

Sticky Leverage

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Abstract

We examine the effects of long-lived nominal debt contracts in a quantitative business cycle model with financial frictions. In our setting, as in reality, firms fund themselves with a mix of nominal defaultable debt and equity securities to issue in every period. Debt is priced fairly taking into account default and inflation risk, but is attractive because of the tax-deductibility of interest payments. Unanticipated inflation changes the real burden of corporate debt and, more significantly, distorts corporate investment and production decisions. These effects are large and very persistent. Interest rates rules stabilize the economy, supporting their popularity with policy makers. (Key words: Debt deflation, debt overhang, monetary non-neutrality)

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1 Introduction

Since the onset of the financial crisis in 2008 monetary policy has staged the most aggressive response in at least 30 years. At the same time, financial markets now occupy a much more prominent role in macroeconomic theory. Typical models of financial frictions focus on debt and identify leverage as both a source, and an important mechanism of transmission, of economic fluctuations.¹ Surprisingly, the fact that debt contracts are almost always denominated in nominal terms is usually not given a lot of attention in the literature.^{2,3} Yet, nominal debt creates an obvious link between inflation and the real economy. This is a potentially important source of monetary non-neutrality, and it creates a role for monetary policy even with fully flexible prices.

Our goal in this paper is to develop a tractable general equilibrium model that captures the interplay between nominal debt, inflation, and real aggregates, and examine its main quantitative implications. Our model embeds the dynamic investment and financing decisions of firms into a general equilibrium macroeconomic environment which is subject to real and nominal shocks.

In our setting, as in reality, firms fund themselves by choosing the appropriate mix of nominal defaultable debt and equity securities to issue in every period. Debt is priced fairly by bondholders, who take into account default and inflation risk, but is attractive to issue because of the tax-deductibility

¹Some examples include Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), Cooley, Marimon and Quadrini (2004), Gourio (2012), Jermann and Quadrini (2012), and Gomes and Schmid (2013).

²Among the very rare exceptions are Dopke and Schneider (2006), Christiano, Motto, and Rostagno (2009), Fernandez-Vilaverde (2010), Bhamra et al (2011), and De Fiore et al. (2011).

³At the end of 2012 U.S. non-financial businesses alone had nearly 12.5 trillion dollars in outstanding credit market debt - about 75% of GDP. Nearly all of these instruments are in the form of nominal liabilities, often issued at fixed rates of interest.

of interest payments. Macroeconomic quantities are obtained by aggregating across the optimal decisions of each firm and by ensuring consistency with the consumption and savings choices of a representative household/investor. As a result, our model endogenously links movements in aggregate quantities such as investment and output to changes in corporate leverage and defaults.

We have two main results. First, because debt contracts are written in nominal terms, unanticipated changes in inflation, regardless of their source, always have real effects, even without sticky prices or wages. In particular, lower than expected inflation increases the real value of debt, worsens firms' balance sheets, and makes them more likely to default. If defaults and bankruptcies have resource costs, this immediately and adversely impacts output and consumption.

More importantly, however, when debt is long-lived there is an important *debt overhang*. Even surviving firms begin to cut future investment and production plans, as the increased (real) debt lowers the expected rewards to their equity owners. This phenomenon, which is missing from many models with only short term debt, has been emphasized in several empirical studies of financial crisis and significantly propagates the initial change in inflation, accounting for most of its effects on the economy.⁴ Crucially, however, in our model debt overhang emerges and persists *endogenously* over several periods even though debt can be freely adjusted in every period.

Second, by adding a monetary policy rule linking short term nominal interest rates with inflation and output, our setting also offers important insights into the ongoing monetary stimulus around the world. We find that a standard Taylor rule parameterization implies that Central Banks should try to produce significant inflation in response to adverse real shocks, such as declines

⁴Some recent examples include Reinhart and Rogoff (2011) and Mian and Sufi (2011).

in productivity and in wealth.

Although probably not suited to understand the response of the economy to all shocks, we believe our environment with long-term nominal debt contracts offers a clearer understanding of financially driven recessions than traditional models emphasizing sticky prices and wages. While those models often imply that adverse shocks would be mitigated if prices were allowed to fall, our setting suggests the exact opposite. As Fisher (1933) suggests, deflation would only magnify the real burden of debt and further worsen economic activity. The monetary policy implications are also subtly different. In our model, Central Banks should respond to episodes of excessive leverage not necessarily by lowering the effective real interest rate in the economy but by actively creating immediate inflation.

While the notion that a debt deflation may have significant macroeconomic consequences goes back at least to Fisher (1933), it has not been incorporated into the modern quantitative macroeconomic literature until quite recently. Kang and Pflueger (2012) study the asset pricing implications of allowing for nominal corporate debt in a model driven by productivity and inflation shocks. Their empirical analysis supports the view that inflation uncertainty raises corporate default rates and bond risk premiums. Their model assumes constant labor and considers only two-period debt.

Fernandez-Villaverde (2010) considers nominal government debt, and Christiano, Motto, and Rostagno (2009) embed nominal entrepreneurial loans into a medium-scale DSGE model. Both use short-term debt and emphasize the role of sticky prices. De Fiore et al. (2011) examine optimal monetary policy when firms have nominal debt with default risk. Eggertson and Krugmann (2012) study leverage and debt overhang with short term debt, but their focus is on sticky prices and the role of fiscal policy, and investment plays no role in

the analysis. Neither offers a quantitative analysis and both rely on short-term debt. Occhino and Pescatori (2012a,b) examine debt overhang with one-period debt. Jermann and Yue (2013) study interest rate swaps in conjunction with short-term nominal debt and default. Other macro economic analyses using only real long-term debt and default include Gomes and Schmid (2013) and Miao and Wang (2012).

Debt overhang has been extensively studied in the corporate finance literature, usually in partial models that focus only on optimal firm level decisions. Examples include Myers (1977) and, more recently, Hennessy (2004), Moyen (2005), and Chen and Manso (2010). Like most corporate finance papers, their focus is on real quantities and there is no role for nominal debt.

More broadly, our paper also expands on the growing literature on the macroeconomic effects of financial frictions. This includes Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), Cooley, Marimon and Quadrini (2004), Gertler and Karadi (2011) and Jermann and Quadrini (2012).

The next two sections describe our model and show some of its key properties regarding the real effects of inflation. Section 4 discusses the calibration of our baseline model, and Section 5 shows our quantitative findings. The final section contains concluding remarks.

2 The Model

To be able to focus on the novel mechanisms associated with long-term nominal debt financing and investment, the model is kept as parsimonious as possible. Firms own the productive technology and the capital stock in this economy. They are operated by owners or equity holders but partially financed by de-

faultable debt claims. The firms' optimal choices are distorted by taxes and default costs. Households consume the firms' output and invest any savings in the securities issued by firms. The government plays a minimal role: it collects taxes on corporate income and rebates the revenues to the households in lump-sum fashion.

2.1 Firms

We now describe the behavior of firms and its investors in detail. At any point in time production and investment take place in a continuum of measure one of firms, indexed by j . Some of these firms will default on their debt obligations, in which case they are restructured before resuming operations as before. This means that firms remain on-going concerns at all times, so that their measure remains unchanged through time. Although this is not an essential assumption, it greatly enhances tractability to use an environment where all firms make identical choices.⁵

2.1.1 Technology

Each firm produces according to the function:

$$y_t^j = A_t F(k_t^j, n_t^j) = A_t (k)^{\alpha} n^{1-\alpha}. \quad (1)$$

where A_t is aggregate productivity. Solving for the static labor choice to get the firms' operating profit:

$$R_t k_t^j = \max_{n_t^j} A_t F(k_t^j, n_t^j) - w_t n_t^j \quad (2)$$

⁵Gilchrist, Sim and Zakrajsek (2010) and Gomes and Schmid (2013) offer a detailed model where the cross-section of firms moves over time with entry and default events.

where $R_t = \alpha y_t / k_t$ is the implicit equilibrium “rental rate” on capital. Given constant returns to scale, all firms chose identical ratios k^j / n^j , so R_t is identical across firms.

Firm level profits are also subject by additive idiosyncratic shocks, $z_t^j k_t^j$, so that operating profits are equal to:

$$(R_t - z_t^j) k_t^j \quad (3)$$

We assume that z_t^j is i.i.d. across firms and time, has mean zero, and cumulative distribution $\Phi(z)$ over the interval $[\underline{z}, \bar{z}]$, with $\int_{\underline{z}}^{\bar{z}} \phi(z) dz = \int d\Phi(z)$. We think of these as direct shocks to firms’ operating income and not necessarily output. They summarize the overall firm specific component of their business risk. Although they average to zero in the cross section, they can potentially be very large for any individual firm.

Finally, firm level capital accumulation is given by the identity:

$$k_{t+1}^j = (1 - \delta + i_t^j) k_t^j = g(i_t^j) k_t^j \quad (4)$$

where i_t^j denotes the investment to capital ratio.

2.1.2 Financing

Firms fund themselves by issuing both equity and defaultable nominal bonds. Let B_t^j denote the stock of outstanding defaultable nominal bonds at the beginning of period t .

To capture the fact that outstanding debt is of finite maturity, we assume that in every period t a fraction λ of the principal is paid back, while the remaining $(1 - \lambda)$ remains outstanding. This means that the debt has an

expected life of $1/\lambda$. In addition to principal amortization, the firm is also required to pay a periodic coupon c per unit of outstanding debt.

Letting p_t^j denote the market price of one unit of debt in terms of consumption goods during period t , it follows that the (real) market value of new debt issues during period t is given by:

$$p_t^j(B_{t+1}^j - (1 - \lambda)B_t^j)/P_t = p_t^j(b_{t+1}^j - (1 - \lambda)b_t^j/\mu_t) \quad (5)$$

where $b_t^j = B_t^j/P_{t-1}$, P_t is the overall price level in period t , and we define $\mu_t = P_t/P_{t-1}$ as the economy wide rate of inflation between period $t - 1$ and t . We will work with the real value of these outstanding liabilities throughout the remainder of the paper.

2.1.3 Dividends and Equity Value

In the absence of new debt issues, (real) distributions to shareholders will be equal to:

$$(1 - \tau)(R_t - z_t^j)k_t^j - ((1 - \tau)c + \lambda)\frac{b_t^j}{\mu_t} - i_t^j k_t^j + \tau \delta k_t^j$$

The first term captures the firm's operating profits, from which we deduct the required debt repayments and investment expenses and add the tax shields accrued through depreciation expenditures. This expression for equity distributions is consistent with the fact that only interest payments may be tax deductible.

It follows that the value of the firm to its shareholders, denoted by the function $E(\cdot)$, is the present value of these distributions plus the value of any

new debt issues. It is useful to write this value function in two parts, as follows:

$$E(k_t^j, b_t^j, z_t^j, \mu_t) = \max \left[0, (1 - \tau) (R_t - z_t^j) k_t^j - ((1 - \tau) c + \lambda) \frac{b_t^j}{\mu_t} + V(k_t^j, b_t^j, \mu_t) \right] \quad (6)$$

where the continuation value $V(\cdot)$ obeys the following Bellman equation:

$$V(k_t^j, b_t^j, \mu_t) = \max_{b_{t+1}^j, k_{t+1}^j} \left\{ p_t^j \left(b_{t+1}^j - (1 - \lambda) \frac{b_t^j}{\mu_t} \right) - (i_t^j - \tau \delta) k_t^j + E_t M_{t,t+1} \int_{\underline{z}}^{\bar{z}} E(k_{t+1}^j, b_{t+1}^j, z_{t+1}^j, \mu_{t+1}) d\Phi(z_{t+1}) \right\} \quad (7)$$

and summarizes the effects of the decisions about future investment and financing on equity values.

Several observations about the value of equity (6) will be useful later. First, limited liability implies that equity value, $E(\cdot)$, is bounded and will never fall below zero. This implies that equity holders will default on their credit obligations whenever their idiosyncratic profit shock z_t^j is above a cutoff level $z_t^* \leq \bar{z}$, defined by the expression:

$$(1 - \tau) (R_t - z_t^{j*}) k_t^j - ((1 - \tau) c + \lambda) \frac{b_t^j}{\mu_t} + V(k_t^j, b_t^j, \mu_t) = 0 \quad (8)$$

It is this value z_{t+1}^{j*} that truncates the integral in the continuation value of (7).

Second, the stochastic discount factor $M_{t,t+1}$ is exogenous to the firm and must be determined in equilibrium, in a manner consistent with the behavior of households/investors. Third, the value function is homogenous of degree one in capital k_t^j and debt b_t^j and so is the default cutoff z_t^{j*} .⁶

Finally, the equity value $E(\cdot)$ is decreasing in the value of outstanding real

⁶We will see below that the equilibrium prices p and M are homogenous of degree zero in these variables.

debt obligations b_t^j/μ_t . In particular:

$$\begin{aligned}\frac{\partial E(\cdot)}{\partial b_t^j} &= -((1-\tau)c + \lambda) \frac{1}{\mu_t} + \frac{\partial V(\cdot)}{\partial b_t^j} \\ &= -((1-\tau)c + \lambda + p_t^j(1-\lambda)) \frac{1}{\mu_t} \leq 0\end{aligned}$$

Intuitively, higher debt reduces both the value of the current distribution to equity holders and, with $\lambda < 1$, the continuation value, $V(\cdot)$.

2.1.4 Optimal Policies and the Debt Overhang

Given the expression for the value function of the owners of the firm we can describe the optimal decisions regarding investment and borrowing by the following first order conditions:

$$1 = E_t M_{t,t+1} \frac{\partial E(k_{t+1}^j b_{t+1}^j, z_{t+1}^j, \mu_{t+1})}{\partial k_{t+1}^j} \quad (9)$$

$$\frac{\partial p_t^j}{\partial b_{t+1}^j} \left(b_{t+1}^j - (1-\lambda) \frac{b_t^j}{\mu_t} \right) + p_t^j = -E_t M_{t,t+1} \frac{\partial E(k_{t+1}^j b_{t+1}^j, z_{t+1}^j, \mu_{t+1})}{\partial b_{t+1}^j} \quad (10)$$

We assume throughout the paper that first order conditions are sufficient for an optimum. For the case with one period debt, $\lambda = 1$, it is straightforward to derive conditions on the distribution for the idiosyncratic shock $\Phi(\cdot)$ to guarantee that this is true.

The condition for optimal investment is quite standard. It equates the required reduction in current equity distributions, associated with a marginal increase in next period's stock of capital, k_{t+1} , with the discounted expected future marginal increase in equity value.

The equation for optimal debt, b_{t+1} , is more novel and is at the heart of our results. Simply stated, it compares the marginal benefit of issuing new debt,

on the left hand side, with the expected reduction in future equity values on the right.

Crucially, the expression for the marginal benefit in equation (10) recognizes that the debt price, p , will change whenever new debt is issued. This is because an increase in the amount of debt outstanding will likely increase the probability of default. In turn, the magnitude of this effect depends on the (real) value of currently outstanding liabilities, $(1 - \lambda)b_t^j/\mu_t$, so that it is in effect cheaper to issue new debt when the existing stock is already high. It is this feature that generates persistence (stickyness) in debt and our overhang results.

2.1.5 Default and Credit Risk

We now turn to the problem facing the firm's creditors. These agents buy corporate liabilities, at price p_t^j , and collect regular coupon and principal payments, $(c + \lambda) \frac{b_{t+1}^j}{\mu_{t+1}}$, until the firm defaults.

In default, shareholders walk away from the firm, while creditors take over and restructure the firm. Creditors become the sole owners and investors of the firm and are entitled to collect the current after-tax operating income $(1 - \tau)(R_{t+1} - z_{t+1}^j)k_{t+1}^j$. After this restructuring, creditors resume their customary role by selling off the equity portion to new owners while continuing to hold the remaining debt. This means that in addition to the current cash flows, the creditors have a claim that equals the total enterprise, or asset, value, $V(k_{t+1}^j, b_{t+1}^j) + p_{t+1}^j(1 - \lambda)b_{t+1}^j$.⁷

Restructuring however is not entirely costless and there is a separate loss,

⁷This is only one of several equivalent ways of describing the bankruptcy procedures that yields the same payoffs for shareholders and creditors upon default. Equivalently we could assume that they sell debt and continue to run the firm as the new equity holders.

in the amount ξk_{t+1}^j , with $\xi \in [0, 1]$, that must be born.⁸

With these assumptions, the creditor's valuation of their holdings of corporate debt at the end of period t is:⁹

$$b_{t+1}^j p_t^j = E_t M_{t,t+1} \left\{ \begin{aligned} & \Phi(z_{t+1}^{j*}) [c + \lambda + (1 - \lambda) p_{t+1}^j] \frac{b_{t+1}^j}{\mu_{t+1}} + \int_{z_{t+1}^{j*}}^{\bar{z}} [(1 - \tau) (R_{t+1} - z_{t+1}^j) k_{t+1}^j] \\ & + V(k_{t+1}^j, b_{t+1}^j, \mu_{t+1}) + (1 - \lambda) \frac{p_{t+1}^j b_{t+1}^j}{\mu_{t+1}} - \xi k_{t+1}^j \end{aligned} \right] d\Phi(z_{t+1}) \quad (11)$$

The right hand side of this expression can be divided in two parts. The first term reflects the cash flows received if no default takes place, while the integral contains the payments in default, net of the restructuring charges.

It is immediate to establish that this market value of corporate debt is decreasing in restructuring losses, ξ , and the default probability, implied by the cutoff z^{j*} . It can also be shown that bond prices are declining in the expected rate of inflation - since equity values increase in μ_{t+1} . Finally, note that p_t^j is also homogeneous of degree zero in k_{t+1}^j and b_{t+1}^j .

All together, our assumptions ensure that when the restructuring process is complete a defaulting firm is indistinguishable from a non-defaulting firm. All losses take place in the current period and are absorbed by the creditors. Since all idiosyncratic shocks are i.i.d. and there are no adjustment costs, default has no further consequences. As a result, both defaulting and non-defaulting firms adopt the same optimal policies and look identical at the beginning of the next period.

⁸We can think of these costs as including legal fees, but also other efficiency losses and frictions associated with the bankruptcy and restructuring processes. These costs represent a collective loss for bond and equity holders, and may also imply a loss of resources for the economy as a whole.

⁹Note that creditors discount the future using the same discount factor as shareholders, $M_{t,t+1}$. This is consistent with our assumption that they belong to the same risk-sharing household.

2.2 Households

To complete our general equilibrium model we now need to describe the household sector. This is made of a single representative family that owns all securities and collects all income in the economy, including a rebate on corporate income tax revenues. Preferences are time-separable over consumption C and hours worked, N :

$$U = E \left\{ \sum_{t=0}^{\infty} \beta^t \frac{[u(C_t, N_t)]^{1-\sigma} - 1}{1-\sigma} \right\} \quad (12)$$

where the parameters $\beta \in (0, 1)$ and $\sigma > 0$ are tied to the rate of inter temporal preference and household risk aversion. We further assume that momentary utility is described by the Cobb-Douglas function:

$$u(C_t, N_t) = C_t^{1-\theta} (3 - N_t)^\theta \quad (13)$$

where the value of θ will be linked to the elasticity of labor supply.

As is common in the literature, we find it useful to assume that each member of the family works or invests independently in equities and bonds, and all household income is then shared when making consumption and savings decisions.

2.3 Equilibrium and Aggregation

Given the optimal decisions of firms and households implied by the problems above we can now characterize the general dynamic competitive equilibrium in this economy. As stated above, the nature of the problem means that, outside default, this equilibrium is symmetric, in the sense that all firms make

identical decisions at all times. The only meaningful cross-sectional difference concerns the realization of the shocks z_t^j which induce default for a subgroup of firms with mass $1 - \Phi(z^*)$. Default implies a one-time restructuring charges for firms, but these temporary losses have no further impact on the choices concerning future capital and debt. Thus all firms remain ex-ante identical in all periods. This means that we can drop all subscripts j for firm specific variables.

It follows that aggregate output in the economy, Y_t , can be expressed as:

$$Y_t = y_t - [1 - \Phi(z^*)] \xi^r \xi k_t \quad (14)$$

As discussed above, ξk_t captures the loss that creditors suffer in bankruptcy. Some of these losses may be in the form of legal fees and might be recouped by to other members of the representative family. But some may represent a genuine destruction of resources. The relative balance between these two alternatives is governed by the parameter $\xi^r \in [0, 1]$. In the special case where $\xi^r = 0$, default entails no loss of resources at the aggregate level.

Since all firms make identical choices, the aggregate capital stock is equal to $K_t = k_t$ and its law of motion is simply:

$$K_{t+1} = (1 - \delta) K_t + I_t \quad (15)$$

where again aggregate investment is simply $I_t = i_t k_t$.

To complete the description of the economy we require that both goods and labor market clear. This is accomplished by imposing the aggregate resource constraint:

$$Y_t = C_t + I_t \quad (16)$$

and the labor market consistency condition:

$$N_t = n_t \tag{17}$$

3 Characterization

Our model implies that *any* movements in the price level, or the inflation rate, μ_t , will have real effects on the economy. In this sense, changes in the inflation rate work to propagate and possibly amplify any underlying shocks to the economy, *regardless* of their origin. This section seeks to isolate and understand this transmission by showing how optimal nominal debt and real investment respond to exogenous changes in the inflation rate under very general conditions.

The quantitative importance of these effects and their impact on the rest of the economy are examined in the next section, which also endogenizes the movements in the inflation rate.

3.1 Normalized Equity and Debt Values

The constant returns to scale nature of the technology and costs allows us to rewrite the entire model in terms of ratios to the capital stock. Specifically we can rewrite the expression for the value of equity (6) as:

$$E/k = e(\omega, z, \mu) = \max \left\{ 0, (1 - \tau)(R - z) - ((1 - \tau)c + \lambda) \frac{\omega}{\mu} + v(\omega, \mu) \right\} \tag{18}$$

where $\omega = b/k$ is a measure of the leverage ratio. Similarly, we scale equation (7) to express the continuation function, $v(\cdot) = V/k$, as:

$$v(\omega, \mu) = \max_{\omega', i} \left\{ p \left(\omega' g(i) - (1 - \lambda) \frac{\omega}{\mu} \right) - i + \tau \delta + g(i) EM' \int_{\underline{z}}^{z^{*'}} [(1 - \tau)(R' - z')] - ((1 - \tau)c + \lambda) \frac{\omega'}{\mu'} + v(\omega', \mu') \right\} d\Phi(z') \quad (19)$$

where we use primes to denote future values, and the definition $g(i) = (1 - \delta + i)k$.

The market value of the outstanding debt (11) can be expressed as:

$$\omega' p = EM' \left\{ \begin{aligned} & \Phi(z^{*'}) [c + \lambda] \frac{\omega'}{\mu'} + (1 - \lambda) \frac{p' \omega'}{\mu'} \\ & + (1 - \Phi(z^{*'})) [(1 - \tau)R' - \xi + v(\omega', \mu')] - (1 - \tau) \int_{z^{*'}}^{\bar{z}} z' d\Phi(z) \end{aligned} \right\} \quad (20)$$

3.2 Optimal Default

The response of the default rate is an important ingredient in our results below. To understand its behavior we can look at the response of the optimal default cutoff level, z^* , implied by the limited liability condition on equity holders. Expressed as a function of the leverage ratio, ω , the optimal default threshold is

$$z^* = R - c \frac{\omega}{\mu} - \frac{\lambda}{(1 - \tau)} \frac{\omega}{\mu} + \frac{1}{(1 - \tau)} v(\omega, \mu) \quad (21)$$

Differentiating this expression with respect to outstanding leverage ω we get:

$$\frac{\partial z^*}{\partial \omega} = - \left(c + \frac{\lambda}{1 - \tau} \right) \frac{1}{\mu} + \frac{1}{(1 - \tau)} \frac{\partial v(\omega, \mu)}{\partial \omega} < 0 \quad (22)$$

Intuitively, an increase in outstanding debt increases the required principal and coupon payments, and by reducing the cut-off z^* , makes default more likely.

In addition, the envelope condition implies that:

$$\frac{\partial v(\omega, \mu)}{\partial \omega} = -p \frac{1 - \lambda}{\mu} \leq 0 \quad (23)$$

so that when debt maturity exceeds one period ($\lambda < 1$) an increase in outstanding debt also decreases the (expected) future payments to equity holders and thus the value of equity itself, again encouraging default.

3.3 Debt Overhang and the Impact of Inflation

To understand the impact of changes in the inflation rate it is useful to examine the behavior of firm leverage and investment to these shocks.

The optimal leverage ratio follows from the first order condition for the normalized value function (19):

$$pg(i) + \frac{\partial p}{\partial \omega'} \left(\omega' g(i) - (1 - \lambda) \frac{\omega}{\mu} \right) = - (1 - \tau) g(i) EM' \Phi(z^{*'}) \frac{\partial z^{*'}}{\partial \omega} \quad (24)$$

Similarly the optimal investment policy obeys:

$$\begin{aligned} 1 &= p\omega' + EM' \int_{\underline{z}}^{z^{*'}} \left[(1 - \tau) (R' - z') - ((1 - \tau) c + \lambda) \frac{\omega'}{\mu'} + v(\omega', \mu') \right] d\Phi(z') \\ &= p\omega' + EM' \int_{\underline{z}}^{z^{*'}} (1 - \tau) (z^{*'} - z') d\Phi(z') \end{aligned} \quad (25)$$

These conditions are simply more informative versions of the abstract expressions (9) and (10).

To help us isolate the effects of *any* changes in the rate of inflation it is useful to start by assuming that μ_t is exogenous and follows a simple process with no persistence. The next section considers the more realistic case when the inflation rate changes endogenously as a result of real and monetary shocks.

First we show that the effect of unanticipated inflation on leverage depends crucially on the maturity of debt. Proposition 1 establishes this result under general conditions.

Proposition 1. *Consider an economy where optimal choices are described by the optimality conditions (24)-(25), and where:*

- *there are no aggregate resource costs associated with bankruptcy, i.e., $\xi^r = 0$; and*
- *all realizations of exogenous shocks have been zero for a long time so that at time $t - 1$ $\mu_{t-1} = \mu$ and $\omega_t = \omega$*

Suppose that at time t the economy experiences a temporary decline in the inflation rate so that $\mu_t < \mu_{t-1}$. Then $\omega_{t+1} \geq \omega_t$, with equality if and only if $\lambda = 1$.

Proof. With $\lambda = 1$ the current inflation rate, μ_t , has no direct effect on the choice of $\omega' = \omega_{t+1}$ in (24). Moreover, since

- μ_t is i.i.d., there is no effect on the debt pricing equation (20) and the equilibrium price of debt, p ;
- $\xi^r = 0$, there are no resource costs and neither aggregate consumption, C , nor the stochastic discount factor, M' , are affected;

It follows that there are no indirect general equilibrium effects either, and the optimal choice of leverage, ω_{t+1} is unaffected by the shock.

However, when $\lambda < 1$ a decline in μ raises the marginal benefit of issuing new debt (since $\frac{\partial p}{\partial \omega'} \leq 0$) and ω' will increase accordingly. ■

Proposition 1 shows that temporary movements in inflation will be propagated over time as long as debt maturity is not instantaneous. This is because a change in the existing leverage has a direct impact on the marginal cost of issuing new debt. An unanticipated decline in the rate of inflation μ_t increases the (real) value of currently outstanding liabilities, $(1 - \lambda)b_t^j/\mu_t$, so that it is in effect relatively cheaper to issue new debt.¹⁰ Economically, with more outstanding debt, shareholders can shift more of the burden of the default costs onto the existing debtholders. This encourages the firm to maintain a persistently elevated level of ω - so that nominal debt is effectively “sticky”. In other words, nominal debt does not move to fully offset the shock to inflation.

In a sense, this response of leverage reflects the fact that debt prices act like an (endogenous) convex adjustment cost that slows down the response to shocks. As shown in equation (25), the term representing the overall *marginal* price effect, $\frac{\partial p}{\partial \omega'} \left(\omega' g(i) - (1 - \lambda) \frac{\omega}{\mu} \right)$, is increasing with debt issued $\left(\omega' g(i) - (1 - \lambda) \frac{\omega}{\mu} \right)$. This also means that leverage is a state variable not just due to its obvious impact on equity values, but also because it directly affects future leverage.

The extreme assumptions in Proposition 1 also highlight how persistent movements in leverage can occur even when $\lambda = 1$. This occurs if either the underlying inflation movements are persistent or if there are some resource costs associated with default ($\xi^r \neq 0$).

We now turn to discuss the effects of inflation on real investment decisions. Note however that the first order condition (25) only pins down the required equilibrium value of the rental rate, R' and not the optimal level of the capital stock. The constant returns scale nature of the problem implies that investment is determined only in general equilibrium.

¹⁰Alternatively, it is more expensive to retire the outstanding debt.

Proposition 2 shows that the equilibrium required return on capital must rise following a decline in inflation.

Proposition 2. *Consider the economy of Proposition 1. Suppose that at time t the economy experiences a temporary decline in the inflation rate so that $\mu_t < \mu_{t-1}$. Then $R_{t+1} \leq R_t$, with equality if and only if $\lambda = 1$.*

Proof. See Appendix ■

Proposition 2 shows that the effect of a decline in inflation is to push up the required rate of return on capital, a result essentially identical to that of a tightening in financing constraints. This is in effect a type of debt overhang, by which firm's investment is adversely impacted when corporate debt rises.¹¹

These two propositions establish the key real effects of the movements on the inflation rate in our model. They show that under all but the most extreme set of assumptions about debt maturities and restructuring costs, changes in inflation generate long-lived movements in corporate leverage which in turn change firm investment and, in general equilibrium, aggregate consumption and welfare.

In the language of wedges, changes in the inflation rate distort consumption-investment allocations and, when default has aggregate costs, also impact aggregate productivity and output. This last effect is present even if inflation is i.i.d. and $\lambda = 1$, as long as $\xi^r > 0$. In a general model it adds both persistence and amplification to inflation shocks.

¹¹Again, because of constant returns to scale we cannot establish a direct effect on capital accumulation. However it is clear that this increase in the required rental rate requires a decline in the capital-labor ratio. The net effect on the stock of capital can only be established in a quantitative model, or one where labor is essentially fixed.

4 Parameterization

The model is calibrated at quarterly frequency. While we choose parameters to match steady state targets whenever feasible, we use model simulations to pin down parameters that determine the stochastic properties of the model economy. Whenever possible, the steady state targets correspond to empirical moments computed over the 1955.*I* – 2012.*IV* period. In the first part of our quantitative analysis we will work with an exogenous inflation process and try to assess the dynamic effects and the relative importance of changes in inflation.

4.1 Inflation and Productivity Processes

We start by constructing estimates of the joint behavior of innovations to productivity and inflation. We begin by assuming the following general VAR(1) representation for the stationary component of productivity and inflation:

$$\begin{bmatrix} a_t \\ \pi_t \end{bmatrix} = \Gamma \begin{bmatrix} a_{t-1} \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^a \\ \varepsilon_t^\pi \end{bmatrix}.$$

where $a_t = \ln A_t$, $\pi_t = \ln \mu_t - \ln \bar{\mu}$, and $\bar{\mu}$ is the average (gross) rate of inflation during this period.¹² To do this, we first construct series for Solow residuals and inflation using data on GDP, hours, capital stock and the GDP deflator from the BEA and the BLS. Estimating this autoregressive system yields empirical measures of the standard deviations σ_a and σ_π of the productivity and inflation shocks, as well as their cross-correlation, $\rho_{\pi a}$.

¹²We normalize $\ln(\bar{A}) = 0$.

For our sample period, we find that:

$$\Gamma = \begin{bmatrix} 0.98 & -0.094 \\ 0.012 & 0.85 \end{bmatrix}$$

and $\sigma_a = 0.0074$ and $\sigma_\pi = 0.0045$, and $\rho_{\pi a} = -0.19$. We call this the VAR specification of our shocks. We also consider a more restrictive AR(1) specification where $\rho_{\pi a} = 0$ and $\Gamma_{12} = \Gamma_{21} = 0$. In this case, the diagonal elements of Γ are 0.97 and 0.85, respectively, with $\sigma_a = 0.007$ and $\sigma_\pi = 0.0040$. This version of the model allows us to examine the case where the exogenous inflation shocks have no real effects, other than those on firm leverage and offers more reliable variance decomposition results.

4.2 Idiosyncratic Profit Shocks

Instead of adopting a specific distribution for the p.d.f. $\phi(z)$ we use a general quadratic approximation of the form:

$$\phi(z) = \eta_1 + \eta_2 z + \eta_3 z^2 \tag{26}$$

The distribution is assumed symmetric with $\bar{z} = -\underline{z} = 1$. Our other assumptions about this distribution's mean imply that $\eta_2 = 0$, and η_3 is tied to our choice of the only free parameter, η_1 . The value for η_1 is picked to ensure that our model matches the unconditional volatility of the leverage ratio, a key variable in our model.

Together with the average leverage ratio, ω , and expected debt life, $1/\lambda$, the value for η_1 is an important determinant of the persistence of inflation shocks. Intuitively, this is because when the mass of firms accumulated around

the default, $\phi(z^*)$, is sizable, any shock will have a large impact on the default probability, $\Phi(z^*)$, and on bond prices, $p(\omega)$. This matters because the sensitivity of debt prices, $\partial p/\partial \omega$, effectively determines the magnitude of the implicit costs to adjusting the stock of debt in equation (24). Thus, when $\phi(z^*)$ - governed by the choice of η_1 - is large, debt will be more persistent and the effects on the real economy will last longer.

4.3 Technology and Preferences

Our choices for the capital share α , depreciation rate δ , and the subjective discount factor β correspond to fairly common values and pin down the capital-output, investment-output, and the average rate of return on capital in our economy. As long as they remain in a plausible range, the quantitative properties of the model are not very sensitive to these parameter values. The preference parameter θ is chosen so that in steady state working hours make up one third of the total time endowment, with σ set to deliver a plausible level of risk aversion.

4.4 Institutional Parameters

The parameter λ pins down average debt maturity. This is an important parameter for determining the propagation of inflation shocks. Our benchmark calibration implies an average maturity of about 4 years, and an actual duration of 3 years. These values are similar to initial maturities of industrial and commercial loans, but significantly shorter than those for corporate bonds. Given the importance of this parameter, we prefer to err on the save side and focus on the quantitative results when debt maturity is conservatively calibrated. We will document how the results change with alternative average

maturities.

The tax wedge τ is chosen so that average firm leverage matches the observed leverage ratio for the U.S. non-financial business sector. Average leverage is 0.42 in the period since 1955.¹³

Finally, the default loss parameter, ξ , is chosen to match the average default rate.¹⁴ We set the aggregate loss parameter $\xi^r = 1$, so that all restructuring charges involve a deadweight resource loss.

Table 1 summarizes our parameter choices for the benchmark calibration. The model is quite parsimonious and requires only 10 structural parameters, in addition to the stochastic process for the shocks. Table 2 shows the implications of these choices for the first and second (unconditional) moments of a number of key variables. As discussed above, our calibration ensures that the model matches the key financial variables associated with the optimal capital structure of the firm. These include, a leverage ratio of 42%, and a quarterly default rate of around 40 basis points. This choice of parameters also implies a plausible average credit spread of about 0.4% per quarter.

The second panel in Table 2 shows that our quantitative model shares many of the properties of other variations of the stochastic growth model. All the main aggregates, have plausible volatilities, except for labor.

¹³Alternatively, we could fix the tax parameter by attempting to match the (statutory) wedged implied by the interaction between corporate income rates and the tax treatment individual interest and equity income. Depending on the time period, this method would imply a value of τ around 25%.

¹⁴Because our preferences do not exhibit much risk aversion, matching empirical credit spreads also requires large default losses. We prefer not to introduce more complex specifications for the utility function $U(\cdot)$ because they could mask our key insights.

Table 1: **Calibration**

Parameter	Description	Value
β	Subjective Discount Factor	0.99
γ	Risk Aversion	1
θ	Elasticity of Labor	0.63
α	Capital Share	0.36
δ	Depreciation Rate	0.025
λ	Debt Amortization Rate	0.06
ξ^r	Fraction of Resource Cost	1
ξ	Default Loss	0.38
τ	Tax Wedge	0.40
η_1	Distribution Parameter	0.6617

5 Quantitative Analysis

We now investigate the quantitative importance of the frictions induced by nominal long-term debt. First, we are interested in determining how the model responds to inflation shocks and how much endogenous propagation can plausibly be generated by the combination of our sticky leverage and debt overhang mechanisms. Next, we show how the model can be modified so that the inflation rate is determined endogenously and investigate how a standard monetary policy rule can help stabilize the economy following different shocks.

5.1 Model with Exogenous Inflation

5.1.1 Impulse Responses

As we have seen above, our model predicts that under very general conditions, even exogenous i.i.d. movements in inflation, can induce prolonged movements in corporate leverage and investment, and, in equilibrium output

Table 2: **Aggregate Moments**

	Data	Model	Model
		AR(1)	VAR(1)
First Moments			
Investment/Output, I/Y	0.21	0.24	0.24
Leverage, ω	0.42	0.42	0.42
Default Rate, $1 - \Phi(z^*)$	0.42%	0.42%	0.42%
Credit Spread	0.39%	0.39%	0.39%
Second Moments			
σ_Y	1.66%	1.58%	1.67%
σ_I/σ_Y	4.12	4.22	4.48
σ_C/σ_Y	0.54	0.41	0.43
σ_N/σ_Y	1.07	0.49	0.54
σ_ω	1.7%	1.5%	1.7%

and consumption. Figure 1 shows how these responses would look like in a plausible quantitative version of the model, when inflation follows the exogenous AR(1) process specified above, which is assumed to be uncorrelated with productivity.

We can see that following lower than expected inflation, the default rate increases as the real value of outstanding corporate liabilities increases. This increase in the default rate immediately produces output losses since restructuring costs are not rebated to households and represent real deadweight losses.

As Proposition 1 implies, leverage - we report its market value, $p\omega'$ - rises and remains elevated for a long time even though inflation quickly returns to its long run mean. This persistence in leverage contributes to a prolonged, and significant contraction in investment spending - the debt overhang result.

Initially at least, there is an important change in the intertemporal allocation of resources, as consumption actually rises, reflecting the fact that the

required rental rate on capital has increased. Soon however, the lower capital stock further contributes to lowering output, and consumption declines as well. Labor initially mirrors the behavior of consumption, as households seek to smooth their leisure decisions as well. Over time, reduced capital contributes to lowering the marginal product of labor.¹⁵

Interestingly, Figure 1 also shows that the response of output and other aggregates is hump-shaped, a feature that is also present when we look at productivity shocks below. This propagation result is tied to the slow response of leverage to the shocks. As we have seen above, the multi period nature of debt implies an endogenous form of “adjustment costs”, the magnitude of which is tied to $\partial p / \partial \omega$. This sensitivity of debt prices depends in turn on how sensitive the default rate is to shocks, as measured by $\phi(z^*)$. The hump shaped response occurs only when the mass of firms near the default boundary is relatively large.

5.1.2 Variance Decompositions

The quantitative importance of fluctuations in the inflation rate are documented in Tables 3 and 4. Here we perform a variance decomposition of movements in the key macroeconomics aggregates in the baseline model driven by *independent* AR(1) shocks to inflation and productivity. This ensures that the measured contributions of the inflation shocks are entirely due to their effects on the endogenous variables. The Tables focus on the role of two key ingredients: the average leverage ratio, ω , and the expected life of the bond, $1/\lambda$.

Table 3 investigates the importance of the average leverage ratio to our

¹⁵With GHH preferences, consumption and labor do not move at all on impact and then immediately decline.

results. For the baseline case, where leverage matches the data for the period since 1955, inflation movements account for a little over 1/3 of the variation in output and hours and nearly 2/3 of that in consumption and investment. Even when the leverage ratio is only 32%, inflation still accounts for 35% of the movements in investment. When the leverage ratio matches the value observed in the period since 2005, which is 52%, inflation shocks dominate business cycle fluctuations and account for 60% of the variance of output.

Table 4 examines the role of debt maturity and highlights the importance of multi-period debt. With one period debt, real quantities are almost unaffected by movements in the inflation rate, and we essentially recover monetarily neutrality. There are two reasons for the much weaker real effects of inflation relative to the benchmark case. First, for a given inflation process, percentage gains and losses on bonds produced by inflation are smaller the shorter the maturity. Second, the debt overhang channel is absent with one-period debt.

Overall, these findings appear quite striking to us. After all this is a model with exogenous inflation shocks that by assumption have no direct impact on real quantities. However, despite the fact that the price level is entirely flexible, inflation has potentially very large real effects.

5.2 Model with Endogenous Inflation

We now consider the case of endogenous inflation changes. While there are several possible alternatives, we follow the popular practice of using a monetary policy rule of the form

$$\ln(r_t/\bar{r}) = \rho_r \ln(r_{t-1}/\bar{r}) + (1 - \rho_r)v_\mu(\ln(\mu_t/\bar{\mu}) + (1 - \rho_r)v_y(\ln(Y_t/\bar{Y})) + \zeta_t, \quad (27)$$

Table 3: **Variance Decomposition and Leverage**

	Y	Inv	Cons	Hrs	Lev	Default
Bench., $\bar{\omega} = 0.42$						
TFP shock a	0.63	0.37	0.39	0.60	0.10	0.05
Inflation shock μ	0.37	0.63	0.61	0.40	0.90	0.95
Low Lev, $\bar{\omega} = 0.32$						
TFP shock a	0.84	0.65	0.83	0.89	0.10	0.01
Inflation shock μ	0.16	0.35	0.17	0.11	0.90	0.99
High Lev, $\bar{\omega} = 0.52$						
TFP shock a	0.40	0.21	0.29	0.55	0.03	0.03
Inflation shock μ	0.60	0.79	0.71	0.45	0.97	0.97

where ζ_t is an exogenous monetary policy shock and r_t is the nominal one period interest rate which satisfies the Euler equation:

$$r_t = E \frac{1}{M_{t,t+1} \mu_{t+1}}, \quad (28)$$

with the bars denoting the steady-state values of the relevant variables¹⁶. We follow the literature and set the monetary policy responses $\rho_y = 0.5$ and $\rho_\mu = 1.5$. The smoothing parameter is $\rho_r = 0.6$.

To summarize, in this version of the model, inflation is driven either by endogenous monetary policy responses to movements in output, or by exogenous shocks to monetary policy itself. We investigate these two possibilities in turn.

¹⁶Another common alternative is to assume a money demand equation of the form $M_t/P_t = L(Y_t, r_t)$, arising from a cash in advance constraint or money in the utility function and where money growth, ΔM_t , is controlled by the monetary authority and possibly stochastic.

Table 4: **Variance Decomposition and Maturity**

	Y	Inv	Cons	Hrs	Lev	Default
Bench., $\lambda = 0.06$						
TFP shock a	0.63	0.37	0.39	0.60	0.10	0.05
Inflation shock μ	0.37	0.63	0.61	0.40	0.90	0.95
Long Maturity, $\lambda = 0.03$						
TFP shock a	0.45	0.26	0.65	0.80	0.83	0.01
Inflation shock μ	0.55	0.74	0.35	0.20	0.17	0.99
One-period debt, $\lambda = 1$						
TFP shock a	1	1	1	1	0.01	0.02
Inflation shock μ	0.00	0.00	0.00	0.00	0.99	0.98

5.2.1 Monetary Policy and Endogenous Inflation

Figure 2 shows how the exogenous behavior of the inflation rate in Figure 1 can be thought as an endogenous response to discretionary changes in monetary policy. Specifically, we now consider the effects of an exogenous shock ζ_t to the policy rule. The shock is set so as to produce an inflation response in the second panel that closely resembles the exogenous inflation shock considered above.¹⁷

As can be seen from this figure, the responses of the key variables are similar regardless of whether the inflation movements are exogenous or induced by the monetary policy rule. What matters for the response of the real economy is the actual behavior of the inflation rate itself.

¹⁷Specifically, this is accomplished by assuming that ζ_t follows an AR(1) with a persistence parameter of 0.99 and setting the interest smoothing parameter $\rho_r = 0.3$.

5.2.2 Productivity Shocks

With endogenous monetary policy, the inflation rate will also change when the economy is hit by real shocks. Put another way, our monetary policy rule changes the response of the real economy to the underlying shocks by generating more or less inflation.

Figure 3 documents the impact of a shock to (the logarithm of) total factor productivity, a_t , with and without a monetary policy response. The green lines show the baseline responses to the productivity shock without the monetary policy rule, so that inflation is unaffected by the shock. In this case, we observe the patterns common to other quantitative equilibrium models, with the notable exception that our setting generates much more persistence. In particular, output now displays a clear hump shaped response to the shock.

This is due to the amplification induced by our financial friction and the debt overhang behavior. With long-lived debt, firms are discouraged from investing and accumulate less capital. This further increases leverage and generates a prolonged decline.

The blue lines represent the responses when the nominal interest rate follows the monetary policy rule (27). Here, output, investment, labor, and to some extent consumption, move much less. This is because the monetary policy rule increases the inflation rate and reduces the real burden of outstanding debt. As discussed above (in reverse), this increase in inflation then lowers corporate defaults, and positively impacts investment and the other real vari-

ables.¹⁸

This experiment emphasizes that our debt overhang result is entirely driven by the long-lived nature of our debt contracts and does not rely on nominal frictions. However, when debt is nominal, inflation significantly helps to eliminate this overhang problem.

5.2.3 Wealth Shocks

Figure 4 examines the case of a shock to the stock of capital in the economy. This experiment can be seen as capturing some aspects of the contraction seen since 2007/08, with sharply declining real estate values.

We think of this as a rare one off event. Formally, this is implemented by an unexpected decrease in the value of the capital stock k of 5% through a one time increase in the depreciation rate, δ . On impact, this destruction of the capital stock lowers both overall firm and equity values. This leads to an immediate spike in corporate defaults, and an increase in the leverage ratio.

When the inflation rate remains constant (green lines), persistence in leverage and debt overhang produce long lasting real declines in investment and output. However, when monetary policy responds according to the rule (27) (blue lines), the inflation rate increases immediately. This reduces the burden of outstanding liabilities. The effects of this shock on the real aggregates are significantly mitigated, and the economy recovers a lot faster. With a 5% reduction in the capital stock, the model's implied inflation increases by a total of 3.6% above its steady state level over the first year after the shock.

¹⁸Interestingly, the lower output in the policy rule, everything else equal, would lead to a lower interest rate. However, in this model, the nominal interest rate actually increases because inflation increases too. See Cochrane (2011) for a detailed analysis of how inflation is determined by Taylor rules. In particular, the unique stable solution of the difference equation for inflation implied by the Taylor rule typically produces opposing movements in current inflation and output.

A priori, this policy is consistent with a popular policy prescription from several macroeconomists in the immediate aftermath of the crisis and summarized in the following quote:

“I’m advocating 6 percent inflation for at least a couple of years,” says Rogoff, 56, who’s now a professor at Harvard University. “It would ameliorate the debt bomb and help us work through the deleveraging process.” (Bloomberg, May 19, 2009).

6 Conclusion

In this paper we have presented a general equilibrium model with nominal long-term debt that can help us better understand the monetary non-neutralities associated with Irving Fisher’s (1933) debt deflation. The model also sheds some light on the ongoing financial crisis and possible monetary policy responses. Unlike other popular models of monetary non-neutralities, we eschewed price rigidities. Yet our model is capable of generating very large and persistent movements in output and investment.

Almost unavoidably, our attempt to write a parsimonious and tractable model leaves out many important features. In particular, we ignore nominal debt contracts other than those held by firms, even though household debt is roughly equal in magnitude and subject to similarly large restructuring costs.

Our analysis also abstracts from the role of movements in credit risk premia and the behavior of asset prices in general. In addition, while convenient, the assumption of constant returns to scale, which nearly eliminates firm heterogeneity and renders the model so tractable, also limits our ability to study firm behavior more comprehensively.

These and other simplifying assumptions can and should be further ex-

plored in future work. Nevertheless we believe none is essential to the main ideas in the paper.

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7 Appendix

This appendix describes the steps necessary for the proof of proposition 2.

First we characterize the total value of the outstanding claims on the firm,
 $v(\omega, \mu) + p(1 - \lambda) \frac{\omega}{\mu}$.

Lemma The total or enterprise value of the *firm* is given by

$$q = v + (1 - \lambda) \frac{p\omega}{\mu} = 1 - \delta(1 - \tau) \quad (29)$$

Proof. Use bond pricing equation (20) to replace $p\omega'$ in the expression for equity value (19) and rearrange to obtain:

$$v(\omega, \mu) + p(1 - \lambda) \frac{\omega}{\mu} = \max_{\omega', i} \left\{ \begin{aligned} &g(i)EM' \left[\int_{\underline{z}}^{z^{*'}} \frac{(c+\lambda)\omega'}{\mu'} d\Phi(z') + (1 - \lambda) \frac{p'\omega'}{\mu'} + \int_{z^{*'}}^{\bar{z}} [(1 - \tau)(R' - z') \right. \\ &+ v(\omega', \mu') - \xi] d\Phi(z') \Big] + g(i)EM' \int_{\underline{z}}^{z^{*'}} [(1 - \tau)(R' - z') + v(\omega', \mu') \\ &\quad \left. - ((1 - \tau)c + \lambda) \frac{\omega'}{\mu'} \right] d\Phi(z') + \tau\delta - i \end{aligned} \right.$$

The result then follows by imposing the optimality condition (25) and the assumptions that:

$$\begin{aligned} \int_{\underline{z}}^{\bar{z}} z' d\Phi(z') &= 0 \\ \int_{\underline{z}}^{\bar{z}} d\Phi(z') &= 1 \end{aligned}$$

■

Next we construct an alternative expression for the optimal choice of leverage, ω'

Lemma The optimal leverage decision (24) obeys:

$$g(i)EM' \left\{ \Phi(z^{*'}) \frac{\tau c}{\mu'} + \frac{\partial z^{*'}}{\partial \omega'} \phi(z^{*'}) \left(\tau c \frac{\omega'}{\mu'} + \xi \right) \right\} = \frac{\partial p}{\partial \omega'} (1 - \lambda) \frac{\omega}{\mu} \quad (30)$$

Proof. Take derivatives of the debt pricing equation (20) to get:

$$p + \omega' \frac{\partial p}{\partial \omega'} = E_t M' \left\{ \begin{aligned} & \Phi(z^{*'}) (c + \lambda) \frac{1}{\mu'} + (1 - \lambda) \frac{p'}{\mu'} + (1 - \Phi(z^{*'})) \frac{\partial v'}{\partial \omega'} \\ & + \frac{\partial z^{*'}}{\partial \omega'} \phi(z^{*'}) \left((c + \lambda) \frac{\omega'}{\mu'} - (1 - \tau) R' + \xi - v' + (1 - \tau) z^{*'} \right) \end{aligned} \right\}$$

Use the expressions for z^* and $\frac{\partial z^{*'}}{\partial \omega'}$ to get:

$$\begin{aligned} p + \omega' \frac{\partial p}{\partial \omega'} &= E_t M' \left\{ \begin{aligned} & \Phi(z^{*'}) (c + \lambda + (1 - \lambda) p') \frac{1}{\mu'} \\ & + \frac{\partial z^{*'}}{\partial \omega'} \phi(z^{*'}) \left((c + \lambda) \frac{\omega'}{\mu'} + \xi - ((1 - \tau) c + \lambda) \frac{\omega'}{\mu'} \right) \end{aligned} \right\} \\ &= EM' \left\{ \Phi(z^{*'}) \frac{\tau c}{\mu'} - (1 - \tau) \Phi(z^{*'}) \frac{\partial z^{*'}}{\partial \omega'} + \frac{\partial z^{*'}}{\partial \omega'} \phi(z^{*'}) \left(\tau c \frac{\omega'}{\mu'} + \xi \right) \right\} \end{aligned}$$

Equation (30) follows from replacing this expression in (24) ■

Finally we rewrite the equilibrium rental rate on capital, R and establish Proposition 2.

Lemma The equilibrium rental rate, R obeys the Euler equation:

$$1 = EM' \left[(1 - \tau) R' + 1 - \delta(1 - \tau) - (1 - \Phi(z^{*'})) \xi + \Phi(z^{*'}) \frac{c \tau \omega'}{\mu'} \right] \quad (31)$$

Proof. This follows directly from replacing the market value of debt (20) in the optimality condition for investment (25) and using the definition of the market value of the firm (29). ■

Proof of Proposition 2 Taking derivatives of (31) and using (30) yields:

$$\begin{aligned}
0 &= EM' \frac{\partial \omega'}{\partial \mu} \left\{ (1 - \tau) \frac{\partial R'}{\partial \omega'} \right\} + EM' \frac{\partial \omega'}{\partial \mu} \left\{ \frac{\partial z^{*'}}{\partial \omega'} \phi(z^{*'}) \left(\tau c \frac{\omega'}{\mu'} + \xi \right) + \Phi(z^{*'}) \frac{\tau c}{\mu'} \right\} \\
&= EM' \frac{\partial \omega'}{\partial \mu} \left\{ (1 - \tau) \frac{\partial R'}{\partial \omega'} + \frac{\partial p}{\partial \omega'} \frac{(1 - \lambda) \omega}{g(i) \mu} \right\}
\end{aligned}$$

Since $\frac{\partial p}{\partial \omega'} < 0$ it follows that $\frac{\partial R'}{\partial \omega'} > 0$

Figure 1: An exogenous inflation shock. This figure shows the effect of an unanticipated decline in the inflation rate μ_t on the key variables of the model.

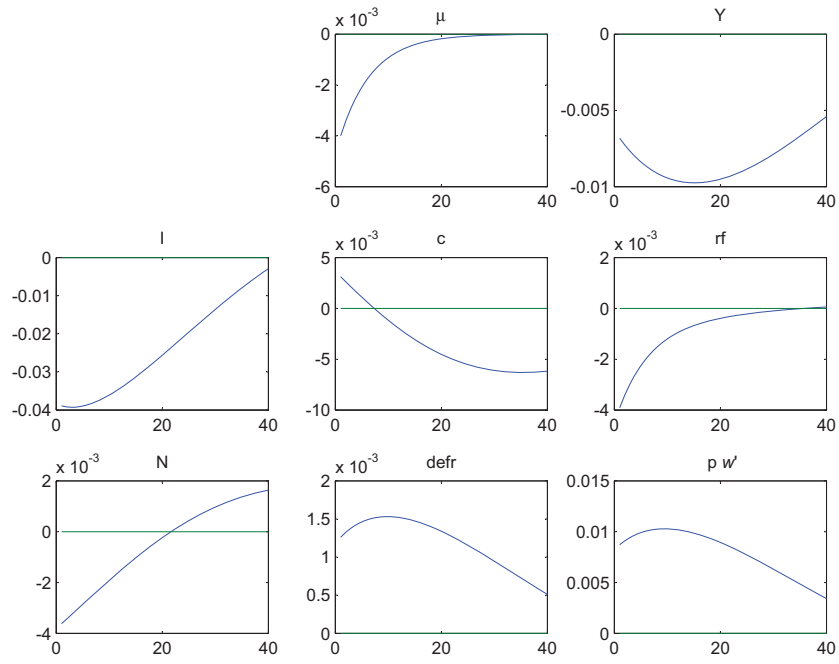


Figure 2: An endogenous inflation shock. This figure shows the effect of an exogenous shock to the monetary policy rule on the key variables of the model. The blue line illustrates the response to an exogenous change in monetary policy, ζ , while the green line shows the response in the baseline case with exogenous inflation.

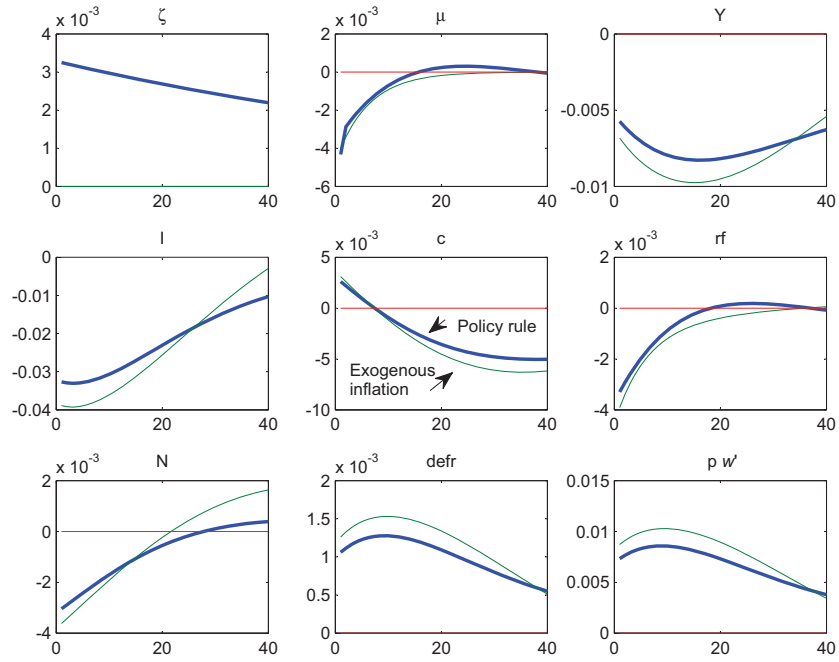


Figure 3: A productivity shock. This figure shows the effect of an exogenous shock to productivity, A . It compares the effects when inflation is exogenous (green line) and when it adjusts endogenous as a consequence of a monetary policy rule (blue line).

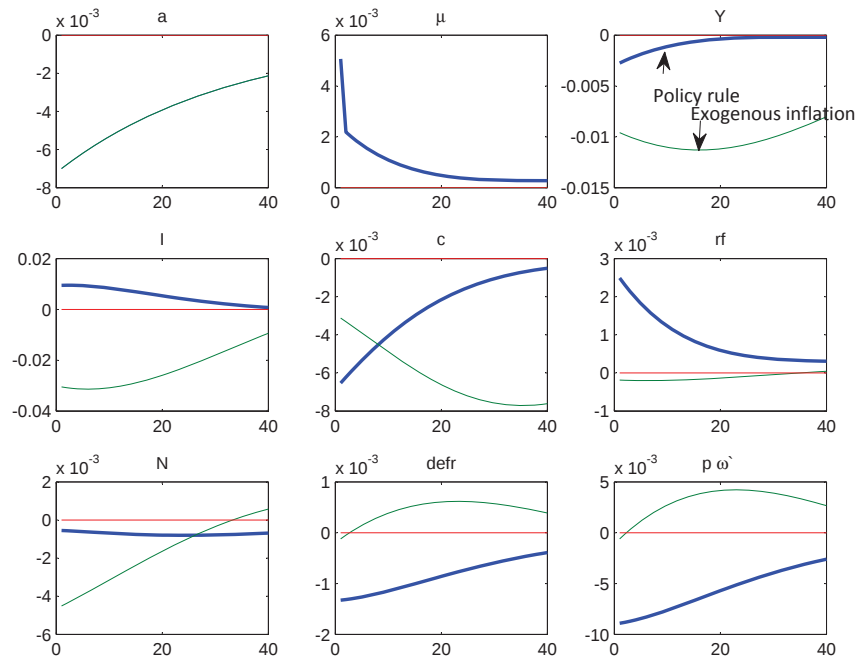


Figure 4: A wealth shock. This figure shows the effect of an exogenous destruction of the capital stock, k , through a temporary increase in the depreciation rate, δ . It compares the effects when inflation is exogenous (green line) and when it adjusts endogenous as a consequence of a monetary policy rule (blue line).

