

International Comovement through Endogenous Long Run Risk

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Abstract

The international macroeconomics literature has had a hard time capturing the joint comovement of quantities and asset prices across countries. We introduce recursive preferences in an endogenous growth model of innovation and technology adoption through trade in varieties and provide an explanation of these comovements. In our model, growth is risky, and it affects current and future expected profits through its effect on the equilibrium marginal rate of substitution. We calibrate the model to data on productivity, R&D and trade in intermediate goods and show that the comovement in quantities across countries is mainly determined by the comovement in current profits, whereas the comovement in asset prices is mainly determined by the comovement in future expected profits across countries. With such mechanism, we can therefore reconcile a low comovement in quantities with a high comovement in asset prices.

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1 Introduction

The international macroeconomics literature has had a hard time reconciling the joint dynamics of equilibrium quantities and asset prices. How can we reconcile the fact that, even for pairs of very well integrated countries, cross-country correlations in macroeconomic aggregates are so much lower than cross-country correlations in asset prices? And for the observed level of correlation in quantities, why do we observe such a low volatility of exchange rates? These are only a few of what Obstfeld and Rogoff (2000) call “truly perplexing puzzles”.

The literature has tackled these problems in various ways. The introduction of frictions in otherwise standard business cycle models has shed some light on these puzzles. Frictions in real goods markets — trade costs, non tradable goods — and the presence of home bias in preferences help reconciling the dynamics of international macroeconomic quantities.¹ Intuitively, real frictions impose a wedge on the perfect alignment of the marginal rate of substitutions across agents in the economy. On the other hand, models with nominal and financial frictions — sticky prices and wages, incomplete financial markets — have obtained some success in adequately explaining asset pricing puzzles.²

In this paper, we address jointly the quantity and the asset pricing puzzles through the lense of a multicountry general equilibrium model of endogenous growth and cross-country spillovers in productivity that result from the international adaption of foreign developed varieties. A crucial ingredient of our model is *growth risk*. The main result is that, while *realized growth in each country* is the main driver of the modest cross-country comovements in macroeconomic quantities, it is the *future expected global growth* that matters for the high comovements in asset prices. For future expected global growth to matter, agents in our model must fear variations in the long-run prospects of the economy which, in turn, depend both on domestic and foreign innovation.

The details of our model are as follows. Growth is driven by the accumulation of technology through endogenous innovation. Innovations are embodied in intermediate goods, which spread across countries through international trade. In each country, a final producer uses both domestic and foreign intermediate goods with a CES production function that features a love-for-variety effect: the more the intermediate goods used in production, the higher the productivity of the country. Each of these intermediate goods is produced by monopolistic

¹For a comprehensive summary of the literature see, among others Obstfeld and Rogoff (2000), Lewis (1995), and references therein.

²Among others, see Ghironi and Melitz (2005), and Corsetti, Dedola, and Leduc (2010) and references therein.

competitive firms that buy the right to use a new technology to produce the good. New technologies are introduced endogenously after an innovator invests resources to come up with a new idea, which eventually becomes an intermediate good. Technologies then diffuse around the world through trade in the intermediate goods that embody them. The final result of this mechanism is that the productivity of a country depends not only on its own innovations, but also on the foreign innovations embodied in the imported intermediate products.

In each country, a representative consumer takes consumption and saving decisions. Preferences are recursive, so that consumers care for the timing of resolution of uncertainty and fear variation in long-run expected growth. Endogenous innovation paired with recursive preferences make the equilibrium endogenous growth risky, through its effect on the present discounted value of future profits of all the firms in the economy. We close the model with the assumption of complete international financial markets: the international risk sharing condition establishes a one to one mapping between the depreciation of the real exchange rate and the ratio of the domestic and foreign marginal rates of substitution.

We then calibrate the model using data on output, R&D, and trade in varieties. The processes of innovation and technology adoption are calibrated using data on R&D expenditures (domestic innovation) and bilateral trade data (technology adoption) at a high level of disaggregation (HS-6 digit product level of disaggregation). Our results show that the main economic mechanism of our model is quantitatively significant and can deliver a mild comovement in international macroeconomic quantities and a much higher international synchronization of asset prices.

The main mechanism works as follows. Risky growth through endogenous innovation and recursive preferences determines the optimal level of R&D, and therefore the level of current and future expected growth. Innovations then spread across countries through a process of technology adoption that we measure with trade in varieties. Risky growth has a first order impact on the stock market and governs its international correlation structure. This is because aggregate dividends in one country express the present discounted value of the future profits of all the firms that are selling in the country, both domestic and foreign. A similar mechanism drives the high cross-country correlation of real interest rates. On the other hand, realized quantities, such as consumption and output, are primarily driven by realized levels of innovation and technology adoption of foreign innovations on the current output growth. These, in turn, are driven by the only source of uncertainty in the model: an exogenous TFP

shock, independent across countries.

In a nutshell, the risk sharing mechanism is composed of two parts: (i) one that affects current growth, which then affects mainly the cross-country correlations of quantities, and (ii) one that affects the prospects of growth, which determines the cross-country correlations of asset prices. We can then use our mechanism to reconcile the low international correlation of quantities with the high international correlation of asset prices.

Our paper is related to several strands on literature. First the literature on endogenous growth through innovation, as in Romer (1990). In our model, technological progress increases with the number of intermediate goods that embody technology. Kung and Schmid (2011) extend Romer (1990) to include recursive preferences and capture asset prices fluctuations that are consistent with the empirical literature. We develop our model along these lines, and we extend it to an international setting. Closer to this paper is Croce, Nguyen, and Schmid (2012), who study international spillovers of volatility and entropy.

The second strand of literature is the one on technology adoption and innovation through trade in varieties (Broda, Greenfield, and Weinstein (2006) and Santacreu (2014)). These papers use highly disaggregated trade data to measure the extent to which foreign innovations adopted through trade in varieties affect positively the growth rate of the trading partners. We introduce recursive preferences into a version of Santacreu (2014) to be able to capture correlations of asset prices.

Finally, the paper is related to the literature on asset pricing with long run risk, starting from the seminal one-country model of Bansal and Yaron (2004), and later applied to the international setting by Bansal and Shaliastovich (2009), Colacito and Croce (2011), and Colacito and Croce (2013). As we show in the next Section, the general equilibrium mechanism of our paper is capable of generating, endogenously, dynamics of within-country and cross-country expected growth that are consistent with the long run risk literature.

The rest of the paper proceeds as follows. In Section 2, we develop a reduced form model that captures, exogenously, the main mechanism of cross-country correlation of productivity. In Section 3 we present a full-fledged model that endogenizes such mechanism. Section 4 presents the calibration and the quantitative results, and Section 5 concludes.

2 A Reduced Form Model

Following the seminal paper of Bansal and Yaron (2004), the long-run risk literature of asset pricing has forcefully argued in favor of the existence of a small predictable risk factor driving the growth prospects of the economy. A long-run-risk explanation of asset pricing puzzles requires this factor to be: i) highly autocorrelated *within* the country, and ii) highly correlated *across* countries. When paired with recursive preferences, a strong persistence *within*-country delivers a large equity premium with a sensible risk aversion parameter (Bansal, Kiku, and Yaron (2012)). On the other hand, a high correlation *across* countries is instrumental in obtaining a high cross-country correlation in the marginal rate of substitution — and thus an exchange rate process that is not *too* volatile — while still keeping the cross-country correlation of realized consumption growth close to the low levels we observe in the data (Brandt, Cochrane, and Santa-Clara (2006)).

In this section, we describe a reduced-form model that captures our novel mechanism of international spillovers of long-run risk. In Section 3, we introduce our benchmark general equilibrium model, in which we endogenize such mechanism. Let the home and foreign long run risk processes be described by the following two-dimensional autoregressive process $z_t \equiv (z_t^H, z_t^F)'$:

$$z_t = \Phi z_{t-1} + \Sigma \epsilon_t \quad . \quad (1)$$

Consistently with our benchmark model, we restrict our attention to a symmetric calibration in which the autocorrelation matrix Φ and the variance-covariance matrix Σ are given by

$$\Phi = \begin{pmatrix} \varphi & \varphi_H^F \\ \varphi_H^F & \varphi \end{pmatrix} ; \quad \Sigma = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix} \quad .$$

For stationarity, the eigenvalues of Φ must be within the unit circle. The $\epsilon \equiv (\epsilon^H, \epsilon^F)'$ innovations are *i.i.d.* and normally distributed with zero mean and unit variance. We allow for a non zero cross-country correlation in innovations and define $\rho_z \equiv \text{Corr}(\epsilon^H, \epsilon^F)$. To showcase the mechanism of our paper, we analyze two versions of equation (1), both of which generate the required correlation structure of long run risk. We argue that the second describes a more plausible economics mechanism.

First, we consider a *univariate* long run risk specification, in which we shut down the off-diagonal elements of the autocorrelation matrix Φ , by imposing

$\varphi_H^F = 0$. In this case, lagged values of z^F (z^H) do *not* affect current values of z^H (z^F). With this restriction in place, it is straightforward to show that $Corr(z_t^H, z_{t-1}^H) = Corr(z_t^F, z_{t-1}^F) = \varphi$ and $Corr(z_t^H, z_t^F) = \rho_z$. Therefore, for long run risk to be highly correlated both *within* and *across* countries, it must be that $\varphi \approx 1$ and $\rho_z \approx 1$. Many papers have studied the nature of the autocorrelation coefficient φ and its effect on asset prices. Here, for the univariate case, we focus on the cross-country correlation coefficient ρ_z . A high cross-country correlation between the home and foreign long-run risk components can be obtained exclusively by assuming a high cross-country correlation in the long run innovations, ϵ^H and ϵ^F . This is the solution adopted by Bansal and Shaliastovich (2009) and Colacito and Croce (2011). In sum, for a univariate specification to work, one must assume both a high autocorrelation coefficient and a high cross-country correlation in long-run innovations.

The second specification we consider a *bivariate* long run risk process, which represents the reduced form of the mechanism in our benchmark model. This specification features an international spillover effect, which we capture by allowing for non-zero off diagonal elements of Φ ($\varphi_H^F \neq 0$). Therefore, lagged values of z^F (z^H) *do* affect current realizations of z^H (z^F). Now, to isolate the effect of the interaction term on the correlation structure of home and foreign long run risk, we shut down the cross-country correlation in the ϵ shocks and impose $\rho_z = 0$. With this exercise we want to answer the following question: How strong must the interaction effect be in order to obtain the high cross-country correlation between z^H and z^F that is prescribed by the long run risk literature? The answer will give us an idea of the quantitative impact of the international spillover mechanism that we propose in our benchmark model.³

We find that with a bivariate long run risk specification we can achieve a high cross country correlation for relatively small values of the interaction parameter. For values as small as 0.05, the cross-country correlation is 0.80. For values larger than 0.30, the correlation is above 0.96, as typically assumed by the international finance literature on long run risks.

In our general equilibrium model, international adoption of domestically developed intermediate goods endogenously generates a strong comovement between home and domestic total factor productivity. Crucially, the only exogenous shocks of our model, which we label short run productivity shocks, are independent across countries, as captured by the restriction $\rho_z = 0$ in the reduced form example of this section.

³While conducting the exercise, we tweak the φ parameter, our only degree of freedom, to obtain a very high within country autocorrelation of z^H and z^F , as prescribed by the long run risk literature.

3 The Model: The Endogenous Mechanism

In this section, we present a model of innovation and technology adoption, that endogenizes the correlation of output growth across countries.

3.1 Households

Domestic households have recursive preferences of the form

$$U_{d,t} = \left\{ (1 - \beta) C_{d,t}^\theta + \beta \left(E_t \left(U_{d,t+1}^{1-\gamma} \right) \right)^{\frac{\theta}{1-\gamma}} \right\}^{\frac{1}{\theta}}$$

where γ is the CRRA, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$ and $\psi \equiv \frac{1}{1-\theta}$ is the IES. We assume that $\psi > \frac{1}{\gamma}$, that is there is a preference for early resolution of uncertainty.

The SDF in this case is

$$M_{d,t+1} = \beta \left(\frac{C_{d,t+1}}{C_{d,t}} \right)^{\theta-1} \left(\frac{U_{d,t+1}}{E_t \left(U_{d,t+1}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}} \right)^{1-\gamma-\theta}$$

Consumers consume, supply labor and capital inelastically to the final producers, make investment decisions and save. We assume complete international asset markets. Agents have access to complete set of state-contingent securities that are traded internationally.

3.2 Final Good Producers.

Final producers are perfectly competitive, and use capital, $K_{d,t}$, labor, $L_{d,t}$ (we assume that labor and capital are immobile across countries but perfectly mobile across sectors within a country), and a composite of domestic and foreign intermediate goods, $G_{d,t}$, to produce a non-traded final good $Y_{d,t}$ according to

$$Y_{d,t} = \left(K_{d,t}^\alpha (\Omega_{d,t} L_{d,t})^{(1-\alpha)} \right)^{(1-\xi)} G_{d,t}^\xi$$

where $\alpha, \xi \in (0, 1)$ and

$$G_{d,t} = \left[\int_0^{N_{d,t}^d} (X_{d,i,t}^d)^\nu di + \int_0^{N_{f,t}^d} (X_{f,j,t}^d)^\nu dj \right]^{\frac{1}{\nu}}$$

with $i \in (0, N_{d,t}^d)$ and $j \in (0, N_{f,t}^d)$; $\frac{1}{1-\nu}$ is the elasticity of substitution across intermediate goods with $\nu < 1$, $X_{d,i,t}^d$ is the amount of domestically produced

intermediate good i that is used for final production in the domestic economy, and $X_{f,j,t}^d$ is the amount of foreign produced intermediate good j that is used for final production in the domestic economy.

Final producers choose capital, labor, investment, and intermediate goods to maximize shareholder value subject to the production technology. Intermediate goods may be domestically produced ($X_{d,i,t}^d$) or produced abroad and adopted domestically ($X_{f,j,t}^d$). Formally,

$$\max_{\{I_{d,t}, L_{d,t}, K_{d,t+1}, X_{d,i,t}^d, X_{f,j,t}^d\}_{i \geq 0, i \in [0, N_{d,t}]}} E_0 \left[\sum_{t=0}^{\infty} M_{d,t} D_{d,t} \right]$$

Dividends are given by

$$D_{d,t} = Y_{d,t} - I_{d,t} - W_{d,t} L_{d,t} - \int_{i=0}^{N_{d,t}^d} P_{d,i,t}^d X_{d,i,t}^d di - \int_{i=0}^{N_{f,t}^d} P_{f,j,t}^d X_{f,i,t}^d dj$$

where $W_{d,t}$ is the wage rate, $I_{d,t}$ is capital investment, $P_{d,i,t}^d$ is the price of a domestically produced intermediate good, and $P_{f,j,t}^d$ is the price of a foreign produced intermediate good, which has been adopted domestically. Both prices are expressed in units of the domestic producer's final good (see more below). The law of motion for capital, taking into account adjustment costs, is given by

$$K_{d,t+1} = (1 - \delta)K_{d,t} + \Lambda \left(\frac{I_{d,t}}{K_{d,t}} \right) K_{d,t}$$

where

$$\Lambda_{d,t} \equiv \Lambda \left(\frac{I_{d,t}}{K_{d,t}} \right) = \frac{\alpha_1}{\zeta} \left(\frac{I_{d,t}}{K_{d,t}} \right)^{\zeta} + \alpha_2$$

so that

$$\Lambda'_{d,t} = \alpha_1 \left(\frac{I_{d,t}}{K_{d,t}} \right)^{\zeta-1}$$

as in Jermann (1998).

3.3 Intermediate Good Producers

In each country there is a set of monopolistic competitive firms that produce a differentiated good using final output according to a CRS production function (one unit of final output is used to produce one unit of the intermediate good). All producers produce with the same efficiency. Intermediate producers take as given the demand by final producers and then set a price that is a constant markup over the marginal cost (which is given by the final good price since

this is the only factor of production in the economy). Intermediate producers produce for the domestic and foreign market. To export the good, they face an additional cost given by a unit trade cost τ_t .

Formally, monopolists solve the following static profit maximization problem each period

$$\begin{aligned}
\max_{P_{d,i,t}^d, P_{d,j,t}^{f*}} \Pi_{d,i,t} &\equiv \max_{P_{d,i,t}^d, P_{d,j,t}^{f*}} (\pi_{d,i,t}^d + \pi_{d,i,t}^f) \\
&= \max_{P_{d,i,t}^d} P_{d,i,t}^d X_{d,i,t}^d(P_{d,i,t}^d) - X_{d,i,t}^d(P_{d,i,t}^d) \\
&\quad + \max_{P_{d,j,t}^{f*}} \underbrace{(P_{d,j,t}^f Q_t)}_{P_{d,j,t}^{f*}} X_{d,j,t}^f(\underbrace{P_{d,j,t}^f Q_t}_{P_{d,j,t}^{f*}}) - X_{d,j,t}^f(\underbrace{P_{d,j,t}^f Q_t}_{P_{d,j,t}^{f*}})
\end{aligned}$$

where $P_{d,j,t}^{f*} = P_{d,j,t}^f Q_t$ is the price, in domestic good unites, of a domestically produced intermediate good that is being exported, and Q_t is the exchange rate, defined as units of domestic final goods per one unit of foreign final good.

In a symmetric equilibrium, it must be that

$$X_{d,i,t} \equiv X_{d,t} \quad \text{and} \quad P_{d,i,t}^d \equiv P_{d,t}^d = \frac{1}{\nu} \quad \text{and} \quad P_{d,j,t}^{f*} \equiv P_{d,t}^{f*} = \frac{1}{\nu} \tau_t$$

where $\frac{1}{\nu}$ is the markup.

We express real prices of the intermediate goods in units of the importers' final good. In particular, when the domestic (foreign) intermediate good is used for the production of the foreign (domestic) final output, we have

$$P_{d,t}^f \equiv \frac{1}{\nu} \tau_t Q_t^{-1}$$

$$P_{f,t}^d \equiv \frac{1}{\nu} \tau_t Q_t$$

τ_t is an iceberg transport cost from selling the good abroad, and Q_t is the exchange rate, expressed in units of domestic goods per unit of foreign good.

3.4 Innovation and Adoption

3.4.1 Innovation

In each country an innovator invests resources (final output) to come up with a new prototype of a product. If the innovator is successful, it starts producing

the new good as an intermediate producer. The law of motion for innovations is

$$N_{d,t+1}^d = \vartheta_{d,t} S_{d,t} + (1 - \phi) N_{d,t}^d$$

where

$$\vartheta_{d,t} = \frac{\chi N_{d,t}^d}{S_{d,t}^{1-\eta} (N_{d,t}^d)^\eta}$$

as in Comin and Gertler (2006), $S_{d,t}$ is R&D expenditure (in terms of the domestic final good) and where ϕ is the probability that a new variety becomes obsolete.

Innovators choose $S_{d,t}$ to maximize the present discounted value of a new prototype which is what they expect to get from selling the good in the domestic and foreign markets. They maximize the present discounted value of the profits that they expect to obtain from selling the good in the future to both domestic and foreign producers.

$$V_{d,i,t} = \Pi_{d,i,t} + (1 - \phi) E_t[M_{d,t+1} V_{d,i,t+1}]$$

where overall profits from firm i evolves according to

$$\Pi_{d,i,t} N_{d,t}^d = \pi_{d,i,t}^d N_{d,t}^d + \pi_{d,i,t}^f N_{d,t}^f$$

In equilibrium,

$$V_{d,i,t} \equiv V_{d,t} \quad , \quad \Pi_{d,i,t} \equiv \Pi_{d,t} \quad , \quad \pi_{d,i,t}^d \equiv \pi_{d,t}^d \quad and \quad \pi_{d,i,t}^f \equiv \pi_{d,t}^f$$

It is useful to define In growth rates, the law of motion for new innovations becomes

$$g_{N_{d,t}} \equiv \log \left(\frac{N_{d,t+1}}{N_{d,t}} \right) = \log \left(\chi \left(\frac{S_{d,t}}{N_{d,t}} \right)^\eta + (1 - \phi) \right) \approx \phi - \chi \left(\frac{S_{d,t}}{N_{d,t}} \right)^\eta$$

In steady state, the growth rate is constant and therefore the amount of final output that is spent in R&D grows at the same rate of the number of newly invented technologies. And since one unit of output is spent to produce one new variety, that investment in R&D grows at the rate of final output and at the rate of newly invented technologies:

$$g_y = g_{Nd} = g_S$$

3.4.2 Adoption

In the benchmark model we assume that adoption is exogenous and that every period a fraction ϑ_f^d of foreign goods is imported. In CGS endogenous adoption is the channel that genereates endogenous propagation of shocks.⁴ In this paper it is recursive preferences because we want to introduce the role of risk.

$$N_{f,t+1}^d = \vartheta_f^d(N_{f,t+1} - \phi N_{f,t}^d) + \phi N_{f,t}^d$$

where N_{ft} is the mass of foreign produced intermediate goods that are used for final production and N_{ft}^d is the number of imported goods by the domestic economy. This expression says that every period country d imports a fraction of the goods from country f which have not been adopted previously and which have not become obsolete.

3.5 Market clearing condition

Final output is used for consumption, intermediate goods production and investment in R&D.

$$Y_t = C_t + I_t + S_{d,t} + N_{d,t}^d X_{d,t}^d + N_{d,t}^f X_{d,t}^f$$

3.6 First order conditions

3.6.1 Final Producers

We set up the Lagrangian of the final producers, in real terms, as:

$$\begin{aligned} \mathcal{L}_d = & E_0 \left\{ \sum_{t=0}^{\infty} M_{d,t} (Y_{d,t} - W_t L_{d,t} - I_{d,t} - \int_{i=0}^{N_{d,t}^d} P_{d,i,t}^d X_{d,i,t}^d di - \int_{j=0}^{N_{d,t}^f} P_{f,j,t}^d X_{f,j,t}^d dj \right. \\ & \left. + q_{d,t} ((1 - \delta) K_{d,t} + \Lambda \left(\frac{I_{d,t}}{K_{d,t}} \right) K_{d,t} - K_{d,t+1}) \right\} \end{aligned}$$

where $q_{d,t}$ is the lagrange multiplier of the capital accumulation equation. The first order conditions are:

$$q_{d,t} = \frac{1}{\Lambda'_{d,t}}$$

⁴The assumption of exogenous international adoption is relaxed in the Appendix.

$$1 = E_t \left[M_{d,t+1} \left\{ \frac{1}{q_{d,t}} \left(\alpha(1-\xi) \frac{Y_{d,t+1}}{K_{d,t+1}} + q_{d,t+1}(1-\delta) - \frac{I_{d,t+1}}{K_{d,t+1}} + q_{d,t+1}\Lambda_{d,t+1} \right) \right\} \right]$$

$$W_{d,t} = (1-\alpha)(1-\xi) \frac{Y_{d,t}}{L_{d,t}}$$

$$P_{d,i,t}^d = \xi Y_{d,t} (X_{d,i,t}^d)^{\nu-1} G_{d,t}^{-\nu}$$

$$P_{f,i,t}^d = \xi Y_{d,t} (X_{f,i,t}^d)^{\nu-1} G_{d,t}^{-\nu}$$

3.6.2 TFP

Recall that intermediate good prices are

$$P_{d,t}^f \equiv \frac{1}{\nu} \tau_t Q_t^{-1}$$

$$P_{f,t}^d \equiv \frac{1}{\nu} \tau_t Q_t$$

Substitute these prices in the FOCs for the final producer to get

$$X_{d,t}^d = \left(\xi \nu Y_{d,t} G_{d,t}^{-\nu} \right)^{\frac{1}{1-\nu}}$$

and

$$X_{f,t}^d = \left(\xi \nu Y_{d,t} G_{d,t}^{-\nu} \right)^{\frac{1}{1-\nu}} (\tau_t Q_t)^{\frac{1}{\nu-1}} = X_{d,t}^d (\tau_t Q_t)^{\frac{1}{\nu-1}}$$

Note that in a symmetric equilibrium, we can express G_t as

$$\begin{aligned} G_{d,t} &= \left[\int_0^{N_{d,t}^d} (X_{d,i,t}^d)^\nu di + \int_0^{N_{f,t}^d} (X_{f,j,t}^d)^\nu dj \right]^{\frac{1}{\nu}} \\ &= \left(N_{d,t}^d (X_{d,t}^d)^\nu + N_{f,t}^d (X_{f,t}^d)^\nu \right)^{\frac{1}{\nu}} \end{aligned}$$

In the above expression for G_t , substitute the expressions for $X_{d,t}^d$ and $X_{f,t}^d$ to get

$$G_t = \xi \nu Y_{d,t} \left[N_{d,t}^d + (\tau_t Q_t)^{\frac{\nu}{\nu-1}} N_{f,t}^d \right]^{\frac{1-\nu}{\nu}}$$

and then substituting into the final production to get

$$Y_{d,t} = \left(K_{d,t}^\alpha (\Omega_{d,t} L_{d,t})^{1-\alpha} \right) (\xi \nu)^{\frac{\xi}{1-\xi}} \left[N_{d,t}^d + (\tau_t Q_t)^{\frac{\nu}{\nu-1}} N_{f,t}^d \right]^{\frac{\xi(1-\nu)}{\nu(1-\xi)}}$$

For endogenous growth we need that the production function is homogeneous of degree 1 in the accumulating factors, then

$$\alpha + \frac{\frac{\xi}{\nu} - \xi}{1 - \xi} = 1$$

We can write it as in KS

$$Y_{d,t} = K_{d,t}^\alpha (Z_{d,t} L_{d,t})^{1-\alpha}$$

where domestic TFP is

$$Z_{d,t} \equiv \Omega_t (\bar{A})^{\frac{1}{1-\alpha}} \left[N_{d,t}^d + (\tau_t Q_t)^{\frac{\nu}{\nu-1}} N_{f,t}^d \right]$$

and $\bar{A} \equiv (\xi \nu)^{\frac{\xi}{1-\xi}}$.

In logs,

$$\log Z_{d,t} = \log \Omega_t + \log \left\{ (\bar{A})^{\frac{1}{1-\alpha}} \left[N_{d,t}^d + (\tau_t Q_t)^{\frac{\nu}{\nu-1}} N_{f,t}^d \right] \right\}$$

$$\log(TFP) = \log(TFP^{EXO}) + \log(TFP^{ENDO})$$

Note that the endogenous component of TFP depends on the number of varieties that have been produced domestically, $N_{d,t}^d$, and the number of varieties that have been produced in the foreign country $N_{f,t}^d$. Therefore $N_{d,t}^d$ is the domestic component of endogenous TFP and $(\tau_t Q_t)^{\frac{\nu}{\nu-1}} N_{f,t}^d$ is the foreign component of endogenous TFP.

3.6.3 Innovation

We assume that every innovation that is produced in a country is immediately adopted by the final producers in that country. Hence,

$$N_{d,t} = N_{d,t}^d$$

The value of an innovation (or market price for an innovation) is given by the expected discounted profits that the innovator expects to get from selling

the good to the domestic and foreign final producers.

$$V_{d,t+1} = \Pi_{d,t} + (1 - \phi)E_t[M_{t+1}V_{d,t+1}]$$

Discounted future profits on patents are the payoff to innovators. Since the R&D sector is competitive, the optimality condition for R&D investment is

$$S_{d,t} = E_t[M_{d,t+1}V_{d,t+1}](N_{d,t+1} - (1 - \phi)N_{d,t})$$

or, equivalently,

$$\frac{1}{\vartheta_{d,t}} = E_t[M_{d,t+1}V_{d,t+1}]$$

3.6.4 Adoption

Every period t , a fraction of the goods that were invented in the country will be adopted by the foreign economy in period $t + 1$.

3.6.5 International risk sharing condition

Given complete markets the real exchange rate Q_t evolves according to

$$\frac{Q_{t+1}}{Q_t} = \frac{M_{t+1}^*}{M_{t+1}}$$

3.6.6 The Stock Market

Stocks are claims to all future dividends of all the firms in the economy. Aggregate dividends are thus the net payout of the production sector

$$\mathcal{D}_{d,t} = D_{d,t} + N_{d,t}^d \pi_{d,t}^d + N_{f,t}^d \pi_{f,t}^d - S_{d,t}$$

To be completed.

3.7 The mechanism

To be added.

4 Quantitative results

To be added.

5 Conclusion

To be added.

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Appendix A: A bivariate LRR Example

Consider the following processes

$$\begin{aligned} z_t &= \rho_z z_{t-1} + \rho_z^{z^*} z_{t-1}^* + \sigma_z \epsilon_{t+1}^z \\ z_t^* &= \rho_z^* z_{t-1} + \rho_z^{z^*} z_{t-1} + \sigma_z^* \epsilon_{t+1}^{z^*} \end{aligned} \quad ,$$

where $\epsilon \equiv (\epsilon^z, \epsilon^{z^*})' \sim N(i.i.d., (0, \Sigma))$. We allow for cross-country correlation in the shocks and define $\rho_{\epsilon^z \epsilon^{z^*}}^{\epsilon^z \epsilon^{z^*}}$ the off-diagonal elements of $\Sigma = (1, \rho_{\epsilon^z \epsilon^{z^*}}^{\epsilon^z \epsilon^{z^*}}; \rho_{\epsilon^z \epsilon^{z^*}}^{\epsilon^z \epsilon^{z^*}}, 1)$.

5.1 Moments

Assuming stationarity, we have the following.

- Conditional mean

$$\begin{aligned} E_{t-1}(z_t) &= \rho_z z_{t-1} + \rho_z^{z^*} z_{t-1}^* \\ E_{t-1}(z_t^*) &= \rho_z^* z_{t-1} + \rho_z^{z^*} z_{t-1} \end{aligned}$$

- Unconditional means

$$\begin{aligned} E(z_t) &= EE_{t-1}(z_t) = 0 \\ E(z_t^*) &= EE_{t-1}(z_t^*) = 0 \end{aligned}$$

- Conditional variances

$$\begin{aligned} Var_{t-1}(z_t) &= \sigma_z^2 \\ Var_{t-1}(z_t^*) &= \sigma_z^{*2} \end{aligned}$$

- Unconditional variances

$$\begin{aligned} Var(z_t) &= EVar_{t-1}(z_t) + VarE_{t-1}(z_t) = \frac{\sigma_z^2 + (\rho_z^{z^*})^2 Var(z_t^*) + 2\rho_z \rho_z^{z^*} cov(z_t, z_t^*)}{1 - \rho_z^2} \\ Var(z_t^*) &= EVar_{t-1}(z_t^*) + VarE_{t-1}(z_t^*) = \frac{\sigma_z^{*2} + (\rho_z^{z^*})^2 Var(z_t) + 2\rho_z^* \rho_z^{z^*} cov(z_t, z_t^*)}{1 - \rho_z^{*2}} \end{aligned}$$

- Conditional cross-country covariance

$$Cov_{t-1}(z_t, z_t^*) = \sigma_z \sigma_z^* \rho_{\epsilon^z \epsilon^{z^*}}^{\epsilon^z \epsilon^{z^*}}$$

- Unconditional cross-country covariance

$$\begin{aligned} Cov(z_t, z_t^*) &= ECov_{t-1}(z_t, z_t^*) + Cov(E_{t-1}(z_t), E_{t-1}(z_t^*)) \\ &= \frac{\sigma_z \sigma_z^* \rho_{\epsilon z}^{z^*} + \rho_z \rho_z^* Var(z_t) + \rho_z^* \rho_z^{z^*} Var(z_t^*)}{1 - \rho_z \rho_z^* - \rho_z^{z^*} \rho_z^{z^*}} \end{aligned}$$

Note the that the two conditional variances and the unconditional cross-country covariance are a 3by3 system of equation.

- Unconditional cross-country correlation

$$Corr(z_t, z_t^*) = \frac{Cov(z_t, z_t^*)}{\sqrt{Var(z_t)} \sqrt{Var(z_t^*)}}$$

- Autocovariance

$$\begin{aligned} Cov(z_t, z_{t-1}) &= \rho_z Var(z_t) + \rho_z^{z^*} Cov(z_t, z_t^*) \\ Cov(z_t^*, z_{t-1}^*) &= \rho_z^* Var(z_t^*) + \rho_z^{z^*} Cov(z_t, z_t^*) \end{aligned}$$

Note: the autocovariance depends on the cross country covariance which, in turn is affected (among other things) by the cross-country parameters $\rho_z^{z^*}$ and $\rho_z^{z^*}$.

- Autocorrelation

$$\begin{aligned} Corr(z_t, z_{t-1}) &= \frac{Cov(z_t, z_{t-1})}{Var(z_t)} \\ Corr(z_t^*, z_{t-1}^*) &= \frac{Cov(z_t^*, z_{t-1}^*)}{Var(z_t^*)} \end{aligned}$$

5.2 Special cases: Symmetry

Symmetry means that all parameters are symmetric across countries, that is: $\rho_z = \rho_z^*$, $\rho_z^{z^*} = \rho_z^* z_{t-1}$ and $\sigma_z = \sigma_z^*$. The expressions above simplify accordingly.

- Conditional mean

$$\begin{aligned} E_{t-1}(z_t) &= \rho_z z_{t-1} + \rho_z^{z^*} z_{t-1}^* \\ E_{t-1}(z_t^*) &= \rho_z z_{t-1}^* + \rho_z^{z^*} z_{t-1} \end{aligned}$$

- Unconditional means

$$E(z_t) = E(z_t^*) = 0$$

- Conditional variances

$$Var_{t-1}(z_t) = Var_{t-1}(z_t^*) = \sigma_z^2$$

- Unconditional variances

$$Var(z_t) = Var(z_t^*) = \frac{\sigma_z^2 + (\rho_z^{z^*})^2 Var(z_t) + 2\rho_z \rho_z^{z^*} cov(z_t, z_t^*)}{1 - \rho_z^2}$$

- Conditional cross-country covariance

$$Cov_{t-1}(z_t, z_t^*) = \sigma_z^2 \rho_{\epsilon^z}^{z^*}$$

- Unconditional cross-country covariance

$$Cov(z_t, z_t^*) = \frac{\sigma_z^2 \rho_{\epsilon^z}^{z^*} + 2\rho_z \rho_z^{z^*} Var(z_t)}{1 - \rho_z^2 - (\rho_z^{z^*})^2}$$

- Unconditional cross-country correlation

$$Corr(z_t, z_t^*) = \frac{Cov(z_t, z_t^*)}{\sqrt{Var(z_t)} \sqrt{Var(z_t^*)}}$$

- Autocovariance

$$Cov(z_t, z_{t-1}) = Cov(z_t^*, z_{t-1}^*) = \rho_z Var(z_t) + \rho_z^{z^*} Cov(z_t, z_t^*)$$

- Autocorrelation

$$Corr(z_t, z_{t-1}) = Corr(z_t^*, z_{t-1}^*) = \frac{Cov(z_t, z_{t-1})}{Var(z_t)}$$

5.2.1 Univariate case

Keep the assumption of symmetry and impose $\rho_z^{z^*} = \rho_z^{z^*} = 0$. We retrieve the univariate case of Colacito and Croce (2013) with, in particular

- Unconditional variances

$$Var(z_t) = Var(z_t^*) = \frac{\sigma_z^2}{1 - \rho_z^2}$$

- Unconditional cross-country covariance

$$Cov(z_t, z_t^*) = \frac{\sigma_z^2 \rho_{\epsilon^z}^{z^*}}{1 - \rho_z^2}$$

- Unconditional cross-country correlation

$$Corr(z_t, z_t^*) = \frac{Cov(z_t, z_t^*)}{\sqrt{Var(z_t)} \sqrt{Var(z_t^*)}} = \rho_{\epsilon^z}^{z^*}$$

- Autocovariance

$$Cov(z_t, z_{t-1}) = Cov(z_t^*, z_{t-1}^*) = \rho_z Var(z_t)$$

- Autocorrelation⁵

$$Corr(z_t, z_{t-1}) = Corr(z_t^*, z_{t-1}^*) = \frac{Cov(z_t, z_{t-1})}{Var(z_t)} = \rho_z$$

5.2.2 Bivariate case

Keep the assumption of symmetry and allow for $\rho_z^{z^*} = \rho_{z^*}^z \neq 0$. We want to ask how strong the spillover effects across z and z^* must be in order to obtain a cross-country correlation $Corr(z_t, z_t^*)$ that is as high as required by Colacito-Croce (0.900 or 1.000). To emphasise the spillover channel, we shut down the “exogenous” cross-country correlation coming from the ϵ shocks, that is we impose $\rho_{\epsilon z}^{z^*} = 0$. While performing the exercise for difference values of $\rho_z^{z^*}$, we adjust the level of ρ_z to obtain a within country autocorrelation consistent with the long run risk literature (0.979).

Appendix B: Equilibrium and Solution

Appendix C: Data

In this section, we describe the data that we use to measure the strength of our mechanism. We measure domestic innovation with R&D expenditures over GDP. These data are available in the OECD STAN database. We then use data from the UN COMTRADE database to obtain an indirect measure of international adoption. Assuming that technology is embodied in a product, we can measure the international adoption mechanism with the extensive margin of trade, that is, the number of products that are traded across countries. The idea is the following: Goods embody R&D that is then spread across countries through international trade. The UN COMTRADE database contains bilateral trade data at the 5- and 6- digit level of disaggregation for a large sample of countries and time period. These are annual data, and we can classify the products into consumption, intermediate and capital goods. For the purpose of our analysis we focus on intermediate and capital goods, since we are interested in technology spillovers. To get a better understanding of the amount of R&D that is spread across countries through trade in varieties, we obtain the R&D content of trade for the countries in our sample. We then regress the correlation of output growth and TFP growth per pair of countries againsts our measure of R&D content of imports. We

⁵The Bansal and Yaron (2004) calibration is $\rho_z = 0.979$ (or 0.985, or 0.987 like in Colacito and Croce (2011), and Colacito and Croce (2013)). Also, the cross-country correlation across shocks in Colacito and Croce (2011) is $\rho_{\epsilon z}^{z^*} = 0.900$ (or 1.000 like in Colacito and Croce (2013)).

obtain a positive connection between these two variables. We can do this at various frequencies.