

Optimal Taxation with Endogenous Default under Incomplete Markets *

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Abstract

In a dynamic economy, we characterize the fiscal policy of the government when it levies distortionary taxes and issues defaultable bonds to finance its stochastic expenditure. Households predict the possibility of default, generating endogenous debt limits that hinder the government's ability to smooth shocks using debt. Default is followed by temporary financial autarky. The government can only exit this state by paying a fraction of the defaulted debt. Since this payment may not occur immediately, in the meantime, households trade the defaulted debt in secondary markets; this device allows us to price the government debt before and during the default.

JEL codes: H3, H21, H63, D52, C60.

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1 Introduction

For many governments, debt and tax policies are conditioned by the possibility of default. For emerging economies, default is a recurrent event and is typically followed by a lengthy debt-restructuring process, in which the government and bond holders engage in renegotiations that conclude with the government paying a fraction of the defaulted debt.¹

We find that emerging economies exhibit lower levels of indebtedness and higher volatility of government tax revenue than do industrialized economies—where, contrary to emerging economies, default is not observed in our dataset—.² Also, emerging economies, exhibit higher interest rate spreads, especially for high levels of domestic debt-to-output ratios, than industrialized economies. In fact, industrialized economies exhibit interest rate spreads that are low and roughly constant for different levels of domestic debt-to-output ratios. Moreover, in emerging economies, the highest interest rate spreads are observed after default and during the debt-restructuring period.³ Finally, we find that in our dataset, higher spreads are associated with more volatile tax revenues.

These empirical facts indicate that economies that are more prone to default display different government tax policy, as well as different prices of government debt, before default and during the debt-restructuring period. Therefore, the option to default, and the actual default event, will affect the utility of the economy’s residents: Indirectly, by affecting the tax policy and debt prices, but also directly, by not servicing the debt in the hands of the economy’s residents during the default event.

Our main objective is to understand how the possibility of default and the actual default event affect tax policy, debt prices—before default and during financial autarky—, and welfare of the economy.⁴ For this purpose, we analyze the dynamic taxation problem of a benevolent government in a closed economy under incomplete markets which has access to distortionary labor taxes and non-state-contingent debt. We assume, however, that the government cannot commit to pay the debt, and in case the government defaults, the economy enters temporary financial autarky and faces exogenous offers to pay a fraction of the defaulted debt that occur at an exogenous rate.⁵ The government has the option to accept the offer—and, thus, exit financial

¹See Pitchford and Wright (2008) and Benjamin and Wright (2009).

²To measure “indebtedness”, I am using government domestic debt-to-output ratios, where domestic debt is the debt issued under domestic law (see Panizza (2008)). We use domestic and not total government debt because our model is a closed economy. As a proxy of tax policy, we use government revenue-to-output ratio or inflation tax.

³Examples of this are Argentina 2001, Ecuador 1997, and Russia 1998.

⁴Throughout this paper, we will also refer to the restructuring period as the financial autarky.

⁵In this model, financial autarky is understood as the period during which the government is precluded from

autarky—or to stay in financial autarky until a new offer comes along. During temporary financial autarky, the defaulted debt still has positive value because it is going to be paid in the future with positive probability. Hence, households can trade the defaulted debt in a secondary market from which the government is excluded; the equilibrium price in this market is used to price the debt during a period of default. Finally, we assume that the government commits itself to a path of taxes when the economy is not in financial autarky.

In the model, the government has three policy instruments: (1) distortionary taxes, (2) government debt, and (3) default decisions that consist of: (a) whether to default on the outstanding debt and (b) whether to accept the offer to exit temporary financial autarky.

The government faces a trade-off between levying distortionary taxes to finance the stochastic process of expenditures and not defaulting, or issuing debt and thereby increasing the exposure to default risk. The option to default introduces some degree of state contingency on the payoff of the debt since the financial instrument available to the government becomes an option, rather than a non-state-contingent bond. This option, however, does not come free of charge: Households accurately predict the possibility of default, and the equilibrium incorporates it into the pricing of the bond; this originates a “Laffer curve” type of behavior for the debt income, thereby implying endogenous debt limits. In this sense, our model generates “debt intolerance” endogenously.⁶

The main insight of the paper is that this borrowing limits hinder the government’s ability to smooth shocks using debt, thus rendering tax policy more volatile, and implying higher interest rate spreads. The possibility of default introduces a trade-off between the cost of the lack of commitment to repay the debt, reflected in the price of the debt, and the flexibility that comes from the option to default and partial payments, reflected in the pay-off of the debt.

In a benchmark case, with quasi-linear utility, and a Markov process for the government expenditure but allowing for offers of partial payments to exit financial autarky, we characterize, analytically, the determinants of the optimal default decision and its effects on the optimal taxes, debt and allocations. In particular, we first show that default is more likely when the government’s expenditure or debt is higher, and that the government is more likely to accept any given offer to pay a fraction of the defaulted debt when the level of defaulted debt is lower. Second, by imposing additional restrictions, we show that prices — both outside and during financial autarky — are non-increasing on the level of debt, thus implying that spreads are non-decreasing. Third, we show that the law of motion of the optimal government tax policy departs from the standard martingale-type behavior found in Aiyagari et al. (2002); in partic-

issuing new debt/savings.

⁶A term coined by Reinhart et al. (2003).

ular, we show that the law of motion of the optimal government tax policy is affected, on the one hand, by the benefit from having “more state-contingency” on the payoff of the bond, but, on the other hand, by the cost of having the option to default.⁷

Finally, we calibrate a more complete model; the model is qualitatively consistent with the differences observed in the data between emerging and industrialized economies. In terms of welfare policy, the numerical simulations suggest a nonlinear relationship between welfare and the probability of receiving an offer of partial payments. In particular, increasing the probability of receiving offers for exiting autarky decreases welfare when this probability is low/medium to begin with, but increases it when the probability is high.

The paper is organized as follows. We first present the related literature. Section 2 presents some stylized facts. Section 3 introduces the model. Section 4 presents the competitive equilibrium, and section 5 presents the government’s problem. Section 6 derives analytical results that characterize the optimal government policies for a simple example. Section 7 contains some numerical exercises. Section 8 briefly concludes. All proofs are gathered in the appendices.

1.1 Related Literature

The paper builds on and contributes to two main strands in the literature: endogenous default and optimal taxation.

Regarding the first strand, we model the strategic default decision of the government as in Arellano (2008) and Aguiar and Gopinath (2006), which, in turn, are based on the seminal paper by Eaton and Gersovitz (1981). Our model, however, differs from theirs in several ways. First, we consider distortionary taxation; Arellano (2008) and references therein implicitly assume lump-sum taxes. Second, in our model, the government must pay at least a positive fraction of the defaulted debt to exit financial autarky through a “debt-restructuring process”; in Arellano (2008) and references therein, the government is exempt from paying the totality of the defaulted debt upon exit of autarky. We model this “debt-restructuring process” exogenously, indexing it by two parameters, because we are interested in studying only the consequences of this process on the optimal fiscal policy and welfare.⁸ Third, our economy is closed—i.e., “creditors” are the representative household—; Arellano (2008) and references therein assume an open economy with foreign creditors. This allows me to capture the direct impact of the default event in the residents of the economy. Empirical evidence seems to suggest that government default has a

⁷See also Farhi (2010) for an extension of Aiyagari et al. (2002) results to an economy with capital.

⁸See Benjamin and Wright (2009), Pitchford and Wright (2008) and Yue (2010) for ways of modeling the debt-restructuring process endogenously.

non negligible direct impact on domestic residents; either because a considerable portion of the foreign debt is in the hands of domestic residents, or because the government also defaults on domestic debt.⁹ Ideally, a model should consider both type of lenders; and although outside the scope of this paper, this could be an interesting avenue for future research.¹⁰

Regarding the second strand, we base our work on Aiyagari et al. (2002), where, in a closed economy, the benevolent infinitely-lived government chooses distortionary labor taxes and non-state-contingent *risk-free* debt, taking into account restrictions from the competitive equilibria, to maximize the households' lifetime expected utility. Our work relaxes this last assumption and, as a consequence, the option to default creates endogenous debt limits, reflected in the equilibrium prices.

In their work, by imposing non-state-contingent debt, AMSS reconcile the behavior of optimal taxes and debt observed in the data with the theory developed in the seminal paper of Lucas and Stokey (1983), in which the government has access to state-contingent debt. These papers assume full commitment on taxes and risk-free debt. Our work relaxes this last assumption and, as a consequence, the option to default creates endogenous debt limits, reflected in the equilibrium prices. It is worth to note that all these papers (and ours) take market incompleteness as exogenous, since the goal is study the implications of this assumption. Albeit outside the scope of this paper, it would be interesting to explore ways of endogenizing market incompleteness; the paper by Hopeynhan and Werning (2009) seems a promising avenue for this.

Following the aforementioned literature, we assume that, although the government can commit itself to a tax policy outside temporary financial autarky, during this period, taxes are set mechanically so that tax revenues finance the government expenditure. This feature is related to Debortoli and Nunes (2010). Here the authors study the dynamics of debt in the Lucas and Stokey (1983) setting but with the caveat that at each time t , with some given probability, the government can lose its ability to commit to taxes; the authors refer to this as “loose commitment.” Thus, our model provides a mechanism that “rationalizes” this probability of “loosing commitment” by assuming that the government is not committed to paying debt and can default at any time. It is worth noting that, in their model, the budget constraint during the no-commitment stage remains essentially the same, whereas ours does not.

Finally, in recent independent papers, Doda (2007), Cuadra et al. (2010), study the procyclicality of fiscal policy in developing countries by solving an optimal fiscal-policy problem. Their

⁹For Argentina's default in 2001, almost 50 percent of the face value of debt to be restructured (about 53 percent of the total owed debt from 2001) is estimated to be in the hands of Argentinean residents; Local pension funds alone held almost 20 percent of the total defaulted debt (see Sturzenegger and Zettelmeyer (2006)). See Reinhart and Rogoff (2008) for a discussion and stylized facts on domestic debt defaults.

¹⁰See Broner et al. (2010) for a paper studying this issue in a more stylized setting.

work differs from ours in two main aspects. They assume, first, an open small economy (i.e., foreign lenders) and, second, no secondary markets.¹¹

2 Stylized Facts

In this section, we present stylized facts regarding the domestic government debt-to-output ratio and central government revenue-to-output ratio of several countries: Industrialized economies (IND, henceforth), emerging economies (EME, henceforth) and a subset of these: Latin American (LAC, henceforth).¹²

In the dataset set, IND do not exhibit default events, whereas EME/LAC (LAC in particular) do exhibit several defaults.¹³ Thus, we take the former group as a proxy for economies with access to risk-free debt and the latter group as a proxy for economies without commitment. It is worth to point out that we are not implying that IND economies are a type of economy that will never default; we are just using the fact that in *my dataset* IND economies do not show default events, to use them as a proxy for the type of economy modeled in AMSS (i.e., one with risk-free debt). There is still the question of what type characteristics of an economy will prompt it to behave like IND or EME/LAC economy. A possible explanation is that for IND default is more costly, due to a higher degree of financial integration. That is, default — and the posterior period of financial autarky — could have a larger impact on the financing of the firms, thus lowering the productivity of the economy. We delve more into this question, in the context of the model in section 7.

The main stylized facts that we found are, first, that EME/LAC economies have higher default risk than IND economies and that within the former group, the default risk is much higher for economies with high levels of debt-to-output ratio. Second, EME and LAC economies exhibit tighter debt ceilings than economies that do not default (in this dataset, represented by IND). Third, economies with higher default risk exhibit more volatile tax revenues than economies with low default risk, and this fact is particularly notable for the group of EME/LAC economies (where defaults are more pervasive).

As shown below, our theory predicts that endogenous borrowing limits are more active for a

¹¹Aguiar et al. (2008) also allow for default in a small open economy with capital where households do not have access to neither financial markets nor capital and provide labor inelastically. The authors' main focus is on capital taxation and the debt "overhang" effect.

¹²For the latter ratios, we used the data in Kaminsky et al. (2004), and for the first ratio, we used the data in Panizza (2008).

¹³For LAC, in our sample, four countries defaulted, and most notable, Argentina defaulted repeatedly.

high level of indebtedness. That is, when the government debt is high (relative to output), the probability of default is higher, thus implying tighter borrowing limits, higher spreads and higher volatility of taxes. But when this variable is low, default is an unlikely event, thereby implying slacker borrowing limits, lower spreads and lower volatility in the taxes. Hence, implications in the upper tail of the domestic debt-to-output ratio distribution can be different from those in the “central part” of it. Therefore, the mean and even the variance of the distribution are not too informative, as they are affected by the central part of the distribution; quantiles are better suited for recovering the information in the tails of the distribution.¹⁴

Figure D.1 plots the percentiles of the domestic government debt-to-output ratio and of a measure of default risk for three groups: IND (black triangle), EME (blue square) and LAC (red circle).¹⁵ The X-axis plots the time series averages of domestic government debt-to-output ratio, and the Y-axis plots the values of the measure of default risk.¹⁶ For each group, the last point on the right correspond to the 95 percentile, the second to last to the 90 percentile and so on; these are comparable between groups as all of them represent a percentile of the corresponding distribution. EME and LAC have lower domestic debt-to-output ratio levels than IND, in fact the domestic debt-to-output ratio value that amounts for the 95 percentile for EME and LAC, only amounts for (approx.) 85 percentile for IND (which in both cases is only about 50 percent of debt-to-output ratio).¹⁷ *Thus, economies that are prone to default (EME and LAC) exhibit tighter debt ceilings than economies that do not default (in this dataset, represented by IND).*

Table 2(A) compares the measure of default risk between IND and EME matching them across low and high debt-to-output ratio levels. That is, for both groups (IND and EME) we select economies with debt-to-output ratio below the 25th percentile (these are economies with low debt-to-output) and for these economies we compute the average risk measure; we do the same for those economies with debt-to-output ratio above the 75th percentile (these are economies with high debt-to-output). For the case of low debt-to-output ratio, the EME group presents higher (approx. twice as high) default risk than the IND group; however, for high debt-to-

¹⁴I refer the reader to Koenker (2005) for a thorough treatment of quantiles and quantile-based econometric models.

¹⁵This type of graph is not the conventional QQplot as the axis have the value of the random variable which achieves a certain quantile and not the quantile itself. For our purposes, this representation is more convenient.

¹⁶I constructed the measure of default risk as the spread using the EMBI+ real index for countries for which it is available and using the 3-7 year real government bond yield for the rest, minus U.S. bond return. This is an imperfect measure of default risk for domestic debt since EMBI+ considers mainly foreign debt. However, it is still informative since domestic default are positively correlated with defaults on sovereign debt, at least for the period of 1950's onwards, see Fig. 10 in Reinhart and Rogoff (2008).

¹⁷I obtain this by projecting the 95 percentile point of the EME and LAC onto the X-axis and comparing with the 85 percentile point of IND.

Table 1: (A) Measure of default risk for EME and IND groups for different levels of debt-to-output ratio; (B) Std. Dev. of ctral. government revenue over GDP (%) for EME and IND groups for different levels of default risk.

(A)			(B)		
Debt/GDP	EME	IND	DEF. RISK	EME	IND
< 0.25%	5.4	2.0	< 0.25%	0.9	1.4
> 0.75%	10.7	2.9	> 0.75%	2.5	1.7

output ratio economies, this difference is multiplied by a factor four. *Thus, economies that are prone to default (EME and LAC) exhibit higher default risk than economies that do not default (in this dataset, represented by IND), and, moreover, the default risk is much higher for economies in the former group that have high levels of debt-to-output ratio.*

Table 2(B) compares the standard deviation of the central government revenue-to-output ratio between IND and EME matching them across low and high default risk levels. It shows that for IND there is little variation of the volatility across low and high levels of default risk. For EME, however, there standard deviation of the central government revenue-to-output ratio is higher for economies with high default risk.¹⁸ It is worth noting, that all the EME with high default risk levels defaulted at least once during our sample. *Thus, economies with higher default risk exhibit more volatile tax revenues than economies with low default risk.* This is particularly notable for the group of EME/LAC economies.

These stylized facts establish a link between (a) default risk/default events, (b) debt ceilings and (c) volatility of tax revenues. In particular, the evidence suggests that economies that show higher default risk, also exhibit lower debt ceilings and more volatile tax revenues. The theory below sheds a light upon the forces driving these facts.¹⁹

3 The Model

In this section we describe the stochastic structure of the model, the timing and policies of the government and present the households problem.

¹⁸I looked also at the inflation tax as a proxy for tax policy; results are qualitatively the same.

¹⁹It is important to note that we are not arguing any type of causality; we are just illustrating co-movements. In fact, in the model below, all three features are endogenous outcomes of equilibrium.

3.1 The setting

Let time be indexed as $t = 0, 1, \dots$. Let (g_t, δ_t) be the exogenous government expenditure at time t and the fraction of the defaulted debt which is re-paid when exiting autarky, resp. These are the exogenous driving random variables of this economy. Let $\omega_t \equiv (g_t, \delta_t) \in \mathbb{G} \times \bar{\Delta}$, where $\mathbb{G} \subset \mathbb{R}$, $\bar{\Delta} \equiv \Delta \cup \{1\} \cup \{\bar{\delta}\}$ and $\Delta \subset [0, 1)$ are compact, and in order to avoid technical difficulties, we assume $|\mathbb{G}|$ and $|\Delta|$ are finite.²⁰ The set Δ models the offers — as fractions of outstanding debt — to repay the defaulted debt; $\{1\}$ represents the case where the government services the totality of its debt, and $\bar{\delta}$ is such that that the government rejects it in every possible state of the world, is designed to capture situations where the government does not receive an offer to repay.²¹

Finally, we denote histories as $\omega^t \equiv (\omega_0, \omega_1, \dots, \omega_t) \in \Omega^t \equiv (\mathbb{G} \times \bar{\Delta})^t$ but we use $\omega \in \Omega$ to denote ω^∞ .

3.2 The government policies and timing

Let $\mathbb{B} \subseteq \mathbb{R}$ be compact. Let B_{t+1} be the choice of debt at time t to be paid at time $t + 1$; τ_t is the labor tax; d_t is the default decision, it takes value 1 if the government decides to default and 0 otherwise; finally, let a_t is the decision of accepting an offer to repay the default debt, it takes value 1 if the offer is accepted and 0 otherwise.

The timing for the government is as follows. At each time t , the government can levy distortionary linear labor taxes, and allocate one-period, non-state-contingent bonds to the households to cover the expenses g_t . The government, after observing the present government expenditure and the outstanding debt to be paid this period, has the option to default on 100 percent of this debt—i.e., the government has the option to refuse to pay the totality of the maturing debt.

As shown in figure D.2, if the government opts to exercise the option to default (node (B) in figure D.2), nature plays immediately, and with some probability, sends the government to temporary financial autarky, where the government is precluded from issuing bonds in that period. If this does not occur, the government enters a stage in which nature draws a fraction δ of debt to be repaid, and the government has the option to accept or reject this offer. If

²⁰For a given set, $|S|$ is the cardinal of the set.

²¹An alternative way of modeling this situation is to work with $\bar{\Delta} \equiv \Delta \cup \{1\} \cup \{\emptyset\}$ where $\emptyset$ indicates no offer. Another alternative way is to add an additional random variable, $\iota \in \{0, 1\}$ that explicitly indicates if the government received an offer ($\iota = 1$) or not ($\iota = 0$) and let $\bar{\Delta} \equiv \Delta \cup \{1\}$.

the government accepts, it pays the new amount (the outstanding debt times the fraction that nature chose), and it is able to issue new bonds for the following period. If the government rejects, it goes to temporary financial outage (bottom branch in figure D.2).

Finally, if the government is not in financial autarky—because it either chooses not to default, or it accepts the partial payment offer—then in the next period, it has the option to default, with new values of outstanding debt and government expenditure. If the government is in temporary financial autarky, then in the next period, it will face a new offer for partial payments with probability λ .

The next assumption formalizes the probability model mentioned above.

ASSUMPTION 3.1. $\Pr(g_t \in G | \delta_t, \omega^{t-1}) \equiv \pi_{\mathbb{G}}(G | g_{t-1})$ for any $G \in \mathbb{G}$, $\Pr(\delta_t \in D | g_t, \omega^{t-1}) \equiv \Pr\{\delta_t \in D | d_t\}$ for any $D \in \bar{\Delta}$, where:²²

$$\Pr\{\delta_t \in D | d_t\} = \begin{cases} 1\{1 \in D\} & \text{if } d_t = 0 \\ (1 - \lambda)1\{\{\bar{\delta}\} \in D\} + \lambda\pi_{\Delta}(D) & \text{if } d_t = 1 \end{cases}$$

Essentially, this assumption imposes a Markov restriction on the probability and also additional restrictions across the variables. In particular, given g , $1 - \lambda$ is the probability of $\delta = \bar{\delta}$ (i.e., not receiving an offer) and $\pi_{\Delta}(\cdot)$ is a probability over Δ . Finally, we use Π to denote the probability distribution over Ω generated by assumption 3.1, and $\Pi(\cdot | \omega^t)$ to denote the conditional probability over Ω , given ω^t .

The next definition formalizes the concept of government policy and the government budget constraint. In particular, it formally introduces the fact that debt is non-state contingent (i.e., B_{t+1} only depends on the history up to time t , ω^t).

Definition 3.1. A government policy is a sequence $(\sigma_t)_t$ where $\sigma_t \equiv (B_{t+1}, \tau_t, d_t, a_t) : \Omega \rightarrow \mathbb{B} \times [0, 1] \times \{0, 1\}^2$ only depends on the history up to time t , ω^t .²³

Let $(p_t)_t$ be a stochastic process (p_t depends on ω^t) that denotes the price of one unit of government debt, at time t . We refer to this process as a price schedule.

Definition 3.2. A government plan or policy σ^∞ is attainable (given a price schedule and initial debt B_0) iff for all t

$$g_t + \phi_t \delta_t B_t = \kappa_t \tau_t n_t + \phi_t p_t B_{t+1}.$$

with $\phi_t \equiv (1 - d_t) + d_t a_t$.²⁴

²²It is easy to generalize this to a more general formulation such as λ and π_{Δ} depending on g .

²³See appendix A for technical details.

²⁴As defined, the government policy and prices depend on the particular history ω , so the equalities are understood to hold for all $\omega \in \Omega$.

We define $\kappa_t \equiv \kappa_t(\sigma^\infty)$ as the productivity process. For simplicity we restrict it to be non-random, and following the sovereign default literature we set it to $\kappa_t = 1$ if $(1 - d_t) + d_t a_t = 1$ (i.e., either no default, or the country defaulted but accepted repayment offer) and $\kappa_t \equiv \kappa < 1$ otherwise, representing direct output costs of being in financial autarky. Also, observe that if the government defaults ($d_t = 1$) and rejects the offer of repayment ($a_t = 0$), its budget constraint boils down to $g_t = \tau_t n_t$, and if the government does default, $d_t = 1$ but accepts the offer to pay the defaulted debt, $a_t = 1$, then it has liabilities to be repaid for $\delta_t B_t$ and can issue new debt.

A few final remarks about the “debt-restructuring process” are in order. This process is defined by (λ, π_Δ) . These parameters capture the fact that debt-restructuring is time-consuming but, generally, at the end, a positive fraction of the defaulted debt is honored.²⁵²⁶ This debt-restructuring process intends to capture the fact that, after defaults (over domestic or international debt, or both), economies see their access to credit severely hindered. For, instance, this fact is well-documented for *sovereign* defaults; also, the data suggests that in many instances default on domestic debt and sovereign debt happen simultaneously.²⁷ Hence, the debt restructuring process intends to capture, up to some extent, this observed feature of the data.

3.3 The Household Problem

Households are price takers and homogeneous; they have time-separable preferences for consumption and labor processes. They also make debt/savings decisions by trading government bonds.

Given a government plan σ^∞ , let $\varrho_t(\cdot; \sigma^\infty) : \Omega \rightarrow \mathbb{R}$ be the time t payoff of one unit of government debt, given that the government acts according to σ^∞ ; i.e.,

$$\varrho_t(\omega; \sigma^\infty) = (1 - d_t(\omega)) + d_t(\omega)\{a_t(\omega)\delta_t(\omega) + (1 - a_t(\omega))q_t(\omega)\}$$

²⁵See Yue (2010), and Pitchford and Wright (2008) and Benjamin and Wright (2009) for two different ways of modeling this process as renegotiation between the government and the debt holders).

²⁶I could also allow for, say, $\pi_\Delta(\cdot | g_t, B_t, d_t, d_{t-1}, \dots, d_{t-K})$ some $K > 0$, denoting that possible partial payments depend on the credit history and level of debt. See Reinhart et al. (2003), Reinhart and Rogoff (2008) and Yue (2010) for an intuition behind this structure.

²⁷For instance Argentina defaulted three times on its domestic debt between 1980 and 2001. Two of these defaults coincided with external defaults (1982 and 2001). Also, in Reinhart and Rogoff (2008) figure 10 shows the probability of external default versus the comparable statistic for domestic default either through inflation or explicit default, one can see that after 1950's there is a close co-movement.

where $q_t(\omega)$ is just notation for the price of selling one unit of government debt in the secondary market at time t ; given that the government acts according to σ^∞ and history ω .

A few remarks about ϱ_t are in order. Since the household takes government actions as given, from the point of view of the households the government debt is an asset with payoff that depends on the state of the economy. That is, if the government decides not to default ($d_t = 1$) then $\varrho_t = 1$; if the government decides to default ($d_t = 0$) but then accepts to repay a fraction δ_t , the household receives $\varrho_t = \delta_t$; finally, if the governments default and rejects the repayment option, the household can sell the unit of government debt in the secondary market and obtain $\varrho_t = q_t$. Observe that in cases where the government never repays a positive fraction (e.g., the model by Arellano (2008)), $\varrho_t = q_t = 0$. Finally, the dependence of ϱ on the state clearly illustrates, that default decisions add certain degree of state contingency to the government debt.

Definition 3.3. *A household allocation is a (c^∞, n^∞) such that $c_t : \Omega \rightarrow \mathbb{R}_+$ and $n_t : \Omega \rightarrow [0, 1]$ depend only on the partial history up to time t , ω^t . A household debt plan is a b^∞ such that for all t , $b_{t+1} : \Omega \rightarrow [\underline{b}, \bar{b}]$ depends only on the partial history up to time t , ω^t .²⁸*

The household problem is given by: Given a (ω_0, b_0) ,

$$\sup_{(c^\infty, n^\infty, b^\infty) \in \mathbb{B}(g_0, b_0; \sigma^\infty)} E_{\Pi(\cdot|g_0)} \left[\sum_{t=0}^{\infty} \beta^t u(c_t(\omega), 1 - n_t(\omega)) \right]$$

where $\mathbb{B}(g_0, b_0; \sigma^\infty)$ is the set of household allocations and debt plans, such that for all t

$$c_t - (1 - \tau_t)\kappa_t n_t + p_t b_{t+1} = \varrho_t b_t,$$

and $b_{t+1} \leq b_t$, if $\phi_t = 0$. This restriction implies that, during financial autarky, when only secondary markets are open, the household cannot “print” debt.²⁹

4 Competitive Equilibrium with Government

We now define a competitive equilibrium, for a given government policy and derive the equilibrium taxes and prices.

Definition 4.1. *Given a $s_0 \equiv (g_0, B_0 = b_0, \phi_{-1})$, a competitive equilibrium with government is a government policy, σ^∞ , a household allocation, (c^∞, n^∞) , a household debt plan, b^∞ , and a price schedule p^∞ such that:*

²⁸I assume $b_{t+1} \in [\underline{b}, \bar{b}]$ and $[\underline{b}, \bar{b}] \supseteq \mathbb{B}$ so in equilibrium these restrictions will not be binding. See appendix A for more technical details regarding the debt and allocations.

²⁹Observe that, by definition, allocations, prices and debt plans depend on ω , so all equalities and inequalities are understood to hold for all $\omega \in \Omega$; for instance $b_{t+1} \leq b_t$ should be understood as $b_{t+1}(\omega) \leq b_t(\omega)$ for all $\omega \in \Omega$ and so on.

1. Given (g_0, b_0) , the government policy and the price schedule, the household allocation and debt plan solve the household problem.
2. Given B_0 and the price schedule, σ^∞ is attainable.
3. For all t , $c_t + g_t = \kappa_t n_t$.
4. For all t , $B_{t+1} = b_{t+1}$, and $B_{t+1} = B_t$ if $1 - d_t + d_t a_t = 0$.

We use $CEG(s_0)$ to denote the set of all competitive equilibrium with government. Observe that, the market clearing for debt indicates that $B_{t+1} = b_{t+1}$. However, if the economy is in financial autarky — where the government cannot issue debt, and thus agents must only trade among themselves —, we impose $B_{t+1} = B_t$ which implies $b_t = b_{t+1}$, i.e., agents do not change their debt positions.

4.1 Equilibrium Prices and Taxes

In this section we present the expressions for equilibrium taxes and prices of debt. The former quantity is standard (e.g. Aiyagari et al. (2002) and Lucas and Stokey (1983)); the latter quantity, however, incorporates the possibility of default of the government. The following assumption is standard and ensures that u is smooth enough to compute first order conditions.

ASSUMPTION 4.1. (i) $u \in \mathbb{C}^2(\mathbb{R}_+ \times [0, 1], \mathbb{R})$ with $u_c > 0$, $u_{cc} < 0$, $u_l > 0$ and $u_{ll} > 0$, and $\lim_{l \rightarrow 0} u_l(l) = \infty$.³⁰

From the first order conditions of the optimization problem of the households, the following equations follow³¹

$$\frac{u_l(c_t, 1 - n_t)}{u_c(c_t, 1 - n_t)} = (1 - \tau_t)\kappa_t(\sigma^\infty), \quad (1)$$

and

$$p_t = E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(c_{t+1}, 1 - n_{t+1})}{u_c(c_t, 1 - n_t)} \varrho_{t+1} \right]. \quad (2)$$

³⁰ $\mathbb{C}^2(X, Y)$ is the space of twice continuously differentiable functions from X to Y . The assumption $u_{cc} < 0$ could be relaxed to include $u_{cc} = 0$ (see the section 6 below).

³¹As before, we omit dependence on ω to ease the notational burden. For the more detailed expression and the complete derivations, please see appendix B.

From the definition of ϱ , and the restrictions on Π , equation 2 implies, for $d_t = 0$ or $a_t = 1$,

$$\begin{aligned} p_t = & \beta \int_{\mathbb{G}} \left(\frac{u_c(c_{t+1}, 1 - n_{t+1})}{u_c(c_t, 1 - n_t)} (1 - d_{t+1}) \right) \pi_{\mathbb{G}}(dg_{t+1}|g_t) \\ & + \beta \int_{\mathbb{G}} \lambda d_{t+1} \int_{\Delta} \frac{u_c(c_{t+1}, 1 - n_{t+1})}{u_c(c_t, 1 - n_t)} a_{t+1} \delta_{t+1} \pi_{\Delta}(d\delta_{t+1}) \pi_{\mathbb{G}}(dg_{t+1}|g_t) \\ & + \beta \int_{\mathbb{G}} \frac{u_c(c_{t+1}, 1 - n_{t+1})}{u_c(c_t, 1 - n_t)} d_{t+1} \left\{ \lambda \int_{\Delta} (1 - a_{t+1}) \pi_{\Delta}(d\delta_{t+1}) + (1 - \lambda) \right\} q_{t+1} \pi_{\mathbb{G}}(dg_{t+1}|g_t), \quad (3) \end{aligned}$$

where q_t denotes the price p_t for $d_t = 1$ and $a_t = 0$, and is given by

$$\begin{aligned} q_t = & \beta \int_{\mathbb{G}} \lambda \int_{\Delta} \frac{u_c(c_{t+1}, 1 - n_{t+1})}{u_c(c_t, 1 - n_t)} a_{t+1} \delta_{t+1} \pi_{\Delta}(d\delta_{t+1}) \pi_{\mathbb{G}}(dg_{t+1}|g_t) \\ & + \beta \int_{\mathbb{G}} \frac{u_c(c_{t+1}, 1 - n_{t+1})}{u_c(c_t, 1 - n_t)} \left\{ \lambda \int_{\Delta} (1 - a_{t+1}) \pi_{\Delta}(d\delta_{t+1}) + (1 - \lambda) \right\} q_{t+1} \pi_{\mathbb{G}}(dg_{t+1}|g_t). \quad (4) \end{aligned}$$

Each term in the equation 3 corresponds to a “branch” of the tree depicted in figure D.2. The first line represents the value of one unit of debt when the government chooses to honor the entire debt. The second line represents the value of the debt if the government decides not to pay the debt, but ends up in partial default. The third line captures the value of the debt when the government defaults on 100 percent of the debt, but the households can sell it in the secondary markets.

If $\lambda = 0$ and $u_c = 1$, then the last two terms vanish and the price is analogous to the one obtained in Arellano (2008). Also observe that, if $\lambda = 0$, it follows that $q_t = \int_{\mathbb{G}} \left(\frac{u_c(c_{t+1}, 1 - n_{t+1})}{u_c(c_t, 1 - n_t)} \right) q_{t+1} \pi_{\mathbb{G}}(dg_{t+1}|g_t)$, which by substituting forward and standard transversality conditions, yields $q_t = 0$.

The novelty of these pricing equations with respect to the standard sovereign default model, e.g., Arellano (2008) and Aguiar and Gopinath (2006) is the presence of secondary market prices, q_t .³² By imposing positive repayment (with some probability), the model is able to deliver a price of defaulted debt during the financial autarky period. In sections 6 and 7, we shed some light on the pricing implications of this model and how it relates with the data. Delving more on the pricing implications of equations 3 - 4, albeit outside the scope of this paper, seems like a promising avenue for future research.

5 The Government Problem

The government maximizes the welfare of the representative household by choosing the policies. The government, however, cannot commit to repaying the debt, but while having access to

³²See also Chatterjee and Eyingungor (2012) for the equilibrium prices in the presence of long term debt.

financial markets, commits to future tax promises. That is, as long as the government has access to financial markets, it honors past promises of taxes; when the government defaults and enters financial autarky ought to choose taxes to balance the budget by assumptions. Once the government exits financial autarky, it starts anew, without any outstanding tax promises.

It is worth to point out that, in the case where there is repayment for any state of the economy, the fiscal authority is the Ramsey problem studied by Aiyagari et al. (2002).³³ There, we also provide a succinct expression for the Bellman equation that defines the value function described in section 5.2 and the associated optimal policy functions, I also discuss and characterize the relevant state space which is an endogenous object; see Kydland and Prescott (1980) and Chang (1998).

Below, we first described the so-called implementability conditions for the government and then present the recursive formulation of the government's problem.

5.1 The Implementability Constraints

Recall that $\phi_t \equiv (1 - d_t) + d_t a_t$. By using the first order conditions 1 and 2, to replace taxes and prices in the government budget constraint, $\kappa_t \tau_t n_t - g_t + \phi_t \{p_t B_{t+1} - \delta_t B_t\} \geq 0$ (and $B_{t+1} = B_t$ if $\phi_t = 0$), we obtain

$$\left(\kappa_t - \frac{u_l(\kappa_t n_t - g_t, 1 - n_t)}{u_c(\kappa_t n_t - g_t, 1 - n_t)} \right) n_t - g_t + \phi_t \{p_t B_{t+1} - \delta_t B_t\} \geq 0.$$

Letting, $\mu_t \equiv u_c(\kappa_t n_t - g_t, 1 - n_t)$, then the display above can be cast as

$$Z_{\phi_t}(\mu_t, n_t, g_t) + \phi_t \{P_t B_{t+1} - \delta_t \mu_t B_t\} \geq 0 \quad (5)$$

where $p_t = P_t/\mu_t$ is given by the expression in equation 2 and

$$Z_{\phi_t}(\mu_t, n_t, g_t) \equiv (\kappa_t \mu_t - u_l(\kappa_t n_t - g_t, 1 - n_t)) n_t - \mu_t g_t.$$

The variable μ_t should be viewed as the marginal utility of consumption at time t , that was promised at time $t-1$. The intuition behind this variable is that that CEG can be characterized by a sequence of equations (given by first order conditions and budget constraints) and each one connects only periods of “today” and “tomorrow”. Moreover, from the perspective of any period, a CEG can be seen as the current policies and allocations, together with a “promise”

³³Also, in the case the government had access to lump-sum taxes, it will set distortionary taxes to zero and thus this model would be akin to that of Arellano (2008).

of policies for next period; this is being captured by μ . See Kydland and Prescott (1980) and Chang (1998) for a more thorough discussion in similar settings.

In principle, μ_t should be specified for each $\omega_t \equiv (g_t, \delta_t)$, we thus use $\vec{\mu}_t$ to denote a function from $\mathbb{G} \times \bar{\Delta}$ to $\mathbb{R}_+$. Thus, $\vec{\mu}_t(g_t, \delta_t)$ denotes the function $\vec{\mu}_t$ evaluated at (g_t, δ_t) . Also, observe that, from equation 2, equilibrium prices at time t are only a function of $(g_t, \vec{\mu}_{t+1})$ and of the default and debt-repayment strategies, i.e., $\mathcal{P}_t = \mathcal{P}(g_t, \vec{\mu}_{t+1}, d_{t+1}, a_{t+1})$ (henceforth, unless needed we leave the dependence on the default and debt-repayment strategies implicit, to ease the notational burden).

Thus, equation 5 imposes the following set of constraints: Given any (g, B, μ, ϕ) ,

$$\begin{aligned} \bar{\Gamma}(g, B, \mu, \phi) \equiv \{ & (n, B', \vec{\mu}') \in [0, 1] \times \mathbb{B} \times \mathbb{M} : \mu = u_c(\kappa_\phi n - g, 1 - n), \\ & Z_\phi(\mu, n, g) + \phi \{ \mathcal{P}(g, \vec{\mu}')B' - B\mu \} \geq 0, \text{ and if } \phi = 0, B' = B \}, \end{aligned} \quad (6)$$

where $\kappa_\phi \equiv \kappa(1 - \phi) + \phi$. It is clear by the last equality, that μ imposes restrictions on the choice of n . In fact, if $u_c(\kappa n - g, 1 - n)$ is monotonic as a function of n (e.g., if u is separable in leisure and consumption and increasing in the latter) then there exists only one possible n given (μ, g) . The set $\mathbb{M}$ summarizes the a-priori restrictions on $\vec{\mu}$ and it is given by $\mathbb{M} \equiv \cup_{g \in \mathbb{G}} \mathbb{M}(g)$ where $\mathbb{M}(g) \equiv \{m : \exists n \in [0, 1], \text{ s.t. } m(g) = u_c(n - g, 1 - n)\}$.

5.2 The Government Optimization Problem

The government problem is divided in two parts: an “initial problem” (i.e., the problem at time 0) and the “continuation problem” at (i.e., at time $t \geq 1$). We start with the latter which can be cast recursively. For any $(g, B, \vec{\mu}, \phi)$, let $\mathbf{V}^*(g, B, \vec{\mu}, \phi)$ be the value of having the option to default (if $\phi = 1$) or having the option to repay the default debt (if $\phi = 0$), given g , an outstanding level of debt B and a profile $\vec{\mu}$. Also, $\mathbf{d}^*$ and $\mathbf{a}^*$ denote the optimal policy functions for default and for repayment of a fraction of the defaulted debt, resp. For expositional purposes, we separate the study of this problem into two cases: the case where the economy is in financial access ($\phi = 1$) and the one where the economy is in financial autarky ($\phi = 0$).

Financial Access Case: In this case (where $\phi = 1$), the government has the option to default on the debt. Thus, for any $(g, B, \vec{\mu}, 1) \in \mathbf{R}^*$,

$$\mathbf{V}^*(g, B, \vec{\mu}, 1) = \max \{ \mathcal{V}_1^*(g, B, \vec{\mu}(g, 1)), \mathbf{V}^*(g, B, \vec{\mu}) \} \quad (7)$$

where

$$\mathbf{V}^*(g, B, \vec{\mu}) \equiv \lambda \int_{\Delta} \max \{ \mathcal{V}_1^*(g, \delta B, \vec{\mu}(g, \delta)), \mathcal{V}_0^*(g, B) \} \pi_{\Delta}(d\delta) + (1 - \lambda) \mathcal{V}_0^*(g, B). \quad (8)$$

The set $\mathbf{R}^*$ is the set of states, $(g, B, \vec{\mu}, \phi)$ for which there exists a CEG that takes these as initial states. The value $\mathcal{V}_1^*(g, \delta B, \mu)$ is the value of repaying a fraction δ of the outstanding debt, B , given (g, μ) , and the value $\mathcal{V}_0^*(g, B)$ is the value of defaulting on the outstanding debt. The function $\mathbb{V}^*$ is the value function of the government that defaulted ($d = 1$) and is awaiting the lottery to receive an offer of repayment (and has the option to reject it).

The “max” in equation 7 stems from the fact that the default authority optimally compares the value of not defaulting and paying the totality of the outstanding debt (i.e., $\mathcal{V}_1^*(g, B, \vec{\mu}(g, 1))$) or defaulting, and waiting for an offer of repayment (i.e., $\mathbb{V}^*(g, B, \vec{\mu})$). The max in equation 8 arises from the fact that the default authority optimally compares the value of accepting the offer of repayment (given a fraction δ at hand) and the value of rejecting.

Therefore, the optimal policy functions are: ³⁴ For any $(g, B, \vec{\mu}, \phi)$,

$$\mathbf{d}^*(g, B, \vec{\mu}, \phi) = 1 \{ \mathcal{V}_1^*(g, B, \vec{\mu}(g, 1)) < \mathbb{V}^*(g, B, \vec{\mu}) \} \text{ if } \phi = 1$$

(if $\phi = 0$, the government is “forced” to choose $d = 1$), and, for any (g, δ, B, μ)

$$\mathbf{a}^*(g, \delta, B, \mu) = 1 \{ \mathcal{V}_1^*(g, \delta B, \mu) \geq \mathcal{V}_0^*(g, B) \} \text{ if } \delta \neq \bar{\delta}$$

(if $\delta = \bar{\delta}$ —which occurs with probability $1 - \lambda$ — the government is “forced” to choose $a = 0$).

Finally, the function $\mathcal{V}^*$ is given by:

$$\mathcal{V}_\phi^*(g, B, \mu) = \max_{(n, B', \vec{\mu}') \in \Gamma(g, B, \mu, \phi)} \left\{ u(\kappa_\phi n - g, 1 - n) + \beta \int_{\mathbb{G}} \mathbf{V}^*(g', B', \vec{\mu}', \phi) \pi_{\mathbb{G}}(dg' | g) \right\} \quad (9)$$

for any (g, B, μ, ϕ) .

Observe that, when optimally choosing $(n, B', \vec{\mu}')$, the government takes as given that the (future) default authority acts according to $(\mathbf{d}^*, \mathbf{a}^*)$. This implies that equilibrium prices depend on B' ; this is a feature of sovereign default models, see Arellano (2008).

Financial Autarky Case: In this case ($\phi = 0$), the government is in financial autarky. It cannot issue new debt (or equivalently, the new debt ought to coincide with the outstanding defaulted debt). Additionally, since the economy is already in default, the government does not have to decide whether to default or not (or, equivalently, it is forced to choose $d = 1$).

³⁴As defined, we are imposing that if indifferent the government chooses not to default. This is just a normalization that is standard in the literature; it is easy to see that $\mathbf{d}^*$ could be defined as a correspondence, taking any value in $[0, 1]$ if $\mathcal{V}_1^*(g, B, \vec{\mu}(g, 1)) = \mathbb{V}^*(g, B, \vec{\mu})$.

In this case, $\mathbf{V}^*$, for any $(g, B, \vec{\mu}, 0) \in \mathbf{R}^*$, is given by,

$$\begin{aligned} \mathbf{V}^*(g, B, \vec{\mu}, 0) &= \mathbb{V}^*(g, B, \vec{\mu}) \\ &= \lambda \int_{\Delta} \max \{ \mathcal{V}_1^*(g, \delta B, \vec{\mu}(g, \delta)), \mathcal{V}_0^*(g, B) \} \pi_{\Delta}(d\delta) + (1 - \lambda) \mathcal{V}_0^*(g, B). \end{aligned} \quad (10)$$

Observe that in this case (of $\phi = 0$), $\bar{\Gamma}(g, B, \mu, \phi) = \{(n, B', \vec{\mu}) : B' = B, Z_{\phi}(\mu, n, g) \leq 0 \text{ and } \mu = u_c(\kappa n - g, 1 - n)\}$. Therefore, n ought to be such that the budget is balanced (i.e., $Z(u_c(\kappa n - g, 1 - n), n, g) = 0$); denote such n as $n_A(g)$ for any g . Moreover, μ does not impose any restrictions on $(B', \vec{\mu}')$ because, first, the government cannot issue new debt while in financial autarky and ought to keep track of the defaulted debt (i.e., $B' = B$); second, since in financial autarky there is no issuance of new debt, the government does not have any “outstanding past promising” of consumption. This implies that

$$\mathcal{V}_0^*(g, B) = u(\kappa n_A(g) - g, 1 - n_A(g)) + \beta \max_{\vec{\mu}' \in \mathbb{M}} \int_{\mathbb{G}} \mathbf{V}^*(g', B, \vec{\mu}', 0) \pi_{\mathbb{G}}(dg' | g).$$

That is, the government receives the payoff of running a balance budget, $u(\kappa n_A(g) - g, 1 - n_A(g))$, and next period will have the option to repay the outstanding debt, without any past tax promises, hence $\mathbf{V}^*(g, B, \vec{\mu}', 0)$ is being maximize over all (feasible) $\vec{\mu}'$ (the expression for $\mathbf{V}^*$ is below, in equation 10). Observe that the RHS of the equation does not depend on $\vec{\mu}$ or δ , thus, for case $\phi = 0$, $\mathcal{V}^*$ does not depend on $\vec{\mu}$.

To conclude, we present the problem of the government at time $t = 0$. The crucial difference is that the government starts the period without any outstanding past consumption promises and both (g_0, B_0, ϕ_{-1}) are given as parameters.³⁵ That is, $\mathbf{V}_o^*(g_0, B_0, \phi_{-1}) \equiv \max_{\vec{\mu} \in \mathbb{M}} \mathbf{V}^*(g_0, B_0, \vec{\mu}, \phi_{-1})$.

Below we present some particular cases where the value function gets simplified.

5.2.1 Example I: The case of $\lambda = 0$

Under this assumption, equation 7 boils down to

$$\mathbf{V}^*(g, B, \mu, 1) = \max \{ \mathcal{V}_1^*(g, B, \mu), \mathcal{V}_0^*(g) \}.$$

Observe that there is no need to have the whole function $\vec{\mu}$ as part of the state, only $\vec{\mu}(g, 1)$ — which without loss of generality we denoted as μ —. Also, there is no need to keep B as part

³⁵There exists the restriction that (g_0, B_0, ϕ_{-1}) and the solution of $\vec{\mu}$ ought to be such that, taking these quantities as starting values, a CEG exists. That is, there is a continuation sequence that satisfy the restrictions for a CEG.

of the state during financial autarky, since defaulted debt is never repaid. Hence, $\mathcal{V}_0^*$ is given by

$$\mathcal{V}_0^*(g) = u(\kappa n_A(g) - g, 1 - n_A(g)) + \beta \int_{\mathbb{G}} \mathcal{V}_0^*(g') \pi_{\mathbb{G}}(dg'|g).$$

And

$$\mathcal{V}_1^*(g, B, \mu) = \max_{(n, B', \vec{\mu}') \in \bar{\Gamma}(g, B, \mu, 1)} \left\{ u(n - g, 1 - n) + \beta \int_{\mathbb{G}} \mathbf{V}^*(g', B', \vec{\mu}'(g'), 1) \pi_{\mathbb{G}}(dg'|g) \right\},$$

5.2.2 Example II: The case of $u_c = 1$ and π_{Δ} degenerate at 0.

Under this assumption, μ can be dropped as a state variable (since $u_c = 1$ and thus it does not affect the pricing equation). Now, equation 7 boils down to

$$\mathbf{V}^*(g, B, 1) = \max\{\mathcal{V}_1^*(g, B), \lambda \max\{\mathcal{V}_1^*(g, 0), \mathcal{V}_0^*(g)\} + (1 - \lambda) \mathcal{V}_0^*(g)\},$$

and

$$\mathcal{V}_0^*(g) = u(\kappa n_A(g) - g, 1 - n_A(g)) + \beta \int_{\mathbb{G}} (\lambda \max\{\mathcal{V}_1^*(g', 0), \mathcal{V}_0^*(g')\} + (1 - \lambda) \mathcal{V}_0^*(g')) \pi_{\mathbb{G}}(dg'|g).$$

As in the previous example, there is no need to keep B as part of the state during financial autarky, since defaulted debt is never repaid. Finally,

$$\mathcal{V}_1^*(g, B) = \max_{(n, B') \in \bar{\Gamma}(g, B)} \left\{ u(n - g, 1 - n) + \beta \int_{\mathbb{G}} \mathbf{V}^*(g', B', 1) \pi_{\mathbb{G}}(dg'|g) \right\},$$

where $\bar{\Gamma}(g, B) \equiv \bar{\Gamma}(g, B, 1)$.

This example is closely related to the models by Arellano (2008) and Aguiar and Gopinath (2006) (without distortionary taxes).³⁶

5.2.3 Example III: The case of $\lambda = 0$ and no default.

Consider an economy where there is no default (this is imposed ad-hoc), then the value function boils down to

$$\mathbf{V}^*(g, B, \mu) = \max_{(n, B', \vec{\mu}') \in \bar{\Gamma}(g, B)} \left\{ u(n - g, 1 - n) + \beta \int_{\mathbb{G}} \mathbf{V}^*(g', B', \vec{\mu}'(g')) \pi_{\mathbb{G}}(dg'|g) \right\},$$

and now $\mathbf{d}^*$ is set to never default, also note that $\mathbf{V}^*$ does not depend on ϕ , since trivially $\phi = 1$ always. This is precisely the type of model studied in Aiyagari et al. (2002).

³⁶There is a slight difference in the timing; these models define $\mathbf{V}^*(g, B, 1) = \max\{\mathcal{V}_1^*(g, B), \mathcal{V}_0^*(g)\}$. Our model could be adapted to replicate this timing.

6 Analytical Results

In this section we present some analytical results for a benchmark model that is characterized by quasi-linear per-period utility and stochastically ordered process for g .³⁷ The proofs for the results are gathered in appendix C.

ASSUMPTION 6.1. $u(c, n) = c + H(1 - n)$ where $H \in \mathbb{C}^2((0, 1), \mathbb{R})$ with $H'(0) = \infty$, $H'(l) > 0$, $H'(1) < \kappa$, $2H''(l) < H'''(l)(1 - l)$

This assumption imposes that the per-period utility of the households is quasi-linear and it is analogous to, say, assumption in p. 10 in AMSS. As noted above, under this assumption, μ can be dropped as a state variable. This implies that $\mathbf{V}^*$ is only a function of (g, B, ϕ) and the same holds true for the optimal policy functions. It is also clear from the previous section that $\mathbf{d}^*(g, B, \phi)$ is only non-trivial if $\phi = 1$; thus, henceforth, we omit the dependence of ϕ .

Moreover, to further simplify the technical details, we assume, unless stated otherwise, that $\mathbb{B}$ has only finitely many points each.³⁸

6.1 Characterization of optimal default decisions

The next theorem characterizes the optimal decisions of default and acceptance offer to repay the defaulted debt as “threshold decisions”; it is analogous to the one in Arellano (2008), but extended to this setting. Recall that, $\mathbf{d}^*(g, B)$ and $\mathbf{a}^*(g, \delta, B)$ are the optimal decision of default and acceptance offer respectively, given the state (g, δ, B) .

THEOREM 6.1. *Suppose assumption 6.1 holds and suppose $\kappa = 1$ and $H'' < 0$. Then, there exists $\bar{\lambda}$ such that for all $\lambda \in (0, \bar{\lambda})$, the following holds:*

1. *There exists a $\bar{\delta} : \mathbb{G} \times \mathbb{B} \rightarrow \Delta$ such that $\mathbf{a}^*(g, \delta, B) = 1\{\delta \leq \bar{\delta}(g, B)\}$ and $\bar{\delta}(g, B)$ non-increasing as a function of B .³⁹*
2. *If, in addition, for any $g_1 \leq g_2$, $\pi_{\mathbb{G}}(\cdot|g_1) \leq_{FOSD} \pi_{\mathbb{G}}(\cdot|g_2)$, there exists a $\bar{g} : \mathbb{B} \rightarrow \mathbb{G}$ such that $\mathbf{d}^*(g, B) = 1\{g \geq \bar{g}(B)\}$ and $\bar{g}$ non-increasing.*

³⁷By stochastically ordered, we mean that the transition probability of g satisfies that for any $g_1 \leq g_2$, $\pi_{\mathbb{G}}(\cdot|g_1) \leq_{FOSD} \pi_{\mathbb{G}}(\cdot|g_2)$; where, for two probability measures P and Q , $P \leq_{FOSD} Q$ means that the corresponding cdf, $F_P(X \leq t) \geq F_Q(Y \leq t)$ for any t , where F_P (F_Q) is the cdf associated to the probability measure P (Q).

³⁸This assumption is made for simplicity. It can be relaxed to allow for general compact subsets, but some of the arguments in the proofs will have to be change slightly. Also, the fact that $\mathbb{B} \equiv \{B_1, \dots, B_{|\mathbb{B}|}\}$ is only imposed for the government; the households can still choose from convex sets; only in equilibrium we impose $\{B_1, \dots, B_{|\mathbb{B}|}\}$.

³⁹It turns out, that the first part of the statement holds for any λ .

This result shows that default is more likely to occur for high levels of debt, but so are rejections of offers to exit financial autarky. The latter result implies that the average fraction of repaid debt, conditional on the offer being accepted, is decreasing with the level of debt.⁴⁰ It also follows that other things equal, higher debt levels are, on average, associated with longer financial autarky periods. Thus, these two results imply a positive co-movement between the (observed) average haircut and the average length of financial autarky.⁴¹

Finally, the result holds for $\lambda \in (0, \bar{\lambda})$, because for $0 < \lambda \leq \bar{\lambda}$, i.e., λ 's that are small enough, the variation of the continuation value of financial autarky with respect to the outstanding defaulted debt can be controlled and is dominated by the variation of the per-period payoff. This allows us to show that the value functions satisfy a single crossing condition that is the key to show the results.

6.2 Implications for equilibrium Prices and Taxes

We now study the implications of the above results on prices and taxes. We take the assumptions needed for theorem 6.1 as given throughout the section.

Observe that

$$\begin{aligned} p(g_t, B_{t+1}) = & \beta \int_{\mathbb{G}} 1\{g' \leq \bar{g}(B_{t+1})\} \pi_{\mathbb{G}}(dg'|g_t) + \beta \lambda \int_{\mathbb{G}} 1\{g' > \bar{g}(B_{t+1})\} D(g', B_{t+1}) \pi_{\mathbb{G}}(dg'|g_t) \\ & + \beta \int_{\mathbb{G}} 1\{g' > \bar{g}(B_{t+1})\} (\lambda \alpha(g', B_{t+1}) + (1 - \lambda)) q(g', B_{t+1}) \pi_{\mathbb{G}}(dg'|g_t), \end{aligned} \quad (11)$$

where $\alpha(g, B) \equiv \int_{\Delta} 1\{\delta > \bar{\delta}(g, B)\} \pi_{\Delta}(d\delta)$, $D(g, B) \equiv \int_{\Delta} 1\{\delta \leq \bar{\delta}(g, B)\} \delta \pi_{\Delta}(d\delta)$, and

$$q(g_t, B_t) = \beta \lambda \int_{\mathbb{G}} D(g', B_t) \pi_{\mathbb{G}}(dg'|g_t) + \beta \int_{\mathbb{G}} (\lambda \alpha(g', B_t) + (1 - \lambda)) q(g', B_t) \pi_{\mathbb{G}}(dg'|g_t). \quad (12)$$

Compare this with the case that there is no defaulted-debt repayment, $p^o(g_t, B_{t+1}) \equiv \beta \int_{\mathbb{G}} 1\{g' \leq \bar{g}(B_{t+1})\} \pi_{\mathbb{G}}(dg'|g_t)$. From analogous calculations to those in theorem 6.1, it follows that $p^o(g, B)$ is non-increasing as a function of B , given raise to endogenous debt limits; see Arellano (2008). By inspection of equation 11, it is easy to see that, other things equal, this result is attenuated by the presence of (potential) defaulted debt payments and secondary markets. Although, for a general π_{Δ} and $\pi_{\mathbb{G}}$ is hard to further characterize p and q , the next theorem shows that q and p are both non-increasing on the level of debt, once we impose additional restrictions on π_{Δ} and $\pi_{\mathbb{G}}$.

⁴⁰According to the theorem, this quantity equals $E_{\pi_{\mathbb{G}}}[\int_{\delta \in \Delta} \delta 1\{\delta \leq \bar{\delta}(g, B)\} \pi_{\Delta}(d\delta)]$.

⁴¹This last fact seems to be consistent with the data; see Benjamin and Wright (2009) Fact 3 in the paper. It is important to note, however, that we derived the implications by looking at *exogenous* variation of the debt level; in the data this quantity is endogenous and, in particular varies with g . This endogeneity issue should be taken into account if one would like to perform a more thorough test of the aforementioned implications.

THEOREM 6.2. *Suppose assumptions of theorem 6.1 hold. Suppose further that $\pi_\Delta = \mathbf{1}_{\delta_0}(\delta)$ and $g \sim iid\pi_\mathbb{G}$. Then: (1)*

$$q(g, B) = q(B) = \frac{\beta\lambda\delta_0 A(B)}{1 - \beta + \beta\lambda A(B)}, \text{ where } A(B) = \pi_\mathbb{G}(\{g: \delta_0 \leq \bar{\delta}(g, B)\}),$$

and is non-increasing as a function of B . (2) $p(g, B)$ is non-increasing as a function of B .

This theorem shows that high levels of debt are associated with higher return on debt, both, before and during financial autarky. Moreover, it also shows that prices are increasing on the fraction of repayment (δ_0) and on the probability of receiving an offer λ . This result and those in theorem 6.1 imply a positive relationship between the return, the (observed) average recovery rate and the length of financial autarky. Whether these results hold once we relax the strong assumptions over π_Δ and $\pi_\mathbb{G}$, is unknown to me. In the numerical simulation, however, we allow for more general formulations, and find that prices are still non-increasing in B (see figure D.3).

In the next theorem, we show that under some additional assumptions there exists a finite level of debt such that $p(g, B)B$ is maximal as a function of B , and thus an endogenous debt limit exists.

THEOREM 6.3. *Suppose assumptions of theorem 6.2 hold, $\mathbb{B} = [\underline{B}, \bar{B}]$ such that (i) $\underline{B} \leq 0$, (ii) $p(\bar{B}) = q(\bar{B}) = 0$, and (iii) $p(g, B)B$ is continuous as a function of B . Then there exists a $B^* \in (\underline{B}, \bar{B})$ such that the optimal level of debt belongs to $(\underline{B}, B^*)$.*

A few remarks about this theorem are in order. First, assumptions (i)-(ii) are designed to control $p(g, B)B$ at the boundary. In particular, assumption (i) ensures that the derivative is positive in $\underline{B}$ and (ii) ensures that the revenue from debt is naught at $\bar{B}$. Since the payoff obtained from tax revenues is bounded, one could always choose $\bar{B}$ to be so high that the government cannot levy enough taxes to pay it. Second, this result in conjunction with theorem 6.2, shed some light on the evidence regarding debt-to-output and default risk measures presented in section 2.

In order to analyze the ex-ante effect of default risk on taxes, we consider the case $\lambda = 0$ (i.e., autarky is an absorbing state) to simplify the analysis. By theorem 6.1, the default decision is a threshold decision, so, for each history $\omega \in \Omega$ we can define $T(\omega) = \inf\{t : g_t \geq \bar{g}(B_t(\omega))\}$ (it could be infinity) such that for all $t \leq T(\omega)$ the economy is not in financial autarky. For such t , the implementability constraint is given by (omitting the dependence on ω)

$$B_t u_c(n_t - g_t, 1 - n_t) + g_t u_c(n_t - g_t, 1 - n_t) \leq (u_c(n_t - g_t, 1 - n_t) - u_l(n_t - g_t, 1 - n_t)) n_t + p_t u_c(n_t - g_t, 1 - n_t) B_{t+1}.$$

Let $\nu_t(\omega)$ be the Lagrange multiplier associated to this restriction in the optimization problem of the government, given (t, ω) . Observe that if $\nu_t = 0$, from the first display it is easy to see that $\tau_t = 0$.

From the FONC of the government it follows (see appendix C.1 for the derivation) ⁴²

$$u_c(n_t - g_t, 1 - n_t) - u_l(n_t - g_t, 1 - n_t) - \nu_t \frac{dA(n_t, g_t, B_t)}{dn_t} = 0,$$

where $A(n, g, B) \equiv (u_c(n - g, 1 - n) - u_l(n - g, 1 - n))n - (g + B)u_c(n - g, 1 - n)$. And

$$\nu_t \left(1 + \frac{dp_t}{dB_{t+1}} \frac{B_{t+1}}{p_t} \right) = \int_{\mathbb{G}} \nu_{t+1} \frac{1\{g' \leq \bar{g}(B_{t+1})\}}{\int_{\mathbb{G}} 1\{g' \leq \bar{g}(B_{t+1})\} \pi_{\mathbb{G}}(dg'|g_t)} \pi_{\mathbb{G}}(dg'|g_t). \quad (13)$$

The Lagrange multiplier associated with the implementability condition is constant in Lucas and Stokey (1983) and, thus, trivially a martingale. In Aiyagari et al. (2002) the Lagrange multiplier associated with the implementability condition is a martingale with respect to the probability measure $\pi_{\mathbb{G}}$.⁴³ Equation 13 implies that the law of motion of the Lagrange multiplier differs in two important aspects. First, the expectation is computed under the default-adjusted measure; this stems from the fact that the option to default adds “some” degree of state-contingency to the payoff of the government debt; this effect lowers the marginal cost of the debt. Second, the aforementioned expectation is multiplied by $\left(1 + \frac{dp_t(B_{t+1})}{dB_{t+1}} \frac{B_{t+1}}{p_t(B_{t+1})}\right)$, which can be interpreted as the “markup” that the government has to pay for having this option to default; this effect increases the marginal cost of the debt.

The next lemma studies further what happens to taxes and production on the eve of default. Namely, it establishes that, for states (g, B) for which the government chooses to default, labor (and thus production) in financial autarky is higher than what it would have been under financial access, and also, if H is concave (convex), taxes in financial autarky are lower (higher) than what it would have been under financial access.

Let $\mathbf{n}_C^*(g, \delta, B)$ be the optimal choice of labor under access to financial markets, given state (g, δ, B) . Let $\tau_A^*(g)$ and $\mathbf{n}_A^*(g)$ be the optimal choice of tax and labor under financial autarky, given state g . ⁴⁴

LEMMA 6.1. *Suppose assumption 6.1 holds, suppose $\kappa = 1$ and $H'' \neq 0$. Suppose that $\mathcal{D} \equiv \{(g, B) : \mathbf{d}^*(g, B) = 1 \text{ and } \mathbf{n}_C^*(g, 1, B) < n_1(1)\} \neq \{\emptyset\}$. Then there exists a $\bar{\lambda}$ such that for all $0 < \lambda \leq \bar{\lambda}$:*

⁴²This derivation assumes that $\mathbb{B}$ is at least a convex set, so as to make sense of differentiation. It, also, assumes differentiability of $\bar{g}$.

⁴³The martingale property is also preserved if capital is added to the economy; see Farhi (2010). This property, however, changes if we allow for ad-hoc borrowing limits (see Aiyagari et al. (2002))

⁴⁴Observe that τ_A^* and $\mathbf{n}_A^*$ do not depend on B , because the latter is obtained from balancing the budget, and the former is a function of the latter.

(1) $\mathbf{n}_C^*(g, 1, B) < \mathbf{n}_A^*(g)$, for all $(g, B) \in \mathcal{D}$.

(2) If $H'' < (>)0$ then $\tau_A^*(g, B) < (>)\tau_C^*(g, 1, B)$, for all $(g, B) \in \mathcal{D}$.

The condition that $\mathbf{n}_C^*(g, 1, B) < n_1(1)$ is an interiority assumption, and it implies that $\tau_C^*(g, 1, B) < 1$ for all $(g, B) \in \mathcal{D}$ and is needed to ensure that the per-period payoff is decreasing.⁴⁵ This result stems from the fact that, if default is chosen, it must be true that the per-period payoff under financial autarky is high enough to compensate the government for exercising the option value of deferring default for one period.

7 Numerical Results

Throughout this section, we run a battery of numerical exercises in order to assess the performance of the model. We compare our findings with an economy in which the option to default is not present—precisely the model considered in Aiyagari et al. (2002). We denote the variables associated with this model with a (sub)superscript “AMSS”; variables associated to our economy are denoted with a (sub)superscript “ED” (short for Economy with Default).

In the dataset IND economies are proxies of the AMSS model and EME/LAC are proxies of our model. As discussed before, IND do not exhibit default events in the dataset. Thus, we take IND as a proxy for economies modeled in AMSS. There is the question of what characteristics of the economy will prompt it to behave like AMSS- or ED-type economies. One possible explanation is that by factors extraneous to the model, such as political instability, ED presents lower discount factor from the government and thus are more likely to default. Another possible explanation is that for AMSS-type/IND economy, default is more costly because these economies are financially more integrated, and a default — and the posterior period of financial autarky — could have a larger impact on financing of the firms, thus lowering the productivity (in the model represented by a lower κ). The next lemma shows that, for a simplified version of the economy, higher κ imply lower likelihood of default⁴⁶

LEMMA 7.1. *Suppose the assumptions 6.1 - 3.1 hold, $H'' < 0$ and $\lambda = 0$. Let $\kappa < \kappa'$ and let $\mathcal{D}_\kappa \equiv \{(g, B) : \mathbf{d}^*(g, B) = 1\}$, then $\mathcal{D}_\kappa \subseteq \mathcal{D}_{\kappa'}$.*

For all the simulations the utility function is given by $u(c, 1 - n) = c + C_1 \frac{(1-n)^{1-\sigma}}{1-\sigma}$. The process for $(g_t)_t$ follows a 5 state discrete state space Markov chain “induced” by $g_{t+1} = \mu_g(1 - \rho_1) + \rho_1 g_t + \sigma_g \sqrt{1 - \rho_1^2} \varepsilon_{t+1}$, $\varepsilon_{t+1} \sim N(0, 1)$. By “induced”, we mean that the transition probability follows from applying Tauchen’s (Tauchen (1986)) results to the AR(1) process.

⁴⁵ $n_1(1)$ is defined in lemma C.1 and is the value of n such that makes tax revenue equal to zero.

⁴⁶This finding is still present in the numerical simulations that allow for more general parametrization.

We calibrate the parameters of the model as follows. We choose $\beta = 0.9$, $\sigma = 3$, $\kappa = 1$ and $C_1 = 0.01$. This choice is taken from AMSS; the authors choose $C_1 = 1$ but work with 100 as the unit of time — to be split between leisure and labor — whereas we work with 1. The state space is given by $\mathbb{B} = [0, 0.5]$ and $\mathbb{G} = [0, 0.1]$.⁴⁷ For the benchmark parametrization (column (I)) we choose $\rho_1 = 0$, $\mu_g = 0.05$, $\sigma_g = 0.045$, $\lambda = 0.25$, and $\Delta = \{0.4, 0.6, 0.8\}$ where the probability π_Δ puts uniform probability over the interval. We choose β , λ and Δ to match: a default frequency between 3-6 percent, a recovery rate of (approx.) 50 percent and a autarky spell between 10-15 periods.⁴⁸

We perform 1000 MC iterations, each consisting of sample paths of 1000 observations for which only the last 250 observations were considered in order to eliminate the effect of the initial values.⁴⁹

Table 2: In the table, E and std denote the mean and standard deviation across time, respectively. All quantities are averaged across MC simulations. In parenthesis the 5% and 95% percentile of the MC sample. All results (except Default spell) are in percentages.

Whole sample	(I)		(II)	(III)	(IV)	(V)
	AMSS	ED	ED	ED	ED	ED
$E(b_t/n_t)$	6.1	3.4	3.5	5.8	3	0.4
	(5.1,7)	(3.3,8)	(3,4)	(5,6.8)	(2.8,3.5)	(0.3,0.5)
$E(\tau_t)$	7.0	6.5	6.5	6.7	6.5	6.4
	(6.5,7.6)	(6,7)	(6,7)	(6.2,7.2)	(6,6.7)	(5,7.8)
$Std(\tau_t)$	2.5	4.2	4.1	3.5	4.4	4.2
	(2.2,2.8)	(4,4.4)	(3.8,4.4)	(3.3,3.7)	(4.1,4.6)	(3.8,4.6)
$E(Spread)$		18	14	10	3.2	13
		(15,21)	(11,17)	(8.8,11.3)	(2.5,3.7)	(9,15)
E(Def. Spell)		9.3	10.5	2.7	17	38.4
E(Rec. Rate)		49.5	60	55	54	56
$Pr(Def)$		6.4	5.5	18.7	4.8	3.5
		(4.6,8.6)	(4.4,7.5)	(16,21.5)	(2.8,6.7)	(0.8,6.3)

Table 2 reports the results for the whole sample and tables 3 report the results for the “financial autarky” sample and “financial access” sample, resp. I constructed these latter sub-samples by separating, for each MC iterations, the periods in which the ED economy was in autarky from

⁴⁷In this parametrization $|\mathbb{B}| = 50$.

⁴⁸The default recovery rate is taken from Yue (2010), where 35 percent is the recovery rate for Argentina and 65 percent is the recovery rate for Ecuador. See Pitchford and Wright (2008) for more details for the default spell.

⁴⁹To solve the model we use standard techniques to iterate over value functions and policy functions and an “outer” loop that iterates on prices until convergence.

those in which the economy was not.

Table 3: In the table, E and std denote the mean and standard deviation across time, respectively. All quantities are averaged across MC simulations. In parenthesis the 5% and 95% percentile of the MC sample. All results are in percentages.

	Financial Autarky						Financial Access					
	(I)	(II)	(III)	(IV)	(V)		(I)	(II)	(III)	(IV)	(V)	
	AMSS	ED	ED	ED	ED		AMSS	ED	ED	ED	ED	
$E(b_t/n_t)$	7.2	4.4	5.2	8.1	3.5	0.4	3.6	1.4	1.5	3.7	1	0.2
	(6.3,8.1)	(4.3,4.6)	(5.2,5.2)	(7.9,2)	(2.9,4)	(0.3,0.5)	(2.6,4.7)	(1.2,1.6)	(0.9,1.4)	(3.1,4.1)	(0.8,1.2)	(0.1,0.2)
$E(\tau_t)$	7.8	7.0	7.0	7.4	6.8	6.6	5.7	5.6	5.8	6.0	5.5	3.3
	(7.2,8.3)	(6.4,7.6)	(6.5,7.7)	(6.8,8)	(6.3,7.3)	(5.3,7.9)	(4.9,6.2)	(5,6.3)	(5.2,6.3)	(5.5,6.5)	(4.9,6.1)	(2.2,4.6)
$Std(\tau_t)$	2.5	4.4	4.4	3.6	4.5	4.2	2.4	3.6	3.6	3.3	3.9	3.3
	(2.2,2.9)	(4.1,4.7)	(4.1,4.7)	(3.3,3.9)	(4.2,4.7)	(3.8,4.6)	(2.0,2.7)	(3.3,3.9)	(3.3,3.8)	(3,3.6)	(3.5,4.2)	(2.8,5.6)
$E(Spread)$		27	24	15	4.2	140		2.2	1.0	4.2	0.2	0.04

Column (I) (in all the tables) reports the result for the aforementioned parametrization which we use as benchmark. The average debt-to-output ratio (row 1) for the whole sample is around 6 percent for the AMSS economy; in the ED economy, however, it is around 3.5 percent because of the presence of the endogenous borrowing limits arising from the possibility of default. This level is low compared to what is observed in the data: a ratio of approximately 23 percent for Argentina (1990-2005).⁵⁰ For the financial autarky sub-sample, the average debt-to-output ratio is actually the *defaulted* debt-to-output ratio, this provides additional evidence of endogenous borrowing limits being “active” in higher levels of debt and is consistent with the stylized facts presented in section 2.

The average tax rate (row 2) is slightly higher in the AMSS than in the ED economy, across all three samples. The volatility of the tax rate (row 3), however, is higher in the ED economy, especially in the financial autarky sub-sample, where is almost twice as high. A noteworthy remark is that taxes are more volatile in the ED economy, even during the financial access sub-sample, this is due to the presence of endogenous borrowing limits. This fact shows that the model is able to generate the corresponding stylized fact presented in section 2. Finally, when the ED economy is in autarky, the government is precluded from issuing debt, rendering taxes more volatile than in the other sub-samples.

In order to study the pricing implications, we compute the spread as $\frac{r_{t+1}}{p_t} - \frac{1}{\beta}$ where $r_{t+1} = 1$ if not in default and $r_{t+1} = \delta_{t+1}$ if an offer (of δ_{t+1}) was accepted and $r_{t+1} = q_{t+1}$ otherwise.

⁵⁰For the default period (2001-2005), this ratio was (approx.) 45 percent.

The spread of the model around 18 percent, and during financial autarky is highest, around 27 percent. This feature — higher spreads during financial autarky — is (at least qualitatively) consistent with what we observe in the data (see the discussion in section 7.1); the model however, generates higher spreads than those observe for the whole sample.

Finally, the welfare in AMSS is 6.2 whereas in the ED economy is 2.1.⁵¹

Column (II) (in all the tables) reports the case where $\underline{\delta} = \bar{\delta} = 0.6$. This parametrization has the same (unconditional) mean for Δ as the one in column (I), but no variance. Thus, it allows the model to shed light on the importance of the range of available offers to repay the debt and to quantify the selection bias coming from the fact that repayments of defaulted debt are *chosen* by the government.

Observe that the recovery rate in (I) is lower than the one in (II), and the expected duration spell is higher than in (I). These outcomes illustrate the fact that the government only accepts low offers of δ , thereby biasing downwardly the sample.

Column (III) (in all the tables) reports the case where $\lambda = 1$. A direct implication of this parametrization is that average default spell is reduced (relative to the model (I)) and the probability of default is higher. Also, the spread is lower for the financial autarky sub-sample; this outcomes illustrates the fact that *once* in default, the government re-pays (at least a fraction) of the defaulted debt in a shorter period of time. Finally, the welfare for this configuration is 3.2.

Column (IV) (in all the tables) reports the case where $\lambda = 0.15$.⁵² The average default spell increases (relative to both (I) and (III)); this directly follows from a lower λ . Consequently, financial autarky becomes more costly and this implies lower frequency of default. For this case, the welfare is 1.4.

It is interesting to note that, at least for the current parametrization, *conditional on having default in equilibrium*, debt-restructuring process that yield a higher frequencies of offers (a higher λ), are preferred.

Column (V) (in all the tables) considers the case where $\rho_1 = 0.9$ and $\mu_g = \frac{0.05}{1-\rho_1}$ (the rest of the parameters are as in case (I)). For this case, the average default spell is around 38 periods (around 4 times as long as the one for (I)) and the default frequency drops to 3 percent. These facts reflect that default tends to occur when government expenditure is high, whereas acceptance of offers of repayment occur when the government expenditure is low. When the

⁵¹Welfare is computed as the expectation of the value function with respect to the long run distribution for each model.

⁵²For values of λ lower than 0.15 default was not present in the MC sample.

government expenditure is highly persistent, once in financial autarky, the government expenditure remains high thus prompting the government to reject more offers of repayment (relative to the case where expenditure is low). This effect implies that the default spell is longer and also that autarky is more costly, thus delivering less defaults in equilibrium. Interestingly, the average recovery rate is around 55 percent thus presenting a lower downward bias than the one in (I). This stems from the fact that, for this parametrization, the government rejects *any* offer when government expenditure is high and accept almost all offers when government expenditure is low. Thus, since the realization of δ and g are independent, the sample of *accepted* offers presents a low selection bias. Also, observe that the spread for the financial autarky subsample is an order of magnitude higher than the previous one, this reflects the long autarky spell. Welfare for this case is around 0.5.

7.1 Impulse Response Functions

In this subsection, we draw a particular path for g_t given by

$$g_t = \begin{cases} 0 & \text{if } t < T \\ 0.075 & \text{if } t \in [T, T + 4] \\ 0 & \text{if } t \geq T + 5 \end{cases} \quad (14)$$

This choice is completely arbitrary, chosen to showcase all the features of the model.

Figure D.4 presents the results. The dotted line in all the panels is the path d_t . The economy enters default during the third period of high government expenditure and stays in financial autarky for two periods. The upper-left panel shows the debt for both economies (ED solid and AMSS dashed); the endogenous borrowing limits present in the ED economy render lower levels of debt during “bad times.” During autarky, since we keep track of the defaulted debt, we have a plateau; then, the economy leaves default by paying part of the outstanding debt.

The lower-left panel shows the tax path for both economies (ED solid and AMSS dashed); for the ED economy, the path is more volatile and for the AMSS taxes return more slowly to zero since they ought to finance the high levels of debt. The lower-right panel depicts consumption for both economies (ED solid and AMSS dashed). Finally, the upper-right panel shows the spread. The spread increases before the default event (during the periods of high expenditure and debt before default) and is maximal during financial autarky.

This last result is consistent with the data that shows that during the debt-restructuring period, the measure of default risk stays significantly higher than during “no default periods.” For instance, for Argentina, this measure was around 5 percent during 1997-2000 and 2005-2006

but around 60 percent during 2001-2005; for Russia, it was around 4 percent during 2000-2006 but around 20 percent during 1998-1999; finally, for Ecuador, it was around 9 percent during 1997 and 2001-2006 and around 21 percent during 1998-2000.

In brief, the aforementioned figures show a summary of the dynamics generated by this model: endogenous debt limits, higher volatility of taxes, and higher spreads due to default risk, especially, during a default period.

8 Conclusion

We study a government's problem, in a closed economy, that consists of choosing distortionary taxes with only non-state-contingent government debt, but allowing for partial defaults on the debt.

First, we provide an explanation for the lower debt-to-output ratios and more volatile tax policies observed in emerging economies, vis-à-vis industrialized economies. This stems from the fact that the holders of government debt forecast the possibility of default, imposing endogenous debt limits. These limits restrict the ability of the government to smooth shocks using debt, thus rendering taxes more volatile.

Second, we propose a device to price the debt during temporary financial autarky. Numerical simulations show that the spread during the default period is higher than for the rest of the sample; this characteristic is consistent with data for defaulters—e.g., Argentina, Ecuador and Russia.

Third, and last, the numerical simulations suggest that increasing the probability of receiving offers for exiting autarky decreases welfare when this probability is low/medium to begin with, but increases it when the probability is high.

Although this model does a good job of explaining qualitatively the facts observed in the data, it does not do very well in matching the data quantitatively. A line of future research should delve further into the production side of this economy and its driving shocks, and also on developing the pricing implications.⁵³

⁵³See Aguiar and Gopinath (2006) and Mendoza and Yue (2012), and Pouzo and Presno (2012) exploring some of these issues.

References

- Aguiar, M., Amador, M., and Gopinath, G. (2008). Investment cycles and sovereign debt overhang. *Mimeo, dept. of Economics, Univ. of Rochester, Stanford Univ. and Harvard*.
- Aguiar, M. and Gopinath, G. (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics*, 69(1):64–83.
- Aiyagari, R., Marcet, A., Sargent, T., and Seppälä, J. (2002). Optimal taxation without state contingent debt. *Journal of Political Economy*, 110:1220–1254.
- Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *American Economic Review*, 98(3):690–713.
- Benjamin, D. and Wright, M. (2009). Recovery before redemption: A theory of delays in sovereign debt renegotiations. *Mimeo, Dept of Economics, UCLA*.
- Broner, F., Martin, A., and Ventura, J. (2010). Sovereign risk and secondary markets. *American Economic Review*, 100(4):1523–1555.
- Chang, R. (1998). Credible monetary policy in a infinite horizon model: Recursive approaches. *Journal of Economic Theory*, 81:431–461.
- Chatterjee, S. and Eyingungor, B. (2012). Maturity, indebtedness, and default risk. *American Economic Review*, 102(6):2674–99.
- Cuadra, G., Sanchez, J. M., and Saprizza, H. (2010). Fiscal policy and default risk in emerging markets. *Review of Economic Dynamics*, 13(2):919–933.
- Debortoli, D. and Nunes, R. (2010). Fiscal policy under loose commitment. *Journal of Economic Theory*, 145(3).
- Doda, B. (2007). Optimal fiscal policy with sovereign default. *Mimeo, Dept of Economics, Univ. of Toronto*.
- Eaton, J. and Gersovitz, M. (1981). Debt with potential repudiation: Theoretical and empirical analysis. *Review of Economic Studies*, 48(2):289–309.
- Farhi, E. (2010). Capital taxation and ownership when markets are incomplete. *Journal of Political Economy*, 118:908–948.
- Hopeynhan, H. and Werning, I. (2009). Equilibrium default. *Mimeo UCLA*.

- Kaminsky, G., Reinhart, C., and Vegh, C. (2004). When it rains, it pours: Procyclical capital flows and macroeconomic policies. NBER Working Papers 10780, National Bureau of Economic Research, Inc.
- Koenker, R. (2005). *Quantile Regression*. Econometric Society Monograph Series. Cambridge University Press.
- Kydland, F. and Prescott, E. (1980). Dynamic optimal taxation, rational expectations and optimal control. *Journal of Economic Dynamics and Control*, 2:79–91.
- Lucas, R. and Stokey, N. L. (1983). Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics*, 12(1):55–93.
- Mendoza, E. and Yue, V. (2012). A general equilibrium model of sovereign default and business cycles. *Quarterly Journal of Economics*, 127(2):889–946.
- Panizza, U. (2008). Domestic and external public debt in developing countries. Technical Report 188, United Nations conference on trade and development.
- Pitchford, R. and Wright, M. (2008). Restructuring the sovereign debt restructuring mechanism. *Mimeo, dept. of Economics, Univ. of Sidney and UCLA*.
- Pouzo, D. and Presno, I. (2012). Sovereign default risk and uncertainty premia. *Mimeo, UC Berkeley*.
- Reinhart, C. and Rogoff, K. (2008). The forgotten history of domestic debt. Working Paper 13946, National Bureau of Economic Research.
- Reinhart, C., Rogoff, K., and Savastano, M. (2003). Debt intolerance. Working Paper 9908, National Bureau of Economic Research.
- Sturzenegger, F. and Zettelmeyer, J. (2006). *Debt Defaults and Lessons from a Decade of Crises*. The MIT press.
- Tauchen, G. (1986). Finite state markov-chain approximations to univariate and vector autoregressions. *Economics Letters*, 20:177–181.
- Yue, V. (2010). Sovereign default and debt renegotiation. *Journal of International Economics*, 80(2):176–187.

A Notation and Stochastic Structure of the Model

Throughout the appendix, for a generic mapping f from a set S to T , we use $s \mapsto f(s)$ or $f : S \rightarrow T$ to denote it. For the case that a mapping depends on many variables, the notation $s_1 \mapsto f(s_1, s_2)$ is used to denote the function f *only* as a function of s_1 , keeping s_2 fixed.

For a generic X , let $X^\infty : \Omega \times \{0, 1, 2, \dots\} \rightarrow \mathbb{R}$ be an stochastic process; $X_t(\omega) \equiv X(\omega, t)$ is the value of the stochastic process at time t and given ω ; $X_t(\cdot, \omega^{t-1})$ is understood as a function from $\mathbb{G} \times \bar{\Delta}$ to $\mathbb{R}$ and is the function implied by the stochastic process, at time t , given the *past* history ω^{t-1} ; X_t^s is an stochastic process given by the values $\omega \mapsto (X_t(\omega), \dots, X_s(\omega))$; similarly, $X_t^s(\cdot, \omega^t)$ is an stochastic process given by the values $\tilde{\omega} \in \{\Omega : \tilde{\omega}^t = \omega^t\} \mapsto (X_t(\tilde{\omega}), \dots, X_s(\tilde{\omega}))$.

As noted throughout the text, for stochastic processes, X^∞ and Y^∞ , the equality $X_t = Y_t$ is understood as $X_t(\omega) = Y_t(\omega)$ for all $\omega \in \Omega$.

We denote $\mathcal{O} \equiv \mathcal{G} \times \mathcal{D}$ denote the product σ -algebra, where $\mathcal{G}$ and $\mathcal{D}$ are the σ -algebra attached to $\mathbb{G}$ and $\bar{\Delta}$ resp. We use $\mathcal{O}^t$ to denote the σ -algebra generated by ω^t , for all $t \in \{0, 1, \dots\}$.

With this notation in place, definition 3.1 implies that a government policy $(\sigma_t)_t$ is such that $B_{t+1} : \Omega \rightarrow \mathbb{B}$ is $\mathcal{O}^t$ -measurable; $\tau_t : \Omega \rightarrow [0, 1]$ is $\mathcal{O}^t$ -measurable; $d_t : \Omega \rightarrow \{0, 1\}$ is $\mathcal{G} \times \mathcal{O}^{t-1}$ -measurable; $a_t : \Omega \rightarrow \{0, 1\}$ is $\mathcal{O}^t$ -measurable. Observe that the measurability restriction on d_t represents the fact that the government decides to default or not, before observing the (eventual) fraction of debt to be repaid and whether it receives such an offer or not, in case of default.

The price schedule is defined to be $\mathcal{O}^t$ -measurable. At times, we will be explicit about the dependence of prices on σ^∞ and use $p_t(\omega; \sigma^\infty)$ to denote the price at time t , history ω and given policy σ^∞ .

Similarly, definition 3.3 implies that $c_t : \Omega \rightarrow \mathbb{R}_+$ and $n_t : \Omega \rightarrow [0, 1]$ are $\mathcal{O}^t$ -measurable and $b_{t+1} : \Omega \rightarrow [\underline{b}, \bar{b}]$ is $\mathcal{O}^t$ -measurable.

B Optimization Problem for the Households

The Lagrangian associated to the household problem is given by

$$\begin{aligned} \mathcal{L}(c^\infty, n^\infty, b^\infty, \lambda^\infty, \mu^\infty, \psi^\infty) \equiv & \sum_{t=0}^{\infty} \beta^t E_{\Pi(\cdot|\omega_0)} [\{u(c_t(\omega), 1 - n_t(\omega)) - \lambda_t(\omega) \{c_t(\omega) - (1 - \tau_t(\omega))\kappa_t(\sigma^\infty)n_t(\omega) + p_t(\omega; \sigma^\infty)b_{t+1}(\omega) - \varrho_t(\omega; \sigma^\infty)b_t(\omega)\} \\ & + \Psi_t(\omega)c_t(\omega) + \psi_{1t}(b_{t+1} - \underline{b}) + \psi_{2t}(\bar{b} - b_{t+1})\}], \end{aligned}$$

where λ_t is the Lagrange multiplier associated to the budget constraint and Ψ_t is the Lagrange multipliers associated to the restrictions that $c_t \geq 0$, and ψ_{it} $i = 1, 2$ is the Lagrange multiplier associated to the debt restrictions.

Assuming interiorty of the solutions, the first order conditions (FONC) are given by:

$$\begin{aligned} c_t &: u_c(c_t(\omega), 1 - n_t(\omega)) - \lambda_t(\omega) = 0 \\ n_t &: -u_l(c_t(\omega), 1 - n_t(\omega)) - \lambda_t(\omega)(1 - \tau_t(\omega))\kappa_t(\omega; \sigma^\infty) = 0 \\ b_{t+1} &: p_t(\omega; \sigma^\infty)\lambda_t(\omega) - E_{\Pi(\cdot|\omega^t)}[\beta\lambda_{t+1}(\omega)\varrho_{t+1}(\omega)] = 0. \end{aligned}$$

Then

$$\frac{u_l(c_t(\omega), 1 - n_t(\omega))}{u_c(c_t(\omega), 1 - n_t(\omega))} = (1 - \tau_t(\omega))\kappa_t(\omega; \sigma^\infty), \quad (\text{B.15})$$

and

$$p_t(\omega; \sigma^\infty) = E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(c_{t+1}(\omega), 1 - n_{t+1}(\omega))}{u_c(c_t(\omega), 1 - n_t(\omega))} \varrho_{t+1}(\omega) \right]. \quad (\text{B.16})$$

From the definition of ϱ , equation B.16 implies, for $d_t = 0$ and $a_t = 1$,

$$\begin{aligned} p_t(\omega; \sigma^\infty) &= E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(c_{t+1}(\omega), 1 - n_{t+1}(\omega))}{u_c(c_t(\omega), 1 - n_t(\omega))} (1 - d_{t+1}(\omega)) \right] \\ &+ E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(c_{t+1}(\omega), 1 - n_{t+1}(\omega))}{u_c(c_t(\omega), 1 - n_t(\omega))} d_{t+1}(\omega) a_{t+1}(\omega) \delta_{t+1} \right] \\ &+ E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(c_{t+1}(\omega), 1 - n_{t+1}(\omega))}{u_c(c_t(\omega), 1 - n_t(\omega))} d_{t+1}(\omega) (1 - a_{t+1}(\omega)) q_{t+1}(\omega; \sigma^\infty) \right]. \end{aligned}$$

For $d_t = 1$ and $a_t = 0$,

$$\begin{aligned} p_t(\omega; \sigma^\infty) &= E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(c_{t+1}(\omega), 1 - n_{t+1}(\omega))}{u_c(c_t(\omega), 1 - n_t(\omega))} a_{t+1}(\omega) \delta_{t+1} \right] \\ &+ E_{\Pi(\cdot|\omega^t)} \left[\beta \frac{u_c(c_{t+1}(\omega), 1 - n_{t+1}(\omega))}{u_c(c_t(\omega), 1 - n_t(\omega))} (1 - a_{t+1}(\omega)) q_{t+1}(\omega; \sigma^\infty) \right]. \end{aligned}$$

The next lemma shows that the FONC of the households are sufficient.

LEMMA B.1. *Suppose assumption 4.1(i) holds. Then first order conditions, B.15 - B.16, are also sufficient.*

C Proof of Lemmas in section 6

Throughout, we assume that $\mathbb{G} \equiv \{g_1, \dots, g_{|\mathbb{G}|}\}$, $\mathbb{B} \equiv \{B_1, \dots, B_{|\mathbb{B}|}\}$ and $\Delta \equiv \{\delta_1, \dots, \delta_{|\Delta|}\}$ with $|\mathbb{G}|$, $|\mathbb{B}|$ and $|\Delta|$ all finite (for a generic set A , $|A|$ denotes the cardinality of a set).

Under assumption 6.1, it is easy to see that the revenue of the government, as a function of leisure and productivity, is $R(\kappa, n) = (\kappa - H'(1 - n))n$. Moreover, the optimal level of labor belongs to $[n_2(\kappa), n_1(\kappa)]$

where $R(\kappa, n_1(\kappa)) = 0$ and $R'(\kappa, n_2(\kappa)) = 0$, and in this domain $R' < 0$; see lemma C.1(1) for the proof. Let, for any given κ , $R \mapsto n(\kappa, R)$ be the inverse mapping of $n \mapsto R(\kappa, n)$.

Let W be the per-period payoff, i.e.,

$$W(R) = \kappa n(\kappa, R) - g + H(1 - n(\kappa, R)) \asymp \kappa n(\kappa, R) + H(1 - n(\kappa, R)).$$

Lemma C.1(2) establishes that W is differentiable, uniformly bounded, non-increasing (decreasing over all R such that $(-\kappa + H'(1 - n(\kappa, R))) < 0$) function.

The government budget constraint becomes: If $\mathbf{d}(g, B) = 0$, $g + \delta B - R \leq p(g, B'; \mathbf{e})B'$, where $\mathbf{e} \equiv (\mathbf{d}, \mathbf{a})$ and

$$\begin{aligned} p(g, B'; \mathbf{e}) = & \beta \int_{\mathbb{G}} (1 - \mathbf{d}(g', B')) \pi_{\mathbb{G}}(dg'|g) + \lambda \int_{\mathbb{G}} \mathbf{d}(g', B') \int_{\Delta} \mathbf{a}(g', \delta', B') \delta' \pi_{\Delta}(d\delta') \pi_{\mathbb{G}}(dg'|g) \\ & + \int_{\mathbb{G}} \mathbf{d}(g', B') \left(\int_{\Delta} \{ \lambda(1 - \mathbf{a}(g', \delta', B')) + (1 - \lambda) \} \pi_{\Delta}(d\delta') \right) q(g', B'; \mathbf{e}) \pi_{\mathbb{G}}(dg'|g), \end{aligned}$$

with

$$q(g, B'; \mathbf{e}) = \beta \int_{\mathbb{G}} \left\{ \int_{\Delta} \{ \lambda \mathbf{a}(g', \delta', B') \delta' + \{ \lambda(1 - \mathbf{a}(g', \delta', B')) + 1 - \lambda \} q(g', B'; \mathbf{e}) \} \pi_{\Delta}(d\delta') \right\} \pi_{\mathbb{G}}(dg'|g).$$

If $\mathbf{d}(g, B) = 1$, $g - R \leq 0$. Henceforth, we omit d, a from the prices.

Let

$$\begin{aligned} V(g, B) = & (1 - \mathbf{d}^*(g, B)) V_C(g, B) \\ & + \mathbf{d}^*(g, B) \left[\lambda \int_{\Delta} \{ \mathbf{a}^*(g, \delta', B) V_C(g, \delta' B) + (1 - \mathbf{a}^*(g, \delta', B)) V_A(g, B) \} \pi_{\Delta}(d\delta') + (1 - \lambda) V_A(g, B) \right], \end{aligned}$$

where ⁵⁴

$$V_C(g, \delta B) = W(g + \delta B - p(g, \mathbf{B}^*(g, \delta B); \mathbf{d}^*, \mathbf{a}^*) \mathbf{B}^*(g, \delta B)) + \beta \int_{\mathbb{G}} V(g', \mathbf{B}^*(g, \delta B)) \pi_{\mathbb{G}}(dg'|g),$$

and

$$\begin{aligned} V_A(g, B) = & W(g) + \beta \int_{\mathbb{G}} \left(\lambda \int_{\Delta} \{ \mathbf{a}^*(g, \delta, B) V_C(g', \delta B) + (1 - \mathbf{a}^*(g, \delta, B)) V_A(g', B) \} \pi_{\Delta}(d\delta) + (1 - \lambda) V_A(g', B) \right) \\ & \times \pi_{\mathbb{G}}(dg'|g). \end{aligned}$$

The policy functions are given

$$\mathbf{B}^*(g, B) = \arg \max_{B'} \left\{ W(g + B - p(g, B'; \mathbf{e}^*) B') + \beta \int_{\mathbb{G}} V(g', B') \pi_{\mathbb{G}}(dg'|g) \right\},$$

⁵⁴Formally, W is defined only in the interval $[0, R(\kappa, n_2(\kappa))]$. Hence, it must be $g + B - p(g, B') B' \in [0, R(\kappa, n_2(\kappa))]$ for some $B' \in \mathbb{B}$. If $g + B - p(g, B') B' < 0$, it implies that the revenue from taxes is negative, i.e., the government is issuing subsidies. This can easily be handled by allowing lump-sum transfers. The case where $g + B - p(g, B') B' > R(\kappa, n_2(\kappa))$ is more problematic since it entails that there are not enough resources to cover the deficit. To ensure this does not happen: we can define the feasible set of debt choices to be $\{B' \in \mathbb{B} : g + B - p(g, B') B' \leq R(\kappa, n_2(\kappa))\}$, if for a given (g, B) the set is empty, we set the per-period payoff to a large negative value (but finite) and set $d(g, B) = 1$.

$\mathbf{a}^*(g, \delta, B) = \arg \max \{V_C(g, \delta B), V_A(g, B)\}$ and $\mathbf{d}^*(g, B) = \arg \max \{V_C(g, B), \bar{V}(g, B)\}$, where

$$\bar{V}(g, B) = \lambda \int_{\Delta} \{\mathbf{a}^*(g, \delta', B) V_C(g, \delta' B) + (1 - \mathbf{a}^*(g, \delta', B)) V_A(g, B)\} \pi_{\Delta}(d\delta') + (1 - \lambda) V_A(g, B).$$

Proof of Theorem 6.1. Part (1). By lemma C.4, $\delta \mapsto V_C(g, \delta B)$ is non-increasing, provided $B > 0$ (but this is the only case it matters since the government will never default on savings $B < 0$). On the other hand $V_A(g, B)$ is constant with respect to δ . Therefore if for some $\delta \in \Delta$, $\mathbf{a}^*(g, \delta, B) = 1$, then for all $\delta_1 \leq \delta$ the same must hold. Thus, there exists a $\bar{\delta} : \mathbb{G} \times \mathbb{B} \rightarrow [0, 1]$ such that $1\{\delta \leq \bar{\delta}(g, B)\}$ iff $\mathbf{a}^*(g, \delta, B) = 1$.

To show that $B \mapsto \bar{\delta}(g, B)$ is non-increasing. It suffices to show that for all δ such that $\delta > \bar{\delta}(g, B_1)$ then $\delta > \bar{\delta}(g, B_2)$ for any $B_1 < B_2$. Since $\delta > \bar{\delta}(g, B_1)$, it follows that $V_C(g, \delta B_1) < V_A(g, B_1)$, $\forall (g, B_1, \delta) : \delta > \bar{\delta}(g, B_1)$.

Let $\epsilon(g, B_1, \delta) \equiv V_A(g, B_1) - V_C(g, \delta B_1) > 0$. Since g, B_1 and δ belong to discrete sets, there exists a $\epsilon > 0$ such that $\epsilon \leq \epsilon(g, B_1, \delta) \forall (g, B_1, \delta) : \delta > \bar{\delta}(g, B_1)$.

Since $B \mapsto V_C(g, B)$ is non-increasing (see lemma C.4), it follows that $V_C(g, \delta B_2) \leq V_C(g, \delta B_1)$, $\forall (g, \delta) \in \mathbb{G} \times \Delta$. Therefore, $\forall (g, B_1, B_2, \delta) : \delta > \bar{\delta}(g, B_1)$,

$$\begin{aligned} V_C(g, \delta B_2) - V_A(g, B_2) &\leq V_C(g, \delta B_1) - V_A(g, B_2) \\ &\leq V_C(g, \delta B_1) - V_A(g, B_1) + \{V_A(g, B_1) - V_A(g, B_2)\}. \end{aligned}$$

Hence, if $\{V_A(g, B_1) - V_A(g, B_2)\} < \epsilon$, it follows that $V_C(g, B_2, \delta) - V_A(g, B_2) < 0$ and the desired result follows. To show that $\{V_A(g, B_1) - V_A(g, B_2)\} < \epsilon$, observe that

$$V_A(g, B_1) - V_A(g, B_2) = \beta \int_{\mathbb{G}} \{\bar{V}(g', B_1) - \bar{V}(g', B_2)\} \pi_{\mathbb{G}}(dg' | g).$$

And by lemma C.2, for any $\varepsilon > 0$, there exists a $\lambda(\varepsilon)$, such that

$$V_A(g, B_1) - V_A(g, B_2) < \varepsilon, \quad \forall \lambda \in [0, \lambda(\varepsilon)].$$

Choosing $\varepsilon = \epsilon$ the result thus follows.

This implies that $V_A(g, B_2) < V_C(g, B_2, \delta)$, which means that δ is rejected, and by the first part of the argument, $\delta \geq \bar{\delta}(g, B_2)$.

Part (2). Following Arellano (2008) we show the result in two parts. Also, to simplify notation we omit $\mathbf{e}^*$ from p .

Step 1. We show that for any $B_1 < B_2$, $\mathbb{S}(B_1) \subseteq \mathbb{S}(B_2)$ where $\mathbb{S}(B) = \{g : \mathbf{d}^*(g, B) = 1\}$. If $\mathbb{S}(B_1) = \{\emptyset\}$ the proof is trivial, so we proceed with the case this does not hold and let $\bar{g} \in \mathbb{S}(B_1)$. If B_2 is not feasible, in the sense that there does not exist any B' such that $\bar{g} + B_2 - p(g, B')B' - R \leq 0$, then $\mathbb{S}(B_2) = \mathbb{G}$. And the result holds trivially, so we proceed with the case that B_2 is feasible, given $\bar{g}$.

It follows (since we assume that under indifference, the government chooses not to default) $V_C(\bar{g}, B_1) < \bar{V}(\bar{g}, B_1)$, and the same holds for any $(B, g) \in \text{Graph}(\mathbb{S})$. Since $B \mapsto V_C(\bar{g}, B)$ is no-increasing (see lemma C.4), it follows that

$$V_C(\bar{g}, B_2) \leq V_C(\bar{g}, B_1), \quad \forall g \in \mathbb{G} \text{ and } B_1 < B_2.$$

Therefore, for $(B_2, B_1, \bar{g}) \in \mathbb{B} \times \text{Graph}(\mathbb{S})$

$$\begin{aligned} V_C(\bar{g}, B_2) - \bar{V}(\bar{g}, B_2) &\leq V_C(\bar{g}, B_1) - \bar{V}(\bar{g}, B_2) \\ &\leq V_C(\bar{g}, B_1) - \bar{V}(\bar{g}, B_1) + \{\bar{V}(\bar{g}, B_1) - \bar{V}(\bar{g}, B_2)\}. \end{aligned}$$

We know that $V_C(\bar{g}, B_1) - \bar{V}(\bar{g}, B_1) \equiv -\epsilon(\bar{g}, B_1) < 0$. Thus, if $\bar{V}(\bar{g}, B_1) - \bar{V}(\bar{g}, B_2) < \epsilon(\bar{g}, B_1)$, then $V_C(\bar{g}, B_2, 1) < \bar{V}(\bar{g}, B_2)$ and the desired result follows.

Observe that $|\mathbb{B} \times \text{Graph}(\mathbb{S})| < \infty$, so there exists $\epsilon > 0$ such that $\epsilon \leq \epsilon(\bar{g}, B_1)$. By lemma C.2, there exists a $\lambda(\epsilon) > 0$ such that

$$|\bar{V}(g, B_1) - \bar{V}(g, B_2)| < \epsilon, \quad \forall \lambda \in [0, \lambda(\epsilon)] \text{ and } (g, B_1, B_2) \in \mathbb{G} \times \mathbb{B}^2.$$

Hence, $V_C(\bar{g}, B_2) - \bar{V}(\bar{g}, B_2) < 0$, thereby implying that $\bar{g} \in \mathbb{S}(B_2)$.

Step 2. We show that, for any B and any $g_1 < g_2$, if g_1 is such that $\mathbf{d}^*(g_1, B) = 1$, then $\mathbf{d}^*(g_2, B) = 1$.

Let n_i^C be the optimal choice of labor, when $g = g_i$; n_i^A and B_i^C are defined similarly. Let $BL(n, g) \equiv R(1, 1 - n) - g$. Since $g_2 > g_1$, $BL(n_1^C, g_2) \leq BL(n_1^C, g_1) = B - p(g, B_1^C)B_1^C$ and $BL(n_2^C, g_1) \geq BL(n_2^C, g_2) = B - p(g, B_2^C)B_2^C$. Let $\tilde{n}$ be such that $BL(\tilde{n}, g_1) = BL(n_2^C, g_2)$. Since in the relevant domain $n \mapsto BL(n, g)$ is non-increasing, it follows that $\tilde{n} \geq n_2^C$; moreover, $(\tilde{n}, B_2^C)$ are feasible for (B, g_1) . It is also true that $n_1^A < n_2^A$ since $g_1 < g_2$. Also, since in (g_1, B) the government defaults, it follows from the proof of lemma 6.1 that $B - p(g, B_2^C)B_2^C \geq 0$ — this follows because $(\tilde{n}, B_2^C)$ are feasible for (B, g_1) and it cannot roll over its debt — and thus $\tilde{n} \leq n_1^A$ since $n \mapsto BL(n, g)$ is non-increasing.

Therefore,

$$BL(n_2^C, g_2) = B - p(g, B_2^C)B_2^C = BL(\tilde{n}, g_1) \iff R(1, 1 - n_2^C) - g_2 = R(1, 1 - \tilde{n}) - g_1$$

iff $R(1, 1 - n_2^C) - R(1, 1 - n_2^A) = R(1, 1 - \tilde{n}) - R(1, 1 - n_1^A)$. Observe that $n_2^A > n_1^A$ implies $\tilde{n} > n_2^C$, and $\tilde{n} \leq n_1^A$ implies that $R(1, 1 - n_2^C) - R(1, 1 - n_2^A) \geq 0$, and thus $n_2^C \leq n_2^A$. There could be two possibilities (a) $n_2^C \leq n_2^A \leq \tilde{n} \leq n_1^A$ (with at least inequalities strict) or (b) $n_2^C < \tilde{n} \leq n_2^A < n_1^A$. We now analyze (a). By our assumptions $n \mapsto BL(n, g)$ is non-increasing and strictly concave (since $R''(1, 1 - n) = 2H''(1 - n) - H'''(1 - n)n < 0$ by assumption 6.1). thus

$$\frac{R(1, 1 - n_2^C) - R(1, 1 - n_2^A)}{n_2^C - n_2^A} > \frac{R(1, 1 - \tilde{n}) - R(1, 1 - n_1^A)}{\tilde{n} - n_1^A}$$

which implies $\tilde{n} - n_1^A > n_2^C - n_2^A$. (b) is analogous and will not be repeated here.

Let $F(n, g) \equiv n - g + H(1 - n)$, since $\pi_{\mathbb{G}}(\cdot | g_1)$ is FOSD by $\pi_{\mathbb{G}}(\cdot | g_2)$, $g \mapsto V_C(B, g)$ is non-increasing (see lemma C.4), and the fact that $(\tilde{n}, B_2^C)$ is feasible for (g_1, B) and $g_2 \in \mathbb{G}$,

$$F(\tilde{n}, g_1) - F(n_2^C, g_2) \leq V_C(B, g_1) - V_C(B, g_2).$$

Since $n \mapsto F(n, g) \equiv n - g + H(1 - n)$ is concave it follows that $\frac{F(n_2^A, g_2) - F(n_2^C, g_2)}{n_2^A - n_2^C} > \frac{F(n_1^A, g_1) - F(\tilde{n}, g_2)}{n_1^A - \tilde{n}}$, and since $\tilde{n} - n_1^A > n_2^C - n_2^A$, $F(n_2^A, g_2) - F(n_2^C, g_2) > F(n_1^A, g_1) - F(\tilde{n}, g_2)$. Therefore,

$$F(n_1^A, g_1) - F(n_2^A, g_2) < F(\tilde{n}, g_1) - F(n_2^C, g_2) \leq V_C(B, g_1) - V_C(B, g_2).$$

By the same arguments in the proof of lemma C.2, for any $\epsilon > 0$, there exists a $\bar{\lambda}$, such that $\forall \lambda \in [0, \bar{\lambda})$

$$\bar{V}(g_2, B) - \bar{V}(g_1, B) \leq \epsilon.$$

By choosing $\epsilon > 0$, smaller than $V_C(g_1, B) - V_C(g_2, B) - \{F(n_1^A, g_1) - F(n_2^A, g_2)\} > 0$ (since there are finitely many g_i and B , choose the minimal one), it follows that $\bar{V}(g_1, B) - \bar{V}(g_2, B) < V_C(B, g_1) - V_C(B, g_2)$, since $V_C(g_1, B) - \bar{V}(g_1, B) \leq 0$, this implies that $V_C(g_2, B) < \bar{V}(B, g_2)$, as desired.

Hence, step 2 establishes that $\mathbf{d}^*$ is of the threshold type, since it shows that, for any B , if $\mathbf{d}^*(g, B) = 1$, the same is true for any $g' > g$. That is $\{g : \mathbf{d}^*(g, B) = 1\}$ is of the form $\{g : g \geq \bar{g}(B)\}$. Step 1 shows that the $\bar{g}$ ought to be non-increasing. $\square$

Proof of Theorem 6.2. Part 1. First, by equation 12 it is easy to see that when $g \sim iid \pi_{\mathbb{G}}$, $q(g, B) = q(B)$. Moreover,

$$q(B_t) = \beta \lambda \int_{\mathbb{G}} D(g', B_t) \pi_{\mathbb{G}}(dg') + \left(\beta \int_{\mathbb{G}} (\lambda \alpha(g', B_t) + (1 - \lambda)) \pi_{\mathbb{G}}(dg') \right) q(B_t),$$

and after simple algebra,

$$\begin{aligned} q(B_t) &= \frac{\beta \lambda \int_{\mathbb{G}} D(g', B_t) \pi_{\mathbb{G}}(dg')}{1 - \beta (\lambda \int_{\mathbb{G}} \alpha(g', B_t) \pi_{\mathbb{G}}(dg') + (1 - \lambda))} \\ &= \frac{\beta \lambda \delta_0 \pi_{\mathbb{G}}(\{g : \delta_0 \leq \bar{\delta}(g, B_t)\})}{1 - \beta (1 - \lambda \pi_{\mathbb{G}}(\{g : \delta_0 \leq \bar{\delta}(g, B_t)\}))} \end{aligned}$$

where the last equality follows from the fact that, since $\pi_{\Delta} = \mathbf{1}_{\delta_0}(\delta)$, $D(g, B) \equiv \int_{\Delta} 1\{\delta \leq \bar{\delta}(g, B)\} \delta \pi_{\Delta}(d\delta) = \delta_0 1\{\delta_0 \leq \bar{\delta}(g, B)\}$ and $\int_{\Delta} 1\{\delta \leq \bar{\delta}(g, B)\} \pi_{\Delta}(d\delta) = 1\{\delta_0 \leq \bar{\delta}(g, B)\}$.

From theorem 6.1(1), $B \mapsto \bar{\delta}(g, B)$ is non-increasing. Therefore, $\{g : \delta_0 \leq \bar{\delta}(g, B_1)\} \supseteq \{g : \delta_0 \leq \bar{\delta}(g, B_2)\}$, for $B_1 \leq B_2$. Hence $B \mapsto \pi_{\mathbb{G}}(\{g : \delta_0 \leq \bar{\delta}(g, B)\})$ is non-increasing. It is easy to see that q is of the form $q(B) = \psi \circ \pi_{\mathbb{G}}(\{g : \delta_0 \leq \bar{\delta}(g, B)\})$ where $\psi(t) = \frac{\lambda \beta \delta_0 t}{1 + \beta \lambda t - \beta}$ and ψ is increasing. So $B \mapsto q(B)$ is non-increasing.

Part 2. From part 1, note that $q(B) \leq \delta_0$. Also, observe that under assumptions

$$\begin{aligned}
p(g_t, B_{t+1}) &= p(B_{t+1}) = \beta \int_{\mathbb{G}} 1\{g < \bar{g}(B_{t+1})\} \pi_{\mathbb{G}}(dg) + \beta \lambda \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_{t+1})\} D(g, B_{t+1}) \pi_{\mathbb{G}}(dg) \\
&\quad + \beta \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_{t+1})\} (\lambda \alpha(g, B_{t+1}) + (1 - \lambda)) \pi_{\mathbb{G}}(dg) q(B_{t+1}) \\
&= \beta \int_{\mathbb{G}} 1\{g < \bar{g}(B_{t+1})\} \pi_{\mathbb{G}}(dg) + \beta(1 - \lambda) \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_{t+1})\} \pi_{\mathbb{G}}(dg) q(B_{t+1}) \\
&\quad + \beta \lambda \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_{t+1})\} F(g, B_{t+1}) \pi_{\mathbb{G}}(dg)
\end{aligned}$$

where $F(g, B) \equiv 1\{\delta_0 \leq \bar{\delta}(g, B)\} \delta_0 + (1 - 1\{\delta_0 \leq \bar{\delta}(g, B)\}) q(B) = q(B) + 1\{\delta_0 \leq \bar{\delta}(g, B)\} [\delta_0 - q(B)]$. First note that $B \mapsto F(g, B)$ is non-increasing: Let $B_1 \leq B_2$, then

$$\begin{aligned}
F(g, B_1) &\geq 1\{\delta_0 \leq \bar{\delta}(g, B_2)\} \delta_0 + (1 - 1\{\delta_0 \leq \bar{\delta}(g, B_2)\}) q(B_1) \\
&\geq 1\{\delta_0 \leq \bar{\delta}(g, B_2)\} \delta_0 + (1 - 1\{\delta_0 \leq \bar{\delta}(g, B_2)\}) q(B_2) = F(g, B_2),
\end{aligned}$$

where the first line follows from the fact that $B \mapsto 1\{\delta_0 \leq \bar{\delta}(g, B)\}$ is non-increasing and $\delta_0 - q(B) \geq 0$; the second line follows from the fact that $B \mapsto q(B)$ is non-increasing. Second, note that $F(g, B) \in [0, \delta_0]$.

Now consider $B_1 \leq B_2$,

$$\begin{aligned}
p(B_1) &\geq \beta \int_{\mathbb{G}} 1\{g < \bar{g}(B_2)\} \pi_{\mathbb{G}}(dg) + \beta(1 - \lambda) \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_2)\} \pi_{\mathbb{G}}(dg) q(B_1) \\
&\quad + \beta \lambda \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_2)\} F(g, B_1) \pi_{\mathbb{G}}(dg) \\
&\geq \beta \int_{\mathbb{G}} 1\{g < \bar{g}(B_2)\} \pi_{\mathbb{G}}(dg) + \beta(1 - \lambda) \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_2)\} \pi_{\mathbb{G}}(dg) q(B_2) \\
&\quad + \beta \lambda \int_{\mathbb{G}} 1\{g \geq \bar{g}(B_2)\} F(g, B_2) \pi_{\mathbb{G}}(dg) = p(g_t, B_2)
\end{aligned}$$

where the first line follows from the fact that $B \mapsto 1\{g < \bar{g}(B)\}$ is non-increasing and $1 > (1 - \lambda)q(B_1) + \lambda F(g, B_1)$; the second line follows from the fact that both q and $F(g, \cdot)$ are non-increasing. $\square$

Proof of Theorem 6.3. First, assumption (i) implies that $\pi_{\mathbb{G}}(g \leq \bar{g}(\underline{B})) = 1$. Because either $\underline{B} < 0$ and the government never defaults or $\underline{B} = 0$ and (at most) the government is indifferent between defaulting or not, and by assumption it does not do so.

If $\underline{B} < 0$, then $\frac{dp(g, \underline{B}) \underline{B}}{d\underline{B}}$ is clearly positive. If $\underline{B} = 0$, then

$$\begin{aligned}
\frac{dp(g, 0)}{dB} &= (\beta - \beta(1 - \lambda)q(0)) \frac{d\pi_{\mathbb{G}}(g \leq \bar{g}(0))}{dB} + \beta \lambda \frac{d \int 1\{g > \bar{g}(B)\} F(g, B) \pi_{\mathbb{G}}(dg)}{dB} \Big|_{B=0} \\
&= (\beta - \beta(1 - \lambda)q(0)) \frac{d\pi_{\mathbb{G}}(g \leq \bar{g}(0))}{dB} + \beta \lambda \int 1\{g > \bar{g}(0)\} \frac{dF(g, B)}{dB} \Big|_{B=0} \pi_{\mathbb{G}}(dg) \\
&\quad + \beta \lambda \int \frac{d1\{g > \bar{g}(B)\}}{dB} \Big|_{B=0} F(g, 0) \pi_{\mathbb{G}}(dg) \\
&= (\beta - \beta(1 - \lambda)q(0)) \frac{d\pi_{\mathbb{G}}(g \leq \bar{g}(0))}{dB} + \beta \lambda \int \frac{d1\{g > \bar{g}(B)\} \pi_{\mathbb{G}}(dg)}{dB} \Big|_{B=0} F(g, 0),
\end{aligned}$$

where the third line follows from the fact that $g > \bar{g}(\underline{B})$ for all g . Since $B \mapsto 1\{g > \bar{g}(B)\}$ is nondecreasing and $F > 0$, the second term is non-negative. By analogous argument, the same holds true for the first term. Therefore, $\frac{dp(g,0)}{dB} > 0$, thus implying $\frac{dp(g,B)B}{dB} \big|_{B=0} \geq p(0) > 0$.

Note that revenue from zero debt is zero. Therefore, taking an “infinitesimal” amount of positive debt increases the revenue from debt. On the other hand, by assumption (ii), $p(g, \bar{B}) = 0$. So, there exists an amount of debt in $(\underline{B}, \bar{B})$ that yields maximal $p(g, B)B$, we denote this value as B^* (does not depend on g , because p does not depend).

We claim that the government never chooses a level of debt above the value that achieves the highest point. To show this, suppose not. Take any $B' > B^*$. By continuity of $B \mapsto p(B)B$ and the fact that B^* is maximal, there exists a $B'' \in [0, B^*]$ such that $p(B'')B'' = p(B')B'$ and $V_C(g, B'') \geq V_C(g, B')$ (by lemma C.4, $B \mapsto V_C(g, B)$ is non-increasing); hence the government will never choose (optimally) B' over B'' . $\square$

Proof of Lemma 6.1. Since $\mathcal{D}$ is non-empty; there exists at least a pair $(g, B) \in \mathcal{D}$. By construction $V_C(g, 1, B) < \bar{V}(g, B)$ (by assumption, if indifferent, the government does not default) where

$$\bar{V}(g, B) \equiv \lambda \int_{\Delta} \{\mathbf{a}^*(g, \delta', B)V_C(g, \delta', B) + (1 - \mathbf{a}^*(g, \delta', B))V_A(g, B)\} \pi_{\Delta}(d\delta') + (1 - \lambda)V_A(g, B).$$

By lemma C.1(2) W is decreasing, provided that $H'(1 - \mathbf{n}_C^*(g, 1, B)) < \kappa$. Since $H'' \neq 0$, then H' is monotonic, so the only point (in the relevant domain) for which $H'(1 - n) = \kappa$ is $n = n_1(\kappa)$. Since by construction of $\mathcal{D}$, $1 - \mathbf{n}_C^*(g, 1, B) \neq n_1(1)$, it follows that W is decreasing.

Step 1. We first show that for $(g, B) \in \mathcal{D}$, $B - p(g, \mathbf{B}^*(g, B); \mathbf{e}^*)\mathbf{B}^*(g, B) \geq 0$. Suppose not, that is, $B - p(g, \mathbf{B}^*(g, B); \mathbf{e}^*)\mathbf{B}^*(g, B) < 0$. Thus, for any $(g, B) \in \mathcal{D}$,

$$W(g + B - p(g, \mathbf{B}^*(g, B); \mathbf{e}^*)\mathbf{B}^*(g, B)) - W(g) \equiv \eta(g, B) > 0.$$

Since $\mathbb{G} \times \mathbb{B}$ are a finite collection of points, so is $\mathcal{D}$. Hence, it follows that there exists a $\eta > 0$ such that $\eta \leq \eta(g, B)$ for all $(g, B) \in \mathcal{D}$.

Also, note that for any $B' \neq B$

$$V(g, B') - \bar{V}(g, B) = V(g, B') - \bar{V}(g, B') + \{\bar{V}(g, B') - \bar{V}(g, B)\}.$$

By lemma C.2, for any $\epsilon > 0$, there exists a $\lambda(\epsilon)$ such that

$$V(g, B') - \bar{V}(g, B) \geq V(g, B') - \bar{V}(g, B') - \epsilon, \quad \forall (B, B', g).$$

By choosing $\epsilon \equiv \beta^{-1}0.5\eta$ (and set $\bar{\lambda} \equiv \lambda(\beta^{-1}0.5\eta)$), it follows that, for all $(g, B) \in \mathcal{D}$.

$$\begin{aligned} V_C(g, B) - V_A(g, B) &= \eta(g, B) + \beta \int_{\mathbb{G}} \{V(g, \mathbf{B}^*(g, B)) - \bar{V}(g, B)\} \pi_{\mathbb{G}}(dg'|g) \\ &\geq \eta - \beta\epsilon + \beta \int_{\mathbb{G}} \{V(g, \mathbf{B}^*(g, B)) - \bar{V}(g, \mathbf{B}^*(g, B))\} \pi_{\mathbb{G}}(dg'|g) \\ &\geq 0.5\eta > 0 \end{aligned}$$

where the last line follows from our previous calculations and the fact that $V(g, \mathbf{B}^*(g, B)) - \bar{V}(g, \mathbf{B}^*(g, B)) \geq 0$ (because $V(g, B) = \max\{V_C(g, B), \bar{V}(g, B)\}$). This implies that $V_C(g, B) > V_A(g, B)$. Therefore, since $\lambda < 1$, it implies that $V(g, B) > \bar{V}(g, B)$. But this is a contradiction to the fact that $\mathbf{d}^*(g, B) = 1$.

Step 2. By step 1, it follows that $B \geq p(g, \mathbf{B}^*(g, B); \mathbf{e}^*)\mathbf{B}^*(g, B)$ for all $(g, B) \in \mathcal{D}$. This implies that the revenue under financial access ought to be strictly greater than the revenue under financial autarky, i.e.,

$$R(1, \mathbf{n}_C^*(g, 1, B)) = g + B - p(g, \mathbf{B}^*(g, B); \mathbf{e}^*)\mathbf{B}^*(g, B) > g = R(\kappa, \mathbf{n}_A^*(g)).$$

Since $\kappa = 1$ and $R' < 0$ (see lemma C.1(1)) it follows that $\mathbf{n}_C^*(g, 1, B) < \mathbf{n}_A^*(g)$. Since $H'' < 0$, the previous statement implies that $H'(1 - \mathbf{n}_C^*(g, 1, B)) < H'(1 - \mathbf{n}_A^*(g))$ iff $1 - \tau_C^*(g, B, 1) < 1 - \tau_A^*(g) \iff \tau_A^*(g) < \tau_C^*(g, B, 1)$. $\square$

C.1 Derivation of Equation 13

By our characterization of the default rule. In this setting, to default or not, boils down to choosing a T (contingent on ω) such that for all $t < T$ there is no default and for $t \geq T$ there is financial autarky. Thus, the optimal T , n^∞ and B^∞ must solve the following program: $\sup_T \sup_{(n^\infty, B^\infty) \in \Gamma(T)} \mathcal{L}(\omega_0; n^\infty, T)$, where:

$$\mathcal{L}(\omega_0; n^\infty, T) \equiv \int_{\Omega} \left(\sum_{t=0}^{T(\omega)-1} \beta^t u(n_t(\omega) - g_t(\omega), 1 - n_t(\omega)) + \sum_{t=T(\omega)}^{\infty} \beta^t u(n_t^A(\omega) - g_t(\omega), 1 - n_t^A(\omega)) \right) \Pi(d\omega | \omega_0).$$

Where $n_t^A(\omega)$ is such that $\left(\kappa_t(\omega) - \frac{u_l(\kappa_t(\omega)n_t(\omega) - g_t(\omega), 1 - n_t(\omega))}{u_c(\kappa_t(\omega)n_t(\omega) - g_t(\omega), 1 - n_t(\omega))} \right) n_t(\omega) = g_t(\omega)$ for all t and ω , and

$$\Gamma(T) \equiv$$

$$\{(n^\infty, B^\infty) : \forall \omega \in \Omega, 0 \leq A(n_t(\omega), g_t(\omega), B_t(\omega)) + \mathcal{P}_t(\omega; B_{t+1})B_{t+1}(\omega) \ \forall t < T \text{ and } n_t(\omega) = n_t^A(\omega) \ \forall t \geq T(\omega)\}$$

where $A(n, g, B) = (u_c(n - g, 1 - g) - u_l(n - g, 1 - n))n - (g + B)u_c(n - g, 1 - g)$ and $\mathcal{P}_t(\omega; B_{t+1}) \equiv p(\omega; B_{t+1})u_c(n_t(\omega) - g_t, 1 - n_t(\omega))$.

By assumption, the solution of B_{t+1} is in the interior. Given a particular T , and for (t, ω) such that $t < T(\omega)$, the optimal choice for (n^∞, B^∞) ought to satisfy

$$u_c(n_t(\omega) - g_t(\omega), 1 - n_t(\omega)) - u_l(n_t(\omega) - g_t(\omega), 1 - n_t(\omega)) - \nu_t(\omega) \frac{dA(n_t(\omega), g_t(\omega), B_t(\omega))}{dn_t} = 0$$

and

$$-\nu_t(\omega) \left\{ p_t(\omega; B_{t+1}) + \frac{p_t(\omega; B_{t+1})}{B_{t+1}} B_{t+1} \right\} + \beta \int_{\{\tilde{\omega} \in \Omega : \tilde{\omega}^t = \omega^t\}} \nu_{t+1}(\tilde{\omega}) \Pi(d\tilde{\omega} | \omega^t) = 0.$$

Observe that $\int_{\{\tilde{\omega} \in \Omega : \tilde{\omega}^t = \omega^t\}} \nu_{t+1}(\tilde{\omega}) \Pi(d\tilde{\omega} | \omega^t) = \int_{\mathbb{G}} \nu_{t+1}(\omega^t, g_{t+1}) \pi_{\mathbb{G}}(dg_{t+1} | g_t)$.

Where $(t, \omega) \mapsto \nu_t(\omega)$ is the Lagrange multiplier of $0 \leq A(n_t(\omega), g_t(\omega)) + p_t(\omega; B_{t+1})B_{t+1}(\omega)$ and is non-negative. Also, note that if $\nu_t(\omega) = 0$, then

$$u_c(n_t(\omega) - g_t(\omega), 1 - n_t(\omega)) - u_l(n_t(\omega) - g_t(\omega), 1 - n_t(\omega)) = 0$$

which implies that $\tau_t(\omega) = 0$.

C.2 Supplementary Lemmas

LEMMA C.1. *Suppose assumption 6.1 holds. Then:*

(1) $R(\kappa, \cdot) : (0, 1] \rightarrow \mathbb{R}$ is such that the optimal level of labor belongs to $[n_2(\kappa), n_1(\kappa)]$ where $R(\kappa, n_1(\kappa)) = 0$ and $R'(\kappa, n_2(\kappa)) = 0$, and in this domain $R' < 0$.

(2) $W : [0, R(\kappa, n_2(\kappa))] \rightarrow \mathbb{R}$ is differentiable, uniformly bounded, non-increasing (decreasing over all R such that $(-\kappa + H'(1 - n(\kappa, R))) < 0$) function. Where, for any κ , $R \mapsto n(\kappa, R)$ is the inverse of $R(\kappa, \cdot)$ (by part (1) exists in the relevant domain).

LEMMA C.2. *Suppose assumption 6.1 holds. For any $\epsilon > 0$, there exists a $\lambda(\epsilon) > 0$ such that, for all B and $B + h$ in $\mathbb{B}$ and $g \in \mathbb{G}$, $|\bar{V}(g, B) - \bar{V}(g, B + h)| \leq \epsilon$.*

LEMMA C.3. *Suppose assumption 6.1 hold. Then $V_C \in L^\infty(\mathbb{G} \times \mathbb{B})$ and $V_A \in L^\infty(\mathbb{G} \times \mathbb{B})$.*

LEMMA C.4. *Suppose assumption 6.1 hold. Then:*

1. V_C is non-increasing in B (and thus in δ , whenever $B > 0$).
2. V_A is non-increasing in B .
3. If $\pi_{\mathbb{G}}(\cdot|g_1) <_{FOSD} \pi_{\mathbb{G}}(\cdot|g_2)$ for $g_1 < g_2$, V_A and V_C are non-increasing as a function of g .

D Figures and Tables

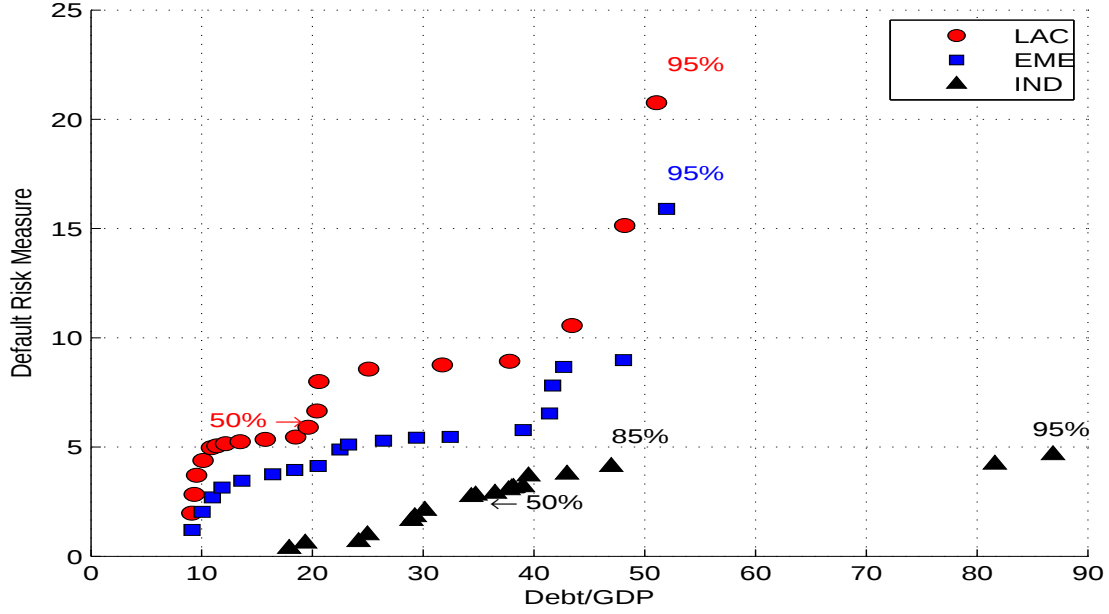


Figure D.1: The percentiles of the domestic government debt-to-output ratio and of a measure of default risk for three groups: IND (black triangle), EME (blue square) and LAC (red circle)

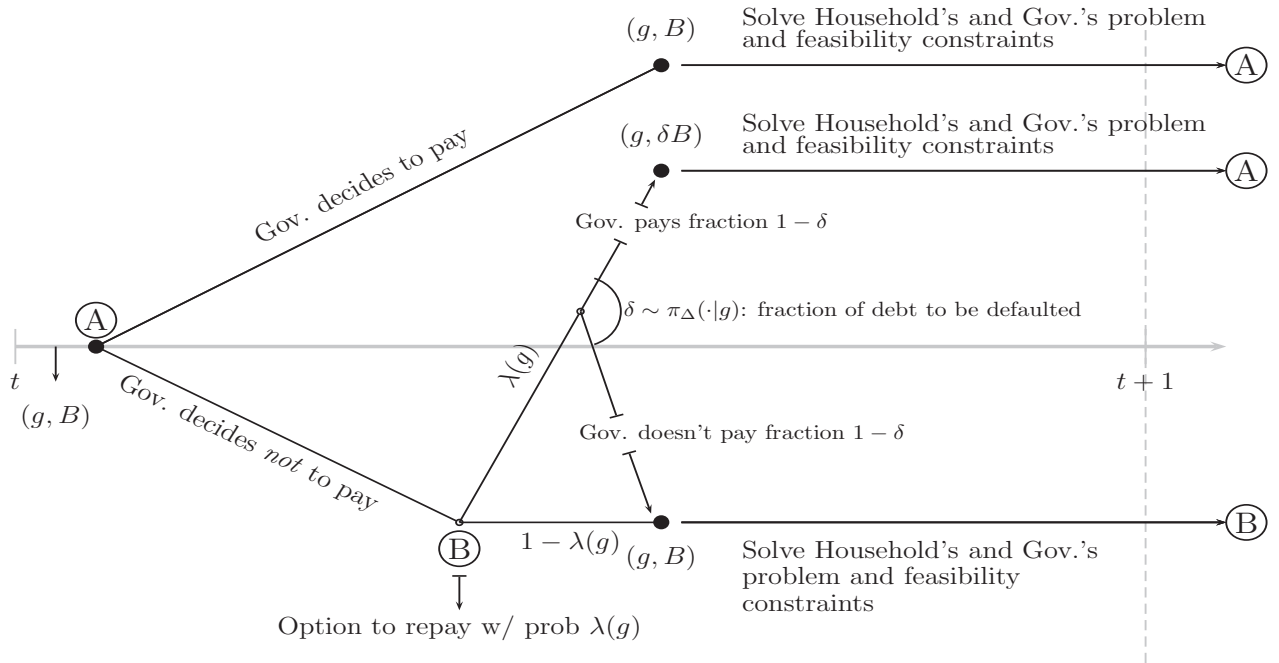


Figure D.2: Timing of the Model

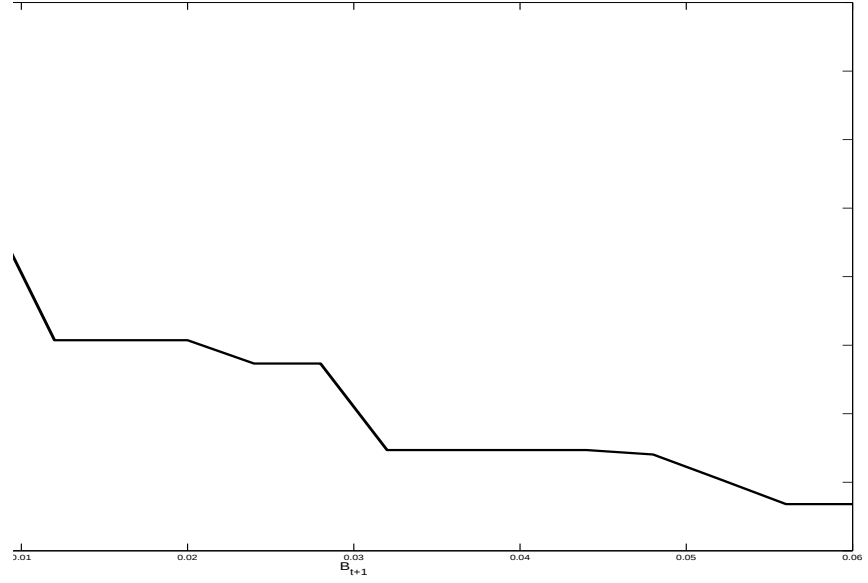


Figure D.3: Equilibrium price as a function of future debt.

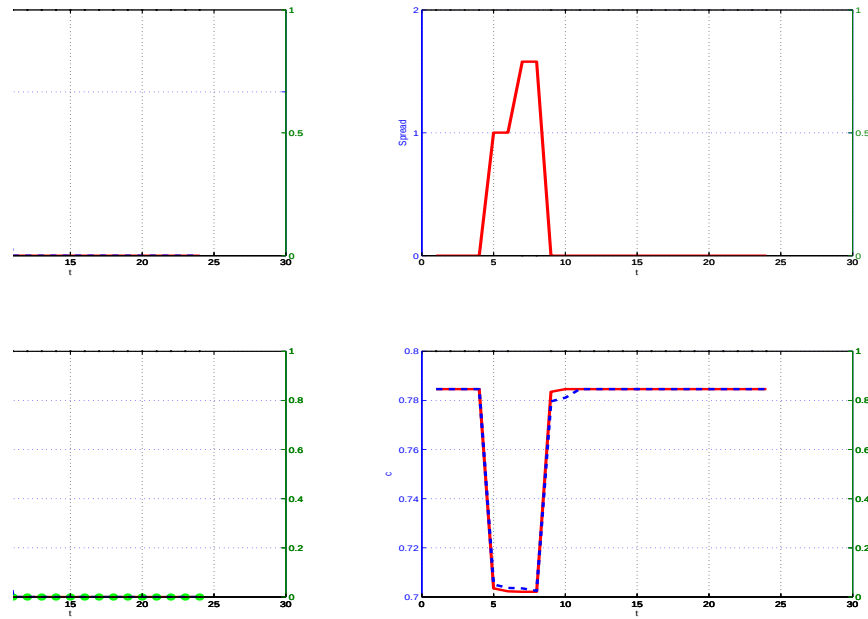


Figure D.4: Impulse Responses for a particular realization of $(g_t)_{t=0}^{25}$. In all the panels, solid red line belongs to the ED Economy and the dashed blue line belongs to the AMSS Economy.