

Inflation Dynamics and Marginal Costs: the Crucial Role of Hiring and Investment Frictions

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Abstract

We embed convex hiring and investment costs and their interaction in a New Keynesian DSGE model with Nash wage bargaining. We explore the implications with respect to inflation dynamics. We estimate hiring frictions to explain about 60% of the variation in marginal costs, the labor share to explain around 30%, while the remaining 10% is accounted for by intrafirm bargaining. These results have been obtained with moderate total and marginal adjustment costs. Labor market frictions are thus far more important than the labor share in driving marginal costs at business cycle frequencies, in sharp contrast to results in the literature.

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1 Introduction

New Keynesian models posit that inflation dynamics are driven by current and expected future real marginal costs². Because the real marginal cost equals the labor share of income, the labor market is implicitly given a key role in driving inflation. However, the characterization of the labor market abstracts from frictions, which implies that there is no independent role for unemployment as a determinant of inflation over and above that of total hours worked. The absence of unemployment from models of inflation is in stark contrast with its prominence in monetary policy debates and strategies. At the time of writing, central Banks such as the Federal Reserve and the Bank of England have publicly committed to anchor their interest rate policies to the dynamic behavior of the unemployment rate. These monetary strategies make it even more important to achieve a better understanding of the linkages between unemployment rate and inflation rate dynamics.

In recent years, various studies have embedded frictional models of the labor market into the standard New-Keynesian framework to shed light on the channels by which the modelling of unemployment could affect inflation dynamics. Early work by Walsh (2005) and Trigari (2009) sparked some enthusiasm about the possibility that explicitly modelling search frictions could help understand inflation dynamics. Both studies found that accounting for unemployment increases inflation persistence. However, the robustness of these results have been questioned by Gali (2011), while Heer and Maußner (2010), have shown that they disappear when capital is introduced in the model. As shown in Krause, Lopez-Salido and Lubik (2008), the explicit modelling of labor market frictions changes the notion of real marginal costs, and thereby potentially generates an independent role for unemployment to affect inflation. However, these authors find that unemployment is unlikely to affect quantitatively the transmission of shocks to inflation dynamics. Gali (2011) finds similar results, and concludes that such frictions are too small to affect the behavior of marginal costs. So the bottom line is that modelling explicitly unemployment complicates the model without affecting inflation dynamics. Other studies by Krause and Lubik (2007) and Faccini, Millard and Zanetti (2013) have shown that the explicit modelling of search frictions generates an additional undesirable outcome, that is, it breaks the mapping between wage stickiness and inflation persistence that arises in simple versions of the New-Keynesian model with competitive labor markets.

All the aforementioned studies share the common feature that the costs associated with hiring are considered in isolation, abstracting from the interactions with investment dynamics. A large body of evidence, as reviewed by Yashiv (2013), has documented the existence of frictions both in

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²See Woodford (2003), Gali (2008) and Christiano et al (2010) for surveys and discussions.

the capital and in the labor market. Existing work by Merz and Yashiv (2007) and Yashiv (2013) has shown the importance of accounting for their interactions in explaining the joint behavior of hiring and investment as well as the behavior of stock prices. Mumtaz and Zanetti (2012), provide additional evidence in this direction by means of a Bayesian estimation of the general equilibrium model.

In this paper we explicitly model labor and capital frictions, allowing for convex hiring and investment costs to jointly affect marginal costs and explore the implications with respect to inflation dynamics. The specification of the adjustment cost function follows work by Merz and Yashiv (2007) and Yashiv (2013), and allows us to explore the behavior of the model when the frictions operate one at a time and when they interact. Simulating the model for different specifications of the cost function and drawing randomly from a distribution of parameter values, we reach the following conclusions. In the absence of capital frictions we recover the result by Gali (2011), and show that labor market frictions are irrelevant for the response of marginal costs and inflation to technology shocks. However, we show that this result does not arise because search frictions are too small to matter, but because the contributions of the labor share and labor frictions add up nearly to the same response of marginal costs that would be generated in a competitive New-Kenesian model. More importantly, we show that when labor market frictions and capital frictions are jointly active, their interaction has the potential to affect the response of marginal costs and inflation substantially, by boosting the response of real marginal costs to technology shocks. It turns out that when hiring and investment costs are allowed to interact in the adjustment cost function, labor market frictions contribute more than the labor share to explain the impact response of marginal costs. So we conclude that adding capital frictions to the model breaks the irrelevance of labor frictions for inflation dynamics when technology shocks are the only source of fluctuations. While Gali (2011) focuses on technology and monetary shocks, we allow for a variety of shocks in our model. We show that the result that labor market frictions are irrelevant for the impulse response of inflation extends beyond technology and monetary shocks, and also hold when labor supply, preference and investment shocks are a source of fluctuations, provided that hiring costs are the only cost of adjustment. More importantly, we find that allowing for labor and capital frictions to interact breaks this irrelevance result and can be potentially important for the transmission of shocks. Whether it is so in practice, is an empirical question that goes beyond our simulation exercise.

We then take the model to the data and carry out a Bayesian estimation of various specifications of the model, where we restrict the adjustment cost function so as to shut down different frictions one at a time. Looking at the conditional marginal data density for the inflation series, we find that adding separately either labor or capital frictions to the New-Kenesian competitive benchmark helps explain inflation dynamics, by increasing the ability of the estimated model to capture the cross-correlation of inflation with the other series that are made available to the estimation. More interestingly, the likelihood of inflation increases markedly when both capital and labor market frictions are considered jointly, and it increases even more if we account for the complementarity of

hiring and investment. The statistical improvement in explaining inflation dynamics obtained by introducing both hiring and investment adjustment costs is validated by the out-of-sample forecast analysis. At the one-quarter, one-year and two-year horizon, the model that allows for both hiring and investment frictions outperforms the competitive model by short of 10%, and the improvement is significant at conventional levels of confidence.

Importantly, the estimated decomposition of changes in marginal costs in the model with both hiring and capital frictions reveals that the frictional component of marginal costs is far more important than the labor share of income in explaining inflation dynamics at business cycle frequencies. In other words, our model generates a dominant role for labor market frictions in driving inflation dynamics. Because the frictional component of marginal cost is the ratio of marginal profits to the marginal product of labor, our results indicate that the response of profits to changes in the aggregate state of the economy is a key determinant of inflation. Interestingly, the role of profits has been found to be central in any theory that can successfully account for unemployment fluctuations at business cycle frequencies. By incorporating labor frictions into a New-Keynesian model, we are able to extend the importance of fluctuations in profits to inflation dynamics.

Our paper is also related to the strand of literature that investigates the role of intrafirm wage bargaining in macroeconomic models of the labor market. In an application of the bargaining protocol proposed by Stole and Zwiebel (1996) to search and matching models, Cahuc and Wasmer (2001) argue that when firms employ a large number of workers, they should anticipate the impact of their hiring policy on the negotiated wage. We show that accounting for intrafirm bargaining in our New-Keynesian setting alters the notion of marginal costs by introducing an additional term above the labor share of income and the component related to search frictions. Our simulation exercise shows that the contribution of this intrafirm bargaining component to changes in marginal costs is potentially non-trivial. Moreover, in the cross-equation restriction derived from the assumption of capital frictions, intrafirm bargaining affects future marginal costs by making wages respond to the rents associated with the accumulation of capital. As a result of our Bayesian estimation it turns out that the estimated effect of hiring policies on the negotiated wage bill matters little for inflation dynamics, while the hold up problem that arises in the capital market has important implications for the expected behavior of marginal costs.

In what follows Section 2 presents the model, Section 3 the calibration and the simulation exercise, Section 4 the data and the estimation, Section 5 concludes. Technical matters are relegated to appendices.

2 The Model

Our modeling strategy is to introduce Merz and Yashiv (2007)-type of adjustment costs into the simplest New-Keynesian model. Our starting point is therefore the framework developed by Gali (2011), who introduces a very stylized labor adjustment cost function into a simple New-Keynesian

model as a way to capture search and matching frictions.³ We minimize deviations from Gali (2011), from which we depart in the following dimensions: i) we include a larger number of shocks, because unlike Gali (2011), we are also interested in the full estimation of the model; ii) we abstract from the labor force participation margin to streamline the model and the estimation as much as possible; iii) we allow for intrafirm bargaining to affect the negotiated wage, in the spirit of Cahuc and Wasmer (2001). Following Stole and Zwiebel (1996), Cahuc and Wasmer (2001) have argued that when firms are ‘large’, in the sense that they employ a positive measure of workers, and wages are negotiated through Nash bargaining, the firm should anticipate the impact of their hiring policy on the negotiated wage. This modification of the standard bargaining problem is known as intrafirm bargaining. Cahuc, Marque and Wasmer (2008) have shown that accounting for intrafirm bargaining can have important implications for the equilibrium of the labor market. We show that when the framework by Gali (2011), is duly extended to account for intrafirm bargaining, this modification in wage formation affects the notion of marginal costs. In the estimation of the model we will be able to assess whether the role of intrafirm bargaining is quantitatively important for inflation dynamics.

In what follows we look in detail at households, three types of firms, the monetary authority and the aggregate economy. The model features three sources of frictions: price adjustment costs, costs of hiring workers and costs of installing capital. It has five shocks: a shock to the trend growth of technology, a preference shock, an investment shock, a labor supply shock and a monetary policy shock.

2.1 Households

The representative household comprises a unit measure of workers searching for jobs in a frictional labour market. At the end of each time period workers can be either employed or unemployed. The household enjoys utility from the aggregate consumption index C_t and disutility from employment, N_t . Employed workers earn the nominal wage W_t and hold nominal bonds denoted by B_t . Both variables are expressed in units of consumption, which is the numeraire. The budget constraint is:

$$P_t^C C_t + \frac{B_{t+1}}{R_t} = W_t N_t + B_t + \Upsilon_t, \quad (1)$$

where $R_t = (1 + i_t)$ is the gross nominal interest rate, P_t^C is the price of the consumption good and Υ_t is a lump sum component of income which includes dividends from ownership of firms and government transfers.

The labour market is frictional and workers who are unemployed at the beginning of each period t are denoted by U_t^0 . It is assumed that these unemployed workers can start working in the same period if they find a job with probability $x_t = \frac{H_t}{U_t^0}$, where H_t denotes the total number of matches. It follows that the workers who remain unemployed for the rest of the period, denoted by U_t , is

³In fact, Gali (2011) shows that his model with a simple adjustment cost function is isomorphic to a more complicated model where search frictions are endogenized by the introduction of a matching function.

$U_t = (1 - x_t)U_t^0$. Consequently, the evolution of aggregate employment N_t is:

$$N_t = (1 - \delta_N)N_{t-1} + x_t U_t^0, \quad (2)$$

where δ_N is the separation rate.

The intertemporal problem of the households is to maximize the discounted present value of current and future utility:

$$\max E_t \sum_{j=0}^{\infty} \beta^j \vartheta_{t+j} \left(\ln C_{t+j} - \frac{\chi_{t+j}}{1+\varphi} N_{t+j}^{1+\varphi} \right),$$

subject to the budget constraint (1) and the law of motion of employment (2). The parameter $\beta \in (0, 1)$ denotes the discount factor, φ is the inverse Frisch elasticity of labour supply, χ_t is a labor supply shock and ϑ_t denotes a preference shock, which are assumed to follow a first order autoregressive process $\ln \chi_t = \rho_\chi \ln \chi_{t-1} + e_t^\chi$ and $\ln \vartheta_t = \rho_\vartheta \ln \vartheta_{t-1} + e_t^\vartheta$, with $e_t^\vartheta \sim N(0, \sigma_\vartheta)$ and $e_t^\chi \sim N(0, \sigma_\chi)$.

Denoting by λ_t the Lagrange multiplier associated with the budget constraint, the first order conditions with respect to C_t , B_{t+1} , and N_t are:

$$\lambda_t = \frac{\vartheta_t}{P_t^C C_t}, \quad (3)$$

$$\frac{1}{R_t} = \beta E_t \frac{P_t^C C_t}{P_{t+1}^C C_{t+1}} \frac{\vartheta_{t+1}}{\vartheta_t}, \quad (4)$$

$$\mathcal{V}_t^N = \frac{W_t}{P_t^C} - \chi_t N_t^\varphi C_t - \frac{x_t}{1-x_t} \mathcal{V}_t^N + \beta (1 - \delta_N) E_t \frac{\vartheta_{t+1}}{\vartheta_t} \frac{C_t}{C_{t+1}} \mathcal{V}_{t+1}^N. \quad (5)$$

In the competitive benchmark, where employment generates no rents and thus $\mathcal{V}_t^N = 0$, the marginal rate of substitution equals the real wage: $\frac{W_t}{P_t^C} = \chi_t N_t^\varphi C_t$, which determines labour supply.

2.2 Firms

We assume three types of firms: intermediate good producers, final good producers and retailers. Intermediate producers hire labour and invest in capital to produce a homogeneous product, which is then sold to final producers in perfect competition. Final producers transform each unit of the homogeneous product into a unit of a differentiated product facing price rigidities a la Rotemberg (1982). This separation between intermediate and final firms is often assumed to get around the difficulties that arise whenever the bargaining problem and the price setting decisions are concentrated in the same firm.

We also assume that retailers buy a bundle of differentiated goods from the final producers and transform it into homogeneous consumption and investment goods. In turn, these goods are sold

in perfect competition to consumers and intermediate firms, respectively. A key assumption is that the transformation technology may differ in the consumption and in the investment sector, which generates a relative price for investment goods.

Retailers

We assume that there is a continuum of retailers that buy the final output good and converts it into consumption and investment. The final good is a Dixit-Stiglitz aggregator of a bundle of differentiated goods:

$$Y_t = \left(\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)},$$

where ϵ denotes the elasticity of substitution across varieties. Denoting by $P_t^Y(i)$ the price of a variety produced by a monopolistic competitor i , the expenditure minimizing price index associated with the output index Y_t is:

$$P_t^Y = \left(\int_0^1 P_t^Y(i)^{1-\epsilon} di \right)^{1/(1-\epsilon)}.$$

The retailers, which are defined on the zero to one interval in each sector, are assumed to operate under perfect competition. The transformation technology in the consumption sector is $C_t = Y_t^C$, where Y_t^C denotes the amount of final good that is used in the production of the consumption good. The maximization problem of the representative consumption retailer is:

$$\max_{Y_t^C} P_t^C Y_t^C - P_t^Y Y_t^C, \quad (6)$$

which implies that

$$P_t^C = P_t^Y. \quad (7)$$

The transformation technology in the investment sector is $I_t = q_t Y_t^I$, where Y_t^I denotes the amount of final good that is used in the production of the investment good and q_t follows the stochastic process $\ln q_{t+1} = \rho_q \ln q_t + e_q$, and $e_q \sim N(0, \sigma_q)$, where e_q is an investment specific technology shock. The maximization problem reads:

$$\max_{Y_t^I} P_t^I q_t Y_t^I - P_t^Y Y_t^I, \quad (8)$$

which, together with equation (7) implies that $\frac{P_t^I}{P_t^Y} = \frac{P_t^I}{P_t^C} = \frac{1}{q_t}$.

Intermediate producers

A unit measure of intermediate producers sell homogeneous goods to final producers in perfect competition. Intermediate firms combine physical capital, K , and labour, N , in order to produce intermediate output goods, Z . The constant returns to scale production function is

$f(A_t N_t, K_{t-1}) = (A_t N_t)^\alpha K_{t-1}^{1-\alpha}$, where A_t is a labor-augmenting productivity factor that is assumed to follow the stochastic process $\ln(A_t/A_{t-1}) = \rho_a \ln(A_{t-1}/A_{t-2}) + (1 - \rho_a)\mu + e_t^a$, with $e_t^a \sim N(0, \sigma_a)$. Note that μ denotes the economy's growth rate on the balanced growth path. It is assumed that hiring and investment are expensive activities. Hiring costs include advertising, screening and training. Investment costs include installation costs, learning the use of new equipment, etc. All these activities, which imply some disruption in the production process, are captured by the adjustment cost function $g(I_t, K_{t-1}, H_t, N_t)$, where I denotes investment, and H denotes hires. These adjustment costs are thought of as forgone output. Following Merz and Yashiv (2007) and Yashiv (2013) we assume that this function is constant returns to scale and is increasing in each of the firm's decision variables. We also allow for the interaction of investment and hiring adjustment costs. In particular, we assume the following explicit functional form:

$$g(I_t, K_{t-1}, H_t, N_t) = \left[\frac{e_1}{\eta_1} \left(\frac{I_t}{K_{t-1}} \right)^{\eta_1} + \frac{e_2}{\eta_2} \left(\frac{H_t}{N_t} \right)^{\eta_2} + \frac{e_3}{\eta_3} \left(\frac{I_t}{K_{t-1}} \frac{H_t}{N_t} \right)^{\eta_3} \right] (A_t N_t)^\alpha K_{t-1}^{1-\alpha} \quad (9)$$

The net output of a representative firm at time t is:

$$Z_t = f(A_t N_t, K_{t-1}) - g(I_t, K_{t-1}, H_t, N_t).$$

In every period t , the existing capital stock depreciates at the rate δ_K and is augmented by new investment:

$$K_t = (1 - \delta_K)K_{t-1} + I_t, \quad 0 \leq \delta_K \leq 1. \quad (10)$$

Similarly, the number of a firm's employees decreases at the rate δ_N and it is augmented by new hires H_t . The law of motion for employment reads:

$$N_t = (1 - \delta_N)N_{t-1} + H_t, \quad 0 \leq \delta_N \leq 1, \quad (11)$$

where we have assumed that new hires are immediately productive.

At the beginning of each period, firms hire new workers and invest in capital. Next, wages are negotiated following a standard Nash bargaining criterion. When maximizing its market value, defined as the present discounted value of future cash flows, the representative producer anticipates the impact of its hiring and investment policy on the bargained wage. The intertemporal maximization problem of the firm reads as follows:

$$\begin{aligned} \max E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \{ & mc_{t+j} [f(A_{t+j} N_{t+j}, K_{t+j-1}) - g(I_{t+j}, K_{t+j-1}, H_{t+j}, N_{t+j})], \\ & - \frac{W_{t+j}(I_{t+j}, K_{t+j-1}, H_{t+j}, N_{t+j})}{P_{t+j}^C} N_{t+j} - \frac{P_{t+j}^I}{P_{t+j}^C} I_{t+j} \}, \end{aligned}$$

subject to the laws of motion for capital (10) and labour (11), where $\Lambda_{t,t+j} = \beta^j \frac{C_t}{C_{t+j}} \frac{\partial_{t+j}}{\partial_t}$ is the real discount factor of the households who own the firms and $mc_t \equiv P_t^Z / P_t^C$ is the relative price of the intermediate firm's good. This relative price will equal the real marginal cost for a final goods producer since, as discussed later, producers transform one unit of intermediate good into one unit of final good that, in equilibrium, they will sell at $P_t^Y = P_t^C$.

The first-order conditions for dynamic optimality are:

$$\mathcal{Q}_t^K = E_t \Lambda_{t,t+1} \left[\left(mc_{t+1} (f_{K,t+1} - g_{K,t+1}) - \frac{W_{K,t+1}}{P_{t+1}} N_{t+1} \right) + (1 - \delta_K) \mathcal{Q}_{t+1}^K \right] \quad (12)$$

$$\mathcal{Q}_t^K = mc_t g_{I,t} + \frac{W_{I,t}}{P_t^C} N_t + \frac{P_t^I}{P_t^C}, \quad (13)$$

$$\mathcal{Q}_t^N = mc_t (f_{N,t} - g_{N,t}) - \frac{W_t}{P_t^C} - \frac{W_{N,t}}{P_t^C} N_t + (1 - \delta_N) E_t \Lambda_{t,t+1} \mathcal{Q}_{t+1}^N \quad (14)$$

$$\mathcal{Q}_t^N = mc_t g_{H,t} + \frac{W_{H,t}}{P_t^C} N_t, \quad (15)$$

where $\mathcal{Q}_t^K$ and $\mathcal{Q}_t^N$ are the Lagrange multipliers associated with the capital and the employment laws of motion, respectively.

Substituting for $\mathcal{Q}_t^K$ and $\mathcal{Q}_t^N$, the four equations above can be rewritten:

$$\begin{aligned} mc_t g_{I,t} + \frac{W_{I,t}}{P_t^C} N_t + \frac{P_t^I}{P_t^C} &= E_t \Lambda_{t,t+1} \left\{ mc_{t+1} (f_{K,t+1} - g_{K,t+1}) - \frac{W_{K,t+1}}{P_{t+1}} N_{t+1} \right. \\ &\quad \left. + (1 - \delta_K) (mc_{t+1} g_{I,t+1} + \frac{W_{I,t+1}}{P_{t+1}^C} N_{t+1} + \frac{P_{t+1}^I}{P_{t+1}^C}) \right\} \end{aligned} \quad (16)$$

$$\begin{aligned} mc_t g_{H,t} + \frac{W_{H,t}}{P_t^C} N_t &= mc_t (f_{N,t} - g_{N,t}) - \frac{W_t}{P_t^C} - \frac{W_{N,t}}{P_t^C} N_t \\ &\quad + (1 - \delta_N) E_t \Lambda_{t,t+1} \left(mc_{t+1} g_{H,t+1} + \frac{W_{H,t+1}}{P_{t+1}^C} N_{t+1} \right), \end{aligned} \quad (17)$$

which offer two dynamic equations for the final goods producers' real marginal cost.

In the competitive benchmark there are no adjustment costs and there is no room for bargaining. As a result, equation (17) implies that the marginal rate of substitution equals the marginal revenue product of employment: $W_t / P_t^C = mc_t f_{N,t}$, while equation (16) implies that the user cost of capital equals the marginal revenue product of capital:

$$E_t \frac{P_t^I}{P_t^C} R_t / \pi_{t+1} = E_t \left[mc_{t+1} f_{K,t+1} + (1 - \delta_K) \frac{P_{t+1}^I}{P_{t+1}^C} \right]$$

Final good producers

There is a unit measure of monopolistically competitive final good firms indexed by $i \in [0, 1]$. Each firm i transforms $Z(i)$ units of the intermediate good into $Y(i)$ units of a differentiated good, where $Z(i)$ denotes the amount of intermediate input used in the production of good i . Monopolistic competition implies that each final firm i faces the following demand for its own product:

$$Y_t(i) = \left(\frac{P_t^Y(i)}{P_t^Y} \right)^{-\epsilon} Y_t, \quad (18)$$

where Y_t denotes aggregate demand from the retailers.

Following Krause, Lopez-Salido and Lubik (2008), we assume price stickiness à la Rotemberg, meaning firms maximize current and expected discounted profits subject to quadratic price adjustment costs.

Final good firms maximize the following expression:

$$\max E_t \sum_{s=0}^{\infty} \Lambda_{t,t+s} \frac{P_t^Y}{P_{t+s}^Y} \left[(P_{t+s}^Y(i) - P_{t+s}^Y mc_{t+s}) Y_{t+s}(i) - \frac{\zeta}{2} \left(\frac{P_{t+s}^Y(i)}{P_{t+s-1}^Y(i)} - 1 \right)^2 P_{t+s}^Y Y_{t+s} \right],$$

subject to the demand function (18). The first order conditions with respect to $P_t^Y(i)$ and $Y_t(i)$ read as follows:

$$\begin{aligned} Y_t(i) - \zeta \left(\frac{P_t^Y(i)}{P_{t-1}^Y(i)} - 1 \right) \frac{1}{P_{t-1}^Y(i)} P_t^Y Y_t &= \eta_t \varepsilon \frac{Y_t}{P_t^Y} \left(\frac{P_t^Y(i)}{P_t^Y} \right)^{-\varepsilon-1} \\ -\Lambda_{t,t+1} \frac{P_t^Y}{P_{t+1}^Y} \zeta \left(\frac{P_{t+1}^Y(i)}{P_t^Y(i)^2} \right) \left(\frac{P_{t+1}^Y(i)}{P_t^Y(i)} - 1 \right) P_{t+1}^Y Y_{t+1}, \end{aligned} \quad (19)$$

and

$$\eta_t = P_t^Y(i) - P_t^Y \cdot mc_t, \quad (20)$$

where η_t is the Lagrange multiplier associated with eq. (18).

Since all firms set the same price and therefore produce the same output at equilibrium, equations (19) and (20) can be combined to obtain the following law of motion for inflation:

$$\pi_t(1 + \pi_t) = \frac{1 - \varepsilon}{\zeta} + \frac{\varepsilon}{\zeta} mc_t + E_t \frac{1}{1 + r_t} (1 + \pi_{t+1}) \pi_{t+1} \frac{y_{t+1}}{y_t}, \quad (21)$$

where r_t denotes the real interest rate and we have used $E_t \Lambda_{t,t+1} = \frac{1}{(1+i_t)/(1+\pi_{t+1})} = \frac{1}{1+r_t}$, where i_t and r_t denote the net nominal and real interest rates, respectively. Equation (21) specifies that inflation depends on marginal costs as well as expected future inflation. Solving forward equation (21), it is possible to show that inflation depends on current and expected future real marginal

costs.

2.3 Hiring, investment frictions and marginal costs

In order to understand the driving forces of inflation in this model, it is worth solving the FOCs for capital and employment in equations (12) and (14) for real marginal cost. Rearranging the dynamic optimality condition for employment in (14), we get the following expression:

$$mc_t = \frac{\frac{W_t}{P_t^C}}{f_{N,t} - g_{N,t}} + \frac{\frac{W_{N,t}}{P_t^C} N_t}{f_{N,t} - g_{N,t}} + \frac{Q_t^N - (1 - \delta_N) E_t \Lambda_{t,t+1} Q_{t+1}^N}{f_{N,t} - g_{N,t}}. \quad (22)$$

The first term in the above equation is the real unit labour cost, expressed as the ratio of real wages to the net marginal product of labour. Because the production function is Cobb-Douglas, in the competitive benchmark where $g_{N,t} = 0$, the unit labor cost equals the labor share of income $W_t N_t / P_t Y_t$. The second term is a correction for intra-firm bargaining. Since the marginal product of labour is decreasing with the size of the firm, the marginal worker will decrease the marginal product of labour and the wage bargained by all the intra-marginal workers. Correctly anticipating the effect of hiring policies on the negotiated wage bill generates an incentive to increase hiring. In our model, this intrafirm bargaining effect has an impact on the marginal cost: the marginal cost of expanding output by raising employment, decreases with the negative effect of firm size on the negotiated wage bill. In a model where the labor market is competitive such an effect is absent because there is no room for bargaining.

The third term shows that with frictions in the labor market, marginal costs depend on expected changes in the value of employment, a point already made by Krause, Lopez-Salido and Lubik (2008). Because employment is a valuable asset, the cost of expanding output at the margin by increasing employment has to be netted out of the expected changes in its value. In this model, changes in the marginal value of employment are driven by changes in marginal hiring cost evaluated at the shadow value of output, and by changes in the impact of the marginal hire on the negotiated wage bill, see equation (15). When the labor market is competitive, the value of employment is always zero, so the third term in equation (22) generates no dynamics.

There are reasons to believe that this component might have potentially important implications for marginal costs. From equation (14) it is possible to notice that the numerator of this expression is the flow profit of a match. A large literature that started following the work by Shimer (2005), has developed a range of models to overcome the inability of the standard textbook search and matching model to match the volatility of unemployment at business cycle frequencies. While there is yet no consensus on what model is most appropriate to explain unemployment dynamics, frictional models of the labor market can account for the volatility of unemployment only if the volatility of marginal profits is very high.

In our model marginal costs are also related to the dynamics of capital. Rearranging equation

(12) to solve for expected real marginal cost next period, one gets the following expression:

$$E_t mc_{t+1} = E_t \frac{\frac{W_{K,t+1}}{P_{t+1}} N_{t+1}}{A_{t+1} f_{K,t+1} - g_{K,t+1}} + \frac{1}{\Lambda_{t,t+1}} E_t \frac{\mathcal{Q}_t^K - \Lambda_{t,t+1} (1 - \delta_K) \mathcal{Q}_{t+1}^K}{A_{t+1} f_{K,t+1} - g_{K,t+1}}. \quad (23)$$

The first term in equation (23) is a correction term for intrafirm bargaining. Higher capital makes workers more productive, thereby increasing the expected marginal product of labour and the bargained wage. In equation (12), the presence of this term reflects a typical hold-up problem: because workers appropriate parts of the rents generated by employment, the capital effect on wages decreases the value of capital, leading to under-investment. Rearranging equation (12) to solve for the expected marginal cost, we can look at the effect of capital on wages as a determinant real marginal cost: the marginal cost of expanding production by raising capital increases with the positive effect of capital on wages.

The second term shows that expected changes in Tobin's Q will also affect real marginal cost: the marginal cost of expanding output by raising capital is netted out of the expected gains in the value of capital. In the competitive benchmark, the term related to intrafirm bargaining washes out, as there is no room for bargaining. But the second term still remains, and drives the dynamics of marginal costs to the extent that expected changes in the relative price of the investment good generate expected gains or losses in the value of capital. Adjustment costs will give rise to richer dynamics in the value of capital, which will be driven, as dictated by equation (13), not only by changes in the price of investment, but also by changes in marginal investment costs, evaluated at the shadow value of output, and by the impact of investment on the negotiated wage bill.

It is worth noting that hiring and investment frictions interact in general equilibrium, even absent interactions within the adjustment cost function. Replacing (15) into (13) we get:

$$\mathcal{Q}_t^K = \frac{\mathcal{Q}_t^N - \frac{W_{H,t}}{P_t^C} N_t}{g_{H,t}} g_{I,t} + \frac{W_{I,t}}{P_t^C} N_t + \frac{P_t^I}{P_t^C},$$

while substituting (13) into (15) we obtain:

$$\mathcal{Q}_t^N = \frac{\mathcal{Q}_t^K - \frac{W_{I,t}}{P_t^C} N_t - \frac{P_t^I}{P_t^C}}{g_{I,t}} g_{H,t} + \frac{W_{H,t}}{P_t^C} N_t.$$

Quite clearly, the value of capital and employment, which reflect the existence of investment and hiring frictions as per equations (13) and (15), directly affect each other.

2.4 Wage Bargaining

Wages are assumed to maximize a geometric average of the household's and the firm's surplus weighted by the parameter γ , which denotes the bargaining power of the households:

$$W_t = \arg \max \left\{ (\mathcal{V}_t^N)^\gamma (\mathcal{Q}_t^N)^{1-\gamma} \right\}, \quad (24)$$

The first order condition to this problem leads to the Nash sharing rule:

$$(1 - \gamma)\mathcal{V}_t^N = \gamma\mathcal{Q}_t^N. \quad (25)$$

Substituting (5) and (14) into the above equation and using the sharing rule (25) to eliminate the terms in $\mathcal{Q}_{t+1}^N$ and $\mathcal{V}_{t+1}^N$ one gets the following expression for the real wage:

$$\frac{W_t}{P_t^C} = \gamma mc_t (f_{N,t} - g_{N,t}) - \gamma \frac{W_{N,t}}{P_t^C} N_t + (1 - \gamma) \left[\chi_t C_t N_{it}^\varphi + \frac{x_t}{1 - x_t} \frac{\gamma}{1 - \gamma} \mathcal{Q}_t^N \right]. \quad (26)$$

Assuming a Cobb-Douglas production function and the adjustment cost function in (9), the solution to the differential equation in (26) reads as follows:

$$\begin{aligned} \frac{W_t}{P_t^C} = mc_t A_t^\alpha K_{t-1}^{1-\alpha} & \left\{ \alpha \left[1 - \frac{e_1}{\eta_1} \left(\frac{I_t}{K_{t-1}} \right)^{\eta_1} \right] A_1 N_t^{\alpha-1} + \left(1 - \frac{\alpha}{\eta_2} \right) e_2 H_t^{\eta_2} A_2 N_t^{\alpha-1-\eta_2} \right. \\ & \left. + \left(1 - \frac{\alpha}{\eta_3} \right) e_3 \left(\frac{H_t I_t}{K_{t-1}} \right)^{\eta_3} A_3 N_t^{\alpha-1-\eta_3} \right\} \\ & (1 - \gamma) \left[\chi_t C_t N_{it}^\varphi + \frac{x_t}{1 - x_t} \frac{\gamma}{1 - \gamma} \left(mc_t g_{H,t} + \frac{W_{H,t}}{P_t^C} N_t \right) \right], \end{aligned} \quad (27)$$

where A_1 , A_2 , and A_3 are parameters, which are reported in the Appendix A.2 together with the full derivation.

Notice that in the special case in which workers have no bargaining power, i.e., $\gamma = 0$, the real wage equals the marginal rate of substitution between consumption and leisure, as in the standard New-Keynesian model with a perfectly competitive labour market.

2.5 Aggregation

Aggregating output demand in units of the consumption good implies the following relationship between the consumption and investment demand from the representative retailer and aggregate supply of the final good:

$$C_t + \frac{P_t^I}{P_t^C} I_t = Y_t = \left(\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)}. \quad (28)$$

Aggregating on the supply side of the economy implies the following relationship between final goods and intermediate inputs:

$$Z_t = f(A_t N_t, K_{t-1}) - g(I_t, K_{t-1}, H_t, N_t) = \int_0^1 Y_t(i) di = Y_t \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} di = Y_t, \quad (29)$$

where $\int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} di = 1$ since with Rotemberg pricing there is no price dispersion in equilibrium.

Combining the expressions in (28) and (29) implies that:

$$C_t + \frac{P_t^I}{P_t^C} I_t = f(A_t N_t, K_{t-1}) - g(I_t, K_{t-1}, H_t, N_t). \quad (30)$$

2.6 The Monetary Authority

The monetary authority sets the nominal interest rate following the Taylor rule:

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\rho_r} \left[\left(\frac{1 + \pi_t}{1 + \pi^*} \right)^{r_\pi} \left(\frac{\tilde{y}_t}{\tilde{y}^*} \right)^{r_y} \right]^{1-\rho_r} \xi_t, \quad (31)$$

where $\tilde{y}_t = \log(Y_t/A_t)$, π_t measures the rate of inflation of the consumption good, and an asterisk superscript denotes the steady-state values of the associated variables. The parameter ρ_r represents interest rate smoothing, and r_y and r_π govern the response of the monetary authority to deviations of output and inflation from their steady-state values. The term ξ_t captures a monetary policy shock, which is assumed to follow the process $\ln \xi_t = \rho_\xi \ln \xi_{t-1} + e_t^\xi$, with $e_t^\xi \sim N(0, \sigma_\xi)$.

3 Hiring and Investment Frictions and Inflation Dynamics: An Impulse Response Analysis

This section investigates whether hiring and investment frictions can potentially affect the transmission of shocks to inflation dynamics. In order to do so, we compare how marginal cost and inflation respond to a variety of shocks in the model with unrestricted adjustment costs and in alternative versions obtained by shutting down one friction at a time. If neither hiring or investment frictions mattered for inflation dynamics, we would observe that for any given parameterization of the model, any shock would generate the same response of marginal costs and inflation, independently of the precise nature of adjustment costs.

So, to test for the irrelevance of frictions for the transmission of shocks to inflation dynamics, we simulate the impact impulse responses of marginal costs and inflation for various parameterizations of the model and different specifications of the adjustment cost function. To generate a range of plausible parameterizations, we start this Section by calibrating the steady-state equilibrium of

the model that allows for interactions between hiring and investment costs. Given this benchmark parameterization, we can define stochastic distributions for the parameters that are centered around the calibrated values. For each parameterization, we compute impulse responses for the benchmark model with unrestricted adjustment costs, and for the restricted specifications. Specifically, we obtain the version of the model that excludes interactions between hiring and investment costs, by restricting $e_3 = 0$, everything else equal. Similarly, the model with hiring frictions only is obtained by restricting both $e_1 = 0$ and $e_3 = 0$, the model with investment frictions only is obtained by restricting both $e_2 = 0$ and $e_3 = 0$, while the competitive neoclassical benchmark corresponds to the case of $e_1 = e_2 = e_3 = 0$. For each of the five shocks in our model, namely shocks to technology, labor supply, investment, preferences and monetary policy, we simulate impulse responses for the restricted and unrestricted models, based on 5,000 random parameterizations. We then compute averages and standard deviations for the impact responses of inflation, marginal costs and its sub-components generated by each specification of the model and inspect whether allowing for various types of frictions makes any difference.

3.1 Calibration

We calibrate the parameters that affect the stationary equilibrium of the model using two sources of information: some are either normalized or set using *a priori* information, while the remaining ones are selected so as to match U.S. data.

Table 1

The discount factor β equals 0.99 implying a quarterly interest rate of 4%. The quarterly job separation rate δ_N is set 0.13 as in Yashiv (2013) and measures separations from employment into either unemployment or inactivity. The capital depreciation rate δ_K is set to 0.02, to match a quarterly investment/capital ratio of 0.024. The inverse Frisch elasticity φ is set equal to 3, in line with the range of estimates by Domeji and Floden (2006) and in between the value of 5 used in Gali (2010) and the more standard value of 1 as in Christiano Eichenbaum and Trabandt (2013a) among many others. The elasticity of demand is set to 11, implying a steady-state markup of 10% as in Krause, Lopez Salido and Lubik (2008).

The exponential terms η_1 , η_2 and η_3 in the adjustment cost function are set to equal 2, 2 and 1, respectively, as in Yashiv (2013). The parameter capturing trend growth in labour augmenting technological progress μ is set to equal 0.454%, which is the quarterly growth rate of our measure of output used in the estimation.

This leaves us with five parameters to calibrate, the elasticity of output to the labour input α , the bargaining power γ and three parameters in the adjustment cost functions, e_1 , e_2 , and e_3 . These parameters are calibrated to match: i) a labor share of $2/3$;⁴ ii) an unemployment

⁴In this model the elasticity of output to the labor income does not correspond exactly to the labor share of income, but these two values are close at the calibrated stationary equilibrium.

rate of approximately 10%. This value is approximately equal to the average of the expanded unemployment series measured by Hall (2005), which was constructed to include inactive workers who are marginally attached to the labor force, consistently with our measure of the separation rate; iii) marginal hiring costs, $g_H/(f/N)$, equal to 0.25, approximately equal to four weeks of wages; iv) marginal investment costs, $g_I/(f/K)$, equal to 0.8; v) total adjustment costs equal to 2.5% of output. As discussed extensively in Yashiv (2013), our calibration of adjustment costs is conservative, in the sense that the target values for marginal and total adjustment costs lie at the lower end of the spectrum of estimates reported in the literature.

It is worth noting that our calibration allows labor frictions to account for a higher share of marginal costs than Gali (2011) and Krause, Lopez-Salido and Lubik (2008). Both studies assume that average and marginal hiring costs equal nearly 5% of quarterly wages, following empirical evidence by Silva and Toledo (2009). This figure is reasonable if hiring costs are interpreted only as the costs incurred in posting vacancies. Our functional form for adjustment costs allows for hiring costs to be interpreted in a wider sense, which also includes training as well as any other source of forgone output associated with hiring. As reported by Silva and Toledo (2009), these costs are an order of magnitude higher than job advertisement costs. In our calibration we prefer to err on the conservative side, and assume that real marginal hiring costs are approximately only 25% of the average product of labor. As discussed extensively in Yashiv (2013), this value lies at the lower end of plausible estimates in the empirical literature. This calibration target for real marginal costs translates into marginal recruiting costs that equal nearly 36% of quarterly wages, about one month of wages.

3.2 Simulation

Table 2 reports the stochastic distributions for the parameter values used in the simulation of the model. The distributions of some parameter values that affect the steady state are relatively tight around the calibrated mean to ensure that the non-linear steady-state solver always converges to a sensible root. For the same reason, the discount factor, the job separation rate and the depreciation rate of capital are fixed to their calibrated mean. We adopt wider distributions for those parameters that do not change the steady-state. All parameters are uniformly distributed.

Table 2

The model has five shocks, but we focus first on technology shocks to compare with the calibration exercise presented by Gali (2011). Table 3 reports the average response of inflation, marginal costs and its components to a 1% temporary increase in the growth rate of technology. The table implies the following key results: first, having only hiring adjustment costs on top of the benchmark competitive model leaves the response of marginal costs and inflation virtually unaffected as seen in comparing the L-Gali and L models to the Competitive model. This is akin to the result found

by Gali (2011), although our interpretation is different. Gali (2011) argues that because search frictions are small relative to the labor share of income, then they are unlikely to contribute to fluctuations in marginal costs. The results of the simulations reported in Table 3 show that the contributions of both the frictional and the intrafirm bargaining components are substantial: on average, the frictional component accounts for nearly 30% of the impact response in marginal costs and the intrafirm bargaining component accounts for almost 20%, while the labor share explains around 50%. These contributions of the L model differ from the L-Gali model due to different calibration.

The results indicate that in a model with hiring frictions, both the labor share and the frictional components increase following a positive technology shock, while the intrafirm bargaining component responds negatively.⁵

Thus, because the response of these three components adds up nearly to the same response that we observe in the competitive benchmark, hiring frictions overall do not matter for the transmission of technology shocks to inflation.

It is also worth noting that the result that labor frictions matter in the decomposition of marginal costs is only partially driven by our broader interpretation of labor adjustment costs. In Table 3 we show – for the L-Gali model – the contribution of labor frictions to impact changes in marginal costs derived by calibrating the model with only labor frictions following Gali (2011). The table shows that the average contribution of profits to changes on impact in marginal costs is 22%, which is lower than in our parameterization, the L model, but not negligible.

Second, introducing only investment adjustment costs in the competitive New-Keynesian model, i.e. restricting only $e_2 = e_3 = 0$ but allowing for $e_1 > 0$, does affect the behavior of marginal costs and inflation: both variables respond more on impact, relative to the competitive benchmark, as shown in the K-model.

Third, introducing both investment and hiring frictions into the model, and abstracting for now from interactions in the adjustment cost function, i.e. restricting only $e_3 = 0$, magnifies the impact response of marginal costs and inflation relative to the case where only capital frictions are active, as shown in the KL model. This reveals that the effect of capital and labor frictions on marginal costs and inflation is not additive, but generates interesting interactions in general equilibrium, even if they do not interact in the adjustment cost function. Furthermore, it is worth noting that introducing both hiring and investment adjustment costs increases the average response of labor frictions and its importance in driving marginal costs relative to the case in which only hiring frictions are active. Going further and introducing interactions between capital and labor frictions in the adjustment cost function – the KL interactions model – magnifies this effect and now frictions induce a bigger response than the labor share.

⁵Because Table 3 reports the percentage contribution of labor share, frictions and intrafirm bargaining to impact changes in marginal costs, it conceals the sign of the impulse responses. Our simulation exercise shows that for every draw from the parameter space in Table 2, a positive technology shock always induces a positive response of labor share and frictions, and a negative response of the intrafirm bargaining component.

We can summarize our numerical findings from the simulations in the following conclusions:

Conclusion 1 *Introducing only investment adjustment costs in a perfectly competitive New-Keynesian model can produce quantitatively substantial effects on the transmission of technology shocks to marginal costs and inflation; introducing only hiring adjustment costs only can not.*

Conclusion 2 *Allowing for the coexistence of both hiring and investment adjustment costs but not for their interactions in the adjustment cost function, magnifies the impact response of marginal costs and inflation to technology shocks relative to the competitive benchmark and to the case where only investment frictions are active.*

Conclusion 3 *Allowing for both hiring and investment adjustment costs but not for their interactions in the adjustment cost function, increases the importance of labor frictions relative to the labor share as a driver of inflation on the impact of a technology shock relative to the case where only labor frictions are active.*

Conclusion 4 *Allowing for interactions between hiring and investment costs in the adjustment cost function magnifies both the impact of technology shocks on marginal costs and inflation and the importance of labor frictions as a determinant of changes in real marginal costs.*

Table 3

Moving beyond technology shocks, we simulate impulse responses for marginal costs and inflation to monetary, labor supply, preference and investment shocks. This is shown in Tables 4-7.

Tables 4-7

Inspecting the tables we draw the following conclusions:

Conclusion 5 *Introducing only investment adjustment costs in a perfectly competitive New-Keynesian model can produce quantitatively substantial effects on the transmission of monetary, labor supply, preference and investment shocks to marginal costs and inflation; introducing only hiring adjustment costs only can not.*

Conclusion 6 *Following a monetary, labor supply, preference or investment shock, the contribution of labor frictions to impact changes in marginal costs is substantial in any specification of the model that allows for hiring adjustment costs.*

Conclusion 7 *Allowing for both hiring and investment frictions can substantially affect the transmission of shocks to inflation dynamics relative to the case where only one of these frictions is active, except in the case of labor supply shocks.*

The exercises above show that in our model hiring and investment frictions have the potential to affect the transmission of shocks to inflation dynamics. Whether they do so in a way that helps explain inflation is an empirical question, to which we turn in the following Section.

4 Bayesian Estimation of the Full Model

We estimate the model with Bayesian methods.⁶ First, we take a first-order approximation of the system of equations around a deterministic steady-state with zero inflation. We then solve the model and apply the Kalman filter to evaluate the likelihood function of the observable variables. The likelihood function and the prior distribution of the parameters are combined to obtain the posterior distributions. The posterior kernel is simulated numerically using the Metropolis-Hasting algorithm.

We estimate the model using the minimum set of observables and shocks that are required to assess whether hiring and investment adjustment costs help explain and predict inflation dynamics in the context of New Keynesian models. Inflation, output and the interest rate, are typically considered the very minimum set of observables when estimating any model of inflation dynamics. Given that our theoretical mechanism affects marginal costs through the interplay of hiring and investment dynamics, we add series of gross investment and hiring flows to this set of observables. To keep both the model and the estimation as simple as possible, we have abstracted from the government sector. Our dataset will therefore include only observables pertaining to the private sector in the spirit of Gali (1999), thus not confounding the analysis with government output, inflation, hiring and investment.

This section begins by describing the data used for the analysis, and next discusses the choice of priors. Finally, we present our results in three steps: first we compare how different versions of the model, obtained by restricting various combinations of the adjustment cost parameters, perform in terms of fitting inflation, looking at the conditional marginal data density of this series. We show that the version of the model that allows for interactions between hiring and investment adjustment costs statistically outperforms the other restricted specifications of the model, including the benchmark competitive benchmark. We then compare our preferred specification of the frictional model and the perfectly competitive benchmark in terms of out-of-sample forecast performance. Finally, we illustrate the estimated decomposition of marginal costs in our preferred model specification by breaking down the contribution of its three components: labor share, search frictions and intrafirm bargaining.

4.1 The Data

The model is estimated on quarterly US data over the period 1976 Q1 to 2011 Q4. The data pertain to the U.S. private sector and have been downloaded from the Federal Reserve Economics Data (FRED) set. We make the following mapping between model variables and observable data series: output growth in the model is the rate of growth of non-farm business sector output scaled by civilian non-institutional population; inflation is the corresponding implicit price deflator; the interest rate is the effective federal funds rate; investment growth corresponds to the growth of real

⁶See An and Schorfheide (2007) for a discussion.

gross private domestic investment scaled by civilian non-institutional population; the hiring rate is gross hiring flows scaled by civilian non-institutional population.⁷ The hiring rate, the inflation rate and the interest rate are demeaned prior to estimation.

4.2 Bayesian Estimation

We use five shocks to match the behavior of our five observable series: a preference shock, a labor augmenting technology shock, a monetary policy shock, a labor supply shock and an investment technology shock. All shocks are assumed to follow a first-order autoregressive process with i.i.d. normal errors, with the exception of the labor augmenting technology shock, which features a unit root.

The model contains 26 structural parameters, excluding the shock parameters. For the parameters that affect the steady state, the prior mean of the estimated parameters and the value of the parameters that are kept fixed in the estimation, is assigned following the calibration discussed in Section 3.1. Turning to the remaining parameters that have no impact on the stationary equilibrium, we set the coefficients on the Taylor rule following Gali (2011): the response to inflation and output is 1.5 and 0.125, respectively – while the degree of interest rate smoothing captured by the parameter ρ_r is set to equal 0.75. The parameter governing price stickiness is set to equal 120, also following Gali’s (2011) calibration, and the standard deviation around the prior is the same as in Krause, Lopez-Salido and Lubik (2008). The slope of the Phillips curve implied by our calibration is the same as in a Calvo model where the average duration of prices is one year. This value is in line with microdata evidence reported by Nakamura and Steinsson (2008) on the mean and median duration of prices that exclude sales, ranging between 9.0 to 11.7 months for the mean, and with the macro-estimates in Christiano Eichenbaum and Trabandt (2013b, page 25).⁸

The prior value for the autocorrelation coefficients of the shocks is very loose around 0.5.

In terms of distributions for our priors, we use the beta distribution for priors that take values between zero and one, the gamma distribution for parameters restricted to be positive and the inverse gamma distribution for the standard deviation of the shocks. Table 4 reports the parameters that are kept fixed during the estimation. Global sensitivity analysis following Iskrev (2010) reveals that the parameter e_3 is not identified. Similarly, the elasticity of substitution ε and the coefficient of price stickiness ζ are not jointly identified. Hence we keep both e_3 and ε fixed in the estimation.

⁷The classification codes of the series used in the estimations are the following: FEDFUNDS for the effective federal funds rate, IPDNBS for the price deflator, OUTNFB for output, GPDIC96 for investment, and CNP16OV for the population series. The computation of the hiring series first builds on the flows between E (employment), U (unemployment) and N (not-in-the-labor-force) that correspond to the E,U,N stocks published by CPS. The methodology of adjusting flows to stocks is taken from BLS, and is given in Frazis et al (2005). This methodology, applied by BLS for the period 1990 onward, produces a dataset that appears in http://www.bls.gov/cps/cps_flows.htm. Here the series have been extended back to 1976.

⁸We have checked that our result that adjustment cost matter for inflation is robust to using a prior for ζ equal to 62, which generates a Phillips curve equivalent to a Calvo model where average price duration is three quarters, as in Gertler, Sala and Trigari (2008).

A key feature of our estimation strategy is that we select priors of the adjustment cost function to be tight around the calibrated mean. A reason why we do so is because we are interested in understanding whether conservative parameterizations of the adjustment costs can help explain and predict inflation in the context of New-Keynesian models, so we want to enforce posterior estimates of adjustment costs to remain on the lower end of the range of reasonable values.

This strategy allows us to directly confront the findings here with the results by Gali (2011), whose main point was that adjustment costs are too small to matter for inflation dynamics. For the same reason, as summarized in Table 4, we keep fixed in the estimation a number of parameters that have a direct impact on the steady-state, and could therefore change the size of adjustment costs relative to output. We can think about these parameters as being estimated with infinitely tight priors. A technical reason for either fixing or selecting tight priors for the parameters affecting the steady-state is that doing so ensures convergence of the steady-state non-linear solver, which is only possible if the starting values are not too far from the solution. So for these reasons, the workers' separation rate δ_N and the capital depreciation rate δ_K are fixed, as in Gertler, Sala and Trigari (2008), and the priors around the bargaining power parameter γ and trend output growth μ , are tight. We select instead looser priors for the parameters that do not affect the steady-state, such as the coefficients of the Taylor rule. We have also estimated versions of the model where we estimate a larger number of parameters and/or assume looser priors on the parameters affecting the steady-state. Going for this alternative estimation strategy tends to increase the ability of adjustment costs to explain inflation dynamics. However, these estimations imply larger adjustment costs and hence are less suited to address Gali's (2011) critique.

Table 8 reports the values of the parameters that are kept fixed in the estimation, while tables 9 and 10 fully characterize the priors used for the estimation as well as the posterior estimates.⁹ Overall, the posterior estimates of the parameters appear to be tightly estimated across all specifications. An interesting feature of our results is that all models estimate a very inelastic labor supply, in line with estimates based on micro data. *Another* result that is common across models is that the degree of interest rate smoothing in the Taylor rule is estimated to be lower than the prior mean.

Models with adjustment costs, tend to estimate a larger labor share of income than assumed in the calibration, around 75% in the specification that allows for interactions between hiring and investment adjustment costs. This order of magnitude is the same as estimated in the New-Keynesian model with search frictions by Christiano, Eichenbaum and Trabandt (2013a).

Other findings that are in line with the latter paper results, are that models with adjustment costs estimate a low response of monetary policy to changes in output, and a small sensitivity of interest rates to output in the Taylor rule. Turning to the shocks, all models estimate that

⁹Each estimation of the model is based on five parallel chains, each one consisting of 250,000 draws from the Metropolis algorithm, half of which are discarded as burn-in. Brooks and Gelman (1998) diagnostics provide evidence on convergence. Acceptance rates for all the models were between 25 and 35 percent.

preferences and labor supply shocks are highly persistent, while shocks to trend growth are not. Investment shocks are instead estimated to be more persistent in those specifications that assume some form of investment adjustment cost.

Tables 8-10

4.3 Results

We now evaluate whether adjustment costs affect marginal costs in a way that is useful to explain inflation, and to disentangle what specific form of friction matters. To this end, we first estimate the full model described in Section 2 using the priors and the shocks discussed above. Next, using the same shocks, the same observables and the same priors for all the parameters, we estimate versions of the same model, where we restrict the parameters e_1 , e_2 , and e_3 so as to shut down one friction at a time. We estimate four versions of the model with adjustment costs: the unrestricted model, the model that shuts down interactions between hiring and investment costs in the adjustment cost function, $e_3 = 0$, the model that allows only for hiring costs, restricting $e_1 = e_3 = 0$, and the model that allows only for investment costs, restricting $e_2 = e_3 = 0$. We will denote these four models with $\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{M}_3$, and $\mathcal{M}_4$, respectively. Each of these four versions is compared to the benchmark competitive New Keynesian model, obtained by restricting $e_1 = e_2 = e_3 = 0$ and denoted by $\mathcal{M}_{comp}$.

Table 11 shows the estimated marginal and total adjustment costs for different specifications of the adjustment cost function, evaluated at the posterior means reported in Tables 9 and 10.

Table 11

4.3.1 Marginal Data Density and Out of Sample forecasts

Following Fernandez-Villaverde and Rubio-Ramirez (2004) we rely on the marginal data density (MDD) as the measure of fit. The MDD is computed for each model using the modified harmonic mean estimator introduced by Geweke (1999). Considering that this criterion penalizes overparametrization, models with less restrictions on the adjustment costs function would rank better only if the extra parameters are informative in explaining the data. The relative fit of inflation across models is then assessed as in Neri and Ropele (2012) by computing the conditional marginal data density for the inflation rate, which is the likelihood of observing the inflation rate series, conditional on the entire set of observables used in the estimation and the various specifications of the model $\mathcal{M}_j$ for $j \in \{1, 2, 3, 4, comp\}$.

We denote the set of observables excluding inflation as D_T and the inflation series as Π_T . It is possible to show that the conditional MDD of inflation for a model with some form of adjustment costs $\mathcal{M}_j$ is:

$$p(\Pi_T | D_T, \mathcal{M}_j) = \frac{p(D_T, \Pi_T | \mathcal{M}_j)}{p(D_T | \mathcal{M}_j)}, \quad j \in \{1, 2, 3, 4\},$$

where the numerator of the ratio on the right hand side is the MDD of the adjustment cost model estimated on the full sample, and the denominator is the MDD of the same model estimated on the restricted sample (the full sample less inflation). Similarly, the conditional MDD of inflation in the competitive New-Keynesian model is:

$$p(\Pi_T | D_T, \mathcal{M}_{comp}) = \frac{p(D_T, \Pi_T | \mathcal{M}_{comp})}{p(D_T | \mathcal{M}_{comp})}.$$

The conditional MDD can be written as:

$$p(\Pi_T | D_T, \mathcal{M}_j) = \int p(\Pi_T | \Theta, D_T, \mathcal{M}_j) p(\Theta | D_T, \mathcal{M}_j) d\Theta,$$

where $p(\Pi_T | \Theta, D_T, \mathcal{M}_j)$ is the conditional likelihood and $p(\Theta | D_T, \mathcal{M}_j)$ is the conditional distribution of the parameters Θ .

Table 12 reports the results.

Table 12

As shown in the table, the conditional MDD **of inflation** in the competitive model is 335 log points. Introducing investment and hiring adjustment costs separately increases the conditional MDD to 445 and 449 log points, respectively. As we discuss below, the improvement of 110 and 114 log points respectively, is very strong evidence that both hiring and investment frictions considered independently, help the New-Keynesian model fit inflation. Interestingly, the model with hiring frictions only has a much higher likelihood than the competitive benchmark, even if the impulse responses of inflation generated by the two models is virtually the same for all shocks, as shown in Section 3.1. This result arises because the model with hiring frictions only is better able to capture the cross-correlations of the inflation series with the other variables made observables to the estimation.

When both frictions are allowed to operate jointly, the conditional MDD of inflation jumps to 492 log points, an improvement of 157 points with respect to the competitive benchmark. Allowing for complementarities between hiring and investment in the adjustment cost function increases the conditional marginal data density to 504 log points, an increment of 12 log points with respect to the model that restricts only the interaction term e_3 to equal zero. Such a difference corresponds to an odds ratio of e^{12} in favor of the unrestricted model, and provides significant evidence that accounting for interactions in the adjustment cost function improves the fit of inflation. Based on this analysis, we select the unrestricted model as our preferred specification of adjustment costs.

If we now compare the MDD of the models estimated on the full sample, which is the likelihood of observing **the whole set of observables** conditional on a model $\mathcal{M}_j$, we see that adding hiring and investment frictions separately increases the MDD from 2118 to 2272 and 2210, respectively. This implies that introducing either hiring or investment frictions into a competitive benchmark

helps explain both inflation and the joint behavior of our five observable series, hiring, investment, inflation, output and the interest rate. Allowing for hiring and investment adjustment costs to operate jointly increases the MDD of the model estimated on the full sample even further, to 2396 log points, while allowing for interactions between hiring and investment costs adds another 9 log points to the overall fit of the model.

To provide some intuition on how large the estimated differences in log-points are, we follow Melosi (2013). We perform a Bayesian test of the null hypothesis that a model with adjustment cost is at odds with the behavior of inflation by comparing the conditional MDD associated with a particular specification of the adjustment cost and with the competitive model. As shown by Shorfheide (2000), under a 0-1 loss function the null is rejected if the adjustment cost model has a larger posterior probability than the competitive model. The posterior probability of a model with adjustment costs denoted by $p(\mathcal{M}_j|\Pi_T)$ is:

$$p(\mathcal{M}_j|\Pi_T) = \frac{p(\Pi_T|D_T, \mathcal{M}_j) p_0(\mathcal{M}_j)}{p(\Pi_T|D_T, \mathcal{M}_j) p_0(\mathcal{M}_j) + p(\Pi_T|D_T, \mathcal{M}_{comp}) p_0(\mathcal{M}_{comp})},$$

where $p(\Pi_T|D_T, \mathcal{M}_j)$ and $p(\Pi_T|D_T, \mathcal{M}_{comp})$ are the conditional MDD defined above, while $p_0(\mathcal{M}_j)$ and $p_0(\mathcal{M}_{comp}) = 1 - p_0(\mathcal{M}_j)$ are the prior probabilities in favor of the two models. The null *can be rejected* so long as the prior probability in favor of the hiring cost only and investment cost only are not less than 3.1E-50 and 1.7E-48, respectively. Similarly, for the unrestricted model, the null *can be rejected* so long as the prior probability in its favor is no less than 4.1E-74. So, one can conclude that *the null cannot be rejected* only if one's prior is *virtually closed* to the possibility that any form of adjustment cost model be statistically superior. Similarly, *one cannot reject the null* that the interaction term does not help fit inflation if the prior in favor of the unrestricted model and against the specification restricting $e_3 = 0$ is no less than 6E-6, which is substantial evidence in favor of the unrestricted model.

As an additional check for whether adjustment costs can help explain inflation in the context of New Keynesian models, we estimate the competitive and the unrestricted model with adjustment costs over a rolling sample of the observables and investigate out-of-sample forecast performance. Each estimation sample comprises 61 quarters. The first sample starts in 1976Q1 and ends in 1991Q1. We then roll the starting point and the end point of the sample by one quarter and repeat the estimation until we reach 2011Q3. So each model is estimated 83 times, using the priors reported in Table 9. For each estimation we compute the inflation forecast one, four, and eight quarters ahead as well as the root mean squared forecast errors (RMSFE). The results reported in Table 13 show that the model with adjustment costs performs nearly 9% better than the competitive model at the 1-quarter and 2-years horizon, and nearly 10% better at the 1-year horizon. The Diebold and Mariano (1995) test shows that the difference between squared residuals in the adjustment cost and the competitive model is negative and significant at 10% confidence for the 1-quarter and 1-year horizon and it is negative and significant at 5% confidence at 1-year.

4.3.2 Decomposition of Marginal Costs

In order to understand what drives the different behavior of inflation in the two models we investigate the determinants of the estimated marginal costs series. In what follows we restrict our attention to the competitive benchmark and our preferred specification of the model with adjustment costs, which is the one that allows for the interaction between hiring and investment costs in the adjustment cost function. As reported in Table 11, in the latter model the marginal hiring and investment costs are 0.26 and 0.86 at the estimated equilibrium, while total adjustment costs are around 2.8% of GDP, which are moderate values. The smoothed series for estimated adjustment costs relative to output oscillates between 2.7% and 2.9% in sample, and marginal adjustment costs exhibit moderate fluctuations, so we are not estimating large swings in adjustment costs.

Marginal costs can be decomposed using either the optimality condition for employment or capital, which we report again for convenience in equations (32) and (33), respectively:

$$mc_t = \frac{\frac{W_t}{P_t^C}}{f_{N,t} - g_{N,t}} + \frac{\frac{W_{N,t}}{P_t^C} N_t}{f_{N,t} - g_{N,t}} + \frac{\mathcal{Q}_t^N - (1 - \delta_N) E_t \Lambda_{t,t+1} \mathcal{Q}_{t+1}^N}{f_{N,t} - g_{N,t}}, \quad (32)$$

$$E_t mc_{t+1} = E_t \frac{\frac{W_{K,t+1}}{P_{t+1}} N_{t+1}}{A_{t+1} f_{K,t+1} - g_{K,t+1}} + \frac{1}{\Lambda_{t,t+1}} E_t \frac{\mathcal{Q}_t^K - \Lambda_{t,t+1} (1 - \delta_K) \mathcal{Q}_{t+1}^K}{A_{t+1} f_{K,t+1} - g_{K,t+1}}. \quad (33)$$

In the competitive model, the marginal cost coincides with the labor share, as the second and third term in equation (32) disappear. Equation (33) dictates that expected future marginal costs must equal the sum of the positive expected impact of capital on wages and the expected change in the value of capital. In the competitive benchmark there is no intra-firm bargaining, which implies that the expected next-period marginal cost only reflects expected changes in the value of capital. In turn, this term only reflects expected changes in the relative price of the investment good, as the first two terms in equation (13) wash out. Allowing for both hiring and investment adjustment costs, can potentially modify the dynamics of the marginal cost series by allowing for frictions and intrafirm bargaining to matter both in the optimality conditions for hiring and for capital.

Figure 1 shows the estimated dynamics of the marginal cost series in the competitive model and in the model with frictions.

Figure 1

Visual inspection of the two series tells that their behavior is similar. However, because in the competitive model all the fluctuations in the marginal costs are accounted for by the labor share, the estimated fluctuations in the labor share are implausibly large. Figure 2 decomposes marginal costs in the preferred adjustment-cost specification:

Figure 2

In this model, as shown the figure, most of the fluctuations are explained by the frictional component, which implies that fluctuations of the labor share are much more moderate.

This result is remarkable, because the labor share of income is not an input to the estimation, so the result that fluctuations in the labor share are much more reasonable is not directly implied by the estimation. Specifically, we estimate hiring frictions to explain about 60% of the variation in marginal costs, the labor share to explain around 30%, while the remaining 10% is accounted for by the intrafirm bargaining component.

Note, too, that these results have been obtained with moderate total and marginal adjustment costs, as reported in Table 11. The result that labor frictions are far more important than the labor share in driving marginal costs at business cycle frequencies, stands in sharp contrast to the one reached by Gali (2011) and Krause, Lopez-Salido and Lubick (2008), but should not come as a surprise in the light of the numerical properties of the model described in Section (3.1). This is an important result since New-Keynesian models of the labor market have so far identified marginal costs with the labor share of income, thereby implying that the labor share is the sole determinant of inflation dynamics.

Let's denote the marginal profit of expanding hiring at the margin as $\Pi_t \equiv mc_t(f_{N,t} - g_{N,t}) - \frac{W_t}{P_t^C} - \frac{W_{N,t}}{P_t^C} N_t$. Then rearranging equation (32), the frictional component of marginal costs can be interpreted as the ratio of marginal profits to the marginal product of labor.

$$\frac{\mathcal{Q}_t^N - (1 - \delta_N)E_t\Lambda_{t,t+1}\mathcal{Q}_{t+1}^N}{f_{N,t} - g_{N,t}} = \frac{\Pi_t}{f_{N,t} - g_{N,t}}$$

A high elasticity of profits to the marginal product of labor has been shown to be key for matching unemployment fluctuations at business cycle frequencies (*Shimer (2005)*).

Because gross hiring flows are observable in estimation, the estimated behavior of profits in this model is consistent with the dynamics of hiring. It turns out that in the context of New Keynesian models with frictional labor markets, the estimated behavior of profits relative to the marginal product is also key to explain changes in real marginal costs and, as a result, inflation. Specifically, Figure 2 shows that following a slowdown in economic activity, the expected fall in the value of employment contributes positively to changes in marginal costs. This pattern clearly emerges following the recessionary episodes of the late 1970s - early 1980s, the economic slowdown of the early 2000s and in the great recession, while it is less marked in the contraction of the early 1990s. This result suggests that accounting for frictions is important to explain why inflation tends to increase above its long term average following a recession.

As to the intrafirm bargaining component in the optimality condition for hiring, it appears to play only a minor role in the decomposition of marginal costs. So we find little quantitative importance for the channel by which the marginal hire can exploit decreasing returns to labor, and the associated fall in the wage bill for all the intra-marginal workers to increase the negotiated wage. Figure 3 plots the decomposition of the estimated marginal costs series using the optimality

condition for investment in equation (33).

Figure 3

The two determinants of expected marginal costs exhibit nearly perfect positive correlation, meaning that at times where the value of capital is expected to decrease, negotiated wages are most sensitive to changes in the capital stock, making the hold-up problem related to the accumulation of capital more severe. Quantitatively, we find that the fluctuations in the price of capital and the hold up problem that arises with the effect of capital on wages, are equally important in explaining the dynamics of expected marginal costs.

5 Conclusions

TO BE COMPLETED

References

- [1] An, S. and Schorfheide, F. (2007). "Bayesian Analysis of DSGE Models," **Econometric Reviews** 26, 2-4, 113-72.
- [2] Brooks, S. P. and Gelman, A. (1998). "General Methods for Monitoring Convergence of Iterative Simulations". **Journal of Computational and Graphical Statistics**, 7, p. 434–455.
- [3] Cahuc, P. and Wasmer, E. (2001). "Does Intrafirm Bargaining Matter In The Large Firm'S Matching Model?," **Macroeconomic Dynamics** 5(05), 742-747.
- [4] Cahuc, P., Marque, F. and Wasmer, E. (2008). "A Theory Of Wages And Labor Demand With Intra-Firm Bargaining And Matching Frictions," **International Economic Review**, 49(3), pages 943-72.
- [5] Christiano, L. J., Trabandt, M. and Walentin, K. (2010). "DSGE Models for Monetary Policy Analysis," Chapter 7 in Benjamin M. Friedman and Michael Woodford (eds.) **Handbook of Monetary Economics Vol 3A**, 285-367, Elsevier, Amsterdam.
- [6] Christiano, L. J., Eichenbaum, M. S., and Trabandt, M. (2013a). "Unemployment and Business Cycles," NBER Working Papers 19265, National Bureau of Economic Research.
- [7] Christiano, L. J., Eichenbaum, M. S., and Trabandt, M. (2013b). "Understanding the Great Recession", manuscript
- [8] Diebold, F. X., and Mariano, R. S. (1995). "Comparing Predictive Accuracy," **Journal of Business & Economic Statistics**, American Statistical Association, vol. 13(3), pages 253-63, July.
- [9] Domeij, D., and Floden, M. (2006). "The labor-supply elasticity and borrowing constraints: why estimates are biased", **Review of Economic Dynamics** 9, 242–262.
- [10] Faccini, R., Millard, S. and Zanetti, F. (2013). "Wage rigidities in an estimated DSGE model of the UK labour market, " **Manchester School** 81, S1, 66–99.
- [11] Fernandez-Villaverde, J. and . Rubio-Ramirez, J. F. (2004). "Comparing dynamic equilibrium models to data: a Bayesian approach", **Journal of Econometrics**, 123,153-187.
- [12] Frazis, H. J., Robison, Edwin L., Evans, T. D. and Duff, M. A. (2005). "Estimating Gross Flows Consistent with Stocks in the CPS," **Monthly Labor Review**, September, 3-9.
- [13] Krause, M. U., Lopez-Salido, D. and Lubik, T. A. (2008). "Inflation dynamics with search frictions: A structural econometric analysis," **Journal of Monetary Economics** 55, 892–916.

- [14] Gali, J. (2008). **Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework**. Princeton University Press, Princeton.
- [15] Gali, J. (2011). "Monetary policy and unemployment" in Benjamin M. Friedman and Michael Woodford (eds.) **Handbook of Monetary Economics Vol. 3A**, 487-546, Elsevier, Amsterdam.
- [16] Gali, J., and Gertler, M. (1999). "Inflation dynamics: a structural econometric analysis." **Journal of Monetary Economics** 44, 195–222.
- [17] Gertler, M., Sala, L. and Trigari, A. (2008). "An estimated monetary DSGE model with unemployment and staggered nominal wage bargaining." **Journal of Money, Credit and Banking**, 40, 1,713-64.
- [18] Geweke, J. (1999). "Using simulation methods for bayesian econometric models: inference, development, and communication," **Econometric Reviews**, 18, 1, 1-73.
- [19] Hall, R. E. (2006). "Job loss, job finding, and unemployment in the US economy over the past fifty years", in: **NBER Macroeconomics Annual 2005**, 20, 101-166, National Bureau of Economic Research.
- [20] Heer, B., and Maussner, A. (2010). "Inflation and Output Dynamics in a Model with Labor Market Search and Capital Accumulation," **Review of Economic Dynamics**, 13(3), 654-686.
- [21] Iskrev, N. (2010). "Local identification in DSGE models," **Journal of Monetary Economics**, Elsevier, vol. 57(2), pages 189-202, March.
- [22] Melosi, L. (2013), "Signaling effects of monetary policy", Federal Reserve Bank of Chicago, mimeo.
- [23] Merz, M. and Yashiv, E. (2007). "Labor and the market value of the firm," **American Economic Review**, 97, 1, 419-31.
- [24] Mumtaz, H. and Zanetti, F. (2012). "Factor adjustment costs: a structural investigation", Bank of England Working Paper No. 467.
- [25] Nakamura, E., and J. Steinsson (2008). "Five Facts About Prices: A Reevaluation of Menu Cost Models," **Quarterly Journal of Economics**, 123(4), 1415-1464.
- [26] Neri, S. and Ropele, T. (2012). "Imperfect Information, Real-Time Data and Monetary Policy in the Euro Area," **Economic Journal** 122(561), 651-674.
- [27] Rotemberg, J. (1982). "Monopolistic price adjustment and aggregate output", **Review of Economic Studies** 49, 517-31.

- [28] Sbordone, A. (2005). “Do expected future marginal costs drive inflation dynamics?” **Journal of Monetary Economics**, 49, 1,183-97.
- [29] Schorfheide, F. (2000). “Loss Function-Based Evaluation of DSGE Models,” **Journal of Applied Econometrics**, 15(6), 645–670.
- [30] Shimer, R. (2005). “The Cyclical Behavior of Equilibrium Unemployment and Vacancies, ” **American Economic Review**, 95(1), pp. 25-49.
- [31] Silva, J., and Toledo, M. (2009). “Labor Turnover Costs and the Cyclical Behavior of Vacancies and Unemployment,” **Macroeconomic Dynamics** 13 (S1), 76-96.
- [32] Smets, F., and Wouters, R. (2007). “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” **American Economic Review**, 97, 3, 586-606.
- [33] Stole, L. A. and Zwiebel, J. (1996). “Intra-firm Bargaining under Non-binding Contracts,” **Review of Economic Studies** 63(3), 375-410.
- [34] Trigari, A. (2009). “Equilibrium Unemployment, Job Flows, and Inflation Dynamics,” **Journal of Money, Credit and Banking**, 41(1), 1-33.
- [35] Walsh, C. E. (2005). “Labor Market Search, Sticky Prices, and Interest Rate Policies,” **Review of Economic Dynamics** 8(4),829-849.
- [36] Woodford, M. (2003). **Interest and Prices: Foundations of a Theory of Monetary Policy**. Princeton University Press, Princeton.
- [37] Yashiv, E. (2013). “Capital Values and Job Values and the Joint Behavior of Hiring and Investment,” working paper

Appendix A1

Solving for the Wage Function with Intrafirm Bargaining

Remark 8 *why is all centered? RF: No longer centered now.*

We rewrite below for convenience the wage sharing rule consistent with Nash bargaining as derived in equation (25):

$$(1 - \gamma)\mathcal{V}_{it}^N = \frac{\gamma \mathcal{Q}_{jt}^N}{1 - \tau}, \quad (34)$$

where we make use of subscripts i and j to denote a particular household i and firm j bargaining over the wage W_{jt} .

Substituting (5) and (14) into the above equation one gets:

$$\begin{aligned} \gamma \left\{ \left[mc_{jt} (f_{N,jt} - g_{N,jt}) - \frac{W_{jt}}{P_t^C} - \frac{W_{N,jt}}{P_t^C} N_{jt} \right] + \beta(1 - \delta_N) E_t \frac{C_t}{C_{t+1}} \frac{\mathcal{Q}_{jt+1}^N}{1 - \tau} \right\} = \\ (1 - \gamma) \left[\frac{W_{jt}}{P_t^C} - \chi_t N_{it}^\varphi C_t - \frac{x_{t+j}}{1 - x_{t+j}} \mathcal{V}_t^N + \beta(1 - \delta_N) E_t \frac{C_t}{C_{t+1}} \mathcal{V}_{it+1}^N \right]. \end{aligned}$$

Using the sharing rule in (34) to cancel out the terms in $\mathcal{Q}_{jt+1}^N$ and $\mathcal{V}_{it+1}^N$ we obtain the following expression for the real wage:

$$\frac{W_{jt}}{P_t^C} = \gamma mc_{jt} (f_{N,jt} - g_{N,jt}) - \gamma \frac{W_{N,jt}}{P_t^C} N_{jt} + (1 - \gamma) \left[\chi_t C_t N_{it}^\varphi + \frac{x_{t+j}}{1 - x_{t+j}} \frac{\gamma}{1 - \gamma} \frac{\mathcal{Q}_t^N}{1 - \tau} \right]. \quad (35)$$

Ignoring the term in square brackets, which is independent of N_{jt} , and dropping all subscripts from now onward with no risk of ambiguity, we can rewrite the above equation as follows:

$$W_N + \frac{1}{\gamma N} W - P^C mc \left(\frac{f_N}{N} - \frac{g_N}{N} \right) = 0 \quad (36)$$

The solution of the homogeneous equation:

$$W_N + \frac{1}{\gamma N} W = 0, \quad (37)$$

is

$$W(N) = CN^{-\frac{1}{\gamma}}, \quad (38)$$

where C is a constant of integration of the homogeneous equation. Assuming that C is a function of N and deriving (38) w.r.t. N , yields:

$$W_N = C_N N^{-\frac{1}{\gamma}} - \frac{1}{\gamma} C N^{-1-\frac{1}{\gamma}}. \quad (39)$$

Substituting (38) and (39) into (36) one gets:

$$C_N = N^{\frac{1-\gamma}{\gamma}} P^C mc (f_N - g_N). \quad (40)$$

Integrating (40) yields:

$$C = P^C mc \int_0^N z^{\frac{1-\gamma}{\gamma}} (f_z - g_z) dz + D, \quad (41)$$

where D is a constant of integration. Let's solve for the two integrals in f_z and g_z , one at a time. Assuming that $f(Az, K) = (Az)^\alpha K^{1-\alpha}$, we can write:

$$P^C mc \int_0^N z^{\frac{1-\gamma}{\gamma}} f_z dz = P^C mc \alpha \frac{\gamma}{1-\gamma(1-\alpha)} A^\alpha N^{\frac{1-\gamma(1-\alpha)}{\gamma}} K^{1-\alpha}. \quad (42)$$

Given our assumptions on the functional form of g as in (9), the function g_N can be rearranged as follows:

$$\begin{aligned} g_N = & -A^\alpha K^{1-\alpha} \left[e_2 H^{\eta_2} N^{\alpha-1-\eta_2} + e_3 \left(\frac{HI}{K} \right)^{\eta_3} N^{\alpha-1-\eta_3} \right] \\ & + \alpha A^\alpha K^{1-\alpha} \left[\frac{e_1}{\eta_1} \left(\frac{I}{K} \right)^{\eta_1} N^{\alpha-1} + \frac{e_2}{\eta_2} H^{\eta_2} N^{\alpha-1-\eta_2} + \frac{e_3}{\eta_3} \left(\frac{HI}{K} \right)^{\eta_3} N^{\alpha-1-\eta_3} \right] \end{aligned} \quad (43)$$

Integrating separately the two additive terms in the first row of the above equation yields:

$$P^C mc \int_0^N z^{\frac{1-\gamma}{\gamma}} A^\alpha K^{1-\alpha} e_2 H^{\eta_2} z^{\alpha-1-\eta_2} dz = P^C mc e_2 H^{\eta_2} A^\alpha K^{1-\alpha} \frac{\gamma}{1-\gamma+\gamma(\alpha-\eta_2)} N^{\frac{1-\gamma+\gamma(\alpha-\eta_2)}{\gamma}}, \quad (44)$$

$$P^C mc \int_0^N z^{\frac{1-\gamma}{\gamma}} A^\alpha K^{1-\alpha} e_3 \left(\frac{HI}{K} \right)^{\eta_3} z^{\alpha-1-\eta_3} dz = P^C mc e_3 \left(\frac{HI}{K} \right)^{\eta_3} A^\alpha K^{1-\alpha} \frac{\gamma}{1-\gamma+\gamma(\alpha-\eta_3)} N^{\frac{1-\gamma+\gamma(\alpha-\eta_3)}{\gamma}}, \quad (45)$$

Integrating separately the three terms on the second row of equation (43) yields:

$$-P^C mc \int_0^N z^{\frac{1-\gamma}{\gamma}} \alpha A^\alpha K^{1-\alpha} \frac{e_1}{\eta_1} \left(\frac{I}{K} \right)^{\eta_1} z^{\alpha-1} dz = -P^C mc \frac{e_1}{\eta_1} \left(\frac{I}{K} \right)^{\eta_1} \alpha A^\alpha K^{1-\alpha} \frac{\gamma}{1-\gamma(1-\alpha)} N^{\frac{1-\gamma(1-\alpha)}{\gamma}}, \quad (46)$$

$$-P^C mc \int_0^N z^{\frac{1-\gamma}{\gamma}} \alpha A^\alpha K^{1-\alpha} \frac{e_2}{\eta_2} H^{\eta_2} z^{\alpha-1-\eta_2} dz = -P^C mc \frac{e_2}{\eta_2} H^{\eta_2} \alpha A^\alpha K^{1-\alpha} \frac{\gamma}{1-\gamma+\gamma(\alpha-\eta_2)} N^{\frac{1-\gamma+\gamma(\alpha-\eta_2)}{\gamma}}, \quad (47)$$

$$-P^C mc \int_0^N z^{\frac{1-\gamma}{\gamma}} \alpha A^\alpha K^{1-\alpha} \frac{e_3}{\eta_3} \left(\frac{HI}{K} \right)^{\eta_3} z^{\alpha-1-\eta_3} dz = -P^C mc \frac{e_3}{\eta_3} \left(\frac{HI}{K} \right)^{\eta_3} \alpha A^\alpha K^{1-\alpha} \frac{\gamma}{1-\gamma+\gamma(\alpha-\eta_3)} N^{\frac{1-\gamma+\gamma(\alpha-\eta_3)}{\gamma}}. \quad (48)$$

Denoting $A_1 \equiv \frac{\gamma}{1-\gamma(1-\alpha)}$, $A_2 \equiv \frac{\gamma}{1-\gamma+\gamma(\alpha-\eta_2)}$ and $A_3 \equiv \frac{\gamma}{1-\gamma+\gamma(\alpha-\eta_3)}$, we can now rewrite (41) as follows:

$$C = D + P^C mc A^\alpha K^{1-\alpha} \left\{ \alpha A_1 N^{1/A_1} + e_2 H^{\eta_2} A_2 N^{1/A_2} + e_3 \left(\frac{HI}{K} \right)^{\eta_3} A_3 N^{1/A_3} - \alpha \frac{e_1}{\eta_1} \left(\frac{I}{K} \right)^{\eta_1} A_1 N^{1/A_1} - \alpha \frac{e_2}{\eta_2} H^{\eta_2} A_2 N^{1/A_2} - \alpha \frac{e_3}{\eta_3} \left(\frac{HI}{K} \right)^{\eta_3} A_3 N^{1/A_3} \right\}. \quad (49)$$

Plugging (49) into (38) one gets:

$$W(N) = DN^{-\frac{1}{\gamma}} + P^C mc A^\alpha K^{1-\alpha} \left\{ \alpha \left[1 - \frac{e_1}{\eta_1} \left(\frac{I}{K} \right)^{\eta_1} \right] A_1 N^{\alpha-1} + \left(1 - \frac{\alpha}{\eta_2} \right) e_2 H^{\eta_2} A_2 N^{\alpha-1-\eta_2} + \left(1 - \frac{\alpha}{\eta_3} \right) e_3 \left(\frac{HI}{K} \right)^{\eta_3} A_3 N^{\alpha-1-\eta_3} \right\}. \quad (50)$$

In order to eliminate the constant of integration D we assume that $\lim_{N \rightarrow 0} NW(N) = 0$. The solution to (35) therefore is:

$$\begin{aligned} \frac{W_t}{P_t^C} &= \gamma mc_t A_t^\alpha K_{t-1}^{1-\alpha} \left\{ \alpha \left[1 - \frac{e_1}{\eta_1} \left(\frac{I_t}{K_{t-1}} \right)^{\eta_1} \right] A_1 N_t^{\alpha-1} + \left(1 - \frac{\alpha}{\eta_2} \right) e_2 H_t^{\eta_2} A_2 N_t^{\alpha-1-\eta_2} \right. \\ &\quad \left. + \left(1 - \frac{\alpha}{\eta_3} \right) e_3 \left(\frac{H_t I_t}{K_{t-1}} \right)^{\eta_3} A_3 N_t^{\alpha-1-\eta_3} \right\} \\ &\quad + (1-\gamma) \left[\chi_t C_t N_t^\varphi + \frac{x_t}{1-x_t} \frac{\gamma}{1-\gamma} \left(mc_t g_{H,t} + \frac{W_{H,t}}{P_t^C} N_t \right) \right]. \end{aligned} \quad (51)$$

Appendix A2

The stationary model

Let $\tilde{C}_t = C_t/A_t$ for any variable C_t . The stationary model is characterized by the following equations:

The Euler equation (4):

$$\frac{1}{R_t} = \beta E_t \frac{P_t^C \tilde{C}_t}{P_{t+1}^C \tilde{C}_{t+1}} \frac{A_t}{A_{t+1}} \frac{\vartheta_{t+1}}{\vartheta_t}.$$

The production function $f(A_t N_t, K_{t-1}) = (A_t N_t)^\alpha K_{t-1}^{1-\alpha}$:

$$\frac{f(A_t N_t, K_{t-1})}{A_t} \equiv \tilde{f}_t = N_t^\alpha \tilde{K}_t^{1-\alpha} \left(\frac{A_t}{A_{t-1}} \right)^{\alpha-1}$$

The capital law of motion in eq. (10):

$$\tilde{K}_t = (1 - \delta_{K,t}) \tilde{K}_{t-1} \left(\frac{A_{t-1}}{A_t} \right) + \tilde{I}_t \quad (52)$$

The employment law of motion in eq. (11):

$$N_t = (1 - \delta_{N,t}) N_{t-1} + H_t$$

The adjustment cost function eq. (9)

$$\frac{g(I_t, K_{t-1}, H_t, N_t)}{A_t} \equiv \tilde{g}_t = \left[\frac{e_1}{\eta_1} \left(\frac{I_t}{K_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_1} + \frac{e_2}{\eta_2} \left(\frac{H_t}{N_t} \right)^{\eta_2} + \frac{e_3}{\eta_3} \left(\frac{I_t}{K_{t-1}} \frac{A_t}{A_{t-1}} \frac{H_t}{N_t} \right)^{\eta_3} \right] N_t^\alpha \tilde{K}_t^{1-\alpha} \left(\frac{A_t}{A_{t-1}} \right)^{\alpha-1}$$

The derivative of the adjustment cost function with respect to hiring:

$$\frac{g_H(I_t, K_{t-1}, H_t, N_t)}{A_t} \equiv \tilde{g}_{H,t} = \tilde{f}_t \left[\left(\frac{H_t}{N_t} \right)^{\eta_2-1} \frac{e_2}{N_t} + \tilde{I}_t \frac{A_t}{A_{t-1}} \left(\frac{H_t}{N_t} \frac{\tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_3-1} \frac{e_3}{\tilde{K}_{t-1}} N_t \right]$$

The derivative of the adjustment cost function with respect to investment:

$$g_I(I_t, K_{t-1}, H_t, N_t) = \tilde{f}_t \frac{A_t}{A_{t-1}} \left[\left(\frac{\tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_1-1} \frac{e_1}{\tilde{K}_{t-1}} + H_t \left(\frac{H_t}{N_t} \frac{\tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_3-1} \frac{e_3}{\tilde{K}_{t-1}} N_t \right]$$

The derivative of the adjustment cost function with respect to employment:

$$\frac{g_N(I_t, K_{t-1}, H_t, N_t)}{A_t} \equiv \tilde{g}_{N,t} = \tilde{f}_t \frac{H_t}{N_t^2} * \\ * \left[- \left(\frac{H_t}{N_t} \right)^{\eta_2-1} e_2 - \frac{\tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \left(\frac{H_t}{N_t} \frac{\tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_3-1} (\eta_3 - 1) e_3 \right] + \alpha \tilde{g}_t \frac{1}{N_t}$$

The derivative of the adjustment cost function with respect to capital:

$$g_K(I_t, K_{t-1}, H_t, N_t) = \tilde{f}_t \left(\frac{A_t}{A_{t-1}} \right)^2 \frac{\tilde{I}_t}{\tilde{K}_{t-1}^2} * \\ * \left[- \left(\frac{\tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_1-1} e_1 - \frac{H_t}{N_t} \left(\frac{H_t}{N_t} \frac{\tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_3-1} (\eta_3 - 1) e_3 \right] + (1 - \alpha) \tilde{g}_t \frac{A_t}{A_{t-1}} \frac{1}{\tilde{K}_{t-1}}$$

The resource constraint in eq. (30):

$$\tilde{C}_t + \frac{P_t^I}{P_t^C} \tilde{I}_t = \tilde{f}_t - \tilde{g}_t \quad (53)$$

The first order condition with respect to capital in equation (16):

$$\left(mc_t g_{I,t} + W_{I,t}^r N_t + \frac{P_t^I}{P_t^C} \right) = E_t \Lambda_{t,t+1} \left\{ mc_{t+1} (f_{K,t+1} - g_{K,t+1}) - W_{K,t+1}^r N_{t+1} \right. \\ \left. + (1 - \delta_K) (mc_{t+1} g_{I,t+1} + W_{I,t+1}^r N_{t+1} + \frac{P_{t+1}^I}{P_{t+1}^C}) \right\}, \quad (54)$$

where the superscript r denotes real variables and

$$\Lambda_{t,t+1} = \beta \frac{\tilde{C}_t}{\tilde{C}_{t+1}} \frac{A_t}{A_{t+1}} \frac{\vartheta_{t+1}}{\vartheta_t}.$$

The first order condition with respect to employment in eq. (17):

$$mc_t \tilde{g}_{H,t} + \tilde{W}_{H,t}^r N_t = mc_t \left(\tilde{f}_{N,t} - \tilde{g}_{N,t} \right) - \tilde{W}_t^r - \tilde{W}_{N,t} N_t \\ + (1 - \delta_N) E_t \Lambda_{t,t+1} \left(mc_{t+1} \tilde{g}_{H,t+1} + \tilde{W}_{H,t+1}^r N_{t+1} \right) \frac{A_{t+1}}{A_t}, \quad (55)$$

The Phillips curve eq. (21):

$$\pi_t(1 + \pi_t) = \frac{1 - \varepsilon}{\zeta} + \frac{\varepsilon}{\zeta} mc_t + E_t \frac{1}{1 + r_t} (1 + \pi_{t+1}) \pi_{t+1} \frac{(\tilde{f}_{t+1} - \tilde{g}_{t+1})}{(\tilde{f}_t - \tilde{g}_t)} \frac{A_{t+1}}{A_t}, \quad (56)$$

The Taylor rule eq. (31):

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\rho_r} \left[\left(\frac{1 + \pi_t}{1 + \pi^*} \right)^{r_\pi} \left(\frac{\tilde{f}_t - \tilde{g}_t}{f^* - g^*} \right)^{r_y} \right]^{1 - \rho_r} \xi_t, \quad (57)$$

The real wage eq. (27), using eq.(51):

$$\begin{aligned} \tilde{W}_t^r = mc_t \tilde{K}_{t-1}^{1-\alpha} \left(\frac{A_t}{A_{t-1}} \right)^{\alpha-1} & \left\{ \alpha \left[1 - \frac{A_t}{A_{t-1}} - \frac{e_1}{\eta_1} \left(\frac{\tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_1} \right] A_1 N_t^{\alpha-1} + \left(1 - \frac{\alpha}{\eta_2} \right) e_2 H_t^{\eta_2} A_2 N_t^{\alpha-1-\eta_2} \right. \\ & + \left(1 - \frac{\alpha}{\eta_3} \right) e_3 \left(\frac{H_t \tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_3} A_3 N_t^{\alpha-1-\eta_3} \Big\} \\ & + (1 - \gamma) \left[\chi_t \tilde{C}_t N_t^\varphi + \frac{x_t}{1 - x_t} \frac{\gamma}{1 - \gamma} \left(mc_t \tilde{g}_{H,t} + \tilde{W}_{H,t}^r N_t \right) \right], \quad (58) \end{aligned}$$

The derivative of the wage function with respect to investment:

$$\begin{aligned} W_{I,t}^r = mc_t \left(\frac{A_t}{A_{t-1}} \right)^\alpha \tilde{K}_{t-1}^{-\alpha} & \left\{ \alpha A_1 N_t^{\alpha-1} \left[-e_1 \left(\frac{\tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_1-1} \right] \right. \\ & + \left(1 - \frac{\alpha}{\eta_3} \right) e_3 A_3 N_t^{\alpha-1-\eta_3} \eta_3 \left(\frac{H_t \tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_3-1} H_t \Big\} \end{aligned}$$

The derivative of the wage function with respect to hiring:

$$\begin{aligned} \tilde{W}_{H,t}^r = mc_t \left(\frac{A_t}{A_{t-1}} \right)^{\alpha-1} \tilde{K}_{t-1}^{1-\alpha} & \left[\left(1 - \frac{\alpha}{\eta_2} \right) e_2 A_2 N_t^{\alpha-1-\eta_2} \eta_2 H_t^{\eta_2-1} \right. \\ & + \left(1 - \frac{\alpha}{\eta_3} \right) e_3 A_3 N_t^{\alpha-1-\eta_3} \eta_3 \left(\frac{H_t \tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_3} \frac{\tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \Big] \end{aligned}$$

The derivative of the wage function with respect to capital:

$$W_{K,t}^r = (1 - \alpha) \left\{ \tilde{W}_t^r - (1 - \gamma) \left[\chi_t \tilde{C}_t N_t^\varphi - \frac{x_t}{1 - x_t} \frac{\gamma}{1 - \gamma} \left(mc_t \tilde{g}_{H,t} + \frac{W_{H,t}^r}{P_t^C} N_t \right) \right] \right\} \frac{A_t}{A_{t-1}} \frac{1}{\tilde{K}_{t-1}} + mc_t \left(\frac{A_t}{A_{t-1}} \right)^{1+\alpha} *$$

$$\tilde{K}_{t-1}^{-\alpha} \tilde{I}_t \left\{ \alpha A_1 N_t^{\alpha-1} e_1 \left(\frac{\tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_1-1} - \left(1 - \frac{\alpha}{\eta_3} \right) e_3 A_3 N_t^{\alpha-1-\eta_3} \eta_3 \left(\frac{H_t \tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_3-1} \frac{H_t}{\tilde{K}_{t-1}} \right\}$$

The derivative of the wage function with respect to employment:

$$\begin{aligned} \tilde{W}_{N,t}^r = & mc_t \left(\frac{A_t}{A_{t-1}} \right)^{\alpha-1} \tilde{K}_{t-1}^{1-\alpha} \left\{ \alpha \left[1 - \frac{e_1}{\eta_1} \left(\frac{\tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_1} \right] A_1 (\alpha - 1) N_t^{\alpha-2} \right. \\ & \left. + \left(1 - \frac{\alpha}{\eta_2} \right) e_2 H_t^{\eta_2} A_2 (\alpha - \eta_2 - 1) N_t^{\alpha-\eta_2-2} + \left(1 - \frac{\alpha}{\eta_3} \right) e_3 \left(H_t \frac{\tilde{I}_t}{\tilde{K}_{t-1}} \frac{A_t}{A_{t-1}} \right)^{\eta_3} A_3 (\alpha - 1 - \eta_3) N_t^{\alpha-2-\eta_3} \right\} \end{aligned}$$

Appendix A3

The Steady State Equilibrium

Normalization of labour force participation:

$$1 = N + U \quad (59)$$

Using (2):

$$\delta_N N = \frac{x}{1-x} U \quad (60)$$

Using (55):

$$[1 - \beta(1 - \delta_N)] \left(mc\tilde{g}_H + \tilde{W}_H^r N \right) = mc(\tilde{f}_N - \tilde{g}_N) - \tilde{W}_N^r N - \tilde{W}^r, \quad (61)$$

where the superscript r denotes real variables.

Using (54):

$$\left[1 - (1 - \delta_K) \frac{\beta}{\exp(\mu)} \right] (1 + mcg_I + W_I^r N) = \frac{\beta}{\exp(\mu)} [mc(f_K - g_K) - W_K^r N] \quad (62)$$

Using (27):

$$\tilde{W}^r = \gamma mc(\tilde{f}_N - \tilde{g}_N) - \gamma \tilde{W}_N^r N + (1 - \gamma) \left[\chi_t \tilde{C} N^\varphi + \frac{x_t}{1 - x_t} \frac{\gamma}{1 - \gamma} (mc_t \tilde{g}_H + \tilde{W}_H^r N) \right] \quad (63)$$

Using (53):

$$\tilde{f}(K, N) - \tilde{g} = \tilde{C} + \tilde{I} \quad (64)$$

Using (52):

$$\tilde{K} = \frac{\tilde{I}}{1 - (1 - \delta_K) \exp(\mu)^{-1}} \quad (65)$$

by definition of job finding rate:

$$x = \frac{H_t}{U_t / (1 - x)} \quad (66)$$

and using (21)

$$mc = \frac{\varepsilon - 1}{\varepsilon}. \quad (67)$$

This system solves for $N, U, H, \tilde{I}, \tilde{W}^r, \tilde{C}, \tilde{K}, x, mc$ once the derivatives of the wage function with respect to $H, \tilde{I}, N$, and $\tilde{K}$ are substituted.

Table 1: Calibrated parameters and steady state values

<i>Description</i>	<i>Parameter</i>	<i>Value</i>
Discount factor	β	0.99
Separation rate	δ_N	0.133
Capital depreciation rate	δ_K	0.02
Elasticity of output to labor input	α	0.66
Adj. cost function	η_1	2
Adj. cost function	η_2	2
Adj. cost function	η_3	1
Adj. cost function investment term	e_1	70
Adj. cost function hiring term	e_2	3.1
Adj. cost function interaction term	e_3	-7
Elasticity of substitution	ϵ	11
Trend growth of technology	μ	0.00454
Workers' bargaining power	γ	0.28
<i>Panel B: Steady State Values</i>		
<i>Definition</i>	<i>Expression</i>	<i>Value</i>
Total adjustment cost/ output	g_t/y_t	0.025
Marginal investment adjustment cost	$g_{I,t}/(y_t/k_t)$	0.8
Marginal hiring adjustment cost	$g_{H,t}/(y_t/n_t)$	0.25
Labor share of income	$\frac{(w/p)}{[(F-g)/n]}$	2/3
Unemployment rate	u_t	0.10

Table 2: Parameter values used in the simulation of the model

<i>description of the parameters</i>		<i>range</i>
Elasticity of output to labor input	α	[0.66,0.70]
Workers' bargaining power	γ	[0.27,0.29]
Inverse Frish elasticity	φ	[1,5]
Adjustment cost function (investment)	e_1	[69,75]
Adjustment cost function (hiring)	e_2	[2.5,3.5]
Adjustment cost function (interactions)	e_3	[-6.8,-7.2]
Price stickiness	ζ	[10,120]
Elasticity of substitution	ε	[9,13]
Taylor rule coefficient on inflation	r_π	[1.1,3.0]
Taylor rule coefficient on output	r_y	[0,0.130]
Interest rate smoothing	ρ_r	[0,0.75]
Autoregressive parameter on technology shocks	ρ_a	[0,0.80]
Long-run technology growth (%)	μ	[0.44,0.46]

Notes: Parameter values are randomly drawn from uniform distributions.

**Table 3: Simulated average responses to technology shocks:
inflation and mc decomposition**

	π	mc	Labor share	Labor frictions	Intrafirm
	Response on impact		Contributions to changes in mc (%)		
Competitive	0.16 (0.19)	0.56 (1.02)	—	—	—
L-Gali	0.16 (0.19)	0.58 (1.08)	0.70 (0.17)	0.22 (0.18)	0.08 (0.03)
L	0.16 (0.21)	0.62 (1.23)	0.54 (0.11)	0.29 (0.15)	0.17 (0.08)
K	0.20 (0.19)	0.84 (0.73)	—	—	—
KL	0.26 (0.27)	1.16 (1.12)	0.48 (0.10)	0.33 (0.13)	−0.18 (0.04)
KL interactions	0.31 (0.30)	1.75 (1.76)	0.39 (0.08)	0.45 (0.10)	−0.16 (0.03)

Notes: This Table reports average responses to a one percent temporary increase in technology growth, obtained by simulating our model using six different specifications of the adjustment cost function. Each model has been simulated using 5,000 draws from the parameter distributions in Table 2. The first two columns report marginal costs and inflation in percentage deviations from the steady state. Columns three to five report percent contributions to changes in marginal cost accounted for by the labor share, labor market frictions and intrafirm bargaining. Standard deviations are reported in brackets.

**Table 4: Simulated average responses to monetary shocks:
inflation and mc decomposition**

	π	mc	Labor share	Labor frictions	Intrafirm
	Response on impact		Contributions to changes in mc (%)		
Competitive	−0.42 (0.39)	−1.96 (2.18)	—	—	—
L-Gali	−0.42 (0.39)	−1.96 (2.17)	0.70 (0.04)	0.20 (0.04)	0.10 (0.01)
L	−0.44 (0.42)	−2.04 (2.38)	0.29 (0.03)	0.58 (0.04)	0.13 (0.01)
K	−0.30 (0.28)	−0.95 (0.79)	—	—	—
KL	−0.35 (0.33)	−1.25 (1.15)	0.35 (0.03)	0.52 (0.03)	0.13 (0.01)
KL interactions	−0.40 (0.38)	−1.74 (1.84)	0.33 (0.03)	0.53 (0.03)	0.14 (0.01)

Notes: This Table reports average responses to a 25bp shock to the interest rate, obtained by simulating our model using six different specifications of the adjustment cost function. Each model has been simulated using 5,000 draws from the parameter distributions in Table 2. The first two columns report marginal costs and inflation in percentage deviations from the steady state. Columns three to five report percent contributions to changes in marginal cost accounted for by the labor share, labor market frictions and intrafirm bargaining. Standard deviations are reported in brackets.

**Table 5: Simulated average responses to investment shocks:
inflation and mc decomposition**

	π	mc	Labor share	Labor frictions	Intrafirm
	Response on impact		Contributions to changes in mc (%)		
Competitive	−0.44 (0.17)	−3.26 (2.54)	—	—	—
L-Gali	−0.44 (0.17)	−3.26 (2.54)	0.60 (0.10)	0.30 (0.20)	0.10 (0.01)
L	−0.47 (0.19)	−3.39 (2.69)	0.13 (0.06)	0.75 (0.06)	0.12 (0.02)
K	−0.11 (0.04)	−0.58 (0.32)	—	—	—
KL	−0.12 (0.04)	−0.68 (0.41)	0.16 (0.06)	0.71 (0.06)	0.13 (0.02)
KL interactions	−0.17 (0.06)	−1.20 (0.77)	0.25 (0.05)	0.59 (0.03)	0.16 (0.03)

Notes: This Table reports average responses to a one percent increase in the relative price of investment, obtained by simulating our model using six different specifications of the adjustment cost function. Each model has been simulated using 5,000 draws from the parameter distributions in Table 2. The first two columns report marginal costs and inflation in percentage deviations from the steady state. Columns three to five report percent contributions to changes in marginal cost accounted for by the labor share, labor market frictions and intrafirm bargaining. Standard deviations are reported in brackets.

**Table 6: Simulated average responses to labor supply shocks:
inflation and mc decomposition**

	π	mc	Labor share	Labor frictions	Intrafirm
	Response on impact		Contributions to changes in mc (%)		
Competitive	0.01 (0.02)	0.03 (0.06)	—	—	—
L-Gali	0.01 (0.01)	0.03 (0.02)	0.48 (0.08)	0.47 (0.06)	0.05 (0.02)
L	0.004 (0.01)	0.01 (0.04)	0.42 (0.03)	0.50 (0.03)	0.08 (0.02)
K	0.04 (0.02)	0.18 (0.11)	—	—	—
KL	0.00 (0.01)	−0.04 (0.04)	0.39 (0.04)	0.53 (0.02)	0.08 (0.02)
KL interactions	−0.02 (0.01)	−0.20 (0.10)	0.31 (0.06)	0.59 (0.04)	0.10 (0.03)

Notes: This Table reports average responses to a one percent positive labor supply shock, obtained by simulating our model using six different specifications of the adjustment cost function. Each model has been simulated using 5,000 draws from the parameter distributions in Table 2. The first two columns report marginal costs and inflation in percentage deviations from the steady state. Columns three to five report percent contributions to changes in marginal cost accounted for by the labor share, labor market frictions and intrafirm bargaining. Standard deviations are reported in brackets.

**Table 7: Simulated average responses to preference shocks:
inflation and mc decomposition**

	π	mc	Labor share	Labor frictions	Intrafirm
	Response on impact		Contributions to changes in mc (%)		
Competitive	0.02 (0.03)	0.05 (0.12)	—	—	—
L-Gali	0.02 (0.03)	0.05 (0.13)	0.51 (0.13)	0.44 (0.11)	0.05 (0.03)
L	0.02 (0.03)	0.04 (0.16)	0.44 (0.09)	0.47 (0.08)	0.09 (0.03)
K	0.12 (0.05)	0.61 (0.36)	—	—	—
KL	0.17 (0.08)	0.92 (0.64)	0.44 (0.14)	0.47 (0.14)	0.09 (0.04)
KL interactions	0.20 (0.10)	1.14 (0.94)	0.34 (0.12)	0.54 (0.15)	0.11 (0.11)

Notes: This Table reports average responses to a one percent preference shock, obtained by simulating our model using six different specifications of the adjustment cost function. Each model has been simulated using 5,000 draws from the parameter distributions in Table 2. The first two columns report marginal costs and inflation in percentage deviations from the steady state. Columns three to five report percent contributions to changes in marginal cost accounted for by the labor share, labor market frictions and intrafirm bargaining. Standard deviations are reported in brackets.

Table 8: Non-Estimated Parameters in the adjustment costs models

Panel A: Parameters

Definition	Parameter	Value	Target/Source
Discount factor	β	0.99	Annual interest rate 4%
Separation rate	δ_N	0.133	Yashiv (2012)
Capital depreciation rate	δ_K	0.02	Yashiv (2012)
Adj. cost function	η_1	2	normalization
Adj. cost function	η_2	2	normalization
Adj. cost function	η_3	1	normalization
Adj. cost function interaction term	e_3	-7	calibration
Elasticity of substitution	ϵ	11	Krause et al. (2008)

Table 9: Prior and posterior distribution for the structural parameters and the shocks

			<i>Prior dist.</i>		<i>Posterior dist.</i>
<i>Model</i>				$\mathcal{M}_{comp}$	$\mathcal{M}_1$
<i>Parametric restrictions</i>				$e_1 = e_2 = e_3 = 0$	<i>none</i>
		Distr.	Mean,Std	Mean,Std	Mean,Std
<i>Structural Parameters</i>					
Adj. Fn investment	e_1	G	70, 2.0	-	72.7,0.26
Adj. Fn hiring	e_2	G	3.1, 0.1	-	3.26,0.01
Workers' bargaining power	γ	B	0.28, 0.1	-	0.283,0.002
Inverse Frisch elasticity	φ	G	3, 0.2	4.3, 0.03	3.6,0.02
Elasticity of output to labor	α	B	0.66, 0.02	0.68, 0.002	0.77,0.004
Price stickiness	ζ	G	120, 5	104, 1.0	130,0.01
Technology growth (%)	μ	B	0.45, 0.01	0.45,0.001	0.46,0.001
Taylor response to infl	r_π	G	1.5, 0.15	2.1,0.01	1.46,0.01
Taylor response to output	r_y	G	0.125, 0.05	0.11,0.003	0.02,0.003
Interest rate smoothing	ρ_r	B	0.75, 0.15	0.12,0.01	0.58,0.02
<i>Shocks: autoregressive parameters</i>					
Technology trend	ρ_a	B	0.5, 0.2	0.22,0.03	0.20,0.02
Technology investment	ρ_q	B	0.5, 0.2	0.50,0.02	0.95,0.02
Monetary policy	ρ_ξ	B	0.5, 0.2	0.75,0.03	0.16,0.03
Preferences	ρ_ϑ	B	0.5, 0.2	0.96,0.01	0.89,0.02
Labor supply	ρ_χ	B	0.5, 0.2	0.96,0.02	0.97,0.02
<i>Shocks: standard deviations</i>					
Technology trend	σ_a	IG	0.01, 0.1	0.015,0.0008	0.014,0.001
Technology investment	σ_q	IG	0.01, 0.1	0.009,0.0005	0.017,0.0008
Monetary policy	σ_ξ	IG	0.01, 0.1	0.006,0.0002	0.003,0.0002
Preferences	σ_ϑ	IG	0.01, 0.1	0.025,0.003	0.014,0.002
Labor supply	σ_χ	IG	0.01, 0.1	0.012,0.002	0.009,0.001

Table 10: Prior and posterior distribution for the structural parameters and the shocks

			<i>Prior dist.</i>		<i>Posterior dist.</i>	
<i>Model</i>				$\mathcal{M}_2$	$\mathcal{M}_3$	$\mathcal{M}_4$
<i>Parametric restrictions</i>				$e_3 = 0$	$e_1 = e_3 = 0$	$e_2 = e_3 = 0$
		Distr.	Mean,Std	Mean,Std	Mean,Std	Mean,Std
<i>Structural Parameters</i>						
Adj. Fn investment	e_1	G	70, 2.0	69.8, 0.64	—	72.9, 0.30
Adj. Fn hiring	e_2	G	3.1, 0.1	3.1, 0.02	2.7, 0.01	—
Workers' bargaining power	γ	B	0.28, 0.1	0.31, 0.002	0.27, 0.001	—
Inverse Frisch elasticity	φ	G	3, 0.2	3.4, 0.03	3.5, 0.02	4.2, 0.04
Elasticity of output to labor	α	B	0.66, 0.02	0.77, 0.006	0.70, 0.03	0.67, 0.002
Price stickiness	ζ	G	120, 5	129, 0.65	114, 0.65	106, 1.5
Technology growth	μ	B	0.45, 0.01	0.45, 0.003	0.46, 0.001	0.0046, 0.001
Taylor response to infl	r_π	G	1.5, 0.15	1.4, 0.02	1.8, 0.02	1.47, 0.008
Taylor response to output	r_y	G	0.125, 0.05	0.014, 0.004	0.02, 0.01	0.01, 0.003
Interest rate smoothing	ρ_r	B	0.75, 0.15	0.49, 0.03	0.24, 0.01	0.36, 0.02
<i>Shocks: autoregressive parameters</i>						
Technology trend	ρ_a	B	0.5, 0.2	0.22, 0.02	0.08, 0.01	0.06,
Technology investment	ρ_q	B	0.5, 0.2	0.90, 0.02	0.76, 0.02	0.90
Monetary policy	ρ_ξ	B	0.5, 0.2	0.31, 0.03	0.19, 0.01	0.32
Preferences	ρ_ϑ	B	0.5, 0.2	0.90, 0.01	0.97, 0.005	0.92
Labor supply	ρ_χ	B	0.5, 0.2	0.96, 0.02	0.90, 0.01	0.92
<i>Shocks: standard deviations</i>						
Technology trend	σ_a	IG	0.01, 0.1	0.015, 0.0009	0.015, 0.0009	0.018, 0.001
Technology investment	σ_q	IG	0.01, 0.1	0.019, 0.001	0.006, 0.0005	0.013, 0.001
Monetary policy	σ_ξ	IG	0.01, 0.1	0.004, 0.0002	0.006, 0.0003	0.004, 0.0002
Preferences	σ_ϑ	IG	0.01, 0.1	0.014, 0.001	0.027, 0.002	0.018, 0.002
Labor supply	σ_χ	IG	0.01, 0.1	0.011, 0.001	0.018, 0.001	0.020, 0.001

Table 11: Adjustment costs implied by the Bayesian estimation

Specification		$\frac{g}{f}$		$\frac{gI}{fK}$		$\frac{gH}{fN}$	
		mean	std.	mean	std.	mean	std.
$\mathcal{M}_1$	both hiring and investment with interactions	0.028	0.003	0.86	0.25	0.26	0.02
$\mathcal{M}_2$	both hiring and investment without interactions	0.048	0.006	1.7	0.24	0.42	0.001
$\mathcal{M}_3$	hiring costs only	0.024	0.005	—	—	0.36	0.001
$\mathcal{M}_4$	investment costs only	0.023	0.006	1.8	0.26	—	—

Table 12: Log Marginal Likelihood

		MDD full sample	Conditional MDD for inflation
<i>Specification</i>			
$\mathcal{M}_1$	both hiring and investment with interactions	2405 (1 : e^{287})	504 (1 : e^{169})
$\mathcal{M}_2$	both hiring and investment without interactions	2396 (1 : e^{278})	492 (1 : e^{157})
$\mathcal{M}_3$	hiring costs only	2272 (1 : e^{154})	449 (1 : e^{114})
$\mathcal{M}_4$	investment costs only	2210 (1 : e^{92})	445 (1 : e^{110})
$\mathcal{M}_{comp}$	no adjustment costs	2118 (1 : 1)	335 (1 : 1)

Notes: Posterior odds $M_{comp} : M_i$ are reported in brackets.

Table 13: Out of sample inflation forecast errors

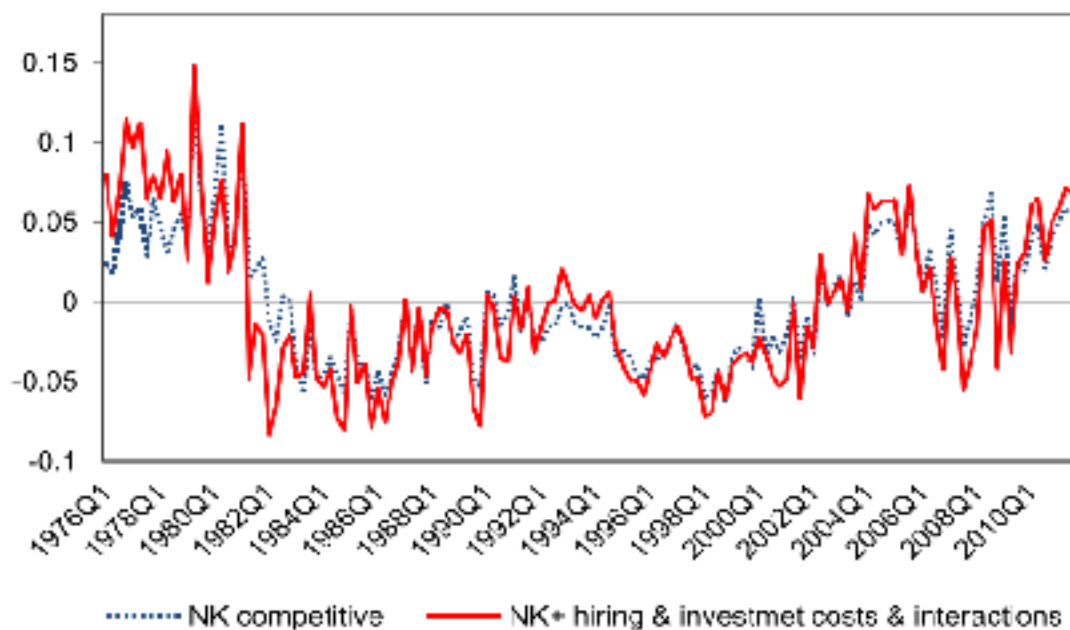
	Competitive	Both hiring and investment with interactions
<i>Out of sample</i>		
RMSFE 1 quarter ahead	0.372	0.340
RMSFE 4 quarters ahead	0.467	0.423
RMSFE 8 quarters ahead	0.512	0.466

Notes: all numbers are expressed in percentage points.

Figure 1

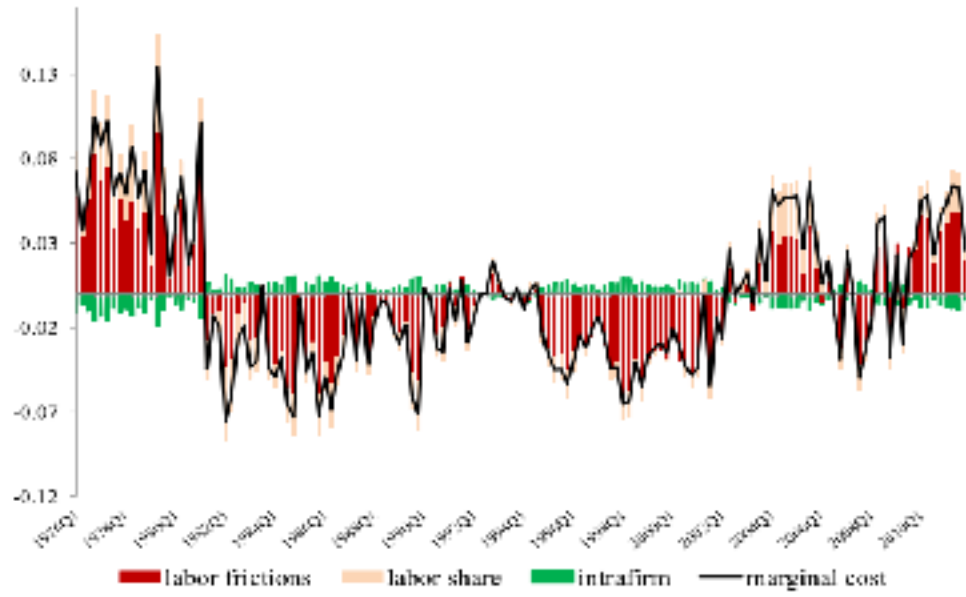
Estimated marginal costs: competitive vs. adjustment cost model

Remark 9 *I have a problem here; can u re-insert the figure? RF: I think there is a bug in scientific with pdf figures. As you noticed, something funny was going on with this picture. I have now saved all the figures as .png files. Scientific still cannot compile well these images in the final pdf, you need to use win-edt for this purpose.*



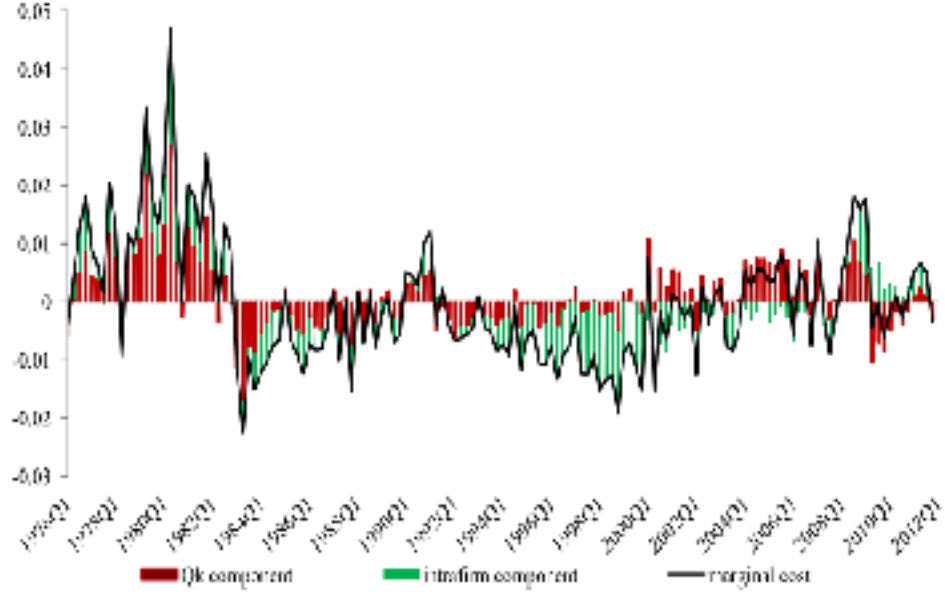
Notes: this figure plots the smoothed estimate of marginal costs in the competitive model and the model with hiring and investment costs and interactions.

Figure 2
Estimated marginal costs decomposition: hiring FOC



Notes: this figure plots contributions to changes in marginal costs estimated in the model that allows for both hiring and investment costs as well as their interaction. The decomposition is based on the dynamic optimality condition for employment in equation (22), and marginal costs are expressed in percentage deviations from the steady state.

Figure 3
Estimated marginal costs decompositions: investment FOC



Notes: this figure plots the decomposition of contributions to changes in expected marginal costs estimated in the model that allows for both hiring and investment costs as well as their interaction. The decomposition is based on the dynamic optimality condition for capital in equation (23), and marginal costs are expressed in percentage deviations from the steady state.