

Optimal Contract, Ownership Structure and Asset Pricing*

Hae Won (Henny) Jung

Qi Zeng

University of Melbourne

University of Melbourne

hae.jung@unimelb.edu.au

qzeng@unimelb.edu.au

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Abstract

This paper studies a dynamic equilibrium asset pricing model with managerial moral hazard. A representative, all-equity firm is owned by two separate types of constant absolute risk-aversion (CARA) investors—a large shareholder and a continuum of small shareholders. The large shareholder holds a long-term view and trades infrequently. From her controlling equity stake in the firm, the large shareholder plays a dominant role in hiring a manager and implementing the managerial compensation scheme. Small dispersed and competitive shareholders, however, trade shares of the firm continuously. We obtain a closed-form solution that characterizes the firm's ownership structure, managerial contract and equity return. As a benchmark case, we consider the first-best case with observable managerial effort in which the equilibrium solution arises solely from optimal risk sharing incentives. Through both the analytical and numerical characterizations of the model, we find that (i) The presence of moral hazard leads to a larger equity stake in the firm held by the large shareholder; (ii) Both the expected stock return and stock return volatility are lower under moral hazard because of risk sharing effects of managerial incentives; (iii) The risk aversion parameters of the manager and investors have different effects on the equilibrium outcome; (iv) The interactions among pay-performance sensitivity, large shareholder ownership, and expected stock return/stock return volatility can be positive or negative.

Keywords: ownership structure, optimal managerial contracts, equilibrium asset pricing

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1 Introduction

It has long been recognized that it is generally not possible to achieve unconstrained Pareto-optimal market allocation in the presence of moral hazard (see, for example, Arrow, 1971). A typical remedy for the moral hazard problem is to provide optimal contracts: a principal designs a compensation contract to induce the desired level of the agent's effort that affects the firm's output (Grossman and Hart, 1983, Holmstrom and Milgrom, 1987). Accordingly, one would expect to observe a direct relation between a firm's output and its manager's compensation. The asset price process, which itself is a function of the firm's output, should be also related to the compensation given to the manager (Ou-Yang, 2005).

In this paper we consider a continuous-time equilibrium model of asset pricing and executive compensation. There is a representative firm and three types of players in the economy: a large shareholder, a manager, and a continuum of small shareholders. Given a contract, a manager chooses his level of effort to maximize his expected utility. The large shareholder, holding a block of shares, hires the manager by offering an optimal contract to maximize the expected utility from the dividends of the shares. The equilibrium asset price is determined competitively through the optimal portfolio and consumption choices of small shareholders. Since the equilibrium asset price depends on the anticipated managerial effort induced by the manager's compensation contract which has been determined by the large shareholder, the price is also a function of the large shareholder's ownership stake. Finally, the large shareholder optimally chooses the ownership stake in the firm while taking into account the effects of that choice on the equilibrium asset price.

The two types of shareholders represent different types of governance structure. On the one hand, small shareholders act competitively in the stock market. They represent passive monitoring and "vote with their feet". Namely, if they believe that the firm's management and board of directors do not perform well, they will simply sell their shares and thus force the equilibrium stock price down. The large shareholders, on the other hand, undertake active monitoring and control (Tirole, 2006). They

take a long term view when purchasing a block of shares and thus trade infrequently. In our model, the large shareholder is equipped with the power to hire a manager and set his compensation packages. Our assumption of the longer investment horizon of a firm’s large shareholders is supported by existing empirical evidence (Tirole, 2006).

Our model separates the strategic principal-agent relationship from the competitive asset pricing process. The basic idea has been explored in the literature (DeMarzo and Urošević, 2006; Burkart et al., 1997). In these two studies, large shareholders directly control the firm’s output either by exerting a monitoring effort themselves or by choosing corporate investment projects. Thus these large shareholders act as the managers of the firm. In our setting, however, the large shareholder plays a dominant role in the board of directors especially for the decisions on CEO selection and compensation.

This separation contrasts sharply with other studies that look at the link between agency contracts and asset pricing (Gorton et al., 2013; Ou-Yang 2005). In those studies, shareholders act competitively in the stock market, yet they strategically coordinate together to determine the manager’s compensation contracts. We model the procedure more explicitly by incorporating a representative large shareholder, who acts strategically both in the security market and in the corporate decision process.

Our setup with a CARA-normal framework enables us to obtain a closed-form solution in which we fully characterize the optimal managerial contract, the equilibrium stock price and the ownership structure under moral hazard. We link the compensation contract to the firm’s output and stock return. Consequently, we are able to define two alternate pay-performance sensitivity (PPS) measures of the manager—the sensitivity of CEO pay to the firm’s total output and the sensitivity of CEO pay to the excess dollar return on the stock.

To provide further implications, in addition to the analytical characterization of the model, we calibrate it by matching the equilibrium variables to empirical counterparts in the historical U.S. data and then perform a comparative static exercise. By comparing the equilibrium solution under moral hazard with the first-best solution, we find that large shareholder ownership is greater under moral

hazard. Moreover, both the expected stock return and stock return volatility are lower under moral hazard because of risk sharing effects of managerial incentives. Further, the risk aversion parameters of the players have differing effects on the equilibrium outcome: managerial risk aversion can lead to a higher equity risk premium and greater volatility, whereas, if investors are more risk-averse, the stock return increases, but the stock return volatility decreases because higher managerial incentives are offered through the manager's optimal contracts. Lastly, the interactions among pay-performance sensitivity, large shareholder ownership, and expected stock return/stock return volatility can be positive or negative.

Our paper is based on two streams of literature. The first stream is the recent development of continuous-time contracting models (e.g., Sannikov, 2008; Edmans et al., 2012; Cvitanic and Zhang, 2013). The active research in this field over the past several years enables us to solve dynamic contracting problems explicitly. An important distinction of our setting from those studies is that investors as well as the manager are risk-averse so that we take into account the risk-aversion of the principal. Williams (2009) is an exception from which we benefit most. In contrast to his paper, we account for firm ownership structure and asset return and show how they are linked to optimal managerial contracts.

The other stream of literature examines moral hazard and adverse selection problems with different ownership structures (DeMarzo and Urošević, 2006; Burkart et al., 1997). This literature takes the view that large shareholders have control over the firm through their costly monitoring. They focus on the effects of ownership structure on the equilibrium stock price and the conflict of interests between large and small shareholders, but do not consider how ownership structure shapes the equilibrium solution through its impact on optimal managerial contracts.

Our paper is closely related to a few studies that incorporate agency problems into asset pricing. Gorton et al. (2013) is the most closely related to ours. As mentioned earlier, in contrast to their study, we explicitly separate the strategic principal-agent relationship from the competitive trading behavior

of shareholders. Moreover, we solve for the general optimal contracting problem, whereas they assume that the manager's compensation is implemented by his equity share. Ou-Yang (2005) pursues the same goal as we do in this paper but he solves a problem with multiple firms, whereas we consider only a representative firm. As in Gorton et al. (2013), he focuses solely on competitive shareholders. Further, in his setting, shareholders consider their expected utility from the terminal wealth, even though firms produce outputs dynamically. We study an equilibrium model in which all the intermediate outputs are split among shareholders and the manager. Also, he assumes the functional form of an incentive contract (as a function of stock prices), whereas we endogenously derive the optimal incentive contracts by solving the general contracting problem.

There is extensive theoretical and empirical literature on the association between ownership structure distinguished by large shareholders and the stock market over the past two decades. By introducing the informational role of stock prices, Holmstrom and Tirole (1993) document the effects of market liquidity on monitoring incentives and managerial incentives. In our setting, we do not consider informational heterogeneity between small and large shareholders. In contrast to their study, we also attempt to solve for the optimal compensation contract endogenously, rather than assuming a specific form, and consider implications of the principal's risk aversion to obtain quantitative asset pricing implications. In the asset pricing literature, there are also several studies that have examined possible implications of large shareholders (Cvitanic, 1997; El Karoui, Peng and Quenez, 1997; Cuoco and Cvitanic, 1998). In this literature, however, the effects that trading by this type of investors have on the asset prices are assumed to be exogenous, which distinguishes ours from the literature because we endogenously derive the effects of the large shareholder's optimal decision on asset pricing through optimal contracting.

Finally, our paper is also linked to the literature that studies the general problem of the competitive market with moral hazard (Prescott and Townsend, 1984; Zame, 2007). This literature incorporates the traditional problems of moral hazard and adverse selection into standard competitive general equi-

librium models and determines the general conditions under which the equilibrium exists. Our paper provides a specific example of such an economy.

Our paper contributes to the literature in two respects. First, we provide a general framework to incorporate the dynamic contracting problem to the asset pricing literature. We should point out that, because of the separation of two types of shareholders, we greatly reduce the complication of solving for optimal consumption and portfolio choices *and* the dynamic contracting problem simultaneously, as indicated by our closed-form results. While the solution for the optimal dynamic contract can be obtained by the recent development of dynamic contracting theory, solving for the equilibrium stock prices has still been a challenge. Second, we provide a unified theory that links managerial incentive contracts, equilibrium stock prices, and ownership structure.

The remainder of the paper is as follows. In the next section we describe the general setup of our model. Using a CARA-normal framework, we then obtain the closed-form solution for the optimal managerial contract, equilibrium stock price and ownership structure under moral hazard. This is followed by the analytical characterization of the equilibrium in comparison with the the first-best solution as a benchmark case and further predictions from a quantitative comparative statistic analysis based on the calibrated model. We discuss several potential extensions from the model proposed in this paper and then conclude.

2 The Model

We study a continuous-time principal-agent problem in an asset pricing framework with a large controlling shareholder. We consider a representative all-equity firm in the economy. As in Holmstrom and Tirole (1993), Admati et. al (1994), and DeMarzo and Urošević (2006), the firm’s shareholders are composed of two groups: “large” shareholders or blockholders who hold a block ownership in the firm for a relatively long-time horizon, and a continuum of competitive small (dispersed) shareholders who

collectively hold the remaining equity stake. In reality, major strategic corporate decisions require the approval of corporate boards that are significantly influenced by large shareholders.¹

In our model, the large shareholder plays a dominant role in hiring the manager to operate the firm and determining the manager's optimal compensation contracts. For simplicity, we ignore strategic behavior among different blockholders and refer to the firm's group of blockholders by a single representative large shareholder. Based on prior research on firm ownership including the aforementioned studies, we consider different holding periods of these two groups of shareholders. More specifically, we assume that the large shareholder continues to hold her equity stake in the firm for a finite-time horizon, whereas small shareholders trade shares of the firm continuously. We later discuss the extension of the model briefly that allows for the ownership dynamics of the large shareholder over time as in DeMarzo and Urošević (2006). In addition to the firm's shares, both types of investors can trade a risk-free bond (savings account) continuously.

As in traditional principal-agent models with moral hazard (see Laffont and Martimort (2002)), the manager can affect the firm's output process by exerting a costly effort. The firm's investors do not observe the manager's effort so that they cannot distinguish it from the underlying exogenous shock in the firm's output. The large shareholder induces the manager to choose a desired level of effort only indirectly by offering incentive contracts that are contingent on the firm's output. For simplicity, the manager, unlike investors, is assumed to trade no securities so that he consumes solely from the compensation paid by the firm. We now describe the various elements of the model in detail.

2.1 Firm's Output Process

Time is continuous over a finite horizon $t \in [0, T]$. A standard Brownian motion Z_t on a complete probability space $(\Omega, \mathcal{F}, P)$ with the information filtration $\mathbb{F} = \{\mathcal{F}_t, 0 \leq t \leq T\}$ is the source of

¹See Shleifer and Vishny (1997), Tirole (2006), Holderness (2009) and Cronqvist and Fahlenbrach (2009) for detailed discussion. In particular, the last study shows significant blockholder effects on a number of important corporate decisions such as investment, financial, and executive compensation policies.

uncertainty in the firm's output process. As in the standard principal-agent model, we assume that the distribution of the firm's cumulative output is affected by the manager's effort or action, a_t . The firm's cumulative output X_t evolves as follows:

$$dX_t = a_t dt + \sigma dZ_t, \tag{1}$$

where a_t is the manager's effort choice and $\sigma > 0$ is a constant that represents the exogenous noise level in the firm's output. As in the standard principal-agent problem, we assume that managerial effort does not affect the volatility of the firm's output process.

2.2 Managerial Preference and Contracts

The firm hires a risk-averse manager under the control of its large shareholder. The manager chooses an effort level $a_t \in [\underline{a}, \bar{a}]$ at time $t \in [0, T]$ by incurring effort cost $\Psi(a_t)$ measured in the same unit as his consumption, which is strictly increasing, convex, and twice continuously differentiable.

The manager's effort, which affects the firm's total output process X_t as shown by (1), is unobservable to the firm's shareholders. The manager receives dynamic incentives through his optimal contracts that are explicitly contingent on the contractible output process. As in traditional principal-agent models with moral hazard, it is convenient to augment the definition of the manager's contract to also include the manager's effort process $\{a_t, t \in [0, T]\}$. Formally, a contract $\Gamma \equiv [c_M, C_M, a]$ is a stochastic process describing the manager's compensation $c_{M,t}$ at time $t < T$ and C_M at time T and his effort level a_t at time t . If $\mathbb{F}_t$ denotes the information filtration generated by the history of the firm's output process up to time t , the process Γ is $\mathbb{F}_t$ -adapted. We require that the manager's contract be incentive compatible and feasible as shown below.

As is standard, we assume that the manager's trading in the firm's shares (insider trading) is restricted. We also do not consider the manager's saving, so that the only source of income for the

manager's consumption process is his compensation paid by the firm.² Given the manager's contract $\Gamma = [c_M, C_M, a]$, the manager's total expected utility (or his value function) at time $t \in [0, T]$ is given by

$$W_{M,t}(C_M, c_M, a) \equiv e^{\delta_M t} V_{M,t}(C_M, c_M, a) = e^{\delta_M t} E_{M,t}^a \left[\int_t^T u^M(\tau, c_{M,\tau}, a_\tau) d\tau + U^M(T, C_{M,T}) \right], \quad (2)$$

where u^M and U^M represent the manager's utility functions which are discounted to time zero and continuously differentiable with the following properties: $u_c^M \equiv \partial u^M / \partial c > 0$, $u_{cc}^M \equiv \partial^2 u^M / \partial c^2 < 0$, $u_a^M \equiv \partial u^M / \partial a < 0$, $u_{aa}^M \equiv \partial^2 u^M / \partial a^2 < 0$, $U_c^M \equiv \partial U^M / \partial c > 0$, and $U_{cc}^M \equiv \partial^2 U^M / \partial c^2 < 0$. Note that the above expectation is taken with respect to the probability distribution determined by the manager's effort process a .

A contract $\Gamma = [c_M, C_M, a]$ must be incentive compatible (IC) for the manager, that is, the recommended effort level in the contract maximizes the manager's value function as shown by

$$a = \arg \max_{a'} V_{M,t}(C_M, c_M, a'). \quad (3)$$

In addition, a contract Γ must be feasible, that is, the manager's expected utility at any time is at least as great as his reservation utility $\underline{V}^M$:

$$V_{M,t}(C_M, c_M, a) \geq \underline{V}^M. \quad (4)$$

For simplicity, we assume that the above individual rationality (IR) constraint is imposed only at $t = 0$ when the manager is hired and that she will commit to work afterwards until $t = T$.

²See Edmans et. al (2012) for the impacts of the manager's private saving on his incentives and optimal contracts.

2.3 Large and Small Shareholders

The firm has a representative large shareholder indexed by L and a continuum of small, dispersed shareholders indexed by S (uniformly indexed over the unit interval, that is, $S \in [0, 1]$). We normalize the total number of the firm's shares outstanding to one. We denote the large shareholder's equity stake in the firm by Θ , so that the total number of shares collectively held by small shareholders is $1 - \Theta$. While the large shareholder continues to hold its equity stake chosen at time zero, small shareholders continuously trade the firm's shares at P_t at time $t \geq 0$. In addition to the firm's shares, a risk-free bond is available to investors. We assume the perfectly elastic supply of the risk-free bond so that it pays a continuously compounded constant return $r > 0$. This assumption can be relaxed by alternatively assuming the zero supply of the risk-free bond which endogenously determines the risk-free rate over time. The investors are initially endowed with wealth $Y_{i,0} \geq 0$, for $i = L, S$.

We characterize these investors' preferences by $(u^i(t, c_{i,t}), U^i(C_{i,T}))$ for $i = L, S$, where $c_{i,t}$ and $C_{i,T}$ represent their consumption at time t and T , respectively, and all the functions are also discounted to time zero and continuously differentiable with the following properties: $u_c^i \equiv \partial u^i / \partial c > 0$, $u_{cc}^i \equiv \partial^2 u^i / \partial c^2 < 0$, $U_c^i \equiv \partial U^i / \partial c > 0$, and $U_{cc}^i \equiv \partial^2 U^i / \partial c^2 < 0$. Then their value functions at time t are given by

$$W_{i,t} \equiv e^{\delta_i t} V_{i,t}(C_i, c_i) \equiv e^{\delta_i t} E_{i,t}^{\hat{a}} \left[\int_t^T u^i(\tau, c_{i,\tau}) d\tau + U^i(T, C_i) \right], \quad \text{for } i = L, S. \quad (5)$$

Note that the above expectation is taken with respect to the manager's action that is anticipated by investors, $\hat{a}$.

We first consider the large shareholder's problem. She chooses her equity stake Θ in the firm at time zero, determines the manager's optimal contract $\Gamma = [c_M, C_M, a]$, and continuously adjusts its

consumption $(C_{L,T}, c_{L,t})$ and risk-free savings account $B_{L,t}$. The budget constraint is then given by

$$dB_{L,t} = (rB_{L,t} - c_{L,t})dt + \Theta(dX_t - c_{M,t}dt), \quad (6)$$

which reflects the fact that, in a time interval $(t, t + dt)$, the large shareholder receives a dividend payment from its shares Θ which amounts to the change in the firm's cumulative output minus the manager's compensation payment within the time interval.

The managerial contract is determined in a way that maximizes the large shareholder's value function subject to the IC and IR constraints for the manager as specified in (3) and (4). Following the dynamic contracting literature (Williams, 2009; Sannikov, 2008; Cvitanic and Zhang, 2013), we represent the manager's (time-shifted) value function as the following stochastic process:

$$dV_{M,t} = -u^M(t, c_{M,t}, a_t)dt + G_{M,t}\sigma dZ_t, \quad (7)$$

where $G_{M,t}$ is a $\mathbb{F}_t$ -adapted process that represents the sensitivity of the manager's utility to the exogenous shock in the firm's output and plays a key role in the provision of incentives as will become clear below. The manager's optimal contract process Γ can then be written in terms of the manager's total utility process $W_{M,t}$. As the aforementioned studies show with a detailed proof (see, for example, Williams (2009)), the IC constraint for the manager can be replaced by the following local IC constraint under certain technical conditions:

$$G_{M,t} = -u_a^M(t, c_{M,t}, a_t), 0 \leq t \leq T, \text{ a.s.}, \quad (8)$$

which corresponds to the first-order approach in static principal-agent problems. At the optimal effort, we can substitute the above condition for $G_{M,t}$ in (7).

Small shareholders, however, do not influence the firm's contracting decision, but trade the shares of

the firm competitively as a price taker by rationally anticipating the large shareholder's equity stake $\hat{\Theta}$ and the manager's effort $\hat{a}_t$. A small shareholder S's budget constraint within a time interval $(t, t + dt)$ is then given by

$$dB_{S,t} = (rB_{S,t} - c_{S,t})dt + \theta_{S,t}(dX_t - c_{M,t}dt) - P_t d\theta_{S,t}, \quad (9)$$

where $B_{S,t}$ and $\theta_{S,t}$ are his holdings in the risk-free bond and the firm's stock. The small shareholder's wealth process $Y_{S,t}$ from his portfolio holdings is given by $Y_{S,t} = B_{S,t} + \theta_{S,t}P_t$, which thus evolves according to

$$dY_{S,t} = (rY_{S,t} - c_{S,t})dt + \theta_{S,t}(dP_t + dX_t - c_{M,t}dt - rP_tdt). \quad (10)$$

The investor maximizes his value function, that is, his expected utility from the remaining consumption process, by making an optimal consumption and portfolio decision at each point in time.

2.4 Equilibrium Characterization

An equilibrium for this economy is given by the optimal managerial contract process $\Gamma = [C_M, c_M, a]$, the optimal consumption and portfolio choices of the large and small shareholders $(C_i, c_i, B_i, \Theta, \theta_S)$ for $i = L, S$, and the stock price P such that

- The manager exerts effort a that maximizes the manager's expected utility for a given (C_M, c_M) .
- (Γ, Θ) maximizes the large shareholder's expected utility subject to the IC and IR constraints for the manager.
- Small shareholders rationally anticipate the large shareholder's ownership stake and the manager's effort level: $\hat{\Theta} = \Theta$; $\hat{a} = a$.
- The large and small shareholders optimally choose their portfolio holdings and consumption process, $(C_i, c_i, B_i, \Theta, \theta_S)$ for $i = L, S$, given the risk-free rate r and stock price P .
- For any Θ chosen by the large shareholder, the stock price P clears the market: $\int_0^1 \theta_{S,t} dS = 1 - \Theta$.

3 Equilibrium Solution with CARA Preferences

In order to fully solve for the equilibrium, we assume the following for the rest of the paper. First, all the players have constant absolute risk aversion (CARA) preferences over consumption with different risk-aversion parameters, γ_i for $i = L, M, S$. Second, their subjective discount rates are identical and denoted by δ . Third, the manager's disutility of effort is given by a quadratic function: $\Psi(a) = \frac{1}{2}\varphi a^2$ where $\varphi > 0$ is a constant. More specifically, their utility functions evaluated at time zero are

$$\begin{aligned} u^M(t, c_{M,t}, a_t) &= -\frac{1}{\gamma_M} \exp\left(-\delta t - \gamma_M \left(c_{M,t} - \frac{1}{2}\varphi a_t^2\right)\right), \quad U^M(T, C_{M,T}) = -\frac{1}{\gamma_M} \exp(-\delta T - \gamma_M C_{M,T}), \\ u^i(t, c_{i,t}) &= -\frac{1}{\gamma_i} \exp(-\delta t - \gamma_i c_{i,t}), \quad U^i(T, C_{i,T}) = -\frac{1}{\gamma_i} \exp(-\delta T - \gamma_i C_{i,T}) \quad \text{for } i = L, S. \end{aligned} \quad (11)$$

With the assumptions above, we now turn to our main analysis to derive the manager's optimal contract and stock market equilibrium under moral hazard in the presence of a large shareholder. First, for any given equity stake Θ by the large shareholder, we solve her value function maximization problem, while taking into account the manager's incentive constraint. Second, we solve small shareholders' consumption and portfolio choices which determine the equilibrium share price of the firm. Finally, we consider the large shareholder's ownership decision at time zero. All the detailed proofs are provided in the Appendix.

3.1 The Large Shareholder's Problem and Manager's Contract

Suppose that the large shareholder holds an equity stake of Θ in the firm. The large shareholder's value function maximization problem at time t is given by

$$W_{L,t} = e^{\delta t} V_{L,t} = e^{\delta t} \sup E_{L,t}^a \left[\int_t^T u^L(\tau, c_{L,\tau}) d\tau + U^L(T, C_L) \right], \quad (12)$$

where her consumption and managerial contract are optimally determined subject to the dynamics of two state variables. The first one is the dynamics of the large shareholder's risk-free bond balance $B_{L,t}$ in (6), and the other one is that of the manager's expected utility $W_{M,t}$ in (7), which, along with (8), leads to

$$dW_{M,t} = \left[\delta W_{M,t} - e^{\delta t} u^M(t, c_{M,t}, a_t) \right] dt - e^{\delta t} u_a^M(t, c_{M,t}, a_t) \sigma dZ_t. \quad (13)$$

We solve the above problem using the Hamilton-Jacobi-Bellman (HJB) approach and summarize the result in the following proposition.

Proposition 1

Given the large shareholder's equity stake Θ , its optimal value function has the form of

$$W_{L,t}^* = -k_1 (-W_{M,t}^*)^{-\frac{\gamma_L}{\gamma_M} \Theta} e^{-\gamma_L r B_{L,t}}. \quad (14)$$

The optimal policies are thus

$$\begin{aligned} a^* &= k_2, \\ c_L^* &= -\frac{1}{\gamma_L} \ln(\gamma_L r k_1) + \frac{\Theta}{\gamma_M} \ln(-W_{M,t}^*) + r B_{L,t}, \\ c_M^* &= -\frac{1}{\gamma_M} \ln(k_3) - \frac{1}{\gamma_M} \ln(-W_{M,t}^*) + \frac{1}{2} \varphi(a^*)^2, \end{aligned} \quad (15)$$

where k_1, k_2 , and k_3 are determined by

$$\frac{\gamma_L}{\gamma_M} k_3 - \gamma_L r - \gamma_L^2 r \Theta \sigma (\sigma \varphi k_2 k_3) + \gamma_L \left(1 + \frac{\gamma_L}{\gamma_M} \Theta \right) (\sigma \varphi k_2 k_3)^2 = 0, \quad (16)$$

$$\frac{\gamma_L}{\gamma_M} \varphi k_2 k_3 - \gamma_L r - \frac{\gamma_L^2}{\gamma_M} r \Theta \sigma (1 + \gamma_M \varphi k_2^2) \sigma \varphi k_3 + \frac{\gamma_L}{\gamma_M} \left(1 + \frac{\gamma_L}{\gamma_M} \Theta \right) (1 + \gamma_M \varphi k_2^2) \sigma^2 \varphi^2 k_2 k_3^2 = 0, \quad (17)$$

$$\begin{aligned} r \ln(\gamma_L r k_1) &= -\delta + \frac{\gamma_L}{\gamma_M} \Theta \left(\frac{k_3}{\gamma_M} - \delta \right) - \frac{\gamma_L}{\gamma_M} r \Theta \ln(k_3) + \frac{1}{2} \gamma_L r \Theta \varphi k_2^2 \\ &\quad - \gamma_L r \Theta k_2 - \frac{\gamma_L^2}{\gamma_M} r \Theta^2 \sigma (\sigma \varphi k_2 k_3) + \frac{1}{2} \frac{\gamma_L}{\gamma_M} \Theta \left(1 + \frac{\gamma_L}{\gamma_M} \Theta \right) (\sigma \varphi k_2 k_3)^2 + \frac{1}{2} (\gamma_L r \Theta \sigma)^2 + r. \end{aligned} \quad (18)$$

Under the optimal strategy above, the manager's expected utility process from his remaining optimal compensation contract follows

$$dW_M^* = \mu_W^* W_M^* dt - \sigma_W^* W_M^* dZ_t, \quad \text{or} \quad (19)$$

$$d \ln(-W_{M,t}^*) = \left(\mu_W^* - \frac{1}{2} (\sigma_W^*)^2 \right) dt - \sigma_W^* dZ_t, \quad (20)$$

where

$$\mu_W^* = \delta - \frac{k_3}{\gamma_M}; \quad \sigma_W^* = \sigma \varphi k_2 k_3. \quad (21)$$

By Proposition 1, the manager's effort desired by the large shareholder is a constant that is determined by the large shareholder's ownership stake Θ as well as model parameters. The large shareholder's consumption and the manager's compensation are linear in the log of the manager's expected utility process $W_{M,t}$. The linear forms are consistent with the prior dynamic contracting study with a CARA-normal setting. Since the agent's expected utility is based on the optimal compensation contract $\Gamma = [C_M, c_M, a]$, we can infer some implications of the contract by looking at how his total expected utility evolves over time. More specifically, by (19), the stochastic state variable follows a geometric Brownian motion with constant drift μ_W^* and volatility σ_W^* , which are also deterministic as shown by (21). By (15), (20) and (1), we obtain the manager's incremental compensation at time t by

$$dc_{M,t}^* = -\frac{1}{\gamma_M} \left(\mu_W^* - \frac{1}{2}(\sigma_W^*)^2 + \frac{\sigma_W^*}{\sigma} a_t^* \right) dt + \frac{\sigma_W^*}{\gamma_M \sigma} dX_t. \quad (22)$$

By (22), we define the first measure of pay-performance sensitivity (PPS) as the dollar change in CEO pay for a dollar change in the firm's cumulative output X_t :

$$PPS_1^* = \frac{dc_{M,t}^*}{dX_t} = \frac{\sigma_W^*}{\gamma_M \sigma} = \frac{\varphi k_2 k_3}{\gamma_M}, \quad (23)$$

where the last equality follows from (21). It is important to note that this PPS measure, as well as another measure that will be shown later, is positively associated with the volatility term σ_W^* of the manager's expected utility process. To put it differently, managerial incentives provided through the optimal contract effectively affects the volatility term in his optimal utility process.

3.2 Small Shareholders' Problem and Equilibrium Stock Price

Given an anticipated ownership by the large shareholder, small shareholders can forecast the manager's optimal compensation contract, including his optimal effort level that affects the firm's output process. They trade the shares of the firm competitively, that is, they take the stock price as given in making

their portfolio choice. We similarly define the value function maximization of a small shareholder S at time t as:

$$W_{S,t} = e^{\delta t} V_{S,t} = \sup E_{S,t}^a \left[\int_t^T u_S(\tau, c_{S,\tau}) d\tau + U_S(T, C_S) \right], \quad (24)$$

where his optimal trading strategy and consumption are determined. The following proposition characterizes the stock market equilibrium derived from the above problem along with the stock market clearing condition.

Proposition 2

For any Θ chosen by the large shareholder, there exists a competitive stock market equilibrium that endogenously determines the stock price by

$$P_t^* = \lambda + \frac{1}{\gamma_M r} \ln(-W_{M,t}^*) \text{ where} \\ r\lambda = \frac{1}{\gamma_M r} \left(\mu_W^* - \frac{1}{2}(\sigma_W^*)^2 \right) + k_2 - \frac{1}{2}\varphi k_2^2 + \frac{1}{\gamma_M} \ln(k_3) - (1 - \Theta)\gamma_S r \left(\sigma - \frac{\sigma_W^*}{\gamma_M r} \right)^2. \quad (25)$$

In addition, the optimal value function of a small shareholder S is given by

$$W_{S,t}^* = -\frac{\nu}{\gamma_S} e^{-\gamma_S r Y_{S,t}}, \text{ where } r \ln(r\nu) = r - \delta - \frac{1}{2} \left[(1 - \Theta)\gamma_S r \left(\sigma - \frac{\sigma_W^*}{\gamma_M r} \right) \right]^2, \quad (26)$$

where μ_W^* and σ_W^* are given by (21).

As shown in Propositions 1 and 2, the equilibrium share price P_t^* and the manager's optimal compensation $c_{M,t}^*$ are positively and negatively related to the state variable, $\ln(-W_{M,t}^*)$, respectively. This observation suggests a negative relation between CEO pay and contemporaneous share price, which appears because the current stock price reflects the costs of compensating the manager that are shared by shareholders.

Further, we define the excess dollar return for holding one share of the firm's stock within the time interval $(t, t + dt)$ as

$$\begin{aligned} dR_t^* &\equiv dP_t^* + dX_t - c_{M,t}^* dt - rP_t^* dt = (1 - \Theta)\gamma_S r \left(\sigma - \frac{\sigma_W^*}{\gamma_M r} \right)^2 dt + \left(\sigma - \frac{\sigma_W^*}{\gamma_M r} \right) dZ_t \\ &= \left[(1 - \Theta)\gamma_S r \left(\sigma - \frac{\sigma_W^*}{\gamma_M r} \right)^2 - \left(\sigma - \frac{\sigma_W^*}{\gamma_M r} \right) \frac{k_2}{\sigma} \right] dt + \frac{1}{\sigma} \left(\sigma - \frac{\sigma_W^*}{\gamma_M r} \right) dX_t. \end{aligned} \quad (27)$$

By combining the above definition of the excess dollar return on the stock and (22), we derive another measure of managerial incentives as the dollar change in CEO pay for a dollar change in the excess dollar return on the stock:

$$PPS_2 = \frac{dc_{M,t}^*}{dR_t^*} = \frac{r\sigma_W^*}{\gamma_M r\sigma - \sigma_W^*}. \quad (28)$$

We now consider the stock market equilibrium variables. The drift and volatility terms in the first equation of (27) represent the expected stock return and stock return volatility in equilibrium:

$$\mu_R^* = (1 - \Theta)\gamma_S r \left(\sigma - \frac{\sigma_W^*}{\gamma_M r} \right)^2, \quad (29)$$

$$\sigma_R^* = \sigma - \frac{\sigma_W^*}{\gamma_M r}, \quad (30)$$

which implies the positive relation between these two variables as follows:

$$\mu_R^* = (1 - \Theta)\gamma_S r \sigma_R^{*2}. \quad (31)$$

Note that this mean-variance relation arises because the collective demand for the stock by small shareholders with CARA preferences is competitively determined by the expected dividends adjusted for the investor's risk premium, $\mu_R^*/(\gamma_S r \sigma_R^{*2})$, which, in turn, must equal the supply of the shares, $1 - \Theta$, in order for the stock market clearing condition to hold. In other words, the large shareholder's ownership has an influence on the expected stock return μ_R^* both directly and indirectly. On the one hand, its ownership stake reduces the stock liquidity available to small shareholders, thereby increasing the current stock price and, therefore, lowering the expected stock return. By Proposition 1, on the other hand, its ownership also affects the volatility $\sigma_W(\Theta)$ of the manager's expected utility process. As implied by the two measures of managerial incentives, (23) and (28), the volatility term is effectively chosen by the large shareholder through her implementation of the manager's contract. That is, a higher sensitivity of CEO pay to the firm's output or stock return leads to more volatile compensation

payments and, therefore, more volatile expected utility process for the manager. Since shareholders receive the dividend streams net of the manager's compensation payments, more volatile CEO pay implies less volatile residual dividend streams and, thereby, lower stock return volatility.

3.3 The Large Shareholder's Ownership Choice

Using the equilibrium share price determined above, we now solve for the large shareholder's optimal equity stake Θ^* . At time zero, the large shareholder's value function (14) can be rewritten as a function of the level of the large shareholder ownership Θ ,

$$W_{L,0} = -k_1(\Theta) \exp[-\gamma_L r (Y_{L,0} - \Theta \lambda(\Theta))] = -\exp \left[-\gamma_L r \left(Y_{L,0} + \frac{1}{\gamma_L r} \left(\ln(\gamma_L r) + \frac{\delta}{r} - 1 \right) + \Omega_L(\Theta) \right) \right],$$

$$\text{where } \Omega_L(\Theta) = \Theta(1 - \Theta) \gamma_S \left(\sigma - \frac{\sigma_W}{\gamma_M r} \right)^2 - \frac{1}{2} \gamma_L \Theta^2 \left(\sigma - \frac{\sigma_W}{\gamma_M r} \right)^2 \quad (32)$$

In the above, the first equality is obtained by her budget constraint at time zero, $\Theta P_0 + B_{L,0} = Y_{L,0}$, and the stock price equilibrium (25), and the second equality is by the derivation of the two constants k_1 and λ as a function of Θ in (18) and (25), which leads to the following proposition.

Proposition 3

The large shareholder's optimal ownership policy at time zero solves

$$\Theta^* = \arg \max_{\Theta} W_{L,0} = \arg \max_{\Theta} \Omega_L(\Theta) \left(= \frac{1}{r} \Theta \left[\mu_R^*(\Theta) - \frac{1}{2} \gamma_L r \Theta \sigma_R^*(\Theta)^2 \right] = \Lambda(\Theta) \sigma_R^*(\Theta)^2 \right), \quad (33)$$

where $\Lambda(\Theta) = \Theta [(1 - \Theta) \gamma_S - \frac{1}{2} \Theta \gamma_L]$.

The large shareholder's objective function above consists of two different parts. The first one is $\Lambda(\Theta)$, a quadratic function of Θ that depends on the risk aversions of the large and small shareholders. The second one, more importantly, is $\sigma_R(\Theta)^2$, which, as discussed above, reflects the influence the large shareholder's ownership has on the firm's stock return volatility through her control over the manager's compensation contract. If the second force did not exist, that is, there were no impact of the large

shareholder's ownership on the firm's stock return volatility σ_R , her optimal ownership would be simply

$$\bar{\Theta} \equiv \frac{1}{2 + \gamma_L/\gamma_S}, \quad (34)$$

which maximizes $\Lambda(\Theta)$. This ownership level is, in fact, what the large shareholder would choose if there were no contracting problem with a manager and, therefore, no influence of the large shareholder on the firm's stock return. In the presence of the principal-agent problem, however, the large shareholder, as anecdotal and empirical evidence suggest, plays a dominant role in implementing executive compensation in our model, which suggests the following corollary:

Corollary 1

If there exists an interior solution for $\Theta^ \in (0, 1)$ under moral hazard, then the solution satisfies*

$$-\frac{\sigma_R^{*'}(\Theta^*)}{\sigma_R^*(\Theta^*)} \left(= \frac{\sigma_W^{*'}(\Theta^*)}{\gamma_M r \sigma - \sigma_W^*(\Theta^*)} \right) = \frac{\Lambda'(\Theta^*)}{2\Lambda(\Theta^*)}. \quad (35)$$

where σ_W^* is given by (21).

Under a reasonable set of parameter values, as will be shown later, the equilibrium stock return volatility declines with large shareholder ownership, that is, $\sigma_R^{*'}(\Theta) < 0$, so that the optimal ownership stake chosen by the large shareholder is determined at a level lower than $\bar{\Theta}$ in the case of no principal-agent problem. Given the optimal ownership stake Θ^* by the large shareholder, the small shareholder's value function (24) at time zero can be similarly derived as

$$\begin{aligned} W_{S,0} &= -\exp \left[-\gamma_S r \left(Y_{S,0} + \frac{1}{\gamma_S r} \left(\ln(\gamma_S r) + \frac{\delta}{r} - 1 \right) + \Omega_S(\Theta^*) \right) \right], \\ \text{where } \Omega_S(\Theta^*) &= \frac{1}{r} (1 - \Theta^*) \left[\mu_R(\Theta^*) - \frac{1}{2} \gamma_S r (1 - \Theta^*) \sigma_R^*(\Theta^*)^2 \right]. \end{aligned} \quad (36)$$

4 Results and Implications

We now derive our main results by analyzing the properties of the equilibrium. We first compare the first-best and second-best equilibrium solutions analytically and then provide additional implications

of the model by calibrating it to empirical moments for baseline parameter values and performing a sensitivity analysis from the calibrated model.

4.1 Comparison with the First-best Solution

In what follows, we illustrate the first-best solution to analytically compare with the equilibrium solution under moral hazard. The procedure to solve for the first-best solution is largely similar to the approach employed to obtain the second-best solution under moral hazard except that the manager's IC constraint (8) is not binding. The manager's utility process $V_{M,t}$ simply follows (7), where the volatility term $G_{M,t}$ is freely chosen by the large shareholder because no incentive provision is required when the manager's choice of action is observable and verifiable. The following proposition shows the closed-form solution for the first-best case.

Proposition 4

Given the large shareholder's equity stake Θ , her optimal value function is of the form

$$W_{L,t}^{FB} = -k^{FB}(-W_{M,t}^{FB})^{-\frac{\gamma_L}{\gamma_M}\Theta} e^{-\gamma_L r B_{L,t}}, \quad (37)$$

and the optimal policies from her value function maximization are given by

$$\begin{aligned} a_t^{FB} &= \frac{1}{\varphi}, \\ c_{L,t}^{FB} &= -\frac{1}{\gamma_L} \ln(\gamma_L r k^{FB}) + \frac{\Theta}{\gamma_M} \ln(-W_{M,t}^{FB}) + r B_{L,t}, \\ c_{M,t}^{FB} &= -\frac{1}{\gamma_M} \ln(\gamma_M r) - \frac{1}{\gamma_M} \ln(-W_{M,t}^{FB}) + \frac{1}{2} \varphi (a_t^{FB})^2, \\ \hat{G}_{M,t} &\equiv e^{\delta t} G_{M,t} = \frac{\gamma_L r \Theta}{\left(1 + \frac{\gamma_L}{\gamma_M} \Theta\right)} (-W_{M,t}^{FB}). \end{aligned} \quad (38)$$

Under the optimal strategy, the manager's utility process also follows a geometric Brownian motion with drift μ_W^{FB} and volatility σ_W^{FB} :

$$\mu_W^{FB} = \delta - r; \quad \sigma_W^{FB} = \frac{\gamma_L r \Theta \sigma}{\left(1 + \frac{\gamma_L}{\gamma_M} \Theta\right)}. \quad (39)$$

The equilibrium stock price from the small shareholders' value function maximization is

$$P_t^{FB} = \lambda^{FB} + \frac{1}{\gamma_M r} \ln(-W_{M,t}^{FB}), \text{ where}$$

$$r\lambda^{FB} = \frac{1}{\gamma_M r} \left(\mu_W^{FB} - \frac{1}{2}(\sigma_W^{FB})^2 \right) + \frac{1}{2\varphi} + \frac{1}{\gamma_M} \ln(\gamma_M r) - (1 - \Theta)\gamma_S r \left(\sigma - \frac{\sigma_W^{FB}}{\gamma_M r} \right)^2. \quad (40)$$

Finally, the large shareholder's optimal equity stake in the firm is given by

$$\Theta^{FB} = \frac{1}{2 + \gamma_L \left(\frac{1}{\gamma_M} + \frac{1}{\gamma_S} \right)}. \quad (41)$$

Note that, with the observability of the manager's effort or action, its optimal level is simply determined by the parameter φ of the manager's effort cost as usual in a CARA-normal framework. It is worth noting that the optimal policies as well as the equilibrium stock price above have the same form as in the second-best case, but the constants specifying these variables are determined at a different level. Further, compared to (34) in the case of no principal-agent problem, the large shareholder's optimal equity stake is smaller in the first-best case, which is due to the additional cost of risk sharing with the manager. In a similar manner to the second-best case, we derive the two measures of managerial incentives as well as the expected stock return and stock return volatility in the following corollary.

Corollary 2

The sensitivities of CEO pay to the firm's total output and to the stock return, respectively, are given by

$$PPS_1^{FB} = \frac{dc_{M,t}^{FB}}{dX_t} = \frac{\sigma_W^{FB}}{\gamma_M \sigma} = \frac{\left(\frac{\gamma_L}{\gamma_M} \Theta^{FB} \right) r}{\left(1 + \frac{\gamma_L}{\gamma_M} \Theta^{FB} \right)}, \quad (42)$$

$$PPS_2^{FB} = \frac{dc_{M,t}^{FB}}{dR_t^{FB}} = \frac{\sigma_W^{FB} r}{\gamma_M r \sigma - \sigma_W^{FB}} = \left(\frac{\gamma_L}{\gamma_M} \Theta^{FB} \right) r. \quad (43)$$

The resulting expected stock return and stock return volatility are

$$\mu_R^{FB} = (1 - \Theta^{FB})\gamma_S r \left(\sigma - \frac{\sigma_W^{FB}}{\gamma_M r} \right)^2 = \frac{r(1 - \Theta^{FB})\gamma_S \sigma^2}{\left(1 + \frac{\gamma_L}{\gamma_M} \Theta^{FB} \right)^2}, \quad (44)$$

$$\sigma_R^{FB} = \sigma - \frac{\sigma_W^{FB}}{\gamma_M r} = \frac{\sigma}{\left(1 + \frac{\gamma_L}{\gamma_M} \Theta^{FB} \right)}, \quad (45)$$

where Θ^{FB} is given by (41).

As in the standard principal-agent model, the first-best solution with the risk-averse principal and agent is pinned down by the principal's optimal risk sharing incentives. As one can see from (42) and (43) along with (41), the PPS measures increase with the large shareholder's risk aversion γ_L , but decrease with the manager's risk aversion γ_M . In our setup, however, there is another party sharing the firm's risk—small shareholders. Note that the manager's contract would be effectively deterministic if Θ were small enough, which implies risk bearing by small shareholders. Yet, since small shareholders could rationally anticipate the large shareholder's decision which would be reflected in the equilibrium stock price, the large shareholder's optimal ownership choice would take into account the risk-aversion of small shareholders. The following proposition summarizes the analytical comparison between the first-best and second-best solutions, whose proof is provided in the Appendix.

Proposition 5

- *The manager's optimal effort level under moral hazard is lower than the first-best effort level.*
- *The drift of the manager's expected utility process is greater in the second-best case.*
- *If $\frac{\sigma_W^* '(\Theta^{FB})}{\gamma_M \tau \sigma - \sigma_W(\Theta^{FB})} < \frac{\gamma_L}{\gamma_M + \gamma_L \Theta^{FB}}$ where Θ^{FB} is given by (41), under moral hazard,*
 - *the large shareholder's optimal ownership stake is greater,*
 - *the volatility of the manager's expected utility process is greater,*
 - *both measures of managerial incentives (pay-performance sensitivity (PPS)) are higher, and*
 - *the expected excess dollar return from holding a share of the stock within the unit time interval and its volatility are lower.*

The condition in the last argument implies that, under moral hazard, the large shareholder would increase the volatility of the manager's utility process at a lower rate as she increases her share in the firm, compared to the first-best case. In order to compensate for the risk from holding the additional share in the firm, the large shareholder would effectively increase the volatility through the optimal managerial contract. Due to incentive provisions under moral hazard, however, it is reasonable to assume that that adjustment can be smaller in the second-best case than in the first-best case. In our calibration of the model in Section 4.2, the condition holds under the baseline values of parameters so that the analytical comparison between the first-best and second-best solutions in the last argument is also confirmed quantitatively.

4.2 Quantitative Analysis

In this section, we derive additional implications of the model through a quantitative analysis. We first calibrate the model to obtain a reasonable set of baseline parameter values and then examine the quantitative effects of the model parameters by varying each of them about their baseline values in the sensitivity analysis.

4.2.1 Model Calibration

We set the risk-free rate r to 2% (the average real interest rate per annum from the 6-month commercial paper rate that is reported in Guvenen (2009)) and the time discount rate δ to 0.0513 (so that the annual discount factor $e^{-\delta} = 0.95$). We take the volatility of the firm's cash flow $\sigma = 0.25$ from He (2011).

Table I lists the remaining parameters of the model whose baseline values are chosen to match key empirical moments. The empirical moments we attempt to match in our calibration exercise include (i) the median shares (37%) owned by large shareholders (or blockholders) in a randomly selected, CRSP- and Compustat-listed corporation as reported in Holderness (2009); (ii) the dollar-dollar CEO incentives (a manager's wealth rise of about \$3 for a \$1,000 increase in firm value; that is, 0.003) in Edmans et al. (2008); (iii) and (iv) the mean and volatility of annual real stock return (8.2% and 19.4%) from Standard and Poor's 500 index that are reported in Guvenen (2009).

For a candidate vector of parameter values, we compute the model-predicted values of the equilibrium variables in the second-best case under moral hazard. The baseline values of the parameters are those that minimize the distance between the model-predicted and observed values of the moments. As shown in Table II, the model is able to match the empirical moments reasonably well. The calibrated parameters in Table III suggest that, in order to match the empirical moments, the manager as well as small shareholders are significantly more risk averse than the large shareholder, which is consistent with anecdotal and empirical evidence. It should be noted, however, that this calibration exercise is

necessary to ensure that our subsequent quantitative analysis provides directional predictions that are likely to be empirically relevant, not to suggest the unique set of model parameters that reconcile the model with the data.

Figure 1 compares the equilibrium solutions in the first-best and second-best cases for different levels of the large shareholder's ownership Θ . As is standard, the large shareholder's optimal risk sharing incentive determines the extent to which managerial compensation is sensitive to the firm's output or its stock return. As her equity stake increases, the optimal sensitivity of CEO pay becomes larger, thereby reducing the volatility of the firm's residual output and, therefore, its stock return volatility. In addition to risk sharing incentives, the second-best pps measures capture the additional sensitivities of CEO pay required to induce the desired effort level indirectly through the manager's contract when the agent's effort is not observable. We also observe that, for a given level of large shareholder ownership, the expected stock return and stock return volatility are lower in the second-best case because the residual cash flow net of managerial compensation payments is less volatile due to greater sensitivity of CEO pay for incentive provision under moral hazard. The last two graphs show the values of $\Omega_L(\Theta)$ and $\Omega_S(\Theta)$ in the large and small shareholders' value functions at time zero, given by (32) and (36), respectively. The large shareholder's optimal level of Θ is determined at a level that maximizes the concave function of the former graph, from which we see that a slightly higher equity stake is chosen by the large shareholder under moral hazard compared to the first-best level.

4.2.2 Sensitivity Analysis

To provide further implications of the model, we now explore the quantitative effects of the model parameters using the baseline parameter values. More specifically, we vary each of the parameters—the underlying exogenous randomness σ in the firm's output, the cost of managerial effort φ , and each player's risk aversion γ_i for $i = L, M, S$ —about their baseline values, respectively, and look at both the first-best and second-best equilibrium solutions. We report in Figures 2-6 the variations in the

manager's optimal effort level, the drift and volatility of the manager's expected utility process, two measures of managerial incentives, the large shareholder's optimal ownership choice, and the expected stock return and stock return volatility.

Figures 2 and 3 display the effects of the exogenous noise level and the cost of managerial effort. An increase in either of these parameters makes the provision of managerial incentives more costly, thereby lowering the PPS measures and optimal managerial effort level. In either case, the large shareholder's ownership stake declines, whereas the expected stock return and stock return volatility increase. In the case of an increase in σ , on the one hand, despite lower incentive provisions offered through the managerial contract, the more volatile underlying firm output itself increases the volatility of the manager's expected utility process, which also makes dividend stream and, therefore, stock return more volatile. An increase in φ , on the other hand, leads to higher stock return volatility only through optimal contracting. Due to higher effort cost, lower incentives are provided to the manager, which leads to more volatile residual dividend streams and, therefore, higher stock return volatility.

In addition, we report quantitative effects of either of risk aversion parameters in Figures 4-6. First, it is noticeable that the key driving force that determines the large shareholder's second-best equity stake is her risk sharing incentives in the sense that its variations are largely consistent with those in the first-best case. More specifically, the large shareholder's optimal ownership stake declines with γ_L , but largely increases with the other players' risk aversions (γ_S and γ_M). We observe that investors' risk aversion (γ_L or γ_S) has a positive impact on the manager's optimal effort level, which is due to higher incentive provisions implemented by the large shareholder for the purpose of risk sharing. Second, in contrast to the standard principal-agent model with moral hazard, the manager's optimal effort level moves non-monotonically with the manager's absolute risk aversion γ_M . When γ_M is small enough, an increase in this parameter lowers PPS measures sharply, thereby lowering his optimal effort level. When γ_M is large enough, however, there are two conflicting forces. It is, on the one hand, costly for the large shareholder to provide managerial incentives as managerial risk aversion increases, which lowers

PPS measures and managerial effort level. The large shareholder, on the other hand, also effectively increases her equity stake in the firm because of her risk aversion relatively lower than the manager, which gives the large shareholder to increase PPS measures in implementing managerial contract. As a result, PPS measures decline but managerial effort increases with respect to the manager’s risk aversion.

Further, we look at the variations in the stock market equilibrium. We first note that, in accord with the analytical characterization in Proposition 5, both the expected stock return and stock return volatility are lower under moral hazard than in the first-best case. This observation, however, is interesting because it is contrary to the conventional wisdom that agency problems and frictions can result in more volatile stock price and/or higher risk premium. In our model, managerial optimal incentive provision plays a significant risk-sharing role which effectively lowers the stock return volatility. These stock return variables, however, still increase as the agency problem becomes more severe (that is, a higher σ , φ , or γ_M). The expected stock return increases as investors become more risk averse, but its volatility decreases. It is also interesting to see that the manager’s risk aversion can increase the expected stock return and stock return volatility. By considering the manager’s trading for managerial incentives, Gorton et al. (2013) show that managerial trading results in more volatile stock price and higher risk premium only when the manager is more risk-averse than investors. However, in their model, this finding only holds with dispersed small investors, not with large shareholders.

Lastly, it is worth emphasizing that the three endogenous variables in equilibrium—large shareholder ownership, managerial incentives, and stock return—can be positively or negatively associated with each other, depending on the exogenous structural change, which might explain some mixed empirical findings on these relations. For instance, the relation between large shareholder ownership (or outside block ownership) and the pay-performance sensitivity (PPS) of CEO compensation has been examined by different studies in the literature on corporate governance. Mehran (1995) and others (e.g., Bertrand and Mullainathan (2001)), on the one hand, argues by showing a negative association between incentive-based compensation and the size of external block ownership that monitoring

by large shareholders appears to be a substitute with incentive compensation for aligning incentives. More recent studies such as Almazan, Hartzell and Starks (2005) and Kim (2010) show that the sensitivity of CEO compensation to firm performance is positively associated with the level of outside block ownership. Whether these two major governance mechanisms are complements or substitutes should be addressed in a framework of general equilibrium interactions as in our study.

5 Further Extensions

There are several directions in which our current model can be generalized. First, we adopt a simple CARA-normal setup in this paper. This enables us to obtain closed-form solutions for the optimal contract, equilibrium asset price and ownership structure. There are two theoretical limitations to the generalization of the framework: As Sannikov (2008) presents numerical solutions to the contracting problems with different non-CARA preferences, it will be hard to find a closed-form solution even only to the optimal contracting problem. In addition, under different preferences and cumulative payoff structures, it will be hard to guess and confirm the form of the equilibrium price process. More work would be required to obtain the general form of the price process.

Second, in our current setup, we allow the large shareholder to trade only once, and the manager is hired at time zero and committed to work until the terminal date. As in DeMarzo and Urošević (2006), we can allow the large shareholder to trade on a finite set of times in a continuous time framework. Each time, the large shareholder determines a new compensation contract for the manager that lasts until the next trading time. The manager accepts the contract and agrees to fulfil it until the end of the stipulated period, as long as it satisfies his IR and IC constraints. The process for solving this problem would be almost identical to the process that we use in our basic model except that the boundary conditions might be different. However, with a CARA-normal setup, there are no wealth effects in the optimal consumption and portfolio choice problems, so that we might be able to use similar forms of

the solutions for HJB equations.

Finally, we assume in this paper that there exists a representative firm. It would be interesting to extend this simple model to a general framework of multiple firms that would allow us to propose a CAPM with optimal managerial contracts. We shall pursue the general model in another paper.

6 Conclusion

We propose a unified theoretical framework that examines the interactions among ownership structure, managerial compensation and asset price. Using a simple CARA-normal framework, we obtain a closed-form solution to the moral hazard problem in which we can fully characterize the endogenous determination of these variables. Using the baseline parameter values obtained by the calibration exercise, we perform a sensitivity analysis to provide quantitative implications of the model.

By comparing the equilibrium solution under moral hazard with the first-best solution, we find that large shareholder ownership is greater under moral hazard. Moreover, both the expected stock return and stock return volatility are lower under moral hazard because of risk sharing effects of managerial incentives. Further, the risk aversion parameters of the players have differing effects on the equilibrium outcome: managerial risk aversion can lead to a higher equity risk premium and greater volatility, whereas, if investors are more risk-averse, the stock return increases, but the stock return volatility decreases because higher managerial incentives are offered through the manager's optimal contracts. Finally, the relation between managerial incentives and large shareholder ownership as well as the relation between managerial incentives and expected stock returns can be both positive and negative depending on changes in the economy.

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Appendix: Proofs

Proof of Proposition 1

To simplify notation, we denote in this proof as well as in subsequent proofs the large shareholder, each small shareholder, and the manager by L , S , and M , respectively. Let Θ be the equity stake chosen by L at time zero which is held constant until the terminal date T . We solve the L 's optimal consumption and managerial contracting problem using the HJB approach. There are two stochastic state variables in this optimization problem: the L 's risk-free account balance $B_{L,t}$ and the M 's expected utility $W_{M,t}$. On the one hand, by (6) and (1), the L 's risk-free account evolves according to

$$dB_{L,t} = (rB_{L,t} - c_{L,t} - \Theta c_{M,t} + \Theta a) dt + \Theta \sigma dZ_t. \quad (46)$$

By (13), the M 's expected utility process $W_{M,t}$, on the other hand, evolves as follows:

$$\begin{aligned} dW_{M,t} &= \left(\delta W_{M,t} - e^{\delta t} u_M(t, c_{M,t}, a_t) \right) dt - \sigma e^{\delta t} \partial_a u_M(t, c_{M,t}, a_t) dZ_t \\ &= \left(\delta W_{M,t} + \frac{1}{\gamma_M} e^{-\gamma_M(c_{M,t} - \frac{1}{2}\varphi a_t^2)} \right) dt + \sigma \varphi a_t e^{-\gamma_M(c_{M,t} - \frac{1}{2}\varphi a_t^2)} dZ_t, \end{aligned} \quad (47)$$

where the second equation is obtained by (11).

The L 's value function V_L is thus a deterministic function of the two state variables, which is denoted by $F(t, B, W)$ (where the subscripts of the state variables are, for expositional convenience, omitted for the analysis here). The HJB equation can thus be written as

$$\begin{aligned} F_t + \max_{c_L, c_M, a} \left[F_W (\delta W + \frac{1}{\gamma_M} H) + F_B (rB - c_L - \Theta c_M + \Theta a) + \frac{1}{2} F_{BB} \Theta^2 \sigma^2 \right. \\ \left. + F_{BW} \Theta \sigma (\sigma \varphi a H) + \frac{1}{2} F_{WW} (\sigma \varphi a H)^2 + u_L(c_L, t) \right] = 0, \end{aligned} \quad (48)$$

where $H \equiv e^{-\gamma_M(c_M - \frac{1}{2}\varphi a^2)}$. The first order conditions (FOCs) for (c_L, c_M, a) are then

$$\begin{aligned} -F_B + e^{-\delta t - \gamma_L c_L} &= 0, \\ F_W H + F_B \Theta + F_{BW} \Theta \sigma^2 \gamma_M \varphi a H + F_{WW} \gamma_M (\sigma \varphi a H)^2 &= 0, \\ F_W \varphi a H + F_B \Theta + F_{BW} \Theta \sigma^2 \varphi (1 + \gamma_M \varphi a^2) H + F_{WW} \sigma^2 \varphi^2 a (1 + \gamma_M \varphi a^2) H^2 &= 0. \end{aligned} \quad (49)$$

It is easy to verify that the L 's value function is of the following form,

$$F(t, B, W) = -k_1 (-W)^{-\frac{\gamma_L}{\gamma_M} \Theta} e^{-\delta t - \gamma_L r B}, \quad (50)$$

where k_1 is a constant to be determined. The derivatives of $F(t, B, W)$ are given by

$$\begin{aligned} F_B &= -\gamma_L r F, \\ F_W &= \frac{\gamma_L}{\gamma_M} \Theta (-W)^{-1} F, \\ F_{BB} &= \gamma_L^2 r^2 F, \\ F_{BW} &= -\frac{\gamma_L^2}{\gamma_M} r \Theta (-W)^{-1} F, \\ F_{WW} &= \frac{\gamma_L}{\gamma_M} \Theta \left(1 + \frac{\gamma_L}{\gamma_M} \Theta \right) (-W)^{-2} F. \end{aligned} \quad (51)$$

Then there exists the following set of optimal policies:

$$\begin{aligned} a^* &= k_2, \\ c_L^* &= -\frac{1}{\gamma_L} \ln(\gamma_L r k_1) + \frac{\Theta}{\gamma_M} \ln(-W) + rB, \\ c_M^* &= -\frac{1}{\gamma_M} \ln(k_3) - \frac{1}{\gamma_M} \ln(-W) + \frac{1}{2} \varphi(a^*)^2, \end{aligned} \quad (52)$$

where k_2 and k_3 are constants to be determined.

Using the above solutions, one can see that $H = k_3(-W)$ and that the first FOC in (49) immediately holds. Substituting the optimal solutions (52) and the derivatives of F (51) into the second and third FOCs in (49) leads to the following equations for k_2 and k_3 :

$$\frac{\gamma_L}{\gamma_M} k_3 - \gamma_L r - \gamma_L^2 r \Theta \sigma (\sigma \varphi k_2 k_3) + \gamma_L \left(1 + \frac{\gamma_L}{\gamma_M} \Theta\right) (\sigma \varphi k_2 k_3)^2 = 0, \quad (53)$$

$$\frac{\gamma_L}{\gamma_M} \varphi k_2 k_3 - \gamma_L r - \frac{\gamma_L^2}{\gamma_M} r \Theta \sigma (1 + \gamma_M \varphi k_2^2) \sigma \varphi k_3 + \frac{\gamma_L}{\gamma_M} \left(1 + \frac{\gamma_L}{\gamma_M} \Theta\right) (1 + \gamma_M \varphi k_2^2) \sigma^2 \varphi^2 k_2 k_3^2 = 0. \quad (54)$$

Multiplying (53) and (54) by $-\frac{1}{\gamma_M} (1 + \gamma_M \varphi k_2^2)$ and k_2 , respectively, and summing them up, we obtain

$$k_3 = \gamma_M r + \gamma_M^2 r k_2 (\varphi k_2 - 1). \quad (55)$$

The constant k_1 is determined by substituting the optimal policies (52) and the derivatives of F (51) into the L's HJB equation (48):

$$\begin{aligned} r \ln(\gamma_L r k_1) &= -\delta + \frac{\gamma_L}{\gamma_M} \Theta \left(\frac{k_3}{\gamma_M} - \delta \right) - \frac{\gamma_L}{\gamma_M} r \Theta \ln(k_3) + \frac{1}{2} \gamma_L r \Theta \varphi k_2^2 \\ &\quad - \gamma_L r \Theta k_2 - \frac{\gamma_L^2}{\gamma_M} r \Theta^2 \sigma^2 \varphi k_2 k_3 + \frac{1}{2} \frac{\gamma_L}{\gamma_M} \Theta \left(1 + \frac{\gamma_L}{\gamma_M} \Theta\right) (\sigma \varphi k_2 k_3)^2 + \frac{1}{2} (\gamma_L r \Theta \sigma)^2 + r. \end{aligned} \quad (56)$$

Under the optimal solutions, the manager's remaining utility process in (47) follows a geometric Brownian motion as shown by

$$dW_M^* = \mu_W^* W_M^* dt - \sigma_W^* W_M^* dZ_t, \quad (57)$$

where

$$\mu_W^* \equiv \delta - \frac{k_3}{\gamma_M}; \quad \sigma_W^* \equiv \sigma \varphi k_2 k_3. \quad (58)$$

Applying Ito's Lemma gives

$$d \ln(-W_M^*) = \left(\mu_W^* - \frac{1}{2} (\sigma_W^*)^2 \right) dt - \sigma_W^* dZ_t. \quad (59)$$

Proof of Proposition 2

We need to derive the equilibrium stock price. Due to the optimal consumption policies that are linear functions of the state variable, $\ln(-W_{M,t})$, as shown in the proof of Proposition 1, we conjecture and verify the following equilibrium stock price process,

$$P_t = \lambda + \frac{1}{\gamma_M r} \ln(-W_{M,t}), \quad (60)$$

where λ is a constant that will be determined below. By (59), it follows that

$$dP_t = \frac{1}{\gamma_M r} d \ln(-W_{M,t}) = \frac{1}{\gamma_M r} \left[\left(\mu_W^* - \frac{1}{2}(\sigma_W^*)^2 \right) dt - \sigma_W^* dZ_t \right]. \quad (61)$$

We now consider the S's value function in (24). By (10), the investor's wealth $Y_{S,t}$ (we ignore arguments and time subscripts below for expositional convenience) evolves according to

$$dY = \left(rY - c_S + \theta_S \left(\frac{1}{\gamma_M r} (\mu_W^* - \frac{1}{2}(\sigma_W^*)^2) + a - c_M - rP \right) \right) dt + \theta_S \left(\sigma - \frac{\sigma_W^*}{\gamma_M r} \right) dZ_t. \quad (62)$$

Let $Q(t, Y)$ be the S's value function which is a deterministic function of the state variable Y . The S's HJB equation is then

$$Q_t + \max_{c_S, \theta_S} \left\{ Q_Y \left[rY - c_S + \theta_S \left(\frac{1}{\gamma_M r} (\mu_W^* - \frac{1}{2}(\sigma_W^*)^2) + a - c_M - rP \right) \right] \right. \\ \left. + \frac{1}{2} Q_{YY} \theta_S^2 \left(\sigma - \frac{\sigma_W^*}{\gamma_M r} \right)^2 + u_S(t, c_S) \right\} = 0, \quad (63)$$

from which we obtain the first order conditions with respect to c_S and θ_S as follows:

$$Q_Y = e^{-\delta t - \gamma_S c_S}, \quad (64)$$

$$\theta_S = \frac{Q_Y \left(\frac{1}{\gamma_M r} (\mu_W^* - \frac{1}{2}(\sigma_W^*)^2) + a - c_M - rP \right)}{(-Q_{YY}) \left(\sigma - \frac{\sigma_W^*}{\gamma_M r} \right)^2}. \quad (65)$$

It is easy to verify that the above value function has the following form of

$$Q(t, Y) = -\frac{\nu}{\gamma_S} e^{-\delta t - \gamma_S r Y}, \quad (66)$$

where ν is an additional constant to be determined. The first FOC (64) determines the S's optimal consumption by

$$c_S = -\frac{1}{\gamma_S} \ln(r\nu) + rY. \quad (67)$$

The second FOC (65), along with the optimal policies from the L's value function maximization (52) and the stock market clearing condition, $\theta_S = 1 - \Theta$, determines λ in the equilibrium stock price (60) as follows:

$$r\lambda = \frac{1}{\gamma_M r} \left(\mu_W^* - \frac{1}{2}(\sigma_W^*)^2 \right) + k_2 - \frac{1}{2} \varphi k_2^2 + \frac{1}{\gamma_M} \ln(k_3) - (1 - \Theta) \gamma_S r \left(\sigma - \frac{\sigma_W^*}{\gamma_M r} \right)^2. \quad (68)$$

Substituting the derivatives of $Q(t, Y)$ and the optimal policies (52) into the HJB equation (63) determines the constant ν as follows:

$$r \ln(r\nu) = r - \delta - \frac{1}{2} \gamma_S^2 r^2 (1 - \Theta)^2 \left(\sigma - \frac{\sigma_W^*}{\gamma_M r} \right)^2. \quad (69)$$

Note that μ_W^* and σ_W^* are given by (58).

Proof of Proposition 3

At time zero, the L's value function (50) is given by

$$V_{L,0} = -k_1(-W_{M,0})^{-\frac{\gamma_L}{\gamma_M}\Theta} e^{-\gamma_L r B_{L,0}}. \quad (70)$$

Using its budget constraint at time zero, $\Theta P_0 + B_{L,0} = Y_{L,0}$, and the stock price equilibrium (60), we can rewrite (70) as

$$V_{L,0}(\Theta) = -k_1(\Theta) \exp(-\gamma_L r (Y_{L,0} - \Theta \lambda(\Theta))) = -\exp\left(-\gamma_L r \left(Y_{L,0} - \Theta \lambda(\Theta) - \frac{1}{\gamma_L r} \log k_1(\Theta)\right)\right). \quad (71)$$

Using the equations (56) and (68) for k_1 and λ , we obtain

$$\begin{aligned} -\Theta \lambda(\Theta) - \frac{1}{\gamma_L r} \log k_1(\Theta) &= \Theta(1 - \Theta) \gamma_S \left(\sigma - \frac{\sigma_W^*}{\gamma_M r}\right)^2 - \frac{\gamma_L}{2} \Theta^2 \left(\sigma - \frac{\sigma_W^*}{\gamma_M r}\right)^2 \\ &= \frac{\Theta}{r} \mu_R^*(\Theta) - \frac{1}{2} \gamma_L \Theta^2 \sigma_R^*(\Theta)^2, \end{aligned} \quad (72)$$

where μ_R^* and σ_R^* are defined by (29) and (30).

Proof of Proposition 4

The procedures in this proof is largely similar to the previous proofs for the second-best solution under moral hazard. We first define the time-shifted variables as follows:

$$W_{M,t} \equiv e^{\delta t} V_{M,t}, \quad \widehat{G} \equiv e^{\delta t} G_{M,t}. \quad (73)$$

By (7) and (73), one can see that $W_{M,t}$ evolves as

$$dW_M = \left(\delta W_M - e^{\delta t} u_M(t, c_M, a)\right) dt + \sigma \widehat{G} dZ_t. \quad (74)$$

The L's value function maximization is similarly based on the two stochastic state variables B_L and W_M whose dynamics are given by (46) and (74). Note that we omit their subscripts for expositional convenience below. Then the L's HJB equation can be written as

$$\begin{aligned} F_t + \max_{c_L, c_M, a, \widehat{G}} \left[F_W \left(\delta W + \frac{1}{\gamma_M} H \right) + F_B (rB - c_L - \Theta c_M + \Theta a) + \frac{1}{2} F_{BB} \Theta^2 \sigma^2 \right. \\ \left. + F_{BW} \Theta \sigma^2 \widehat{G} + \frac{1}{2} F_{WW} \sigma^2 \widehat{G}^2 + u_L(t, c_L) \right] = 0, \end{aligned} \quad (75)$$

where $H = e^{-\gamma_M(c_M - \frac{1}{2}\varphi a^2)}$. The first order conditions (FOCs) with respect to the respective control variables are given by

$$\begin{aligned} -F_B + e^{-\delta t - \gamma_L c_L} &= 0, \\ F_W H + F_B \Theta &= 0, \\ F_W \varphi a H + F_B \Theta &= 0, \\ F_{WW} \widehat{G} + F_{BW} \Theta &= 0. \end{aligned} \quad (76)$$

By the second and third FOCs above, the M's first-best effort level is given by

$$a = \frac{1}{\varphi}. \quad (77)$$

In addition, the fourth FOC leads to the optimal volatility choice,

$$\hat{G} = -\frac{F_{BW}}{F_{WW}}\Theta. \quad (78)$$

We similarly guess and verify the L's value function of the form

$$F(t, B, W) = -k^{FB}(-W)^{-\frac{\gamma_L}{\gamma_M}\Theta} e^{-\delta t - \gamma_L r B}. \quad (79)$$

Using the derivatives of $F(t, B, W)$ and the FOCs in (76), we obtain the optimal policies in the first-best case as follows:

$$\begin{aligned} a^{FB} &= \frac{1}{\varphi}, \\ c_L^{FB} &= -\frac{1}{\gamma_L} \ln(\gamma_L r k^{FB}) + \frac{\Theta}{\gamma_M} \ln(-W) + rB, \\ c_M^{FB} &= -\frac{1}{\gamma_M} \ln(\gamma_M r) - \frac{1}{\gamma_M} \ln(-W) + \frac{1}{2}\varphi a^2, \\ \hat{G} &= \frac{\gamma_L r \Theta}{\left(1 + \frac{\gamma_L}{\gamma_M}\Theta\right)}(-W), \end{aligned} \quad (80)$$

where the constant k^{FB} is determined by substituting the above optimal policies into the HJB equation (75) back. Under the optimal policies, the log of the M's remaining utility process W_M in (74) evolves according to

$$d \ln(-W_M) = \left(\mu_W^{FB} - \frac{1}{2}(\sigma_W^{FB})^2 \right) dt - \sigma_W^{FB} dZ_t, \quad (81)$$

where

$$\mu_W^{FB} \equiv \delta - r; \quad \sigma_W^{FB} \equiv \frac{\gamma_L r \Theta \sigma}{\left(1 + \frac{\gamma_L}{\gamma_M}\Theta\right)}. \quad (82)$$

The derivation of the stock market equilibrium remains largely the same except that the constants defining the equilibrium stock price and the S's value function, denoted by λ^{FB} and ν^{FB} , respectively, are differently determined because the M's effort, the constant term in the M's compensation payments, and the drift and volatility of the M's utility process in the first-best case above are different from their corresponding values in the second-best case. Finally, the L's optimal ownership choice problem at time zero also reduces to

$$\Theta^{FB} = \arg \max_{\Theta} \frac{\Theta}{r} \mu_R^{FB}(\Theta) - \frac{1}{2} \gamma_L \Theta^2 \sigma_R^{FB}(\Theta)^2, \quad (83)$$

where $\mu_R^{FB}(\Theta)$ and σ_R^{FB} are obtained by replacing σ_W^* in (29) and (30) with σ_W^{FB} in (82). It is straightforward to show that there exists a unique solution for Θ^{FB} in the above problem as follows:

$$\Theta^{FB} = \frac{1}{2 + \gamma_L \left(\frac{1}{\gamma_M} + \frac{1}{\gamma_S} \right)}. \quad (84)$$

Proof of Proposition 5

Suppose that the L's optimal ownership stake is Θ . Equation (55) implies that if $k_2 < (>) \frac{1}{\varphi}$, then $k_3 < (>) \gamma_M r$. We rearrange equation (53) as

$$\frac{k_3}{\gamma_M} - r + (\sigma \varphi k_2 k_3) \left(\left(1 + \frac{\gamma_L}{\gamma_M} \Theta \right) (\sigma \varphi k_2 k_3) - \gamma_L r \Theta \sigma \right) = 0, \quad (85)$$

which leads to the finding of $k_2 < \frac{1}{\varphi}$; $k_3 < \gamma_M r$. Since the drift μ_W^{FB} of the M's stochastic utility process in the first-best case is constant and equal to $\delta - r$, it immediately follows that

$$\mu_W^* = \delta - \frac{k_3}{\gamma_M} > \mu_W^{FB} = \delta - r, \quad (86)$$

which holds at any Θ .

The remaining observations in Proposition 5 unambiguously hold only if the L's optimal ownership stake Θ^* is larger under moral hazard than Θ^{FB} in the first-best case. We first compare the volatility of the manager's expected utility process in the second-best case with the first-best level. By (53), (58) and the observation of $k_3 < \gamma_M r$, we find that

$$\sigma_W^* = \frac{\gamma_L r \Theta^* \sigma + \left[(\gamma_L r \Theta^* \sigma)^2 - 4 \left(1 + \frac{\gamma_L}{\gamma_M} \Theta^* \right) \left(\frac{k_3}{\gamma_M} - r \right) \right]^{\frac{1}{2}}}{2 \left(1 + \frac{\gamma_L}{\gamma_M} \Theta^* \right)} > \frac{\gamma_L r \Theta^* \sigma}{\left(1 + \frac{\gamma_L}{\gamma_M} \Theta^* \right)}. \quad (87)$$

If Θ^* is chosen at a level greater than Θ^{FB} specified in (84), the right-hand side of (87) at Θ^* is greater than σ_W^{FB} , thereby implying the following observation: $\sigma_W^* > \sigma_W^{FB}$. By (23), (29), (30), and (28), it is straightforward to see that, if $\Theta^* > \Theta^{FB}$, the two measures of pay-performance sensitivity (PPS) are higher under moral hazard than in the first-best case, whereas the expected stock return and stock return volatility in the presence of moral hazard are lower than in the first-best case.

Table I: Parameters of the Model

Parameter	Description
σ	Firm output (or cash flow) uncertainty
φ	Managerial effort cost
γ_L	Large shareholder's absolute risk aversion
γ_M	Manager's absolute risk aversion
γ_S	Small shareholder's absolute risk aversion

Table II: Observed and Model-Predicted Moments

	Large shareholder's ownership stake (Θ^*)	Sensitivity of CEO pay to stock return (PPS_2^*)	Expected stock return (μ_R^*)	Volatility of stock return (σ_R^*)
Observed	0.37	0.003	0.082	0.194
Model-Predicted (Second-best)	0.3697	0.0058	0.0821	0.194

Table III: Baseline Parameter Values

Parameters				
σ	φ	γ_L	γ_M	γ_S
0.25	1.542	73.35	167.70	172.98

Figure 1: First-best and Second-best Solutions for Different Shareholdings of the Large Shareholder, Θ

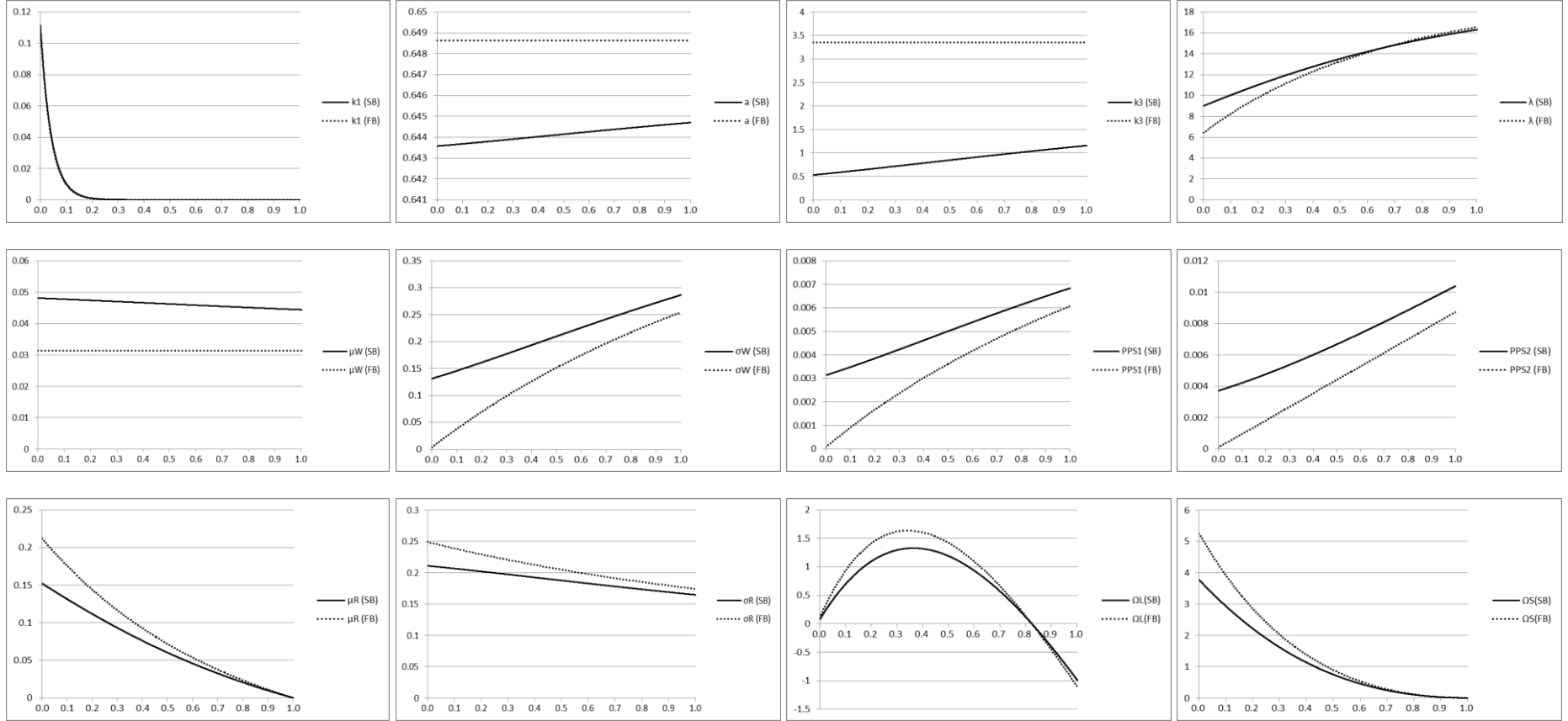


Figure 2: Effects of Firm Output Uncertainty, σ

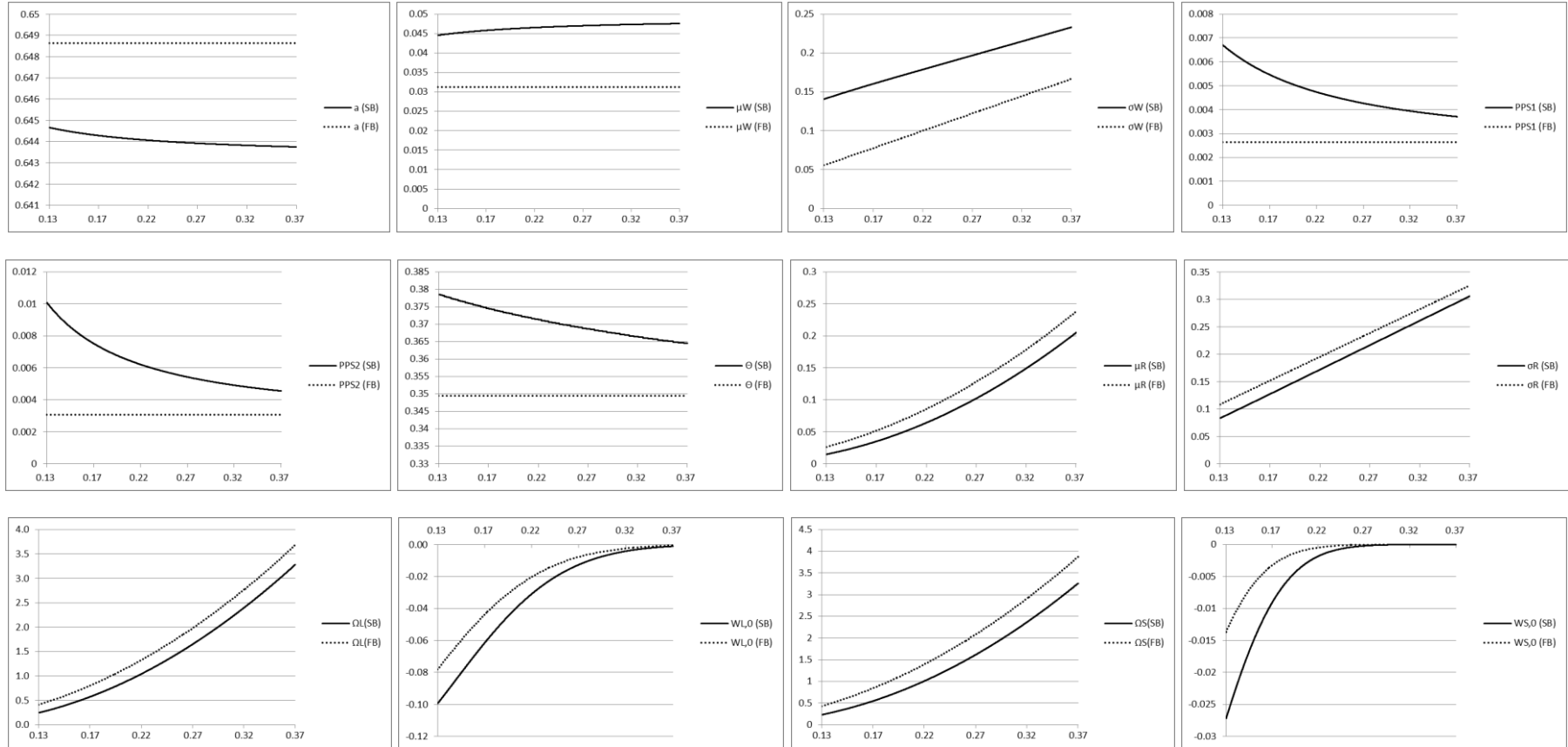


Figure 3: Effects of Manager's Effort Cost, φ

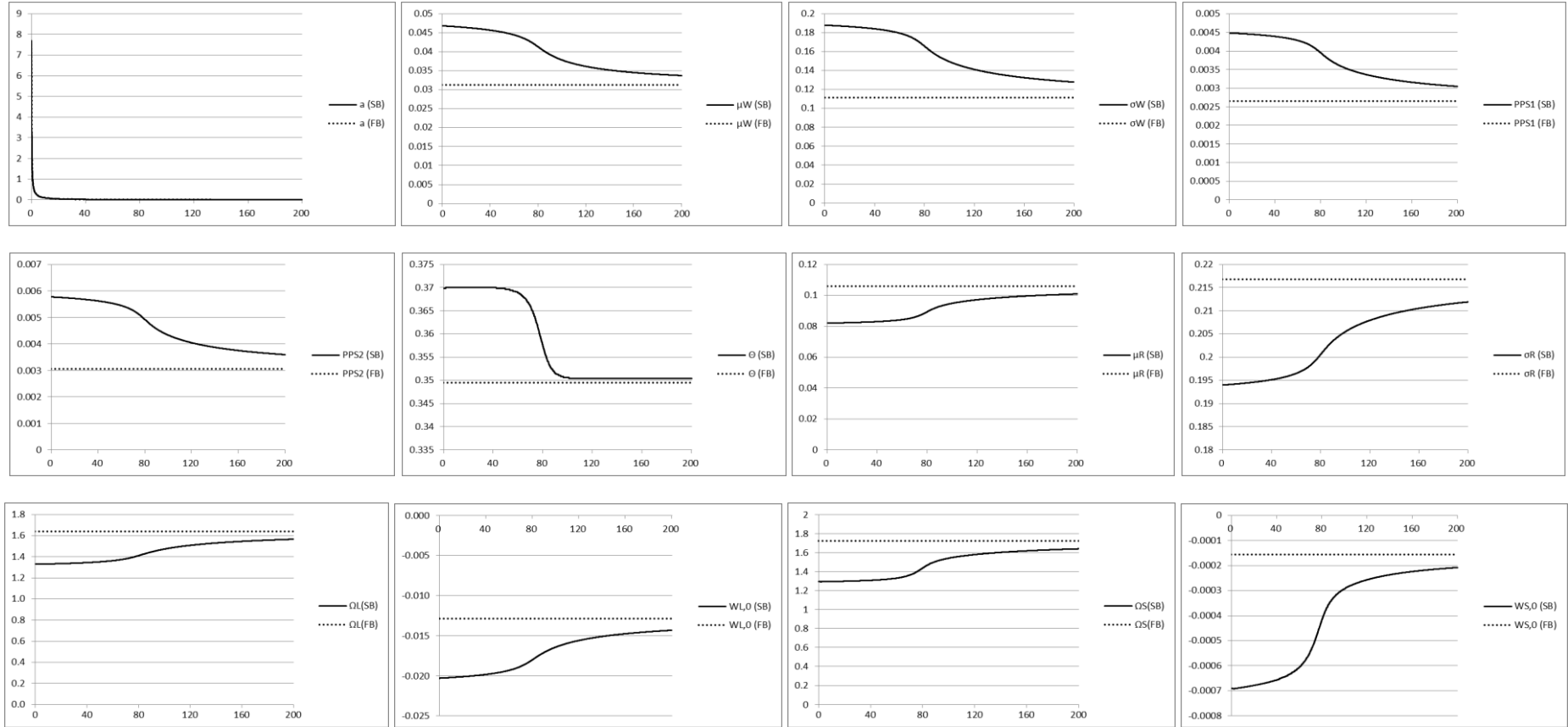


Figure 4: Effects of Large Shareholder's Absolute Risk Aversion, γ_L

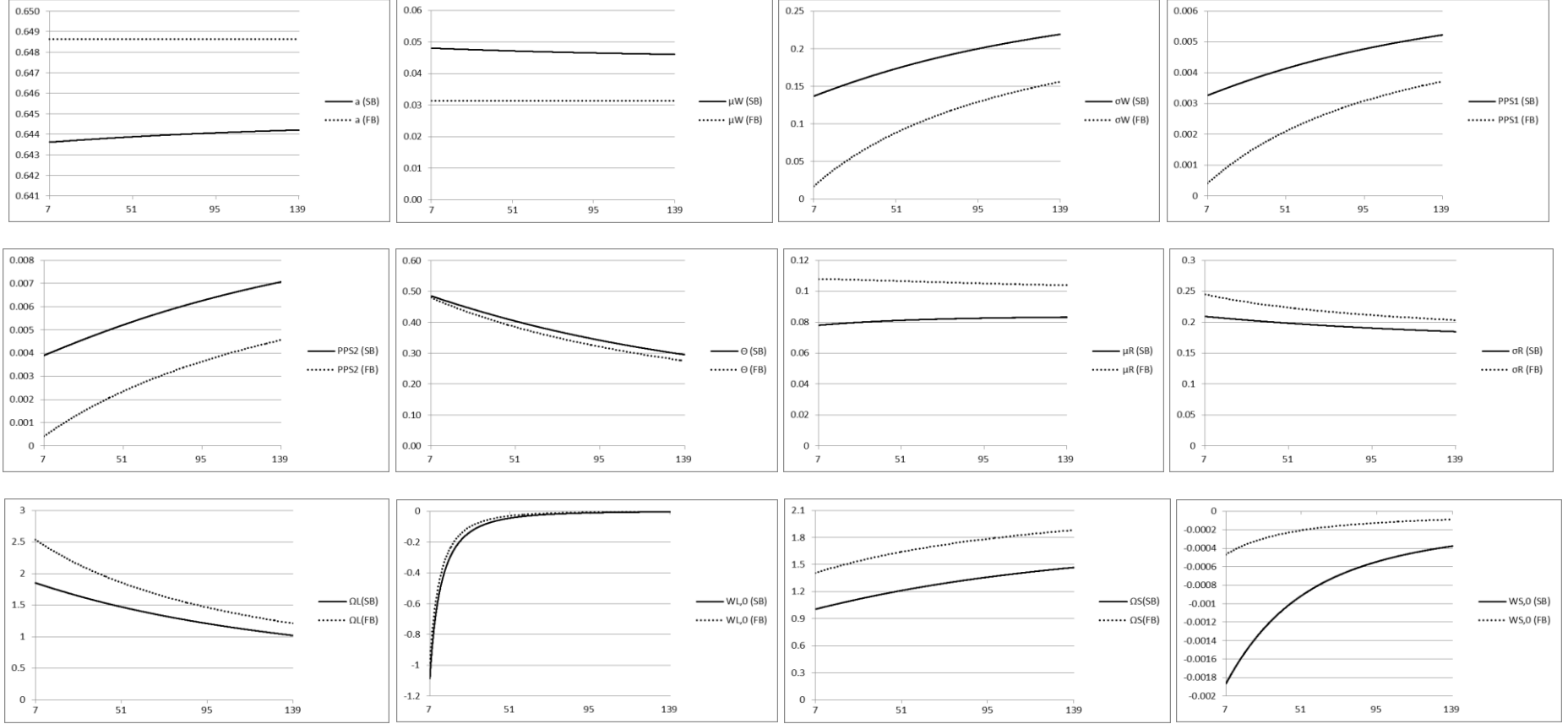


Figure 5: Effects of Small Shareholders' Absolute Risk Aversion, γ_S

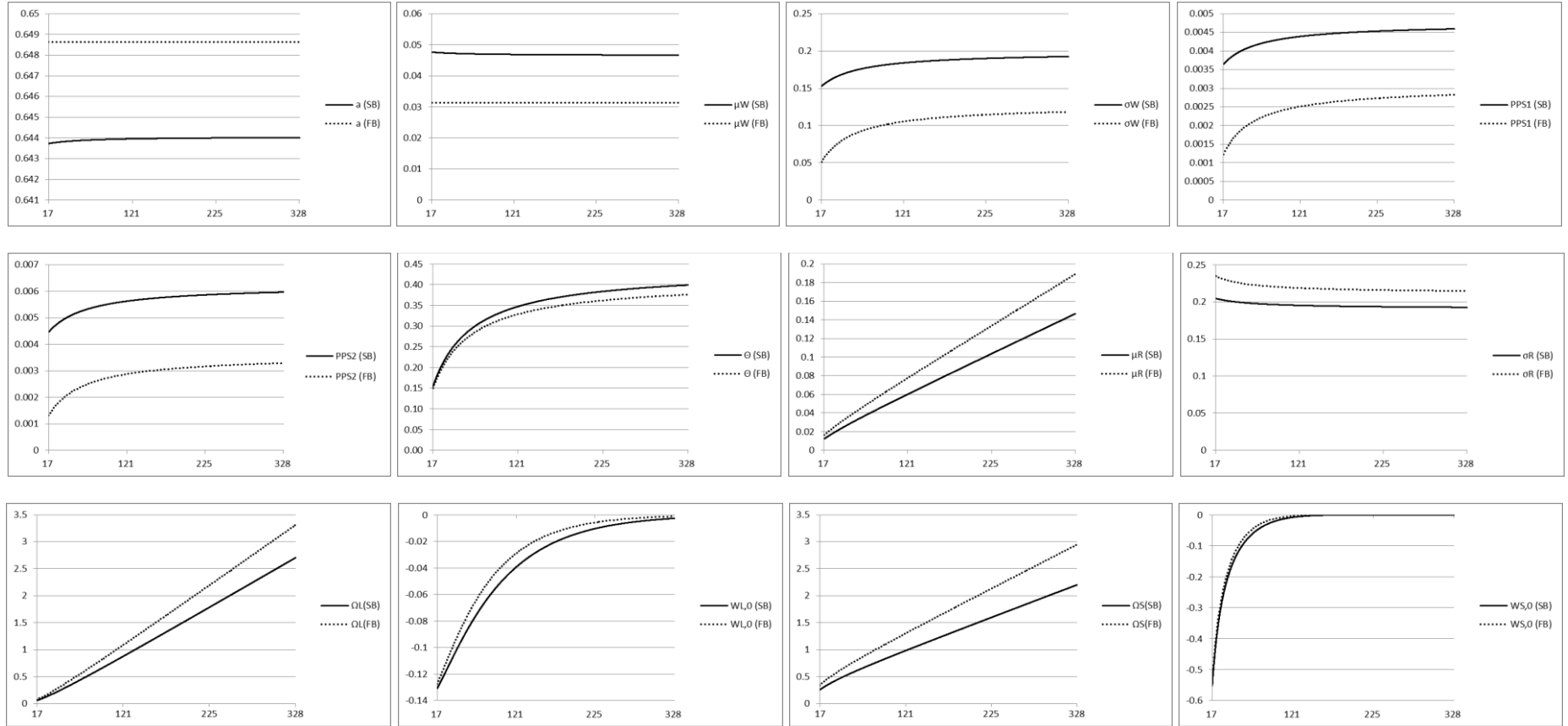


Figure 6: Effects of Manager's Absolute Risk Aversion, γ_M

