed cost space is partitioned into the following regions

$$\begin{aligned} Q_1 &= \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 1/6, f_{21} \geq 2/3\} \\ Q_2 &= \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 1/6 - 1/12 f_{21}, f_{21} \leq 1/5\} \\ Q_3 &= \{(f_{12}, f_{21}) \in R_+^2 : f_{21} \geq 2/3 - 1/3 f_{12}, f_{12} \leq 2/19\} \\ Q_4 &= \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 18/19(1/9 - 1/18 f_{21}), f_{21} \geq 9/10(2/9 - 1/9 f_{12})\} \\ Q_0 &= R_+^2 \setminus \{Q_1 \cup Q_2 \cup Q_3 \cup Q_4\} \end{aligned}$$

Then combining these two partitions of the fixed-cost space, we get the following regions:

$$\begin{aligned}
A_{11} &= P_1 \cap Q_1 = \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 1/6, f_{21} \geq 2/3\} \\
A_{13} &= P_1 \cap Q_3 = \{(f_{12}, f_{21}) \in R_+^2 : 1/12 \leq f_{12} \leq 2/19, f_{21} \geq 2/3\} \\
A_{33} &= P_3 \cap Q_3 = \{(f_{12}, f_{21}) \in R_+^2 : f_{21} \geq 2/3 - 1/3f_{12}, f_{12} \leq 2/31\} \\
A_{22} &= P_2 \cap Q_2 = \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 1/6 - 1/12f_{12}, f_{21} \leq 1/5\} \\
A_{24} &= P_2 \cap Q_4 = \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 1/12 - 1/24f_{21}, f_{21} \leq 1/5\} \\
&\quad \cap \{(f_{12}, f_{21}) : f_{12} \leq 18/19(1/9 - 1/18f_{21}), f_{21} \leq 9/10(2/9 - 1/9f_{12})\} \\
A_{44} &= P_4 \cap Q_4 = \{(f_{12}, f_{21}) \in R_+^2 : f_{21} \leq 9/10(2/9 - 1/9f_{12}), f_{12} \geq 30/31(2/30 - 1/30f_{12})\} \\
A_{10} &= P_1 \cap Q_0 = \{(f_{12}, f_{21}) \in R_+^2 : 2/19 \leq f_{12} \leq 1/6, f_{21} \geq 2/3\} \\
A_{20} &= P_2 \cap Q_0 = P_2 \setminus \{A_{24} \cup A_{22}\} \\
A_{03} &= P_0 \cap Q_3 = Q_3 \setminus \{A_{33} \cup A_{13}\} \\
A_{04} &= P_0 \cap Q_4 = Q_4 \setminus \{A_{44} \cup A_{24}\} \\
A_{12} &= A_{14} = A_{21} = A_{23} = A_{31} = A_{32} = A_{34} = A_{41} = A_{42} = A_{43} = \{\} \\
A_{01} &= A_{02} = A_{30} = A_{40} = \{\} \\
A_{00} &= R_+^2 \setminus \{A_{11} \cup A_{13} \cup A_{33} \cup A_{22} \cup A_{24} \cup A_{44} \cup A_{10} \cup A_{20} \cup A_{03} \cup A_{04}\}
\end{aligned}$$

Here A_{ij} means that pre-reform, the equilibrium is Case i and post-reform, the equilibrium is Case j for any $(f_{12}, f_{21}) \in A_{ij}$. Note that there are many sets A_{ij} that are empty, in which case no exact equilibria exist, both pre- and post-reform. There are two regions A_{ij} where $i \neq j$ and $i, j \neq 0$: A_{24} and A_{13} . These are interesting because they represent the equilibria where a change in entry patterns following the policy reform. A_{13} means that after lowering the iceberg cost to export from 2 to 1, the entry pattern changed from Case 1, i.e. $(e_{12}, e_{21}) = (0, 0)$, to Case 3, i.e. $(e_{12}, e_{21}) = (1, 0)$, so the policy change induces country 2 to start exporting the differentiated good to 1, even though firms from 1 still do not export to 2. The A_{24} equilibrium is analogous in that the policy induces firms from 2 to start exporting, but in this case, both before and after the reform, firms from country 1 export to country 2. There are four nonempty sets A_{ii} where there is no change in the entry patterns after the reform, so we can expect the effect of the extensive margin to be minimal. The other non-empty sets A_{ij} have either $i = 0$ or $j = 0$; this means that before or after the reform, equilibria may fail to exist.

9.5 The Algorithm at Work: An Approximate Equilibrium

As before, suppose there are two countries $i = 1, 2$. Each has a measure $\theta_i = w_i L_i$ of firms that are considering which markets to enter. For simplicity consider only an environment with trade (i.e. no multinational production). An example allowing for multinational production is available upon request. International trade is subject to both iceberg and fixed costs, which can be asymmetric across country pairs. Consider a policy reform that lowers the iceberg costs for firms from 2 to export to 1 from $\tau_{12} = 2$ to $\tau'_{12} = 1$. Let $\tau_{21} = \tau'_{21} = \tau_{11} = \tau'_{11} = \tau_{22} = \tau'_{22} = 1$. The trade elasticity is $\sigma = 2$ and the expenditure share for the differentiated goods sector is $\mu = 1/2$. The other exogenous parameters are given below

$$w_1 = 1, \quad w_2 = 1, \quad L_1 = 1, \quad L_2 = 1, \quad \phi_1 = 2, \quad \phi_2 = 1$$

This implies

$$p_{11} = p'_{11} = 1, \quad p_{22} = p'_{22} = 2, \quad p_{12} = \frac{\sigma}{\sigma - 1} \frac{w_2 \tau_{12}}{\phi_2} = 4, \quad p_{21} = \frac{\sigma}{\sigma - 1} \frac{w_1 \tau_{21}}{\phi_1} = 1, \quad p'_{12} = \frac{\sigma}{\sigma - 1} \frac{w_2 \tau'_{12}}{\phi_2} = 2$$

Define

$$\begin{aligned} Q_{11} &= \mu \frac{1}{\sigma} \left(\frac{p_{11}}{P_1} \right)^{1-\sigma} = \mu \frac{1}{\sigma} \left(\frac{P_1}{p_{11}} \right) = \frac{1}{4} \\ Q_{21} &= \mu \frac{1}{\sigma} \left(\frac{p_{21}}{P_2} \right)^{1-\sigma} = \mu \frac{1}{\sigma} \left(\frac{P_2}{p_{21}} \right) = \frac{1}{6} \\ Q_{22} &= \mu \frac{1}{\sigma} \left(\frac{p_{22}}{P_2} \right)^{1-\sigma} = \mu \frac{1}{\sigma} \left(\frac{P_2}{p_{22}} \right) = \frac{1}{12} \end{aligned}$$

Then

$$\pi = \frac{\pi_{11} + \pi_{21} + \pi_{22} - 3\epsilon}{2} = \frac{\left(\frac{1}{4} + \frac{1}{6} + \frac{1}{12}\right)(1 + \pi) - 3\epsilon}{2} = \frac{1}{3} - 2\epsilon$$

Hence the restrictions are given by

$$\pi_{ij} > \epsilon \Leftrightarrow Q_{ij}(1 + \pi) > \epsilon \Leftrightarrow Q_{ij} \left(\frac{4}{3} - 2\epsilon \right) > \epsilon \Leftrightarrow \epsilon < \frac{\frac{4}{3}Q_{ij}}{1 + 2Q_{ij}}$$

Applied to the three relevant cases, this yields the restrictions

$$\begin{aligned}\epsilon &< \frac{\frac{1}{3}}{1 + \frac{1}{2}} = \frac{2}{9} \\ \epsilon &< \frac{\frac{2}{9}}{1 + \frac{1}{3}} = \frac{1}{6} \\ \epsilon &< \frac{\frac{1}{9}}{1 + \frac{1}{6}} = \frac{2}{21}\end{aligned}$$

Let $\epsilon = \frac{1}{20}$. Then we have $\pi = \frac{1}{3} - \frac{1}{10} = \frac{7}{30}$. Then the baseline parameterization is

$$f_{12} = \pi_{12} + \epsilon = \frac{61}{480}, \quad f_{11} = f_{22} = f_{21} = \epsilon = \frac{1}{20}$$

To show that there is no exact equilibrium after reform, note that either $e'_{12} = 1$ or $e'_{12} = 0$.

Consider first the case with $e'_{12} = 1$. We have $\pi' \in [\frac{1}{3} - \frac{8}{3}\frac{1}{20}, \frac{1}{3}]$, and $P_1 \in \{2, \frac{2}{3}\}$, depending on the entry decisions of other agents. If $P'_1 = 2$ and $e'_{11} = 0$, we must have $\pi'_{11} < f_{11}$ and $\pi'_{12} > f_{12}$. But $\pi'_{11} = \frac{1}{4}\frac{2}{1}(1 + \pi') = \frac{1}{2}(1 + \pi') \geq \frac{1}{2}(\frac{4}{3} - \frac{8}{3}\frac{1}{20}) = \frac{9}{15} > \frac{1}{20} = f_{11}$, a contradiction. If, on the other hand, $P'_1 = \frac{2}{3}$ and $e'_{11} = 1$, we must have $\pi'_{11} > f_{11}$ and $\pi'_{12} > f_{12}$. But in this case we have $\pi'_{12} = \frac{1}{4}\frac{2}{2}(1 + \pi') = \frac{1}{2}(1 + \pi') \leq \frac{1}{2}\frac{4}{3} = \frac{1}{3} < \frac{61}{480} = f_{12}$, a contradiction. Hence there is no exact equilibrium when $e'_{12} = 1$.

Then for exact equilibrium to exist, we must have $e'_{12} = 0$. In this case we need only consider the case when $P' = 1$ and $e'_{11} = 1$ because otherwise $P'_1 = 0$ (when $e'_{11} = 0$) and demand is not well-defined. If $e'_{12} = 0$, $P' = 1$, and $e'_{11} = 1$, we must have $\pi'_{11} > f_{11}$ and $\pi'_{12} < f_{12}$. But $\pi'_{12} = \frac{1}{4}\frac{1}{2}(1 + \pi') \geq \frac{1}{8}(\frac{4}{3} - 2\frac{1}{20}) = \frac{1}{8}\frac{37}{30} = \frac{74}{480} > \frac{61}{480} = f_{12}$, a contradiction. Hence, exact equilibrium does not exist given this parameterization.

With no exact equilibrium, I now proceed to compute an approximate equilibrium. As in the algorithm, define the sets

$$\begin{aligned}\Omega_{i1} &= \{j : \tau'_{ij} = \tau_{ij}, e_{ij} = 0\}, & \Omega_{i2} &= \{j : \tau'_{ij} = \tau_{ij}, e_{ij} = 1\} \\ \Omega_{i3} &= \{j : \tau'_{ij} < \tau_{ij}, e_{ij} = 0\}, & \Omega_{i4} &= \{j : \tau'_{ij} < \tau_{ij}, e_{ij} = 1\}\end{aligned}$$

There are $4N = 8$ sets. For this example these sets are

$$\begin{aligned}\Omega_{11} &= \{\}, & \Omega_{12} &= \{1\}, & \Omega_{13} &= \{\}, & \Omega_{14} &= \{2\} \\ \Omega_{21} &= \{\}, & \Omega_{22} &= \{1, 2\}, & \Omega_{23} &= \{\}, & \Omega_{24} &= \{\}\end{aligned}$$

Suppose the initial cutoffs are $k_{ij} = 100$, $\forall i = 1, 2$ and $j = 1, 2, 3, 4$. Given the cutoffs we have that the guess for the post-reform entry patterns are given by

$$E = \begin{bmatrix} e'_{11} & e'_{12} \\ e'_{21} & e'_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

This yields $P'_1 = \frac{2}{3} = P'_2$ and $\pi' = \frac{1}{3} - \frac{8}{3}\epsilon = \frac{1}{5}$. This in turn yields

$$\begin{aligned}\pi'_{11} &= \frac{1}{6}(1 + \pi') = \frac{1}{5} > \frac{1}{20} = f_{11} \\ \pi'_{12} &= \frac{1}{12}(1 + \pi') = \frac{1}{10} < \frac{61}{480} = f_{12} \\ \pi'_{21} &= \frac{1}{6}(1 + \pi') = \frac{1}{5} > \frac{1}{20} = f_{21} \\ \pi'_{22} &= \frac{1}{12}(1 + \pi') = \frac{1}{10} > \frac{1}{20} = f_{22}\end{aligned}$$

Hence the equilibrium is only 75% accurate.

There are no updates to the cutoffs of sets that are empty: $k_{11}^{(2)} = k_{13}^{(2)} = k_{21}^{(2)} = k_{23}^{(2)} = k_{24}^{(2)} = 100$ because $\hat{\Omega}_{11}^n = \hat{\Omega}_{13}^n = \hat{\Omega}_{21}^n = \hat{\Omega}_{23}^n = \hat{\Omega}_{24}^n = \{\}$. Further, $\hat{\Omega}_{12}^n = \{1\}$, $\hat{\Omega}_{14}^n = \{\}$, $\hat{\Omega}_{22}^n = \{1, 2\}$ yields $k_{12}^n = 100$, $k_{14}^n = 0$, $k_{22}^n = 100$. Let the Newton step be $\lambda = 0.2$. Then the guess for the next iteration is $k_{12}^{(2)} = \lambda k_{12}^n + (1 - \lambda)k_{12}^{(1)} = 100$, $k_{14}^{(2)} = \lambda k_{14}^n + (1 - \lambda)k_{14}^{(1)} = 80$, $k_{22}^{(2)} = 100$.

Because $k_{12}^{(2)} = 80$, we have

$$E^{(2)} = \begin{bmatrix} e'^{(2)}_{11} & e'^{(2)}_{12} \\ e'^{(2)}_{21} & e'^{(2)}_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Which yields $P_1'^{(2)} = 1$, $P_2'^{(2)} = \frac{2}{3}$ and $\pi' = \frac{7}{30}$. This in turn yields

$$\begin{aligned}\pi'_{11} &= \frac{1}{4}(1 + \pi') = \frac{37}{120} > \frac{1}{20} = f_{11} \\ \pi'_{12} &= \frac{1}{8}(1 + \pi') = \frac{74}{480} > \frac{61}{480} = f_{12} \\ \pi'_{21} &= \frac{1}{6}(1 + \pi') = \frac{37}{180} > \frac{1}{20} = f_{21} \\ \pi'_{22} &= \frac{1}{12}(1 + \pi') = \frac{37}{360} > \frac{1}{20} = f_{22}\end{aligned}$$

As before, the equilibrium is only 75% accurate.

Likewise, there are no updates to the cutoffs of sets that are empty: $k_{11}^{(3)} = k_{13}^{(3)} = k_{21}^{(3)} = k_{23}^{(3)} = k_{24}^{(3)} = 100$ because $\hat{\Omega}_{11}^n = \hat{\Omega}_{13}^n = \hat{\Omega}_{21}^n = \hat{\Omega}_{23}^n = \hat{\Omega}_{24}^n = \{\}$. Further, $\hat{\Omega}_{12}^n = \{1\}$, $\hat{\Omega}_{14}^n = \{2\}$, $\hat{\Omega}_{22}^n = \{1, 2\}$ yields $k_{12}^n = 100$, $k_{14}^n = 100$, $k_{22}^n = 100$. Then the guess for the next iteration is $k_{12}^{(3)} = \lambda k_{12}^n + (1 - \lambda)k_{12}^{(2)} = 100$, $k_{14}^{(3)} = \lambda k_{14}^n + (1 - \lambda)k_{14}^{(2)} = 84$, $k_{22}^{(3)} = 100$.

Because $k_{12}^{(3)} = 84$, we have

$$E^{(3)} = \begin{bmatrix} e_{11}'^{(3)} & e_{12}'^{(3)} \\ e_{21}'^{(3)} & e_{22}'^{(3)} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Resulting in the same price indices and dividend per share as the second iteration: $P_1'^{(3)} = 1$, $P_2'^{(3)} = \frac{2}{3}$ and $\pi' = \frac{7}{30}$. Then as above we obtain $k_{12}^{(4)} = 100$, $k_{14}^{(4)} = \lambda k_{14}^n + (1 - \lambda)k_{14}^{(3)} = 87.2$, $k_{22}^{(4)} = 100$.

Iterating in this fashion, we find $k_{14}^{(t)} = \lambda \cdot 100 + (1 - \lambda)k_{14}^{(t-1)}$ which implies

$$\begin{aligned}k_{14}^{(t)} - k_{14}^{(t-1)} &= 0.2(100 - k_{14}^{(t-1)}) \\ \Rightarrow k_{14}^{(t)} &> k_{14}^{(t-1)}, \quad \|k_{14}^{(t+1)} - k_{14}^{(t)}\| < \|k_{14}^{(t)} - k_{14}^{(t-1)}\|\end{aligned}$$

So that $\|k_{14}^{(t)} - k_{14}^{(t-1)}\| \rightarrow 0$ as $t \rightarrow \infty$. Suppose we set the tolerance to be 0.1. Then when does the iteration stop? We have

$$\begin{aligned} k_{14}^{(5)} &= \lambda k_{14}^n + (1 - \lambda)k_{14}^{(4)} = 89.60 \\ k_{14}^{(6)} &= \lambda k_{14}^n + (1 - \lambda)k_{14}^{(5)} = 91.81 \\ k_{14}^{(7)} &= \lambda k_{14}^n + (1 - \lambda)k_{14}^{(6)} = 93.45 \\ &\dots \\ k_{14}^{(19)} &= \lambda k_{14}^n + (1 - \lambda)k_{14}^{(18)} = 99.55 \\ k_{14}^{(20)} &= \lambda k_{14}^n + (1 - \lambda)k_{14}^{(19)} = 99.64 \end{aligned}$$

And because $\|k_{14}^{(20)} - k_{14}^{(19)}\| < 0.1$, the process stops and we settle on an approximate equilibrium that is 75% accurate, which in this case is the highest percentage possible given the number of available country pairs. It is easy to verify that the cutoffs for the other sets remain unchanged in the iterations leading up to algorithmic termination.