

Labor Mobility within Currency Unions*

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Comments Welcome
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Abstract

We study the effects of labor mobility within a currency union suffering from nominal rigidities. When the demand shortfall in depressed region is mostly internal, migration may not help regional macroeconomic adjustment. When external demand is also at the root of the problem, migration out of depressed regions may produce a positive spillover for stayers. We consider a planning problem and compare its solution to the equilibrium. We find that the equilibrium is generally constrained inefficient, although the welfare losses may be small if the economy suffers mainly from internal demand imbalances.

1 Introduction

Mundell (1961) is famously cited for his exaltation of labor mobility as a precondition for optimal currency areas. This idea, which quickly settled as a cornerstone of the growing Optimal Currency Area (OCA) literature, seems broadly consistent with the precious few experiences we have to date.¹ The United States enjoys relatively high mobility and

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¹See [Dellas and Tavlas \(2009\)](#) for a review of the OCA literature. Important precedents to [Mundell \(1961\)](#) are [Friedman \(1953\)](#), [Meade \(1957\)](#), and [Scitovsky \(1958\)](#). Mundell emphasized that labor mobility may be imperfect across regions within national borders, so that this OCA condition may not hold even for a single country, thereby weakening Friedman's argument for flexible exchange rates at the national level.

has proven to be a successful currency union. Mobility is arguably much lower within the Eurozone, which sunk into trouble scarcely ten years after its inauguration.²³

Intuitive as Mundell's notion may be, we know of no formal study connecting mobility with macroeconomic adjustment within a currency union. To remedy this, we set up a currency union model featuring nominal rigidities and incorporate labor mobility across the different regions (or countries) that compose the currency union. We use this simple model to tackle two related questions. First, does mobility help stabilize macroeconomic conditions across regions in a union? Second, is equilibrium mobility socially optimal?

Our findings do not fully validate the Mundellian view, but they are consistent with a potential important role for mobility. Workers migrating away from depressed regions naturally benefit from the option to pick up and go somewhere better. The interesting and less obvious question is whether their exodus also helps those that stay behind. That is, whether it aids in the macroeconomic adjustment of regions. A major insight of our analysis is that the answer to this question is subtle because workers leaving a region depart not only with their labor, but also with their purchasing power.

Indeed, we provide a benchmark case where migration has no effect on the per-capita allocations across regions. For this benchmark, the entire demand shortfall in depressed regions is internal, located within the non-tradable sector. When workers migrate out of a depressed region local labor supply is reduced, but so is the demand non-traded goods, which, in turn, lowers the demand for labor. The two effects cancel, leaving the situation for stayers unchanged.

Away from this neutral benchmark, depressed regions might also suffer from external demand shortfalls. When this is the case, migration out of depressed regions may help improve the region's macroeconomic outcome. For example, at the opposite end of the spectrum, suppose regions only produce traded goods and that there is no home bias in the demand for these goods. The demand for each region's product is then determined entirely by external demand at the union level, and internal demand plays no special role. In this case, migration out of a depressed region improves the outcome of stayers by increasing their employment, income and consumption.

Overall, these results highlight that the macroeconomic spillover benefits from mobility are not straightforward. In particular, the degree of economic openness (how much regions trade with one another) turns out to be a key parameter. Openness was proposed by McKinnon (1963) as another precondition for an optimal currency area. Our findings

²For example, according to Bonin et al. (2008) annual interstate mobility in the US was 2-2.5% in 2005 and 2006, while cross-border moves within Europe are around 0.1%.

³The OCA literature has isolated other factors for a union's success, including fiscal and product market integration, which also differ between the US and the Eurozone.

thus reveal an interesting interaction between these two separate notions discussed in the OCA literature.

Turning to the second, normative, question we find that the equilibrium with free mobility is not generally constrained efficient. Typically, there is too little mobility from a social welfare perspective.⁴ This is the case because the macroeconomic outcome of a depressed region weakly improves when some workers migrate away from it. One may think of this effect as a macroeconomic externality not internalized by private agents. A social planner, in contrast, internalizes these benefits, leading her to promote greater mobility.

What parameters affect the size of the inefficiency? Is the equilibrium inefficiency likely to be quantitatively significant? To answer these questions it is useful to note that the equilibrium turns out to be efficient in the benchmark case with internal demand imbalances where migration does not affect per capita regional outcomes. This case features no macroeconomic spillovers or externalities, aligning private and social benefits and costs. This suggests that the welfare losses incurred by the equilibrium, vis a vis the planner's solution, depend on just how far actual economies are from this benchmark. The inefficiencies may be small if economies are relatively closed and intraregional trade is a small fraction of production.

In reality, nominal rigidities may be present in prices, in wages or in both. In the policy arena, concerns over wage rigidity seem to dominate those over price rigidity, while the academic literature is more balanced. Indeed, in standard models the source of rigidity makes little difference to the conclusions. Mainly for expositional reasons, we begin our analysis with price rigidity. This allows us to sidestep some thorny issues that arise with wage rigidity, such as the rationing assumption for employment. We also explicitly consider rigid wages for a benchmark rationing rule that delivers the same results as rigid prices. All of our results go through for this case. In ongoing work we explore other rationing rules for employment that may introduce additional effects.

2 Internal Demand Imbalances

Our first model builds on the traded and non-traded goods setup introduced by [Obstfeld and Rogoff \(1995\)](#) and developed by [Farhi and Werning \(2012\)](#) for the study of fiscal transfers within currency unions. Here we extend these settings by incorporating mobility of workers.

⁴With agent heterogeneity we believe it may be possible to construct examples where mobility is socially pernicious. We abstract from these cases. We don't know how realistic such cases may be.

A finite number of regions indexed by $i \in I$ form a currency union. Our focus is on trade and mobility within the union. Consequently, we abstract from non-members and assume that either the union comprises the entire world or that it is closed to the rest of the world. There is a traded good, a non-traded good and labor. The traded good is supplied inelastically and traded competitively, with its price adjusting to clear the world market. In each region, non-traded goods are produced from labor by monopolistic firms.

The fundamental source of inefficiency that we introduce is nominal rigidities in the form of price or wage stickiness. Shocks induce variations in productivities and preferences across regions, but because of these rigidities, and because the currency union rules out adjustments in exchange rates, the equilibrium allocation may be distorted away from the flexible price outcome. Some regions may end up with prices for their non-traded good that are “too high” hurting the demand for their product and leading to a depressed labor market. Other regions may end up with prices that are “too low”, enjoying high demand and leading to a hot labor market.

For simplicity take the prices of non-traded goods (or wages) as given and consider one-time unanticipated shocks. However, our results generalize to considering the ex-ante decision of firms (or workers) that set prices of non-traded goods (or wages) before the realization of some state of the world s with probability $\pi(s)$, but cannot change them in the ex-post stage when the state of the world s is realized. At this ex-post stage, prices (or wages) are fixed and the analysis is similar to the one we undertake here.

2.1 Preferences, Technology and Markets

Agents. There is a continuum of agents with a finite number of types $j \in J$ each with mass μ^j . Let $\mu^{i,j} \in [0, \mu^j]$ denote the mass of agents of type j who end up living in region i , satisfying the adding up constraint

$$\mu^j = \sum_{i \in I} \mu^{i,j}. \quad (1)$$

An agent of type j living in region i maximizes utility from consumption of traded goods $C_T^{i,j}$, non-traded goods $C_{NT}^{i,j}$ and labor $N^{i,j}$

$$U^{i,j} = \max_{C_T^{i,j}, C_{NT}^{i,j}, N^{i,j}} U^{i,j}(C_T^{i,j}, C_{NT}^{i,j}, N^{i,j}), \quad (2)$$

subject to the budget constraint

$$P_T C_T^{i,j} + P_{NT,i} C_{NT}^{i,j} \leq W_i N^{i,j} + E_T^j + T_i + \sum_{k \in I} \pi^{j,k} \Pi_k, \quad (3)$$

where P_T is the price of the traded good, $P_{NT,i}$ is the price of the non-traded good in region i , W_i is the wage in region i , $\pi^{j,k}$ is the share of profits Π_k from region k that accrue to agents of type j (satisfying $\sum_{j \in J} \mu^j \pi^{j,k} = 1$), and T_i is a type-independent lump sum rebate to agents in region i , and E_T^j is the endowment of traded goods of agents of type j .

Note that we assume that both the agent type $j \in J$ and region $i \in I$ affects utility. This allows us to capture differences in tastes for regional location and heterogenous relocation costs. For example, suppose there are two regions

$$I = \{\text{Spain, Germany}\}.$$

A simple model may then be to assume two agent types corresponding to their previous residence

$$J = \{\text{Spaniard, German}\}.$$

Additionally, we may imagine that within each region agents differ in their degree of mobility, say, because they have a different costs of moving, tastes for living abroad, or family attachments to their original region. We can capture this by expanding the set of agent types

$$J = \{\text{Mobile Spaniard, Immobile Spaniard, Mobile German, Immobile German}\}.$$

Thus, our framework could flexibly accommodate these and other considerations.

Turning to the budget constraints, agents can only work and consume in their region of residence. Their endowments of the traded good E_T^j is inalienable and does not depend on the region in which they locate. Agents are also allowed to own shares of firms from all regions. For our basic equilibrium analysis we shall assume that both taxes and transfers are region specific but do not depend on the agent type. We have in mind a union where regions do not discriminate agents based on past residence.

The agents first order conditions are

$$\frac{U_{C_T}^{i,j}}{P_T} = \frac{U_{C_{NT}}^{i,j}}{P_{NT,i}}, \quad (4)$$

$$-\frac{U_N^{i,j}}{W_i} = \frac{U_{C_{NT}}^{i,j}(s)}{P_{NT,i}}. \quad (5)$$

If agents are free to choose in which region i to live, then we have the additional condition that

$$\mu^{i,j} = 0 \quad \text{if} \quad U^{i,j} < \max_{i' \in I} U^{i',j}. \quad (6)$$

Mobility preferences and costs are implicitly incorporated in the utility functions $U^{i,j}$, as discussed above.

The traded good is traded competitively across regions. Define the average endowment of traded goods by

$$E_T = \frac{\sum_{j \in J} \mu^j E_T^j}{\sum_{j \in J} \mu^j}.$$

Firms. Non-traded goods are produced in each region i by competitive firms that combine a continuum of non-traded varieties indexed by $l \in [0, 1]$ using the constant returns to scale CES technology

$$Y_{NT,i} = \left(\int_0^1 Y_{NT,i,l}^{1-\frac{1}{\varepsilon}} dl \right)^{\frac{1}{1-\frac{1}{\varepsilon}}},$$

with elasticity $\varepsilon > 1$.

Each variety is produced by a monopolist using a linear technology:

$$Y_{NT,i,l} = A_i N_{i,l}.$$

Each monopolist hires labor in a competitive market with wage W_i , but pays $W_i(1 + \tau_{L,i})$ net of a region specific tax on labor $\tau_{L,i}$.

We assume that the prices of intermediate goods are given and fixed.⁵ All intermediate goods are symmetric, with prices $P_{NT,i,l} = P_{NT,i}$. This guarantees a symmetric equilibria

⁵As mentioned at the beginning of this section, we could also think that monopolists set prices $P_{NT,i,l}$ in advance, at an ex ante stage, before the realization of the state of the world s is known. We are studying the ex-post stage, after the shock.

with $Y_{NT,i,l} = Y_{NT,i}$ and $N_{i,l} = N_i$, satisfying

$$Y_{NT,i} = A_i N_i. \quad (7)$$

Aggregate profits from intermediate goods in region i are

$$\Pi_i = (1 - \tau_{\pi,i}) \left(P_{NT,i} - \frac{1 + \tau_{L,i}}{A_i} W_i \right) Y_{NT,i}, \quad (8)$$

where $\tau_{\pi,i}$ is the profit tax. We assume that intermediate firms hire labor to meet demand at the fixed price $P_{NT,i}$. They will have an incentive to do so as long as their profit margin is positive $P_{NT,i} - \frac{1 + \tau_{L,i}}{A_i} W_i > 0$. We assume throughout that this is the case.

Government. The government budget constraint in each region is

$$\sum_{j \in J} \mu^{i,j} T_i = \tau_{L,i} W_i N_i + \tau_{\pi,i} \left(P_{NT,i} - \frac{1 + \tau_{L,i}}{A_i} W_i \right) Y_{NT,i}. \quad (9)$$

In principle, we could have considered a a single union-wide budget constraint (equal to the sum across regions of the regional budget constraints) that allowed for transfers across regions, as in a fiscal union. Instead we choose to work with tax and transfer rules that ensure budget balance at the regional level.

Equilibrium definition. Given prices $P_{NT,i}$, endowments E_T^j , profit shares $\pi^{j,i}$, and taxes $\tau_{L,i}$ and $\tau_{\pi,i}$, an *equilibrium without free mobility* is a set of masses $\mu^{i,j}$ satisfying the adding up constraint (1), a price for traded goods P_T , a set of wages W_i , consumptions and labor supplies $C_T^{i,j}$, $C_{NT}^{i,j}$ and $N^{i,j}$, outputs $Y_{NT,i}$ and labor demands N_i , profits Π_i , and taxes T_i , such that consumers maximize, firms meet demand, hire labor, pay taxes and distribute profits, the government's budget constraint holds and markets clear. Formally, we impose the budget constraint (3) with equality, the first-order conditions (4)–(5), equations (7)–(9) and the market clearing conditions

$$\sum_{i \in I, j \in J} \mu^{i,j} C_T^{i,j} = \sum_{j \in J} \mu^j E_T, \quad (10)$$

$$\sum_{j \in J} \mu^{i,j} C_{NT}^{i,j} = Y_{NT,i}, \quad (11)$$

$$\sum_{j \in J} \mu^{i,j} N^{i,j} = N_i, \quad (12)$$

for all regions $i \in I$. An *equilibrium with free mobility* requires in addition that condition (6) hold for all agent types $j \in J$ and regions $i \in I$.

2.2 Additional Assumptions on Preferences, Taxes, and Endowments

To make the problem tractable, we make the following additional assumptions on preferences, taxes, and profit shares.

First, we assume that profits are full taxed $\tau_{\pi,i} = 1$, so that $\Pi_i = 0$, and transferred to all residents equally

$$T_i = \frac{P_{NT,i}Y_{NT,i} - W_iN_i}{\mu_i}, \quad (13)$$

where μ_i denotes the mass of agents living in region i

$$\mu_i = \sum_{j \in J} \mu^{i,j}. \quad (14)$$

We also assume that the endowment of traded goods is independent of the agent type j so that $E_T^j = E_T$. These assumptions ensure that all agents living in region i have the same non-labor income $P_T E_T + T_i$. This is convenient, as we do not then have to keep track of the wealth distribution.

Second, we assume that in any region i , the utility functions $U^{i,j}$ represent the same preference ordering for all agent types j . In other words, individual utility functions $U^{i,j}$ are monotone transformations of some underlying regional utility function $\tilde{U}^i(C_T, C_{NT}, N)$. This simplifying assumption helps with aggregation, since we do not need to keep track of preference heterogeneity within regions. Note that we still allow preferences to depend on location.

Additionally, we assume that these preferences are separable between consumption and labor, and homothetic in consumption goods. Specifically,

$$\tilde{U}^i(C_T^{i,j}, C_{NT}^{i,j}, N^{i,j}) = \hat{U}^i(\tilde{u}^i(C_T^{i,j}, C_{NT}^{i,j}), N^{i,j}),$$

where the sub-utility function $\tilde{u}^i$ is assumed to be homogeneous of degree one, increasing, concave, strictly quasi-concave and twice continuously differentiable. Homotheticity of preferences over consumption goods is convenient because it implies that, given the relative price $p_i = \frac{P_T}{P_{NT,i}}$ of traded goods in region i , agents of type j in region i choose to consume traded and non traded goods in fixed proportions

$$C_{NT}^{i,j} = \alpha^i(p_i)C_T^{i,j},$$

for some function $\alpha^i(\cdot)$ that is increasing and differentiable. This conveniently encapsulates the restriction implied by the first order condition (4). This condition is crucial because the stickiness of non-traded prices, together with the lack of monetary independence, places restrictions on the possible variability across regions i in the relative price p_i .

These assumptions on preferences also imply that all agent types j within a region i choose to work the same amount $N^{i,j} = N_i/\mu_i$. Total (non-labor and labor) income is then $P_T E_T + P_{NT,i} Y_{NT,i}/\mu_i$, which is independent of agent type j . As a result, the allocation over consumption and labor that any agent enjoys if living in region i is independent of the agent's type j . This greatly facilitates the analysis.

3 Equilibrium and Optimum with Domestic Demand Imbalances

We now study the previous model from a positive and normative perspective. We first describe a positive property of the equilibrium. We then define and characterize the problem of a planner that can control mobility decisions.

3.1 Equilibrium

Implementability conditions. With the additional assumptions on preferences, taxes, profit shares and endowments stated in Section 2.2, equilibria have a simple structure. Consumption of traded goods, non-traded goods, and labor of agents of type j in region i are given by

$$C_T^{i,j} = E_T, \quad (15)$$

$$C_{NT}^{i,j} = \alpha^i(p_i) E_T, \quad (16)$$

$$N^{i,j} = \alpha^i(p_i) \frac{E_T}{A_i}. \quad (17)$$

Total output and labor in region i are then simply determined by

$$Y_{NT,i} = \mu_i \alpha^i(p_i) E_T,$$

$$N_i = \mu_i \alpha^i(p_i) \frac{E_T}{A_i}.$$

We can then determine wages W_i and transfers T_i from equations (5) and (13).

Proposition 1 (Implementability). *Given a price P_T for traded goods, there exists a unique equilibrium with free mobility (up to the indifference of agents in their location decisions). Given in addition regional population sizes μ_i and masses of agents $\mu^{i,j}$ that satisfy (14), there exists a unique equilibrium without free mobility.*⁶

A useful measure of demand imbalance at the regional level, is the labor wedge

$$\tau_i = 1 + \frac{1}{A_i} \frac{U_N^{i,j}}{U_{CNT}^{i,j}},$$

where the right hand side is independent of the agent type j . The labor wedge is zero at a first-best allocation. A positive labor wedge $\tau_i > 0$ indicates that region i is in a bust. Conversely, a negative labor wedge $\tau_i < 0$ indicates that region i is in a boom.

Impact of movers on stayers. We now state a crucial property of the equilibrium that addresses the question that we raised in the Introduction regarding the impact of movers on stayers.

Proposition 2 (Independence of per-capita allocations on location decisions). *Given a price P_T for traded goods, the allocation of agents of type j in region i is identical and given by equations (15)-(17) in all equilibria with or without free-mobility.*

Given a price for traded goods P_T , the allocation of agents of type j in region i is totally independent of the distribution of agents across regions. When some agents moves out of a region i , reducing μ_i , the aggregate demand for non-traded goods in region i is reduced, which reduces the demand for labor in region i . The move also reduces the supply of labor in region i . The net impact of these two effects on stayers is null—their allocation remains unchanged. Of course movers achieve a different allocations. But their move has absolutely no impact on stayers. For example, the labor wedge τ_i remains unaffected.

We will explore the normative consequences of Proposition 2 on the social efficiency of mobility decisions in Section 3.2. For now, consider the equilibrium with free mobility. Consider a depressed region i . Workers migrating away from this region naturally benefit

⁶Under the interpretation that the shock is one-time unanticipated, we can simply take prices as given and we do not have to concern ourselves with the dependence of the ex-ante price setting stage on the ex-post equilibrium. If instead we think that the shock and a nonzero ex-ante probability and that monopolists set prices in advance, at an ex-ante stage, before the realization of the state of the world s , then no matter what equilibrium allocation $C_T^{i,j}$, $C_{NT}^{i,j}$, $N^{i,j}$, $Y_{NT,i}$, N_i and wages W_i arises in each state of the world s in the ex-post stage (the dependence of these variables on s is suppressed in our notation), we can always adjust the *non state contingent* tax $\tau_{L,i}$ and the taxes T_i , so that any desired price $P_{NT,i,l} = P_{NT,i}$ independent of the state of the world s is indeed chosen by firms at the ex-ante stage.

from the option to move to some region i' with higher economic activity. Overall, the level of economic activity in the union increases. In this sense, mobility helps macroeconomic adjustment in a currency union, which can be seen as a vindication of the view associated with [Mundell \(1961\)](#). However, Proposition 2 introduces an important qualification. Indeed, migration out of region i and into region i' does not improve the lot of stayers in the region i of origin (nor does it hurt the lot of agents in the region i' of destination).

3.2 Social Optimum

We consider a planning problem that allows us to characterize constrained Pareto efficient allocations. The planning problem is indexed by a set of nonnegative Pareto weights λ^j . By varying these Pareto weights, we can trace out the entire constrained Pareto frontier. The planning problem seeks to maximize a weighted average of the agents' utilities over the set of equilibria *without* free mobility. In other words, we are assuming extreme powers of relocation for the planner.⁷ Our main result is that constrained efficient allocations are consistent with free mobility.

In order to study the planning problem, it is useful to first define the indirect utility function

$$V^{i,j}(C_T^{i,j}, p_i) = U^{i,j} \left(C_T^{i,j}, \alpha^i(p_i) C_T^{i,j}, \frac{\alpha^i(p)}{A_i} C_T \right).$$

The price of traded goods P_T can be controlled with union-wide monetary policy, which consists in our model of setting the numeraire. Because our results on mobility do not depend on whether monetary policy is chosen optimally, we consider two planning problems, a restricted planning problem and a full planning problem. The restricted planning problem takes monetary policy (the price of traded goods P_T) as given and seeks to optimally allocate agents over regions⁸

$$W(P_T) = \max_{\mu^{i,j}} \sum_{i \in I, j \in J} \lambda^j \mu^{i,j} V^{i,j} \left(E_T, \frac{P_T}{P_{NT,i}} \right), \quad (18)$$

⁷ Alternative implementations of the same allocations could be achieved by introducing additional agent and location specific lump sum taxes

⁸ Under the interpretation that the shock has a nonzero ex-ante probability and that monopolists set prices in advance, at an ex-ante stage, before the realization of the state of the world s , then we could also study an ex-ante problem that maximizes expected welfare across states s . The only difference is that the *non state contingent* prices $P_{NT,i}$ would be choice variables, and there would be corresponding additional optimality conditions. Because the prices $P_{NT,i}$ are not state contingent, it would typically not be possible to achieve the first best in every state s . Given these prices $P_{NT,i}$, the analysis would be unchanged.

s.t. for all $j \in J$,

$$\sum_{i \in I} \mu^{i,j} = \mu^j. \quad (19)$$

The full planning problem allows for flexible monetary policy and characterizes jointly the optimal allocation of agents across regions and optimal monetary policy. The only difference with the restricted planning problem (18) is that P_T is a choice variable instead of a parameter, so that the maximization takes place over P_T and $\mu^{i,j}$. This planning problem can be solved recursively, solving first for the optimal allocation of agents across regions for a given P_T as characterized by the restricted planning problem (18), and then maximizing over P_T

$$\max_{P_T} W(P_T). \quad (20)$$

We call the solutions of the restricted planning problem constrained efficient given monetary policy, and the solutions of the full planning problem constrained efficient.

3.3 Optimal Mobility Given Monetary Policy

In this section, we characterize constrained efficient allocations given monetary policy—the solutions of the restricted planning problem (18). The first order condition for $\mu^{i,j}$ yields

$$\lambda^j U^{i,j} - \nu^j = \rho^{i,j}, \quad (21)$$

where ν^j is the multiplier on the constraint (19), $\rho^{i,j} = 0$ if $\mu^{i,j} \in (0,1)$, $\rho^{i,j} \geq 0$ if $\mu^{i,j} = 1$ and $\rho^{i,j} \leq 0$ if $\mu^{i,j} = 0$, and where with a slight abuse of notation, we replace $V^{i,j}$ by $U^{i,j}$. The first order conditions (21) can be rewritten as the condition that for all $i \in I$ and $j \in J$,

$$\mu^{i,j} = 0 \quad \text{if} \quad U^{i,j} < \max_{i' \in I} U^{i',j},$$

and coincides with the first order condition for free mobility (6). It is therefore not necessary to intervene in mobility decisions.

Proposition 3 (Optimal Mobility). *Constrained efficient allocation given monetary policy P_T are consistent with free mobility.*

The intuition for Proposition 3 is straightforward. Since there are no external effects of agents' location decisions on other agents, free mobility is optimal.

3.4 Optimal Monetary Policy

In this section, we characterize constrained efficient allocations—the solution of the full planning problem (20). The derivative of the indirect utility function with respect to the relative price of non-traded goods can be computed to be

$$V_p^{i,j}(C_T^{i,j}, p_i) = \frac{\alpha_p^i}{p^i} C_T^{i,j} U_{C_T}^{i,j} \tau_i.$$

Using this observation, the first order condition for P_T can be written as

$$\sum_{i \in I, j \in J} \lambda^j \mu^{i,j} \alpha_p^i E_T U_{C_T}^{i,j} \tau_i = 0. \quad (22)$$

Proposition 4 (Optimal Monetary Policy). *Constrained efficient allocations are such that a weighted average of the labor wedge τ_i across regions i is equal to zero, where the weight on τ_i is given by $\sum_{j \in J} \lambda^j \mu^{i,j} \alpha_p^i E_T U_{C_T}^{i,j}$ as in condition (22).*

Proposition 4 establishes that optimal union-wide monetary policy targets a weighted average across regions for the labor wedge τ_i . Because of our assumption of nominal rigidities, all the labor wedges τ_i cannot in general be set to zero so that perfect stabilization is generally impossible. However, at the union level the economy is stabilized in the sense that the weighted average for the labor wedge across regions is set to zero. This implies that optimal union-wide monetary policy ensures that some regions are in a boom and some in a bust.

The optimal allocation of agents across regions is still characterized by Proposition 3. Of course P_T influences the value of the endogenous variables $U^{i,j}$, but not the conditions for optimal mobility (??), which given these variables, coincide with the conditions for free mobility (6).

3.5 Sticky Wages

Our arguments rely on nominal rigidities. We have chosen to expose the main result with sticky prices and flexible wages, but this is inessential. Indeed, consider the same model but assume now that prices are flexible, but that wages are sticky.

We take the set of wages W_i as given. We consider a simple symmetric allocation rule which specifies that labor N_i in region i is distributed equally among all the agents living

in region i . Optimal price setting dictates that

$$P_{NT,l,i} = P_{NT,i} = \frac{\epsilon}{\epsilon - 1} (1 - \tau_{L,i}) \frac{W_i}{A_i}. \quad (23)$$

Given this set of prices $P_{NT,i}$, the analysis is then exactly the same as in the model with sticky prices and flexible wages.

Proposition 5. *Consider the model with sticky wages set at W_i and consider the associated equilibrium and optimal allocations. These are identical to those of a model with sticky prices set by (23). In particular, Propositions 2, 3, and 4 apply to the model just outlined with flexible prices and sticky wages.*

4 External Demand Imbalances

As in Section 2, there is a finite number of regions $i \in I$ forming a currency union. Each region produces a single differentiated final good. We index goods by their region of origin i . Although production is specialized, consumption is not. All goods are consumed throughout the union, with arbitrary differences in spending patterns across goods from different regions depending on the region of residence.

As in Section 2, the key friction is nominal rigidities. Shocks induce variations in productivities and preferences. However, due to nominal rigidities, and because regions are in a currency union, prices (or wages) cannot vary with the shocks, distorting the allocation away from the flexible price outcome. Some regions then end up with a boom and a hot labor market, others with a bust and a depressed labor market.

For simplicity, we proceed as in Section 2. We take prices (or wages) as given, and focus on a one-time unanticipated shock. However, our results generalize to a setting where firms (or workers) set prices (or wages) in an ex-ante stage before the realization of some state of the world s with probability $\pi(s)$, but cannot change them in the ex-post stage when the state of the world s is realized. At this ex-post stage, prices (or wages) are fixed. We would then focus on a particular state s and suppress from our notation any explicit dependence on the state.

4.1 Preferences, Technology and Markets

Agents. There is a continuum of agents of a finite number of different types $j \in J$. The mass of agents of type j is μ^j . We denote by $\mu^{i,j} \in [0, \mu^j]$ the mass of agents of type j who

live in region i , satisfying

$$\mu^j = \sum_{i \in I} \mu^{i,j}. \quad (24)$$

An agent of type j living in region i maximizes utility from consumption $\{C_k^{i,j}\}_{k \in I}$ of the final goods and labor $N^{i,j}$

$$U^{i,j} = \max_{C_k^{i,j}, N^{i,j}} U^{i,j}(\{C_k^{i,j}\}, N^{i,j}), \quad (25)$$

subject to a budget constraint

$$\sum_{k \in I} P_k C_k^{i,j} + \leq W_i N^{i,j} + T_i + \sum_{k \in I} \pi^{j,k} \Pi_k, \quad (26)$$

where P_k is the price of final good k , W_i is the wage in region i , $\pi^{j,k}$ is the share of profits Π_k from region k of agents of type j , and T_i is a type-independent lump sum transfer to agents in region i .

The agents first order conditions are that for all k ,

$$\frac{U_{C_k}^{i,j}}{U_N^{i,j}} = -\frac{P_k}{W_i}. \quad (27)$$

If agents are free to choose the region they live in, then we have the additional condition

$$\mu^{i,j} = 0 \quad \text{if} \quad U^{i,j} < \max_{i' \in I} U^{i',j}. \quad (28)$$

Mobility costs are implicitly incorporated in the utility functions $U^{i,j}$.

Firms. Final goods are produced in each region i by competitive firms that combine a continuum of non-traded varieties indexed by $l \in [0, 1]$ using the constant returns to scale CES technology

$$Y_i = \left(\int_0^1 Y_{i,l}^{1-\frac{1}{\varepsilon}} dl \right)^{\frac{1}{1-\frac{1}{\varepsilon}}},$$

with elasticity $\varepsilon > 1$.

Each variety is produced by a monopolist with linear technology

$$Y_{i,l} = A_i N_{i,l}.$$

Each monopolist hires labor in a competitive market with wage W_i , but pays $W_i(1 + \tau_{L,i})$

net of a region specific tax on labor $\tau_{L,i}$. The prices $P_{i,l} = P_i$ set by monopolists are sticky, and are taken as given. All intermediaries in a given region then hire the same amount of labor N_i implying

$$Y_i = A_i N_i. \quad (29)$$

The common price of intermediate goods P_i is then also the price of the final good produced in region i .⁹

Aggregate profits from intermediate good production in region i equals

$$\Pi_i = (1 - \tau_{\pi,i}) \left(P_i - \frac{1 + \tau_{L,i}}{A_i} W_i \right) Y_i, \quad (30)$$

where $\tau_{\pi,i}$ is the profit tax. Profit shares must satisfy for every region i

$$\sum_{j \in J} \mu^j \pi^{j,i} = 1. \quad (31)$$

Government. The government budget constraint in each region imposes that each region balances its budget

$$\sum_{j \in J} \mu^{i,j} T_i = \tau_{L,i} W_i N_i + \tau_{\pi,i} \left(P_i - \frac{1 + \tau_{L,i}}{A_i} W_i \right) Y_i. \quad (32)$$

Equilibrium definition. Given prices P_i , profit shares $\pi^{j,i}$ and taxes $\tau_{L,i}$ and $\tau_{\pi,i}$, an *equilibrium without free mobility* is a set of masses $\mu^{i,j}$, wages W_i , consumptions and labor supplies $C_k^{i,j}$ and $N^{i,j}$, outputs Y_i and labor demands N_i , profits Π_i , and taxes T_i , such that the conditions (24)-(27) and (29)-(32) are verified, and markets clear so that for all $k \in I$,

$$\sum_{i \in I, j \in J} \mu^{i,j} C_k^{i,j} = Y_k, \quad (33)$$

$$\sum_{j \in J} \mu^{k,j} N^{k,j} = N_k. \quad (34)$$

An *equilibrium with free mobility* requires in addition that condition (28) hold.

⁹If the shock is one-time unanticipated, this poses no difficulty. As mentioned at the beginning of this section, we could also think that monopolists set prices in advance, at an ex ante stage, before the realization of the state of the world s . Prices $P_{i,l}$ of intermediate goods must be set before the realization of s is known.

4.2 Additional Assumptions on Preferences and Taxes

Just as for the previous model, we make some additional assumptions here to make the problem tractable.

First, we assume that profits are fully taxed $\tau_{\pi,i} = 1$ and rebated to local agents

$$T_i = \frac{P_i Y_i - W_i N_i}{\mu_i}, \quad (35)$$

where μ_i the mass of agents living in region i ,

$$\mu_i = \sum_{j \in J} \mu^{i,j}. \quad (36)$$

These assumptions ensure that all agents living in region i have the same income, given by the value $\frac{P_i Y_i}{\mu_i}$ of the final goods produced in region i . Moreover, they ensure that each region i runs a balanced budget since

$$\mu_i T_i = \tau_{L,i} W_i N_i + \tau_{\pi,i} \left(P_i - \frac{1 + \tau_{L,i} W_i}{A_i} \right) Y_i.$$

Second, we assume that in any region i , the utility functions $U^{i,j}$ represent the same preference ordering for all agent types j . Moreover, we assume that these preferences are separable between consumption and labor, and homothetic in consumption. We denote by $\alpha_k^i \in (0, 1)$ the spending share on good k , with $\sum_{k \in I} \alpha_k^i = 1$. This choice of preference is flexible enough to allow for any arbitrary degree of home bias in consumption. We denote by P^i the corresponding price index, and by $C^{i,j}$ the consumption index. With some abuse of notation, we can write the utility of an agent of type j in region i as

$$U^{i,j}(C^{i,j}, N^{i,j}).$$

5 Equilibrium and Optimum with External Demand Imbalances

We now turn to characterizing the equilibrium and contrast it to the problem of a planner that can control mobility directly.

5.1 Equilibrium

Implementability conditions. With the additional assumption on preferences, taxes and profit shares stated in Section 4.2, equilibria take a simple form. Aggregate income in region i is given by $P_i Y_i$. Total demand for good k from region i is then

$$\alpha_k^i \frac{P_i}{P_k} Y_i$$

Adding up across regions gives total demand and hence income for country k

$$\sum_{i \in I} \alpha_k^i P_i Y_i = P_k Y_k. \quad (37)$$

Proposition 6 (Implementability). *In any equilibrium with or without free mobility, regional production Y_i must satisfy the implementability condition (37). Conversely, given any regional productions Y_i satisfying the implementability condition (37), regional population sizes μ_i and masses of agents $\mu^{i,j}$ satisfying (36), there exists a unique equilibrium without free mobility with regional production Y_i .¹⁰*

We have already proved the first part of the proposition. For the second part of the proposition, consider $Y_i > 0$ that satisfy the implementability condition (37). The unique equilibrium such that the aggregate productions are given by Y_i can be constructed as follows. We have

$$C_k^{i,j} = \frac{1}{\mu_i} \alpha_k^i \frac{P_i}{P_k} Y_i, \quad (38)$$

$$N^{i,j} = \frac{1}{\mu_i} \alpha_k^i \frac{P_i}{P_k} Y_i. \quad (39)$$

We can also compute the consumption index

$$C^{i,j} = \frac{1}{\mu_i} \frac{P_i}{P^i} Y_i.$$

We then compute N_i , wages W_i , profits Π_i , and taxes T_i from equations (34), (27), (30), and (35).

¹⁰Under the interpretation that the shock is one-time unanticipated, we can simply take prices as given and we do not have to concern ourselves with the dependence of the ex-ante price setting stage on the ex-post equilibrium. If instead we think that the shock and a nonzero ex-ante probability and that monopolists set prices in advance, at an ex-ante stage, before the realization of the state of the world s , then no matter what equilibrium allocation $C_k^{i,j}$, $N^{i,j}$, Y_i , N_i , and wages W_i arises in each state of the world s in the ex-post stage (the dependence of these variables on s is suppressed in our notation), we can always adjust the *non state contingent* tax $\tau_{L,i}$ and the taxes T_i , so that any desired price $P_{i,l} = P_i$ independent of the state of the world s is indeed chosen by firms at the ex-ante stage.

We can also compute the labor wedge τ_i , which represents a useful measure of demand imbalance at the regional level

$$\tau_i = 1 + \frac{U_N^{i,j}}{U_C^{i,j}} \frac{P^i}{P_i A^i},$$

where the right hand side is independent of the agent type j . The labor wedge is zero at a first-best allocation. A positive labor wedge $\tau_i > 0$ indicates that region i is in a bust. Conversely, a negative labor wedge $\tau_i < 0$ indicates that region i is in a boom.

Demand structure. We now establish that the linear system of equations given by (37) admits a unique positive solution, up to constant of proportionality.

Proposition 7 (Demand Structure). *There exists a set of strictly positive regional productions Y_i^* such that for any set of regional productions Y_i satisfying the implementability condition (37), there exists $\lambda > 0$ such that $Y_i = \lambda Y_i^*$ for all i .*

Proof. Let A be the matrix $[a_{ki}]$ where $a_{ki} = \alpha_k^i \frac{P_i}{P_k}$. We have $a_{ki} > 0$ for all i, k . We can therefore apply the Perron-Frobenius theorem. It is easy to see that 1 is an eigenvalue of A . Indeed, we can consider the matrix $I - A$, multiply each line by P_k , sum them, and get 0. Hence $(P_1, P_2, \dots, P_I)$ is a left eigenvector of A with eigenvalue 1. Because $P_k \geq 0$ for all k , this proves that 1 is the Perron-Frobenius eigenvalue of the transpose A' of A . By implication 1 is also the Perron-Frobenius eigenvalue of A . This proves that there exists a right eigenvector $(Y_1^*, Y_2^*, \dots, Y_I^*)'$ of A with eigenvalue 1 and $Y_k^* > 0$. Moreover, the vector space associated with the eigenvalue 1 is one-dimensional. $\square$

The proportionality constant λ is a union-wide aggregate demand shifter that we treat as a dimension of policy. In a richer model λ would be determined by monetary policy at the currency union.

Impact of movers on stayers. We now state a simple property of the model that crucially differentiates it from the model of Section 2.

Proposition 8 (Dependence of per-capita allocations on location decisions). *Given a value for $\lambda > 0$, in all equilibria with or without free mobility, the consumption and labor allocation of agents of type j in region i is given by equations (38)-(39). It depends on the equilibrium only through the sufficient statistic of the population size μ_i of region i , to which it is inversely proportional.*

Proposition 8 should be contrasted with Proposition 2. Given λ , the allocation of agents of type j in region i is inversely proportional to the population size μ_i of region

i . When some agents moves out of a region i , reducing μ_i , the aggregate consumption $\lambda \frac{P_i}{p_i} Y_i^*$ and labor supply $\lambda \frac{1}{A_i} Y_i^*$ of region i remain unchanged, and are shared among fewer people due to the outward migration. Of course, as in the model of Sections 2-3.2, movers achieve a different allocations, but the key difference is that their move now has an impact on stayers, increasing their consumption and labor supply in proportion of the ratio of the migration outflow to the population size μ_i .

We will explore the normative consequences of Proposition 8 on the social efficiency of mobility decisions in Section 3.2. For now, let us focus on the equilibrium with free mobility. Consider a depressed region i . As in the model of Sections 2, workers migrating away from this region naturally benefit from the option to move to some region i' with higher economic activity. This stimulates the economy of the region i of origin in per-capita terms and cools off the economy of the destination region i' in per-capita terms. Mobility may therefore have extra benefits for macroeconomic adjustment in a currency union when demand imbalances are external rather than internal, reinforcing the view associated with Mundell (1961).

5.2 Social Optimum

Once again, we consider a planning problem indexed by a nonnegative Pareto weights λ^j . By varying these Pareto weights we can trace out the entire constrained Pareto frontier. The planner maximizes over the set of equilibria *without* free mobility. As in Section 3.2, we are assuming extreme powers of relocation for the planner.¹¹ As we shall see, in contrast to the model with internal imbalances of Sections 2-3.2, constrained efficient allocations are not consistent with free mobility.

Because our results on mobility do not depend on whether union-wide aggregate demand management λ is set optimally, we consider two planning problem, a restricted planning problem and a full planning problem. The restricted planning problem takes union-wide aggregate demand management λ as given and seeks to optimally allocate

¹¹ Alternative implementations of the same allocations could be achieved by introducing additional agent and location specific lump sum taxes

agents over regions. Using Propositions 6 and 7, we can write the planning problem as¹²

$$W(\lambda) = \max_{\mu_i, \mu^{ij}} \sum_{i \in I, j \in J} \lambda^j \mu^{ij} U^{ij} \left(\lambda \frac{P_i}{P^i} \frac{Y_i^*}{\mu_i}, \lambda \frac{Y_i^*}{A_i \mu_i} \right), \quad (40)$$

subject to, for all $j \in J$

$$\sum_{i \in I} \mu^{ij} = \mu^j,$$

and for all $i \in I$

$$\sum_{j \in J} \mu^{ij} = \mu_i.$$

The full planning problem allows for flexible monetary policy and characterizes jointly the optimal allocation of agents across regions and optimal monetary policy. The only difference with the first planning problem (40) is that λ is a choice variable instead of a parameter, so that the maximization takes place over λ , μ_i and μ^{ij} . This planning problem can be solved recursively, solving first for the optimal allocation of agents across regions for a given λ as characterized by the restricted planning problem (40), and then maximizing over λ

$$\max_{\lambda} W(\lambda). \quad (41)$$

We call the solutions of the restricted planning problem constrained efficient given union-wide aggregate demand management, and the solutions of the full planning problem constrained efficient.

5.3 Optimal Mobility

In this section, we characterize the constrained efficient allocation given union-wide aggregate demand management—the solutions of the restricted planning problem (40). We break down the restricted planning problem into two steps. In the first step, we solve

$$V(\{\mu_i\}, \lambda) = \max_{\mu^{ij}} \sum_{i \in I, j \in J} \lambda^j \mu^{ij} U^{ij} \left(\lambda \frac{P_i}{P^i} \frac{Y_i^*}{\mu_i}, \lambda \frac{Y_i^*}{A_i \mu_i} \right), \quad (42)$$

¹²Under the interpretation that the shock has a nonzero ex-ante probability and that monopolists set prices in advance, at an ex-ante stage, before the realization of the state of the world s , then we could also study an ex-ante problem that maximizes expected welfare across states s . The only difference is that the *non state contingent* prices P_i would be choice variables, and there would be corresponding additional optimality conditions. Because the prices P_i are not state contingent, it would typically not be possible to achieve the first best in every state s . Given these prices P_i , the analysis would be unchanged.

subject to, for all $j \in J$

$$\sum_{i \in I} \mu^{i,j} = \mu^j, \quad (43)$$

and for all $i \in I$

$$\sum_{j \in J} \mu^{i,j} = \mu_i. \quad (44)$$

In the second step, we solve

$$W(\lambda) = \max_{\mu_i} V(\{\mu_i\}, \lambda), \quad (45)$$

subject to

$$\sum_{i \in I} \mu_i = 1.$$

For all $i \in I$ and $j \in J$, we have the following first order condition for $\mu^{i,j}$ in the first-step planning problem 42:

$$\lambda^j U^{i,j} - v^j - \gamma_i = \rho^{i,j}, \quad (46)$$

where γ_i is the multiplier on the constraint (44), v^j is the multiplier on the constraint (43), $\rho^{i,j} = 0$ if $\mu^{i,j} \in (0, 1)$, $\rho^{i,j} \geq 0$ if $\mu^{i,j} = 1$ and $\rho^{i,j} \leq 0$ if $\mu^{i,j} = 0$. This condition characterizes the optimal location of agents of different types across the different regions.

The envelope theorem implies that for all $i \in I$,

$$V_{\mu_i} = - \sum_{j \in J} \lambda^j \frac{\mu^{i,j}}{\mu_i} \lambda \frac{P_i}{P^i} \frac{Y_i^*}{\mu_i} U_C^{i,j} \tau_i + \gamma_i.$$

The right hand side is independent of the agent type j .

The first order conditions for the second-step planning problem (45) are for all $i \in I$,

$$V_{\mu_i} = \gamma,$$

that is

$$\gamma_i = \gamma + \sum_{j \in J} \lambda^j \frac{\mu^{i,j}}{\mu_i} \lambda \frac{P_i}{P^i} \frac{Y_i^*}{\mu_i} U_C^{i,j} \tau_i. \quad (47)$$

The sign of $\gamma_i - \gamma$ coincides with the sign of the labor wedge τ_i . If region i is in a boom, we have $\tau_i < 0$ and $\gamma_i < \gamma$. Conversely, if region i is in a bust, we have $\tau_i > 0$ and $\gamma_i > \gamma$.

The first order conditions (46) can be rewritten as the condition that additional condi-

tion that for all $i \in I$ and $j \in J$,

$$\mu^{i,j} = 0 \quad \text{if} \quad U^{i,j} - \frac{\gamma_i}{\lambda^j} < \max_{i' \in I} U^{i',j} - \frac{\gamma_{i'}}{\lambda^j}, \quad (48)$$

and should be contrasted with the first order condition for free mobility (28). In general the maximand over i of $U^{i,j} - \frac{\gamma_i}{\lambda^j}$ is different from the maximand of $U^{i,j}$. It is therefore necessary to intervene in mobility decisions. One way to characterize this intervention is through an implicit additive utility wedge in the comparison between utility in region i and utility in region i' . This utility wedge is given by $\frac{\gamma_{i'} - \gamma_i}{\lambda^j}$.

Proposition 9 (Optimal Mobility). *Constrained efficient allocation given union-wide aggregate demand management are in general inconsistent with free mobility. The utility wedge in the comparison between utility in region i and utility in region i' . This utility wedge is given by $\frac{\gamma_{i'} - \gamma_i}{\lambda^j}$, where γ_i is defined by equation (47).*

Proposition 9 shows that from a social perspective, in an equilibrium with free mobility, agents tend to locate too little in regions that are in a relative boom (regions with a low value of γ_i) and too much in regions that are in a relative bust (regions with a high value of γ_i). In other words, agents do not move out enough from regions in a relative bust towards regions in a relative boom.

A simple intuition for Proposition 9 is that agents do not internalize that by moving out of a region in a bust, they increase the consumptions and labor supplies of the agents that remain in that region proportionately—the overall aggregate consumption $\lambda \frac{P_i}{P^i} Y_i^*$ and labor supply $\lambda \frac{1}{A_i} Y_i^*$ in region i both remain unchanged, and are shared among fewer people due to the outward migration. The impact of the staying agents' utilities is commensurate with the labor wedge τ_i in that region. Indeed, suppose that a mass $-d\mu_i > 0$ of agents leaves region i . Then the change in utility of staying agents of type j is given by

$$\begin{aligned} dU^{i,j} &= \frac{-d\mu_i}{\mu_i} \lambda \frac{P_i}{P^i} \frac{Y_i^*}{\mu_i} U_C^{i,j} + \frac{-d\mu_i}{\mu_i} \lambda \frac{1}{A_i} \frac{Y_i^*}{\mu_i} U_N^{i,j} \\ &= -\frac{d\mu_i}{\mu_i} \lambda \frac{P_i}{P^i} \frac{Y_i^*}{\mu_i} U_C^{i,j} \tau_i. \end{aligned}$$

These effects of an agent's mobility decision on other agents' utilities are not internalized. Hence the need for corrective government intervention in mobility decisions. The

relevant sufficient statistic for these interventions is the multiplier

$$\gamma_i = - \sum_{j \in J} \lambda^j \mu^{i,j} \frac{dU^{i,j}}{d\mu_i} = \sum_{j \in J} \lambda^j \frac{\mu^{i,j}}{\mu_i} \lambda \frac{P_i}{P^i} \frac{Y_i^*}{\mu_i} U_C^{i,j} \tau_i,$$

which aggregates these external effects through a weighted average across agent types j with weights $\lambda^j \mu^{i,j}$.

The result in Proposition 9 can be contrasted with that in Proposition 3. In the model with internal demand imbalances of Sections 2-3.2, constrained efficient allocations are consistent with free mobility, and agents' mobility decisions have no external effect on other agents' utilities. Indeed the allocation of an agent of type j in region i is independent of the distribution of agents across regions. By contrast, in the model with external imbalances of this section, constrained efficient allocations are in general not consistent with free mobility, and agents' mobility decisions have external effects on other agents' utilities.

The key difference can be traced back to the positive effects analyzed in Sections 3.1 and 5.1. In the model with internal demand imbalances, for a given stance of monetary policy P_T , migration out of a region reduces the total demand for the region's non-traded goods, and hence total demand for labor in the region by the same amount as it decreases the total supply of labor in the region. As a result, the labor supply of stayers is unchanged, and so is their consumption. By contrast, in the model with external imbalances, for a given stance of union-wide aggregate demand management λ , migration out of a region does not reduce the total demand for the region's goods, which must then be met by an increase in the labor supply of stayers, raising their income and in turn their consumption.

5.4 Optimal Union-Wide Aggregate Demand Management

In this section, we characterize constrained efficient allocations—the solutions of the full planning problem 41. The first order condition for λ is

$$\sum_{i \in I, j \in J} \lambda^j \mu^{i,j} \frac{P_i}{P^i} \frac{Y_i^*}{\mu_i} U_C^{i,j} \tau_i = 0, \quad (49)$$

where the right hand side is independent of $j \in J$. This condition characterizes the optimal level of union-wide aggregate demand in the currency union.

Proposition 10 (Optimal Union-Wide Aggregate Demand). *Constrained efficient allocations*

are such that a weighted average of the labor wedge τ_i across regions i is equal to zero, where the weight on τ_i is given by $\sum_{j \in J} \lambda^j \mu^{i,j} \frac{P_i}{P_i^*} \frac{Y_i^*}{\mu_i} U_C^{i,j}$ as in condition (49).

Proposition 10 is similar to Proposition 4 and has a similar interpretation. The choice of the price of the aggregate demand shifter λ , which can be thought of as union-wide aggregate demand management, plays the same role as union-wide monetary policy P_T .

The optimal allocation of agents across regions is still characterized by Proposition 9. Of course λ influences the value of the endogenous variables $U^{i,j}$ and γ_i , but not the conditions for optimal mobility (??), which given these variables, does not coincide with the conditions for free mobility (6).

5.5 Sticky Wages

Our arguments go through if wages are sticky but prices are flexible. We take the set of wages W_i as given. We consider the same simple symmetric allocation rule as in Section 3.5 which specifies that labor N_i in region i is distributed equally among the all the agents living in region k . Optimal price setting dictates that

$$P_{i,l} = P_i = \frac{\epsilon}{\epsilon - 1} (1 - \tau_{L,i}) \frac{W_i}{A_i}, \quad (50)$$

where consistent with our previous analysis $\tau_{L,i}$ cannot be adjusted and is taken here to be a parameter. Given this set of prices P_i , the analysis is then exactly the same as in the model with sticky prices and flexible wages.

Proposition 11. *Consider the model with sticky wages set at W_i and consider the associated equilibrium and optimal allocations. These are identical to those of a model with sticky prices set by (50). In particular, Propositions 10 and 9 apply to the model just outlined with flexible prices and sticky wages.*

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