Advertised Prices in Decentralized Markets

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February 15, 2015

Abstract

A model of a decentralized market is developed that features search frictions, advertised prices and bargaining. Sellers can post ask prices to attract buyers through a process of directed search, but *ex post* there is the possibility of renegotiation. Similarly, buyers can advertise negotiable bid prices to attract sellers. Even though transaction prices often differ from quoted prices, advertised bid and ask prices play a crucial role in directing search and reducing trading frictions. The features and predictions of the model align well with aspects of the secondary market for transferable taxicab license plates in Toronto. This provides a useful and unique context for studying the relationships between advertised and actual prices in a decentralized market.

Journal of Economic Literature Classification: D40, G12, L10

Keywords: Bid and Ask Prices, Search Frictions, Price Commitment.

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1 Introduction

In this paper I develop a dynamic model of a decentralized market that features search frictions, advertised bid and ask prices, and transaction prices determined by bargaining. Buyers and sellers have the capacity to post advertisements to attract a trading partner through a process of directed search. The advertisement communicates commitment to a bid or ask price, but the counterparty that has not engaged in pre-match price publication maintains the ability to trigger *ex post* renegotiation. In a setting where there is *ex ante* uncertainty about a trader's relative strength at the bargaining table, bid and ask prices are chosen strategically as a means of directing search. The theory provides insight about how advertised prices and transaction prices are determined in equilibrium when there is limited commitment by only one party to an advertised price. The possibility of unfavorable outcomes in the bargaining procedure justifies the prevalence of posted prices even if advertised bid and ask prices tend to be different from transaction prices.

The model characterizes an asset market with trading frictions. The trading process has three features: pre-match communication, search frictions, and a strategic method of price determination. The model assumes that traders on either side of the market have the opportunity to communicate by posting a public advertisement. Many decentralized markets feature some form of public medium for advertising market participation and attracting trading partners by including a *bid price* (a buyer's offer-to-purchase price) or an *ask price* (a seller's quoted price). Accordingly, the communication stage of the trading process is modeled as publicly advertised bid and ask prices. Next, buyers and sellers meet stochastically according to a matching technology. This feature reflects the presence of search frictions; it is necessary to first find and contact a trading partner before a transaction can take place. The model assumes that matches occur between traders that advertise a price and those that instead decide to actively search. Search is *directed* in the sense that searching traders can observe all price announcements and target a particular advertisement in their search.

Transaction prices are determined by bargaining between a matched buyer and seller, where the advertised price is interpreted as the initial offer in an alternating offer bargaining game. The relative bargaining strength of the seller is modeled as a match-specific random variable, and therefore bargaining ensues in equilibrium whenever renegotiation favors the trader that is not constrained by a commitment to an advertised price. In such settings, buyers' and sellers' advertised prices can provide incentives for potential trading partners to direct their search towards them. If the seller's expected bargaining strength is sufficiently high, for example, an ask price effectively limits the seller's share of the surplus and can therefore be chosen strategically as a means of attracting a buyer. The strategic role of an ask price is therefore similar to that in Chen and Rosenthal (1996a,b), Arnold (1999), and Lester, Visschers, and Wolthoff (2014), where a seller sets an asking price to effect a price ceiling which encourages buyers to visit and incur the cost of inspecting the item for sale. The results here do not rely on *ex ante* heterogeneity or idiosyncratic valuations (observable or otherwise). Instead, an appropriately chosen ask price is appealing to buyers because it insures them against unfavorable outcomes in the bargaining game. An advertised bid price permits a buyer to implement an analogous technique for seducing sellers.

The model is a hybrid of random and directed search models. In the literature on random search, the terms of trade are typically determined *ex post* by means of a bargaining protocol (Diamond, 1982; Mortensen, 1982a,b; Pissarides, 1984, 1985; Mortensen and Pissarides, 1994). In contrast, the terms of trade are publicly posted in a competitive search model akin to those studied by Montgomery (1991), Peters

(1991), and Moen (1997), and market participants have the ability to commit not to renegotiate. In many markets, the actual transaction prices often differ from the advertised terms of trade because of *ex post* negotiation. The search model proposed in this paper can account for this and at the same time establishes a link between an advertised price and the trader's matching probability. In equilibrium, ask prices exceed the average transaction price, whereas bid prices lie below the average sale price. These predictions differ from directed search models that adopt a reserve price interpretation of the advertised price, as in Julien, Kennes, and King (2000); Albrecht, Gautier, and Vroman (2006, 2010), and the strategic renegotiation process in Camera and Selcuk (2009). A novel and appealing part of the theory relative to the existing directed search literature is that the set of traders advertising prices is not imposed exogenously. Rather, the decision to advertise a bid or ask price is determined in equilibrium and depends on the details of the bargaining procedure.

As an interesting application of the model, I consider the secondary market for transferable standard taxicab licenses (STLs) in Toronto. These are license plates that can not only be used by an owner-driver to operate a taxicab vehicle in Toronto, but can also be leased, rented to shift drivers or transferred to a new owner by means of a transaction in the secondary market. This market provides a unique context that is particularly appropriate for examining the importance of search frictions and the role of advertised prices because the assets being exchanged (*i.e.*, the standard taxicab licenses) are homogeneous, but are nevertheless traded in a decentralized manner; buyers and sellers publish prices and search for each other by means of an online platform for classified advertisements. The advertised prices and STL transaction data support the theoretical implications insofar as advertised ask prices are typically higher than transaction prices, while advertised bid prices are lower, which suggests that final prices are determined by an *ex post* bargaining procedure between a buyer and a seller.

The remainder of the paper is organized as follows. Section 2 describes the model environment. The equilibrium strategies for bargaining and price posting are characterized in Section 3. Section 3 also defines a steady state directed search equilibrium with advertised prices. Section 4 provides an overview of the secondary market for STLs and describes the recent transaction data. Section 5 presents the advertised price data and discusses the role of bid and ask prices in directing search and the incidence of renegotiation. Section 6 concludes.

2 The Environment

Agents, endowments and preferences. Time is discrete and denoted by $t = 0, 1, 2, \ldots$. There is a large number of infinitely-lived agents. Some of them are initially endowed with one indivisible asset. Let A denote the number of agents endowed with the asset, or equivalently, the total number of assets in the economy. For simplicity, assume that each agent can hold at most one unit of the asset.[1] The asset yields a dividend d each period.

Agents are risk-neutral with common discount factor $\beta \in (0, 1)$. An agent that owns an asset is subject to a random preference shock that can reduce his valuation of the dividend from d to $(1 - x)d$, with $x > 0$. Conditional on holding an asset, the preference shock arrives with probability δ each period. Once an agent experiences this shock, his valuation of the dividends from his particular asset will remain low forever. This captures the idea that some agents that own an asset might develop a need for selling it, which generates churning in the market.

[1]This simplifies the bargaining problem and avoids having to solve for a more complicated distribution of asset holdings in equilibrium.

Trading process. Ownership of the asset can be transferred in a decentralized market subject to search frictions described below. Once the asset is sold, the seller exits the market with the revenue from the sale. Traders therefore transition to a different trading status depending on their asset holdings and their valuation of the asset. There are three different stages that occur sequentially: (i) buyers (n) do not own an asset; (ii) owners (m) have the asset and value its stream of dividends; and (iii) sellers (s) have the asset, but have experienced the preference shock and no longer fully value its dividends. Denote the measure of traders of different types at time t as n_t, m_t, and s_t.

Let $\{V_t^n, V_t^m, V_t^s\}$ denote the value functions associated with buying, owning, and selling the asset. These values represent the solution to the system of Bellman equations derived below. The net surplus from a transaction at time t at price P is therefore $V_t^m - V_t^n - P$ for a buyer, and $P - V_t^s$ for a seller.

Search and matching. The meeting process between buyers and sellers is subject to frictions, but search can be *directed* by advertised prices in the following sense: buyers have the opportunity to advertise a bid price, whereas sellers can post an ask price. Traders that do not post prices then observe all advertised prices and search for a trading partner with a particular bid or ask price. Let p denote the advertised price, and let $s_t^a(p)$ and $n_t^a(p)$ (or $n_t^b(p)$ and $s_t^b(p)$) indicate the numbers of traders posting and searching for ask price (or bid price) p. The superscript indicates whether the advertised price is a seller's ask price, or a bid price quoted by a buyer. In what follows, the superscripts a and b are suppressed whenever an expression applies to both cases.

Buyers and sellers are matched according to a bilateral meeting technology given by a matching function, $\mathcal{M}(n_t(p), s_t(p))$. The function $\mathcal{M}$ exhibits constant returns to scale, is increasing and strictly concave in both arguments, and is continuously differentiable. It must also be the case that the number of matches is less than the number of traders on the short side of the market, $\mathcal{M}(n_t(p), s_t(p)) \leq \min\{n_t(p), s_t(p)\}$. It is useful to define $\theta_t(p) \equiv n_t(p)/s_t(p)$, which is referred to as market *tightness*. Given the properties of $\mathcal{M}$, the probability that a buyer will meet a seller in period t may be written as a function of $\theta_t(p)$:

$$\lambda(\theta_t(p)) = \frac{\mathcal{M}(n_t(p), s_t(p))}{n_t(p)} = \mathcal{M}(1, 1/\theta_t(p)). \tag{1}$$

Let $\gamma(\theta_t(p))$ denote the analogous matching probability from the perspective of a seller:

$$\gamma(\theta_t(p)) = \frac{\mathcal{M}(n_t(p), s_t(p))}{s_t(p)} = \mathcal{M}(\theta_t(p), 1) = \theta_t(p)\lambda(\theta_t(p)). \tag{2}$$

Traders direct their search by posting or targeting a particular p knowing the matching probabilities, given the matching function and their beliefs about the price posting strategies and search behavior of other traders.

Price determination We assume that buyers and sellers have the capacity to commit to honoring their quoted price. A price poster's counterparty, on the other hand, has not publicly announced any such commitment and therefore reserves the right to renegotiate. Absent agreement to transact at the advertised price, the transaction price of the asset is determined by means of an alternating offer bargaining game. Following Binmore, Rubinstein, and Wolinsky (1986), the buyer and seller bargain strategically when there is a perceived risk that the bargaining process will terminate in disagreement between offers. The trader that has rejected the posted price is the first to make a counteroffer.

For example, suppose the seller has advertised an ask price of p, which the buyer has opted not to accept. In the first round of the renegotiation game, the buyer offers p_1 which the seller can either accept or reject. If the offer is accepted, bargaining game ends and ownership of the asset is transferred to the buyer in exchange for payment p_1. If p_1 is rejected, negotiations breakdown with probability $1 - \exp(-(1-\phi)\Delta)$ and both parties continue searching for a trading partner the following period. If bargaining continues, the seller proposes a price p_2 to be accepted or rejected by the buyer. If rejected, negotiations can again be terminated, this time with probability $1 - \exp(-\phi\Delta)$. The bargaining game continues indefinitely until either negotiations are aborted or a mutually agreeable offer is proposed and accepted. Figure 1 displays the timing of the alternating offer bargaining game. Figure 2 displays the renegotiation process when a seller approaches a buyer that has advertised a bid price.

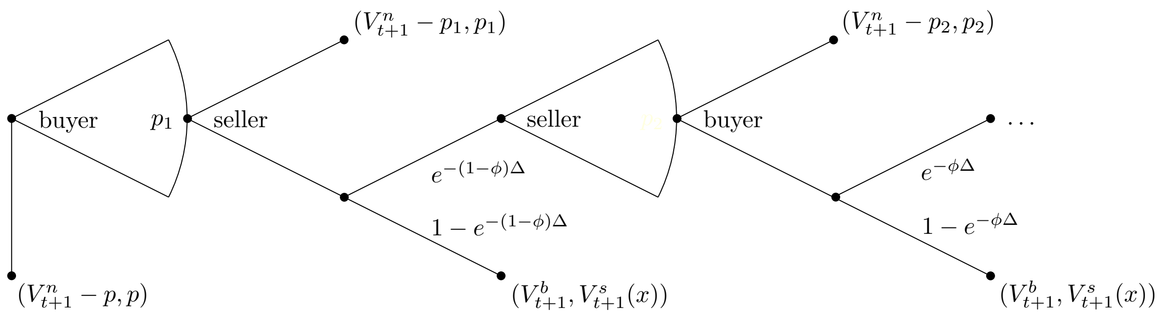

Figure 1: Price determination when the seller is approached by a buyer.

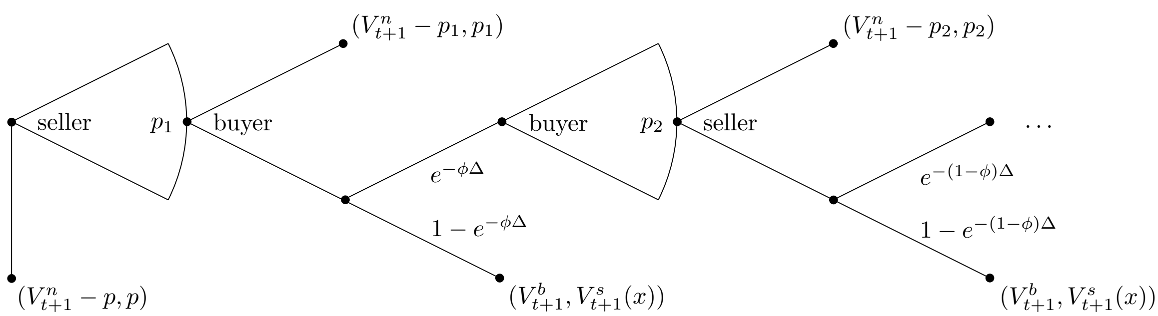

Figure 2: Price determination when the buyer is approached by a seller.

Consider the limiting equilibrium outcome by letting the length of the bargaining rounds, $\phi\Delta$ and $(1-\phi)\Delta$, decrease to zero for a constant $\phi \in [0, 1]$, as in Binmore

(1980) and Binmore, Rubinstein, and Wolinsky (1986). This allows the dynamic strategic model of bargaining to be collapsed into a single time period of the dynamic search model. Here, the perceived risk of breakdown can be thought of as a proxy for commitment; the higher (lower) is ϕ, the higher (lower) is the seller's (buyer's) commitment to his offer as the *final* offer, even if rejected. By taking the limit as $\Delta \to 0$, what is important is the bargainer's relative power of commitment, even though neither player can terminate the bargaining process in disagreement with positive probability. This follows Schelling's (1956) interpretation of strategic bargaining and the model based on commitment in Section 8.7 of Myerson (1991).

When a buyer and a seller match, there might be uncertainty about the bargaining opponent's capacity to commit to offers. To capture this, let ϕ be a match specific random variable drawn from a distribution G with support $[0, 1]$. This uncertainty establishes a potential role for advertised prices as a commitment to an initial offer in the bargaining game. When a seller advertises a price, for example, the buyer can accept the ask price, but retains the right to renegotiate depending on the realization of ϕ. A lower ask price is appealing to buyers because it could be lower than the price that would otherwise result from the bargaining procedure. In such cases, the buyer could simply accept the seller's ask price and avoid bargaining altogether. It might therefore be possible for a seller to set an ask price strategically to attract buyers. Similarly, advertising a high bid price might be an effective strategy for a buyer to draw the attention of a seller by providing the opportunity to capture a minimum share of the match surplus.

Free entry. Participation in the market requires forgoing other potential investment opportunities. An exogenous per period opportunity cost is denoted $c \in (0, d)$. It is worthwhile to enter the market as a buyer at time t as long as the expected net surplus from a possible purchase is positive.

3 Equilibrium

Bargaining outcome. The following lemma combines Propositions 3 and 5 from Binmore, Rubinstein, and Wolinsky (1986). It says that in the absence of agreement to transact at the advertised price, the unique subgame perfect equilibrium outcome of the bargaining game converges to the asymmetric generalization of the Nash solution. The net surplus captured by the seller is a fixed share of the total surplus from a transaction, where the share is determined by the parameter ϕ for the seller's relative bargaining strength.

Lemma 1. *If the buyer rejects the advertised ask price and engages the seller in the bargaining game, there exists a unique subgame perfect equilibrium. The existence and uniqueness results similarly hold if the seller rejects the advertised bid price and engages the buyer in the bargaining game. In the limit as $\Delta \to 0$, the equilibrium outcome in both cases is*

$$\hat{p}_t = (1-\phi)V^s_{t+1} + \phi\left[V^m_{t+1} - V^n_{t+1}\right]. \tag{3}$$

Optimal renegotiation. Upon matching with a seller advertising an ask price of p, the buyer's optimal strategy is to bargain if $\hat{p}_t < p$ and to accept p otherwise. When a seller targets a buyer with advertised bid price p, the seller elects to renegotiate in order to receive $\max\{p, \hat{p}_t\}$. These optimal strategies are stated in the following lemma using the solution to the bargaining problem in (3).

Lemma 2. *An ask price of p is accepted by the buyer whenever*

$$\phi > \frac{p - V^s_{t+1}}{V^m_{t+1} - V^n_{t+1} - V^s_{t+1}} \equiv \Phi_t(p), \tag{4}$$

whereas a bid price of p is accepted whenever $\phi < \Phi_t(p)$.

The expected transaction price in a match between a buyer and a seller posting ask price p is therefore

$$\begin{aligned} \mathrm{E}^a[P_t|p] = {} & [1 - G(\Phi_t(p))]\, p + G(\Phi_t(p)) V^s_{t+1} \\ & + \left[V^m_{t+1} - V^n_{t+1} - V^s_{t+1}\right] \int_0^{\Phi_t(p)} \phi dG(\phi) \end{aligned} \tag{5}$$

When a seller targets a buyer with advertised bid price p, the expected price conditional on a match is

$$\begin{aligned} \mathrm{E}^b[P_t|p] = {} & G(\Phi_t(p))p + [1 - G(\Phi_t(p))]\, V^s_{t+1} \\ & + \left[V^m_{t+1} - V^n_{t+1} - V^s_{t+1}\right] \int_{\Phi_t(p)}^1 \phi dG(\phi) \end{aligned} \tag{6}$$

Search and advertised prices. To study the search decisions and price posting problems of buyers and sellers, it is necessary to first derive the Bellman equations for V^m_t, V^n_t and V^s_t. The present discounted value of owning the asset at time t, V^m_t, satisfies

$$V^m_t = d - c + \beta\left[(1-\delta)V^m_{t+1} + \delta V^s_{t+1}\right]. \tag{7}$$

Equation (9) states that the value of owning the asset is equal to the current dividend, π, less the opportunity cost, c, plus the expected present discounted value next period; this will either be the value of maintaining ownership, V^m_{t+1}, which occurs with probability $(1-\delta)$, or the value after the preference shock, which occurs with probability δ. In this case, the trader captures the present discounted value of selling the asset, V^s_{t+1}. The values associated with trying to sell or buy an asset at time t,

V_t^s and V_t^n, satisfy

$$V_t^s = (1-x)d - c + \beta \max_{i\in\{a,b\},p} \left\{\gamma(\theta_t^i(p))\mathrm{E}^i[P_t|p] + (1-\gamma(\theta_t^i(p)))V_{t+1}^s\right\} \tag{8}$$

$$V_t^n = -c + \beta \max_{i\in\{a,b\},p} \left\{\lambda(\theta_t^i(p))\left(V_{t+1}^m - \mathrm{E}^i[P_t|p]\right) + (1-\lambda(\theta_t^i(p)))V_{t+1}^n\right\}, \tag{9}$$

where maximizing with respect to $i \in \{a, b\}$ reflects the optimal choice between price posting and actively searching. Maximizing with respect to p in (8), for example, reflects the optimal choice of ask price for a seller if $i = a$, and the optimal search decision among buyers with advertised bid prices if $i = b$. In (7) it reflects the optimal choice of bid price or search decision of a buyer. When there is a match between a buyer and seller, the actual transaction price depends on the subsequent bargaining game given the advertised price. The expected payments, denoted $\mathrm{E}^a[P_t|p]$ and $\mathrm{E}^b[P_t|p]$, take into account the equilibrium outcome of the bargaining game as well as the optimal renegotiation strategies of traders as per equations (5) and (6). In terms of timing, notice that when a buyer and seller match in period t and agree to the terms of a transaction, the buyer becomes the owner of the asset and pays the seller in period $t+1$.

Definition 1. *Given $V_{t+1}^m - V_{t+1}^n - V_{t+1}^s > 0$, a directed search equilibrium at time t with advertised prices is a set of ask prices $\mathbb{P}_t^a \subset \mathbb{R}_+$, a set of bid prices $\mathbb{P}_t^b \subset \mathbb{R}_+$, functions for market tightness $\theta_t^i : \mathbb{R}_+ \to \mathbb{R}_+ \cup \infty$ for $i \in \{a, q\}$, and a pair of values $\bar{V}_t^n, \bar{V}_t^s \in \mathbb{R}_+$ satisfying*

1. *optimal use of advertised prices: define*

$$\bar{V}_t^s(a) = (1-x)d - c + \beta \max_{p\in\mathbb{P}_t^a} \left\{\gamma(\theta_t^a(p))\mathrm{E}^a[P_t|p] + (1-\gamma(\theta_t^a(p)))V_{t+1}^s\right\}$$

$$\bar{V}_t^n(a) = -c + \beta \max_{p\in\mathbb{P}_t^a} \left\{\lambda(\theta_t^a(p))\left(V_{t+1}^m - \mathrm{E}^a[P_t|p]\right) + (1-\lambda(\theta_t^a(p)))V_{t+1}^n\right\},$$

or $\bar{V}_t^s(a) = (1-x)d - c + \beta V_{t+1}^s$ *and* $\bar{V}_t^n(a) = -c + \beta \bar{V}_{t+1}^n$ *if* $\mathbb{P}_t^a$ *is empty, and*

$$\bar{V}_t^s(b) = (1-x)d - c + \beta \max_{p \in \mathbb{P}_t^b} \left\{ \gamma(\theta_t^b(p)) \mathrm{E}^b[P_t|p] + (1-\gamma(\theta_t^b(p)))V_{t+1}^s \right\}$$
$$\bar{V}_t^n(b) = -c + \beta \max_{p \in \mathbb{P}_t^b} \left\{ \lambda(\theta_t^b(p)) \left(V_{t+1}^m - \mathrm{E}^b[P_t|p] \right) + (1-\lambda(\theta_t^b(p)))V_{t+1}^n \right\},$$

or $\bar{V}_t^n(b) = -c + \beta \bar{V}_{t+1}^n$ *and* $\bar{V}_t^s(b) = (1-x)d - c + \beta V_{t+1}^s$ *if* $\mathbb{P}_t^b$ *is empty. Then*

$$\bar{V}_t^s = \max \left\{ \bar{V}_t^s(a), \bar{V}_t^s(b) \right\}$$
$$\bar{V}_t^n = \max \left\{ \bar{V}_t^n(a), \bar{V}_t^n(b) \right\}.$$

2. *optimal price posting: for* $p \in \mathbb{R}_+$,

$$\bar{V}_t^s \geq (1-x)d - c + \beta \left[\gamma(\theta_t^a(p)) \mathrm{E}^a[P_t|p] + (1-\gamma(\theta_t^a(p)))V_{t+1}^s \right],$$

with equality if $p \in \mathbb{P}_t^a$, *and*

$$\bar{V}_t^n \geq -c + \beta \left[\lambda(\theta_t^b(p)) \left(V_{t+1}^m - \mathrm{E}^b[P_t|p] \right) + (1-\lambda(\theta_t^b(p)))V_{t+1}^n \right],$$

with equality if $p \in \mathbb{P}^b$.

3. *optimal search: for* $p \in \mathbb{R}_+$,

$$\bar{V}_t^s \geq (1-x)d - c + \beta \left[\gamma(\theta_t^b(p)) \mathrm{E}^b[P_t|p] + (1-\gamma(\theta_t^b(p)))V_{t+1}^s \right],$$

with equality if $\theta_t^b(p) < \infty$, *and*

$$\bar{V}_t^n \geq -c + \beta \left[\lambda(\theta_t^a(p)) \left(V_{t+1}^m - \mathrm{E}^a[P_t|p] \right) + (1-\lambda(\theta_t^a(p)))V_{t+1}^n \right],$$

with equality if $\theta_t^a(p) > 0$.

Optimal search ensures that active buyers and sellers contact a potential trading partner to maximize their value from searching. For prices not advertised in equilibrium, part 3 of Definition 1 guarantees that beliefs about tightness are consistent with a notion of subgame perfection. Market tightness θ_t^a (θ_t^b) adjusts so that buyers (sellers) achieve $\bar{V}_t^n$ ($\bar{V}_t^s$) for any ask price (bid price), including prices not advertised in equilibrium.[2] Optimal price posting ensures that buyers and sellers advertise prices

[2]If p is such that a buyer (seller) cannot achieve $\bar{V}_t^n$ ($\bar{V}_t^s$) for any finite (positive) buyer-seller

to maximize their value from directing search, given the equilibrium relationships between market tightness and posted prices. The optimal use of advertised prices is a novel part of the definition of a directed search equilibrium. It states that buyers and sellers engage in price posting if and only if it is worthwhile to do so.

In order to attract a trading partner, the ask or bid price must deliver at least the same value that could be achieved by advertising or searching elsewhere. An optimal ask price at time t must therefore solve the price posting problem in the seller's Bellman equation subject to the constraint that buyers searching for a seller advertising that exact ask price achieve their equilibrium value $\bar{V}_t^n$. Similarly, an optimal bid price must solve the buyer's price posting problem subject to sellers receiving their equilibrium value $\bar{V}_t^s$. These optimization problems and their usefulness in characterizing a directed search equilibrium are presented in Proposition 1.

Proposition 1. *If* $\{\mathbb{P}_t^a, \mathbb{P}_t^b, \theta_t^a, \theta_t^b, \bar{V}_t^n, \bar{V}_t^s\}$ *is an equilibrium, then*

1. *any* $p \in \mathbb{P}_t^a$ *and* $\theta = \theta_t^a(p)$ *solve*

$$\max_{p,\theta} \gamma(\theta)\mathrm{E}^a[P_t|p] + (1-\gamma(\theta))V_{t+1}^s \tag{10}$$

subject to

$$\bar{V}_t^n = -c + \beta\left[\lambda(\theta)\left(V_{t+1}^m - \mathrm{E}^a[P_t|p]\right) + (1-\lambda(\theta))V_{t+1}^n\right], \tag{11}$$

2. *and any* $p \in \mathbb{P}_t^b$ *and* $\theta = \theta_t^b(p)$ *solve*

$$\max_{p,\theta} \lambda(\theta)\left(V_{t+1}^m - \mathrm{E}^b[P_t|p]\right) + (1-\lambda(\theta))V_{t+1}^n \tag{12}$$

subject to

$$\bar{V}_t^s = (1-x)d - c + \beta\left[\gamma(\theta)\mathrm{E}^b[P_t|p] + (1-\gamma(\theta))V_{t+1}^s\right]. \tag{13}$$

ratio, then $\theta_t^a = \infty$ ($\theta_t^b = 0$).

Conversely, if some $\{p^a, \theta^a\}$ and $\{p^b, \theta^b\}$ solve problems 1 and 2, then there exists an equilibrium $\{\mathbb{P}_t^a, \mathbb{P}_t^b, \theta_t^a, \theta_t^b, \bar{V}_t^n, \bar{V}_t^s\}$ such that either $p^a \in \mathbb{P}_t^a$ and $\theta^a = \theta_t^a(p^a)$, or $p^b \in \mathbb{P}_t^b$ and $\theta^b = \theta_t^b(p^b)$.

The next proposition characterizes the expected transaction price conditional on a match between a buyer and a seller. This expected price takes into account the equilibrium price posting strategies of buyers and sellers as per Proposition 1.

Proposition 2. *If $\{\mathbb{P}_t^a, \mathbb{P}_t^b, \theta_t^a, \theta_t^b, \bar{V}_t^n, \bar{V}_t^s\}$ is an equilibrium, then the expected transaction price for any buyer-seller match satisfies*

$$\mathrm{E}[P_t] = \big(1 - \eta(\theta)\big)\left[V_{t+1}^m - V_{t+1}^n\right] + \eta(\theta) V_{t+1}^s, \tag{14}$$

where $\eta(\theta) = \theta\gamma'(\theta)/\gamma(\theta)$.

Condition (14) states that the expected transaction price satisfies the Hosios (1990) condition. Although it does not reveal the advertised price or identify the side of the market engaged in price posting, it does imply that any price quoted in equilibrium implements the same division of the total surplus (in expectation). If sellers advertise prices in equilibrium, the equilibrium ask price is determined by equating (14) and (5). The former equation ensures optimal price posting and search, while the latter takes into account the distribution G for the bargaining parameter ϕ, the bargaining solution in Lemma 1, and the buyer's optimal decision about whether to negotiate or accept the advertised ask price according to Lemma 2. Similarly, equating (14) and (6) characterizes the unique equilibrium bid price at time t whenever bid prices are advertised in equilibrium.

For a given distribution G, the side of the market posting prices is determined endogenously according to part 1 of Definition 1. Because search is directed, a change in the distribution G will not affect the *average* transaction price; the strategic choice of bid and ask prices among buyers and sellers and the equilibrium decisions by their counterparties about when to renegotiate appropriately restrict the set of possible transaction prices such that the optimal trade-off is achieved between asset liquidity and the expected private gains from a transaction. The following proposition establishes uniqueness of the equilibrium posted price, given that traders are *ex ante* identical and assets are homogeneous.

Proposition 3. *There are no incentives for buyers or sellers to advertise prices if the distribution G from which the bargaining parameter, ϕ, is drawn satisfies*

$$\int_0^1 \phi dG(\phi) = 1 - \eta(\theta_t^a(V_{t+1}^m - V_{t+1}^n)) = 1 - \eta(\theta_t^b(V_{t+1}^s)). \tag{15}$$

Otherwise, there is a unique advertised price in a directed search equilibrium. It is an ask price if and only if G satisfies

$$\int_0^1 \phi dG(\phi) > 1 - \eta(\theta_t^a(V_{t+1}^m - V_{t+1}^n)) = 1 - \eta(\theta_t^b(V_{t+1}^s)). \tag{16}$$

If neither (15) nor (16) hold, sellers capture little of the expected surplus in a transaction. Consequently, they are unwilling to reduce their expected gains further by committing to an ask price for the purposes of directing buyers' search. Instead, bid prices are advertised by the demand side of the market and sellers search for buyers.

Evolution of stocks. The number of sellers evolves according to

$$s_{t+1} = s_t - \gamma(\theta_t)s_t + \delta m_t \tag{17}$$

In light of the uniqueness result in Proposition 3, the a or b superscripts and the argument p have been suppressed for notational convenience. Since every owner and every seller owns exactly one asset, the total number of sellers and owners must equal the total number of assets in the economy:

$$m_t + s_t = A. \tag{18}$$

Finally, market tightness in every period satisfies the free entry condition:

$$V_t^n = -c + \beta \left[\lambda(\theta) \left(V_{t+1}^m - \mathrm{E}[P_t]\right) + (1 - \lambda(\theta))V_{t+1}^n\right] = 0. \tag{19}$$

where $\mathrm{E}[P_t]$ is the expected price conditional on equilibrium price posting as per Proposition 2.

3.1 Steady State Equilibrium

This economy has a steady state equilibrium in which buyers and sellers behave optimally given consistent beliefs about matching probabilities conditional on advertised prices. In the steady state, all values and stock variables that are determined endogenously within the model are constant over time. The steady state measures of buyers, sellers and owners satisfy the stationary versions of (17) and (18), and the definition of market tightness:

$$\mathcal{M}(b, s) = \delta m. \tag{20}$$

$$s = A - m \tag{21}$$

$$n = s\theta. \tag{22}$$

The steady state equilibrium versions of (7), (8), and (9) are

$$(1-\beta)V^m = d - c - \beta\delta\left[V^m - V^s\right] \tag{23}$$

$$(1-\beta)V^s = (1-x)d - c + \beta\gamma(\theta)\left[1-\eta(\theta)\right]\left[V^m - V^n - V^s\right] \tag{24}$$

$$(1-\beta)V^n = -c + \beta\lambda(\theta)\eta(\theta)\left[V^m - V^n - V^s\right]. \tag{25}$$

Solving for V^n and equating it to zero yields the free entry condition:

$$\beta\lambda(\theta)\eta(\theta)xd = \left[1 - \beta\left(1 - \delta - \gamma(\theta)(1-\eta(\theta))\right)\right]c. \tag{26}$$

The advertised price p satisfies

$$\left[1 - G(\Phi(p))\right]\Phi(p) + \int_0^{\Phi(p)} \phi dG(\phi)) = 1 - \eta(\theta), \tag{27}$$

if p is an ask price quoted by sellers, or

$$G(\Phi(p))\Phi(p) + \int_{\Phi(p)}^{1} \phi dG(\phi)) = 1 - \eta(\theta), \tag{28}$$

if p is a bid price advertised by buyers. Equations (27) and (28) combine the steady state version of (14) with steady state versions of (5) and (6). They therefore reflect the equilibrium price posting behaviour of sellers and buyers that correctly anticipate the optimal renegotiation strategies of their counterparties. The steady state equilibrium renegotiation rule is

$$\Phi(p) = \frac{p - V^s}{V^m - V^s}. \tag{29}$$

Definition 2. *The steady state directed search equilibrium with advertised prices for this market is a set of values* $\{V^n, V^m, V^s\}$*; an advertised price,* p*; a renegotiation* *rule, $\Phi(p)$; market tightness, θ; and a distribution of traders across states $\{m, s, n\}$ such that:*

(i) the values $\{V^m, V^s, V^n\}$ satisfy equations (23)*,* (24) *and* (25)*;*

(ii) searching traders follow an optimal renegotiation strategy: $\Phi(p)$ satisfies (29)*;*

(iii) traders strategically advertise prices: p satisfies either (27) *or* (28)*, subject to $\Phi(p) \in [0, 1]$;*

(iv) market tightness is the result of free entry of buyers: market tightness, θ, satisfies (26)*; and*

(v) the distribution of traders across states is stationary: $\{m, s, n\}$ satisfy (20)*,* (21)*, and* (22)*.*

4 Toronto Standard Taxicab Licenses

Toronto's taxicab licensing system includes, among other classes of licenses, 3,451 *standard taxicab licenses* (STLs). This license type can be used for owner-operated taxicabs, leased to a licensed taxicab driver, rented to shift drivers either directly or through intermediaries, or transferred to a new owner by means of a transaction in a decentralized secondary market. Since a transfer of ownership must be approved and comply with the guidelines of the Municipal Code, every transaction is recorded by the city. This section presents the data related to STL transactions, focusing on recent transfers between March and August 2014.

4.1 STL Transaction Data

Figure 3 plots the price in current CAD for each STL transfer from March to August 2014. These "transactions" include transfers to family members for prices close to zero,[3] as well as many within-family transactions that have prices further from zero

[3]The City of Toronto's 2012 review of the taxicab industry states that "standard taxicabs cannot be inherited; however, they are often sold to family members for a token amount of 1 dollar."

but potentially less than market value. In order to focus on market transactions between buyers and sellers that must first search for each other in a decentralized market, STL transfers for amounts less than or equal to 2 are hereinafter excluded from the sample.[4] Transactions between a buyer and a seller that share a common surname are also removed from the sample. The shaded dots in Figure 3 identify STL transfers that satisfy one or both of these sample exclusion criteria. The solid line in Figure 3 tracks the monthly average price for the restricted sample of market transactions. Table 1 contains descriptive statistics of the STL transaction data.

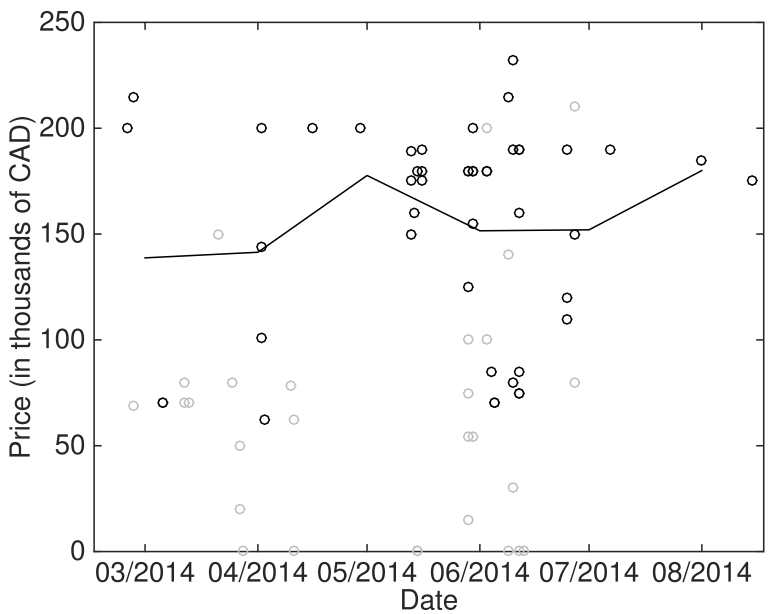

Figure 3: STL Transaction Prices and Monthly Averages

[4]In the full sample of 78 transactions, there are 2 transactions with a price of 1 and 6 transactions with a price of 2. The minimum price among the remaining transactions is substantially higher at 15,000.

Table 1: Summary Statistics of the STL Transaction Data

	obs.	mean	st. error
transaction price	49	155,469	6,936
number of transactions per month	6	8.1667	3.3007

Notes: This table displays summary statistics of the restricted sample of STL transactions from March to August 2014. Prices are in current CAD.

4.2 STL New Issues Data

It would be interesting to study time on the market for each buyer and seller. Unfortunately, the STL transaction data do not reveal the length of the search process for either party. Nevertheless, investigation of the database of Business Licence Renewals and New Issues maintained by Municipal Licensing & Standards provides some clues about the severity of search frictions in the secondary market for STLs. In particular, the time elapsed between an estate transfer and a subsequent market transaction is recorded in this database. As described in the 2014 report of the taxicab industry prepared by the City of Toronto, the STL is first transferred to the estate of a deceased license holder, but must then be sold to a licensed taxicab driver. The time between these changes in ownership likely reflects (i) the time required to sort out the deceased license holder's estate, (ii) an administrative delay, and (iii) the difficulty in finding a buyer. All three sources of delay are potentially relevant when the STL is sold in the decentralized market, whereas only the first two are important when the STL is transferred to a family member. Comparing the time between transfers for these two groups should provide insight about time on the market for STL sellers.

Figure 4 plots the empirical distribution functions for the time elapsed between two consecutive changes in STL ownership when the first transfer follows the death of a license holder. Data for Figure 4 include all estate sales recorded between August

2013 and August 2014. The sample is divided into two groups according to market transactions and family transfers.[5] As noted above, there is reason to suspect that longer estate ownership duration for market transactions can be attributed to the time required to advertise or sort through classified ads, contact a potential trading partner, and negotiate the terms of trade. A procedure for deriving an estimate of the average time on the market for sellers from estate ownership duration data is detailed in Appendix A. The results imply that the average time required to find a buyer in the decentralized market is approximately 4 months.

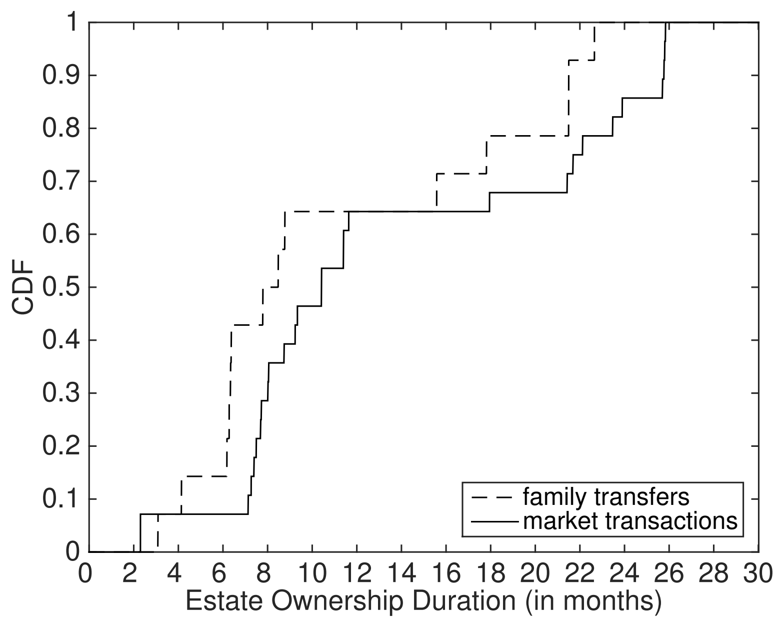

Figure 4: Distribution of Time until New Ownership following Death of STL Owner

[5]A *family transfer* satisfies at least one of the following two conditions: (i) the recorded price for the transaction is less than or equal to 2, or (ii) the new owner has the same surname as the deceased.

5 Advertised Prices

Both sellers and buyers post classified advertisements on `Kijiji.ca`, an online classified service owned by eBay, to convey market participation and attract potential trading partners on the opposite side of the market. Online classified advertisement data were collected from March until August 2014. Each ad is one of two types: either an "I am offering" ad for those offering an item for sale, or an "I want" ad for those on the demand side of the market. Many advertisers on both sides of the market publish a price as part of the ad, although some advertisers omit the price and instead select the option to display "Please Contact" after the "Price" heading. The ad also contains the date listed, a title, a message, and sometimes information about the seller including address and phone number.

Between March and August 2014, messages posted to `Kijiji.ca` include 73 seller ads and 138 buyer ads. A few of these ads omit prices, but 60 of the 73 ads posted by sellers have ask prices ranging from 95,000 to 500,000, and 82 of the 138 want ads contain bid prices between 40,000 and 235,000. Table 2 contains these summary statistics.

Table 2: Summary Statistics of the STL Classified Advertisement Data

	obs.	mean	st. error	min	max
number of seller ads per month	6	12.1667	1.9221	7	20
advertised ask price	60	257,790	11,245	95,000	500,000
number of buyer ads per month	6	23.0000	4.4347	9	38
advertised bid price	82	119,892	5,385	40,000	235,000

Notes: This table displays summary statistics of the STL classified advertisement data from March to August 2014.

For buyers and sellers that advertise their participation in the secondary market for STLs, a link between each classified ad and the corresponding transaction recorded by the city is unfortunately not possible in most cases. Relating the data to the theoretical predictions is futher complicated by the fact that the the decision to advertise a price and the optimal ask or bid price should depend on the details of the bargaining procedure (*i.e.*, the particular distribution G from which the bargaining parameter, ϕ, is drawn) according to Proposition 3. Even without knowing the precise details of the renegotiation process or trader-specific time on the market, the distributions of advertised and transaction prices may nonetheless provide insight about the role that advertised prices play in directing search and their informativeness about market value.

An equilibrium implication of the model is that ask prices should be above the average transaction price, whereas bid prices should be below. The advertised prices in the secondary market for STLs feature 55 out of 60 (over 90 percent) advertised ask prices and 62 out of 82 advertised bid prices (over 75 percent) satisfying these requirements. The classified ads and transaction data therefore provide some qualitative evidence that advertised prices are used to attract potential trading partners despite being subject to renegotiation. Figure 5 displays the distributions of bid prices, ask prices, and actual transaction prices between March and August, 2014.

It is worth remarking that these relationships between advertised and actual price distributions differ from those predicted by existing directed search models that adopt a reserve price interpretation of a posted price. For example, one would expect prices advertised by sellers to be lower than transaction prices when sellers are capacity constrained and there is *ex post* competition among buyers (*e.g.*, Julien, Kennes, and King (2000); Albrecht, Gautier, and Vroman (2006, 2010), and the strategic renegotiation process in Camera and Selcuk (2009)). Similarly, a directed search model that features pre-match price posting by buyers and post-match price quoting by competing sellers would imply transaction prices in equilibrium below those advertised by the demand side of the market (*e.g.*, Julien, Kennes, and King (2000); **?**); **?**); **?**. These comparisons with the existing directed search literature highlight another novel and appealing feature of the model presented in Section 2; namely, the set of market participants advertising prices is determined in equilibrium rather than assumed exogenously.

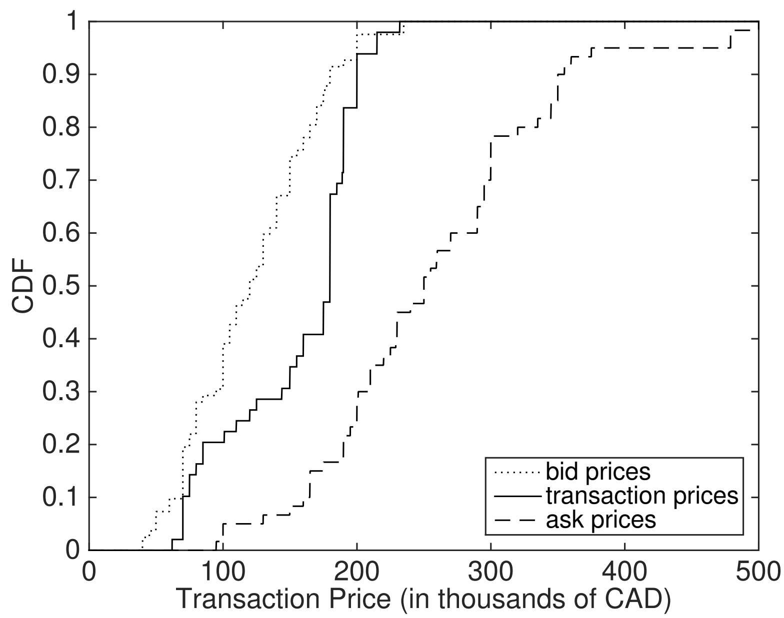

Figure 5: Distributions of Bid Prices, Ask Prices and Transaction Prices

6 Conclusion

In this paper I develop a theoretical model of a decentralized asset market with trading frictions. The trading process has three important features: pre-match communication, search frictions, and a strategic method of price determination. Traders on both sides of the market have the opportunity to communicate by posting a public advertisement with a bid or ask price. Traders that do not advertise a price instead observe all price announcements and search for a trading partner by targeting a particular advertisement. Transaction prices are then determined by bargaining between a matched buyer and seller, where the advertised price is interpreted as the initial offer in an alternating offer bargaining game. The advertisement therefore conveys commitment to a bid or ask price, but the counterparty that has not engaged in pre-match price publication maintains the ability to trigger *ex post* renegotiation. In a setting where there is *ex ante* uncertainty about a trader's relative strength in negotiations, the advertised price provides incentive for potential trading partners to direct their search. The possibility of unfavourable outcomes in the bargaining procedure therefore supports the commonplace strategy of posting a bid or ask price even when advertised prices tend to be different from transaction prices.

The decentralized secondary market for STLs exhibits many of the features modeled in Section 2; there is evidence of search frictions, pre-match price announcements and *ex post* bargaining, as discussed in Sections 4 and 5. Moreover, the assets being traded are essentially identical, which rules out alternative interpretations of advertised prices that rely on idiosyncratic values and costly inspection.[6] This application is therefore well-suited for examining the severity of search frictions and the link between advertised prices and transaction prices in a decentralized asset market with very limited heterogeneity in the asset. One might expect the role of advertised prices in directing search and reducing trading frictions to be even more essential in a market for idiosyncratic assets (for example, housing markets) in which buyers are likely to visit multiple sellers before a purchase eventuates.

[6]For example, Chen and Rosenthal (1996a,b) and Arnold (1999) propose that asking prices provide incentives for buyers to incur an inspection cost to learn their idiosyncratic valuation of the item for sale. The asking price, which represents commitment to a price ceiling, guarantees additional surplus to the buyer whenever the inspection reveals to both parties the buyer's high willingness to pay.

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A Estimating Time on the Market from Estate Ownership Duration

Suppose there is an administrative delay of a periods associated with the official transfer of ownership of a STL, and that sorting out a deceased STL holder's estate is a process that terminates each period with probability q. Afterwards, absent a family transfer, a suitable buyer is found and ownership of the STL is transferred from the estate to the buyer by means of a transaction in the decentralize market each period with probability $\gamma(\theta)$. Let d_i^j denote the duration of estate ownership (*i.e.*, time elapsed between transfer to and from the estate of the deceased STL owner) for STL $i \in \{1, 2, \ldots, A\}$ when transferred to a family member ($j = F$) or sold in the decentralized market ($j = M$). Duration d_i^F is therefore a geometrically distributed random variable with CDF

$$\text{Prob}\left\{d_i^F \leq t\right\} = q \sum_{k=0}^{t-a-1} (1-q)^k = 1 - (1-q)^{t-a}, \tag{30}$$

if $t > a$ and $\text{Prob}\left\{d_i^F \leq t\right\} = 0$ otherwise. Duration d_i^M, on the other hand, is a random variable with CDF

$$\begin{aligned}\text{Prob}\left\{d_i^M \leq t\right\} &= q\gamma(\theta) \sum_{k=0}^{t-a-2} \sum_{h=0}^{t-a-2-k} (1-q)^k (1-\gamma(\theta))^h \\ &= 1 - \frac{q(1-\gamma(\theta))^{t-a} - \gamma(1-q)^{t-a}}{q-\gamma(\theta)}. \end{aligned} \tag{31}$$

if $t > a+1$ and $\text{Prob}\left\{d_i^M \leq t\right\} = 0$ otherwise. The extent of the administrative delay, a, and probabilities q and $\gamma(\theta)$ can then be estimated from the estate ownership duration data by maximizing the following log-likelihood function:

$$\begin{aligned}\log \mathcal{L}(a,q,\gamma) = &\sum_{d\in D^F} \log\left(q(1-q)^{d-a-1}\right)\\ &+ \sum_{d\in D^M} \log\left(\frac{q\gamma}{q-\gamma}\left[(1-\gamma)^{d-a-1}-(1-q)^{d-a-1}\right]\right)\end{aligned} \tag{32}$$

where D^F represents the set of estate ownership durations recorded for family transfers, and D^M is the same for market transactions. With ownership duration measured in days, this procedure yields parameter estimates $\hat{a}_{\text{seq}} = 52$, $\hat{q}_{\text{seq}} = 3.8455\times 10^{-3}$ and $\hat{\gamma}_{\text{seq}} = 8.5738\times 10^{-3}$. The corresponding distribution functions for market transactions and family transfers are displayed in Figure 6. The implication for expected time on the market for sellers (*i.e.*, average time spent searching/waiting for a buyer) is therefore $1/\hat{\gamma}_{\text{seq}} = 117$ days, or approximately 3.8 months.

This procedure may in fact understate trading frictions in the decentralized market by assuming that the administrative delays and trading delays occur sequentially. It is entirely possible that an STL is advertised for sale, for example, before other non-search-related sources of delay are resolved by the representative of a deceased STL owner. As a useful benchmark for comparison, consider an alternative assumption that both the sorting of the estate and the search process occur simultaneously. A market transaction occurs only after both activities have terminated. In this setting, d_i^M is a random variable with CDF

$$\begin{aligned}\text{Prob}\left\{d_i^M \le t\right\} &= \left(q\sum_{k=0}^{t-a-1}(1-q)^k\right)\left(\gamma(\theta)\sum_{k=0}^{t-a-1}(1-\gamma(\theta))^k\right)\\ &= \left[1-(1-q)^{t-a}\right]\left[1-(1-\gamma)^{t-a}\right].\end{aligned} \tag{33}$$

if $t > a$ and $\text{Prob}\left\{d_i^M \le t\right\} = 0$ otherwise. The appropriate log-likelihood function to be maximized is

$$
\begin{aligned}
\log \mathcal{L}(a, q, \gamma) = & \sum_{d \in D^F} \log \left(q(1-q)^{d-a-1} \right) \\
& + \sum_{d \in D^M} \log \left(\begin{array}{l} q(1-q)^{d-a-1} \left[1-(1-\gamma)^{d-a} \right] \\ \quad + \gamma(1-\gamma)^{d-a-1} \left[1-(1-q)^{d-a-1} \right] \end{array} \right)
\end{aligned} \tag{34}
$$

This procedure with simultaneous delays yields parameter estimates $\hat{a}_{\text{sim}} = 53$, $\hat{q}_{\text{sim}} = 3.7047 \times 10^{-3}$ and $\hat{\gamma}_{\text{sim}} = 4.4736 \times 10^{-3}$. The CDFs are displayed in Figure 7; they look very similar to the ones in Figure 6, however the average time on the market for sellers is $1/\hat{\gamma}_{\text{sim}} = 224$ days, or approximately 7.3 months, which is substantially longer than under the sequential delay assumption.

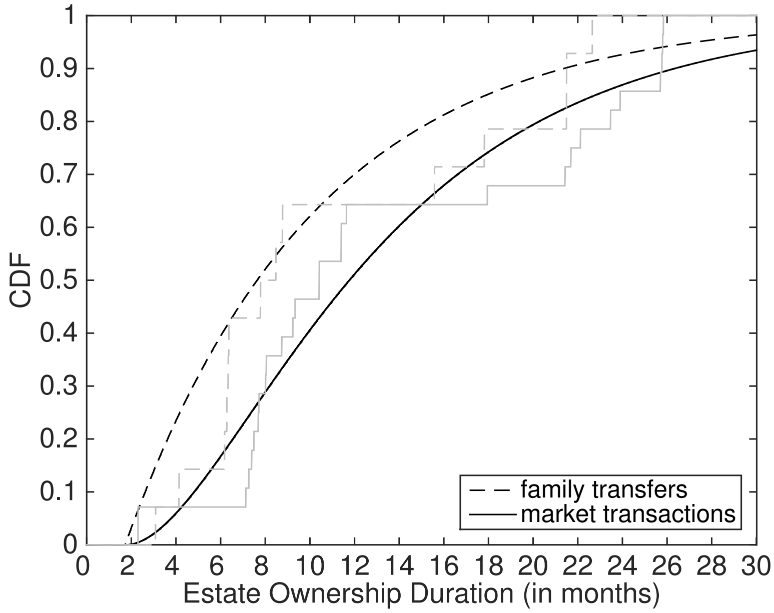

Figure 6: Estimated CDF of Time until New Ownership following Death of STL Owner (Sequential Delay)

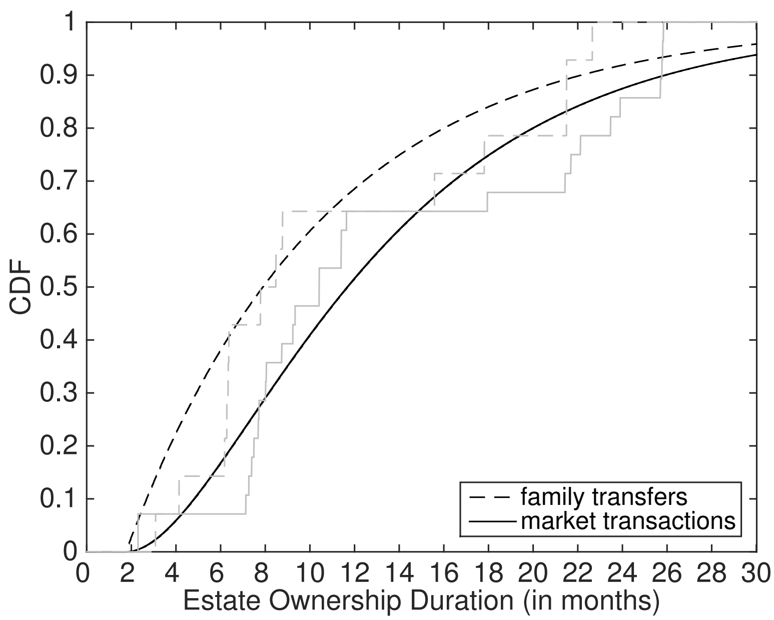

Figure 7: Estimated CDF of Time until New Ownership following Death of STL Owner (Simultaneous Delay)