

Lack of Selection and Poor Management Practices: Firm Dynamics in Developing Countries*

[PRELIMINARY]

Ufuk Akcigit[†]
University of Pennsylvania

Michael Peters[‡]
LSE

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Abstract

As recently shown by Hsieh and Klenow (2012), firm dynamics differ substantially across countries. While firms in the US experience substantial growth during their life-cycle, firms in developing countries, especially in India, barely expand. We present a tractable micro-founded endogenous growth model to explain these differences. At the heart of the theory are two sources of heterogeneity across countries. First, we explicitly allow firms to register as formal firms or stay in the informal sector. While informality comes with the benefit of not being subject to taxes and regulation, informal firms are subject to government audits and the risk of being shut down. This lowers the marginal return of technology adoption and informal firms have an incentive to stay small. Second, we incorporate the recent state-of-the-art advances from Bloom and Van Reenen (2010) that managerial practices differ across countries and incorporate managers as a necessary input to run multi-product establishments. Better managers will induce a steeper life-cycle profile as it allows firms to scale up easily and to expand into new product lines. While the model has rich implications for firms' life-cycle, it still has a tractable analytic solution, which we can easily confront with the micro-evidence and calibrate successfully to the data of Hsieh and Klenow (2012).

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[†]Email address: uakcigit@econ.upenn.edu.

[‡]Email address: m.peters1@lse.ac.uk.

1 Motivation

Why do some countries stagnate while others grow? While there is a broad consensus in the literature that these differences are due to countries differing in their processes for aggregate productivity (Banerjee and Duflo, 2005; Caselli, 2005; Klenow and Rodriguez-Clare, 1997), there is less agreement on the fundamental reasons for this heterogeneity. This is due to both theoretical and empirical shortcomings. A fruitful approach to answering this macroeconomic question has to be a micro-to-macro approach, which focuses on the factors that affect microeconomic agents, i.e., individual producers. Fortunately, the last decade has seen an advent of both new data and new theory, which allows us to speak to this underlying microstructure. Thanks to two highly influential recent papers by Chang-Tai Hsieh and Peter Klenow, a number of important empirical micro facts have been uncovered from hard-to-find census data sets of all manufacturing firms in the US, Mexico, China and India, which have arguably important consequences for aggregate productivity. Hsieh and Klenow (2009) showed that the cross-sectional dispersion of marginal products is much higher in India than in the US, so that aggregate productivity is low because of misallocation. In more recent work, Hsieh and Klenow (2012) showed that the life-cycle of firms also differs remarkably across countries.

1. In terms of employment, firms in the US experience high growth over their life-cycle, while there is no evidence of growth in India. Firms in Mexico fall somewhere in between.
2. In terms of productivity, US and Mexican firms experience similar dynamics, while Indian firms' productivity increases very little over the life-cycle.

While many recent papers have tried to provide a theoretical foundation for the cross-country differences in misallocation,¹ there has been little theoretical work explaining why firm dynamics differ so much across countries.² In this paper, we take a step in this direction by providing a tractable framework for a *comparative approach to firm dynamics*.

To do so, we want to put the productivity process, both at the micro and macro level, at center stage, and hence, we turn to the recent generation of micro-founded models of growth, in particular Klette and Kortum (2004). While such models have been built to study firm dynamics in developed economies (Lentz and Mortensen (2008), Acemoglu et al. (2012), Akcigit and Kerr (2010)), this is not the case for developing countries.³ In this paper we provide such a framework and use it to diagnose the main obstacles that firms in India and Mexico are facing and that prevent them from growing like their US counterparts.

¹The seminal papers for the recent literature on misallocation are Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), with Hopenhayn and Rogerson (1993) being an early contribution. As far as theories are concerned, there is now a sizable literature on credit market frictions (Buera et al., 2011; Moll, 2010; Midrigan and Xu, 2010), size-dependent policies (Guner et al., 2008), monopolistic market power (Peters, 2011) and adjustment costs (Collard-Wexler et al., 2011). A synthesis of the literature is also contained in Hopenhayn (2012) and Jones (2011).

²A notable exception is Cole et al. (2012), who argue that cross-country differences in the financial system will affect the type of technologies that can be implemented. While US firms will be able to adopt technologies with large growth potential over their life-cycle, firms in India will not be able to do so given their worse financial system.

³Exceptions are de Mel et al. (2009), who consider a static model of innovation to study innovation incentives of firms in Sri Lanka, and Peters (2011), who applies a dynamic Schumpeterian model to firm-level data in Indonesia.

Endogenous technical change models are a natural environment to study this question as they focus on firms' productivity-enhancing investment decisions. We believe that models of endogenous growth have been underutilized in the development literature partly due to a lack of data to discipline these models and partly due to the fact that the early models of endogenous growth have been mainly constructed to model innovation decisions of the firms in developed countries.⁴ Hence, these early models have been harmonized with terminologies such as innovation, R&D, patent protection, and innovation policy, which do not seem to capture the reality of firms in developing countries well. For the remainder of this paper we therefore refer to innovation in a broad sense, capturing not only the implementation of new ideas but rather a variety of costly productivity-enhancing activities, encompassing also training, reorganization or the acquisition of high-quality complementary factors. By doing so we hope to show that the recent advances on the theory and empirical sides provide a very useful combination for interpreting the empirical regularities and identifying the main obstacles that developing economies are facing at the firm level, which can then be used in rigorous calibration/estimation and counterfactual policy analysis.

We explicitly introduce two dimensions of cross-country heterogeneity into the theory. First, we follow Bloom and Van Reenen (2010, 2007), who document that *management practices* differ greatly across countries. In particular, they show that

3. While US firms are managed best, Indian firms are managed worst.

Second, a well-known fact in the literature is that the size of the informal sector differs greatly across countries. While informal firms are of minor economic importance in the US, Hsieh and Klenow (2012) report that

4. The share of employment in the informal sector is 80% in India and 30% in Mexico.

We embed these aspects in a micro-founded model of endogenous growth and argue that these empirical regularities are very naturally related, i.e., cross-country differences in the efficiency of managerial practices and the regulation of the informal sector affect firms' innovation and entry incentives in a way to qualitatively generate the observed life-cycle differences. More specifically, we build a two-sector model of firm dynamics, where a final good is produced by a continuum of intermediate good producers. Upon entry, each producer can choose to operate either as a formal or an informal firm.⁵ The advantage of being an informal firm is to not having to pay taxes, at the risk of being caught by government officials and having to shut down operations and lose the whole business. In our model, both the taxes and screening technology are among the policy instruments that the government controls and we take those as structural parameters, which we calibrate using the theory. As in Klette and Kortum (2004), we model firm dynamics as the outcome of creative destruction, where firms expand into new product lines by investing in productivity-enhancing investment activities. However, firms have to hire managers to run multiple product lines, so that the efficiency of managerial factors determines firms' returns to scale and hence innovation incentives. This aspect of the model allows us to link our predictions to the empirical findings of Bloom and Van Reenen (2010) and Bloom et al. (2010)

⁴A major impediment to bringing the first generation models of endogenous growth to the data is that these were aggregate models, which do not have direct implications at the firm level (Romer, 1990; Aghion and Howitt, 1992; Grossman and Helpman, 1991).

⁵For a recent experimental study to estimate the "demand for formality," see De Mel et al. (Forthcoming).

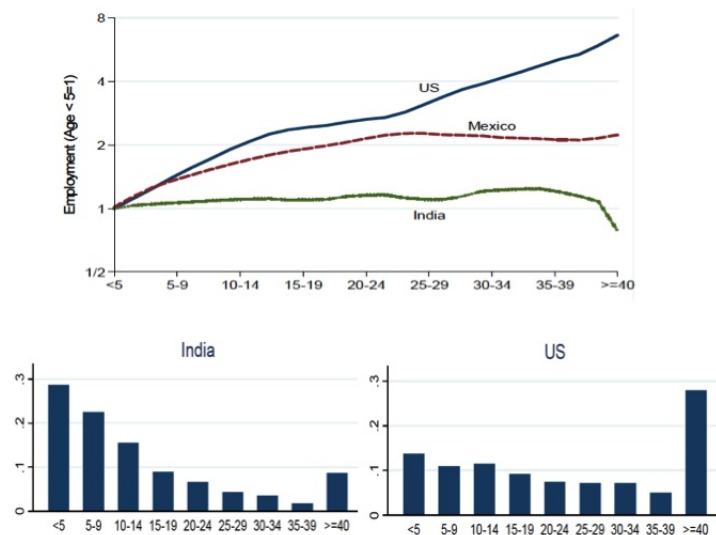
Even though the model has a rich microstructure of sector-specific firm dynamics, it is tractable enough to solve analytically and to derive closed-form expressions for firms' life-cycle growth. This allows for a transparent discussion of the two main questions we started with: What are the economic forces that lead to the observed differences in firm dynamics across countries? What are the consequences for aggregate productivity? To answer these questions quantitatively, we calibrate our model to the data of Hsieh and Klenow (2012). Despite its analytic tractability, our baseline model is successful in matching the targeted moments about the life-cycle of US and Indian firms. The main lessons we draw from our baseline model are as follows: The steepness of the life-cycle growth trajectory of US firms (conditional on survival) reflects large incumbent innovation incentives, which are driven by an efficient managerial technology that allows firms to scale up seamlessly. The small share of informal sector firms in the US is due to a tighter monitoring policy, which raises the value of formality despite non-trivial costs for taxes, social security and other regulatory overhang. The pattern of US aggregate productivity growth is mostly driven by incumbents' innovation incentives, which makes successful firms grow fast and unsuccessful firms exit early. Indian firms by contrast simply earn too little infra-marginal rents to generate sufficient innovation incentives for steep life-cycle growth. Through the lens of our model, this is due to an inefficient managerial technology. That these low innovation incentives do not necessarily translate into abysmal aggregate productivity growth is due to the other margin of growth, namely, entry in the informal sector. Compared to the US, being an informal producer in India is relatively more valuable as the inefficient regulatory environment makes it easy to escape government auditing by "staying small." Hence, while average life-cycle growth is considerably smaller than in the US, this does not necessarily lead to counterfactually large differences in aggregate productivity growth.

The rest of the paper is organized as follows. Section 2 describes the main empirical regularities that are fundamental in our analysis. Section 3 presents the theoretical model. Section 4 provides the details of our quantitative analysis and its results. Section 5 concludes.

2 Two facts: management and firm dynamics across countries

The purpose of this paper is to develop a flexible, micro-founded model of endogenous growth, which can be parametrized to match the pervasive firm-level facts in both rich and poor countries. Using census data from India, Mexico and the US, Hsieh and Klenow (2012) study the life-cycle of manufacturing establishments and report striking differences across these three countries. We focus on one particular aspect of their analysis, namely, the cross-sectional age-size relationship. For completeness, consider Figure 1 below, which contains two graphs from Hsieh and Klenow (2012), and against which we will calibrate our theory. In the top panel, we depict the simple cross-sectional relationship of average firm-size by age. While US firms grow by 400% within their first 30 years of existence, firms in India remain small during their life-cycle. Firms in Mexico are an intermediate case.⁶ We will interpret the relationship depicted

⁶Interpreting the cross-sectional relationship as being informative about the life-cycle is of course not innocuous, as it reflects both the life-cycle growth of a given cohort and differential effects upon entry across cohorts. Using the panel dimension of their data, Hsieh and Klenow (2012) argue that the cross-sectional relationship is indeed a good approximation to firms' life-cycle. Our theory will be stationary, so that - by construction - no cohort effects are present. Hence, through the lens of our model, the cross-sectional relationship will indeed fully reflect firm dynamics, i.e., the life-cycle.



Source: Hsieh and Klenow (2012)

Figure 1: Firm dynamics across countries: The age-size relationship in the cross-section

in Figure 1 as the dynamic outcome of firms' incentives to grow their product base. Hence, conditional on survival, US firms are big, because they have been able to outcompete their rivals and hence acquire many products in their portfolio. In contrast, firms in India do not grow as the incentives to add products are low. In such a world, the cross-country differences in firms' growth trajectories are therefore highly informative about the intensity of creative destruction induced by incumbent firms.

In an economy where firms grow at the expense of other firms, it is important to realize that the top panel of Figure 1 is conditional on survival. Hence, it might be that the surviving firm in the US is indeed very large, but the probability of surviving for three decades is sufficiently small for old plants to not be economically important in an aggregate sense. The lower panel in Figure 1 addresses this issue by reporting the share of aggregate employment being accounted for by plants of different age. It is clearly seen that survival rates are sufficiently similar in the US and India that the differential life-cycle patterns manifest themselves also in the aggregate data: while in the US old firms account for the bulk of economic activity, this is far less pronounced in India.⁷

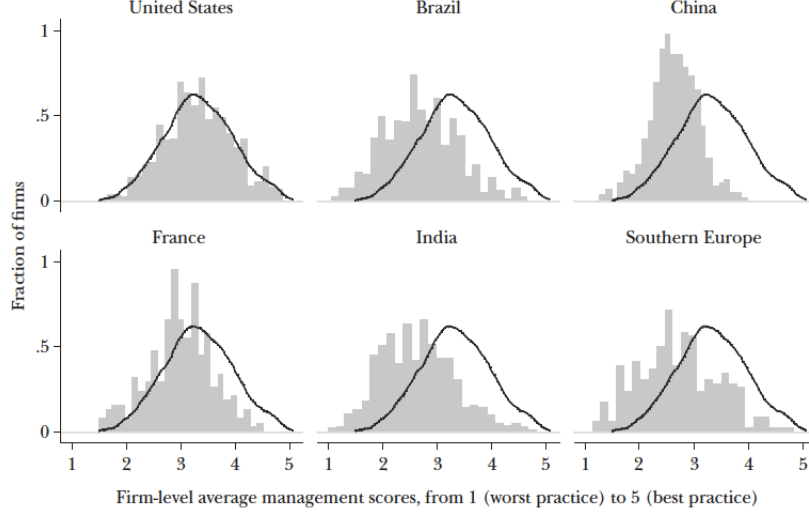
Arguing that the firm dynamics underlying the data in Figure 1 reflect the differential innovation rates of firms is of course only a proximate answer - the whole premise of a model of endogenous growth is of course that firms' innovation incentives are endogenous choices and hence informative about the underlying differences across countries. One of the differences between the US and India, which we think is of first-order importance, is the availability and quality of managerial resources. Incorporating managerial factors into an endogenous growth model is attractive for two reasons. First, there is a long tradition in economics, dating back

⁷We report only the US and Indian data as it is these two countries we are most interested in, in our quantitative exercise.

to Chandler (1977) and Penrose (1959), that links firms' expansion potential to managerial resources. Second, recent work has shown that managerial practices do differ markedly across countries and are systematically related to economic development. As an example consider Figure 2, taken from Bloom and Van Reenen (2010).

Figure 2

Management Practice Scores across Firms



Source: Bloom, Genakos, Sadun, and Van Reenen (2009).

Notes: Bars are the histogram of the actual density. The line is the smoothed (kernel) of the U.S. density for comparison. Southern Europe combines Greece and Portugal.

Source: Bloom and Van Reenen (2010)

Figure 2: Managerial practices across countries

The figure shows the distribution of management scores across firms relative to the US. It is clearly seen that less developed countries have a large tail of relatively badly managed firms. In the next section we will therefore explicitly introduce a managerial technology in an endogenous growth model and argue that underdeveloped managerial practices might be an important contributor to the differences in firm dynamics across countries.

3 The model

In this section we lay out the baseline version of our model of firm dynamics. As in Klette and Kortum (2004) we model firms as a collection of *products* that they supply to the market. Firms can spend resources on innovation to offer other producers' products at lower prices and thereby replace incumbent firms. Hence, firms' life cycle is crucially dependent on how successful the typical firm is in growing its product portfolio over time. For old firms to be much larger than young firms (as in the US), it has to be the case that firms manage to "accumulate" products quickly, as they age. Focusing on this extensive margin of firm growth is not only useful in that it provides an extremely tractable framework of firm dynamics but also because

Goldberg et al. (2010) and De Loecker et al. (2012) have recently provided empirical evidence on the slow expansion of the product base of Indian firms.

We start out with a parameterization of the model, which is tractable enough so that we can solve the entire model (including the stationary firm-size distribution) in closed form. This allows for a very transparent link between the theory and the data and for an intuitive interpretation of our baseline calibration. Furthermore, it provides a convenient structure to guide the subsequent extensions of the theory in section 5, where we will add more degrees of freedom. The two main features of the theory are as follows. First, we assume that managing a large portfolio of products requires resources. We think of these as “management costs,” which the firm has to pay irrespective of the quantity sold (per product). It is these management costs that will crucially determine the marginal return to innovation. Second, we want to be able to talk about the coexistence of formal and informal firms, which is arguably important in many developing economies. We assume that formal and informal firms compete in the product market and the focus on the legal differences between these two types of firms. While formal firms are disadvantaged in that they have to pay taxes (which we think of in a broad sense, i.e., also incorporating various forms of “red tape”), informal firms are subject to stochastic audits by the government authority, which can lead to informal firms being forced to cease operating in some of their product lines. We think of the likelihood of auditing as an important structural parameter across countries and will calibrate it from our theory.

3.1 Preferences and technology

We start out with a standard endogenous growth model in continuous time. On the demand side, there is a representative household, with standard preferences

$$U_0 = \int_0^\infty \exp(-\rho t) \ln C_t dt, \quad (1)$$

where, as usual, $\rho > 0$ is the discount factor. Given the unitary intertemporal elasticity of substitution, the Euler equation along the balanced growth path is simply given by

$$g = r - \rho,$$

where g is the growth rate of the economy and r is the interest rate.

Households supply labor inelastically and can be employed as either production workers or managers. The final good, which we take as the numeraire of the economy, is a composite of a continuum of products, which for simplicity takes the Cobb-Douglas form

$$\ln Y_t = \int_0^1 \ln y_{jt} dj, \quad (2)$$

and y_{jt} is the amount of product j produced at time t . Production takes place by heterogeneous firms. In particular, the production function for good j at time t is given by

$$y_{jf} = q_{jf} l_{jf} \quad (3)$$

where q_{jf} is the firm-product specific production technology and l_{jf} is the number of workers employed for producing intermediate good j . The distribution of efficiencies $q_{j,t}$ will evolve endogenously through firms’ choices of innovation spending and will determine which firm

produces which product.⁸ Besides the input of labor, production of each good requires a fixed overhead cost of ϕ workers. We think of these as management costs (which we allow to vary across countries), i.e., workers being employed in the internal organization of the firm and the distribution of products to the market. Hence, the labor market clearing condition is simply

$$1 = L_t^P + L_t^M, \quad (4)$$

where we normalize the size of the labor force to unity and L_t^P and L_t^M denote the labor demand for production workers and managers, respectively. As workers are homogeneous and there are no frictions on the labor market, there will be a single equilibrium wage w_t . Given this structure, it is useful to define the aggregate efficiency index Q_t as the Cobb-Douglas composite of individual efficiencies

$$\ln Q_t \equiv \int_0^1 \ln q_{jt} dj.$$

3.2 Product market equilibrium

Now consider the equilibrium in the product market. At each point in time, each product line j is populated by a set of firms that can produce this good with productivity $[q_{jt}^f]$, where f identifies the firm. Clearly, the most productive firm (which we will sometimes refer to as the (quality) leader) will be the sole producer of product j . The Cobb-Douglas structure in (2) implies that the demand for an individual product will have unitary demand elasticity. Hence, the leader will always be forced to engage in limit pricing. To make things simple, we assume that there is imperfect diffusion of technology, i.e., if the leader in product j can produce the product with efficiency q_{jt} , the remaining firms can produce it with efficiency $\frac{q_{jt}}{\lambda}$ for some $\lambda > 1$. This assumption allows us to sidestep any issues of mark-up heterogeneity, which we do not think to be of first-order importance to understand differences in the life-cycle of firms across countries.⁹ Given this assumption, the equilibrium price for product j is given by

$$p_{ij} = \frac{\lambda w_t}{q_{jt}}, \quad (5)$$

as $\frac{\lambda w_t}{q_{jt}}$ are exactly the competitive fringe's marginal costs of producing product j . Equation (2) then implies that the demand for product j is given by

$$y_{jt} = \frac{Y_t}{p_{ij}} = \frac{q_{jt} Y_t}{\lambda w_t}. \quad (6)$$

Similarly, the allocation of labor demand is simply

$$l_{jt} = \frac{y_{jt}}{q_{jt}} = \frac{Y_t}{\lambda w_t} = \bar{l}_t, \quad (7)$$

⁸We will be using the terms efficiency and productivity interchangeably when referring to q . Our model does not feature any frictions in the input market and mark-ups will be constant. Hence, the usual ambiguity between TFPQ, i.e., efficiency or (physical) productivity, and TFPR, i.e., (revenue) productivity, does not feature in our theory.

⁹See Peters (2011) for a related model that focuses on heterogeneous mark-ups.

i.e., the allocation of labor is equalized *across products*. This of course does not imply that the allocation of labor is also equalized across firms - as some firms will (endogenously) have more products than other firms, the distribution of employment is fully driven by the distribution of products. This tight link between firm-level employment and firms' product portfolio is not only analytically attractive but also conceptually useful in that it clarifies that our model attributes firm dynamics to a single mechanism: why do countries differ in the speed at which firms accumulate (and lose) products along their life-cycle.

The resulting profit (before paying the managers) for the producer of variety j is then simply

$$\tilde{\pi}_{jt} = \left[\frac{\lambda w_t}{q_{jt}} - \frac{w_t}{q_{jt}} \right] \frac{q_{jt} Y_t}{\lambda w_t} = \frac{(\lambda - 1)}{\lambda} Y_t.$$

The fact that profits are equalized across products is of course driven by our Cobb-Douglas structure, which implies that total spending is equalized across products (and hence does not depend on frontier quality q_j)

Substituting (6) into (2), we can solve for the equilibrium wage, as

$$w_t = \frac{1}{\lambda} Q_t.$$

Hence, equilibrium wages are fully determined by aggregate productivity Q_t and do not depend on the allocation of labor across managers and production workers. As workers are equalized across products and there is a unit measure of products, it follows directly that $\bar{l}_t = L_t^P$. Hence, total output in the economy is given by

$$Y_t = \exp \left(\int_0^1 \ln(q_{jt} l_{jt}) dj \right) = Q_t L_t^P. \quad (8)$$

Similarly, the total number of managers is trivially given by $L_t^M = L^M = \phi$, as each product requires a manager to oversee the operation of the firm.¹⁰ Together with the market clearing condition in (7) this implies that $L_t^P = 1 - \phi$, so that total profits per product (including the payments for managers) are

$$\pi_{jt} = \left(\frac{\lambda - 1}{\lambda} - \phi \right) Q_t \equiv \pi Q_t \quad (9)$$

where $\pi = \frac{\lambda - 1}{\lambda} - \phi$ governs firms' rents (per productivity unit Q_t). Expectedly, it is increasing in the mark-up and decreasing in the cost of overhang.

3.3 Innovation and dynamics

As explained above, the implied firm dynamics in our theory are driven by firms losing products to competitors and breaking into new markets by being able to supply existing products at higher quality. To do so, firms need to spend resources, which we refer to as innovation. Following the usual convention in a Schumpeterian model, we assume that innovations arrive stochastically and that firms spend resources to increase the flow rate at which such successful innovations arrive. In the case where an innovating firm is successful, it

¹⁰This follows from our assumption of no economies of scale in managing products. Assuming this linear structure, where managerial resources per product are constant, is analytically very attractive. We will drop this assumption in Section 5, where we will consider some extensions.

1. gains one of the product lines (say j') randomly, and
2. increments the productivity in that particular product line by $\omega \geq \lambda$

$$q_{j'}^{new} = \omega q_{j'}^{old}. \quad (10)$$

Note that, given our assumption, profits are equalized across products (see (9)). Hence, firms are indifferent as to which product line to target, so that the random allocation of products to new producers would be an equilibrium even if we allowed firms to engage in directed innovation (see also Lentz and Mortensen (2008).) Note also that we can allow the step-size of innovation ω and the leader's productivity advantage over the competitive fringe λ to be potentially different.¹¹ This is conceptually useful, because they have very different economic implications. While λ governs firms' innovation incentives through its effect on profits, ω links firms' innovation choices to the realized growth rate (see (21) below). Hence, while ω is a technological parameter of the innovation technology, we think of λ more as an institutional parameter governing the protection of intellectual property rights.¹² To be precise, suppose that ω was equal across countries but λ is close to unity in poor economies as technologies can easily be copied. Then, the equilibrium growth rate will be low in poor countries because there are no innovation incentives even though the actual innovation technology might be very similar to that in the US. We will show below that in our model, firm dynamics are driven *only* by λ and do not depend on ω .

To generate innovations, firms have to spend resources. We assume that if a firm with n products wants to achieve a flow rate of innovation (per product) of x and if the aggregate rate of innovation is given by $\bar{x}$, it needs to spend

$$z_t^n(x) = \eta n Q_t \bar{x} x \quad (11)$$

units of the final good. Here η is a parameter that parameterizes the cost of innovation. The assumption that $z_t(n)$ is proportional to n and Q_t is the usual scaling condition for the model to be consistent with a balanced growth path and Gibrat's law.¹³ Maybe the most unusual part of our parameterization is the explicit recognition of negative externalities in research, i.e., the fact that a higher *aggregate* innovation intensity $\bar{x}$ increases innovation costs for an individual firm. We do this for tractability. As we will show below, these externalities uniquely determine the innovation incentives while keeping the problem of the individual firm linear. This is analytically attractive and has all the economic insights we want to stress.¹⁴

¹¹Of course we need to assume that $\omega \geq \lambda$ as the comparative advantage of the new quality leader is clearly bounded by the productivity increase over the erstwhile producer that was just replaced.

¹²See Acemoglu and Akcigit (2012) for an analysis of IPR protection in a Schumpeterian model.

¹³To see why innovation costs have to be proportional to Q_t , recall that wages and profits are proportional to Q_t . Without such normalization, innovation would become relatively cheap and innovation incentives would not be stationary. Note that such scaling would not be necessary if R&D were to use workers in production as wages are proportional to Q_t . The reason we did not go that route in our basic model is that it would require us to solve for the labor market equilibrium to determine firms' profits (see (8)). For now we opted for the easier approach to simply assume this appropriate normalization. The fact that innovation costs are proportional to n is required for Gibrat's law. If there were (dis)economies of scale in innovation, firms of different sizes would have different innovation incentives and hence growth rates. While there is some debate on the empirical validity of Gibrat's law for small firms, we feel comfortable with this assumption.

¹⁴We also show in the Appendix that a convex cost function of the form $z_t^n(x) = \eta n Q_t x^\xi$ will have very similar results. In particular, the model still has the same analytical solution conditional on the optimal innovation intensity x , which is uniquely determined but more tedious to characterize.

Furthermore, our calibration strategy does not use (11) explicitly but targets the equilibrium innovation intensities directly. We will be more precise about this in section 4.3, where we discuss the identification of the model.¹⁵

3.4 Formal and informal firms

While the model so far is a relatively standard endogenous growth model, we now introduce a novel feature that is important when thinking about firm-dynamics in underdeveloped economies: informal firms. By informal establishments we mean producers that “sail under the radar” of government authorities to save on taxes and social security contributions and do not want to subject themselves to regulation. The informal sector is of immense importance in developing countries. Hsieh and Klenow (2012), for example, report that roughly 80% of workers in India are employed in the informal sector of the economy. Hence, we do not want to speak to this moment in the data but we want to have a framework that can help us to understand how the existence of these firms affects the growth incentives of formal firms. While formal and informal firms differ in a variety of dimensions, we want to focus here purely on the institutional aspects. The first-order reason to *not* register a business formally is clearly one of tax evasion. As noted above, we interpret “taxes” rather broadly here and subsume under the term taxes all formal expenses, including regulatory costs, higher firing costs, other forms of red tape and social security contributions. In the theory, we represent these expenses as a tax-parameter, κ , that formal firms have to pay (and which are redistributed to the household in a lump-sum fashion). The net profit of a formal firm is therefore given by

$$\pi_{jt}^f = (1 - \kappa) \pi_{jt} = (1 - \kappa) \left(\frac{\lambda - 1}{\lambda} - \phi \right) Q_t \equiv \pi^f Q_t,$$

where π_{jt} is given in (9).

But for both formal and informal firms to exist in equilibrium there also have to be costs of not registering a firm. We think of the main disadvantage of being in the informal sector as having to “lay low under the regulator’s radar” to not attract attention. In a recent survey about informal entrepreneurship in China, The Economist, for example, argued that “this form of [informal] business has inherent limits. To the extent that firms operate outside the law, they are vulnerable to shakedowns from local officials and mood-swings in Beijing. Although success brings praise, too much of it can invite envy and scrutiny. Each new list compiled of China’s greatest tycoons is often accompanied by stories about those on earlier lists who later fell foul of the law” (The Economist (2010)). We think of this particular institutional feature as particularly important in our context as it reduces the incentives of informal firms to grow big. Hence, it precisely affects the margin we are interested in, namely, firms’ life-cycle growth. In our theory, we take a reduced-form approach. Specifically, firms are subject to (random) audits by the government. Audits take place at flow rate β , and if an informal firm is caught producing some product j , it loses this particular product. Once a product is lost, the second-most productive firm in the respective product line fills the gap and resumes production.¹⁶

¹⁵In particular, we show explicitly on page 23 that our calibration is insensitive to the particular form of (11).

¹⁶As profits are independent of the level of quality in the particular sector, this assumption is isomorphic to simply assume that confiscated product lines are simply allocated randomly (according to the equilibrium distribution) to formal and informal firms.

3.5 Entry

In our model, the cross-sectional churning of products is driven by quality improvements of existing firms, but we also want to allow new firms to enter the market. This is not only necessary for realism and because we think that cross-country differences in the extent of entry are important, but also methodologically - without having new firms entering the market, there will not be a stationary firm-size distribution.¹⁷ We model entry by the following (somewhat unusual) entry process. There is a fixed amount (measure 1) of entry labor.¹⁸ It is allocated either for formal or informal entry. To generate “curvature” in the entrants’ problem we follow our approach in (11) and assume that there is a crowding out. In particular, if there is aggregate entry at the rate $\bar{E}$ into sector s , the rate of entry for an individual firm is $(\iota \bar{E})^{-1}$ per worker. Letting W_{nt}^f and W_{nt}^i be the values of having n products in the formal and informal sector, respectively (we will solve for these value function below), the entrant’s problem in sector s is

$$\max_{e^s} \left\{ \frac{e^s}{\iota \bar{E}^s} W_{1t}^s - w^e e^s \right\}, \quad (12)$$

where e^s is the number of workers hired to engage in entry and w^e is the equilibrium wage for entry labor. Note also that (12) implies that entrants are always entering with one product. The equilibrium levels of the sector-specific entry intensities are therefore given by

$$\bar{E}^i = \frac{W_{1t}^i}{w^e \iota} \text{ and } \bar{E}^f = \frac{W_{1t}^f}{w^e \iota}.$$

Market clearing for entry labor requires that $1 = \iota (\bar{E}^f + \bar{E}^i)$, so that the sector-specific equilibrium entry rates are

$$\bar{E}^i = \frac{W_{1t}^i}{[W_{1t}^f + W_{1t}^i] \iota} \text{ and } \bar{E}^f = \frac{W_{1t}^f}{[W_{1t}^f + W_{1t}^i] \iota}. \quad (13)$$

Hence, the relative entry rates reflect exactly the relative (endogenous) values of entering the particular sector.

3.6 Equilibrium

Given this environment, we can now turn to the characterization of the equilibrium in this economy. We focus on equilibria that admit a balanced growth path (BGP).

Definition 1 *Consider the economy described above. A BGP equilibrium consists of time paths of aggregate variables $[C_t, Y_t, X_t, L_t^P, L_t^M, w_t, Q_t]_t$, prices and labor allocations $[p_{jt}, l_{jt}]_{j,t}$, entry rates $[E_t^i, E_t^f]$, innovation rates $[x_n^f, x_n^i]_n$, a distribution of firms across products in both sectors $[\{\mu_{nt}^i\}_n, \{\mu_{nt}^f\}]_{nt}$ and a measure of active firms such that $[F_t^i, F_t^f]_t$, such that*

¹⁷Intuitively, firms exit once they lose all their products. If there was no mechanism for new firms to enter (or alternatively for these firms to give it a second try), the economy would be populated by a single firm in the long-run. Formally, having exited the market would be an absorbing state.

¹⁸Having a specific factor for entry is mainly for simplicity as we do not have to solve for the labor market equilibrium in that case. We will reconsider this assumption in Section 5 below.

- *firms maximize profits*
- *households maximize utility*
- *markets clear*
- *the growth rate, the number of firms and the number of entrants are constant*
- *the distributions μ are stationary and consistent with the stochastic process generated by firms' innovation and entry choices*

The main missing object to characterize the equilibrium is the optimal sector-specific innovation rates x_n^f and x_n^i . The choice of how much to innovate is obviously a forward-looking decision. Hence, we have to determine the value function of a firm with n products. Letting W_{nt}^f denote the value function by a formal firm, standard arguments imply that W_{nt}^f solves the HJB equation

$$rW_{nt}^f - \dot{W}_t^f = \max_x \left[n\pi^f Q_t - \eta n Q_t \bar{x}^f x + (x + \nu) n \left[W_{n+1}^f - W_n^f \right] + n\tau \left[W_{n-1}^f - W_n^f \right] \right] \quad (14)$$

where τ is the flow rate at which one of the firm's product is stolen by another competitor ("creative destruction"), x is the innovation rate and ν denotes the flow rate of receiving a product from the audited informal firms. Clearly, both τ and ν are endogenous variables, but they are taken as given by each firm.

Similarly, the value of an informal firm is

$$rW_{nt}^i - \dot{W}_t^i = \max_x \left[n\pi Q_t - \eta n Q_t \bar{x}^i x + (x + \nu) n \left[W_{n+1}^f - W_n^f \right] + n(\tau + \beta) \left[W_{n-1}^f - W_n^f \right] \right]. \quad (15)$$

These equations concisely show the trade-off between being formal and informal: Informal firms earn higher profits per product as $\pi^f = (1 - \kappa)\pi$, but they also face a larger probability of losing products - their effective rate of destruction is $\tau + \beta$.

Lemma 1 *The value functions are given by*

$$\begin{aligned} W_{nt}^f &= \frac{\pi^f}{\rho + \tau - \nu} n Q_t \equiv V^f n Q_t \\ W_{nt}^i &= \frac{\pi}{\rho + \tau + \beta - \nu} n Q_t \equiv V^i n Q_t. \end{aligned} \quad (16)$$

Proof. See the Appendix. ■

Given these closed-form expressions for the value function, the optimal innovation rates are easy to characterize. Consider the formal firms. Substituting (16) into (14) we get that

$$\begin{aligned} x^f &= \arg \max_x \left[n\pi Q_t - \eta n Q_t \bar{x} x + (x + \nu) n V^f Q_t - n(\tau + \beta) V^f Q_t \right] \\ &= \arg \max_x \left[-\eta \bar{x} x + x V^f \right], \end{aligned}$$

which implies that

$$x^f = \bar{x} = \frac{1}{\eta} V^f = \frac{1}{\eta} \frac{\pi^f}{\rho + \tau - \nu}. \quad (17)$$

Similarly, the innovation rate of informal firms is

$$x^i = \bar{x}^i = \frac{1}{\eta} V^i = \frac{1}{\eta} \frac{\pi}{\rho + \tau + \beta - \nu}. \quad (18)$$

Hence, as usual, innovation incentives are increasing in the flow profits (π^f and π) and decreasing in both the discount rate ρ and the rate of creative destruction τ . More novel, innovation incentives are also affected by the regulatory environment, i.e., the flow rate of auditing β . The direct effect affects only informal firms: a more efficient regulatory agency reduces innovation incentives by exposing informal firms to the risk of a shake-up. This formalizes the notion of informal firms choosing to remain small to not attract the attention of regulatory watch-dogs. However, there are also indirect, less-obvious equilibrium effects. Not only does β affect the rate of creative destruction τ , but also the possibility of actually benefitting from government auditing captured by ν . Holding the size of the informal sector fixed, a higher rate of auditing β will increase the rate at which formal and non-audited informal firms remain the producers of their product as the current informal quality leader was forced to leave the market. This *increases* innovation incentives (especially for formal firms that do not suffer from the direct effect of auditing) and captures the intuition that more efficient auditing provides a level playing field, which is essential to provide formal firms with the right incentives to engage in productivity-enhancing activities.

Given these analytic expressions for the respective innovation rates, we can solve for the full set of equilibrium allocations. Note first that (13), (17) and (18) imply that

$$\bar{E}^f = \frac{1}{\iota} \frac{x^f}{x^f + x^i} \text{ and } \bar{E}^i = \frac{1}{\iota} \frac{x^f}{x^f + x^i}, \quad (19)$$

i.e. given $x = (x^f, x^i)$ the entry flow rates are fully determined. The equilibrium rate of creative destruction, i.e. the flow rate at which the frontier of technologies expands, is simply given by

$$\tau = \sum_{s=i,f} (E^s + X^s), \quad (20)$$

where X^s is the aggregate innovation effort generated in sector s and E^s is the flow of entry in sector s .¹⁹ This also implies that the equilibrium growth rate is given by

$$g = \ln(\omega) \tau, \quad (21)$$

where recall that ω denotes the step-size of quality improvement. To characterize the aggregate innovation rates X^s , the composition of the economy between formal and informal firms is important. Hence let α_t^f be the endogenous share of products that are produced by formal

¹⁹Note that (20) implicitly assumes that the technologies of audited informal firms is not lost but that it is used by the replacing firm. If we were to assume that the technologies are lost, the rate of creative destruction would be given by $\bar{\tau} = \tau - \beta$. We prefer the former interpretation for two reasons. Methodologically, we do not want to introduce a mechanical correlation between the rate of growth and our policy parameter β . Economically, we think that firms will find a way to appropriate the surplus of the audited technology - the audited informal firm could either sell the technology to a new firm or simply give it to a cousin, who resumes the operation under a new name.

producers a time t . Along the BGP, this share will be constant, i.e. $\alpha_t^f = \alpha^f$. Given α^f , the aggregate innovation rates are simply

$$X^f = \alpha^f x^f \text{ and } X^i = (1 - \alpha^i) x^i$$

as x^s denotes the innovation rate per product and α^f and $(1 - \alpha^f)$ are the respective measures of products in the respective sectors. Furthermore, the aggregate flow rate of audited products ν is simply given by

$$\nu = (1 - \alpha^f) \beta, \quad (22)$$

as each product is audited with rate β , a share $(1 - \alpha^f)$ of which is informal.

An important equilibrium object that we still need to characterize is the distribution of *firms* across *products*. This is clearly a central object for us as it will tell us how successful firms are in building a large product base. Consider again the formal firms first. For $n \geq 2$, this distribution μ_n^f evolves in an interval Δt according to

$$\mu_{n,t+\Delta t}^f F_{t+\Delta t}^f = \mu_{n,t}^f F_t^f + \left[\mu_{n-1,t}^f F_t^f (x^f + \nu) (n-1) + \mu_{n+1,t}^f F_t^f \tau (n+1) \right] \Delta t - \left[\mu_{n,t}^f F_t^f (x^f + \tau + \nu) n \right] \Delta t. \quad (23)$$

Here, F_t^f denotes the mass of formal firms, $\mu_{n-1,t}^f (x_{n-1}^f + \nu) (n-1)$ and $\mu_{n+1,t}^f \tau (n+1)$ denote the flow into the state n (per firm) and $\mu_{n,t}^f (x_n^f + \tau + \nu) n$ the flow out of the state n . Similarly,

$$\mu_{1,t+\Delta t}^f F_{t+\Delta t}^f = \mu_{1,t}^f F_t^f + \left[E_t^f + 2F_t^f \mu_{2,t}^f \tau \right] - \left[\mu_{1,t}^f F_t^f (x^f + \tau + \nu) \right], \quad (24)$$

where E_t^f is the flow rate of entry of formal firms. For μ_n^f to be stationary (and F_t^f to be constant), we therefore require that

$$\mu_2^f = \frac{\mu_1^f (x^f + \tau + \nu) - \frac{E^f}{F^f}}{2\tau} \quad (25)$$

$$\mu_{n,t}^f = \frac{\mu_{n-1}^f (x^f + \tau + \nu) (n-1) - \mu_{n-2}^f (x^f + \nu) (n-2)}{\tau n} \text{ for } n \geq 3. \quad (26)$$

For the informal firms we get the exact same expression except that the flow rate of destruction is given by $\tau + \beta$ instead of τ . Given (25) and (26), we find the following.

Lemma 2 *The respective stationary distributions and number of firms in each sector are given by*

$$\mu_n^f = \left(\frac{x^f + \nu}{\tau} \right)^n \frac{1}{n} \frac{1}{\ln \left(\frac{\tau}{\tau - x^f - \nu} \right)} \quad (27)$$

$$\mu_n^i = \left(\frac{x^i + \nu}{\tau + \beta} \right)^n \frac{1}{n} \frac{1}{\ln \left(\frac{\tau + \beta}{\tau + \beta - x^i - \nu} \right)}. \quad (28)$$

and

$$F^f = \frac{E^f}{x^f + \nu} \ln \left(\frac{\tau}{\tau - x^f - \nu} \right) \quad (29)$$

$$F^i = \frac{E^i}{x^i + \nu} \ln \left(\frac{\tau + \beta}{\tau + \beta - x^i - \nu} \right). \quad (30)$$

Given the closed form for the distributions (μ_n^i, μ_n^f) , we can now solve for the share of formal products (which does not of course equal the share of formal firms), if formal firms have more products on average. Note first that by definition.

$$\alpha^f = F^f \sum_{n=1}^{\infty} \mu_n^f n \equiv F^f \bar{n}^f, \quad (31)$$

i.e., the total number of formal *products* α^f is equal to the number of firms F^f times the number of products by firm. Substituting for F^f and μ_n^f we get that

$$\alpha^f = \sum_{n=1} F^f \mu_n^f n = E^f \frac{1}{\tau} \sum_{n=1} \left(\frac{x^f + \nu}{\tau} \right)^{n-1} = E^f \frac{1}{\tau - x^f - \nu}. \quad (32)$$

Using (32) (and the analogous expression for $\alpha^i = 1 - \alpha^f$) we can express the rate of creative destruction as

$$\begin{aligned} \tau &= E^f + E^i + \alpha^f x^f + (1 - \alpha^f) x^i \\ &= \frac{\tau - \nu}{\tau - x^f - \nu} E^f + \frac{\tau + \beta - \nu}{\tau + \beta - x^f - \nu} E^i. \end{aligned} \quad (33)$$

To determine the existence and uniqueness of the BGP equilibrium, we therefore only have to establish that there is indeed a unique rate of destruction τ , which is implied by firms' innovation incentives and the resulting firm-size distributions. To see that this is the case, let the *net* flow rate of destruction

$$\hat{\tau} \equiv \tau - \nu$$

be given. The equilibrium innovation rates x^f and x^i are then fully determined from (17) and (18). Similarly, the mass of entrants E^f and E^i is fully determined from (19) and α^f follows from (32). Hence, using (33), we get that

$$\begin{aligned} \hat{\tau} &= \frac{\tau - \nu}{\tau - x^f - \nu} E^f + \frac{\tau + \beta - \nu}{\tau + \beta - x^f - \nu} E^i - \nu \\ &= \frac{\hat{\tau}}{\hat{\tau} - x^f} E^f + \frac{\hat{\tau} + \beta}{\hat{\tau} + \beta - x^f} E^i - \frac{1}{\hat{\tau} + \beta - x^i} E^i \beta, \\ 1 &= . \end{aligned}$$

which implies that the equilibrium $\hat{\tau}$ solves the equation²⁰

$$\iota = \frac{1}{x^f + x^i} \left(\frac{x^f}{\hat{\tau} - x^f} + \frac{x^i}{\hat{\tau} + \beta - x^i} \right).$$

²⁰We still have to formally establish uniqueness. Numerically, we always recover a unique equilibrium.

3.7 Equilibrium firm dynamics

We are mostly interested in the model's implication for the process of firm dynamics, especially how they differ across countries and what the important determinants are. The two aspects we are interested in are:

1. firms' average employment conditional on age
2. the probability of survival conditional on age.

These moments have a convenient closed-form expression, which allows for an easy interpretation.

The key to deriving these moments is to realize that the process of firm dynamics is closely linked to the flow equations governing the evolution of the industry as a whole. Letting $p_n^s(t)$ be the probability that a firm in sector s , which entered the market at time 0 (with one product), has n products at time t , we get that for $n \geq 1$,

$$\dot{p}_n^s(t) = (n-1)p_{n-1}^s(t)\chi^s + (n+1)p_{n+1}^s(t)\tau^s - np_n^s(t)(\chi^s + \tau^s),$$

where χ^s and τ^s are the sector-specific flow rate of gaining and losing a product, i.e.,

$$\hat{\tau}^s = \begin{cases} \tau & \text{if } s = f \\ \tau + \beta & \text{if } s = i \end{cases} \quad (34)$$

$$\chi^s = \begin{cases} x^f + \nu & \text{if } s = f \\ x^i + \nu & \text{if } s = i \end{cases}. \quad (35)$$

Note that this is exactly the same equation as for the aggregate economy. The difference is the remaining state of losing the last product. While for the economy at large, new firms replace old firms and there is no absorbing state, this is not the case for an individual firm. Once a firm loses its last product, the firm irrevocably exits. Formally,

$$\dot{p}_0^s(t) = p_1^s(t)\tau^s. \quad (36)$$

This set of equations fully determines the function $p_n^s(t)$. In particular, let us define the function

$$\vartheta^s(t) = \frac{\chi^s(1 - e^{-(\tau^s - \chi^s)t})}{\tau^s - \chi^s e^{-(\tau^s - \chi^s)t}}.$$

Then, $p_n^s(t)$ can be expressed recursively as

$$\begin{aligned} p_0^s(t) &= \frac{\tau^s(1 - e^{-(\tau^s - \chi^s)t})}{\tau^s - \chi^s e^{-(\tau^s - \chi^s)t}} \\ p_1^s(t) &= (1 - p_0^s(t))(1 - \vartheta^s(t)) \\ p_n^s(t) &= p_{n-1}^s(t)\vartheta^s(t). \end{aligned} \quad (37)$$

Using these equations, our two moments of interest can easily be calculated. Consider first the age-size relationship, which depends on the distribution of products *conditional on survival* (i.e., conditional on having at least one product). Using (37), this density is given by

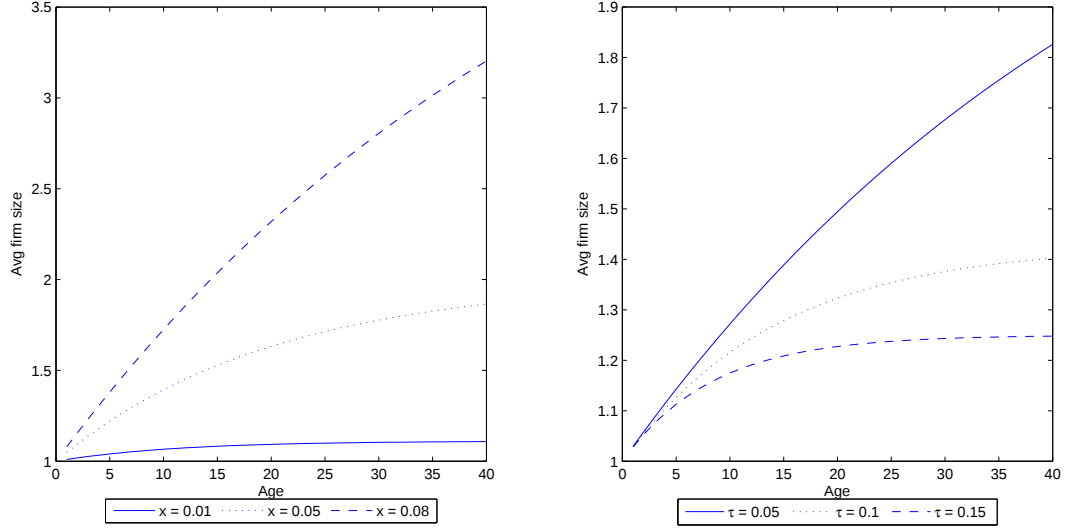


Figure 3: The cross-sectional age-size relationship

$$\frac{p_n^s(t)}{1 - p_0^s(t)} = (1 - \vartheta^s(t)) \vartheta^s(t)^{n-1}.$$

Letting $\bar{n}^s(t)$ denote the average number of products (and hence workers) of a firm in sector s at time t (i.e., at age t), we get that²¹

$$\begin{aligned} \bar{n}^s(t) &= \sum_{n=1}^{\infty} n \frac{p_n^s(t)}{1 - p_0^s(t)} = \sum_{n=1}^{\infty} n (1 - \vartheta^s(t)) \vartheta^s(t)^{n-1} = (1 - \vartheta^s(t)) \sum_{n=1}^{\infty} n \vartheta^s(t)^{n-1} \\ &= \frac{1}{1 - \vartheta^s(t)}. \end{aligned} \quad (38)$$

This moment is depicted in Figure 3 below. We plot $\bar{n}^s(t)$ for different levels of the innovation intensity x (left panel) and the aggregate rate of destruction τ (right panel). The figure clearly shows the two mechanisms through which firms stay small in India but grow in the US. Either, the innovation intensity is higher in the US (right panel) or the rate of creative destruction is higher in India (left panel). Now consider our other moment of interest, the (unconditional) probability of survival. The empirical fact that old firms in the US account for a large share of aggregate activity requires that US firms see more growth as they age and that firms in the US tend to be more likely to survive. In Figure 4 below, we depict the unconditional probability of survival, which is given by

$$q^s(t) = 1 - p_0^s(t) = \frac{(\tau^s - \chi^s) e^{-(\tau^s - \chi^s)t}}{\tau^s - \chi^s e^{-(\tau^s - \chi^s)t}}.$$

²¹Note that $(1 - \vartheta) \sum_{n=1}^{\infty} n \vartheta^{n-1} = 1 + \vartheta + \vartheta^2 + \vartheta^3 = \frac{1}{1 - \vartheta}$, so that $\sum_{n=1}^{\infty} n \vartheta^{n-1} = \left(\frac{1}{1 - \vartheta}\right)^2$.

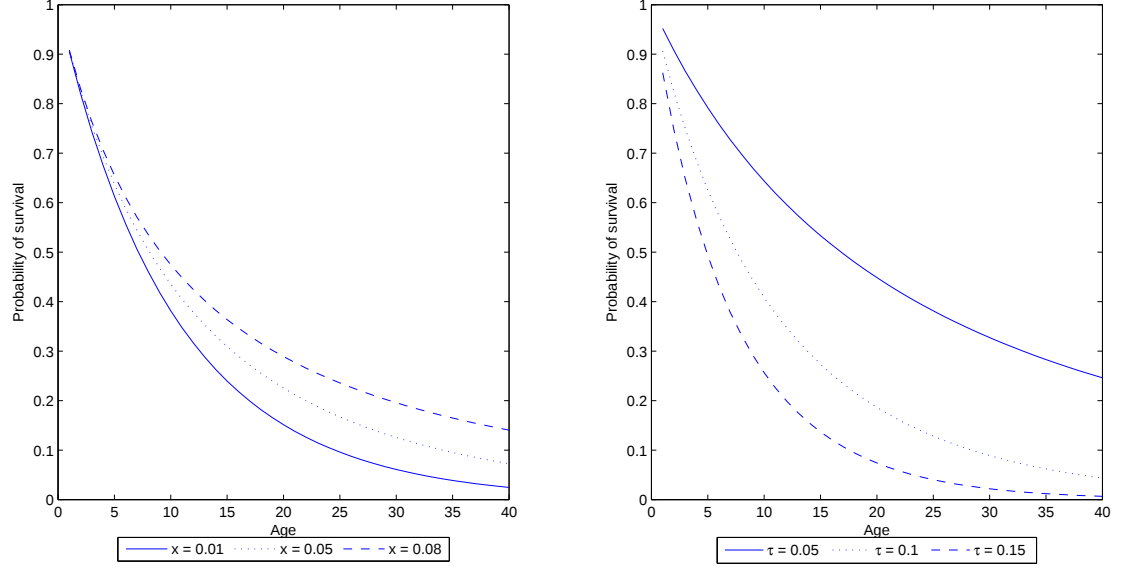


Figure 4: The probability of survival

From the point of view of the individual the same characteristics share the extensive and intensive margin of growth: a higher innovation intensity and a low rate of destruction increases longevity and life-cycle growth. Crucially, however, firms' innovation incentives and the aggregate rate of creative destruction are jointly determined in equilibrium. Hence, the partial equilibrium comparative statics depicted in Figure 3 do not have a direct counterpart in the theory. The question is which variation in underlying structural parameters can generate both relatively low innovation incentives and a high rate of creative destruction in India and whether the theory is even capable of generating these life-cycle differences quantitatively. We turn to this question in Section (4), where we calibrate the model. Before doing so, however, we require a corollary to $\bar{n}^s(t)$ derived above. In the calibration we want to target the average age-size profile in the population of firms, i.e., not conditional on the status of the firm (because we do not see that in the data). Hence, we are interested in

$$\bar{n}(t) = \bar{n}^f(t) \omega(t) + \bar{n}^i(t) (1 - \omega(t)), \quad (39)$$

where $\omega(t)$ denotes the share of formal firms conditional on age t . To derive $\omega(t)$, let there be E^f formal entrants at time 0. The mass of formal firms at time t is then $E^f (1 - p_0^f(t))$ as $p_0^f(t)$ is exactly the probability that a formal firm has zero products at time t . Hence,

$$\omega(t) = \frac{E^f (1 - p_0^f(t))}{E^f (1 - p_0^f(t)) + E^i (1 - p_0^i(t))}. \quad (40)$$

If $p_0^f(t) = p_0^i(t)$, we therefore had $\omega(t) = \frac{E^f}{E^f + E^i} = \omega(0)$. But in general we expect that $p_0^f(t) < p_0^i(t)$ so that we have selected attrition over time - informal firms are more likely to die so that as time progresses, more and more formal firms will populate the economy. Using (38), (39) and (40) we get that

$$\bar{n}(t) = \frac{1}{1 - \vartheta^f(t)} \frac{E^f (1 - p_0^f(t))}{E^f (1 - p_0^f(t)) + E^i (1 - p_0^i(t))} + \frac{1}{1 - \vartheta^i(t)} \frac{E^i (1 - p_0^i(t))}{E^f (1 - p_0^f(t)) + E^i (1 - p_0^i(t))}.$$

This is the expression we will use as a target for the calibration.

4 Calibration

We are now going to take the basic model to the data. Our quantitative aim is twofold. On the positive side, we want to show that the model is able to match the basic facts we set out to explain. On the methodological side, the quantitative exercise will turn out to be very useful in showing where the model still needs to be amended to make sense of the data.

4.1 Sources of variation

For our basic calibration, we posit that the US and India are heterogeneous in three dimensions.²² In terms of the underlying technologies, we mainly think of differences in the managerial costs ϕ . As seen above (see, e.g., (17) and (18)) managerial costs determine firms' infra-marginal rents and hence crucially affect the returns to innovation. As firms' growth along their life-cycle is determined through their innovation incentives, the slow growth of firms in India suggests that managerial costs are relatively high. The other two differences pertain to the market environment. Specifically, we allow for cross-country differences in the amount of wasteful red tape that formal firms face, i.e. the implicit tax rate κ , and in the efficiency of the auditing technology β . The tax rate κ affects only the relative profit flow between formal and informal firms and hence both relative innovation incentives (again see (17) and (18)) and relative entry (see (19)). The intensity of auditing β also affects these endogenous decisions as it effectively shortens the time horizon of informal firms, but it also affects firm dynamics directly. So κ and β are not only independently identified but the effect of the auditing technology β on the implied firm-dynamics is in fact quite nuanced, precisely because of its direct and indirect effects. We will therefore discuss in detail the implied comparative statics with respect to β in the calibrated model below. We also set two structural parameters, namely, the step-size λ and the discount rate ρ exogenously. While we consider setting ρ to be relatively uncontroversial, the reasoning for λ is more subtle. As explained above, λ plays two roles. It not only directly affects the equilibrium growth rate by parameterizing the implied change in productivity of each innovation, but it also affects firms' profit margin, i.e., the mark-up in the economy. With the data at hand, we do not have sufficient information to empirically distinguish between the two. Hence, we decided to fix λ so that it does not render the equilibrium growth rate as a pure nuisance parameter. We think this is helpful in terms of disciplining the model to not allow for an arbitrarily large disconnect between the rate of churning in the microdata (i.e., τ^{CD}) and the rate of aggregate productivity growth.

²²We will formally establish the identification of our parameters in Section 4.3 below.

Moments	Data	Main parameter
US		
Growth contribution of incumbents	0.75	ι
Share of informal workers	0.02	κ_{US}
Average growth rate	0.03	ϕ_{US}
Relative size at age 30	4	β_{US}
India		
Share of informal workers	0.80	κ_{IN}
Average growth rate	0.03	ϕ_{IN}
Relative size at age 30	1.5	β_{IN}

Table 1: Moments from Hsieh-Klenow (2012) [Version April 2012]

Notes: The share of informal workers is taken from Hsieh-Klenow (April 2012, Table 2, p. 26). The relative size of firms at age 30 years is taken from data provided by Chang-Tai Hsieh and Pete Klenow. See also Figure 1.

4.2 Moments

To learn about these moments and to calibrate the remaining parameters, we target the moments reported in Table 1 below. The three main moments that we think are informative about the three sources of variation are the average growth rate, the share of informal workers in the economy and the size-age relationship in the cross-section. As seen, the salient features of the two economies we want to match are the difference in the life-cycle and the importance of informal firms in the respective economies. In the baseline calibration we aim for the same equilibrium growth rate. We think of this more as a disciplining device, as a large share of the faster aggregate growth in India is surely due to the standard neoclassical effect of convergence, i.e., an element that we do not have in our theory.

As seen above, our theory is sufficiently tractable to essentially deliver closed-form expressions for these moments. Consider first the contribution of incumbent firms to aggregate productivity growth. The aggregate growth rate in this economy is simply the gross rate of creative destruction τ^{CD} , which was given in (20) as

$$\tau^{CD} = E^f + E^i + X^i + X^f = E^f + E^i + \alpha^f x^f + (1 - \alpha^f) x^i. \quad (41)$$

Hence, the growth contribution of incumbent firms is simply given by

$$g^I = \frac{X^f + X^i}{\tau^{CD}} = \frac{\alpha^f x^i + (1 - \alpha^f) x^f}{\tau^{CD}}. \quad (42)$$

To solve for the aggregate labor share of informal firms, simply note that - in this model - each product has the same amount of workers. Hence, the share of labor in the informal sector is exactly equal to the share of products produced by informal firms, i.e.,

$$S_L^i = 1 - \alpha^f. \quad (43)$$

The expression for the equilibrium rate of growth implied by the model is standard and given by

$$g = \ln(\lambda) \tau^{CD}, \quad (44)$$

where λ is the step-size of innovation. Finally, consider the age-size relationship in terms of employment. As for (43), this only requires us to determine the distribution of the number of products conditional on age. As shown above (see (38), (39) and (40)), the average number of products at time t is given by

$$\bar{n}(t) = \frac{\omega(t)}{1 - \vartheta^f(t)} + \frac{1 - \omega(t)}{1 - \vartheta^i(t)}, \quad (45)$$

where $\omega(t)$ is the share of formal firms at time t and $\frac{1}{1 - \vartheta^s(t)}$ is the average firm size in sector s . By calibrating the model to the four moments contained in (42)-(45) we can study how much we can learn about the underlying environment from the cross-country microdata on firm dynamics. Which of the structural parameters are identified is the question we turn to now.

4.3 Identification

To analyze the identification of the model, recall that we showed above that the equilibrium is unique. Hence, let the innovation rates x^f and x^i be given. As both the innovation rates (x^f, x^i) and the entry rates (E^f, E^i) are proportional to the value functions (V^s, V^i) (see (17), (18) and (19)), we get that

$$E^f = E^f(\iota, x) \text{ and } E^i = E^i(\iota, x) \quad (46)$$

where $x = (x^f, x^i)$. The share of formal products α^f was given in (32) as

$$\alpha^f = \frac{E^f}{\tau - x^f - (1 - \alpha^f)\beta}.$$

It is easy to show that this equation has a unique solution, which - by using (41) and (46) - is given by

$$\alpha^f = \alpha^f(\iota, x, \beta).$$

The rate of creative destruction is therefore given by

$$\tau^{CD} = \tau^{CD}(\iota, x, \beta).$$

As the whole process of firm dynamics depends only on $(\beta, \alpha^f, \tau, x^i, x^f, E^f, E^i)$, the equations above show that the quadruple (x^i, x^f, β, ι) is a sufficient statistic for the economy as far as dynamic moments are concerned. Hence, from the four moments above (or in fact any other moments we could build from either the firm dynamics (e.g., exit rates) or the cross-section (e.g. the cross-sectional age profile at other ages), we can only identify the two structural parameters (β, ι) and the two endogenous outcomes (x^f, x^i) . Given (x^f, x^i) we can then use the firms' first-order conditions

$$x^f = \frac{1}{\eta} \frac{(1 - \kappa) \left(\frac{\lambda - 1}{\lambda} - \phi \right)}{\rho + \tau - \nu} \quad (47)$$

$$x^i = \frac{1}{\eta} \frac{\frac{\lambda - 1}{\lambda} - \phi}{\rho + \tau - \nu + \beta} \quad (48)$$

Moment	Data	Model
Growth contribution of incumbents g^I (see (42))	75%	75%
Share of informal workers (see (43))	2%	2%
Equilibrium growth rate (see (44))	3%	3%
Relative size at age 30 (see (45))	400%	400%

Parameter/Outcome	Symbol	Value
Efficiency of entrants	ι	35
Flow rate of auditing	β	1.4
Innovation intensity of formal firms	x^f	0.08
Innovation intensity of informal firms	x^i	0.33

Table 2: Calibration of structural parameters: US

to learn about the deep parameters (η, κ, ϕ) .

Clearly, the two conditions in (47) do not identify the three parameters (η, κ, ϕ) . Intuitively, a high innovation intensity in the informal sector x^i can be due to either an efficient innovation technology (low η) or large infra-marginal rents (low ϕ). By just focusing on the firm dynamics we cannot identify these two demand shifters for firms' innovation incentives. To do so we would need additional moments, e.g. firms' total expenditures on innovation resources or the managerial wage bill. Not only do we lack this information from the microdata but it would also require us to be much more specific about the precise mechanism on what we call, for example, managerial resources. Do we think of these as only the required overhead to efficiently produce output given a technology (ϕ)? Or are managerial resources also important to generate growth (η)? Through the lens of an aggregate model like ours, we do not feel that this distinction is particularly enlightening and hence we prefer to treat the innovation intensities (x^f, x^i) as the main proximate object of interest and use the optimality conditions (47) as a way to think about the fundamental determinants. If we were to take a stand on the particular (η, ϕ) -combination that gives rise to the equilibrium x^i , we can then determine κ from (47).

This discussion also shows why our calibration exercise is insensitive to the particular assumptions about the R&D technology embedded in (11). As long as firms innovate with a constant innovation intensity per product (which will be the case as long as the innovation technology has constant returns to scale with respect to innovation inputs x and the number of products n), all our results are unchanged. It is only when we want to identify the fundamental technology parameters that give rise to (x^f, x^i) as an equilibrium outcome that we have to take stand on particular functional forms.

4.4 Calibration results

Given this discussion, we now turn to the quantitative calibration. We first consider the US, where we calibrate the two parameters (β, ι) and the two equilibrium outcomes (x^f, x^i) to match the four moments mentioned above. The results of this exercise are contained in Table 2 below.

In the first panel, we simply show that the model has enough independent variation to match the 4 moments perfectly.²³ In the lower panel, we report the implied calibrated parameters. To see what these parameters imply, note first that the implied rate of creative destruction in the US is given by $\tau_{US}^{CD} = 0.114$. The average instantaneous flow growth rate of formal and informal firms is therefore given by

$$\begin{aligned}\gamma^f &= x^f + (1 - \alpha^f)\beta - \tau = -0.006 \\ \gamma^i &= x^i + (1 - \alpha^f)\beta - (\tau + \beta) = -1.15.\end{aligned}\tag{49}$$

Note that these are the *unconditional* growth rates. Hence, informal firms contract very quickly as the auditing technology in the US is estimated to be very efficient so that informal firms lose products very quickly. A more intuitive way to study the implications of the model is contained in Figure 5 below, which depicts various moments from the implied firm dynamics in the US and which we want to contrast with the results reported in Hsieh and Klenow (2012). Our main interest lies in the age-size relationship in the top left and the employment distribution by age (the bar graphs in the middle right). As in the data, we see a steady rise in average employment across the life-cycle. Hence, while formal incumbents contract slightly on average (see (49)), firms clearly grow *conditional on survival*. This growth experience translates into the age-employment relationship, where old firms account for a third of total employment. This number is of course much smaller than the average size of old firms, because not many firms survive until age 30. This ex-ante probability of survival is depicted in the panel on the top right: the probability of surviving 30 years is roughly 25% for formal firms in the US. This figure also shows the main difference between formal and informal firms in the US. Informal firms are small and short-lived - the probability of surviving until age five is essentially zero for small firms. Through the lens of the model this is precisely due to the efficient audit technology in the US, which detects informal establishments quickly, i.e., before they even have a chance to grow. Looking at the cross-section, this implies that the firm-size distribution of formal firms dominates that of informal firms in a first-order stochastic dominance sense as seen in the panel on the bottom right.

Taking these results as our benchmark for the US, let us now turn to the model's implications for India. As seen in Table 1, we discipline ourselves by requiring the entry technology ι to be the same as in the US. This leaves us with the audit technology β and the two equilibrium outcomes (x^f, x^i) to match the three Indian moments. The results are contained Table 3 below.

First, note that the model is not able to match the data exactly. The reason is that the theory implies stringent relations between these three equilibrium outcomes. We think of these restrictions as being economically quite meaningful and it is for this reason that we tied our hands by neither allowing the step-size λ nor the efficiency of entry resources ι to vary across countries. To see where the discipline comes from, note that the total amount of entry is given by ι^{-1} . Hence, productivity growth due to entry is given by $\frac{\log(\lambda)}{\iota} \approx 0.8\%$. Hence, when targeting an average growth rate of 3% in both economies, we have to have the same rate of incumbents' technological progress in both economies. This, however, is hard to reconcile with

²³We want to note that this is not automatically the case, as the theory imposes restrictions on the different moments we target. When we turn to the calibration for India, it will be seen that we will not be able to match the moments perfectly.

US

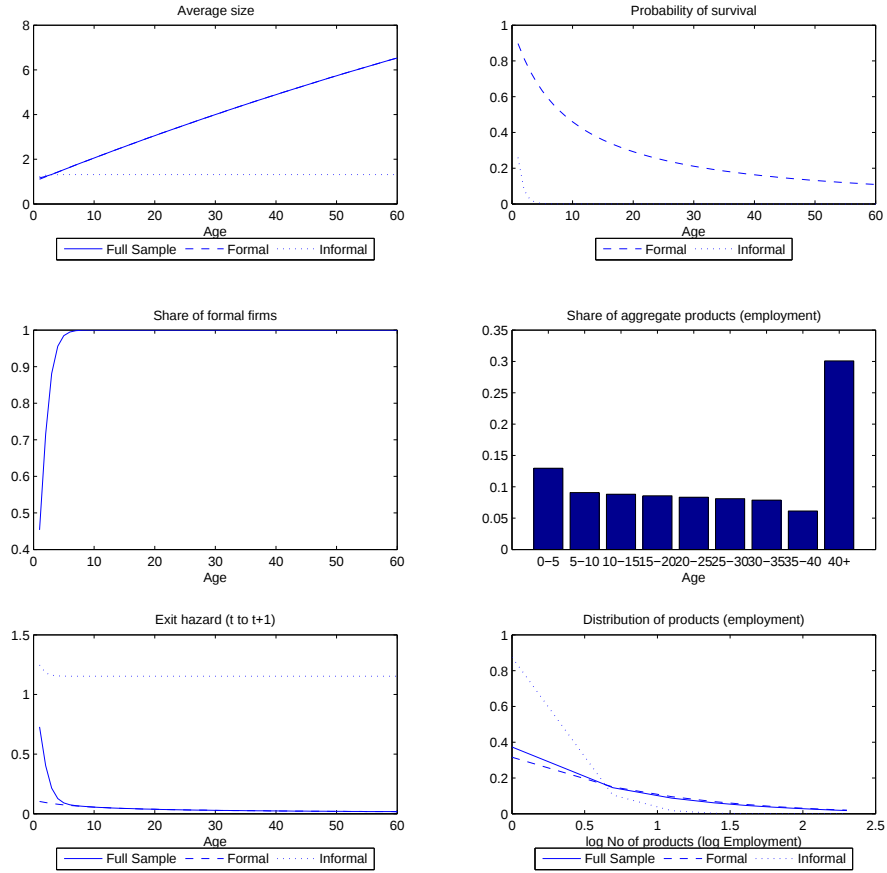


Figure 5: Firm dynamics in the US

Moment	Data	Model
Share of informal workers (see (43))	80%	70%
Equilibrium growth rate (see (44))	3%	1.3%
Relative size at age 30 (see (45))	130%	160%

Parameter/Outcome	Symbol	Value
Flow rate of auditing	β	0.014
Innovation intensity of formal firms	x^f	0.011
Innovation intensity of informal firms	x^i	0.024

Table 3: Calibration of structural parameters: India

INDIA

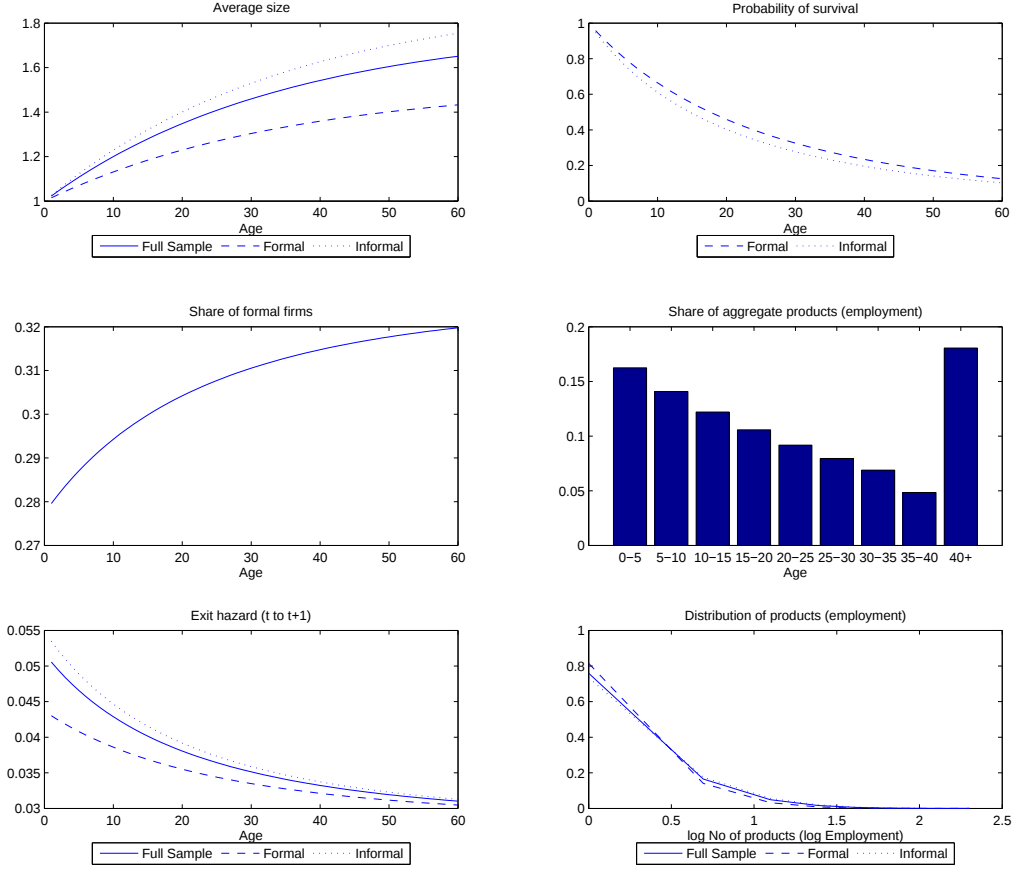


Figure 6: Firm dynamics in India

the shallow size-age profile in India and the high share of informal workers. With an equal weighting matrix, the model delivers a compromise in which the growth is underestimated and the size-age profile is slightly too steep. Hence, the model has quite specific predictions on where we need additional degrees of freedom to match the experience of the Indian economy and we will come back to this in Section 5 below.

Akin to (49) the respective unconditional growth rates for India are given by $\gamma^f = -2.83\%$ and $\gamma^i = -2.85\%$, i.e. formal and informal firms grow (or contract) almost at the same rate. The implied firm dynamics for India are contained in Figure 6 below. In contrast to the US, there is very little growth along the life-cycle - after 50 years, firms have grown by only 50% conditional on survival. This is in sharp contrast with the experience of the US, where surviving firms are 600% bigger when they turn 50 years old. The fact that (conditional on survival) informal firms have a steeper life-cycle profile stems from selection. As seen in the top-right panel, informal firms have a slightly lower survival probability, which generates

the lower average growth rate. The combination of exit rates and growth experience will give rise to the Indian age-employment profile shown in the bar plots. Compared to the US, older firms are much less important in the aggregate. To a large degree this is driven by the intensive margin of growth (conditional on survival) and less so by differential exit rates. The survival probabilities are actually slightly *lower* in the US (which of course reduces the aggregate importance of old firms in the economy). But the much faster life-cycle growth of US plants more than compensates for this effect of more intense churning. Finally, the firm-size distribution, which in our model is simply the distribution of products, differs across countries in that it has a fatter tail in the US.

5 Extensions

In this section, we discuss some extensions that we want to consider. Within the general structure of our modeling approach there are two classes of extensions: those that will affect the current calibration exercise and those that will not (but which will alter the economic interpretation of our results). The reason for this dichotomy is our calibration strategy, which focuses directly on the endogenous innovation intensities x^f and x^i . How these endogenous outcomes will be “implemented as an equilibrium” will of course depend on the precise microstructure, and any counterfactual policy experiments will have to take these into account. But given the data and a model that implies constant innovation intensities, the calibration will be unchanged. Hence, consider first the set of extensions that will affect the calibration of the model.

5.1 Innovative capacity versus monopoly power

As usual in Schumpeterian models of growth, our model makes a clear distinction between the step-size of new innovations and the monopolistic scope producers have. While each innovation increases the productivity of the current producer by ω (see (10)), the flow profits of producing firms are fully determined by the productivity advantage over the competitive fringe, which we parameterized as $\lambda \leq \omega$ (see (5)). While many models equate these two parameters, they are of course conceptually distinct. To see this, consider the equilibrium growth rate, which is given by $g = \ln(\omega)\tau$, where τ does not depend on ω . Hence, the innovation step-size affects equilibrium growth directly, while λ has only indirect effects through firms’ innovation and entry incentives. In our benchmark calibration, we not only assumed that $\omega = \lambda$, but furthermore that λ also does not vary across countries. Allowing these structural parameters to differ by country might be important. In particular, suppose that $\omega_{US} = \omega_{IN} = \lambda_{US} > \lambda_{IN}$. In this case, we would continue to assume that countries are alike in terms of their technological possibilities (ω) but that they might differ in the breadth of their protection of property rights (λ). Through the lens of the model, this would make it easier to match the low innovation incentives in India (which are essential to generate the slow life-cycle growth) while not forcing aggregate productivity growth in India to be even lower than it is in the current calibration. To discipline the calibration of λ , however, we would need measures of firms’ mark-ups (or other moments of profitability), which we do not have at the moment.

5.2 The intensive margin of firm size

As stressed above, our model generates both growth (in the panel dimension) and dispersion in firm size (in the cross-section) only through firms' extensive margin in the number of products. In the model, this is a direct implication of the Cobb-Douglas demand structure that forced sales and employment to be independent of productivity. If we had assumed a more general CES demand function, this would no longer be the case but both sales and employment would be increasing in productivity. Allowing for such more general price effects would probably strengthen the link between firm size and innovative activity. Hence, the more innovative economy would likely have both a more dispersed firm-size distribution and a steeper life-cycle profile. Having to rely on the Cobb-Douglas structure, however, is not just a convenience. If profits were a function of productivity q , firms would direct their innovation expenditures toward specific product lines and we would need to impose additional assumptions to make the model consistent with balanced growth. These could either come in the form of particular innovation cost functions (as in Atkeson and Burstein (2010)) or by simply assuming that innovation is undirected (as in Acemoglu et al. (2012)).

5.3 Opportunity costs, the entry margin and a uni-model firm-size distribution

While the benchmark model is able to match the salient features of firm dynamics, it does not generate enough dispersion in average firm size across countries. Specifically, while in the data, the smallest firms in India are much smaller than the smallest firms in the US, the model predicts a substantial mass of one-product firms in both countries. Moreover, the modal size of products is - by construction - one in both countries. This is also counterfactual. Through the lens of the model, US firms must not have an incentive to enter with a single product, or more generally, the size of the marginal entrant has to be lower in India. This would in fact be a natural implication of the theory, if we were to assume that (a) firms had a choice to enter with multiple products and (b) the opportunity costs of being an entrepreneur were higher in the US. Suppose, for example, that the India has a frictional labor market with involuntary unemployment. The opportunity costs for the marginal entrepreneur in India would then be zero and he would enter with a single product as long as this product is profitable. In contrast, in the US, the opportunity cost of running a firm would be equal to the equilibrium wage. Hence, a US entrepreneur would not want to spend his time entering the market with a single product, but the marginal entrepreneur would be of larger size.²⁴

5.4 Incentives to grow and incentives to enter

One of the main shortcomings of the structure so far is the right link between entry and innovation incentives: in our model both the equilibrium entry and innovation rates are proportional to the (constant) value per product. This implies that the model will not be able to generate a pattern in which the incentives to enter, say, the informal sector are high, but the innovation incentives conditional on being an informal firm are low. We do not have sufficiently rich

²⁴In fact, such friction would not only induce variation in the size-upon-entry but would also alter innovation incentives ex-post through the usual market-size effect: if the size of the labor force was relatively small, equilibrium profits and hence innovation incentives would be relatively low.

data to empirically investigate whether this pattern is actually present in the data, but both anecdotal evidence and some microdata suggest that such asymmetry might be important.²⁵ To generate these effects in the theory we would need a convex cost structure that makes the efficient scale small. There are essentially two ways to generate this using the mechanisms we have introduced so far. One concerns the structure of the management costs, which is discussed below. The other is related to the auditing mechanism. To keep the theory tractable, we have assumed so far that conditional on being audited by the government, informal firms lose a particular product. However, it might be more realistic to assume that the whole firm is closed whenever a particular product has been audited. This would imply a more than proportional increase in the risk of being caught, which would limit the incentives to innovate (i.e., growth) relative to the incentives to enter.

5.5 How to model management?

Our formalization of management so far is at best rudimentary. Currently, we simply introduce management as a fixed cost of production at the product level. We feel that this is unsatisfactory for two reasons. First, the assumption of constant returns in management (at the firm level) is very special and rules out some interesting economics. Suppose, for example, that the returns to scale in management were higher in the US than in India.²⁶ This would make the returns to innovation in the US high and capture the idea that firms in India stay small precisely because managerial technologies represent a bottleneck to growth - firms in India simply run into decreasing returns relatively quickly as they try to break into new product markets. We think that this notion of management is very attractive and in fact corresponds very closely to the one proposed by Edith Penrose in her landmark study “A Theory of Firm Growth” (Penrose, 1959). However, allowing for such decreasing returns makes the model much harder to solve, mostly because the convexity of the management cost function causes the value function to not scale linearly in the number of products n , which would in turn imply a decreasing innovation intensity per product. This is a model we can only solve numerically and we plan to do so in the future. The second reason concerns the substitutability of human capital across different sectors. In our benchmark model, workers can be managers and vice versa. This - by construction - rules out an interpretation in which Indian firms do not grow simply because the supply of appropriate managers is low. If, however, managers are a specific factor in running large firms and managers in poor countries are scarce, the flat life-cycle of Indian firms could simply be a consequence of Indian firms not being able to recruit enough (high-quality) managers to produce at a large scale. In fact, this interpretation seems to be suggested by Bloom et al. (2010). Allowing for a richer substitution pattern between production workers and managers is among our future work.

²⁵We looked, for example, at a large cross-sectional data set about informal firms in Indonesia. The Integrated Survey of Cottage and Small-scale Firms (SUSI) contains data on small (non-directory) and cottage/household firms in all (non-agricultural) sectors and samples roughly 60,000 firms. While the data have only cross-sectional information and hence do not lend themselves to studying firm dynamics, entrepreneurs were asked about their employment growth within the last several years and we see very little life-cycle growth.

²⁶While in our current model the management costs are $c_M = w\phi n$, an extension would be to assume that $c_M = w\phi n^\theta$ with $\theta_{US} < \theta_{IN}$.

6 Conclusion

In this paper our goal was to shed light on the important question of why some countries grow faster than others through a comparative approach of firm dynamics. We performed this comparison through the lens of a micro-founded general equilibrium model of growth, which is both analytically tractable and has rich implications for firm dynamics and hence can easily be disciplined with firm-level data. This feature has been particularly helpful in identifying the structural parameters by calibrating to the firm-level empirical moments of Hsieh and Klenow (2012). In the benchmark model, we allow for two sources of heterogeneity across countries, both of which we consider to be of first-order importance for the context of developing economies. On the one hand, we explicitly allow firms to choose to operate in either the formal or the informal sector. While informality comes with the benefit of not being subject to taxes and regulation, informal firms are subject to government audits and the risk of being shut down. This lowers the marginal return of innovation and informal firms have an incentive to stay small to not trigger attention from government officials. On the other hand, we incorporate the lesson from Bloom and Van Reenen (2010, 2007) that managerial practices differ across countries and incorporate managers as a necessary input to run multi-product establishments. Our analysis so far suggests the following conclusions:

First, the steepness of the life-cycle growth trajectory of US firms (conditional on survival) reflects larger incumbent innovation incentives in the US that are driven by an efficient managerial technology that allows firms to scale up easily. The fact that US firms are better at managing multiple production units simultaneously allows them to expand into new products without running into increasing marginal managerial costs.

Second, US aggregate productivity growth is mostly driven by incumbents' innovation incentives that make successful firms grow fast and unsuccessful firms exit early. Indian firms, by contrast, simply earn too little infra-marginal rents to generate sufficient innovation incentives for steep life-cycle growth. Through the lens of our model, this is due to an inefficient managerial technology. That these low innovation incentives do not necessarily translate into abysmal aggregate productivity growth is due to another margin of growth, namely, entry in the informal sector.

Third, the smaller share of the informal sector in the US is due to an efficient monitoring technology. This raises the relative value of operating as a formal firm despite the non-trivial costs of taxes, social security and other regulatory overhang that are being paid by formal firms.

Finally, being an informal producer in India is valuable (relative to the US), as the inefficient regulatory environment makes it easy to escape government auditing by "sailing under the radar." This causes the average life-cycle growth to be considerably smaller than in the US but does not necessarily lead to large differences in aggregate productivity growth.

Our future work plan includes various extensions to our current analysis. First, we aim to investigate the differential implications of firms' innovative capacity versus their monopoly power, which we so far have modeled as being isomorphic. Second, we considered only the extensive margin of firm growth, i.e., resulting only from firms' expansion into new product lines. However, another important source of growth is the more standard channel of getting better at producing a given good by reducing its cost of production - a mechanism that we plan to include. The third important extension is a variable entry size. In the current model, all entry occurs with a single product. However, the average size at entry differs across countries,

which might be an important source of observed heterogeneity in firm dynamics.

Our analysis has provided an example of how valuable and convenient the standard tools of *endogenous growth theory* are for understanding the data in a *macroeconomic* general equilibrium framework that incorporates the *microeconomic* decisions of firms. We firmly believe that further work in development economics through the lens of endogenous growth theory could bring many new, empirically founded insights about the sources of income and productivity differences across regions, countries and nations.

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7 Appendix

7.1 Proof of Lemma (Equation (16))

The value function W_n solves

$$rW_{nt}^f - \dot{W}_t^f = \max_x \left[n\pi^f Q_t - \eta n Q_t \bar{x}^f x + (x + \nu) n \left[W_{n+1}^f - W_n^f \right] + n\tau \left[W_{n-1}^f - W_n^f \right] \right].$$

Conjecture that $W_{n,t} = \varphi n Q_t$. Then

$$\begin{aligned} rW_{nt}^f - \dot{W}_{n,t}^f &= Q_t n \max_x \left[\pi^f - \eta \bar{x}^f x + (x + \nu) \varphi - \tau \varphi \right] \\ \left(r - \frac{\dot{W}_{n,t}^f}{W_{nt}^f} \right) W_{nt}^f &= Q_t n \left(\pi^f + \nu \varphi - \tau \varphi \right) \\ \rho \varphi Q_t n &= Q_t n \left(\pi^f + \nu \varphi - \tau \varphi \right). \end{aligned}$$

Hence, $\rho \varphi = \pi^f + \nu \varphi - \tau \varphi$ so that

$$\varphi = \frac{\pi^f}{\rho + \tau - \nu}.$$