

Capital Unemployment, Financial Shocks, and Investment Slumps*

JOB MARKET PAPER

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January 27, 2015

Abstract

Recoveries from financial crises are characterized by low investment rates and declines in capital stocks. This paper constructs an equilibrium framework in which financial shocks have a persistent effect on aggregate investment. The key assumption is that physical capital is traded in a decentralized market with search frictions, generating “capital unemployment.” After a negative financial shock, the share of unemployed capital is high, and the economy dedicates more resources to absorbing existing unemployed capital into production, and less to accumulating new capital. An estimation of the model for the U.S. economy using Bayesian techniques shows that the model can generate the investment persistence and half of the output persistence observed in the Great Recession. Investment search frictions also lead to a different interpretation of the sources of business-cycle fluctuations, with a larger role for financial shocks, which account for 33% of output fluctuations. Extending the model to allow for heterogeneity in match productivity, the framework also provides a mechanism for procyclical capital reallocation, as observed in the data.

JEL Classification: E22, E23, E44, E32, D53

Keywords: Financial Shocks, Investment Dynamics, Search Frictions, Business Cycles, Capital Reallocation, Great Recession

*I thank my advisors, Martin Uribe, Stephanie Schmitt-Grohé, Guillermo Calvo, Michael Woodford, and Ricardo Reis for guidance and support. For very useful comments and suggestions, I thank Jennifer La’O, Emi Nakamura, Jaromir Nosal, Bernard Salanié, Jon Steinson, Stefania Albanesi, Sushant Acharya, Robert Barro, Javier Bianchi, Saki Bigio, Patrick Bolton, Ryan Chahrour, Bianca De Paoli, Keshav Dogra, Valentina Duque, Axelle Ferriere, Ana Fostel, Mariana Garcia, Marc Giannoni, Linda Goldberg, Fatih Karahan, Thomas Klitgaard, Sergey Kolbin, Wataru Miyamoto, Tommaso Monacelli, Thuy Lan Nguyen, Friederike Niepmann, Nate Pattison, Diego Perez, Paolo Pesenti, Laura Pilossoph, Benjamin Pugsley, Aysegul Sahin, Argia Sbordone, Joseph Stiglitz, Ernesto Talvi, Andrea Tambalotti, and seminar participants at Columbia University, the Federal Reserve Bank of New York and the Federal Reserve Bank of Minneapolis. This paper was partly written at the Federal Reserve Bank of New York, while I was a summer intern. I am especially grateful for their hospitality. Finally, I thank William Wheaton and *Jones Lang LaSalle* for sharing their vacancy-rate data.

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1 Introduction

The U.S. Great Recession was followed by a persistent investment slump: Five years after the trough, investment rates remain below their historical average, and the stock of capital continues to fall with respect to its trend, constituting the most important contributor to persistently low economic activity (see [Hall, 2014](#), and [Figure 1](#)). The low levels of aggregate investment observed during the recovery from the U.S. Great Recession are challenging from the points of view of real and monetary models (see [Kydland and Zarazaga, 2012](#); [Del Negro, Giannoni and Patterson, 2012](#)). According to these large classes of models, the recovery should be characterized by high investment rates and rising stocks of physical capital.

The low-investment pattern exhibited by the recovery from the U.S. Great Recession is a salient characteristic of financial-crisis episodes across time and space. [Figure 1](#) shows evidence from a sample of 100 post-war recession episodes in advanced economies. Recoveries from financial crises are characterized by investment rates below the historical average and by a fall in capital stock with respect to its trend – as observed in the U.S. Great Recession.¹ This pattern is not, in fact, characteristic of the average “regular” recession episode, in which investment rates recover with output and capital stock stabilizes close to its trend.

Motivated by this evidence, this paper constructs a general equilibrium framework in which financial shocks lead to investment slumps. The key idea in the model is that the production of new capital is affected by existing “capital unemployment” (i.e., owners of idle units of capital unable to find a firm willing to buy or rent these units to produce). After a negative financial shock (i.e., shocks to the net worth of the business sector or the risk of business projects), the share of unemployed capital is high; the economy, then, can achieve a better allocation by directing more resources to absorb existing unemployed capital into the production process and directing fewer resources to the accumulation of new capital, leading to low investment rates even after the shock has dissipated. The model’s main assumption, which leads to equilibrium capital unemployment, is that trade in physical capital occurs in a decentralized market characterized by search frictions, capturing costs that firms face when matching capital to business projects.

¹A “financial crisis” is defined as a recession episode in which a banking crisis event (as defined in [Reinhart and Rogoff, 2009a](#)) takes place between the output peak and recovery point. [Appendix A](#) describes the sample construction and data used. The finding of investment lagging behind output recovery has been documented by [Calvo, Izquierdo and Talvi \(2006\)](#) in a sample of emerging-market sudden-stop crisis episodes. For other empirical studies characterizing financial-crisis episodes, see [Cerra and Saxena \(2008\)](#) and [Reinhart and Rogoff \(2009b, 2014\)](#).

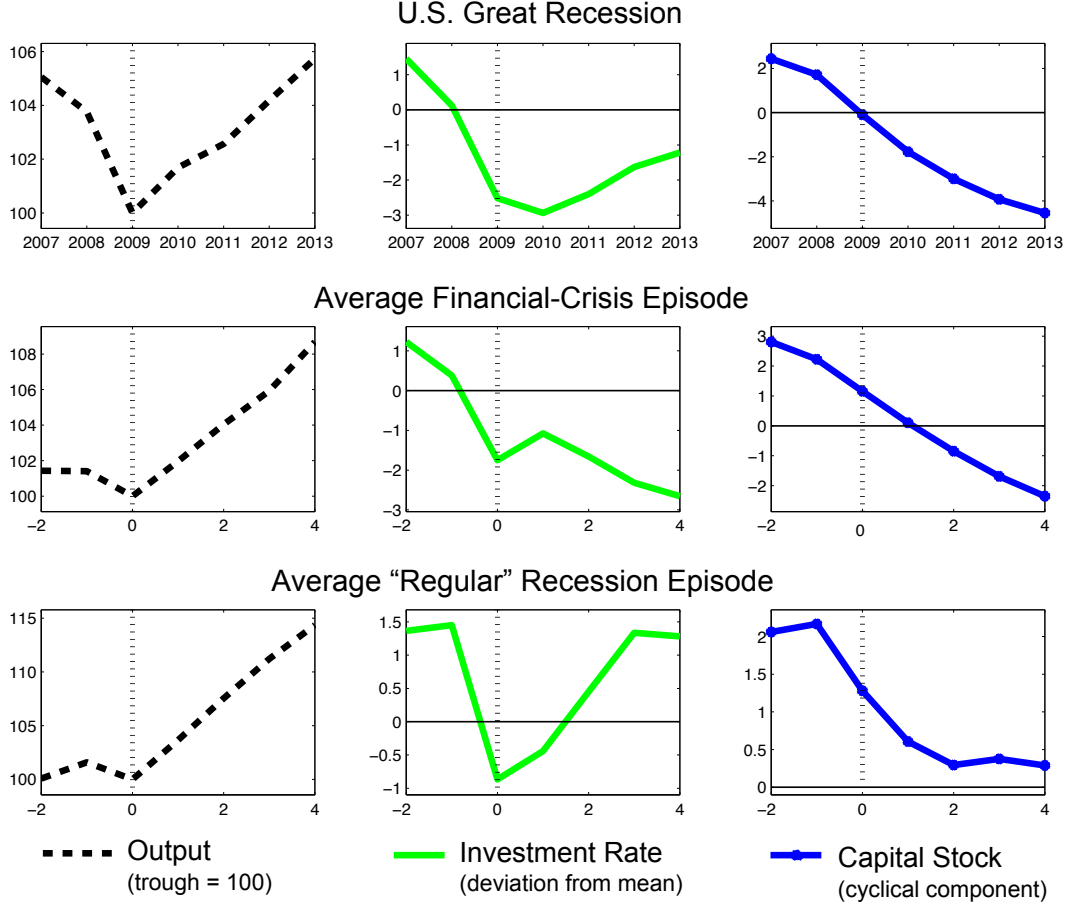


FIGURE 1: FINANCIAL CRISES AND INVESTMENT SLUMPS.

Note: Output refers to real, per capita, gross domestic product; investment rate refers to the ratio of gross fixed capital formation to gross domestic product; capital stock refers to the net stock of fixed assets. The sample of recession episodes includes 20 “financial crisis” episodes and 80 “regular” recession episodes for 22 advanced economies in the period 1950–2006. For each country included in the sample, investment rates are expressed in percent deviation from the mean over 1950–2013; the capital stock is expressed in percent deviation from a log-quadratic trend. In each recession episode, $t = 0$ denotes the output trough. The time unit for average episodes is a year. See Appendix A for a description of the sample and data.

To assess the quantitative importance of the proposed mechanism, the paper constructs a stochastic business-cycle model with investment search frictions and capital unemployment. The model is estimated for the U.S. economy using Bayesian techniques and data prior to the U.S. Great Recession. It is shown that following a sequence of shocks such as those experienced by the U.S. economy in 2008 – and with no further shocks – the model predicts the persistence of aggregate investment and at least half of the output persistence observed in the aftermath of the U.S. Great Recession. Conducting the same exercise in a benchmark model without investment search frictions, the model predicts that both investment and output should be significantly higher than the levels observed in the data, as noted in the previous literature.

The estimated model is also used to interpret the sources of U.S. business-cycle fluctuations. Results indicate that investment search frictions and capital unemployment are a relevant propagation mechanism for financial shocks: While these shocks account 33% of output fluctuations in the model with investment search frictions, they only account for 1% of output fluctuations in the benchmark real model without investment search frictions. Real models with financial frictions that distort firm purchases of capital can only assign a small role to financial shocks primarily because observed fluctuations in aggregate investment do not imply large fluctuations in the stock of capital, which is the input to the production function (as discussed, for example in [Schwartzman, 2012](#); [Bigio, 2014](#)). In the framework developed in the present paper, the input to the production function is employed capital, which does fluctuate significantly in response to firm purchases of capital following financial shocks. The estimated model disciplines the fluctuations in capital unemployment with data on commercial real estate vacancy rates (office, retail, and industrial space). As shown in [Figure 2](#), the level and fluctuations in this measure of capital unemployment are comparable to those of U.S. labor unemployment.² The estimation attributes most of the fluctuations in capital unemployment to financial shocks, which have a large effect on firms' capital demand.

In the model search is directed, in the sense that sellers and buyers can search offers at a particular price, and the probability of finding a match depends on this price (see, for example, [Shimer, 1996](#); [Moen, 1997](#)). Search frictions in the physical capital market were first studied in a random search environment by [Kurmann and Petrosky-Nadeau \(2007\)](#). In a calibrated version of their model they show that these frictions are not a quantitatively relevant propagation mechanism of TFP shocks. The most important difference from their quantitative framework is the inclusion of financial shocks, that in the present paper account for most of the fluctuations in market tightness. In fact, if the present paper included only TFP shocks, it would also have concluded that search frictions in investment are not a relevant quantitative propagation mechanism once output fluctuation is matched, a result reminiscent to that found in [Shimer \(2005\)](#) for the labor market.

The directed-search framework for the physical-capital market developed in the present paper builds on those developed for the labor market in [Shi \(2009\)](#), [Menzio and Shi \(2010, 2011\)](#), [Schaal \(2012\)](#) and [Kaas and Kircher \(2013\)](#). Studying these frictions for the physical

²Figure 13 of Appendix B shows that measures of capital unemployment available to Euro economies experiencing deep financial crises (Greece, Ireland, Portugal, and Spain) also show a large increase in capital unemployment, comparable to that of labor unemployment.

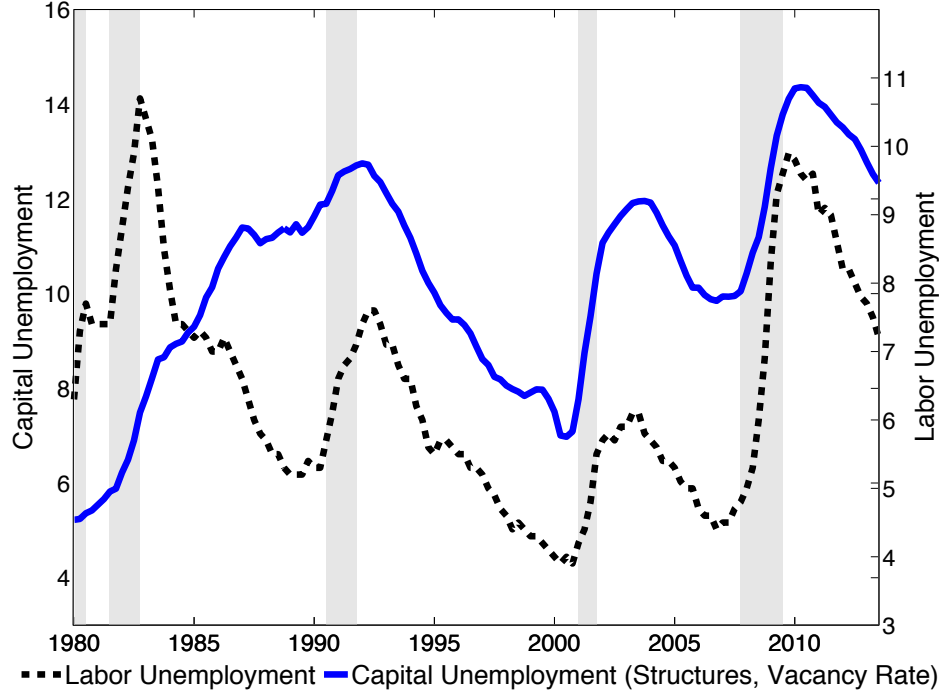


FIGURE 2: U.S. UNEMPLOYMENT OF PHYSICAL CAPITAL AND LABOR, 1980–2013.

Note: Capital unemployment (structures) constructed based on vacancy rates of office, retail and industrial units. Data source: CBRE and REIS. See Appendix A for details. Labor unemployment refers to the civilian unemployment rate. Data source: Federal Reserve of Saint Louis. Data is expressed in percent. Shadow areas denote NBER (peak to trough) recession dates.

capital market provides two novel mechanisms: First, it provides a new interaction between the production of capital and capital unemployment. The existence of high capital unemployment leads to a lower accumulation of new capital goods, while existing units are absorbed into production. This mechanism is not present in labor-market models in which population is generally assumed to be constant or exogenous. Second, because physical capital is not only a factor of production, but can also be used by firms as collateral for loans (see, for example, [Kiyotaki and Moore, 1997](#); [Geanakoplos, 2010](#)), fluctuations in capital unemployment interact with financial shocks in a way not seen in the labor market.

The framework developed in this paper can also be used to study capital reallocation. This is done by extending the model to allow for heterogeneity in capital match-specific productivity. This extension allows a characterization not only of the transition of capital from unemployment to employment, but of the transition of capital from employment to employment, since it adds a motive for trading unmatched capital while it remains employed (similar to “on the job search” in the labor-market literature). As shown in [Shi \(2009\)](#) and

[Menzio and Shi \(2011\)](#) for the labor market, the directed-search structure of the model is especially suitable to studying employment–employment transitions resulting from heterogeneity in match-specific productivity. The paper shows that capital reallocation is procyclical in this framework, as in the data (see [Ramey and Shapiro, 1998](#); [Eisfeldt and Rampini, 2006](#)). This is because negative shocks are associated with fewer capital purchases, making it harder for sellers of employed capital to find buyers.

Layout. The remainder of the paper is organized as follows. Section 2 discusses the relationship with the literature. Section 3 introduces investment search frictions and capital unemployment into a simple neoclassical growth model, and presents the main mechanism relating capital unemployment to capital accumulation. Section 4 builds a quantitative business-cycle model including search frictions in investment. Section 5 presents the model estimation and the quantitative results. Section 6 studies capital reallocation in the framework of the model. Section 7 concludes and discusses possible extensions.

2 Relationship with the Literature

This section discusses the contribution of the present paper from the perspective of four strands of the literature.

Financial Shocks and Macroeconomic Fluctuations. This paper builds on the growing body of literature that studies the effect of financial shocks on macroeconomic fluctuations. The study of the implications of financial frictions has a long tradition in macroeconomics (for a recent survey, see [Brunnermeier, Eisenbach and Sannikov, 2012](#)). Following the Great Recession, a number of studies have shown that shocks that affect the severity of financial frictions can have a large impact on aggregate fluctuations (see, for example, [Mendoza, 2010](#); [Arellano, Bai and Kehoe, 2012](#); [Jermann and Quadrini, 2012](#); [Gertler and Kiyotaki, 2013](#); [Christiano, Motto and Rostagno, 2014](#)).

The present paper contributes to this literature with a new financial-shock propagation mechanism by introducing the possibility of capital unemployment, whose fluctuations are mostly driven by this type of shock. The propagation mechanism proposed for financial shocks in this paper provides two novel dimensions to this literature. First, the relevant role assigned to financial shocks does not rely on price or wage stickiness, and holds in the context of a real

model that would assign a small role to financial shocks in the absence of investment search frictions. The role of financial shocks is a key discussion in the business-cycle literature and an important source of discrepancy between real and monetary models, with the latter attributing a much larger effect to these shocks than the former (as discussed in [Christiano, Motto and Rostagno, 2014](#)). The present paper shows that an important part of the discrepancy between these two branches of the literature can be reconciled by introducing investment search frictions. In a second contribution to this literature, the present paper provides a mechanism whereby financial shocks are followed by investment slumps, as documented in Figure 1.³

Investment Dynamics. By studying investment slumps following financial shocks, this paper relates to the large body of literature studying aggregate investment dynamics (see, for example, [Caballero, 1999](#); [Cooper and Haltiwanger, 2006](#)). In particular, the mechanism of this paper is consistent with the empirical findings that attribute a key role to financial factors in aggregate investment (see for example [Gilchrist, Sim and Zakrajšek, 2014](#)).

Since the set of financial shocks studied in this paper include a shock to the idiosyncratic cross-sectional uncertainty of the quality of capital, the findings of this paper are also related to the recent branch of the literature studying the effect of uncertainty shocks on aggregate investment and economic activity (see, for example, [Bloom, Bond and Van Reenen, 2007](#); [Bloom, 2009](#); [Bloom et al., 2012](#)). In these papers, uncertainty leads firms to adopt a “wait-and-see” strategy, contracting investment until uncertainty is revealed. The difference with these papers is that the wait-and-see strategy only implies a short-lived pause in investment: investment recovers after uncertainty dissipates. The present paper studies a mechanism by which, if these shocks lead to a significant increase in capital unemployment, the effects in investment can be persistent, as observed in the U.S. Great Recession and the typical financial crisis episode.

In a recent independent work, [Rognlie, Shleifer and Simsek \(2014\)](#) also study persistent falls in investment such as the one following the U.S. Great Recession. The key ingredients to their model are an overbuilding of residential capital, nominal rigidities, and the zero lower bound.⁴ Therefore the mechanism of their paper and that of the present paper are

³[Queralto \(2013\)](#) constructs a quantitative framework in which financial crises have persistent effects on economic activity. Since that mechanism relies on endogenous TFP growth, the findings are complementary to those presented in the present paper.

⁴See also [Schmitt-Grohé and Uribe \(2012b\)](#) and [Eggertsson and Mehrotra \(2014\)](#) for related papers studying the persistence of the Great Recession associated to the zero lower bound.

complementary interpretations of the investment slump following the U.S. Great Recession. The result of the present paper also applies to financial crises in which monetary policy is not constrained by the zero lower bound and to those in which a residential overbuilding does not take place.

Search Frictions. By modeling capital unemployment in a search theoretical framework, this paper relates to the extensive literature studying search frictions in labor, assets, and goods markets. The relationship with the literature on search frictions in the labor market was discussed in Section 1. Given that physical capital is both a good and an asset, the search frictions studied in this paper are also related to those of goods markets or other asset markets. With regard to goods markets, [Bai, Rios-Rull and Storesletten \(2012\)](#) recently studied search frictions that affect the purchase of investment goods, as in the present paper. Unlike the present paper, these frictions only affect the flow of production and not the stock of existing capital units (which is the main feature of capital unemployment).

In other asset markets, a number of contributions have shown how search frictions affect the liquidity and returns of assets (see, for example, [Lagos and Rocheteau, 2008, 2009](#)). In the housing market, search frictions have been used to explain fluctuations in prices, trading and vacancy rates (see, for example, [Wheaton, 1990](#); [Krainer, 2001](#); [Caplin and Leahy, 2011](#); [Piazzesi, Schneider and Stroebe, 2013](#)). The main difference with respect to these contributions is that the physical capital considered in the present paper is a productive asset, and therefore fluctuations in its unemployment have a direct relationship with economic activity and firms' investment.

Capital Utilization. The effect of capital unemployment on economic activity is related to that studied in the literature on variable capital utilization (for surveys on capital utilization, see [Winston, 1974](#); [Betancourt and Clague, 2008](#)). However, capital unemployment and capital utilization are two different concepts, related to different economic mechanisms. To clarify the difference between the two concepts, it is useful to define a set of categories to classify capital stock, similar to those used to classify the status of the labor force, summarized in Figure 3 (see, for example, [Bureau of Labor Statistics, 2014](#)). An unemployed unit of capital is a unit of capital that has not been used for production within the period and whose owners have actively searched to sell or rent the capital unit. Therefore, while

			Owner of Capital Searching to Sell or Rent	
			No	Yes
Capital Held by	Agent without Production Technology	Idle		1.
				2.
	Agent with Production Technology	Idle		3.
				4.
		Not Idle	5.	6.
			7.	8.

FIGURE 3: A CLASSIFICATION OF CAPITAL STOCK STATUS.

Note: The capital stock status is classified defining eight categories similar to those in the labor market (see, for example, [Bureau of Labor Statistics, 2014](#)). Employed capital (Regions 5, 6, 7 and 8) includes units of capital that have been used for production within a period. This includes capital temporarily idle as part of regular business operations, such as shift changes. Unemployed capital (Regions 2 and 4) includes units of capital that have not been used for production within the period and their owners have actively searched to sell or rent the capital unit. Employed and unemployed capital constitute the “capital force.” Capital outside the capital force (Regions 1 and 3) includes idle units that have not been used for production within the period and whose owners are not seeking buyers or renters.

capital utilization describes the intensity with which capital is used by firms that own or rent capital (a consumption decision), capital unemployment describes whether owners of idle capital are unable to sell or rent it (an investment decision). The difference between capital unemployment and capital utilization then parallels that between labor unemployment and labor hoarding (see, for example, [Burnside, Eichenbaum and Rebelo, 1993](#); [Sbordone, 1996](#)).

Being two different concepts, capital utilization and capital unemployment can have different empirical measures. For instance, standard empirical measures of capital utilization relate to firms’ use of their production capacity.⁵ Empirical measures of capital unemployment would instead relate the share of physical capital (owned by either firms or households) that is idle and available in the market for sale or rent, such as this paper’s data collected

⁵For the U.S., the Federal Reserve Board estimates capacity utilization for industries in manufacturing (see [Federal Reserve Board, 2014](#), for a description of the methodology). [Gorodnichenko and Shapiro \(2011\)](#) use the Survey of Plant Capacity from the [U.S. Census Bureau \(2014\)](#) to construct data on capital utilization. [Basu, Fernald and Kimball \(2006\)](#), [Basu et al. \(2013\)](#), and [Fernald \(2009\)](#) provide estimates of factor utilization for the U.S. economy, capturing labor effort and the work week of capital.

from the commercial real estate market (see Figure 2). As illustrated in Appendix B (Figure 14) for recent U.S. recession episodes, these empirical measures of capital unemployment and capital utilization can have significantly different behaviors.

Capital utilization and capital unemployment can also be modeled differently. Models of capital utilization typically treat it as a control variable whose choice, related to utilization costs, can be described as an intensive margin (e.g., a higher utilization rate causes greater depreciation, as in Calvo, 1975; Greenwood, Hercowitz and Huffman, 1988, Regions 5 and 7 of Figure 3) or as an extensive margin (e.g., less-productive units are left idle, as in Cooley, Hansen and Prescott, 1995; Gilchrist and Williams, 2000, Region 3 of Figure 3). Recent contributions using search frictions in the product market show that this variable can also be related to the probability of a firm finding customers (see, for example, Petrosky-Nadeau and Wasmer, 2011; Bai, Rios-Rull and Storesletten, 2012; Michaillat and Saez, 2013). In the present paper’s model, capital unemployment is a state variable. The key margins affecting the flows of unemployed capital to employment are the price of capital posted by sellers and the mass of capital buyers are willing to purchase at a given price (transition from Regions 2 and 8 to Regions 5 and 7 of Figure 3).

For this reason, this paper will show that different factors affect fluctuations in capital utilization and capital unemployment and that different implications follow from explicitly modeling capital unemployment (such as the a low rate of investment when capital unemployment is high). Nevertheless, the concepts of capital utilization and capital unemployment can be seen as complementary. In fact, once the model with capital unemployment is extended to study capital reallocation (transitions from Regions 6 and 8 to Regions 5 and 7 of Figure 3), changes in the probability of selling capital units will affect firms’ capital utilization rates.

3 Investment Search Frictions: Basic Framework

This section introduces investment search frictions into a simple neoclassical growth model. The framework abstracts from uncertainty, endogenous labor supply and other frictions – which will be later introduced in the quantitative model – to make the mechanism clear. Policy functions and transitional dynamics are studied, showing how capital accumulation is affected by existing capital unemployment. In the standard neoclassical growth model, the process of convergence from an initial capital stock below the steady state is characterized by a monotonic increase in the capital stock. By contrast, in the model with investment search frictions, if the initial total capital stock is below the steady state and the rate of capital unemployment is sufficiently high, the transitional dynamics for the capital stock are not monotonic, featuring an initial decrease and a subsequent increase. Therefore, if a shock leads to a sufficiently high level of capital unemployment, recovery is characterized by an investment slump, as documented in Figure 1.

3.1 Environment

Time is discrete and infinite, with four-stage periods. There is no aggregate uncertainty.

Goods. There are consumption and capital goods: Consumption goods are perishable; capital goods depreciate at a constant rate, $\delta > 0$. Capital can be traded in either of two states: matched or unmatched. Only matched capital can be used as input in the production of consumption goods.

Agents. The economy is populated by a unit mass of identical households and a unit mass of entrepreneurs. Households consume, produce unmatched physical capital and (inelastically) supply labor. The representative household has a continuum of infinitely lived members, a positive fraction of whom are entrepreneurs. Within each household there is perfect consumption insurance.⁶ Entrepreneurs have access to a technology to produce consumption goods, using matched capital and labor as inputs, and to a search technology to transform unmatched capital into matched capital. Capital produced by households begins

⁶The assumption of large families follows Merz (1995), Andolfatto (1996) and, more recently, Gertler and Karadi (2011) and Christiano, Motto and Rostagno (2014). This assumption facilitates the work in Section 4, when financial frictions are introduced explicitly and entrepreneurs are endowed with net worth. In the current section, this assumption plays no role and is not different from a framework in which a representative firm produces consumption goods.

unmatched. Only entrepreneurs can store matched capital. Capital held by entrepreneurs is denoted *employed capital*, and capital held by households is denoted *unemployed capital*.

Each period, entrepreneurs have a probability $\psi > 0$ of retiring from entrepreneurial activity. The fraction ψ of entrepreneurs who retire from entrepreneurial activity is replaced by a new identical mass of entrepreneurs from the households' members, so the population of entrepreneurs is constant. Retiring entrepreneurs' capital becomes unmatched and is transferred to the household. Dividends from entrepreneurial activity, resulting from capital purchases and production, are transferred each period to the household.

Physical capital markets. Trade of unmatched capital between entrepreneurs and households occurs in a decentralized market with search frictions. Search is directed, following a structure similar to those in [Menzio and Shi \(2010, 2011\)](#) for the labor market and in [Menzio, Shi and Sun \(2013\)](#) for the money market. In particular, this market is organized in a continuum of submarkets indexed by the price of unmatched capital, denoted x . Sellers (households) and buyers (entrepreneurs) can choose which submarket to visit. In each submarket, the market tightness, denoted $\theta(x)$, is defined as the ratio between the mass of capital searched by entrepreneurs and the mass of unemployed capital offered in that submarket. Households face no search cost. Visiting submarket x in period t , they face a probability $p(\theta_t(x))$ of finding a match, where $p : \mathbb{R}_+ \rightarrow [0, 1]$ is a twice continuously differentiable, strictly increasing, strictly concave function that satisfies $p(0) = 0$ and $\lim_{\theta \rightarrow \infty} p(\theta) = 1$. Entrepreneurs face a cost per unit searched denominated in consumption goods and denoted $c_s > 0$. Visiting submarket x in period t , they face a probability $q(\theta_t(x))$ of finding a match, where $q : \mathbb{R}_+ \rightarrow [0, 1]$ is a twice continuously differentiable, strictly decreasing function that satisfies $q(\theta) = \frac{p(\theta)}{\theta}$, $q(0) = 1$ and $\lim_{\theta \rightarrow \infty} q(\theta) = 0$. The cost of a unit of capital for entrepreneurs in submarket x is denoted Q^x (which includes two components: the price paid to the seller, x , and the search cost in submarket x).

Timing. Each period is divided into four stages: production, separation, search, and investment. In the *production stage*, entrepreneurs produce consumption goods using matched capital from the previous period; employed and unemployed capital depreciates. In the *separation stage*, a fraction ψ of entrepreneurs retires and their capital becomes unmatched. An identical mass of entrepreneurs begins entrepreneurial activity with no initial capital. In the

search stage, entrepreneurs who do not retire and new entrepreneurs purchase unmatched capital from households, and net dividends in terms of consumption goods are transferred. In the *investment stage*, households produce physical capital and consume, and retired entrepreneurs transfer their capital to households.

3.2 Households

Household preferences are described by the lifetime utility function

$$\sum_{t=0}^{\infty} \beta^t U(C_{i,t}), \quad (1)$$

where $C_{i,t}$ denotes consumption of household i in period t , $\beta \in (0, 1)$ is the subjective discount factor, and $U : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a twice continuously differentiable, strictly increasing, strictly concave function.

Unemployed capital held by household i evolves according to the law of motion

$$K_{i,t+1}^u = \int_0^{(1-\delta)K_{i,t}^u} (1 - p(\theta_t(x_{k,i,t}))) dk + \psi(1 - \delta)K_t^e + I_{i,t}, \quad (2)$$

where $K_{i,t}^u$ denotes the stock of unemployed capital held by household i at the beginning of period t , K_t^e denotes the stock of employed capital at the beginning of period t , $I_{i,t}$ denotes the household's investment in period t , and $x_{k,i,t}$ denotes the submarket in which unemployed capital unit k is listed by household i in period t . The first term of the right-hand side of equation (2) represents the depreciated mass of capital which was unemployed at the beginning of period t and was not sold to entrepreneurs for a given market tightness, $\theta_t(x)$, and submarket choice $x_{k,i,t}$. The second term of the right-hand side of equation (2) represents the mass of employed capital transferred from retired entrepreneurs to households. The third term represents the addition (subtraction) to unemployed capital stock from investment.

The sequential budget constraint of household i is given by

$$C_{i,t} + I_{i,t} = \int_0^{(1-\delta)K_{i,t}^u} p(\theta_t(x_{k,i,t}))x_{k,i,t} dk + W_t \bar{h} + \tilde{\Pi}_{i,t}, \quad (3)$$

where W_t denotes the wage rate in period t , $\bar{h}$ denotes the household (inelastic) supply of hours of work to the labor market, and $\tilde{\Pi}_{i,t}$ denotes net transfers in terms of consumption goods from entrepreneurs to household i in period t – described further in the next section. The

left-hand side of equation (3) represents the uses of income: consumption and investment. The right-hand side of the equation represents the sources of income: selling unmatched capital in the decentralized market, labor income, and transfers from entrepreneurs.

Household i 's problem is then to choose plans for $C_{i,t}$, $I_{i,t}$, $K_{i,t+1}^u$, and $x_{k,i,t}$ that maximize utility (1), subject to the sequence of budget constraints (3), the accumulation constraints for unemployed capital (2), given the initial levels of capital, $K_{i,0}^u$ and K_0^e , the given sequence of net transfers, $\tilde{\Pi}_{i,t}$, and the given sequence of market-tightness functions, $\theta_t(x)$. Denoting $\Lambda_{i,t}$ the Lagrange multiplier associated with the budget constraint (3), in an interior solution, the optimality conditions are (2) and (3), and the first-order are conditions

$$\Lambda_{i,t} = U'(C_{i,t}), \quad (4)$$

$$\Lambda_{i,t} = \beta \Lambda_{i,t+1} (1 - \delta) [p(\theta_{t+1}(x_{i,t+1}^u)) x_{i,t+1}^u + (1 - p(\theta_{t+1}(x_{i,t+1}^u)))], \quad (5)$$

$$-p(\theta(x_{i,t}^u)) = p'(\theta_t(x_{i,t}^u)) \theta'_t(x_{i,t}^u) (x_{i,t}^u - 1), \quad (6)$$

where $x_{i,t}^u$ denotes household i 's choice of submarket for unmatched capital in period t , the unit of capital subindex, k , has been dropped because the optimality condition with respect to the choice of submarket, $x_{i,t}$, is the same for all units of capital.

3.3 Entrepreneurs

Entrepreneurs have access to a technology to produce consumption goods that uses matched capital as input:

$$Y_{j,t} = A_t F(K_{j,t}^e, h_{j,t}) = A_t (K_{j,t}^e)^\alpha (h_{j,t})^{1-\alpha}, \quad (7)$$

where $Y_{j,t}$ denotes output produced by entrepreneur j in period t , $K_{j,t}^e \geq 0$ denotes the stock of matched capital held by entrepreneur j at the beginning of period t , $h_{j,t}$ denotes hours of work employed by entrepreneur j in period t , A_t is an aggregate productivity factor affecting the production technology in period t .

The entrepreneur's objective is to maximize the present discounted value of dividends distributed to households:

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \Pi_{j,t+s}^e, \quad (8)$$

where $\Pi_{j,t}^e$ denotes net dividends paid by entrepreneur j to the household in period t , E_t

denotes the expectation conditional on the information set available at time t (the expected value is over the idiosyncratic retirement shock), and the household's subindex, i , in the shadow value Λ_t has been dropped since the first-order conditions of the household's problem are the same for all households. Net dividends of entrepreneur j are defined by the flow-of-funds constraint:

$$\Pi_{j,t}^e = A_t F(K_{j,t}^e, h_{j,t}) - W_t h_{j,t} - (1 - \psi_{j,t}) \left[\int_x Q_t^x \iota_{j,t}^{e,x} dx \right] + \psi_{j,t} (1 - \delta) K_{j,t}^e, \quad (9)$$

where $\iota_{j,t}^{e,x} \geq 0$ denotes the mass of capital purchased by entrepreneur j in submarket x in period t , and the stochastic variable $\psi_{j,t} \in \{0, 1\}$ takes the value 1 if entrepreneur j retires from entrepreneurial activity in period t and 0 otherwise, and satisfies $E_{t-1}(\psi_{j,t}) = \psi \forall t, j$. The three terms in the right-hand side of equation (9) represent the sources of net dividends transferred from entrepreneurs to households: The first term represents the output in terms of consumption goods produced by entrepreneur j in period t . The second term denotes the net purchase of physical capital, expressed in consumption units, that entrepreneur j makes in the case of not retiring in period t . The last term represents the transfer of unmatched capital that entrepreneur j makes to households in the case of retiring in period t . The first two terms define the net transfer, in terms of consumption goods, that entrepreneur j makes to households in period t : $\tilde{\Pi}_{j,t}^e \equiv A_t F(K_{j,t}^e) - W_t h_{j,t} - (1 - \psi_{j,t}) \left[\int_x Q_t^x \iota_{j,t}^{e,x} dx \right]$ (see the household's budget constraint (3)).

By the law of large numbers, the cost per unit of capital, of mass $\iota_{j,t}^{e,x}$, purchased in the submarket x of the decentralized market is given by

$$Q_t^x = x + \frac{c_s}{q(\theta_t(x))}. \quad (10)$$

The right-hand side of equation (10) represents the two components of the cost of a unit of capital in the decentralized market: the price paid to the seller, x , and the search cost, $\frac{c_s}{q(\theta_t(x))}$.

The stock of matched capital for entrepreneur j , who has the opportunity to invest in period t , evolves according to the law of motion

$$K_{j,t+1}^e = (1 - \delta) K_{j,t}^e + \int_x \iota_{j,t}^{e,x} dx. \quad (11)$$

Denote the period in which entrepreneur j enters entrepreneurial activity as t_{0j} , and assume entrepreneurs enter entrepreneurial activity with no initial matched capital; that is, $K_{j,t_{0j}}^e = 0 \forall t_{0j} \geq 0$.⁷

Entrepreneur j 's problem, is then to choose plans for $K_{j,t+1}^e$, $\iota_{j,t}^{e,x}$, and $h_{j,t}$ that maximize the present discounted value of dividends (8) subject to the sequence of flow-of-funds constraints (9), the accumulation constraints for matched capital (11), and the nonnegativity constraints for capital purchases in the decentralized market ($\iota_{j,t}^{e,x} \geq 0$), given the initial level of matched capital, $K_{j,t_{0j}}^e$, the given sequence of aggregate productivity A_t , the given sequence of prices, W_t , and the given sequence of market-tightness functions, $\theta_t(x)$. Denoting the Lagrange multiplier associated with the budget constraint (11) in period $t+s$ with $Q_{j,t+s} \frac{\Lambda_{t+s}}{\Lambda_t}$ and by $\Xi_{j,t+s}^x \frac{\Lambda_{t+s}}{\Lambda_t}$ the Lagrange multiplier associated with the nonnegativity constraint for capital purchases in submarket x in period $t+s$, the optimality conditions are (9), (11), $\iota_t^{e,x} \geq 0$, the first-order conditions – which, after some operations, can be expressed as

$$h_{j,t} = \left(\frac{(1-\alpha)A_t}{W_t} \right)^{\frac{1}{\alpha}} K_{j,t+1}^e \quad (12)$$

$$\Lambda_t Q_{j,t} = \beta \Lambda_{t+1} \left[r_t^k + (1-\delta)(\psi + (1-\psi)Q_{j,t+1}) \right], \quad (13)$$

$$Q_t^x = Q_{j,t} + \Xi_{j,t}^x, \quad (14)$$

and the complementary slackness conditions,

$$\Xi_{j,t}^x \geq 0, \quad \iota_{j,t}^{e,x} \Xi_{j,t}^x = 0, \quad (15)$$

for all x , where the net revenues from production per unit of matched capital are defined by $r_t^k \equiv \alpha \left(\frac{1-\alpha}{W_t} \right)^{\frac{1-\alpha}{\alpha}} A_t^{\frac{1}{\alpha}}$.

3.4 Equilibrium

The entrepreneurs' optimality conditions, (14) and (15), imply that, in equilibrium, any submarket visited by a positive number of entrepreneurs must have the same cost per unit of capital, and entrepreneurs will be indifferent among them. Formally, for all x ,

$$\theta_t(x) \left(x + \frac{c_s}{q(\theta_t(x))} - Q_t \right) = 0. \quad (16)$$

⁷A mass one of entrepreneurs starts period 0 with a stock of matched capital K_0^e .

where the entrepreneur's subindex, j , has been dropped in the shadow value Q_t because the optimality conditions (13)–(15) are the same for all entrepreneurs. This condition determines the equilibrium market-tightness function: For all $x < Q_t$,

$$\theta_t(x) = q^{-1} \left(\frac{c_s}{Q_t - x} \right). \quad (17)$$

For all $x \geq Q_t$, $\theta_t(x) = 0$: capital units listed above the value of capital for entrepreneurs remain unmatched.

Using the definition of market tightness, the law of large numbers, and the fact that a household's choice of submarket, $x_{k,i,t}$ is the same for all units of capital, k , and all households, i , the flow of capital that transitions from unemployment to employment is given by $p(\theta_t(x_t^u))(1 - \delta) \int_0^1 K_{i,t}^u di = \int_0^1 \iota_{j,t}^{e,x^u} dj = \int_0^1 \int_x \iota_{j,t}^{e,x} dx dj$. Aggregating the entrepreneurs' capital-accumulation constraints provides a law of motion for employed capital:

$$K_{t+1}^e = (1 - \psi)(1 - \delta)K_t^e + p(\theta_t(x_t^u))(1 - \delta)K_t^u. \quad (18)$$

where the aggregate stock of employed and unemployed capital in period t are defined, respectively, by $K_t^e \equiv \int_0^1 K_{j,t}^e dj$ and $K_t^u \equiv \int_0^1 K_{i,t}^u di$.

From the household's capital-accumulation constraint (2), and using again the law of large numbers and the fact that the choice of submarket, $x_{k,i,t}$, is the same for all units of capital, k , and all households, i , a law of motion for unemployed capital is obtained:

$$K_{t+1}^u = (1 - p(\theta_t(x_t^u)))(1 - \delta)K_t^u + \psi(1 - \delta)K_t^e + I_t, \quad (19)$$

where aggregate investment is defined by $I_t \equiv \int_0^1 I_{i,t} di$.

The capital-unemployment rate at the beginning of period t can then be defined as

$$k_t^u \equiv \frac{K_t^u}{K_t}, \quad (20)$$

where $K_t \equiv K_t^e + K_t^u$ denotes total aggregate capital stock at the beginning of period t .

Labor-market clearing requires $\int_j h_{j,t} dj = \bar{h}$. Aggregating the households' budget constraints and the entrepreneurs' flow-of-funds constraints and using the entrepreneurs' optimality conditions and the laws of motion for employed and unemployed capital provides the

economy's resource constraint:

$$C_t + I_t + c_s \theta_t(x_t^u)(1 - \delta)K_t^u = A_t F(K_t^e, \bar{h}). \quad (21)$$

where aggregate consumption is defined by $C_t \equiv \int_0^1 C_{i,t} di$.

The competitive equilibrium in this economy can then be defined as follows.

Definition 1 (Competitive equilibrium). *Given initial conditions for employed and unemployed capital, K_0^e and K_0^u , and sequences of aggregate productivity, A_t , a competitive equilibrium is a sequence of individual allocations and shadow values $\{(C_{i,t}, I_{i,t}, K_{i,t+1}^u, x_{i,t}^u)_{i \in [0,1]}, (K_{j,t+1}^e, \ell_{j,t}^{e,x}, h_{j,t})_{j \in [0,1]}\}$, $\{(\Lambda_{i,t})_{i \in [0,1]}, (Q_{j,t})_{j \in [0,1]}\}$, aggregate allocations $\{C_t, I_t, K_{t+1}^e, K_{t+1}^u\}$, prices $\{W_t\}$, and market-tightness functions $\{\theta_t(x)\}$ such that*

- (i) *The individual allocations and shadow values solve the household's and entrepreneur's problems at the equilibrium prices and equilibrium market-tightness functions for all i and j .*
- (ii) *The market-tightness function satisfies (16) for all x .*
- (iii) *The labor market clears.*

3.5 Characterizing Equilibrium

Efficiency. Given the directed-search structure of the decentralized market, it can be shown that the competitive equilibrium is efficient in the sense that its allocation coincides with the solution a social planner would select when facing the same technological constraints as those faced by private agents, including search effort. Efficiency is defined and established in the following definition and proposition.

Definition 2 (Efficient allocation). *A sequence of allocations, $\{C_t, I_t, K_{t+1}, k_{t+1}^u, \theta_t^u\}$, is efficient if it solves the following social planner's problem.*

$$\max_{\{C_t, I_t, K_{t+1}, k_{t+1}^u, \theta_t^u\}} \sum_{t=0}^{\infty} \beta^t U(C_t), \quad (22)$$

$$s.t. \ C_t + I_t + c_s \theta_t^u(1 - \delta)k_t^u K_t = A_t F((1 - k_t^u)K_t, \bar{h}), \quad (23)$$

$$K_{t+1} = (1 - \delta)K_t + I_t, \quad (24)$$

$$(1 - k_{t+1}^u)K_{t+1} = [(1 - \psi)(1 - k_t^u) + p(\theta_t^u)k_t^u](1 - \delta)K_t, \quad (25)$$

given initial conditions for capital stock and capital-unemployment rate, K_0 and k_0^u , and sequences of aggregate productivity, A_t .

Proposition 1. *The competitive equilibrium is efficient.*

Proof. See Appendix D. ■

Denoting the Lagrange multiplier of resource constraint (23) as Λ_t^{sp} , and the Lagrange multiplier for employed-capital law of motion (25) as $(Q_t^{\text{sp}} - 1)\Lambda_t^{\text{sp}}$, the optimality conditions of the social planner's problem are (23)–(25), and the first-order conditions, that after operating, can be expressed as

$$U'(C_t) = \Lambda_t^{\text{sp}}, \quad (26)$$

$$c_s = p'(\theta_t^u)(Q_t^{\text{sp}} - 1), \quad (27)$$

$$\Lambda_t^{\text{sp}} Q_t^{\text{sp}} = \beta \Lambda_{t+1}^c \{A_{t+1} F_1(K_{t+1}^e, \bar{h}) + (1 - \delta)[\psi + Q_t^{\text{sp}}(1 - \psi)]\}, \quad (28)$$

$$\Lambda_t^{\text{sp}} = \beta \Lambda_{t+1}^{\text{sp}} (1 - \delta) \{(1 - p(\theta_{t+1})) + Q_{t+1}^{\text{sp}} p(\theta_{t+1}) - c_s \theta_{t+1}\}. \quad (29)$$

Equation (26) states that the social planner equates the marginal utility of consumption with the social shadow value of wealth, Λ_t^{sp} . Equation (27) states that the planner equates the social marginal costs and benefits of increasing market tightness: The left-hand side of equation (27) represents the social marginal cost of increasing the market tightness, which is given by the cost c_s per unit searched; the right-hand side of equation (27) represents the social marginal benefit of increasing market tightness, which is the product of two terms: the marginal increase in the probability of matching unemployed capital, given by $p'(\theta_t^u)$, and the shadow value of employed capital in consumption units, $(Q_t^{\text{sp}} - 1)$. Equation (28) states that the planner equates the social marginal cost of increasing the employment rate of capital in period t , given by $\Lambda_t^{\text{sp}} Q_t^{\text{sp}}$, with the expected discounted social marginal benefit of increasing the capital employment rate in period $t + 1$, given by the right-hand side of equation (28), which includes the marginal product of employed capital (given by $A_{t+1} F_1(K_{t+1}^e, \bar{h})$) and the expected depreciated value of a unit of employment capital (given by $(1 - \delta)[\psi + Q_t^{\text{sp}}(1 - \psi)]$). Finally, equation (29) states that the social planner equates the social marginal cost of increasing the capital stock in period t , given by Λ_t^{sp} , with the expected discounted social marginal benefit of increasing the capital stock in period $t + 1$ given by the right-hand side of equation (29). Since newly produced capital is unemployed, the marginal benefit is that

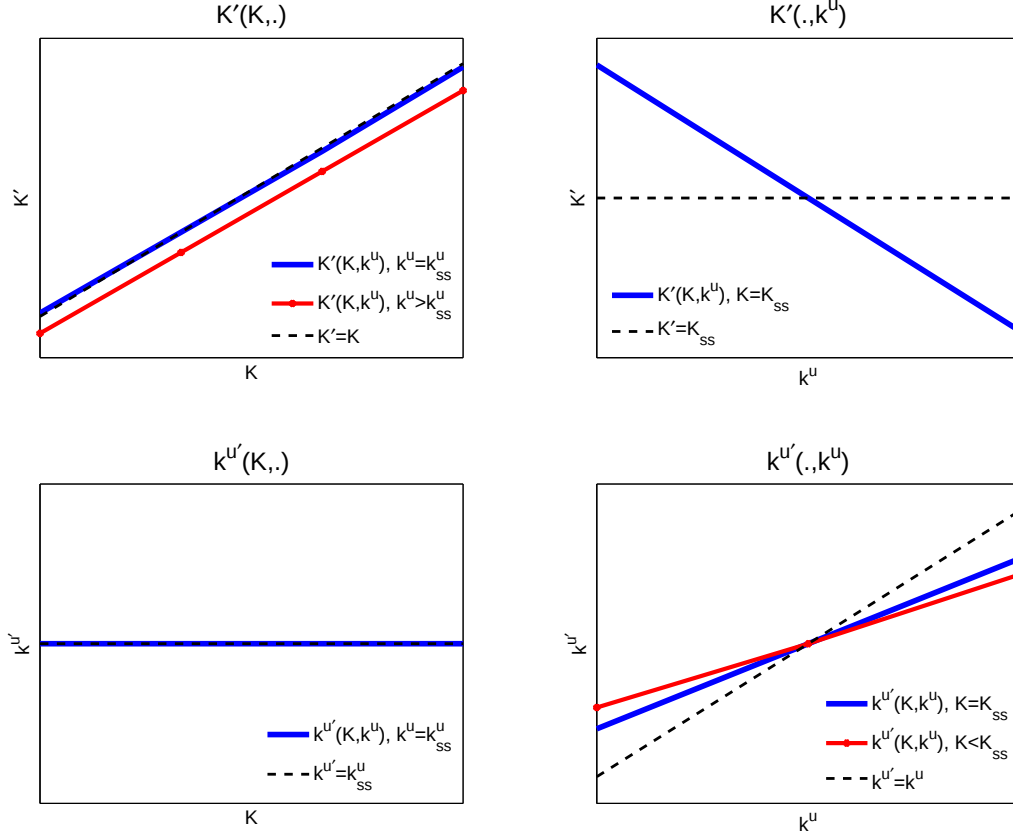


FIGURE 4: POLICY FUNCTIONS.

Note: Decision rules in the social planner's problem (22). In each panel, only one state variable varies on the horizontal axis. The other state variables is fixed at a given specified value.

of a consumption unit with probability $[1 - p(\theta_{t+1}^u)]$, and that of an employed unit of capital (given by $\Lambda_{t+1}^{sp} Q_t^{sp}$) with probability $p(\theta_{t+1}^u)$, net of search costs (given by $c_s \theta_{t+1}^u$).

Policy functions and transitional dynamics. This section studies the policy functions of the social planner's problem (22), and the resulting process of convergence from an initial capital stock and capital-unemployment rate to the steady-state path, assuming that aggregate technology is constant over time.

Figure 4 shows decision rules for next-period capital stock and next-period capital-unemployment rate, as a function of the two state variables: current capital stock and current capital-unemployment rate.⁸ In each panel, one state variable varies on the horizontal axis and the others are fixed at a given specified value. If the current share of unemployed capital is at its steady-state level, the planner's decision rules for next-period capital are similar to those

⁸Functional forms used were those of Section 5. Parameter values were set to those used as priors in the quantitative analysis of Section 5.

of the standard neoclassical growth model: Increasing the capital stock for levels of current capital stock below the steady state, decreasing the capital stock for current values of capital above the steady-state level, as depicted in the top-left panel of Figure 4.

This pattern no longer holds if the capital-unemployment rate is above its steady-state level. As shown on the top-left panel of Figure 4, for a sufficiently high level of the current capital-unemployment rate, next-period's optimal capital stock is below its current level even for levels below the steady state. The reason for this is that, as depicted on the top-right panel of Figure 4, next-period capital stock is a decreasing function of the current share of unemployed capital. For instance, if the stock of capital is at its steady-state level, but the share of unemployed capital is above its steady-state level, the social planner chooses to decrease the capital stock. This is because, if the share of unemployed capital is above its steady state, the social planner wants to reduce next-period share of unemployed capital (see bottom-right panel of Figure 4). In the framework of the present paper, the production of new capital goods only increases the stock of unemployed capital (see equation (2)). For a given level of consumption, by reducing the stock of capital, the social planner can dedicate more resources to matching, and reduce the share of unemployed capital.

As implied by the policy functions, the transitional dynamics to the steady state, starting from a stock of capital below the steady state depends on the initial share of capital unemployment. As shown in Figure 5, starting from an initial share of unemployed capital equal to the steady-state level, the stock of capital increases monotonically, as it would in the standard neoclassical growth model. However, when the initial share of capital unemployment is sufficiently high, the stock of capital first decreases, and then increases to catch up with the steady-state level. Capital unemployment provides a reason why the recovery from a negative shock can be characterized by an investment slump (as shown in Figure 1 1). The remaining task is then to study which shocks can lead to a significant increase in the capital-unemployment rate. This will be analyzed quantitatively in Section 5.

To further study the economic mechanism induced by the investment search friction, Appendix C considers a prototype economy with time-varying wedges (in the spirit of Chari, Kehoe and McGrattan, 2007), and maps the equilibrium of the economy with search frictions in investment to wedges in the prototype economy.

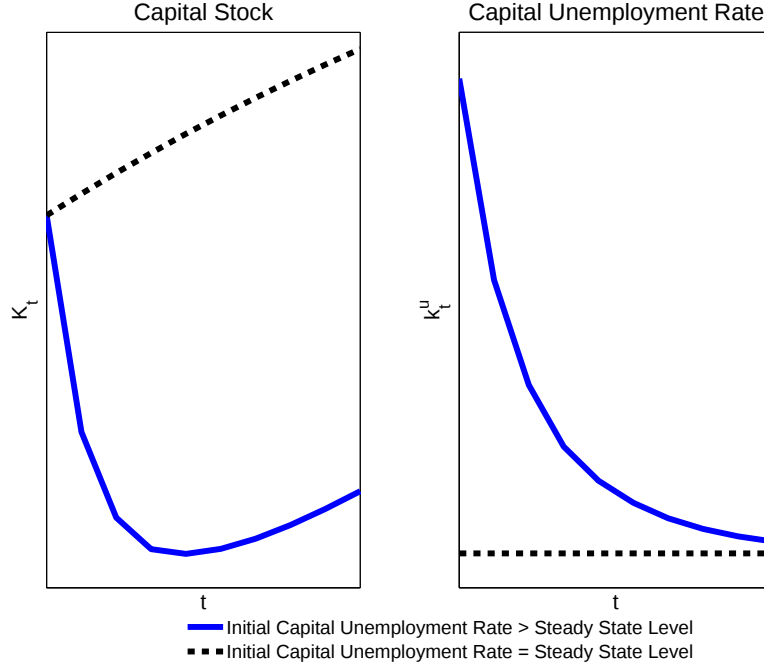


FIGURE 5: TRANSITIONAL DYNAMICS AND INITIAL CAPITAL-UNEMPLOYMENT RATE.

Note: Transitional dynamics from initial capital stock (K_0) below the steady-state level for two alternative values of the initial capital-unemployment rate level (k_0^u).

4 A Quantitative Business-Cycle Model with Investment Search Frictions

This section extends the basic framework of Section 3 to a stochastic business-cycle environment to quantitatively study the proposed mechanism. The model includes financial frictions and two shocks related to the severity of the financial frictions that have been studied in the literature as having an important role in U.S. business cycles and in the Great Recession: shocks to the cross-sectional idiosyncratic uncertainty and to the business sector's net worth (Christiano, Motto and Rostagno, 2014; Jermann and Quadrini, 2012). It also features other frictions and shocks that the literature has shown to be relevant sources of business-cycle fluctuation in the U.S. economy (see Smets and Wouters, 2007; Justiniano, Primiceri and Tambalotti, 2010, 2011; Schmitt-Grohé and Uribe, 2012a). In particular, the model incorporates investment-adjustment costs, variable capital utilization, internal habit formation in consumption, and four other structural shocks: neutral productivity, investment-specific productivity, government spending and preferences.

4.1 Environment

Goods. As in Section 3, consumption goods are perishable, and capital goods depreciate at a rate $\delta > 0$. Capital goods can be traded in either of two states: matched or unmatched. Only matched capital can be used as input in the production of consumption goods.

Agents. The economy is populated by a unit mass of identical households, a unit mass of entrepreneurs, and an arbitrary large number of financial intermediaries (see Figure 6). Households consume, supply labor, produce unmatched physical capital and purchase bonds issued by financial intermediaries. As in Section 3, the representative household has a continuum of infinitely lived members, with a positive fraction of them being entrepreneurs. Within each household, there is perfect consumption insurance. Entrepreneurs have access to a technology to produce consumption goods, using matched capital and labor as inputs, and to a search technology to transform unmatched capital into matched capital. Capital produced by households begins unmatched. Only entrepreneurs can store matched capital.

Unlike in Section 3, entrepreneurs cannot finance their purchases of capital with direct transfers from households. Instead, entrepreneurs purchase capital each period by borrowing from financial intermediaries and by using their own net worth.

Each period, an entrepreneur has a probability $\bar{\psi} > 0$ of retiring from entrepreneurial activity. The fraction $\bar{\psi}$ of entrepreneurs that retires from entrepreneurial activity each period is replaced by a new equal mass of entrepreneurs from the households' members. New entrepreneurs start entrepreneurial activity with an exogenous and stochastic stock of net worth transferred from the households. Retiring entrepreneurs' capital becomes unmatched and is traded with households, and their net worth, after selling the unmatched capital, is transferred to their households.

An unrestricted mass of financial intermediaries can enter the economy each period. They can sell bonds to households and lend to entrepreneurs for capital-good purchases. Additionally, the economy includes a government that conducts fiscal policy.

Markets. The economy has four competitive markets: goods, labor, physical capital and credit (see Figure 6). The goods and labor markets are frictionless. The market for physical capital is characterized by search frictions. The credit market is characterized by frictions associated with asymmetric information in lending. Further details on the frictions that char-

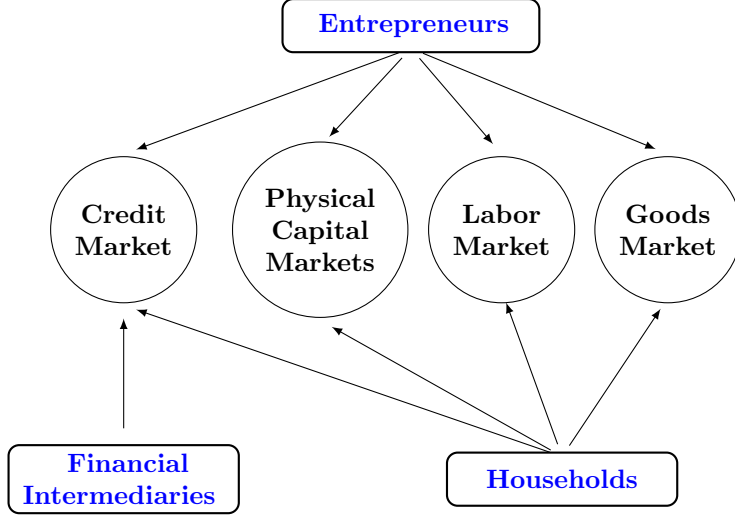


FIGURE 6: AGENTS AND MARKETS.

acterize the credit and physical-capital markets are provided below.

Credit market. Lending to entrepreneurs is assumed to entail an agency problem associated with asymmetric information and costly state verification (Townsend, 1979). In particular, entrepreneurs face an idiosyncratic shock whose realization is private information and can only be known by the lender through costly verification.

Following Bernanke, Gertler and Gilchrist (1999), it is assumed that the idiosyncratic shock is an i.i.d. shock to the quality of capital, denoted ω , whose realization is known by neither entrepreneurs nor financial intermediaries when lending occurs. Entrepreneurs finance the purchase of capital partly by borrowing and partly from their own net worth. The set of contracts offered to entrepreneurs, (Z_{t+1}, D_{t+1}) , specifies an aggregate state-contingent interest rate, Z_{t+1} , for each loan amount, D_{t+1} , to be repaid in case of no default. In the case of default, the financial intermediary seizes the entrepreneur's assets, paying a proportional recovery cost, μ_m . It is further assumed that the capital held by the entrepreneur becomes unmatched in the event of default. This form of contract implies in each period that a cutoff value exists for the realization of ω , denoted $\bar{\omega}_t$, below which entrepreneurs default. This formulation also implies that all entrepreneurs choose the same level of leverage, leading to an aggregation result by which it is not necessary to keep track of the distribution of net worth among entrepreneurs (which is particularly suitable for quantitative analysis). Each period $t + 1$, the realization of ω is drawn from a distribution $F_{\omega,t}(\omega, \sigma_t)$, where σ_t is an

exogenous shock to the cross-sectional dispersion of idiosyncratic shocks.

On the other side of the market, it is assumed that financial intermediaries obtain funds by issuing one-period, non-state-contingent bonds, purchased by households (similar to deposits). Financial intermediaries are diversified across idiosyncratic shocks and have free entry.

Physical capital markets. As in Section 3, trade of unmatched capital between entrepreneurs and households occurs in a decentralized market with search frictions. In addition, this section also includes two centralized markets in which matched capital can be traded between entrepreneurs at price Q^c , and unmatched capital can be traded between households, financial intermediaries and retired entrepreneurs at price J^u . Including these two markets is convenient for technical reasons. In particular, the centralized market in which matched capital can be traded between entrepreneurs allows the analysis to focus on an equilibrium that does not depend on the distribution of capital among entrepreneurs.⁹ The centralized market in which unmatched capital can be traded facilitates the study of financial intermediaries, who, in the event of default, seize the entrepreneur's capital (recall that the entrepreneur's capital becomes unmatched in the event of default). Figure 7 summarizes these three markets for capital, with the participants and forms of trade that characterize each market.

Search frictions that characterize the decentralized market for unmatched capital are identical to those in Section 3. In particular, search is directed: The market is organized in a continuum of submarkets indexed by the price of unmatched capital, denoted x , and sellers (households) and buyers (entrepreneurs) can choose which submarket to visit. In each submarket, the market tightness, denoted $\theta(x)$ is defined as the ratio between the mass of capital searched by entrepreneurs and the mass of unemployed capital offered in that submarket. Households face no search cost. Visiting submarket x , they face a probability $p(\theta(x))$ of finding a match, where $p : \mathbb{R}_+ \rightarrow [0, 1]$ is a twice continuously differentiable, strictly increasing, strictly concave function that satisfies $p(0) = 0$ and $\lim_{\theta \rightarrow \infty} p(\theta) = 1$. Entrepreneurs face a cost per unit searched, $c_s > 0$, denoted in terms of consumption goods. Visiting submarket x , they face a probability $q(\theta(x))$ of finding a match, where $q : \mathbb{R}_+ \rightarrow [0, 1]$

⁹A key assumption in Bernanke, Gertler and Gilchrist (1999) for the result that all entrepreneurs choose the same level of leverage is the existence of a market in which entrepreneurs can trade physical capital. This aggregation result, which is particularly convenient for quantitative analysis, can be extended to the framework of the present paper if entrepreneurs are allowed to trade matched capital in a centralized market. Studying an economy in which a centralized market for trading matched capital does not exist is left for future research.

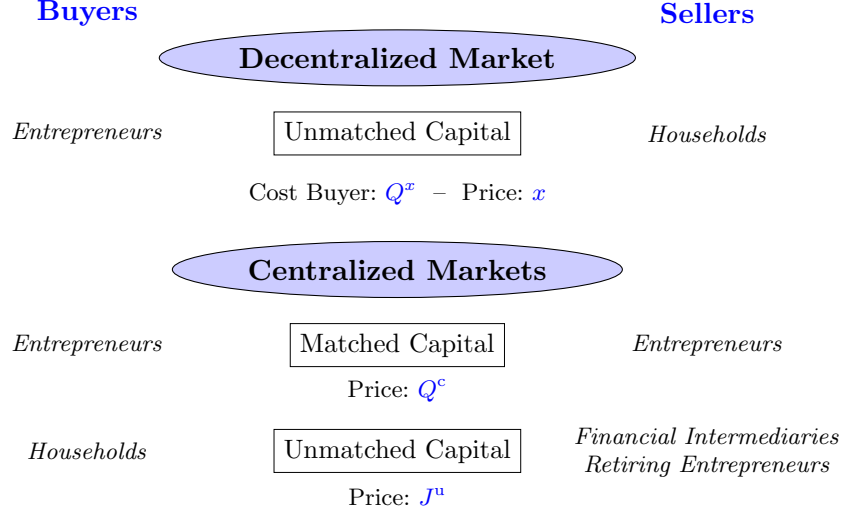


FIGURE 7: STRUCTURE OF CAPITAL MARKETS, QUANTITATIVE MODEL.

is a twice continuously differentiable, strictly decreasing function that satisfies $q(\theta) = \frac{p(\theta)}{\theta}$, $q(0) = 1$ and $\lim_{\theta \rightarrow \infty} q(\theta) = 0$. The cost of a unit of capital for entrepreneurs in submarket x is denoted Q^x (which includes two components: the price paid to the seller, x , and the search cost in submarket x).

Timing. Time is discrete and infinite, with each period divided into six stages: production, repayment, separation, borrowing, search and investment. In the *production stage*, entrepreneurs produce consumption goods using capital matched in the previous period. In the *repayment stage*, entrepreneurs repay their loans from the previous period or default; in case of default their capital becomes unmatched and financial intermediaries monitor and seize the entrepreneur's production and capital. In the *separation stage*, a fraction $\bar{\psi}$ of entrepreneurs that have not defaulted retires and their capital becomes unmatched. A new mass of entrepreneurs begins entrepreneurial activity with no initial capital and with an exogenously determined net worth. In the *borrowing stage*, entrepreneurs who do not retire and new entrepreneurs borrow from financial intermediaries, and financial intermediaries sell bonds to households. In the *search stage*, the remaining entrepreneurs purchase unmatched capital from households and matched capital from other entrepreneurs. In the *investment stage*, households produce unmatched physical capital and consume; retired entrepreneurs transfer their net worth, including unmatched capital, to their households; and financial intermediaries sell seized unmatched capital to households.

4.2 Households

Household preferences are described by the lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \{U(C_{i,t} - \rho_c C_{i,t-1}) - V(h_{i,t}; \varphi_t)\}, \quad (30)$$

where $C_{i,t}$ denotes consumption of household i in period t , $h_{i,t}$ denotes hours worked by household i in period t ; $\beta \in (0, 1)$ is the subjective discount factor; $\rho_c \in [0, 1)$ is a parameter governing the degree of internal habit formation; φ_t denotes an exogenous and stochastic preference shock in period t (labeled a *labor-wedge shock*); for every realization of φ_t , $V(\cdot; \varphi_t) : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a twice continuously differentiable, strictly increasing, strictly convex function; $U : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a twice continuously differentiable, strictly increasing, strictly concave function; and E_t denotes the expectation conditional on the information set available at time t .

The stock of unemployed capital held by household i evolves according to

$$K_{i,t+1}^u = \int_0^{(1-\delta)K_{i,t}^u} (1 - p(\theta_t(x_{k,i,t}))) dk + l_{i,t}^h + A_t^I \left[I_{i,t} - \Phi\left(\frac{I_{i,t}}{K_t}\right) K_t \right], \quad (31)$$

where $K_{i,t}^u$ denotes the stock of unemployed capital held by household i at the beginning of period t , $x_{k,i,t}$ denotes the submarket in which unemployed capital unit k is listed by household i in period t , $l_{i,t}^h$ denotes the units of unmatched capital purchased by households in the centralized market in period t , $I_{i,t}$ denotes investment by household i in period t , K_t denotes aggregate capital stock at the beginning of period t (taken as given by household i), A_t^I denotes an exogenous aggregate shock that affects the production of capital from investment goods in period t (as in [Justiniano, Primiceri and Tambalotti, 2011](#), labeled an *investment-specific technology shock*), and $\Phi : \mathbb{R} \rightarrow \mathbb{R}$ is a twice continuously differentiable, strictly convex function that introduces investment-adjustment costs. The first term of the right-hand side of equation (31) represents the depreciated mass of capital unemployed at the beginning of period t and not sold to entrepreneurs for a given market-tightness function $\theta_t(x)$ and choice of submarket $x_{k,i,t}$. The second term of the right-hand side of equation (31) represents the mass of employed capital purchased by the households from retired and defaulting entrepreneurs. The third term represents the addition (subtraction) to unemployed-capital stock from investment, net of adjustment costs.

Households have access to a one-period, non-state-contingent bond issued by financial

intermediaries. The household's sequential budget constraint is given by

$$C_{i,t} + I_{i,t} + J_t^u l_{i,t}^h + T_t + B_{i,t} = R_{t-1} B_{i,t-1} + W_t h_{i,t} + \int_0^{(1-\delta)K_{i,t}^u} p(\theta_t(x_{k,i,t})) x_{k,i,t} dk + \Pi_t, \quad (32)$$

where $B_{i,t}$ denotes the one-period bond holdings chosen by household i at the beginning of period t , which pays a gross non-state-contingent interest rate, R_t ; W_t denotes the wage rate; Π_t denotes net transfers from entrepreneurs and financial intermediaries to households in period t – described further in the next sections; and T_t represents a lump-sum government tax (subsidy) in period t .

Household i 's problem is then to choose the state-contingent sequences of $C_{i,t}$, $h_{i,t}$, $I_{i,t}$, $l_{i,t}^h$, $K_{i,t+1}^u$, $B_{i,t}$ and $x_{k,i,t}$ that maximize the expected utility (30), subject to the sequence of budget constraints (32) and the accumulation constraints for unemployed capital (31), for the given initial levels of capital and consumption ($K_{i,0}^u$, K_0^e , K_0 , and $C_{i,-1}$), the given sequence of prices (W_t , J_t^u and R_t), the given sequence of dividends and taxes (Π_t and T_t), the given sequence of market-tightness functions ($\theta_t(x)$), and the given sequence of labor wedges (φ_t) and investment-specific productivities (A_t^I). Denoting the Lagrange multiplier associated with the budget constraint (32) as $\Lambda_{i,t}$, the optimality conditions in an interior solution are (32), (31), and the first-order conditions:

$$\Lambda_{i,t} = U'(C_{i,t} - \rho_c C_{i,t-1}) - \beta \rho_c E_t U'(C_{i,t+1} - \rho_c C_{i,t}), \quad (33)$$

$$\Lambda_{i,t} J_t^u = \beta \Lambda_{i,t+1} (1 - \delta) [p(\theta_{t+1}(x_{i,t+1}^u)) x_{i,t+1}^u - (1 - p(\theta_{t+1}(x_{i,t+1}^u))) J_{t+1}^u], \quad (34)$$

$$1 = J_t^u A_t^I [1 - \Phi' \left(\frac{I_{i,t}}{K_t} \right)], \quad (35)$$

$$\Lambda_{i,t} = \beta R_t E_t \Lambda_{i,t+1}, \quad (36)$$

$$-p(\theta(x_{i,t}^u)) = p'(\theta(x_{i,t}^u)) \theta'_t(x_{i,t}^u) (x_{i,t}^u - J_t^u), \quad (37)$$

$$V'(h_{i,t}; \varphi_t) = \Lambda_{i,t} W_t, \quad (38)$$

where $x_{i,t}^u$ denotes household i 's choice of submarket for unmatched capital in period t , and the unit of capital subindex, k , has been dropped because the optimality condition with respect to the choice of submarket, $x_{k,i,t}$, is the same for all units of capital.

4.3 Financial Intermediaries

Financial intermediaries sell one-period non-state-contingent bonds to households and lend to entrepreneurs. The set of contracts offered to entrepreneur j specifies an aggregate, state-contingent interest rate, $Z_{j,t+1}$, for each loan amount, $D_{j,t+1}$, to be repaid in case of no default. In case of default, the financial intermediary seizes the entrepreneur's assets, with a recovery value of $\mathcal{R}_{j,t+1}(\omega)$, and pays a proportional monitoring cost, μ_m . Debt schedules available for entrepreneur j include all contracts $(Z_{j,t+1}, D_{j,t+1})$ that allow a financial intermediary to repay in all states the risk-free bond sold to households, after diversifying idiosyncratic risk:¹⁰

$$D_{j,t+1}R_t = [1 - F_\omega(\bar{\omega}_{j,t+1}; \sigma_t)]Z_{j,t+1}D_{j,t+1} + (1 - \mu_m) \int_0^{\bar{\omega}_{j,t+1}} \mathcal{R}_{j,t+1}(\omega) dF_\omega(\omega; \sigma_t), \quad (39)$$

where $\bar{\omega}_{j,t+1}$ denotes the default threshold in period $t+1$ for entrepreneur j with outstanding debt $D_{j,t+1}$ and stock of matched capital $K_{j,t+1}^e$ – to be discussed in detail in the next section. The left-hand side of equation (39) represents the obligations assumed by the financial intermediary selling the risk-free bond to households. The right-hand side of equation (39) represents the resources obtained by the financial intermediary from lending, after diversifying over idiosyncratic risk. It includes two terms, representing resources from entrepreneurs who do not default and resources from those who do.

It is assumed that in the default state financial intermediaries monitor and seize the entrepreneur's production and capital. Hence,

$$\mathcal{R}_{j,t+1}(\omega) = [r_{j,t+1}^k + (1 - \delta)J_{t+1}^u]\omega K_{j,t+1}^e, \quad (40)$$

where $r_{j,t+1}^k$ denotes the net revenues from production per unit of effective capital, $\omega K_{j,t+1}^e$ – to be described in detail in the next section.

4.4 Entrepreneurs

Entrepreneurs have access to technology to produce consumption goods using labor and matched capital as inputs. In particular, the output produced by an effective unit of matched

¹⁰For formulations of debt contracts similar to the one presented in this section, see [Arellano, Bai and Zhang \(2012\)](#) and [Christiano, Motto and Rostagno \(2014\)](#).

capital, ℓ , employing $\tilde{h}_{\ell,t}$ hours of work, is given by¹¹

$$y_{\ell,t} = A_t \left(\tilde{h}_{\ell,t} \right)^{1-\alpha}, \quad (41)$$

where $y_{\ell,t}$ denotes output in units of matched capital ℓ in period t and A_t is an exogenous aggregate productivity shock affecting the production technology in period t (labeled the *neutral-technology shock*).

Each period, entrepreneurs face an i.i.d. shock to the quality of their matched capital, denoted ω , drawn from a log-normal distribution with c.d.f. $F_\omega(\omega; \sigma_t)$ and satisfying $E_t(\omega_{t+1}) = 1 \forall t$ and $\text{Var}_t(\log(\omega_{t+1})) = \sigma_t^2 \forall t$, where σ_t is an exogenous aggregate shock to the cross-sectional dispersion of idiosyncratic shocks (labeled the *risk shock*, as in [Christiano, Motto and Rostagno, 2014](#)). Output produced by entrepreneur j with a mass of matched capital $K_{j,t}^e$, with $\tilde{h}_{j,t}$ hours worked in each of these units of capital and a utilization rate of $u_{j,t}$, denoted $Y_{j,t}$, is then given by

$$Y_{j,t} = A_t \left(\tilde{h}_{j,t} \right)^{1-\alpha} u_{j,t} \omega_{j,t} K_{j,t}^e, \quad (42)$$

where $\omega_{j,t}$ denotes the realization of the exogenous and stochastic variable ω for entrepreneur j in period t . The term $\omega_{j,t} K_{j,t}^e$ denotes the effective mass of matched capital held by entrepreneur j at the beginning of period t .

Entrepreneurs pay wage rate W_t per hour worked and face convex costs on the utilization rate. It follows that net revenues from production per unit of effective matched capital for entrepreneur j are given by

$$r_{j,t}^k = \left(A_t \left(\tilde{h}_{j,t} \right)^{1-\alpha} - W_t \tilde{h}_{j,t} \right) u_{j,t} - \mathcal{C}_u(u_{j,t}), \quad (43)$$

where $\mathcal{C}_u(u) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a twice continuously differentiable, strictly increasing, strictly convex function. Note that $r_{j,t}^k$ is independent of the mass of matched capital held by entrepreneur j , $K_{j,t}^e$, and independent of the realization of the idiosyncratic shock for entrepreneur j , $\omega_{j,t}$.

In this setup, all entrepreneurs face an expected linear rate of return per unit of capital

¹¹This production technology is similar to one in which production is carried out in a continuum of plants, as, for example, in [Cooley, Hansen and Prescott \(1995\)](#). In this framework, it can be shown that the aggregate production function of the economy displays constant returns to scale.

purchased:

$$R_{j,t+1}^{k,m} \equiv \frac{r_{j,t+1}^k + (1 - \delta) [\bar{\psi} J_{t+1}^u + (1 - \bar{\psi}) Q_{t+1}^c]}{Q_t^m}, \quad (44)$$

for $m \in \{x, c\}$. The denominator of the right-hand side of (44) represents the price at which the effective unit of matched capital was purchased. The numerator of the right-hand side of (44) represents the sources of revenue per unit of effective matched capital. The first component of the numerator represents net revenue from production. The second component represents the expected revenue from selling the depreciated unit of effective matched capital. If the entrepreneur retires (with probability $\bar{\psi}$), this effective unit of matched capital is traded unmatched at a price J_{t+1}^u . If the entrepreneur does not retire (with probability $1 - \bar{\psi}$), this effective unit of matched capital is traded matched at Q_{t+1}^c .

Entrepreneurs purchase capital using their net worth and borrowing from financial intermediaries. This means that, at the end of each period t and for any entrepreneur j , the entrepreneur's balance sheet follows

$$\int_x Q_t^x \tilde{K}_{j,t+1}^x dx + Q_t^c \tilde{K}_{j,t+1}^c = D_{j,t+1} + N_{j,t+1}, \quad (45)$$

where $D_{j,t+1} \geq 0$ denotes debt contracted by entrepreneur j in period t , to be paid in period $t + 1$, $N_{j,t+1} \geq 0$ denotes the net worth of entrepreneur j at the end of period t , $\tilde{K}_{j,t+1}^x \geq 0$ denotes the stock of capital held by entrepreneur j at the end of period t , purchased in submarket x of the decentralized market at a cost per unit Q_t^x , and $\tilde{K}_{j,t+1}^c \geq 0$ denotes the stock of capital held by entrepreneur j at the end of period t purchased in the centralized market at a cost Q_t^c per unit. The latter case also includes the stock of capital held by entrepreneur j from the previous period, which is equivalent to selling and repurchasing the unit in the centralized market at price Q_t^c . Note that $\int_x \tilde{K}_{j,t+1}^x dx + \tilde{K}_{j,t+1}^c = K_{j,t+1}^e$. The left-hand side of equation (45) represents the entrepreneur's assets, given by the value of the matched capital. The right-hand side of equation (45) represents the entrepreneur's liabilities and equity, given by debt with financial intermediaries and net worth.

As in Section 3, by the law of large numbers, the cost per unit of capital of mass $\tilde{K}_{t+1}^x$ purchased in the submarket x of the decentralized market is given by

$$Q_t^x = x + \frac{c_s}{q(\theta_t(x))}. \quad (46)$$

The right-hand side of equation (46) represents the two components of the cost of a unit of capital in the decentralized market: the price paid to the seller, x , and the search cost, $\frac{c_s}{q(\theta_t(x))}$.

To solve the entrepreneur's problem, it is useful to define the entrepreneur's leverage and "portfolio weights," from the components of the entrepreneur's balance sheet (45). Leverage for entrepreneur j at the end of period t is defined by

$$L_{j,t} \equiv \frac{\int_x Q_t^x \tilde{K}_{j,t+1}^x dx + Q_t^c \tilde{K}_{j,t+1}^c}{N_{j,t+1}}. \quad (47)$$

The portfolio weight of each asset considered in the left-hand side of equation (45) is

$$w_{j,t}^m \equiv \frac{Q_t^m \tilde{K}_{j,t+1}^m}{L_{j,t} N_{j,t+1}}, \quad (48)$$

for $m \in \{x, c\}$. From (45) and the nonnegativity constraint of capital holdings ($\tilde{K}_{j,t+1}^x \geq 0$ for $m \in \{x, c\}$), it follows that $w_{j,t}^m \in [0, 1] \forall m \in \{x, c\}$.

Entrepreneurs are risk neutral and their objective is to maximize their expected net worth, given at the end of period t by¹²

$$E_t \left\{ \int_{\bar{\omega}_{j,t+1}}^{\infty} \left[\omega \tilde{R}_{j,t+1}^k L_{j,t} N_{j,t+1} - Z_{j,t+1} D_{j,t+1} \right] dF_{\omega}(\omega; \sigma_t) \right\}, \quad (49)$$

where the portfolio return, denoted $\tilde{R}_{j,t+1}^k$, is defined by $\tilde{R}_{j,t+1}^k \equiv \int_x w_{j,t}^x R_{j,t+1}^{k,x} dx + w_{j,t}^c R_{j,t+1}^{k,c}$. The first term in the objective function (49) represents the revenue that will be received in period $t + 1$ by entrepreneur j . The second term represents debt repayments to financial intermediaries. Given that the entrepreneur receives revenue and performs debt repayment only in case of not defaulting, these terms are integrated over the realizations of $\omega_{j,t}$ above $\bar{\omega}_{j,t+1}$.

From the objective function (49), it follows that the expected value for entrepreneur j of repaying debt $D_{j,t+1}$ in the repayment stage of period $t + 1$ is given by

$$V_{j,t+1}^R = \omega_{j,t+1} \tilde{R}_{j,t+1}^k L_{j,t} N_{j,t+1} - Z_{j,t+1} D_{j,t+1}. \quad (50)$$

¹²The assumption that entrepreneurs are risk neutral maximize their expected net worth follows the quantitative literature implementing the costly state-verification framework. For a recent study relaxing this and other assumptions of the standard implementation of costly state verification used in this paper, see [Dmitriev and Hoddenbagh \(2013\)](#).

Given that the expected value of defaulting is equal to zero, equation (50) implies that the optimal default threshold, $\bar{\omega}_{j,t+1}$, is implicitly defined by

$$\bar{\omega}_{j,t+1} \tilde{R}_{j,t+1}^k L_{j,t} N_{j,t+1} = Z_{j,t+1} D_{j,t+1}. \quad (51)$$

Using (47) and (51) in (49), entrepreneur j 's objective function can be reexpressed as

$$E_t \left\{ \left[\int_{\bar{\omega}_{j,t+1}}^{\infty} \omega dF_{\omega}(\omega; \sigma_t) - (1 - F_{\omega}(\bar{\omega}_{j,t+1}; \sigma_t)) \bar{\omega}_{j,t+1} \right] \tilde{R}_{j,t+1}^k L_{j,t} N_{j,t+1} \right\}, \quad (52)$$

which is proportional to net worth $N_{j,t+1}$.

Similarly, substituting (51) and (47) into (39) and (40), the financial intermediaries' participation constraint is

$$\frac{L_{j,t} - 1}{L_{j,t}} R_t = [1 - F_{\omega}(\bar{\omega}_{j,t+1}; \sigma_t)] \bar{\omega}_{j,t+1} \tilde{R}_{j,t+1}^k + (1 - \mu_m) \int_0^{\omega_{j,t+1}} \omega dF_{\omega}(\omega; \sigma_t) \tilde{R}_{j,t+1}^{k,\psi}, \quad (53)$$

where the portfolio return conditional on separation is defined by $\tilde{R}_{j,t+1}^{k,\psi} \equiv \int_x w_{j,t}^x R_{j,t+1}^{k,\psi,x} dx + w_{j,t}^c R_{j,t+1}^{k,\psi,c}$, and $R_{j,t+1}^{k,\psi,m}$ denotes the return of an effective unit of separated capital, which, similar to (44), is defined by

$$R_{j,t+1}^{k,\psi,m} \equiv \frac{r_{j,t+1}^k + (1 - \delta) J_{t+1}^u}{Q_t^m}, \quad (54)$$

for $m \in \{x, c\}$. The combinations $(\bar{\omega}_{j,t+1}, L_{j,t}, \tilde{h}_{j,t}, u_{j,t}, w_{j,t}^x, w_{j,t}^c)$ that satisfy (53) define a menu of $(t+1)$ -contingent debt contracts offered to entrepreneurs equivalent to those defined in (39). Let $\mathcal{D}_t(\tilde{h}_{j,t}, u_{j,t}, w_{j,t}^x, w_{j,t}^c)$ denote the set of debt schedules $(\bar{\omega}_{j,t+1}, L_{j,t})$ offered to entrepreneurs by financial intermediaries.

Entrepreneur j 's problem is to choose the state-contingent plans $\tilde{h}_{j,t}$, $u_{j,t}$, $L_{j,t}$ and $\omega_{j,t+1}$, $w_{j,t}^x$, and $w_{j,t}^c$, with $(L_{j,t}, \omega_{j,t+1}) \in \mathcal{D}_t(\tilde{h}_{j,t}, u_{j,t}, w_{j,t}^x, w_{j,t}^c)$ that maximize the expected net worth (52) subject to the sequence of technological constraints, (43), return constraints, (44) and (54), and nonnegativity constraint for portfolio weights ($w_{j,t}^m \geq 0$ for $m \in \{x, c\}$) for the given sequence of prices $(W_t, Q_t^c$ and $J_t^u)$, debt schedules $(\mathcal{D}_t(\tilde{h}_{j,t}, u_{j,t}, w_{j,t}^x, w_{j,t}^c))$, market-tightness functions $(\theta_t(x))$, risk (σ_t) , and neutral-technology shocks (A_t) . With $\Lambda_{j,t+1}^e$ as the Lagrange multiplier on the financial intermediary's participation constraint, and $\Xi_{j,t}^m$ as the Lagrange multiplier associated with nonnegativity constraint for portfolio weights ($w_{j,t}^m \geq 0$),

the optimality conditions are (43), (44), (53), (54), and

$$A_t(1 - \alpha) \left(\tilde{h}_t \right)^{-\alpha} = W_t, \quad (55)$$

$$\alpha A_t \tilde{h}_t^{(1-\alpha)} = \mathcal{C}'_u(u_t), \quad (56)$$

$$E_t \left\{ \left[1 - \Gamma_t(\bar{\omega}_{t+1}) \right] \frac{R_{t+1}^k}{R_t} - \Lambda_{t+1}^e \left[\frac{R_{t+1}^k}{R_t} \left(\Gamma_t(\bar{\omega}_{t+1}) - g_t(\bar{\omega}_{t+1}) + (1 - \mu_m) g_t(\bar{\omega}_{t+1}) \frac{R_{t+1}^{k,\psi}}{R_{t+1}^k} \right) - 1 \right] \right\} = 0, \quad (57)$$

$$\Lambda_{t+1}^e = \frac{\Gamma'_t(\bar{\omega}_{t+1})}{\Gamma'_t(\bar{\omega}_{t+1}) - \mu g'_t(\bar{\omega}_{t+1}) + (1 - \mu_m) g'_t(\bar{\omega}_{t+1}) \left(\frac{R_{t+1}^{k,\psi}}{R_{t+1}^k} - 1 \right)}, \quad (58)$$

$$Q_t^m = Q_t + \Xi_t^m, \quad (59)$$

and the complementary slackness conditions

$$\Xi_t^m \geq 0, \quad w_t^m \Xi_t^m = 0, \quad \text{for } m \in \{x, c\}, \quad (60)$$

where $\Gamma_t(\bar{\omega}_{t+1}) \equiv [1 - F_\omega(\bar{\omega}_{t+1}; \sigma_t)] \bar{\omega}_{t+1} + g_t(\bar{\omega}_{t+1})$, $g_t(\bar{\omega}_{t+1}) \equiv \int_0^{\bar{\omega}_{t+1}} \omega dF_\omega(\omega; \sigma_t)$, and $Q_t \equiv \frac{r_{t+1}^k + (1-\delta) [\bar{\psi} J_{t+1}^u + (1-\bar{\psi}) Q_{t+1}^c]}{R_{t+1}^k}$. The entrepreneur's subindex, j , has been dropped because the objective function is linear in the net worth of entrepreneur j and does not appear in any of the constraints. Therefore, all entrepreneurs will choose the same plans $(h_t, u_t, L_t$ and $\omega_{t+1})$, independent of net worth.

4.5 Government

The government is assumed to consume a stochastic amount of consumption goods, financed each period by levying lump-sum taxes on households. The government budget constraint is given by

$$G_t = T_t, \quad (61)$$

where G_t is government spending in period t (labeled the *government-spending shock*).

4.6 Equilibrium

In equilibrium, all centralized markets clear. For the centralized market for unmatched capital, equilibrium then requires that

$$\int_0^1 \iota_{i,t}^h di = \psi_t (1 - \delta) K_t^e, \quad (62)$$

where $K_t^e \equiv \int_0^1 K_{j,t}^e dj$ denotes the aggregate stock of employed capital at the beginning of period t , and $\psi_t \equiv (1 - g_{t-1}(\bar{\omega}_t))\bar{\psi} + g_{t-1}(\bar{\omega}_t)$ denotes the total share of employed capital that was separated in period t as a result of entrepreneurs' retirement and default. The left-hand side of (62) represents households' purchases in the market for unmatched capital. The right-hand side of (62) represents the mass of capital sold in the market for unmatched capital, from retired entrepreneurs and financial intermediaries that seized capital of defaulting entrepreneurs (see Figure 7).

Replacing (62) in (31) and using the law of large numbers and the fact that the choice of submarket, $x_{k,i,t}$, is the same for all units of capital, k , and all households, i , and the choice of investment, $I_{i,t}$ is the same for all households i , the law of motion for unemployed capital is

$$K_{t+1}^u = (1 - p(\theta_t(x_t^u)))(1 - \delta)K_t^u + \psi_t(1 - \delta)K_t^e + A_t^I \left[I_t - \Phi\left(\frac{I_t}{K_t}\right) K_t \right], \quad (63)$$

where $K_t^u \equiv \int_0^1 K_{i,t}^u di$ denotes the aggregate stock to unemployed capital at the beginning of period t , and $I_t = \int_0^1 I_{i,t} di$ denotes aggregate investment.

Given that matched capital is homogeneous, no arbitrage between centralized and decentralized markets of matched capital requires $Q_t = Q_t^c$. Moreover, entrepreneurs' optimality conditions (59) and (60) imply that, in equilibrium, any submarket visited by a positive number of entrepreneurs must have the same cost per unit of capital, and entrepreneurs will be indifferent among them. Formally, for all x ,

$$\theta_t(x) \left(x + \frac{c_s}{q(\theta_t(x))} - Q_t \right) = 0. \quad (64)$$

This condition determines the equilibrium market-tightness function: For all $x < Q_t$,

$$\theta_t(x) = q^{-1} \left(\frac{c_s}{Q_t - x} \right). \quad (65)$$

For all $x \geq Q_t$, $\theta_t(x) = 0$.

Using the definition of market tightness, the law of large numbers, and the fact that a household's choice of submarket, $x_{k,t}$ is the same for all units of capital k , the flow of capital that transitions from unemployment to employment is given by $p(\theta_t(x_t^u))(1 - \delta)K_t^u = \int_0^1 \tilde{K}_{j,t+1}^{x^u} dj = \int_0^1 \int_x \tilde{K}_{j,t+1}^x dx dj$. Aggregating the entrepreneurs' capital-accumulation constraints and imposing market clearing in the centralized market provides a law of motion for

employed capital:

$$K_{t+1}^e = (1 - \psi_t)(1 - \delta)K_t^e + p(\theta_t(x_t^u))(1 - \delta)K_t^u. \quad (66)$$

The capital-unemployment rate at the beginning of period t can be then defined as

$$k_t^u \equiv \frac{K_t^u}{K_t}, \quad (67)$$

where $K_t \equiv K_t^e + K_t^u$ denotes total aggregate capital stock at the beginning of period t .

Labor-market clearing requires $\int_0^1 \tilde{h}_{j,t} u_{j,t} \omega_{j,t} K_{j,t}^e dj = h_t$, where $h_t \equiv \int_0^1 h_t di$. Aggregating production functions (42) across entrepreneurs, using the fact that all entrepreneurs choose the same level of hours worked and utilization for each unit of effective capital and imposing the labor-market-clearing condition yields,

$$Y_t = A_t (u_t K_t^e)^\alpha (h_t)^{(1-\alpha)}, \quad (68)$$

where Y_t denotes aggregate output in period t .

Let ζ_t denote the exogenous aggregate net transfer from households to entrepreneurs in period t (labeled the *equity shock*). Aggregate net worth then evolves following the law of motion

$$N_{t+1} = (1 - \bar{\psi})[1 - \Gamma_{t-1}(\bar{\omega}_t)]R_t^{k,c}Q_{t-1}K_t^e + \zeta_t, \quad (69)$$

where N_{t+1} denotes aggregate net worth at the end of period t , and $R_t^{k,c}$ denotes the return of an effective unit of capital that does not separate in period t , which, similar to (44) and (54), is defined by $R_t^{k,c} \equiv \frac{r_t^k + (1-\delta)Q_t^c}{Q_{t-1}}$. The first term on the right-hand side of (69) represents the aggregate return obtained from effective matched capital employed in period t by entrepreneurs who did not default in the default stage and did not retire in the separation stage. The second term on the right-hand side of (69) represents the exogenous aggregate transfer from households to new entrepreneurs. The return obtained from effective matched capital employed in period t by entrepreneurs who did not default in the default stage, but did retire in the separation stage is transferred to households. It follows that the net transfer from entrepreneurs to households is given by

$$\Pi_t = \bar{\psi}[1 - \Gamma_{t-1}(\bar{\omega}_t)]R_t^{k,\psi}Q_{t-1}K_t^e - \zeta_t. \quad (70)$$

where $R_t^{k,\psi} \equiv \frac{r_t^k + (1-\delta)J_t^u}{Q_{t-1}}$ denotes the return of an effective unit of capital that separates in period t .

Starting from the households' budget constraint (32) and replacing the government budget constraint (61), the market-clearing condition for unmatched capital (62), the market-clearing condition for the credit and labor markets, the definition of net revenues from production (43) and the participation constraints of financial intermediaries (39) aggregated across entrepreneurs, the expression for aggregate transfers from entrepreneurs (70) yields the economy's resource constraint,

$$C_t + I_t + G_t = Y_t - c_s \theta_t (1 - \delta) K_t^u - \Omega_t - \mathcal{C}_u(u_t) K_t^e, \quad (71)$$

where $\Omega_t \equiv \mu g_{t-1}(\bar{\omega}_t) R_t^{k,\psi} Q_{k,t-1} K_t^e$ and aggregate consumption is defined by $C_t \equiv \int_0^1 C_{i,t} di$.

Let $S_t^x \equiv [A_t, A_t^I, G_t, \varphi_t, \sigma_t, \zeta_t]$ define the aggregate exogenous state vector of the economy. The competitive equilibrium in this economy can then be defined as follows.

Definition 3 (Competitive equilibrium). *Given initial conditions for employed and unemployed capital, K_0^e and K_0^u , consumption C_{-1} , and a state-contingent sequence of aggregate exogenous states, S_t^x , a competitive equilibrium is a state-contingent sequence of individual allocations and shadow values $\{(C_{i,t}, h_{i,t}, I_{i,t}, \ell_{i,t}^h, K_{i,t+1}^u, B_{i,t}, x_{i,t}^u)_{i \in [0,1]}, (\tilde{h}_{j,t}, u_{j,t}, L_{j,t}, \omega_{j,t+1}, w_{j,t}^x, w_{j,t}^c)_{j \in [0,1]}\}$, $\{(\Lambda_{i,t})_{i \in [0,1]}, (Q_{j,t})_{j \in [0,1]}\}$, aggregate allocations $\{C_t, I_t, h_t, K_{t+1}^e, K_{t+1}^u, N_t, \Pi_t\}$, prices $\{Q_t^c, J_t^u, W_t\}$, debt schedules $\{\mathcal{D}_t(\tilde{h}_{j,t}, u_{j,t}, w_{j,t}^x, w_{j,t}^c)\}$, and market-tightness functions $\{\theta_t(x)\}$, such that:*

- (i) *Individual allocations and shadow values solve the household's and entrepreneur's problems at the equilibrium prices, equilibrium market-tightness functions, and debt schedules, for all i and j .*
- (ii) *Debt schedules satisfy financial intermediaries' participation constraint (53).*
- (iii) *The market-tightness function satisfies (65) for all x .*
- (iv) *Centralized markets clear.*

5 Quantitative Analysis

This section conducts a quantitative study of the role of search frictions in investment based on the model presented in Section 4. It begins by specifying assumptions for functional forms and stochastic processes contained in the model. It then discusses the empirical methodology for calibration and estimation of the model's parameters for the U.S. economy, presents estimation results, and conducts exercises based on the estimation related to the U.S. Great Recession and business cycles.

5.1 Model Estimation

Functional forms. The assumptions made on functional forms are standard in the related literature. For the households' period utility function,

$$\begin{aligned} U(c) &= \frac{c^{1-v}}{1-v} \\ V(h; \varphi) &= \varphi \frac{h^{1+\frac{1}{\phi}}}{1+\frac{1}{\phi}}, \end{aligned}$$

where $v > 0$ is the inverse of the intertemporal elasticity of substitution and $\phi > 0$ is the Frisch elasticity of labor supply.

Investment-adjustment costs are assumed to take a quadratic form:

$$\Phi\left(\frac{I_t}{K_t}\right) = \frac{\kappa}{2} \left(\frac{I_t}{K_t} - \delta\right)^2,$$

where $\kappa > 0$ is a parameter governing the degree of investment-adjustment costs.

Utilization costs are assumed to take the form

$$\mathcal{C}_u(u) = \alpha \bar{h}^{(1-\alpha)} \left[e^{c_u(u-1)} - 1 \right] \frac{1}{c_u},$$

where $c_u > 0$, and $\bar{h}$ is the steady-state level of hours worked per unit of employed capital, defined by $\bar{h} \equiv \frac{\bar{h}}{\bar{K}^e}$, where $\bar{h}$ and $\bar{K}^e$ are the steady-state level of hours worked and employed capital. As in [Christiano, Motto and Rostagno \(2014\)](#), this functional form is chosen to obtain a steady-state unity utilization rate independent of the parameter c_u .

The matching function is assumed to take a CES function, yielding the finding probabil-

ities:

$$\begin{aligned} p(\theta) &= \theta \left(1 + \theta^\xi\right)^{-1/\xi}, \\ q(\theta) &= \left(1 + \theta^\xi\right)^{-1/\xi}, \end{aligned}$$

where $\xi > 0$. This functional form has been used in quantitative studies of directed search in the labor market (see, for example, [Schaal, 2012](#)).

Stochastic processes. The six aggregate shocks are modeled as first-order autoregressive processes:

$$\begin{aligned} \log A_t &= \rho_A \log A_{t-1} + \epsilon_t^A, \\ \log A_t^I &= \rho_{A^I} \log A_{t-1}^I + \epsilon_t^I, \\ \log G_t &= (1 - \rho_G) \log \bar{G} + \rho_G \log G_{t-1} + \epsilon_t^G, \\ \log \varphi_t &= (1 - \rho_\varphi) \log \bar{\varphi} + \rho_\varphi \log \varphi_{t-1} + \epsilon_t^\varphi, \\ \log \sigma_t &= (1 - \rho_\sigma) \log \bar{\sigma} + \rho_\sigma \log \sigma_{t-1} + \epsilon_t^\sigma, \\ \zeta_t &= (1 - \rho_\zeta) \bar{\zeta} + \rho_\zeta \zeta_{t-1} + \epsilon_t^\zeta, \end{aligned}$$

where $\bar{G} > 0$ denotes steady-state government spending, $\bar{\varphi} > 0$ is a parameter that determines steady-state hours worked, $\bar{\sigma} > 0$ denotes the steady-state cross-sectional dispersion of idiosyncratic shocks, $\bar{\zeta}$ denotes steady-state lump-sum transfers from households to entrepreneurs, and it is assumed that $\epsilon_t^i \sim N(0, \sigma^i) \forall t$ and $i \in \{A, A^I, G, \varphi, \sigma, \zeta\}$.

Data. The model is estimated using U.S. quarterly data prior to the Great Recession, from 1980:Q1 to 2007:Q4.¹³ The data include six time series: real per capita GDP, real per capita consumption, real per capita nonresidential private investment, per capita hours worked, credit spreads, and commercial, nonresidential real estate vacancy rates. Data on GDP, consumption and investment were log-linearly detrended. Credit spreads were measured by the difference between the interest rate on BAA corporate bonds and the three-month U.S. government bond rate. Appendix A provides more detailed information about the sources and construction of these data.

¹³The estimation period begins in 1980 due to the availability of commercial-real-estate vacancy rates.

Including data on GDP, consumption, investment, and hours is standard in the empirical business-cycle literature. Including credit spreads is relevant to discipline the financial friction and financial shocks (see [Christiano, Motto and Rostagno, 2014](#)). The counterpart of this variable in the model is the difference between the interest rate paid by entrepreneurs, Z_t , and the risk-free rate R_t . Including data on the commercial-real-estate vacancy rate (see Figure 2) is a novel feature of the present paper and is aimed at disciplining the search friction in investment – specifically, the two parameters related to search frictions, the curvature of the matching function, ξ , and the search cost, c_s . The counterpart of this variable in the model is the capital-unemployment rate, k_t^u .

It is assumed that all series are observed with measurement error. Measurement error in output, consumption, investment, hours worked, credit spreads and vacancy rates, denoted $\epsilon_{Y,t}^{\text{me}}$, $\epsilon_{C,t}^{\text{me}}$, $\epsilon_{I,t}^{\text{me}}$, $\epsilon_{h,t}^{\text{me}}$, $\epsilon_{s,t}^{\text{me}}$ and $\epsilon_{k^u,t}^{\text{me}}$, are assumed to be i.i.d. innovations with mean zero and standard deviation $\sigma_i^{\text{me}} \forall i \in \{Y, C, I, h, s, k^u\}$.

Empirical strategy. From the assumed functional forms and stochastic processes in the previous sections, the model features 27 structural parameters. Let Θ be a vector containing all the parameters of the model. This vector also includes the six nonstructural parameters representing the standard deviations of the measurement errors on the observables, as discussed in the previous section. The model parameters are partitioned into two sets: $\Theta = [\Theta_1, \Theta_2]$. The first set,

$$\Theta_1 \equiv [\beta, v, \phi, \alpha, \delta, \bar{\psi}, \mu_m, \bar{G}, \bar{\varphi}, \bar{\sigma}],$$

contains 10 calibrated or fixed a priori parameters. The remaining 23 parameters,

$$\Theta_2 \equiv [\rho_c, \kappa, c_u, \xi, c_s, \rho_A, \rho_{A^I}, \rho_G, \rho_\varphi, \rho_\sigma, \rho_\zeta, \sigma_A, \sigma_{A^I}, \sigma_G, \sigma_\varphi, \sigma_\sigma, \sigma_\zeta, \sigma_Y^{\text{me}}, \sigma_C^{\text{me}}, \sigma_I^{\text{me}}, \sigma_h^{\text{me}}, \sigma_s^{\text{me}}, \sigma_{k^u}^{\text{me}}],$$

are estimated using Bayesian methods surveyed in [An and Schorfheide \(2007\)](#). The following sections discuss the values assigned to parameters fixed a priori and the estimation of the remaining parameters.

TABLE I
CALIBRATED PARAMETERS

Parameter	Value	Description
β	0.99	Subjective discount factor
ν	2	Intertemporal elasticity of substitution
ϕ	1	Frisch elasticity of labor supply
α	0.4	Capital share
δ	0.025	Depreciation rate of capital
ψ	0.028	Retirement rate of entrepreneurs
$\bar{G}$	0.2	Steady-state share of government spending
$\bar{h}$	1	Steady-state hours worked
$\bar{s}$	0.02	Steady-state annual spreads
$\bar{\mu}$	0.2	Steady-state loss in default
$F(\bar{\omega})$	0.075	Default rate

Note: The time unit is one quarter.

Benchmark model without investment search frictions. To put the results of the estimated model from Section 4 into perspective, a benchmark model for the U.S. economy is also estimated. This benchmark model, detailed in Appendix E, is identical to the model of Section 4 except for the search friction in investment considered in this paper. The same empirical strategy described in the previous section is used for the benchmark model. The only differences are that the set of parameters Θ_2 does not include the parameters related to the search friction (i.e., ξ and c_s), and that the structure vacancy data are not included in the estimation as an observable. Henceforth, the model in Section 4 is labeled as the “Model with Search Frictions” and the benchmark model as “Model No Search Frictions.”

Calibrated parameters. Table I displays the values assigned to the calibrated parameters, contained in the vector Θ_1 or related targets. The subjective discount factor, β , the inverse of the intertemporal elasticity of substitution, ν , the Frisch elasticity of labor supply, ϕ , the aggregate capital share, α , and the depreciation rate, δ , are set to 0.99, 2, 1, 0.4, and 0.025, respectively, standard values in related business cycle literature. The labor disutility parameter $\bar{\varphi}$ is set at a value consistent with a steady-state level of hours worked of one. The value of $\bar{\psi}$ is set to 0.027, which is consistent with the average annual exit rate of establishments in the United States for the period 1980–2007 of 11%. This value is also in line with the death rate of entrepreneurs in quantitative implementations of the costly state-verification framework. The value of the steady-state share of government spending, $\bar{G}$, was set at 0.2, a standard value in business-cycle studies for the U.S. economy.

The values used for the parameters related to the financial friction (μ_m , $\bar{\omega}$, and $\bar{\zeta}$) are close to those used in previous quantitative studies of the costly state verification. In particular, the values of $\bar{\omega}$ and $\bar{\zeta}$ were set to target values of annual default rate and annual spreads of 3% and 200 basis points, respectively, which correspond to the U.S. historical averages (used for example in [Bernanke, Gertler and Gilchrist, 1999](#)). To set the value of the parameter μ_m , note that in the framework of the present paper, financial intermediaries in the state of default face a loss of the return of capital R_t^k not only related to monitoring costs (as in previous models with costly state verification, but without search frictions in investment), but also related to the fact that capital becomes unmatched in the event of default (and has a return of $R_t^{k,\psi}$ instead of R_t^k ; see Sections 4.3 and 4.4). To make the loss in default comparable to those of previous studies – e.g., between 0.2 and 0.36 in [Carlstrom and Fuerst \(1997\)](#); 0.12 in [Bernanke, Gertler and Gilchrist \(1999\)](#) – μ_m was set to target a value of steady-state loss in default, $\bar{\mu}$, of 0.2, where the steady-state loss in default is defined as $\bar{\mu} \equiv 1 - (1 - \mu_m) \left(\frac{\bar{R}^{k,\psi}}{\bar{R}^k} \right)$, with $\bar{R}^{k,\psi}$ and $\bar{R}^k$ denoting the steady-state values of $R_t^{k,\psi}$ and R_t^k .¹⁴

Estimated parameters. Table II presents the assumed prior distributions of the estimated parameters contained in the vector Θ_2 , denoted $P(\Theta_2)$. For the two parameters related to the search friction in investment – namely, the curvature of the matching function, ξ , and the search cost, c_s , for which, to my knowledge, estimates are not available – inverse gamma distributions were chosen. The mean of the distribution of the curvature of the matching function (ξ) was set at the value of 1. The mean of the distribution of the search cost parameter, c_s was set to 0.06 to target a steady-state level of capital under the mean of the prior distributions equal to the one observed in the data. For the other parameters, prior distributions were chosen following the related literature estimating models for the U.S. economy ([Smets and Wouters, 2007](#); [Schmitt-Grohé and Uribe, 2012a](#); [Christiano, Motto and Rostagno, 2014](#)).

In particular, the standard errors of the innovations are assumed to follow an inverse-gamma distribution with a mean of 0.1 and a standard deviation of 2; the persistence of the autoregressive stochastic processes, a beta distribution with mean 0.5 and standard deviation of 0.2; the parameter that governs internal habit formation (ρ_c), a beta distribution with mean 0.5 and standard deviation of 0.2; the parameter that governs investment adjustment costs

¹⁴In the benchmark model without search frictions, $\mu_m = \bar{\mu}$.

(κ), a gamma distribution with mean 3 and standard deviation of 2; and the parameter that governs the curvature of capital utilization costs (c_u), an inverse-gamma distribution with mean 2.5 and standard deviation of 2. Finally, uniform prior distributions were chosen for the innovations of the measurement error. These variables are restricted to account for at most 6% of the variance of the corresponding observable time series.

Given the prior parameter distribution, $P(\Theta_2)$, the Metropolis–Hastings algorithm was used to obtain draws from the posterior distribution of Θ_2 , denoted $\mathcal{L}(\Theta_2|Y)$ where Y is the data sample (see, for example, [An and Schorfheide, 2007](#)). Table II presents the posterior estimates of the model parameters with search frictions in investment.

Model fit. The predictions of the model regarding standard deviations, correlation with output and serial correlations of the six time series included in the estimation as observables are presented in Table III, together with their data counterparts. The predictions of the

TABLE II
ESTIMATED PARAMETERS ON U.S. DATA - MODEL WITH SEARCH FRICTIONS

		Prior distribution			Posterior distribution	
Parameter	Description	Distribution	Mean	St. dev	Mean	St. dev
A. Economic parameters						
ρ_c	Habit parameter	Beta	0.5	0.2	0.62	0.04
κ	Investment-adj costs	Gamma	3	2	4.3	0.3
ξ	Curvature-matching tech	Inv Gam	1	0.1	0.50	0.02
c_s	Search cost	Inv Gam	0.06	0.005	0.08	0.01
c_u	Curvature-utilization	Inv Gam	2.5	2	3.1	0.26
B. Stochastic processes						
Autocorrelations						
ρ_A	Neutral technology	Beta	0.5	0.2	0.98	0.01
ρ_{A^1}	Investment-specific tech	Beta	0.5	0.2	0.91	0.05
ρ_G	Government spending	Beta	0.5	0.2	0.91	0.02
ρ_φ	Labor wedge	Beta	0.5	0.2	0.986	0.004
ρ_σ	Risk	Beta	0.1	0.5	0.78	0.04
ρ_ζ	Equity	Beta	0.1	0.5	0.83	0.05
Standard deviation innovation						
σ_A	Neutral technology	Inv Gam	0.1	2	0.005	0.0004
σ_{A^1}	Investment-specific tech	Inv Gam	0.1	2	0.03	0.003
σ_G	Government spending	Inv Gam	0.1	2	0.02	0.002
σ_φ	Labor wedge	Inv Gam	0.1	2	0.02	0.002
σ_σ	Risk	Inv Gam	0.1	2	0.09	0.01
σ_ζ	Equity	Inv Gam	0.1	2	0.05	0.005

Note: The time unit is one quarter. Bayesian estimates are based on 500,000 draws from the posterior distribution.

TABLE III
SECOND MOMENTS: DATA AND MODEL

	Standard deviations					
	σ_Y	$\frac{\sigma_C}{\sigma_Y}$	$\frac{\sigma_I}{\sigma_Y}$	$\frac{\sigma_h}{\sigma_Y}$	$\frac{\sigma_s}{\sigma_Y}$	$\frac{\sigma_{k^u}}{\sigma_Y}$
Data	4.1	0.82	2.80	1.39	0.45	0.55
Model with search	3.9	0.84	5.93	1.14	0.89	0.56
Model no search	2.8	0.99	4.02	1.08	0.45	
	Correlations with output					
	$\rho(C,Y)$	$\rho(I,Y)$	$\rho(h,Y)$	$\rho(s,Y)$	$\rho(k^u,Y)$	
Data	0.90	0.51	0.78	-0.53	-0.28	
Model with search	0.36	0.80	0.65	-0.40	-0.21	
Model no search	0.63	0.70	0.49	0.17		
	Autocorrelations					
	$\rho(Y_t, Y_{t-1})$	$\rho(C_t, C_{t-1})$	$\rho(I_t, I_{t-1})$	$\rho(h_t, h_{t-1})$	$\rho(s_t, s_{t-1})$	$\rho(k_t^u, k_{t-1}^u)$
Data	0.98	0.99	0.98	0.99	0.87	0.99
Model with search	0.94	0.99	0.89	0.92	0.84	0.98
Model no search	0.95	0.97	0.93	0.94	0.75	

Note: Columns labeled Y , C , I , h , s , and k^u refer, respectively, to output, consumption, investment, hours worked, credit spreads, and capital unemployment in the model. Data counterparts described in Appendix A. The time unit is one quarter. Data corresponds to the period 1962–2013, except for capital unemployment, which corresponds to the period 1980–2013.

benchmark model without search frictions in investment are also presented in Table III for comparison.

Overall the predictions of the estimated models are in line with empirical second moments. The predicted standard deviations of the model with search frictions are in general larger than the one of the model without search frictions. For output, consumption, hours worked and capital unemployment the predictions of the model with search friction are similar to those observed in the data; for investment and credit spreads the model with search frictions predicts a higher volatility than the one observed in the data. The correlations with output and autocorrelations predicted by the estimated models are in general in line with those observed in the data. For the case of credit spreads, while the estimated model without search frictions predicts a positive correlation with output, the model with search frictions in investment predicts a negative correlation with output, as observed in the data.

5.2 Quantitative Results

This section presents two exercises based on the estimated model to study the quantitative relevance of the proposed mechanism. The first relates to the Great Recession, which is an example of a deep financial crisis of the sort that motivated this theoretical framework

(see Section 1). The second exercise studies the role of financial shocks in U.S. business cycle fluctuations in the presence of search frictions in investment. Both exercises proceed by comparing the results from the model presented in Section 4 to a benchmark model without investment search frictions, as presented in the previous section and detailed in Appendix E.

Recovery from the U.S. Great Recession. The estimated model is used to ask whether, following a sequence of shocks such as those experienced by the U.S. economy in 2008, and without any further shock, the model can predict an investment slump such as the one observed following the U.S. Great Recession – that, as discussed in Section 1, is an empirical regularity of financial-crisis episodes. To answer this question, the estimated model is used to smooth the shocks experienced by the U.S. economy through the last quarter of 2008. Beginning in the first quarter of 2009, the predicted response of the economy is computed: All shocks are set to zero, and the driving stochastic processes are only driven by their estimated autoregressive components; states evolve endogenously.

Results from this exercise are displayed in Figure 8 and indicate that the model with investment search frictions predicts a slump of investment following the U.S. Great Recession even larger than the one observed in the data. The same exercise in the benchmark model without investment search frictions predicts that both investment and output should be significantly higher than the levels observed in the data, as noted in the previous literature (see Section 1). The right panel of Figure 8 also shows that the proposed model with search frictions in investment can account for 50% of the difference between the observed recovery and the recovery predicted by the benchmark model without search frictions.¹⁵

The Role of Financial Shocks in U.S. Business Cycles. The estimated model can also be used to interpret the sources of U.S. business-cycle fluctuations. Table IV compares the variance decomposition predicted by the model with investment search frictions to the variance decomposition predicted by the benchmark model without search frictions. The most remarkable result is the difference between the two models in term of the contribution of financial shocks. The benchmark model without investment search frictions assigns a small role to financial shocks, and attributes most of the predicted movements in output and

¹⁵It is worth noting that the model’s prediction for capital unemployment is in line the data on vacancy rates observed in the Great Recession. This variable and the prediction for the rest of the observables are included in Appendix B, showing that for all variables the model with search frictions in investment predict less recovery than the model without search frictions in investment.

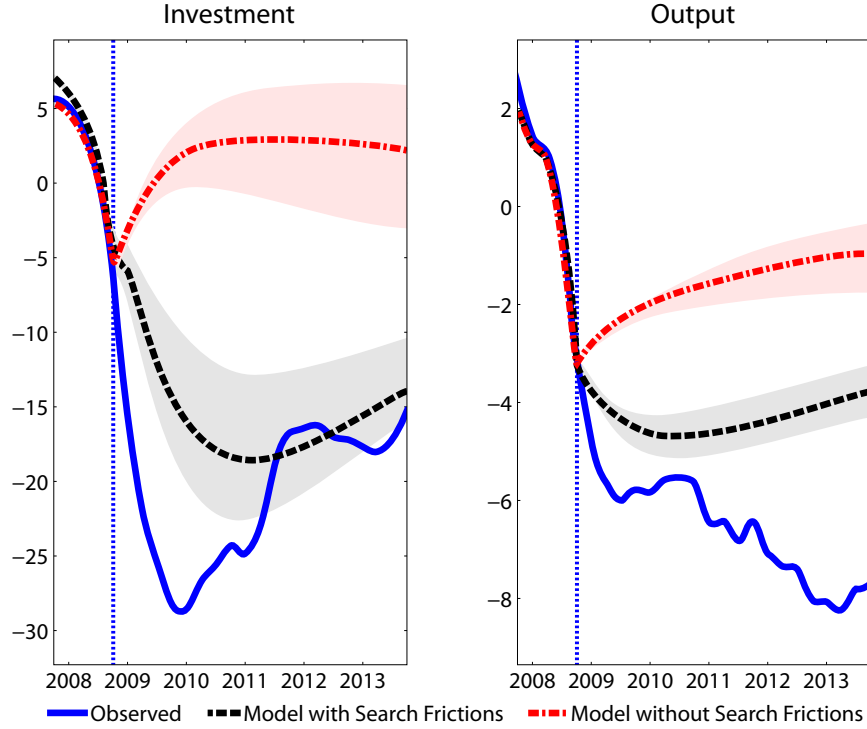


FIGURE 8: U.S. GREAT RECESSION: PREDICTED RECOVERY.

Note: Time-series labeled *Observed* correspond to the data on real per capital investment and output, log-linearly detrended (see Appendix A for details). Time-series labeled *Model with Search Frictions* and *Model No Search Frictions* refer, respectively, to predictions from the model presented in Section 4, and to predictions from the benchmark model presented in Appendix E. Model predictions computed since 2009, following the sequence of shocks smoothed from the estimated models for the period 1980–2007. Shadow areas represent equal tail probability 90 % credible sets associated with the posterior distributions. The time unit is one quarter. For details on the models’ estimations see Section 5.1 and Appendix E.

investment to technology shocks (neutral and investment-specific) and to labor wedge shocks. The model with search frictions developed in this paper attributes a relevant role to financial shocks, which account for 33% of output fluctuations and 56% of investment fluctuations.

This result is of interest since the role of financial shocks is a key discussion in the business-cycle literature and an important source of discrepancy between real and monetary models, with the latter attributing a much larger effect to these shocks than the former (as discussed in [Christiano, Motto and Rostagno, 2014](#)). The present paper shows that an important part of this discrepancy between these two branches of the literature can be reconciled by introducing investment search frictions. To understand this result, note that in the model with search frictions in investment, 63% of the predicted movements in capital unemployment are

TABLE IV
VARIANCE DECOMPOSITION

Shock		Y	C	I	h	s	k^u
Model no search							
Neutral technology	A	31.8	33.1	13.2	15.9	1.3	
Investment-specific technology	A^I	24.5	20.0	55.9	19.4	42.5	
Labor wedge	φ	42.0	45.3	15.8	60.0	1.6	
Government spending	G	0.8	1.1	4.6	3.2	0.6	
Risk	σ	0.1	0.1	3.3	0.4	44.7	
Equity	ζ	0.7	0.5	7.2	1.1	9.5	
Model with search							
Neutral technology	A	17.4	18.8	7.0	6.0	0.1	0.8
Investment-specific technology	A^I	4.1	5.4	23.5	11.6	5.6	34.6
Labor wedge	φ	44.6	56.9	11.9	54.8	0.2	1.3
Government spending	G	0.5	0.7	1.4	1.6	0.0	0.3
Risk	σ	0.5	0.3	2.0	0.4	39.1	15.0
Equity	ζ	33.0	17.9	54.2	25.6	55.0	47.9

Note: Columns labeled Y , C , I , h , s , and k^u refer, respectively, to output, consumption, investment, hours worked, credit spread, and capital unemployment in the model. Data counterparts described in Appendix A.

explained by financial shocks. Studying impulse-response functions, the next section comes back to this result.

Impulse responses. To further study the quantitative findings presented in this section, Figure 9 shows the impulse response of capital unemployment, investment, and output to a one-standard-deviation negative neutral-technology shock and a one-standard-deviation negative equity shock. The responses of investment and output in the benchmark model without investment search frictions are also included for comparison.¹⁶ While a negative neutral-technology shock generates a decrease of capital unemployment, a negative equity shock generates an increase in capital unemployment. Moreover the response of capital unemployment is 10 times larger in absolute value in response to a one-standard-deviation equity shock than in response to a one-standard-deviation neutral-technology shock. For this reason, the responses of investment and output are more different in the case of the financial shock than in the case of a neutral-technology shock. The impulse-response functions also indicate a large and persistent effect on investment and output following a negative financial shock that is not present in the benchmark model without investment search frictions.

¹⁶Standard deviations refer to those of the model with search frictions in investment. Appendix B shows the impulse response for all shocks and for the six observables included in the estimation.

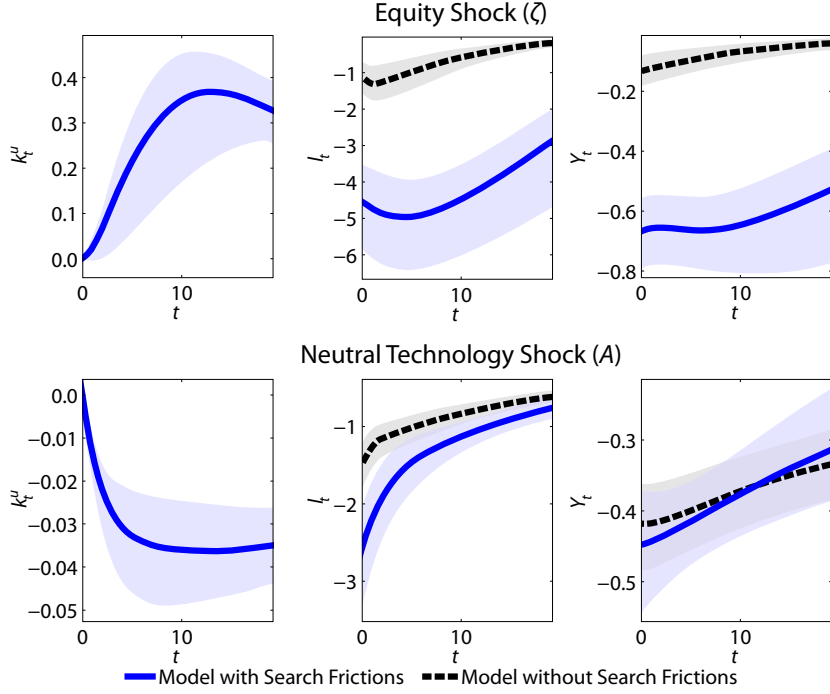


FIGURE 9: IMPULSE-RESPONSES TO CONTRACTIONARY SHOCKS.

Note: Response of capital unemployment, investment, and output to a one-standard-deviation negative equity shock (ζ) and a neutral-technology shock (A). Label “Model with Search Frictions in Investment” and “Model No Search Frictions in Investment” refer, respectively, to the model responses presented in Section 4 and the benchmark model in Appendix E. Impulse responses expressed in percent deviations from steady state. Horizontal axes display quarters after the shock. Shadow areas represent equal tail probability 90 % credible sets associated with the posterior distributions.

6 Capital Reallocation

This section shows that the analytical framework with investment search frictions developed in this paper can also be used to study capital reallocation. It begins by extending the model to allow for heterogeneity in capital match-specific productivity. This extension allows a characterization not only of the transition of capital from unemployment to employment, but of the transition of capital from employment to employment, since it adds a motive for trading capital while it remains employed (similar to “on the job search” in the labor-market literature; see [Menzio and Shi, 2011](#)). A quantitative analysis of the extended model shows that the model’s predictions regarding capital reallocation are in line with those observed in the data. The model also has predictions regarding misallocation during crises.

6.1 Extended Model with Capital Reallocation

The basis of the analytical framework developed in this section is the quantitative model developed in Section 4. The section begins by describing the extended model's new assumptions regarding production technology and the market structure of physical capital. It then discusses the problem of selling employed capital in the decentralized market, the entrepreneur's problem, and equilibrium in the extended framework. The notation used in this section is the same as that presented in Section 4.

Production technology. As in Section 4, it is assumed that entrepreneurs have access to technology to produce consumption goods using labor and matched capital as inputs. Unlike in Section 4, each unit of employed capital has a match-specific productivity. This match-specific productivity is revealed after an unmatched unit of capital becomes matched, and does not vary until the specific match is destroyed. The output produced by an effective unit of capital i , with match-specific productivity z_i , and employing $\tilde{h}_{it}$ hours of work, is given by

$$y_{i,t} = A_t z_{i,t} \left(\tilde{h}_{i,t} \right)^{1-\alpha}, \quad (72)$$

where $z_i \in \mathcal{Z} = \{z_1, z_2, \dots, z_{N_z}\}$, $N_z \geq 2$ and $\mathcal{Z} \gg 0$.

Physical capital markets. As in Section 4, capital held by entrepreneurs is denoted employed capital, and capital held by households is denoted unemployed capital. Households can only hold unmatched capital. Trade of unmatched capital between entrepreneurs (buyers) and households (sellers) occurs in a decentralized market with search frictions. The search frictions that characterize the decentralized market for unmatched capital are identical to those in Sections 3 and 4. Unlike in Section 4, entrepreneurs now also have access to the decentralized market as sellers, where they can sell an employed unit of capital as unmatched capital to other entrepreneurs. When a unit of capital employed with match-specific productivity z_i is traded in the decentralized market, a new match-specific productivity is drawn from the set $\mathcal{Z}$, with a probability mass function $f_Z(z) : \mathcal{Z} \rightarrow [0, 1]$, assumed to be the same for all t . Let $\bar{z}$ denote the expected match-specific productivity of a new match (i.e. $\bar{z} \equiv E(z_i)$). It is assumed that $\bar{z} \in \mathcal{Z}$.

As in Section 4, entrepreneurs also have access to a centralized market in which they

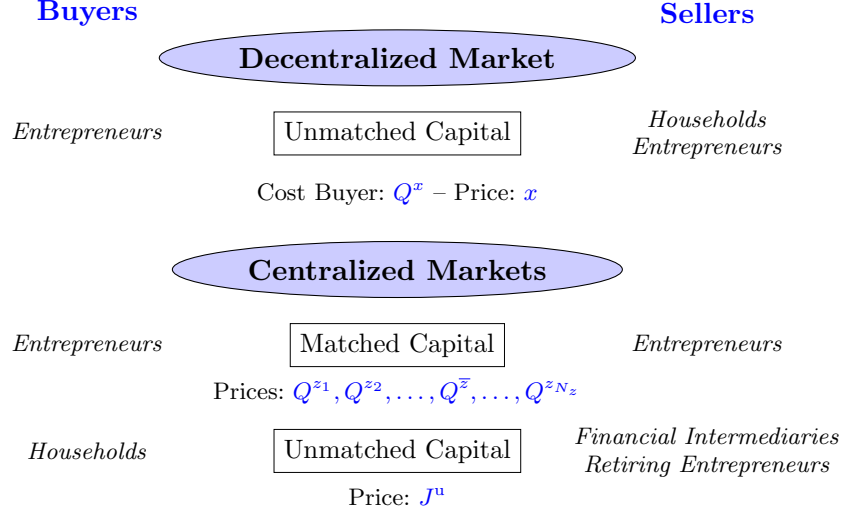


FIGURE 10: STRUCTURE OF CAPITAL MARKETS, MODEL WITH CAPITAL REALLOCATION.

trade matched capital. When a unit of employed capital is traded in the centralized market it maintains its match-specific productivity. The match-specific productivity of any unit of capital is common knowledge. The difference with respect to Section 4 is that now units of capital matched at different match-specific productivities will be traded at different prices. The price in the centralized market of a unit of capital with match-specific productivity z_i is denoted Q^{z_i} . The price in the centralized market of a unit of capital matched at the average productivity $\bar{z}$ will be denoted $Q^{\bar{z}}$.

Finally, as in Section 4 there is also a centralized market in which unmatched capital can be sold by financial intermediaries and retired entrepreneurs to households at price J^u . Figure 10 summarizes these three markets for capital, with the participants and forms of trade that characterize each market.

Seller's problem for employed capital. An entrepreneur that holds a unit of employed capital matched at productivity z_i can choose to sell this unit in the decentralized market – as an unmatched unit of capital – just as households do with their units of unemployed capital. The only difference between entrepreneurs and households when visiting the decentralized market as sellers is that in the event of not finding a buyer the price of a unit of matched capital is different from the price of a unit of unmatched capital. Therefore, entrepreneurs who visit the decentralized market as sellers and households will typically search in different submarkets. For the same reason entrepreneurs holding units of capital at different match-

specific productivities will also search in different submarkets. Formally, the *seller's problem* for an entrepreneur holding a unit of employed capital matched at productivity z_i is given by

$$\max_{x_t^{z_i}} \{p(\theta_t(x_t^{z_i}))x^{z_i} + (1 - p(\theta_t(x_t^{z_i})))Q_t^{z_i}\}, \quad (73)$$

where $x_t^{z_i}$ denotes the submarket visited by an entrepreneur that holds a unit of capital matched at productivity z_i .

Entrepreneur's problem. As in Section 4, entrepreneurs purchase capital using their net worth and borrowing from financial intermediaries. Including match-specific productivity into the framework developed in Section 4 implies that the entrepreneur's balance sheet now includes different types of assets purchased in the centralized market. At the end of each period, t , equation (74) describes entrepreneur j 's balance sheet:

$$\int_x Q_t^x \tilde{K}_{j,t+1}^x dx + \sum_i Q_t^{z_i} \tilde{K}_{j,t+1}^{z_i} = D_{j,t+1} + N_{j,t+1}, \quad (74)$$

where $\tilde{K}_{t+1}^x$ denotes the stock of matched capital held by entrepreneur j at the end of period t , purchased in the submarket x of decentralized market, at a cost Q_t^x per unit of capital; and $\tilde{K}_{j,t+1}^{z_i}$ denotes the stock of capital matched with productivity z_i held by entrepreneur j at the end of period t purchased in the centralized market at price $Q_t^{z_i}$. The latter case also includes the stock of capital matched with productivity z_i held by entrepreneur j from the previous period, which is equivalent to selling and repurchasing the unit in the centralized market at price $Q_t^{z_i}$.

As in Section 4, to solve the entrepreneur's problem, it is useful to define the entrepreneur's leverage and "portfolio weights," from the components of the entrepreneurs balance sheet (74). The entrepreneur's leverage in period t is defined as

$$L_{j,t} \equiv \frac{\int_x Q_t^x \tilde{K}_{j,t+1}^x dx + \sum_i Q_t^{z_i} \tilde{K}_{j,t+1}^{z_i}}{N_{j,t+1}}. \quad (75)$$

The portfolio weight of each asset considered in the left side of equation (74) is given by

$$w_{j,t}^m \equiv \frac{Q_t^m \tilde{K}_{j,t+1}^m}{L_{j,t} N_{j,t+1}}, \quad (76)$$

for $m \in \{x, z_1, z_2, \dots, z_{N_z}\}$.

As in Section 4, the expected rate of return per unit of matched capital for the assets considered in the left-hand side of equation (74) is defined by

$$R_{j,t+1}^{k,z_i} \equiv \frac{r_{j,t+1}^{k,z_i} + (1 - \delta) [\bar{\psi} J_{t+1}^u + (1 - \bar{\psi}) Q_{t+1}^c]}{Q_t^{z_i}}, \quad (77)$$

for $i \in \{1, 2, \dots, N_z\}$, and

$$R_{j,t+1}^{k,x} \equiv \frac{\sum_i r_{j,t+1}^{k,z_i} f_z(z_i) + (1 - \delta) [\bar{\psi} J_{t+1}^u + (1 - \bar{\psi}) Q_{t+1}^c]}{Q_t^x}, \quad (78)$$

where similar to equation (43) in Section 4, net revenues from production per unit of effective capital matched at productivity z_i are defined by

$$r_{j,t}^{k,z_i} = \left(A_t z_i \left(\tilde{h}_{j,t}^{z_i} \right)^{1-\alpha} - W_t \tilde{h}_t^{z_i} \right) u_{j,t}^{z_i} - \mathcal{C}_u(u_{j,t}^{z_i}). \quad (79)$$

The entrepreneurs' objective function (equation (49) in Section 4) can then be expressed as

$$E_t \left\{ \int_{\bar{\omega}_{t+1}}^{\infty} \left[\omega \tilde{R}_{j,t+1}^k L_{j,t} N_{j,t+1} - Z_{j,t+1} D_{j,t+1} \right] dF_{\omega}(\omega, \sigma_t) \right\}. \quad (80)$$

where, similar to Section 4, the portfolio return, denoted $\tilde{R}_{j,t+1}^k$, is defined by $R_{j,t+1}^k \equiv \int_x w_{j,t}^x R_{j,t+1}^{k,x} dx + \sum_i w_{j,t}^{z_i} R_{j,t+1}^{k,z_i}$.

Similarly, the financial intermediary's participation constraint (equation (53) in Section 4) can be expressed as

$$D_{j,t+1} R_t = [1 - F_{\omega}(\bar{\omega}_{t+1}, \sigma_t)] Z_{j,t+1} D_{j,t+1} + (1 - \mu_m) \int_0^{\omega_{t+1}} \omega dF_{\omega}(\omega, \sigma_t) \tilde{R}_{j,t+1}^{k,\psi} L_{j,t} N_{j,t+1}, \quad (81)$$

where $\tilde{R}_{j,t+1}^{k,\psi}$ denotes the portfolio return of *separated* capital, which, similar to Section 4, is defined by $\tilde{R}_{j,t+1}^{k,\psi} \equiv \int_x w_{j,t}^x R_{j,t+1}^{k,x,\psi} dx + \sum_i w_{j,t}^{z_i} R_{j,t+1}^{k,z_i,\psi}$.

From this, the entrepreneur's problem can proceed as in Section 4.

Equilibrium. As in Section 4, any submarket visited by a positive number of buyers must have the same price for capital in equilibrium, and buyers will be indifferent among them.

Formally, for all x ,

$$\theta_t(x) \left(x + \frac{c_s}{q(\theta_t(x))} - Q_t^{\bar{z}} \right) = 0. \quad (82)$$

This condition determines the equilibrium market-tightness function: For all $x < Q_t$,

$$\theta_t(x) = q^{-1} \left(\frac{c_s}{Q_t^{\bar{z}} - x} \right). \quad (83)$$

For all $x \geq Q_t^{\bar{z}}$, $\theta_t(x) = 0$.

The mass of capital that transitions from employment to employment, denoted I_t^{ee} , is defined by

$$I_t^{\text{ee}} = \sum_{z_i} (1 - \psi_t) p(\theta_t(x_t^{z_i})) (1 - \delta) K_t^{z_i},$$

where $K_t^{z_i}$ denotes the stock of employed capital matched at productivity z_i in period t . This object will be the main focus of the next section, when studying the quantitative implications of this model for capital reallocation.

Similar to Section 4, market clearing in centralized markets for capital imply employed capital matched at productivity level z_i evolves then according to the law of motion,

$$K_{t+1}^{z_i} = K_t^{z_i} (1 - \psi_t) (1 - p(\theta_t(x_t^{z_i}))) + [I_t^{\text{ue}} + I_t^{\text{ee}}] f_z(z_i),$$

where I_t^{ue} denotes the mass of capital that transitions from unemployment to employment, that using the definition of market tightness, the law of large numbers, and the fact that a household's choice of submarket, $x_{i,t}$ is the same for all units of capital i , is given by $I_t^{\text{ue}} = p(\theta_t(x_t^{\text{u}})) (1 - \delta) K_t^{\text{u}}$.

The remaining equilibrium conditions are similar to the model presented in Section 4.

6.2 Quantitative Analysis

This section studies some quantitative implications of the model regarding capital reallocation, using the estimated parameters values from Section 5. The only new functional form is that associated to the distribution of match-specific productivities. The discrete set of match-specific productivities is assumed to have three values, labeled low-, medium-, and high-

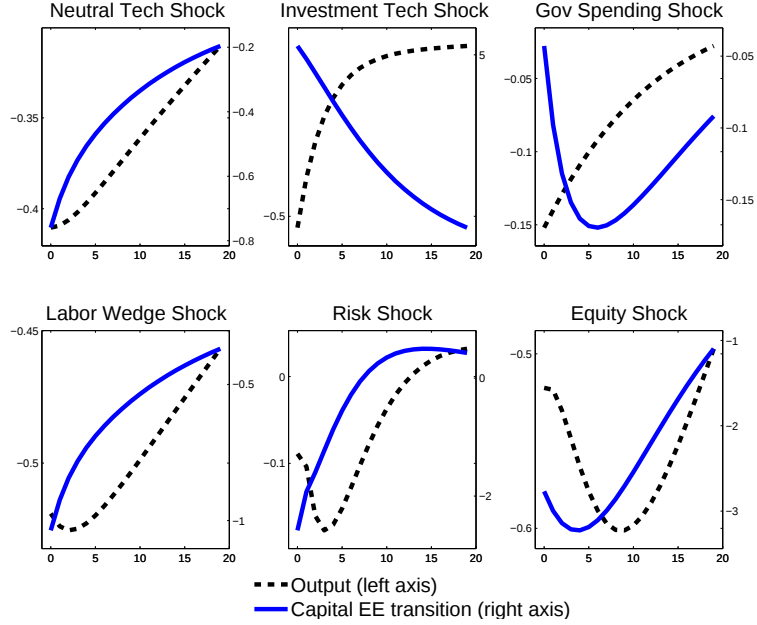


FIGURE 11: IMPULSE RESPONSES TO CONTRACTIONARY SHOCKS.

Note: Response of capital employment-to-employment transitions and output to one-standard-deviation contractionary shocks. Labels *Neutral Tech Shock*, *Investment Tech Shock*, *Gov Spending Shock*, *Labor Wedge Shock*, *Risk Shock*, and *Equity Shock*, refer, respectively, to shocks to the variables A_t , A_t^I , G_t , φ_t , σ_t , and ζ_t presented in Section 4. Impulse responses expressed in percent deviations from steady state. Horizontal axes display quarters after the shock.

match specific productivity. The steady-state distribution of match-specific productivities is assumed to be uniform. The dispersion between low- and high-match specific productivity is set to target a steady state value of capital reallocation of 0.9% per quarter, the average of the range reported in [Eisfeldt and Rampini \(2006\)](#).

Procyclical capital reallocation. A well-documented stylized fact is that capital reallocation in the U.S. economy is procyclical (see [Ramey and Shapiro, 1998](#); [Eisfeldt and Rampini, 2006](#)). The model presented in this section predicts a correlation between the mass of capital that transitions from employment to employment and output of 33.8%, in line with the range between 43.1% and 51.1% correlation between capital reallocation and output reported in [Eisfeldt and Rampini \(2006\)](#).

Figure 11 shows that, in response to most contractionary shocks, the mass of capital that transitions from employment to employment tends to fall, explaining the procyclical nature of capital reallocation. The explanation of this result through the lens of the model is that contractionary shocks are generally associated with less demand of capital from entrepreneurs,

TABLE V
VARIANCE DECOMPOSITION

Shock		I^{ee}
Neutral technology	A	1.2
Investment-specific technology	A^I	49.8
Government spending	G	0.1
Labor wedge	φ	3.2
Risk	σ	5.8
Equity	ζ	39.9

which leads sellers visit submarkets with less favorable terms, both in terms of price of the units of capital and in terms of the probability of finding a buyer. Therefore, the same factors that lead to a countercyclical capital unemployment lead to a procyclical capital reallocation.

The estimated model can also be used to interpret the sources of fluctuations in capital reallocation. Table V shows the variance decomposition predicted by the model for the transition of capital from employment to employment and shows that most of the predicted capital-reallocation movements can be accounted by investment-specific productivity shocks (49.8%) and financial shocks (45.6%) . These findings are consistent those of Section 5, (most of the variation of capital unemployment can be explained by investment-specific shocks and financial shocks) and with those of previous literature explaining procyclical capital reallocation (Cui, 2013).

Misallocation. Empirical evidence points out that recession episodes, and in particular financial crises, are periods of misallocation (see, for example, Midrigan and Xu, 2014).

The predictions of the model presented in this section are also consistent with this empirical finding. Figure 12 shows the response to contractionary shocks of the mass of capital employed with a low match-specific productivity and the mass of capital employed at a high match-specific productivity. The share of capital employed in match-specific productivity increases, especially in response to a negative equity shock (ζ_t). This is because reallocation is especially concentrated in units of capital employed at low match-specific productivity. Therefore, through the lens of this model, capital misallocation during crises is the other side of procyclical capital reallocation.

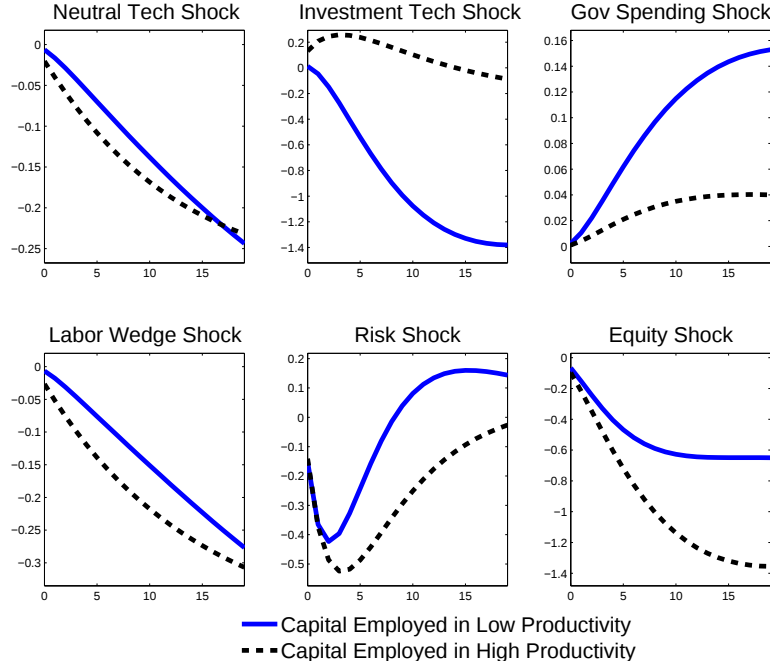


FIGURE 12: IMPULSE RESPONSES TO CONTRACTIONARY SHOCKS.

Note: Response to one-standard-deviation contractionary shocks of the mass of capital employed with a low match-specific productivity ($K_t^{z_1}$, labeled *Low Productivity*) and the mass of capital employed at a high match-specific productivity ($K_t^{z_{Nz}}$ labeled *High Productivity*) predicted by the model presented in Section 6. Labels *Neutral Tech Shock*, *Investment Tech Shock*, *Gov Spending Shock*, *Labor Wedge Shock*, *Risk Shock*, and *Equity Shock*, refer, respectively, to shocks to the variables A_t , A_t^I , G_t , φ_t , σ_t , and ζ_t presented in Section 4. Impulse responses expressed in percent deviations from steady state. Horizontal axes display quarters after the shock.

7 Conclusion and Future Research

This paper presented a model with investment search frictions in which financial shocks have a sizable effect in macroeconomic variables through capital unemployment. An estimated version of the model for the U.S. economy shows that the proposed mechanism can lead to investment slumps such as the one observed during the Great Recession. This result is relevant because slow investment recoveries typically characterize financial crisis episodes.

Using the estimated version of the model to interpret the sources of business-cycle fluctuations in the U.S. economy, the model assigns a large role (33% of output fluctuations) to financial shocks, in the context of a real model that would have assigned a negligible role to these shocks (1% of output fluctuations). This result is relevant because an important source of discrepancy between real and monetary business-cycle models is the role assigned to financial shocks. This paper shows that incorporating investment search frictions can reconcile an

important part of this discrepancy. Finally, the paper shows that the framework can be used to explain capital reallocation and misallocation during crises, as documented by previous empirical literature.

The findings of this paper suggest that two related areas of future research could be promising to develop. The first area is normative. As shown in the paper, the directed-search framework studied leads to an efficient allocation. However, combining the search frictions considered in this paper with asymmetric information would lead to a scope for policy related to asset purchases and subsidy programs as shown in [Guerrieri and Shimer \(2014\)](#).

The second area for future research is empirical. In particular, future research could explore more direct evidence of investment search frictions. For instance, it would be possible to investigate the existence of a “Beveridge curve” in the physical-capital market, using data from capital-intermediary firms. It would also be possible to study the testable implications developed from the model in this paper regarding the relationship between capital unemployment, economic activity and investment. This could be done, for instance, using geographical data of the sort used in this paper to measure capital unemployment. These extensions are planned for future research.

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Appendices

A Data Appendix

A.1 Financial-Crises, Investment, and Capital Stock

To study investment recovery during financial crises, I construct a sample of post-WWII recession episodes in advanced economies. The sample includes annual data from 1950 to 2013 for 22 countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Taiwan, United Kingdom, United States. Only recessions prior to 2007 were considered.

A recession event is identified by a period of contraction in annual real GDP per capita (a similar empirical strategy is followed, for example, in [Calvo, Izquierdo and Talvi, 2006](#)). Given a contraction in GDP per capita, the output peak is defined as the period prior to the beginning of a recession episode; the recovery point is defined as the period in which output per capita recovers its precrisis level; the output trough is defined as the period with the lowest level of GDP per capita between output peak and recovery point.¹⁷

Recession episodes are then classified into *financial crises* and *regular recession episodes*. Following [Calvo, Coricelli and Ottonello \(2012\)](#) a financial crisis is defined as a recession episode in which a banking-crisis event (as defined in [Reinhart and Rogoff, 2009a](#)) took place between the output peak and the recovery point. Regular recession episodes are recession episodes not classified as financial crises. With this methodology, a sample of 100 recession episodes is obtained, with 20 financial crises and 80 regular recession episodes (see Table VI). For each recession episode $t = 0$ is defined as the output trough. Variables of interests are then averaged in a window around $t = 0$ (from $t = -2$ to $t = 4$).

The source of the data used to identify recession episodes and construct the time series of average recession episodes shown in Figure 1 was [Feenstra, Inklaar and Timmer \(2013\)](#), *Penn World Tables*, downloaded from <http://www.ggd.net/pwt>. In particular, the following data were used:

1. Real GDP: Real GDP at constant 2005 national prices.

¹⁷More formally, for each country i the algorithm to identify recession episodes can be described as follows. Let y_{it} denote GDP per capita of country i in period t .

- (i) Set $t_0 = 1950$.
- (ii) Let $\Gamma_p = \{\tau \in [t_0, 2007] : y_{i,\tau} < y_{i,\tau-1}\}$. If $\Gamma_p = \emptyset$ country i has no more recession episodes. If $\Gamma_p \neq \emptyset$ set $p = \min\{\Gamma_p\} - 1$. Let $\Gamma_r = \{\tau \in [p, 2007] : y_{i,\tau} > y_{i,p}\}$. If $\Gamma_r = \emptyset$ country i has no more recession episodes. If $\Gamma_r \neq \emptyset$ set $r = \min\{\Gamma_r\}$. Denote with p the recession peak and with r the recession trough.
- (iii) Set $t_0 = r$ and repeat from (ii) until the country has no more recession episodes.

TABLE VI
SAMPLE OF RECESSION EPISODES

Financial crises		Other episodes							
Country	Peak	Country	Peak	Country	Peak	Country	Peak	Country	Peak
Australia	1989	Australia	1950	Denmark	1973	Ireland	1951	Spain	1974
Canada	1981	Australia	1955	Denmark	1979	Ireland	1955	Spain	1992
Denmark	1987	Australia	1960	Finland	1952	Ireland	1965	Sweden	1976
Denmark	1992	Australia	1973	Finland	1956	Ireland	1974	Sweden	1980
Finland	1990	Australia	1976	Finland	1976	Ireland	1982	Switzerland	1951
France	1992	Australia	1981	France	1974	Italy	1974	Switzerland	1957
Greece	1989	Austria	1951	Germany	1966	Italy	2001	Switzerland	1974
Greece	1991	Austria	1974	Germany	1974	Japan	1973	Switzerland	1981
Iceland	1982	Austria	1977	Germany	1981	Netherlands	1957	Switzerland	1990
Iceland	1987	Austria	1980	Germany	1992	Netherlands	1974	Switzerland	2001
Italy	1992	Austria	1983	Germany	2001	Netherlands	1980	Taiwan	2000
Japan	1992	Austria	1992	Greece	1961	Netherlands	2001	United Kingdom	1957
Japan	1997	Belgium	1957	Greece	1973	Norway	1981	United Kingdom	1968
Norway	1987	Belgium	1974	Greece	1979	Portugal	1957	United Kingdom	1979
Spain	1978	Belgium	1980	Iceland	1950	Portugal	1973	United States	1953
Spain	1980	Belgium	1992	Iceland	1956	Portugal	1982	United States	1957
Sweden	1990	Canada	1953	Iceland	1960	Portugal	1992	United States	1969
United Kingdom	1973	Canada	1956	Iceland	1966	Portugal	2002	United States	1973
United Kingdom	1990	Canada	1989	Iceland	1974	Spain	1952	United States	1979
United States	1990	Denmark	1954	Iceland	2001	Spain	1958	United States	1981

2. Real Capital Stock: Capital stock at constant 2005 national prices.

3. Population.

4. Real Per Capita GDP: Constructed as (4) = (1) / (3).

5. Real Per Capita Capital Stock: Constructed as (5) = (2) / (3).

6. Share of gross capital formation at current PPPs

The time series used in Figure 1 were (4), (5) (for each country, expressed in percent deviation from a log-quadratic trend), and (6) (for each country, expressed in percent deviation from its mean 1950–2013).

For the U.S. Great Recession the following time series were used for Figure 1:

1. Nominal GDP: Gross domestic product, billions of dollars, seasonally adjusted at annual rates. Source: U.S. Bureau of Economic Analysis (BEA, <http://www.bea.gov>), National Income and Product Accounts Tables (Table 1.1.5).
2. Nominal Investment: Gross private domestic fixed investment. Source: BEA, National Income and Product Accounts Tables (Tables 1.1.5).
3. Real GDP: Gross domestic product, billions of chained (2009) dollars, seasonally adjusted at annual rates. Source: BEA, National Income and Product Accounts Tables (Table 1.1.6).

4. Real Capital Stock: Private fixed assets, chain-type quantity indexes. Source: BEA, Fixed Assets Accounts Tables (Table 2.2).
5. Civilian Noninstitutional Population, 16 years and over. Source: Bureau of Labor Statistics, Labor Force Statistics from the Current Population Survey.
6. Real Per Capita GDP: Constructed as $(6) = (3) / (5)$.
7. Real Per Capita Capital Stock: Constructed as $(7) = (4) / (5)$.
8. Investment Rate: Constructed as $(8) = (2) / (1)$.

The time series used in Figure 1 were (6), (7) (expressed in percent deviation from a log-quadratic trend) and (8) (expressed in percent deviation from its mean 1950–2013).

A.2 Capital Unemployment

The data on capital unemployment for structures in the U.S. economy – used in Figure 2 and in the model estimation of Section 5 – were constructed as a weighted average of quarterly vacancy rates of office space, retail space, and industrial space. Data were obtained from CBRE (<http://www.cbre.com/EN/Pages/Home.aspx>) and REIS (<https://www.reis.com/>). Weights for office space, retail space, and industrial space were defined using data on Current-Cost Net Stock of Private Fixed Assets, Equipment, Structures, and Intellectual Property Products by Type, source U.S. Bureau of Economic Analysis (BEA, <http://www.bea.gov>), Table 2.1. The following items were included to compute the weight. For Office space: “Office”; for retail space: “Multimerchandise shopping”, “Food and beverage establishments”, “Commercial warehouses”, and “Other commercial”; for industrial space: “Manufacturing”, “Power and communication”, and “Mining exploration, shafts, and wells.” These items jointly represent 60.5% of nonresidential structures. The weights were computed as the average of the share of each item over the period 1980–2012, which is similar to the period for which the data on vacancy rates is available (1980–2013).

A.3 Bayesian Estimation

The following data for the U.S. economy were used to construct the quarterly time series used in the model estimation of Section 5:

1. Nominal GDP: Gross domestic product, billions of dollars, seasonally adjusted at annual rates. Source: U.S. Bureau of Economic Analysis (BEA, <http://www.bea.gov>) National Income and Product Accounts Tables (Table 1.1.5).

2. Nominal Consumption: Sum of personal consumption expenditures, durable goods and services, billions of dollars, seasonally adjusted at annual rates. Source: BEA, National Income and Product Accounts Tables (Table 1.1.5).
3. Nominal Investment: Sum of gross private domestic fixed nonresidential investment in structures, equipment and software. Source: BEA, National Income and Product Accounts Tables (Tables 1.1.5 and 5.3.5).
4. Real GDP: Gross domestic product, billions of chained (2009) dollars, seasonally adjusted at annual rates. Source: BEA, National Income and Product Accounts Tables (Table 1.1.6).
5. GDP Deflator: constructed as $(5) = (1) / (4)$.
6. Nonfarm Business Hours Worked. Source: Bureau of Labor Statistics (BLS, <http://www.bls.gov/>), Major Sector Productivity and Costs.
7. Civilian Noninstitutional Population, 16 years and over. Source: BLS, Labor Force Statistics from the Current Population Survey.
8. 3-Month Treasury Bill, Secondary Market Rate. Source: Federal Reserve Bank of St. Louis, Federal Reserve Economic Data (FRED, <http://www.stlouisfed.org/>).
9. Moody's Seasoned Baa Corporate Bond Yield. Source: FRED.
10. Real Per Capita GDP: constructed as $(10) = (4) / (7)$.
11. Real Per Capita Consumption: constructed as $(11) = ((2) / (5)) / (7)$.
12. Real Per Capita Investment: constructed as $(12) = ((3) / (5)) / (7)$.
13. Per Capita Hours Worked: constructed as $(13) = (6) / (7)$.
14. Credit Spreads: constructed as $(14) = (1 + (9)) / (1 + (8))$.
15. Capital Unemployment: constructed based on data on vacancy rates of nonresidential commercial real estate (office, retail, and industrial sectors). Methodology detailed in A.2.

The six time series used in the Bayesian estimation were (10) , (11) , (12) , (13) , (14) and (15) , with (10) , (11) , (12) log-linearly detrended.

B Additional Figures

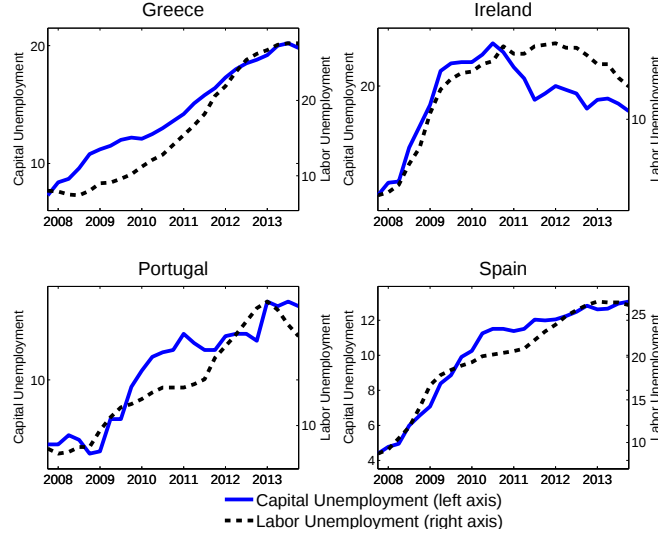


FIGURE 13: UNEMPLOYMENT OF CAPITAL AND LABOR, EURO ECONOMIES, 2007–2013.
Note: Capital unemployment (structures) refers to the vacancy rates of office space (<http://www.jll.eu/emea/en-gb/>). Labor unemployment refers to the unemployment rate (<http://epp.eurostat.ec.europa.eu/portal/page/portal/eurostat/home/>). Data is expressed in percent.

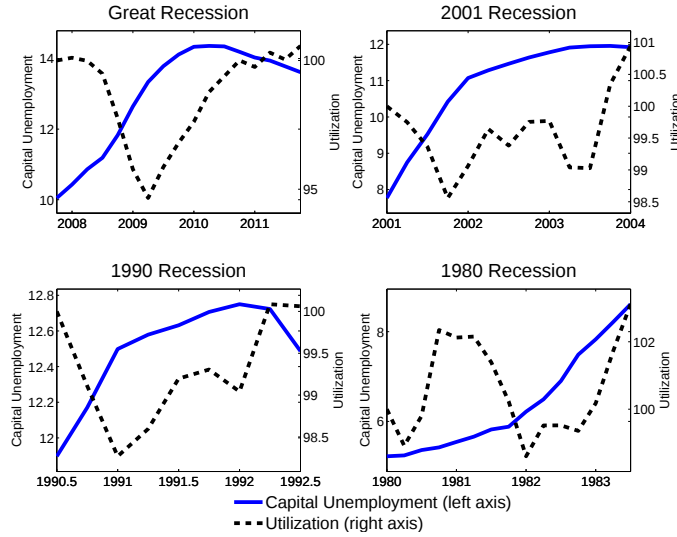


FIGURE 14: UNEMPLOYMENT OF CAPITAL AND UTILIZATION, U.S. RECESSION EPISODES.
Note: Capital unemployment (structures) constructed based on vacancy rates of office, retail and industrial units. Data source: CBRE and REIS. See Appendix A for details. Data on utilization refers to estimates of factor utilization for the U.S. economy in Fernald (2009), capturing labor effort and the work week of capital. Capital Unemployment expressed in percent. For each recession episode, utilization index = 100 at the output peak.

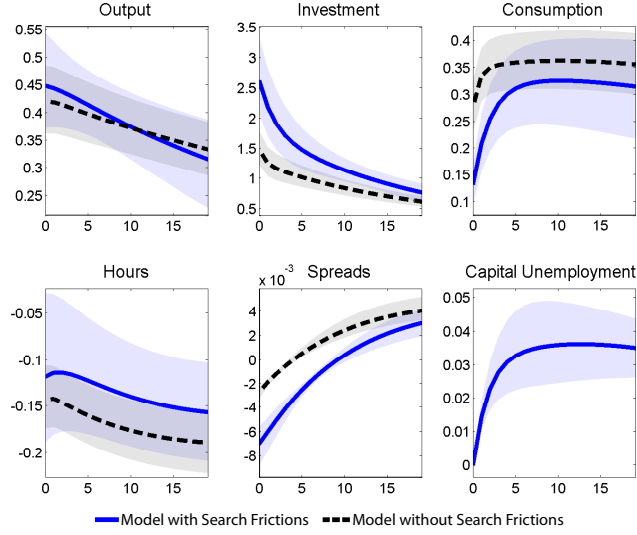


FIGURE 15: IMPULSE-RESPONSES TO A NEUTRAL-TECHNOLOGY SHOCK.

Note: Response of output, investment, consumption, hours worked, credit spreads, and capital unemployment to a one-standard-deviation neutral-technology shock (A). Label “Model with Search Frictions in Investment” and “Model No Search Frictions in Investment” refer, respectively, to the model responses presented in Section 4 and the benchmark model in Appendix E. Impulse responses expressed in percent deviations from steady state. Shadow areas represent equal tail probability 90 % credible sets associated with the posterior distributions. Horizontal axes display quarters after the shock.

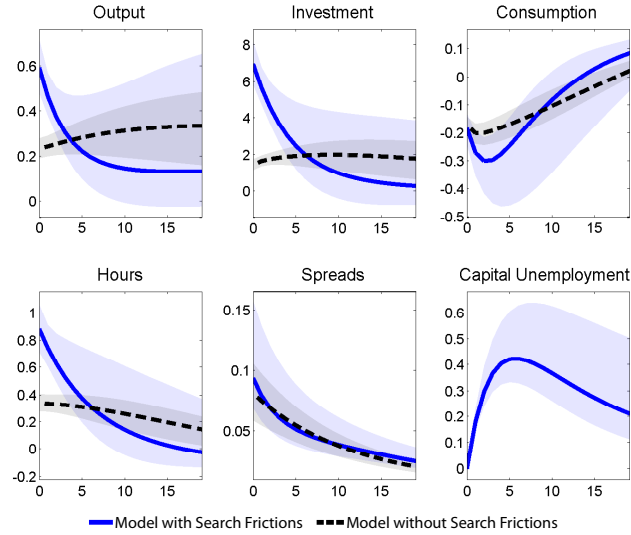


FIGURE 16: IMPULSE-RESPONSES TO AN INVESTMENT-SPECIFIC TECHNOLOGY SHOCK.

Note: Response of output, investment, consumption, hours worked, credit spreads, and capital unemployment to a one-standard-deviation investment-specific technology shock (A^I). Label “Model with Search Frictions in Investment” and “Model No Search Frictions in Investment” refer, respectively, to the model responses presented in Section 4 and the benchmark model in Appendix E. Impulse responses expressed in percent deviations from steady state. Shadow areas represent equal tail probability 90 % credible sets associated with the posterior distributions. Horizontal axes display quarters after the shock.

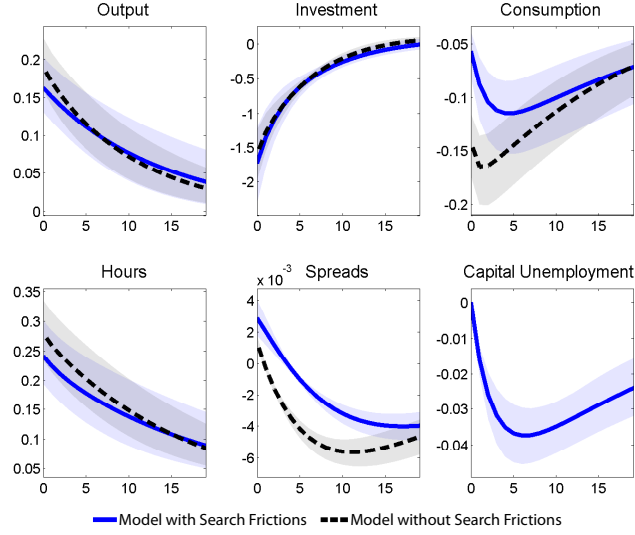


FIGURE 17: IMPULSE-RESPONSES TO A GOVERNMENT SPENDING SHOCK.

Note: Response of output, investment, consumption, hours worked, credit spreads, and capital unemployment to a one-standard-deviation government spending shock (G). Label “Model with Search Frictions in Investment” and “Model No Search Frictions in Investment” refer, respectively, to the model responses presented in Section 4 and the benchmark model in Appendix E. Impulse responses expressed in percent deviations from steady state. Shadow areas represent equal tail probability 90 % credible sets associated with the posterior distributions. Horizontal axes display quarters after the shock.

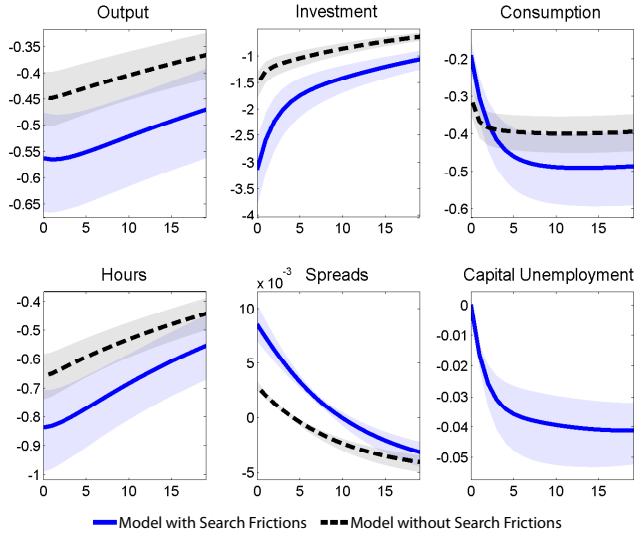


FIGURE 18: IMPULSE-RESPONSES TO A LABOR-WEDGE SHOCK.

Note: Response of output, investment, consumption, hours worked, credit spreads, and capital unemployment to a one-standard-deviation labor-wedge shock (φ). Label “Model with Search Frictions in Investment” and “Model No Search Frictions in Investment” refer, respectively, to the model responses presented in Section 4 and the benchmark model in Appendix E. Impulse responses expressed in percent deviations from steady state. Shadow areas represent equal tail probability 90 % credible sets associated with the posterior distributions. Horizontal axes display quarters after the shock.

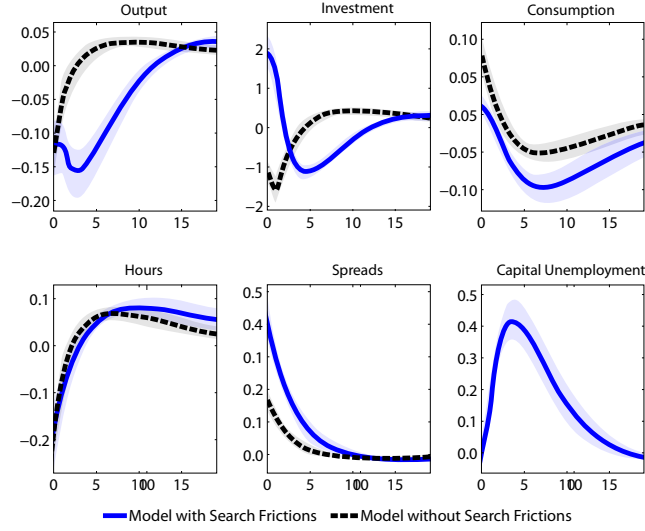


FIGURE 19: IMPULSE-RESPONSES TO A RISK SHOCK.

Note: Response of output, investment, consumption, hours worked, credit spreads, and capital unemployment to a one-standard-deviation risk shock (σ). Label “Model with Search Frictions in Investment” and “Model No Search Frictions in Investment” refer, respectively, to the model responses presented in Section 4 and the benchmark model in Appendix E. Impulse responses expressed in percent deviations from steady state. Shadow areas represent equal tail probability 90 % credible sets associated with the posterior distributions. Horizontal axes display quarters after the shock.

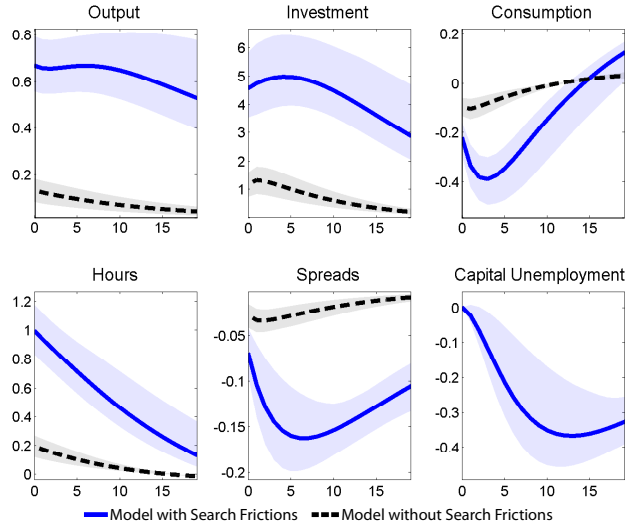


FIGURE 20: IMPULSE-RESPONSES TO AN EQUITY SHOCK.

Note: Response of output, investment, consumption, hours worked, credit spreads, and capital unemployment to a one-standard-deviation equity shock (ζ). Label “Model with Search Frictions in Investment” and “Model No Search Frictions in Investment” refer, respectively, to the model responses presented in Section 4 and the benchmark model in Appendix E. Impulse responses expressed in percent deviations from steady state. Shadow areas represent equal tail probability 90 % credible sets associated with the posterior distributions. Horizontal axes display quarters after the shock.

C Mapping from Investment Search Frictions to Wedges

To further study the economic mechanism induced by the search friction in investment, this section considers a prototype economy with time-varying wedges (in the spirit of [Chari, Kehoe and McGrattan, 2007](#)), and maps the equilibrium of the economy presented in Section 3 with search frictions in investment to wedges in the prototype economy.

The *prototype economy* corresponds to a neoclassical growth model, with no disutility from labor, and with time-varying exogenous productivity, taxes on capital income, and government consumption. Agents in this economy are households, firms, and the government. The household's problem in the prototype economy is given by

$$\max_{\{\hat{C}_t, \hat{I}_t, \hat{K}_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(\hat{C}_t), \quad (84)$$

$$\text{s.t. } \hat{C}_t + \hat{I}_t + \hat{T}_t = (1 - \hat{\tau}_t^k) \hat{r}_t^k \hat{K}_t + \hat{W}_t \bar{h} + \hat{\Pi}_t^f, \quad (85)$$

$$\hat{K}_{t+1} = (1 - \delta) \hat{K}_t + \hat{I}_t, \quad (86)$$

where “hats” represent variables in the prototype economy; $\hat{C}_t$ denotes consumption in period t , $\hat{I}_t$ denotes consumption in period t , $\hat{K}_t$ denotes the stock of capital held by households in period t , $\hat{r}_t^k$ denotes the rental rate of capital in period t taken as given by households, $\hat{\tau}_t^k$ denotes a capital-income-tax in period t , $\hat{W}_t$ denotes the wage rate in period t taken as given by households, $\bar{h}$ denotes the household (inelastic) supply of hours of work to the labor market, $\hat{T}_t$ denotes lump-sum taxes levied by the government on households in period t , and $\hat{\Pi}_t^f$ denote lump-sum transfers from the entrepreneurs to households in period t taken as given by households.

Firms rent capital and employ labor from households each period, in competitive markets, to maximize profits, given by $\hat{\Pi}_t^f \equiv \hat{A}_t F(\hat{K}_t, \hat{h}_t) - \hat{r}_t^k \hat{K}_t - \hat{W}_t \hat{h}_t$, where $\hat{A}_t$ denotes aggregate productivity in period t .

The government budget constraint in the prototype economy is given by

$$\hat{G}_t = \hat{T}_t, \quad (87)$$

where $\hat{G}_t$ denotes an exogenous government consumption in period t .

Definition 4 (Equilibrium in the prototype economy with wedges). *Given initial conditions for capital, $\hat{K}_0$, and a sequence of wedges $\{\hat{A}_t, \hat{G}_t$ and $\hat{\tau}_t^k\}$, an equilibrium in the prototype economy with wedges is a sequence of allocations $\{\tilde{C}_t, \tilde{I}_t, \tilde{K}_{t+1}\}$ such that three*

conditions are satisfied:

$$U'(\hat{C}_t) = \beta U'(\hat{C}_{t+1}) \left[(1 - \hat{\tau}_{t+1}^k) \hat{A}_{t+1} F_1(\hat{K}_{t+1}, \bar{h}) + (1 - \delta) \right] \quad (88)$$

$$\hat{K}_{t+1} = (1 - \delta) \hat{K}_t + \hat{I}_t \quad (89)$$

$$\hat{C}_t + \hat{G}_t + \hat{I}_t = \hat{A}_t F(\hat{K}_t, \bar{h}). \quad (90)$$

Equation (88) is the standard intertemporal optimality condition for the household's problem, with the rental rate of capital r_t^k replaced by its equilibrium value, $\hat{r}_t^k = \hat{A}_t F_1(\hat{K}_t, \bar{h})$. Equation (90) is the resource constraint of the prototype economy obtained by aggregating the households', firms', and government's budget constraints and using the definition of firms profits.

To establish the mapping with the economy with investment search frictions, let the efficiency wedge in the prototype economy, $\hat{A}_t$, be given by

$$\hat{A}_t = A_t (1 - k_t^u)^\alpha, \quad (91)$$

where variables without “hat” denote allocations in the economy with investment search frictions (Definition 2). Let the capital-income tax in the prototype economy be implicitly defined by

$$(1 - \hat{\tau}_t^k) \hat{A}_t F_1(\hat{K}_t, \bar{h}) = p(\theta_t^u)(Q_t^{\text{sp}} - 1) - c_s \theta_t^u. \quad (92)$$

Let government consumption in the prototype, $\hat{G}_t$, be given by

$$\hat{G}_t = c_s \theta_t^u (1 - \delta) k_t^u K_t. \quad (93)$$

Then the following equivalence result can be established.

Proposition 2. *Let $\{C_t, I_t, K_{t+1}, k_{t+1}^u, \theta_t^u\}$ denote equilibrium allocations of the economy with investment search frictions (Definition 2), for given initial conditions for capital stock and capital-unemployment rate, K_0 and k_0^u , and sequences of aggregate productivity, A_t . If the efficiency wedge is given by (91), the capital-income-tax wedge is given by (92), and the government consumption wedge is given by (93), the allocations $\{C_t, I_t, K_{t+1}\}$ constitute an equilibrium of the prototype economy (Definition 4).*

Proof. See Appendix D. ■

From this proposition, it follows that the investment search frictions proposed in this section manifest themselves as three wedges in a neoclassical growth model without search

frictions. First, an efficiency wedge, as shown in (91), is the direct result of capital unemployment, the fact that only a fraction, $1 - k_t^u$, is used for production in period t in the economy with investment search friction. Second, an investment wedge, as shown in (92), relates the marginal benefits of saving to the shadow value of employed capital, net of search costs. Third, a government spending wedge, as shown in (93), subtracts search costs from the resources available to the economy each period. It is relevant to note that the wedges of the prototype economy without search frictions, defined in (91)–(93) depend on the evolution of the endogenous state variable, k_t^u . Therefore, the allocation of other models with friction that manifest themselves as efficiency, investment or government spending wedges will generally differ from the allocation in the model economy presented in this section.

D Proofs

D.1 Proof of Proposition 1

Using the equilibrium market-tightness function (17), the first-order condition for households (6) can be expressed as

$$-c_s \theta_t(x_t^u) q'(\theta_t(x_t^u)) = p'(\theta_t(x_t^u)) q(\theta_t(x_t^u)) (x_t^u - 1), \quad (94)$$

From Definition 1 and equation (94), it follows that sequences $\{C_t, I_t, K_{t+1}^e, K_{t+1}^u, \Lambda_t, Q_t, \theta_t^u, x_t^u\}$ are a competitive equilibrium if and only if they satisfy the following conditions

$$U'(C_t) = \Lambda_t, \quad (95)$$

$$\Lambda_t = \beta \Lambda_{t+1} (1 - \delta) \{p(\theta_{t+1}^u) x_{t+1} - (1 - p(\theta_{t+1}^u))\}, \quad (96)$$

$$-c_s \theta_t^u q'(\theta_t^u) = p'(\theta_t^u) q(\theta_t^u) (x_t - 1), \quad (97)$$

$$\Lambda_t Q_t = \beta \Lambda_{t+1} [A_{t+1} F_1(K_{t+1}^e, \bar{h}) + (1 - \delta) (\psi + (1 - \psi) Q_{t+1})], \quad (98)$$

$$Q_t = x_t + \frac{c_s}{q(\theta_t^u)}, \quad (99)$$

$$K_{t+1}^e = (1 - \psi)(1 - \delta) K_t^e + A_t^e p(\theta_t^u) (1 - \delta) K_t^u, \quad (100)$$

$$K_{t+1}^u = (1 - \theta_t^u)(1 - \delta) K_t^u + \psi(1 - \delta) K_t^e + I_t, \quad (101)$$

$$A_t F(K_t^e, \bar{h}) = C_t + I_t + c_s \theta_t^u (1 - \delta) K_t^u. \quad (102)$$

To show that the competitive equilibrium is efficient, it must be shown that if sequences $\{C_t, I_t, K_{t+1}^e, K_{t+1}^u, \Lambda_t, Q_t, \theta_t^u, x_t^u\}$ satisfy (95)–(102), they also satisfy the social planner's optimality conditions (23)–(29). Replacing the definitions of capital-unemployment rate (20) and total capital stock in (100), (101), and (102), and operating, equations (23), (24), and (25) are obtained. Pick $\Lambda_t = \Lambda_t^{\text{sp}}$; replacing in (95), equation (26) is obtained. Pick $Q_t = Q_t^{\text{sp}}$;

replacing in (98), equation (28) is obtained. Replacing (99) in (97), and operating, equation (28) is obtained. Finally, replacing (99) in (96), equation (29) is obtained.

D.2 Proof of Proposition 2

To establish the mapping between the economy with investment search frictions and the prototype economy with wedges, it must be shown that if sequences $\{C_t, I_t, K_{t+1}, k_{t+1}^u, \theta_t^u\}$ satisfy the social planner's optimality conditions (23)–(29), and wedges are defined by (91)–(92), then the allocations $\{C_t, I_t, K_{t+1}\}$ also satisfy (88)–(90).

Replacing the definition of the efficiency wedge, (91), and the definition of the government consumption wedge, (93), on the resource constraint of the social planner's problem, (23), the resource constraint of the prototype economy, (90), is obtained. Replacing equation (26) and the definition of the capital-income-tax wedge on the planner's optimality condition (89), equation (23) is obtained. Finally, the social planner's capital-accumulation constraint (24) coincides with the prototype economy's capital-accumulation constraint, (88). Therefore, equations (88)–(90) are satisfied.

E Benchmark Business Cycle Economy

This section presents the benchmark business-cycle model used in Section 5 for comparison with the model developed in Section 4. The only difference between the two models is that the benchmark economy does not include investment search frictions. The notation used in this section is the same as that presented in Section 4.

Goods. As in Section 4, there are perishable consumption goods, and capital goods that depreciate at a rate $\delta > 0$. Unlike 4, there is no distinction between matched and unmatched capital.

Agents. As in Section 4, the economy is populated by a large number of identical households, entrepreneurs and financial intermediaries (see Figure 6).

Markets. As in Section 4, the economy has four competitive markets: goods, labor, physical capital and credit (see Figure 6). The goods and labor markets are frictionless. Unlike Section 4, the market for physical capital is also frictionless. In this market, households and entrepreneurs trade capital at the price Q_t . The credit market is characterized by frictions associated with asymmetric information in lending as described in Section 4.

Households. Household i 's problem is

$$\begin{aligned} \max_{\{C_{i,t}, I_{i,t}, B_{i,t}, h_{i,t}\}} E_0 \sum_{t=0}^{\infty} \beta^t \{U(C_{i,t} - \rho_c C_{i,t-1}) - V(h_{i,t}; \varphi_t)\}, \\ \text{s.t. } C_{i,t} + I_{i,t} + T_t + B_{i,t} = R_{t-1} B_{i,t-1} + W_t h_{i,t} + Q_t A_t^I \left[I_{i,t} - \Phi \left(\frac{I_{i,t}}{K_t} \right) K_t \right] + \Pi_t. \end{aligned}$$

The only difference with respect to the household's problem presented in Section 4 is that households sell their capital stock to entrepreneurs in a centralized market at the price Q_t .

Entrepreneurs. Entrepreneur j 's problem is

$$\begin{aligned} \max_{\{\tilde{h}_{j,t}, u_{j,t}, L_{j,t}, \omega_{j,t+1}\}} E_t \left\{ \left[\int_{\bar{\omega}_{j,t+1}}^{\infty} \omega dF_{\omega}(\omega; \sigma_t) - (1 - F_{\omega}(\bar{\omega}_{j,t+1}; \sigma_t)) \bar{\omega}_{j,t+1} \right] R_{j,t+1}^k L_{j,t} N_{j,t+1} \right\} \\ \text{s.t. } \frac{L_{j,t} - 1}{L_{j,t}} R_t = [1 - F_{\omega}(\bar{\omega}_{j,t+1}; \sigma_t)] \bar{\omega}_{j,t+1} R_{j,t+1}^k + (1 - \mu_m) \int_0^{\omega_{j,t+1}} \omega dF_{\omega}(\omega; \sigma_t) R_{j,t+1}^k, \\ r_{j,t}^k = \left(A_t (\tilde{h}_{j,t})^{1-\alpha} - W_t \tilde{h}_{j,t} \right) u_{j,t} - \mathcal{C}_u(u_{j,t}), \\ R_{j,t+1}^k = \frac{r_{j,t+1}^k + (1 - \delta) Q_{t+1}}{Q_t}. \end{aligned}$$

The difference with respect to the entrepreneur's problem presented in section 4 is that entrepreneurs only purchase capital in a centralized market at the price Q_t .

Equilibrium. In equilibrium all markets clear. Similar to Section 4, aggregate net worth evolves following the law of motion

$$N_{t+1} = [1 - \Gamma_{t-1}(\bar{\omega}_t)] R_t^k Q_{t-1} K_t + \zeta_t. \quad (103)$$

The net transfer from entrepreneurs to households is given by

$$\Pi_t = [1 - \Gamma_{t-1}(\bar{\omega}_t)] R_t^k Q_{t-1} K_t - \zeta_t. \quad (104)$$

The aggregate capital stock evolves following the law of motion

$$K_{t+1} = (1 - \delta) K_t + A_t^I \left[I_t - \Phi \left(\frac{I_t}{K_t} \right) K_t \right]. \quad (105)$$

The economy's resource constraint is given by

$$C_t + I_t + G_t = A_t (K_t)^{\alpha} (h_t)^{(1-\alpha)} - \Omega_t - \mathcal{C}_u(u_t) K_t, \quad (106)$$

where $\Omega_t \equiv \mu g_{t-1}(\bar{\omega}_t) R_t^k Q_{k,t-1} K_t$.

An equilibrium in the benchmark economy can then be defined as follows.

Definition 5 (Competitive equilibrium). *Given initial conditions for capital, K_0 and consumption C_{-1} , and a state-contingent sequence of aggregate exogenous states, S_t^x , a competitive equilibrium is a state-contingent sequence of individual allocations and shadow values, $\{(C_{i,t}, h_{i,t}, I_{i,t}, l_{i,t}^h, K_{i,t+1}^u, B_{i,t}, x_{i,t}^u)_{i \in [0,1]}, (\tilde{h}_{j,t}, u_{j,t}, L_{j,t}, \omega_{j,t+1})_{j \in [0,1]}\}$, $\{(\Lambda_{i,t})_{i \in [0,1]}, (Q_{j,t})_{j \in [0,1]}\}$, aggregate allocations, $\{C_t, I_t, h_t, K_{t+1}^e, K_{t+1}^u, N_t, \Pi_t\}$, prices, $\{Q_t^e, J_t^u, W_t\}$, and debt schedules $\{\mathcal{D}_t(\tilde{h}_{j,t}, u_{j,t})\}$, such that:*

- (i) *Individual allocations and shadow values solve the household's and entrepreneur's problems at the equilibrium prices and debt schedules, for all i and j .*
- (ii) *Debt schedules satisfy financial intermediaries' participation constraint (53).*
- (iii) *All markets clear.*