

# Optimal Unemployment Insurance and Cyclical Fluctuations

(Preliminary and Incomplete)

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## Abstract

In this paper, we investigate the design of optimal unemployment insurance in an environment with moral hazard and cyclical fluctuations. The optimal unemployment insurance contract balances the insurance motive to provide consumption for the unemployed with the provision of incentives to search for a job. This balance is affected by aggregate conditions, as recessions are characterized by reductions in job finding rates. We show how benefits should vary with aggregate conditions in an optimal contract. In a special case of the model, the optimal contract can be solved in closed form. We show how this contract can be implemented in a rather simple way: by allowing unemployed workers to borrow and save in a risk-free bond, providing flow payments which are constant over an unemployment spell but vary with the aggregate state, and giving additional lump sum payments (or charges) upon finding a job or when the aggregate state switches. We then consider a calibrated version of the model and study the quantitative impact of changing from the current unemployment system to the optimal one. In a recession, the optimal system reduces unemployment rates by roughly 2.5 percentage points and shortens the duration of unemployment by roughly 50%.

## 1 Introduction

In the United States, unemployment insurance is implemented at the state level, with a typical program providing 26 weeks of benefits, at a level determined as a fraction of previous earnings (the replacement rate) up to a cap. The average replacement rate in the US between 1975 and 2004 was 36% of the claimant's average weekly wage.<sup>1</sup> However in times of recession, the federal government has typically provided extended unemployment benefits, in different tiers which kick in once state benefits are exhausted. During the most recent recession, a number of extended

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<sup>1</sup>These figures come from [Kletzer and Rosen \[2006\]](#).

benefits programs with different tiers have been instituted, allowing for a maximum of 99 weeks of unemployment benefits in high unemployment states. During the most recent recession, in June 2008 Congress instituted the Emergency Unemployment Compensation program. This provided an additional 34 weeks of benefits to all states (in two tiers), a third tier providing a further 13 weeks for states with a 3-month average unemployment rate of 6% or greater, and a fourth tier with an additional 6 weeks for states with a 3-month average unemployment rate of 8.5% or greater.<sup>2</sup> Then in 2009, a federal-state Extended Benefits program was instituted (and extended in July 2010), which provided an additional 20 weeks by essentially adding 13 weeks to tier three and 7 weeks to tier four. Thus in high unemployment states, benefits were provided for a maximum of 99 weeks. It is intuitive that unemployment insurance should change with changes in the labor market, as economic slowdowns naturally lead to longer unemployment spells. However the observed pattern of extensions and the threshold unemployment rates of the different tiers are rather arbitrary, and the whole program has been subject to uncertainties in its implementation. In this paper, we study the design of optimal unemployment insurance when the economy experiences cyclical fluctuations in job finding and job loss rates.

In particular, we build on the previous work by [Shavell and Weiss \[1979\]](#) and [Hopenhayn and Nicolini \[1997\]](#), who studied optimal unemployment insurance with moral hazard.<sup>3</sup> As in their papers, a risk averse unemployed worker puts forth effort to search for a new job, with effort increasing the likelihood of finding a job, but being costly in utility terms. A risk-neutral unemployment agency provides unemployment benefits in order to help the worker smooth his consumption over the unemployment spell, and seeks to minimize the cost of providing a given level of expected utility to the worker. However the agency cannot observe the level of search effort, and so must structure the benefits in order to provide appropriate search incentives.

Our model adds business cycle fluctuations by having the job finding rate switch according to an exogenous Markov process. When the economy enters a recessionary state, job finding rates fall for all levels of search effort, while finding rates rise once the recession ends. We also allow for changes in job loss rates over the business cycle, although the recent literature such as [Shimer \[2012\]](#) has suggested that these are less important. We analyze how the cyclical fluctuations affect the optimal level and duration of benefits provided over an unemployment spell, and how those benefits change when the aggregate state of the economy changes. We also compare the optimal unemployment program with a version of the current system to evaluate potential gains from reform.

Similar issues have been addressed recently in the literature, but ours is the most complete analysis of the optimal contracting problem in unemployment insurance design, which complements the recent work which is more applied and empirical. A similar discrete time model was analyzed by [Sanchez \[2008\]](#), who showed that benefits decrease faster in booms than recessions. However he

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<sup>2</sup>The information on the unemployment programs comes from the Department of Labor website, <http://ows.doleta.gov/unemploy>.

<sup>3</sup>[He \[2012\]](#) develops a related continuous time model of management compensation.

did not characterize the differences in benefit levels across states. Kroft and Notowidigdo [2011] analyze how unemployment benefits (but not durations) should vary in a related job search model. Landais et al. [2012] study a general equilibrium matching model with search effort and focus on characterizing the optimal benefit level over the cycle. They also consider an extension where benefits expire.

## 2 The Model

In this section, we layout the model. The model is essentially a continuous time version of Hopenhayn and Nicolini [1997] with multiple unemployment spells. In particular, we assume that an infinitely-lived worker transits over time between employment and unemployment. When he is in an unemployment spell, he may exert effort to find a job, with effort being costly to him but increasing the arrival rate of a job. An unemployed worker does not have any income except perhaps a minimal amount of consumption,  $\alpha \geq 0$ .<sup>4</sup> For simplicity, we assume that all jobs are identical and pay a constant wage  $\omega$ . Employed workers lose their jobs according to an exogenous separation rate.<sup>5</sup> When employed, the worker does not take effort. In addition, we assume that the economy switches between a *good* (or boom) state and a *bad* (or recession) state. In the good state, job finding rates are higher and separation rates are lower. We denote the aggregate state  $s_t \in G, B$  for the good and bad states.

Let  $a_t$  be the search effort of the worker at  $t$  and chosen from  $[0, \bar{a}]$  by the worker at  $t$ .<sup>6</sup> If the economy is currently in state  $s$  and an unemployed worker puts forth search effort  $a_t$ , the arrival intensity of a job offer is

$$q_s(a_t) \equiv q_{s0} + q_{s1}a_t \text{ with } q_{s0}, q_{s1} > 0.$$

To interpret this, suppose that the state does not change and the agent puts forth a constant effort  $a_0$ . Then the waiting time until a job offer arrives is an exponentially distributed random variable with rate  $q_s(a_0)$ . Or, equivalently, over a short time interval of length  $\Delta > 0$  in which  $a_0$  is taken, the probability that the worker gets a job in this period is approximately  $q_s(a_0)\Delta$ . We assume that

$$q_B(\hat{a}) < q_G(\hat{a}) \text{ for all } \hat{a} \in [0, \bar{a}].$$

Note that since  $q_G(\cdot)$  and  $q_B(\cdot)$  are affine, this assumption is equivalent to that  $q_{B0} < q_{G0}$  and  $q_{B0} + q_{B1} < q_{G0} + q_{G1}$ .

For simplicity, we assume that an employed worker loses his job with an exogenous separation rate  $p_s$  with  $p_B > p_G$ . The aggregate state of the economy switches between the two states

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<sup>4</sup>We introduce this minimal consumption level to allow for bounded utility levels with utility functions that are unbounded below.

<sup>5</sup>An extension of the model, following Wang and Williamson [1996] would incorporate costly effort to retain the job.

<sup>6</sup>We set an upper bound of effort,  $\bar{a}$ , to make the utility maximization problem we are going to discuss well defined. In all the examples,  $\bar{a}$  is sufficiently large so that the optimal effort is always interior.

according to a two-state Markov switching process with switching rate  $\lambda_s$ .

## 2.1 Preferences and Incentive Compatible Contracts

If the worker consumes  $c$  and takes effort  $a$ , his instantaneous utility is  $u(c, a) = \bar{u}(c + \alpha, a)$  where  $u$  is strictly increasing and concave in  $c$  and strictly decreasing and convex in  $a$  with  $u(c, 0) = u(c)$ . Note that we incorporate the lower bound on consumption  $\alpha \geq 0$  here and thus require  $c > 0$ . We also assume that workers die stochastically, which is governed by an idiosyncratic simple Poisson process with constant arrival rate  $\kappa > 0$ . When the shock hits, the worker dies, receives zero utility forever and the contract is terminated. The subjective discount rate of the worker is  $\hat{\rho} > 0$ , but incorporating the death probability the effective discount rate is  $\rho = \hat{\rho} + \kappa$ .

We assume that an insurance agency (“the principal”) provides unemployment insurance to help the worker (“the agent”) smooth his consumption. The worker’s employment status and the aggregate state are publicly observable, but the search effort taken by the worker is not observable to the insurance agency. So moral hazard arises and the agency needs to offer an unemployment insurance contract which induces the worker to take appropriate search effort. Here, a contract, denoted as a pair of processes  $(c, a) \equiv (\{c_t\}_{t \in [0, \infty)}, \{a_t\}_{t \in [0, \infty)})$ . Specifically,  $c$  is the consumption process with  $c_t$  being the amount of instantaneous consumption of the worker *promised* by the contract at  $t$ . We assume that  $c_t \in [0, \bar{c}]$  for all  $t \in [0, \infty)$ . The lower bound is due to the requirement that worker’s minimal consumption level,  $\alpha$ , must be satisfied and the upper bound is due to the resource constraint of the insurance agency. The process  $a$  is the effort process with  $a_t$  being the instantaneous effort level of the worker *suggested* by the contract, with  $a_t = 0$  if the worker is employed. The contract is history dependent in the sense that  $c_t$  and  $a_t$  depend on the entire history of the worker’s employment status and the aggregate state up to time  $t$ . Technically, if we denote the filtration generated by the history as  $\{\mathcal{F}_t\}_{t \in [0, \infty)}$ , then  $(c, a)$  is  $\mathcal{F}_t$ -adapted.

We can now describe the worker’s utility maximization problem. Under a contract  $(c, a)$ , note that the true effort level is not verifiable, so the worker effort to maximize his life time expected utility:

$$\max_{\hat{a} \in A} E^{\hat{a}}[\rho \int_0^\infty e^{-\rho t} u(c_t, \hat{a}_t) dt] \quad (1)$$

Here,  $A$  is the set of all  $\mathcal{F}_t$ -adapted processes with value in  $[0, \bar{a}]$ , namely, the set of all feasible effort processes.  $E^{\hat{a}}[\cdot]$  is the expectation operator induced by the effort process  $\hat{a}$  with  $\hat{a} \in A$ . A contract is *incentive compatible* if and only if  $a$  is the optimal choice of the worker solving the problem (1).

If the worker is unemployed, to promise consumption  $c_t$  the agency needs to deliver  $c_t$  to the worker, however if he is employed, the agency needs only to deliver  $c_t - \omega$ . The discount rate of the agency is also  $\hat{\rho}$  and so the effective rate is  $\rho$ . Although most studies of social insurance consider a risk-neutral agency, we allow for the agency to have a cost function  $v$  which is increasing and

convex. Thus the objective of the agency is to design an insurance contract as follows.

$$\max_{(c,a) \in C} E^a[-\rho \int_0^\infty e^{-\rho t} v(c_t - \mathbf{1}(\{\text{the worker is employed}\})\omega) dt]$$

such that

$$(c, a) \text{ is incentive compatible}$$

and

$$E^a[\rho \int_0^\infty e^{-\rho t} u(c_t, a_t) dt] \geq \bar{W}.$$

Here,  $C$  is the set of all feasible contracts,  $\mathbf{1}(\cdot)$  is the indicator function of an event and  $\bar{W}$  is the reservation utility of the worker.<sup>7</sup> The first constraint is the incentive constraint and the second is the participation constraint. In this paper, we assume that the enforcement is complete meaning that once the contract is signed neither party can leave it until the worker dies.

### 3 The Optimal Contract

We now derive the law of motion for the agent's promised utility, the key endogenous state variable in our setting, then find the Hamilton-Jacobi-Bellman equations determining the optimal contract.

#### 3.1 The Worker's Incentives and Promised Utility

We first show how the worker chooses his optimal search effort under a contract and how the agency provides incentives to induce appropriate effort. Under a contract, the consumption provided to the worker at each instant can in general depend on the worker's entire employment history. Therefore traditional dynamic programming methods cannot be applied directly because of the lack of a recursive structure. Therefore, as in [Sannikov \[2008\]](#) and [Schättler and Sung \[1993\]](#), we use a martingale approach to formulate the key incentive compatibility condition. This allows us to show that the agent's promised utility captures the relevant history, and allows us to use recursive methods.<sup>8</sup> The approach for controlled Markov jump process is developed by [Boel et al. \[1975\]](#) and [Davis \[1976\]](#).

We have already defined the aggregate state  $s_t \in \{G, B\}$  and now we assign the states numerical values  $G = 0$  and  $B = 1$ . Similarly, we define  $j_t \in \{E, U\}$  as the worker's job status, with  $E$  being employed (numerical value of zero) and  $U$  unemployed (numerical value of 1). All the information observable to the insurance agency is characterized by the two processes  $j$  and  $s$ , which are two-state Markov jump processes with *unpredictable* jumps between 0 and 1.<sup>9</sup> The associated compensated

<sup>7</sup>A contract  $(c, a)$  is *feasible* if  $c$  and  $a$  are  $\mathcal{F}_t$ -adapted and  $c_t \in [0, \bar{c}]$  and  $a_t \in [0, 1]$  for all  $t \in [0, \infty)$ .

<sup>8</sup>The same results hold using a stochastic maximum principle as in [Williams \[2011\]](#).

<sup>9</sup>See [Elliott \[1982\]](#) for details on jump processes. For technical reason, we assume that their trajectories are continuous from right with probability one.

jump martingales,  $\{m_t^J\}_{t \in [0, \infty)}$  and  $\{m_t^S\}_{t \in [0, \infty)}$ , are defined as:

$$m_t^J = \int_0^t [-(1-j_t)((1-s_t)q_G(a_t) + s_tq_B(a_t)) + j_t((1-s_t)p_G + s_t p_B)]d\hat{t} + j_t$$

and

$$m_t^S = \int_0^t [(1-s_t)\lambda_G + s_t p_G]d\hat{t} + j_t.$$

Equivalently

$$dm_t^J = [-(1-j_t)(1-s_t)q_G(a_t) + s_tq_B(a_t)) + j_t(1-s_t)p_G + s_t p_B]dt + \Delta j_t$$

and

$$dm_t^S = [-(1-s_t)\lambda_G + j_t\lambda_B]dt + \Delta s_t.$$

with  $\Delta j_t$  and  $\Delta s_t$  being the indicators of job finding or loss events and switches of the aggregate state. For example, if at date  $t$  the worker is unemployed ( $j_t = 1$ ) and the economy is in a boom ( $s_t = 0$ ), then:

$$dm_t^J = q_G(a_t)dt + \Delta j_t.$$

Thus the martingale  $m_t^J$  has a positive drift but a negative jump when the worker finds a job, so that its expectation is zero.

Given an unemployment insurance contract  $(c, a)$  and arbitrary effort process  $\hat{a}$ , we define the promised utility of the worker as

$$W_t \equiv E^{\hat{a}}[\rho \int_t^\infty e^{-\rho t} u(c_t, \hat{a}_t) dt | \mathcal{F}_t] \text{ for } t \in [0, \infty]$$

which is the expected utility of the worker under the contract from date  $t$  on. We then have the following result which is derived from a martingale representation theorem.

**Proposition 1** *Under a contract  $(c, a)$ , suppose that the effort process  $\hat{a}$  is chosen. Then, there exist two  $\mathcal{F}_t$ -predictable processes  $\{g_t^J\}_{t \in [0, \infty)}$  and  $\{g_t^S\}_{t \in [0, \infty)}$  such that*

$$E^{\hat{a}}[\int_0^\infty e^{-\rho t} g_t^J dt] < \infty \text{ and } E^{\hat{a}}[\int_0^\infty e^{-\rho t} g_t^S dt] < \infty \quad (2)$$

and

$$dW_t = \rho(W_t - u(c_t, \hat{a}_t))dt + \rho g_t^J dm_t^J + \rho g_t^S dm_t^S \text{ for } t \in [0, \infty]. \quad (3)$$

**Proof.** See Appendix A.1. ■

Here,  $\{g_t^J\}_{t \in [0, \infty)}$  is the sensitivity of promised utility to changes in his employment status while  $\{g_t^S\}_{t \in [0, \infty)}$  is the sensitivity to changes in the aggregate state. In fact, the worker's promised utility of the worker encodes all the relevant information in the observable history. With this state variable, we make the model recursive and so can use traditional dynamic programming methods.

Since search effort is costly but affects the worker's employment status,  $\{g_t^J\}_{t \in [0, \infty)}$  governs the incentive to take search effort. This is demonstrated by the following Proposition.

**Proposition 2** *Suppose that  $\{g_t^J\}_{t \in [0, \infty)}$  is the process given in Proposition 1 under the contract  $(c, a)$ . Then the contract is incentive compatible if and only if at any date  $t$  when the worker is unemployed:*

$$a_t \in \arg \max_{\tilde{a} \in [0, \bar{a}]} g_t^J q_{st}(\tilde{a}) + u(c_t, \tilde{a}) \quad (4)$$

**Proof.** See Appendix A.2 ■

Thus the proposition shows that the local or instantaneous incentive constraint (4) is sufficient to characterize full incentive compatibility. To see how the incentive constraint captures the worker's tradeoff between the utility cost of effort against the benefit of an increase in promised utility, suppose that the optimal effort choice from (4) is interior. Then we have:

$$g_t^J q_{s1} = -u_a(c_t, a_t)$$

By (3), the worker's promised utility increases by  $\rho g_t^J$  if he finds a job at  $t$ . Increases in search effort make job finding more likely, and the expected marginal benefit is  $\rho g_t^J q_{G1}$ , which is equated to the marginal cost  $-\rho u_a(c_t, a_t)$ . Thus  $g_t^J$ , the increase in promised utility when an unemployed worker finds a job, is the main channel by which the contract provides search incentives.

## 3.2 The Value Functions and the Optimal Contract

In this section, we derive the conditions which the value function of the insurance agency must satisfy, which is key to solving for the optimal contract. Define  $V(W, j, s)$  as the maximal expected payoff that the insurance agency can attain under an incentive compatible contract which promises utility  $W$  to the worker when his employment status is  $j$  and the current aggregate state is  $s$ . Since  $j$  and  $s$  are binary variables, we can effectively think of four separate value functions  $V(\cdot, j, s)$  for  $j = E, U$  and  $s = G, B$ .

### 3.2.1 The Boundary Points of the Value Functions

As a first step in the construction of the value functions, we compute their boundary points. The left boundary points are generated by the harshest contracts, which promise the lowest expected utility to the worker, and the right boundary points are generated by the most generous contracts, which promise the highest expected utility to the worker.

The left boundary points are denoted by the pairs  $(W_l^{js}, V(W_l^{js}, j, s))$  with  $W_l^{js}$  being the lowest expected utility that the insurance agency can promise to the worker in the four different cases. Since the worker's utility is strictly increasing and consumption is bounded by  $\alpha$ , the harshest punishment to the worker provides zero (additional) consumption when he is unemployed and takes away all his wage income when he is employed. Under this contract the worker does not have incentive to take search effort because his consumption is constant at  $\alpha$ , regardless his employment

status. His expected utility is

$$\underline{u} \equiv \rho \int_0^\infty e^{-\rho t} u(0, 0) dt = \bar{u}(\alpha)$$

constantly. Under this contract, the insurance agency's income is 0 when the worker is unemployed and  $\omega$  if he is employed. Furthermore, in good state, the job finding rate is  $q_{G0}$  and separation rate is  $p_G$ ; in the bad state, the finding rate is  $q_{G0}$  and separation rate is  $p_B$ . Therefore, the agency's expected payoff in the four cases can be computed as in Proposition 4 in the appendix.

Symmetrically, the right boundary points are denoted  $(W_r^{js}, V(W_r^{js}, j, s))$  with  $W_r^{js}$  the highest expected utility that the insurance agency can promise to the worker. The most generous reward to the worker is providing the maximal consumption  $\bar{c}$  constantly. Therefore, the agency's value is always  $-v(\bar{c})$ . In an unemployment spell, the worker's consumption is  $\alpha + \bar{c}$  and in an employment spell it is  $\alpha + \bar{c} + \omega$ . Therefore, the worker has incentive to take search effort when unemployed. The details of the calculation of the worker's expected utility is given by Proposition 4.

### 3.2.2 The Hamilton-Jacobi-Bellman Equations

With the boundaries established, we now characterize the value functions in the interior of their domains. We derive the appropriate Hamilton-Jacob-Bellman (HJB) equations they satisfy, which allows us to construct the optimal contract by solving a system of differential equations.

First, it is convenient to change the control variables. Given a contract  $(c, a)$ , let  $\{g_t^J\}_{t \in [0, \infty)}$  and  $\{g_t^S\}_{t \in [0, \infty)}$  be the sensitivity processes defined in Proposition 1. By (3), if an unemployed worker finds a job at time  $t$  then  $\Delta j_t = 1$  and his promised utility jumps up by  $\rho g_t^J$ . Similarly, when an employed worker loses his job,  $\Delta j_t = -1$  and his promised utility falls by  $\rho g_t^J$ . So if we define  $W_t^J$  as the worker's promised utility immediately after the change of his job status, then:

$$g_t^J \Delta j_t = \frac{W_t^J - W_t}{\rho} \quad (5)$$

Then we can re-write the incentive constraint (4) as:

$$\max_{\hat{a}} \frac{W_t^J - W_t}{\rho} q_s(\hat{a}) + u(c_t, \hat{a}). \quad (6)$$

Note that this incentive constraint effectively ties down one of the potential degrees of freedom of the contract, as  $a$  and  $W^J$  as they are linked by the incentive constraint. We now define a function  $a^s(W, W^J)$  as the unique solution of (6). So current promised utility and consumption, by controlling  $W_t^J$  the agency can induce an effort level.

In addition, all workers experience a change in promised utility when the aggregate state switches. By (3), promised utility jumps by  $\rho g_t^S$  if the economy transits from boom into recession ( $\Delta s_t = 1$ ) and falls  $-\rho g_t^S$  if the economy goes from a recession to a boom. So if  $W_t^S$  is the promised utility right after the switch in the aggregate state then

$$g_t^S \Delta s_t = \frac{W_t^S - W_t}{\rho}.$$

Thus we now view the agency's instruments in designing a contract as:  $c_t$ , consumption above the minimal level;  $W_t^J$ , promised utility after a change in the worker's employment status; and  $W_t^S$ , promised utility after a change in the aggregate state.

Since the proofs of the existence, differentiability and concavity of the value functions are highly technical, and follow standard methods, we assume these properties without proof. The following proposition characterizes the HJB equations that the value functions satisfy.

**Proposition 3** *Suppose that the value functions  $V(\cdot, j, s)$  exist, are differentiable, and concave. Then their left and right boundary points satisfy the conditions in Proposition 4. Furthermore, they satisfy the following set of HJB equations:*

$$\begin{aligned} \rho V(W, s, u) = & \max_{\substack{\hat{c} \in [0, \bar{c}] \\ W^J \in [W_l^{se}, W_r^{se}] \\ W^S \in [W_l^{s'u}, W_r^{s'u}]} -\rho v(\hat{c}) + \rho V_W(W, s, u) (W - u(\hat{c}, a^s(W, W^J)) \\ & - q_s(a^s(W, W^J)) \frac{W^J - W}{\rho} - \lambda_s \frac{W^S - W}{\rho}) \\ & + q_s(a^s(W, W^J)) (V(W^J, s, e) - V(W, s, u)) + \lambda_s (V(W^S, s', u) - V(W, s, u)) \end{aligned} \quad (7)$$

for any  $W \in [W_l^{su}, W_r^{su}]$  and  $s = G, B$ ;

$$\begin{aligned} \rho V(W, s, e) = & \max_{\substack{\hat{c} \in [0, \bar{c}] \\ W^J \in [W_l^{su}, W_r^{su}] \\ W^S \in [W_l^{s'e}, W_r^{s'u}]} -\rho v(\hat{c} - \omega) + \rho V_W(W, s, e) \left( W - u(\hat{c}) - p_s \frac{W^J - W}{\rho} - \lambda_s \frac{W^S - W}{\rho} \right) \\ & + p_s (V(W^J, s, u) - V(W, s, e)) + \lambda_s (V(W^S, s', e) - V(W, s, e)) \end{aligned} \quad (8)$$

for any  $W \in [W_l^{se}, W_r^{se}]$  and  $s = G, B$ .

**Proof.** See Appendix A.4. ■

We now show how the worker's promised utility reacts in response to the switches of the aggregate state, which are out of his control of the worker. In fact, the jumps in promised utility follow the "slope matching" rule documented in Piskorski and Tchistyi [2011] and Li [2012].

**Corollary 1** *Suppose that the value functions exist, are differentiable and concave. Given the current state  $s$ , the worker's employment status  $j$  and promised utility  $W_t$ , then  $W_t^S$  the promised utility after the aggregate state switches to  $s'$  satisfies:*

$$\begin{aligned} W_t^S &= W_l^{s'j} \text{ if } V_W(W_t, s, j) \geq V_W(W_l^{s'j}, s', j) \\ W_t^S &= W_r^{s'j} \text{ if } V_W(W_t, s, j) \leq V_W(W_r^{s'j}, s', j) \end{aligned}$$

or else  $W_t^S$  solves

$$V_W(W_t, s, j) = V_W(W_t^S, s', j).$$

**Proof.** In each HJB equation listed in Proposition 3,  $W_t$  independently solves the following:

$$\max_{W^S} -\rho V_W(W, s, j) \lambda_s \frac{W^S - W}{\rho} + \lambda_s V(W^S, s', j),$$

whose solution gives the result. ■

## 4 A Solvable Special Case

While the results above hold for general utility functions, the optimal contracts typically require numerical methods to solve for the optimal contracts. In this section we consider a special case which allows for explicit solutions. This allows us to gain insight on the structure of the optimal contract and how consumption and effort should respond to cyclical fluctuations.

In particular, we focus on a single unemployment spell with permanent jobs, so we set  $p_s = 0$  for  $s = G, B$ . We also assume the job finding rate is linear,  $q_s(a) = q_s a$ . That is, we  $q_{0s} = 0$  and we drop the subscript “1” on  $q_{1s}$ . We also assume that both workers and the unemployment agency have exponential utility. Thus workers’ preferences are given by:

$$u(c, a) = -\exp(-\theta_A(c - h(a)))$$

where  $h$  is increasing and convex with  $h(0) = 0$ . The agency’s cost function is given by:

$$v(c) = \exp(\theta_P c).$$

Of these special assumptions, the most important are the exponential utility and cost functions, which often give explicit solutions (see [Holmstrom and Milgrom \[1987\]](#) and [Williams \[2011\]](#), for example). Models of optimal social insurance typically consider risk neutral cost functions for the government, as we will also do below in our quantitative study. However the unemployment agency may well care about the variability of unemployment insurance payments in order to making budgeting decisions. Below we will see how variation in  $\theta_P$ , the agency’s coefficient of absolute risk aversion, affects the structure of the contract.

### 4.1 The Value of an Employed Worker

With permanent employment, the agency’s value of an employed worker is easy to establish. First, note that the HJB equation (8) simplifies to:

$$\rho V(W, e, s) = \max_c -\rho v(c - \omega) + \rho V_W(W, e, s)[W - u(c)]$$

The optimality equation for  $c$  is thus:

$$-v'(c - \omega) = V_W(W, e, s)u_c(c).$$

Since the worker values consumption smoothing and there is no more risk or need to provide incentives once a worker is employed, it is easy to see that the optimal contract provides constant consumption for an employed worker. Thus agency’s value function is independent of the aggregate state, with promised utility simply determining how much consumption the agency must provide. In particular, we show in [Appendix B.1](#) that the value function is given by:

$$V(W, e, s) = V(W, e) = -\exp(-\theta_P \omega)(-W)^{-\frac{\theta_P}{\theta_A}}$$

and the consumption of an employed agent is:

$$c(W, e) = -\frac{1}{\theta_A} \log(-W).$$

Thus we just invert the utility function in order to determine how much consumption is needed to deliver the promised utility to the agent.

## 4.2 Full Information

Although we did not consider it above, in order to analyze the effect of moral hazard in unemployment insurance contracts, it is useful to first analyze the case when the agent's effort is observable and contractible.

With full information, we can simply dispense with the incentive constraint in our analysis above. Therefore the HJB equation (7) for the agency's value function for an unemployed worker in this case is:

$$\begin{aligned} \rho V(W, u, s) = & \max_{c, a, W^J, W^S} -\rho v(c) + \rho V_W(W, u, s) \left[ W - u(c, a) - q_s a \frac{W^J - W}{\rho} - \lambda_s \frac{W^S - W}{\rho} \right] \\ & + q_s a [V(W^J, e) - V(W, u, s)] + \lambda_s [V(W^S, u, s') - V(W, u, s)] \end{aligned}$$

The optimality conditions for  $c$  and  $a$  are:

$$\begin{aligned} -v'(c) &= V_W(W, u, s) u_c(c, a) \\ \rho V_W(W, u, s) \left[ u_a(c, a) + q_s \frac{W^J - W}{\rho} \right] &= q_s [V(W^J, e) - V(W, u, s)]. \end{aligned}$$

The optimality conditions for  $W^J$  and  $W^S$  imply the “slope matching” conditions:

$$\begin{aligned} V_W(W, u, s) &= V_W(W^J, e) \\ V_W(W, u, s) &= V_W(W^S, u, s'). \end{aligned}$$

In Appendix B.2 we show that the value function for an unemployed worker takes the same form as that for an employed worker, but with a different leading constant that depends on the aggregate state. That is:

$$V(W, u, s) = -V_u(s)(-W)^{-\frac{\theta_P}{\theta_A}}.$$

We also show that the optimal choice of effort is independent of  $W$ , that is  $a = \bar{a}(s)$ . In the appendix, we use the optimality condition for effort and the HJB equation to derive the four equations which determine the four unknowns  $(\bar{a}(G), V_u(G), \bar{a}(B), V_u(B))$ . Our results also imply that the optimal consumption and the adjustments in promised utility can be written:

$$\begin{aligned} c(W, u, s) &= \frac{\log(V_u(s)) + \theta_A h(\bar{a}(s))}{\theta_P + \theta_A} - \frac{1}{\theta_A} \log(-W) \\ W^J(W, s) &= \left( \frac{V_u(s)}{\exp(-\theta_P \omega)} \right)^{-\frac{\theta_A}{\theta_P + \theta_A}} W \\ W^S(W, s) &= \left( \frac{V_u(s)}{V_u(s')} \right)^{-\frac{\theta_A}{\theta_P + \theta_A}} W. \end{aligned}$$

Thus the consumption function again inverts the utility function, now also compensating the worker for putting forth costly effort. In addition, the agency's marginal value of providing consumption varies with the aggregate state  $s$  through  $V_u(s)$ , so that changes the consumption delivery. Upon finding a job or having a switch in the aggregate state, there is a multiplicative adjustment in promised utility, capturing changes in the relative costs of providing utility to the worker.

### 4.3 Moral Hazard

Under moral hazard,  $a$  is no longer a free choice variable for the unemployment agency, but instead must satisfy the incentive constraint (6). The agent's optimality condition captures the incentive constraint here:

$$-u_a(c, a) = q_s \frac{W^J - W}{\rho}.$$

As discussed above, this gives a relation between  $(W^J, a)$ , which above we used to determine  $a^s(W, W^J)$ . However here is easiest to solve for  $W^J$ :

$$W^J = W - \frac{\rho}{q_s} u_a(c, a)$$

Imposing the incentive constraint, the agency's value function for an unemployed worker satisfies the HJB equation (7), which now can be written:

$$\begin{aligned} \rho V(W, u, s) = & \max_{c, a, W^S} -\rho v(c) + \rho V_W(W, u, s) \left[ W - u(c, a) + a u_a(c, a) - \lambda_s \frac{W^S - W}{\rho} \right] \\ & + q_s a \left[ V \left( W - \frac{\rho}{q_s} u_a(c, a), e \right) - V(W, u, s) \right] + \lambda_s [V(W^S, u, s') - V(W, u, s)] \end{aligned}$$

The optimality condition for  $c$  is then:

$$-v'(c) = u_c(c, a) V_W(W, u, s) - a u_{ac}(c, a) \left[ V_W(W, u, s) - V_W \left( W - \frac{\rho}{q_s} u_a(c, a), e \right) \right],$$

while the optimality condition for  $a$  is:

$$\rho a u_{aa}(c, a) \left[ V_W(W, u, s) - V_W \left( W - \frac{\rho}{q_s} u_a(c, a), e \right) \right] = q_s \left[ V(W, u, s) - V \left( W - \frac{\rho}{q_s} u_a(c, a), e \right) \right].$$

In Appendix B.3 we show that the value function takes the same form as in the full information case, but with a different leading constants. That is:

$$V(W, u, s) = -V^*(s)(-W)^{-\frac{\theta_P}{\theta_A}}.$$

We also show that the optimal choice of effort is again independent of  $W$ :  $a = a^*(s)$ . Previously we derived the optimal consumption policy in terms of this constant, but now the optimality condition for  $c$  is significantly more complicated, which makes that difficult. So instead we also show that the consumption policy function is of the same form as in the full information case, that is:

$$c(W, u, s) = c^*(s) - \frac{1}{\theta_A} \log(-W).$$

The appendix shows how to determine the six constants  $(V^*(s), a^*(s), c^*(s))$  for  $s = G, B$ , using the optimality conditions for  $c$  and  $a$  and the HJB equation. We also show that the adjustments in continuation utility are again multiplicative and can be written:

$$\begin{aligned} W^J(W, u, s) &= w_J(s)W \\ W^S(W, u, s) &= w_S(s)W. \end{aligned}$$

The consumption function captures the same factors as under full information: delivering utility to the agent, compensating him for putting forth effort, and reflecting variations in the agency's costs. However now the constant term  $c^*(s)$  also reflects the incentive effects and the corresponding compensation for the additional employment risk the worker must bear. The utility adjustment factor  $w_S$  in response to the exogenous change in the aggregate state is as before, reflecting changes in the agency's marginal cost of providing utility. However now the adjustment term  $w_J$  captures the incentive effects of increases in utility upon finding a job.

Therefore under the optimal contract, promised utility when unemployed evolves as:

$$\begin{aligned} dW_t &= \rho \left[ W_t - u(c_t, a_t) - q_{s_t} a_t \frac{W_t^J - W_t}{\rho} - \lambda_{s_t} \frac{W_t^S - W_t}{\rho} \right] dt + (W_t^J - W_t) \Delta s_t^J + (W_t^S - W_t) \Delta s_t^S \\ &= W_t [\rho - \rho u(c^*(s_t), a^*(s_t)) - q_{s_t} a^*(s_t)(w_J(s_t) - 1) - \lambda_{s_t}(w_S(s_t) - 1)] dt \\ &\quad + W_t(w_J(s_t) - 1) \Delta s_t^J + W_t(w_S(s_t) - 1) \Delta s_t^S \\ &= W_t [\mu_W(s_t) dt + (w_J(s_t) - 1) \Delta s_t^J + (w_S(s_t) - 1) \Delta s_t^S], \end{aligned}$$

where the last line defines  $\mu_W$ . For later use we define a new variable  $X = -\log(-W)$ , which converts promised utility to physical units and evolves as:

$$dX_t = \mu_W(s_t) dt + \log(w_J(s_t)) \Delta s_t^J + \log(w_S(s_t)) \Delta s_t^S.$$

Under our implementation below  $X_t$  will be tied to the worker's wealth. Thus under the contract an unemployed worker's promised utility grows at a constant rate in each aggregate state, and experiencing proportional jumps when the state switches or when the worker finds a job.

## 4.4 Illustration

Illustrate results, comparative statics of key constants, discuss how contract works.

Contrast full info with moral hazard

## 5 Implementation of the Optimal Contract

We now show how to implement the optimal contract via some simple instruments. Previous studies of optimal unemployment insurance contracts have focused on a direct implementation, where the unemployment agency tracks the worker's promised utility and makes payments conditional upon

it. Promised utility and consumption decline over an unemployment spell, providing a rationale for declining unemployment insurance benefits. However in this section we show that the optimal contract can also be implemented by allowing the worker to save and borrow, providing benefits which are constant in each aggregate state, providing a re-employment bonus, and giving an additional bonus or charge when the aggregate state changes. Thus our implementation mixes elements of the current system of constant benefits with the unemployment insurance savings accounts of [Feldstein and Altman \[2007\]](#), and the re-employment bonuses which were tried in some states US states and whose effects were studied by [Robins and Spiegelman \[2001\]](#). Cyclical fluctuations are new to our study, and necessitate an additional payment or charge when the state of the economy switches to capture changes in incentives and costs of insurance. Our implementation is also related to [Shimer and Werning \[2008\]](#), who find that constant benefits are optimal with exponential utility in a sequential search model when agents can save and borrow. Although our setting differs, our results are similar and provide the same distinction between benefits and consumption, and thus insurance and liquidity, as in their paper.

## 5.1 A Worker's Consumption-Savings-Effort Problem

We will consider an implementation of the optimal contract via a consumption-savings-effort problem of the worker. The worker has wealth  $x_t$  and has access to a bond with a state-dependent instantaneous rate of return  $r(s_t)$  when unemployed and  $r^e$  when employed. In addition, the worker gets a flow payment each instant, which is time-independent but depends on the worker's employment status and the aggregate state. Let  $b^e$  be the constant flow payment when employed and  $b^u(s_t)$  be the payment when unemployed. Finally, when the worker is unemployed he gets a lump sum payment of  $B(s_t)$  when he finds a job and  $A(s_t, x_t)$  when the aggregate economy switches state. Note that although all of these terms are referred to as "payments" they may be negative, in which case they are charges or taxes the agent must pay. When the agent is employed wealth evolves as:

$$dx_t = [r^e x_t - c_t + b^e]dt. \quad (9)$$

When the worker is unemployed his wealth follows:

$$dx_t = [r(s_t)x_t - c_t + b^u(s_t)]dt + B(s_t)\Delta s_t^J + A(s_t, x_t)\Delta s_t^s. \quad (10)$$

The goal of our implementation is to choose the parameters  $(r^e, b^e, r(s_t), b^u(s_t), B(s_t), A(s_t, x_t))$  so that the agent's optimal choices agree with those given above under the contract. We now consider the problem of a worker who chooses his consumption and effort optimally, given the wealth evolution above. We begin with an employed worker, then turn to an unemployed worker.

## 5.2 An Employed Worker

We denote the worker's value function  $J(x, e)$  and note that it satisfies the HJB equation:

$$\rho J(x, e) = \max_c \rho u(c) + J_x(x, e)[r^e x - c + b^e],$$

and the first order condition is:

$$\rho u'(c) = J_x(x, e).$$

In Appendix C.1, we show that the value function takes the form  $J(x, e) = -J_e \exp(-r^e \theta_A x)$ , where:

$$J_e = \frac{\rho}{r^e} \exp\left(\frac{r^e - \rho - r^e \theta_A b^e}{r^e}\right).$$

We also show that the consumption function is:

$$c(x, e) = \frac{\rho - r^e}{\theta_A r^e} + b^e + r^e x$$

Now in order for this decentralization to implement the optimal contract, we need to ensure that consumption and wealth are constant (as they are under the optimal contract), and that the levels of consumption and expected utility agree with those in the contract. In order for the promised utility levels to agree we need:

$$W = J(x, e) = -J_e \exp(-r^e \theta_A x)$$

and therefore note that we have:

$$c(W(x), e) = -\frac{1}{\theta_A} \log(-J(x, e)) = -\frac{1}{\theta_A} \log(J_e) + r^e x.$$

Thus for  $c(W(x), e) = c(x, e)$  we need  $r^e = \rho$ , and thus the interest rate must equal to the rate of time preference. In this case:

$$c(x, e) = b^e + \rho x$$

so that wealth  $x$  is indeed constant. In addition, we can simplify the agent's value function to:

$$J(x, e) = -\exp(-\theta_A (b^e + \rho x)).$$

Thus there is an indeterminacy in the implementation at this state, as to deliver a given level of promised utility  $W$  we can trade off the constant payment  $b^e$  with the initial wealth  $x$ . Note that we can interpret  $b^e$  as the after-tax wage, so one implementation simply sets  $x = 0$  and sets the labor income tax (or subsidy) in order to deliver the appropriate level of utility. But a different implementation would set  $b^e = 0$  and so would completely tax away labor income, and have the agent finance his consumption when employed out of an appropriately chosen stock of wealth  $x$ . We resolve this indeterminacy below when we consider the unemployed.

### 5.3 An Unemployed Worker

We now suppose that once the worker finds a job, he receives a constant payment  $b^e$  which is tied to his wealth and the aggregate state at the date he becomes employed. That is, if the worker finds a job at some date  $T$  and his wealth upon beginning the job is  $x_T$  (after he receives the bonus

$B(s_T)$  as in (10)), then in employment he gets the constant payment  $b^e = (r(s_T) - \rho)x_T + b_0(s_T)$ . Thus:

$$J(x_T, e) = -\exp(-\theta_A(b^e(x_T) + \rho x_T)) = -\exp(-\theta_A(r(s_T)x_T + b_0(s_T))) = -J^*(s_T) \exp(-\theta_A r(s_T)x_T),$$

where the last equality defines  $J^*(s)$ . Note that once employed, the worker's wealth is constant  $x_t = x_T$  for  $t > T$ , so the above expression determines  $J(x, e)$ . We also specify that the payment upon a switch of the aggregate state  $A(s, x)$  takes the following form:

$$A(s, x) = \left( \frac{r(s)}{r(s')} - 1 \right) x + \frac{r(s)}{r(s')} \hat{A}(s)$$

for some constants  $\hat{A}(s)$ . Thus upon the switch of the aggregate state, the worker is given a payment  $\hat{A}$  independent of wealth, as well as additional term which accounts for the effective gain or loss to the worker from the change in interest rates. We now consider the problem of an unemployed worker, who must choose consumption and effort over his unemployment spell, with wealth evolution given by (10) and post-employment value function determined above. The HJB equation for the unemployed worker when the current aggregate state is  $s_t^J = s$  is then:

$$\begin{aligned} \rho J(x, u, s) &= \max_{c, a} \rho u(c, a) + J_x(x, u, s)[r(s)x - c + b^u(s)] \\ &\quad + q_s a [J(x + B(s), e) - J(x, u, s)] + \lambda_s [J(x + A(s, x), u, s') - J(x, u, s)]. \end{aligned}$$

The first order condition for  $c$  is as above:

$$\rho u_c(c, a) = J_x(x, u, s),$$

while the first order condition for  $a$  is:

$$-\rho u_a(c, a) = q_s [J(x + B(s), e) - J(x, u, s)].$$

In Appendix C.2, we show that the value function takes the form  $J(x, u, s) = -J_u(s) \exp(-r(s)\theta_A x)$  and that effort is  $a = a(s)$  independent of  $x$ . The optimality condition for  $a$  and the HJB equation provide two sets of equations which determine the constants  $(a(s), J_u(s))$ . We also show that the optimal consumption function can be written:

$$c(x, u, s) = -\frac{1}{\theta_A} \log \frac{r(s)J_u(s)}{\rho} + h(a(s)) + r(s)x.$$

We now must set the parameters of the policy  $(b_0(s), b^u(s), \hat{A}(s), B(s), r(s))$  in order to implement the optimal contract. To do so, we must clearly have the same effort choices and so the optimality condition for  $a$  (27) in the appendix must hold for  $a(s) = a^*(s)$ :

$$r(s)\theta_A h'(a^*(s)) = q_s \left[ 1 - \frac{J^*(s)}{J_u(s)} \exp(-r\theta_A B(s)) \right]. \quad (11)$$

We must have the utility levels match as well, so:

$$W = -J_u(s) \exp(-r(s)\theta_A x),$$

and thus we have:

$$c(W(x), u, s) = \hat{c}(s) - \frac{1}{\theta_A} \log(J_u(s)) + r(s)x.$$

So to have the consumption policies agree  $c(W(x), u, s) = c(x, u, s)$  we must have:

$$\hat{c}(s) = h(a^*(s)) - \frac{1}{\theta_A} \log \frac{r(s)}{\rho}. \quad (12)$$

Finally, the evolution of  $x_t$  must agree with the evolution of promised utility above. Note that we have:

$$x_t = \frac{1}{r(s_t)\theta_A} [\log J_u(s_t) - \log(-W_t)]$$

and therefore:

$$\begin{aligned} dx_t &= \frac{1}{r(s_t)\theta_A} dX_t + \left( \frac{1}{r(s'_t)\theta_A} [\log J_u(s'_t) - \log(-W_t)] - x_t \right) \Delta s_t^S \\ &= \frac{1}{r(s_t)\theta_A} \mu_W(s_t) dt + \log(w_J(s_t)) \Delta s_t^J + \left( \log(w_S(s_t)) + \frac{1}{r(s'_t)\theta_A} [\log J_u(s'_t) - \log(-W_t)] - x_t \right) \Delta s_t^S. \\ &= \frac{1}{r(s_t)\theta_A} \mu_W(s_t) dt + \log(w_J(s_t)) \Delta s_t^J \\ &\quad + \left( \log(w_S(s_t)) + \left( \frac{r(s_t)}{r(s'_t)} - 1 \right) x_t + \frac{1}{r(s'_t)\theta_A} [\log J_u(s'_t) - \log J_u(s_t)] \right) \Delta s_t^S \end{aligned}$$

At the same time, under the optimal policies derived here we have:

$$dx_t = \left[ \frac{1}{\theta_A} \log \frac{r(s_t)J_u(s_t)}{\rho} - h(a^*(s_t)) + b^u(s_t) \right] dt + B(s_t) \Delta s_t^J + A(s_t, x_t) \Delta s_t^S.$$

Thus we must have:

$$b^u(s) = \frac{1}{r(s)\theta_A} \mu_W(s) - \frac{1}{\theta_A} \log \frac{r(s)J_u(s)}{\rho} + h(a^*(s)) \quad (13)$$

$$B(s) = \log(w_J(s)) \quad (14)$$

$$\hat{A}(s) = \frac{r(s')}{r(s)} \log(w_S(s)) + \frac{1}{r(s)\theta_A} [\log J_u(s') - \log J_u(s)]. \quad (15)$$

Therefore we have that this policy implements the optimal contract. The parameters are all determined jointly, as  $a^*$  and  $J_u$  depend on all the other parameters of the policy. However loosely speaking we can think of (11) as determining  $b_0(s)$  and thus  $J^*(s)$ , (12) determining  $r$ , (13) determining  $b^u(s)$ , (14) determining  $B(s)$ , and (15) determining  $\hat{A}(s)$ .

## 5.4 Illustration

Repeat the same type of calculations for the solvable contract, showing the implementation parameters.

## 6 A Quantitative Example

While the previous sections provided useful insight into the structure of the contract and how it can be implemented, the assumptions we made there were special and differed from the literature. In this section we study the quantitative implications of the optimal contract under more standard assumptions in a calibrated version of the model. We reintroduce job loss, and calibrate the model to match key labor market indicators by simulating the employment dynamics of a large population of workers.

We now assume workers' preferences are additively separable and of the form:

$$u(c, a) = u(c) - h(a) = \frac{(c_t + \alpha)^{1-\gamma}}{1-\gamma} - \frac{a_t^{1+\theta}}{1+\theta}$$

with  $\gamma, \theta > 0$ . Here  $\gamma > 0$  is the coefficient of relative risk,  $\alpha > 0$  is the minimal consumption of the worker, and  $\theta$  governs the elasticity of job search. Here  $\alpha$  can be interpreted as the consumption the agent derives from sources other than unemployment insurance. For example [Hopenhayn and Nicolini \[1997\]](#) consider variants where the agent has assets which yield a constant flow of income for consumption.<sup>10</sup> We also now assume that the unemployment agency is risk neutral, so  $v(c) = c$ . In addition to having job loss ( $\lambda_s > 0$ ), we reintroduce the (small) constant terms  $q_{0s}$  in the job finding rate in order to ease computation.

### 6.1 The Benchmark Contract

In order to measure the effects of switching to the optimal unemployment insurance system, we also consider a stylized version of the current system, which we call the benchmark contract. Under the benchmark contract, an unemployed worker receives the constant benefit  $c^B$  from the insurance agency for a fixed length of time. In order to capture the regular extensions of benefits during recessions, we assume that the duration of benefits is state dependent. Specifically, the worker receives benefits for at most  $T_G$  weeks in a boom and  $T_B > T_G$  weeks in a recession. Once the benefits have expired, the agency provides no further consumption.

We now show how to calculate the worker's utility levels and search effort under the benchmark contract. First, let  $W_s^e$  be the expected utility of the worker under the benchmark contract when he is employed in state  $s$ . We suppose these levels are given, and now show how to determine the evolution of promised utility over an unemployment spell. First, we consider the period after  $T_B$ , in which all unemployment benefits are expired in both states, and denote the worker's promised utility by  $W_s^{u3}$  in this region. Suppose at some date  $\bar{t} > T_B$ , the economy is in a boom and let  $\tau$  be the first date of a switch in either job status or the aggregate state. That is  $\tau = \tau^S \wedge \tau^J$  with  $\tau^S$  the time at which the economy enters a recession and  $\tau^J$  the time at which the worker finds a job. So the worker chooses his optimal effort to solve the following problem:

$$W_G^{u3} = \max_{a \in A} E_{\bar{t}}^a \left[ \rho \int_{\bar{t}}^{\tau} e^{-\rho(t-\bar{t})} (u(\alpha) - h(a_t)) dt + e^{-\rho(\tau-\bar{t})} (\mathbf{1}(\{\tau = \tau^J\}) W_G^e + \mathbf{1}(\{\tau = \tau^S\}) W_B^{u3}) \right].$$

---

<sup>10</sup>Technically, with  $\alpha$  being positive, we prevent the utility level from being  $-\infty$  if consumption is 0 when  $\gamma > 1$ .

Following the same steps as in deriving the HJB equations above, we see that the values  $W_s^{u3}$  solve the HJB equations for  $s = G, B$ :

$$\rho W_s^{u3} = \max_{\hat{a} \in [0, \bar{a}]} \rho(u(\alpha) - h(\hat{a})) + q_s(\hat{a})(W_s^e - W_s^{u3}) + \lambda_s(W_{s'}^{u3} - W_s^{u3}).$$

Now, we turn to the time interval  $[T_G, T_B]$  when the worker only receives benefits in a recession, but not in a boom. Denote the continuation value of the worker during this period by  $W_s^{u2}(t)$ , where the values are now time dependent due to the impending termination of benefits. Similar to the previous case, these values satisfy the following pair of HJB equations:

$$\begin{aligned} \rho W_G^{u2}(t) - \frac{d}{dt} W_G^{u2}(t) &= \max_{\hat{a} \in [0, \bar{a}]} \rho(u(\alpha) - h(\hat{a})) + q_G(\hat{a})(W_G^e - W_G^{u2}(t)) + \lambda_G(W_B^{u2}(t) - W_G^{u2}(t)) \\ \rho W_B^{u2}(t) - \frac{d}{dt} W_B^{u2}(t) &= \max_{\hat{a} \in [0, \bar{a}]} \rho(u(c^B + \alpha) - h(\hat{a})) + q_B(\hat{a})(W_B^e - W_B^{u2}(t)) + \lambda_B(W_G^{u2}(t) - W_B^{u2}(t)) \end{aligned}$$

with boundary conditions  $W_G^{u2}(T_B) = W_G^{u3}$  and  $W_B^{u2}(T_B) = W_B^{u3}$ .

Finally, in the time interval  $[0, T_G]$ , the worker receives unemployment benefits in both booms and recessions. Again, promised utility which we denote by  $W_s^{u1}(t)$  is time dependent and satisfies the HJB equation for  $s = G, B$ :

$$\rho W_s^{u1}(t) - \frac{d}{dt} W_s^{u1}(t) = \max_{\hat{a} \in [0, \bar{a}]} \rho(u(c^B + \alpha) - h(\hat{a})) + q_s(\hat{a})(W_s^e - W_s^{u1}(t)) + \lambda_s(W_{s'}^{u1}(t) - W_s^{u1}(t))$$

with boundary conditions  $W_s^{u1}(T_G) = W_s^{u2}(T_G)$ .

## 6.2 Data and Calibration

We take a time period to be one week. First, we fix a few parameters following the literature. Following [Hopenhayn and Nicolini \[1997\]](#), we set the coefficient of risk aversion to be  $\gamma = 0.5$  and the weekly discount rate  $\rho = 0.001$ , which corresponds to annual discount rate of 5%. Since utility is bounded below by zero, we set  $\alpha = 0$ , which is interpretable as the worker having no outside assets or other sources of income. We set the constants in the job finding rate both to very small numbers  $q_{s0} = 10^{-5}$ , which prevents some singularity problems but has no impact on our main results. We set the maximum compensation that the UI agency can give to a worker (in addition to his wage if any), at  $\bar{c} = \omega$ , the wage. Thus the worker can consume at most  $\omega$  when unemployed and  $2\omega$  when unemployed. This is simply used to determine the upper bounds on the worker's utility and the agency's costs. The data set for our estimation below ends in 2007, and the median annual wage then was \$25,737 so with \$495 the corresponding weekly value.<sup>11</sup> From this, we deduct 15% for taxes as do [Landais et al. \[2012\]](#) and [Gruber \[1997\]](#), so set  $\omega = 420$ .

For the benchmark contract, we set the benefit length to be  $T_G = 26$  weeks in booms, which is the average duration across US states, and  $T_B = 39$  weeks in recessions, corresponding to the length of the regular federal extended unemployment benefits program. We consider one example below

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<sup>11</sup>This data come from the SSA at <http://www.ssa.gov/oact/cola/central.html>.

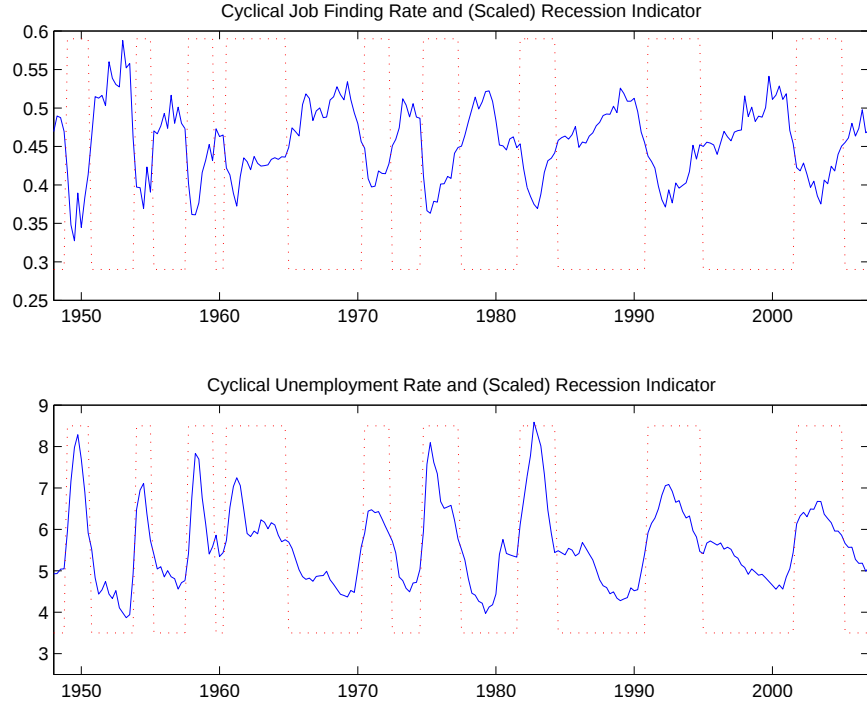


Figure 1: Cyclical job finding and unemployment rates, along with the estimated recession indicator.

where we extend benefits to  $T_B = 99$  weeks, the maximal length with the emergency benefits extension during the most recent recession. We set the unemployment benefit to  $c^b = 0.47\omega$ , consistent with the 47% average replacement ratio in the US in fiscal 2009.<sup>12</sup>

In order to obtain the estimates of the driving Markov switching processes for the aggregate state, and the corresponding job finding and job loss rates, we use the data from Shimer [2012]. This data set consists of quarterly averages of monthly job finding and separation rates from 1948-2006. As Shimer emphasized, most of the cyclical component in the data comes from the job finding rates. To focus on this cyclical component, we use an H-P filter to remove a low frequency trend from the job finding rate data. Then we estimate a two-state Markov chain on this data, following the approach of Hamilton [1989]. That is, letting the H-P filtered job finding rate be  $f_t$  we estimate:

$$f_t = m_{s_t} + \epsilon_t,$$

where  $s_t$  is the Markov-switching recession indicator at a quarterly frequency and  $\epsilon_t$  is an error. We obtain the estimates  $m_G = 0.487$  and  $m_B = 0.411$  for the mean job finding rates in booms and recessions, and diagonals of the transition matrix are 0.933 and 0.911. Thus, for example, the probability of remaining in a boom in the next quarter conditional on being in a boom in the current quarter is 0.933. These transition probabilities imply switching rates of the aggregate state of  $\lambda_G = 0.0173$  and  $\lambda_B = 0.0233$  on a weekly basis.

<sup>12</sup>See [http://workforcesecurity.doleta.gov/unemploy/ui\\_replacement\\_rates.asp](http://workforcesecurity.doleta.gov/unemploy/ui_replacement_rates.asp)

|                                  | Benchmark |        | Optimal |        |
|----------------------------------|-----------|--------|---------|--------|
|                                  | Boom      | Recess | Boom    | Recess |
| Unemp Rate (%)                   | 5.33      | 6.57   | 3.61    | 3.98   |
| Unemp Duration (weeks)           | 6.22      | 7.33   | 4.42    | 4.65   |
| Finding Rate (month)             | 0.486     | 0.409  | 0.642   | 0.615  |
| Separation Rate (month)          | 0.0334    | 0.0349 | 0.0334  | 0.0349 |
| Net Cost/Worker (% of $\omega$ ) | 2.50      | 3.08   | 0.38    | 0.58   |

Table 1: Summary statistics

The cyclical job finding rates and the estimated recession indicator, which is one when the smoothed (full-sample) probability of a recession is greater than 0.5, are shown in the top panel of Figure 6.2. Using the recession indicator estimated from the job finding rates, we then read off the mean separation rates in booms and recessions from the H-P filtered separation rate data. These are 0.0337 and 0.0353 in booms and recessions, respectively, which imply job loss rates of  $p_G = 0.0085$  and  $p_B = 0.0089$ . Although we do not use it in the calibration, we also extracted the cyclical unemployment rate, which is shown in the bottom panel of Figure 6.2. There we see that the recession indicator from the finding rate data corresponds quite well to periods of high unemployment. In particular, we find that average unemployment rate is 4.93 in booms and 6.56 in recessions.

We calibrate the rest of the parameters by simulating a population of 2 million workers and computing the average job finding rates as well as directly calculating the elasticities of unemployment duration with respect to an increase in benefits. For this simulation, we assume a mortality rate of 0.8% per year, as in US data, which means that we terminate and re-start the contracts for 0.8% of our population at an annual rate. For the elasticities, as discussed by Landais et al. [2012] and Chetty [2008], the typical range of estimates is 0.5-1, and we target an elasticity in the middle of this range at 0.7. This is midway between the value of 0.9 of Meyer [1990] which has been commonly used in the literature and the more recent estimates around 0.55 by Kroft and Notowidigdo [2011] and Chetty [2008]. We compute this elasticity in our simulated data by computing the average unemployment duration under our benchmark contract as well as a contract with benefits increased by 1%. The job average job finding rates are largely driven by the parameters governing the impact of job search on finding rates, and we find that  $q_{G1} = 0.0038$  and  $q_{B1} = 0.0035$  match the data quite well. The elasticity is largely driven by the effort cost function parameter, and we find that  $\theta = 0.16$  gives us an elasticity of 0.72.

### 6.3 Quantitative Implications

Simulate recession and compare benefits extension.

Benchmark: 5.3%  $\Rightarrow$  6.7%, 99-Week: 5.3%  $\Rightarrow$  6.8%.

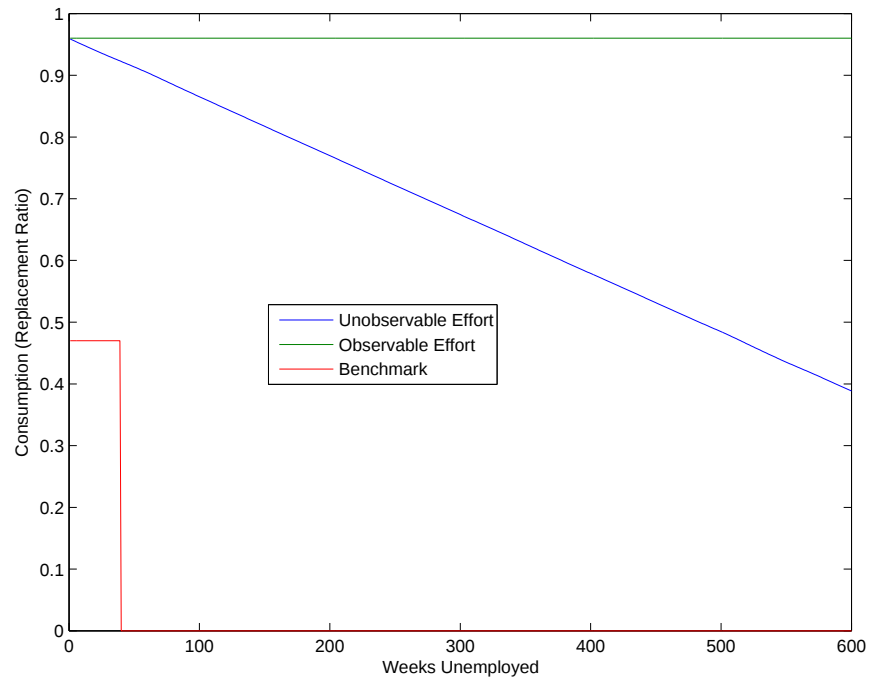


Figure 2: Consumption over an unemployment spell in a recession

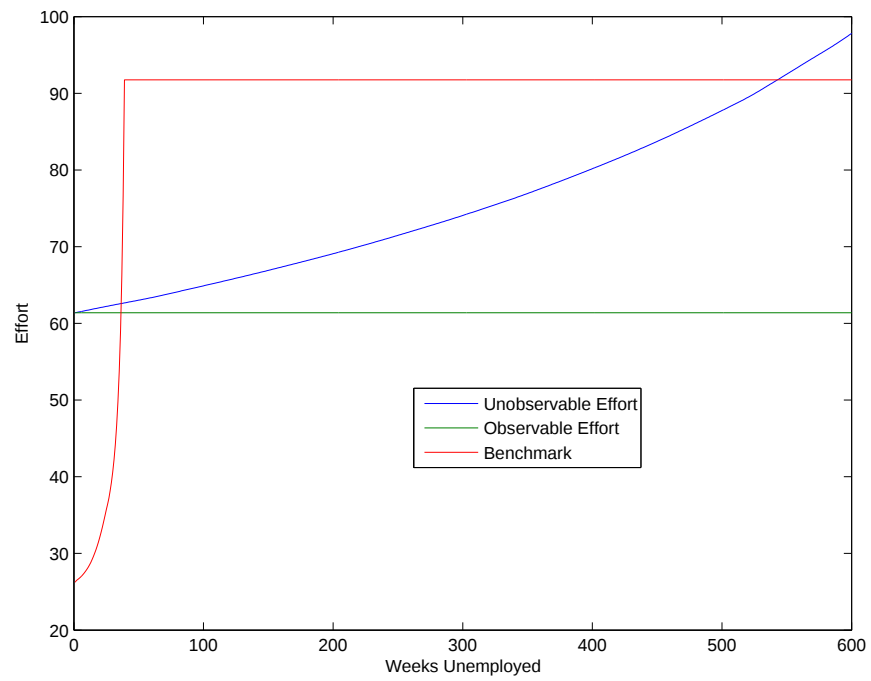


Figure 3: Effort over an unemployment spell in a recession

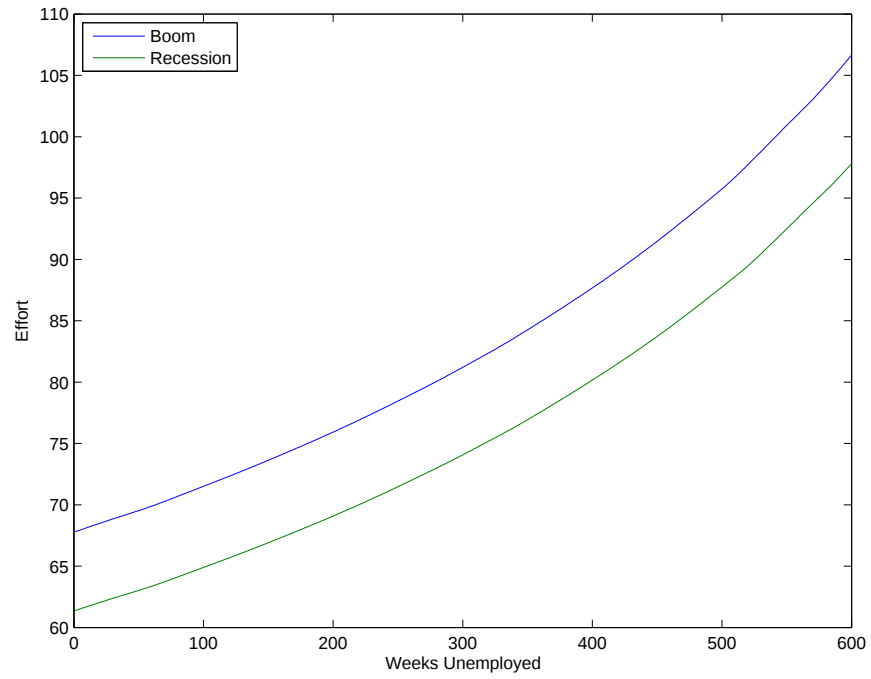


Figure 4: Effort over an unemployment spell

Optimal: 3.6%  $\Rightarrow$  4.0%.

## 7 Conclusion

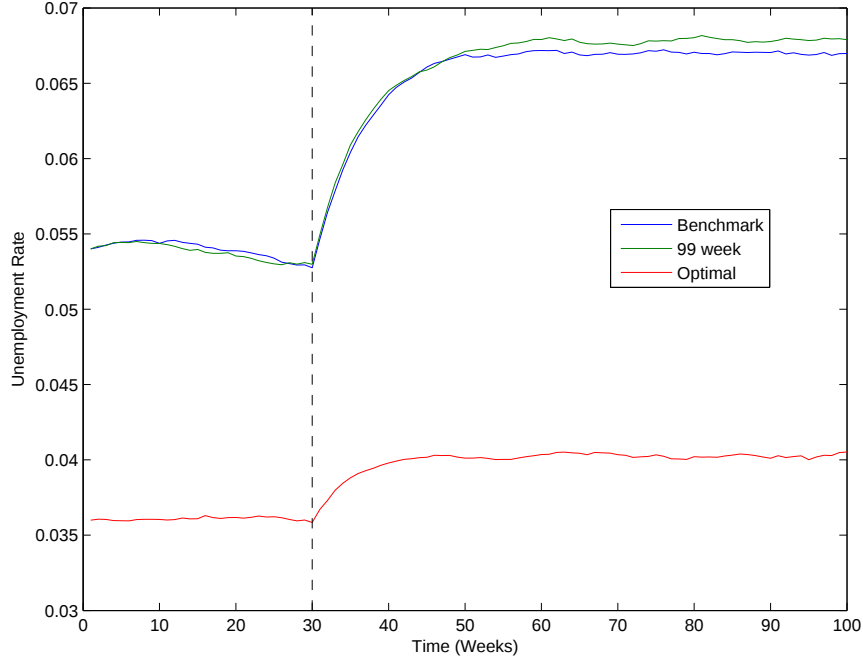


Figure 5: Unemployment rates in a long recession under the optimal contract, the benchmark contract, and a contract with benchmark contract with benefits extended to 99 weeks.

## A Proofs

### A.1 Proof of Proposition 1

Let

$$\begin{aligned}\phi_t(c, a) &= E_t^a[\rho \int_0^\infty e^{\rho \hat{t}} u(c_{\hat{t}}, a_{\hat{t}}) d\hat{t} | \mathcal{F}_t] \\ &= \rho \int_0^t e^{-\rho \hat{t}} u(c_{\hat{t}}, a_{\hat{t}}) d\hat{t} + e^{-\rho t} W_t.\end{aligned}\tag{16}$$

In words,  $\phi_t(c, a)$  is the conditional expected total utility of the worker based on the information unfolded up to time  $t$ . Therefore, process  $\{\phi_t(c, a)\}_{t \in [0, \infty)}$  is a  $\mathcal{F}_t$ -adapted martingale. According to the *martingale representation theorem*,<sup>13</sup> there exist two  $\mathcal{F}_t$ -predictable and square integrable (equation (2)) processes  $\{g_t^J\}_{t \in [0, \infty)}$  and  $\{g_t^S\}_{t \in [0, \infty)}$  such that

$$d\phi_t(c, a) = \rho e^{-\rho t} g_t^J dm_t^J + \rho e^{-\rho t} g_t^S dm_t^S.\tag{17}$$

So (17) and (16) imply (3).

### A.2 Proof of Proposition 2

Suppose not, such that the process  $a$  does not satisfy the equations in Proposition 2 for some  $\tilde{t} > 0$  with strictly positive probability. Let  $\{W_t\}_{t \in [0, \infty)}$  be the promised utility process generated by  $a$  under the contract  $(c, a)$ . We

<sup>13</sup>Note that,  $u(c_t, a_t)$  is bounded in compact intervals,  $\{\phi_t(c, a)\}_{t \in [0, \infty)}$  is a uniformly integrable martingale, so the martingale representation theorem is valid here. See Elliott [1982] for technical details.

define

$$\hat{\phi}_t(a') = \rho \int_0^t e^{-\rho s} u(c_s, a'_s) ds + e^{-\rho t} W_t \text{ for } t \in [0, \infty)$$

for some alternative feasible effort process  $a'$ . In fact,  $\hat{\phi}_t(a')$  is the conditional expected utility of the worker based on the history up to time  $t$ , under the contract  $(c, a)$ , if he chooses  $a'$  from 0 to  $t$  and then switches to  $a$  at  $t$  permanently. Obviously,  $\hat{\phi}_0(a') = W_0$ , the expected utility of the worker if he chooses  $a$  from the beginning. According to (3), we have

$$\begin{aligned} d\hat{\phi}_t(a') &= \rho e^{-\rho t} u(c_t, a'_t) dt - \rho e^{-\rho t} W_t dt + e^{-\rho t} dW_t \\ &= \rho e^{-\rho t} \{[u(c_t, a'_t) - u(c_t, a_t)] dt + g_t^J dm_t^J + g_t^S dm_t^S\}. \end{aligned}$$

Without loss of generality, we assume that the worker is unemployed at  $t$ , and the results for an employed worker can be proved similarly. Let  $m_t'^J$  be the compensated jump martingale associated with  $j_t$ , the employment status indicator process, induced by the effort process  $a'$ . Then

$$dm_t^J = \{[(1 - s_t)q_G(a'_t) + s_t q_B(a'_t)] - [(1 - s_t)q_G(a_t) + s_t q_B(a_t)]\} dt + dm_t'^J\}.$$

Therefore

$$\begin{aligned} d\hat{\phi}_t(a') &= \rho e^{-\rho t} \{[u(c_t, a'_t) - u(c_t, a_t)] \\ &\quad + g_t^J ((1 - s_t)q_G(a'_t) + s_t q_B(a'_t)) - g_t^J ((1 - s_t)q_G(a_t) + s_t q_B(a_t))] dt \\ &\quad + g_t^J dm_t'^J + g_t^S dm_t^S\}. \end{aligned}$$

Note that, under the measure generated by  $a'$ ,  $\{m_t'^J\}_{t \in [0, \infty)}$  and  $\{m_t^S\}_{t \in [0, \infty)}$  are two martingales. The drift term has the same sign as

$$[g_t^J ((1 - s_t)q_G(a'_t) + s_t q_B(a'_t)) + u(c_t, a'_t)] - [g_t^J ((1 - s_t)q_G(a_t) + s_t q_B(a_t)) + u(c_t, a_t)].$$

So we choose  $a'$  such that (4) is satisfied. Then  $\{\hat{\phi}_t(a')\}_{t \in [0, \infty)}$  is a sub-martingale under the measure generated by  $a'$ . Therefore

$$E^{a'}[\hat{\phi}_{\tilde{t}}(a')] > \hat{\phi}_0(a') = W_0$$

which implies that  $a$  is dominated by the effort plan that adopting  $a'$  from 0 to  $\tilde{t}$  and then switching to  $a$ . So  $a$  is not optimal.

To prove the other direction, suppose that  $a$  satisfies the incentive compatibility conditions in Proposition 2. Then, by definition,  $\{\hat{\phi}_t(a')\}_{t \in [0, \infty)}$  is a super-martingale under the measure induced by  $a'$  for any alternative effort process  $a'$ . Since  $c_t$ ,  $a_t$ , and  $a'_t$  are bounded in compact intervals according to the feasibility conditions,  $\hat{\phi}_\infty(a')$  is bounded. Then

$$W_0 = \hat{\phi}_0(a') \geq E^{a'}[\hat{\phi}_\infty(a')]$$

and  $a$  dominates  $a'$ .

### A.3 Proposition 4

**Proposition 4** *The lower bounds of the expected utility of the worker are  $W_l^{js} = \underline{u}$  for  $j = E, U$  and  $s = G, B$  and the corresponding values of the insurance agency,  $V(W_l^{js}, j, s)$ , satisfy the following:*

$$\rho V(W_l^{Us}, U, s) = \lambda_s(V(W_l^{Us'}, U, s') - V(W_l^{Us}, U, s)) + q_{s0}(V(W_l^{Es}, E, s) - V(W_l^U, U, s)) \quad (18)$$

$$\begin{aligned} \rho V(W_l^{Es}, E, s) &= \rho v(\omega) + \lambda_s(V(W_l^{Es'}, E, s') - V(W_l^{Es}, E, s)) \\ &\quad + p_s(V(W_l^{Us}, U, s) - V(W_l^{Es}, E, s)) \end{aligned} \quad (19)$$

for  $s = G, B$  where  $s' \neq s$ . The upper bounds of the expected utility of the worker  $W_r^{js}$  satisfy the following:

$$\rho W_r^{Us} = \max_{\hat{a} \in [0, \bar{a}]} \rho u(\bar{c}, a) + \lambda_s (W_r^{Us'} - W_r^{Us}) + q_s(\hat{a})(W_r^{Es} - W_r^{Us}) \quad (20)$$

$$\rho W_r^{Gs} = \rho u(\bar{c} + \omega) + \lambda_s (W_r^{Es'} - W_r^{Es}) + p_G (W_r^{Us} - W_r^{Es}) \quad (21)$$

and the corresponding expected values of the insurance agency are  $V(W_r^{js}, j, s) = -v(\bar{c})$ .

For the proof, we start with the results of the left boundary points, and focus on the case when the worker is unemployed. Let  $\tau = \tau^S \wedge \tau^J$  with  $\tau^S$  being the time of the next switch in the aggregate state and  $\tau^J$  the time of the next job offer. Then, for any  $\Delta > 0$ , note that the current instantaneous payoff is 0 and we have

$$\begin{aligned} V(W_l^{Us}, U, s) &= e^{-\rho(\Delta \wedge \tau)} \Pr(\tau > \Delta) V(W_l^{Us}, U, s) + e^{-\rho(\Delta \wedge \tau)} [\Pr(\Delta < \tau, \tau = \tau^S) V(W_l^{Us'}, U, s') \\ &\quad + \Pr(\Delta < \tau, \tau = \tau^J) V(W_l^{Es}, E, s)] \end{aligned}$$

or

$$\begin{aligned} V(W_t^{Us}, U, s) &= e^{-\rho(\Delta \wedge \tau)} e^{-(\lambda_s + q_{s0})\Delta} V(W_l^{Us}, U, s) + e^{-\rho(\Delta \wedge \tau)} (1 - e^{-(\lambda_s + q_{s0})\Delta}) [ \\ &\quad \frac{\lambda_s}{\lambda_s + q_{s0}} V(W_l^{Us'}, U, s') + \frac{q_{s0}}{\lambda_s + q_{s0}} V(W_l^{Es}, E, s)]. \end{aligned}$$

So

$$\begin{aligned} 0 &= e^{-\rho(\Delta \wedge \tau)} e^{-(\lambda_s + q_{s0})\Delta} (V(W_l^{Us'}, U, s') - V(W_l^{Us}, U, s)) \\ &\quad + e^{-\rho(\Delta \wedge \tau)} (1 - e^{-(\lambda_s + q_{s0})\Delta}) [\frac{\lambda_s}{\lambda_s + q_{s0}} V(W_l^{Us'}, U, s') + \frac{q_{s0}}{\lambda_s + q_{s0}} V(W_l^{Es}, E, s)]. \end{aligned}$$

Let  $\Delta \rightarrow 0$ , then we have (18). (19) can be proved similarly.

Now, we prove the result on the right boundary point, focusing first on an unemployed worker who solves the following problem:

$$\begin{aligned} \max_{a \in A} \quad & E_0 [\rho \int_0^{\Delta \wedge \tau} e^{-\rho t} u(\bar{c}, a) dt] + e^{-\rho(\Delta \wedge \tau)} [\Pr(\tau > \Delta) W_r^{Us} \\ & + \Pr(\tau > \Delta, \tau = \tau^S | a_t) W_r^{Us'} + \Pr(\tau > \Delta, \tau = \tau^J | a_t) W_r^{Es}] \end{aligned}$$

or equivalently:

$$\begin{aligned} \max_{a \in A} \quad & E_0 [\rho \int_0^{\Delta \wedge \tau} e^{-\rho t} u(\bar{c}, a) dt] + e^{-\rho(\Delta \wedge \tau)} [e^{-(\lambda_s + q_s(a_t))\Delta} W_r^{Us} \\ & + (1 - e^{-(\lambda_s + q_s(a_t))\Delta}) (\frac{\lambda_s}{\lambda_s + q_s(a_t)} W_r^{Us'} + \frac{q_s(a_t)}{\lambda_s + q_s(a_t)} W_r^{Es})] \end{aligned}$$

The optimized value is  $W_r^{Us}$ , so for any  $\hat{a} \in [0, \bar{a}]$  and  $\Delta > 0$  we have:

$$\begin{aligned} W_r^{Us} &\geq E_0 [\rho \int_0^{\Delta \wedge \tau} e^{-\rho t} u(\bar{c}, \hat{a}) dt] + e^{-\rho(\Delta \wedge \tau)} [e^{-(\lambda_s + q_s(\hat{a}))\Delta} W_r^{Us} \\ &\quad + (1 - e^{-(\lambda_s + q_s(\hat{a}))\Delta}) (\frac{\lambda_s}{\lambda_s + q_s(\hat{a})} W_r^{Us'} + \frac{q_s(\hat{a})}{\lambda_s + q_s(\hat{a})} W_r^{Es})] \end{aligned}$$

and

$$\begin{aligned} 0 &\geq \frac{1}{\Delta} E_0 [\rho \int_0^{\Delta \wedge \tau} e^{-\rho t} u(\bar{c}, \hat{a}) dt] + \frac{1}{\Delta} (e^{-\rho(\Delta \wedge \tau)} e^{-(\lambda_s + q_s(\hat{a}))\Delta} W_r^{Us} - W_r^{Us}) \\ &\quad + \frac{1}{\Delta} e^{-\rho(\Delta \wedge \tau)} (1 - e^{-(\lambda_s + q_s(\hat{a}))\Delta}) (\frac{\lambda_s}{\lambda_s + q_s(\hat{a})} W_r^{Us'} + \frac{q_s(\hat{a})}{\lambda_s + q_s(\hat{a})} W_r^{Es}). \end{aligned}$$

Let  $\Delta \rightarrow 0$ , then we have

$$0 \geq \rho u(\bar{c}, \hat{a}) - W_r^{Us} + \lambda_s(W_r^{Us'} - W_r^{Us}) + q_s(\hat{a})(W_r^{Es} - W_r^{Us})$$

and the equality holds only if  $\hat{a}$  is optimal. So we have (20).

If the worker is employed, similarly, for any  $\Delta > 0$ , we have

$$\begin{aligned} W_r^{Es} &= E_0[\rho \int_0^{\Delta \wedge \tau} e^{-\rho t} u(\bar{c} + \omega, \hat{a}) dt] + e^{-\rho(\Delta \wedge \tau)} [e^{-(\lambda_s + p_s)\Delta} W_r^{Es} \\ &\quad + e^{-\rho(\Delta \wedge \tau)} (1 - e^{-(\lambda_s + p_s)\Delta}) (\frac{\lambda_s}{\lambda_s + p_s} W_r^{Es'} + \frac{p_s}{\lambda_s + p_s} W_r^{Us})] \end{aligned}$$

or

$$\begin{aligned} 0 &= \frac{1}{\Delta} E_0[\rho \int_0^{\Delta \wedge \tau} e^{-\rho t} u(\bar{c} + \omega, \hat{a}) dt] + \frac{1}{\Delta} (e^{-\rho(\Delta \wedge \tau)} e^{-(\lambda_s + p_s)\Delta} W_r^{Es} - W_r^{Es}) \\ &\quad + \frac{1}{\Delta} e^{-\rho(\Delta \wedge \tau)} (1 - e^{-(\lambda_s + p_s)\Delta}) (\frac{\lambda_s}{\lambda_s + p_s} W_r^{Es} + \frac{p_s}{\lambda_s + p_s} W_r^{Us}) \end{aligned}$$

and let  $\Delta \rightarrow 0$  we have (21).

## A.4 Proof of Proposition 3

We suppose that the economy is in a boom and the worker is unemployed. The other cases can be proved similarly. At time  $t$ , we assume that the promised utility of the worker is  $W_t = W \in (W_l^{Gu}, W_r^{Gu})$ . Let  $c$  be a consumption process. Let  $\{W_t^J\}_{t \in [0, \infty)}$  and  $\{W_t^S\}_{t \in [0, \infty)}$  be the processes which indicate the utility levels just after the arrival of a job offer and the change of the aggregate state at each point of time. The induced effort process is  $a$ . As before, let  $\tau = \tau^S \wedge \tau^J$  with  $\tau^S$  be the time of the next switch into recession and  $\tau^J$  be the time of the arrival of the next job offer. At time  $t$ , according to the definition of  $V(\cdot, u, G)$ , for any  $\Delta > 0$ , we have

$$\begin{aligned} V(W_t, u, G) &\geq -E_t[\rho \int_t^{(t+\Delta) \wedge \tau} e^{-\rho \hat{t}} v(c_{\hat{t}}) d\hat{t} | \mathcal{F}_{t-}] \\ &\quad + e^{-\rho((t+\Delta) \wedge \tau)} \Pr(\tau > t + \Delta | a) V(W_{t+\Delta}, u, G) \\ &\quad + e^{-\rho((t+\Delta) \wedge \tau)} \Pr(\tau > t + \Delta, \tau = \tau^J | a) V(W_{t+\Delta}^J, e, G) \\ &\quad + e^{-\rho((t+\Delta) \wedge \tau)} \Pr(\tau > t + \Delta, \tau = \tau^S | a) V(W_{t+\Delta}^S, u, B). \end{aligned}$$

In particular, we let  $\Delta$  be very small and  $W_t^J = W_{\tilde{t}} + \xi$  such that  $W_{\tilde{t}} + \xi \in [W_l^{Ge}, W_r^{Ge}]$  for some  $\xi$  and  $\tilde{t} \in [t, t + \Delta]$ . Since  $W_t$  is interior, so such  $\xi$  exist. Then the effort is constant in  $[t, t + \Delta]$  and equal to  $a^G(W_t, W_t^J)$ . Then we have

$$\begin{aligned} 0 &\geq -\frac{1}{\Delta} E_t[\rho \int_t^{(t+\Delta) \wedge \tau} e^{-\rho \hat{t}} v(c_{\hat{t}}) d\hat{t} | \mathcal{F}_{t-}] \\ &\quad + \frac{1}{\Delta} (e^{-\rho((t+\Delta) \wedge \tau)} e^{-(\lambda_G + q_G(a^G(W_t, W_t^J)))\Delta} V(W_{t+\Delta}, u, G) - V(W_t, u, G)) \\ &\quad + \frac{1}{\Delta} e^{-\rho((t+\Delta) \wedge \tau)} (1 - e^{-(\lambda_G + q_G(a^G(W_t, W_t^J)))\Delta}) \frac{q_G(a^G(W_t, W_t^J))}{\lambda_G + q_G(a^G(W_t, W_t^J))} V(W_{t+\Delta}^J, e, G) \\ &\quad + \frac{1}{\Delta} e^{-\rho((t+\Delta) \wedge \tau)} (1 - e^{-(\lambda_G + q_G(a^G(W_t, W_t^J)))\Delta}) \frac{\lambda_G}{\lambda_G + q_G(a^G(W_t, W_t^J))} V(W_{t+\Delta}^S, u, B). \end{aligned}$$

Let  $\Delta \rightarrow 0$  and we have

$$\begin{aligned} 0 &\geq -\rho(v(c_t) + V(W_t, u, G)) + V_W(W_t, u, G)\rho(W_t - u(c_t, a_t)) \\ &\quad - q_G(a^G(W_t, W_t^J)) \frac{W_t^J - W_t}{\rho} - \lambda_G \frac{W_t^S - W_t}{\rho} \\ &\quad + q_G(a^G(W_t, W_t^J))(V(W_t^J, e, G) - V(W_t, u, G)) + \lambda_G(V(W_t^S, u, B) - V(W_t, u, G)). \end{aligned}$$

The equality holds if  $c_t$ ,  $W_t^J$  and  $W_t^S$  are optimally chosen, so we have (7).

## B Calculations for the Solvable Special Case

### B.1 Employed Workers

It is easy to see that the value function is independent of  $s$  and we guess that it is of the form:

$$V(W, e, s) = V(W, e) = -V_e(-W)^{-\frac{\theta_P}{\theta_A}}.$$

It is actually easy to verify this directly by using the fact that  $c$  is constant under employment, but here we use the guess and verify approach because many similar calculations will be repeated below. Note that this implies:

$$V_W(W, e) = -V_e \frac{\theta_P}{\theta_A} (-W)^{-\frac{\theta_P}{\theta_A} - 1}.$$

Substituting this expression into the optimality condition for  $c$  implies:

$$\begin{aligned} -\theta_P \exp(\theta_P(c - \omega)) &= -V_e \frac{\theta_P}{\theta_A} (-W)^{-\frac{\theta_P}{\theta_A} - 1} \theta_A \exp(-\theta_A c) \\ \Rightarrow \exp((\theta_P + \theta_A)c) &= V_e \frac{\theta_P}{\theta_A} (-W)^{-\frac{\theta_P + \theta_A}{\theta_A}} \theta_A \exp(\theta_P \omega) \\ \Rightarrow c &= \frac{1}{\theta_P + \theta_A} \log(V_e) - \frac{1}{\theta_A} \log(-W) + \frac{\theta_P}{\theta_P + \theta_A} \omega \end{aligned}$$

Therefore we have:

$$v(c - \omega) = V_e^{\frac{\theta_P}{\theta_P + \theta_A}} \exp\left(-\frac{\theta_A \theta_P}{\theta_A + \theta_P} \omega\right) (-W)^{-\frac{\theta_P}{\theta_A}},$$

and:

$$u(c) = -V_e^{-\frac{\theta_A}{\theta_P + \theta_A}} \exp\left(-\frac{\theta_A \theta_P}{\theta_A + \theta_P} \omega\right) (-W).$$

Thus we can write the HJB equation as:

$$\begin{aligned} -V_e(-W)^{-\frac{\theta_P}{\theta_A}} &= -V_e^{\frac{\theta_P}{\theta_P + \theta_A}} \exp\left(-\frac{\theta_A \theta_P}{\theta_A + \theta_P} \omega\right) (-W)^{-\frac{\theta_P}{\theta_A}} \\ &\quad - V_e \frac{\theta_P}{\theta_A} (-W)^{-\frac{\theta_P + \theta_A}{\theta_A}} \left[-1 + V_e^{-\frac{\theta_A}{\theta_P + \theta_A}} \exp\left(-\frac{\theta_A \theta_P}{\theta_A + \theta_P} \omega\right)\right] (-W) \end{aligned}$$

The terms in  $W$  cancel and we see that  $V_e$  solves:

$$\begin{aligned} -V_e &= -V_e^{\frac{\theta_P}{\theta_P + \theta_A}} \exp\left(-\frac{\theta_A \theta_P}{\theta_A + \theta_P} \omega\right) - V_e \frac{\theta_P}{\theta_A} \left[-1 + V_e^{-\frac{\theta_A}{\theta_P + \theta_A}} \exp\left(-\frac{\theta_A \theta_P}{\theta_A + \theta_P} \omega\right)\right] \\ \Rightarrow -\left(1 + \frac{\theta_P}{\theta_A}\right) &= -V_e^{-\frac{\theta_A}{\theta_P + \theta_A}} \exp\left(-\frac{\theta_A \theta_P}{\theta_A + \theta_P} \omega\right) \left(1 + \frac{\theta_P}{\theta_A}\right) \\ \Rightarrow V_e &= \exp(-\theta_P \omega) \end{aligned}$$

Substituting this into the expression for  $c$  gives the other result.

### B.2 Full Information

Using the conjectured form of the value function in the first slope matching condition gives:

$$-V_u(s) \frac{\theta_P}{\theta_A} (-W)^{-\frac{\theta_P + \theta_A}{\theta_A}} = -\exp(-\theta_P \omega) (-W)^{-\frac{\theta_P + \theta_A}{\theta_A}},$$

so therefore we have:

$$W^J = \left( \frac{V_u(s)}{\exp(-\theta_P \omega)} \right)^{-\frac{\theta_A}{\theta_P + \theta_A}} W.$$

In turn this implies:

$$V(W^J, e) - V(W, u, s) = [-V_u(s) \frac{\theta_P}{\theta_P + \theta_A} V_e^{\frac{\theta_A}{\theta_A + \theta_P}} + V_u(s)](-W)^{-\frac{\theta_P}{\theta_A}}$$

Similarly, for the second slope matching condition we get

$$-V_u(s) \frac{\theta_P}{\theta_A} (-W)^{-\frac{\theta_P + \theta_A}{\theta_A}} = -V_u(s') \frac{\theta_P}{\theta_A} (-W^S)^{-\frac{\theta_P + \theta_A}{\theta_A}}$$

and thus we have:

$$W^S = \left( \frac{V_u(s)}{V_u(s')} \right)^{-\frac{\theta_A}{\theta_P + \theta_A}} W,$$

and therefore:

$$V(W^S, u, s') - V(W, u, s) = [-V_u(s') \left( \frac{V_u(s)}{V_u(s')} \right)^{\frac{\theta_P}{\theta_P + \theta_A}} + V_u(s)](-W)^{-\frac{\theta_P}{\theta_A}}.$$

From the optimality condition for  $c$  we then have:

$$c = \frac{1}{\theta_P + \theta_A} \log(V_u(s)) - \frac{1}{\theta_A} \log(-W) + \frac{\theta_A}{\theta_P + \theta_A} h(\bar{a}(s))$$

Therefore we have in addition:

$$\begin{aligned} u(c, \bar{a}(s)) &= -V_u(s)^{-\frac{\theta_A}{\theta_P + \theta_A}} \exp\left(\frac{\theta_A \theta_P}{\theta_A + \theta_P} h(\bar{a}(s))\right) (-W) \\ u_a(c, \bar{a}(s)) &= -\theta_A h'(\bar{a}(s)) V_u(s)^{-\frac{\theta_A}{\theta_P + \theta_A}} \exp\left(\frac{\theta_A \theta_P}{\theta_A + \theta_P} h(\bar{a}(s))\right) (-W) \\ v(c) &= V_u(s)^{\frac{\theta_P}{\theta_P + \theta_A}} \exp\left(\frac{\theta_A \theta_P}{\theta_A + \theta_P} h(\bar{a}(s))\right) (-W)^{-\frac{\theta_P}{\theta_A}} \end{aligned}$$

Then we can write the optimality condition for  $a$  as:

$$\begin{aligned} & -\rho V_u(s) \frac{\theta_P}{\theta_A} (-W)^{-\frac{\theta_P + \theta_A}{\theta_A}} \left( -\theta_A h'(\bar{a}(s)) V_u(s)^{-\frac{\theta_A}{\theta_P + \theta_A}} \exp\left(\frac{\theta_A \theta_P}{\theta_A + \theta_P} h(\bar{a}(s))\right) (-W) \right) \\ & + \rho V_u(s) \frac{\theta_P}{\theta_A} (-W)^{-\frac{\theta_P + \theta_A}{\theta_A}} \frac{q_s}{\rho} \left( \left( \frac{V_u(s)}{\exp(-\theta_P \omega)} \right)^{-\frac{\theta_A}{\theta_P + \theta_A}} - 1 \right) (-W) \\ & = q_s [-V_u(s)^{\frac{\theta_P}{\theta_P + \theta_A}} V_e^{\frac{\theta_A}{\theta_A + \theta_P}} + V_u(s)] (-W)^{-\frac{\theta_P}{\theta_A}} \end{aligned}$$

Thus we see that  $W$  cancels out and we get:

$$V_u(s)^{-\frac{\theta_A}{\theta_P + \theta_A}} \left( \rho \theta_P h'(\bar{a}(s)) \exp\left(\frac{\theta_A \theta_P}{\theta_A + \theta_P} h(\bar{a}(s))\right) + \left(1 + \frac{\theta_P}{\theta_A}\right) q_s V_e^{\frac{\theta_A}{\theta_A + \theta_P}} \right) = q_s \left(1 + \frac{\theta_P}{\theta_A}\right) \quad (22)$$

Substituting all of the above results into the HJB equation gives:

$$\begin{aligned} & -\rho V_u(s) (-W)^{-\frac{\theta_P}{\theta_A}} = -\rho V_u(s)^{\frac{\theta_P}{\theta_P + \theta_A}} \exp\left(\frac{\theta_A \theta_P}{\theta_A + \theta_P} h(\bar{a}(s))\right) (-W)^{-\frac{\theta_P}{\theta_A}} \\ & -\rho V_u(s) \frac{\theta_P}{\theta_A} (-W)^{-\frac{\theta_P + \theta_A}{\theta_A}} \left( W - V_u(s)^{-\frac{\theta_A}{\theta_P + \theta_A}} \exp\left(\frac{\theta_A \theta_P}{\theta_A + \theta_P} h(\bar{a}(s))\right) (-W) \right) \\ & -\rho V_u(s) \frac{\theta_P}{\theta_A} (-W)^{-\frac{\theta_P + \theta_A}{\theta_A}} \left[ \frac{q_s \bar{a}(s)}{\rho} \left( \left( \frac{V_u(s)}{\exp(-\theta_P \omega)} \right)^{-\frac{\theta_A}{\theta_P + \theta_A}} - 1 \right) + \frac{\lambda_s}{\rho} \left( \left( \frac{V_u(s)}{V_u(s')} \right)^{-\frac{\theta_A}{\theta_P + \theta_A}} - 1 \right) \right] (-W) \\ & + q_s \bar{a}(s) [-V_u(s)^{\frac{\theta_P}{\theta_P + \theta_A}} V_e^{\frac{\theta_A}{\theta_A + \theta_P}} + V_u(s)] (-W)^{-\frac{\theta_P}{\theta_A}} + \lambda_s [-V_u(s') \left( \frac{V_u(s)}{V_u(s')} \right)^{\frac{\theta_P}{\theta_P + \theta_A}} + V(s)] (-W)^{-\frac{\theta_P}{\theta_A}} \end{aligned}$$

Therefore again the terms in  $W$  cancel. After simplifying and again canceling a common  $(1 + \frac{\theta_P}{\theta_A})$  factor we get:

$$V_u(s)^{-\frac{\theta_A}{\theta_P + \theta_A}} \left( \rho \exp\left(\frac{\theta_A \theta_P}{\theta_A + \theta_P} h(\bar{a}(s))\right) + q_s \bar{a}(s) V_e^{\frac{\theta_A}{\theta_A + \theta_P}} + \lambda_s V_u(s')^{\frac{\theta_A}{\theta_P + \theta_A}} \right) = \rho + q_s \bar{a}(s) + \lambda_s \quad (23)$$

Therefore we have 4 equations, (22) and (23) which each must hold for  $s = G, B$ , in the 4 unknowns  $(\bar{a}(G), V_u(G), \bar{a}(B), V_u(B))$ .

### B.3 Moral Hazard

To ease some of the derivations, redefine the constant in the consumption function as  $c^*(s) + h(a^*(s))$ . Then given the forms of the guesses for  $c$  and  $a$  we can evaluate:

$$\begin{aligned}
v(c) &= \exp(\theta_P(c^*(s) + h(a^*(s))))(-W)^{-\frac{\theta_P}{\theta_A}} \\
u(c, a^*(s)) &= -\exp(-\theta_A c^*(s))(-W) \\
u_c(c, a^*(s)) &= \theta_A \exp(-\theta_A c^*(s))(-W) \\
u_a(c, a^*(s)) &= -\theta_A h'(a^*(s)) \exp(-\theta_A c^*(s))(-W) \\
u_{aa}(c, a^*(s)) &= -[\theta_A^2 h'(a^*(s))^2 + \theta_A h''(a^*(s))] \exp(-\theta_A c^*(s))(-W) \\
u_{ac}(c, a^*(s)) &= \theta_A^2 h'(a^*(s)) \exp(-\theta_A c^*(s))(-W)
\end{aligned}$$

In addition, we can evaluate:

$$W^J = W - \frac{\rho}{q_s} (-\theta_A h'(a^*(s)) \exp(-\theta_A c^*(s)))(-W) = [1 - \frac{\rho}{q_s} \theta_A h'(a^*(s)) \exp(-\theta_A c^*(s))]W.$$

So therefore we have:

$$\begin{aligned}
V(W^J, e) &= -V_e [1 - \frac{\rho}{q_s} \theta_A h'(a^*(s)) \exp(-\theta_A c^*(s))]^{-\frac{\theta_P}{\theta_A}} (-W)^{-\frac{\theta_P}{\theta_A}} \\
V_W(W^J, e) &= -\frac{\theta_P}{\theta_A} V_e [1 - \frac{\rho}{q_s} \theta_A h'(a^*(s)) \exp(-\theta_A c^*(s))]^{-\frac{\theta_A + \theta_P}{\theta_A}} (-W)^{-\frac{\theta_P + \theta_A}{\theta_A}}
\end{aligned}$$

The first order conditions and solution for  $W^S$  are of the same form as in the full information case, and thus we can write:

$$W^S = \left( \frac{V^*(s)}{V^*(s')} \right)^{-\frac{\theta_A}{\theta_P + \theta_A}} W,$$

and therefore:

$$V(W^S, u, s') - V(W, u, s) = [-V^*(s') \left( \frac{V^*(s)}{V^*(s')} \right)^{\frac{\theta_P}{\theta_P + \theta_A}} + V^*(s)](-W)^{-\frac{\theta_P}{\theta_A}}.$$

The optimality condition for  $c$  then can be written, canceling the terms in  $W$ :

$$\begin{aligned}
\theta_P \exp(\theta_P(c^*(s) + \theta_P h(a^*(s)))) &= -V^*(s) \frac{\theta_P}{\theta_A} [-\theta_A \exp(-\theta_A c^*(s)) + a^*(s) \theta_A^2 h'(a^*(s)) \exp(-\theta_A c^*(s))] \\
&+ a^*(s) \theta_A^2 h'(a^*(s)) \exp(-\theta_A c^*(s)) \frac{\theta_P}{\theta_A} V_e [1 - \frac{\rho}{q_s} \theta_A h'(a^*(s)) \exp(-\theta_A c^*(s))]^{-\frac{\theta_A + \theta_P}{\theta_A}}.
\end{aligned}$$

Simplifying this gives:

$$\begin{aligned}
\exp((\theta_P + \theta_A)c^*(s) + \theta_P h(a^*(s))) &= V^*(s) [1 - a^*(s) \theta_A h'(a^*(s))] \\
&+ a^*(s) \theta_A h'(a^*(s)) V_e \left[ 1 - \frac{\rho}{q_s} \theta_A h'(a^*(s)) \exp(-\theta_A c^*(s)) \right]^{-\frac{\theta_A + \theta_P}{\theta_A}}
\end{aligned} \tag{24}$$

The optimality condition for  $a$  can be written, again canceling the terms in  $W$ :

$$\begin{aligned}
a^*(s) [\theta_A^2 h'(a^*(s))^2 + \theta_A h''(a^*(s))] \exp(-\theta_A c^*(s)) \\
\frac{\theta_P}{\theta_A} \left[ -V^*(s) + V_e \left( 1 - \frac{\rho}{q_s} \theta_A \exp(-\theta_A c^*(s)) h'(a^*(s)) \right)^{-\frac{\theta_A + \theta_P}{\theta_A}} \right] \\
= \frac{q_s}{\rho} \left[ -V_e \left( 1 - \frac{\rho}{q_s} \theta_A \exp(-\theta_A c^*(s)) h'(a^*(s)) \right)^{-\frac{\theta_P}{\theta_A}} + V^*(s) \right]
\end{aligned}$$

Simplifying yields:

$$\begin{aligned}
& a^*(s)\theta_P \left( \theta_A h'(a^*(s))^2 + h''(a^*(s)) \right) \left[ V^*(s) - V_e \left( 1 - \frac{\rho}{q_s} \theta_A \exp(-\theta_A c^*(s)) h'(a^*(s)) \right)^{-\frac{\theta_A + \theta_P}{\theta_A}} \right] \\
& = \exp(\theta_A c^*(s)) \frac{q_s}{\rho} \left[ V^*(s) - V_e \left( 1 - \frac{\rho}{q_s} \theta_A \exp(-\theta_A c^*(s)) h'(a^*(s)) \right)^{-\frac{\theta_P}{\theta_A}} \right]
\end{aligned} \tag{25}$$

The final equation comes from substituting the results into the HJB equation and simplifying, which gives:

$$\begin{aligned}
0 = & (\rho + q_s a^*(s) + \lambda_s) V^*(s) - \rho \exp(\theta_P (c^*(s) + h(a^*(s)))) \\
& + \rho \frac{\theta_P}{\theta_A} V^*(s) \left[ 1 - \exp(-\theta_A c^*(s)) + a^*(s) \theta_A \exp(-\theta_A c^*(s)) h'(a^*(s)) - \frac{\lambda_s}{\rho} \left( \left( \frac{V^*(s')}{V^*(s)} \right)^{\frac{\theta_A}{\theta_A + \theta_P}} - 1 \right) \right] \\
& - q_s a^*(s) V_e \left( 1 - \frac{\rho}{q_s} \theta_A \exp(-\theta_A c^*(s)) h'(a^*(s)) \right)^{-\frac{\theta_P}{\theta_A}} - \lambda_s V^*(s')^{\frac{\theta_A}{\theta_A + \theta_P}} V^*(s)^{\frac{\theta_P}{\theta_A + \theta_P}}
\end{aligned} \tag{26}$$

Thus for  $s = G, B$  the equations (24) - (26) determine the six constants  $(V^*(s), a^*(s), c^*(s))$ .

## C Calculations for the Implementation

### C.1 An Employed Worker

Using the guess in the first order condition we can find  $c$  as:

$$c(x, e) = -\frac{1}{\theta_A} \log \frac{r^e J_e}{\rho} + r^e x,$$

and so we have:

$$u(c) = -\frac{r^e J_e}{\rho} \exp(-r^e \theta_A x).$$

Substituting this into the HJB equation, and canceling the terms in  $J_e \exp(-r^e \theta_A x)$  gives:

$$-\rho = -r^e + r^e \log \frac{r^e J_e}{\rho} + r^e \theta_A b^e$$

Solving for  $J_e$  gives the result in the text. Substituting the resulting  $J_e$  into the consumption function gives the other result.

### C.2 An Unemployed Worker

Using the guess of the value function and the structure of  $A$  we have:

$$\begin{aligned}
J(x + B(s), e) - J(x, u, s) &= [J_u(s) - J^*(s) \exp(-r \theta_A B(s))] \exp(-r(s) \theta_A x) \\
J(x + A(s, x), u, s') - J(x, u, s) &= J_u(s) \exp(-r(s) \theta_A x) - J_u(s') \exp(-r(s') \theta_A (x + A(s, x))) \\
&= [J_u(s) - J_u(s') \exp(-r \theta_A \hat{A}(s))] \exp(-r \theta_A x)
\end{aligned}$$

The first order condition for  $c$  then implies the consumption function in the text. The first order condition for  $a$  gives:

$$\rho \theta_A h'(a) \exp(-\theta_A (c - h(a))) = q_s [J_u(s) - J^*(s) \exp(-r(s) \theta_A B(s))] \exp(-r(s) \theta_A x).$$

Using the result for  $c$  and canceling terms in  $x$  gives:

$$r(s) \theta_A J_u(s) h'(a(s)) = q_s [J_u(s) - J^*(s) \exp(-r(s) \theta_A B(s))], \tag{27}$$

which verifies that effort is independent of  $x$ .

Using the previous results, and canceling the terms in  $x$ , the HJB equation then becomes:

$$\begin{aligned} -\rho J_u(s) = & -r(s)J_u(s) + r(s)\theta_A J_u(s) \left[ \frac{1}{\theta_A} \log \frac{r(s)J_u(s)}{\rho} - h(a(s)) + b^u(s) \right] \\ & + q_s a(s) [J_u(s) - J^*(s) \exp(-r(s)\theta_A B(s))] + \lambda_s [J_u(s) - J_u(s') \exp(-r(s)\theta_A \hat{A}(s))]. \end{aligned}$$

This can be simplified to yield:

$$\begin{aligned} r(s) - \rho - q_s a(s) - \lambda_s = & r(s)\theta_A \left[ \frac{1}{\theta_A} \log \frac{r(s)J_u(s)}{\rho} - h(a(s)) + b^u(s) \right] \\ & - q_s a(s) \frac{J^*(s)}{J_u(s)} \exp(-r(s)\theta_A B(s)) - \lambda_s \frac{J_u(s')}{J_u(s)} \exp(-r(s)\theta_A \hat{A}(s)). \end{aligned} \quad (28)$$

Thus (27) and (28) for  $s = G, B$  determine the constants  $(a(s), J_u(s))$ .

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