

Central Bank Purchases of Private Assets

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September 29, 2013

Abstract

A model is constructed in which consumers and banks have incentives to fake the quality of collateral. Conventional central banking policy can exacerbate these problems, in that lower nominal interest rates make asset prices higher, which makes faking collateral more profitable, thus increasing haircuts and interest rate differentials. Central bank purchases of private mortgages can increase welfare by bypassing incentive problems associated with private banks, increasing asset prices, and relaxing collateral constraints. However, this may exacerbate incentive problems in the mortgage market.

1 Introduction

During and since the global financial crisis, central banks have engaged in unconventional purchases of long-maturity assets, on a very large scale, particularly in the United States. For the Fed, such purchases, often referred to as quantitative easing (QE), have included purchases of long-maturity government debt in exchange for reserves, swaps of short-maturity government debt for long-maturity government debt, and purchases of mortgage-backed securities and agency securities. The focus of this paper is on the effects of the latter types of purchases. While asset purchases by the central bank involving only government debt act to change the composition of the outstanding consolidated government debt, central bank purchases of what are essentially private assets puts monetary policy in potentially different territory.

In purchasing private assets, the central bank needs to be concerned with the quality of the assets it purchases, and with the incentive effects of central bank actions. The fact that the central bank is a willing buyer of private assets may make it the victim of sellers of low-quality assets, and the changes in asset prices brought on by central bank actions may create incentives to cheat in the private sector. Further, if the central bank engages in private asset purchases, it needs to understand the relationship between its conventional monetary policy actions and its unconventional ones.

In this paper, we construct a model of asset exchange and monetary policy, in which economic agents have an incentive to cheat on the quality of collateral. The basic structure comes from Lagos and Wright (2005) and Rocheteau and Wright (2005), and some details of the model are closely-related to models constructed in Williamson (2012, 2013), particularly in terms of the structure of financial intermediation and the relationship between fiscal and monetary policy.

In the model, the basic assets are currency, reserves, government debt, and housing. Assets are necessary for exchange to take place. Indeed, housing is a private asset which can be useful in exchange, but in the model it is more efficient for consumers to own houses and take out mortgages, which are held by financial intermediaries. Consumers then use financial intermediary liabilities and currency in decentralized exchange. An efficient financial intermediation arrangement, in the spirit of Diamond-Dybvig (1983) (and as in Williamson 2012, 2013), is a type of insurance arrangement, and banks act to efficiently allocate liquid assets in exchange.

Limited commitment in the model requires that private debt be secured. Consumers secure mortgage debt with housing, and banks need to secure deposit liabilities with mortgage loans, government debt, and reserves. But, a key element in the model is that consumers, at a cost, can fake the quality of houses posted as collateral. Similarly, banks can fake the quality of mortgage debt at a cost. In this way, we capture elements that we think were important during the financial crisis, and in the period leading up to it. In the model, the incentive problems faced by consumers and banks are similar to the counterfeiting problem captured by Li, Rocheteau, and Weill (2012), though the technical features of how we deal with the incentive problems here are somewhat different.

We first study the behavior of the model under conventional monetary policy. A key feature of the equilibrium we study is that collateral is scarce in the aggregate. This scarcity creates inefficiency in exchange, and is reflected in a low real interest rate. Given the fiscal policy rule, treated as given, a Friedman rule allocation is not feasible.

We determine conditions under which the incentive problems in the model matter. Essentially, incentive constraints will bind in equilibrium if the costs of faking the quality of collateral are sufficiently low, and if asset prices are sufficiently high. An important feature of a secured debt contract when there is a binding incentive constraint, is that there is an endogenous haircut. That is, to convince lenders that the collateral is not faked, the borrower does not borrow up to the full value of the collateral.

Without private asset purchases by the central bank, the incentive problems of consumers and banks can matter. If incentive problems kick in, that happens when the central bank sets a low nominal interest rate. A lower nominal interest rate can increase the margin between the mortgage interest rate and the interest rate on government debt, while increasing the haircuts on houses (in mortgage lending) and on mortgages (as bank collateral). This can act to reduce welfare, and at the extreme an equilibrium may not exist at the zero lower bound on the nominal interest rate.

Can private asset purchases by the central bank improve matters? That depends. First, even if incentive constraints do not bind in the absence of central bank asset purchases, such purchases can be beneficial. If the central bank purchases mortgages at a price in excess of the value of such assets in the private market, then this can increase welfare by increasing the value of the aggregate stock of collateral, and relaxing collateral constraints. Rocheteau and Rodrigues-Lopez (2013) obtain a related result. It can also be advantageous for the central bank to purchase mortgages at private market prices, to bypass banks when banks' incentive constraints bind. However, in circumstances where consumers' incentive constraints bind, central bank purchases of mortgages will exacerbate incentive problems. In such circumstances, the nominal interest rate is positive at the optimum.

Related work on central bank intervention and collateral includes Kiyotaki and Moore (2012), Gertler and Kiyotaki (2012), Gertler and Karadi (2012), and Venkateswaran and Wright (2013). Geromichalos, Herrenbrueck and Salyer (2013) provides an explanation for the term premium on long-maturity assets.

In the second section the model is constructed, and an equilibrium is characterized and analyzed in Section 3. The fourth section contains an analysis of central bank purchases of private assets, and the final section is a conclusion.

2 Model

The basic structure in the model is related to Lagos and Wright (2005) and Rocheteau and Wright (2005). Time is indexed by $t = 0, 1, 2, \dots$, and in each period there are two sub-periods – the centralized market (CM) followed by the decentralized market (DM). There is a continuum of buyers and a continuum of sellers, each with unit mass. An individual buyer has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + F_t + u(x_t)],$$

where H_t is labor supply in the CM , F_t is consumption of housing services in the CM , x_t is consumption in the DM , and $0 < \beta < 1$. Assume that $u(\cdot)$ is strictly increasing, strictly concave, and twice continuously differentiable with $u'(0) = \infty$, $u'(\infty) = 0$, and $-x \frac{u''(x)}{u'(x)} < 1$. Each seller has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t (X_t - h_t),$$

where X_t is consumption in the CM , and h_t is labor supply in the DM . Buyers can produce in the CM , but not in the DM , and sellers can produce in the DM , but not in the CM . One unit of labor input produces one unit of the perishable consumption good, in either the CM or the DM .

In the DM , there are random matches between buyers and sellers, with each buyer matched with a seller. All DM matches have the property that

there is no memory or recordkeeping, so that a matched buyer and seller have no knowledge of each others' histories. A key assumption is limited commitment – no one can be forced to work – and so lack of memory implies that there can be no unsecured credit. If any seller were to extend an unsecured loan to a buyer, the buyer would default.

Following Williamson (2012, 2013), assume limitations on the information technology that imply that currency will be the means of payment in some *DM* transactions, and some form of credit (here it will be financial intermediary credit) will be used in other *DM* transactions. Suppose that, in a fraction ρ of *DM* transactions – denoted *currency transactions* – there is no means for verifying that the buyer possesses any assets other than currency. Thus, in these meetings, the seller can only verify the buyer's currency holdings, and so currency is the only means of payment accepted in exchange. However, in a fraction $1 - \rho$ of *DM* meetings – denoted *non-currency transactions* – the seller can verify the entire portfolio held by the buyer. Assume that, in any *DM* meeting, the buyer makes a take-it-or-leave-it offer to the seller. At the beginning of the *CM*, buyers do not know what type of match they will have in the subsequent *DM*, but they learn this at the end of the *CM*, after consumption and production have taken place. A buyer's type (i.e. whether they will need currency to trade in the *DM* or not) is private information, and at the end of the *CM*, a buyer can meet at most one other economic agent of his or her choice.¹

In addition to currency, there are three other assets in the model: nominal government bonds, reserves and housing.” A government bond sells for z_t^b units of money in the *CM* of period t , and pays off one unit of money in the *CM* of period $t + 1$. One unit of reserves can be acquired in exchange for z_t^m units of money in the *CM* in period t , and pays off one unit of money in the *CM* of period $t + 1$. Housing is in fixed supply, with a continuum of houses with unit mass in existence forever. If a buyer holds a_t units of housing at the beginning of the *CM* of period t , then that buyer receives $a_t y$ units of housing services, where $y > 0$. Only buyers receive utility from consuming housing services, and only the owner of a house can consume the services. Houses sell in the *CM* at the price ψ_t .

2.1 Asset Exchange and Banking

Our approach is to conjecture an equilibrium financial arrangement involving financial intermediation, and a particular pattern of asset-holding. Ultimately, when we construct the equilibrium, we will check to make sure that deviations from that arrangement are suboptimal. In equilibrium, it will be optimal for buyers to hold the entire stock of housing (since they are the agents who receive utility from housing services). But it will also be efficient for houses to be used in exchange. For that to happen, buyers will take out mortgage loans from

¹Type is private information, and trading opportunities are limited at the end of the *CM* so as to prevent the unwinding of bank deposit contracts. See Jacklin (1987) and Wallace (1988).

financial intermediaries – banks – with houses used as collateral. Buyers will then trade the deposit liabilities of banks in the *DM*.

In the spirit of Diamond-Dybvig (1983), banks play an insurance role. Buyers could in principle acquire a portfolio of currency, government bonds, and housing in the *CM*. Then, in the *DM*, if the seller accepts only currency, the buyer exchanges currency for goods, and government bonds and housing are of no use in exchange. But if the seller can verify the existence of all assets in the buyer's portfolio, then government bonds and housing can be used as collateral to obtain a loan from the seller. Currency can be traded as well in this circumstance, but in general it will be inefficient, ex post, to have acquired currency as opposed to a higher-yielding asset in the preceding *CM*. A bank is able to insure buyers against the need for different types of assets in different types of exchange. The bank's deposit contract will allow the depositor to withdraw currency as needed, and to trade bank deposits backed by assets when that is feasible in the *DM*.

2.2 Buyer's problem

Quasi-linear preferences for the buyer allows us to separate the buyer's contracting problem vis-a-vis the bank from his or her decisions about the remaining portfolio. In the *CM*, the buyer acquires housing a_t , at a price ψ_t , and holds this quantity of housing until the next *CM*, when the buyer receives the payoff $\psi_{t+1} + y$ (market value of the housing, plus the payoff in terms of housing services). As well, the buyer receives mortgage loans from banks. A mortgage which is a promise to pay l_t^h units of consumption goods in the *CM* of period $t + 1$ sells at the price q_t , in units of the *CM* good in period t . Further, a mortgage loan must be secured with housing assets, otherwise the buyer would abscond. But the buyer is able to produce "counterfeit housing," i.e. a buyer can produce assets that are indistinguishable to the bank from actual housing, at a cost of γ_h per unit of counterfeit housing. This stands in for incentive problems related to asset appraisals, or private information associated with the buyer's ability to service the mortgage debt.

In equilibrium, the buyer will not produce counterfeit housing (e.g. see Li, Rocheteau, and Weill 2012). For the buyer to keep himself or herself honest may require that the buyer not borrow up to the full value of the collateral, so let $\theta_t^h \in [0, 1]$ denote the fraction of the housing assets of the buyer that the lender is permitted to seize if the buyer defaults.

Then, the buyer solves

$$\max_{l_t^h, a_t, \theta_t^h} [q_t l_t^h - \beta l_t^h - \psi_t a_t + \beta(\psi_{t+1} + y)a_t] \quad (1)$$

subject to

$$-l_t^h + (\psi_{t+1} + y)a_t \theta_t^h \geq 0 \quad (2)$$

$$-\gamma^h + \lambda_t^h(\psi_{t+1} + y)\theta_t^h \leq 0 \quad (3)$$

Here, (1) is the objective function for the buyer, the net return on the buyer's portfolio, (2) is the buyer's collateral constraint, and (3) is the buyer's incentive

constraint. In constraint (3), the marginal payoff to creating counterfeit housing must be non-positive. If the buyer creates one unit of counterfeit housing, he or she bears the cost of counterfeiting, γ^h , and relaxes the collateral constraint (3), where λ_t^h is the multiplier associated with the constraint (3).

2.3 Bank's Problem

In the *CM*, when a bank writes deposit contracts with buyers, a buyer does not know his or her type, i.e. whether or not he or she will need currency to trade in the subsequent *DM*. Once the buyer learns his or her type, at the end of the *CM*, type remains private information to the buyer. The bank contract specifies that the buyer will deposit k_t units of goods with the bank in the *DM*, and gives the depositor one of two options. First, at the end of the period, the depositor can visit the bank and withdraw c_t in currency, in units of *CM* consumption goods, and have no other claims on the bank. Alternatively, if the depositor does not withdraw currency, he or she can have a claim to d_t units of consumption goods in the *CM* of period $t + 1$, and these claims can be traded in the intervening *DM*. In equilibrium, a bank maximizes the expected utility of its representative depositor, subject to the constraint that it earn a nonnegative net payoff, and satisfy a collateral constraint and an incentive constraint. If the bank did not solve this problem in equilibrium, then another bank could enter the industry, make depositors better off, and still earn a nonnegative expected payoff. As with a buyer, a bank must collateralize its deposit liabilities, though we assume that the bank can commit (say, by putting cash in the ATM) to meeting its commitments to cash withdrawals. For the bank, collateral consists of mortgage loans, government bonds, and reserves. Further, the bank can create counterfeit loans in its asset portfolio, and in equilibrium the bank must have the incentive not to do that.

Thus, in equilibrium, the bank solves

$$\max_{k_t, l_t, c_t, d_t, b_t, m_t, \theta_t} \left[-k_t + \rho u \left(\beta \frac{\phi_{t+1}}{\phi_t} c_t \right) + (1 - \rho) u(\beta d_t) \right] \quad (4)$$

subject to

$$k_t - z_t^m m_t - z_t^b b_t - \rho c_t - q_t l_t - \beta(1 - \rho) d_t + \beta \frac{\phi_{t+1}}{\phi_t} (m_t + b_t) + \beta l_t \geq 0, \quad (5)$$

$$-(1 - \rho) d_t + \frac{\phi_{t+1}}{\phi_t} (m_t + b_t) + \theta_t l_t \geq 0 \quad (6)$$

$$-\gamma + \theta_t \lambda_t \leq 0 \quad (7)$$

The objective function in (4) is the expected utility of the depositor. The buyer deposits k_t with the bank in the *CM*, and with probability ρ exchanges currency worth $\frac{\phi_{t+1}}{\phi_t} c_t$ in the *CM* of period $t + 1$ with a seller, as the result of a take-it-or-leave-it offer by the buyer. With probability $1 - \rho$ the buyer

meets a seller who will accept claims on the bank, and the buyer makes a take-it-or-leave-it offer which nets βd_t in DM consumption goods from the seller. Inequality (5) states that the net payoff to the bank must be nonnegative, where $k_t - z_t^m m_t - z_t^b b_t - \rho c_t - q_t l_t$ denotes the payoff in the period- t CM from acquiring deposits and purchasing reserves, government bonds, currency, and loans. The quantity $-\beta(1-\rho)d_t + \beta \frac{\phi_{t+1}}{\phi_t} (m_t + b_t) + \beta l_t$ is the discounted payoff in the CM in period $t+1$, from paying off the holders of deposit claims and collecting the payoffs on reserves, government bonds, and loans. Inequality (6) is the bank's collateral constraint, which states that the bank's remaining deposit liabilities in the CM of period $t+1$ cannot exceed the value to the bank of the assets pledged as collateral against deposits. Here, $\theta_t \in [0, 1]$ is the fraction of loans pledged as collateral, which is a choice variable for the bank. Finally, inequality (7) is the incentive constraint for the bank, which states that the net marginal payoff from acquiring a counterfeit loan must be non-positive, where the net marginal payoff is minus the cost of producing a counterfeit loan plus the value of relaxing the collateral constraint (6). Note that λ_t is the multiplier associated with (6).

2.4 Government

For now, we will not separate monetary and fiscal policy, and all activities and constraints considered in this section apply to the consolidated government. When we characterize equilibria, we will make explicit assumptions about the powers of the monetary and fiscal authorities, and the policy rules they follow. The consolidated government issues currency, reserves, and nominal bonds, denoted by, respectively, C_t , M_t , and B_t , in nominal terms. The government issues liabilities and purchases assets only in the CM . As well, the government makes a lump-sum transfer τ_t to each buyer in the CM in period t .

Thus, the consolidated government budget constraints are given by

$$\phi_0 (C_0 + z_0^m M_0 + z_0^b B_0) - \tau_0 = 0 \quad (8)$$

$$\phi_t [C_t - C_{t-1} + z_t^m M_t - M_{t-1} + z_t^b B_t - B_{t-1}] - \tau_t = 0, \quad t = 1, 2, 3, \dots \quad (9)$$

3 Equilibrium

To solve for an equilibrium, we will first characterize the solutions to the buyer's and bank's problems. Then, we will make some assumptions about policy rules, and solve for a stationary equilibrium.

We will assume that the buyer's collateral constraint (2) binds, and later check that the constraint binds in equilibrium. Then, from the buyer's problem, (1) subject to (??) and (3), the first-order conditions for an optimum are

$$q_t - \beta - \lambda_t^h = 0 \quad (10)$$

$$-\psi_t + \beta(\psi_{t+1} + y) + \lambda_t^h(\psi_{t+1} + y)\theta_t^h = 0 \quad (11)$$

$$-l_t^h + (\psi_{t+1} + y)a_t^h \theta_t^h = 0 \quad (12)$$

$$\begin{aligned} \text{If } -\gamma^h + \lambda_t^h(\psi_{t+1} + y) &\leq 0, \text{ then } \theta_t^h = 1, \\ \text{otherwise } \theta_t^h &= \frac{\gamma^h}{\lambda_t^h(\psi_{t+1} + y)} \end{aligned} \quad (13)$$

Here, condition (13) gives the optimal determination of θ_t^h , i.e. the extent to which the buyer borrows against his or her housing. If the incentive constraint (3) is satisfied for $\theta_t^h = 1$, then $\theta_t^h = 1$ is optimal, as increasing θ_t^h relaxes the buyer's collateral constraint (2) and does not enter the objective function. However, if the incentive constraint (3) does not hold for $\theta_t^h = 1$, then θ_t^h needs to be reduced until the incentive constraint holds with equality. Thus, if the cost of producing a “counterfeit” house is sufficiently low, the buyer cannot borrow fully against collateral, otherwise purchasers of the buyer's mortgage debt correctly infer that the collateral is no good.

As with the buyer, we will assume for now that the bank's collateral constraint (6) binds, and check later that this holds in equilibrium. From the bank's problem (??) subject to (5)-(7), we can first conclude that (5) must bind, otherwise the bank could increase the value of the objective function without violating any constraints. Then, the first-order conditions for an optimum are

$$-z_t^b + \beta \frac{\phi_{t+1}}{\phi_t} + \lambda_t \frac{\phi_{t+1}}{\phi_t} = 0 \quad (14)$$

$$-q_t + \beta + \lambda_t \theta_t = 0 \quad (15)$$

$$\beta \frac{\phi_{t+1}}{\phi_t} u' \left(\beta \frac{\phi_{t+1}}{\phi_t} c_t \right) - 1 = 0 \quad (16)$$

$$\beta u'(\beta d_t) - \beta - \lambda_t = 0 \quad (17)$$

$$-(1 - \rho)d_t + \frac{\phi_{t+1}}{\phi_t} (m_t + b_t) + \theta_t l_t = 0 \quad (18)$$

$$\text{If } -\gamma + \lambda_t \leq 0, \text{ then } \theta_t = 1, \text{ otherwise } \theta_t = \frac{\gamma}{\lambda_t}. \quad (19)$$

In equilibrium, assets markets clear in the CM , so the representative bank's demands for currency, government bonds, and reserves are equal to the respective supplies coming from the government, i.e.

$$\rho c_t = \phi_t C_t, \quad (20)$$

$$b_t = \phi_t B_t, \quad (21)$$

$$m_t = \phi_t M_t. \quad (22)$$

As well, the demand for loans from banks equals the quantity supplied by buyers,

$$l_t = l_t^h, \quad (23)$$

and buyers' demand for assets is equal to the supply,

$$a_t = 1. \quad (24)$$

We will construct stationary equilibria, in which real quantities are constant forever, and all nominal quantities grow at the constant gross rate μ forever, so that the gross rate of return on money, $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ for all t . Then, from the government's budget constraints (8) and (9), and (20)-(22),

$$\rho c + z^m m + z^b b - q l^g = \tau_0 \quad (25)$$

$$V \left(1 - \frac{1}{\mu} \right) + \frac{m}{\mu} (z^m - 1) + \frac{b}{\mu} (z^b - 1) - l^g \left(\frac{q}{\mu} - \beta \right) = \tau \quad (26)$$

We will assume that the fiscal authority fixes the real value of the transfer in period 0, $\tau_0 = V$, i.e. V is exogenous. Then, in (26), the tax on buyers τ in each period $t = 1, 2, 3, \dots$, is determined by (26), where τ is endogenous. The fiscal policy rule is thus fixed in this sense, and the job of the central bank is to optimize treating the fiscal policy rule as given. So, in determining an equilibrium, all we need to take into account is equation (25) with $\tau_0 = V$, or

$$\rho c + z^m m + z^b b = V. \quad (27)$$

Then, from (10)-(24) and (27), we get the following in a stationary equilibrium, which is a partial solution to our problem:

$$\lambda^h = \beta \theta [u'(\beta d) - 1] \quad (28)$$

$$\psi = \frac{\beta y \left\{ \theta \theta^h u'(\beta d) + 1 - \theta \theta^h \right\}}{1 - \beta \left\{ \theta \theta^h u'(\beta d) + 1 - \theta \theta^h \right\}} \quad (29)$$

$$-l^h + (\psi + y) a^h \theta^h = 0 \quad (30)$$

$$\text{If } -\gamma^h + \frac{\beta \theta [u'(\beta d) - 1] y}{1 - \beta \{ \theta u'(\beta d) + 1 - \theta \}} \leq 0, \text{ then } \theta^h = 1, \quad (31)$$

$$\text{otherwise } \theta^h = \frac{\gamma^h (1 - \beta)}{\beta \theta [u'(\beta d) - 1] (y + \gamma^h)}$$

$$z^b = \frac{\beta u'(\beta d)}{\mu} \quad (32)$$

$$q = \beta [\theta u'(\beta d) + 1 - \theta] \quad (33)$$

$$\frac{\beta}{\mu} u' \left(\frac{\beta}{\mu} c \right) - 1 = 0 \quad (34)$$

$$-(1 - \rho) d + \frac{V - \rho c}{\beta u'(\beta d)} + \theta l = 0 \quad (35)$$

$$\begin{aligned} \text{If } -\gamma + \beta [u'(\beta d) - 1] &\leq 0, \text{ then } \theta = 1, \\ \text{otherwise } \theta &= \frac{\gamma}{\beta [u'(\beta d) - 1]}. \end{aligned} \quad (36)$$

The characteristics of the equilibrium will depend critically on whether the buyer's incentive constraint (3) and/or the bank's incentive constraint (7) bind. We will first explore the four relevant cases, without any intervention by way of private asset purchases by the central bank, and then look at the effects of central bank asset purchase policies.

3.1 Non-binding Incentive Constraints for Buyers and Banks

Non-binding incentive constraints implies that $\theta^h = \theta = 1$. Then, from (23), (29), (30), (33), and (35), we get

$$\psi = \frac{\beta y u'(\beta d)}{1 - \beta u'(\beta d)} \quad (37)$$

$$q = \beta u'(\beta d) \quad (38)$$

$$\frac{\beta}{\mu} u' \left(\frac{\beta}{\mu} c \right) - 1 = 0 \quad (39)$$

$$-(1 - \rho)d + \frac{V - \rho c}{\beta u'(\beta d)} + \frac{y}{1 - \beta u'(\beta d)} = 0 \quad (40)$$

Equations (37)-(40), and another equation describing how monetary policy is set (to be discussed below) determine the price of housing ψ , the price of a mortgage claim, q , the real quantity of currency c held by each buyer who trades currency in the DM , the value of deposit claims d traded by buyers in non-currency meetings in the DM , and the gross inflation rate μ . Note that, in this equilibrium, the gross real rates of return on government bonds, housing, and mortgages, are all equal to $\frac{1}{\beta u'(\beta d)}$. Further, all assets carry a liquidity premium, in the sense that gross rates of return are lower than the “fundamental” gross rate of return $\frac{1}{\beta}$ determined by pricing the discounted future payoffs on the assets, so long as $u'(\beta d) > 1$. Note from (17) and (28) that the incentive constraints for the bank and the buyer, respectively, bind if and only if $u'(\beta d) > 1$, i.e. if and only if exchange in DM non-currency trades is inefficient in the sense of not maximizing surplus. Surplus in a DM meeting is maximized when consumption of the buyer is equal to x^* , where $u'(x^*) = 1$.

Next, to make the problem more straightforward, let x_1 and x_2 denote, respectively, consumption in DM meetings in which currency is traded, and deposits are traded. Then, from (32), and (37)-(40), we get

$$z^b = \frac{u'(x_2)}{u'(x_1)} \quad (41)$$

$$-(1 - \rho)x_2 u'(x_2) + V - \rho x_1 u'(x_1) + \frac{y \beta u'(x_2)}{1 - \beta u'(x_2)} = 0 \quad (42)$$

$$\psi = \frac{\beta y u'(x_2)}{1 - \beta u'(x_2)} \quad (43)$$

$$q = \beta u'(x_2) \quad (44)$$

$$\mu = \beta u'(x_1) \quad (45)$$

Here, the central bank sets the price of a nominal bond, z^s , exogenously, and then (41) and (42) determine x_1 and x_2 .

Finally, for this to be an equilibrium requires, from (31) and (36), that

$$-\gamma^h + \frac{\beta [u'(x_2) - 1] y}{1 - \beta u'(x_2)} \leq 0, \quad (46)$$

$$-\gamma + \beta [u'(x_2) - 1] \leq 0. \quad (47)$$

3.2 Buyer's Incentive Constraint Binds, Bank's Incentive Constraint Does Not Bind

In this case, $\theta^h \leq 1$, $\theta = 1$, and from (29)-(35), we obtain (38), (39) and

$$\theta^h = \frac{\gamma^h (1 - \beta)}{\beta [u'(\beta d) - 1] (y + \gamma^h)}, \quad (48)$$

$$\psi = \frac{\beta y + \gamma^h}{1 - \beta}, \quad (49)$$

$$-(1 - \rho)d + \frac{V - \rho c}{\beta u'(\beta d)} + \frac{\gamma^h}{\beta [u'(\beta d) - 1]} = 0. \quad (50)$$

Then, as in the previous subsection, if we solve for an equilibrium in terms of x_1 and x_2 , then we obtain, (41), (44), (45), and

$$-(1 - \rho)x_2 u'(x_2) + V - \rho x_1 u'(x_1) + \frac{\gamma^h u'(x_2)}{u'(x_2) - 1} = 0. \quad (51)$$

Then, given z^b , equations (41) and (51) solve for x_1 and x_2 , and from (31) and (36), we require

$$-\gamma^h + \frac{\beta [u'(x_2) - 1] y}{1 - \beta u'(x_2)} > 0, \quad (52)$$

and (47). In this equilibrium the gross real rates of return on government bonds and mortgages are equal in equilibrium, that is

$$\frac{1}{q} = \frac{1}{\beta u'(x_1) z^b} = \frac{1}{\beta u'(x_2)},$$

just as in the previous section. But here, because of the binding incentive constraint for the buyer, the gross rate of return on housing is given by

$$\frac{\psi + y}{\psi} = \frac{y + \gamma^h}{\beta y + \gamma^h} > \frac{1}{\beta u'(x_2)}$$

where the inequality in the expression above follows from (52). Therefore, the rate of return on housing is higher than the rate of return on mortgages and on bonds, because of the binding no-counterfeiting constraint for the buyer. Further, the gap between the rates of return on housing and other assets gets larger the smaller is x_2 , i.e. the lower is the rate of return on mortgages and government bonds.

3.3 Buyer's Incentive Constraint Does Not Bind, Bank's Incentive Constraint Binds

This case has $\theta^h = 1$, and $\theta \leq 1$. Then, from (29)-(35), we obtain (39) and

$$q = \gamma + \beta \quad (53)$$

$$\theta = \frac{\gamma}{\beta [u'(\beta d) - 1]}, \quad (54)$$

$$\psi = \frac{y(\gamma + \beta)}{1 - \gamma - \beta}, \quad (55)$$

$$-(1 - \rho)d + \frac{V - \rho c}{\beta u'(\beta d)} + \frac{\gamma y}{\beta [u'(\beta d) - 1] (1 - \gamma - \beta)} = 0. \quad (56)$$

Then, solving for an equilibrium in terms of x_1 and x_2 , we obtain, (41), (45), (53), (55), and

$$\theta = \frac{\gamma}{\beta [u'(x_2) - 1]} \quad (57)$$

$$-(1 - \rho)x_2 u'(x_2) + V - \rho x_1 u'(x_1) + \frac{\gamma y u'(x_2)}{(1 - \gamma - \beta) [u'(x_2) - 1]} = 0. \quad (58)$$

Then, given z^b , equations (41) and (58) solve for x_1 and x_2 , and from (31) and (36), we require

$$\gamma^h \geq \frac{\gamma y}{1 - \beta - \gamma}, \quad (59)$$

and

$$\gamma < \beta [u'(x_2) - 1] \quad (60)$$

In this equilibrium, the gross real rate of return on government debt is less than the gross rates of return on mortgages and houses, i.e.

$$\frac{1}{\mu z^s} < \frac{1}{q} = \frac{\psi + y}{\psi},$$

or

$$\frac{1}{\beta u'(x_2)} < \frac{1}{\gamma + \beta},$$

where the inequality follows from (60). Note that in this case, in which the incentive problem that presents itself is associated with the behavior of banks, the wedge appears between the rate of return on safe assets and the rates of return on mortgages and houses. In the previous subsection, the incentive problem was with respect to household behavior, and the wedge was between the rates of return on safe assets and mortgages, and the rate of return on housing.

3.4 Buyer's and Bank's Incentive Constraints Bind

This case has $\theta^h < 1$ and $\theta < 1$. Then, from (29)-(35), we obtain (39), (53), (54) and

$$\theta^h = \frac{\gamma^h(1 - \beta)}{\gamma(y + \gamma^h)}, \quad (61)$$

$$-(1 - \rho)d + \frac{V - \rho c}{\beta u'(\beta d)} + \frac{\gamma^h}{\beta [u'(\beta d) - 1]} = 0. \quad (62)$$

$$\psi = \frac{\beta y + \gamma^h}{1 - \beta}, \quad (63)$$

Then, solving for an equilibrium in terms of x_1 and x_2 , we obtain, (41), (45), (53), (57), (61), (63), and

$$-(1 - \rho)x_2 u'(x_2) + V - \rho x_1 u'(x_1) + \frac{\gamma^h u'(x_2)}{[u'(x_2) - 1]} = 0. \quad (64)$$

Given z^b , equations (41) and (64) solve for x_1 and x_2 , and from (31) and (36), we require

$$\gamma^h < \frac{\gamma y}{1 - \beta - \gamma}, \quad (65)$$

and (60).

In this equilibrium, the gross real rate of return on government debt is less than the gross rate of return on mortgages, which in turn is less than the gross rate of return on houses, i.e.

$$\frac{1}{\mu z^s} < \frac{1}{q} < \frac{\psi + y}{\psi},$$

or

$$\frac{1}{\beta u'(x_2)} < \frac{1}{\gamma + \beta} < \frac{y + \gamma^h}{\beta y + \gamma^h},$$

where the first inequality follows from (60), and the second from (65). Thus, in this case both incentive problems present themselves. The incentive problem for the bank drives a wedge between the safe rate of interest and the mortgage rate, and the household's incentive problem gives an additional wedge between the mortgage rate and the rate of return on housing.

3.5 Conventional Monetary Policy

From our analysis in the previous four subsections, we can use inequalities (46), (47), (52), (59), (60), and (65) to construct Figures 1 and 2. Those figures show, given x_2 (the quantity of consumption in equilibrium by buyers in *DM* meetings where currency is not exchanged) which incentive constraints bind given γ and γ^h , the costs of counterfeiting assets.

First, Figure 1 shows the case where $1 - \beta u'(x_2) > 0$, which implies that the parameter space is divided into four regions, one where neither incentive

constraint binds, in the upper right, one where both constraints bind, in the lower left, one where the bank's incentive constraint binds and the buyer's does not, in the upper left, and one where the bank's incentive constraint does not bind and the buyer's does. An interesting feature here is that, if $\gamma^h < \frac{\beta[u'(x_2)-1]}{1-\beta u'(x_2)}$, then the buyer's incentive constraint does not bind for low γ , but binds for high γ . This is because the buyer's incentive to fake the asset depends on two things: the price of housing and the cost of faking a house. But the price of housing depends on the bank's incentive to fake a mortgage loan. When the cost of faking a mortgage loan is low, the bank effectively takes a haircut on the mortgage loan in equilibrium, and this tends to reduce the demand for mortgages and their price, which in turn reduces the price of houses below what it would otherwise be. Since the price of houses is low when γ is low, this reduces the buyer's incentive to fake a house.

Next, in Figure 2, we show the configuration when $1 - \beta u'(x_2) \leq 0$. In this case, there are only three regions in the parameter space – the one in which neither incentive constraint binds has disappeared. When $1 - \beta u'(x_2) \leq 0$, from (41) and (45) the real rate of return on government debt is low, and all asset prices tend to be high, which gives banks and buyers a greater incentive to fake assets. As a result, no matter how high the costs of faking assets are in this case, there cannot exist an equilibrium where neither incentive constraint binds.

The next step is to determine, given counterfeiting costs (γ, γ^h) , what equilibrium exists given x_2 , and to then characterize equilibria so that we can do policy analysis. From Figures 1 and 2, it will be critical whether counterfeiting costs satisfy

$$\gamma^h \geq \frac{\gamma y}{1 - \beta - \gamma}, \quad (66)$$

or

$$\gamma^h < \frac{\gamma y}{1 - \beta - \gamma}. \quad (67)$$

3.5.1 Large Counterfeiting Costs for Buyers

Suppose that (66) holds, and define $\hat{x}$ as the solution to

$$\gamma = \beta [u'(\hat{x}) - 1]$$

Then, given (66), (46), (47), (59), and (60),

1. If $x_2 \geq \hat{x}$, then neither the buyer's nor the bank's incentive constraints bind, and so (x_1, x_2) must satisfy (42).
2. If $x_2 < \hat{x}$, then the buyer's incentive constraint does not bind, but the bank's incentive constraint binds, so (x_1, x_2) must satisfy (58).

Therefore, if (66) holds,

$$-(1-\rho)x_2 u'(x_2) + V - \rho x_1 u'(x_1) + I(x_2) \left[\frac{y\beta u'(x_2)}{1 - \beta u'(x_2)} \right] + [1 - I(x_2)] \left\{ \frac{\gamma y u'(x_2)}{(1 - \gamma - \beta)[u'(x_2) - 1]} \right\} = 0, \quad (68)$$

where $I(x_2) = 1$ if $x_2 \geq \hat{x}$, and $I(x_2) = 0$ otherwise. Then, in equilibrium, x_1 and x_2 are determined by (68) and (41), given z^b , where

$$z^b \leq 1, \quad (69)$$

where (69) is the zero lower bound on the nominal interest rate on government debt.

In doing policy analysis, we will assume that

$$x^* > V + \frac{y\beta}{1-\beta}, \quad (70)$$

which implies that the total value of the consolidated government debt, plus housing collateral, valued at housing's "fundamental" price (the present value of the flow utility from housing for a buyer), is insufficient to support efficient trade in the *DM*. Assumption (70) implies that fiscal policy is inefficient – the fiscal authority adheres to a policy rule which requires that the central bank forego the Friedman rule, which would otherwise be efficient in this context.

If we add expected utilities across agents in this economy, then in equilibrium our welfare measure is

$$W = \rho[u(x_1) - x_1] + (1 - \rho)[u(x_2) - x_2] \quad (71)$$

Proposition 1 *If (66) holds and*

$$\frac{\hat{x}(\gamma + \beta)}{\beta} - V - \frac{(\gamma + \beta)y}{1 - \gamma - \beta} \leq 0 \quad (72)$$

then incentive constraints do not bind in equilibrium for buyers or banks.

Proof. From our analysis above, if (66) holds then from (68), incentive constraints will never bind in equilibrium if and only if the zero lower bound is attained for $x_2 = x_1 \geq \hat{x}$. This requires, from (68), that the solution x to

$$xu'(x) - V - \frac{y\beta u'(x)}{1 - \beta u'(x)} \leq 0,$$

has the property $x \geq \hat{x}$. Further $x \geq \hat{x}$ if and only if (72) holds. ■

Now, suppose that (70) and (72) hold. What is an optimal monetary policy in this case, i.e. what is the optimal $z^b \leq 1$? We can obtain a simple characterization if we assume that

$$u(x) = \frac{x^{1-\alpha}}{1-\alpha}, \quad (73)$$

where $0 < \alpha < 1$, so the coefficient of relative risk aversion is constant. The optimal monetary policy problem in this case, given the objective function (71) is then

$$\max_{x_1, x_2, z^s} \left\{ \rho \left(\frac{x_1^{1-\alpha}}{1-\alpha} - x_1 \right) + (1 - \rho) \left(\frac{x_2^{1-\alpha}}{1-\alpha} - x_2 \right) \right\} \quad (74)$$

subject to

$$-(1-\rho)x_2^{1-\alpha} + V - \rho x_1^{1-\alpha} + \frac{y\beta x_2^{-\alpha}}{1-\beta x_2^{-\alpha}} = 0, \quad (75)$$

$$z^b = \left(\frac{x_1}{x_2}\right)^\alpha \quad (76)$$

$$z^b \leq 1 \quad (77)$$

Differentiating (74), the derivative of an “indifference curve” is

$$\frac{\partial x_2}{\partial x_1} = -\frac{\rho[x_1^{-\alpha} - 1]}{(1-\rho)[x_2^{-\alpha} - 1]}. \quad (78)$$

Similarly, differentiating the bank’s incentive constraint (in equilibrium), equation (75), we get

$$\frac{\partial x_2}{\partial x_1} = -\frac{\rho(1-\alpha)x_1^{-\alpha}}{(1-\rho)(1-\alpha)x_2^{-\alpha} + \frac{y\beta\alpha x_2^{-\alpha-1}}{[1-\beta x_2^{-\alpha}]^2}} > -\frac{\rho x_1^{-\alpha}}{(1-\rho)x_2^{-\alpha}} \geq -\frac{\rho[x_1^{-\alpha} - 1]}{(1-\rho)[x_2^{-\alpha} - 1]}$$

where the weak inequality follows from (76) and (77). Therefore, the zero-lower-bound constraint (77) binds at the optimum. So, even though a Friedman rule is not feasible here ($x_1 = x_2 = x^*$), it is nevertheless optimal for the nominal interest rate to be zero.

Next, suppose that (72) does not hold, so

$$\frac{\hat{x}(\gamma + \beta)}{\beta} - V - \frac{(\gamma + \beta)y}{1 - \gamma - \beta} > 0. \quad (79)$$

Proposition 2 *If (66), (79), and (73) hold, and an equilibrium exists with $z^b = 1$, then $z^b < 1$ is optimal.*

Proof. If (79) holds, then from (68), $z^b < \hat{x}$ is feasible. Thus, if an equilibrium with $z^b = 1$ exists, from (68) it solves

$$-(1-\rho)x_2 u'(x_2) + V - \rho x_1 u'(x_1) + \frac{\gamma y u'(x_2)}{(1-\gamma-\beta)[u'(x_2) - 1]} = 0, \quad (80)$$

and (41). Then, if we differentiate (80) for the constant relative risk aversion case (73), evaluate the derivative for $z^b = 1$, i.e. $x_1 = x_2$, we obtain

$$\frac{\partial x_2}{\partial x_1} = -\frac{\rho(1-\alpha)x_1^{-\alpha}}{(1-\rho)(1-\alpha)x_1^{-\alpha} - \frac{\gamma y \alpha x_1^{-\alpha-1}}{[x_1^{-\alpha} - 1]^2}} < -\frac{\rho}{(1-\rho)},$$

if

$$(1-\rho)(1-\alpha)x_1^{-\alpha} - \frac{\gamma y \alpha x_1^{-\alpha-1}}{[x_1^{-\alpha} - 1]^2} > 0 \quad (81)$$

where, from (78), $-\frac{\rho}{1-\rho}$ is the derivative of the objective function at the zero lower bound. Therefore, welfare can be increased by increasing the nominal interest rate from the zero lower bound in this case. In the case where (81) does not hold, then it is immediate that welfare increases with an increase in the nominal interest rate from the zero lower bound. ■

The reason for the result in the above proposition is that, when cheating starts to matter, reducing the nominal interest rate tends to increase the price of houses, and the price of mortgages, making the incentive problem for the bank more severe. At the zero lower bound there is no efficiency gain at the margin through the usual channel that is typically reflected in a positive nominal interest rate. However, the marginal incentive effect at the zero lower bound is negative from reducing the nominal rate, so for the central bank to move away from the zero lower bound is optimal.

To illustrate some other possible characteristics of equilibrium outcomes when (66) holds, consider the case where the utility function is given by (73) and $\alpha = \frac{1}{2}$. Then, look for an equilibrium in which $x_2 < \hat{x}$. From (41), and (68), x_1 and x_2 solve

$$(z^b)^2 = \frac{x_1}{x_2}, \quad (82)$$

$$-(1-\rho)x_2^{\frac{1}{2}} + V - \rho x_1^{\frac{1}{2}} + \frac{\gamma y x_2^{-\frac{1}{2}}}{(1-\gamma-\beta) \left[x_2^{-\frac{1}{2}} - 1 \right]} = 0. \quad (83)$$

So, given z^b , we can solve for x_2 to get

$$x_2^{\frac{1}{2}} = \frac{-(1-\gamma-\beta)(1-\rho+\rho z^b+V) \pm \left[\frac{(1-\gamma-\beta)^2(1-\rho+\rho z^b+V)^2}{-4(1-\gamma-\beta)(1-\rho+\rho z^b)(1-\gamma-\beta+\gamma y)} \right]^{\frac{1}{2}}}{2(1-\gamma-\beta)(1-\rho+\rho z^b)}, \quad (84)$$

and then we can solve for x_1 from (82). Now, suppose $z^b = 1$. Then from (84), we get

$$x_2^{\frac{1}{2}} = \frac{-(1-\gamma-\beta)(1+V) \pm [(1-\gamma-\beta)^2(1+V)^2 - 4(1-\gamma-\beta)(1-\gamma-\beta+\gamma y)]^{\frac{1}{2}}}{2(1-\gamma-\beta)}.$$

The zero lower bound is not attainable if

$$(1-\gamma-\beta)(1+V)^2 - 4(1-\gamma-\beta+\gamma y) < 0,$$

which holds for sufficiently small V . Thus, if the supply of government debt is sufficiently small, then there is no zero-lower-bound equilibrium in this case, which is depicted in Figure 3. In the figure, IC is equation (68), and ZLB denotes the zero lower bound on the nominal interest rate. Note that IC is continuous but not differentiable at $x_2 = \hat{x}$, and there is no solution at the zero lower bound.

Equation (84) also implies that, for given z^b , there can be more than one equilibrium, as in Figure 4. In the figure, at point B the inflation rate is

higher, the real interest rate on government debt is lower, and the haircut on loans acquired by banks is higher than at point A . Thus, at point B the value of collateralizable wealth is lower than at point A , so less consumption is supported (in both currency trades and non-currency trades) in the DM .

3.5.2 Small Counterfeiting Costs for Buyers

Next, assume that (67) holds, i.e. the costs of counterfeiting for buyers are small, in a well-defined sense, relative to the costs of counterfeiting for banks. Define $\hat{x}$ to be the solution to

$$\gamma^h = \frac{\beta[u'(\hat{x}) - 1]y}{1 - \beta u'(\hat{x})}. \quad (85)$$

Then, given (66), (46), (47), (59), and (60),

1. If $x_2 \geq \hat{x}$, then neither the buyer's nor the bank's incentive constraints bind, and so (x_1, x_2) must satisfy (42).
2. If $\hat{x} \leq x_2 < \tilde{x}$, then the buyer's incentive constraint binds, but the bank's incentive constraint does not, so (x_1, x_2) must satisfy (51).
3. If $x_2 < \hat{x}$, then the buyer's and the bank's incentive constraints bind, so (x_1, x_2) must satisfy (64).

Therefore, if (67) holds,

$$-(1-\rho)x_2u'(x_2)+V-\rho x_1u'(x_1)+I(x_2)\left[\frac{y\beta u'(x_2)}{1-\beta u'(x_2)}\right]+[1-I(x_2)]\left\{\frac{\gamma^h u'(x_2)}{u'(x_2)-1}\right\}=0, \quad (86)$$

where $I(x_2) = 1$ if $x_2 \geq \hat{x}$, and $I(x_2) = 0$ otherwise.

Thus, for this case, the analysis goes through in exactly the same fashion, qualitatively, as for the case where (66) holds. That is, if V is sufficiently large, then incentive constraints never bind in equilibrium, and it is optimal for the nominal interest rate on government debt to be zero. But, if V is sufficiently small, then incentive constraints can bind in equilibrium, and the zero lower bound is not optimal. Further, the zero lower bound may not be attainable, and there can exist multiple equilibria for a given nominal interest rate.

4 Private Asset Purchases by the Central Bank

We now want to analyze what happens if the central bank chooses to purchase mortgage loans. Assume that the government intervenes in the CM by purchasing mortgages at the price q_t . It will be important to consider three possible types of intervention. With the first type of intervention, the central bank and private banks are both active in the market for mortgage loans, and the central bank and private banks pay the same price for these loans. With the

second type of intervention, the central bank purchases all the mortgages that are forthcoming at the price q_t , and q_t is no larger than the price that private banks would pay for mortgages. Finally, with the third type of intervention, the central bank purchase all mortgages forthcoming at the price q_t , and this price is set in excess of what private banks would pay for the same mortgages.

We need to add private asset purchases to the consolidated government's budget constraints (8) and (9). Letting l_t^g denote the loans acquired by the central bank in period t , we obtain

$$\phi_0 (C_0 + z_0^m M_0 + z_0^b B_0) - q_0 l_0^g - \tau_0 = 0 \quad (87)$$

$$\phi_t [C_t - C_{t-1} + z_t^m M_t - M_{t-1} + z_t^b B_t - B_{t-1}] - q_t l_t^g + l_{t-1}^g - \tau_t = 0, \quad t = 1, 2, 3, \dots \quad (88)$$

As before, we confine attention to stationary equilibria, and a fiscal policy rule with $\tau_0 = V$, a constant, under which taxes in periods $t = 1, 2, 3, \dots$ respond passively. Then, in equilibrium, instead of (27) we get

$$\rho c + z^m m + z^b b = V + q l^g. \quad (89)$$

4.1 Central Bank and Private Banks Both Active in the Mortgage Market

In equilibrium, $l^h = l + l^g$, where l^h is the supply of mortgage loans from households, and l is loan demand from banks. Then, from (35) and (89),

$$-(1 - \rho)\beta du'(\beta d) + V - \rho c + \beta u'(\beta d)\theta l + (1 - \theta)\beta l^g = 0. \quad (90)$$

Thus, we can say that, in equilibrium, asset purchases by the central bank in this type of regime are irrelevant if $\theta = 1$, i.e. if the bank's incentive constraint does not bind. If there is no incentive problem in equilibrium for the bank, then mortgages and government bonds sell at the same price. The central bank can swap reserves for mortgages, and it will make no difference.

Therefore, the only cases where central bank mortgage purchases can work are ones where $\theta < 1$ in equilibrium. First, consider the case where the bank's incentive constraint binds, and the buyer's does not. Then, the equilibrium is described by (41), (45), (53), (55), (57), and instead of (58),

$$-(1 - \rho)x_2 u'(x_2) + V - \rho x_1 u'(x_1) + \frac{\gamma y u'(x_2)}{(1 - \gamma - \beta)[u'(x_2) - 1]} + \left\{ \frac{\beta [u'(x_2) - 1] - \gamma}{u'(x_2) - 1} \right\} l^g = 0. \quad (91)$$

Therefore, since (60) is a necessary condition for the equilibrium to exist, increasing l^g always shifts the locus described by (91) in a direction that will increase welfare. That is, fixing x_2 , if l^g increases, then (91) is satisfied for a larger value of x_1 . Thus, increasing asset purchases in this regime is always optimal, so the central bank should purchase all of the mortgages that are forthcoming at the market price q . Therefore, at the optimum (91) becomes

$$-(1 - \rho)x_2 u'(x_2) + V - \rho x_1 u'(x_1) + \frac{(\gamma + \beta)y}{1 - \gamma - \beta} = 0. \quad (92)$$

Similarly, consider an equilibrium in which the bank's and the buyer's incentive constraints bind. Then, an equilibrium is described by (39), (53), (54), (61), and (62) is replaced by

$$-(1-\rho)x_2u'(x_2) + V - \rho x_1u'(x_1) + \frac{\gamma^h u'(x_2)}{[u'(x_2) - 1]} + \left\{ \frac{\beta [u'(x_2) - 1] - \gamma}{u'(x_2) - 1} \right\} l^g = 0. \quad (93)$$

So, just as in the previous case, it is optimal for the central bank to take all the mortgage loans forthcoming at the price that private banks would pay for such loans, so (93) becomes

$$-(1-\rho)x_2u'(x_2) + V - \rho x_1u'(x_1) + \frac{(\gamma + \beta)\gamma^h}{\gamma} = 0 \quad (94)$$

4.2 Central Bank Purchases All Mortgages, and Pays a Premium

With this type of policy intervention, the central bank purchases mortgages at a price q , which is greater than the price $\tilde{q}$ that private banks are willing to pay in the market. Further, the central bank purchases the entire supply of mortgages forthcoming from buyers at the price q . The central bank collateralizes loans in the same manner as do private banks.

The consolidated government budget constraint gives (89), and then the incentive constraint for banks in equilibrium gives

$$-(1-\rho)x_2u'(x_2) + V - \rho x_1u'(x_1) + ql^g = 0. \quad (95)$$

As well, from (10)-(13),

$$-\psi + \beta(\psi + y) + (q - \beta)(\psi + y)\theta^h = 0 \quad (96)$$

$$-l^h + (\psi + y)\theta^h = 0 \quad (97)$$

$$\text{If } -\gamma^h + (q - \beta)(\psi + y) \leq 0, \text{ then } \theta^h = 1, \quad (98)$$

$$\text{otherwise } \theta^h = \frac{\gamma^h}{(q - \beta)(\psi + y)}$$

In equilibrium,

$$l^h = l^g, \quad (99)$$

i.e. mortgage loans supplied by buyers are equal to loans purchased by the central bank. As well, $q \geq \tilde{q}$, so, from (15), (17), and (19),

$$q \geq \min[\beta u'(x_2), \beta + \gamma] \quad (100)$$

First, consider the case where the incentive constraint does not bind for the buyer, so $\theta^h = 1$, and from (96),

$$\psi = \frac{qy}{1 - q} \quad (101)$$

Then, from (97), (95), and (99), the incentive constraint for the bank in equilibrium becomes

$$-(1 - \rho)x_2u'(x_2) + V - \rho x_1u'(x_1) + \frac{qy}{1 - q} = 0, \quad (102)$$

and from (98) and (100), q must satisfy

$$\min[\beta u'(x_2), \beta + \gamma] \leq q \leq \frac{\beta y + \gamma^h}{y + \gamma^h} \quad (103)$$

Thus, as long as q falls in the range specified in (103), then increasing q , which will increase the price of houses, from (101), and induce a larger supply of mortgage loans, shifts the relationship (102) in a welfare-improving direction. This result might seem surprising, as the welfare improvement can potentially occur even if incentive constraints do not bind for buyers, or for banks at the price they are willing to pay for mortgage loans.

Then, so long as

$$\min[\beta u'(x_2), \beta + \gamma] \leq \frac{\beta y + \gamma^h}{y + \gamma^h} \quad (104)$$

as (103) holds, it is optimal for the central bank to set

$$q = \frac{\beta y + \gamma^h}{y + \gamma^h}$$

in this regime, and to purchase all mortgage loans forthcoming at that price, so at the optimum (102) becomes

$$-(1 - \rho)x_2u'(x_2) + V - \rho x_1u'(x_1) + \frac{\beta y + \gamma^h}{1 - \beta} = 0, \quad (105)$$

and for (105) to hold in equilibrium under this optimal policy, from (104),

$$\text{If } \gamma \leq \beta[u'(x_2) - 1], \text{ then } \gamma^h \geq \frac{\gamma y}{1 - \beta - \gamma} \quad (106)$$

$$\text{If } \gamma > \beta[u'(x_2) - 1], \text{ then } \gamma^h \geq \frac{\beta[u'(x_2) - 1]y}{1 - \beta u'(x_2)} \quad (107)$$

Next, suppose that the household's incentive constraint binds, so that $0 < \theta^h < 1$. Then, from (96)-(98),

$$\psi = \frac{\beta y + \gamma^h}{1 - \beta}, \quad (108)$$

$$l^h = \frac{\gamma^h}{q - \beta}, \quad (109)$$

and so (95), (99), and (109) give

$$-(1 - \rho)x_2u'(x_2) + V - \rho x_1u'(x_1) + \frac{q\gamma^h}{q - \beta} = 0. \quad (110)$$

Therefore, from (110), increases in q in this regime are welfare-reducing, as they reduce the stock of collateralizable wealth. Increasing q worsens the incentive problem for buyers, and has no effect on the price of housing, from (108), serving only to increase the size of the haircut on housing collateral, and lowering the quantity of mortgage loans.

4.3 Conventional and Unconventional Monetary Policy

From the previous two subsections, if central bank asset market purchases are conducted optimally, then there are three possibilities:

1. The bank's and buyer's incentive constraints bind, and the central bank purchases all of the mortgage loans forthcoming at the price that a private bank would pay. We get this case if (60) and (65) hold, in which case the bank's incentive constraint in equilibrium is (94).
2. The buyer's incentive constraint does not bind, and the central bank purchases all of the mortgage loans forthcoming at a price greater than what private banks are willing to pay. Here, (106) and (107) apply, and the bank's incentive constraint in equilibrium is (105).
3. The bank's incentive constraint does not bind, and the buyer's constraint does, in which case central bank asset purchases are suboptimal. For this case, (47) and (52) hold, and the incentive constraint for the bank in equilibrium is (51).

In Figures 5, and 6, we show how, given x_2 , the counterfeiting costs γ^h and γ determine the central bank's optimal purchase intervention. In Figure 5, constructed for the case where $1 - \beta u'(x_2) > 0$, for any value of γ there is always a value of γ^h sufficiently large that the central bank should purchase all mortgages forthcoming, at a premium relative to what private banks would pay for mortgages. In this region of the parameter space, the critical incentive effect the central bank needs to be concerned with is paying too high a price for mortgages and inducing buyers to produce fake assets. In Figure 5, when the cost of faking a mortgage is high for a private bank, and the cost of faking a house is low for a buyer, then it is optimal for the central bank not to intervene. In this case, the incentive problem for buyers is so severe that intervention by the central bank via mortgage purchases would only aggravate the problem. In Figure 5, if the cost of faking a house is low for a buyer, and the cost of faking a mortgage is in a middle range for a bank, then the central bank should purchase mortgages, but at the market price.

In Figure 6, constructed for the case where $1 - \beta u'(x_2) \leq 0$, the key difference from Figure 5 is that, for any γ^h , there exists a sufficiently high value for γ such

that the central bank should not intervene through purchases of mortgages. This is because x_2 is small in this case, so asset prices are high, which aggravates incentive problems.

4.3.1 Large Counterfeiting Costs for Buyers: Optimal Asset Purchases and Open Market Operations

We will analyze equilibrium in a similar manner to what was done in the case with no asset purchases. First, suppose that (66) holds. For this case, parameters are always in the top region in Figure 5, or the top left region in Figure 6, and the central bank intervenes by purchasing mortgages at a premium. The bank's incentive constraint in equilibrium is then given by (105). We can compare this to what we get with no intervention, which is an incentive constraint in equilibrium for the bank given by (69). The central bank chooses z^b , then (41) and (105) determine x_1 and x_2 , and asset prices are determined by (100) and (101). The determination of (x_1, x_2) is depicted in Figure 5, where IC is equation (105), and " $z^s = \text{constant}$ " is equation (41).

Proposition 3 *If (66) holds, then for any choice of z^b by the central bank, welfare is higher with optimal central bank purchases than with no asset purchases by the central bank.*

Proof. If $x_2 \geq \hat{x}$, then without central bank purchases, (x_1, x_2) solves, from (69),

$$-(1 - \rho)x_2u'(x_2) + V - \rho x_1u'(x_1) + \frac{y\beta u'(x_2)}{1 - \beta u'(x_2)} = 0, \quad (111)$$

whereas with optimal private asset purchases by the central bank, (x_1, x_2) solves (105). Then,

$$\frac{\beta y + \gamma^h}{1 - \beta} \geq \frac{y(\gamma + \beta)}{1 - \beta - \gamma} \geq \frac{y\beta u'(x_2)}{1 - \beta u'(x_2)}, \quad (112)$$

where the first inequality follows from (66), and the second from the fact that $x_2 \geq \hat{x}$. Further, the first inequality is strict if (66) is a strict inequality, and the second inequality is strict if $x_2 > \hat{x}$. Therefore, from (112), with optimal private asset purchases by the central bank, for any x_2 , x_1 is at least as large as in the case with no private asset purchases. Further, x_1 is strictly larger with optimal asset purchases, except in the case where $x_2 = \hat{x}$ and (66) holds with equality. Next, suppose that $x_2 < \hat{x}$, in which case (x_1, x_2) solves, from (69),

$$-(1 - \rho)x_2u'(x_2) + V - \rho x_1u'(x_1) + \frac{\gamma y u'(x_2)}{(1 - \gamma - \beta)[u'(x_2) - 1]} = 0. \quad (113)$$

In this case,

$$\frac{\beta y + \gamma^h}{1 - \beta} \geq \frac{y(\gamma + \beta)}{1 - \beta - \gamma} > \frac{\gamma y u'(x_2)}{(1 - \gamma - \beta)[u'(x_2) - 1]}, \quad (114)$$

where the second inequality follows from $x_2 < \hat{x}$. So, similar to the first case, welfare must be higher with optimal central bank asset purchases than without. ■

From the above proposition, comparing equilibrium determination of (x_1, x_2) under (105) to (68), we can get something like Figure 6, where IC_1 is the incentive constraint for the bank in equilibrium without central bank asset purchases, i.e. equation (68), and IC_2 is equation (105). Thus welfare is unambiguously larger in equilibrium with optimal asset purchases by the central bank, no matter what nominal interest rate the central bank chooses in this case.

So, if (66) holds, then it is always optimal for the central bank to intervene by purchasing all mortgages that are forthcoming, at a price that makes buyers indifferent to taking out a mortgage against a fake house. Then, we want to ask what optimal conventional monetary policy is under these circumstances. How should the central bank set z^b ? The central bank's problem is to solve

$$\max_{z^b, x_1, x_2} \rho[u(x_1) - x_1] + (1 - \rho)[u(x_2) - x_2] \quad (115)$$

subject to (41), (105), and

$$z^b \leq 1 \quad (116)$$

Proposition 4 *If (66) holds, $-\frac{xu''(x)}{u'(x)} = \alpha < 1$, and the central bank conducts optimal purchases of private assets, then $z^b = 1$ at the optimum.*

Proof. Differentiating a level surface of the objective function in (115), we obtain

$$\frac{\partial x_2}{\partial x_1} = -\frac{\rho[u'(x_1) - 1]}{(1 - \rho)[u'(x_2) - 1]}$$

As well, differentiating (105) gives

$$-\frac{\rho u'(x_1)}{(1 - \rho)u'(x_2)},$$

if $-\frac{xu''(x)}{u'(x)} = \alpha < 1$. Then, since

$$-\frac{\rho[u'(x_1) - 1]}{(1 - \rho)[u'(x_2) - 1]} < -\frac{\rho u'(x_1)}{(1 - \rho)u'(x_2)}$$

for $x_1 > x_2$, and

$$-\frac{\rho[u'(x_1) - 1]}{(1 - \rho)[u'(x_2) - 1]} = -\frac{\rho u'(x_1)}{(1 - \rho)u'(x_2)}$$

for $x_1 = x_2$, therefore welfare can always be increased if $z^b < 1$ by increasing z^b , from (41). Therefore $z^b = 1$ is optimal. ■

From the above proposition, we can conclude that, when (66) is satisfied, the equilibrium allocation under an optimal monetary policy is $x_1 = x_2 = x$, where from (105) x satisfies

$$xu'(x) = V + \frac{\beta y + \gamma^h}{1 - \beta}. \quad (117)$$

On the left-hand side of (117), the function $xu'(x)$ is strictly increasing in x , given our assumption in the above proposition. The right-hand side of (117) is the total value of the debt issued by the fiscal authority, V , plus the value of housing collateral. Note that higher x implies higher welfare, as surplus increases in all DM trades with an increase in x . Thus at the optimum, how much the central bank can do to increase welfare is limited by the cost of counterfeiting housing collateral, γ^h . The larger is γ^h , the more the central bank can increase the value of housing collateral above its fundamental value $\frac{\beta y}{1 - \beta}$, by purchasing mortgages at a price higher than the private sector's shadow price.

Figure 7 illustrates optimal policy when (66) holds. In the figure, ZLB is the zero lower bound constraint on the nominal interest rate, IC_1 is the bank's incentive constraint in equilibrium without central bank asset purchases, (68), and IC_2 is (105), the bank's incentive constraint in equilibrium when the central bank conducts optimal asset purchases. With optimal purchases of mortgages by the central bank, welfare is higher for any short-term nominal interest rate, and with optimal asset purchases by the central bank the optimal allocation is at A , where the nominal interest rate is zero. In the figure, I_1 , I_2 , and I_3 denote "indifference curves" along which net surplus in the DM is constant.

4.3.2 Small Counterfeiting Costs for Buyers: Optimal Monetary Policy

Next, consider the case where (67) holds. Here, for large x_2 , parameters are in the top region in Figure 5, where the central bank should purchase private assets at a price in excess of the market price. For a middle range of values for x_2 , parameters are in the lower right region in Figure 5, or the region on the right in Figure 6, where it is optimal for the central bank not to purchase private assets. Finally, for small values of x_2 , parameters are in the lower middle region in Figure 5, or the middle region in Figure 6, where the central bank should intervene by accommodating all the available supply of private mortgages at the market price.

Specifically, from (51), (94), and (105), the incentive constraint for the bank in equilibrium is given by

$$-(1-\rho)x_2u'(x_2)+V-\rho x_1u'(x_1)+I_1(x_2)\left[\frac{\beta y + \gamma^h}{1 - \beta}\right]+I_2(x_2)\left[\frac{\gamma^h u'(x_2)}{u'(x_2) - 1}\right]+I_3(x_2)\left[\frac{(\gamma + \beta)\gamma^h}{\gamma}\right] = 0, \quad (118)$$

where

$$I_1(x_2) = 1 \text{ if } x_2 \geq \hat{x}, \text{ and } 0 \text{ otherwise.}$$

$$I_2(x_2) = 1 \text{ if } \hat{x} \leq x_2 \leq \tilde{x}, \text{ and } 0 \text{ otherwise.}$$

$I_3(x_2) = 1$ if $x_2 \leq \hat{x}$, and 0 otherwise.

Then, an equilibrium solution for (x_1, x_2) is determined from (118) and (41) given z^b . Since $z^b \leq 1$, i.e. the zero lower bound on the nominal interest rate must be respected, from (118), and given (70), we get the following:

1. Case 1: If

$$\frac{\tilde{x}}{\beta} \left(\frac{\gamma^h + \beta y}{\gamma^h + y} \right) - \frac{\beta y + \gamma^h}{1 - \beta} \leq V < x^* - \frac{\beta y}{1 - \beta}, \quad (119)$$

then $x_2 \in [\tilde{x}, x^*]$ in equilibrium.

2. Case 2: If

$$\frac{\hat{x}(\gamma + \beta)}{\beta} - \frac{\gamma^h(\gamma + \beta)}{\gamma} \leq V \leq \frac{\tilde{x}}{\beta} \left(\frac{\gamma^h + \beta y}{\gamma^h + y} \right) - \frac{\beta y + \gamma^h}{1 - \beta}, \quad (120)$$

then $x_2 \in [\hat{x}, \tilde{x}]$ in equilibrium.

3. Case 3: If

$$V < \frac{\hat{x}(\gamma + \beta)}{\beta} - \frac{\gamma^h(\gamma + \beta)}{\gamma}, \quad (121)$$

then $x_2 < \hat{x}$ is feasible in equilibrium.

First, we know from our analysis above that, if (119) holds (case 1), then $z^b = 1$ at the optimum, and the central bank purchases mortgages at a price greater than what private sector banks are willing to offer. For the other two cases, we need the following proposition.

Proposition 5 *If (120) holds and $-\frac{xu''(x)}{u'(x)} = \alpha < 1$, then $z^b < 1$ at the optimum, and purchases of mortgages by the central bank can be suboptimal.*

Proof. If (120) holds, then $z^b = 1$ is achieved in the region where $\hat{x} \leq x_2 \leq \tilde{x}$. From (118), the derivative of (118) for $\hat{x} < x_2 < \tilde{x}$, given $-\frac{xu''(x)}{u'(x)} = \alpha < 1$, is

$$\frac{\partial x_2}{\partial x_1} = \frac{-\rho(1 - \alpha)u'(x_1)}{(1 - \rho)(1 - \alpha)u'(x_2) + \frac{\gamma^h u''(x_2)}{[u'(x_2) - 1]^2}} \equiv \nabla \quad (122)$$

Therefore, for $z^b = 1$, $x_1 = x_2$, in which case either $\nabla > 0$, or $\nabla < 0$ and

$$\nabla < -\frac{\rho[u'(x_1) - 1]}{(1 - \rho)[u'(x_2) - 1]},$$

so welfare increases if z^b is reduced from $z^b = 1$. Further, suppose that $z^b = 1$ is achieved for $x_2 < \tilde{x}$. Then, for γ^h sufficiently small, if we evaluate ∇ for $x_2 = \tilde{x}$, then $\nabla < 0$, and

$$\nabla > -\frac{\rho[u'(x_1) - 1]}{(1 - \rho)[u'(x_2) - 1]},$$

which implies that $x_2 \in (\hat{x}, \tilde{x})$ at the optimum, which implies that x_2 lies in a range where central bank purchases of mortgages are zero at the optimum. ■

Figure 8 shows case 1, where (119) holds. In the figure, IC is the bank's incentive constraint in equilibrium, and ZLB denotes the zero lower bound on the nominal interest rate. At the optimum, A , the nominal interest rate is zero, and the central bank purchases private mortgages at a premium over the price that would be offered by a private bank. Figure 9 shows what can happen in case 2, when (120) holds. In the figure, the optimum is at A , where the nominal interest rate is greater than zero, and the central bank purchases no mortgages.

5 Discussion

Our results show that, in the absence of central bank purchases of private mortgages, there may exist incentive problems in asset markets that can be exacerbated by conventional monetary policy. If cheating is sufficiently low-cost, and asset prices are sufficiently high, then banks may wish to “fake” mortgage loans, and buyers may wish to “fake” houses. In equilibrium, cheating does not occur, but if the potential for cheating matters, then banks and buyers stay honest by giving their collateral a haircut. This implies that the effect of the potential for cheating shows up in a reduction in the size of aggregate collateralizable wealth.

When there are no central bank purchases of private mortgages, and incentive constraints bind at the zero lower bound on the nominal interest rate, it is optimal for the central bank to increase the nominal interest rate above zero, as this reduces asset prices and mitigates the incentive problem. Purchases of mortgages by the central bank have two beneficial effects. First, this allows the central bank to directly intermediate mortgages, and to bypass private banks if these banks have the incentive to cheat. Second, even if the incentive to cheat by private banks is not operational, if the central bank purchases mortgages at a price higher than the value of these mortgages to private banks, this can increase the value of collateralizable wealth, relax incentive constraints, and have a beneficial effect on welfare. However, central bank purchases of mortgages can be detrimental due to the incentive of buyers to cheat, and this problem can be exacerbated if the central bank increases the market price of mortgages.

There are circumstances in which optimal monetary policy, in the context of a low supply of government debt, includes the purchase of private mortgages by the central bank, along with a reduction of the nominal interest rate to zero. In other circumstances, in which the incentive of buyers to cheat matters, it can be optimal for the central bank to forego the purchase of private mortgages, and set the nominal interest rate above zero.

6 Conclusion

We have built a model where there are incentive problems in the mortgage market – banks can fake the quality of mortgage debt, and consumers can fake

the quality of housing which is posted as collateral. These incentive problems, combined with a scarcity of collateralizable wealth, create a role for central bank purchases. Low nominal interest rates can serve to exacerbate incentive problems, as this tends to raise asset prices and to encourage the faking of quality.

The model implies that central bank purchases work for two reasons. First, by intermediating mortgages, the central bank can bypass private-sector incentive problems. Second, by inflating the value of housing wealth, central bank asset purchases relax collateral constraints and improve welfare. However, central bank purchases of private assets can also exacerbate incentive problems. By increasing asset prices, the central bank can encourage consumers to fake the quality of housing when borrowing on the mortgage market. In such circumstances a positive nominal interest rate may be optimal.

The paper did not address an important issue related to private asset purchases by the central bank. In general, such purchases will tend to favor some credit market participants relative to others. If private assets are purchased by the central bank, some choices must be made about which assets to purchase, and which assets not to purchase. If the purchase programs work as intended, this must have redistributive effects, and there are important political economy issues that need to be addressed. It is possible that, even if private asset purchases can have beneficial effects, the costs in terms of central bank independence are too large for these purchases to be desirable.

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Figure 1: Equilibria when $1 - \beta u'(x_2) > 0$

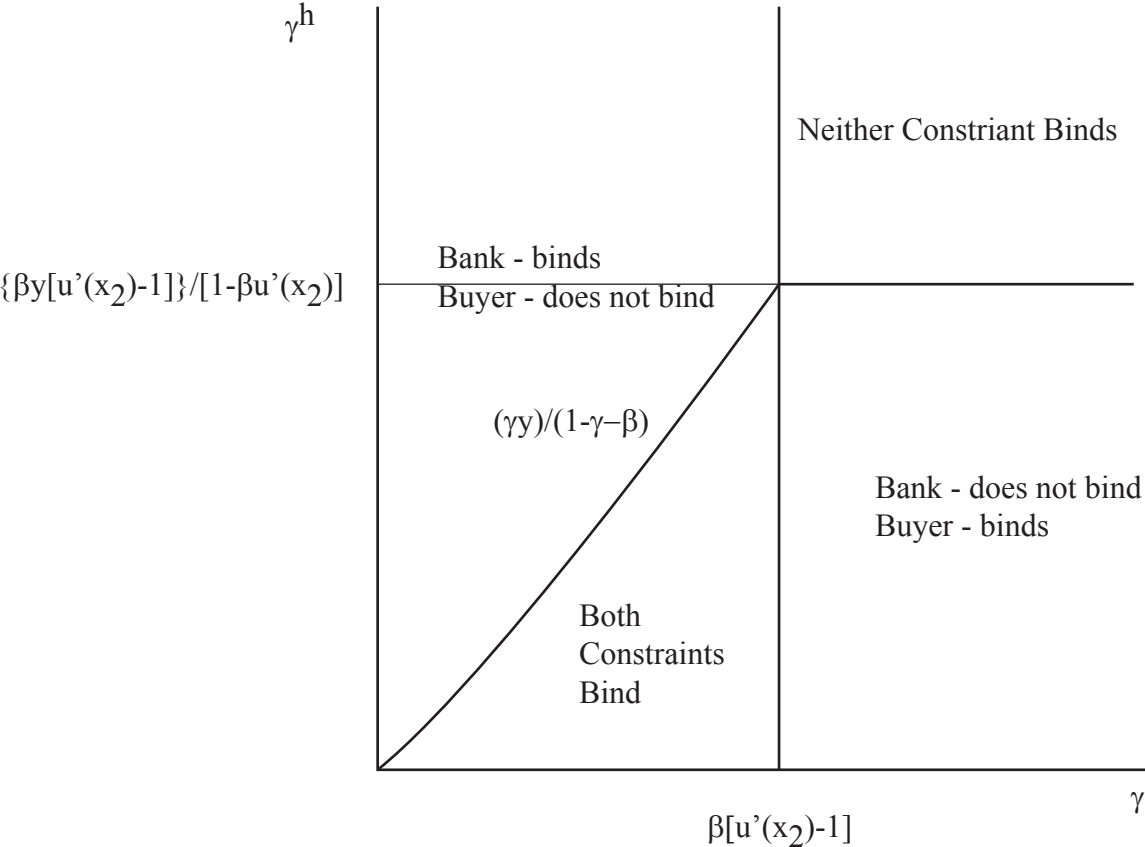


Figure 2: Equilibrium When $1 - \beta u'(x_2) \leq 0$

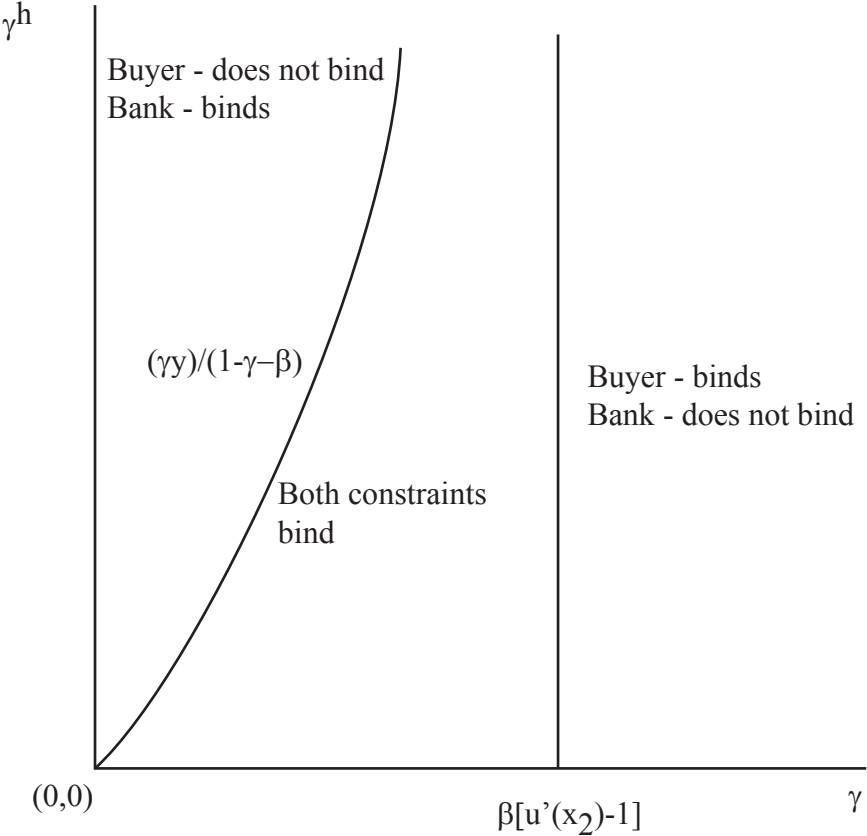


Figure 3: Zero Lower Bound is not Attainable

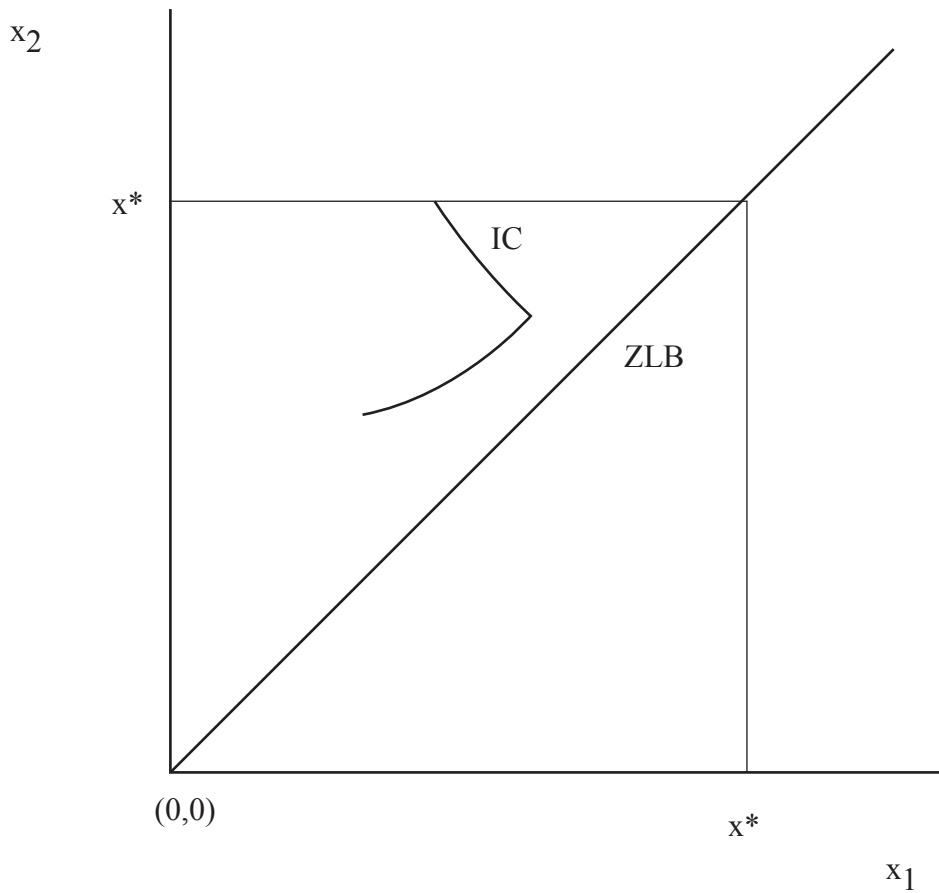


Figure 4: Multiple Equilibria

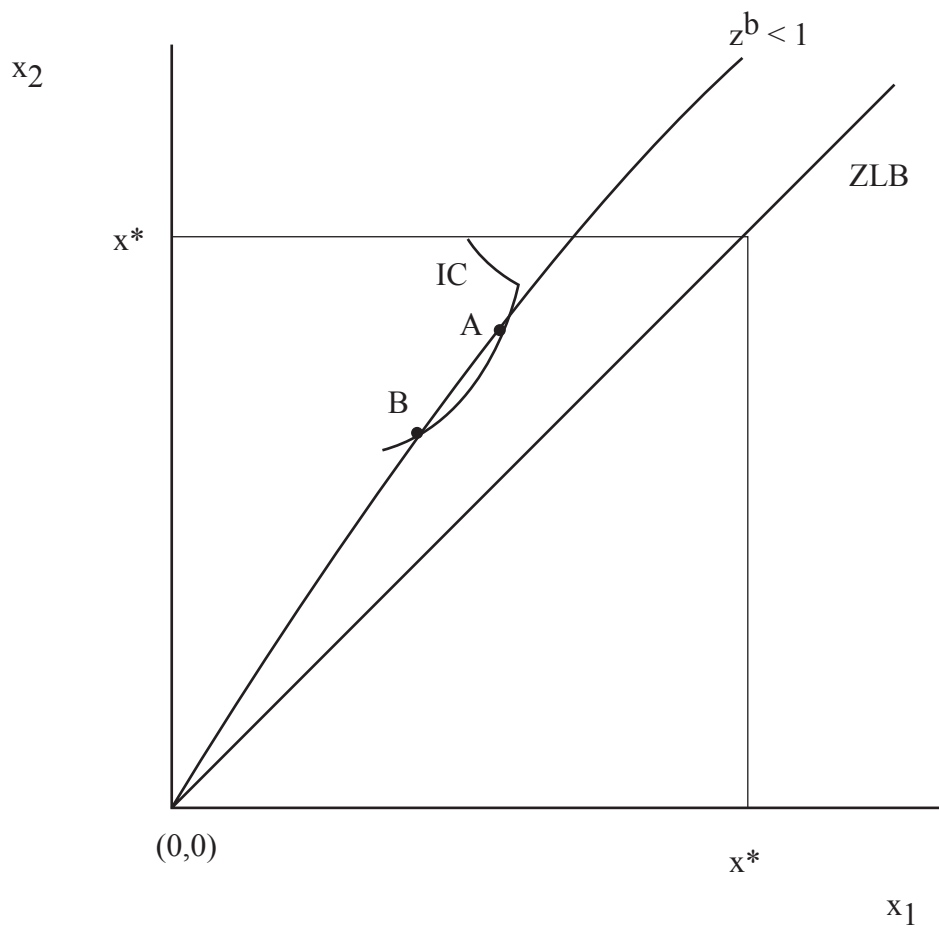


Figure 5: Optimal Central Bank Purchases When $1 - \beta u'(x_2) > 0$

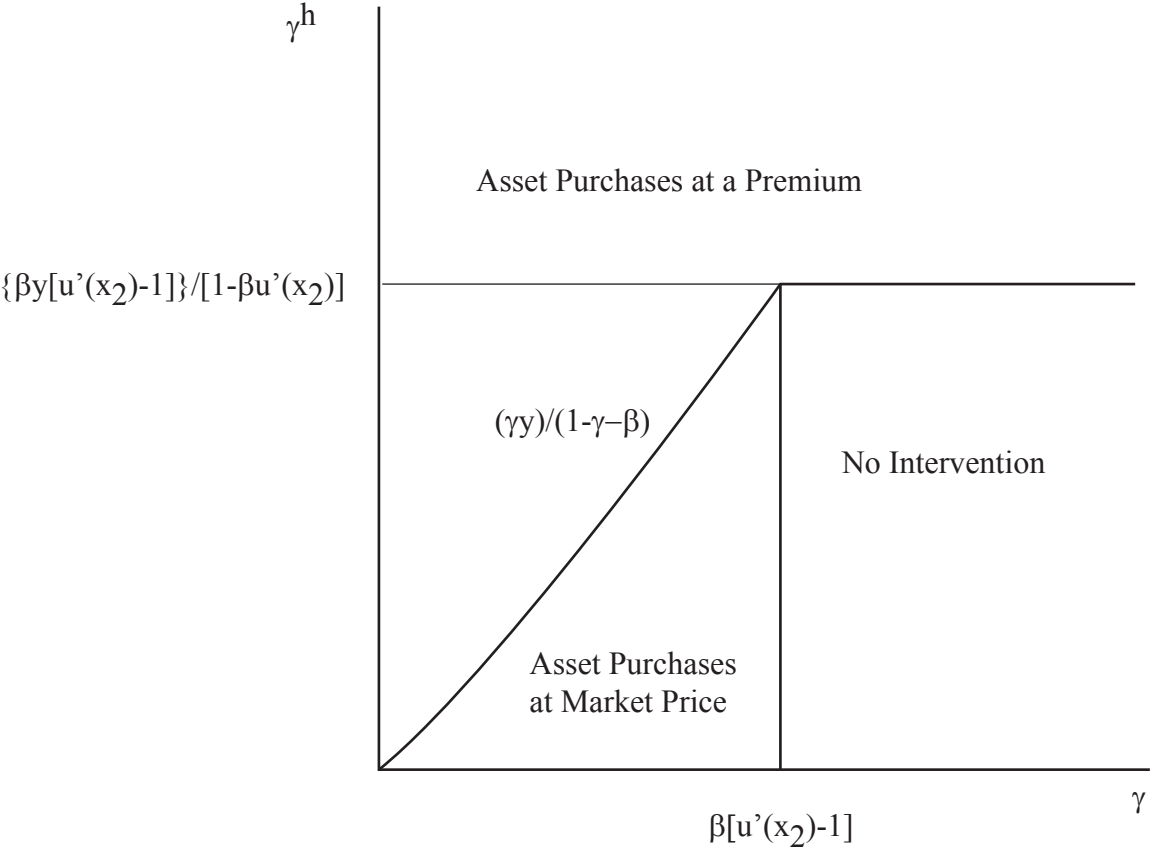


Figure 6: Optimal Central Bank Intervention When $1 - \beta u'(x_2) \leq 0$

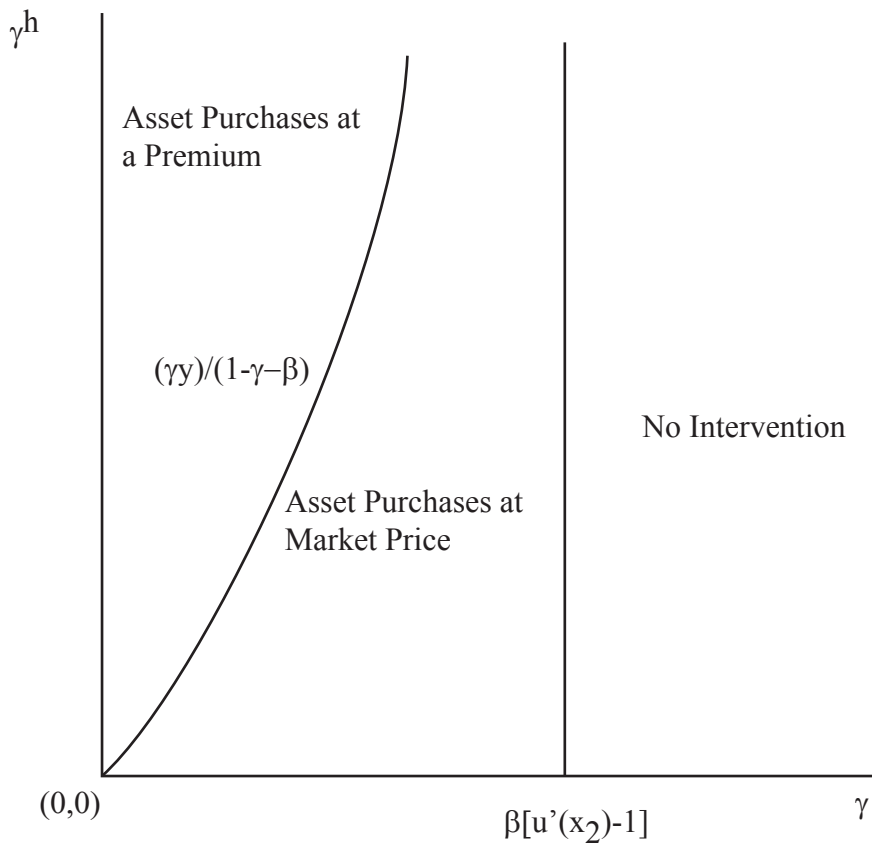


Figure 7: High Cost of Cheating for the Buyer

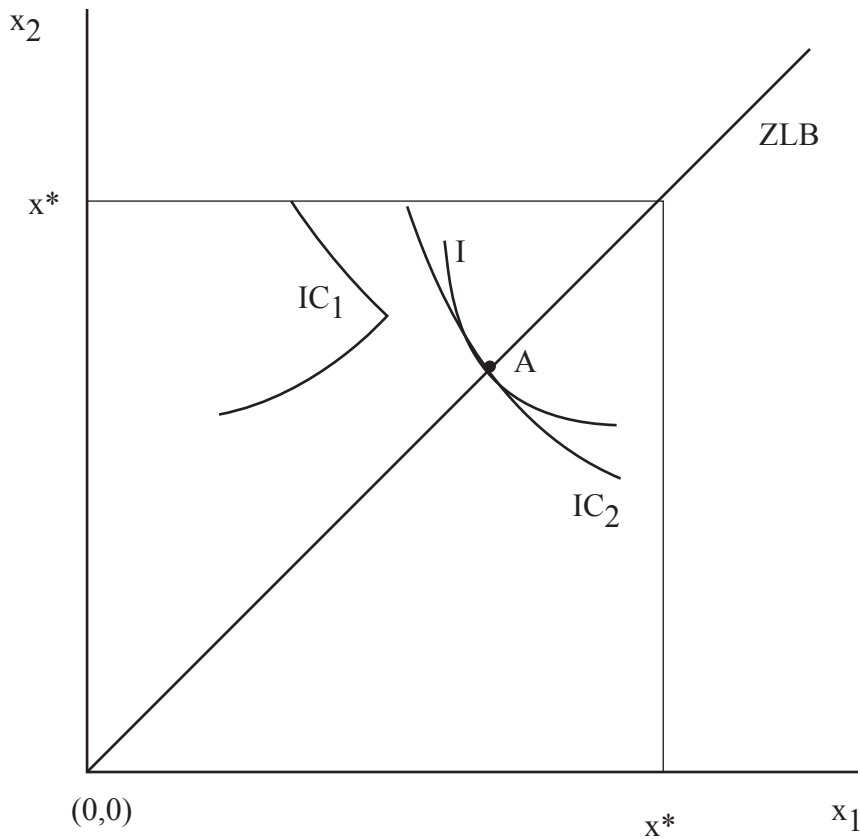
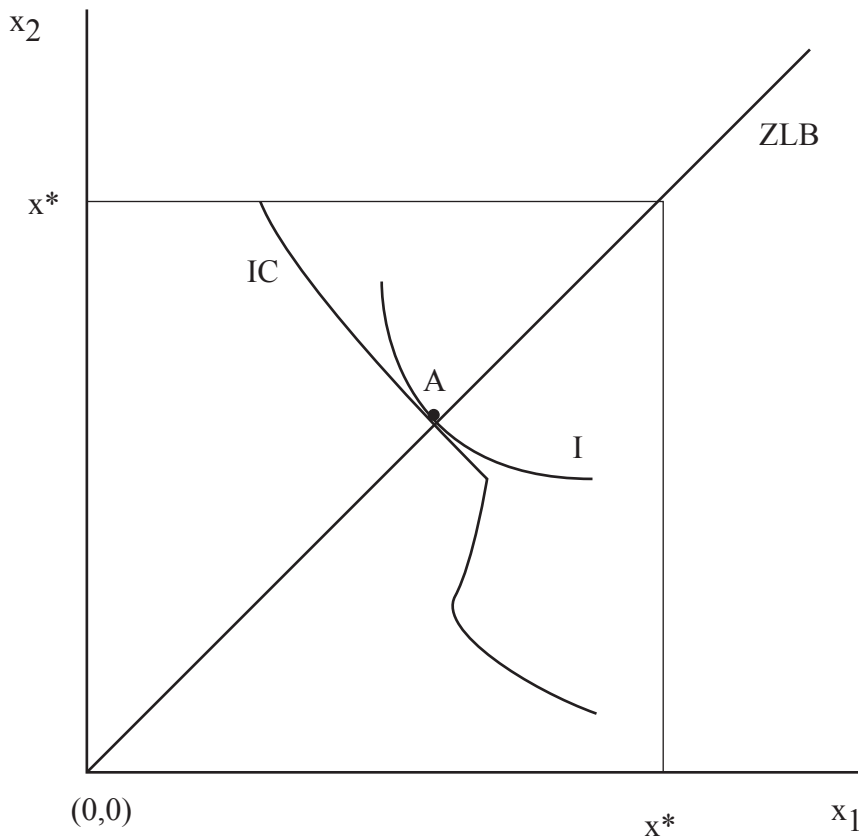


Figure 8: Low Cost of Cheating for the Buyer, Asset Purchases At the Optimum



**Figure 9: Low Cost of Cheating for the Buyer, No Asset Purchases
At the Optimum**

