

Rehypothecation

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Abstract

Rehypothecation refers to the practice of reusing (selling or pledging as collateral) an asset that has already been pledged as collateral for a cash loan. In high inflation economies, rehypothecation improves economic welfare, but there is generally too much of it. We find that regulatory constraints that limit the practice generally serve to improve economic welfare.

Keywords: rehypothecation, money, repo, collateral.

1 Introduction

Consider two parties who agree to a temporary swap of assets, say cash in exchange for a security, as is characteristic of any repo transaction. From one perspective, the transaction looks like a collateralized cash loan. In a broker-client relationship, for example, the client may ask to borrow cash from the broker in order to purchase a security which is held by the broker on behalf of the client. At some point in the future, the client may wish to sell the security to pay off his loan. But before that day arrives, the broker may find it advantageous to sell the client's security (or to re-pledge it as collateral) in another repo arrangement. The practice of selling a security that has been pledged as collateral is called *rehypothecation*.

To the uninitiated, the rehypothecation of assets may seem strange at best and possibly illegal at worst. The practice is, in fact, perfectly legal in most jurisdictions. The rehypothecation right is explicitly stated in most brokerage agreements. In granting the right, the client benefits through a lower interest rate on his cash loan. As a result, the client's pledged collateral can potentially be re-used (re-sold) many times over, circulating as a *de facto* exchange medium. On the surface, at least, it appears that the same collateral is supporting several loans whose aggregate value may far exceed the value of the underlying collateral. To some observers, the outcome resembles a form fractional-reserve banking. As such, it should not be surprising to learn that rehypothecation is often regulated.

One goal of this paper to explore the theoretical implications of a legal restriction that limits (or precludes) the practice of rehypothecation. Of course, to answer this question we need a theory of rehypothecation. To the best of our knowledge, no such theory exists—at least, not one that has been written down in the context of a standard general equilibrium model.

We consider a situation in which two agents are in a trusting relationship—a partnership. While trust is present within the partnership, it does not exist among potential trading partners outside the relationship. Thus, intertemporal trades that occur outside the relationship can only occur through the use of an exchange medium. We assume two possible exchange media, one of which is more liquid than the other. That is, in a subset of transactions, only cash can be used. In another subset of transactions, both cash and an interest-bearing security (a Lucas tree) can be used.

At the beginning of a trading day, partners are allocated to one of the two markets. Common sense suggests that their combined wealth, consisting of money and assets, should be temporarily reassigned—cash should flow to the partner who is travelling to the cash market, and assets should flow to the partner travelling to cash/asset market. From an outsider’s point of view, the transaction looks like a repo—a collateralized cash loan. Of course, it is not at all clear who is lending what here—we could alternatively interpret the transaction as a loan of securities, collateralized by cash. In the model, however, collateral plays no role in supporting trade *within* the partnership because we assume the existence of trust.

But if the security plays no role in securing debt within a partnership, what accounts for the delivery of assets in exchange for a cash loan? The answer is that the security may play some useful role in a future transaction where securities are an acceptable exchange medium. To realize this potential, it is absolutely essential that asset holder is granted the right of rehypothecation. In this manner, the security enhances the commitment power for the agent engaged extra-partnership trades where securities are acceptable exchange media.

In the context of this simple theory of rehypothecation, we explore the economic implications of real-world restrictions that limit the practice. In Canada, for example, rehypothecation is prohibited, while in the U.K. there are no restrictions. The U.S. falls somewhere in between.

In our model, like most monetary models, the optimal policy is the Friedman rule. At a low enough inflation rate (high enough rate of return on money), rehypothecation plays no economic role. For monetary policies away from the Friedman rule, however, cash-only transactions are constrained. Transactions that are supported with collateral assets may or may not be constrained, depending on the supply of available assets. These two features are familiar in many monetary models. What is new here is the demand for rehypothecation when inflation is away from the Friedman rule. Rehypothecation permits cash to flow where it is needed, and the asset to flow where it can potentially be used to support a future loan.

For our numerical examples, we find that restrictions on the extent of rehypothecation are generally welfare *improving*. From the perspective of the theory of the second-best, this result is not surprising. The exact mechanism by which the result emerges, however, is interesting. Evidently, the

regulatory constraint we consider—which is modeled after what we see in reality—has the effect of increasing the demand for cash (there is an incentive to accumulate additional cash to overcome the regulatory restriction). This, in turn, has the effect of increasing the value of cash, which relaxes the cash constraint in the cash-only market. The cost of this is reduced trade in the non-cash market but this latter effect is more than offset by the former.

2 Environment

Time is discrete and the horizon is infinite, $t = 0, 1, 2, \dots, \infty$. Each date t is divided into three subperiods labeled the *morning*, *afternoon*, and *night*, respectively. There are two types of infinitely-lived agents labeled *bankers* and *workers*. There is a continuum of each type of agent, each with population mass normalized to unity.

Each morning, bankers are subject to an idiosyncratic shock that determines whether they have a desire to consume *early* or *late* in the afternoon.¹ Let $0 < \alpha < 1$ denote the probability of an early consumption opportunity, and let $\sigma(1 - \alpha)$ denote the probability of a late consumption opportunity, $0 \leq \sigma \leq 1$. Let $y \geq 0$ denote the nonstorable output produced in the afternoon and let $u(c)$ denote the flow utility payoff from consuming c either early or late. Assume $u'' < 0 < u'$ and $u(0) = 0$.

Each morning, workers are also subject to an idiosyncratic shock that determines whether they have an opportunity to produce early or late (if at all). Let α denote the probability of having an early production opportunity, and let $\sigma(1 - \alpha)$ denote the probability of having a late production opportunity. The flow utility payoff from producing y units of output in the afternoon is denoted $-h(y)$, where $h' > 0$, $h'' \geq 0$ and $h'(0) = h(0) = 0$.

In the evening, all agents have linear preferences defined over a non-storable night-good, $x \in \mathbb{R}$ (negative values of x are interpreted as production). Thus, our model adopts the quasilinear preference structure of Lagos and Wright (2005). Preferences for banker $i \in [0, 1]$ are given by

$$E_0^i \sum_{t=0}^{\infty} \delta^t [u(c_t(i)) + x_t^b(i)] \quad (1)$$

¹One might alternatively model an investment opportunity.

and preferences for worker $i \in [0, 1]$ are given by

$$E_0^i \sum_{t=0}^{\infty} \delta^t [-h(y_t(i)) + x_t^w(i)] \quad (2)$$

where $0 < \delta < 1$.

The environment has a single productive asset—a Lucas tree that generates a constant income flow $\omega \geq 0$ at the beginning of each night.

A Pareto optimal allocation is a feasible allocation that maximizes a weighted sum of *ex ante* utilities (1) and (2). If u is strictly concave, then efficiency dictates $c_t(i) = c_t$ for all bankers that have a desire to consume (and $c_t(i) = 0$ for those that do not). If h is strictly convex, then efficiency dictates $y_t(i) = y_t$ for all workers that have an opportunity to produce (and $y_t(i) = 0$ for those that do not). Since there are equal numbers of bankers and workers early and late each afternoon, the resource constraint implies $c_t = y_t$ for all t . Clearly, the efficient afternoon allocation is stationary and satisfies $u'(y^*) = h'(y^*)$.

The resource constraint at night is given by

$$\int_0^1 x_t^b(i) di + \int_0^1 x_t^w(i) di = \omega \quad (3)$$

One may without loss treat bankers and workers symmetrically, so let $x = x_t^w(i)$ and $z = x_t^b(i)$ for all i and t . The resource constraint is therefore just $x + z = \omega$. The choice of x (and z) serves to only distribute utility. Since the total surplus is proportional to $[u(y^*) - h(y^*)]$, the *ex ante* participation constraints are satisfied for any x such that $h(y^*) < x < u(y^*)$.

Remark 1 *Should check to see that what I just said is correct.*

3 Market structure

Assume for the moment that agents cannot communicate with each other in the morning. For now, bankers and workers simply realize their types in the morning and then move into their afternoon encounters. These encounters, as well as those in the evening, occur in centralized meeting places. We assume competitive spot markets.

If agents could commit to their promises, the solution to the resource allocation problem would be simple: bankers could borrow their afternoon consumption directly from workers, repaying them in the evening. Assume instead that agents are anonymous—that is, they lack commitment and that their personal trading histories are unavailable. In this case, an exchange medium is necessary for trade. In principle, this exchange medium could take the form of equity shares in the Lucas tree. We assume that equity can be used as a payment instrument in late afternoon trades but not in early afternoon trades. Early afternoon trades can only be financed with cash — a fiat money instrument. That is, we impose a cash-in-advance constraint on early afternoon trades, but not on late afternoon trades (where both cash and equity may be used).

Throughout we assume that the supply of fiat money grows at a constant gross rate $\mu > \delta$ and that new money is injected (or withdrawn) via lump-sum transfers (or taxes) in the evening.

4 Decision making

4.1 Bankers

4.1.1 Afternoon trading

Consider a banker that enters the morning with wealth portfolio (m, a) , where m represents money and a represents claims to the asset. Let (m'_e, a'_e) denote the wealth portfolio carried into the night by early bankers. Since the asset is assumed to be illiquid in early afternoon trades, $a'_e = a$. Let p_e denote the money price of early afternoon output y_e . Then $m'_e = m - p_e y_e$, subject to the cash-in-advance constraint $m'_e \geq 0$.

Let $B^e(m, a)$ denote the value of being an early banker and let $V(m', a')$ denote the value of being a banker that enters the night with portfolio (m', a') . These value functions must satisfy the following recursive relationship:

$$B^e(m, a) \equiv \max_{y_e} \{u(y_e) + V(m - p_e y_e, a) + \lambda_e [m - p_e y_e]\} \quad (4)$$

The demand for early afternoon output must satisfy:

$$(1/p_e)u'(y_e) = V_1(m'_e, a) + \lambda_e \quad (5)$$

By the envelope theorem:

$$B_1^e(m, a) = (1/p_e)u'(y_e) \quad (6)$$

$$B_2^e(m, a) = V_2(m'_e, a) \quad (7)$$

Let $B^l(m, a)$ denote the value of being an late banker. Let p_l denote the money price of goods and q_l the money price of the asset in the late afternoon. Then $m'_l = m + q_la - p_ly_l - q_la'_l$, with $m'_l \geq 0$ and $a'_l \geq 0$. Anonymity restricts late bankers to finance purchases entirely out of accumulated wealth: $m + q_la \geq p_ly_l$. Their choice problem is given by:

$$B^l(m, a) \equiv \max_{y_l, a'_l} \{u(y_l) + V(m'_l, a'_l) + \lambda_l [m + q_la - p_ly_l] + \psi a'_l\} \quad (8)$$

The demands for late afternoon goods and assets must satisfy:

$$(1/p_l)u'(y_l) = V_1(m'_l, a'_l) + \lambda_l \quad (9)$$

$$q_l V_1(m'_l, a'_l) = V_2(m'_l, a'_l) + \psi \quad (10)$$

By the envelope theorem:

$$B_1^l(m, a) = (1/p_l)u'(y_l) \quad (11)$$

$$B_2^l(m, a) = (q_l/p_l)u'(y_l) \quad (12)$$

4.1.2 Evening trading

Let $B(m, a)$ denote the value of entering the morning as a banker with portfolio (m, a) ; i.e.,

$$B(m, a) \equiv \alpha B^e(m, a) + \sigma(1 - \alpha)B^l(m, a) + (1 - \sigma)(1 - \alpha)V(m, a) \quad (13)$$

For a banker that enters the evening with portfolio (m', a') , the choice problem is given by:

$$V(m', a') \equiv \max_{m^+, a^+} \{x + \delta B(m^+, a^+)\} \quad (14)$$

subject to:

$$x = (\phi + \omega) a' + (1/p)(m' - m^+) - \phi a^+ - \tau$$

where ϕ denotes the real ex-dividend asset price in the evening, p denotes the evening price-level, and τ represents a lump-sum tax. There are also the non-negativity constraints $m^+, a^+ \geq 0$, but we anticipate that these will not bind for bankers in the evening.² The money and asset demands in the evening must satisfy:

$$(1/p) = \delta B_1(m^+, a^+) \quad (15)$$

$$\phi = \delta B_2(m^+, a^+) \quad (16)$$

By the envelope theorem:

$$V_1(m', a') = (1/p) \quad (17)$$

$$V_2(m', a') = (\phi + \omega) \quad (18)$$

4.2 Workers

4.2.1 Afternoon trading

Early afternoon workers supply product for money:

$$W^e(m, a) \equiv \max_{y_e} \{-h(y_e) + N(m + p_e y_e, a)\} \quad (19)$$

Product supply must satisfy:

$$(1/p_e)h'(y_e) = N_1(m'_e, a) \quad (20)$$

By the envelope theorem:

$$W_1^e(m, a) = (1/p_e)h'(y_e) \quad (21)$$

$$W_2^e(m, a) = N_2(m'_e, a) \quad (22)$$

Late afternoon workers supply product for money and/or assets:

$$W^l(m, a) \equiv \max_{y_l, a'_l} \{-h(y_l) + N(m'_l, a'_l)\}$$

where $m'_l = m + q_l a + p_l y_l + q_l a'_l$. While there are non-negativity constraints to consider here ($m'_l, a'_l \geq 0$), we anticipate that these will not bind as workers

²Bankers will want to rebuild their asset positions in order to finance their consumption expenditures the next day.

will want to accumulate wealth in the afternoon. Consequently, optimal behavior is characterized by:

$$(1/p_l)h'(y_l) = N_1(m'_l, a'_l) \quad (23)$$

$$q_l N_1(m'_l, a'_l) = N_2(m'_l, a'_l) \quad (24)$$

By the envelope theorem:

$$W_1^l(m, a) = (1/p_l)h'(y_l) \quad (25)$$

$$W_2^l(m, a) = q_l N_1(m'_l, a'_l) \quad (26)$$

4.2.2 Evening trading

Let $W(m, a)$ denote the value of entering the morning as a banker with portfolio (m, a) ; i.e.,

$$W(m, a) \equiv \alpha W^e(m, a) + \sigma(1 - \alpha)W^l(m, a) + (1 - \sigma)(1 - \alpha)N(m, a) \quad (27)$$

For a worker that enters the evening with portfolio (m', a') , the choice problem is given by:

$$N(m', a') \equiv \max_{m^+, a^+} \left\{ \begin{aligned} &[(\phi + \omega)a' + (1/p)(m' - m^+) - \phi a^+ - \tau] + \delta W(m^+, a^+) \\ &+ \zeta m^+ + \xi a^+ \end{aligned} \right\} \quad (28)$$

We anticipate that workers will want to dispose of their money and asset holdings in the evening, so here we make explicit the non-negativity constraints $m^+, a^+ \geq 0$. Optimality requires:

$$(1/p) = \delta W_1(m^+, a^+) + \zeta \quad (29)$$

$$\phi = \delta W_2(m^+, a^+) + \xi \quad (30)$$

By the envelope theorem:

$$N_1(m', a') = (1/p) \quad (31)$$

$$N_2(m', a') = (\phi + \omega) \quad (32)$$

4.3 Gathering restrictions

Let's start with the bankers in the evening. Use (13) to derive expressions for B_1 and B_2 and combine these with (15) and (16) to form the expressions:

$$\begin{aligned}(1/p) &= \delta \{ \alpha B_1^e(m^+, a^+) + \sigma(1 - \alpha) B_1^l(m^+, a^+) + (1 - \sigma)(1 - \alpha) V_1(m^+, a^+) \} \\ \phi &= \delta \{ \alpha B_2^e(m^+, a^+) + \sigma(1 - \alpha) B_2^l(m^+, a^+) + (1 - \sigma)(1 - \alpha) V_2(m^+, a^+) \}\end{aligned}$$

Next, use (6), (11), (17) and (7), (12), (18) with the expressions above to form:

$$(1/p) = \delta \{ \alpha(1/p_e^+) u'(y_e^+) + \sigma(1 - \alpha)(1/p_l^+) u'(y_l^+) + (1 - \sigma)(1 - \alpha)(1/p^+) \} \quad (33)$$

$$\phi = \delta \{ \alpha(\phi^+ + \omega) + \sigma(1 - \alpha) q_l^+ (1/p_l^+) u'(y_l^+) + (1 - \sigma)(1 - \alpha)(\phi^+ + \omega) \} \quad (34)$$

Now combine the workers' optimality conditions (20) and (23) with (31) to form:

$$(1/p) = (1/p_e) h'(y_e) \quad (35)$$

$$(1/p) = (1/p_l) h'(y_l) \quad (36)$$

Combining (24) with (31) and (32), we have:

$$(\phi + \omega) = (q_l/p) \quad (37)$$

Next, multiply both sides of (33) by p^+ to get:

$$(p^+/p) = \delta \{ \alpha(p^+/p_e^+) u'(y_e^+) + \sigma(1 - \alpha)(p^+/p_l^+) u'(y_l^+) + (1 - \sigma)(1 - \alpha) \}$$

At this stage, we impose the following stationarity restrictions: $\phi^+ = \phi$, $(p^+/p) = \mu$, $(p^+/p_e^+) = (p/p_e)$, $(p^+/p_l^+) = (p/p_l)$, $y_e^+ = y_e$, $y_l^+ = y_l$. Invoking stationarity, this latter expression becomes:

$$1 = \left(\frac{\delta}{\mu} \right) \{ \alpha(p/p_e) u'(y_e) + \sigma(1 - \alpha)(p/p_l) u'(y_l) + (1 - \sigma)(1 - \alpha) \}$$

Now use (35) and (36) to form:

$$1 = \left(\frac{\delta}{\mu} \right) \{ \alpha L(y_e) + \sigma(1 - \alpha) L(y_l) + (1 - \sigma)(1 - \alpha) \} \quad (38)$$

where

$$L(y) \equiv \left(\frac{u'(y)}{h'(y)} \right)$$

Note that $L'(y) < 0$ and $L(y^*) = 1$.

Lemma 2 *Condition (38) implies that the first-best allocation $y_l = y_h = y^*$ can only be implemented with the Friedman rule $\mu = \delta$.*

Consider next equation (34) with stationarity imposed:

$$\phi = \delta \{ \alpha (\phi + \omega) + \sigma(1 - \alpha)(q_l/p_l)u'(y_l) + (1 - \sigma)(1 - \alpha)(\phi + \omega) \}$$

Using condition (37) we can derive:

$$\frac{q_l}{p_l} = \frac{p}{p_l} \frac{q_l}{p} = \frac{p}{p_l} (\phi + \omega) = \frac{(\phi + \omega)}{h'(y_l)}$$

Combining this latter expression with the former, we derive:

$$\phi = \delta (\phi + \omega) \{ \alpha + \sigma(1 - \alpha)L(y_l) + (1 - \sigma)(1 - \alpha) \} \quad (39)$$

Lemma 3 *Condition (39) implies that the asset possesses a liquidity premium $\phi - \phi^* > 0$ iff trade is constrained in the late afternoon market; $L(y_l) > 1$ ($y_l < y^*$).*

4.4 Equilibrium

In what follows, we assume a monetary policy away from the Friedman rule; i.e., $\mu > \delta$. Because only cash is used in early afternoon trades, $\mu > \delta$ implies that the equilibrium level of production is inefficient, $0 < y_e < y^*$. From condition (5), this in turn implies $\lambda_e > 0$ so that bankers spend all their cash in the early afternoon: $p_e y_e = m$. As $m = M$ in equilibrium, we have:

$$p_e = \frac{M}{y_e} \quad (40)$$

While early afternoon trades are necessarily constrained, the same is not true of late afternoon trades. This is because the banker is permitted to

use his asset as an exchange medium in the late afternoon market. Whether trading is constrained or not in the late afternoon depends on parameters. Recall that the liquidity constraint for the late banker is $m + q_l a \geq p_l y_l$. In any equilibrium we have $m = M$ and $a = 1$, so that:

$$p_l y_l \leq M + q_l \quad (41)$$

with (41) holding with equality if $y_l < y^*$.

Since $\mu > \delta$, we know that $0 < y_e < y^*$. The question is whether trade in the late afternoon is constrained or not. If it is unconstrained, then $y_l = y^*$ and conditions (38) and (39) become:

$$1 = \left(\frac{\delta}{\mu} \right) \{ \alpha L(y_e) + 1 - \alpha \} \quad (42)$$

$$\phi = \delta (\phi + \omega) \quad (43)$$

which implies that the asset is priced at its fundamental value. Condition (41) in this case satisfies $M/p_l + q_l/p_l \geq y^*$, or

$$\frac{M}{p} \frac{p}{p_l} + \frac{q_l}{p} \frac{p}{p_l} \geq y^*$$

Conditions (35) and (40) imply $M/p = y_e h'(y_e)$. Condition (37) implies that $(q_l/p) = (\phi + \omega)$. Condition (36) implies that $(p/p_l) = 1/h'(y_l)$. Combining these results with the expression above yields $y_e h'(y_e) + (\phi + \omega) \geq y^* h'(y^*)$, or

$$\left(\frac{\omega}{1 - \delta} \right) \geq y^* h'(y^*) - y_e h'(y_e) \quad (44)$$

where y_e is determined by (42). Notice that y_e is independent of ω and is a decreasing function of μ . Consequently, $\hat{\omega}(\mu) \equiv (1 - \delta) [y^* h'(y^*) - y_e h'(y_e)]$ is an increasing function of μ , with $\hat{\omega}(\delta) = 0$. In short, it becomes more difficult to implement efficient trades at higher inflation rates.

In “asset poor” economies (ω sufficiently small), condition (44) is violated so that the late banker’s liquidity constraint binds:

$$(\phi + \omega) = y_l h'(y_l) - y_e h'(y_e) \quad (45)$$

In this case, the equilibrium (y_e, y_l, ϕ) is characterized by conditions (38), (39) and (45).

Proposition 4 *For $\mu = \delta$ (Friedman rule), the equilibrium is efficient: $y_e = y_l = y^*$ independent of ω . Given $\mu > \delta$, we have $0 < y_e < y^*$ and $y_l = y^*$ for $\omega \geq \hat{\omega}(\mu)$, with (y_e, ϕ) determined by (42), (43); and $0 < y_e \leq y_l < y^*$ for $\omega < \hat{\omega}(\mu)$ with (y_e, y_l, ϕ) determined by (38), (39) and (45).*

5 Rehypotheccation

The relevant region in the parameter space consist of cases for which $\omega < \hat{\omega}(\mu)$ and $\mu > \delta$. In this region, there is an “asset shortage” in the sense that the Lucas tree commands a liquidity premium. That is, its market price is above its fundamental value, reflecting the scarcity of available exchange media.

Early models of asset scarcity (e.g., see Geromichalos, *et. al.* 2007) stress a general lack of high quality (liquid) assets. In the equilibrium described above, the problem has as much to do with the *distribution* of a given amount of liquidity. In the present context, note that the early bankers in our model possess the asset in their wealth portfolio even though they can make no immediate use of it for making payments. Late bankers, on the other hand, are strapped for liquidity. It seems that there may be room for a deal between bankers.

There is more than one way in which to set things up here. To demonstrate the basic point, we consider the simplest possible scenario. Imagine that bankers are matched pairwise and that they can commit perfectly to arrangements agreed upon between themselves (commitment is still lacking in dealings between bankers and workers). Moreover, to make things even more simple than they need to be, assume that banker types are perfectly negatively correlated within each pairwise match. In short, we coalesce two bankers into one decision-making unit, with each banker alternating stochastically between types, such that the two bankers are always different types (let $\alpha = 1/2$). We discuss later the implications of relaxing commitment power among the bankers.³

³We could also cite the Corbae and Ritter paper.

5.1 Banker coalition

Consider then a pair of bankers—a *coalition*—entering the morning of a period with *combined* wealth portfolio (m, a) . Let (m_l, a_l) denote the portfolio allocated to the late banker. Then the early banker faces the following liquidity constraint,

$$m - m_l - p_e y_e \geq 0 \quad (46)$$

and a flow budget constraint,

$$m + q_l a - m_l - q_l a_l - p_e y_e \geq 0 \quad (47)$$

together with the restrictions,

$$m_l \geq 0 \quad (48)$$

$$a - a_l \geq 0 \quad (49)$$

There are also the restrictions $m_l \leq m$ and $a_l \geq 0$, but we anticipate that these will never bind.⁴

Consider now the late banker. If he has no opportunity to trade, then he simply carries his portfolio into the evening, pooling them with the early banker. If the late banker does have a trading opportunity, he faces the liquidity constraint,

$$m_l + q_l a_l \geq p_l y_l \quad (50)$$

and the flow budget constraint,

$$m_l + q_l a_l - m'_l - q_l a'_l - p_l y_l \geq 0 \quad (51)$$

where (m'_l, a'_l) is the portfolio the late banker carries into the evening. We also impose the restrictions,

$$m'_l \geq 0 \quad (52)$$

$$a'_l \geq 0 \quad (53)$$

Each banker is assumed to own an equal share of the coalition portfolio $(m/2, a/2)$. The value of the coalition lies in the opportunity for “asset

⁴Since the early banker must use cash to finance consumption, it would make no sense to send all of the cash to late banker ($m_l - m$). Likewise, since only the late banker can potentially make use of the asset in a transaction, it makes no sense to allocate the entire asset position to the early banker ($a_l = 0$).

trades” in the morning, conditional on type realizations. Given our setup, a natural exchange would have cash flowing to the early banker and assets flowing to the late banker; i.e.,

$$\begin{aligned} m_l &< m/2 \\ q_l a_l &> q_l a/2 \end{aligned}$$

If $m_l < m/2$, then the late banker is in effect sending $[m/2 - m_l]$ dollars to the early banker. If $a_l > a/2$, then the early banker is in effect sending $q_l [a_l - a/2]$ dollars worth of assets to the late banker. If the value of what is exchanged is equated, then the transaction can be considered either as an outright purchase of assets by the late banker, or as a fully collateralized cash loan to the early banker. (Alternatively, we might view the transaction as a fully collateralized *asset* loan to the late banker.)

Of course, there is no reason to believe that a “balanced” trade $[m/2 - m_l] = q_l [a_l - a/2]$ is necessarily optimal in this relationship. In fact, we anticipate that an optimal intrabank allocation will sometimes have the property $0 < [m/2 - m_l] < q_l [a_l - a/2]$. In this case, the transaction looks like an overcollateralized cash loan (from late to early banker) or, equivalently, and undercollateralized asset loan (from early to late banker).

It is the undercollateralized asset loan that seems to trouble regulators concerned with the rehypothecation of assets. While the rehypothecation of borrowed cash is viewed as natural (what else is one *supposed* to do with borrowed cash?), the rehypothecation of borrowed assets is not.⁵ As such, some jurisdictions place restrictions on the rehypothecation of borrowed securities. One such real-world restriction takes the following form:

$$\theta [m/2 - m_l] \geq q_l [a_l - a/2] \tag{54}$$

for some policy parameter $\theta \geq 1$. Think of θ as the largest leverage ratio permissible in any under collateralized asset loan (in which the collateral can be rehypothecated).⁶ In what follows, we will be interested in examining the economic consequences of this regulatory restriction.

⁵An image sometimes offered of the practice is that of borrowing money against a vehicle, and then discovering that your vehicle has unwittingly been sold under your nose to another agent who takes possession of it. In reality, the rehypothecation right is something a debtor must agree to beforehand.

⁶In Canada, rehypothecation is apparently prohibited; in which case $\theta = 1$. In the U.K., there are apparently no legal limits to rehypothecation; in which case $\theta = \infty$. In the U.S., $\theta = 1.4$.

Let B and V denote the coalition value functions for the morning and evening, respectively. These value functions satisfy the following recursive relationship:

$$\begin{aligned}
B(m, a) \equiv \max_{\{y_e, y_l, m_l, a_l, a'_l, m'_l\}} & u(y_e) + \lambda_e[m - m_l - p_e y_e] + \zeta_e m_l + \xi_e q_l(a - a_l) \\
& + \sigma \left\{ u(y_l) + \lambda_l[m_l + q_l a_l - p_l y_l] \right. \\
& + \psi[m_l + q_l a_l - m'_l - q_l a'_l - p_l y_l] \\
& + \zeta_l m'_l + \xi_l q_l a'_l + \chi[\theta(m/2 - m_l) - q_l(a_l - a/2)] \\
& \left. + V(m', a') \right\} + (1 - \sigma)V(m - p_e y_e, a)
\end{aligned}$$

where

$$\begin{aligned}
m' &\equiv [m - m_l - p_e y_e] + m'_l \\
a' &\equiv [a - a_l] + a'_l
\end{aligned}$$

represents the coalition asset portfolio taken into the evening conditional on a late afternoon trade. Note that the non-negativity constraints (48), (49), (52) and (53) imply $m' \geq 0, a' \geq 0$. The Lagrange multipliers $\lambda_e, \zeta_e, \xi_e, \lambda_l, \psi, \zeta_l, \xi_l, \chi$ are associated with the constraints described above, and the value function V remains as defined in (14). That is, the choice problem of the coalition in the evening is formally equivalent to what each banker faced individually in the previous section. This latter fact implies that the envelope results (17) and (18) remain valid here. Using these latter two conditions, the first-order conditions for the coalition problem in the morning are given by:

$$u'(y_e) - [\lambda_e + 1/p]p_e = 0 \quad (55)$$

$$u'(y_l) - [\lambda_l + \psi]p_l = 0 \quad (56)$$

$$-\lambda_e + \zeta_e + \sigma[\lambda_l + \psi - \theta\chi - 1/p] = 0 \quad (57)$$

$$-q_l \xi_e + \sigma[q_l(\lambda_l + \psi - \chi) - (\phi + \omega)] = 0 \quad (58)$$

$$-\psi + \zeta_l + 1/p = 0 \quad (59)$$

$$-(\psi - \xi_l)q_l + (\phi + \omega) = 0 \quad (60)$$

From the envelope conditions associated with the coalition problem above, we have:

$$B_1(m, a) = \lambda_e + 1/p + \sigma\theta\chi/2 \quad (61)$$

$$B_2(m, a) = q_l \xi_e + (\phi + \omega) + \sigma q_l \chi/2 \quad (62)$$

Again, because the choice problem in the evening remains structurally the same for bankers whether operating together or apart, the first-order conditions (15) and (16) determining portfolio rebalance remain valid here.

Finally, note the decisions facing workers remain unaffected by the coalition structure. Thus, conditions (35), (36) and (37) continue to remain valid here.

5.2 Gathering restrictions

Combine (61) with (15) to form:

$$\frac{1}{p} = \delta [\lambda_e^+ + 1/p^+ + \sigma\theta\chi^+/2] \quad (63)$$

After some derivation (Appendix A1), we get:

$$\mu = \delta[L(y_e) + \sigma\theta p\chi/2] \quad (64)$$

Next, combine (62) with (16) to form:

$$\phi = \delta [q_l^+ \xi_e^+ + (\phi^+ + \omega) + \sigma q_l^+ \chi^+/2] \quad (65)$$

Using (58) and some derivations (Appendix A2), we get:

$$\phi = \delta(\phi + \omega) [\sigma L(y_l) + (1 - \sigma) - \sigma p\chi/2] \quad (66)$$

Next, combine conditions (55) and (56) with (57) to form (see Appendix A3):

$$p\zeta_e = [L(y_e) - 1] - \sigma[L(y_l) - 1] + \sigma\theta p\chi \quad (67)$$

Similarly, combine (56) with (58) and (36) to form, after some manipulation (Appendix A4):

$$p\xi_e = \sigma [L(y_l) - 1] - \sigma p\chi \quad (68)$$

Lemma 5 $y_l = y^*$ only if $\chi = 0$.

Proof. Follows from (68) and $\xi_e \geq 0$. ■

We now invoke the market-clearing condition $m = 2M$. Bankers in the early afternoon spend all of their cash (at the Friedman rule, they weakly

prefer to do so). Thus, (46) holds with equality. Together with the market-clearing condition, we have $2M = p_e y_e + m_l$, which when combined with (35) yields:

$$2M/p - m_l/p = h'(y_e)y_e \quad (69)$$

The liquidity constraint in the late afternoon (50) and condition (36) imply:

$$m_l/p + (\phi + \omega)a_l \geq y_l h'(y_l) \quad (70)$$

Finally, the regulatory constraint needs to be satisfied in equilibrium. Thus,

$$\theta[M/p - m_l/p] \geq (a_l - 1)(\phi + \omega) \quad (71)$$

where here, we have invoked the equilibrium conditions $m = 2M$ and $a = 2$. If this constraint remains slack, then $\chi = 0$; otherwise $\chi > 0$ and the condition above holds with equality.

To characterize the different possible configurations for stationary equilibria, we begin by assuming that the regulatory constraint is slack.

5.3 Regulatory constraint is slack

Assume that the regulatory constraint is slack ($\chi = 0$). Then from (64) we have a condition that determines y_e solely as a function of inflation μ ; i.e., $\mu = \delta L(y_e)$. Once again, the Friedman rule is consistent with first-best implementation, at least, as far as early afternoon production is concerned. Let us assume (and then verify) that $y_l = y^*$ ($\lambda_l = 0$). Then from (66) we have the standard asset-pricing formula for the Lucas tree. Assume that $\mu > \delta$, so that $L(y_e) > 1$. Since $\lambda_l = \chi = 0$, from (67) we have $\zeta_e > 0$. This, in turn, implies $m_l = 0$, so that (69) determines the evening price-level p ; i.e., $2M/p = y_e h'(y_e)$.

We now identify the circumstances under which the assumption $y_l = y^*$ is legitimate. Essentially, we need condition (70) to hold. Since $m_l = 0$, we need $(\phi + \omega)a_l \geq y^* h'(y^*)$. With the asset priced at fundamental value and all assets allocated to the late banker ($a_l = a = 2$), we need the following to be true:

$$\omega \geq 0.5(1 - \delta)y^* h'(y^*) \equiv \hat{\omega}$$

Compare this latter condition with its counterpart under the earlier unit banking structure (condition 44):

$$\omega \geq (1 - \delta) [y^* h'(y^*) - y_e h'(y_e)] \equiv \hat{\omega}(\mu)$$

Recall that $\hat{\omega}(\mu)$ is strictly increasing in $\mu \geq \delta$ and that $\hat{\omega}(\delta) = 0$. Therefore, there exists a $\mu = \mu_0 > \delta$ such that $\hat{\omega}(\mu_0) = \hat{\omega}$.

Proposition 6 *We might say a few things about this. For low inflation rates, it's easier to implement efficiency in the afternoon under unit banking. But welfare is probably higher under the coalition structure anyway (unless some strange things happen in GE).*

Let us now fix $\omega \geq \hat{\omega}$ and identify what is needed to ensure that the regulatory constraint does not bind. Recall that $M/p = 0.5y_e h'(y_e)$, $m_l = 0$, $a_l = 2$ and $(\phi + \omega) = \omega(1 - \delta)^{-1}$. Combining these conditions with (71), we derive:

$$\theta \geq \left[\frac{2\omega}{y_e h'(y_e)(1 - \delta)} \right] \equiv \Theta(\omega, \mu)$$

5.4 Regulatory constraint binds

Suppose that $\omega \geq \hat{\omega}$ and that $\theta < \Theta(\omega, \mu)$ so that $y_l = y^*$ is implementable when the regulatory constraint is slack but not when it binds ($\chi > 0$). From (64) we have:

$$\mu = \delta[L(y_e) + 0.5p\chi\sigma\theta]$$

We know from Lemma X that $\chi > 0$ implies $y_l < y^*$. We must have a result somewhere that says if $y_l < y^*$ then $y_e < y^*$. Assuming this is the case, then the effect of a binding regulatory constraint is to *increase y_e closer to its first-best level*. (This suggests the possibility that the regulatory constraint might improve welfare in some circumstances.) From (61), we see that if $\chi > 0$, then the coalition attaches additional value to cash accumulation because more cash helps to overcome the regulatory constraint on future rehypothecation. The additional cash helps finance more consumption in the early afternoon, at the expense of the late afternoon. (Consumption smoothing).

Again using (64), we can express $\sigma\chi p$ in terms of y_e :

$$\sigma\chi p = 2\theta^{-1}[\mu/\delta - L(y_e)], \tag{72}$$

which allows us to solve the asset price ϕ as a function of (y_e, y_l) :

$$\phi = \frac{\delta\omega F(y_e, y_l)}{1 - \delta F(y_e, y_l)}, \quad (73)$$

where

$$F(y_e, y_l) \equiv 1 + \theta^{-1}[L(y_e) - \mu/\delta] + \sigma[L(y_l) - 1]$$

For any given (m_l, a_l) we can characterize an equilibrium (y_e, y_l, p) using the market clearing condition and the liquidity and regulatory constraints, which both bind. We get:

$$2M/p - m_l/p = h'(y_e)y_e \quad (74)$$

$$m_l/p + \frac{a_l\omega}{1 - \delta F(y_e, y_l)} = h'(y_l)y_l \quad (75)$$

$$\theta[M/p - m_l/p] = \frac{(a_l - 1)\omega}{1 - \delta F(y_e, y_l)}. \quad (76)$$

The Lagrange multipliers on m_l and a_l imply:

$$p\zeta_e = [\mu/\delta - 1] + [\mu/\delta - L(y_e)] - \sigma[L(y_l) - 1] \geq 0 \quad (77)$$

$$p\xi_e = -2\theta^{-1}[\mu/\delta - L(y_e)] + \sigma[L(y_l) - 1] \geq 0. \quad (78)$$

Proposition 7 *If $1 < \theta < \Theta(\omega, \mu)$, with $\omega \geq \hat{\omega}$ and $\mu > \delta$, then: (i) $L(y_e) < \mu/\delta$; (ii) $y_l < y^*$; (iii) $a_l \in (1, 2)$; and (iv) $m_l = 0$ with $\zeta_e > 0$.*

Proof. Note that under the assumptions on θ and ω , $\chi > 0$.

(i) $L(y_e) < \mu/\delta$ follows from (72) and $\chi > 0$.

(ii) $y_l < y^*$, i.e., $L(y_l) > 1$, follows from $\theta < \Theta(\omega, \mu)$.

(iii) First, we need to show that $F(y_e, y_l) \in (1, \delta^{-1})$. $F(y_e, y_l) < \delta^{-1}$ follows from (73) and $\phi > 0$. To show $F(y_e, y_l) > 1$ we use (78). A simple rearrangement implies $\theta^{-1}[L(y_e) - \mu/\delta] + F(y_e, y_l) - 1 \geq 0$. Thus, $F(y_e, y_l) \geq 1 + \theta^{-1}[\mu/\delta - L(y_e)] > 1$, where the last inequality follows from $L(y_e) < \mu/\delta$.

To show $a_l < 2$ (and hence, $\xi_e = 0$), note that (75) implies

$$\frac{[1 - \delta F(y_e, y_l)]m_l/p}{1 - \delta} + \frac{a_l\omega}{1 - \delta} = \frac{[1 - \delta F(y_e, y_l)]h'(y_l)y_l}{1 - \delta}.$$

Given $m_l \geq 0$ and $\omega \geq \hat{\omega}$ we obtain

$$0.5a_l h'(y^*)y^* \leq \frac{a_l \omega}{1 - \delta} \leq \frac{[1 - \delta F(y_e, y_l)]h'(y_l)y_l}{1 - \delta}.$$

Suppose $a_l = 2$. Since $F(y_e, y_l) \in (1, \delta^{-1})$, we get $h'(y^*)y^* < h'(y_l)y_l$, a contradiction with (ii). Thus, $a_l < 2$ and $\xi_e = 0$.

(iv) Suppose $\zeta_e = \xi_e = 0$. Then, (77) and (78) imply

$$\begin{aligned} L(y_e) - 1 &= \frac{2(\theta - 1)}{(\theta - 2)}[\mu/\delta - 1] \\ \sigma[L(y_l) - 1] &= \frac{2}{(2 - \theta)}[\mu/\delta - 1]. \end{aligned}$$

If $\theta > 1$, we need both $\theta < 2$ and $\theta > 2$, so this cannot be an equilibrium. Thus, $\zeta_e > 0$ with implies $m_l = 0$.

Given $m_l = 0$, $a_l > 1$ follows from (76) and $F(y_e, y_l) < \delta^{-1}$. ■

Note that, given $\xi_e = 0$, we obtain $F(y_e, y_l) = f(y_e)$ where:

$$f(y_e) \equiv 1 + \theta^{-1}[\mu/\delta - L(y_e)].$$

Hence, for $\theta > 1$, the equilibrium is characterized by:

$$\begin{aligned} h'(y_l)y_l - 0.5\theta h'(y_e)y_e &= \frac{\omega}{1 - \delta f(y_e)} \\ \mu/\delta - L(y_e) &= 0.5\theta\sigma[L(y_l) - 1], \end{aligned}$$

with $m_l = 0$ and

$$\begin{aligned} p &= \frac{2M}{h'(y_e)y_e} \\ a_l &= \frac{h'(y_l)y_l[1 - \delta f(y_e)]}{\omega}. \end{aligned}$$

With a binding regulatory constraint, the early banker is disposing of all the coalition's cash, while the late banker is unable to utilize all the coalition's assets. Thus, the market value of the cash loan to the early banker exceeds the market value of the asset loan to the late banker. From an outsider's perspective, this transaction would look like an under-collateralized cash loan or, equivalently, an over-collateralized asset loan.

5.5 Comparative statics with a binding regulatory constraint

Suppose $u(y) = \ln y$ and $h(y) = y$. Then, we can solve the model analytically. The expressions are, however, rather complex and cannot be easily interpreted. Consider instead expressing y_e in terms of y_l . We get

$$y_e = \frac{2\delta}{2\mu - \delta\theta\sigma(1/y_l - 1)}.$$

Since $y_l < y^* = 1$ when $\chi > 0$, we have that y_e is decreasing in y_l . Therefore, if we start in an equilibrium where $y_l = y^*$ and the regulatory constraint is just satisfied, i.e., where $\omega = \hat{\omega}$ and $\theta = \Theta(\mu)$, then any parameter perturbation that makes the regulatory constraint bind implies y_e increases and y_l decreases. Since bankers' preferences are strictly concave, any small such perturbation improves their ex-ante welfare (i.e., before their afternoon type is revealed).

The intuition for the welfare result derived above is that the regulatory constraint corrects the inefficiency generated by being away from the Friedman rule. Starting from an equilibrium with $\mu > \delta$ and $y_e < y^*, y_l = y^*$, imposing a binding regulatory constraint increases the rate of return on cash, since it is now valued higher than before due to its ability to relax the regulatory constraint.

Figure 1 shows an example comparing ex-ante banker's welfare for the three arrangements considered in this paper. The parameterization used is: $\beta = 0.96, \sigma = 0.5, \theta = 1.05, \mu = 1.05$. The figure shows welfare as a function of ω . In this particular case, a coalition dominates banks acting unilaterally. However, imposing a limit on rehypothecation increases welfare when the asset's dividend is moderately high. Figure 2 shows which arrangement dominates as we vary μ and ω . As we can see, when a coalition dominates unibanking, imposing a regulatory constraint weakly increases welfare.

Figure 1: Ex-ante banker's welfare - $\mu = 1.05$

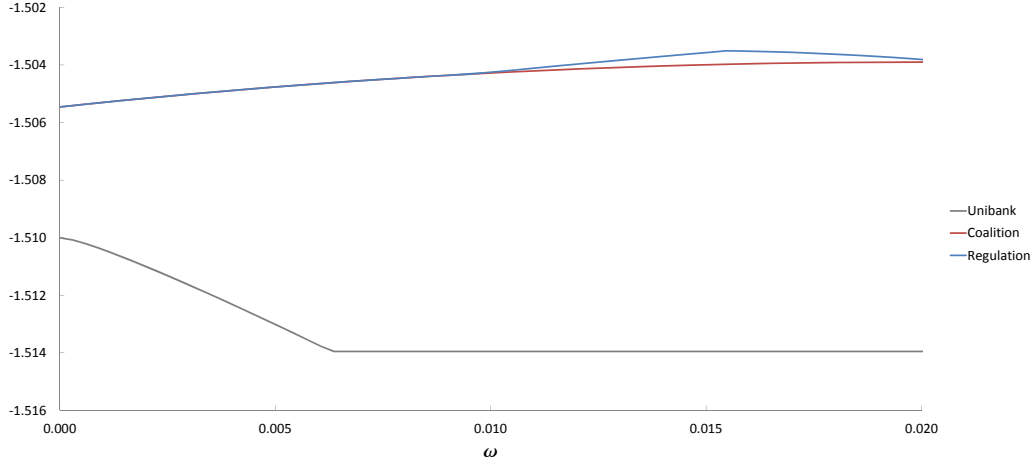
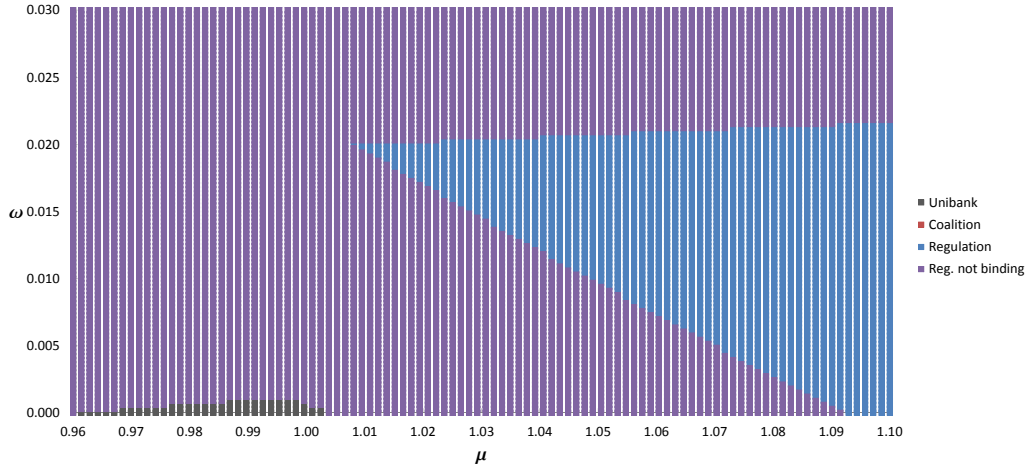


Figure 2: Ex-ante banker's welfare



6 Appendix

6.1 A1

Condition (55) implies $[\lambda_e^+ + 1/p^+] = u'(y_e)/p_e^+$, which when combined with the expression (63) yields:

$$\frac{1}{p} = \delta \left[\frac{1}{p_e^+} u'(y_e) + 0.5 \chi^+ \sigma \theta \right]$$

Now, multiply through by p^+ . By stationarity, $(p^+/p) = \mu$ and $p^+\chi^+ = p\chi$. Moreover, by condition (35), $(p^+/p_e^+) = 1/h'(y_e)$. Combining these restrictions yields (64).

6.2 A2

Condition (58) implies $q_l\xi_e = \sigma[q_l(\lambda_l + \psi - \chi) - (\phi + \omega)]$, which when combined with (65) yields:

$$\begin{aligned}\phi &= \delta [\sigma[q_l^+ (\lambda_l^+ + \psi^+ - \chi^+) - (\phi^+ + \omega)] + (\phi^+ + \omega) + \sigma q_l^+ \chi^+ / 2] \\ \phi &= \delta [\sigma q_l^+ (\lambda_l^+ + \psi^+) + (1 - \sigma)(\phi^+ + \omega) - \sigma q_l^+ \chi^+ / 2]\end{aligned}$$

Using condition (56) $[\lambda_l^+ + \psi^+] = u'(y_l)/p_l^+$, we derive:

$$\begin{aligned}\phi &= \delta \left[\sigma q_l^+ \left(\frac{1}{p_l^+} u'(y_l) \right) + (1 - \sigma)(\phi^+ + \omega) - \sigma q_l^+ \chi^+ / 2 \right] \\ \phi &= \delta \left[\sigma \frac{q_l^+}{p^+} \left(\frac{p^+}{p_l^+} u'(y_l) \right) + (1 - \sigma)(\phi^+ + \omega) - \sigma \frac{q_l^+}{p^+} p^+ \chi^+ / 2 \right]\end{aligned}$$

Now use conditions (36) and (37) to derive:

$$\phi = \delta(\phi^+ + \omega) [\sigma L(y_l) + (1 - \sigma) - \sigma p^+ \chi^+ / 2]$$

Imposing stationarity $\phi = \phi^+$ and $p\chi = p^+\chi^+$ and rearranging yields (66).

6.3 A3

From (57) we have:

$$\zeta_e = \lambda_e - \sigma[\lambda_l + \psi - \theta\chi - 1/p]$$

Using condition (56),

$$\zeta_e = \lambda_e - \sigma\left[\frac{1}{p_l} u'(y_l) - \theta\chi - 1/p\right]$$

From condition (55) we have $\lambda_e = u'(y_e)/p_e - 1/p$ which, when combined with the expression above, yields:

$$\zeta_e = \frac{1}{p_e} u'(y_e) - 1/p - \sigma\left[\frac{1}{p_l} u'(y_l) - \theta\chi - 1/p\right]$$

$$p\zeta_e = \frac{p}{p_e}u'(y_e) - 1 - \sigma\left[\frac{p}{p_l}u'(y_l) - \theta p\chi - 1\right]$$

Now use conditions (35) and (36) to get:

$$\begin{aligned} p\zeta_e &= L(y_e) - 1 - \sigma[L(y_l) - \theta p\chi - 1] \\ p\zeta_e &= [L(y_e) - 1] - \sigma[L(y_l) - 1] + \sigma\theta p\chi \end{aligned}$$

6.4 A4

Condition (58) implies:

$$\begin{aligned} \xi_e &= \sigma \left[(\lambda_l + \psi - \chi) - \frac{(\phi + \omega)}{q_l} \right] \\ \xi_e &= \sigma (\lambda_l + \psi) - \sigma \frac{(\phi + \omega)}{q_l} - \sigma\chi \end{aligned}$$

Use (56) to get:

$$\begin{aligned} \xi_e &= \sigma \frac{1}{p_l}u'(y_l) - \sigma \frac{(\phi + \omega)}{q_l} - \sigma\chi \\ p\xi_e &= \sigma \frac{p}{p_l}u'(y_l) - \sigma \frac{p(\phi + \omega)}{q_l} - \sigma p\chi \\ p\xi_e &= \sigma [L(y_l) - 1] - \sigma p\chi \end{aligned}$$

7 References

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