

A Generalized Model of Stock-Flow Matching

Carlos Carrillo-Tudela*
University of Essex,
CESifo and IZA

William Hawkins†
Yeshiva University

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Abstract

We develop a model of stock-flow matching in the labor market which allows for rich heterogeneity of match quality. The economy consists of many labor markets, and workers and jobs continually flow into each labor market. Within a labor market, potential worker-job matches differ in quality. Accordingly, a worker newly arrived in a labor market may not find any acceptable matches; if so, she becomes part of the stock of unemployed workers and must wait for the arrival of a new vacancy in the flow which offers her a sufficiently high quality match. When labor market conditions change, the set of acceptable matches changes in response. Our model is consistent with several stylized facts about the labor market, such as the importance of flows, as well as stocks, for matching rates, as well as with duration dependence in unemployment. It is tractable enough to be used in business cycle analysis. Finally, it provides a natural explanation for shifts in matching efficiency.

Keywords: Job search, unemployment, stock-flow matching, matching function.

JEL: E24, J63, J64.

*Correspondence: Department of Economics, University of Essex, Wivenhoe Park, Colchester, CO4 3SQ, UK; cocarr(at)essex(dot)ac(dot)uk

†Correspondence: Department of Economics, Yeshiva University, 245 Lexington Ave (LX711), New York, NY 10016, USA; whawkins(at)yu(dot)edu

1 Introduction

Why do unemployment and vacancies coexist? In this paper, we answer: a worker remains unemployed because no acceptable job matches are available to her. But this answer begs the question: what determines whether a job match is acceptable, and how does this vary with labor market conditions and policies? To analyze this, we construct an equilibrium matching model which allows for rich heterogeneity in match quality, while preserving the spirit of the stock-flow matching approach of Taylor (1995), Coles (1999), Lagos (2000), Shimer (2007), and Ebrahimi and Shimer (2010). We characterize the set of acceptable matches between unemployed workers and vacant jobs and show how it changes with aggregate conditions and labor market policy.

The key contribution of our paper is to allow for heterogeneous match quality in a stock-flow matching model. Stock-flow matching is an attractive paradigm, since it provides a natural account of why unemployment exit rates depend on the flows of newly unemployed workers and jobs, as well as the stocks of existing unemployed workers and vacancies (Coles and Smith, 1998; Burdett and Cunningham, 1999; Gregg and Petrongolo, 2005; Coles and Petrongolo, 2008; Kuo and Smith, 2009). But its usefulness for applied analysis has been limited since, for the sake of tractability, previous models of stock-flow matching have needed to limit match quality to take only two values, productive or unproductive. This means that the set of acceptable matches cannot, by construction, change when labor market conditions do. Our approach does not suffer from this limitation.

The economy we study consists of many distinct labor markets, represented by ‘islands.’ Unemployed workers and vacant jobs located on the same island match according to a stock-flow matching process. When a new vacant job is created, it may match with unemployed workers waiting for a job; similarly, when a new unemployed worker arrives on the island, she may match with the stock of vacancies. Different worker-job matches are of different qualities, and not all potential matches are sufficiently productive to be worth forming. This heterogeneity arises because: (i) different workers can perform a different set of jobs, and (ii) workers’ productivities vary across the jobs they can perform. If one interprets islands as industries, for example, the first type of heterogeneity arises due to skill or geographical location mismatch. The second type of heterogeneity arises due to differences in match-specific productivity. Further, islands in our model may also differ, in that they offer workers different sets of jobs and productivities. Worker mobility across islands is due to idiosyncratic reasons and is modelled as an exogenous reallocation process that occurs from time to time.

A worker who arrives in an island and does not find a suitable match (either because she cannot perform any of the available jobs or because she rejects available offers in order to wait for a better employment opportunity) is no longer part of the flow of new unemployed workers, and instead becomes part of the stock. Once in the stock, such a worker exits unemployment only when a new job arrives on the island, and then only if she can perform the job and if she is able to generate the highest output relative to other applicants. Similarly, a newly arrived vacancy becomes part of the stock if it is not immediately filled; if so, it must wait for new arrival unemployed workers to match with.

Given that workers are reallocated randomly across islands, our model encompasses employment to unemployment transitions, unemployment to employment transitions and employment to

employment transitions. The model also generates duration dependence in unemployment due to the heterogeneities within and across islands. The crucial difference with Shimer (2007) is that the probability of workers labor market transitions depends on workers' acceptance decisions as well as firms' acceptance and job creation decisions. In turn these decisions are based on the stocks of unemployed workers and vacant jobs and the degree of match heterogeneity, within and across islands.

Our explanation for why unemployment and vacant jobs coexist is quite different from the one proposed by the Diamond-Mortensen-Pissarides (DMP) framework (Diamond, 1982; Mortensen, 1982; Pissarides, 1985, 2000). In the DMP framework, even though productive matches for an unemployed worker may exist, an exogenously imposed black-box matching friction (perhaps intended to model difficulty in finding out about the identity or location of potential trading partners) prevents immediate pairing. The worker is unemployed not because no suitable matches are available to her, but because she has been unable to contact any. That this is the key friction in the labor market seems somewhat implausible. (If it is, it is hard to account for the small amount of time spent by the unemployed on job search (Krueger and Mueller, 2012).)

Moreover, the reduced-form matching function approach is a shaky foundation for our theories of employment and unemployment for two further reasons. First, as Petrongolo and Pissarides (2001) note in their survey of the matching function: '...its usefulness depends on its empirical viability and on how successful it is in capturing the key implications of the heterogeneities and frictions in macro models' (p. 392). But in reality, this approach has numerous empirical shortcomings. It cannot explain why the flows of unemployed workers and vacancies should matter for matching rates, in addition to the stocks. It cannot account for the rise of long-term unemployment in recessions (Elsby et al., 2010; Pissarides, 2013; Wiczer, 2013), nor for the fall in the measured efficiency of the matching function (Elsby et al., 2010; Şahin et al., 2012).

Further, even if standard matching functions did a better job of describing the process of matching between unemployed workers and vacant jobs in at least some dimensions, the black-box nature of the matching function in standing in for a more microfounded account of 'heterogeneities and frictions' would still be troubling. The Lucas critique should give us caution about deriving policy conclusions when the matching function is treated as an invariant structural object. There is no way of telling whether the matching function will remain invariant if the economic environment changes, either due to cyclical or policy changes. Indeed, the fact that matching efficiency seems to have declined in the most recent recession suggests that assuming it is invariant may more generally have misleading policy and welfare implications. The current paper is a first step in building a microfounded model of matching which may be more immune to such critiques.

2 Basic Framework

Time is continuous and denoted by t . There is a measure M of workers and a measure N_t of firms that populate the economy. All agents are risk neutral and discount the future at rate r . The measure of workers is exogenously fixed over time, while the measure of firms is endogenous and can vary over time. Workers can be either employed or unemployed. When unemployed each worker receives a flow payoff of b . The wages of employed workers will be determined below. The

objective of any worker is to maximize his expected lifetime utility. Any firm consists of only one job, which can be either vacant or filled, and uses labor as the only input. Let γ denote the flow cost of posting a job opening. The objective of any firm is to maximize expected lifetime profits.

The economy is divided into a measure one of distinct labor markets, where unemployed workers and firms with vacant jobs coexist. We refer to these labor markets as matching islands and index them by $i \in [0, 1]$. At any time t , there is a measure u_{it} of unemployed workers and a measure v_{it} of vacancies on island i . (These two measures refer to the stock of unemployed workers and vacant jobs on island i at time t .) There is also an additional island, the ‘employment’ island, on which all production takes place. Once an unemployed worker and a job vacancy decide to form a productive match, they immediately move to this island. For simplicity we assume that once on the employment island, there is no recall of previous islands for any worker-firm pair. Let e_t denote the measure of employed workers on the employment island. Since firms have only one job, e_t also denotes the measure of firms with filled jobs.¹

All agents are subject to exogenous reallocation shocks. Let s denote the common Poisson rate at which these reallocation shocks arrive. When an unemployed worker receives a reallocation shock, this worker leaves his current island (either a matching island or the employment island) and arrives to a new, randomly chosen matching island. When a job receives a reallocation shock, it gets destroyed. If such a job was filled, the worker immediately moves to a randomly chosen island. Let μ denote the inflow rate of workers into a given island i . and λ the inflow rate of vacancies into a given island i . These two rates are endogenous objects that we will determine later.

Matching on any given island i follows a stock-flow process. Newly arrived workers can only match with the stock of existing vacancies, while newly arrived vacancies can only match with the stock of existing unemployed workers. In particular, a newly arrived worker on an island i observes the entire stock of vacancies on that island. However, this worker can only perform productively in a small subset of the available vacant jobs, and then even among these jobs, the quality of the potential matches that can be formed is heterogeneous. To model this, we assume that in an island in which the measure of the stock of existing vacant jobs is v_{it} , the number a particular newly arrived worker can perform is a finite integer k , distributed according to a Poisson distribution with parameter v_{it} . That is, the probability that the worker can perform precisely k jobs is equal to

$$e^{-v_{it}} \frac{v_{it}^k}{k!}.$$

Similarly, a newly arrived firm with a job vacancy on an island i also observes the the entire stock of unemployed workers on that island. In this case, the number of workers that can perform the job is distributed according to a Poisson distribution with parameter u_{it} , so that the probability that the precisely k workers can perform the job is given by

$$e^{-u_{it}} \frac{u_{it}^k}{k!}.$$

From the perspective of workers and vacancies in the stock, the probability that they can form a productive match with the newly arrived vacancy or worker, respectively, is iid across arrivals.

¹We discuss below the usefulness of this assumptions and the limitations in brings to our analysis.

A key assumption in our model is that workers have different match-specific productivities, x , across all those jobs they can perform within an island. Let x be a continuous random variable that is distributed according to some exogenous cdf F_i , where $\underline{x}_i$ and $\bar{x}_i$ denote the infimum and supremum of its support and $b < \underline{x}_i$ for all i . Islands in our economy are ex-ante different from each other through differences in F_i . Once a worker arrives on an island and observes all the vacant jobs he can perform, he draws a match-specific productivity for each of these jobs. The worker then chooses whether to form a match with one of these jobs. Similarly, when a new job arrives and observes the set of workers who can perform the job, each workers draws a match-specific productivity for that job. The firm then decides whether to match with one of these workers. For simplicity we assume no recall of previously rejected jobs or workers. Further, we assume no stock-stock matching.

2.1 Worker Value Function

Consider an island i at time t , on which there are u_{it} workers in the stock of unemployment and v_{it} firms in the stock of vacancies. The expected value of an unemployed worker in the stock, W_u is described by the following Hamiltonian-Jacobi-Bellman (HJB) equation,

$$\begin{aligned} rW_u(u, v) = & b + \lambda \int_{R(u, v)}^{\bar{x}} [W_e(x, u, v) - W_u(u, v)] e^{-u(1-F(x))} f(x) dx \\ & + \left[-\lambda \left(1 - e^{-u(1-F(R(u, v)))} \right) + \mu e^{-v(1-F(R(u, v)))} - su \right] \frac{\partial}{\partial u} W_u(u, v) \\ & + \left[\lambda e^{-u(1-F(R(u, v)))} - \mu \left(1 - e^{-v(1-F(R(u, v)))} \right) - sv \right] \frac{\partial}{\partial v} W_u(u, v) \\ & + s \left[\mathbb{E}_{u', v'} W_n(u', v') - W_u(u, v) \right], \end{aligned} \quad (1)$$

Here we have suppressed the subscripts i and t to save on notation and the expectation is taken with respect to the distribution of pairs (u, v) across all islands i in the economy. Note that in deriving this equation we have already taken into account that the optimal acceptance strategy for any worker on this island is to use a reservation match-productivity cutoff, $R(u, v)$. A worker is willing to form a match with a job that offers a match-specific productivity $x \geq R(u, v)$. Later we will show that such reservation productivity exists and is unique. Note that $W_e(x, u, v)$ denotes the worker's expected value of employment given that the match has a match-specific productivity x and was formed on an island with state (u, v) ; and $W_n(u', v')$ denotes the expected value of being in the flow.

To understand (1), begin by noting that the first term on the right side is the flow unemployment income b . The second term in (1) gives the capital gain associated with the worker becoming employed. This occurs when a new job arrives on the worker's island which he is able to perform (which occurs at flow rate λ). The number of other workers who can also perform the job is a random variable, distributed according to a Poisson distribution with parameter u . The worker under consideration can match with the job only if he draws the highest match quality (among those drawn by the $k + 1$ workers who are able to perform the job), at least if that quality exceeds the reservation productivity, $R(u, v)$. The form of the term in the HJB equation then follows by

writing

$$\begin{aligned}
& \lambda \sum_{k=0}^{\infty} e^{-u} \frac{u^k}{k!} \int_{R(u,v)}^{\bar{x}} [W_e(x, u, v) - W_u(u, v)] F(x)^k f(x) dx \\
&= \lambda \int_{R(u,v)}^{\bar{x}} [W_e(x, u, v) - W_u(u, v)] \left[\sum_{k=0}^{\infty} e^{-u} \frac{(uF(x))^k}{k!} \right] f(x) dx \\
&= \lambda \int_{R(u,v)}^{\bar{x}} [W_e(x, u, v) - W_u(u, v)] e^{-u(1-F(x))} f(x) dx,
\end{aligned} \tag{2}$$

where recall that the number of workers that can perform the new job follows a Poisson distribution with parameter u and that the firm will match with the worker that obtains the highest match-specific productivity, conditional on $x \geq R(u, v)$.

The third term on the right side of (1) describes the change in the value of unemployment due to a change in the stock of unemployment. The latter can happen when (i) at flow rate λ new jobs arrive, but with probability

$$\sum_{k=0}^{\infty} e^{-u} \frac{u^k}{k!} [1 - F(R_u)^k] = [1 - e^{-u(1-F(R_u))}]$$

some other unemployed workers take them, subtracting from the stock of unemployment; (ii) at flow rate μ new workers arrive, but with probability

$$\sum_{k=0}^{\infty} e^{-v} \frac{v^k}{k!} F(R(u, v))^k = e^{-v(1-F(R(u, v)))}$$

all the resulting match-specific draws x are below the reservation threshold, and so increasing the stock of unemployed; and (iii) at rate su existing workers in the stock of unemployment leave the island, subtracting from the stock of unemployment.

The fourth term describes the change in the value of unemployment due to a change in the stock of vacancies. The latter can happen when (i) at flow rate $\lambda e^{-u(1-F(R(u, v)))}$, new jobs arrive and are not accepted, so adding to the stock of vacancies; (ii) at flow rate $\mu [1 - e^{-v(1-F(R(u, v)))}]$, a new workers arrives and matches with vacancies in the stock, subtracting from the stock of vacancies; (iii) at flow rate sv existing vacancies leave. Finally, at flow rate s the worker under consideration leaves to a randomly island. Once this happens the worker becomes part of the flow in the new island.

Now consider an employed worker with match-specific productivity x . In this case, the value of employment, $W_e(x, u, v)$, is described by the following HJB equation

$$rW_e(x, u, v) = w(x, u, v) + s [\mathbb{E}_{u', v'} W_n(u', v') - W_e(x, u, v)], \tag{3}$$

where $w(x, u, v)$ denotes the wage of the worker. The value of employment is stationary as the only shock that can affect workers on the employment island is that the match is destroyed, at rate s , in which case the worker is returned to a randomly-drawn island.

Finally, consider a worker in the flow arriving at an island i that is currently characterized by

u_{it} workers in the stock of unemployment and v_{it} firms in the stock of vacancies. The expected value of being in the flow for this worker is given by

$$\begin{aligned}
W_n(u, v) &= W_u(u, v) + \sum_{k=0}^{\infty} e^{-v} \frac{v^k}{k!} \int_{R(u, v)}^{\bar{x}} [W_e(x, u, v) - W_u(u, v)] k F(x)^{k-1} f(x) dx, \\
&= W_u(u, v) + \int_{R(u, v)}^{\bar{x}} [W_e(x, u, v) - W_u(u, v)] \left[\sum_{k=1}^{\infty} e^{-v} \frac{v^k}{k!} k F(x)^{k-1} \right] f(x) dx \\
&= W_u(u, v) + \int_{R(u, v)}^{\bar{x}} [W_e(x, u, v) - W_u(u, v)] \left[\sum_{k=1}^{\infty} v e^{-v} \frac{v^{k-1}}{(k-1)!} F(x)^{k-1} \right] f(x) dx \\
&= W_u(u, v) + v \int_{R(u, v)}^{\bar{x}} [W_e(x, u, v) - W_u(u, v)] e^{-v(1-F(x))} f(x) dx. \tag{4}
\end{aligned}$$

The value of being a worker in the flow is that you get to observe k potential matches with vacancies in the stock, where k is distributed Poisson with mean v . The worker in the flow will match with the job in which he has the highest match-specific productivity. Given that k matches from the stock are observed, the density of the maximum match-specific productivity at x is given by $k F(x)^{k-1} f(x)$; the factor of k is because any of the k matches is equally likely to be the maximum.

2.2 Firm Value Function

Consider an island i at time t , on which there are u_{it} workers in the stock of unemployment and v_{it} firms in the stock of vacancies. The expected value for firm of having a job opening in the stock of vacancies is given by the following HJB equation,

$$\begin{aligned}
r J_v(u, v) &= -\gamma + \mu \int_{R(u, v)}^{\bar{x}} [J_f(x, u, v) - J_v(u, v)] e^{-v(1-F(x))} f(x) dx \\
&\quad + \left[-\lambda \left(1 - e^{-u(1-F(R(u, v)))} \right) + \mu e^{-v(1-F(R(u, v)))} - s u \right] \frac{\partial}{\partial u} J_v(u, v) \\
&\quad + \left[\lambda e^{-u(1-F(R(u, v)))} - \mu \left(1 - e^{-v(1-F(R(u, v)))} \right) - s v \right] \frac{\partial}{\partial v} J_v(u, v) \\
&\quad - s J_v(u, v), \tag{5}
\end{aligned}$$

where once again we have suppressed the subscripts i and t to save on notation. Notice that the kernel in the integral in the first line involves $\exp(-v(1-F(x)))$ rather than $\exp(-u(1-F(x)))$, as in the case of the unemployed worker, because it is other vacancies who compete with this vacancy in trying to match with an inflowing worker. Also, the relevant inflow rate is μ , the arrival rate of workers, rather than λ , the arrival rate of firms. Note that when a reallocation shock hits the vacant job, the job is destroyed. Otherwise, the derivation of (5) mirrors that of (1).

The expected value of a filled job with match-specific heterogeneity x , form on an island with state (u, v) and paying a wage w is given by the following HJB equation,

$$r J_f(x, u, v) = x - w(x, u, v) - s J_f(x, u, v). \tag{6}$$

where the flow profit of a firm is given by $x - w(x, u, v)$. Similarly, to (3) the value of a filled job is

stationary as the only shock that can affect a filled job on the employment island is that the match is destroyed, at rate s , in which case the job is destroyed.

Now consider the expected value of a new vacancy arriving to island i with u_{it} workers in the stock of unemployment and v_{it} jobs in the stock of vacancies. The expected value of being in the flow for this firm is given by

$$\begin{aligned} J_n(u, v) &= J_v(u, v) + \sum_{k=0}^{\infty} e^{-u} \frac{u^k}{k!} \int_{R(u, v)}^{\bar{x}} [J_f(x, u, v) - J_v(u, v)] k F(x)^{k-1} f(x) dx \\ &= J_v(u, v) + u \int_{R(u, v)}^{\bar{x}} [J_f(x, u, v) - J_v(u, v)] e^{-u(1-F(x))} f(x) dx, \end{aligned} \quad (7)$$

where we have used the same arguments as in (4). The value of being a vacancy in the flow exceeds that of being a vacancy in the stock, since a vacancy in the flow can productively match with k unemployed agents in the stock, where k is distributed Poisson with mean u . The vacancy in the flow will match with the best of these match qualities provided that it exceeds the reservation match quality. Given that k matches from the stock are observed, the density of the maximum match-specific productivity at x is given by $k F(x)^{k-1} f(x)$; the factor of k is because any of the k matches is equally likely to be the maximum. If none of the k matches exceeds the reservation quality, then the job enters the stock of vacancies.

Finally, at any time t , a firm decides whether to create a vacancy in these economy or stay inactive. Creating a vacancy in the economy implies paying a sunk cost c , after which the vacancy is randomly allocated on an island i . We assume that there is free-entry, such that at any point in time, the number of firms in the economy, $N(t)$, is determined by

$$c = \mathbb{E}_{u', v'} W_n(u', v'), \quad (8)$$

where the expectation is taken with respect to the distribution of pairs (u, v) across all islands i in the economy.

2.3 Wage Determination

We assume that a worker-firm pair determines a wage using Nash Bargaining at the moment the match is form on the relevant island. The pair commits to this wage for the duration of the match on the employment island. The agreed wage, $w(x, u, v)$, maximizes the Nash product

$$[W_e(x, u, v) - W_u(u, v)]^\beta [J_f(x, u, v) - J_v(u, v)]^{1-\beta},$$

where β denotes the worker's bargaining power. This implies that the surplus of the match

$$S(x, u, v) \equiv [W_e(x, u, v) - W_u(u, v)] + [J_f(x, u, v) - J_v(u, v)], \quad (9)$$

is divided according to

$$\frac{W_e(x, u, v) - W_u(u, v)}{\beta} = S(x, u, v) = \frac{J_f(x, u, v) - J_v(u, v)}{1 - \beta}. \quad (10)$$

Notice that both the unemployed worker and the vacancy will agree on the reservation match quality $R(u, v)$: we will have that $W_e(R(u, v)) = W_u(u, v)$ and $J_f(R(u, v)) = J_v(u, v)$. The crucial assumption we are making by adopting Nash Bargaining in this way is that when a worker and a vacancy decide to form a match, they do not consider other workers or vacancies in the stock. An alternative is to use auctions to determine wages as here the outside option is to form a match with the next best match alternative.

2.4 Reservation Match-specific Productivity

Combining (3) and (6), the surplus associated with a given match match,

$$S(x, u, v) = \frac{x}{r+s} + \frac{s}{r+s} \mathbb{E}_{u',v'} W_n(u', v') - [W_u(u, v) + J_v(u, v)]. \quad (11)$$

This equation is very useful. First, to characterize the reservation match quality, substitute $x = R(u, v)$ and observe that at the reservation match quality, $S(R(u, v), u, v) = 0$, and rearrange to obtain that

$$R(u, v) = (r+s) [W_u(u, v) + J_v(u, v)] - s \mathbb{E}_{u',v'} W_n(u', v'). \quad (12)$$

Second, subtracting $S(R(u, v), u, v) = 0$ from both sides of (11) gives that

$$S(x, u, v) = \frac{x - R(u, v)}{r+s}. \quad (13)$$

It follows that

$$W_e(x, u, v) - W_u(u, v) = \beta S(x, u, v) = \frac{\beta}{r+s} [x - R(u, v)]$$

and

$$J_f(x, u, v) - J_v(u, v) = (1 - \beta) S(x, u, v) = \frac{1 - \beta}{r+s} [x - R(u, v)]. \quad (14)$$

Third, differentiating (12) with respect to u and v , we have that

$$\begin{aligned} \frac{\partial}{\partial u} R(u, v) &= (r+s) \left[\frac{\partial}{\partial u} W_u(u, v) + \frac{\partial}{\partial u} J_v(u, v) \right] \\ \frac{\partial}{\partial v} R(u, v) &= (r+s) \left[\frac{\partial}{\partial v} W_u(u, v) + \frac{\partial}{\partial v} J_v(u, v) \right]. \end{aligned}$$

Finally, add the two HJB equations (1) and (5) and substitute from all the previous equations to get that

$$\begin{aligned} R(u, v) &= b - \gamma + \frac{\lambda\beta}{r+s} \int_{R(u,v)}^{\bar{x}} [x - R(u, v)] e^{-u(1-F(x))} f(x) dx \\ &\quad + \frac{\mu(1-\beta)}{r+s} \int_{R(u,v)}^{\bar{x}} [x - R(u, v)] e^{-v(1-F(x))} f(x) dx \\ &\quad + \frac{1}{r+s} \left[-\lambda \left(1 - e^{-u(1-F(R(u,v)))} \right) + \mu e^{-v(1-F(R(u,v)))} - su \right] \frac{\partial}{\partial u} R(u, v) \\ &\quad + \frac{1}{r+s} \left[\lambda e^{-u(1-F(R(u,v)))} - \mu \left(1 - e^{-v(1-F(R(u,v)))} \right) - sv \right] \frac{\partial}{\partial v} R(u, v). \end{aligned} \quad (15)$$

This equation gives a PDE which determines $R(u, v)$ in any island i , where note that each island will have different reservation match-specific productivities because each island offers different $F_i(x)$ for workers and firms.

2.5 Worker and Job Flows

On each island i , the evolution of the stock of unemployment and the stock of vacancies is described by the following pair of differential equations,

$$\dot{u} = -\lambda \left(1 - e^{-u(1-F(R(u,v)))}\right) + \mu e^{-v(1-F(R(u,v)))} - su \quad (16)$$

$$\dot{v} = \lambda e^{-u(1-F(R(u,v)))} - \mu \left(1 - e^{-v(1-F(R(u,v)))}\right) - sv. \quad (17)$$

Equation (16) shows that the stock of unemployment on an island decreases when new vacancies arrive and successfully match with the existing stock of unemployed workers or when some of the existing unemployed workers experience a reallocation shock. The stock of unemployment increases when a new worker arrives to the island and he is unsuccessful in matching with the existing stock of vacancies. Similarly, equation (17) shows that the stock of vacancies on an island decreases either because new workers arrive to the island and are successful in matching with some the vacancies in the stock or because some vacancies get destroyed. The stock of vacancies increases when new vacancies arrive and are not successful in matching with the stock of unemployment.

3 Equilibrium

Definition 1. A stock-flow matching equilibrium is a set of value functions W_u, W_e, W_n, J_v, J_f and J_n ; a measure of unemployed workers, u_{it} , and job vacancies v_{it} in the stock; a set of wages $w(x, u_{it}, v_{it})$ for each $x \in [\underline{x}_i, \bar{x}_i]$; a reservation match-specific productivity $R(u_{it}, v_{it})$ for each island $i \in [0, 1]$; the rate at which vacancies flow into new island, λ ; and the measure of $N(t)$ of firms such that:

1. The reservation match-specific productivity $R(u, v)$, the measure of unemployed workers u and vacant firms v in the stock of a given island, and the measure of firms in the economy N , satisfy equations (15), (16), (17), (8), and $\lambda = sN$.
2. The value functions W_u, W_e, W_n, J_v, J_f and J_n , satisfy equations (1), (3), (4), (5), (6) and (7).
3. Wages are determining through Nash Bargaining on every island and solve (10).

3.1 Characterisation of Steady States

We start our analysis by solving for the steady state in a single island.

Because the free entry condition in our economy applies at the level of the entire economy, from the point of view of an individual island, the inflow rates of workers and firms, λ and μ , are exogenous, and in steady state, they are also constant. This means that the steady state, at

the level of the single island, can be characterized as the solution to the following three equations, which are the steady-state versions of (15), (16), and (17):

$$R = b - \gamma + \frac{\lambda\beta}{r+s} \int_R^{\bar{x}} [x - R] e^{-u(1-F(x))} f(x) dx + \frac{\mu(1-\beta)}{r+s} \int_R^{\bar{x}} [x - R] e^{-v(1-F(x))} f(x) dx \quad (18)$$

$$0 = -\lambda \left(1 - e^{-u(1-F(R))}\right) + \mu e^{-v(1-F(R))} - su \quad (19)$$

$$0 = \lambda e^{-u(1-F(R))} - \mu \left(1 - e^{-v(1-F(R))}\right) - sv. \quad (20)$$

Notice that subtracting the last two equations implies a close and intuitive relationship between u and v , specifically, $v - u = (\lambda - \mu)/s$. This is intuitive: since matching between workers and jobs is one-to-one, and since each are exogenously reallocated at rate s , the only difference between their numbers is generated because of their different inflow rates μ and λ respectively.

We can establish the following Proposition

Proposition 1. *Conditional on the inflow rates λ and μ , a unique steady-state equilibrium (R, u, v) exists.*

Proof. See Appendix. □

The proof of existence and uniqueness is constructive, so that it is straightforward to establish comparative statics results

Proposition 2. *Conditional on the inflow rates λ and μ , an increase in b , a decrease in γ , and a change in the distribution of $F(\cdot)$ cause the reservation match quality R and the measures of unemployed workers and vacancies to move in the same direction.*

Proof. See Appendix. □

An increase in b or a decrease in γ reduces the surplus associated with employment matches. This increases the option value of waiting for the best matches, relative to less high quality matches. When workers and firms become more picky about match quality, all else equal, the measures of unmatched workers and firms increase.

The comparative static effects of changes in the arrival rates of jobs and workers, λ and μ , are more difficult to deal with, since changes in the flow arrival rate of jobs (or workers) has several effects. Intuitively:

- A faster arrival rate of matches increases the reservation match quality conditional on u and v , since workers and jobs become more picky about match quality as they expect to be able to sample again from the match quality distribution sooner. (This is clear in (27).)
- A faster arrival rate of jobs should increase the number of vacancies and decrease the number of unemployed workers. However, we do not at this time have a proof of this result. We can show that a faster arrival rate of jobs increases v relative to u ; this is clear from (31). We can also show the following lemma:

Lemma 1. *Conditional on R , an increase in λ increases v and decreases u .*

- But of course, all of the previous point was conditional on R . And adjustment of R is key to the mechanism that we are proposing. We already know that an increase in R , all else equal, leads to an increase in u and v . Thus, the net effect on v of an increase in λ seems likely to be positive: the direct effect is positive, and the indirect effect via R is also positive. The effect on u seems more complicated: now the two effects go in opposite directions.
- Of course, the size of the effect on $F(R)$ of any change in R can be arbitrarily small. For example, imagine that $F(\cdot)$ is a two-point distribution, placing small mass on R^H and large mass on $R^L \ll R^H$. Then first, R will never exceed R^H provided that $b - \gamma < R^H$ so that positive activity is optimal, and second, movements of R within the interior of the interval $[R^L, R^H]$ do nothing to $F(R)$, which remains equal to the mass at R^L for all such R . Thus, the case where there is no change in $F(R)$.
- Similarly, if the distribution of $F(x)$ is very concentrated near R (so that almost all jobs are near the reservation quality, either slightly above or slightly below, then small changes in parameters, for example, λ (I guess, although the usual comparative static is small changes in $F(\cdot)$ itself or in b) will deliver very large changes in $F(R)$.

The effects of μ on u and v are the opposite: that seems immediate from the symmetry of the system in u and v .

3.2 Free Entry

In the previous subsection, we studied equilibrium on a single island. Now consider equilibrium at the level of the whole economy. For simplicity, we assume that all islands are the same, that is, $F_i(x) = F(x)$ for all i . This simplified setting allows us to write the free entry condition in a simple way.

To write the equations of the model under free entry in a steady state, we need to specialize the equilibrium equations already written to the steady state case. In steady state, the HJB equation for the value of a vacancy in the stock is:

$$(r + s)J_v(u, v) = -\gamma + \mu \int_{R(u, v)}^{\bar{x}} [J_f(x, u, v) - J_v(u, v)] e^{-v(1-F(x))} f(x) dx \quad (21)$$

(This is (5) with the change-of-state capital gains terms removed since they are zero in steady state.) Use (14) to eliminate $J_f(x, u, v) - J_v(u, v)$:

$$(r + s)J_v(u, v) = -\gamma + \frac{(1 - \beta)\mu}{r + s} \int_{R(u, v)}^{\bar{x}} [x - R(u, v)] e^{-v(1-F(x))} f(x) dx \quad (22)$$

or

$$J_v(u, v) = -\frac{\gamma}{r + s} + \frac{(1 - \beta)\mu}{(r + s)^2} \int_{R(u, v)}^{\bar{x}} [x - R(u, v)] e^{-v(1-F(x))} f(x) dx$$

When all the islands are identical, the free entry condition, with an entry cost of k , takes the form

$$k = J_n(u, v)$$

which can be simplified to give

$$\begin{aligned} (r + s)k + \gamma &= (1 - \beta)u \int_R^\infty [x - R] e^{-u(1-F(x))} f(x) dx \\ &+ \frac{(1 - \beta)\mu}{(r + s)} \int_R^{\bar{x}} [x - R] e^{-v(1-F(x))} f(x) dx \end{aligned} \quad (23)$$

Or, in the case where $r = 0$, we have

$$\begin{aligned} sk + \gamma &= (1 - \beta)u \int_R^\infty [x - R] e^{-u(1-F(x))} f(x) dx \\ &+ (1 - \beta)M \int_R^{\bar{x}} [x - R] e^{-v(1-F(x))} f(x) dx \end{aligned} \quad (24)$$

This is intuitive. The left side is the flow cost of being a vacancy, taking into account both the user cost of the (capital) entry cost and the flow cost of vacancy posting. The right side is the value, first of the possibility of matching in the stock, then of matching in the flow.

That makes the system of equations characterizing the free entry equilibrium as follows. (We assume that $r = 0$ since this simplifies the form of the equations somewhat.)

$$\begin{aligned} R &= b - \gamma + \beta N \int_R^{\bar{x}} [x - R] e^{-u(1-F(x))} f(x) dx \\ &+ (1 - \beta)M \int_R^{\bar{x}} [x - R] e^{-v(1-F(x))} f(x) dx \\ sk + \gamma &= (1 - \beta)u \int_R^\infty [x - R] e^{-u(1-F(x))} f(x) dx \\ &+ (1 - \beta)M \int_R^{\bar{x}} [x - R] e^{-v(1-F(x))} f(x) dx \\ 0 &= -N \left(1 - e^{-u(1-F(R))} \right) + M e^{-v(1-F(R))} - u \\ 0 &= N e^{-u(1-F(R))} - M \left(1 - e^{-v(1-F(R))} \right) - v. \end{aligned}$$

Two simplifications are possible. First, we can simplify the first two equations somewhat by subtracting the second equation from the first:

$$R = b + sk + [\beta N - (1 - \beta)u] \int_R^\infty [x - R] e^{-u(1-F(x))} f(x) dx.$$

This can be substituted into either the first or second equations to obtain an equation only involving $\int_R^{\bar{x}} [x - R] e^{-v(1-F(x))} f(x) dx$ also.

Second, we can as usual subtract the third and fourth equations, to end up with

$$v - u = N - M,$$

Parameter	b	γ	$\lambda = \mu$	s	r	μ_F	σ_F
Value	0.7	0	10	0.1	0	0	0.5

Table 1: Baseline parameters

which is intuitive given that matching is one-to-one.

We conjecture that it is possible to show that an equilibrium exists and is unique.

There are some interesting comparative statics results that can be seen from the equilibrium equations. Notice that the separation rate s does not affect the system in any way, conditional on the amount of entry. This arises from the fact that the separation rate is the only source of discounting, together with the fact that it affects all agents symmetrically.

We will analyze the steady state equations analytically further.

3.3 Numerical analysis

To study some of the properties of our model in more detail, we now turn to investigate a numerical example. We solve our model for a baseline set of parameters, as given in [Table 1](#).

We parameterize our model as follows. First, we assume that the match quality distribution is lognormal, with mean μ_F and variance σ_F^2 . The lognormal distribution allows us to capture the extensive wage dispersion present empirically due to the large amount of mass it places in its right tail. We assume that $r = 0$ for tractability (small positive values of r do not affect the results). We assume that the model unit of time is quarterly, and set $s = 0.1$ to approximate the quarterly employment-unemployment transition probability in U.S. data ([Shimer, 2005](#)). We set $\lambda = \mu$ for simplicity, and set both equal to 10. (To put this in context, notice that in steady state, if all matching opportunities were rejected, this means that there would be measures of 100 workers and 100 jobs on each matching island. Given that not all matching opportunities will be rejected, more precisely, there are 100 workers and 100 jobs per matching island, but some of these will be located on the employment island instead.) We set $b = 0.7$ and $\gamma = 0$. Only the sum of these parameters matters. That $b + \gamma = 0.7$ implies that, for our baseline match quality distribution, 23 percent of matches are less productive than unemployment, even not allowing for the loss of option value of better future matches generated by taking a job.

For our baseline parameters, the reservation match quality is $R = 2.49$, which is such that around 3.4 percent of matches are above the reservation quality. However, since $\lambda = 10$, the arrival rate of new potential matches is high, and accordingly the unemployment rate is only around 16.0 percent. (That is, there are around a measure 16 of workers and jobs in the stock of unemployed workers and vacant jobs on each matching island, and around a measure 84 of matched worker-job pairs on the employment island.)

We now seek to understand the effect of some of the parameters on the steady state equilibrium in the island. [Figure 1](#) shows the effect of changing $\lambda = \mu$. As can be seen, as the inflow rate rises, so too does the measure of unmatched agents (the number of unemployed agents and the number of vacant jobs is equal in this symmetric setting). However, the unemployment rate falls slightly, as the increase in the flow rates mean that fewer agents are unlucky enough to find no

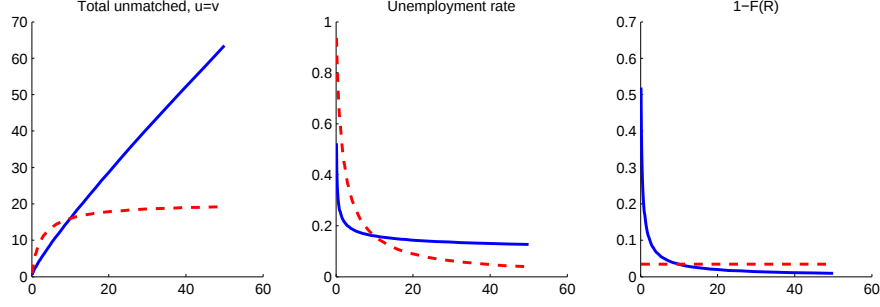


Figure 1: Effect of changes in flow arrival rate of unemployed workers and vacant jobs, $\lambda = \mu$. In each panel, the horizontal axis shows different values of λ . The blue, solid curves show the equilibrium characterized by (18), (19), and (20). The red, dashed curves show what would happen if the reservation match quality were held constant at its baseline level R_0 .

matches that they are suited to. The reservation match quality rises substantially as λ rises; this can be seen in the right panel of Figure 1, which shows the fraction of matches that are acceptable. The change in the reservation match quality has important effects on the equilibrium comparative statics. This can be seen by inspecting the red dashed curves, which show the effect on the three variables plotted of mechanically holding the reservation match quality constant at its baseline. If workers and jobs did not respond to the rise in inflow rates of new potential matches by becoming much pickier, then unemployment and vacancy rates would fall much faster as λ rises.

Figure 2 shows the effect of changing s . Intuitively, a higher s has two effects. First, mechanically, it lowers the number of unemployed agents in the stock, holding constant the reservation match quality. (However, the unemployment rate rises since the expected duration of matches falls.) Second, it lowers the reservation match quality, since it lowers the expected duration of a match, making it less worthwhile to wait for a very high quality match. As can be seen in the Figure, holding the reservation match quality constant at the baseline means that this second effect is shut off, causing the unemployment rate to rise more quickly with s than it did in the true equilibrium.

We can also investigate the effects of other parameters, with intuitive results. Figure 3 shows the effect of increasing the mean of the match quality distribution $F(\cdot)$, and Figure 4 shows the effects of increasing the variance. As can be seen, raising μ_F makes agents less picky about match quality and lowers the unemployment rate, since it is more costly to be unmatched rather than matched, more matches become acceptable, and $1 - F(R)$ rises. Raising σ_F increases the variance of match quality, making it more worthwhile to wait for a very high quality match, lowering the fraction of matches that are above the reservation quality and raising the unemployment and vacancy rates. Finally, raising b lowers the surplus from matching, and so acts like a reduction in μ_F , raising the unemployment rate and lowering the fraction of acceptable match quality draws.

Although we have only analyzed changes in steady states in a single local labor market using exogenous flow rates here, it is intuitive to see how our arguments generalize to understanding cross-sectional dispersion in unemployment and vacancy rates. We can also understand the comparative static effects of aggregate productivity changes through the exercise we did here. However, the effect is complex: a reduction in the productivity of all matches will lower the entry rate of jobs (via the free entry condition), as well as lowering the surplus associated with being matched, thus

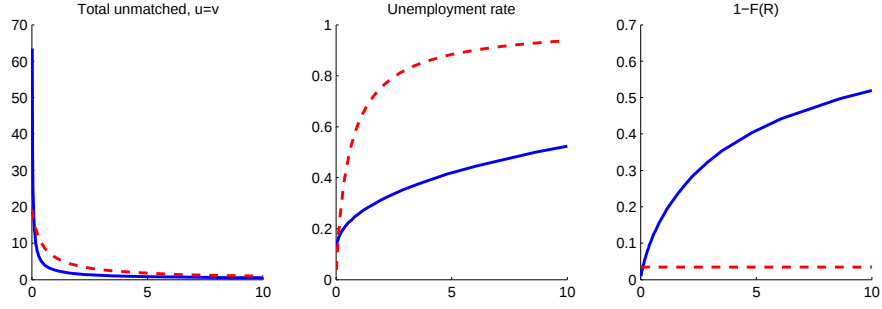


Figure 2: Effect of changes in reallocation rate, s . In each panel, the horizontal axis shows different values of s . The blue, solid curves show the equilibrium characterized by (18), (19), and (20). The red, dashed curves show what would happen if the reservation match quality were held constant at its baseline level R_0 .

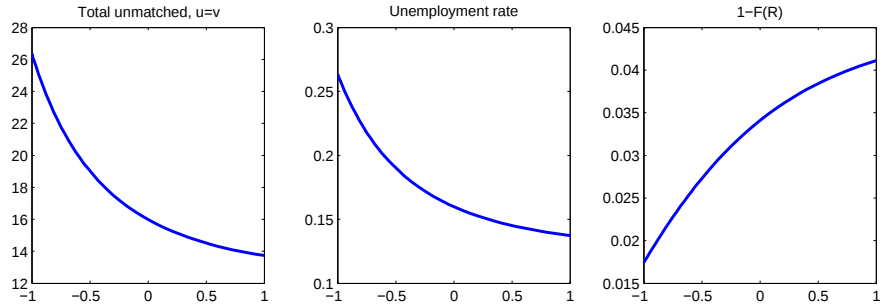


Figure 3: Effect of changes in the mean of log match quality, μ_F .

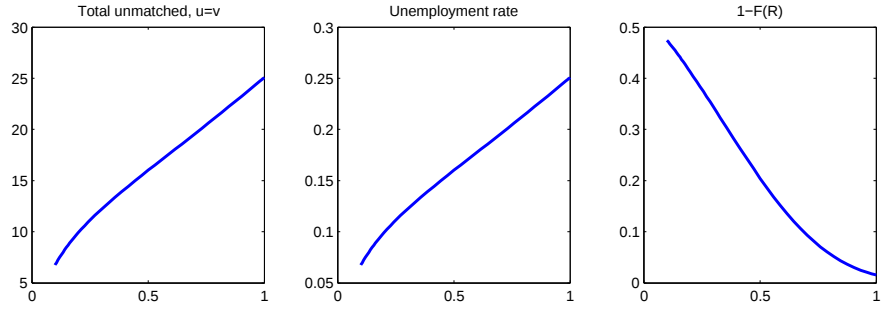


Figure 4: Effect of changes in the standard deviation of log match quality, σ_F .

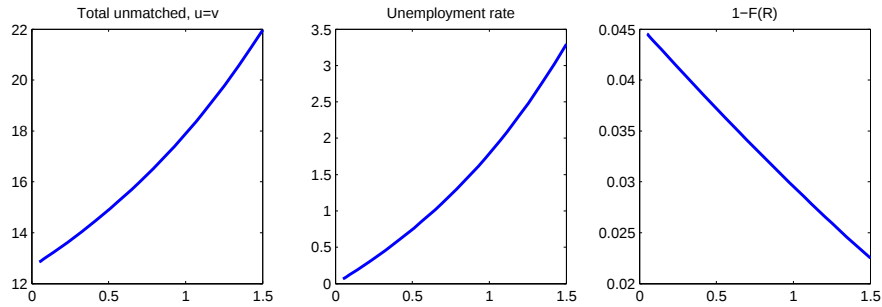


Figure 5: Effect of changes in unemployment income, b .

lowering the fraction of match quality draws which exceed the reservation value.

Notice an interesting feature of our model concerning the efficiency of the matching function. Because of the symmetry of the parametric example considered here ($\lambda = \mu$), by construction we had that there are equal numbers of unemployed workers and vacant jobs in every parametric example shown here. Thus, an econometrician who estimated a reduced-form matching function which took as inputs only the stocks of unemployed workers and vacancies and assumed constant returns, as is standard in implementing the DMP model, would view every change in unemployment and vacancy rates shown above as arising from changes in matching efficiency. It cannot be otherwise: the vacancy-unemployment ratio is constant and equal to one in every example shown here! Thus, the change in the set of acceptable matches, the key mechanism emphasized by our model, seems a very natural candidate for generating shifts in the Beveridge curve at business cycle frequencies, as has been observed in U.S. data by Elsby et al. (2010) and Şahin et al. (2012).²

4 Out-of-Steady State Dynamics

To be written. The model has a nice partial differential equation structure which we can analyze to find the out-of-steady state dynamics.

5 Conclusion

To be written.

A Omitted Proofs

Proof of Proposition 1. We give the proof informally for now.

To gain intuition, we consider the simplest possible case, where $\lambda = \mu$. In this case $v = u$. The equations take the form

$$R = b - \gamma + \frac{\lambda}{r + s} \int_R^{\bar{x}} [x - R] e^{-u(1-F(x))} f(x) dx \quad (25)$$

$$su = \lambda \left[2e^{-u(1-F(R))} - 1 \right]. \quad (26)$$

Conditional on R , (26) is the flow balance equation, which determines $u = v$. The left side of (26) is linear and increasing in u , taking the value 0 at $u = 0$ and increasing as u increases. The right side is decreasing in u , taking the value $+\lambda$ at $u = 0$ and decreasing (provided $1 - F(R) > 0$). It converges to the value $-\lambda$ as $u \rightarrow +\infty$. Therefore conditional on R there is a unique solution for u . This is shown in Figure 6. It is straightforward to see that increasing R decreases $1 - F(R)$ (the probability a match quality is above R). This leads the red curve to shift upwards (from the dashed curve to the dotted curve), so increasing u . That is, in (26), higher R leads to a higher

²We should note that the fact that all shifts in the matching pattern here are associated with shifts in matching efficiency arises from the fact that we constrained the arrival rates of workers and jobs to be equal, $\lambda = \mu$. Changes in λ relative to μ would be interpreted by an econometrician estimating a constant-returns matching function as shifts in labor market tightness, at least in part.

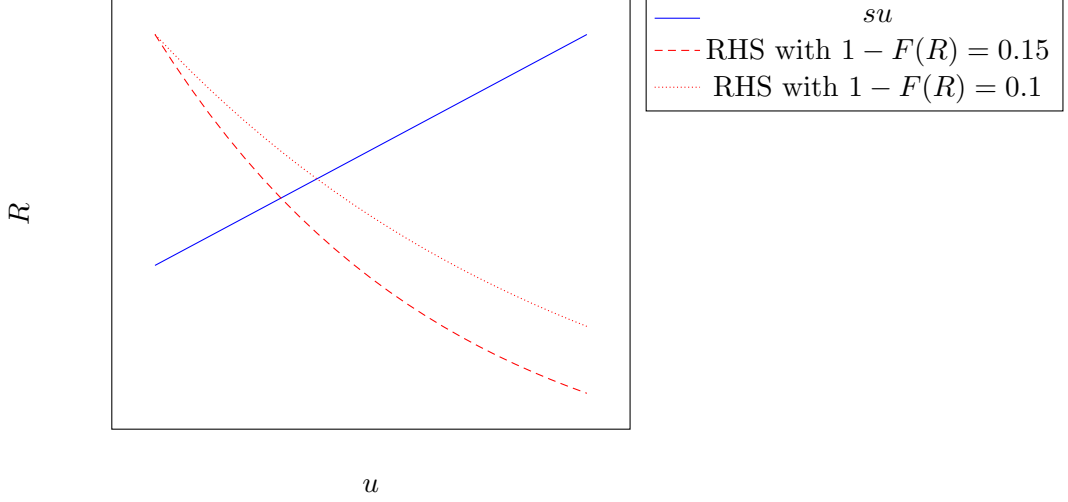


Figure 6: Equation (26)

u . Or, in (R, u) -space, the locus defined by (26) is an upward-sloping curve and strictly upward sloping for R in the support of $F(\cdot)$.

Also, conditional on u , it is similarly easy to analyze (25). This is the reservation wage equation. The left side is linear and increasing in R . The right side is strictly decreasing whenever R is in the support of $F(\cdot)$. Thus, conditional on u , there exists a unique solution for R . To understand the comparative statics, notice that the right side is decreasing in u for any R , that is, the curve defined by the right side of (25) shifts down as u increases. Intuitively, higher $u = v$ means more competition from existing agents in the stock whenever a new match shows up in the flow, reducing the match quality and so reducing how picky each agent is going to be. That is, in (25), higher u leads to a lower R . In (R, u) -space, the locus defined by (26) is a strictly downward-sloping curve whenever R is in the support of $F(\cdot)$. From now on assume that $F(\cdot)$ has connected support on a right-infinite interval, say (by normalization) $[0, \infty)$.

The fact that u is unique conditional on R and that R is unique conditional on u is not sufficient by itself to show that there is a unique solution for (R, u) . However, the fact that the slopes of the two curves are respectively strictly positive and strictly negative is sufficient to show uniqueness. To show existence, it's sufficient to think about the limits of the two locuses.

- When $u \rightarrow +\infty$, the right side of (25) decreases and converges to $b - \gamma$, so $R \rightarrow b - \gamma$. This is intuitive: if there are very many other agents in the stock, you will never match, so you will accept any match delivering positive surplus.
- When $u \rightarrow 0$, the right side of (25) increases to $b - \gamma + \frac{\lambda}{r+s} \int_R^\infty [x - R] dx$, that is, the value of the problem when there is no competition. Call this number $\bar{R}$.
- When $R \rightarrow 0$, the right side of (26) decreases to $\lambda[2e^{-u} - 1]$
- When $R \rightarrow +\infty$, the right side of (26) increases and converges to λ , so that in the limit, $u \rightarrow s/\lambda$. Again, this is intuitive: if no matches are acceptable, all agents in the flow become unmatched, so the unemployment rate is determined by equating the inflow, λ , to the outflow, su .

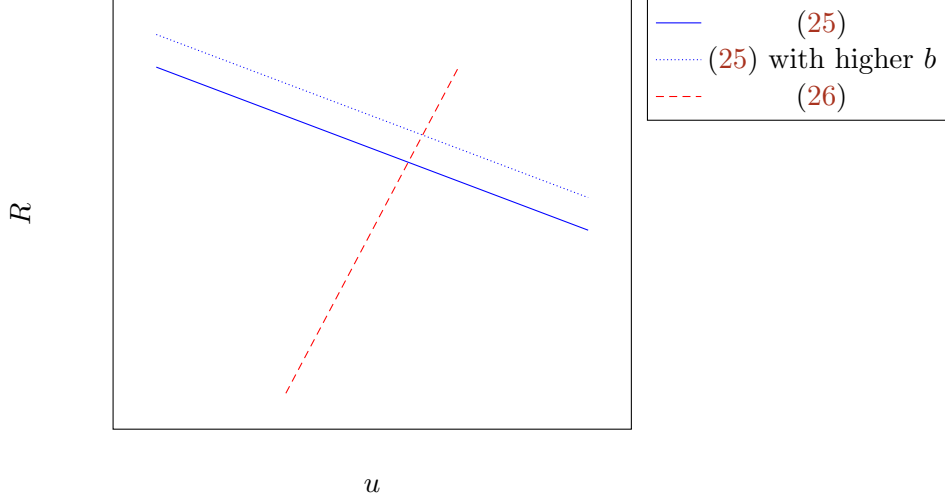


Figure 7: Equilibrium existence and uniqueness

To see that there will exist a solution, consider a graph with u on the horizontal axis and R on the vertical axis. The locus defined by (25) is a downward sloping curve linking $(0, \bar{R})$ to $(+\infty, b - \gamma)$. The locus defined by (26) is an upward sloping curve linking $(\lambda [2e^{-u} - 1], 0)$ to $(\lambda, 0)$. It's immediate by continuity that there is a solution. See Figure 7.

Now return to the case where $\lambda \neq \mu$. Write the equations as

$$R = b - \gamma + \frac{\lambda\beta}{r+s} \int_R^{\bar{x}} [x - R] e^{-u(1-F(x))} f(x) dx + \frac{\mu(1-\beta)}{r+s} \int_R^{\bar{x}} [x - R] e^{-v(1-F(x))} f(x) dx \quad (27)$$

$$0 = \lambda \left(e^{-u(1-F(R))} - 1 \right) + \mu e^{-v(1-F(R))} - su \quad (28)$$

$$v = u + \frac{\lambda - \mu}{s}. \quad (29)$$

Substitute from the last equation to eliminate v from the previous two equations:

$$R = b - \gamma + \frac{\lambda\beta}{r+s} \int_R^{\bar{x}} [x - R] e^{-u(1-F(x))} f(x) dx + \frac{\mu(1-\beta)}{r+s} \int_R^{\bar{x}} [x - R] e^{-\left(u + \frac{\lambda-\mu}{s}\right)(1-F(x))} f(x) dx \quad (30)$$

$$su = \lambda \left(e^{-u(1-F(R))} - 1 \right) + \mu e^{-\left(u + \frac{\lambda-\mu}{s}\right)(1-F(R))}. \quad (31)$$

Compare (30) to (25) and (31) to (26).

Does the proof from before carry over? The left side of (31) is linear and increasing in u as before; the right side is decreasing in u conditional on R and falls from μ to $-\lambda$ as u increases from 0 to $+\infty$, again assuming $1 - F(R) > 0$. So, this part is fine. Similarly, it's again true that the right side of (30) is decreasing in R , while the left side is increasing, and a solution clearly exists. The comparative statics work as before, so the two locuses defined in (R, U) -space by the two equations are still upward- and downward-sloping as before. A very similar argument to before

then establishes existence. $\square$

Proof of Proposition 2. First consider the simplified setting where $\lambda = \mu$. Because $b - \gamma$ and the distribution $F(\cdot)$ only appear in the reservation match quality equation (25), anything that increases the reservation wage conditional on unemployment $u = v$ will move the equilibrium along the upward-sloping locus defined by (26). Thus, in this setting with exogenous flow rates, increases in R and in u go together: any change which doesn't affect (26) will cause u (and v) and R to move in the same direction.

In the more general case with different flow rates, the comparative statics with respect to b , γ , or a shift in the distribution of $F(\cdot)$ are similar. They only occur in (30). Any change, like an increase in b , which increases R conditional on u in that equation, will lead to an increase in both R and u (and therefore in v , which preserves a constant difference $(\lambda - \mu)/s$ from u). $\square$

Proof of Lemma 1. Differentiation gives that

$$0 = \left(e^{-u(1-F(R))} - 1 \right) - \lambda(1 - F(R))e^{-u(1-F(R))} \frac{\partial u}{\partial \lambda} - \mu(1 - F(R))e^{-v(1-F(R))} \frac{\partial v}{\partial \lambda} - s \frac{\partial u}{\partial \lambda}$$

and

$$\frac{\partial v}{\partial \lambda} = \frac{\partial u}{\partial \lambda} + \frac{1}{s}.$$

Use this to eliminate $\partial v/\partial \lambda$ from the preceding equation:

$$0 = \left(e^{-u(1-F(R))} - 1 \right) - \frac{\partial u}{\partial \lambda} \left[(1 - F(R)) \left(\lambda e^{-u(1-F(R))} + \mu e^{-v(1-F(R))} \right) + s \right] - \frac{1}{s} \mu(1 - F(R))e^{-v(1-F(R))}$$

Rearrange:

$$\frac{\partial u}{\partial \lambda} = - \frac{[1 - e^{-u(1-F(R))}] + \frac{1}{s} \mu(1 - F(R))e^{-v(1-F(R))}}{(1 - F(R)) (\lambda e^{-u(1-F(R))} + \mu e^{-v(1-F(R))}) + s}$$

Every expression in the fraction on the right side is positive, so we do get that $\partial u/\partial \lambda < 0$ as conjectured. To show that $\partial v/\partial \lambda > 0$ needs a different simplification. Take the original equation

$$sv = \lambda e^{-u(1-F(R))} + \mu \left[e^{-v(1-F(R))} - 1 \right]$$

and differentiate with respect to λ :

$$s \frac{\partial v}{\partial \lambda} = e^{-u(1-F(R))} - \lambda(1 - F(R))e^{-u(1-F(R))} \frac{\partial u}{\partial \lambda} - \mu(1 - F(R))e^{-v(1-F(R))} \frac{\partial v}{\partial \lambda}$$

Rearranging this gives that

$$\frac{\partial v}{\partial \lambda} = \frac{e^{-u(1-F(R))} [1 - \lambda(1 - F(R)) \frac{\partial u}{\partial \lambda}]}{s + \mu(1 - F(R))e^{-v(1-F(R))}}.$$

Since $\partial u/\partial \lambda < 0$, it is immediate that $\partial v/\partial \lambda > 0$. $\square$

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