

What Shifts the Beveridge Curve?

Recruiting Intensity and Financial Shocks^{*}

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Abstract

Labor market data show a substantial deterioration of aggregate matching efficiency around the Great Recession, even after controlling for compositional changes among job seekers. We augment the multiworker-firm version of the equilibrium random-matching model of the labor market with endogenous firm entry and exit, a choice of recruiting intensity when hiring, and a dividend constraint that induces some firms to borrow and some of those with debt to default. We use the model to study whether aggregate financial shocks can account for the observed drop in matching efficiency—and the ensuing shift in the Beveridge curve—through a reduction in the average recruiting intensity in the economy. Central to this mechanism is the role of young firms which contribute disproportionately to job creation, display the highest recruitment effort per vacancy and, at the same time, are heavily dependant on external finance.

Keywords: Aggregate Matching Efficiency, Beveridge Curve, Financial Shocks, Recruiting Intensity, Unemployment, Vacancies, Young Firms.

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1 Introduction

The Beveridge curve describes the empirical relationship between the unemployment rate and the job vacancy rate of an economy. One of the key stylized facts of macroeconomic fluctuations is the very strong negative correlation, close to -0.9 , between unemployment and vacancies at business cycle frequencies. At the same time, outward shifts of this negatively sloped locus are not uncommon in the historical experience of developed economies (Elsby and Michaels, 2013).

Adverse shifts of the Beveridge curve attract a lot of attention from economists and policy-makers because, seen through the lenses of the standard Diamond-Mortensen-Pissarides model of the labor market, they indicate a deterioration in the degree of matching efficiency of the labor market. One of the building blocks of this model is the aggregate matching function, a production function that takes the total number of jobseekers and the total number of vacant positions open for recruiting as inputs, and the flow of new hires as output. Matching efficiency is a multiplicative shifter of this production function of hires, akin to total factor productivity in production theory. A decline in matching efficiency means that the labor market is less effective in its fundamental role of connecting idle labor with idle jobs.

The last historical episode of outward movement in the Beveridge curve—and one of the most significant for magnitude and duration—occurred in the wake of the Great Recession of 2007-2009. In April 2012, approximately four years after the onset of the recession, the U.S. job openings rate returned to its level of April 2008, after dropping by 50 percent. However, at the same time, the unemployment rate was still three percentage points higher than in April 2008.¹ In this paper, in line with standard theory, we interpret this recent shift as a drop in the aggregate matching efficiency of the labor market and we investigate its causes.²

We begin with a measurement exercise that estimates a time series for aggregate matching efficiency, a variable that in the rest of the paper we label Φ_t . A recent literature (Veracierto, 2011; Barnichon and Figura, 2010; Shimer, 2012; Fujita and Moscarini, 2013; Hall and Schulhofer-Wohl, 2013) has pointed out that compositional changes in the pool of jobseekers—some of them more

¹See Elsby, Hobijn, and Şahin (2010) for a comprehensive account of the U.S. labor market in the aftermath of the Great Recession.

²Recently, Christiano, Eichenbaum, and Trabandt (2013) argued that one does not need any drop in the efficiency parameter of the aggregate matching function to reproduce the joint dynamics of unemployment and vacancies around the Great Recession. We discuss their approach in Section 2.

effective than others in finding employment—can explain a sizable fraction of movements in measured Φ_t , very much like the quality composition of labor input accounts for changes in TFP, when the latter is estimated as a “Solow residual” with standard measures of inputs. We build heavily on this literature by including employed and out-of-the-labor-force jobseekers in the matching function, and by excluding from both the output and the input of the matching function workers on temporary layoff—who have a very high recall rate to the same employer, and hence escape the informational frictions and the heterogeneity that the matching function captures (Fujita and Moscarini, 2013). The resulting time series for Φ_t , after these corrections, drops by 30-35 percentage points during the last recession and recovers only at a very slow pace since then.³ To put this magnitude in context, one should keep in mind that, around the 2001 recession, our estimate of Φ_t falls by only 10 percentage points.

Our proposed explanation for this sharp deterioration of aggregate matching efficiency in the aftermath of the Great Recession builds on three stylized facts on firm dynamics and job creation. First, as Haltiwanger, Jarmin, and Miranda (2012) document, young firms display rich “up-or-out” dynamics: the survival rate of young firms is much lower than for older incumbents but, conditional on survival, young firms grow a lot more rapidly than their mature counterparts. As a result, young firms play a critical role in the creation of new jobs. Start-ups account for close to 20 percent of job creation, in spite of representing only three percent of total employment, and the share of firms younger than 10 year-old in total job creation is close to a half.

Second, the job filling rate—the speed at which a jobseeker fills an open position—displays a lot of heterogeneity in the cross-section of firms. Namely, it increases steeply with employer growth rates (Davis, Faberman, and Haltiwanger, 2013): firms that grow faster also choose to recruit with higher intensity. For example, the job-filling rate almost doubles as monthly employment growth increases from 10 to 20 percent. Presumably, this occurs because these firms spend more in recruitment activities (e.g., through more advertising, higher search effort per vacancy, better screening of applicants, more attractive compensation and working conditions) for vacant position. Since the total number of vacancies measures the input of the aggregate matching function from the firm side, but not through the average recruiting intensity of the

³We confirm the importance of the “compositional factor” that, in our calculations, explains another 15-20 percent drop in measured matching efficiency over the same period.

economy, fluctuations in this factor would show up in the data as movements in aggregate matching efficiency. This is precisely the channel that we explore in this paper.

Third, it is well established that young, small, and fast-growing firms are those that are most dependent on external finance and on credit market conditions (see, e.g., [Hubbard, 1998](#), for an early survey). In particular, as the Kauffman Firm Survey documents, one important source of funding for start-ups is collateralized borrowing against home equity. Thus, adverse shocks to financial intermediaries and to the value of housing, such as the ones witnessed during the last recession, have an especially fierce impact on young firms. In the context of the Great Recession, the evidence on the importance of this channel is abundant. [Chodorow-Reich \(2014\)](#) exploits the property that bank-borrower relationships are sticky and the fact that, before 2007, major lenders had different degrees of exposure to Lehman Brothers to isolate the exogenous component of the credit shock that hit the U.S. economy. He concludes that the withdrawal of credit accounts for at least 1/3 of the employment decline at small and medium-sized firms. At the same time, he cannot reject the null hypothesis of no effect on the largest firms with access to the bond market. [Siemer \(2013\)](#) uses a difference-in-differences approach to identify the heterogeneous effect of the recession on firms belonging to sectors with various degree of external financial dependence. His results imply that external financial constraints account for a 5-10 percentage point reduction of employment growth in small firms relative to large firms during 2007-2009—a result driven predominantly by young firms. [Adelino, Schoar, and Severino \(2013\)](#) and [Fort, Haltiwanger, Jarmin, and Miranda \(2013\)](#) find that a decline in housing prices in a geographical area yields a significant reduction in the differential net job creation rate between start-ups/young and mature businesses in that region.⁴

Taken together, these three facts suggest a novel narrative for the observed deterioration in aggregate matching efficiency and the ensuing shift in the Beveridge curve that occurred throughout the Great Recession and beyond: an aggregate financial shock hits the economy and curbs labor demand—disproportionately so for start-ups and young firms; since these are the the firms displaying the highest recruiting effort and, at the same time, they account for a substantial share of new vacancies, average recruiting intensity in the economy falls. Through

⁴These authors argue that this effect is a symptom of a collapse of collateralized lending to start-ups, and not just a manifestation of deleveraging, as in [Mian and Sufi \(2011\)](#). See also [Mehrotra and Sergeyev \(2013\)](#) for an IV approach to isolate the effect of the fall in housing prices on local job creation rates.

the lens of an aggregate matching function that takes vacancies and unemployment as inputs, the decline in recruiting intensity assumes the form of a drop in measured matching efficiency.

In this paper, we develop a structural equilibrium model to explore the quantitative relevance of this mechanism. Our model is a version of the canonical Diamond-Mortensen-Pissarides random matching framework with decreasing returns in production and nonconvex hiring costs (Cooper, Haltiwanger, and Willis, 2007; Elsby and Michaels, 2013; Acemoglu and Hawkins, Forthcoming). The model simultaneously features a realistic firm life-cycle, as its classic competitive setting counterparts (Jovanovic, 1982; Hopenhayn, 1992), and a frictional labor market with slack on both demand and supply sides—a necessary ingredient of our analysis. We augment this environment in three dimensions. First, we introduce financial frictions: firms face a nonnegativity constraint on dividends that induces some firms to borrow and, some of those with debt, to default. Loan contracts are priced competitively to reflect default risk, in a similar vein to the modelling of financial intermediation in Eaton and Gersovitz (1981) for sovereign debt, Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) for household debt, and Khan, Seng, and Thomas (2014) for firm debt.⁵

Second, we allow for endogenous entry and exit of firms. This is a key element because our mechanism hinges on the severity of the shock for new entrants, a fact that is well supported empirically: according to the Business Dynamics Statistics (BDS), in 2006 there were over 560,000 new firms created, as opposed to about 410,000 in 2011, a decline of about 27 percent, almost 5 times bigger than its counterpart during the downturn of 2001.⁶ A number of papers has focused on explaining the “missing generation of entrants” in the wake of the Great Recession: Siemer (2013); Schott (2013); Sedláček (2014), among others, study the impact of financial shocks on the entry of new firms and the longlasting effects of this missing generation on employment and unemployment, but the implications for aggregate matching efficiency remain unexplored.

Finally, we introduce firms’ choice of recruiting intensity: hiring firms choose the maximum number of open positions that they are willing to fill in each period, and the amount of resources

⁵An alternative approach to adding financial frictions to this class of models introduces a search friction in the financial market as well. See, e.g., Wasmer and Weil (2004) and Petrosky-Nadeau and Wasmer (2013).

⁶In 1999 the number of entrants was roughly 500,000 and, after a drop of 6 percent in 2001, in 2002 the number of start-ups was already back to its pre-recession level.

that they devote to recruitment activities. The sum of all individual firms' recruitment efforts, weighted by their vacancy share, aggregates to the economy's measured matching efficiency. To our knowledge, [Kaas and Kircher \(2011\)](#) is the only other paper that embeds recruiting intensity in a structural equilibrium model of the labor market. It differs from our paper because it is a directed search environment where some of the heterogeneity in job-filling rates across firms derives from the different wages they optimally post to attract jobseekers.⁷ These authors do not use their model to account for the observed recent drop in matching efficiency. While their framework is flexible enough to investigate the role of productivity shocks, it does not yet incorporate financial shocks.

TBC: description of experiments and results

The rest of the paper is organized as follows. In section 2 we measure the magnitude of the drop in aggregate matching efficiency in the aftermath of the Great Recession and, as much as the data allow, we compare it to the 2001 recession. Section 3 outlines the model economy and the stationary equilibrium. Section 4 describes the parameterization of the model, and highlights some cross-sectional features of the economy. Section 5 contains the experiments where we trace the dynamic response of the economy to various shocks and outlines the main results of the paper. Section 6 concludes.

2 Measurement of Aggregate Matching Efficiency

As a starting point, we stipulate the aggregate matching function

$$H_t = \Phi_t V_t^\alpha U_t^{1-\alpha}. \quad (1)$$

where H_t are aggregate hires, V_t vacancies, U_t unemployment, and Φ_t is aggregate matching efficiency, the object of interest. Throughout this measurement exercise, we focus on the sample period 2001:1-2014:2 with monthly frequency. We obtain hires and vacancies from the Job

⁷However, in [Kaas and Kircher \(2011\)](#), quantitatively, most of the heterogeneity needed to explain the [Davis, Faberman, and Haltiwanger \(2013\)](#)'s empirical finding, derives from the specific shape of the labor adjustment cost function. The final functional form adopted in their paper is the same we advocate in Section 3. We have arrived at this conclusion independently.

Openings and Labor Turnover Survey (JOLTS), and unemployment from the Current Population Survey (CPS). In the tradition of the measurement of TFP of the aggregate production function, we obtain Φ_t as a residual, i.e., we fix the elasticity parameter α and derive Φ_t from the ratio $H_t / (V_t^\alpha U^{1-\alpha})$, month by month.

Our model of Section 3 abstracts from heterogeneity in the pool of job-seekers and, thus, from changes in its composition that may affect measured Φ_t from equation (1). Recently, a growing literature has convincingly argued that compositional changes in the pool of job-seekers account for a sizable fraction of the dynamics of matching efficiency [Veracierto \(2011\)](#); [Barnichon and Figura \(2010\)](#); [Fujita and Moscarini \(2013\)](#); [Hall and Schulhofer-Wohl \(2013\)](#). In this section, we build on this literature to identify the component of aggregate matching efficiency that the composition of job-seekers' stock does not account for and, in the rest of the paper, we use our model to study how firms' choices of recruiting intensity over the business cycle contribute to the dynamics of this component.

Our first step is to control for the key sources of heterogeneity in the unemployment pool. [Fujita and Moscarini \(2013\)](#) and [Hall and Schulhofer-Wohl \(2013\)](#) show that, once the unemployed are classified based on the reason of entry into the pool, those on temporary layoff stand out from the rest in terms of their re-employment probability—mainly because of the high recall rate from their previous employer in the first month or two of joblessness. The aggregate matching function view of labor market frictions—that we maintain throughout this paper—relies on the presumption that all hires are the outcome of a costly search process on both sides of the market. However, recalls eliminate search frictions altogether: workers on temporary layoff do not appear to search for other jobs and their recall does not require opening a new vacancy. Hence, we follow [Fujita and Moscarini's](#) approach, and exclude these workers from the argument U_t of the matching function. Symmetrically, we net out from aggregate hires H_t those that originate from workers on layoff.⁸

As [Hall and Schulhofer-Wohl \(2013\)](#) emphasize, over 40 percent of total hires originate from out of the labor force (N_t) and almost as many from employment (E_t). To take these additional

⁸We obtain the number of unemployed workers on layoff from the BLS. Both [Fujita and Moscarini \(2013\)](#) and [Hall and Schulhofer-Wohl \(2013\)](#) estimate from the CPS a monthly recall rate for this class of workers around 0.4, and fairly stable since 2001. We use this rate to estimate the number of hires from unemployed workers on layoff that we subtract from H_t .

categories of job-seekers into account, we generalize our matching function as follows:

$$H_t = \Phi_t \cdot \underbrace{\left(1 + s_t^N \frac{N_t}{U_t} + s_t^E \frac{E_t}{U_t}\right)}_{\text{composition factor}}^{1-\alpha} \cdot V_t^\alpha U^{1-\alpha}, \quad (2)$$

where s_t^N is the fraction of out-of-the labor force job seekers (or, equivalently, the average search intensity of nonparticipants *relative* to that of unemployed job seekers not on layoff). Similarly, s_t^E is the fraction of employed job seekers. [Veracierto \(2011\)](#) shows that, by exploiting the constant-return-to-scale property of the matching function, there is a simple way to identify s_t^N and s_t^E , i.e.,

$$s_t^N = \frac{NE_t/N_t}{UE_t/U_t}, \text{ and } s_t^E = \frac{EE_t/E_t}{UE_t/U_t} \quad (3)$$

and, hence, by using data on out-of-the labor force to employment (NE_t) and employment to employment (EE_t) flows from the CPS, we obtain estimates of s_t^N and s_t^E .⁹ For consistency, also in these formulas, UE_t is measured net of transitions from temporary layoff into employment, and U_t net of jobless workers on layoff. Finally, as standard in this literature, we set $\alpha = 0.5$.

Figure 1 plots both the composition factor in parenthesis in equation (2)—directly measured from the data—and the residual component Φ_t . The log scale makes the two components additive. Overall, the composition factor explains half of the drop in total matching efficiency during the 2001 recession and 1/3 of over the two years 2007-2009. We identify a fall in Φ_t —our object of interest—of around 10 percentage points during the 2001 recession and around 35 percentage points during the 2007-09 downturn. The decline in Φ_t is a lot more persistent in the last recession. Five years after the start of the 2001 downturn, Φ_t is back to its pre-recession level, whereas five years after the Great Recession, Φ_t is still 20 percent lower.

In conclusion, our analysis shows that, even after broadly controlling for compositional changes among job seekers, there is a sizable drop in aggregate matching efficiency that remains to be explained. Clearly, one cannot rule out a *common* reduction in unobserved search intensity across all different types of job seekers: such phenomenon would be observationally equivalent to a lower value for Φ_t .¹⁰ However, the available evidence points to the opposite di-

⁹For job-to-job transitions EE_t , we use the series initially compiled by [Fallick and Fleischman \(2004\)](#), and recently updated.

¹⁰Also a rise in mismatch between unemployment and vacancies across labor markets would translate into

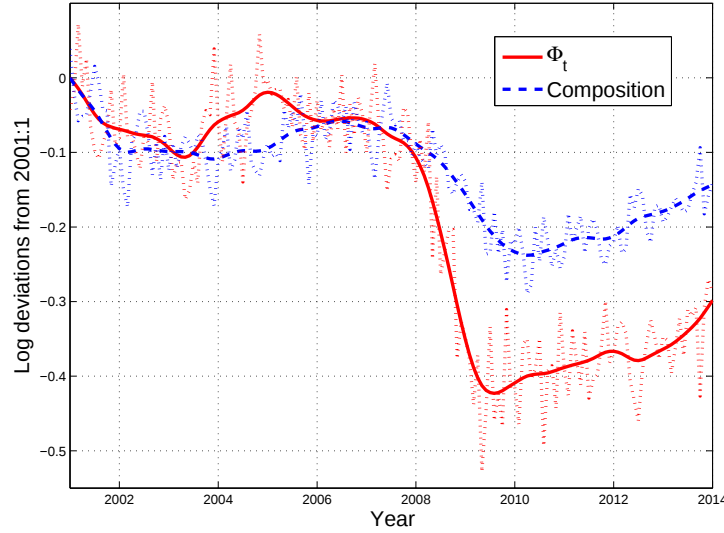


Figure 1: Measured aggregate matching efficiency Φ_t and composition factor. Sample period: 2001:2-2014:4

rection. [Mukoyama, Patterson, and Şahin \(2013\)](#) construct a monthly series of aggregate worker search effort by combining information from the American Time Use Survey (ATUS) and the CPS. Their main finding is that search effort is markedly countercyclical.¹¹

2.1 Relationship with Christiano, Eichenbaum and Trabandt

In a recent influential paper, [Christiano, Eichenbaum, and Trabandt \(2013\)](#), henceforth [CET](#), argue that one can account for the *apparent* shift of the Beveridge curve during the Great Recession without any drop in aggregate matching efficiency. They emphasize that the key step is to fully incorporate dynamics into the equilibrium equation for the Beveridge curve, instead of imposing steady-state at every date t . In a two-state model of the labor market, without on-the-job search, the law of motion for unemployment, together with the matching function, yields the

a drop in Φ_t . According to [Şahin, Song, Topa, and Violante \(2014\)](#), the quantitative importance of this channel around the Great Recession was limited.

¹¹This finding may appear surprising, in light of the generous and repeated extensions of UI benefits implemented by the U.S. government during the post-recession slump. However, the existing evidence seems to point towards the fact that, in order to find large effects of UI benefits on unemployment, one has to resort to its equilibrium impact on job creation ([Hagedorn, Karahan, Manovskii, and Mitman, 2013](#)).

Beveridge curve equation:

$$U_{t+1} = U_t - \Phi_t V_t^\alpha U_t^{1-\alpha} + \delta (1 - U_t), \quad (4)$$

where δ is the separation rate. It is also useful to define the job-finding rate

$$p_t \equiv \frac{H_t}{U_t} = \Phi_t \left(\frac{V_t}{U_t} \right)^\alpha = \Phi_t \theta_t^\alpha \quad (5)$$

and the vacancy yield (or job-filling rate)

$$q_t \equiv \frac{H_t}{V_t} = \Phi_t \left(\frac{U_t}{V_t} \right)^{1-\alpha} = \Phi_t \theta_t^{\alpha-1}. \quad (6)$$

We now run an exercise in the spirit of [CET](#). We use monthly data on unemployment U_t and hires H_t —again, for consistency, net of the temporary layoff components, and the latter also divided by the composition factor—set $\delta = 0.033$ and $\alpha = 0.5$, as in CET, and fix Φ_t to a constant in the three equations (4)-(6). We then obtain residually vacancies $\hat{V}_t$ from equation (4) and trace the Beveridge curve in the U-V space. The aim of this exercise is to show that, if a model is able to generate equilibrium unemployment dynamics similar to those in the data, and equation (4) is one of its equilibrium conditions, then the same model will necessarily be able to explain the shift in the Beveridge curve.¹²

Figure [A1](#) displays the findings of this exercise. The top-left panel shows that, even in absence of a drop in Φ_t , equation (4) can produce an outward shift of similar magnitude to the empirical shift. A more careful observation, though, reveals that the Beveridge curve relation appears too steep relative to the data. In other words, vacancies $\hat{V}_t$ implied by (4) with constant Φ_t are extremely volatile: for a given change in unemployment observed in the data, vacancies move a lot more than in the data.¹³ The vacancy yield plotted in the bottom-left panel confirms this impression. The model's vacancy yield rises twice as much as in data during the Great Recession because of the exaggerated fall in $\hat{V}_t$.

¹²The presentation slides in [CET](#)—but not the published paper—contain this exercise and report almost identical results.

¹³Not surprisingly, the loop in the Beveridge curve implied by the full model in [CET](#) (their Figure 9) is a lot wider than in the data.

Allowing for a deterioration of matching efficiency during 2008-2009 helps limiting the excessive rise in the vacancy yield. To clarify this point, we perform the same exercise, but we feed into equations (4)-(6) our estimated path for Φ_t . Figure A2 collects our findings. The model now matches both the job-finding rate and the vacancy yield quite well.¹⁴ If anything, the top-left panel shows that vacancies $\hat{V}_t$ fall too little in 2008, as the model-implied Beveridge curve is somewhat flatter than its empirical counterpart in the period when Φ_t falls sharply.

2.2 Recruiting Intensity and Aggregate Matching Efficiency

We now describe briefly how, starting from individual hiring decisions at the firm level, we can aggregate into an economy-wide matching function whose efficiency factor has the interpretation of average recruitment intensity in the economy. This derivation follows Davis, Faberman, and Haltiwanger (2013), henceforth DHF.

Any given hiring firm i chooses v_{it} , the number of open positions, ready to be staffed and costly to create, and $e_{it} \in [0, 1]$, recruiting intensity — the probability of filling each open position. Let $v_{it}^* = e_{it}v_{it}$ be the number of *effective* vacancies in firm i . Integrating over all firms we arrive at

$$V_t^* = \int e_{it}v_{it}di, \quad (7)$$

the aggregate number of effective vacancies. Then, under our maintained assumption of a CRS Cobb-Douglas matching function, aggregate hires are given by

$$H_t = (V_t^*)^\alpha U_t^{1-\alpha} = \Phi_t V_t^\alpha U_t^{1-\alpha}, \text{ with } \Phi_t = \left[\int e_{it} \left(\frac{v_{it}}{V_t} \right) \right]^\alpha. \quad (8)$$

Measured aggregate matching efficiency Φ_t is, therefore, an average of firm-level recruitment intensity weighted by individual vacancy shares, raised to the power of α , the economywide elasticity of hires to vacancies.¹⁵ Finally, note that consistency requires that, at the individual level, firm i faces hiring frictions that can be summarized as

$$h_{it} = q(\theta_t^*) e_{it}v_{it}, \quad (9)$$

¹⁴The model-implied job finding rate is, by construction, identical in the two exercises.

¹⁵DHF call (8) the generalized matching function.

where $\theta_t^* = V_t/U_t$ is effective market tightness.¹⁶ Thus, $q(\theta_t^*) = \Phi_t(U_t/V_t)^{1-\alpha}$ is the aggregate job filling rate per effective vacancy, constant across all firms at date t .

In the rest of the paper, we take the empirical estimate of Φ_t from Figure 1 as the target series for aggregate matching efficiency, and quantitatively evaluate whether firms' recruiting intensity can explain its dynamics. Since we specify the aggregate matching function as equation (1), for full consistency, we use data on aggregate hires H_t divided by the composition factor in equation (2) as estimated above.¹⁷

3 Model

Our starting point is the equilibrium random-matching model of the labor market in which firms are heterogeneous in productivity and size, and the hiring process occurs through an aggregate matching function. As discussed in the Introduction, we augment this model in three dimensions—all of which are essential to develop a framework that can address our question. First, our framework features endogenous firm entry and exit. Second, beyond the number of positions to open (vacancies), hiring firms optimally choose their recruiting intensity: by spending more on recruitment resources, they can increase the rate at which they meet job seekers. Third, firms face a dividend constraint that they can overcome, when binding, by borrowing from banks at a premium that reflects their default risk.

In what follows, we present the economic environment in more detail, we outline the model timing, and then we describe the firm, bank, and household problem. Finally, we define a stationary equilibrium for the aggregate economy. Throughout the paper, we use a recursive formulation.

¹⁶In the paper, we are faithful to the notation in this literature and denote measured labor market tightness V_t/U_t as θ .

¹⁷Moreover, recall that, when we construct aggregate hires and unemployment from the data, we subtract hires from workers on temporary layoffs from total hires and unemployed workers on temporary layoff from total unemployment. These are the data series that we use whenever we do model-data comparisons on H_t and U_t .

3.1 Environment

Time is discrete and the horizon is infinite. Three types of agents populate the economy: firms, banks, and households.

Firms. There is a measure λ_0 of potential entrants each period, and the measure of incumbent firms is λ . Firms are heterogeneous in their productivity $z \in Z$, stochastic and i.i.d. across all firms, and operate a decreasing-returns-to-scale (DRS) production technology whose only input is labor $n \in N$. The output of production is an homogeneous final good, whose competitive price is the numeraire of the economy. When active, firms face a non-negativity constraint on dividends. The model timing, explained in detail below, is such that firms must finance wage bills, operating costs, and hiring costs, and pay dividends to households *before* receiving the revenues from current period production.¹⁸ Even though revenues can be stored—relaxing the next period dividend constraint—some firms need to borrow from banks to satisfy the current period constraint.

Potential entrants draw a value of z from the initial distribution $\Gamma_0(z)$ and, conditional on this draw, decide whether to enter and become an incumbent—an action that requires paying the set-up cost χ_0 . Entrant firms receive an initial equity injection from households that covers a fraction ε of the set-up cost and must borrow the rest. This is the only time when firms can obtain funds directly from households—throughout the rest of their lifecycle they must rely on the financial sector.

Incumbents can exit exogenously or endogenously. With probability δ , a destruction shock hits an incumbent firm, forcing it to exit with zero residual value to both shareholders (households) and bondholders (banks). The firms surviving this Poisson shock observe their new value of z , drawn from the conditional distribution $\Gamma(z', z)$, and choose whether to exit or to continue production. Upon choosing to exit, firms can recover a fraction ρ of the firm's last period production flow—the rest being a deadweight loss. Debt is senior to equity in firms' capital structure, thus, if the recoverable fraction of output is enough to repay the existing debt, shareholders can claim the residual; if it is not, the bank recovers part of their loan, but share-

¹⁸De facto, this dividend constraint is a working-capital constraint that forces the firm to pay operating cost, wage bill, and hiring cost before production. If the firm could pay dividends after the revenues from production, then it could finance all its costs with within-period production and, thus, the financial constraint would bind very seldom.

holders get nothing from the liquidated firm. Note that this means that exit can occur either with or without default.

Those incumbents that, after observing z , decide to stay in pay a per-period operating cost χ and choose whether to fire some of their existing employees or hire new workers. Firing is frictionless, but hiring is not: a hiring firm chooses both vacancies v and recruitment effort e with associated hiring cost $C(e, v, n)$. Given (e, v) , the individual hiring function (9) determines next period employment n' , and the aggregate matching technology (8) determines $q(\cdot)$.

Bargaining between every worker and the firm determines wage payments. We simplify this block of the model by assuming that the firm makes a take-it-or-leave-it offer to the jobseeker and, hence, the wage equals ω , the flow value of home production.

Banks. The banking sector is perfectly competitive and has free entry. Banks receive household deposits and extend loans to firms. Both are one-period contracts. Transforming deposits into loans is subject to a cost φ (in terms of the final good) per unit of funds intermediated. Deposits earn the risk-free return $\bar{Q}^{-1}$. The unit price of a loan to a firm with productivity z , employment n' , and desired loan size $b' > 0$ is $Q(n', b', z)$, determined by the equilibrium zero-profit condition that holds for each loan type separately. The loan price is a function of all the firm's state variables that influence the repayment probability next period. In what follows, we use the convention that $Q(n', b', z) = \bar{Q}$ for $b' < 0$ (a deposit). Firms behave competitively in the financial market, and take the price function $Q(\cdot)$ as given.

Households. We envision a representative household with Λ family members, U of which are unemployed. The representative household is risk-neutral and has discount factor $\beta \in (0, 1)$. It trades shares M of the mutual fund comprised of all firms in the economy and makes bank deposits T . It earns dividends D paid by the mutual fund, interests on deposits, and the total wage bill paid by firms to the employed family members. Moreover, unemployed workers, when not searching, produce ω units of the final good at home.

Before describing the firm's problem in detail, we outline the precise timing of the model. Within a period, the events unfold as follows: (i) realization of the productivity and firm destruction shocks, and exogenous exit of incumbents; (ii) exit/default decision by incumbents and entry decision by potential entrants; (iii) new borrowing decisions by incumbents (iv) hiring/firing decisions; (v) labor market matching and firm-worker bargaining; (vi) payment of

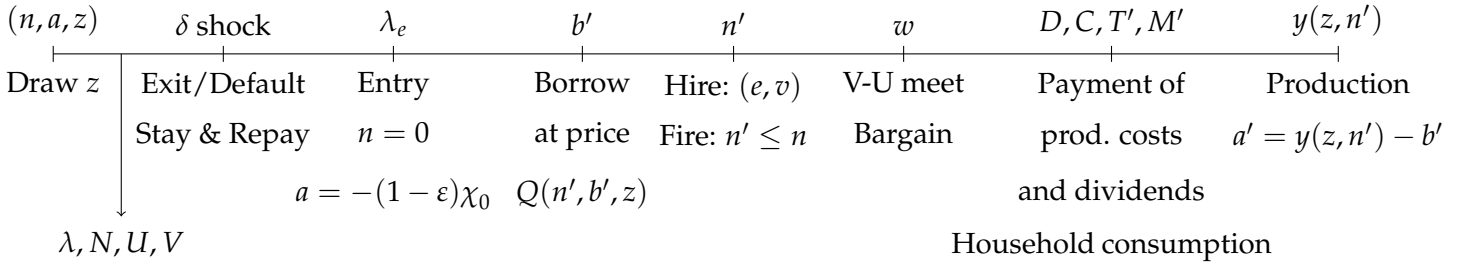


Figure 2: Timeline of the model

operating cost, hiring costs, wage bill, existing debt, and dividends by incumbents and household consumption/saving decisions; (vii) production. Figure 3.1 summarizes this timeline.

For what follows, it is useful to note that the measures of incumbent firms, employment and unemployment are taken at the beginning of the period, before stage (i). Moreover, even though the labor market opens after firms exit or fire, workers who separate in the current period can only start searching in the next one. Finally, since revenues accrue to the firm at the end of the period, after dividends are paid, the net worth of the firm at the beginning of each period—the state variable that matters for all firm’s decisions—is $a = y(z_{-1}, n) - b \in A$, where $y(z_{-1}, n)$ are previous-period revenues from production, and b is the current-period debt.

3.2 Firm problem

We first consider the entry and exit decisions, and then analyze the problem of incumbent firms.

Entry. A potential entrant who has drawn z from $\Gamma_0(z)$ solves the following problem

$$\max \left\{ \mathbb{V}^i(0, -(1 - \varepsilon)\chi_0, z) - \varepsilon\chi_0, 0 \right\}, \quad (10)$$

where $\mathbb{V}^i$ is the value of an incumbent firm. The firm enters if the value of becoming an incumbent with no employment, productivity z , and with initial net worth $-(1 - \varepsilon)\chi_0$ exceeds the cost of the initial equity injection $\varepsilon\chi_0$. Let $i(z) \in \{0, 1\}$ be the entry decision rule. As $\mathbb{V}^i$ is increasing in z , there is an endogenous productivity cut-off z^* such that for all $z \geq z^*$ the firm

chooses to enter. Thus, the measure of entrants is

$$\lambda_e = \lambda_0 \int_Z i(z) d\Gamma_0 = \lambda_0 [1 - \Gamma_0(z^*)]. \quad (11)$$

Exit. After surviving the Poisson destruction shock, an incumbent with n units of labor, post-revenues net worth a , and current and past productivity z and z_{-1} chooses whether to continue production, exit without defaulting, or exit and default by solving

$$\mathbb{V}(n, a, z, z_{-1}) = \max \left\{ \mathbb{V}^i(n, a, z), a - (1 - \rho) y(z_{-1}, n), 0 \right\}. \quad (12)$$

Continuing production has value $\mathbb{V}^i$. Exiting and repaying the debt obligations yields the recoverable portion of output $\rho y(z_{-1}, n)$ net of debt b to the shareholders where, recognizing that $b = y(z_{-1}, n) - a$ yields the expression in the second argument of the max operator. Finally, defaulting implies exit with a firm's residual value of zero to the shareholders. We denote by $x^R(n, a, z, z_{-1}) \in \{0, 1\}$ the exit decision with debt repayment, and by $x^D(n, a, z, z_{-1}) \in \{0, 1\}$ the exit decision with default.

Hire or fire. An incumbent firm (i) with employment, assets, and productivity equal to the triplet (n, a, z) that chooses whether to hire or fire solves

$$\mathbb{V}^i(n, a, z) = \max \left\{ \mathbb{V}^h(n, a, z), \mathbb{V}^f(n, a, z) \right\}, \quad (13)$$

where the two value functions associated with firing (f) and hiring (h) are described below.

The firing firm. A firm that has chosen to fire some of its workers (or not to adjust its work

force) solves

$$\begin{aligned}
\mathbb{V}^f(n, a, z) &= \max_{n', b'} d^f + \beta(1 - \delta) \int_Z \mathbb{V}(n', a', z', z) \Gamma(dz', z) \\
&\text{s.t.} \\
n' &\leq n \\
a' &= y(z, n') - b' \\
d^f &\equiv a - \omega n' - \chi + Q(n', b', z) b' \geq 0
\end{aligned} \tag{14}$$

Firms maximize the shareholder's value and, because of risk-neutrality, they use β as their discount factor, adjusted by the survival rate $(1 - \delta)$. Dividends d^f for a firing firm are given by its initial net worth a , net of the wage bill, and the operating cost plus the additional resources borrowed from the bank. As it is clear from the last equation in (14), firms face a non-negativity constraint on dividends. We let $n'_-(n, a, z)$ denote the firing decision rule.

The hiring firm. The hiring firm chooses the number of vacancies to post $v \in \mathbb{R}_+$, recruitment effort $e \in [0, 1]$ and, by a law of large numbers, understands that its new hires $n' - n$ will be given by the probability that its effective vacancies $v^* = ve$ meet a worker—equal to q and taken as given by the firm—times the number of effective vacancies created, or $n' - n = q(\theta^*)ev$. The firm faces a fixed hiring cost $\bar{\kappa}$ that yields an inaction region (included in the firing decision in the case $n' = n$), and a variable cost function $\mathcal{C}(e, v, n)$, increasing and convex in e and v .

Note that all it matters for the firm continuation value is n' , not the combination of recruiting intensity e and vacant positions v that generates it. As a result, one can split the problem of the hiring firm in two stages. First, the choice of n' . Second, given n' , the choice of the optimal combination (e, v) . This latter problem reduces to the simple static cost-minimization problem:

$$\begin{aligned}
\mathcal{C}^*(n, n') &= \min_{e, v} \mathcal{C}(e, v, n) \\
&\text{s.t.} \\
e &\in [0, 1], v \geq 0 \\
n' - n &= q(\theta^*)ev
\end{aligned} \tag{15}$$

that yields the lowest cost combination $e(n, n')$ and $v(n, n')$ that can deliver n' in a firm of size

n . Let $C^*(n', n)$ be the implied cost function, expressed in terms of target employment n' .

The choice of n' requires solving the dynamic problem

$$\begin{aligned} \mathbb{V}^h(n, a, z) &= \max_{n', b'} d^h + \beta(1 - \delta) \int_Z \mathbb{V}(n', a', z', z) \Gamma(dz', z) \\ &\text{s.t.} \\ n' &> n \\ a' &= y(z, n') - b' \\ d^h &\equiv a - \omega n' - \chi - \bar{\kappa} - C^*(n', n) + Q(n', b', z)b' \geq 0 \end{aligned} \quad (16)$$

The solution of this problem is a decision rule $n'_+(n, a, z)$. Using this function in the solution of (15), we obtain decision rules $e(n, a, z)$ and $v(n, a, z)$ for recruitment effort and vacancies in terms of the current firm states. Finally, recall that, also in this case, dividends d^h must be non-negative.

Given the centrality of the hiring cost function $\mathcal{C}(e, v, n)$ for our analysis, here we briefly discuss its specification. In what follows, we choose the functional form

$$\mathcal{C}(e, v, n) = \left[\frac{\kappa_1}{\gamma_1} e^{\gamma_1} + \frac{\kappa_2}{\gamma_2 + 1} \left(\frac{v}{n} \right)^{\gamma_2} \right] v, \quad (17)$$

with $\gamma_1 \geq 1$ and $\gamma_2 \geq 0$ being the necessary conditions for convexity of $\mathcal{C}$.¹⁹ This cost function implies that the hiring cost per open position, $\mathcal{C}/v$, has two separate components. The first one is increasing in recruiting intensity per vacancy e . The idea is that for any given open position, the firm can choose to spend resources of recruitment activity to make the position more visible or the firm more attractive, to assess more candidates or to better screen them, but all such activities are costly. The second component is the vacancy rate, and it captures the fact that expanding productive capacity is costly in *relative* terms: the implicit presumption is that creating 10 new positions involves a more expensive reorganization of production in a firm with 10 employees than in a firm with 1000 employees.²⁰

¹⁹In the limiting case $\gamma_1 = 1, \gamma_2 = 0$, the model collapses to the standard matching model without recruiting intensity margin: the optimal effort choice is 1 and the job filling rate is equal across all firms.

²⁰When we solve the model, in order to avoid the division by zero for new entrants, we make a small adjustment to the specification in (17): we write the vacancy rate in the second term as $v/(n + n_0)$ and set $n_0 = 1$. One rationale for this correction is that even the smallest start-up has at least an entrepreneur to begin with. [Kaas and Kircher](#)

In Appendix B we derive a number of results for the static hiring problem of the firm under this cost function and derive the exact expression for $\mathcal{C}^*(n, n')$ that we use in the dynamic problem. We show that, by combining first-order conditions, we obtain the optimal choice of e and, hence, the firm-level job filling rate:

$$f(n, n') \equiv q(\theta^*) e(n, n') = \left[\frac{\kappa_2}{\kappa_1} \left(\frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{\frac{\gamma_1}{\gamma_1 + \gamma_2}} \left(\frac{n' - n}{n} \right)^{\frac{\gamma_2}{\gamma_1 + \gamma_2}}. \quad (18)$$

This equation demonstrates that the model implies a log-linear relation between the job filling rate and employment growth at the firm level, with elasticity $\gamma_2 / (\gamma_1 + \gamma_2)$. This log-linear relationship is the main empirical finding of DHF. Appendix B also shows that, once (17) includes the optimal choice of e , $\mathcal{C}$ is equivalent to the hiring cost function that Kaas and Kircher (2011) assume.

Why does firm optimality imply that the job filling rate increases with the growth rate? For two reasons. First, recruiting intensity and the vacancy rate (v/n) are complements in the production of the hires per employee $(n' - n)/n$, the firm's growth rate. Thus, a firm that wants to grow a lot will optimally create more positions and, at the same time, spend more in recruiting effort. Second, the stronger the convexity of $\mathcal{C}$ in the vacancy rate (γ_2), relative to its degree of convexity in effort (γ_1), the more an expanding firm finds it optimal to substitute away from vacancies into recruiting intensity—as the elasticity of the job filling rate with respect to the growth rate implies. In the special case when $\gamma_2 = 0$, recruiting effort is irresponsive to the growth rate, as in Pissarides (2000).

3.3 Bank problem

A firm approaching a bank for a loan of size b' has committed to its employment level n' , which the bank observes along with the firm's current productivity level z . Thus, the firm's type when borrowing is (n', b', z) . The free entry condition induces equality between the expected return on the loan and the risk-free return $\bar{Q}^{-1}$ paid on deposits, thereby determining the unit price $Q(n', b', z)$ of the loan extended to such a firm. Note that, since the household is risk-neutral and infinitely lived, it is easy to anticipate that $\bar{Q} = \beta$ in equilibrium.

(2011) make a similar adjustment.

The expected return on the loan has to be consistent with the repayment decision of the firm, $1 - x^D(n', a', z', z)$: the firm repays whenever it does not default ($x^D(\cdot) = 0$). Recall that, upon endogenous default, the bank can recover a fraction ρ of output $y(z, n')$, whereas when the Poisson destruction shock strikes, with probability δ , all assets of the firm are lost.

Combining all these features, the following pricing rule obtains:

$$Q(n', b', z) = \bar{Q}(1 - \varphi)(1 - \delta) \left[1 - \int_Z x^D(n', y(z, n') - b', z', z) \left(1 - \frac{\rho y(n', z)}{b'} \right) \Gamma(dz', z) \right], \quad (19)$$

where in the second argument of x^D we have used the relationship between post-revenue net worth and debt $a' = y(z', n) - b'$. Even in absence of endogenous default, the loan premium $\bar{Q}/Q(\cdot)$ is positive and equal to $[(1 - \varphi)(1 - \delta)]^{-1}$ because of the intermediation cost and the exogenous firm destruction. Finally, when $\varphi > 0$, the firm *always* finds borrowing costly relative to its discount factor $\beta(1 - \delta)$. This will induce a precautionary saving motive that kicks in as the firm approaches its (frictionless) optimal size.

3.4 Household problem

The representative household solves

$$\begin{aligned} W(U, T, X) &= \max_{A', X', C} C + \beta W(U', T', X') \\ &\text{s.t.} \\ C &> 0, X' \leq 1 \\ C + \bar{Q}T' + PM' &= \omega\Lambda + (D + P)M + T \end{aligned} \quad (20)$$

subject to the law of motion for unemployment that we specify later. In (20), C denotes household consumption, U denotes the number of unemployed members, T are bank deposits, and M are shares of the mutual funds, with the aggregate number of shares normalized to one. The share price is P and owning shares entitles the household to dividends D , the aggregate of all firms' dividends.²¹ Because of the bargaining structure, every period both employed and unemployed household members contribute with ω units of the good to the household budget.

²¹Households consider the initial equity injections into start-ups as negative dividends.

From the first-order conditions for deposits and share holdings we obtain $\bar{Q} = \beta$ and $P = \beta(P + D)$ which imply a constant return of β^{-1} on both deposits and shares and, thus, the household is indifferent over portfolios. Since the household is risk neutral, it is also indifferent over the timing of consumption.

3.5 Stationary equilibrium and aggregation

Let Σ_N , Σ_A , and Σ_Z be the Borel sigma algebras over N and A , and Z . The state space for an incumbent firm is $S = N \times A \times Z^2$, and we denote with s one of its points (n, a, z, z_{-1}) . Let Σ_S be the sigma algebra on the state space, with typical set $\mathcal{S} = \mathcal{N} \times \mathcal{A} \times \mathcal{Z} \times \tilde{\mathcal{Z}}$, and (S, Σ_S) be the corresponding measurable space. Denote with $\lambda : \Sigma_S \rightarrow [0, 1]$ the stationary distribution of incumbent firms at the beginning of the period, before the exogenous exit shock.²²

To simplify the exposition of the competitive equilibrium, it is useful to (i) define the implied decision rule for post-revenue net worth $a'(n, a, z) = y(z, n) - b'(n, a, z)$, the encompassing exit decision rule, including both repayment and default, $x = 1 - (1 - x^R) \cdot (1 - x^D)$, and the encompassing employment rule, including both hiring and firing, $n'(n, a, z) = n_-(n, a, z)$ if $n' \leq n$ and $n'(n, a, z) = n'_+(n, a, z)$ if $n' > n$; (ii) use s as the argument for all decision rules, even though most of them, except the exit decisions, only depend on the triplet (n, a, z) but not on z_{-1} .

A stationary recursive competitive equilibrium is a collection of firms' decision rules $\{i(z), x^R(s), x^D(s)\}$, value functions $\{\mathbb{V}^f, \mathbb{V}^h, \mathbb{V}^i, \mathbb{V}\}$, a measure of entrants λ_e , a pricing function for loans $Q(\cdot)$, a price of safe deposits $\bar{Q}$, share price P and aggregate dividends D , wage function $w(\cdot)$, a distribution of firms λ , and a value for effective labor-market tightness θ^* such that: (i) the decision rules solve the firms problems (10)-(16), $\{\mathbb{V}^f, \mathbb{V}^h, \mathbb{V}^i, \mathbb{V}\}$ are the associated value functions, and λ_e is the mass of entrants at productivity z implied by (11); (ii) the pricing function $Q(\cdot)$ satisfies the free entry condition (19) for every segment (n', b', z) ; (iii) $\bar{Q} = \beta^{-1}$ from household optimization; (iv) the market for shares clears at $X = 1$ with share price

$$P = (1 - \delta) \int_S \mathbb{V}(s) d\lambda + \lambda_0 \int_Z \mathbb{V}^i(0, -(1 - \varepsilon)\chi_0, z) i(z) d\Gamma_0$$

²²Note a subtlety in the timing here. We described the values of the firms *after* the exogenous exit shock, whereas we describe the measure *before* the shock (see the timeline in Figure 3.1). This eases the formulation of the laws of motion of aggregate state variables.

and dividends

$$D = (1 - \delta) \int_S \left\{ [1 - x(s)] d(s) + x^R(s) [a - (1 - \rho) y(z_{-1}, n)] \right\} d\lambda - (\varepsilon \chi_0) \lambda_0 \int_Z i(z) d\Gamma_0$$

(v) the wage function is identically equal to ω , the flow of home production; (vi) the stationary distribution λ is the fixed point of the recursion:

$$\begin{aligned} \lambda(\mathcal{N} \times \mathcal{A} \times \mathcal{Z} \times \tilde{\mathcal{Z}}) &= (1 - \delta) \int_{N \times A \times \tilde{\mathcal{Z}} \times Z} [1 - x(s)] \mathbf{1}_{\{n'(s) \in \mathcal{N}\}} \mathbf{1}_{\{a'(s) \in \mathcal{A}\}} \Gamma(\mathcal{Z}, z) d\lambda \\ &\quad + \lambda_0 \int_{\tilde{\mathcal{Z}}} \mathbf{1}_{\{n'(n_0, -(1-\varepsilon)\chi_0, z) \in \mathcal{N}\}} \mathbf{1}_{\{a'(n_0, -(1-\varepsilon)\chi_0, z) \in \mathcal{A}\}} \Gamma(\mathcal{Z}, z) i(z) d\Gamma_0, \end{aligned} \quad (21)$$

where the first term refers to existing incumbents and the second to new entrants; (v) effective market tightness θ^* is determined by the equilibrium condition

$$\Lambda - N(\theta^*) = \frac{F(\theta^*)}{p(\theta^*)}, \quad (22)$$

where $N(\theta^*) = \int_S n d\lambda$ is aggregate employment, and

$$F(\theta^*) = \delta \int_S n d\lambda + (1 - \delta) \int_S n x(s) d\lambda + (1 - \delta) \int_S (n - n'(s))^- [1 - x(s)] d\lambda, \quad (23)$$

are aggregate separations, which include all the employment lost by firms exiting exogenously and endogenously, plus all the workers fired by shrinking firms, which we have denoted by $(n - n'(s))^-$.²³ In the three equations above, the dependence of N and F from θ^* comes through the decision rules and the stationary distribution, even though, for notational ease, we have omitted θ^* as their explicit argument.

The left-hand side of (22) is the definition of unemployment—labor force minus employment—whereas the right-hand side is the steady-state Beveridge curve, i.e., the law of motion for unemployment

$$U' = U - p(\theta^*) U + F(\theta^*) \quad (24)$$

²³Entrant firms never fire, as they enter with the lowest value on the support for N, n_0 .

in steady-state. Thus, exactly as in [Elsby and Michaels \(2013\)](#), the two sides of (22) are independent equations determining the same variable—unemployment— and, combined, they yield equilibrium market tightness θ^* .²⁴

Clearly, once θ^* is determined, so is U from either side of (22) and, therefore, V^* . Finally, we note that measured aggregate matching efficiency, in equilibrium, is:

$$\Phi = \left[\frac{(1 - \delta) \int_S e(s) v(s) [1 - x(s)] d\lambda + \lambda_0 \int_Z e(0, - (1 - \varepsilon) \chi_0, z) v(0, - (1 - \varepsilon) \chi_0, z) i(z) d\Gamma_0}{V} \right]^\alpha, \quad (25)$$

where measured total vacancies are

$$V = (1 - \delta) \int_S v(s) [1 - x(s)] d\lambda + \lambda_0 \int_Z v(0, - (1 - \varepsilon) \chi_0, z) i(z) d\Gamma_0. \quad (26)$$

4 Parameterization

We begin from the subset of parameters that are calibrated externally. The model period is one month and, thus, we set β to replicate an annualized risk-free rate of 4%. Since the measure of potential entrants λ_0 scales λ —see equation (21)—we choose λ_0 to normalize the total measure of incumbent firms to one. We set the parameter ε that measures the fraction of the start-up cost χ_0 financed by household equity to 0.45 to replicate the total (insider and outsider) equity share of funding for start-ups, measured in the year 2004 before the nascent businesses—a sample of startups in the Kauffman Firm Survey—made any revenues ([Robb and Robinson, 2012](#), Table 5). Finally, to be consistent with the empirical analysis of Section 2, we set the elasticity of aggregate hires to aggregate vacancies in the matching function (α) to 0.5. Table 1 summarizes these parameter values.

This same table lists the remaining 17 parameters of the model that are calibrated internally by minimizing the distance between 17 empirical moments and their equilibrium counterparts in the model. Table 2 lists the targeted moments, their empirical values, and their simulated

²⁴Our computation showed that, typically, $N(\theta^*)$ is increasing in its argument and the right-hand side of (22) is always positive and decreasing. Thus, the crossing point of left- and right-hand side is unique, when it exists. However, an equilibrium may not exist. For example, for very low hiring costs, $N(\theta^*)$ may be greater than Λ . Conversely, for large enough operating or hiring costs, no firms will enter the economy. In this case, there is no equilibrium with market production (albeit there is always some home-production in the economy).

values from the model. Even though every targeted moment is determined simultaneously by all the parameters, in what follows we discuss each of them in relation to the parameter for which, intuitively, that moment yields the most identification power.

We choose the size of the labor force Λ to match an average firm size of 17.5 employees, as computed from the Business Dynamics Statistics (BDS) over the period 2001-2007. We set the flow of home production of the unemployed ω to replicate a steady-state elasticity of measured market tightness θ to a permanent shift in aggregate productivity of 15. This way, the model generates a realistic amount of volatility of labor market variables in response to real shocks.²⁵ We choose the shift parameter of the matching function (a normalization of the value of Φ in steady state) and the exogenous firm destruction rate δ in order to pin down worker flows (net of flows involving temporary layoffs), i.e., a monthly job finding rate of 0.265 and a monthly separation rate of 0.02. Together, these two moments yield a steady-state unemployment rate of 0.07. We assume the revenue function $y(z, n) = zn^\nu$ and choose the extent of decreasing returns to reproduce an aggregate labor share of income equal to 0.67.²⁶ The entry cost χ_0 largely pins down the firm entry rate, roughly 0.10 at the annual frequency in the BDS data. We choose the operating cost χ to reproduce the fraction of firms that survive at least five years since birth, 50 percent in the BDS data. We postulate an AR(1) process for the firm-level productivity shock z . We calibrate the standard deviation and the persistence of the shocks to match the standard deviation of employment growth for continuing establishments in the cross section, 0.42 at the annual frequency (Elsby and Michaels, 2013), and the employment share of large firms (500+ employees), 0.47. The initial productivity distribution for entrants Γ_0 is an exponential distribution $Exp(\zeta)$ in order to give the model a chance to reproduce a fat tail in the size distribution of firms. We choose ζ to reproduce an average size of age-zero firms of 5, or roughly 30 percent of the average firm size for incumbents.²⁷

²⁵Remember that one of our objectives is to explore the differential response of the model to productivity and financial shocks and, to do so, it seems reasonable to start from an economy where productivity shocks induce empirically plausible aggregate fluctuations. Put differently, accounting for the the observed large movements of unemployment and vacancies *along* the Beveridge curve is a necessary starting point for assessing the ability of the model to produce *outward* movements of the Beveridge curve.

²⁶Note a remarkable property of this version of the matching model with decreasing returns in production highlighted by Elsby and Michaels (2013); Acemoglu and Hawkins (Forthcoming): the model allows for large economic fluctuations (because on the margin firms profits are small), and yet is capable to generate a labor share quite far from one (because, on average, firm profits are large).

²⁷Both ζ and χ_0 play an important role in determining entry, but we can separate their effect. Fix all other

Parameter	Value	
A. External		
Household discount factor	β	0.9967
Potential entrants	λ_0	2.3
External equity share of start-up cost	ε	0.45
Elasticity of matching function wrt V_t	α	0.5
B. Internal		
Size of labor force	Λ	
Flow of home production	ω	
Scaling of matching function	$\overline{\Phi}$	
Exogenous exit shock	δ	
DRS parameter	ν	
Entry cost	χ_0	
Operating cost	χ	
S.D. of productivity shocks	Γ	
Persistence of productivity shocks	Γ	
Exponential distribution parameter	Γ_0	
Fixed cost of hiring	$\overline{\kappa}$	
Cost elasticity wrt effort e	γ_1	
Cost elasticity wrt vacancies v	γ_2	
Cost shifter wrt e	κ_1	
Cost shifter wrt v	κ_2	
Financial intermediation wedge	φ	
Recovery parameter (fraction of output)	ρ	

Table 1: Parameter values

We now turn to hiring costs. We choose the fixed cost of hiring $\bar{\kappa}$ to reproduce the well known fact that establishments display a lot of inaction with respect to hiring and firing. Specifically, we choose to reproduce the fact that, annually, 42 percent of establishments have growth rate lower than 0.05 in absolute value. The cost function (17) has four parameter values: the two cost shifters (κ_1, κ_2) and the two elasticities (γ_1, γ_2) . We choose these four parameters to match four moments. First, a cross-sectional elasticity of job filling rates to employment growth rates of 0.82, as DHF estimate.²⁸ Second, the ratio of recruiting intensity of small firms to large

parameters. Given χ_0 , the productivity cutoff z^* is determined. An increase in the exponential parameter ζ flattens the distribution of initial productivity, shifting more mass to the tail. This leads to (i) an increase in the average size of new firms and (ii) an increase in entry. Given ζ , the shape of the initial productivity distribution is determined. An increase in χ_0 raises the productivity cutoff for newborn firms. This leads to (i) an increase in average size of new firms, but (ii) a decrease in entry rates.

²⁸We cannot map directly $\gamma_2 / (\gamma_1 + \gamma_2)$ into the value estimated by DHF for two reasons. First, in DHF the growth rate is the Davis-Haltiwanger growth rate normalized in $[-2, 2]$. Second, (18) assumes an interior solution, whereas there are many firms in the model at the corner $e = 1$ for their optimal choice of recruitment effort.

firms (250+ employees), equal to 1.5 from JOLTS data on hires and vacancies by establishment size. Third, the vacancy share of small firms, 0.34 in JOLTS. Fourth, the total hiring cost as a fraction of monthly wage per hire. We have two independent sources for this statistic: according to [Silva and Toledo \(2009\)](#) this ratio is 0.4. Bersin & Associates (2011), a survey on firm recruitment practices, estimates this number to be around 0.8. In absence of better information, we target 0.6.

The last two parameters in Table 1 are the financial intermediation wedge φ and the fraction of output that can be recovered upon default, ρ . We choose the wedge φ so the borrowing premium for firms with no default risk matches the corporate bond credit spreads estimated by [Gilchrist and Zakrajšek \(2012\)](#) before the onset of the 2007 recession, roughly 2 percent per year. The recovery rate ρ maps into the relative cost of borrowing for small/young firms relative to old firms, larger on average and hence with more recoverable output. We set ρ to replicate the fact that, based on Kauffman Survey data, debt over annual revenues for start-ups in their first year of life (including those firms that exit within a year) is around 1.

4.1 Cross-sectional implications

We now explore some cross-sectional implications of the calibrated model, at its steady-state equilibrium.

TBC

5 Experiments

The aim of our numerical experiments is to account for the dynamics of some key labor market variables during the 2007-09 recession and throughout the slump that followed it. In order to put our findings in context, and to better isolate the role of financial shocks—germane to the last recession—we contrast the predictions of the model for the Great Recession to those for the 2001 downturn. Figure 3 plots the evolution of vacancies, unemployment, job finding rate, vacancy yield, and average recruiting intensity Φ —our index of aggregate matching efficiency—around these two contractionary episodes. Simply put, the data in 2007-09 in the right panel appear like a stretched version of the 2001 time series. This “stretching” is fairly proportional, except

Moment	Data	Model
A. Targeted		
Average firm size	17.5	
Elasticity of θ to aggr. productivity shock	15	
Job finding rate	0.45	
Worker separation rate	0.03	
Aggregate labor share	0.67	
Firm entry rate	0.11	
5-year firm survival rate from birth	0.50	
Standard deviation of n growth	0.42	
Employment share in large firms	0.47	
Average size of age zero firms	5.0	
Annual growth rate < 0.05	0.42	
Elasticity of job-filling rate to growth rate	0.82	
Recr. intensity of small/large firms	1.5	
Vacancy share of small firms	0.34	
Hiring cost/monthly wage per new hire	0.45	
Borrowing premium for large firms	0.02	
Debt/(yearly revenue) ratio for start-ups	1.00	
B. Non-targeted		
TBC		

Table 2: Targeted moments and implications for some additional ones

for Φ , whose recent drop is a lot more severe, as we discussed in detail in Section 2.

Our experiments trace the dynamics of the model economy starting from its initial steady state—whose property we described in Section 4—and transiting deterministically into a new steady state. We are interested in the aggregate response of the series in Figure 3 to three surprises: a real shock to aggregate TFP, and two financial shocks: a temporary rise in the financial wedge φ and a temporary increase in the fraction $(1 - \varepsilon)$ of the set-up cost that start-ups must borrow to begin operating. We will also report the dynamics of the Beveridge curve that each of these model simulations imply.

The increase in φ is meant to capture a disruption of the financial sector that escalates credit spreads, over and above the increase in default risk. Gilchrist and Zakrajšek (2012) estimate credit spreads for the U.S. corporate sector over the past 40 years and separate the component associated to time-varying default risk from the excess bond premium, a component that represents variation in the pricing of default risk, rather than in the risk of default itself, and thus

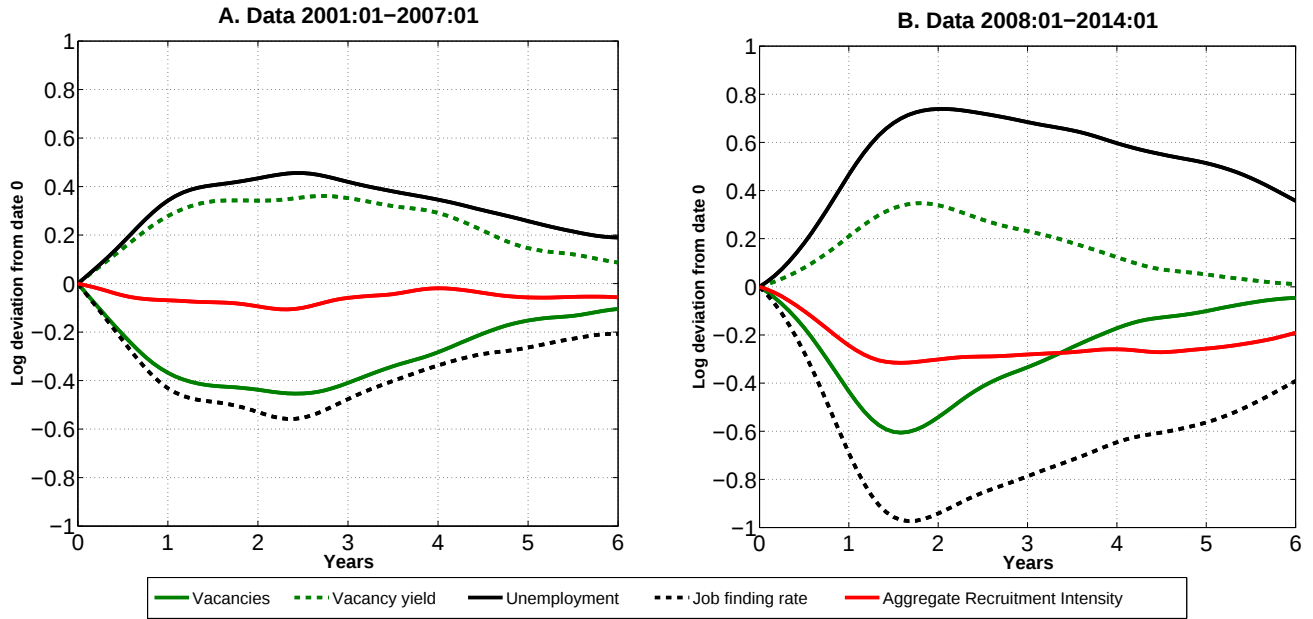


Figure 3: Dynamics of labor market variables around the last two recessions

provides a useful gauge of aggregate credit supply conditions.²⁹ In our model, movements in φ map into changes in the measured excess premium. The fall in ε is meant to summarize another dimension of financial shocks, especially pertinent to the last recession. As argued in the Introduction, startups and young firms do not have an established credit record and, as such, they often rely on personal sources of finance, including home equity. Thus, a fall in housing prices is likely to worsen the entrepreneurial need for external funding in order to start a new business and increase financing costs.

In light of this discussion, in our first experiment, we simulate a recession that mimics the 2001 downturn. We think of this recession as entirely driven by a decline in TFP of about 4 percentage points over 3 years, and a gradual recovery back to its pre-recession level in the next three. In our second experiment, we engineer the same decline in TFP and, on top of it, we increase φ by 400 basis points in the year after the onset of the recession, and we slowly bring them back to their initial level over the next two years. This path replicates the evolution of the

²⁹Gilchrist and Zakrajšek (2012) report, for example, a strikingly high correlation between their measure of excess bond premium and diffusion index of the change in credit standards on commercial and industrial loans at U.S. commercial banks obtained from the Federal Reserve's quarterly Senior Loan Officer Opinion Survey on Bank Lending Practices.

excess bond premium estimated by [Gilchrist and Zakrajšek \(2012\)](#) over the period 2007-2010. In our third experiment, besides the same fall in TFP, we reduce ε enough to replicate a fall of 27 percent in the number of start-up firms. We then slowly increase ε back to its initial level in a way that reproduces the dynamics of the entry rate in the post-recession years.

TBC

6 Conclusions

TBC

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APPENDIX

This Appendix is organized as follows. Section A contains two figures on the replication of Christiano, Eichenbaum, and Trabandt (2013) that we discussed in Section 2. Section B contains the derivations of the cost hiring function that we introduced in Section 4. Section C details the algorithms for the computation of the stationary equilibrium and the transitional dynamics.

A Replication of Christiano, Eichenbaum, and Trabandt (2014)

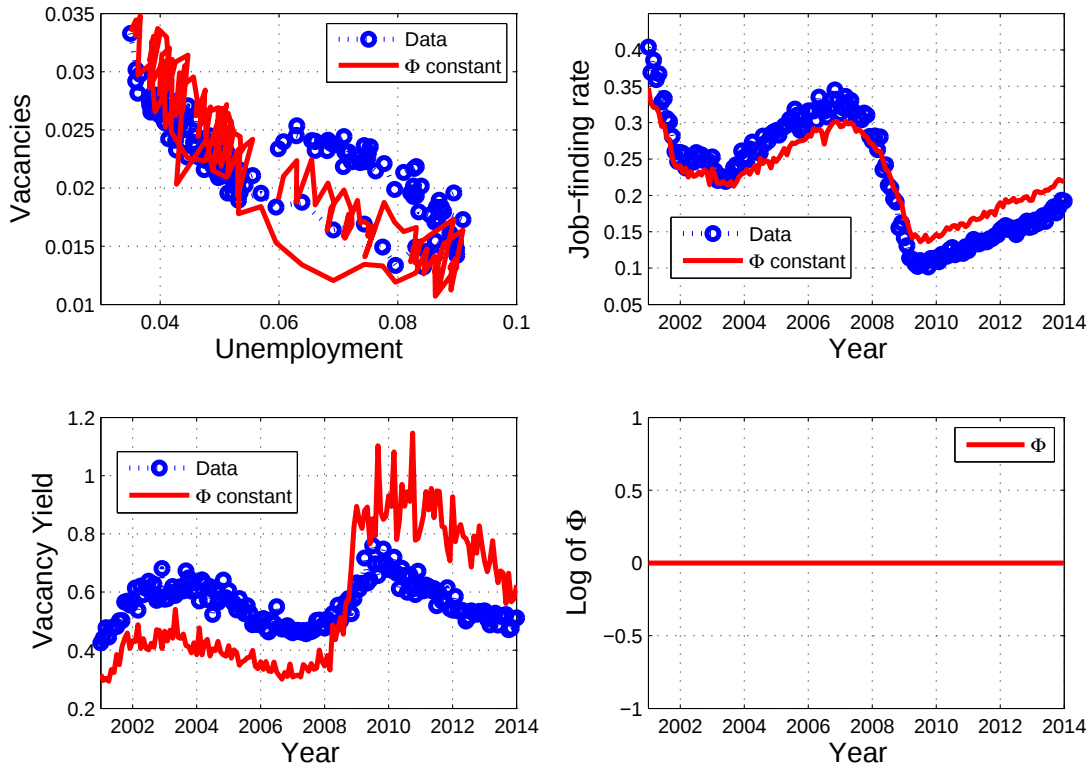


Figure A1: Φ_t fixed as in CET (2014). Top-left panel: Beveridge curve. Top-right: job-finding rate. Bottom-left: vacancy yield. Bottom-right: logarithm of Φ_t . Dotted line: data. Solid line: HP-filtered series.

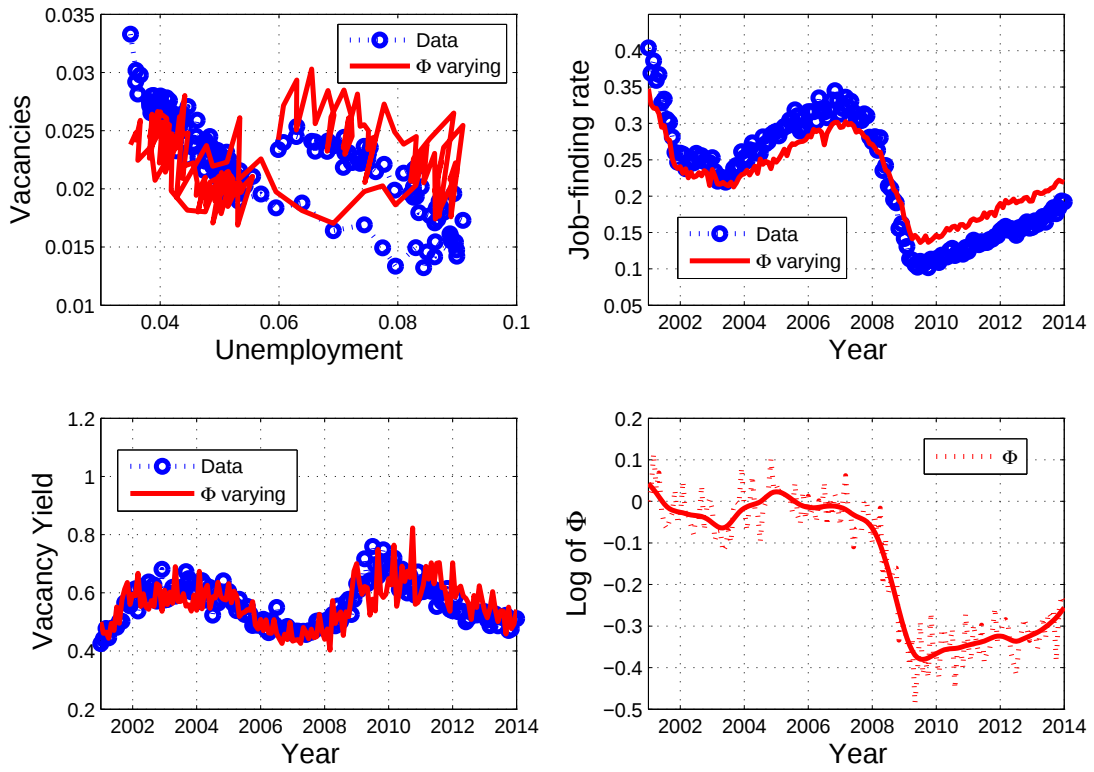


Figure A2: Φ_t varying as estimated in Section 2. Top-left panel: Beveridge curve. Top-right: job-finding rate. Bottom-left: vacancy yield. Bottom-right: logarithm of Φ_t . Dotted line: data. Solid line: HP-filtered series.

B The hiring cost function

In this section we show that, once we postulate the hiring cost function

$$\mathcal{C}(n, e, v) = \left[\frac{\kappa_1}{\gamma_1} e^{\gamma_1} + \frac{\kappa_2}{\gamma_2 + 1} \left(\frac{v}{n} \right)^{\gamma_2} \right] v \quad (\text{B1})$$

then, through firm's optimization we obtain a loglinear cross-sectional relationship between the job-filling rate and the employment growth rate that is consistent with the empirical findings in [DHF](#). Next, we show that our cost function boils down to the one that [Kaas and Kircher \(2011\)](#) choose. Finally, by substituting the firm FOCs into [\(B1\)](#), we derive a formulation of the cost only in terms of (n, n') that we use in the intertemporal problem [\(16\)](#) in the main text.

As we explained in [Section \(3.1\)](#), the firm solves a static cost minimization problem: given a choice of n' , it determines the lowest cost combination of (e, v) that can deliver n' . The hiring firm's cost minimization problem is

$$\begin{aligned} \mathcal{C}(n, n') &= \min_{e, v} \left[\frac{\kappa_1}{\gamma_1} e^{\gamma_1} + \frac{\kappa_2}{\gamma_2 + 1} \left(\frac{v}{n} \right)^{\gamma_2} \right] v \\ \text{s.t.} \quad & n' - n \leq q(\theta^*) e v \\ & e \in [0, 1], \quad v \geq 0 \end{aligned} \quad (\text{B2})$$

Convexity of the cost function [\(B1\)](#) in (e, v) requires $\gamma_1 \geq 1$ and $\gamma_2 \geq 0$. When $\gamma_1 = \gamma_2 = 0$, we have the standard model where every firm sets $e = 1$ and the cost of vacancy creation is linear. After setting up the Lagrangian, and ignoring for now the corner solution $e = 1$, one can easily derive the two FOCs with respect to e and v that, combined together, yield a relationship between the optimal choice of e and the optimal choice of the vacancy rate v/n :

$$e = \left[\frac{\kappa_2}{\kappa_1} \left(\frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1}} \left(\frac{v}{n} \right)^{\frac{\gamma_2}{\gamma_1}}. \quad (\text{B3})$$

Note that, if $\gamma_2 = 0$, as in [Pissarides \(2000\)](#), recruiting intensity is equal to a constant for all firms and it is independent of aggregate labor market conditions—both counterfactual implications. The following changes in parameters (*ceteris paribus*) result in a substitution away from vacancies and towards effort: $\uparrow \kappa_2, \downarrow \kappa_1, \uparrow \gamma_2$, and $\downarrow \gamma_1$. The effect of the cost shifter is

obvious. A higher curvature on the vacancy rate in the cost function ($\uparrow \gamma_2$) makes the marginal cost of creating vacancies rising faster than the marginal cost of recruiting effort; since the gain in terms of additional hires from a marginal unit of effort or vacancies is unaffected by γ_2 , it is optimal for the firm to use relatively more effort.

Now, substituting the law of motion for employment at the firm level into (B3), we obtain the optimal recruitment effort choice, expressed only as a function of the firm-level variables (n, n') :

$$e(n, n') = \left[\frac{\kappa_2}{\kappa_1} \left(\frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{-\frac{\gamma_2}{\gamma_1 + \gamma_2}} \left(\frac{n' - n}{n} \right)^{\frac{\gamma_2}{\gamma_1 + \gamma_2}} \quad (\text{B4})$$

which, in turn implies, for the job filling rate,

$$f(n, n') = q(\theta^*) e(n, n') = \left[\frac{\kappa_2}{\kappa_1} \left(\frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{\frac{\gamma_1}{\gamma_1 + \gamma_2}} \left(\frac{n' - n}{n} \right)^{\frac{\gamma_2}{\gamma_1 + \gamma_2}}. \quad (\text{B5})$$

This equation demonstrates that the model implies a log-linear relation between the job filling rate and employment growth at the firm level, with elasticity $\gamma_2 / (\gamma_1 + \gamma_2) < 1$ as in the data. Moreover, firm-level job filling rates are countercyclical, through their dependence on $q(\cdot)$.

Finally, using (B5) into the firm-level law of motion for employment yields an expression for the vacancy rate

$$\frac{v}{n} = \left[\frac{\kappa_2}{\kappa_1} \left(\frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{-\frac{\gamma_1}{\gamma_1 + \gamma_2}} \left(\frac{n' - n}{n} \right)^{\frac{\gamma_1}{\gamma_1 + \gamma_2}}. \quad (\text{B6})$$

Now, note that, by substituting the optimal choice for recruitment effort (B3) into (B1), we obtain the following formulation for the cost function

$$\mathcal{C}(n, v) = \left[\kappa_2 \frac{\gamma_1 + \gamma_2}{(\gamma_1 - 1)(\gamma_2 + 1)} \left(\frac{v}{n} \right)^{\gamma_2} \right] v, \quad (\text{B7})$$

which is one of the specifications that [Kaas and Kircher \(2011\)](#) invoke.

Finally, if we use (B6) into (B7), we obtain a version of the cost function only as a function

of (n, n') that we can use directly in the dynamic problem (16):

$$\mathcal{C}^*(n, n') = \kappa_2 \left[\frac{\gamma_1 + \gamma_2}{(\gamma_1 - 1)(\gamma_2 + 1)} \right] \left\{ \left[\frac{\kappa_2}{\kappa_1} \left(\frac{\gamma_1}{\gamma_1 - 1} \right) \right]^{\frac{1}{\gamma_1 + \gamma_2}} q(\theta^*)^{-\frac{\gamma_1}{\gamma_1 + \gamma_2}} \left(\frac{n' - n}{n} \right)^{\frac{\gamma_1}{\gamma_1 + \gamma_2}} \right\}^{1 + \gamma_2} n.$$

C Computation of the model