

Municipal Bonds, Default, and Migration in General Equilibrium

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Abstract

Bonds are an important source of funding for municipalities. As financing for big budget construction projects or surprise shortfalls in tax revenue, bonds help smooth tax burden across time. There is good reason for this smoothing: if residents feel their tax burden is excessive, they can migrate. The ability of residents to migrate significantly hampers the ability of local governments to raise taxes, and, in the extreme, can lead to default. We document the relationship between bonds, default, and migration in the data. We then construct an islands model that captures these facts while allowing for endogenous migration, taxation, debt issuance, and default. [TO BE DONE:] We assess the short run, long run, and welfare costs of default in our model and explore macro-prudential policies that can mitigate these costs.

1 Introduction

On October 1, 2013, the city of Detroit defaulted on more than \$600 million. Detroit is just one example of a growing number of municipalities or regional entities facing financial headwinds in the aftermath of the Great Recession (Puerto Rico is a more recent example of this type of struggle). In this paper, we study default by municipalities. To this end, we first document features of the data. We then propose a rich model that is capable of capturing these facts.

Some facts about municipalities in the U.S. are:

- There are two types of municipal bonds, General Obligation (GO) bonds and non-GO bonds. The difference between these two bonds is that a GO bond is backed “by the full faith credit and taxing power of the issuer.”
- While broadly used, GO and non-GO bonds have significantly different default rates.

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- The overall default rate is very low: Since 1970, there have only been 73 defaults.
- While low, the default rate has risen since 2008.
- Recovery rates on defaulted debt range from 40% to 75% or even higher.
- Aggregate municipal debt to Gross State Product (GSP) ratios are around 3% but can be as high as 10%.
- There is substantial migration across municipalities (more precisely, across states).

We build a model to capture these facts. In the spirit of Lucas (1972), we propose a model with a continuum of islands. Each island is a sovereign entity that chooses tax rates and issues debt to maximize welfare of those living on the island. Two critical features of our framework are that 1) residents of an island can choose to leave it and 2) sovereign islands can default on their obligations. Given the high recovery rates on defaulted debt, we explicitly model what happens to defaulted debt.

We characterize the decision of an islander regarding whether to migrate and, if not, their labor-leisure choice. We then characterize the island planner’s problem. Importantly, we show the sovereign’s problem can be setup recursively and looks very similar to the usual sovereign default setup in Arellano (2008). The sovereign’s choice of taxes and default are driven by productivity shocks that differ across islands. A good productivity disturbance to one island can mean bad news for another one since it can trigger migration outflows. With fewer workers around, revenue from taxation could decline forcing the sovereign to rely more on debt. If the “bad” disturbance lingers, the unlucky island may be forced to default. We use a calibrated version of our model to the data and explore the role of fundamentals on default, migration, and the price of debt.

2 Data¹

According to Moody’s, there are two types of municipal bonds. General obligation (GO) bonds are backed “by the full faith credit and taxing power of the issuer.” Some GO bonds have limitations on the implied tax burden, others do not. More than half of municipal bonds, however, are non-GO and are not explicitly backed by taxing power. These bonds are meant to be repaid through proceeds of a revenue-producing venture like a utility enterprise, a new toll road, or a bridge. Non-GO bonds also include bonds backed by lease revenues, appropriations and moral obligations.

While municipalities have equally resorted to GO and non-GO bonds, a large fraction of defaults come from the second type of bonds. According to Moody’s, there have been 73 municipal bond defaults (of bonds rated by Moody’s) since 1970, and 68 (or 93%) have been on non-GO debt. Of

¹For this section, we use data from Moody’s as reported in US Municipal Bond Defaults and Recoveries, 1970-2012.

the five GO defaults, three have occurred since 2008. Given the overall exceedingly low default rate, it is not surprising that interest rates on municipal bonds carry a very low interest rate.

More into the details, the average trailing twelve-month speculative-grade municipal default rate since the financial crisis has increased twofold going from *1 percent* in the period 1991-2007 to *2 percent* since the onset of the Great Recession. It should be noted that the trailing twelve-month municipal default rate had started to increase before the Great Recession. That is the average between 2003 and the start of the recession was already *2.4 percent*. Furthermore, the default rate is low compared to that from corporate defaults. Figure 1 shows the default rates for both municipalities and corporations over the past 40 years. Although the default rate at municipalities have declined in 2012, the lingering consequences of the recession will most likely keep the rates at values above their historical average.

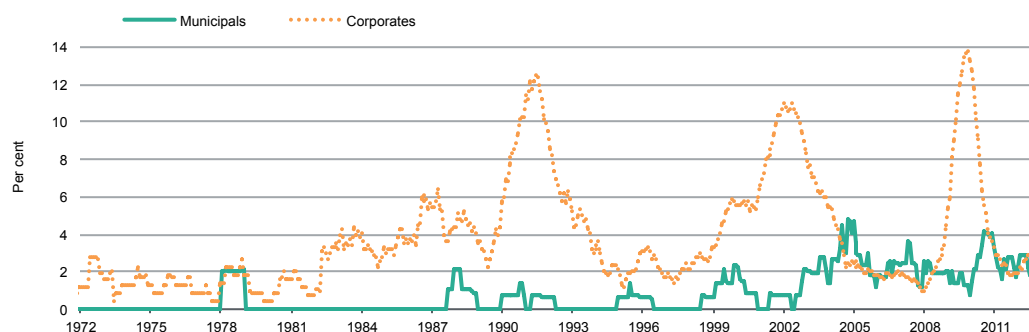


Figure 2.1: Default Rates Municipalities and Corporations (Source: Moody's)

What is the burden of debt on municipalities? We are a bit limited in terms of data, so for this version we rely heavily on state-level figures like gross state output (GSP). Table 1 reports the average and standard deviation of the (annual figures of) debt-to-GSP ratio and the debt service ratio in 2011 and 2012. Figure 2.2 displays the densities.²

	Annual D/GSP	
	Mean	Std
2011	2.96	2.05
2012	2.92	2.12

	Debt Service	
	Mean	Std
2011	5.29	3.24
2012	5.24	3.20

Table 1: Debt, Income, and Debt Service

²Moody's defines the debt service ratio as the fraction of "all state net tax-supported debt as a percentage of pledged revenues."

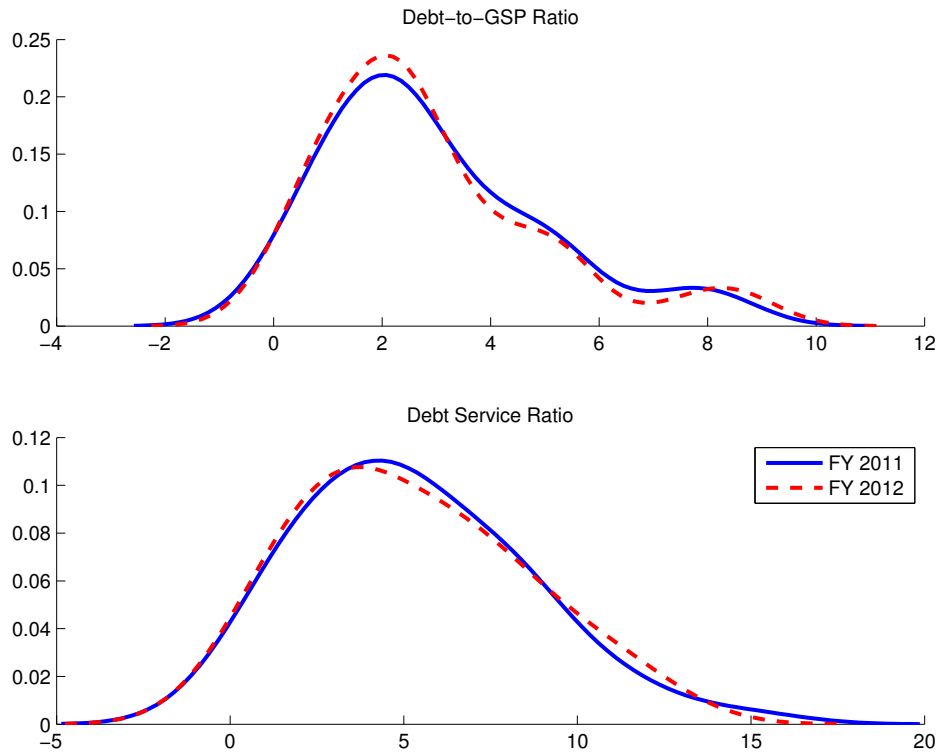


Figure 2.2: Debt, Gross State Product, and Debt Service

How productive are municipalities. Once again, we rely on state data as a proxy. Density annual productivity growth for period 2002 - 2007 from Caliendo et al (2013) “The Impact of Regional and Sectoral Productivity Changes on the U.S. Economy.” The mean and standard deviation of productivity are 1.02 and 0.57, respectively. Clearly, productivity varies substantially across states. Productivity will be a key factor in our model since it drives the decisions of migrate and default.

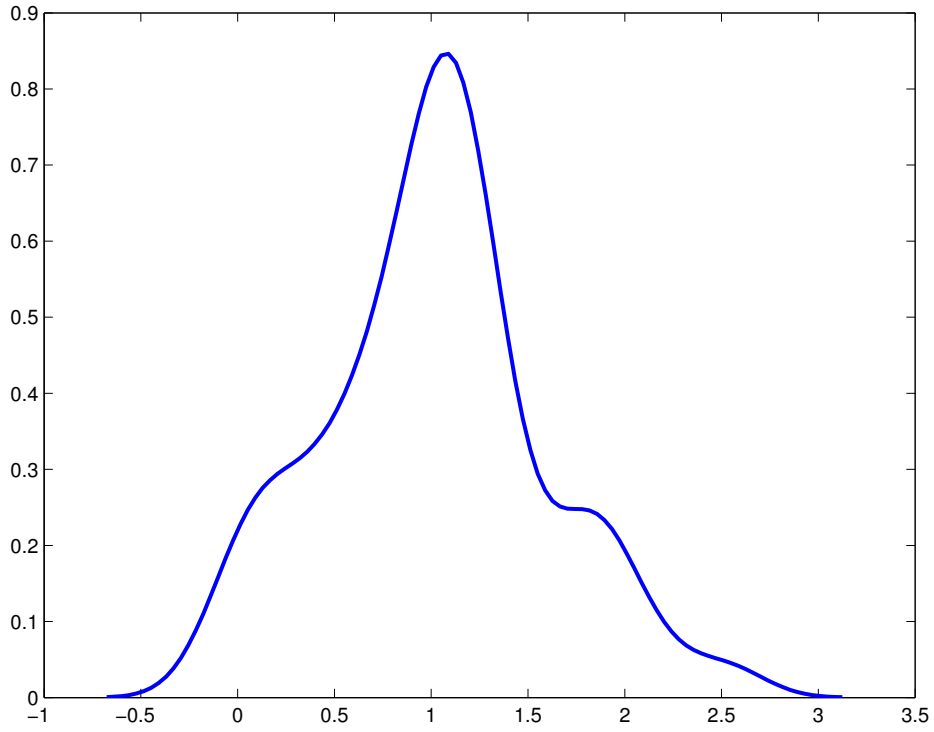


Figure 2.3: State Productivity

We report data on migration flows based on the 2010 Census. Here, we follow the approach outlined in Armenter and Ortega (2010). Specifically, we compute population inflows and outflows for each state in 2010. Figure 2.4 presents the densities for migration. Table 3 presents some statistics of migration in the U.S. expressed as percentage of state population. One can see that an average state sees about 20 percent of its population migrate. Extreme values correspond to the District of Columbia.

	Mean	Std Dev	Skew	Min	Max
Inflow	17.8	10.0	3.9	7.1	74.9
Outflow	18.3	12.0	4.0	5.2	87.1
Net	-0.6	8.8	0.2	-18.7	21.4

Table 2: Migration Flows in the U.S. Some Statistics

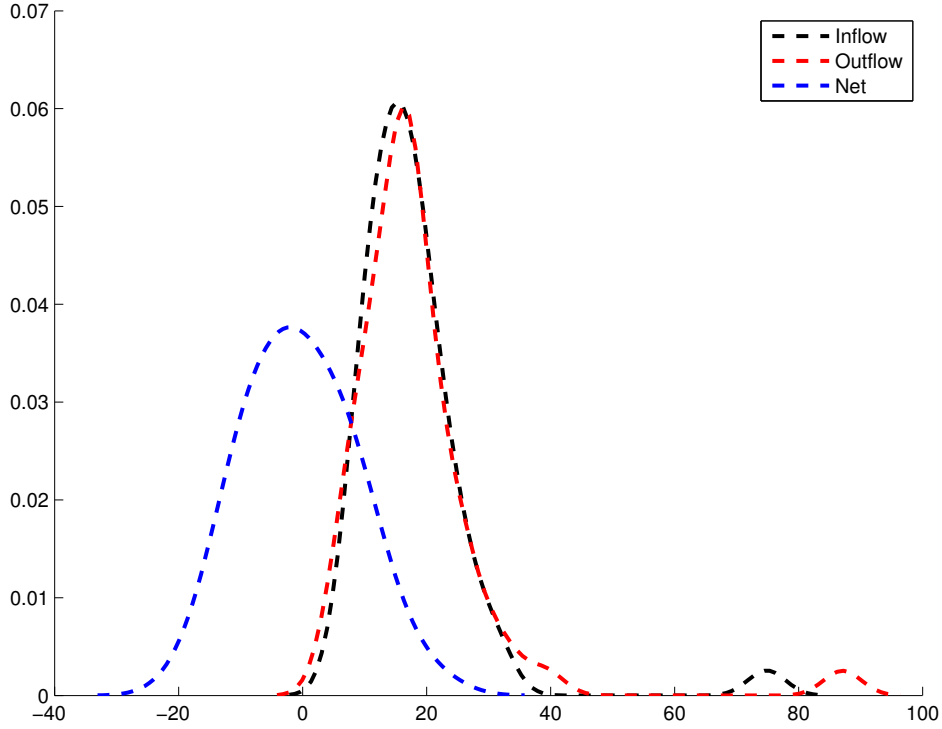


Figure 2.4: Migrations Flow in the U.S.

How much is recovered from a municipal default and how long does settlement take? The evidence varies over time. In an influential paper, Hempel (1971) found that in the 1930s municipal defaults had a conservative recovery rate of 84 percent. Yet recovery on defaulted obligations took several years. Table 3 reports more recent default events. These have exhibited on average lower recovery rates but vary greatly from as little as 40% to as much as 100%.³ There is also a large dispersion in the time to settle with some taking 15 years, most 2-3 years, and some only 3 months.

³They may be gravitating towards the average corporate recovery rate of around 50% (according to Moody's).

City / Institution	Date	Recovery*	Note
Chesapeake Bay Bridge	1970	?	Exited default after 15 years.
Cleveland, OH	1978	100%	\$14 million, resolved in 1980.
Washington Public Power Supply System, WA	1983	40%	
Belfield, ND	1987	55%	Settled in 1991.
Vanceburg, KY	1987	100%	Par+interest received in 1988.
Choate-Symmes Hospitals, MA	1990	61%	Recovered in 8 months.
City of Wenatchee, WA	2012	100%	3 months to resolve.

*Recovery rates are approximate.

Source: Moody's.

Table 3: Select Municipal Bond Defaults

3 Model

3.1 Household problem

Let $\mathbf{x}$ be the state of the island. For now, we need not specify exactly what $\mathbf{x}$ is, only that it has some possibly stochastic law of motion $\mathbf{x}' = \Gamma(\mathbf{x})$.

The moving decision is

$$V(\phi, \mathbf{x}) = \max_{m \in \{0,1\}} (1 - m)S(\mathbf{x}) + m(J - \phi)$$

where S is the value of staying on the island, J is an expected value from leaving the island, and ϕ is the utility cost of moving. We denote the optimal migration policy as

$$m(\phi, \mathbf{x}).$$

The utility conditional on staying is

$$\begin{aligned}
S(\mathbf{x}) &= \max_{c,l} u(c, l) + v(g(\mathbf{x})) + \beta \mathbb{E}_{\mathbf{x}} V(\phi', \mathbf{x}') \\
&\text{s.t.} \\
c &= (1 - \tau(\mathbf{x}))w(\mathbf{x})l + T(\mathbf{x}) \\
c &\geq 0, l \in [0, 1] \\
\mathbf{x}' &= \Gamma(\mathbf{x})
\end{aligned}$$

The budget constraint of the household reflects a proportional tax τ on labor earnings wl as well as a transfer T from the sovereign. Additionally, the utility term $v(g)$ is utility derived from government spending g . For now, we assume it is separable.

J is the expected utility of a random draw from the world of islands: that is, if one decides to

move, one gets a random draw from the distribution of islands. Assuming that the household must stay at that random island for at least one period, we have

$$J = \int S(\mathbf{x}) d\mu(\mathbf{x})$$

where μ is the distribution of islands over island states.

3.2 Firms

Let n be the number of households at the beginning of the period before any migration takes place, and let $\dot{n}$ be the number after migration takes place.

Each island has a firm operating the production function $zL^{1-\alpha}$ and solving

$$\max_{L \geq 0} zL^{1-\alpha} - wL$$

Let L^d denote labor demand. It is given by

$$w = (1 - \alpha)z(L^d)^{-\alpha}.$$

Total labor supply on the island is $\dot{n}l$, so in equilibrium it must be the case that

$$L^d = \dot{n}(\mathbf{x})l(\mathbf{x})$$

where $\dot{n}(\mathbf{x})$ is labor after migration takes place and $l(\mathbf{x})$ is the optimal labor choice of households on an island with state $\mathbf{x}$.⁴ Note zero labor supply cannot be an equilibrium: any finite wage results in strictly positive labor demand.

Note that one could incorporate fixed capital just by changing the interpretation of z (interpreting it as zK^α).

Since capital is not present (or is fixed) there is potentially a perverse incentive to reduce the population on the island: it increases the wage.

3.3 Equilibrium given planner instruments

Define $\mathbf{x} = (z, b, n, f)$ as an island's state.

A steady state equilibrium is $c, l, m, L^d, \mu, T, \tau, g, b', d$ such that

1. (Household optimization) $c(\mathbf{x})$, $l(\mathbf{x})$, and $m(\phi, \mathbf{x})$ are optimal (and feasible) taking instruments and prices as given.

⁴Note that as $\dot{n}$ goes to 0, the wage goes to ∞ . But, because we don't let households choose where to go and there is a continuum of islands, this does not necessarily increase J . Nevertheless, if $n > 0$, then $\dot{n} > 0$ because even if all households migrate, some get returned to the island they left.

2. (Firm optimization) $L^d(\mathbf{x}) \geq 0$ is optimal taking w as given
3. (Labor market clearing) $L^d(\mathbf{x}) = \dot{n}(\mathbf{x})l(\mathbf{x})$
4. (Government) $T \geq 0$, $\tau \in (0, 1)$, $b' = 0$ when $d = 1$ or $f = 1$, $g \geq 0$, $d = 0$ when $f = 1$, and the budget constraint is satisfied:

$$g(\mathbf{x}) + q(b'(\mathbf{x}), \dot{n}(\mathbf{x}), z)b'(\mathbf{x}) \leq b(1 - d(\mathbf{x})) + w(\mathbf{x})\tau(\mathbf{x})\dot{n}(\mathbf{x})l(\mathbf{x}) - T(\mathbf{x})\dot{n}(\mathbf{x})$$

for all $\mathbf{x}$.

We specify free disposal so that the planner may throw away resources if desired (in order to reduce the number of people attracted to his island).

Note that the way services appears in the budget constraint treats it like a public good: every household enjoys the same amount of g , implying $g\dot{n}$ is enjoyed; however, only g is spent. This is not necessary. We could have the expenditures on amenities/services be $g\dot{n}$ or $\theta g\dot{n} + (1 - \theta)g$ controlling how many people benefit for a level of expenditure.

5. (Debt pricing)

$$q(b', n', z) = \bar{q}\mathbb{E}_z(1 - d(z', b', n', f' = 0))$$

Note: this formula is implicitly using $b'(\mathbf{x})$ to obtain $\mathbf{x}'$ and also captures the effect of policies on n' .

6. (Consistency) μ and Γ are consistent with stochastic transitions and policy functions.

In the above

$$\dot{n}(\mathbf{x}) := n - n \left(\int m(\phi, \mathbf{x}) dF(\phi) \right) + \int \tilde{n} \left(\int m(\phi, \tilde{\mathbf{x}}) dF(\phi) \right) d\mu(\tilde{\mathbf{x}}).$$

where $\tilde{n}$ is part of $\tilde{\mathbf{x}}$.

Necessary conditions for equilibrium include

- $l(\mathbf{x}) > 0$ for all $\mathbf{x}$.
- $\dot{n}(\mathbf{x}) > 0$ for all $\mathbf{x}$.

Clearly, for some instruments, an equilibrium may not exist.

3.4 Island planner problem

Each island has a planner who cannot commit to an optimal policy. Rather, the planner reoptimizes each time the state of the island changes (and takes into account that they will reoptimize in the

future). In this sense, the planner is short-sighted. He seeks to maximize the welfare of those currently on the island in a utilitarian fashion.

The key advantage of this approach, rather than having a planner for each island who can commit at time zero, is that there is one optimal policy conditional on $\mathbf{x}$ rather than a separate optimal policy for each island (which may differ even for the same $\mathbf{x}$)—in other words, it makes the policy memoryless. The planner takes μ and J as given (since we are focused on steady state equilibria, μ does not appear below but is implicit).

The planner's problem is

$$\begin{aligned} \max_{d(\mathbf{x}), \tau(\mathbf{x}), T(\mathbf{x}), g(\mathbf{x}), b'(\mathbf{x})} & \int V(\phi, \mathbf{x}) dF(\phi) \\ \text{s.t.} & \end{aligned}$$

$$d(\mathbf{x}), \tau(\mathbf{x}), T(\mathbf{x}), g(\mathbf{x}), b'(\mathbf{x}) \text{ satisfy the government budget constraint}$$

This formulation of the problem implicitly assumes that if there are multiple equilibria, the planner can select the best one.

Note that the planner always has a feasible option: default, set tax rates to zero, and set services to zero.

The specified objective function is equivalent to saying the planner maximizes the utility of those staying on the island. To see this, note the objective function can be written as

$$\int V(\phi, \mathbf{x}) dF(\phi) = \int \max\{S(\mathbf{x}), J - \phi\} dF(\phi)$$

Since J and ϕ are outside the planner's control, it is equivalent for him to maximize $S(\mathbf{x})$.

3.5 Political Equilibrium

A steady state political equilibrium is policies $d(\mathbf{x}), \tau(\mathbf{x}), T(\mathbf{x}), g(\mathbf{x}), b'(\mathbf{x})$ that are optimal for each island and induce J and μ when the initial distribution of islands is μ .

4 Theoretical Results

4.1 Basic characterization

4.1.1 Reservation strategy for moving

Note that since S and J are independent of ϕ , we immediately see m is characterized by a reservation level $R(\mathbf{x})$ in ϕ :

$$m(\phi, \mathbf{x}) = \begin{cases} 1 & \text{if } \phi < R(\mathbf{x}) \\ 0 & \text{if } \phi > R(\mathbf{x}) \\ \text{anything} & \text{if } \phi = R(\mathbf{x}) \end{cases}$$

Further, $R(\mathbf{x})$ is given by

$$R(\mathbf{x}) = J - S(\mathbf{x}).$$

Using this, the measure leaving an island is

$$n_t \int m(\phi, \mathbf{x}_t) dF(\phi) = n_t F(R(\mathbf{x}_t))$$

and the measure entering an island is

$$\int \tilde{n}_t \left(\int m(\phi, \tilde{\mathbf{x}}_t) dF(\phi) \right) d\mu(\tilde{\mathbf{x}}_t) = \int \tilde{n}_t F(R(\tilde{\mathbf{x}}_t)) d\mu(\tilde{\mathbf{x}}_t)$$

So,

$$\dot{n}_t = n_t(1 - F(R(\mathbf{x}_t))) + \int \tilde{n}_t F(R(\tilde{\mathbf{x}}_t)) d\mu(\tilde{\mathbf{x}}_t)$$

Note that in steady state, we have

$$n^{in} = \int \tilde{n}_t F(R(\tilde{\mathbf{x}}_t)) d\mu(\tilde{\mathbf{x}}_t),$$

i.e. n^{in} is constant.

Also, we have

$$\int V(\phi, \mathbf{x}) dF(\phi) = \int_{-\infty}^{R(\mathbf{x})} (J - \phi) dF(\phi) + \int_{R(\mathbf{x})}^{\infty} S(\mathbf{x})$$

or equivalently

$$\int V(\phi, \mathbf{x}) dF(\phi) = S(\mathbf{x}) + F(R(\mathbf{x})) (J - S(\mathbf{x})) - F(R(\mathbf{x})) \mathbb{E}(\phi | \phi \leq R(\mathbf{x})).$$

4.1.2 The household problem solution

Choosing a utility function for the household of

$$u(c, l) = \frac{(c^\theta(1-l)^{1-\theta})^{1-\sigma}}{1-\sigma}$$

we can get closed form solutions for c^* and l^* (note $l^* = 1$ is never optimal with these preferences unless it is the only feasible choice). They are

$$l^* = \max\{0, \theta - \frac{(1-\theta)T}{(1-\tau)w}\}$$

and

$$c^* = (1-\tau)wl^* + T$$

Since there is no heterogeneity, if $l^* = 0$, there is also no output on an island. This may be optimal for the planner since he can borrow.

If the optimal taxes are such that $l^* = 0$, then $c^* = T$ and τ is irrelevant. Moreover, the indirect utility function $u(c^*, l^*) = T^{\theta(1-\sigma)}/(1-\sigma)$ (we must impose that the planner guarantees the household budget constraint is non-empty).

If the optimal taxes are such that $l^* > 0$, then we have

$$c^* = \theta((1-\tau)w + T),$$

$$l^* = \theta - \frac{(1-\theta)T}{(1-\tau)w}$$

and

$$1 - l^* = (1-\theta)\frac{(1-\tau)w + T}{(1-\tau)w}$$

Note this is very intuitive because (a) $(1-\tau)w$ is the price of leisure and (b) the household takes its total endowment $(1-\tau)w + T$ and distributes it according to θ between consumption and leisure. At any rate, the indirect utility function is

$$u(c^*, l^*) = \frac{\left(\theta((1-\tau)w + T) \left((1-\tau)w\right)^{\theta-1} (1-\theta)^{1-\theta} \theta^\theta\right)^{1-\sigma}}{1-\sigma}$$

We can characterize this further. Consider that the equilibrium wage in the interior case is $w^* = z(1-\alpha)(\dot{n})^{-\alpha}(l^*)^{-\alpha}$. Plugging in l^* as a function of w^* , τ , and T , we have

$$w^* = z(1-\alpha)(\dot{n})^{-\alpha} \left(\theta - \frac{(1-\theta)T}{(1-\tau)w^*} \right)^{-\alpha}$$

Implicit function theorem to determine $\partial w^*/\partial T$ and $\partial w^*/\partial \tau$. Is there a unique wage here? Does an equilibrium wage exist?

$$w^* = z(1 - \alpha)(\dot{n})^{-\alpha}(l^*(w^*))^{-\alpha}$$

$$LHS(w^*) := \left(\frac{1}{z(1 - \alpha)(\dot{n})^{-\alpha}} \right)^{-1/\alpha} w^{*-1/\alpha} = l^*(w^*) =: RHS(w^*)$$

Note the derivative

$$l^{*'}(w^*) = \frac{(1 - \theta)T}{(1 - \tau)} w^{*-2}$$

For $T = 0$,

- $l^*(w^*) = \theta$ (income and substitution effects cancel).

For $T > 0$,

- $l^{*'}(w^*) > 0$ (substitution effect dominates)
- $l^* < \theta$,
- if $w^* \leq \frac{T}{1-\tau} \frac{1-\theta}{\theta}$ then $l^*(w^*) = 0$

For $T < 0$,

- $T \geq -(1 - \tau)w$ or a feasible choice does not exist for the household. Equivalently, $w \geq -T/(1 - \tau)$.
- $l^* > \theta$
- $l^{*'}(w^*) < 0$ (income effect dominates).
- $\lim_{w^* \downarrow -\frac{T}{1-\tau}} l^*(w^*) = 1$
- $\lim_{w^* \uparrow \infty} l^*(w^*) = \theta$

The LHS is a decreasing function of w^* that is that approaches ∞ as $w^* \downarrow 0$ and declines monotonically to 0 as $w^* \uparrow \infty$.

The only problematic case is if $T < 0$. In this case, an equilibrium exists, but it is not necessarily unique. For uniqueness, a sufficient condition is $\frac{dw^{*-1/\alpha}}{dw^*} < l^{*'}(w^*)$ (both are < 0) since in the limit of $w^* = \infty$ the LHS is 0 and the RHS is θ . But I have not shown this yet.

Our restriction that the planner have $T \geq 0$ is equivalent to saying the substitution effect dominates the income effect. This guarantees a unique equilibrium w^* and l^* and, moreover, $l^* > 0$.

4.1.3 Simplifying the planner problem

The the planner's problem is

$$\max_{d, \tau, T, g, b'} S(\mathbf{x})$$

where $S(\mathbf{x})$ is consistent with equilibrium. If there are multiple equilibria, this objective function is not well-defined. To handle possible multiplicity, we assume that, if there are multiple equilibria, the one with the highest wage is selected. We show below that there is exactly one equilibrium associated with an equilibrium wage.

The planner's problem can be written as

$$\max_{d, \tau, T, g, b'} \left(\max_{c, l} u(c, l, g) + \beta \mathbb{E}_{z, f} V(\phi', b', n', z', f') \right)$$

subject to constraints. Since there are no intertemporal decisions by households, this then becomes

$$\max_{d, \tau, T, g, b'} \left(\max_{c, l} u(c, l, g) \right) + \beta \mathbb{E}_{z, f} V(\phi', b', n', z', f')$$

Household decisions are only impacted directly by τ, w, T and, if not separable, g . Write the policy functions as household labor supply as $c(w, \tau, T, g)$ and $l(w, \tau, T, g)$ and indirect utility function as $U(w, \tau, T, g)$. The problem is then

$$\max_{d, \tau, T, g, b'} U(w, \tau, T, g) + \beta \mathbb{E}_{z, f} V(\phi', b', n', z', f')$$

Taking into account the reservation strategy by households, an equilibrium conditioned on the planner's instruments is $(w, \dot{n}, R)$ such that

$$\begin{aligned} J - R &= U(w, \tau, T, g) + \beta \mathbb{E}_{z, f} V(\phi', b', \dot{n}, z', f') \\ \dot{n} &= n(1 - F(R)) + n^{in} \\ w &= (1 - \alpha)z(\dot{n}l(w, \tau, T, g))^{-\alpha} \end{aligned}$$

We cannot show there is a unique equilibrium because of the continuation utility's dependence on $\dot{n}$. However, for every equilibrium w , there is exactly one $(\dot{n}, R)$ pair consistent with it. This pair can be solved for directly via

$$\dot{n}(w, \tau, T, g) := \left(\frac{w}{(1 - \alpha)z} \right)^{-1/\alpha} l(w, \tau, T, g)^{-1}$$

and

$$R(w, \tau, T, g) := J - U(w, \tau, T, g) - \beta \mathbb{E}_{z, f} V(\phi', b', \dot{n}, z', f').$$

Then the set of equilibria wages are

$$W^*(b', d, \tau, T, g) := \{w | \dot{n}(w, \tau, T, g) = n(1 - F(R(w, \tau, T, g))) + n^{in}\}.$$

and the equilibrium wage resulting from the planners instruments is $w^*(b', d, \tau, T, g) = \max W^*(b', d, \tau, T, g)$ (b' and d enter through the continuation utility).⁵

Consequently, our equilibrium selection of assuming the highest equilibrium w is chosen implies that only one equilibrium is selected and hence the planner's objective function is well-defined.

Looking at the planner problem again, we have

$$\begin{aligned} S(b, n, z, f) &= \max_{d, \tau, T, g, b'} U(w^*, \tau, T, g) + \beta \mathbb{E}_{z, f} V(\phi', b', \dot{n}, z', f') \\ &\text{s.t.} \\ w^* &= w^*(b', d, \tau, T, g) \\ \dot{n} &= \dot{n}(w^*, \tau, T, g) \\ g + q(b', \dot{n}, z)b' &\leq b(1 - d) + \tau w^* \dot{n} l(w^*, \tau, T, g) - T \dot{n} \end{aligned}$$

As is customary in the default literature, this problem can be broken up into two problems, one for default (D) and the other for no default (N).

$$S(b, n, z, f = 0) = \max_d S^N(b, n, z)(1 - d) + S^D(n, z)d$$

(if repayment is feasible, o/w $S = S^D$) and

$$S(b = 0, n, z, f = 1) = S^D(n, z)$$

where the value of repaying is

$$\begin{aligned} S^N(b, n, z) &= \max_{\tau, T, g, b'} U(w^*, \tau, T, g) + \beta \mathbb{E}_z V(\phi', b', \dot{n}, z', f' = 0) \\ &\text{s.t.} \\ w^* &= w^*(b', d = 0, \tau, T, g) \\ \dot{n} &= \dot{n}(w^*, \tau, T, g) \\ g + q(b', \dot{n}, z)b' &\leq b + \tau \dot{n} w^* l(w^*, \tau, T, g) - T \dot{n} \end{aligned}$$

⁵If $\beta = 0$ and $\partial l / \partial w \geq 0$, there is at most one equilibrium: U is strictly increasing in w (via the envelope theorem); $\dot{n}$ is decreasing in w if $\partial l / \partial w \geq 0$; R is decreasing in $\dot{n}$ (and consequently increasing in w).

and the value of defaulting is

$$\begin{aligned}
S^D(n, z) = \max_{\tau, T, g} & U(w^*, \tau, T, g) + \beta \mathbb{E}_z \left(\begin{array}{c} \delta V(\phi', b' = 0, \dot{n}, z', f' = 1) \\ + (1 - \delta) V(\phi', b' = 0, \dot{n}, z', f' = 0) \end{array} \right) \\
& \text{s.t.} \\
& w^* = w^*(b', d = 1, \tau, T, g) \\
& \dot{n} = \dot{n}(w^*, \tau, T, g) \\
& g \leq (1 - \gamma) (\tau \dot{n} w^* l(w^*, \tau, T, g)) - T \dot{n}
\end{aligned}$$

The fraction $\gamma \in [0, 1]$ is the amount of tax dollars used to repay obligations in default. Note also that a γ fraction is not taken of $T \dot{n}$ because these are transfers, not tax revenue. If $\gamma = 0$, then this is totally unsecured debt. If $\gamma > 0$, then some resources are transferred to creditors. Finally, we have

$$V(\phi, b, n, z, f = 0) = \max\{S(b, n, z, f = 0), J - \phi\}$$

and

$$V(\phi, b = 0, n, z, f = 1) = \max\{S^D(n, z), J - \phi\}.$$

The additional restrictions that the planner must face are

$$\tau \in [0, 1], T \geq 0, g \geq 0, b' \in B.$$

The price schedule is

$$q(b', n', z) = \bar{q} \mathbb{E}_z [1 - d(b', n', z') + d(b', n', z') q^r(b', n', z')]$$

where the value of defaulted debt (what can be recovered) is

$$q^r(b, n, z) = p + \bar{q} \delta \mathbb{E}_z q^r(b + p, \dot{n}^D(n, z), z')$$

and the repayment in a given period is

$$p = \min\{-b, \gamma \tau^D(n, z) \dot{n}^D(n, z) w^D(n, z) l^D(n, z)\}.$$

Note that the value of defaulted debt is only non-zero if $\gamma > 0$ and $b < 0$. This builds on the work of Chatterjee and Gordon (2012) who examine garnishment in a consumer default context. Since each loan is made to a continuum of islands, the return of the contracts is certain (via a law of large numbers).

Note that S^N and consequently S are weakly increasing in b . This is that if any policy feasible for (b, n, z, f) is also feasible for $(b + \epsilon, n, z, f)$. Moreover, the default decision is weakly decreasing in

b . This then implies, in the case of no garnishment, that q is increasing in b' . These are all standard results in a novel context. A last result that I thought should also be true but computationally appears to not be is that the policy b' is monotone in b . Perhaps the fact that the period utility function is not necessarily increasing in g is playing a role here. The policy does appear to be almost monotone.

5 Calibration

The most interesting process for z has large innovations once in a while so that

$$\log z_{t+1} = \rho \log z_t + \epsilon_t$$

with $\rho \in (0, 1)$ and

$$\epsilon_t = \begin{cases} 0 & \text{w.p. } p \\ N(0, \sigma^2) & \text{w.p. } 1 - p \end{cases}$$

for $p \in (0, 1)$. With this sort of process, we capture boom-bust cycles of cities (in particular thinking of manufacturing towns) wherein they have high productivity for decades but suddenly switch to low productivity.

Our calibration strategy is the following:

1. Choose the mean of ϕ to match average migration flows.
2. Choose the (average) default cost γ to match the average default rate.
3. Choose the discount factor β to match the average debt-output ratio.
4. Choose the utility parameter controlling utility from g to match average tax revenue.

6 Quantitative Results

To be completed.

7 Conclusion

To be completed.

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