

Information Frictions, Nominal Shocks, and the Role of Inventories in Price-Setting Decisions*

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Abstract

Models with information frictions display output and inflation dynamics that are consistent with the empirical evidence. However, an assumption in the existing literature is that pricing managers do not interact with production managers within firms. If this assumption were relaxed, nominal shocks would not have real effects on the economy. In this paper, I present a model with information frictions, output inventories, and perfect communication within firms where nominal shocks have real effects. In this model, final goods firms observe aggregate variables with one period lag but observe their nominal input price and demand at all times. Hence, firms will accumulate inventories as long as they think that they are facing a low input price (cost-smoothing role of inventories). After a contractionary nominal shock, the nominal input price goes down, and firms accumulate inventories because the probability of a good productivity shock is positive. This prevents firms' prices from decreasing, which distorts relative prices and makes current profits and households' income go down. As a consequence, the aggregate demand falls. I found that nominal shocks have a delayed effect on output, and that the responses to nominal shocks are significant, persistent, and hump-shaped. Output, capital, and total investment decrease by 0.5%, 1.8% and 3.4%, respectively, after a 1% increase in the nominal interest rate. Moreover, the peak of most of the responses is two quarters after the shock. When the model is simulated, it displays moments that are closer to the data than a comparable model with perfect information.

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1 Introduction

In the past decade, much progress has been made on models studying the impact of information frictions on aggregate supply. Models with sticky information, rational inattention, or dispersed information display output and inflation dynamics that are consistent with the empirical evidence: inflation exhibits inertia, responses to monetary shocks are delayed and persistent, and anticipated disinflations do not result in booms (Ball, Mankiew & Reis, 2005; Klenow & Willis, 2007; Mankiew & Reis 2002, 2010; Nimark, 2008; Woodford, 2002).

However, an assumption in the existing literature is that pricing managers do not interact with production managers within firms. As stated by Hellwig and Venkateswaran (2012), if this assumption were relaxed, nominal shocks would not have real effects on the economy because firms' input prices and demand contain all the information that is relevant to infer the firm's best response. In this paper, I also posit that this result extends to models in which firms take investment decisions such as in Angeletos and La'O (2012). The reason is very simple. Firms follow a rule of thumb: charge a markup times the marginal cost of production. Hence, when firms observe their nominal marginal cost, firms' nominal prices move in the same direction and magnitude as aggregate nominal variables.

Given that this assumption may not be realistic, one challenge for the existing literature is to explain why nominal shocks may have real effects when there is perfect communication within firms and prices are flexible. Explaining this link is relevant for central banks as it explains the transition mechanism of the monetary policy. In other words, if there is "something" within firms that prevents pricing managers from fully adjusting their prices after a nominal shock even under perfect communication within firms and flexible prices, that "something" would contain important information about the impact, duration, delay, and persistence of nominal shocks. I am contributing to the literature by providing an explanation as to this link.

In this paper, I present a model with information frictions, output inventories, and perfect communication within firms where nominal shocks have real effects on the economy. This model is based on the island model of Lucas (1972) and incorporates features from the inventory model of Khan and Thomas (2007). Final goods firms observe aggregate variables with a lag but receive information on their nominal input prices and demand in real time. Final goods producers face idiosyncratic shocks, and as a consequence cannot perfectly infer the state of the economy. Final goods firms set their output prices, determine production, and take inventories decisions based on their information set.

In this model, inventories are the link between information friction, perfect communication

within firms, and non-neutrality of nominal shocks. This is because inventories have a role in pricing and production decisions, which help to explain why nominal shocks have real effects: inventories help to smooth cost shocks. This is not new and has been extensively documented in the literature (Bils & Kahn, 2000; Khan & Thomas, 2007; Ramsey & West, 1999). In almost every model with inventories, firms accumulate inventories when marginal cost goes down, increasing current marginal cost and smoothing marginal cost through time. In this model, I show that this also implies that firms' prices are smoothed through time when monopolistic competition is assumed.

In this model, the cost-smoothing role of inventories helps to explain the non-neutrality of nominal shocks for the following reason: given that firms only observe their nominal input price and demand, they will accumulate inventories as long as they think that they are facing a low input price. Hence, after a contractionary nominal shock, firms observe a lower nominal input price. They do not know what the source of this change is, but they know that it could be due to a positive productivity innovation or due to a nominal shock. Since positive shocks have a positive probability, firms will increase their stock of inventories. This will prevent firms' prices from decreasing, which will distort relative prices, and will make current profits and households' income go down. As a consequence, the aggregate demand falls.

According to the results of this paper, after a negative nominal shock, the negative effect on output (decline in demand) offsets in the first period the positive effect (increase in inventory investment). Hence, nominal shocks have a delayed effect on output. I also found that the responses to nominal shocks are significant, persistent, and hump-shaped. Output, capital, and total investment decrease by 0.5%, 1.8% and 3.4%, respectively, after a 1% increase in the nominal interest rate. Moreover, the peak of most of the responses is two quarters after the shock.

I compared the model with information frictions with a model with perfect and complete information, and I found that assuming this friction makes the model more consistent with the empirical evidence. In a model with complete and perfect information, inventory investment is pro-cyclical; and the standard deviations of output, consumption, hours worked, and investment are small. In contrast, in the model with information frictions, inventory investment is counter-cyclical; and the standard deviations of output, consumption, hours worked, and investment are closer to the estimation by Cooley and Hansen (1995). Also, given the role of inventories, prices and inflation are more stable in absolute terms and relative to output.

This paper is related to work on information frictions and aggregate supply (e.g. Acharya,

2012; Angeletos & La'O, 2012; Ball, Mankiew & Reis, 2005; Klenow & Willis, 2007; Lucas, 1972; Mankiew & Reis, 2002, 2010; Nimark, 2008; Paciello & Wiederhold, 2011; Woodford, 2002). The idea that firms face a signal extraction problem goes back to Lucas (1972), and has been the cornerstone of the recent imperfect information literature. This work departs from previous work by providing a model with perfect communication within firms, in which nominal shocks have real effects.

This work is also part of a recent literature studying monetary models with inventories. Jung and Yun (2013) show that the relationship between current inflation and marginal cost of production weakens in a model with inventories and Calvo-type nominal rigidities. Krytsov and Midrigan (2013) point out that countercyclical markups produced by inventories, rather than nominal rigidities, can account for much of the real effects of monetary policy. Even though markups are not studied in this document, I also find that the relationship between prices and the marginal cost of production breaks down when firms can accumulate inventories. When a firm's cost increases drastically, the firm reduces production and sells a fraction of its inventory holdings. In this situation, the firm's price will also depend on the decrease in the stock of inventories. In contrast to previous work, inventories in my model are crucial to explaining why there are real responses to monetary shocks. This is not true in Jung and Yun (2013) and Krytsov and Midrigan (2013), which both assume some type of price rigidity, so that monetary policy is effective even without inventories.

Finally, this work is related to previous studies about inventories, which point out the cost production smoothing motive of inventory investment (e.g Bills & Kahn, 2000; Eichenbaum, 1989; Khan & Thomas, 2007a, 2007b). In contrast to the existing literature, this work will study the role of inventories in price decisions by introducing monopolistically competitive firms. This will be relevant in order to understand what makes prices more or less responsive to monetary shocks.

This paper is divided into five sections. In section two, I present the model setup and discuss some properties of the decision rules. In section three, I solve this model when all the agents have perfect and complete information. Then, in section four, I solve this model when a particular information friction is assumed. Finally, section five concludes.

2 Model

The model is based on Lucas (1972) and incorporates features from the inventory model of Khan and Thomas (2007). There are three agents in this economy: a representative

household, intermediate goods producers, and final goods firms. Households supply labor and capital to the intermediate goods producers, and they purchase differentiated consumption and investment goods from the final goods firms. Intermediate goods producers are perfectly competitive and supply their output to the final goods firms. Intermediate goods are used by final goods firms to produce output, which is used for consumption and capital accumulation. Final goods firms sell their output in a monopolistic market to households and can accumulate output inventories.

Households derive utility from consumption and leisure and discount future utility by β . The consumption good is a Dixit-Stiglitz aggregator of the final goods of the economy. Households supply labor and capital to the intermediate goods producers in a perfectly competitive market, and they own all intermediate and final goods firms. Capital depreciates at rate δ_K and can be augmented by using final goods as investment: $K_t = (1 - \delta_K)K_{t-1} + X_t$. Investment, X , is also a Dixit-Stiglitz aggregator of the final goods of the economy.

I assume a continuum of differentiated industries with measure one and indexed by j . Each industry is represented by an intermediate goods producer and a final goods firm.

Each intermediate firm produces with capital, k , and labor, h , through a constant returns to scale production function.¹ Its output is $m = \bar{b} \cdot b \cdot k^\alpha h^{1-\alpha}$, where $\bar{b}$ is exogenous aggregate total factor productivity, and b is idiosyncratic productivity. For simplicity, I assume that intermediate goods cannot be stored.

Final goods firms in industry j produce using only the intermediate good of their industry, $m(j)$, through a concave, decreasing returns to scale production function, $y = Am^\gamma$. Their output is denoted by y , and A is constant and equal for all industries. Each final goods firm can accumulate output inventories, which depreciate at rate δ_I . I provide an explicit motive for inventory accumulation by assuming that final goods firms face idiosyncratic demand shocks. I also assume that intermediate producers face idiosyncratic productivity shocks, which makes the price of the intermediate good idiosyncratic and provides an additional motive for inventory accumulation. At the beginning of each period, a final good firm is identified by its inventory holdings, I , its current demand, d , and its current input price, p_m . A final goods firm sets its output price and determines current production by selecting m . Output is devoted to sales and inventory investment.

Finally, the aggregate total factor productivity, $\bar{b}$, and the nominal interest rate, i , follow an AR(1) process in logs, and these are the only sources of aggregate uncertainty in the

¹Even though intermediate producers are matched one-to-one with final goods firms, assuming that intermediate firms are competitive is not problematic. This is because assuming a continuum of identical firms is equivalent to assuming one representative firm in this context.

model.

2.1 Household's Problem

The representative household owns all the economy's firms, and supplies labor and capital to the intermediate goods producers. Each period, the household allocates its total income between consumption and investment, in order to maximize its expected discounted lifetime utility. Hence, the household's problem reads:

$$U = \max_{\{C_t, H_t, K_t, X_t\}} \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \Psi \frac{H_t^{1+\eta}}{1+\eta} \right) \quad (1)$$

s.t.

$$P_t C_t + P_t X_t \leq W_t H_t + i_{t-1} P_{t-1} K_{t-1} + \Pi_t^F \quad (2)$$

$$K_t = (1 - \delta_K) K_{t-1} + X_t \quad (3)$$

H_t is the labor supply, W_t is the nominal wage, i_t is the gross nominal interest rate, and Π^F is aggregate nominal dividends from the economy's firms. At the end of period t , intermediate producers agree to pay an interest rate i_t on the nominal value of capital ($P_t K_t$) at the beginning of period $t+1$. Fixed capital investment, X_t , and the consumption good, C_t , are Dixit-Stiglitz aggregators of the final goods of this economy:

$$C_t = \left(\int_0^1 \chi(j)_t^{\frac{1}{\epsilon}} c(j)_t^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \quad (4)$$

$$X_t = \left(\int_0^1 \chi(j)_t^{\frac{1}{\epsilon}} x(j)_t^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \quad (5)$$

$c(j)_t$ is the consumption of good j in period t , and $x(j)_t$ is the amount of good j invested in period t . $\chi(j)$ is a good-specific demand shock that is heterogeneous across sectors and independent of all other shocks. $\log(\chi(j))$ is distributed normal with zero mean and variance σ_χ^2 . By cost minimization, $c(j)$, $x(j)$, and the aggregate price index, P_t , are given by:

$$c(j)_t = \chi(j) \left[C_t \left(\frac{P_t}{p(j)_t} \right)^\epsilon \right] \quad (6)$$

$$x(j)_t = \chi(j) \left[X_t \left(\frac{P_t}{p(j)_t} \right)^\epsilon \right] \quad (7)$$

$$P_t = \left(\int_0^1 \chi(j) p(j)_t^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \quad (8)$$

Hence, firm j 's output demand, $s(j)$, is given by

$$s(j)_t = c(j)_t + x(j)_t \quad (9)$$

$$s(j)_t = \chi(j)_t \left[(C_t + X_t) \left(\frac{P_t}{p(j)_t} \right)^\epsilon \right] \quad (10)$$

Notice that firm j takes $d(j)_t \equiv \chi(j)_t [(C_t + X_t)P_t^\epsilon]$ as given. Throughout this paper, I define $d(j)_t$ as firm j 's nominal demand in period t . Therefore, it is convenient to re-write $s(j)_t$ as follows:

$$s(j)_t = d(j)_t p(j)_t^{-\epsilon} \quad (11)$$

Finally, from the household's first order conditions, we get:

$$C_t^{-\sigma} = \beta \mathbb{E} \left[\left(\frac{i_t}{P_{t+1}/P_t} + (1 - \delta_K) \right) C_{t+1}^{-\sigma} \right] \quad (12)$$

$$\Psi H_t^\eta = \frac{W_t}{P_t} C_t^{-\sigma} \quad (13)$$

2.2 Intermediate Goods Firms Problem

In each industry j , there is a representative intermediate producer that supplies the input $m(j)$ in a competitive market to the final good firm of industry j . Each intermediate producer chooses current employment and capital, in order to maximize its nominal value, $V(j)^m$. The cost of borrowing one unit of capital in period t is given by the gross nominal interest rate i_{t-1} , and the nominal wage is given by W_t .

$$V(j)_0^m = \max_{\{k_{t-1}, h_t\}} E \sum_{t=0}^{\infty} Q_{0,t} \pi(j)_t^m \quad (14)$$

s.t.

$$\pi(j)_t^m = q(j)_t m(j)_t - i_{t-1} P_{t-1} k(j)_{t-1} - W_t h(j)_t \quad (15)$$

$$m(j)_t = \bar{b}_t b(j)_t k(j)_{t-1}^\alpha h(j)_t^{1-\alpha} \quad (16)$$

$\pi(j)_t^m$ is the firm j 's current nominal profits, $q(j)_t$ is the price of $m(j)$, and $Q_{0,t}$ is the stochastic discount for the economy's firms:

$$Q_{0,t} = \beta \frac{u'(C_t)/P_t}{u'(C_0)/P_0} \quad (17)$$

$b(j)$ is the productivity of the intermediate producer j . $b(j)$ is heterogeneous across sectors at any point in time, and $\log(b(j))$ is distributed normal with zero mean and variance σ_b^2 . $\bar{b}$ is the aggregate total factor productivity of the economy and is common to all intermediate goods firms. $\bar{b}$ is stochastic and follows an AR(1) process in logs.

$$\log(b(j)) \sim N(0, \sigma_b) \quad (18)$$

$$\log(\bar{b}_t) = (1 - \rho_b)\log(\bar{b}) + \rho_b\log(\bar{b}_{t-1}) + \varepsilon_{b,t} \quad (19)$$

$$\varepsilon_b \sim N(0, \sigma_B) \quad (20)$$

Hence, in equilibrium:

$$W_t = (1 - \alpha)q(j)_t \frac{m(j)_t}{h(j)_t} \quad (21)$$

$$i_{t-1}P_{t-1} = \alpha q(j)_t \frac{m(j)_t}{k(j)_{t-1}} \quad (22)$$

$$q(j)_t = \frac{1}{\bar{b}_t b(j)_t} \left(\frac{i_{t-1}P_{t-1}}{\alpha} \right)^\alpha \left(\frac{W_t}{1 - \alpha} \right)^{1-\alpha} \quad (23)$$

$$\pi(j)_t^m = 0 \quad (24)$$

Combining equations (21) and (22) we get that:

$$\frac{k(j)_{t-1}}{h(j)_t} = \frac{K_{t-1}}{H_t} = \left(\frac{W_t}{i_{t-1}P_{t-1}} \right) \left(\frac{\alpha}{1 - \alpha} \right) \quad (25)$$

2.3 Final Goods Firms Problem

Define $V(I, q, d)_t^y$ as the current nominal value of a final goods firm with current inventory stock I , nominal input price q , and nominal demand d . The problem for a final goods firm

reads:

$$V(I_0, q_0, d_0)_0^y = \max_{\{p_t, s_t, y_t, m_t, I_{t+1}\}} E_0 \sum_{t=0}^{\infty} Q_{0,t} \pi_t^y \quad (26)$$

s.t.

$$\pi_t^y = p_t s_t - q_t m_t \quad (27)$$

$$s_t = d_t p_t^{-\epsilon} \quad (28)$$

$$y_t = s_t + I_{t+1} - I_t(1 - \delta_I) \quad (29)$$

$$y_t = A m_t^\gamma \quad (30)$$

$$I_{t+1} \geq 0 \quad (31)$$

π_t^y is the current nominal profit, and p is the price of the final good. Equation (28) is the firm's demand, which was defined in equations (10) and (11). The above problem is strictly concave, and the following first-order conditions pin down the firm's optimal decisions:²

$$p_t = \left(\frac{\epsilon}{\epsilon - 1} \right) \left(\frac{q_t}{\gamma A^{\frac{1}{\gamma}}} \right)^{\frac{1-\gamma}{\gamma}} y_t^{\frac{1-\gamma}{\gamma}} \quad (34)$$

$$\left(\frac{q_t}{\gamma A^{\frac{1}{\gamma}}} \right)^{\frac{1-\gamma}{\gamma}} y_t^{\frac{1-\gamma}{\gamma}} \geq E \left[Q_{t,t+1} \left(\frac{q_{t+1}}{\gamma A^{\frac{1}{\gamma}}} \right)^{\frac{1-\gamma}{\gamma}} y_{t+1}^{\frac{1-\gamma}{\gamma}} \right] \quad (35)$$

Equation (34) states that the firm's price is equal to a markup times the marginal cost of production regardless of the production allocation. On the other hand, according to equation (35), inventories are used to smooth the marginal cost of production through time, and this equation holds with equality if $I_{t+1} > 0$. Suppose, for example, that a firm expects its marginal cost to go up in future periods due to an increase in the price of the input m . In anticipation, the firm could increase its production in the current period, in order to sell those additional units when q goes up. This would make the current and future marginal cost move in opposite directions, smoothing the firm's marginal cost. We have a similar story

² Notice that the firm's problem could be written as follows:

$$V(I_t, q_t, d_t)_0^y = \max_{\{y_t, I_{t+1}\}} E_0 \sum_{t=0}^{\infty} Q_{0,t} \left[d_t (y_t + I_t(1 - \delta_I) - I_{t+1})^{\frac{\epsilon-1}{\epsilon}} - q_t \left(\frac{y_t}{A} \right)^{\frac{1}{\gamma}} \right] \quad (32)$$

s.t.

$$I_{t+1} \geq 0 \quad (33)$$

Since $\epsilon > 1$ and $\gamma \leq 1$, the first term in (32) is strictly concave, and the second term is convex. Hence, this problem is strictly concave.

when a firm expects its demand, d , to increase. For the purposes of this work, the following lemmas will be useful.

Lemma 1. p_t is decreasing in I_t

Proof. First, notice that $V(I, q, d)_t^y$ is a strictly increasing and concave function in I .³ Then, using the envelope theorem:

$$\frac{\partial V(I, q, d)_t^y}{\partial I_t} \propto p_t > 0 \quad (36)$$

$$\frac{\partial^2 V(I, q, d)_t^y}{\partial I_t^2} \propto \frac{\partial p_t}{\partial I_t} < 0 \quad (37)$$

□

In order to understand Lemma 1, suppose that the stock of inventories of a firm increases unexpectedly. Therefore, given that the firm will eventually sell those additional units, the firm's price will have to decrease at some point in order to induce consumers to buy more.

Lemma 2. Assuming that $\epsilon > 1$ and that $\gamma \leq 1$, the optimal decision rules for p_t and I_{t+1} have the following properties:

- The current optimal price (p_t^*) is strictly increasing in the firm's current demand (d_t) and input price (q_t).
- The current optimal price (p_t^*) is weakly increasing in the firm's future demand (d_{t+1}) and input price and q_{t+1} .
- The optimal next period's stock of inventories (I_{t+1}^*) is weakly decreasing in the firm's current demand (d_t) and input price and q_t . Moreover, if the initial stock of inventories is positive ($I_t > 0$), I_{t+1}^* is strictly decreasing in d_t and q_t .
- The optimal next period's stock of inventories (I_{t+1}^*) is weakly increasing in the firm's future demand (d_{t+1}) and input price (q_{t+1}).

Proof. This comes directly from the envelope theorem and the symmetry of the second derivatives. □

³Using the envelope theorem, we get that $V(I, q, d)$ is strictly increasing in I . Similarly, in footnote 2, I showed that the firm's problem is strictly concave, which means that $V(I, q, d)$ is strictly concave in I .

Intuitively, given that inventories are used to smooth cost shocks, a firm will reduce its production and will sell inventories when its demand or input price increase. Similarly, if a firm expects its demand or input price to go up in the future, it will accumulate inventories by increasing its production. This will make the marginal cost, and as a consequence the firm's output price, go up.

Lemma 3. *At the firm level, inventories impose an upper bound for the increase in the firm's price. In particular,*

$$1 \geq \mathbb{E} \left[Q_{t,t+1} \frac{p_{t+1}}{p_t} \right] \quad (38)$$

This comes directly from multiplying both sides of equation (35) by $\epsilon/(\epsilon-1)$. This lemma implies that, with monopolistic competition, inventories smooth not only the marginal cost of production but also firms' prices. Intuitively, suppose that a firm expects its price to go up in the following period and that $p_t < E[Q_{t,t+1}p_{t+1}]$ so that $E \left[Q_{t,t+1} \frac{p_{t+1}}{p_t} \right] > 1$. Notice that this firm could increase its profits by producing more today and selling those extra units in the next period. On the one hand, the increase in current production would make the current marginal cost go up, increasing p_t . On the other hand, according to lemma 1, the increase in the stock of next period's inventories will make p_{t+1} decrease. As a consequence, the firm will accumulate inventories up to the point where $p_t = E[Q_{t,t+1} \cdot p_{t+1}]$. In that situation, the marginal benefit of selling one extra unit today (p_t) will be equal to the marginal benefit of selling one extra unit in the next period ($E[Q_{t,t+1} \cdot p_{t+1}]$).

2.4 Nominal Shocks

For simplicity, I assume that the nominal interest rate follows the exogenous process:

$$\log(i_t) = (1 - \rho_i)\log(1/\beta) + \rho_i\log(i_{t-1}) + \varepsilon_{i,t} \quad (39)$$

$$\varepsilon \sim N(0, \sigma_i^2) \quad (40)$$

This implies that in the deterministic steady state, when $\varepsilon_i = 0$ and $i_t = i_{t-1}$ for all t , the inflation rate is zero.

3 Solving the Model with Complete and Perfect Information

In this section, I solve this model assuming perfect and complete information. As a result, nominal shocks do not have real effect on this economy. However, the optimal decision rules depicted in this subsection will help to explain why nominal shocks have real effects when a particular information friction is introduced. I start by defining the competitive equilibrium of this economy and stating clearly that, under these assumptions, this economy exhibits the classical dichotomy. Then, I report the impulse response functions to a productivity shock and compare them with those generated by two alternative models: (i) one in which there is no heterogeneity across sectors and firms cannot accumulate inventories, and (ii) one model in which there is heterogeneity across firms but firms cannot accumulate inventories.

3.1 Competitive Equilibrium with Perfect and Complete Information

Definition: A competitive equilibrium with perfect and complete information in this economy is a sequence of prices $\{P_t, W_t, q(j)_t, p(j)_t\}$, allocations $\{C_t, K_t, I_t, Y_t, X_t, H_t, y(j)_t, m(j)_t, h(j)_t, k(j)_t\}$, a distribution of final goods firms $\{\lambda(I, q, d)_t\}$, and exogenous variables $\{i_t, \bar{b}_t\}$, such that given the initial conditions $K_0, \lambda(I, q, d)_0, P_{-1}, i_{-1}$:

1. *Households optimize taking prices, exogenous variables, the distribution of final goods firms and initial conditions as given.* The sequence $\{C_t, K_t, X_t, H_t\}$ satisfies equations (12), (13), (2), and (3) along with the transversality condition:

$$\lim_{t \rightarrow \infty} \beta u'(C_t) K_t = 0. \quad (41)$$

2. *Intermediate producers optimize taking prices, exogenous variables, the distribution of final goods firms and initial conditions as given.* The sequence $\{W_t, i_t, P_t, q(j)_t\}$ satisfies equations (21), (22) and (23) for all j .
3. *Final goods producers optimize taking $\{P_t, W_t, q(j)_t, \{p(z)_t\}_{z \neq j}\}$, exogenous variables, the distribution of final goods firms, and initial conditions as given.* The sequence $\{y(j)_t, I(j)_t, m(j)_t, p(j)_t\}$ satisfies equations (34), (35), (29), and (30) along with the

transversality condition:

$$\lim_{t \rightarrow \infty} \beta u'(C_t) I_t = 0. \quad (42)$$

4. The distribution of firms evolves according to

$$\lambda(I', q', d')_{t+1} = \int \mathbf{1}_{\{I(I, q, d)=I'\}} \cdot pr(q' \wedge d'|q, d) \cdot d\lambda(I, q, d)_t \quad (43)$$

Where $\mathbf{1}_{\{I(I, q, d)=I'\}}$ is an indicator function that is equal to 1 if a firm with initial stock of inventories I , input price q , and demand d , chooses a stock of inventories for the next period equal to I' .

5. *Markets Clear*

$$H_t = \int h(j) dj \quad (44)$$

$$K_t = \int k(j) dj \quad (45)$$

$$Y_t = C_t + X_t + I_{t+1} - (1 - \delta_I) I_t \quad (46)$$

6. The nominal interest rate and the total factor productivity follow:

$$\log(i_t) = (1 - \rho_i) \log(1/\beta) + \rho_i \log(i_{t-1}) + \varepsilon_{i,t} \quad (47)$$

$$\log(\bar{b}_t) = (1 - \rho_B) \log(B) + \rho_B \log(\bar{b}_{t-1}) + \varepsilon_{b,t} \quad (48)$$

Proposition 1. *The set of real allocations $\{C_t, K_t, I_t, Y_t, X_t, H_t, y(j)_t, m(j)_t, h(j)_t, k(j)_t\}$ and distribution of final goods firms $\{\lambda(I, q, d)_t\}$ that are consistent with the existence of a competitive equilibrium is independent of the path for the nominal interest rate $\{i_t\}$ and, as a consequence, of the functional form in equation (39).*

Proof. Notice that we can re-write the set of equations that describe the competitive equilibrium in a form that does not involve the nominal interest rate. To see this, we need to define the real interest rate $r_t = i_{t-1} P_{t-1} / P_t$, the real wage rate $w_t = W_t / P_t$, and the relative prices $\tilde{p}(j)_t = p(j)_t / P_t$ and $\tilde{q}(j)_t = q(j)_t / P_t$. Also, the stochastic discount factor becomes: $\tilde{Q}_{0,t} = \beta u'(C_t) / u'(C_0)$. By defining and replacing these variables in the set of equations that describe the competitive equilibrium, we get a system of equations that are independent of the nominal interest rate. $\square$

Hence, this economy exhibits the classical dichotomy. As long as prices are flexible and all agents in this economy have perfect and complete information, real and nominal variables can be analyzed separately.

3.2 Parameter Values

Table 1 presents the parameter values that I used in order to solve this model numerically.⁴ The time period for this model is one quarter. Parameters γ , α , δ_K , ρ_B , σ_B , β were set to 0.8, 0.374, 0.017, 0.9, and 0.03 following Khan and Thomas (2007). For simplicity, I fixed A , B and δ_I to 1, 0.8 and 0. The standard deviation of the idiosyncratic shocks (σ_b and σ_χ) were set to 0.1. This is less than 10 times the standard deviation of the aggregate shock (σ_B), which is a standard number in the information frictions literature. The demand elasticity (ϵ), the intertemporal elasticity of substitution (σ), and the inverse of the Frich elasticity (η) are equal to 4, 2, and 3, which are standard numbers in the literature. Finally, parameter Ψ was set such that h is equal to 0.3 in steady state.

3.3 Economy Dynamics with Perfect and Complete Information

Given this set of parameters, I find the deterministic steady state, which is reported in table 2.⁵ The stock of inventories is very relevant in this economy as it represents 110% of the total output. This value is above the estimation of 80% by Ramey and West (1999). This implies that the value for σ_b and σ_χ have to be smaller in order to match this moment.

On the other hand, Figure 2 displays the firms decision rules for different levels of the nominal demand $d(j)$ and the input price $q(j)$, and the first panel of Figure 3 shows the ergodic distribution for this model. As stated in Lemmas 1 and 2, the price decision rule is strictly decreasing in the initial stock of inventories. Also, notice that firms accumulate inventories when they face a low input price or demand because in those situations the marginal cost of production is low. Another feature of this figure is that the higher the stock of inventories is, the smaller the impact of a shock on the firm's price is. In other words, when a firm's input price increases, the impact on the firm's price can be smoothed as long as the firm has a positive initial stock of inventories. According to the ergodic distribution, 25% of the firms do not have inventories, and two-thirds have an initial stock of inventories between zero and 1.

⁴All the tables and figures can be found at the end of the document.

⁵This is done by assuming that $\sigma_B = \sigma_i = 0$.

Now, in order to compute the dynamics of this model, I take a first order approximation of the economy around the deterministic steady state, following the methodology proposed by Reiter (2009).⁶ This methodology allows a higher order representation of the cross-sectional distribution in the state vector and has the advantage that the solution is fully non-linear in the idiosyncratic (presumably large) shocks but linear in the aggregate (presumably small) shocks.⁷

Figure 5 plots the impulse response functions to a 1% increase in the aggregate total factor productivity, $\bar{b}$. This figure displays some interesting features that will help to explain why nominal shocks have real effects on this economy when a particular information friction is introduced.

Notice that after a productivity shock, many of the relevant variables for the final goods firms are changing. First, the increase in the total aggregate productivity creates an incentive to accumulate more inventories for those firms that are also facing a positive idiosyncratic productivity shock. In contrast, those firms that are facing a negative shock *know* that they will face a better shock with a good probability in the next period, and, therefore, they have more incentives to sell their stock of inventories in the current period. Second, firms expect total demand to keep increasing for another three quarters, which creates incentives to accumulate inventories in the current period. Third, notice that the peak in the input price is in quarter 2 due to an increase in capital accumulation during the first periods. Therefore, given that firms *know* this, and recalling lemma 2, firms have incentives to wait until next period to accumulate inventories. Fourth, total demand is increasing significantly. Hence, firms have incentives to use their stock of inventories in the current period in order to keep their prices relatively constant and take advantage of the increase in aggregate demand. As a result of these effects, most of the firms decide to sell a fraction of their inventories and wait until next period to accumulate inventories, making the inventory investment countercyclical. However, inventory investment is procyclical in the data (e.g. Ramey & West, 1999; Bils & Kahn, 2000; Khan & Thomas, 2007). As I will discuss in the next section, one important assumption behind these functions is that firms *know* what is happening in the economy. Once I modify this assumption the inventory investment becomes procyclical.

On the other hand, notice that total output has a peak in quarter 2. In the first quarter, firms sell a fraction of their inventories, which prevents output and firms' prices to increase.

⁶Appendix B discusses in detail how I solved the model.

⁷For the purposes of this work, this will be particularly useful when computing firm's expectations. Given the linearity of the aggregate variables, firms can use a linear filter, such as the Kalman-Filter, in order to compute their expectations.

However, during the second quarter, most of the firms do not have enough inventories to reduce the impact of the increase in total demand, and the input price is lower, which makes the demand for inventories go up (Figure 2).

Given that two important assumptions in this model are heterogeneity across firms and output inventories, I now compare these impulse response functions with those generated by two alternative models in order to assess how important and relevant these assumptions are. Figure 6 plots the impulse response function to a 1% positive productivity shock for three different models. The black solid lines are the impulse response functions of the baseline model (Full model), and they are equal to those presented in Figure 5. The black dashed lines are the impulse response function of a model in which firms cannot accumulate inventories, but there is heterogeneity across firms (No Inv). The grey lines are the impulse response functions of a model with no heterogeneity and no inventories (Simple).⁸ Even though the dynamics seem similar, the magnitudes and peaks are significantly different.

First, in all cases the aggregate price index goes down because the economy is now able to produce more goods at a lower price. The decline in the price index in the model with no inventories (No Inv) is greater than the decline in the model with no inventories and no heterogeneity (Simple). This is because of a substitution effect. In the model with heterogeneous firms, a fraction of the firms face an even better shock. Given that those firms cannot accumulate inventories, they lower their price and the demand for their product increases. This puts more weight on the more productive firms, which makes the aggregate price index be lower. When we allow firms to accumulate inventories, the aggregate price index decreases even more (Full Model). This is because the less productive firms (those that are facing a negative idiosyncratic productivity shock) sell a fraction of their inventories in order to take advantage of the increase in current demand, and because they know that they are facing a negative shock. This prevents their prices to go up and makes the aggregate price index go down.

Second, consumption and capital increase in the three models. But the increase is greater in the model with inventories and heterogeneous firms (Full Model) and smaller in simple model. The reasons are the same. First, the existence of heterogeneous firms induces a substitution effect. The more productive firms produce more at a lower price, which makes consumption and capital increase more. Then, when firms are allowed to accumulate inventories, the less productive firms sell a fraction of their stocks at a lower price, which makes further induces capital and consumption to increase more.

⁸In the simple model $\sigma_\chi = 0$ and $\sigma_b = 0$.

Finally, the output response is greater in the model with no inventories and no heterogeneity. This is because the less productive firm cannot desaccumulate inventories in order to smooth the negative idiosyncratic shock that they are facing. Hence, when inventories are introduced, those firms produce less and sell more inventories, which prevents output to increase. However, it is worth noticing that the output response of the full model is the only one that is hump shaped. In the other two cases, the maximum response is in the first quarter.

4 Solving the Model with Information Frictions

I now introduce a particular information friction in this economy. I assume that final goods firms observe aggregate variables with a lag of $\mathcal{T}$ periods but receive information about their input prices and demand in real time. For simplicity, I set $\mathcal{T}$ equal to 1, which implies that firms do not observe the current level of the aggregate variables. As stated before, one contribution of this paper is to provide a model with perfect communication within firms in which nominal shocks have real effects. The following proposition shows why this is important:

Proposition 2. *Suppose that all agents in the economy except firms have perfect and complete information. Moreover, assume that final goods producers cannot hold inventories, so their problem becomes:*

$$V(q_0, d_0)_0^y = \max_{\{p_t, s_t, y_t, m_t\}} E \sum_{t=0}^{\infty} Q_{0,t} (p_t s_t - q_t m_t) \quad (49)$$

s.t.

$$s_t = d_t p_t^{-\epsilon} \quad (50)$$

$$y_t = s_t \quad (51)$$

$$y_t = A m_t^{\gamma} \quad (52)$$

If prices are flexible, and if there is perfect communication within firms such that pricing managers perfectly observe their input prices and demand, then nominal shocks do not have real effects on the economy regardless of the information friction on aggregate variables.

Proof. In Proposition 1, I showed that the set of equations that describe the competitive equilibrium under perfect information could be written in a form that does not involve the nominal variables. Since the only equations that change under information frictions are those

involving final goods firms, I only need to show that those equations can be written in a form that does not involve nominal variables. First, notice that the final goods firms problem can be re-stated as:

$$V(q_0, d_0)_0^y = \max_{\{p_t\}} E \sum_{t=0}^{\infty} Q_{0,t} \left(p_t^{1-\epsilon} d_t - \frac{q_t}{A^{\frac{1}{\gamma}}} (d_t p_t^{-\epsilon})^{\frac{1}{\gamma}} \right) \quad (53)$$

Hence, from the first order condition, we get that:

$$p_t^* = \left[\left(\frac{\epsilon}{\epsilon - 1} \right) \frac{q_t}{A^{\frac{1}{\gamma}} \gamma} d_t^{\frac{1-\gamma}{\gamma}} \right]^{\frac{\gamma}{\gamma + \epsilon(1-\gamma)}} \quad (54)$$

And, using the definition of d_t :

$$p_t^* = P_t \cdot \left[\left(\frac{\epsilon}{\epsilon - 1} \right) \left(\frac{q_t}{P_t} \right) \frac{(\chi_t(C_t + X_t))^{\frac{1-\gamma}{\gamma}}}{A^{\frac{1}{\gamma}} \gamma} \right]^{\frac{\gamma}{\gamma + \epsilon(1-\gamma)}} \quad (55)$$

Therefore, the firm's relative price, (p_t^*/P_t) , is independent of the nominal variables, and therefore so is the set of allocations that are consistent with the existence of a competitive equilibrium. $\square$

Proposition 2 is not new in the literature. Hellwig and Venkateswaran (2012) have the same results for a simpler model.⁹ This is, assuming that firms do not accumulate inventories or capital, as long as firms observe their current demand and input prices, information frictions are irrelevant. The intuition is simple, final goods firms only need their current demand and input price in order to infer their best response. Notice that equations (54) and (55) are equivalent. Therefore, a final goods firms does not need to know the actual value for C , X , P or even their demand shock χ , because d and q contain all the information that is relevant.

This Proposition implies, for example, that the models of Mankiw and Reis (2002), Paciello and Wiederhold (2011), and Klenow and Willis (2007) would not display real responses to monetary shock if perfect communications within firms was allowed. Also, even though Angeletos and La'O (2012) discuss in detail nominal and real frictions, and they present a model with real frictions in which firms take investment decisions, their model does not survive to introducing perfect communication within firms. In their model, the price for the investment goods is the aggregate price index. Hence, if we assume that firm observe their

⁹This is formalized in their Proposition 1.

input prices, this implies that firms observe the aggregate price level. As a consequence, they can distinguish between real and nominal variables.

It is worth noticing that Proposition 2 does not hold when final goods firms can accumulate inventories. To see this, recall the firm's optimal price:

$$p_t^* = \left(\frac{\epsilon}{\epsilon - 1} \right) \left(\frac{q_t}{\gamma A^{\frac{1}{\gamma}}} \right) y_t^{*\frac{1-\gamma}{\gamma}} \quad (56)$$

Which is equivalent to:

$$p_t^* = \left(\frac{\epsilon}{\epsilon - 1} \right) \left(\frac{q_t}{\gamma A^{\frac{1}{\gamma}}} \right) [d_t p_t^* + I_{t+1}^* - I_t]^{\frac{1-\gamma}{\gamma}} \quad (57)$$

Notice that the optimal price does not only depend on the firm's current demand and input price but also on the inventory investment, which according to (35) depends on firm's expectations. For example, for a given d and q , if a firm expects its input price to go up, I_{t+1}^* and p_t^* will increase. Given that firms do not have access to contemporaneous information about the aggregate variables, firms will make forecast errors after a shock. This will make firms' decision rules deviate from the decision rules under perfect and complete information.

For instance, suppose that the aggregate price index goes down and everything else is constant. If all the agents *know* this, they will react by adjusting all prices down, which will have no effect on the real variables. Nonetheless, if final goods firms only observe aggregate variables with a lag, they will observe their input price going down. Firms *do not know* the source of that movement. They only know that it could be because (i) the economy has been shocked by a positive productivity shock, (ii) the economy has been shocked by a nominal shock, (iii) the firm has been shocked by a positive productivity shock, or (iv) a combination of everything. Therefore, firms' responses will be a combination of the optimal responses for each case. Given that firms accumulate inventories when they are shocked by a positive productivity shock, they will response by accumulating inventories, which has a positive effect on firm's prices. How strong their responses are will depend on their expectations and the probability for each case. This points out why inventories help to explain why nominal shocks have real effects when perfect communication within firms is assumed.

4.1 Recursive Competitive Equilibrium

Given the information friction that was introduced, it is convenient to define the competitive equilibrium in recursive form. Denote $\mathbf{E}$ as the vector of aggregate state variables, which will

be defined latter. The household's recursive optimization problem is:

$$U(K_{-1}, \mathbf{E}) = \max_{K, C, H, X} \frac{C^{1-\sigma}}{1-\sigma} - \Psi \frac{H^{1+\eta}}{1+\eta} + \beta E[U(K, \mathbf{E}')] \quad (58)$$

s.t.

$$PC + PX \leq WH + i_{-1}P - K_{-1} + \Pi^F \quad (59)$$

$$K = (1 - \delta_K)K_{-1} + X \quad (60)$$

$$\mathbf{E}' = \omega^h(\mathbf{E}) \quad (61)$$

Where equation (61) is the perceived law of motion of $\mathbf{E}$. The solution to this problem is given by decision rules $K(K_{-1}, \mathbf{E})$, $C(K_{-1}, \mathbf{E})$, $H(K_{-1}, \mathbf{E})$, $X(K_{-1}, \mathbf{E})$. Similarly, the final goods firms recursive optimization problem is:

$$V(I, q, d, \mathbf{E}_{-1})^y = \max_{p, s, y, m, I'} \pi^y + E_{\{q', d', \mathbf{E}, Q|q, d, \mathbf{E}_{-1}\}} [QV(I', q', d', \mathbf{E})] \quad (62)$$

s.t.

$$\pi^y = ps - qm \quad (63)$$

$$s = dp^{-\epsilon} \quad (64)$$

$$y = s + I' - I(1 - \delta_I) \quad (65)$$

$$y = Am^\gamma \quad (66)$$

$$I' \geq 0 \quad (67)$$

$$\mathbf{E}' = \omega^F(\mathbf{E}) \quad (68)$$

Where equation (68) is the perceived law of motion of $\mathbf{E}$. Since firms observe aggregate variables with one period lag, the firm's problem depends on $\mathbf{E}_{-1}$ and not on $\mathbf{E}$ as in the household's problem. Hence, the solution in this case is given by decision rules $p(I, q, d, \mathbf{E}_{-1})$, $s(I, q, d, \mathbf{E}_{-1})$, $y(I, q, d, \mathbf{E}_{-1})$, $m(I, q, d, \mathbf{E}_{-1})$, $I(I, q, d, \mathbf{E}_{-1})$.

Given the assumed information friction the vector of aggregate state variables will be given by:

$$\mathbf{E} = [i, \bar{b}, \Lambda, K_{-1}, i_{-1}, \bar{b}_{-1}, \Lambda_{-1}, K_{-2}]' \quad (69)$$

The household's decision rule for capital accumulation along with the firms' decision rules for inventories induce a law of motion for the aggregate variables $\omega(\mathbf{E})$. In the recursive expectations equilibrium the actual and the perceived law of motions are equal. To economize

on notation, I henceforth let $x(\cdot)$ denote the decision rule for x .

Definition: A recursive competitive equilibrium is defined by pricing functions $\{P(\mathbf{E}), W(\mathbf{E}), q(\mathbf{E})\}$, a law of motion for the aggregate variables $\omega(\mathbf{E})$, and a set of decision rules $\{C(\cdot), K(\cdot), H(\cdot), X(\cdot), s(\cdot), y(\cdot), m(\cdot), I(\cdot), p(\cdot)\}$ with associated value functions $\{U(K_{-1}, \mathbf{E}), V(I, q, d, \mathbf{E}_{-1})\}$ such that:

1. $K(\cdot), C(\cdot), H(\cdot), X(\cdot)$ and $U(K_{-1}, \mathbf{E})$ solve the household's recursive optimization problem, taking as given $P(\mathbf{E}), W(\mathbf{E})$, and $\omega(\mathbf{E})$.
2. $p(\cdot), s(\cdot), y(\cdot), m(\cdot), I(\cdot)$ and $V(I, q, d, \mathbf{E}_{-1})$ solve the final goods firms problem, taking as given $q(\mathbf{E}), P(\mathbf{E}), W(\mathbf{E})$, and $\omega(\mathbf{E})$.
3. $q(\mathbf{E}), P(\mathbf{E}), W(\mathbf{E})$ satisfy (21), (22) and (23).
4. Markets clear

$$H(\cdot) = \int h(j) dj \tag{70}$$

$$K(\cdot) = \int k(j) dj \tag{71}$$

$$Y_t = C(\cdot) + X(\cdot) + I(\cdot) - (1 - \delta_I)I \tag{72}$$

5. The perceived law of motion for the aggregate variables is consistent with the actual law of motion

$$\omega(\mathbf{E}) = \omega^h(\mathbf{E}) = \omega^F(\mathbf{E}) \tag{73}$$

6. The distribution of firms evolves according to

$$\lambda(I', q', d') = \int \mathbf{1}_{\{I(I, q, d)=I'\}} \cdot pr(q' \wedge d'|q, d) \cdot d\lambda(I, q, d) \tag{74}$$

Where $\mathbf{1}_{\{I(I, q, d)=I'\}}$ is an indicator function that is equal to 1 if a firm with initial stock of inventories I , input price q , and demand d , chooses a stock of inventories for the next period equal to I' .

4.2 Computation with Information Frictions

I solve this problem for small deviations around the steady state by following the methodology of Reiter (2009). This has an important implication: the law of motion of the aggregate variables is linear. Denoting $\mathbf{Y}$ as the vector of jumps variables, this economy can be described by the following two equations:

$$\widehat{\mathbf{E}}' = F\widehat{\mathbf{E}} + \mathbf{V} \quad (75)$$

$$\widehat{\mathbf{Y}} = G\widehat{\mathbf{E}} \quad (76)$$

$\widehat{x}$ denotes deviation around the steady state. F and G are coefficient matrices, and $\mathbf{V} \equiv [\varepsilon_i, \varepsilon_b, \mathbf{O}_{1 \times (2 \times ni \times nz + 4)}]'$ is the vector of iid shocks. ni is the number of grid points for the stock of inventories and nz is equal to the number of grid points for the idiosyncratic shocks.

To find the equilibrium of this economy, I start with a guess for matrices F and G . Given this guess, the household's and firm's decision rules induce a law of motion and two new matrices $F^{(new)}$ and $G^{(new)}$. In equilibrium, these matrices have to be equal. If they are not, I update these matrices as follows until the difference is small:

$$F^{(new)} = \theta F^{(old)} + (1 - \theta) F^{(new)} \quad (77)$$

$$G^{(new)} = \theta G^{(old)} + (1 - \theta) G^{(new)} \quad 0 < \theta < 1 \quad (78)$$

On the other hand, it is worth noticing that the final goods firms face a signal extraction problem. They observe their current input price (q) and demand (d) but do not have information about the current aggregate variables. Final goods firms need to form expectations about the evolution of their input prices and demand in order to take their pricing and inventories decisions. To see this notice that:

$$d = \chi D \quad (79)$$

$$q = b^{-1} \bar{q} \quad (80)$$

Where $D \equiv (C + X)P^\epsilon$ is the aggregate nominal demand, and $\bar{q} \equiv \frac{1}{b} \left(\frac{i_{-1}P-1}{\alpha} \right)^\alpha \left(\frac{W}{1-\alpha} \right)^{1-\alpha}$ is the average (aggregate) nominal input price. Since the law of motion for the aggregate variables is linear, I use the Kalman Filter to compute the expectations of the final goods

firms. Taking logs in equations (79) and (80) we get:

$$\log(d) = \log(D^{ss}) + D^{ss}\widehat{D} + \log(\chi) \quad (81)$$

$$\log(q) = \log(\bar{q}^{ss}) + \bar{q}^{ss}\widehat{\bar{q}} - \log(b) \quad (82)$$

x^{ss} denotes the value of x in steady state. Notice that firms observe $\log(d)$ and $\log(q)$, but they do not observe $\widehat{D}$, $\widehat{\bar{q}}$, χ , b . Therefore, this can be express as:

$$\begin{bmatrix} \log(d) \\ \log(q) \end{bmatrix} = \begin{bmatrix} \log(D^{ss}) \\ \log(\bar{q}^{ss}) \end{bmatrix} + \begin{bmatrix} G_D \\ G_{\bar{q}} \end{bmatrix} \widehat{\mathbf{E}} + \begin{bmatrix} \chi \\ b \end{bmatrix} \quad (83)$$

$$\widehat{\mathbf{E}}' = F\widehat{\mathbf{E}} + \mathbf{V} \quad (84)$$

Where G_D and $G_{\bar{q}}$ are the rows of matrix G associated with the jumps variables D and $\bar{q}$. Hence, this can be solved by using the Kalman Filter.

4.3 Economy Dynamics with Information Frictions

Using the same values for the parameters (Table 1), I report the steady state for this economy in Table 3 and the ergodic distribution of inventories in the second panel of Figure 3. The only significant difference between the steady state with perfect and complete information and the steady state with information frictions is the amount of inventories. The stock of inventories represents now 132% of total output. Given that the aggregate uncertainty is greater with information frictions and that the final goods firms Value ($V(I, q, d, \mathbf{E}_{-1})^y$) is strictly concave, firms have more incentives to invest in inventories. Even though the fraction of firms with no inventories increased from 25% to 33%, when a firm faces a positive shock, it accumulates more inventories than before. This is because inventories are an insurance against negative shocks from the point of view of a firm.¹⁰ This is why the fraction of firms with more than 1 stock of inventories increased from 36% to 42%.

4.3.1 Productivity shock

Figure 7 plots the impulse response functions to a 1% increase in the total aggregate productivity.

One of the most striking results is that inventories increase after the productivity shock in the model with information frictions. To explain this, suppose for simplicity that the

¹⁰Using the notation from the consumer theory, firms have a precautionary motive for holding inventories.

idiosyncratic productivity b is distributed uniform.¹¹ This implies that the nominal input price q also distributes uniform between $[q^l, q^u]$ with mean $\bar{q}$ as shown in Figure 1. Firms located between $[b^l, \bar{q}]$ have more incentives to accumulate inventories than those located between $[\bar{q}, q^u]$. After a productivity shock, the average input price $\bar{q}$ decreases to $\bar{q} - \phi$, where $\phi > 0$. Figure 1 also shows how the distribution shifts. Given that firms do not know that the economy has been shocked by a positive productivity innovation, all the firms have incentives to accumulate more inventories. Firms located between $[\bar{q}, q^u - \phi]$ of the new distribution (part a in Figure 1) are on the right hand side of the distribution. However, even though they are at the tail of new distribution, they *think* that the distribution has not change, or at least, they do not have enough evidence to reject that hypothesis. As a consequence, those firms do not sell as much inventories as they should. In the model with perfect information, those same firms *knew* that the economy had been shocked, they *knew* that their input price was relatively high, and they *knew* that the average input price was going to keep decreasing. For those reasons, those firms sell inventories when the economy is shocked by a positive productivity innovation in a model with perfect information. Similarly, firms facing an input price between $[q^l, \bar{q}]$ think that they are facing a good input price with respect to the whole distribution. Therefore, they accumulate more inventories. Finally, firms between $[b^l - \phi, b^l]$ (part c in Figure 1) know that the input price distribution has changed, since their input price has probability zero under the old distribution. Hence, those firms accumulate inventories not only because they know that their input price is low, but also because they have better expectations about the evolution of the economy, and they know that the aggregate demand will keep increasing for another couple of periods. Hence, given that more firms are accumulating inventories, total output increases.

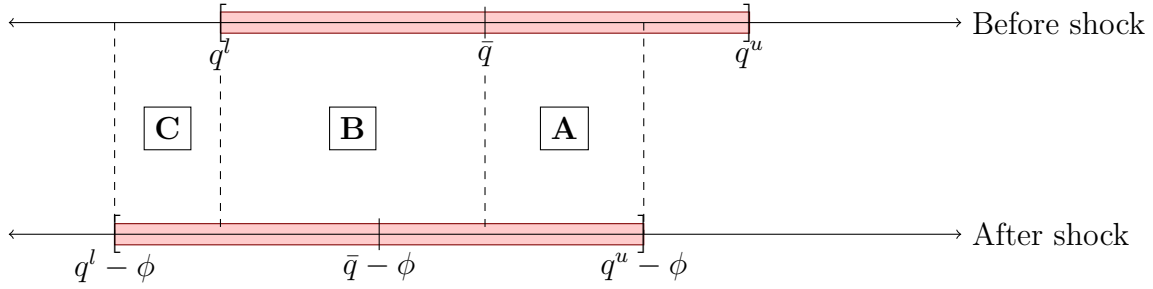
On the other hand, the aggregate price index goes down as the economy is able to produce more goods at a lower price. However, in comparison with the model with perfect information, the magnitude is smaller. This is because the firms at the right tail of the distribution are not selling their inventories. Hence, these firms are setting a higher price. Since firm are accumulating more inventories, current profits decline. This explain why the increase in the aggregate demand is smaller, since household's income is not expanding at a lower rate.

4.3.2 Nominal Shock

Figure 8 plots the impulse response functions of this economy after a 1% increase in the nominal interest rate. After the shock, final goods firms observe a decrease in their nominal

¹¹This actually distributes log-normal, but the interpretation of these results are better understood with an uniform distribution.

Figure 1: Distribution For The Input Price q



Note: This Figure illustrates how the distribution of final goods firms changes after a productivity shock. Assuming that the aggregate productivity distributes uniform, the distribution of the input price is also uniform between $[q^l, q^u]$ (Before shock). After a productivity shock, the distribution shifts to the left (After shock). Firms in part A and B, do not have enough evidence to conclude that the economy was shocked, and they think that they have more incentives to accumulate inventories. Only firms in part C conclude that the distribution changed.

input price and nominal demand. They do not know the source of these changes. They only know that they could be facing a positive productivity shock (aggregate or idiosyncratic), a nominal shock, or a combination of both. Given that there is a good probability that they are facing a positive shock, firms accumulate inventories in the first period. As explained section in 4.3.1 and in Figure 1, this is amplified by firms located at the right tail of the distribution that are not selling their stocks of inventories.

The large increase in inventories reduces the amount of current profits (Π^F), and, as consequence, households' income. Since households want to smooth their consumption, they consume a large fraction of their capital and decide to work more. The positive effect on output (investment inventories) is almost offset by the decline in demand. Even though output increases in the first quarter, the magnitude is insignificant. Hence, the nominal shock has no significant impact on output in the first quarter.

In the second quarter, when firms see that the economy was shocked by an increase in the nominal interest rate, they realized that they made a mistake by accumulating inventories. So they reduce their production in order to sell a large fraction their inventories. However, they don't sell everything in quarter two, since that would imply a large reduction in their output prices. This is why the output peak is in quarter 3.

Given the contraction in output, labor demand falls causing the real wage to go down. As a consequence, households' income decreases, and they consume more of the total capital for another quarter.

When inventory investment and fixed capital investment are added, the dynamics of total investment follows the output dynamics. However, the size of the fluctuations are larger.

Notice that output increases 0.03% in the first quarter while investment goes up by 1.39%. The output and investment peaks are in quarter three when output decreases 0.51% and investment falls 3.51%.

Finally, it is worth pointing out that the output response to a nominal shock is delayed. The output response is only significant one quarter after the shock. Also, most of the peaks are two quarter after the shock, and one quarter after the information friction dies. The economy is shocked in quarter 0, but the maximum responses of output, capital, real interest rate, investment, and real wage to this shock are in quarter 2 and not in quarter 1 when firms know about this shock.

4.3.3 Simulations

Following the experiments by Cooley and Hansen (1995), tables 4 and 5 show the standard deviations, cross-correlations with output, and correlations with the nominal interest rate after simulating the main models of this paper: the model with perfect and complete information (table 4), and the model with information frictions (Table 5). For each table, the economy was simulated for 2,100 quarters, and the first 100 observations were dropped. The artificial series were logged and then detrended by using the Hodrick-Prescott filter. The statistics shown in the tables are averages of statistics. To asses these models, I compare this tables with the numbers reported in Table 7.1 in Cooley and Hansen (1995), which presents business cycles statistics for the U.S. economy for the period 1954:1-1991:2.

It is not surprising that the standard deviations increase in the model with information frictions, since this model adds more uncertainty to the final goods firms, and generates real responses to nominal shocks. Also, the standard deviations of output, consumption, and hours become closer to those reported in Cooley and Hansen (1995).¹² Also, total investment, fixed capital investment, and change in inventories become the most volatile variables in the model with information frictions, which is consistent with the empirical evidence. However, they are two times more volatile than in the data, which implies that the standard deviations in the model should change in order to match these moments. Similarly, even though the price level and inflation are very volatile in both models, prices become more stable in the model with imperfect information. The standard deviation of the price level and inflation is smaller, and they are even smaller when compare to output. This is because firms have more inventories to smooth shocks.

The correlations with output in the model with information frictions are also closer to the

¹²1.72% for output, 0.86% for consumption, 1.59% for hours.

data. In particular, inventory investment is now pro-cyclical in the model with information frictions, and total investment is strongly correlated with output. The correlation between output and consumption reduces, and the correlation between output and hours becomes stronger.

Finally, the second panel of Figure 9 shows a simulated demand and input price for a particular final goods firm, and the first panel shows what the price for this firm would be. The black line is the price that a firm that cannot accumulate inventories would charge, the red line is the price that a final goods firm in the model with perfect information would set, and the blue line is the price that a final goods firm in the model with information friction would charge. Also, Table 6 presents some statistics for this simulation.

One of the purposes of this article was to point out the role of inventories in price-setting decision. Notice that firms' prices follow the evolution of the input prices, but the correlation with demand is not very strong in these models because the production function is not very concave. Notice that the correlation between firms' prices and input prices is very strong (0.998) when firms cannot accumulate inventories. In contrast, when firms can accumulate inventories, this correlation decreases almost 40%. Hence, inventories break the strong relationship between current input prices and current output prices. Also, two important features is that inventories add a lot of persistence. The first autocorrelation of the output price increases from almost zero to 0.55%. As pointed out in this article, inventories are used to smooth the marginal cost of production, which also implies price smoothing in the context of monopolistic competition.

5 Conclusions

In the past decade, much progress has been made on models studying the impact of information frictions on aggregate supply. However, an assumption in the existing literature is that pricing managers do not interact with production managers within firms. If this assumption is relaxed, nominal shocks would not have real effects on the economy. In this paper, I present a model with information frictions, output inventories, and perfect communication within firms where nominal shocks have real effects on the economy. In this model, final goods firms observe aggregate variables with a lag but receive information on their nominal input prices and demand in real time.

I proved that output prices are strictly decreasing in the stock of inventories, strictly increasing in the input price and demand, and weakly increasing in the future input price and

demand. Hence, when a final goods firms face a low input price, it accumulates inventories by producing more. This prevents the firm's price to decrease too much in the current period, and makes the next period price go down. I also showed that inventories impose an upper bound for the increase in the firm's price, which makes firms prices be persistent.

I first solve this model by assuming perfect information. I compare the impulse response functions of this model with those generated by two alternative models. One model with no inventories and no heterogeneous firms, and one model with heterogeneous firms and no inventories. According to these results, heterogeneity and inventories have significant impacts on the economy. On the one hand, heterogeneity generates a substitution effect. When the economy is shocked by a positive productivity innovation, the demand for the most productive firms increases more. This has a positive impact on output, consumption and investment. On the other hand, inventories help low productive firm smooth their idiosyncratic shocks. Therefore, after a productivity shock, they sell a larger fraction of their inventories in order to take advantage of the increase in demand. This has an additional positive impact on consumption and investment. However, the output increase is lower because the less productive firms reduce their production.

Then, I proved that this model does not display non-neutrality of nominal shocks when firms cannot accumulate inventories and there is perfect communication within firms. I also explained that inventories are important to understand this non-neutrality. Since firms accumulate inventories according to their expectations, the current input price and demand are not longer enough to infer the firms' best response. For given input price and demand, a final goods firm will accumulate inventories if it expects the nominal input price to go up. Hence, after a nominal shock, a fraction of the firm make a mistake in their forecast and accumulate inventories. This makes current benefits and households' income go down. As a consequence, households consume a fraction of capital, and the aggregate demand decreases.

According to these results, a nominal shock does not have a significant impact on output in the first quarter. This is because the decline in demand offsets the increase in inventory investment. I also found that most of the responses to nominal shocks are significant, persistent, and hump-shaped. The maximum response of output to a nominal shock is 2 quarters after the shock, when output decreases by 0.5%. Similarly, the maximum response of investment, capital, real interest rate, and real wage is two quarters after the shock.

According to my results, the business cycles properties of the model with information frictions are closer to the estimations by Cooley and Hansen (1995) than those generated by the model with perfect information. In the model with information friction, the standard

deviation of output, consumption, and hours worked are close to those reported by Cooley and Hansen (1995). Also, investment is more volatile, and the standard deviation of the price level and inflation are smaller.

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A Tables and Figures

Table 1: Parameter Values

Parameter	Value	Description
A	1	Aggregate Productivity in the final good sector
γ	0.95	Decreasing returns to scale in the final good sector
δ_I	0	Inventories depreciation
α	0.374	Capital share
δ_K	0.017	Capital depreciation
B	0.8	Aggregate productivity in the intermediate sector
ρ_B	0.9	Persistence of the aggregate productivity shocks
σ_B	0.03	Standard deviation of the aggregate productivity shocks
σ_b	0.10	Standard deviation of the idiosyncratic productivity shocks
β	0.984	Intertemporal discount factor
η	3	Inverse of the Frisch elasticity
σ	2	Intertemporal elasticity of substitution
ϵ	4	Demand Elasticity
σ_χ	0.10	Standard deviation of the preference shocks
ρ_i	0.7	Persistence of the interest shocks
σ_i	0.0025	Standard deviation of the interest rate shocks

Note: This table reports the parameter values used to solve the models of this paper. In the model with no heterogeneity, $\sigma_b = \sigma_\chi = 0$.

Table 2: Steady State Values.
Model with Perfect and Complete Information

Variable	Value	Description
Y	0.80	Output
C	0.71	Consumption
I	0.88	Inventories
K	6.42	Capital
P	0.84	Price index
W	1.00	Nominal Wage
H	0.30	Labor Supply
$\frac{I}{Y}$	1.10	Inventories-Output ratio
$\frac{K}{Y}$	8.01	Capital-Output ratio

Note: This table reports the steady state values for the endogenous model variables in the model with perfect and complete information.

Table 3: Steady State Values.
Model with Information Frictions

Variable	Value	Description
Y	0.80	Output
C	0.71	Consumption
I	1.06	Inventories
K	6.44	Capital
P	0.84	Price index
W	1.00	Nominal Wage
H	0.30	Labor Supply
$\frac{I}{Y}$	1.32	Inventories-Output ratio
$\frac{K}{Y}$	8.01	Capital-Output ratio

Note: This table reports the steady state values for the endogenous model variables in the model in which final goods firms do not have information about current aggregate variables.

Table 4: Business Cycles Statistics: Model with Complete and Perfect Information

Variable	SD(%)	Relative	Cross Correlation of Output with								Corr with i	
			x(-4)	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)		x(+4)
Output	0.342	1.000	0.119	0.296	0.528	0.792	1.000	0.792	0.528	0.296	0.119	0.039
Consumption	0.139	0.407	0.291	0.431	0.601	0.784	0.965	0.715	0.419	0.166	-0.021	0.060
Investment	1.915	5.595	0.022	0.214	0.473	0.773	0.989	0.810	0.572	0.359	0.193	0.027
Capital	0.223	0.652	0.422	0.541	0.673	0.804	0.893	0.612	0.305	0.050	-0.134	0.072
Hours	0.026	0.076	-0.389	-0.182	0.126	0.505	0.705	0.689	0.614	0.531	0.448	-0.032
Price level	11.46	33.49	-0.049	-0.066	-0.073	-0.061	-0.063	-0.033	-0.003	0.035	0.056	-0.989
Inflation	11.16	32.62	0.017	0.006	-0.012	0.002	-0.031	-0.031	-0.040	-0.022	-0.014	-0.476
Fixed Inv	0.837	2.447	-0.187	-0.208	-0.202	-0.127	0.502	0.538	0.438	0.312	0.221	0.008
Change Inv	0.761	2.224	0.211	0.279	0.334	0.324	-0.317	-0.398	-0.345	-0.257	-0.197	-0.002
Real Int rate	0.009	0.026	-0.295	-0.181	0.004	0.264	0.759	0.749	0.625	0.483	0.369	-0.007
Nom i	3.328	9.725	0.035	0.049	0.053	0.041	0.039	0.017	-0.005	-0.036	-0.052	1.000

Note: This table presents the standard deviations, cross-correlation with output, and correlations with the nominal interest rate after simulating the model with perfect and complete information. The economy was simulated for 2,100 quarters, and the first 100 observations were dropped. The artificial series were logged and then detrended by using the Hodrick-Prescott filter. "Relative" is the relative standard deviation with respect to output. Fixed Inv- Fixed capital investment; Change Inv- Change in inventories; Real Int rate- Real Interest rate; Nom i - Nominal interest rate.

Table 5: Business Cycles Statistics: Model with Information Frictions

Variable	SD(%)	Relative	Cross Correlation of Output with								Corr with i	
			x(-4)	x(-3)	x(-2)	x(-1)	x	x(+1)	x(+2)	x(+3)		x(+4)
Output	2.223	1.000	-0.120	0.216	0.462	0.738	1.000	0.738	0.462	0.216	0.019	-0.367
Consumption	0.536	0.241	-0.207	-0.050	0.097	0.292	0.573	0.962	0.759	0.490	0.243	-0.965
Investment	18.09	8.140	-0.079	0.253	0.486	0.741	0.965	0.587	0.330	0.122	-0.038	-0.179
Capital	7.688	3.459	-0.244	-0.023	0.168	0.408	0.700	0.983	0.765	0.496	0.250	-0.907
Hours	0.460	0.207	-0.010	0.299	0.505	0.716	0.853	0.286	0.078	-0.051	-0.135	0.167
Price level	10.29	4.631	-0.271	-0.205	-0.103	0.058	0.345	0.869	0.750	0.544	0.339	-0.991
Inflation	10.12	4.552	-0.022	-0.104	-0.163	-0.293	-0.534	0.121	0.210	0.209	0.186	-0.467
Fixed Inv	34.01	15.30	-0.132	-0.267	-0.336	-0.403	-0.380	0.332	0.399	0.358	0.288	-0.705
Change Inv	34.92	15.71	0.125	0.272	0.350	0.427	0.415	-0.296	-0.374	-0.343	-0.282	0.679
Real Int rate	0.177	0.080	0.171	-0.141	-0.378	-0.677	-0.968	-0.779	-0.513	-0.267	-0.065	0.463
Nom i	3.329	1.498	0.268	0.193	0.089	-0.076	-0.367	-0.880	-0.754	-0.539	-0.329	1.000

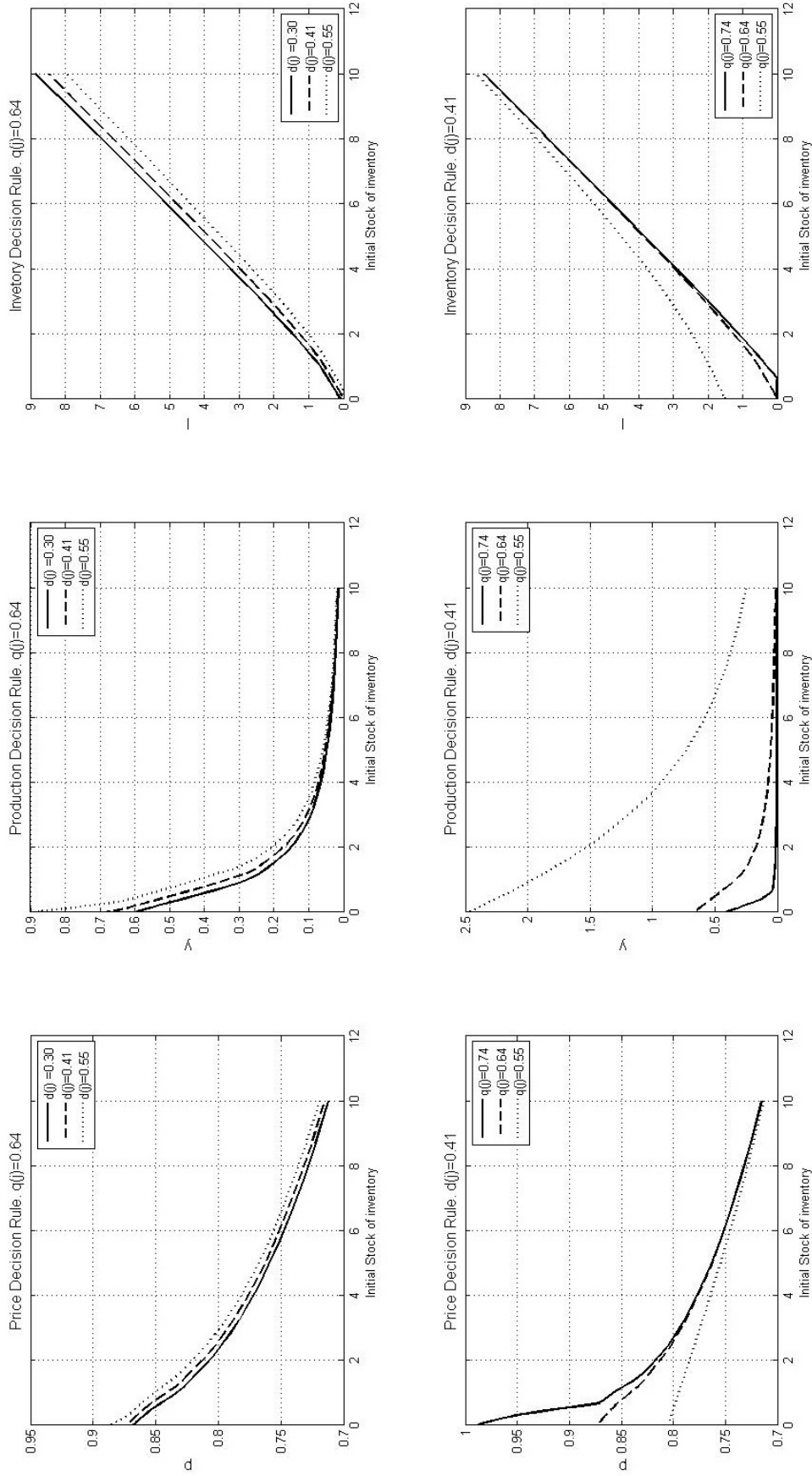
Note: This table presents the standard deviations, cross-correlation with output, and correlations with the nominal interest rate after simulating the model with information frictions. In this model, final goods firms observe aggregate variables with one period lag. The economy was simulated for 2,100 quarters, and the first 100 observations were dropped. The artificial series were logged and then detrended by using the Hodrick-Prescott filter. "Relative" is the relative standard deviation with respect to output. Fixed Inv- Fixed capital investment; Change Inv- Change in inventories; Real Int rate- Real Interest rate; Nom i - Nominal interest rate.

Table 6: Price Statistics for a Simulated Final Goods Firm

Model	Mean	St. Dev	First Auto- correlation	Correlation with q(j)	d(j)
No Inventories	0.878	0.079	0.031	0.998	-0.067
No Information Frictions	0.851	0.062	0.548	0.673	-0.049
Information Frictions	0.845	0.061	0.551	0.626	0.069

Note: This table present the mean, standard deviation, first autocorrelation, and the correlation with the input price and demand for a final goods firm. No inventories- price charged by a final goods firm in a model in which firms cannot accumulate inventories and have perfect information. No Information Frictions- price charged by a final goods firm in a model in which firms can accumulate inventories and have perfect and complete information. Information Frictions- price charged by a final goods firm in a model in which firms observe aggregate variables with a lag and can accumulate inventories.

Figure 2: Final Goods Firms' Decision Rules



Note: This Figure shows the final goods firms decision rules in steady state in a model with perfect and complete information. $q(j)$ - Nominal input price, $d(j)$ - Nominal demand, p - firm's output price, y - Firm's production, i - end of period inventories.

Figure 3: Ergodic distributions

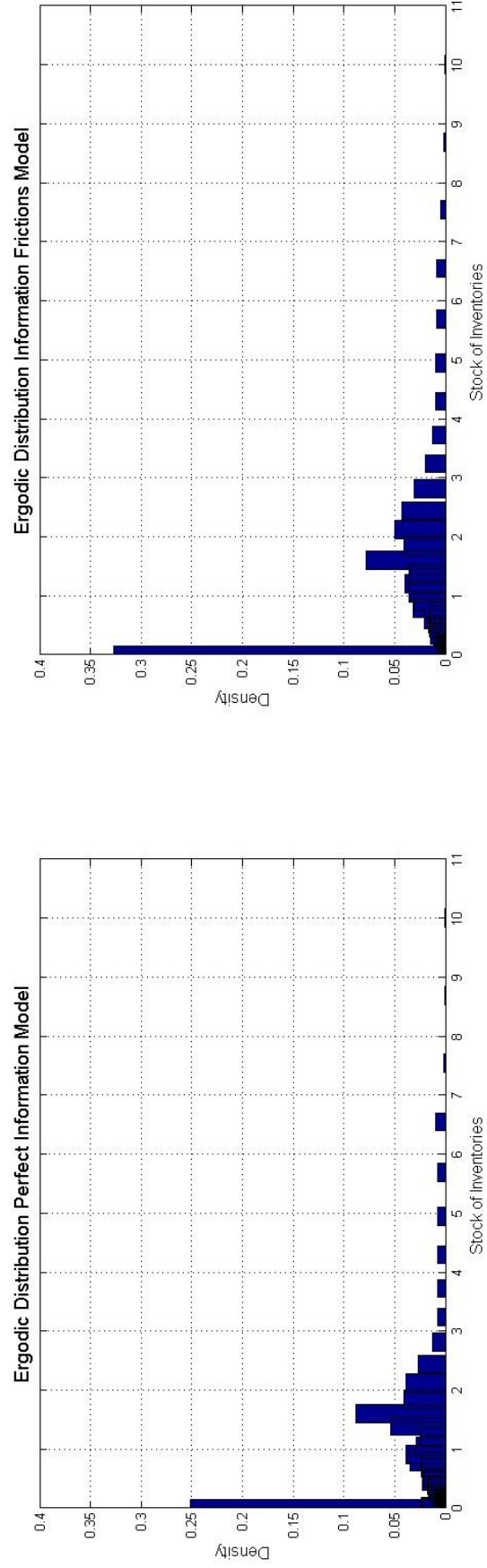
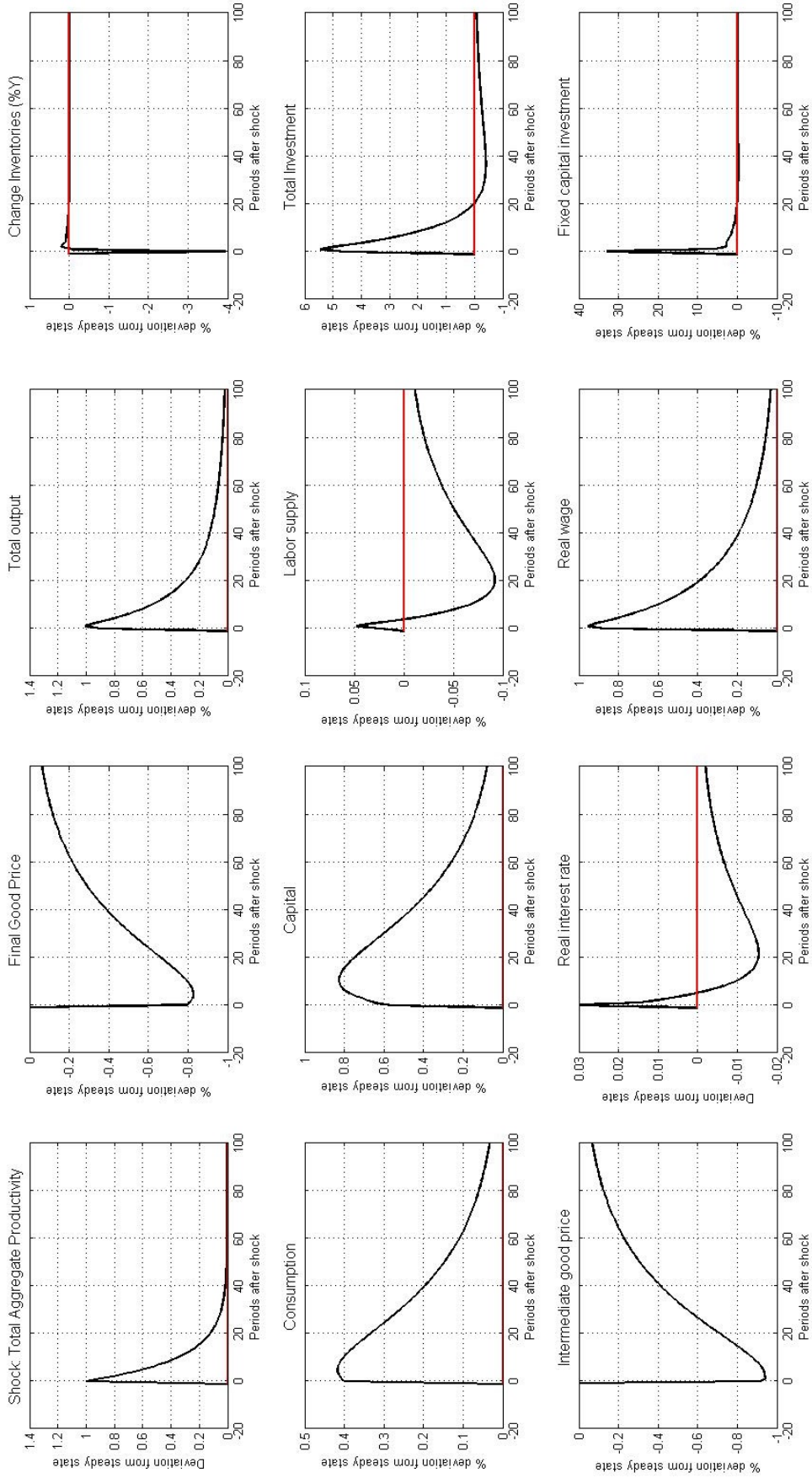


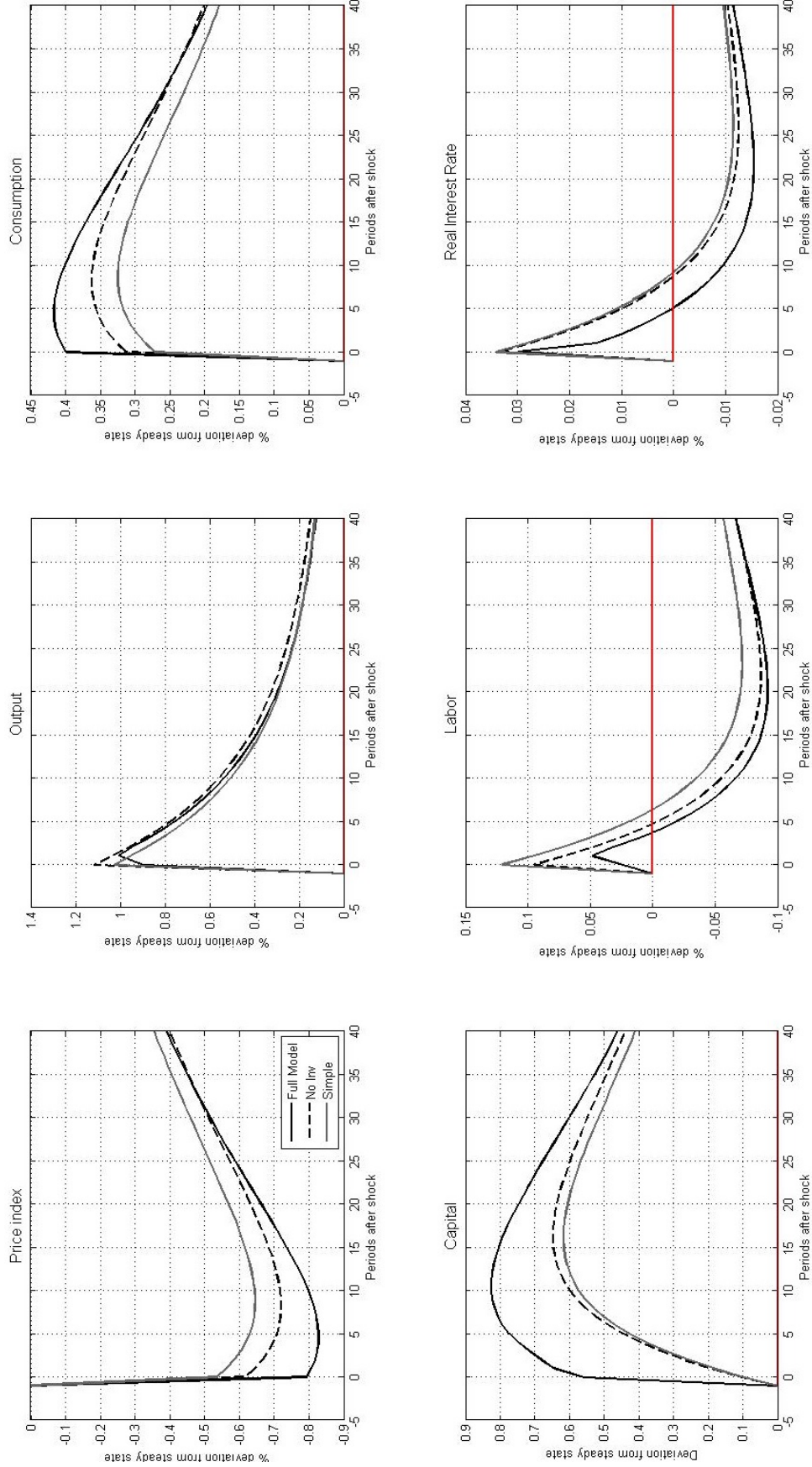
Figure 4: Note: This Figure plots the ergodic distribution of inventories in a model with perfect and complete information (left Figure) and in a model with information frictions (right Figure).

Figure 5: Impulse Response Functions to a Productivity Shock. Model with Perfect and Complete Information



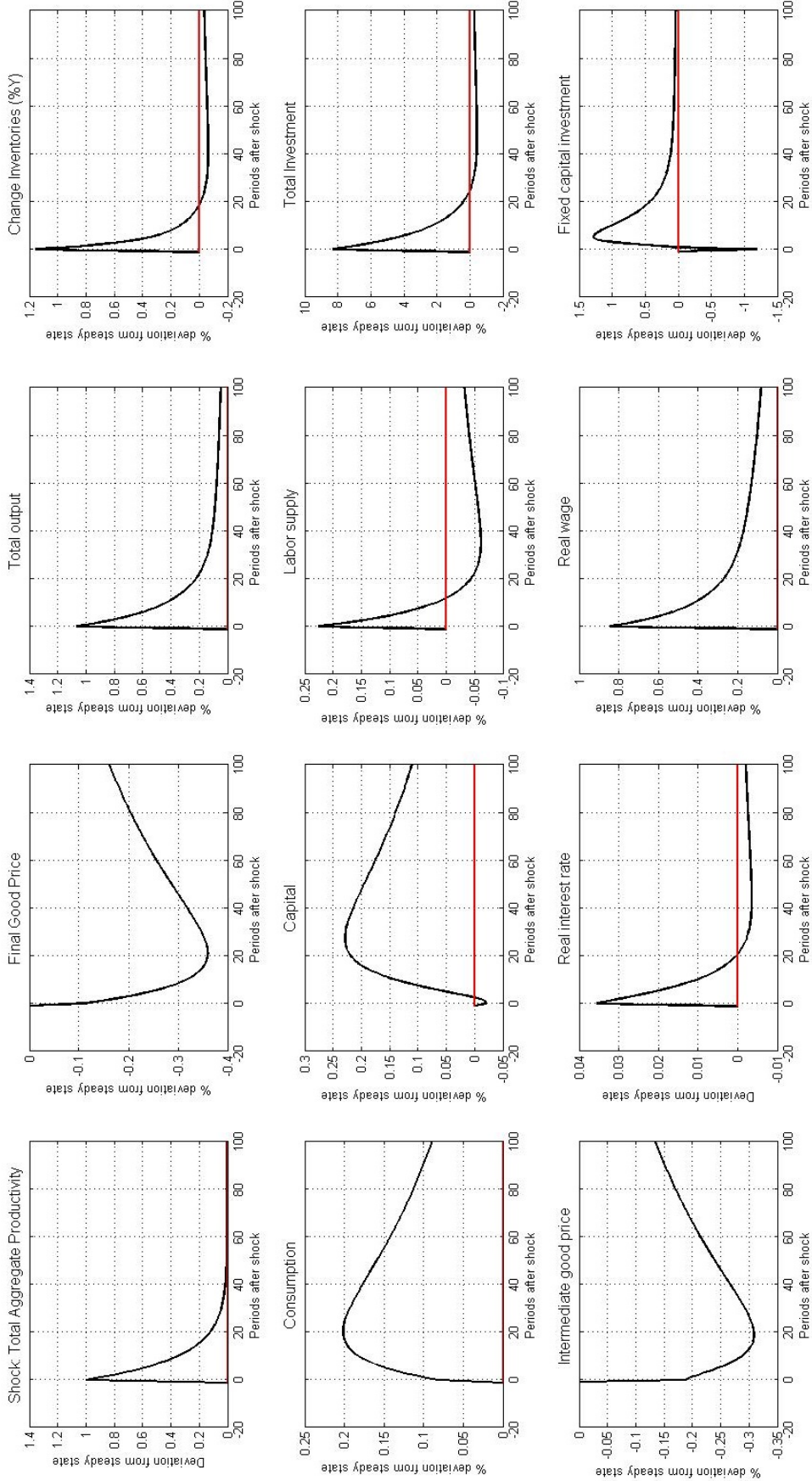
Note: This Figure plot the impulse response functions to a 1% increase in total aggregate productivity, $\bar{b}$ in a model with perfect and complete information. All figures are deviations with respect to the deterministic steady state. Change in inventories is the change in inventories as a fraction of output in steady state. Total investment is the sum of fixed capital investment and inventory investment. Intermediate good price is the average of the input prices.

Figure 6: Impulse Response Functions to a Productivity Shock. Three Different Models with Perfect and Complete Information



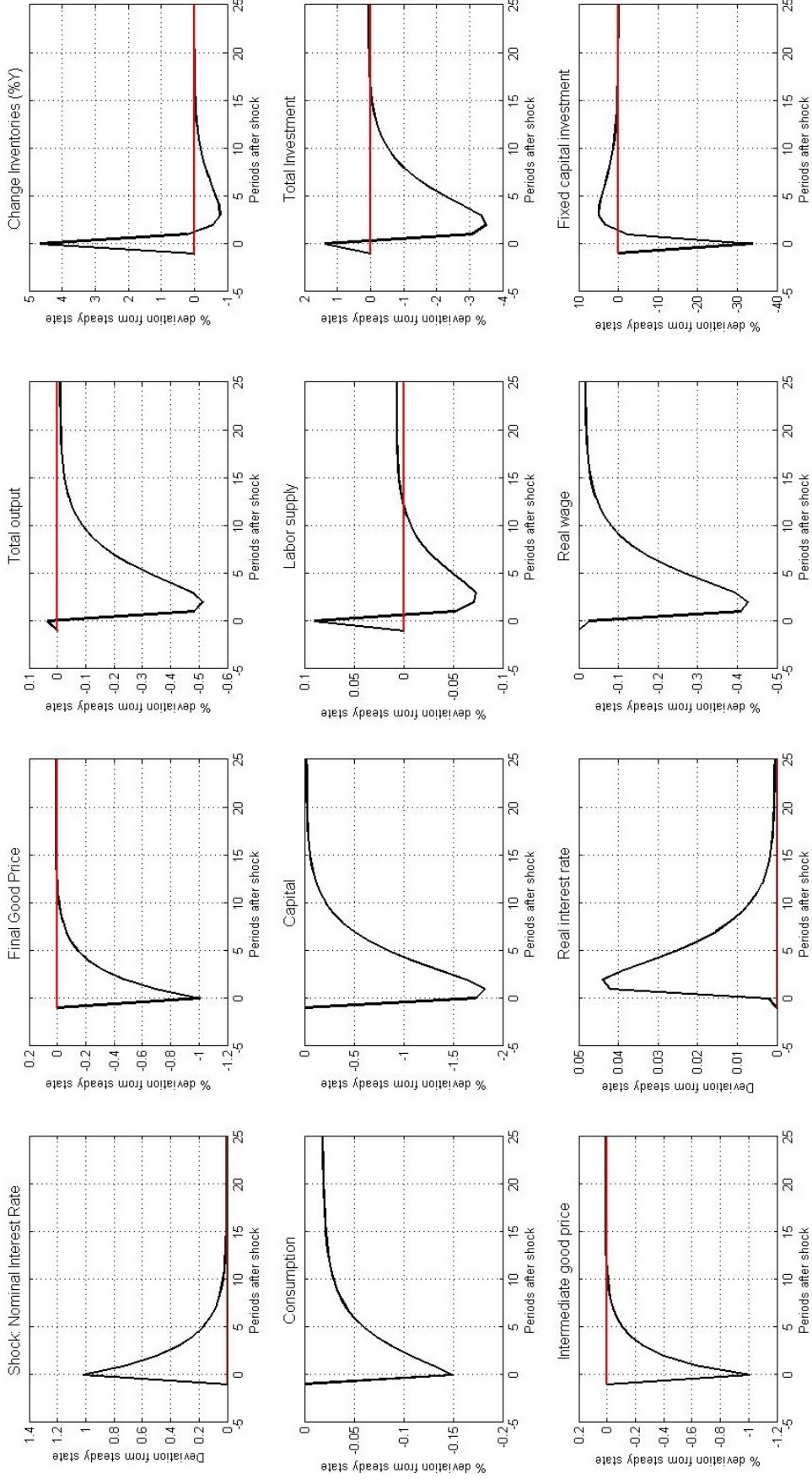
Note: This Figure compares the impulse response function to a 1% increase in the total aggregate productivity for three different models with perfect and complete information. Full model- model with heterogeneous firms and output inventories. No inv- Model with heterogeneous firms and no inventories. Simple- Model with no heterogeneous firms and no output inventories. Figures are deviations with respect to the deterministic steady state.

Figure 7: Impulse Response Functions to a Productivity Shock. Model with Information Frictions



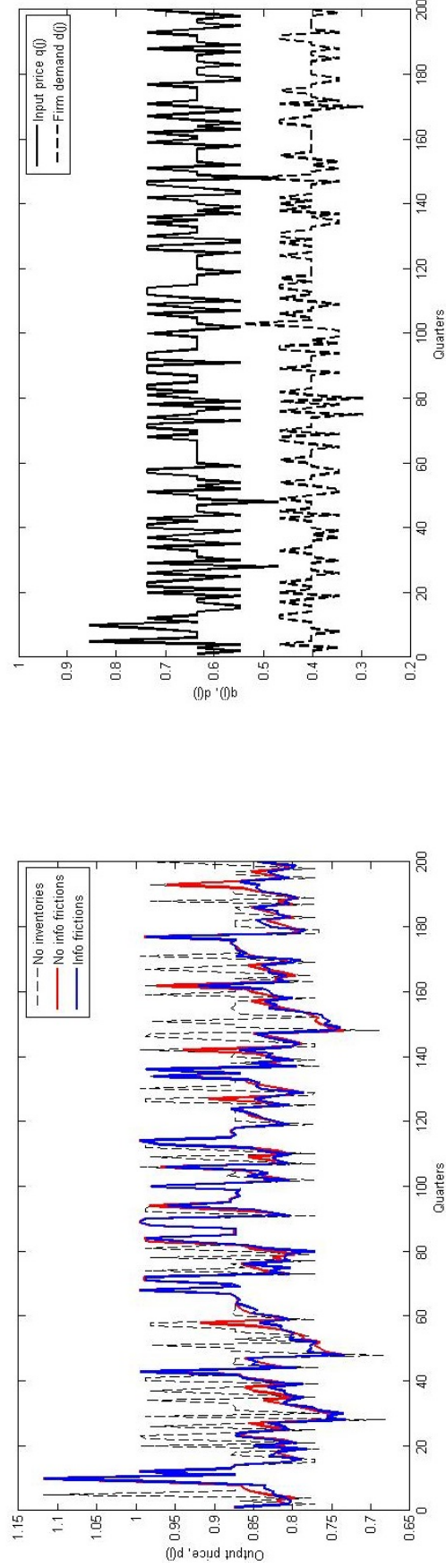
Note: This Figure plots the impulse response functions to a 1% increase in the total aggregate productivity in a model in which final goods firms observe aggregate variables with one period lag. All figures are deviations with respect to the steady state. Change in inventories is the change in inventories as a fraction of output in steady state. Total investment is the sum of fixed capital investment and inventory investment. Intermediate good price is the average of the input prices.

Figure 8: Impulse Response Functions to a Nominal Shock. Model with Information Frictions



Note: This Figure plots the impulse response functions to a 1% increase in the nominal interest rate in a model in which final goods firms observe aggregate variables with one period lag. All figures are deviations with respect to the steady state. Change in inventories is the change in inventories as a fraction of output in steady state. Total investment is the sum of fixed capital investment and inventory investment. Intermediate good price is the average of the input prices.

Figure 9: Simulated Price for Three Different Models



Note: This Figure plots the simulated output price charged by a particular final goods firms in three different models. The right panel plot the simulated input price and demand. The left panel shows the price charged by the final goods firm. No inventories- The final goods firm cannot accumulate inventories. No info frictions- firm can accumulate inventories and has perfect and complete information. Info frictions- firm can accumulate inventories but observes aggregate variables with one period lag.

B Computation of the Model With Perfect and Complete Information

I approximate the model by assuming that the idiosyncratic shocks, b and χ , and the inventories holdings, I , can only take values on the grids $\Gamma^b = \{b^1 \dots b^{nb}\}$, $\Gamma^\chi = \{\chi^1 \dots \chi^{n\chi}\}$, and $\Gamma^I = \{0, I^2 \dots I^{ni}\}$. I find the transition probability matrices Π^b and Π^χ for b and χ using the Tauchen's method. Defining the variable $z \in \Gamma^z = \{z^1 \dots z^{nz \equiv nb \times n\chi}\}$ such that: $z = z^r$ if $b = b^{ceil(r/n\chi)}$ and $\chi = \chi^{mod(r, n\chi)}$, I specify the time varying distribution matrix $\mathbf{\Lambda}_t$ of size $(ni \times nz)$ such that the row l , column r element represents the fraction of firms in state (I^l, z^r) .¹³ Following Costain and Nakov (2011), given the decision rule $I(I, z) = \arg \max_{\{I' \in R^+\}} V(I', I, z)^y$, inventories holdings are kept on the grid Γ^I by rounding $I(I, z)$ up or down stochastically without changing the mean. Specifically, for each $w \in \{1, 2, \dots, nz\}$, define matrix $\mathbf{R}^w$ of size $(ni \times ni)$ as:

$$\mathbf{R}^w = \begin{cases} \frac{I_t^{l(r,w)} - I_t^{*r,w}}{I_t^{l(r,w)} - I_t^{l_t(r,w)-1}} & \text{in column } r, \text{ row } l_t(r, w) - 1 \\ \frac{I_t^{*r,w} - I_t^{l_t(r,w)-1}}{I_t^{l_t(r,w)} - I_t^{l_t(r,w)-1}} & \text{in column } r, \text{ row } l_t(r, w) \end{cases} \quad (85)$$

Where

$$I_t^{*r,w} = \arg \max_{\{I' \in R^+\}} V(I', I = I^r, z = z^w)^y \quad (86)$$

$$I_t^{l_t(r,w)} = \min \{I \in \Gamma^I : I \geq I_t^{*r,w}\} \quad (87)$$

Hence, the evolution of $\mathbf{\Lambda}_t$ can be computed as:

$$vec(\mathbf{\Lambda}_{t+1}) = (\Pi^{z'} \otimes \mathbf{I}_{ni}) \times \mathbf{R} \times vec(\mathbf{\Lambda}_t) \quad (88)$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}^1 & \mathbf{0}_{ni} & \cdots & \mathbf{0}_{ni} \\ \mathbf{0}_{ni} & \mathbf{R}^2 & \cdots & \mathbf{0}_{ni} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{ni} & \mathbf{0}_{ni} & \cdots & \mathbf{R}^{nz} \end{bmatrix} \quad (89)$$

¹³Being more precise, $\Gamma^z = \{(b^1, \chi^1), (b^1, \chi^2) \dots (b^2, \chi^1), (b^2, \chi^2) \dots (b^{nb}, \chi, 1) \dots (b^{nb}, \chi^{n\chi})\}$, and its transition probability matrix is given by $\Pi^z = \Pi^b \otimes \Pi^\chi$

Where $\mathbf{I}_{ni}$ is the identity matrix of size ni .¹⁴ Similarly, the value function $V(I, z)_t^y$ is written as matrix $\mathbf{V}_t^y$ of size $(ni \times nz)$ such that.

$$vec(\mathbf{V}_t^y) = vec(\mathbf{U}_t) + \mathbb{E}_t [Q \times \mathbf{R}' \times (\Pi^z \otimes \mathbf{I}_{ni}) \times vec(\mathbf{\Pi}_{t+1}^F)] \quad (96)$$

Where $\mathbf{U}_t$ is the $(ni \times nz)$ matrix of current profits whose row l , column r element is given by

$$\mathbf{U}(I^l, z^r)_t = (C_t + K_t)\chi(z^r) \left(\frac{P_t}{p_t^{*l,r}} \right)^\epsilon - p_x(z^r)_t x_t^{*l,r} \quad (97)$$

Where $\chi(z^r)_t$ and $p_x(z^r)_t$ are the values of χ and p_x consistent with $z = z^r$. It is worth pointing out that the expectation in equation (96) is over the aggregate shocks of the economy. The expectation over the evolution of z is written explicitly by multiplying by Π^z . Hence, the vector of aggregate variables is given by:

$$\vec{X}_t \equiv \left\{ vec(\mathbf{\Lambda}_{t-1}), K_{t-1}, I_t, \tilde{I}_t, i_t, C_t, Y_t, H_t, \pi_t, P_t, W_t, b_t, r_t, vec(\mathbf{\Pi}_t^F) \right\} \quad (98)$$

Vector $\vec{X}_t$ along with the vector of shocks $\vec{Z}_t = (Z_{i,t}, Z_{b,t})$ consist of $2(ni \times nz) + 11$

¹⁴ define $\tilde{\mathbf{\Lambda}}_t$ such that:

$$vec(\tilde{\mathbf{\Lambda}}_t) = \mathbf{R} \times vec(\mathbf{\Lambda}_t) \quad (90)$$

$$\tilde{\mathbf{\Lambda}}_t^{(w)} = \mathbf{R}^w \times \mathbf{\Lambda}_t^{(w)} \quad (91)$$

Where $\mathbf{X}_t^w$ is the column w of matrix $\mathbf{X}_t$, and $\mathbf{0}_x$ is the zeros matrix of size nz . Hence, the row k , column w element of matrix $\tilde{\mathbf{\Lambda}}_t$ represents the fraction of firms in state $z = z^w$ that, regardless of their initial inventories holdings, have an stock of inventories equal to I^k at the end of period t . Therefore, $\mathbf{\Lambda}_{t+1}$ can also be written as:

$$\mathbf{\Lambda}_{t+1} = \tilde{\mathbf{\Lambda}}_t \times \Pi^z \quad (92)$$

$$vec(\mathbf{\Lambda}_{t+1}) = vec(\tilde{\mathbf{\Lambda}}_t \times \Pi^z) \quad (93)$$

$$vec(\mathbf{\Lambda}_{t+1}) = (\Pi^{z'} \otimes \mathbf{I}_{ni}) \times vec(\tilde{\mathbf{\Lambda}}_t) \quad (94)$$

$$vec(\mathbf{\Lambda}_{t+1}) = (\Pi^{z'} \otimes \mathbf{I}_{ni}) \times \mathbf{R} \times vec(\mathbf{\Lambda}_t) \quad (95)$$

Where $\mathbf{I}_{ni}$ is the identity matrix of size ni .

variables that are determined by the following system of equations

$$C_t^{-\sigma} = \beta \mathbb{E} [r_{t+1} C_{t+1}^{-\sigma}] \quad (99)$$

$$\Psi H_t^\eta = \frac{W_t}{P_t} C_t^{-\sigma} \quad (100)$$

$$\frac{H_t}{K_{t-1}} = \left(\frac{r_t - (1 - \delta_k)}{W_t/P_t} \right) \left(\frac{1 - \alpha}{\alpha} \right) \quad (101)$$

$$C_t + K_t + \tilde{I}_t = Y_t + (1 - \delta_k) K_{t-1} + I_t \quad (102)$$

$$P_t^{1-\epsilon} = \mathbf{e}_{\mathbf{ni}}' [\chi(z) p(I, z)_t^{1-\epsilon} * \mathbf{\Lambda}_t] \mathbf{e}_{\mathbf{nz}} \quad (103)$$

$$I_t = \mathbf{e}_{\mathbf{ni}}' \left[\frac{p(I, z)_t}{P_t} * (\Gamma^I \mathbf{e}_{\mathbf{nz}}) * \mathbf{\Lambda}_t \right] \mathbf{e}_{\mathbf{nz}} \quad (104)$$

$$\tilde{I}_t = \mathbf{e}_{\mathbf{ni}}' \left[\frac{p(I, z)_t}{P_t} * \tilde{I}(I, z)_t * \mathbf{\Lambda}_t \right] \mathbf{e}_{\mathbf{nz}} \quad (105)$$

$$Y_t = \mathbf{e}_{\mathbf{ni}}' \left[\frac{p(I, z)_t}{P_t} * y(I, z)_t * \mathbf{\Lambda}_t \right] \mathbf{e}_{\mathbf{nz}} \quad (106)$$

$$\text{vec}(\mathbf{\Pi}_t^{\mathbf{F}}) = \text{vec}(\mathbf{U}_t) + \mathbb{E}_t \left[\frac{1}{i_t} (\Pi^z \otimes \mathbf{I}_{\mathbf{ni}}) \times \mathbf{R}' \times \text{vec}(\mathbf{\Pi}_{t+1}^{\mathbf{F}}) \right] \quad (107)$$

$$\text{vec}(\mathbf{\Lambda}_{t+1}) = (\Pi^{z'} \otimes \mathbf{I}_{\mathbf{ni}}) \times \mathbf{R} \times \text{vec}(\mathbf{\Lambda}_t) \quad (108)$$

$$\pi_{t+1} = \frac{P_{t+1}}{P_t} \quad (109)$$

$$r_{t+1} = \frac{i_t}{\pi_{t+1}} \quad (110)$$

$$\log(\bar{b}_t) = \log(B) + Z_{\bar{b},t} \quad (111)$$

$$\log(i_t) = \log(1/\beta) + Z_{i,t} \quad (112)$$

$$Z_{\bar{b},t+1} = \rho_b Z_{\bar{b},t} + \varepsilon_{b,t} \quad (113)$$

$$Z_{i,t+1} = \rho_i Z_{i,t} + \varepsilon_{i,t} \quad (114)$$

Following the notation of Costain and Nakov (2011), this set of equations form a first-order system of the form:

$$\mathbb{E}_t \mathcal{F} \left(\vec{X}_{t+1}, \vec{X}_t, \vec{Z}_{t+1}, \vec{Z}_t \right) = 0 \quad (115)$$

This system can linearized by computing numerically the jacobian matrices at the deterministic steady state, in order to express this system as a first-order linear expectational

difference equation system:

$$\mathbb{E}_t \mathcal{A} \Delta \vec{X}_{t+1} + \mathcal{B} \Delta \vec{X}_t + \mathbb{E}_t \mathcal{C} \vec{Z}_{t+1} + \mathcal{D} \vec{Z}_t = 0 \quad (116)$$

Where $\mathcal{A} \equiv D_{\vec{X}_{t+1}} \mathcal{F}^*$, $\mathcal{B} \equiv D_{\vec{X}_t} \mathcal{F}^*$, $\mathcal{C} \equiv D_{\mathbf{E}_{t+1}} \mathcal{F}^*$, $\mathcal{D} \equiv D_{\mathbf{E}_t} \mathcal{F}^*$. Then this system of equations can be solve using the QZ decomposition describe in Klein(2000).