Dynamics of Factor Productivity Dispersions*

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Abstract

This paper documents a new set of stylized facts on the joint distribution of labor and capital productivity across plants. We exploit panel data from Germany, Chile, Colombia and Indonesia and show that the basic patterns are similar in all economies. Decomposing factor productivities into high and low frequency movements, we reveal two new stylized facts. First, factor productivities are positively correlated at high frequency, while they are negatively correlated at low-frequency. Second, differences in factor productivity dispersions across countries are mostly at high frequency, while at low frequency the dissimilarity is rather small. We suggest a new structural explanation for productivity dispersions based on putty-clay technology. Our model implies the coexistence of different technologies at any point in time, which gives rise to productivity dispersions. We demonstrate that this model is able to explain our new stylized facts.

Keywords: Productivity, Putty-clay, Heterogeneous plants.
JEL Classification Numbers: D2, E2, L1, O3, O4.

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1 Introduction

In a frictionless economy, marginal factor productivities should be equal across plants. In reality, however, there are large differences in capital and labor productivities even within the same narrowly-defined industry. A recent economic literature suggests that reallocation of production factors may therefore increase aggregate productivity and that mis-allocation could explain differences in income per capita across countries (see Foster et al. (2001) and Hsieh and Klenow (2009)). In our paper, we contribute to this literature on productivity dispersions, both empirically and theoretically.

Empirically, we decompose observed plant-level factor productivity into movements at high and low frequency. Our frequency decomposition allows us to better understand the structural reasons of productivity dispersions. Transitory differences in productivity should relate to different frictions than persistent ones: For example, time-to-build frictions, price-setting frictions, hiring-costs will all rather affect over a short horizon the optimal size of a plant and its factor mix. Hence, they only lead to short-run (high frequency) deviations from average factor productivities, and, due to their transitory character, the cost of such frictions will be relatively low to the individual plant. By contrast, it should be highly valuable for a plant to eliminate frictions leading to persistent deviations in its marginal factor productivities. Conversely, the theoretical challenge of long-lasting productivity differences is to understand what restricts plants from investing to eliminate the friction. Examples of those lasting frictions could be limited market access, political (dis)favoritism, or missing managerial capacity to run latest technology.

Our empirical analysis uses plant-level data from Germany, Chile, Colombia, and Indonesia. Our decomposition of plant-level factor productivities into high and low frequency movements reveals new stylized facts: First, we find within plant that labor and capital productivities show a negative correlation at low frequency, while they are positively correlated at high frequency. Second, long lasting differences in factor productivities explain the bulk of the total deviations of plant-level factor productivities from the average in all countries. More than 60% (75%) of the productivity variance of labor (of capital) is due to long-lasting deviations.[1] Third, the short lived differences in factor productivities are larger in the developing economies. In other words, differences across countries are at high frequency rather than at low frequency factor productivity dispersions.

Given that low frequency movements explain the bulk of the dispersions in produc-

[1]Previous evidence by Foster et al. (2001) points in a similar direction as our third point.

tivities, the question naturally comes up, why plants do not invest in eliminating these frictions. We link the answer to this question to consumers bearing a part of the burden of the friction and provide a new structural explanation of productivity dispersions building upon the assumption of putty-clay technology. In our model, before opening a new plant, capital and labor inputs are substitutable (putty), while for incumbent plants, technology is fixed (clay). A firm that opens a new plant chooses a technology with a given labor intensity.[2] The labor intensity chosen will reflect their forecast about relative factor prices over the life-time of the plant, which will change over time in response to cyclical shocks and trend growth in the capital intensity of the economy. Consequently, different cohorts of plants will have differently labor intense technologies, and different labor intensities translate into differences in marginal costs. Plants will be closed (and and a new plant will be opened with a new technology) when continuing operations with the old technology is no longer profitable.

We show that this model with putty-clay technology is able to explain all our new findings, while arguably a large class of other models used in the literature misses one or another of the empirical facts. Typically it is the negative low-frequency correlation in capital and labor productivities within a plant which these models have a hard time to explain.

Models with adjustment frictions in production factors may obtain persistent differences in factor productivities across plants - if adjustment is sufficiently costly - but they imply a weakly positive correlation (zero if adjustment costs are on one factor only) across factor productivities and at all frequencies. In these models, firms will typically either over-employ or under-employ both factors relative to factor prices, and adjustment in one factor triggers adjustment of the other (complementary) factor in the same direction, see Dixit (1997). Similarly, models where strategic interactions lead to heterogeneous markups, see Peters (2011), move both factor productivities in the same direction also at low frequency.

While models with financial frictions can generate negative correlations in factor productivities, they have difficulties in making them persistent. There, the negative correlation reflects that a financially constrained firm will typically be less capital intense than an unconstrained firm because the shadow value of its funds is higher and labor intense production aligns revenues and expenses better. Yet, firms tend to out-grow their constraints as shown in Banerjee and Moll (2010) which makes it hard for those models to make productivity differences persistent. Finally, when thinking about the differences

[2]The putty-clay concept goes back to Johansen (1959) and Solow (1962). Recent papers using putty-clay technology include Gilchrist and Williams (2000), Gilchrist and Williams (2005), Kaboski (2005) and Gourio (2011).

in factor productivities in terms of "wedges" as implicit taxes and subsidies, our results imply that revenues are implicitly idiosyncratically taxed with low persistence while implicit capital taxes are strongly persistent.[3] In particular, these implicit distortions on capital ought to be of similar size in Germany and Chile, Colombia, Indonesia.

The theoretical part of our paper contributes to the recently developing literature that tries to model in detail the frictions that give rise to productivity dispersions as endogenous objects. The literature has focused so far on the role of strategic interaction (Peters (2011)), imperfect information (Venkateswaran et al. (2013)), contractual incompleteness (Acemoglu et al. (2007)), and in particular financial frictions (Amaral and Quintin (2010), Banerjee and Moll (2010), Buera et al. (2011), Midrigan and Xu (2013), Moll (2014)). Out of these, most closely related to our paper is Midrigan and Xu (2013), who study the effect of financial frictions in entry and technology adoption decisions and differences in the returns to capital across firms. They assume two sectors with different production technologies, whereas we assume that plants select from a menu of short-run Leontieff production functions. We abstract from financial frictions and focus on the importance of the putty-clay channel. In fact, Gilchrist et al. (2013) shows that differences in usercosts of capital only account for at most one third of the capital productivity dispersion.

Our model has two important implications. First, because of monopolistic competition the technology adoption is inefficient, as part of the higher marginal costs coming from suboptimal technology choice are borne by the consumers through higher prices. Hence, the basic findings of the literature on R&D effort apply – technology adoption is too late compared to first best. Second, because growth is larger in developing economies, the differences in capital intensities between cohorts of plants (in terms of time of technology adoption) will be larger in developing economies, which implies that some of the higher dispersion of factor productivities in developing economies may be an outcome of high growth and not only a cause of low productivity levels.

The remainder of this paper is organized as follows: Section 2 describes our datasets, econometric approach, and provides the results. Section 3 sets up our plant model and analyzes the model mechanisms. Section 4 contains the model calibration and the implied productivity moments of our model. Finally, Section 5 concludes. An Appendix follows which provides more details on the data, robustness results and model solution.

[3] See Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Yang (2012), Bartelsman et al. (2013) for papers in the misallocation literature studying the role of "wedges" in productivity differences.

2 A new set of stylized facts

We first document the differences in factor productivities across plants. In particular, we look at dispersions in the revenue-productivities of capital, K_{it}, and labor, L_{it}, across plants i at time t, where we standardize the factor inputs by calculating their revenue productivity in terms of the value of inputs.

Our measures for labor productivity and capital productivity are

$$\alpha_{it}^{L} := \log \frac{p_{it} y_{it}}{w_{it} L_{it}} \quad \text{and} \quad \alpha_{it}^{K} := \log \frac{p_{it} y_{it}}{(r_{it} + \delta_{it}) K_{it}}, \tag{1}$$

respectively, where $p_{it}y_{it}$, is the plant's value added, $w_{it}L_{it}$ is the plants labor costs, r_{it} is the real interest rate, and δ_{it} is the plant-level depreciation rate, which varies per plant as each plant may employ a different composition of capital goods.

We use revenue productivities of a factor in value terms instead of physical productivities y/L and y/K for three reasons: first, we do not always observe output and input prices, second, because this controls for heterogeneity in inputs and outputs.[4] Outputs as well as factor inputs may have different qualities, and compensating price differentials should control for these, see also Hsieh and Klenow (2009). Third and ultimately, we use these measures because economic theory suggest only for the marginal revenue productivities of input factors in value terms that they should all be equal (to one) in a frictionless economy. The average factor productivities that we defined above are proportional to the marginal productivities in case of Cobb-Douglas production functions. For more general production functions, we view the dispersion in average productivities as an approximation to the dispersion in the marginal ones.

2.1 Frequency decomposition

As we argued in the Introduction, different distortions will lead to differently persistent deviations of plant-level productivities from the cross-sectional average. To understand the relative importance of short-lived and long-lasting distortions as well as their differential impact on labor and capital inputs, we decompose observed factor productivities into movements at high and low frequency within one plant.

For this decomposition we use as our benchmark a moving average filter. Per plant, we create sequences of $J = 5$ ($J = 11$) year sub-panels from the data sets and calculate the average log factor productivity for any plant that we observe in $\{t-(J-1)/2, \ldots, t-$

[4]See Foster et al. (2008) for a discussion on the distinction between revenue and physical productivity measures.

$1, t, t+1, \ldots, t+(J-1)/2\}$ in this panel

$$\bar{\alpha}_{it}^{K,L} = \frac{1}{J} \sum_{j=-(J-1)/2}^{(J-1)/2} \alpha_{it+j}^{K,L}.$$

We only consider plants observed for at least J consecutive years. For each plant in the sample, we obtain a series of centered J-year moving averages, which represent the low frequency movements in factor productivities. We then measure high frequency movements in factor productivities, $\hat{\alpha}_{it}^{K,L}$, as the difference between the original factor productivity series and the moving average

$$\hat{\alpha}_{it}^{K,L} = \alpha_{it}^{K,L} - \bar{\alpha}_{it}^{K,L}.$$

This way we can decompose to what extent high and low frequency movements account for the observed total dispersions in factor productivities. The comparison between 5 and 11 years moving averages may reveal how persistent deviations from average factor productivities are.

We also consider a robustness check, where we decompose the productivity series for each plant employing and HP-filter with $\lambda = 6.25$. We obtain very similar results and as one may expect, the results lie roughly between the ones we obtain under the 5 and 11 year moving average filters. We opt for the MA-filter as a baseline, as this filter is linear and lends itself therefore better to the treatment of measurement error.

2.2 Data description

Our evidence comes from plant/firm-level data from Germany, Chile, Colombia and Indonesia.[5] The balance sheet data base of German firms, USTAN, is a private sector, annual firm-level data available for 26 years (1973-1988) with 30,000 plants per year on average. While in the case of Chile, Colombia and Indonesia, we use plant level data from the ENIA survey from 1995-2007, the EAM census from 1977-2001 and the IBS dataset for 1998-2010, respectively. These datasets are focused on manufacturing sector with the exception of Germany that also contains information on agricultural, wholesale, energy and transport sector. It is crucial for our frequency decomposition that the datasets contain sufficiently many years of observation per plant. Our frequency decomposition requires to have at least five consecutive observations per plant, and hence we abstract

[5]Since 98% of the firms in Germany are one-establishment firms, we abstract from the plant/firm difference throughout the rest of this study.

from plants observed for less periods.

When preparing the data for our analysis, we make sure to treat it in the most comparable way. From each survey, we use plant's four-digit industry code (ISIC), wage bill, value-added and book or current value of capital stock. In order to obtain economically consistent capital series for each plant, we re-calculate capital stocks using the Perpetual Inventory Method (PIM). We exploit that in all cases we have information for capital disaggregated into different types, which allows us to control for heterogeneity in capital composition across plants.

Our capital productivity measure requires information on real interest rate and economic depreciation. For the former, we consider the bank overdraft real interest rate (Germany), the average real lending interest rate (Chile and Colombia), and the average working capital real interest rate (Indonesia). As agents make decisions based on their expectation of the real interest rate, we estimate an AR-1 process of this variable. We do not rely on the depreciation reported by plants, that is potentially biased for tax purposes or other reasons, but instead use economic depreciation rates obtained from National Statistics or external studies if the former is not available.

We identify and remove outliers as follows: First, we drop all observations where either levels or changes of labor, capital stock, value added, and levels of factor productivities (all in logs) exceed three standard deviations from the mean. Second, we drop observations, where the sum of factor expenditures over value added is either below 1/3 or above 3. Thus, we exclude plants that report unreasonably large markups or losses.

Finally, before decomposing factor productivities into high and low frequency movements, we remove predictable differences in factor productivities across plants. Specifically, we remove 2-digit industry-year effects from our variables to focus on differences across plants that are purely idiosyncratic. A more detailed description of the data is provided in Appendix A.

2.3 Empirical Results

Table 1 presents the results our main empirical results. It provides standard deviations and correlations of labor and capital productivity at high and low frequencies for all four countries. Overall, the factor productivity dispersions are smallest in the German sample, in line with previous studies that found relatively less dispersion in developed economies, see Foster et al. (2001). However, most of the difference in deviations between countries is at high-frequency.

More striking than the differences across countries are the structural similarities.

Table 1: Second moments of factor productivities at different frequencies.

	std(a_{it}^L)	std(a_{it}^K)	$\rho(a_{it}^L, a_{it}^K)$	std(a_{it}^L)	std(a_{it}^K)	$\rho(a_{it}^L, a_{it}^K)$
	High frequency (5Y MA)			Low frequency (5Y MA)		
DE	0.067	0.119	0.363	0.234	0.716	-0.169
	(0.002)	(0.006)	(0.036)	(0.006)	(0.047)	(0.019)
CL	0.200	0.307	0.458	0.266	0.798	-0.267
	(0.007)	(0.008)	(0.017)	(0.009)	(0.033)	(0.018)
CO	0.146	0.167	0.568	0.303	0.765	-0.144
	(0.002)	(0.002)	(0.008)	(0.007)	(0.024)	(0.014)
ID	0.227	0.337	0.411	0.297	0.774	-0.219
	(0.003)	(0.004)	(0.007)	(0.005)	(0.014)	(0.008)
	High frequency (11Y MA)			Low frequency (11Y MA)		
DE	0.079	0.151	0.352	0.206	0.603	-0.165
	(0.002)	(0.011)	(0.026)	(0.002)	(0.033)	(0.002)
CL	0.223	0.386	0.348	0.204	0.664	-0.337
	(0.012)	(0.021)	(0.029)	(0.014)	(0.051)	(0.037)
CO	0.167	0.216	0.501	0.252	0.694	-0.193
	(0.005)	(0.007)	(0.017)	(0.011)	(0.042)	(0.027)
ID	0.236	0.400	0.336	0.227	0.683	-0.301
	(0.009)	(0.009)	(0.017)	(0.009)	(0.026)	(0.021)

The results are based on applying a five year moving average filter (5Y MA) and an eleven year moving average filter (11Y MA), respectively. Factor productivities are demeaned by 2-digit industry and year and expressed in logs. Standard errors in parentheses are robust clustered standard errors. ρ denotes correlation. DE: Germany, CL: Chile, CO: Colombia, ID: Indonesia.

There is a significant difference in the co-movement of labor and capital productivity at low and high frequencies in all countries. At low frequency, the correlation of productivity across factors is negative, whereas for high frequency it is positive.[6] The different signs of correlations give us more insights about the type of production function needed.

Equally striking is the persistence of deviations in factor productivities. For all countries, the low frequency dispersions are substantially higher than the high frequency ones. For the developing economies, both low and high frequency dispersions are larger but in particular the high frequency dispersions exceed the German ones by a factor of 2-3, while the low frequency dispersions are only larger by at most 30% for labor and 15% for capital. Our findings do not change substantially between the 5 and 11 year moving averages even though the latter may bias the samples towards larger plants that are more likely observed for 11 consecutive years.

2.4 Measurement error

Plant level data on output and inputs is often plagued by measurement errors which exaggerates the true differences between plants. Hence when constructing our productivity measures we should take these errors into account. Some of them simply stem from typos in digitalizing the data as in any other data set. What makes measurement errors more prevalent in plant level data is the fact that balance sheet items and the economic objects we are interested in do not align perfectly. These mis-alignment measurement errors are particularly important for value added and capital expenses, less so in labor expenses, so we abstract from the latter.

For value added, sources of these errors are in particular incorrect pricing of inventories or of intermediate good inputs. A plant's manager may prefer to raise this period's revenues by attaching an above market price to the end-of-period stock of inventories to smooth revenues. Similarly, intermediate goods used in production may not enter at their marginal current prices, e.g. because they are taken from inventories where they entered at historical prices. Yet, these measurement errors in value added should not be particularly persistent. As a simplification, we therefore assume classical measurement error in value added

$$\log py_{it} = \log py^*_{it} + \epsilon^{py}_{it}, \tag{2}$$

where the true value of value added is indicated by a star. Under this assumption, low frequency movements of factor productivities are hardly affected by this measure-

[6]Our results are robust to using the interquartile range and Spearman's rank correlation coefficient instead.

ment error, while it will inflate both the variance and the covariance of high-frequency movements in capital and labor productivities.

For capital, however, the situation is different. There, measurement error affects most likely net investments, not directly the stock of capital.[7] For example, we use the investment price deflator based on a basket of capital types to deflate a particular capital good that may have a different price. Similarly, we may miss extraordinary depreciation when capital goods become defective, which we do not observe. In this case, we use a wrong actual investment rate when estimating capital stocks using a Perpetual Inventory Method (PIM), where we assume all capital goods to depreciate by their economic depreciation rates. Transitory measurement error in investment translates into non-classical measurement error in capital and we can approximate the latter by noting that

$$\begin{aligned}
\log K^*_{it} &= (1-\delta_{it}) \log K^*_{it-1} + I^*_{it}, \qquad (3)\\
I_{it} &= I^*_{it} + \eta_{it}, \quad \eta_{it} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2_\eta),\\
\log K_{it} &= (1-\delta_{it}) \log K_{it-1} + I_{it} = \log K^*_{it} + \epsilon^K_{it},\\
\epsilon^K_{it} &= (1-\delta_{it})\epsilon^K_{it-1} + \eta_{it},
\end{aligned}$$

where K_{it} denotes the observed capital stock, I_{it} is the observed investment rate, and I^*_{it}, K^*_{it} are the respective true values. We assume that η_{it} is independent of revenues, capital, and labor expenses. Thus, measurement error on investment rate is proportional to the capital stock. Measurement error ϵ^K_{it} is persistent and its auto-correlation is given by $(1-\delta_{it}) \leq 1$. Consequently, dispersions of capital productivities are inflated at all frequencies by measurement error, such that the correlation of capital and labor productivities is shrunk towards zero. In fact, it seems hard to rationalize the magnitude of low frequency capital productivity dispersions by any model. Assuming normality, the estimated standard deviations at low frequency imply that the capital productivity of the 5%-most productive plant is roughly 230-260% larger than the 5% least productive plants. For labor we get still high but more plausible differences in the range of 75-100%.

To gauge the extent of measurement errors ϵ^K_{it}, we match a given correlation ω between value added and expenses for capital subtracting measurement error variance from the observed cross-sectional variance of capital expenditures. For expositional purposes, however, it is useful to assume for now that the $\omega = corr\{\log py, \log wL\}$. In this regard, the assumed correlation between capital and output consist in an upper

[7] Measurement error in capital stocks that does not arise from net investment can arise from wrong deflation of the existing capital stocks.

bound of the true correlation, as we assume that the frictions dis-aligning capital and output, and labor and output are the same at low-frequency, i.e. over longer horizons.

The variance of high-frequency measurement error, η_{it}, can then be obtained as

$$Var(\epsilon^K)[1-(1-\bar{\delta})^2],$$

where $\bar{\delta}$ is the average depreciation rate across all firms and years. We will estimate the extent of measurement error in value added and investments by minimizing the distance of the variance covariance matrix of factor productivities between data and structural model.

The adjusted low frequency dispersions and correlations are reported in Table 2. We find that the corrected capital productivity dispersions are only 40% of its uncorrected value, which suggests that measurement error plays an important role. The low frequency correlation is smaller by a factor of two. Interestingly, the effect of measurement error seems highly similar across countries, such that one of our main findings remains: Low frequency dispersions of factor productivities are large and they are negatively correlated across factors.

Table 2: Second moments of factor productivities after adjusting low frequency std(a_{it}^K) and $\rho(a_{it}^L, a_{it}^K)$ for measurement error.

	std(a_{it}^L)	std(a_{it}^K)	$\rho(a_{it}^L, a_{it}^K)$	std(a_{it}^L)	std(a_{it}^K)	$\rho(a_{it}^L, a_{it}^K)$
	Low frequency (5Y MA)			Low frequency (11Y MA)		
DE	0.234	0.286	-0.423	0.206	0.241	-0.413
CL	0.266	0.350	-0.608	0.204	0.266	-0.847
CO	0.303	0.299	-0.369	0.252	0.244	-0.548
ID	0.297	0.247	-0.551	0.227	0.247	-0.834

The results are based on applying a five year moving average filter (5Y MA) and an eleven year moving average filter (11Y MA), respectively. Factor productivities are demeaned by 2-digit industry and year and expressed in logs. ρ denotes correlation. DE: Germany, CL: Chile, CO: Colombia, ID: Indonesia.

3 Model

In this section, we provide a model that explains the positive high frequency correlation and the negative low frequency correlation between labor and capital productivity. The crucial assumption is putty-clay technology. Ex-ante, before entering the economy, a plant chooses its optimal combination of capital and labor, but ex-post, the capital-labor ratio is fixed, which corresponds to Leontieff production technology.

3.1 Technology

There is a constant mass of firms each operating one plant. Firms decide every period whether to continue operations with the current plant, or shut it down and open a new one. Upon opening a new plant, the firm decides on the capital intensity, which is fixed subsequently. In other words, plants are bound to the capital intensity that was chosen upon entry. This is the putty-clay assumption.

On the other side, we assume that the scale of operations is chosen by the plant. A plant i produces output y_{it} at period t using capital K_{it} and labor N_{it} under a Cobb-Douglas constant returns production function

$$y_{it} = K_{it}^{\alpha} N_{it}^{1-\alpha} = k_{it}^{\alpha} N_{it}, \tag{4}$$

where $k_{it} = \frac{K_{it}}{N_{it}}$ denotes capital intensity. The plant faces an iso-elastic demand function for its differentiated product, such that price p_{it} and revenues $p_{it}y_{it}$ are given by

$$p_{it} = z_{it} \frac{y_{it}^{-\xi}}{1-\xi} \,, \qquad p_{it}y_{it} = z_{it} \frac{\left(k_{it}^{\alpha} N_{it}\right)^{1-\xi}}{1-\xi}, \tag{5}$$

respectively. The markup of the monopolistically competitive plant is given by $1/(1-\xi)$ and z_{it} is a demand shifter.

Profits are given by

$$\pi_{it} = \frac{z_{it} \left(k_{it}^{\alpha} N_{it}\right)^{1-\xi}}{1-\xi} - w_t N_{it} - r_t k_{it} N_{it}, \tag{6}$$

where w_t is the wage-rate and r_t denotes the relative factor price. We set $w_t = 1$ as normalization that expresses the model in labor units.

While plants can in principle choose to produce output with factor mixes according to the production function (5), the capital intensity is predetermined by the firm's initial choice. Meanwhile, as we formulate the model around a steady state growth trend where aggregate capital intensity, output and wages grow by a constant annual rate, the plant's technology, defined as its capital intensity, is decreasing by a constant rate relative to the newest technology available. More formally, relative to the balanced growth path, a plant's technology decreases over time

$$k_{it} = (1-\gamma)k_{it-1},$$

where γ is the constant annual rate growth rate of aggregate capital intensity.

The timing of the model is as follows. In every period of operations, the plant produces output goods and incurs fixed costs of production. It commits to its next period's production scale N with a time-to-build of one period. As profitability shocks z and the relative factor price r follow a first order Markov chain, the plant decides on next period's production scale based on its conditional expectations of next period's z and r. The firm may decide to close the plant only after the plant has produced output goods. If a plant is shut down, the firm may open a new plant for the next period. Opening a new plant is costly, and we denote the installment costs by ψ, but it allows the firm to adjust to a newer technology. Additionally, we assume the profitability shock to be at the plant-level, and a newly opened plant draws profitability from its unconditional distribution.

3.2 Dynamic programming problem

The firm makes profits π in every period and following the timing assumed above, the dynamic programming problem is

$$V(k,N,z,r) = \pi(k,N,z,r) + \beta \max\Big\{V^a(r), V^c(k,N,z,r)\Big\}, \tag{7}$$

where V^c denotes the firm value of continuing operations with the incumbent plant

$$V^c(k,N,z,r) = \max_{N'} E_{z',r'} V(k(1-\gamma), N', z', r'), \tag{8}$$

and V^a is value of adjusting technology by opening a new plant (and closing the incumbent plant)

$$V^a(r) = -\psi + \max_{k',N'} E^*_{z',r'} V(k', N', z', r'), \tag{9}$$

where $'$ denotes next period variables. $E_{z,r}$ is the conditional expectation operator, while $E^*_{z,r}$ is the conditional expectation with respect to r, but the unconditional expectation regarding z. We let ψ denote the installment or entry costs of opening a new plant (and closing the old plant). Notice that the firms decides to open a new plant before observing that new plant's profitability shock z. The firm optimally closes an operating plant and opens a new one whenever the expected continuation value falls below the value of technology adjustment. Before starting operation, the firm chooses the optimal capital intensity of a new plant

$$k^*(z,r) = \arg\max_{k',N'} E^*_{z',r'} V(k', N', z', r'). \tag{10}$$

Since a plant's choice of labor input is conditional on producing next period, the labor choice is a static problem. This property is useful as we can solve for the labor policy function outside the dynamic program. In particular, we rewrite the model by defining $\widetilde{V}(k,z,r) = V(k,N,z,r) - \pi(k,N,z,r)$. The transformed incumbent plant's problem is[8]

$$\widetilde{V}(k,z,r) = \beta \max\Big\{V^a(r), V^c(k,N,z,r)\Big\} \tag{11}$$

$$V^c(k,z,r) = \widetilde{\pi}(k,z,r) + \beta E_{z',r'}\widetilde{V}(k(1-\gamma), z', r') \tag{12}$$

$$V^a(k,z,r) = -\psi + \widetilde{\pi}^*(z,r) + \beta \max_{k'} E^*_{z',r'}\widetilde{V}(k', z', r'). \tag{13}$$

Further, we need to specify laws of motion for the two random variables z_{it} and r_t. We assume them to follow log-normal AR(1) processes

$$\log(z_{it}) = \rho_z \log(z_{it-1}) + \epsilon_{it}\ , \qquad \text{where } \rho_z < 1 \text{ and } \epsilon_{it} \sim N(\mu_z, \sigma_z^2), \tag{14}$$

$$\log(r_t) = \rho_r \log(r_{t-1}) + \eta_t\ , \qquad \text{where } \rho_r < 1 \text{ and } \eta_t \sim N(\mu_r, \sigma_r^2), \tag{15}$$

and we consider z_{it} to be conditionally independent across i and t and r_t conditionally independent across t but perfectly correlated across firms.

The latter implies that there some commonality in the capital intensity choice across all plants that are opened at a given point in time. If, for example, the user cost of capital is particularly high relative to the wage rate, then firms will choose low levels of capital intensity for entering plants. On the contrary if r is low, they will tend to choose high levels of capital intensity. Another important driver is the drift, γ, which creates differences in capital intensities between plants and across age-cohorts.

Details on the model solution are presented in Appendix D. We also show that a

[8] We define $\widetilde{\pi}(k,z,r) \equiv \max_{N'} E_{z',r'}\pi(k(1-\gamma), N', z', r')$ and $\widetilde{\pi}^*(z,r) \equiv \max_{k',N'} E_{z',r'}\pi(k', N', z', r')$.

unique solution to the transformed problem exists. The transformed problem is faster to solve since it involves one less state variable.

3.3 Model implications for productivity dispersions

We previously argued that our model should not only be able to generate factor productivity dispersions, but also that the drift in capital intensities - together with putty-clay technology - should generate a negative correlation between capital and labor productivity. In order to understand better the mechanics of our model in generating factor productivity dispersions at different frequencies, we provide some analytical derivation.

Let us first differentiate expected profits, $E(\pi')$, with respect to N'

$$E(z'|z)N'^{-\xi}k'^{\alpha(1-\xi)} = 1 + E(r'|r)k', \tag{16}$$

which expresses optimal labor supply tomorrow as function of capital intensity tomorrow and the stochastic states today. Using above equation and $z'/E(z'|z) = \exp(\epsilon' - \sigma_\epsilon^2/2)$, we can rewrite factor productivities:

$$\alpha^L = \log\left(\frac{p'y'}{w'N'}\right) = \log\left(\frac{1 + E(r'|r)k'}{\exp(-\epsilon' + \sigma_\epsilon^2/2)(1-\xi)}\right) \tag{17}$$

$$\alpha^K = \log\left(\frac{p'y'}{r'K'}\right) = \log\left(\frac{1 + E(r'|r)k'}{\exp(-\epsilon' + \sigma_\epsilon^2/2)r'k'(1-\xi)}\right) \tag{18}$$

Since profitability shocks, ϵ, have no serial correlation by construction, these shocks are are taken up by high frequency movements of factor productivities. In fact, if we abstract from fluctuations in the relative factor price r, at high frequencies it holds that $std(\alpha^L) = std(\alpha^K) = \widetilde{\sigma}_\epsilon$, where we let $\widetilde{\sigma}_\epsilon$ denote the realized dispersion of profitability shocks. On the other hand, k is highly persistent and follows a determinstic trend such that differences in the capital intensity k are captured by the low frequency movements.

Importantly, z and k imply different co-movements of factor productivities. While an increase in z (beyond expectations) lowers both capital and labor productivity, the deterministic decrease in capital intensity increases labor productivity and decreases capital productivity. We can use this to decompose the covariance of factor shares:

$$\begin{aligned} Cov\left[a^L, a^K\right] \quad &= \widetilde{\sigma}_\epsilon^2 + Cov\big\{\log[1 + E(r'|r)k'], \log[1 + E(r'|r)k'] - \log(k')\big\} \\ &\approx \widetilde{\sigma}_\epsilon^2 - E(r'|r)[1 - E(r'|r)]Var(k'). \end{aligned}$$

Above equation shows formally, what we described before. The covariance of the two factor productivities is driven by profitability shocks that lead to positive comovement and differences in capital intensity lead to negative comovement. The first term is obtained at high frequency while the second term is captured by low frequency difference. Next, we will ask to what extent a realistically calibrated version of our model is able to capture the quantitative structures we found in the data.

4 Results

4.1 Calibration

Our model is at yearly frequency. We calibrate the model setting labor elasticity, $\alpha = 1/3$, and assuming $\beta = 0.95$ such that the risk-free interest rate is approximately 5%. ξ is consistent with a markup of 15%. The user cost of capital, when considering real interest rate on working capital loans and the depreciation rate, is in a range between 0.11 and 0.13 in all four countries, and for simplicity we suppose 0.12 holds for all four countries. δ and γ are estimated from national accounts, where the latter is computed as average growth rate of per capita output in constant prices.

Table 3: Parameter values

		Germany	Chile	Colombia	Indonesia
Assigned parameters					
Capital elasticity	α	0.33	/	/	/
Discount factor	β	0.95	/	/	/
Elasticity of demand	ξ	0.15	/	/	/
Persistence of demand shifter	ρ_z	0.95	/	/	/
Persistence of relative factor price	ρ_r	0.50	/	/	/
Estimated outside the model					
User cost of capital (in %)	r	12.0	/	/	/
Depreciation rate (in %)	δ	9.4	3.5	3.5	6.0
Steady state growth rate (in %)	γ	2.2	2.7	2.2	3.5
Std. of demand shifter	σ_z	0.29	0.40	0.41	0.52
Std. of relative factor price	σ_r	0.06	0.08	0.30	0.26

We set the persistency of the profitability shock to 0.95 and we estimate the conditional dispersion of the AR(1) process following Bachmann and Bayer (2011b), which models fluctuations in idiosyncratic risk as fluctuations in the cross sectional standard deviation of firm-specific Solow residual growth.[9] Additionally, we construct time series of relative factor prices[10] in order to estimate the lognormal AR(1) process specified in the setup of our model for each country. Our estimates of the autocorrelation are roughly 50% for all the countries and for better comparability we impose the same 50% for all four countries.

4.2 Baseline model

Table 4: Plant-age distribution

	Germany		Chile		Colombia		Indonesia	
	Data	Model	Data	Model	Data	Model	Data	Model
1 year	5.5	5.8	6.2	7.2	6.2	8.0	6.5	7.5
2 - 5 years	20.4	19.2	23.8	23.7	23.8	24.7	25.4	24.7
6 - 10 years	17.6	18.5	24.1	21.6	24.1	21.8	24.2	22.2
$\geq$ 10 years	56.5	56.3	45.8	47.4	45.8	45.5	43.9	45.7
Entry cost	-	51	-	45	-	33	-	47
RSS	-	2	-	10	-	9	-	9

Note: Entry costs are expressed as percentage-share of average revenues and the plant-age distribution is given by percentage-shares associated with the four age-groups. RSS denotes the sum of squared residuals between empirical and simulated plant-age distribution.

We estimate the fixed cost of operation using the method of simulated moments. The moments to match are four statistics describing the plant-age distribution. Table 4 presents the empirical and model-implied distributions. The empirical plant-age dis-

[9]The measured Solow residual could reflect true firm productivity but with an error. As in Bachmann and Bayer (2011b), we therefore estimate the process assuming that the Solow residual is composed of the true productivity, which follows a random walk, and a white noise component. We remove firm fixed effects and industry-year effects from the first-difference of Solow residual to focus on idiosyncratic fluctuations. The estimated process assumes a Cobb-Douglas production function, which does not follow the production function that we consider in our model. However, referring to Syverson (2011), this approach holds as a first-order approximation to any production function for computing the TFP.

[10]We obtained the hourly wage series from *Statistisches Bundesamt, Beiheft zur Fachserie 18, Reihe 3* (Germany), *Instituto Nacional de Estadistica, INE* (Chile), *Departamento Nacional de Estadistica, DANE* (Colombia) and *Badan Pusat Statistik, BPS* (Indonesia). We further proceed indexing the series using the consumer price index from each country.

tribution for Germany is constructed from the extensive plant-level ELFLOP dataset collected by the IAB labor institute. In the cases of Colombia and Indonesia, we construct the plant-age distribution directly from our panel datasets. For the former, we use plant-level data from 1977-1982 to construct the plant-age distribution as during that period the survey covered plants of all sizes. As for Chile, we do not have information on the year of foundation of plants and we therefore use the Colombian distribution as proxy. Notice that plants in Germany are on average older compared to the other economies.

Table 5: Productivity dispersions

	Germany		Chile		Colombia		Indonesia	
	Data	Model	Data	Model	Data	Model	Data	Model
High frequency								
$\mathrm{Corr}(\alpha_{it}^L, \alpha_{it}^K)$	0.36	0.90	0.46	0.90	0.57	0.58	0.41	0.69
$\mathrm{Std}(\alpha_{it}^L)$	0.07	0.09	0.20	0.11	0.15	0.13	0.23	0.16
$\mathrm{Std}(\alpha_{it}^K)$	0.12	0.10	0.31	0.12	0.17	0.23	0.34	0.23
Low frequency								
$\mathrm{Corr}(\alpha_{it}^L, \alpha_{it}^K)$	-0.17	-0.76	-0.27	-0.71	-0.14	-0.55	-0.22	-0.65
$\mathrm{Std}(\alpha_{it}^L)$	0.23	0.10	0.27	0.12	0.30	0.11	0.30	0.16
$\mathrm{Std}(\alpha_{it}^K)$	0.72	0.23	0.80	0.24	0.77	0.25	0.77	0.32

Note: All second moments are based on a 5 year moving average filter.

Estimating the baseline model closely fits the empirical plant-age distribution. Despite different calibrations for γ and the stochastic process of r across model economies, and given differences in the empirical plant-age distributions, only estimating the fixed cost of operation obtains a reasonable good match of the plant-age distribution.

The results of the baseline model concerning the productivity moments are shown in Table 5. The model successfully explains the main dynamics found in the data. First, at low frequency the correlation is negative while it is positive at high frequency. It is apparent that the correlations are too large in size, which is arguably driven to some extent by measurement error in the data biasing the correlations toward zero. However, we would like to stress that all empirical correlations are significantly different from zero. Second, we observe substantial differences in factor productivity dispersions between countries at high frequency, which is explained mostly by different calibrations of the profitability shock. Third, the model results also show that the bulk of dispersion in capital productivity is at low frequencies. Labor productivity dispersion is larger at low frequencies although the model results are less distinctive here. Finally, higher growth rates and interest rate fluctuations explain larger dispersions of capital productivity at low frequency in developing countries vis-à-vis developed countries.

4.3 Role of measurement error

A potential reason why the model underpredicts the high frequency factor productivity dispersions is classical measurement error in value added and investment. First, measurement error in value added may explain why the high frequency correlation between capital and labor productivity is small empirically relative to the model. It may further explain the gap in high frequency factor productivity dispersions, which is especially large for the developing economies. Notice that the low frequency movements will capture a small share of the transitory measurement error in value added. Formally,

$$\log py_{it} = \log py^*_{it} + \epsilon^{py}_{it},$$

where py^*_{it} denotes value added as simulated from the model and the measurement error ϵ^{py}_{it} is added.

Second, measurement error in investment translates into persistent measurement error in capital. As in Section 2, we assume

$$\begin{aligned} \log K_{it} &= \log K^*_{it} + \epsilon^K_{it}, \\ \epsilon^K_{it} &= (1-\delta)\epsilon^K_{it-1} + \epsilon^I_{it}, \end{aligned}$$

where K^*_{it} is the capital stock as implied by the model and we add persistent measurement error ϵ^K_{it} that is driven by transitory measurement error in investments ϵ^I_{it}. This type of measurement can first of all account for the gap between low frequency dispersion in capital productivity in the model and the data. Beyond that, it also reduces the difference between the low frequency correlations.

Based on this measurement error framework, we estimate the dispersion of measurement error in revenue and capital by minimizing the distance between the six productivity moments (standard deviations and correlations of capital and labor productivity) from the model and the data counterparts. In Table 6, we present the results of this exercise. In fact, adding two measurement errors crucially improves the model fit and in particular low frequency dispersion in capital productivity and the correlations match

Table 6: Productivity dispersions accounting for measurement error

	Germany		Chile		Colombia		Indonesia	
	Data	Model	Data	Model	Data	Model	Data	Model
High frequency								
Corr($\alpha_{it}^L, \alpha_{it}^K$)	0.36	0.36	0.46	0.44	0.57	0.51	0.41	0.44
Std(α_{it}^L)	0.07	0.13	0.20	0.15	0.15	0.17	0.23	0.18
Std(α_{it}^K)	0.12	0.35	0.31	0.35	0.17	0.38	0.34	0.42
Low frequency								
Corr($\alpha_{it}^L, \alpha_{it}^K$)	-0.17	-0.22	-0.27	-0.18	-0.14	-0.11	-0.22	-0.23
Std(α_{it}^L)	0.23	0.11	0.27	0.13	0.30	0.13	0.30	0.17
Std(α_{it}^K)	0.72	0.60	0.80	0.74	0.77	0.65	0.77	0.76
Measurement error								
Std(ϵ_{it}^{py})	-	0.10	-	0.12	-	0.16	-	0.10
Std(ϵ_{it}^{I})	-	0.29	-	0.25	-	0.22	-	0.29
RSS	-	0.09	-	0.03	-	0.09	-	0.03

Note: All second moments are based on a 5 year moving average filter.

better with their data counterparts. The revenue measurement error is relatively small and accounts for about 10% of total revenue dispersion, while capital dispersion is large. The dispersion of capital expenses including measurement error is about 2-3 times larger then what the putty-clay model implies. This may be seen as suggestive evidence for the potentially large role, measurement error plays in the empirical data.

On the other side, our model is highly stylized and we abstract from further channels of factor productivity dispersion. Financial frictions, wedges in capital usercosts or revenues, differences in the production technology elasticities, may all impact factor productivity dispersions, but these should have relatively little explanatory power for capital productivity dispersions at low frequency. Financial frictions have been shown by Gilchrist et al. (2013) to not account for more than one third of capital productivity dispersion and this ratio does not even refer to deviations at low frequency, which should be smaller following the argumentation that plants save away from their borrowing constraints in the long run. Wedges in revenues, for example heterogeneity in markups, affect both factor productivities equally so it cannot account for the relative magnitude of capital productivity dispersion at low frequency. Differences in production elasticities may well exist in the long run, our empirical results, however, are based on productivity deviations within narrowly-defined industries, which renders large differences in factor expenditure shares less plausible.[11]

5 Conclusion

This paper contributes to the literature on dispersions in factor productivities, which reflect some form of misallocation of factors. We establish three new sets of stylized facts by differentiating between high and low frequency aspects of the dispersions in factor productivities. First, persistent differences in factor productivities explain the bulk of overall factor-productivity dispersions, which shows the importance to understand the dynamic structure of factor productivity dispersions. When we decompose the deviations into a long-lived and a short-lived, more than 60% (75%) of the productivity variance of labor (of capital) is due to long-lived deviations. Second, the difference in factor productivity dispersions across countries is reflected, mostly, at high frequency, while at low frequency the dissimilarity is rather small. Third, we find that the persistent deviations from average factor productivities show negative correlation across factors, while transitory differences from the average are positively correlated across factors.

[11]See Appendix C for a theoretical illustration of the magnitude of revenue elasticities needed to generate the sizeable low frequency dispersion.

We then relate the observed patterns of productivity dispersions to a entry-exit plant model with putty-clay production function. That is, we have a real friction in technology choice by assuming that firms have to select the capital intensity of their production and then operate a Leontieff production function throughout its lifetime. We show that such model is in principle able to rationalize the observed patterns and can predict some of the differences across countries.

Our model implies an alternative explanation to the persistence in productivity dispersion within the same industry. As we model our economy along the balanced growth path, it evinces diverse technologies for firms entering at different periods. The fact that firms in our model have different technologies is able to explain almost half of the low frequency dispersion. Thus, we contribute to the literature in elaborating the important role of technology choice in explaining the observed variation, persistence and correlation of factor productivities.

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A Data

A.1 Description of the Databases

German Firm Data: USTAN

USTAN is itself a byproduct of the Bundesbank's rediscounting and lending activity. The Bundesbank had to assess the creditworthiness of all parties backing promissory notes or bills of exchange put up for rediscounting (i.e. as collateral for overnight lending). It implemented this regulation by requiring balance sheet data of all parties involved, which were then archived and collected, see Bachmann and Bayer (2011b), Stoess (2001), and von Kalckreuth (2003) for details.

We remove observations from East German firms to avoid a break of the series in 1990. This leaves us with a sample of 854,105 firm-year observations, which corresponds to observations on 72,853 firms. The resulting sample covers roughly 70% of the West-German real gross value added in the private non-financial business sector.[12]

[12] We refer to the sectors Agriculture, Energy and Mining, Manufacturing, Construction, and Trade together as the *private non-financial business sector* throughout this paper.

Chilean Plant Data: ENIA

We use the Annual Census of Manufacturing (*Encuesta Nacional Industrial Anual*, ENIA) conducted by the National Institute of Statistics (*Instituto Nacional de Estadsticas*, INE) from 1995 to 2007 to get plant level data from Chile.

The ENIA provides information for all manufacturing plants with total employment of at least ten employees. According to INE statistics, ENIA covers about 50% of total manufacturing employment. The data was used for example by Pavcnik (2002) and Doms and Bartelsman (2000).

Colombian Plant Data: EAM

For Colombian estimates, we exploit the plant-level data from the Colombian Manufacturing Survey (*Encuesta Anual Manufacturera*, EAM) made available through the National Institute of Statistics (*Departamento Administrativo Nacional de Estaditicas*, DANE) for the period 1977 to 1991. The survey covers information from all plants during 1977-1982, while only contains data on plants above 10 employees during 1983-1984 and from 1985 small plants are included in small proportion. See, for example, Das et al. (2007), Fernandes (2007) and Roberts and Tybout (1997) for reference.

Indonesian Plant Data: IBS

The Indonesian estimation is based on the Manufacturing Survey of Large and Medium establishments (*Survei Tahunan Perusahaan Industri Pengolahan*, IBS), provided by the National Institute of Statistics (*Badan Pusat Statistik*, BPS). The survey covers all entities with 20 or more employees. It consist of plant level data on capital stocks, labor costs, output value, intermediate inputs and value added among other variables. The Indonesian data was employed, for example, by Blalock and Gertler (2009), Peters (2011) and Yang (2012).

A.2 Additional cleaning steps in Indonesia

The plant level data provided by the National Institute of Statistics of Indonesia is an extensive dataset that allows us to examine our hypothesis. However, before proceeding with the general cleaning steps we need to implement some specific corrections. For this purpose, we are based on the methodology applied by Blalock and Gertler (2009):

- We have to correct from mistakes due to data keypunching. If the sum of the capital categories is a multiple of 10^n (with n being an integer) of the total capital, we replace the latter with the sum of the categories.

- We drop duplicate observations within the year (i.e. observations which have the same values for all the variables but differ in their plant identification number).
- We have corrected for those cases where value added and total assets are not consistent with the formula provided by BPS.
- The industry survey have changed their industry classification from *ISIC Rev. 2* in 1998 to *ISIC Rev. 3* in 1999. We use United Nations concordance tables to construct the four digit industry classification.

In addition, we have missing values in capital for plants who presented information in previous and/or subsequent years. In order to overcome with this issue, we follow Vial (2006) and we estimate the capital category using the following regression by two-digit sectoral level:

$$\ln K_{it} = \beta_0 + \beta_1 \ln K_{it-1} + \theta \ln X_{it-1} + \mu_i + \epsilon_{it}$$

where K_{it} is the capital category, μ_i is the individual fixed effect and X_{it-1} are a set of explanatory variables (total output, input, employees, wages, fuel costs and expenditures on materials,leasing, industrial services, taxes and others). Given that we have the lagged capital category as explanatory variable, we will only impute the capital for those cases where the plant have reported a value for the previous period. Otherwise, we do not impute it.[13]

Finally, to proceed with the PIM, we need depreciation level from each capital source. However, we have information on depreciation only until 2001. We use the median depreciation rate of each capital stock (by industry) before 2001, and we multiply it with the reported capital after 2001, to construct a measure of depreciation level for the latter period.

A.3 Perpetual inventory method

In order to construct economically consistent measures of capital series for each plant, we re-calculate capital stocks using a perpetual inventory method (PIM). This methodology ensures that aggregated movements in plant-level capital match investment and aggregate yearly depreciation rates, see Caballero et al. (1995) and Bachmann and Bayer (2011a). To begin with, we compute plant-investment ($I_{t,i}$) using the accumulation

[13]For robustness, we proceed by imputing the values using linear interpolation. The main empirical results go into the same direction as the econometric approach proposed.

identity for capital stocks:

$$I_{t,i,j} = K^r_{t+1,i,j} - K^r_{t,i,j} + D^r_{t,i,j}$$

where K^r and D^r are plant's reported capital stock, depreciation and investment. As a second step, we apply a PIM to re-compute the capital stocks for each type of capital. Given that for Germany and Colombia we have book values of capital, while for Chile and Indonesia we have current values of capital, we proceed in different ways. For the first two countries,

$$\begin{aligned} K_{0,i,j} &= \frac{p_0}{p_{base}} K^r_{0,i,j} cf_i \\ K_{t+1,i,j} &= K_{t,i,j}(1-\delta_i) + \frac{p_t}{p_{base}} I_{t,i,j}, \ \ \forall t \in [0,T] \end{aligned}$$

where $\frac{p_t}{p_{base}} I_{t,i,j}$ is the real investments of plant j at time t in capital i and δ_i is the economic depreciation rate of capital i. As the book value of capital is commonly not an accurate estimate of the real productive value of capital stock in the plant, we augment the initial capital stock with a correction factor cf_i that accounts for asset revalorization given the past inflation and the expected period the capital remains in the plant

$$cf_i = \sum_{t_i=0}^{T} (1-\delta_i)^{T-t_i} (1-\pi_{t_i})^{T-t_i} \delta_i,$$

where i denotes buildings, machinery or vehicles. The time length we apply for the correction factor is 30 (buildings), 15 (machinery) and 10 (vehicles).

On the other hand, in the case of Chile and Indonesia, we conduct a different procedure to re-compute the capital stock as it is reported in current prices. In specific,

$$\begin{aligned} K_{0,i,j} &= K^r_{0,i,j} \\ K_{t+1,i,j} &= (1+\pi_{i,t+1}) K_{t,i,j}(1-\delta_i) + I_{t,i,j}, \ \ \forall t \in [0,T] \end{aligned}$$

where $\pi_{i,t+1}$ is the inflation rate of capital i in period $t+1$.

The depreciation reported by the plants are not directly usable to reconstruct the capital stocks as the accounting depreciation is typically higher than the economic depreciation for tax purposes or other reasons. Consequently, we require information on the economic depreciation rate, which we receive from the National Accounts data (*Volkswirtschaftliche Gesamtrechnung*, VGR) in the case of Germany, while for Chile we use a report from Henriquez (2008) which provides an estimate of gross and net consumption of fixed capital by each source for the period 1985-2005. As for Colombia, we use as an indicator for the economic depreciation, the average of the depreciation in Chile for the available period. Finally, for Indonesia we consider the values from van der Eng (2008), who estimated the stock of capital in the country for the period 1950-2007.

As for annual interest rates we consider the bank overdraft real interest rate (Germany), the average real lending interest rate (Chile and Colombia), and the average working capital real interest rate (Indonesia). As agents make decisions based on the expected value of the real interest rate in the future, we estimate an AR-1 process of this variable. In fact, the perfect measure for interest rates in the context of our model would be the plant-level user cost of capital while abstracting from the effects of uncertainty and liquidity (not present in the model). However, such information is not available and we use the best measure available. The average economic depreciation and expected real interest rate are presented in Table 7.

Table 7: Economic Depreciation and Expected Real Interest Rate

	Machinery Depreciation	Buildings Depreciation	Vehicles Depreciation	Expected Real Interest Rate
Germany	15.1%	3.3%	–	4.36%
Chile	6.8%	2.6%	12.0%	6.98%
Colombia	6.6%	2.7%	12.0%	11.39%
Indonesia	6.3%	3.3%	10.0%	7.63%

Finally, we deflate all variables in order to adjust from price changes. For capital, we take an index price by each capital category (when available) using the information of gross fixed capital formation at current and constant prices from National Accounts. And for value added and labor expenses, we use the CPI index or GDP price deflator.

A.4 Robustness

We conduct three robustness checks. First, we apply a moving average filter removing the 4-digit industry year effect (instead of 2-digit) from our variables α. Second, we are also able to test if the financial crisis in Indonesia during the period 1997-1998 affected our main results, splitting the sample before the crisis (before 1997) and after the crisis (from 1999). In summary, our estimations are robust for each different specification. The correlation between labor productivity and capital productivity is positive at high frequencies while at low frequencies is negative. Secondly, most of the dispersion is at low frequency.

Table 8: Second moments of factor productivities at different frequencies. Demeaning using 4-digit industry

	std(a_{it}^L)	std(a_{it}^K)	$\rho(a_{it}^L, a_{it}^K)$	std(a_{it}^L)	std(a_{it}^K)	$\rho(a_{it}^L, a_{it}^K)$
	High frequency (5Y MA)			Low frequency (5Y MA)		
CL	0.202	0.317	0.441	0.258	0.764	-0.229
	(0.006)	(0.007)	(0.016)	(0.009)	(0.033)	(0.018)
CO	0.148	0.180	0.500	0.277	0.720	-0.110
	(0.002)	(0.003)	(0.008)	(0.007)	(0.023)	(0.014)
ID	0.229	0.344	0.389	0.288	0.735	-0.183
	(0.003)	(0.004)	(0.007)	(0.005)	(0.013)	(0.008)
	High frequency (11Y MA)			Low frequency (11Y MA)		
CL	0.223	0.397	0.339	0.198	0.621	-0.304
	(0.013)	(0.021)	(0.030)	(0.011)	(0.047)	(0.037)
CO	0.169	0.230	0.433	0.231	0.639	-0.135
	(0.005)	(0.007)	(0.017)	(0.010)	(0.040)	(0.028)
ID	0.236	0.408	0.322	0.233	0.637	-0.266
	(0.009)	(0.009)	(0.017)	(0.008)	(0.024)	(0.021)

The results are based on applying a five year moving average filter (5Y MA) and an eleven year moving average filter (11Y MA). Factor productivities are demeaned by 4-digit industry and year and expressed in logs. Standard errors are robust clustered standard errors. ρ denotes correlation

A.5 Frequency decomposition using the HP-Filter

Instead of decomposing each firm's labor and capital productivity using a block moving average procedure, we can also apply the HP-Filter. We then define the trend component as low frequency movement and the cyclical component as high frequency movement. In order to avoid problems related with the weakness of the HP filter at the

Table 9: Second moments of factor productivities at different frequencies. Splitting Indonesian panel in before and after the crisis

	std(a_{it}^L)	std(a_{it}^K)	$\rho(a_{it}^L, a_{it}^K)$	std(a_{it}^L)	std(a_{it}^K)	$\rho(a_{it}^L, a_{it}^K)$
	High frequency (5Y MA)			Low frequency (5Y MA)		
ID - Before crisis	0.218 (0.007)	0.323 (0.007)	0.387 (0.018)	0.310 (0.009)	0.761 (0.024)	-0.255 (0.014)
ID - After crisis	0.238 (0.005)	0.340 (0.006)	0.440 (0.011)	0.283 (0.007)	0.792 (0.020)	-0.167 (0.012)

The results are based on applying a five year moving average filter (5Y MA). Factor productivities are demeaned by 2-digit industry and year and expressed in logs. Standard errors are robust clustered standard errors. ρ denotes correlation

boundaries of a time series we restrict our analysis to firms with at least five consecutive observations and drop the first and last observation before computing productivity dispersions and correlations. Given the yearly frequency of our datasets and in line with Ravn and Uhlig (2002), we set the HP multiplier $\lambda = 6.25$. Table A.5 shows the benchmark results under the HP filter decomposition. In fact, our main findings are robust to the decomposition technique. The low frequency dispersion is still much larger the high frequency dispersion, for each country respectively. Further, the low frequency correlation between labor and capital productivity is negative while it is positive at high frequency.

B Measurement error

We argue in Section 2 that in particular the low correlation between capital expenses and value added at low frequency suggests the presence of measurement error in capital stocks. We technically identify the dispersion of true capital expenses, by pre-supposing the correct correlation between true capital expenses and value added, which we denoted by ω. In principle, ω is unknown. In a frictionless model, it should equal one, while in a more realistic setup with adjustment frictions, it may be below one. In order to investigate the sensitivity of results with respect to the assumption on ω, we present

Table 10: Second moments of factor productivities at different frequencies using HP filter

	std(a_{it}^L)	std(a_{it}^K)	$\rho(a_{it}^L, a_{it}^K)$	std(a_{it}^L)	std(a_{it}^K)	$\rho(a_{it}^L, a_{it}^K)$
	High Frequency			Low Frequency		
CL	0.183	0.275	0.466	0.281	0.831	-0.244
	(0.005)	(0.006)	(0.013)	(0.009)	(0.032)	(0.016)
CO	0.135	0.165	0.536	0.312	0.789	-0.139
	(0.002)	(0.002)	(0.006)	(0.007)	(0.023)	(0.012)
ID	0.198	0.306	0.393	0.293	0.783	-0.195
	(0.003)	(0.004)	(0.008)	(0.008)	(0.020)	(0.011)

HP filter decomposition into high and low frequency movements. Factor productivities are demeaned by 2-digit industry and year and expressed in logs. Standard errors are robust clustered standard errors. ρ denotes correlation

three alternatives in Table 11.

Imposing a perfect correlation between true capital expenses and value addded seems too strong an assumption, because it basically takes up all the dispersion in low frequency capital productivity dispersion. Further, the question arises why expenses regarding the more flexible input factor, labor, should be less correlated with value added, than capital expenses. On the other side, imposing a relatively low correlation at low frequency implies a relatively small role of measurement error. We therefore assume that ω equals the correlation observed for labor expenses. This seems in the middle range and also allows for some heterogeneity across data sets reflecting other potential structural differences. A further reason that makes us confident in assuming $\omega = \rho(py, wL)$ is the fact that using a 11 year moving average, the observed correlation between capital expenses and value added remains basically unchanged at about 90% and far from the correlation observed for labor expenses. This suggests that structural reasons, such as capital adjustment frictions, are not the cause of the relatively low correlation, because at 11 years such friction should have a significantly weaker effect on reducing the correlation.

Table 11: Measurement error correction under different ρ specifications

	Data	$(\omega = 0.95)$	$(\omega = \rho(py, wL))$	$(\omega = 1)$
	$sd(RK)$		$sd(rK^*)$	
CL	1.630	1.507	1.463	1.431
CO	1.657	1.547	1.500	1.470
ID	1.468	1.081	1.285	1.248
	$sd(a^K)$		$sd(a^{K*})$	
CL	0.798	0.500	0.350	0.172
CO	0.765	0.483	0.299	0.007
ID	0.774	0.411	0.308	0.016
	$\rho(wL, RK)$		$\rho(wL, rK^*)$	
CL	0.838	0.907	0.933	0.954
CO	0.844	0.904	0.933	0.952
ID	0.797	0.891	0.911	0.938
	$\rho(a^L, a^K)$		$\rho(a^L, a^{K*})$	
CL	-0.267	-0.425	-0.608	-1.238
CO	-0.144	-0.228	-0.369	-15.213
ID	-0.219	-0.413	-0.551	-10.463

Direct and corrected standard deviations of log capital expenditures and the implied capital productivity dispersion. Labor expenditures and capital stocks are expressed in logs. All moments are at low frequency 5 year moving average.

C Summarizing dispersion in wedges

C.1 Wedges in revenue and usercosts of capital

We suppose a monopolistically competitive plant that maximizes profits

$$\begin{aligned}\pi_i &= (1-\tau_i^Y)p_i y_i - wL_i - (1+\tau_i^K)RK_i \\ &= (1-\tau_i^Y)(A_i K_i^\alpha L_i^{1-\alpha})^{1-1/\sigma} - wL_i - (1+\tau_i^K)RK_i,\end{aligned}$$

where τ_i^Y and τ_i^K are a revenue and capital wedge respectively. Using the first order conditions, we can express labor and capital productivity as a function of the capital and output wedges:

$$\begin{aligned}\alpha_i^L &= \frac{p_i y_i}{wL_i} = \frac{1}{1-\alpha}\frac{\sigma}{\sigma-1}\frac{1}{1-\tau_i^Y} = c_{\alpha L}\frac{1}{1-\tau_i^Y} \\ \alpha_i^K &= \frac{p_i y_i}{RK_i} = \frac{1}{\alpha}\frac{\sigma}{\sigma-1}\frac{1+\tau_i^K}{1-\tau_i^Y} = c_{\alpha K}\frac{1+\tau_i^K}{1-\tau_i^Y}\end{aligned}$$

As in Gilchrist et al. (2013), we assume that $\log(1-\tau_i^Y)$ and $\log(1+\tau_i^K)$ are jointly log-normally distributed,

$$\begin{bmatrix} \log(1-\tau_i^Y) \\ \log(1+\tau_i^K) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_Y^2 & \sigma_{KY} \\ \sigma_{KY} & \sigma_K^2 \end{bmatrix}\right),$$

which allows us to estimate the covariance matrix of the wedges directly from the data:

$$\begin{aligned}V[\log(\alpha_i^L)] &= \sigma_Y^2 \\ V[\log(\alpha_i^K)] &= \sigma_Y^2 + \sigma_K^2 - 2\sigma_{YK} \\ V[\log(\tilde{k}_i)] &= V[\log(\frac{RK_i}{wL_i})] = \sigma_K^2\end{aligned}$$

The results are provided in Table 12. For all countries we obtain a usercost wedge dispersion of 90 log points and a revenue wedge dispersion of 30 log points. These imply extreme differences in the usercosts. For example, 60% of plants differ in their usercosts by more than a factor of 6. When using the measurement error-corrected moments to estimate the joint distribution of the two wedges, these differences are still large, but a dispersion of 50 log points seems less unrealistic at least. Aditionally, in order to explain the low frequency dispersion observed in the data, we also need a positive correlation between the wedges. That means a high markup plant tends to have higher user costs of capital and therefore, a lower capital intensity. That means keeping revenues constant, the firm with high markup uses less factor inputs.

	σ_Y	σ_K	ρ_{YK}
	Low frequency		
CL	0.266	0.906	0.528
CO	0.303	0.863	0.482
ID	0.297	0.887	0.525
	Low frequency - corrected for ME		
CL	0.266	0.551	0.860
CO	0.265	0.500	0.833
ID	0.297	0.533	0.983

Table 12: Results of estimating the dispersion of output and capital distortions.

C.2 Wedges in factor shares

Let us now consider an alternative model where plants employ different technologies resulting in different factor shares. Let us take the same model used in the above example, but abstracting from wedges and only changing the production function

$$y_i = K_i^{\alpha_i} L_i^{\beta_i}.$$

We express labor and capital productivity as a function of the factor shares

$$\alpha_i^K = \frac{p_i y_i}{RK_i} = \frac{\sigma}{\sigma - 1}\frac{1}{\alpha_i}$$
$$\alpha_i^L = \frac{p_i y_i}{wL_i} = \frac{\sigma}{\sigma - 1}\frac{1}{\beta_i}$$

and we assume that $\log(\alpha_i)$ and $\log(\beta_i)$ are jointly log-normally distributed

$$\begin{bmatrix} \log(\alpha_i) \\ \log(\beta_i) \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} \sigma_\alpha^2 & \sigma_{\alpha\beta} \\ \sigma_{\alpha\beta} & \sigma_\beta^2 \end{bmatrix} \right).$$

We estimate the covariance matrix from the data as $V[\log(\alpha_i^L)] = \sigma_\alpha^2$, $V[\log(\alpha_i^K)] = \sigma_\beta^2$ and $Cov[\log(\alpha_i^L), \log(\alpha_i^K)] = \sigma_{\alpha\beta}$.

The results are provided in Table 13. We need to rely on extremely heterogeneous capital shares in order to explain the low frequency dispersion of capital productivity in the raw data. However, once we correct from measurement error, the required dispersion in both factor shares is similar. Another implication which emerges from this exercise is that factor shares have to be negatively correlated across plants to explain our empirical findings. This is due to the fact that labor and capital productivity are negatively correlated at low frequency, and consequently $\sigma_{\alpha\beta}$ is negative.

Table 13: Results of estimating the dispersion of revenue elasticities.

	σ_α	σ_β	$corr(\alpha, \beta)$
	Low frequency		
Germany	0.234	0.716	-0.169
Chile	0.266	0.798	-0.267
Colombia	0.303	0.765	-0.144
Indonesia	0.297	0.774	-0.219
	Low frequency - corrected for ME		
Germany	0.234	0.286	-0.423
Chile	0.266	0.350	-0.608
Colombia	0.303	0.299	-0.369
Indonesia	0.297	0.247	-0.551

D Model solution

D.1 Dynamic optimization

In the following, we show the existence and uniqueness of the transformed model in equation (11).

Assumption 1: $\alpha \leq \frac{1}{1+\xi}$.

Lemma 1: The function $\widetilde{\pi}(k, z, r)$ is bounded from above and below in k, $\forall z, r$.

Proof: Since $\widetilde{\pi}(k, z, r)$ is continuous, it is sufficient to show that $\lim_{k \to \infty} |\widetilde{\pi}(k, z, r)| < \infty$.

If this is the case, then $\widetilde{\pi}(k,z,r)$ is bounded for $k' \to \infty$ and by the Weierstrass extreme value theorem, it is bounded for any finite interval $[0,c]$ $\forall c \in \Re$ and hence bounded everywhere. We can maximize out labor (equation 16) and explicitly derive $\widetilde{\pi}(k,z,r)$:

$$\begin{aligned}\widetilde{\pi}(k,z,r) \qquad &= \frac{zE(z)^{\frac{1-\xi}{\xi}}(1+(E(r)+\delta)k)k^{\frac{\alpha(1-\xi^2)}{\xi}}}{(1-\xi)(1+(E(r)+\delta)k)^{\frac{1}{\xi}}} \\ &- \frac{(1-\xi)E(z)^{\frac{1}{\xi}}k^{\frac{\alpha(1-\xi)}{\xi}} - (1-\xi)(r+\delta)k^{\frac{\alpha(1-\xi)+\xi}{\xi}}}{(1-\xi)(1+(E(r)+\delta)k)^{\frac{1}{\xi}}}.\end{aligned}$$

It follows that for $\lim_{k\to\infty} |\widetilde{\pi}(k,z,r)|$ to exist, the following condition needs to be satisfied:

$$\frac{1}{\xi} \geq \frac{\alpha(1-\xi^2)+\xi}{\xi} \;\Leftrightarrow\; \alpha \leq \frac{1}{1+\xi}.$$

The above assumption then implies boundedness of $\widetilde{\pi}(k,z,r)$.

Lemma 2: Let us define the operator T by posing $(Tv)(k,z,r)$ equal to the right-hand-side of equation 11. This operator is defined on the set $\mathcal{B}$ of all real-valued, bounded and continuous functions with domain $\Re_+ \times \Re_{++} \times \Re_{++}$. Then T (i) preserves boundedness, (ii) preserves continuity, and (iii) satisfies Blackwell's sufficient conditions.

Proof: $\widetilde{\pi}(k,z,r)$ is continuous, concave, and bounded in the set of state variables.

(i) Consider $u \in \mathcal{B}$ bounded below by $\underline{u}$ and bounded above by $\overline{u}$. Then $(Tu)(k,z,r)$ is bounded from below and above since $\widetilde{\pi}(k,z,r)$ is bounded as shown before.

(ii) Next, we show that (Tu) is continuous. We note that (Tu) is the maximum of a constant and a function. Since the function is the sum of two continuous functions $\widetilde{\pi}(k,z,r)$ and $u(k,z,r)$, (Tu) is continuous.

(iii) Finally, we need to show that (Tu) satisfies monotonicity and discounting. We note that if $u_1, u_2 \in \mathcal{B}$ and $u_1(k,z,r) < u_2(k,z,r)$ for all k, z and r, then integrating with respect to the distributions of z and r preserves the inequality and hence monotonicity holds.

We can show discounting since for any $u \in \mathcal{B}$ and any constant $a \in \Re$, it holds that

$$(T[u+a])(k,z,r,\phi) = (Tu)(k,z,r,\phi) + \beta a.$$

Blackwell's sufficient conditions for a contraction holds.

Propositon 1: Equation 11 has exactly one solution (in the metric space $\mathcal{B}$).

Proof: We know from Lemma 2 that T defines a contraction mapping on the metric space $\mathcal{B}$ with modulus β. Existence and uniqueness then follow from the Contraction Mapping Theorem.

D.2 Computation

In order to solve the model, we discretize the two lognormal AR(1) processes of the demand shifter and relative factor price. Following Tauchen (1986), we use an equidistant algorithm to discretize both processes using 19 states each. We make use of the property that our model defines a contraction and use value function iteration. In particular, we use few grid points (50) for capital intensity and maximize over the off-grid interpolated value function using spline interpolation. Due to the possibility for firms to cease operations, the value function is non-concave. In face of this issue, we are very careful in solving the problem by combining off-grid methods with a preceding on-grid screen.