

Maturity and Repayment Structure of Sovereign Debt

Yan Bai*

Seon Tae Kim[†]

Gabriel Mihalache[‡]

University of Rochester

ITAM Business School

University of Rochester

PRELIMINARY

November 8, 2013

Abstract

This paper studies the maturity, timing and relative size of repayments for sovereign debt. Using Bloomberg bond data for emerging economies, we document that sovereigns issue debt with shorter maturity but more back-loaded repayments during downturns. To account for this pattern, we study a sovereign-default model of a small open economy which issues a state-uncontingent bond, with a flexible choice of maturity and repayment schedule. In our model, as in the data, during recessions the country prefers its payments to be more back-loaded—delaying relatively larger payments—in order to smooth consumption. However, such back-loaded debt is expensive since payments scheduled later involve higher default risk. To reduce borrowing costs, the country optimally shortens its maturity. We calibrate the model to yearly Brazilian data. The model can rationalize the observed patterns of maturity and repayment structure, as an optimal trade-off between consumption smoothing and endogenous borrowing cost due to lack of enforcement.

*Yan Bai: Department of Economics, University of Rochester, Rochester, NY, 14627. E-mail address: yan.bai@rochester.edu

[†]Seon Tae Kim: Department of Business Administration, Business School, ITAM, Rio Hondo #1, Col. Pregreso Tizapan, Mexico, D.F., Mexico, C.P. 01080. E-mail address: seon.kim@itam.mx

[‡]Gabriel Mihalache: Department of Economics, University of Rochester, Rochester, NY, 14627. E-mail address: gabriel.mihalache@rochester.edu

1 Introduction

At least since Rodrik and Velasco (1999)’s work on the maturity of emerging market debt, international economists have been puzzled by emerging economies’ heavy issuance of short-term debt during crises. Short-term debt tend to hurt consumption smoothing due to roll-over risk. We argue that this is less puzzling than one might think, since countries also adjust the stream of promised payments over time, or the *repayment structure*: while maturity does shorten, payments become more back-loaded. This allows the sovereign to strike a balance between consumption smoothing and its borrowing cost.

To understand how an emerging economy chooses the maturity and, more importantly, the repayment schedule of its external debt, we explore the individual bond data of four emerging markets¹ from the Bloomberg Professional service using panel regressions and document the business cycle behavior of sovereign contracts. We report two major findings on sovereign bonds. First, the promised repayments are more back-loaded during downturns, when output is low and spread is high. Second, the maturity is shorter during periods of low output or high spread, consistent with the evidence presented by Arellano and Ramanarayanan (2012) and Broner, Lorenzoni, and Schmukler (2013).

Our model extends the standard sovereign-default framework of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008) by introducing a flexible choice of repayment schedule. A small, open economy can issue only state-uncontingent bond in the international financial markets. Its government can choose to default over its bond, subject to a punishment of output loss and temporary exclusion from international markets. We depart from the literature and allow the government to issue bonds with different maturities and repayment schedules. For example, the government may issue a T -period, back-loaded (front-loaded) long-term bond. Before the bond matures, the government makes periodic payments increasing (decreasing) over time. To mitigate the curse of dimensionality implicit in specifying rich repayment structures, we restrict the government to hold only one bond at a time. To change its repayment structure, the government must buy back the current bond and issue a new one.

The repayment schedule and maturity of sovereign debt are determined by the interplay of two incentives: (i) smoothing consumption, and (ii) reducing default risk. To smooth consumption, the sovereign would like to align repayments with future output, i.e. larger repayments ought to be scheduled for periods with higher expected output. Given the mean-reverting nature of the output process considered, the growth rate of output decreases

¹The four emerging markets we study are Argentina, Brazil, Mexico, and Russia.

with the current output. Thus, a more back-loaded repayment is preferable during economic downturns since the government can repay the bulk of its obligation in the future, when the economy is expected to recover. Under the consumption-smoothing incentive, the growth rate of repayments and current output should be negatively correlated.

The government must also take into consideration its default risk when making choices over repayment schedule, since high default risk leads to high borrowing cost. A more back-loaded bond is particularly expensive during downturns. Such contract specifies that most repayments are to be made in the far future, which subjects lenders to large losses if the government defaults soon. To reduce borrowing cost while enjoying the consumption-smoothing benefit of back-loaded contracts, the government chooses a shorter maturity in economic downturns. Contracts with shorter maturity allow lenders to receive their investment returns sooner. Lenders therefore bear less default risk and offer a higher bond price.

We calibrate the model to match key moments for the Brazilian economy. Our model generates volatilities of consumption and trade balance similar to the data. The model replicates key features of sovereign debt. The duration is about 6 years in both the model and the data. The average maturity is about 9 years in the model and 8 years in the data. The median growth rate of repayment is 12.2% in the data, which implies that on average countries issue back-loaded bonds. Our model also predicts that on average countries issue back-loaded bonds: repayment growth is 5.3% for the model.

Most importantly, our model matches well the cyclical behavior of maturity, repayment growth, and duration. During economic downturns, the government chooses shorter maturity, but a more back-loaded repayment schedule. Specifically, the correlation between maturity and output is 0.67 in the model and 0.62 in the data, the correlation between repayment growth and output is -0.39 in the model and -0.37 in the data, and the correlation between duration and output is 0.3 in the model and 0.61 in the data. In terms of correlation with spread, both the model and the data predict that maturity does not vary much with spread. The correlation in the model is 0.03 and in the data is -0.01. The correlation between repayment growth and spread is 0.14 in the model and 0.19 in the data. The model, however, overstates the correlation between duration and spread. The model has a correlation of 0.32 while the data has a value of -0.08.

This paper makes two contributions. Empirically, we focus on both the maturity and repayment structure of sovereign debt. We document that the repayment structure does help consumption smoothing during downturns. Most works in the literature, such as Broner, Lorenzoni, and Schmukler (2013), and Arellano and Ramanarayanan (2012), focus on the portfolio choice over short and long duration of debt, ignoring the timing and size of payments.

Theoretically, we model the endogenous choice of repayment schedule and maturity. Our model can match the key stylized fact we document: most of time emerging economies have back-loaded bond contracts, with payments increasing over time. The previous literature, however, features exogenous repayment schedules for sovereign bonds and frequently only models one-period debt. A new line of work studying long-term sovereign debt as in Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), and Hatchondo and Martinez (2009) use perpetuity bonds to avoid the curse of dimensionality. Such perpetuity bonds are restricted to have a front-loaded repayment schedule², opposite to the data. Another line of work studying maturity structure of sovereign debt uses the zero-coupon bond. The bond-level dataset from Bloomberg shows that emerging economies rarely issue such bonds.

The rest of the paper is organized as follows. Section 2 presents the empirical findings on the cyclical behavior of maturity, repayment structure, and duration of sovereign debts. Section 3 introduces the model. Section 4 presents the quantitative analysis over the model. Section 5 concludes.

2 Empirical analysis

Our empirical analysis focuses on how maturity and repayment structure of sovereign debt vary with underlying fundamentals. To this end, we look at a sample of four emerging economies: Argentina, Brazil, Mexico, and Russia.³ The key finding is that emerging economies tend to issue bonds with shorter maturity but more back-loaded repayment schedule during economic downturns when output is low or interest spread is high.

2.1 Data Source

Using the *Bloomberg Professional* database, we extract information on the promised schedule of coupons and principal at the level of individual bond. We use external debt, which are issued in a foreign jurisdiction. The detailed, bond-level data is analyzed in connection with two key aggregates: the short-term credit spread and per-capita real GDP, obtained via IMF’s *eData* service. The short-term spread is measured as the difference in yield-to-maturity between the sovereign’s and U.S. government’s bonds of maturity less than or equal to one year. We interpret this spread as a proxy for the overall market price of the sovereign’s

²For example, in Arellano and Ramanarayanan (2012), one unit of the perpetuity bond promises repayments $\{1, \delta, \delta^2, \dots\}$ and so forth, forever. This requires the decay rate $\delta < 1$, to keep the discounted present value of repayments bounded.

³This is the set of countries considered in Arellano and Ramanarayanan (2012)

use of credit. Per-capita real GDP is, of course, likely to be one of key factors in determining both the price and demand of the sovereign's use of credit. Our quarterly sample covers the period 1995Q1-2011Q4.

Usually, sovereign bonds have a maturity greater than one year and the payments are to be made in foreign, hard currency. At a given point of time, a sovereign has multiple outstanding bonds, of various denominations, coupons rates and frequencies, issuance and maturity dates. Moreover, sovereigns might retire their outstanding debts prematurely by either exercising the call option or buying them back via a reverse auction. Finally, sovereign bonds are sometimes, of course, subject to default and renegotiation. This rich heterogeneity of bonds and portfolios makes it challenging to characterize the complete structure of sovereign bonds precisely.

For our purposes, we need to characterize the repayment structure of sovereign bonds beyond the two usual measures, debt size and maturity, as commonly done in the literature. We construct the cash-flow profile implied by the outstanding bonds, similar to the approach in Dias et al. (2011). Consider an emerging market economy, e.g. Brazil. At a given point in time, this country can have a number of outstanding bonds; for each such bond, we compute the stream of promised repayments, coupons and principal, and convert them to real U.S. dollars by using the (spot) foreign exchange rates and deflating by the U.S. consumer price index (CPI). We then estimate the growth rate and maturity of such promised repayments for an individual bond and aggregate across different bonds using the bond size as the weight: the maturity or payment growth rate of the portfolio of outstanding debt is a weighted average of the bond-level measures, weighted by the size of each bond.

Sovereigns often schedule payments 20 or 30 years in the future. In order to evaluate these promises in terms of real U.S. dollars, several assumptions are necessary. First, we assume that foreign exchange rates are Martingales, which implies the expected future exchange rate equals the current value. Second, for the U.S. CPI, we assume perfect-foresight because the U.S. CPI is quite stable. When the coupon rate is expressed as a spread over the LIBOR rate, e.g. the floating coupon-rate bond, we take as our benchmark the perfect-foresight case. Note that our sample includes bonds with non-fixed coupon rate, e.g. floating and variable coupon-rate bonds, as well as the fixed coupon-rate bond. In contrast, frequently in the literature, non-fixed coupon-rate bonds are excluded from the analysis mainly for convenience rather than for economic reasons. We must address all of these cases consistently in order to produce a coherent picture of payments' timing and size. For example, variable coupon bonds often specifies a rate which rises with the time-to-repayment in a step-wise form; this has important implications for the growth rate of the promised repayments.

Payments are sometimes terminated before the maturity date, for various reasons: exercise of the call option (where applicable), repurchase via a reverse auction, default, etc., of all which we label as “premature termination.” We exclude such prematurely terminated bonds from the promised repayment schedule after, but not until, the termination date.

2.2 Maturity and Repayment Structure

This section discusses how we construct summary statistics for the maturity and repayment structure of sovereign bonds in the data. Consider a sovereign country i in period t . Let $n_t(i)$ denote the number of outstanding bonds issued by sovereign i ; for bond $j \in \{1, 2, \dots, n_t(i)\}$, let $d_t(s, j; i)$ denote the cash flow—in the real U.S. dollars terms—promised by bond j to be paid $s \in \{0, 1, 2, \dots, T_t(j; i)\}$ periods later, where $T_t(j; i)$ refers to the length of time until the last repayment. Throughout this paper, $T_t(j; i)$ is taken as the maturity of the principal or simply as the *maturity* for bond j at time t . The promised cash-flow profile $\{d_t(s, j; i)\}_{s=1}^T$ can be thought of as the distribution of promised repayments $d_t(s, j; i)$ over time, starting next period $t + 1$, analogous to the interest-rate term structure; put differently, we study the term structure of the promised repayment, which refers to the *shape* of the promised repayment as a function of time-to-repayment.

We characterize this term structure by 3 statistics, in addition to the maturity $T_t(j; i)$: total size $l_t(j; i)$, the growth rate $\delta(j; i)$ of repayments, and average length of time-to-repayment $\tau_t(j; i)$. First, the total size of the repayments is measured as their sum (without any discounting):

$$l_t(j; i) = \sum_{s=0}^{T_t(j; i)} d_t(s, j; i) \quad (1)$$

which we take as the measure of the size of bond j at t . The size of bond $l_t(j; i)$ includes the promised payments scheduled for the current period $s = 0$ because it represents the current state of the sovereign’s debt burden. Note that our measure of the bond size $l_t(j; i)$ includes the value of coupons as well as the principal; in comparison, the usual measure of debt size widely used in the literature counts only the principal (ignoring coupons) and reports nominal values. Second, under the assumption that promised repayments grow at a constant annual rate $\delta(j; i)$, we write the term structure of promised repayments $d_t(s, j; i)$ as:

$$\log(d_t(s, j; i)) = \tilde{d}_t(1, j; i) + \frac{s-1}{4} \cdot \log(1 + \delta(j; i)) + \epsilon_t(s, j; i) \quad (2)$$

where s refers to the number of quarters since the current period t , $\tilde{d}_t(1, j; i)$ the trend

component of the logged repayment for $s = 1$, and $\epsilon_t(s, j; i)$ is the error term which captures the deviations of $\log(d_t(s, j; i))$ from the constant growing rate trend. We will estimate $\delta(j; i)$ for each bond j based on the stream of payments promised at issuance, and refer to it as the average growth rate of repayments for bond j . We keep this bond characteristic constant through the life of bond j . Third, the average length of time-to-repayment $\tau_t(j; i)$ in terms of years is written as:

$$\tau_t(j; i) = w_t(s, j; i) \cdot \frac{s}{4}, \quad w_t(s, j; i) \equiv \frac{d_t(s, j; i) \cdot R^{-s/4}}{\sum_s^{T_t(j; i)} \{d_t(s, j; i) \cdot R^{-s/4}\}} \quad (3)$$

where R denotes the gross annual (real) discount rate; we take as benchmark the case of constant R , risk-free and equal to 1.032, i.e., 3.2% discount rate annually; thus, $\tau_t(j; i)$ represents the risk-free version of the Macaulay duration and is referred to simply as the *duration* for bond j .

The bond-level characteristics $(l_t(j; i), \delta(j; i), \tau_t(j; i), T_t(j; i))$ are estimated and then aggregated across outstanding bonds j by using the bond size $l_t(j; i)$ as the weight:

$$\delta_t(i) = \sum_j^{n_t(i)} \omega_t(j; i) \delta(j; i), \quad \tau_t(i) = \sum_j^{n_t(i)} \omega_t(j; i) \tau_t(j; i), \quad T_t(i) = \sum_j^{n_t(i)} \omega_t(j; i) T_t(j; i) \quad (4)$$

where $\omega_t(j; i) \equiv l_t(j; i) / \sum_j^{n_t(i)} l_t(j; i)$. Finally, we measure the total size of outstanding bonds as the sum of bond sizes $l_t(j; i)$ across j :

$$l_t(i) = \sum_j^{n_t(i)} l_t(j; i). \quad (5)$$

The set of such aggregated bond characteristics $(\delta_t(i), \tau_t(i), T_t(i), l_t(i))$ is then combined with two additional aggregate variables, the short-term spread and per-capita real GDP, and pooled across different sovereigns i . This is the aggregate sovereign-quarter panel that we use below.

2.3 Summary Statistics: Case of Brazil

In this section, we discuss the key features of the data on sovereign debt for the case of Brazil, the target country for the quantitative study in Section 4. Throughout this section, we suppress the country index i . We present summary statistics for Brazil at an annual

Table 1: Summary Statistics: Brazil, 1995-2011 Annually

	Spread (%)	Repayment Growth (δ)	Maturity: Years	Duration: Years
Mean	16.99	0.24	7.86	5.99
Median	17.04	0.14	7.57	5.91
Standard Deviation	5.63	0.26	1.59	0.96
Corr. w/ Output	-0.18	-0.37	0.62	0.61
Corr. w/ Spread	1.00	0.19	-0.01	-0.08
Conditional on Output				
< median	16.81	0.30	7.00	5.49
> median	16.29	0.16	8.73	6.51
Conditional on Spread				
< median	13.50	0.15	8.45	6.38
> median	20.49	0.32	7.45	5.73

Note: ‘Repayment Growth (δ)’ refers to the average growth rate of the promised repayment due in the future starting next quarter, ‘Maturity’ the maturity of principals, and ‘Duration’ the Macaulay duration under the risk-free discount rate of 3.2 percent per annum. Output refers to the logged per-capita real GDP, spread refers to the yield spread between the Brazilian and U.S. government bonds with maturities shorter than one year and debt size is the sum of non-discounted logged future repayments (including coupons and principals) for outstanding bonds. Output and debt size are in log terms, while the other variables are in levels. All of the variables are annualized and then adjusted so that changes in the trend component, (estimated by the HP-filter with the smoothing parameter 100), are removed.

frequency, to match the calibration of our quantitative model, where for numerical tractability reasons we take one period to be one year. We convert quarterly series to annual frequency by aggregating over quarters. Then, we remove the trend, which is estimated using the HP filter with the smoothing parameter set to 100; by doing so, the adjusted variables are stationary.

Table 1 presents the summary statistics for Brazil. The (promised) repayment structure is more back-loaded during periods of higher output. Note that the variation in the growth rate of the payments, δ_t , is substantial: the volatility of δ_t is slightly larger than the average level of δ_t . Moreover, the repayment growth δ_t is also greatly correlated with key variables. For instance, consider the change in output from below median to above median. In this case, the repayment growth δ_t drops by 14 percentage points; in comparison, the unconditional average of δ_t is 24 percentage points. Note that the result of more back-loaded repayment schedule when output is low is not driven by the fact that maturity is shorter during downturns. If coupon rate is pretty constant across different maturities, shorter maturity during recession alone could imply a higher growth rate of repayment by our estimation method over repayment growth. The coupon rate, however, varies with maturities. Moreover, as we

discussed earlier, there are also bonds with floating coupon rates.

Both the maturity and duration of the promised repayment covary substantially with the (short-term) spread but their fluctuations are of the moderate magnitude: their standard deviations are less than or equal to about 20 percent relative to their unconditional average levels. Nonetheless, we can characterize the qualitative features of the variations in the maturity and duration of the promised repayment as follows: Both are shorter for higher spread, which is consistent with and extends the results in Arellano and Ramanarayanan (2012).

2.4 Regression: Pooled Samples of Four Countries

We extend the previous case study of Brazil to the full panel of four countries and investigate the systematic relationships among the bond-level characteristics, spread and output. For a given country, the short-term spread is used as a proxy of the overall spreads and output is measured as per-capital real GDP.

Every variable is quarterly and detrended using the HP filter with the smoothing parameter 1,600. Such a detrended variable is denoted by a hatted variable; for instance, $\widehat{\delta}_t(i)$ refers to the repayment growth $\delta_t(i)$ that is detrended. We then pool detrended variables across the four countries. Quantity variables such as output and total size of outstanding bonds $\widehat{l}_t(i)$ are in the logged terms as in the business cycle literature, while the other variables, such as spread, repayment growth, maturity and duration, are kept in levels.

We begin by examining the correlation coefficients among key variables. As shown by Table 2, the repayment growth $\widehat{\delta}_t(i)$ is strongly negatively correlated with output; that is, the sovereign chooses its promised repayment to be more back-loaded when income is low, suggesting a hypothesis according to which a consumption-smoothing incentive might be strong in determining the optimal repayment structure. Moreover, we can also see that both maturity $\widehat{T}_t(i)$ and duration $\widehat{\tau}_t(i)$ are highly negatively correlated with the (short-term) spread. Below we examine those two hypotheses in more detail by conducting a formal statistical analysis.

We write the system of the regression equations of the repayment growth rate $\widehat{\delta}_t(i)$ and maturity $\widehat{T}_t(i)$ as:

$$\widehat{\delta}_t(i) = \beta_0(i) + \beta_1 \widehat{y}_t(i) + \beta_2 \widehat{l}_t(i) + \varepsilon_t(i), \quad (6)$$

$$\widehat{T}_t(i) = \alpha_0(i) + \alpha_1 \widehat{r}_t(i) + \alpha_2 \widehat{l}_t(i) + e_t(i) \quad (7)$$

where $\widehat{y}_t(i)$ refers to output, $\widehat{r}_t(i)$ is the short-term spread, and the coefficients $(\beta_0(i), \alpha_0(i))$

Table 2: Correlation Matrix, 1995-2011 Quarterly: Pooled Samples of Four Countries

	Output	Spread	Repayment Growth (δ)	Maturity	Duration
Output	1.00				
Spread	-0.35	1.00			
Repayment Growth	-0.14	0.02	1.00		
Duration	-0.19	-0.11	-0.41	1.00	
Maturity	-0.04	-0.14	-0.40	0.88	1.00
Observations	271				
Num. Countries	4				

Note: The four countries are Argentina, Brazil, Mexico and Russia. ‘Repayment Growth (δ)’ refers to the average growth rate of the promised repayment due in the future starting next quarter, ‘Maturity’ the maturity of principals, and ‘Duration’ the Macaulay duration under the risk-free discount rate of 3.2 percent per annum. Output refers to the logged per-capita real GDP, spread refers to the yield spread between each country’s bond and U.S. government bonds with maturities shorter than one year. Output is in log-deviation terms, while the other variables are in levels. All of the variables are annualized and then detrended by the HP filter with the smoothing parameter 100.

are country-specific fixed effects. $(\varepsilon_t(i), e_t(i))$ are the error terms. The coefficients of the system of equations (6) and (7) are estimated by the GMM estimator where output, spread, debt size, country dummies, and constant terms are used as instrumental variables commonly for both of the two equations.

We can consider an alternative specification where the duration $\hat{\tau}_t(i)$ is used instead of maturity $\hat{T}_t(i)$. In this case, the system of equations becomes:

$$\hat{\delta}_t(i) = \beta_0(i) + \beta_1 \hat{y}_t(i) + \beta_2 \hat{l}_t(i) + \varepsilon_t(i), \quad (8)$$

$$\hat{\tau}_t(i) = \gamma_0(i) + \gamma_1 \hat{r}_t(i) + \gamma_2 \hat{l}_t(i) + u_t(i) \quad (9)$$

which can be estimated using GMM, with the same instrumental variables as above.

Table 3 includes the estimation results of the maturity and repayment structure of sovereign bonds, together with the regression results for spread and debt size. Regression results are summarized as follows. First, the promised repayment due in the future is more back-loaded, i.e. positive growth rate, during periods of low output and larger outstanding debt than for periods of high output and smaller debt. Second, the maturity of the promised repayment (due in the future) is shorter when the short-term spread is high or the outstanding debt is large. Note that, as shown by Table 3, the spread is significantly countercyclical.

Table 3: Regression Results of Maturity and Repayment Structure of Sovereign Bonds: Pooled Samples of Four Countries, 1995-2011 Quarterly

Dependent Variable	System of Two Eqn.s		System of Two Eqn.s			
	Repay. Growth	Maturity	Repay. Growth	Duration	Spread	Debt Size
Output	-0.303 (0.077)***		-0.267 (0.078)***		-32.66 (4.52)***	0.149 (0.628)
Debt Size	0.057 (0.013)***	-1.620 (0.267)***	0.053 (0.013)***	-0.419 (0.160)***	1.75 (1.29)	0.783 (0.036)***
Spread		-0.051 (0.017)***		-0.028 (0.01)***		-0.007 (0.01)
Constant	0.003 (0.009)	-0.215 (0.175)	0.002 (0.008)	-0.124 (0.126)	-0.968 (0.006)***	-0.001 (0.011)
J-stat. = 1.96, d.f.=2, (p-val. = 0.38) J-stat. = 1.95, d.f.=2, (p-val. = 0.38) R^2 = 0.13 R^2 = 0.64						
Observations	271	271	271	271	271	271

Note: this table provides multivariate regression results of key variables for the pooled samples, during the quarterly period 1995-2011, of four countries: Argentina, Brazil, Mexico and Russia. Robust standard errors are inside the parentheses. ‘***’ indicates the significance at the 1 percentage level. ‘Repay. Growth’ refers to the average growth rate of the promised repayment due in the future starting next quarter denoted by δ , ‘Maturity’ the maturity of principals and ‘Duration’ the Macaulay duration under risk-free discount rate of 3.2 percent per annum. Output refers to the logged per-capita real GDP, spread refers to the yield spread between each country’s bond and U.S. government bonds with maturities shorter than one year and debt size is the sum of non-discounted future repayments (including coupons and principals) for outstanding bonds. All of the variables are annualized and then detrended by the HP filter with the smoothing parameter 100. Output and debt size are in log-deviation terms, while the other variables are in levels. Note that the system of the two regression equations for the repayment growth rate and maturity is estimated by using the GMM estimator where the three variables, (output, spread, debt size), country dummies, and constant terms are used as instrumental variables for both of the regression equations; the J -statistic for the overidentification test is about 1.96 with degrees of freedom of 2 (the p -value is about 0.38), the null hypothesis that the instrumental variables are independent of the error terms is not rejected at the 5 percentage significance level. Similarly, the system of the two regression equations for the repayment growth rate and duration is estimated by using the GMM estimator where the three variables, (output, spread, debt size), country dummies, and constant terms are used as instrumental variables for both of the regression equations. The last two specifications, for spread and debt size, use country-level fixed effects.

Thus, we conclude that both the maturity and duration shorten during downturns.

In this section, we have documented several stylized facts about our measures of maturity and repayment structures, as related to key aggregates: spread and output. We emphasize two main facts: (i) the promised repayment is shifted to be more back-loaded as output drops or interest spread increases, and (ii) the maturity shortens as either output drops or interest spread increases. In addition, we have also found that the spread is highly countercyclical, implying that the maturity shortens during downturns.

3 Model

We study optimal maturity and repayment structure in a dynamic sovereign-default model of a small, open economy. The model consists of a representative consumer and a government borrowing long-term debt from a continuum of competitive lenders. The long-term debt contract is characterized by its maturity and repayment structure. International contracts has limited enforcement in that repayments of debt are state-uncontingent and the sovereign government has the option to default.

3.1 Technology, preference, and international contracts

The economy receives a stochastic income stream y which follows a first-order Markov process with finite support Y and transition matrix Π . The government is benevolent and its objective is to maximize the utility of the representative consumer given by,

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

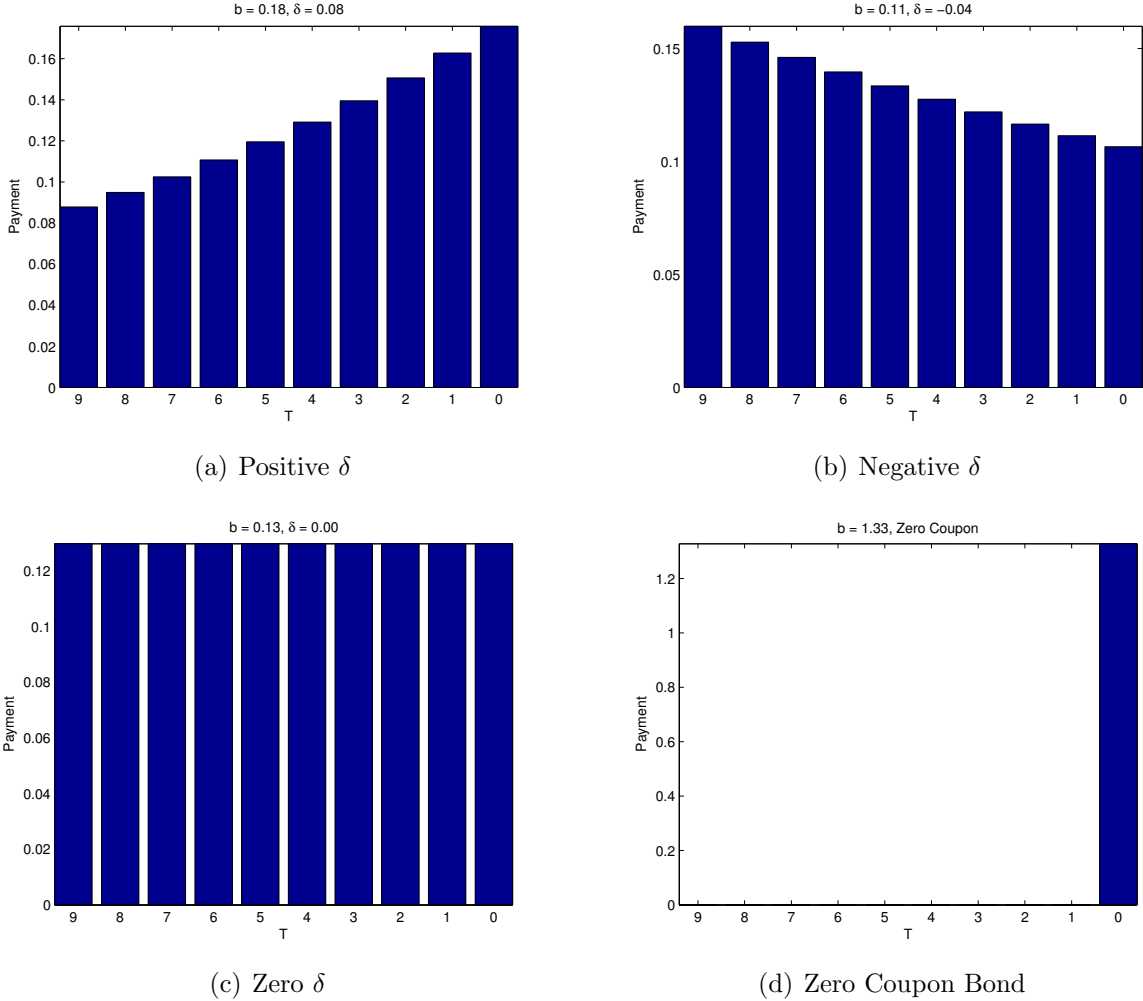
where c_t denotes consumption in period t , $0 < \beta < 1$ the discount factor, and $u(\cdot)$ the period utility function, satisfying the usual Inada conditions. The government borrows from foreign lenders by issuing state-uncontingent bonds. It also decides whether to repay or default on its outstanding debt. All the proceeds of the government are rebated back to the representative consumer in a lump sum fashion.

Conditional on not defaulting, a bond contract is characterized by a maturity T and a growth rate of payments δ . The bond matures in T periods, each period up to maturity $t \leq T$ the government repays $(1 + \delta)^{-t}$. When δ is negative, the repayments are front-loaded, with a downward sloping trend. This is the case covered by the perpetuity bond in Arellano and Ramanarayanan (2012). When δ equals zero, the contract is “flat” as the repayments

are constant over T periods. When δ is positive, the repayments are back-loaded so that the payment is larger later. Figure 1 shows examples of repayment schedules as a function of maturity T , under different values of δ . To make contracts comparable, we pick the number of bond units issued b such that we're financing one unit of consumption today for all cases.

To mitigate the curse of dimensionality implicit in using richer descriptions of debt contracts, we assume that the government can only hold one type bond each period. If the government wants to change its repayment structure, it has to buy back the current bond and issue a new one with different T , δ or units b .

Figure 1: Examples Repayment Schedules



Note: These figures plot repayment schedules of bond contracts for different T and δ . The number of units issued is picked so that all bonds finance 1 unit of consumption today.

While in *good credit standing*, the government has the option to default over its debt.

Following the sovereign default literature, we assume that after default the debt is written off but the government switches to *bad credit standing* and is punished with output losses and temporary exclusion from international financial markets. With probability ϕ , international lenders forgive a government in bad standing and resume lending to it.⁴ Given default risk, lenders charge bond prices which compensate them for losses due to sovereign default.

The state of a government with good credit standing is $s = (T, \delta, b, y)$, including its income shock y and the outstanding units b , with remaining maturity T and growth rate of repayments δ .

3.2 Equilibrium

The government's problem The government in good credit standing chooses whether to default d , with $d = 1$ denoting default:

$$V(s) = \max_{d \in \{0,1\}} \{dV^d(y) + (1-d)V^n(s)\}.$$

where V^d is the defaulting value and V^n repaying value.

If it defaults, the government gets its debt written off but receives a lower output $h(y) \leq y$. Moreover, the government remains in bad credit standing until lenders forgive it with probability ϕ . The defaulting value is given by

$$V^d(y) = u[h(y)] + \beta E \{(1-\phi)V^d(y') + \phi V(0,0,0,y')\}$$

If it repays, the government can continue the current contract, with value V^c , or issuing new debt and receive value V^r . We use $x = 0$ to denote continuing the current contract and $x = 1$ to denote issuing new debt. Specifically, the government's problem under no default is given by,

$$V^n(s) = \max_{x \in \{0,1\}} \{xV^r(s) + (1-x)V^c(s)\}$$

where the continuing welfare is given by

$$V^c(s) = u \left[y - \frac{b}{(1+\delta)^T} \right] + \beta EV(T-1, \delta, b, y'),$$

⁴Our model abstracts from renegotiation. Yue (2010), D'Erasmus (2008), and Benjamin and Wright (2009) study debt renegotiation explicitly. Quantitatively, the model predictions in terms of standard business-cycle statistics of emerging economics are similar as in the work by Arellano (2008) without renegotiation.

and value when choosing a new bond is given by,

$$\begin{aligned} V^r(s) &= \max_{T', \delta', b'} \{u(c) + \beta EV(T', \delta', b', y')\} \\ \text{s.t. } c &= y - \frac{b}{(1+\delta)^T} + q(T', \delta', b', y) b' - q^{rf}(T-1, \delta) b \end{aligned}$$

The bond price schedule for new issuance q depends on the current income shock y and debt structure of the new issuance. We assume that when buying back the old bonds, the government faces the risk-free bond price q^{rf} .⁵

International financial intermediaries Lenders are risk neutral, competitive, and face a world interest rate of r . The bond price schedule makes sure that lenders break even. Without default risk, a contract with (T', δ') pays $(1 + \delta')^{-t}$ for any period t before the maturity T' . We can write the risk-free bond price recursively:

$$q^{rf}(T' - 1, \delta') = \begin{cases} \frac{1}{1+r} \left[\frac{1}{(1+\delta)^{T'-1}} + q^{rf}(T' - 2, \delta) \right] & \text{for } T' \geq 2 \\ \frac{1}{1+r} & \text{for } T' = 1 \end{cases} \quad (10)$$

With default risk, lenders charge a higher implied interest rate to compensate for their losses in the default event. For $T' \geq 2$, the bond price is therefore given by,

$$\begin{aligned} q(T' - 1, \delta', b', y) &= \frac{1}{1+r} E_{y'|y} \{ (1 - d(T' - 1, \delta', b', y')) \times \\ &\quad \left[\frac{1}{(1+\delta)^{T'-1}} + (1 - x(T' - 1, \delta', b', y')) q(T' - 2, \delta', b', y') + x(T' - 1, \delta', b', y') q^{rf}(T' - 2, \delta') \right] \} \end{aligned} \quad (11)$$

and for $T' = 1$ the bond price reduces to the usual one-period bond case

$$q(0, \delta', b', y) = \frac{1}{1+r} E_{y'|y} (1 - d(0, \delta', b', y')) \quad (12)$$

If the government repays next period, lenders receive a payment of $(1 + \delta)^{-(T'-1)}$ per unit outstanding. The repaying government may choose to restructure its debt $x' = 1$ and so repurchase its outstanding debt with risk-free rate q^{rf} . Note that maturity T' and repayment schedule δ' affect the risky bond price in two ways. On the one hand, conditional on not

⁵We plan to conduct sensitivity analysis over the buy-back price in ongoing research.

default, they matter for expected discounted repayment and thus the risk-free part of q . On the other hand, both maturity and repayment schedule matter for future default decisions and thus the risky part of q .

Definition of equilibrium The equilibrium consists of an allocation $\{c, T', \delta', b'\}$, default decision d , restructure decision x , bond schedule $q(\cdot)$, and the risk-free schedule $q^{rf}(\cdot)$ such that given the world interest rate r ,

- (i) The government chooses optimally $\{d, c, T', \delta', b', x\}$ to maximize its welfare given the bond schedules $q(\cdot)$ and $q^{rf}(\cdot)$.
- (ii) Lenders charge bond prices consistent with the government's default behavior, (10) to (12).

4 Quantitative Analysis

This section presents quantitative analysis of the model. We first calibrate the model to the Brazilian economy over the period of 1996 to 2011. We then study the model's implications for standard business cycle statistics of emerging markets, and most importantly the maturity and repayment structure of sovereign debt. Finally, we illustrate the incentives of an emerging economy when choosing maturity and repayment structure for its debt.

4.1 Calibration

We calibrate the parameter values of the model to match key moments in the Brazilian data. The length of one period in the model is set to be one year. The per-period utility function $u(c)$ is chosen as

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma} \quad (13)$$

where σ is the risk aversion parameter. The output of this economy follows an AR(1) process

$$\log(y_t) = \rho \log(y_{t-1}) + \varepsilon_t, \quad (14)$$

where the idiosyncratic shock ε_t follows normal distribution with standard deviation of η . Following Arellano and Ramanarayanan (2012), output of a country in bad credit standing $h(y)$ is given by

$$h(y) = \min \{y, (1-\lambda)E[y]\} \quad (15)$$

where $E[y]$ refers to the unconditional mean of output and $\lambda \in [0, 1]$ captures the default penalty.

To compare the model with the data, we define the *maturity* in the model as T , the *growth rate of repayments* as δ . The *yield to maturity* is the constant interest rate $\hat{r}$ such that the present value of payments, computed using this interest rate, is equal to the market price of the bond, i.e. $\hat{r}$ is implicitly defined by

$$q(T' - 1, \delta', b', y) = \sum_{\tau=T'-1}^0 \exp[-\hat{r}(T' - \tau)] (1 + \delta')^{-\tau}$$

The *spread* s is the difference between the yield to maturity $\hat{r}$ and the risk-free rate r :

$$s(T', \delta', b', y) \equiv \hat{r}(T', \delta', b', y) - r$$

Let $q^{zc}(\tau' - 1, b', y)$ be the price of the *zero coupon* bond of maturity τ' . The *Macauley duration* of the bond is defined as

$$\text{duration}(T', \delta', b', y) = \sum_{\tau=T'}^0 (T' - \tau + 1) \frac{(1 + \delta')^{-\tau} q^{zc}(\tau, b', y)}{q(T', \delta', b', y)}$$

i.e. a weighted average of the maturity of each payment, with the weights given by the value of the payment—interpreted as a zero coupon bond—in the bond price. Note that for $T' = 1$, the bond price reduces to $q(0, \delta', b', y) = \exp(-\hat{r})$, i.e. the usual one period bond price, and duration equals 1, which coincides with the bond's maturity.

Table 4: Benchmark Parameter Values

Parameter	Value	Target
Parameters calibrated independently		
σ Risk-aversion	2.0	Standard value
r Risk-free rate	3.2%	US government bond's yield-to-maturity
ρ Shock persistence	0.9	Brazil GDP
η Shock volatility	0.017	Brazil GDP
ϕ Prob. of return to market	0.154	6.5 years of exclusion since default
Parameters calibrated jointly		
β Discount factor	0.86	Debt service to GDP 5.5%
λ Output loss due to default	0.04	Default frequency 3%

Note: this table provides the benchmark parameter values used in calibrating the model.

Table 4 presents the calibrated parameter values. The risk-aversion parameter σ is set to 2 as standard in the literature. The risk-free interest rate is set to 3.2% to target the average annual yield to maturity for US government bonds. The persistence and volatility of the AR(1) output process are taken from Arellano and Ramanarayanan (2012), who calibrate these two parameters to the HP-filtered Brazilian GDP. The persistence level ρ is 0.9 and the standard deviation η is 0.017. The probability of a defaulting country regaining access to the international financial market ϕ is set to 0.154 to match the observed average 6.5 years of exclusion from international borrowing and lending after default, see Benjamin and Wright (2009). The two remaining parameters, the discount factor β and the output loss parameter λ , are chosen jointly to match the observed debt service-to-GDP ratio and default frequency. We target a default frequency of 3% annually and a debt service-to-GDP ratio of 5.3%.

We solve the model with value function iteration methods. The output process is discretized using the method of Adda and Cooper (2003). We simulate the model for 1,001,000 periods, discard the first 1,000, and obtain the limiting distribution of the model.

Table 5: Key Statistics: Data vs. Model

	Data	Model
Targeted Statistics		
Default frequency	3.0%	2.6%
Debt service/GDP	5.3%	5.5%
Non-targeted Statistics		
SD(consumption)/SD(GDP)	1.1	1.1
SD(trade balance)/SD(GDP)	0.36	0.53
Average spread (%)	4.5	2.9
SD of spread (%)	4.4	3.7
Maturity (years)	7.9	8.9
Duration (years)	6.0	5.9
Repayment growth	12.2%	5.3%

Note: SD denotes standard deviation.

Our model fits the data well in many dimensions. Table 5 reports the model results together with the data statistics for Brazil. In the data, consumption is more volatile than output. This is common in emerging economies, as documented by Neumeyer and Perri (2005). The volatility of consumption is 1.1 times that of output in both the model and the data. The trade balance is normalized by GDP. The model produces a more volatile trade balance than the data: 53% in the model and 36% in the data. The model generates the volatility of spread as 3.7%, while that in the data is 4.4%. The average spread in the model

is lower than that in the data. This is mainly due to the assumption of risk neutral lenders.

The model replicates well features of the sovereign debt. The duration is about 6 years in both the model and the data. The average maturity is about 9 years in the model and 8 years in the data. The median growth rate of repayment is 12.2% in the data, which implies that on average countries issue back-loaded bonds. Our model also predicts that countries issue back-loaded bonds, though it generates repayment growth of 5.3%, lower than the data.

4.2 Bond Price Schedules

The key to our results is the way in which the bond price schedule varies with the maturity and payment growth rate. We first describe the risk-free bond price schedule q^{rf} , which is a function of maturity T' and the repayment growth δ' . We then study the risky bond price schedule q , which incorporates future default risk and depends on maturity, repayment schedule, bond units issued b , and current output y .

Figure 2(a) shows that the risk-free bond price q^{rf} increases with maturity T' . For one bond unit issued today, $b' = 1$, higher maturity is associated with more repayments and thus larger risk-free bond price. This implies that in order to finance one unit of resources today $q^{rf}(T' - 1, \delta')b' = 1$, the per-period payment $b'(1 + \delta')^{-t}$ is lower when T' is higher, for a fixed δ' . As shown in Figure 2(b), q^{rf} decreases with the growth rate of repayment δ' . The reason is that high δ' is associated with back-loaded repayments, which are subject to compounded discounting.

Figure 2: Risk-Free Bond Price q^{rf}

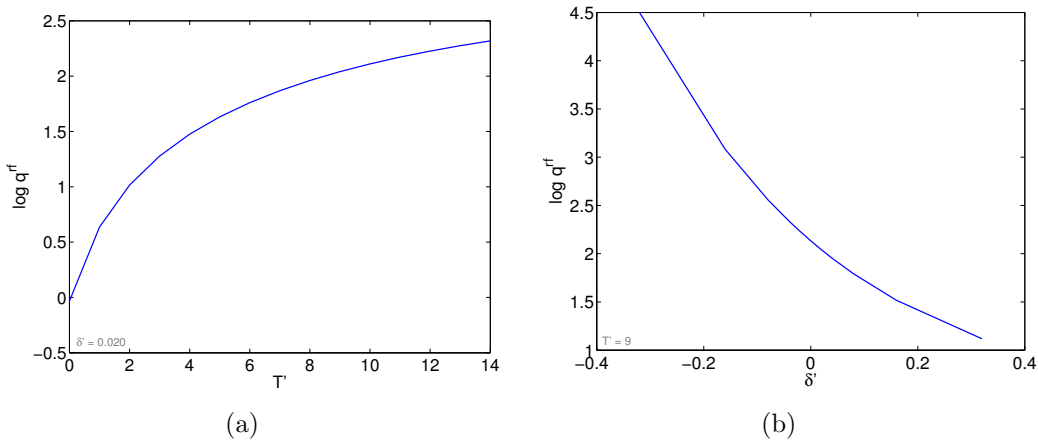
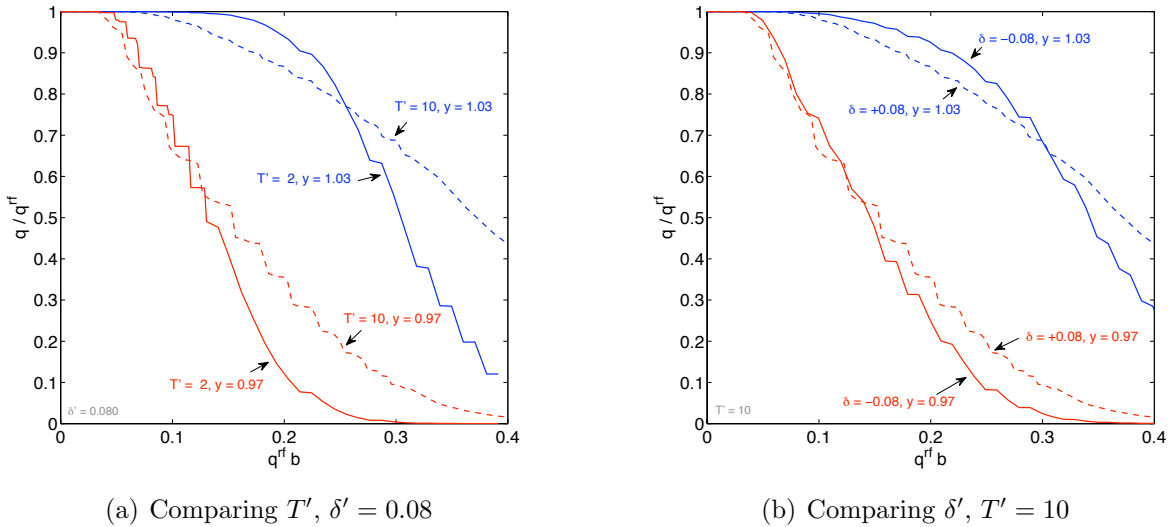


Figure 3 plots the risky bond price schedule $q(T' - 1, \delta', b', y)$ relative to the risk-free bond price schedule $q^{rf}(T' - 1, \delta')$ as a function of bond units, for different levels of T' , δ' , and

the current period output level y . The relative bond prices q/q^{rf} measures the inverse of the default probability. The larger the relative prices, the lower the probability of default. To make contracts comparable, we scale the borrowing b' with risk-free bond price $q^{rf}(T' - 1, \delta')$. Due to default risk, q/q^{rf} decreases with the number of units issued.

Figure 3(a) compares the risky bond price across maturity $T' = 2$ and $T' = 10$ for a fixed δ' but different levels of y . Maturity impacts bond prices through default incentives, via two opposing forces. On one hand, the contract with higher maturity takes longer time to complete and thus involves higher default risks and thus lower bond prices. On the other hand, longer maturity implies a smaller repayment in each period, as discussed above with respect to Figure 2(a). A smaller repayment reduces the government's default incentives in the future and thus shifts up the bond price. As shown by Figure 3(a), when $y = 1.03$ for the debt level below 0.27 the first force dominates and the contract with a shorter maturity ($T' = 2$) offers a higher bond price due to a lower expected default probability. For a lower $y = 0.97$ the threshold for debt under which the first force dominates is lower, 0.11.

Figure 3: Risky Bond Price q



The repayment growth δ' affects the risky bond price via two opposing effects as well. The more back-loaded repayment, i.e. $\delta' > 0$, implies that lenders could suffer from larger losses if the government defaults in the near future. The bond price schedule is therefore tightened for a higher δ' . The more back-loaded repayment, however, can reduce the future per-period repayment $b(1 + \delta)^{-t}$, for a given b . Smaller repayments reduce default incentives later and increase the bond price. With these two opposing effects, Figure 3(b) shows that

under more debt, the second effect dominates and the bond price is lower for low δ' , -0.08 in this case, than for high $\delta' = 0.08$. The region under which the bond price schedule is tighter for low δ' than for high δ' increases as output decreases. The differences in bond price schedules between the low and high δ' shrinks when T' is low.

4.3 Maturity and Repayment Structure

We now turn to our focus: understanding how maturity and repayment structure vary with the business cycle. The results are reported in Table 6.

Our model matches well the cyclicalities of maturity, repayment growth, and duration, when compared to our empirical findings. In downturns, the sovereign government chooses shorter maturity but a more back-loaded repayment schedule. Specifically, the correlation between maturity and output is 0.67 in the model and 0.62 in the data, the correlation between repayment growth and output is -0.39 in the model and -0.37 in the data, and the correlation between duration and output is 0.3 in the model and 0.61 in the data. In terms of the correlation with spread, both the model and the data predict that maturity does not covary much with spread: their correlation is 0.03 in the model and -0.01 in the data. The correlation between repayment growth and spread is 0.14 in the model and 0.19 in the data. The model, however, overstates the correlation between duration and spread. The model has a correlation of 0.32 but the data has a number of -0.08.

We also look at maturity, repayment growth, and duration when output or spread is below the median and above the median. During booms when output is high or spread is low, sovereign debt has a maturity of about 10 years; while during recession, sovereign debt matures in about 5 years. For the repayment, when output is high, the growth rate of repayment is negative 0.8%, meaning that payments shrink over time. When the economy's output is below its median level, the growth rate of repayment is about 4%, i.e. larger repayment due later. The duration is the weighted average of maturity with repayment as weights. It follows the pattern of maturity: shorter duration when the economy is in recession, the duration is about 4 years in recession while 7 years in the boom.

The repayment schedule and maturity of sovereign debt are determined by the interplay of two incentives: (i) smoothing consumption, and (ii) reducing default risk. To smooth consumption, the sovereign would like to align repayments with future output, i.e. larger repayments ought to be scheduled for periods with higher expected output. Given the mean-reverting nature of the output process considered, the growth rate of output decreases with the current output. Thus, a more back-loaded repayment is preferable during economic

Table 6: Maturity and Repayment Structure, Model Prediction

	Maturity	Repayment growth (%)	Duration
Corr. w/ Output	0.67	-0.39	0.30
Corr. w Spread	0.03	0.14	0.32
Conditional on Output			
<median	5.16	3.72	4.87
>median	10.46	-0.80	6.57
Conditional on Spread			
<median	7.18	-0.17	4.65
>median	8.65	2.90	6.86

downturns since the government can repay the bulk of its obligation in the future, when the economy is expected to recover. Under the consumption-smoothing incentive, the growth rate of repayments and current output should be negatively correlated.

The government also takes into consideration the borrowing cost it faces when making choices over repayment schedule. During downturns, when income is low, the range of debt levels for which back-loaded contracts offer better bond prices shrinks, as shown in Figure 3(b). This makes the sovereign more likely to face a tighter bond price if it were to choose a more back-loaded contract. To reduce borrowing cost while enjoying the consumption-smoothing benefit of more back-loaded contracts, the government chooses a shorter maturity in downturns. The reason is that the shorter the maturity, the shorter the pay-back period for lenders and hence the lower the default risk, leading to a more favorable bond price. Additionally, for low maturity the differences in bond price schedules for different δ values are small.

5 Conclusion

It has been a puzzle in the international literature why emerging economies issue bonds with short duration, especially during crises. Our contribution lies in decomposing duration into maturity and a repayment schedule. The maturity shows the same puzzling pattern as found in the literature, countries tend to borrow debt with short maturity during crises. The repayment schedule, however, makes this less of a puzzle since the government chooses more back-loaded repayment and so schedules larger payment further in the future rather than near term. This helps risk sharing during downturns.

References

- Adda, J. and R. W. Cooper (2003, June). *Dynamic Economics: Quantitative Methods and Applications*, Volume 1 of *MIT Press Books*. The MIT Press.
- Aguiar, M. and G. Gopinath (2006). Defaultable debt, interest rates and the current account. *Journal of international Economics* 69(1), 64–83.
- Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *The American Economic Review*, 690–712.
- Arellano, C. and A. Ramanarayanan (2012). Default and the maturity structure in sovereign bonds. *Journal of Political Economy* 120(2), 187–232.
- Benjamin, D. and M. L. Wright (2009). Recovery before redemption: A theory of delays in sovereign debt renegotiations. *unpublished paper, University of California at Los Angeles*.
- Broner, F. A., G. Lorenzoni, and S. L. Schmukler (2013). Why do emerging economies borrow short term? *Journal of the European Economic Association* 11(s1), 67–100.
- Chatterjee, S. and B. Eyigungor (2012). Maturity, indebtedness, and default risk. *American Economic Review* 102(6), 2674–2699.
- D’Erasmus, P. (2008). Government reputation and debt repayment in emerging economies. *Manuscript, University of Texas at Austin*.
- Dias, D. A., C. Richmond, and M. L. J. Wright (2011). The stock of external sovereign debt: Can we take the data at ‘face value’? *working paper*.
- Eaton, J. and M. Gersovitz (1981). Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies* 48(2), 289–309.
- Hatchondo, J. C. and L. Martinez (2009). Long-duration bonds and sovereign defaults. *Journal of International Economics* 79(1), 117–125.
- Neumeyer, P. A. and F. Perri (2005). Business cycles in emerging economies: the role of interest rates. *Journal of Monetary Economics* 52(2), 345–380.
- Rodrik, D. and A. Velasco (1999). Short-term capital flows. Technical report, National Bureau of Economic Research.
- Yue, V. Z. (2010). Sovereign default and debt renegotiation. *Journal of International Economics* 80(2), 176–187.

A Data Appendix

In this appendix, we list the data sources.

- **Bond Issuance Data**

Bond-level data was extracted via the *Bloomberg Professional* service, using the dedicated terminal. For each bond, the following fields were extracted:

- Id: identifier, ISIN, CUSIP;
- Timing: announcement date, date of first coupon payment, maturity date;
- Coupon structure: frequency, type, coupon rate;
- Principal: amount issued, currency of denomination;
- Descriptive notes (containing further details for variable, step, and floating type coupons, in particular time-varying coupon rates and the LIBOR benchmark series used for the floating type).

This enables us to construct the sequence of payments corresponding to each bond and, thus, to compute the growth rate of payments, the maturity, and duration at the individual bond level.

- **Real GDP**

We use the quarterly and annual series for Real GDP from the IMF's *eData* service, the IFS database, as our measure of real output.

- **Nominal Exchange Rates**

Monthly nominal exchange rate series, in foreign currency per USD, were downloaded from the *IMF's eData* website, from the *Balance of Payments Statistics* database.

- **US Price Level**

We use the Federal Reserve Bank of St. Louis' *Federal Reserve Economic Data (FRED)* database to download the monthly *Consumer Price Index for All Urban Consumers: All Items (CPIAUCSL)* series which we use to express all payments in real USD with base year 2005.

- **LIBOR Rates**

For bonds with the floating coupon type, the coupon rate is expressed as a spread over a reference LIBOR rate (of a certain maturity and underlying currency). We download monthly historical data on these LIBOR rates from the *EconStats.com* database.