Global Entropy

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Abstract

The recent fiscal crisis in the EU and the slow-down of the BRICS countries have raised world-wide concerns about future global growth prospects. We examine the role of doubts about both local and foreign economic shocks by constructing an international endogenous growth model with technology diffusion across countries. In this setting, endogenous technology spillovers generate global growth shocks. When agents have concerns for robustness, country-specific shocks (1) alter global entropy, and (2) generate long-term contagion with substantial welfare losses.

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1 Introduction

The world economy is becoming increasingly integrated. Trade barriers are steadily falling and international protection of patents facilitates the global diffusion of new ideas and technologies. This leads to a sort of global-growth-sharing, as improved growth prospects in one country may spill over into other economies through technological diffusions. The flip side of this development is that local bad news can be a source of world-wide uncertainty and may slow down growth globally. Current events provide a clear example of this channel. As we write, indeed, the negative economic outlook about European Union and the more pessimistic expectations about the BRICS countries are generating adverse effects on world-wide growth expectations.

In this paper, we analyze the extent to which uncertainty and doubts about the growth prospects of one specific country can affect global growth. We do this in a setup with two distinguishing features. First, we develop a two-country stochastic model of endogenous global growth with endogenous innovation and technology diffusion across countries. In each country, innovators develop new patents by means of R&D, as in Romer (1990). In the spirit of Santacreu (2011), costly adoption of new patents within and across countries then leads to slow international diffusion of technologies, resulting in equilibrium global growth. We denote this international version of the Romer's model as iRomer.

Second, we assume that agents are uncertain about the joint probability distribution of future growth prospects. More precisely, we assume that agents fear model uncertainty as in Hansen and Sargent (2007) and are willing to optimally slant probabilities towards the worst-case scenario. This feature of the model is important to capture the impact of worst-case scenario considerations on long-term global growth. Both domestic innovation and adoption of foreign technologies depend on risk- and uncertainty-adjusted present value of future expected profits. When agents fear misspecification, both domestic and foreign entropy turn into a crucial endogenous deterrent of world-wide growth.

In this article, we consider various sources of shocks. First of all, we consider orthogonal local transitory shocks to productivity levels and show that they endogenously create a persistent global growth factor. This is consistent with empirical evidence (see, among others, Kose et al. 2003, Kose et al. (2008), Lustig and Verdelhan 2007, and Lustig et al. 2011a, b) and provides an equilibrium foundation to the international long-run risk literature (see, among others, Colacito and Croce 2010, 2011). This result is driven by our equilibrium international diffusion of technology. Preferences for robustness quantitatively enhance this feature.

Second, we consider shocks to productivity volatility (henceforth vol-of-vol shocks) across different countries in order to analyze the impact the regional risk plays in determining global entropy and growth. This exercise is interesting because it shades light on the connection between vol-of-vol shocks and long-term growth, an aspect studied so far in reduce form models (see among others, Bansal and Yaron (2004)). Specifically, we consider country-specific volatility shocks whose magnitude is inversely related to the size of each country. This is particularly relevant when we think about technology diffusion and integration among developed and emerging economies, as they feature very different volatility profiles.

More broadly, this part of our investigation is relevant because it endogenously creates contagion, defined as a rise in international asset prices correlations not immediately accompanied by international macroeconomic co-movements. Our model rationalizes this phenomenon by creating short-run adjustment in asset prices followed by long-term macroeconomic global adjustments. Robustness is actually crucial to produce this result, as it features risk-duration sensitivity, an element not present in standard time-additive preferences. We illustrate this point by studying the dynamic value decomposition (DVD) methods developed by Jaroslav et al. (2011).

Third, we analyze shocks to both the R&D and the adoption sectors, to capture the idea that different countries may experience sudden and persistent boosts to their innovation ability. Although we model these shocks in reduced form, it would be easy to endogenize them and relate them to policy shocks, in the spirit of Croce et al. (2012).

Finally, we study the welfare implications of regional and global shocks for welfare across countries. Following the methodology in Barillas et al. (2009), we link welfare losses associated with worst-case beliefs to the market price of model uncertainty, which we label as the market price of global entropy. Preliminary results suggest that global uncertainty is a first-order determinant of global welfare as it endogenously affects long-term growth.

As a final remark, note that we differ from Santacreu (2011) in several respects. First, our technology structure is novel and, under special calibrations, it collapses to a collection of closed Romer (1990)'s economies. This is relevant to highlight the role of equilibrium international spillovers. Second, we consider robustness corners and take seriously the asset-pricing side of our open economy. Third, we explicitly consider risk and uncertainty when characterizing our economy, as opposed to study it in a neighborhood of the deterministic steady state. Four, endogenous international diffusion of technology provides us general equilibrium guidance on the propagation of vol-of-vol shocks, an element neglected in Santacreu (2011).

The remaining of this manuscript is organized as follows. The next section describes the economy. All computational details necessary to replicate our work are reported in our Appendix.

2 Model

In this section we describe in detail the stochastic model of endogenous growth and technology diffusion we use to examine the link between regional shocks, global entropy and concerns for robustness. As in Romer (1990), sustained productivity growth can be obtained through the accumulation of new patents on innovations that facilitate the production of the final good. In the spirit of Santacreu (2011), growth can also be sustained by promoting a costly adoption of foreign technology.

In this class of models, the speed of patent accumulation, i.e., the growth rate of the economy, depends on the market value of the additional cash flows generated by such innovations. Given that our representative agent has concerns for robustness, the market value of a patent is sensitive to fear about misspecification. Since households price uncertain payoffs using the worst-case distribution, doubts about both future regional cash flows generate a premium for exposure to model uncertainty that affects incentives to innovate and world-wide long-term growth.

For simplicity, we abstract away from physical capital accumulation. The production of the final good is assumed to depend only on three elements: (i) an exogenous stochastic and stationary productivity process, (ii) the international stock of patents, and (iii) the endogenous amount of labor supplied. In our model, labor income is taxed proportionally by the government to finance an exogenous stochastic expenditure stream.

In what follows, we characterize the economy from a home-country perspective only. All equations for the foreign country can be derived in a symmetric way and are detailed in our Appendix. For the sake of simplicity, we drop country-specific indexes wherever they are unnecessary.

2.1 Household

Consumption Bundle In each period, the representative agent consumes a bundle u_t of consumption, C_t, and leisure, $1 - L_t$, defined as follows:

$$u_t = \left[\kappa C_t^{1-\frac{1}{\nu}} + (1-\kappa)[T_t(1-L_t)]^{1-\frac{1}{\nu}}\right]^{\frac{1}{1-\frac{1}{\nu}}}.$$

We let L_t and ν denote labor and degree of complementarity between leisure and consumption, respectively. Leisure is multiplied by T_t, our measure of standards of living, to guarantee balanced growth when $\nu \neq 1$.

Aversion to Model Uncertainty We assume that the representative household has Hansen and Sargent (2007) preferences defined over u_t:

$$U_t = (1-\beta)\ln u_t + \beta \min_{m_{t+1}}[E_t m_{t+1} U_{t+1} + \theta E_t m_{t+1} \ln m_{t+1}],$$

where m_{t+1} is a probability distortion constrained to integrate to unity, and $\theta > 0$ captures confidence in the approximating model. Our agent is afraid that his approximating model is misspecified and considers alternative models that are nearby in the sense of relative entropy ($E_t m_{t+1} \ln m_{t+1}$). This implies that economic decisions are based on a probability measure, $\widetilde{\pi}_{t+1|t}$, optimally slanted towards the worst states. For $\theta = \infty$, there is full trust of the approximating model and these preferences reduce to expected utility. To be precise and fix notation, let $\pi_{t+1|t}$ denote the conditional probability of state s_{t+1} at time t induced by the approximating model. As in Hansen and Sargent (2007), the distorted probability can be linked to $\pi_{t+1|t}$ as follows:

$$\widetilde{\pi}_{t+1|t} = \pi_{t+1|t} \cdot m_{t+1},$$

where the optimal m_{t+1} is

$$m_{t+1} = \frac{e^{-\frac{U_{t+1}}{\theta}}}{E\left[e^{-\frac{U_{t+1}}{\theta}}\right]}.$$

Inserting the optimal distortion m_{t+1} into the preferences of the household delivers the indirect utility function

$$U_t = (1-\beta)\log u_t - \beta\theta \log E_t\left[e^{-\frac{U_{t+1}}{\theta}}\right]. \tag{1}$$

Robustness concerns, θ, is linked to uncertainty sensitivity, $\gamma_U \geq 1$, by imposing $\theta = -\frac{1}{1-\gamma_U}$. When $\gamma_U = 1$, these preferences collapse to standard time-additive log preferences with risk aversion of one.

The parameter θ determines the detection error probabilities, a likelihood-based measure of models' 'proximity'. Let model A be the approximating model and model B the distorted model. Consider N different samples each with T observations. Let $L_{i,j|k}$ be the likelihood of sample $j \in \{1, 2, ..., N\}$ for model $i \in \{A, B\}$ when model $k \in \{A, B\}$ generates the data. We compute error detection probabilities by assigning prior probabilities over model A and B of 0.5:

$$p(\theta^{-1}) = \frac{1}{2}\frac{1}{N}\sum_{j=1}^{N} \mathcal{I}\left(\frac{L_{A,j|A}}{L_{B,j|A}} < 1\right) + \mathcal{I}\left(\frac{L_{A,j|B}}{L_{B,j|B}} > 1\right), \tag{2}$$

where $\mathcal{I}$ is an indicator function. When models A and B are identical, $p(\theta^{-1})$ is 50%. As the two models begin to diverge from each other, $p(\theta^{-1})$ tends toward zero. In our computations, we set $T = 235$ and $N = 100,000$.

Aversion to Risk To better highlight the role of robustness, we also study the welfare implications of global shocks in the alternative case in which the agent has standard time-additive preferences:

$$U_t = \left[(1-\beta)u_t^{1-\gamma_R} + \beta E_t[U_{t+1}^{1-\gamma_R}]\right]^{\frac{1}{1-\gamma_R}}. \tag{3}$$

In this setting, the agent does not care about entropy, and the parameter $\gamma_R > 1$ measures only aversion to consumption risk. In section **??**, we impose $\gamma_U = \gamma_R > 1$ and show that global shocks in economies with high risk aversion and no robustness concern (equation (3)) produce very different welfare costs than in economies with high robustness concerns (equation (1)).

Budget Constraint and Optimality In each period, the household chooses labor, L_t, consumption, C_t, domestic equity shares, Z_t, and domestic public debt holdings, B_t, to maximize utility subject to the following budget constraint:

$$C_t + Q_t Z_t + B_t = (1-\tau_t)W_t L_t + (Q_t + \mathcal{D}_t)Z_{t-1} + (1 + r_{f,t-1})B_{t-1}, \tag{4}$$

where $\mathcal{D}_t$ denotes domestic aggregate dividends (specified in equation (**??**)) and Q_t is the market value of a domestic equity share. Wages, W_t, are taxed at a time-varying rate, τ_t. The intratemporal optimality condition on labor takes the following form:

$$\frac{1-\kappa}{\kappa} A_t^{(1-1/\nu)} \left(\frac{C_t}{1-L_t}\right)^{1/\nu} = (1-\tau_t)W_t \tag{5}$$

and implies that the household's labor supply is directly affected by fiscal policy.

In equilibrium, the following asset pricing conditions hold:

$$\begin{aligned} Q_t &= \mathbb{E}_t[\Lambda_{t+1}(Q_{t+1} + \mathcal{D}_{t+1})], \\ 1 &= \mathbb{E}_t[\Lambda_{t+1}(1 + r_{f,t})], \end{aligned}$$

where Λ_{t+1} is the domestic stochastic discount factor. The representative agent holds the entire supply of domestic equities (normalized to be one for simplicity, i.e., $Z_t = 1 \quad \forall t$) and bonds. To enforce survivorship, we assume that our countries are in financial autarky, i.e., international financial markets are shut-down. Foreign claims are priced using the the foreign discount factor.

Stochastic Discount Factor With robustness, the stochastic discount factor Λ_{t+1} is given by

$$\Lambda_{t+1} = \beta\left(\frac{u_{t+1}}{u_t}\right)^{\frac{1}{\nu}-1}\left(\frac{C_{t+1}}{C_t}\right)^{-1/\nu}\frac{\exp(-U_{t+1}/\theta)}{E_t[\exp(-U_{t+1}/\theta)]}, \tag{6}$$

and it can be decomposed as follows:

$$\Lambda_{t+1} \equiv \Lambda^R_{t+1}\Lambda^U_{t+1},$$

with

$$\Lambda^R_{t+1} \equiv \beta\left(\frac{u_{t+1}}{u_t}\right)^{\frac{1}{\nu}-1}\left(\frac{C_{t+1}}{C_t}\right)^{-1/\nu}$$

and

$$\Lambda^U_{t+1} \equiv \frac{\exp(-U_{t+1}/\theta)}{E_t[\exp(-U_{t+1}/\theta)]}.$$

The first component, Λ^R_{t+1}, is the familiar stochastic discount factor obtained under expected utility with RRA= IES= 1. On the other hand, Λ^U_{t+1} is the minimizing martingale increment associated with the robust agent's problem. When θ approaches infinity ($\gamma_U \to 1$), that component goes to unity, and we recover the stochastic discount factor obtained under expected log utility.

We denote expectations under the true and distorted probability measures as $E[\cdot]$ and $\widetilde{E}[\cdot]$, respectively, so that we can rewrite the standard asset pricing equation for any return R_{t+1} as

$$1 = \mathbb{E}_t[\Lambda_{t+1} R_{t+1}] = \tilde{\mathbb{E}}_t[\Lambda^R_{t+1} R_{t+1}],$$

implying that assets are priced by Λ^R_{t+1} under the worst-case distribution.

In this economy, the maximum conditional Sharpe ratio is $\frac{\sigma_t(\Lambda_{t+1})}{E_t(\Lambda_{t+1})}$, which we decompose and interpret in robustness terms. Specifically, in what follows we refer to $\frac{\sigma_t(\Lambda^R_{t+1})}{E_t(\Lambda^R_{t+1})}$ as the market price of risk, while $\frac{\sigma_t(\Lambda^U_{t+1})}{E_t(\Lambda^U_{t+1})}$ denotes the market price of model uncertainty. We find this terminology more appropriate, as $\sigma_t(\Lambda^U_{t+1})$ goes to zero when the concerns for robustness disappear even though well-defined risks remain. Because in our economy risk is a combination of both local and foreign shocks, in what follows we often refer to the market price of model uncertainty as market price of global uncertainty.

Finally, note that when the agent has time-additive preferences as in equation (3), the stochastic discount factor is

$$\Lambda_{t+1} = \beta \left(\frac{u_{t+1}}{u_t} \right)^{\frac{1}{\nu} - \gamma_R} \left(\frac{C_{t+1}}{C_t} \right)^{-1/\nu},$$

and it does not incorporate any robustness concern.

2.2 Technology, Markets, and Government

Final Good Firm There is a representative and competitive firm that produces the single final output good in the economy, Y_t, using domestic labor, L_t, and a bundle of intermediate goods created within the country, x_{it}, or adopted from abroad, x^*_{it}. We assume that the production function for the final good is specified as follows:

$$Y_t = \Omega_t L_t^{1-\alpha} \left[w \left(\int^{A_t} X_{it}^{\alpha} di \right)^{\frac{\sigma-1}{\sigma}} + (1-w) \left(\int^{A_{HF,t}} X_{it}^{*\alpha} di \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{7}$$

where Ω_t denotes an exogenous stationary stochastic productivity process,

$$\log(\Omega_t) \quad = \quad \rho \cdot \log(\Omega_{t-1}) + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma^2),$$

A_t is the total measure of domestic intermediate goods in use at date t, and $A_{HF,t}$ is the mass of foreign intermediated goods adopted in the home country. The parameter w and σ measure technology home-bias and elasticity of substitution, respectively. When $w \to 1$, our economy collapses to a collection of two closed economies, each one working as in Romer (1990).

This competitive firm takes prices as given and chooses intermediate goods and labor to maximize profits as follows:

$$D_t = \max_{L_t, X_{it}} Y_t - W_t L_t - \int_0^{A_t} P_{it} X_{it} di - \int_0^{A_{HF,t}} P^*_{it} X^*_{it} di,$$

where P_{it} (P^*_{it}) is the price of domestic (foreign) intermediate good i at time t. At the optimum,

$$X_t = \left(\frac{\alpha w \Omega_{H,t}\theta_t}{P_t}\right)^{\frac{1}{1-\alpha}} L_{H,t} \tag{8}$$

$$X_t^* = \left(\frac{\alpha(1-w)\Omega_{H,t}\theta_{HF,t}}{P_t^*}\right)^{\frac{1}{1-\alpha}} L_{H,t} \tag{9}$$

where θ_t and $\theta_{HF,t}$ are composite variables defined in the Appendix which keep track of the relative composition of the intermediate goods bundle. Finally, in the labor market the following holds:

$$W_t = (1-\alpha)\frac{Y_t}{L_t}. \tag{10}$$

Intermediate Goods Firms For the sake of simplicity, we assume that all intermediate goods used in the domestic country are produced within the domestic country border. Same applies to the foreign economy. Hence in our setting there is no imports/exports of products. our countries exchange only patents, that is, intangible assets. Since we follow the literature (Santacreu 2011) in assuming that net-exports are balanced in every period, this assumption produces no loss of generality.

Each domestic (foreign) intermediate good $i \in [0, A_t]$ ($j \in [0, A_{HF,t}]$) is produced by a monopolistic firm. Each firm needs X_{it} (X_{jt}^*) units of the domestic final good to produce X_{it} (X_{jt}^*) units of its respective intermediate good i (j). Given this assumption, the marginal cost of each intermediate good is fixed and equal to one.

Taking the demand schedule of the final good producer as given, each firm chooses its price to maximize profits:

$$\begin{aligned}\Pi_{it} &= \max_{P_{it}} P_{it}X_{it} - X_{it} \quad i \in [0, A_t], \\ \Pi^*_{jt} &= \max_{P^*_{jt}} P^*_{jt}X^*_{jt} - X^*_{jt} \quad j \in [0, A_{HF,t}].\end{aligned}$$

At the optimum, monopolists charge a constant markup over marginal cost:

$$P_{it} = P^*_{jt} \equiv P = \frac{1}{\alpha} > 1.$$

Given the symmetry of the problem for all the monopolistic firms, we obtain:

$$\begin{aligned}X_{it} &= (\alpha^2 w \Omega_{H,t}\theta_t)^{\frac{1}{1-\alpha}} L_{H,t}, \\ \Pi_{it} &= \Pi_t = (\frac{1}{\alpha} - 1)X_t,\end{aligned} \tag{11}$$

and

$$\begin{aligned}X^*_{jt} &= (\alpha^2 (1-w) \Omega_{H,t}\theta_{HF,t})^{\frac{1}{1-\alpha}} L_{H,t} \\ \Pi^*_{jt} &= \Pi^*_t = (\frac{1}{\alpha} - 1)X^*_t.\end{aligned} \tag{12}$$

Using equations (11)–(12), output can be determined as follows:

$$Y_t = \Omega_t L_t^{1-\alpha} \left[w(A_t X_t^{\alpha})^{\frac{\sigma-1}{\sigma}} + (1-w)(A_{HF,t} X_t^{*\alpha})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{13}$$

Since both labor and productivity are stationary, the long-run growth rate of output is determined by the expansion of either the domestic, A_t, or the foreign, $A_{HF,t}$, intermediate goods variety. The expansion of domestic variety originates in the research and development sector, whereas the expansion of foreign varieties depends on the activity in the adoption sector. Next we describe these two sectors.

Research and Development Innovators develop new intermediate goods for the production of final output and obtain patents on them. At the end of the period, these patents are sold to new intermediate goods firms in a competitive market at the price V_{it}. This value is the sum of the present value of the profits obtained by domestic firms, W_{it}, and the profits that can produced if the variety is adopted abroad as well, $J_{it}P_{HF,t}$:

$$V_{it} = W_{it} + J_{it}P_{HF,t},$$

where $P_{HF,t}$ denotes the terms of trade. We specify the recursions of W_{it} and J_{it} later. Here we note that at the symmetric equilibrium, these values are going to be the same across varieties i. To minimize notation, we drop the i index.

Let $1/\vartheta_t$ be the marginal rate of transformation of final goods into new varieties. The free-entry condition in the R&D sector implies that in equilibrium,

$$\frac{1}{\vartheta_t} = E_t\left[\Lambda_{t+1}V_{t+1}\right]. \tag{14}$$

The left-hand side of the free-entry condition measures the cost of producing an extra variety. The right-hand side, in contrast, is equal to the discounted end-of-period market value of the new patent.

Let S_t denote the units of final good devoted to R&D investment. We assume that each existing variety dies, i.e., becomes obsolete, with probability $\delta \in (0, 1)$. In this case, its production is terminated. Given these assumptions, the total mass of domestic-developed varieties evolves according to:

$$A_{t+1} = \vartheta_t S_t + (1 - \delta) A_t,^{1} \tag{15}$$

In the spirit of Comin and Gertler (2006), we impose:

$$\vartheta_t = \chi \left(\frac{S_t}{T_t} \right)^{\eta - 1} \quad \eta \in (0,1), \tag{16}$$

$$T_t = A_t + A_{HF,t} \gamma_{HF}, \tag{17}$$

where T_t denotes the technological frontier of the home country. This specification captures the idea that concepts already discovered make it easier to come up with new varieties, $\partial\vartheta/\partial A > 0$, and that R&D investment has decreasing marginal returns, $\partial\vartheta/\partial S < 0$. When $\gamma_{HF} > 0$, foreign inventions create positive spill overs on domestic innovation activity.

In an extension, we allow for stochastic innovations in the productivity of the R&D sector. We do this by specifying a stochastic process for the parameter χ, ie, by setting $\chi \equiv \chi_{t+1} = \rho_\theta \chi_t + \xi_{t+1}$, where is ξ_t is normally distributed with volatility σ_θ. Note that a positive innovation to χ_t, ie an R&D shock, boosts investment in R&D, and so predictably increases growth. In this sense, allowing for R&D shocks is tantamount to introducing news shocks, that is, innovations to expected growth. Through the adoption channel, domestic news shocks, say, will spillover the foreign country by means of slow technology diffusion. We thus think of R&D shocks as a convenient way of capturing good or bad news about domestic or foreign growth.

[1] This dynamic equation is consistent with the assumption that new patents survive for sure in their first period of life. If new patents are allowed to immediately become obsolete, equations (14) and (15) need to be replaced by $A_{t+1} = (1-\delta)(\vartheta_t S_t + A_t)$ and $\frac{1}{\vartheta_t} = E_t\left[\Lambda_{t+1}(1-\delta)V_{t+1}\right]$, respectively. Our results are not sensitive to this modeling choice.

Adoption. Adoption of domestic technology is assumed to be costless. This implies that innovators can extract all the operating profits from the producers of domestic intermediate goods:

$$W_t = \Pi_t + (1-\delta)E_t\left[\Lambda_{t+1}W_{t+1}\right]. \tag{18}$$

Adoption of foreign technology, instead, is costly and subject to uncertainty. The domestic innovator is able to extra only part of the operating profits realized in the foreign country. To better explain this problem, we find it convenient to look at adoption from a foreign country perspective. Specifically, we assume that for each home-country variety newly produced at time t there exists a foreign adopter willing to pay an initial fee J_t to acquire the right to adopt the technology abroad. After acquiring the right on the patent, the adopter pays an optimally chosen per-period cost $h_{F,t}$ in order to have a positive probability, $\upsilon^A_{F,t}$, of being successful with adoption.

At this point, we are able to price the home-country option value J_t of expanding operations abroad through the adopter:

$$J_t = \max_{h_{F,t}} -h_{F,t} + (1-\delta)\left\{\upsilon_t^{AF}E_t\left[\Lambda^F_{t+1}W^*_{F,t+1}\right] + (1-\upsilon_t^{AF})E_t[\Lambda^F_{t+1}J_{t+1}]\right\}. \tag{19}$$

With probability υ_t^{AF}, the foreign adopter is successful. In this case, the adopter stops paying the adoption investment cost, $h_{F,t}$, and receives the rent, $W^*_{F,t+1}$, e.g., the present value of all future expected foreign operating profits. With probability $(1-\upsilon_t^{AF})$, adoption fails in the current period, but may occur in the following periods. Hence the innovator receives the continuation value J_{t+1}.

The optimal adoption investment per-foreign variety, $h_{F,t}$, is determined by the following first order condition

$$\frac{1}{\upsilon'^{A}_{F,t}} = (1-\delta)E_t[\Lambda^F_{t+1}(W^*_{F,t+1} - J_{H,t+1})].$$

The total adoption investment in the foreign country depends on the total mass of existing home-country technologies not implemented yet, $h_{F,t}(A_{H,t+1} - A_{FH,t})$.

We let the probability of having a successful adoption depend on total adoption investment and is specified as follows:

$$\upsilon_t^{AF} = \alpha_F^A \left(\frac{h_{F,t}(A_{H,t+1} - A_{FH,t})}{Y_{F,t}} \right)^{\gamma_A}. \tag{20}$$

The evolution of the stock of home-country technology adopted in the foreign country is:

$$A_{FH,t+1} = A_{FH,t}(1-\delta) + \upsilon_t^{AF}(A_{H,t+1} - A_{FH,t}). \tag{21}$$

The evolution of the stock of foreign technology adopted in the home mirrors that of the foreign country:

$$A_{HF,t+1} = A_{HF,t}(1-\delta) + \upsilon_t^{AH}(A_{F,t+1} - A_{HF,t}). \tag{22}$$

Since successful adoption is assumed to be *i.i.d.* across adopters, by law of large numbers the probability of adoption υ_t^{AH} corresponds to the fraction of new adopted technologies.

Government The government faces an exogenous and stochastic expenditure stream, G_t, that evolves as follows:

$$\frac{G_t}{Y_t} = \frac{1}{1+e^{-gy_t}}, \tag{23}$$

where

$$gy_t = (1-\rho)\overline{gy} + \rho_g gy_{t-1} + \epsilon_{G,t}, \quad \epsilon_{G,t} \sim N(0, \sigma_{gy}^2).$$

This specification ensures that $G_t \in (0, Y_t) \quad \forall t$. In order to finance these expenditures, the government can use tax income, $T_t = \tau_t W_t L_t$, or public debt according to the following budget constraint:

$$B_t = (1 + r_{f,t-1})B_{t-1} + G_t - T_t. \tag{24}$$

We focus on two tax regimes. Under the first, the government commits to a zero-deficit policy and sets the tax rate, τ_t^{zd}, as follows:

$$\tau_t^{zd} = \frac{G_t/Y_t}{1-\alpha}.$$

In this case, the tax rate perfectly mimics the properties of our exogenous government expenditure process. Under the second regime, in contrast, the government runs surpluses or deficits according to the following rule for the debt-output ratio:

$$\frac{B_t}{Y_t} = \rho_B \frac{B_{t-1}}{Y_{t-1}} + \phi_B \cdot (\log L_{SS} - \log L_t), \tag{25}$$

where L_{SS} is the steady-state level of labor, $\rho_B \in (0,1)$ measures the inverse of the speed of debt repayment, and $\phi_B \geq 0$ is the intensity of the policy. Combining (24) and (25), the tax rate becomes

$$\tau_t(\rho_B, \phi_B) = \tau_t^{zd} + \frac{1}{1-\alpha}\left(\frac{1 + r_{f,t-1}}{Y_t/Y_{t-1}} - \rho_B\right)\frac{B_{t-1}}{Y_{t-1}} + \phi_B \frac{\log L_t - \log L_{SS}}{1-\alpha}. \tag{26}$$

When $\phi_B > 0$, our simple debt policy rule captures the behavior of a government that is concerned about employment and wants to minimize labor fluctuations. In particular, the government cuts labor taxes (increases debt) when labor is below steady state and increases them (reduces debt) in periods of boom for the labor market. The second term on the right-hand side of equation (26) captures the persistent effect that debt repayment has on taxes. The condition $\rho_B < 1$ ensures that the public administration keeps the debt-output ratio stationary. In the language of Leeper et al. (2010), we anchor expectations about debt and rule out unsustainable paths.

Aggregate Resource Constraint and Current Account In the home country, the clearing condition in the final good market implies:

$$Y_t = C_t + S_t + h_t(A_{F,t+1} - A_{HF,t}) + A_t X_t + A_{HF,t} X_t^* + G_t.$$

Final output, therefore, is used for consumption, R&D and adoption investment, production of intermediate goods, and public expenditure.

In the international markets, our countries just exchange the rights to adopt each other technologies. Specifically, at time t the home country generates $\upsilon_{H,t} S_{H,t}$ new varieties and exports adoption rights for a total amount of $\upsilon_{H,t} S_{H,t} J_{H,t}$. Symmetrically, the foreign country sells adoption rights for a total value of $\upsilon_{F,t} S_{F,t} J_{F,t}$. At the equilibrium, the terms of trade, $P_{HF,t}$, adjust to keep international trade balanced in every period:

$$(\upsilon_{H,t} S_{H,t} J_{H,t}) P_{HF,t} = \upsilon_{F,t} S_{F,t} J_{F,t}.$$

3 Calibration

Our analysis is based upon a quantitative evaluation of the model presented above. In this section, we first present and discuss our calibration and then present a discussion of different sources of uncertainty and their impact on global growth by means of impulse responses.

3.1 Parameter Choices

We report our benchmark calibration in table 1. Our benchmark calibration refers to the case of a zero-deficit policy. We will view the US as the domestic country, and examine the joint dynamics of growth and volatility of a variety of specifications of foreign countries vis-a-vis the US, and adjust the parameters for foreign accordingly. Since the main focus of the paper is on the implications of risks, both domestic and foreign, for global growth, we first calibrate the benchmark model to match the level and the volatility of consumption growth in the US. This pins down the scale parameter χ and the volatility of the productivity process, σ. We choose that volatility to match the unconditional volatility of consumption growth observed in the U.S. over the long sample 1929–2008. The implied main statistics of our model are reported in 2.

The parameters for the government expenditure-output ratio are set to have an average share of 10% and an annual volatility of 4%, consistent with U.S. annual data over the sample 1929–2008. Using post–World War II data would make expenditure risk even larger; hence we regard our calibration as conservative.

Relative risk aversion γ_R and subjective discount factor are set to replicate the low historical average of the risk-free rate and the consumption claim risk premium estimated by Lustig et al. (2010). Replicating these asset pricing moments is important because it imposes a strict discipline on the way in which innovations are priced and average growth is determined.

The parameters ν and θ_c control the labor supply and are chosen to yield a steady-state share of hours worked of 1/3 and a steady-state Frisch elasticity of 0.7, respectively. These values are standard in the literature.

Turning to technology parameters, the constant α captures the relative weight of labor and

Table 1: Calibration

Description	Symbol	Value
Preference Parameters		
Consumption-Labor Elasticity	ν	0.7
Utility Share of Consumption	κ	0.095
Discount Factor	β	0.997
Intertemporal Elasticity of Substitution	ψ	1
Risk Aversion	γ_R	7
Technology Parameters (Romer)		
Elast. of Sub. Between Domestic Intermediate Goods	α	0.7
Autocorrelation of Productivity	ρ	0.96
Autocorrelation of R&D Shocks	ρ_θ	0.96
Survival rate of intermediate goods	ϕ	0.97
Elasticity of New Intermediate Goods wrt R&D	η	0.8
Standard of Deviation of Technology Shocks	σ_ω	0.006
Standard of Deviation of R&D Shocks	$\sigma_\theta/\sigma_\omega$	0.66
Adoption parameters		
Elast. of Sub. Domestic/Foreign Intermediate Goods	σ	3.5
Home-bias	w_H	0.75
Elast. of New Foreign Interm. Goods wrt Adoption Investment	γ_A	0.8
Government Expenditure Parameters		
Level of Expenditure-Output Ratio (G/Y)	$\overline{gy}$	-2.2
Autocorrelation of G/Y	ρ_g	0.98
Standard deviation of G/Y shocks	σ_g	0.008

intermediate goods in the production of final goods, and, by equation (11), controls the markup and hence profits in the economy. We choose this parameter to match the empirical share of profits in aggregate income. The parameter η, the elasticity of new intermediate goods with respect to R&D, is within the range of the panel and cross-sectional estimates of Griliches (1990). Since the variety of intermediate goods can be interpreted as the stock of R&D (a directly observable quantity), we can then interpret $1-\phi$ as the depreciation rate of the R&D stock. We set ϕ to 0.97, which corresponds to an annual depreciation rate of about 14%, i.e., the value assumed by the Bureau of Labor Statistics in its R&D stock calculations. The scale parameter χ is chosen to match the average growth rate of the U.S.

Table 2: Main Statistics under Zero-Deficit

	Data	Zero deficit $\phi_B = 0$
$E(\Delta c)$	2.03	2.04
$\sigma(\Delta c)$	2.34	2.54
$ACF_1(\Delta c)$	0.44	0.40
$\sigma(m)$ (%)		33.24
$E(r_f)$	0.93	1.48
$E(r^C - r_f)$		1.09
$corr(m, m^*)$		0.987
$corr(\Delta c, \Delta c^*)$		0.42
$corr(r^C, r^{C,*})$		0.50

economy over the 1929–2008 sample.

Our specification of the stochastic process for R&D shocks is designed to mimic properties of news shocks as estimated in the literature. As a R&D shock boosts innovation, and thus - predictably - growth, such an innovation is indeed a shock to expected growth, just as much as the news shocks considered in the business cycle literature. On the other hand, our model allows to identify a source of such good or bad news about future growth in the R&D sector. Matching estimated properties of news shocks pins down $\rho_\theta = 0.96$ and the relative volatility of σ_θ to σ_ω as 0.66.

The choices of the parameters relating to technology diffusion, σ, γ_A and home-bias, w_H, are based on the literature. Following Colacito and Croce (2011) we set $w_H = 0.75$. Following Santacreu (2011), we choose $\sigma = 3.5$ and $\gamma_A = 0.5$. Our results are qualitatively robust to variations in those choices.

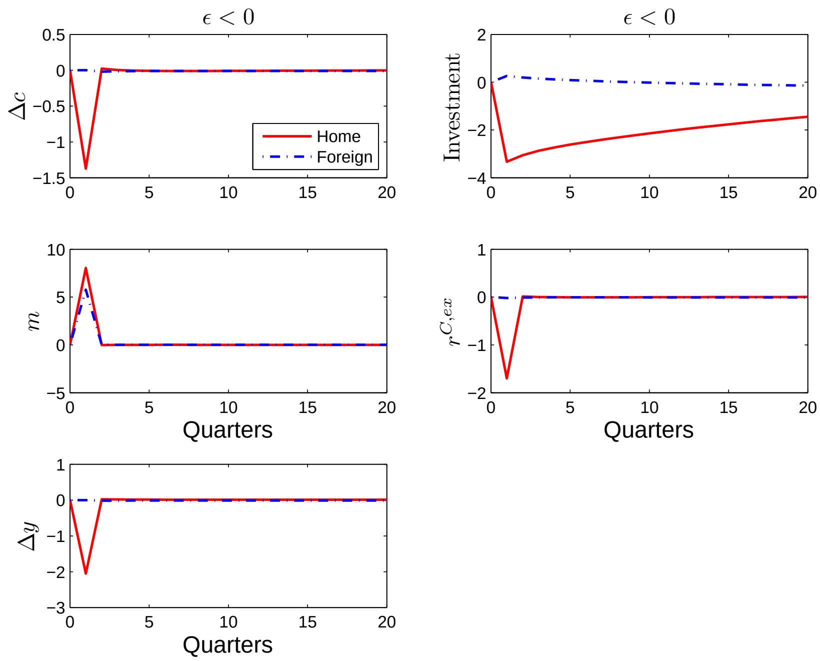


Figure 1: Domestic and Foreign Impulse Responses

The panels document the responses of both domestic and foreign variables to a negative one standard deviation shock to *domestic* productivity.

3.2 Shocks and Responses

Much of our discussion evolves around the notion of contagion - how do movements in foreign performance or news there about affect domestic growth and risk, and thereby global growth? In this section, we provide a preliminary discussion and illustration by means of impulse responses. Figure 1 sets the stage. It displays the responses of both domestic and foreign variables to a bad domestic productivity shock. We focus on the responses of consumption and output growth, R&D investment as well as the stochastic discount factor and the excess returns on consumption claims, both domestic and foreign.

While the responses of the domestic variables are little surprising, our interest is in the response of the foreign variables and the role of technology adoption with doubts. First note that domestically consumption and output growth fall on impact. This just reflects the direct effect of a bad domestic productivity shock on contemporaneous output. Similarly, foreign output and consumption growth do not react in impact. This is not surprising, after all productivity is not affected on impact. More importantly, however, bad news to technology today means lower values of patents today, and does the payoff to innovation decreases. This is reflected in a sharp decrease in domestic R&D investment. Crucially, a fall in domestic R&D investment decreases the variety of patents available for worldwide adoption, in other words, it slows down the growth of the world technology frontier, T_t.

The world technology frontier determines adoption possibility set for the foreign country. Any slowdown in its growth therefore affects foreign growth prospects. With doubts about future growth as with sensitivity to worst case outcomes, foreign consumers are sensitive to movements in the world technology frontier. This becomes clear in the response of the foreign stochastic discount factor to a domestic shock - its movements are almost as pronounced as those in the domestic country, in spite of the absence of a response in foreign consumption growth. Its response thus embodies movements in foreign growth prospects, induced by domestic shocks. It is through this channel that the movements in the world technology frontier generate contagion if agents are sensitive to worst case outcomes. Interestingly, foreign agents attempt to partially offset such contagion by increasing their investment in R&D, as shown in the top right panel.

Qualitatively similar, but quantitatively different behavior is exhibited in response to alternative sources of domestic disturbances, such as volatility, R&D shocks or fiscal news.

4 Entropy Contagion

The goal of this section is to study the international transmission of regional shocks through the diffusion of technologies in order to understand to which extent international technology adoption promotes global growth risk.

In figure 2, we study consumption and output growth risk in economies in which the two countries are ex-ante identical except for the amount of shock volatility that they bear. Specifically, let Σ^H denote the variance-covariance matrix in the home country for the domestic shocks. We assume that the variance-covariance matrix for the foreign shocks is $\Sigma^F = \left(\frac{\sigma_H}{\sigma_F}\right)^2 \Sigma^H$, i.e., just rescaled by a factor that can greater or smaller than one.

When the factor is larger than one, the foreign country is riskier than the domestic one. Since we plot statistics only from the home country point of view, when $\sigma_F/\sigma_H > 1$ we can think of our results as referring to a developed economy interacting with a riskier emerging one. Viceversa, when $\sigma_F/\sigma_H < 1$ our results refer to a situation in which the foreign country is the developed one.

We point out three important results. First, an increase of the relative volatility of the foreign shocks produces a one-to-one increase in the relative volatility of consumption growth across the home and foreign country. This not the case, however, for the long-run component of consumption growth, i.e., its conditional expected value. As depicted in the top-right panel, of figure 2, the foreign country experiences an increase in the volatility of its long-run consumption growth that is lower than that experienced in the home country (the think solid line represents a fortyfive degree line). This is because international trade of technologies plays in favor of the riskier country, meaning that international technology diffusion helps risky countries to stabilize their long-run growth, whereas the opposite is true for the safer countries.

Second, turning our attention to the bottom panels of figure 2, we can see what happens to the total quantity of output volatility as the amount of foreign risk is changed. Overall, the amount of short-run growth volatility remains unchanged, whereas the amount of long-

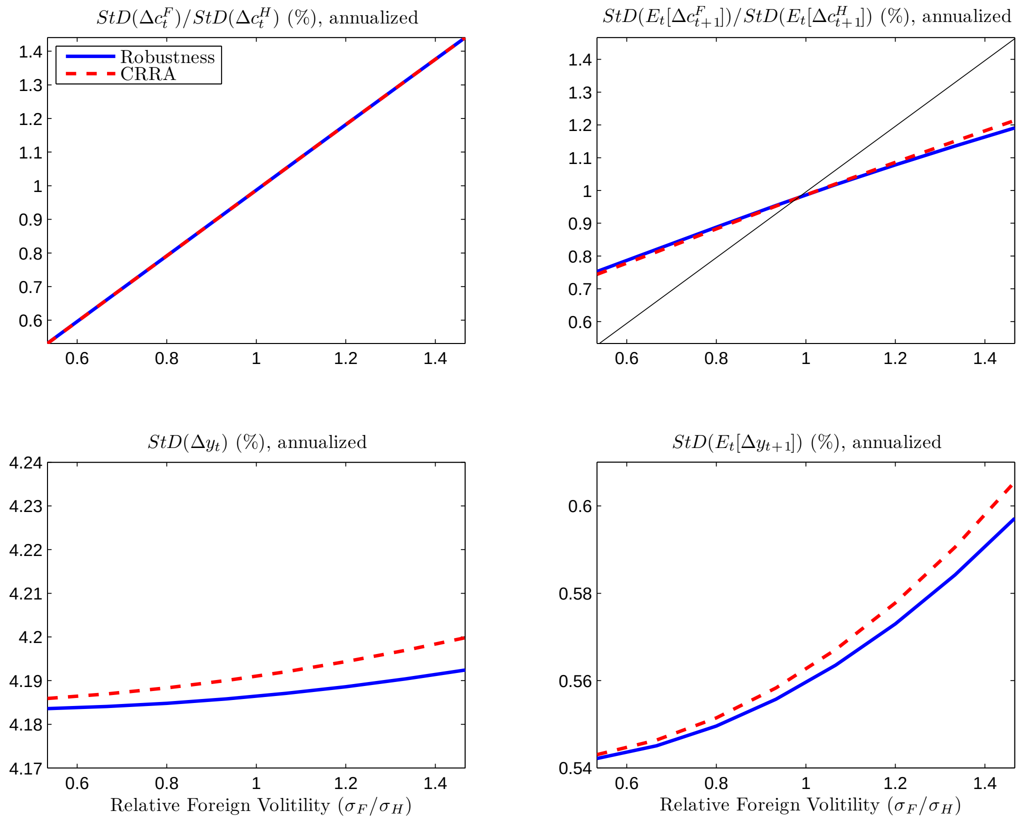


Figure 2: Foreign Risk and Quantities Contagion
The top-left panel shows relative consumption volatility as a function of the rescaling factor for foreign volatility, σ_F/σ_H. The left-bottom panel shows the annualized volatility of the Home-country output growth. The top and bottom right panels focus on the long-run components of consumption and output, respectively. All parameters are set to their benchmark values, except for σ_F/σ_H, which is allowed to differ from one. The CRRA case is obtained by setting $\gamma_R = 1$.

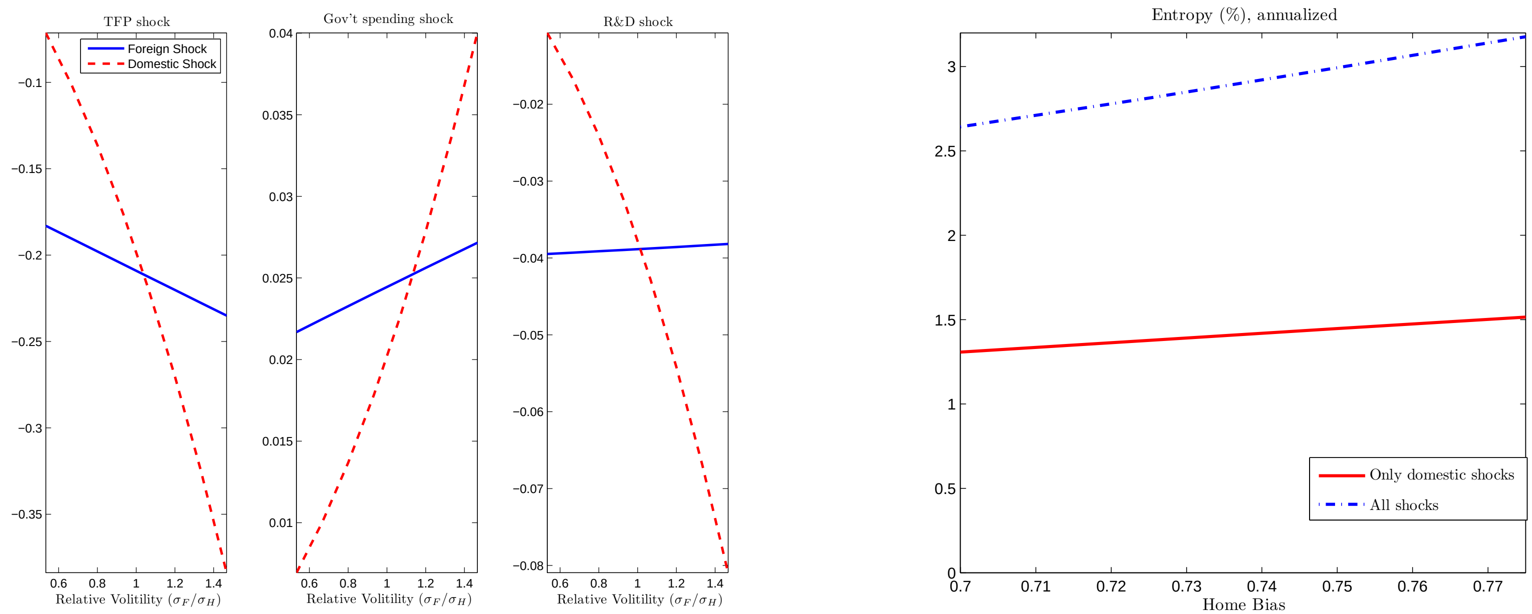
run output growth is very sensitive to σ_F/σ_H. This is a first clear proof that international technology diffusion can turn into a powerful vehicle of contagion, as foreign uncertainty translates into a sizeable amount of domestic long-run growth risk.

Third, by comparing the results with and without robustness concerns, it is easy to see that robustness tend to limit the increase of long-run output risk. This is because long-run uncertainty is a well-known strong source of entropy and hence investments decisions are changed to minimize long-term growth risk.

In figure 3, we study further the concept of contagion through the lenses of entropy and its implied pessimism. Specifically, in figure 3(a) we plot the expected average of the three shocks affecting our countries using the distorted probability measure of the home country. Under the physical measure, these three shocks have a null expected value. Our agent, however, tilts probability toward the worst case scenario and expect negative productivity and R&D shocks and positive expenditure shocks both in the home and foreign country.

Most importantly, in our setting local entropy is the result of global uncertainty, in the sense that it depends strongly on the riskiness of all countries. As we increase the riskiness of the foreign country by making σ_F/σ_H larger, the home country becomes more pessimist not only about foreign shocks but also about local shocks. Although local volatility has not changed in the home country, the fact that its foreign counterpart has become riskier amplifies pessimism. This is what we call entropy contagion. More specifically, Figure 3(b) shows that half of the measured entropy is due to foreign shocks. This result is unchanged in a large range of values for the home bias parameter, ω.

What is even more interesting is that an increase in foreign volatility results in a sharp increase in the pessimist distortion to local shocks, as opposed to foreign shocks. This is the result of technology home bias: as foreign technologies represent a small portion of the local stock of varieties, the worst-case scenario is mostly focused on local shocks, i.e., the most important determinants of local output.



(a) Home-Country Distortions

(b) Domestic vs Total Entropy

Figure 3: Entropy and Pessimism Contagion

Panel A shows the mean of both the domestic and foreign productivity, government expenditure and R&D shocks computed under the Home-country distorted beliefs. All parameters are set to their benchmark values, except the foreign volatility re-scaling factor, σ_F/σ_H. Panel B shows the Home-country total entropy and its national subcomponent as a function of the technology home-bias parameter, ω. All other parameters are set to their benchmark values.

5 Cross-sectional Results

In our model, all countries grow at the same unconditional equilibrium rate. If countries are heterogenous, they simply feature different relative levels for both quantities and prices. Previous literature has focused mainly on heterogeneity induced by technological or policy barrier differences. In contrast to previous studies, in figure 4 we look at the implications of heterogeneity in risk. We focus on relative differences in output, patents value, and subcomponents of the patents value across our two countries.

In figure 4(a), we abstract away from country size and look at quantities normalized by the total stock of local technology. We obtain two intuitive results. First, the most risky country features patents with a lower market value. This is the result of risk/entropy adjustment in the valuation of future profits and it results in a lower level of patents and output compared to the safer country. Second, when looking at size-adjusted variables, preferences for robustness simply amplify the results that we obtain with standard time-additive preferences.

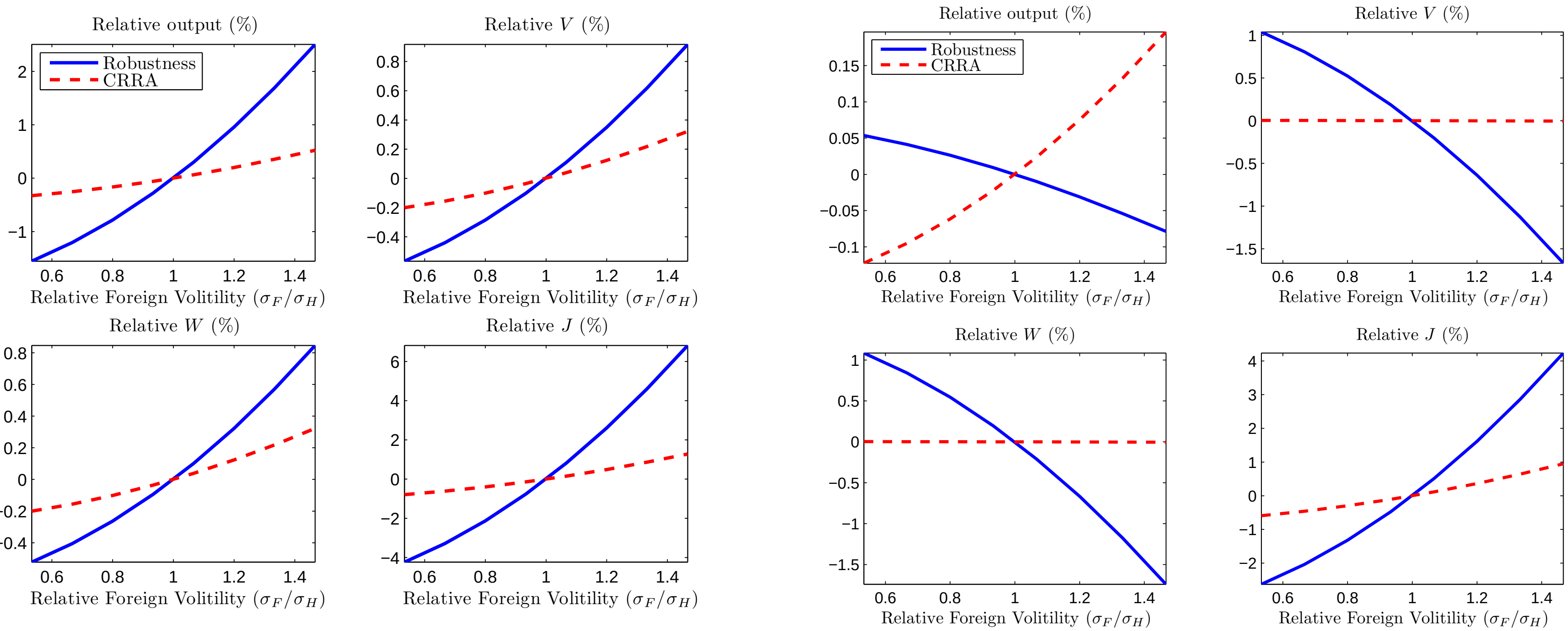
Turning our attention to figure 4(b), we see that most of the abovementioned results are overturned with robust preferences once country size is taken into account. Robustness concerns, in fact, always come with strong precautionary saving motives. Being exposed to stronger shocks gives an incentive to the riskier country to save and hence invest more. The higher investment rate results in a higher share of both world output and wealth. This explains why relative output, relative patents value, and the relative present value of the domestic profits increase when a country is riskier.

Furthermore, once the precautionary savings are taken into account the difference between time-additive and robust preferences is much more dramatic. Specifically, with time-additive preferences the precautionary saving motives are second-order. The relative country size A_H/A_F is not significantly sensitive to changes in σ_F/σ_H and cannot meaningfully change the values reported in figure 3(b). Compared to robustness preferences, time-additive preferences produce uninteresting results.

We conclude this section noting that the qualitative behavior of the adoption value is unchanged by the precautionary motives. Regardless of whether or not we adjust for A_H/A_F, the home country value of expanding operations abroad is increasing in foreign volatility. This result is intuitive once we think in terms of risk-adjusted investment decisions. As the foreign country becomes riskier, the foreign representative agent finds it more and more appealing to invest in the adoption sector, as the home country provides safer technologies. This behavior results in a substantial increase in J_H, i.e. the flow of revenue that the home innovator captures through international adoption. The increase in the size of the foreign country, mitigates this cross-sectional heterogeneity but is not strong enough to make it null.

6 Conclusion

In this paper, we examine the role of technology diffusion and sensitivity towards worst case outcomes in shaping global growth cycles. While the possibility of adopting foreign technologies generates growth options domestically, it also exposes a country's growth prospects to disturbances originating abroad. We show how movements in the world technology frontier contribute to cycles in global growth prospects, and how local shocks shape movements in the latter frontier. In other words, in our model contagion between different countries arises endogenously and is reflected in a global growth factor. When agents are sensitive to worst case outcomes, the dynamics of that endogenous growth factor is a first order determinant of global entropy and thus welfare. Our model thus points to the importance of international policy coordination.



(a) Without accounting for $A_{H,t}/A_{F,t}$ (b) Accounting for $A_{H,t}/A_{F,t}$

Figure 4: Risk and Inequality

In Panel A, relative output, V, W, and J are defined as $\frac{Y_H/A_H}{Y_H/A_F}P_{HF}$, $\frac{V_H}{V_F}P_{HF}$, $\frac{W_H}{W_F}P_{HF}$, $\frac{J_H}{J_F}P_{HF}$, respectively. The total value of a new technology, V, equals the sum of the present value of domestic operating profits, W, and the value of expanding operations abroad through international adoption, J. P_{HF} is the terms of trade. All these relative variables abstract away from relative country size, as measured by the relative technology stock, A_H/A_F. In Panel B, we depict the same objects accounting for differences in country size: $\frac{Y_H}{Y_F}P_{HF}$, $\frac{V_H A_H}{V_F A_F}P_{HF}$, $\frac{W_H A_H}{W_F A_F}$, $\frac{J_H A_H}{J_F A_F}$. All parameters are set to their benchmark value, except for the foreign volatility re-scaling factor σ_F/σ_H. The CRRA case is obtained by setting $\gamma_R = 1$.

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Appendix A: The Model

A.1 Final Production

Home:

$$Y_{H,t} = \Omega_{H,t} L_{H,t}^{1-\alpha} \left[\omega_H \left(\int^{A_{H,t}} x_{H,t}^{\alpha}(i) di \right)^{\frac{\sigma-1}{\sigma}} + (1-\omega_H) \left(\int^{A_{HF,t}} x_{HF,t}^{\alpha}(i) di \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

$$\Pi_{H,t} = P_{H,t} Y_{H,t} - w_{H,t} L_{H,t} - \int^{A_{H,t}} p_{H,t}(i) x_{H,t}(i) di - \int^{A_{HF,t}} p_{HF,t}(i) x_{HF,t}(i) di$$

$$\begin{aligned} \Pi_{H,t} = {} & P_{H,t} \Omega_{H,t} L_{H,t}^{1-\alpha} \left[\omega_H \left(\int^{A_{H,t}} x_{H,t}^{\alpha}(i) di \right)^{\frac{\sigma-1}{\sigma}} + (1-\omega_H) \left(\int^{A_{HF,t}} x_{HF,t}^{\alpha}(i) di \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ & - w_{H,t} L_{H,t} - \int^{A_{H,t}} p_{H,t}(i) x_{H,t}(i) di - \int^{A_{HF,t}} p_{HF,t}(i) x_{HF,t}(i) di \end{aligned} \tag{A1}$$

Maximize (A1) w.r.t. $x_{H,t}$, $x_{HF,t}$ and $\underbrace{w_{H,t}}_{wage}$

$$x_{H,t}: \quad \frac{\sigma}{\sigma-1} \frac{\sigma-1}{\sigma} P_{H,t} \Omega_{H,t} L_{H,t}^{1-\alpha} \omega_H \underbrace{\left[(\cdots) + (\cdots) \right]^{\frac{1}{\sigma-1}} \left(A_{H,t} x_{H,t}^{\alpha} \right)^{-\frac{1}{\sigma}}}_{\theta_{H,t}} \alpha A_{H,t} x_{H,t}^{\alpha-1} - A_{H,t} p_{H,t} = 0$$

$$x_{HF,t}: \ \frac{\sigma}{\sigma-1}\frac{\sigma-1}{\sigma}P_{H,t}\Omega_{H,t}L_{H,t}^{1-\alpha}(1-\omega_H)\underbrace{[(\cdots)+(\cdots)]^{\frac{1}{\sigma-1}}(A_{HF,t}x_{HF,t}^{\alpha})^{-\frac{1}{\sigma}}}_{\theta_{HF,t}}\alpha A_{HF,t}x_{HF,t}^{\alpha-1}-A_{HF,t}p_{HF,t}=0$$

$$x_{H,t}=\left(\frac{\alpha\omega_H P_{H,t}\Omega_{H,t}\theta_{H,t}}{p_{H,t}}\right)^{\frac{1}{1-\alpha}}L_{H,t}$$

$$x_{HF,t}=\left(\frac{\alpha(1-\omega_H)P_{H,t}\Omega_{H,t}\theta_{HF,t}}{p_{HF,t}}\right)^{\frac{1}{1-\alpha}}L_{H,t}$$

$$w_{H,t}:(1-\alpha)\Omega_{H,t}L_{H,t}^{1-\alpha}\frac{1}{L_{H,t}}\left[w\left(\int^{A_{H,t}}x_{H,t}^{\alpha}(i)di\right)^{\frac{\sigma-1}{\sigma}}+(1-w)\left(\int^{A_{HF,t}}x_{HF,t}^{\alpha}(i)di\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}-w_{H,t}=0$$

$$w_{H,t}=(1-\alpha)\frac{Y_{H,t}}{L_{H,t}}$$

Foreign:

$$Y_{F,t} = \Omega_{F,t} L_{F,t}^{1-\alpha} \left[\omega_F \left(\int^{A_{F,t}} x_{F,t}^{\alpha}(i) di \right)^{\frac{\sigma-1}{\sigma}} + (1-\omega_F) \left(\int^{A_{FH,t}} x_{FH,t}^{\alpha}(i) di \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

$$\Pi_{F,t} = P_{F,t} Y_{F,t} - w_{F,t} L_{F,t} - \int^{A_{F,t}} p_{F,t}(i) x_{F,t}(i) di - \int^{A_{FH,t}} p_{FH,t}(i) x_{FH,t}(i) di$$

$$\begin{aligned} \Pi_{F,t} = {} & P_{F,t} \Omega_{F,t} L_{F,t}^{1-\alpha} \left[\omega_F \left(\int^{A_{F,t}} x_{F,t}^{\alpha}(i) di \right)^{\frac{\sigma-1}{\sigma}} + (1-\omega_F) \left(\int^{A_{FH,t}} x_{FH,t}^{\alpha}(i) di \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ & - w_{F,t} L_{F,t} - \int^{A_{F,t}} p_{F,t}(i) x_{F,t}(i) di - \int^{A_{FH,t}} p_{FH,t}(i) x_{FH,t}(i) di \end{aligned} \tag{A2}$$

Maximize (A2) w.r.t. $x_{F,t}$, $x_{FH,t}$ and $\underbrace{w_{F,t}}_{wage}$

$$x_{F,t}: \quad \frac{\sigma}{\sigma-1} \frac{\sigma-1}{\sigma} P_{F,t} \Omega_{F,t} L_{F,t}^{1-\alpha} \omega_F \underbrace{[(\cdots) + (\cdots)]^{\frac{1}{\sigma-1}} (A_{F,t} x_{F,t}^{\alpha})^{-\frac{1}{\sigma}}}_{\theta_{F,t}} \alpha A_{F,t} x_{F,t}^{\alpha-1} - A_{F,t} p_{F,t} = 0$$

$$x_{FH,t}: \quad \frac{\sigma}{\sigma-1} \frac{\sigma-1}{\sigma} P_{F,t} \Omega_{F,t} L_{F,t}^{1-\alpha} (1-\omega_F) \underbrace{[(\cdots) + (\cdots)]^{\frac{1}{\sigma-1}} (A_{FH,t} x_{FH,t}^{\alpha})^{-\frac{1}{\sigma}}}_{\theta_{FH,t}} \alpha A_{FH,t} x_{FH,t}^{\alpha-1} - A_{FH,t} p_{FH,t} = 0$$

$$x_{F,t} = \left(\frac{\alpha \omega_F P_{F,t} \Omega_{F,t} \theta_{F,t}}{p_{F,t}} \right)^{\frac{1}{1-\alpha}} L_{F,t}$$

$$x_{FH,t} = \left(\frac{\alpha(1-\omega_F)P_{F,t}\Omega_{F,t}\theta_{FH,t}}{p_{FH,t}}\right)^{\frac{1}{1-\alpha}} L_{F,t}$$

$$w_{F,t} : (1-\alpha)\Omega_{F,t}L_{F,t}^{1-\alpha}\frac{1}{L_{F,t}}\left[\omega_F\left(\int^{A_{F,t}} x_{F,t}^{\alpha}(i)di\right)^{\frac{\sigma-1}{\sigma}} + (1-\omega_F)\left(\int^{A_{FH,t}} x_{FH,t}^{\alpha}(i)di\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - w_{F,t} = 0$$

$$w_{F,t} = (1-\alpha)\frac{Y_{F,t}}{L_{F,t}}$$

Zero-profit condition:

$$\Pi_{H,t} = 0 = \alpha P_{H,t}Y_{H,t} - A_{H,t}p_{H,t}x_{H,t} - A_{HF,t}p_{HF,t}x_{HF,t} \Rightarrow P_{H,t} = \frac{A_{H,t}p_{H,t}x_{H,t} + A_{HF,t}p_{HF,t}x_{HF,t}}{\alpha Y_{H,t}}$$

$$\Pi_{F,t} = 0 = \alpha P_{F,t}Y_{F,t} - A_{F,t}p_{F,t}x_{F,t} - A_{FH,t}p_{FH,t}x_{FH,t} \Rightarrow P_{F,t} = \frac{A_{F,t}p_{F,t}x_{F,t} + A_{FH,t}p_{FH,t}x_{FH,t}}{\alpha Y_{F,t}}$$

A.2 Intermediate Production

Home:

$$\pi_{H,t} = p_{H,t}x_{H,t} - P_{H,t}x_{H,t} \quad with \quad x_{H,t} = \left(\frac{\alpha\omega_H P_{H,t}\Omega_{H,t}\theta_{H,t}}{p_{H,t}}\right)^{\frac{1}{1-\alpha}} L_{H,t}$$

$$\pi_{H,t} = p_{HF,t}x_{HF,t} - P_{H,t}x_{HF,t} \quad with \quad x_{HF,t} = \left(\frac{\alpha(1-\omega_H)P_{H,t}\Omega_{H,t}\theta_{HF,t}}{p_{HF,t}}\right)^{\frac{1}{1-\alpha}} L_{H,t}$$

Assume $\frac{1}{1-\alpha} = K$, then

$$\max_{\{p_{H,t}\}} \ \pi_{H,t} = p_{H,t}^{1-K}(\alpha\omega_H P_{H,t}\Omega_{H,t}\theta_{H,t})^K - P_{H,t}(\alpha\omega_H P_{H,t}\Omega_{H,t}\theta_{H,t})^K p_{H,t}^{-K}$$

$$\max_{\{p_{HF,t}\}} \ \pi_{HF,t} = p_{HF,t}^{1-K}(\alpha(1-\omega_H)P_{H,t}\Omega_{H,t}\theta_{HF,t})^K - P_{H,t}(\alpha(1-\omega_H)P_{H,t}\Omega_{H,t}\theta_{HF,t})^K p_{HF,t}^{-K}$$

$$p_{H,t}: \ (1-K)p_{H,t}^{-K} = -P_{H,t}Kp_{H,t}^{-K-1} \quad \Rightarrow \quad P_{H,t} = -\left(\frac{1-K}{K}\right)p_{H,t} \quad \Rightarrow \quad p_{H,t} = \frac{1}{\alpha}P_{H,t}$$

$$p_{HF,t}: \ (1-K)p_{HF,t}^{-K} = -P_{H,t}Kp_{HF,t}^{-K-1} \quad \Rightarrow \quad P_{H,t} = -\left(\frac{1-K}{K}\right)p_{H,t} \quad \Rightarrow \quad p_{HF,t} = \frac{1}{\alpha}P_{H,t}$$

$$p_{H,t} = p_{HF,t} = \frac{1}{\alpha}P_{H,t}$$

It follows:

$$\pi_{H,t} = \left(\frac{1}{\alpha} - 1\right)x_{H,t}P_{H,t}$$

$$x_{H,t} = (\alpha^2 \omega_H \Omega_{H,t} \theta_{H,t})^{\frac{1}{1-\alpha}} L_{H,t}$$

$$\pi_{HF,t} = \left(\frac{1}{\alpha} - 1 \right) x_{HF,t} P_{H,t}$$

$$x_{HF,t} = (\alpha^2 (1 - \omega_H) \Omega_{H,t} \theta_{HF,t})^{\frac{1}{1-\alpha}} L_{H,t}$$

where

- $x_{H,t}$ = quantity demanded of domestic tech H in H
- $x_{HF,t}$ = quantity demanded of foreign tech F in H
- $\pi_{H,t}$ = profits of the producer of home tech (i.e. home H produces invented foreign tech H)
- $\pi_{HF,t}$ = profits of the producer of foreign tech (i.e. home H produces adopted foreign tech F)

Foreign:

$$\pi_{F,t} = p_{F,t}x_{F,t} - P_{F,t}x_{F,t} \;\; with \quad x_{F,t} = \left(\frac{\alpha\omega_F P_{F,t}\Omega_{F,t}\theta_{F,t}}{p_{F,t}}\right)^{\frac{1}{1-\alpha}} L_{F,t}$$

$$\pi_{F,t} = p_{FH,t}x_{FH,t} - P_{F,t}x_{FH,t} \;\; with \quad x_{FH,t} = \left(\frac{\alpha(1-\omega_F) P_{F,t}\Omega_{F,t}\theta_{FH,t}}{p_{FH,t}}\right)^{\frac{1}{1-\alpha}} L_{F,t}$$

Assume $\frac{1}{1-\alpha} = K$, then

$$\max_{\{p_{F,t}\}} \pi_{F,t} = p_{F,t}^{1-K}(\alpha\omega_F P_{F,t}\Omega_{F,t}\theta_{F,t})^K - P_{F,t}(\alpha\omega_F P_{F,t}\Omega_{F,t}\theta_{F,t})^K p_{F,t}^{-K}$$

$$\max_{\{p_{FH,t}\}} \pi_{FH,t} = p_{FH,t}^{1-K}(\alpha(1-\omega_F) P_{F,t}\Omega_{F,t}\theta_{FH,t})^K - P_{F,t}(\alpha(1-\omega_F) P_{F,t}\Omega_{F,t}\theta_{FH,t})^K p_{FH,t}^{-K}$$

$$p_{F,t}: \;(1-K)p_{F,t}^{-K} = -P_{F,t}Kp_{F,t}^{-K-1} \quad\Rightarrow\quad P_{F,t} = -\left(\frac{1-K}{K}\right)p_{F,t} \quad\Rightarrow\quad p_{F,t} = \frac{1}{\alpha}P_{F,t}$$

$$p_{FH,t}: \;(1-K)p_{FH,t}^{-K} = -P_{F,t}Kp_{FH,t}^{-K-1} \quad\Rightarrow\quad P_{F,t} = -\left(\frac{1-K}{K}\right)p_{F,t} \quad\Rightarrow\quad p_{FH,t} = \frac{1}{\alpha}P_{F,t}$$

$$p_{F,t} = p_{FH,t} = \frac{1}{\alpha}P_{H,t}$$

It follows:

$$\pi_{F,t} = \left(\frac{1}{\alpha} - 1\right) x_{F,t} P_{F,t}$$

$$x_{F,t} = (\alpha^2 \omega_F \Omega_{F,t} \theta_{F,t})^{\frac{1}{1-\alpha}} L_{F,t}$$

$$\pi_{FH,t} = \left(\frac{1}{\alpha} - 1\right) x_{FH,t} P_{F,t}$$

$$x_{FH,t} = (\alpha^2 (1-\omega_F) \Omega_{F,t} \theta_{FH,t})^{\frac{1}{1-\alpha}} L_{F,t}$$

where (from *foreign* economy point of view)

- $x_{F,t}$ = quantity demanded of domestic F tech in F
- $x_{FH,t}$ = quantity demanded of foreign tech H in F
- $\pi_{F,t}$ = profits of the producer of home tech (i.e. home F produces invented foreign tech F)
- $\pi_{FH,t}$ = profits of the producer of foreign tech (i.e. home F produces adopted foreign tech H)

Remark:

$$\theta_{H,t} = \left[w (A_{H,t} x_{H,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} + (1-w)(A_{HF,t} x_{HF,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} (A_{H,t} x_{H,t}^{\alpha})^{-\frac{1}{\sigma}}$$

$$\theta_{HF,t} = \left[w(A_{H,t} x_{H,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} + (1-w)(A_{HF,t} x_{HF,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} (A_{HF,t} x_{HF,t}^{\alpha})^{-\frac{1}{\sigma}}$$

$$\theta_{F,t} = \left[w(A_{F,t} x_{F,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} + (1-w)(A_{FH,t} x_{FH,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} (A_{F,t} x_{F,t}^{\alpha})^{-\frac{1}{\sigma}}$$

$$\theta_{FH,t} = \left[w(A_{F,t} x_{F,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} + (1-w)(A_{FH,t} x_{FH,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{1}{\sigma-1}} (A_{FH,t} x_{FH,t}^{\alpha})^{-\frac{1}{\sigma}}$$

A.3 TFP process

$$log(\Omega_{H,t}) = \rho log(\Omega_{H,t-1}) + \epsilon_{H,t}$$

$$log(\Omega_{F,t}) = \rho log(\Omega_{F,t-1}) + \epsilon_{F,t}$$

A.4 Innovation

Home:

$$A_{H,t+1} = \upsilon_{H,t} S_{H,t} + (1-\delta) A_{H,t}$$

$$\upsilon_{H,t} = \chi_H T_{H,t}^{-(\eta-1)} S_{H,t}^{\eta-1}$$

$$T_{H,t} = A_{H,t} + A_{HF,t} \underbrace{\gamma_{HF}}_{spillover}$$

NA condition:

$$(1-\tau_{H,t}^{RD}) \frac{1}{\upsilon_{H,t}} = E_t[M_{H,t+1} V_{H,t+1}]$$

where

- $A_{H,t}$ = patents in H (domestically available)
- $A_{HF,t}$= patents adopted in H from F
- $T_{H,t}$ = total patents in H (i.e. invented+adopted)

Foreign:

$$A_{F,t+1} = \upsilon_{F,t} S_{F,t} + (1-\delta) A_{F,t}$$

$$\upsilon_{F,t} = \chi_F T_{F,t}^{-(\eta-1)} S_{F,t}^{\eta-1}$$

$$T_{F,t} = A_{F,t} + A_{FH,t} \underbrace{\gamma_{FH}}_{spillover}$$

NA condition:

$$(1-\tau_{F,t}^{RD}) \frac{1}{\upsilon_{F,t}} = E_t[M_{F,t+1} V_{F,t+1}]$$

where

- $A_{F,t}$ = patents in F (domestically available)
- $A_{FH,t}$= patents adopted in F from H
- $T_{F,t}$ = total patents in F (i.e. invented+adopted)

A.5 Adoption

Cross-country technology accumulation:

Home:

$$A_{HF,t+1} = A_{HF,t}(1-\delta) + \underbrace{\alpha_H^A \left(\frac{h_{H,t}(A_{F,t+1} - A_{HF,t})}{Y_{H,t}}\right)^{\gamma_A}}_{\upsilon_{H,t}^A} (A_{F,t+1} - A_{HF,t})$$

The home-country innovator receives a fee in domestic units of $J_{H,t}\frac{P_{F,t}}{P_{H,t}}$, meaning that the foreign adopter pays a fee $J_{H,t}P_{F,t}$ per each new patent created. Note that this fee is payed once-and-for-all upon the creation of the new technology. This implies that the total fee (in domestic real units) received by the home country is:

$$\upsilon_{H,t} S_{H,t} J_{H,t} \frac{P_{F,t}}{P_{H,t}}.$$

The fee in real-foreign-units is pin down by the profits maximization of the foreign adopter:

$$J_{H,t} = \max_{h_{F,t}} -h_{F,t} + (1-\delta)\left\{\upsilon_{F,t}^A E_t\left[M_{F,t+1} W_{FH,t+1}\right] + (1-\upsilon_{F,t}^A) E_t[M_{F,t+1} J_{H,t+1}]\right\}$$

$$(1-\tau_{F,t}^A)\frac{1}{\upsilon_{F,t}'^A} = (1-\delta)E_t[M_{F,t+1}(W_{FH,t+1} - J_{H,t+1})]$$

Foreign:

$$A_{FH,t+1} = A_{FH,t}(1-\delta) + \underbrace{\alpha_F^A \left(\frac{h_{F,t}(A_{H,t+1} - A_{FH,t})}{Y_{F,t}} \right)^{\gamma_A}}_{\upsilon_{F,t}^A} (A_{H,t+1} - A_{FH,t})$$

The foreign-country innovator receives a fee in real foreign units (i.e., local real units for the foreign country) of $J_{F,t}\frac{P_{H,t}}{P_{F,t}}$, meaning that the home adopter pays a fee $J_{F,t}P_{H,t}$. The fee in foreign units is pin down by the profits maximization of the home adopter:

$$J_{F,t} = \max_{h_H,t} -h_{H,t} + (1-\delta) \left\{ \upsilon_{H,t}^A E_t \left[M_{H,t+1} W_{HF,t+1} \right] + (1 - \upsilon_{H,t}^A) E_t [M_{H,t+1} J_{F,t+1}] \right\}$$

$$(1 - \tau_{H,t}^A) \frac{1}{\upsilon_{H,t}^{\prime A}} = (1-\delta) E_t [M_{H,t+1}(W_{HF,t+1} - J_{H,t+1})]$$

A.6 Valuing Patents

The value of a patent depends on:

1. domestic profits;
2. prob. of selling the patent abroad.

Home:

$$V_{H,t} = W_{H,t} + J_{H,t}\frac{P_{F,t}}{P_{H,t}}$$

$$W_{H,t} = \frac{\pi_{H,t}}{P_{H,t}} + (1-\delta)E_t[M_{H,t+1}W_{H,t+1}]$$

$$W_{HF,t} = \frac{\pi_{HF,t}}{P_{H,t}} + (1-\delta)E_t[M_{H,t+1}W_{HF,t+1}]$$

Foreign:

$$V_{F,t} = W_{F,t} + J_{F,t}\frac{P_{H,t}}{P_{F,t}}$$

$$W_{F,t} = \frac{\pi_{F,t}}{P_{F,t}} + (1-\delta)E_t[M_{F,t+1}W_{F,t+1}]$$

$$W_{FH,t} = \frac{\pi_{FH,t}}{P_{F,t}} + (1-\delta)E_t[M_{F,t+1}W_{FH,t+1}]$$

A.7 Resource Constraints

$$Y_{H,t} = C_{H,t} + S_{H,t} + h_{H,t}(A_{F,t+1} - A_{HF,t}) + A_{H,t}x_{H,t} + A_{HF,t}x_{HF,t} + G_{H,t}\Phi^G$$

$$Y_{F,t} = C_{F,t} + S_{F,t} + h_{F,t}(A_{H,t+1} - A_{FH,t}) + A_{F,t}x_{F,t} + A_{FH,t}x_{FH,t} + G_{F,t}\Phi^G$$

where

$$Y_{H,t} = \Omega_{H,t}L_{H,t}^{1-\alpha}\left[w(A_{H,t}x_{H,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} + (1-w)(A_{HF,t}x_{HF,t}^{\alpha})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

$$Y_{F,t} = \Omega_{F,t}L_{F,t}^{1-\alpha}\left[w(A_{F,t}x_{F,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} + (1-w)(A_{FH,t}x_{FH,t}^{\alpha})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

A.8 Zero Current Account

In each period, trade is balanced. By construction, our countries have zero imports and exports of intermediate goods. Since international transactions refer only to the right of adopting foreign technologies, in each period the zero-balance condition has to be applied to the flow of money exchanged by adopters and the innovators:

$$\upsilon_{H,t}S_{H,t}J_{H,t}P_{F,t} = \upsilon_{F,t}S_{F,t}J_{F,t}P_{H,t} \rightarrow$$

$$\frac{P_{H,t}}{P_{F,t}} = \frac{\upsilon_{H,t}S_{H,t}J_{H,t}}{\upsilon_{F,t}S_{F,t}J_{F,t}}$$

A.9 Households

Home:

$$M_{H,t+1} = \beta \left(\frac{u_{H,t+1}}{u_{H,t}} \right)^{\frac{1}{\nu} - \frac{1}{\psi}} \left(\frac{C_{H,t+1}}{C_{H,t}} \right)^{-\frac{1}{\nu}} \left(\frac{U_{H,t+1}^{1-\gamma}}{E_t[U_{H,t+1}^{1-\gamma}]} \right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}$$

$$\frac{1}{R_{H,t}^f} = E_t \left[M_{H,t+1} \right]$$

$$EV_{H,t} = E_t \left[\left(\frac{U_{H,t+1}}{A_{H,t+1}} \cdot e^{\Delta A_{H,t+1}} \right)^{1-\gamma} \right]$$

$$ev_{H,t} = \log(EV_{H,t})$$

$$u_{H,t} = \left\{ \theta_c C_{H,t}^{1-\frac{1}{\nu}} + (1-\theta_c)[T_{H,t}(1-L_{H,t})]^{1-\frac{1}{\nu}} \right\}^{\frac{1}{1-1/\nu}}$$

$$U_{H,t} = \left[(1-\beta) u_{H,t}^{1-\frac{1}{\psi}} + \beta EV_{H,t}^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}}$$

$$\frac{1-\theta_c}{\theta_c} \left(\frac{C_{H,t}}{T_{H,t}(1-L_{H,t})} \right)^{\frac{1}{\nu}} T_{H,t} = (1-\tau_{H,t}^L) \underbrace{w_{H,t}}_{wage}$$

Foreign:

$$M_{F,t+1} = \beta \left(\frac{u_{F,t+1}}{u_{F,t}}\right)^{\frac{1}{\nu}-\frac{1}{\psi}} \left(\frac{C_{F,t+1}}{C_{F,t}}\right)^{-\frac{1}{\nu}} \left(\frac{U_{F,t+1}^{1-\gamma}}{E_t[U_{F,t+1}^{1-\gamma}]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}}$$

$$\frac{1}{R_{F,t}^f} = E_t\left[M_{F,t+1}\right]$$

$$EV_{F,t} = E_t\left[\left(\frac{U_{F,t+1}}{A_{F,t+1}} \cdot e^{\Delta A_{F,t+1}}\right)^{1-\gamma}\right]$$

$$ev_{F,t} = \log(EV_{F,t})$$

$$u_{F,t} = \left\{\theta_c C_{F,t}^{1-\frac{1}{\nu}} + (1-\theta_c)[T_{F,t}(1-L_{F,t})]^{1-\frac{1}{\nu}}\right\}^{\frac{1}{1-1/\nu}}$$

$$U_{F,t} = \left[(1-\beta)u_{F,t}^{1-\frac{1}{\psi}} + \beta EV_{F,t}^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}$$

$$\frac{1-\theta_c}{\theta_c}\left(\frac{C_{F,t}}{T_{F,t}(1-L_{F,t})}\right)^{\frac{1}{\nu}} T_{F,t} = (1-\tau_{F,t}^L)\underbrace{w_{F,t}}_{wage}$$

A.10 Government

Home:

$$\frac{B_{H,t}}{Y_{H,t}} = \frac{(1 + r^f_{H,t-1})}{e^{\Delta y_{H,t}}}\frac{B_{H,t-1}}{Y_{H,t-1}} + \frac{G_{H,t} - NT_{H,t}}{Y_{H,t}}$$

$$\underbrace{NT_{H,t}}_{\textit{real tax rev.}} = \underbrace{w_{H,t}L_{H,t}\tau^L_{H,t}}_{\textit{labor tax}} - \underbrace{\tau^{RD}_{H,t}S_{H,t}}_{\textit{R\&D subsidy}} - \underbrace{\tau^A_{H,t}h_{H,t}(A_{F,t+1} - A_{HF,t})}_{\textit{adopt. subsidy}}$$

$$\frac{B_{H,t}}{Y_{H,t}} = \rho_B\frac{B_{H,t-1}}{Y_{H,t-1}} + \phi_B(L_{H,t} - L_{ss})$$

$$\frac{G_{H,t}}{Y_{H,t}} = \frac{1}{1 + e^{-gy_{H,t}}}$$

$$gy_{H,t} = (1 - \rho)\overline{gy_H} + \rho_g gy_{H,t-1} + \epsilon^G_{H,t}$$

$$\tau^A_{H,t} = \bar{\tau}^A_H + \epsilon^A_{H,t} \qquad \tau^{RD}_{H,t} = \bar{\tau}^{RD}_H + \epsilon^{RD}_{H,t}$$

Foreign:

$$\frac{B_{F,t}}{Y_{F,t}} = \frac{(1 + r^f_{F,t-1})}{e^{\Delta y_{F,t}}}\frac{B_{F,t-1}}{Y_{F,t-1}} + \frac{G_{F,t} - NT_{F,t}}{Y_{F,t}}$$

$$\underbrace{NT_{F,t}}_{\text{real tax rev.}} = \underbrace{w_{F,t}L_{F,t}\tau^L_{F,t}}_{\text{labor tax}} - \underbrace{\tau^{RD}_{F,t}S_{F,t}}_{\text{R\&D subsidy}} - \underbrace{\tau^A_{F,t}h_{F,t}(A_{H,t+1} - A_{FH,t})}_{\text{adopt. subsidy}}$$

$$\frac{B_{F,t}}{Y_{F,t}} = \rho_B \frac{B_{F,t-1}}{Y_{F,t-1}} + \phi_B(L_{F,t} - L_{ss})$$

$$\frac{G_{F,t}}{Y_{F,t}} = \frac{1}{1 + e^{-gy_{F,t}}}$$

$$gy_{F,t} = (1-\rho)\overline{gy_F} + \rho_g gy_{F,t-1} + \epsilon^G_{F,t}$$

$$\tau^A_{F,t} = \bar{\tau}^A_F + \epsilon^A_{F,t} \qquad \tau^{RD}_{F,t} = \bar{\tau}^{RD}_F + \epsilon^{RD}_{F,t}$$

Appendix B: Normalized Equations

- Normalized variables are denoted by $\sim$. Home (Foreign) variables are normalized by $A_{H,t}$ ($A_{F,t}$).
- Def: $\Gamma_t := \frac{A_{H,t}}{A_{F,t}} = \Gamma_{H,t} = 1/\Gamma_{F,t}$
- Def: $\Xi_{H,t} = \frac{A_{HF,t}}{A_{H,t}}$, $\Xi_{F,t} = \frac{A_{FH,t}}{A_{F,t}}$

// Zero Current Account

$$\frac{P_{H,t}}{P_{F,t}} = \frac{\upsilon_{H,t}\widetilde{S}_{H,t}J_{H,t}}{\upsilon_{F,t}\widetilde{S}_{F,t}J_{F,t}}\Gamma_t \text{ Included} \tag{B3}$$

// Final and Intermediate Sector

$$\theta_{H,t} = \left[\omega_H(x_{H,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\omega_H)(\Xi_{H,t}x_{HF,t}^{\alpha})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}(x_{H,t}^{\alpha})^{-\frac{1}{\sigma}} \text{ Included} \tag{B4}$$

$$\theta_{HF,t} = \left[\omega_H(x_{H,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\omega_H)(\Xi_{H,t}x_{HF,t}^{\alpha})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}(\Xi_{H,t}x_{HF,t}^{\alpha})^{-\frac{1}{\sigma}} \text{ Included} \tag{B5}$$

$$\theta_{F,t} = \left[\omega_F(x_{F,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\omega_F)(\Xi_{F,t}x_{FH,t}^{\alpha})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}(x_{F,t}^{\alpha})^{-\frac{1}{\sigma}} \text{ Included} \tag{B6}$$

$$\theta_{FH,t} = \left[\omega_F(x_{F,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\omega_F)(\Xi_{F,t}x_{FH,t}^{\alpha})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}(\Xi_{F,t}x_{FH,t}^{\alpha})^{-\frac{1}{\sigma}} \text{ Included} \tag{B7}$$

$$x_{H,t} = (\alpha^2\omega_H\Omega_{H,t}\theta_{H,t})^{\frac{1}{1-\alpha}}L_{H,t} \text{ Included} \tag{B8}$$

$$x_{HF,t} = (\alpha^2(1-\omega_H)\Omega_{H,t}\theta_{HF,t})^{\frac{1}{1-\alpha}} L_{H,t} \text{ Included} \tag{B9}$$

$$x_{F,t} = (\alpha^2\omega_F\Omega_{F,t}\theta_{F,t})^{\frac{1}{1-\alpha}} L_{F,t} \tag{B10}$$

$$x_{FH,t} = (\alpha^2(1-\omega_F)\Omega_{F,t}\theta_{FH,t})^{\frac{1}{1-\alpha}} L_{F,t} \text{ Included} \tag{B11}$$

$$\pi_{H,t} = \left(\frac{1}{\alpha} - 1\right) x_{H,t} P_{H,t} \text{ Included} \tag{B12}$$

$$\pi_{HF,t} = \left(\frac{1}{\alpha} - 1\right) x_{HF,t} P_{H,t} \text{ Included} \tag{B13}$$

$$\pi_{F,t} = \left(\frac{1}{\alpha} - 1\right) x_{F,t} P_{F,t} \tag{B14}$$

$$\pi_{FH,t} = \left(\frac{1}{\alpha} - 1\right) x_{FH,t} P_{F,t} \text{ Included} \tag{B15}$$

$$\tilde{Y}_{H,t} = \Omega_{H,t} L_{H,t}^{1-\alpha} \left[\omega_H (x_{H,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\omega_H)(\Xi_{H,t} x_{HF,t}^{\alpha})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \text{ Included} \tag{B16}$$

$$\tilde{Y}_{F,t} = \Omega_{F,t} L_{F,t}^{1-\alpha} \left[\omega_F (x_{F,t}^{\alpha})^{\frac{\sigma-1}{\sigma}} + (1-\omega_F)(\Xi_{F,t} x_{FH,t}^{\alpha})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{B17}$$

// Labor MKT

$$L_{H,t}\tilde{w}_{H,t} = (1-\alpha)\tilde{Y}_{H,t} \text{ Included} \tag{B18}$$

$$L_{F,t}\tilde{w}_{F,t} = (1-\alpha)\tilde{Y}_{F,t} \tag{B19}$$

$$\frac{1-\theta_c}{\theta_c}\left(\frac{\widetilde{C}_{H,t}}{\widetilde{T}_{H,t}(1-L_{H,t})}\right)^{\frac{1}{\nu}}\widetilde{T}_{H,t} = (1-\tau^L_{H,t})\widetilde{w}_{H,t} \text{ Included} \tag{B20}$$

$$\frac{1-\theta_c}{\theta_c}\left(\frac{\widetilde{C}_{F,t}}{T_{F,t}(1-L_{F,t})}\right)^{\frac{1}{\nu}}\widetilde{T}_{F,t} = (1-\tau^L_{F,t})\widetilde{w}_{F,t} \tag{B21}$$

// R&D Sector

$$e^{\Delta a_{H,t+1}} = \upsilon_{H,t}\tilde{S}_{H,t} + (1-\delta) \text{ Included (lagged)} \tag{B22}$$

$$e^{\Delta a_{F,t+1}} = \upsilon_{F,t}\tilde{S}_{F,t} + (1-\delta) \tag{B23}$$

$$\tilde{T}_{H,t} = 1 + \Xi_{H,t}\gamma_{HF} \text{ Included} \tag{B24}$$

$$\tilde{T}_{F,t} = 1 + \Xi_{F,t}\gamma_{FH} \tag{B25}$$

$$\upsilon_{H,t} = \chi_H \tilde{T}_{H,t}^{-(\eta-1)}\tilde{S}_{H,t}^{\eta-1} \text{ Included} \tag{B26}$$

$$\upsilon_{F,t} = \chi_F \tilde{T}_{F,t}^{-(\eta-1)} \tilde{S}_{F,t}^{\eta-1} \tag{B27}$$

$$(1-\tau_{H,t}^{RD})\frac{1}{\upsilon_{H,t}} = E_t[\tilde{M}_{H,t+1} V_{H,t+1}] \text{ Included} \tag{B28}$$

$$(1-\tau_{F,t}^{RD})\frac{1}{\upsilon_{F,t}} = E_t[\tilde{M}_{F,t+1} V_{F,t+1}] \tag{B29}$$

$$V_{H,t} = W_{H,t} + J_{H,t}\frac{P_{F,t}}{P_{H,t}} \text{ Included} \tag{B30}$$

$$W_{H,t} = \frac{\pi_{H,t}}{P_{H,t_1}} + (1-\delta)E_t[\tilde{M}_{H,t+1} W_{H,t+1}] \text{ Included} \tag{B31}$$

$$V_{F,t} = W_{F,t} + J_{F,t}\frac{P_{H,t}}{P_{F,t}} \tag{B32}$$

$$W_{F,t} = \frac{\pi_{F,t}}{P_{F,t}} + (1-\delta)E_t[\tilde{M}_{F,t+1} W_{F,t+1}] \tag{B33}$$

// Adoption Sector

$$\Xi_{H,t} = \Xi_{H,t-1}\frac{(1-\delta)}{e^{\Delta a_{H,t}}} + \upsilon_{H,t-1}^{A}\left(\frac{1}{\Gamma_t} - \Xi_{H,t-1}\frac{1}{e^{\Delta a_{H,t}}}\right) \text{ Included} \tag{B34}$$

$$\Xi_{F,t} = \Xi_{F,t-1}\frac{(1-\delta)}{e^{\Delta a_{F,t}}} + \upsilon_{F,t-1}^{A}\left(\Gamma_{H,t} - \Xi_{F,t-1}\frac{1}{e^{\Delta a_{F,t}}}\right) \tag{B35}$$

$$\upsilon^A_{H,t} = \alpha^A_H \left(\frac{h_{H,t}(e^{\Delta a_{H,t+1}} \frac{1}{\Gamma_t} - \Xi_{H,t})}{\tilde{Y}_{H,t}} \right)^{\gamma_A} \text{ Included} \tag{B36}$$

$$\upsilon^A_{F,t} = \alpha^A_F \left(\frac{h_{F,t}(e^{\Delta a_{F,t+1}} \Gamma_t - \Xi_{F,t})}{\tilde{Y}_{F,t}} \right)^{\gamma_A} \tag{B37}$$

$$W_{HF,t} = \frac{\pi_{HF,t}}{P_{H,t}} + (1-\delta) E_t[\tilde{M}_{H,t+1} W_{HF,t+1}] \text{ Included} \tag{B38}$$

$$W_{FH,t} = \frac{\pi_{FH,t}}{P_{F,t}} + (1-\delta) E_t[\tilde{M}_{H,t+1} W_{FH,t+1}] \text{ Included} \tag{B39}$$

$$J_{H,t} = -h_{F,t} + (1-\delta) \left\{ \upsilon^A_{F,t} E_t \left[\widetilde{M}_{F,t+1} W_{FH,t+1} \right] + (1-\upsilon^A_{F,t}) E_t[\widetilde{M}_{F,t+1} J_{H,t+1}] \right\} \text{ Included} \tag{B40}$$

$$(1-\tau^A_{F,t}) \frac{1}{\upsilon'^A_{F,t}} = (1-\delta) E_t[\widetilde{M}_{F,t+1}(W_{FH,t+1} - J_{H,t+1})] \text{ Included} \tag{B41}$$

$$J_{F,t} = -h_{H,t} + (1-\delta) \left\{ \upsilon^A_{H,t} E_t \left[\widetilde{M}_{H,t+1} W_{HF,t+1} \right] + (1-\upsilon^A_{H,t}) E_t[\widetilde{M}_{H,t+1} J_{F,t+1}] \right\} \tag{B42}$$

$$(1-\tau^A_{H,t}) \frac{1}{\upsilon'^A_{H,t}} = (1-\delta) E_t[\widetilde{M}_{H,t+1}(W_{HF,t+1} - J_{H,t+1})] \tag{B43}$$

// International Size Distribution

$$\Gamma_t = e^{(\Delta a_{H,t} - \Delta a_{F,t})} \cdot \Gamma_{t-1} \text{ Included} \tag{B44}$$

// Aggregate Resource Constraint

$$\tilde{Y}_{H,t} = \tilde{C}_{H,t} + \tilde{S}_{H,t} + h_{H,t}(e^{\Delta a_{H,t+1}} \frac{1}{\Gamma_t} - \Xi_{H,t}) + x_{H,t} + \Xi_{H,t} x_{HF,t} + \tilde{G}_{H,t} \Phi^G \text{ Included} \tag{B45}$$

$$\tilde{Y}_{F,t} = \tilde{C}_{F,t} + \tilde{S}_{F,t} + h_{F,t}(e^{\Delta a_{F,t+1}} \Gamma_t - \Xi_{F,t}) + x_{F,t} + \Xi_{F,t} x_{FH,t} + \tilde{G}_{F,t} \Phi^G \tag{B46}$$

// Household side (EZ eq.s)

$$\widetilde{M}_{H,t+1} = \beta \left(\frac{\widetilde{u}_{H,t+1}}{\widetilde{u}_{H,t}} e^{\Delta a_{H,t+1}} \right)^{\frac{1}{\nu} - \frac{1}{\psi}} \left(\frac{\widetilde{C}_{H,t+1}}{\widetilde{C}_{H,t}} e^{\Delta a_{H,t+1}} \right)^{-\frac{1}{\nu}} \left(\frac{(\widetilde{U}_{H,t+1} e^{\Delta a_{H,t+1}})^{1-\gamma}}{E_t[(\widetilde{U}_{H,t+1} e^{\Delta a_{H,t+1}})^{1-\gamma}]} \right)^{\frac{\frac{1}{\psi} - \gamma}{1-\gamma}} \text{ Included} \tag{B47}$$

$$\frac{1}{R^f_{H,t}} = E_t \left[\widetilde{M}_{H,t+1} \right] \text{ Included} \tag{B48}$$

$$\widetilde{EV}_{H,t} = E_t \left[\left(\tilde{U}_{H,t+1} \cdot e^{\Delta a_{H,t+1}} \right)^{1-\gamma} \right] \text{ Included} \tag{B49}$$

$$ev_{H,t} = \log(EV_{H,t}) \text{ Included} \tag{B50}$$

$$\tilde{u}_{H,t} = \left\{\theta_c \widetilde{C}_{H,t}^{1-\frac{1}{\nu}} + (1-\theta_c)[\widetilde{T}_{H,t}(1-L_{H,t})]^{1-\frac{1}{\nu}}\right\}^{\frac{1}{1-1/\nu}} \text{ Included} \tag{B51}$$

$$\widetilde{U}_{H,t} = \left[(1-\beta)\widetilde{u}_{H,t}^{1-\frac{1}{\psi}} + \beta \widetilde{EV}_{H,t}^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} \text{ Included} \tag{B52}$$

$$\widetilde{M}_{F,t+1} = \beta \left(\frac{\widetilde{u}_{F,t+1}}{\widetilde{u}_{F,t}} e^{\Delta a_{F,t+1}}\right)^{\frac{1}{\nu}-\frac{1}{\psi}} \left(\frac{\widetilde{C}_{F,t+1}}{\widetilde{C}_{F,t}} e^{\Delta a_{F,t+1}}\right)^{-\frac{1}{\nu}} \left(\frac{(\widetilde{U}_{F,t+1} e^{\Delta a_{F,t+1}})^{1-\gamma}}{E_t[(\widetilde{U}_{F,t+1} e^{\Delta a_{F,t+1}})^{1-\gamma}]}\right)^{\frac{\frac{1}{\psi}-\gamma}{1-\gamma}} \tag{B53}$$

$$\frac{1}{R_{F,t}^f} = E_t\left[\widetilde{M}_{F,t+1}\right] \tag{B54}$$

$$\widetilde{EV}_{F,t} = E_t\left[\left(\tilde{U}_{F,t+1} \cdot e^{\Delta a_{F,t+1}}\right)^{1-\gamma}\right] \tag{B55}$$

$$ev_{F,t} = \log(EV_{F,t}) \tag{B56}$$

$$\tilde{u}_{F,t} = \left\{\theta_c \widetilde{C}_{F,t}^{1-\frac{1}{\nu}} + (1-\theta_c)[\widetilde{T}_{F,t}(1-L_{F,t})]^{1-\frac{1}{\nu}}\right\}^{\frac{1}{1-1/\nu}} \tag{B57}$$

$$\widetilde{U}_{F,t} = \left[(1-\beta)\widetilde{u}_{F,t}^{1-\frac{1}{\psi}} + \beta \widetilde{EV}_{F,t}^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} \tag{B58}$$

// Productivity

$$log(\Omega_{H,t}) = \rho log(\Omega_{H,t-1}) + \epsilon_{H,t} \text{ Included} \tag{B59}$$

$$log(\Omega_{F,t}) = \rho log(\Omega_{F,t-1}) + \epsilon_{F,t} \tag{B60}$$

// Government

$$\tilde{B}_{H,t} = \frac{R^f_{H,t-1}}{e^{\Delta a_{H,t}}}\tilde{B}_{H,t-1} + \tilde{G}_{H,t} - \tilde{NT}_{H,t} \text{ Included} \tag{B61}$$

$$\tilde{NT}_{H,t} = \tilde{w}_{H,t}L_{H,t}\tau^L_{H,t} - \tau^{RD}_{H,t}\tilde{S}_{H,t} - \tau^A_{H,t}\tilde{h}_{H,t}(e^{\Delta a_{H,t+1}}\frac{1}{\Gamma_t} - \Xi_{H,t}) \text{ Included} \tag{B62}$$

$$\frac{\tilde{B}_{H,t}}{\tilde{Y}_{H,t}} = \rho_B\frac{\tilde{B}_{H,t-1}}{\tilde{Y}_{H,t-1}} + \phi_B(L_{H,t} - L_{ss}) \text{ Included} \tag{B63}$$

$$\frac{\tilde{G}_{H,t}}{\tilde{Y}_{H,t}} = \frac{1}{1 + e^{-gy_{H,t}}} \text{ Included} \tag{B64}$$

$$gy_{H,t} = (1 - \rho_g)\overline{gy_H} + \rho_g gy_{H,t-1} + \epsilon^G_{H,t} \text{ Included} \tag{B65}$$

$$\text{sub}^A_{H,t} = \overline{\text{sub}^A_H} + \epsilon^A_{H,t} \text{ Included} \tag{B66}$$

$$\tau^A_{H,t} = \frac{1}{1 + \exp(-\text{sub}^A_{H,t})} \text{ Included} \tag{B67}$$

$$\text{sub}_{H,t}^{RD} = \overline{\text{sub}_{H}^{RD}} + \epsilon_{H,t}^{RD} \text{ Included} \tag{B68}$$

$$\tau_{H,t}^{RD} = \frac{1}{1+\exp(-\text{sub}_{H,t}^{RD})} \text{ Included} \tag{B69}$$

$$\tilde{B}_{F,t} = \frac{R_{F,t-1}^{f}}{e^{\Delta a_{F,t}}} \tilde{B}_{F,t-1} + \tilde{G}_{F,t} - \tilde{NT}_{F,t} \tag{B70}$$

$$\tilde{NT}_{F,t} = \tilde{w}_{F,t} L_{F,t} \tau_{F,t}^{L} - \tau_{F,t}^{RD} \tilde{S}_{F,t} - \tau_{F,t}^{A} \tilde{h}_{F,t} (e^{\Delta a_{F,t+1}} \Gamma_t - \Xi_{F,t}) \tag{B71}$$

$$\frac{\tilde{B}_{F,t}}{\tilde{Y}_{F,t}} = \rho_B \frac{\tilde{B}_{F,t-1}}{\tilde{Y}_{F,t-1}} + \phi_B (L_{F,t} - L_{ss}) \tag{B72}$$

$$\frac{\tilde{G}_{F,t}}{\tilde{Y}_{F,t}} = \frac{1}{1+e^{-gy_{F,t}}} \tag{B73}$$

$$gy_{F,t} = (1-\rho)\overline{gy_F} + \rho_g gy_{F,t-1} + \epsilon_{F,t}^{G} \tag{B74}$$

$$\text{sub}_{F,t}^{A} = \overline{\text{sub}_{F}^{A}} + \epsilon_{F,t}^{A} \tag{B75}$$

$$\tau_{F,t}^{A} = \frac{1}{1+\exp(-\text{sub}_{F,t}^{A})} \tag{B76}$$

$$\text{sub}_{F,t}^{RD} = \overline{\text{sub}_F^{RD}} + \epsilon_{F,t}^{RD} \tag{B77}$$

$$\tau_{F,t}^{RD} = \frac{1}{1 + \exp(-\text{sub}_{F,t}^{RD})} \tag{B78}$$

Endog. Variables:

Private Economy: $\Omega_{H,t}$, $\Omega_{F,t}$, x_H, x_F, x_{HF}, x_{FH}, π_H, π_{HF}, π_F, π_{FH}, $\tilde{Y}_H$, $\tilde{Y}_F$, P_H, P_F, p_H, p_F, L_H, L_F, $\tilde{w}_H$, $\tilde{w}_F$, θ_H, θ_{HF}, θ_F, θ_{FH}, $\tilde{S}_H$, $\tilde{S}_F$, Δa_H, Δa_F, $\tilde{T}_H$, $\tilde{T}_H$, V_H, V_F, W_H, W_F, W_{HF}, W_{FH}, J_H, J_F, $\tilde{C}_H$, $\tilde{C}_F$, h_H, h_F, υ_H, υ_F, υ_F^A, υ_H^A, Γ, Ξ_H, Ξ_F, $\tilde{EV}_H$, $\tilde{EV}_F$, $\tilde{ev}_H$, $\tilde{ev}_F$, $\tilde{u}_H$, $\tilde{u}_F$, $\tilde{U}_H$, $\tilde{U}_F$, R_H^f, R_F^f, $\tilde{M}_H$, $\tilde{M}_F$

Government: $\tilde{B}_H$, $\tilde{NT}_H$, $\tilde{B}_F$, $\tilde{NT}_F$, τ_H^L, τ_F^L, τ_H^{RD}, τ_F^{RD}, τ_H^A, τ_F^A, gy_H, gy_F, $\tilde{G}_H$, $\tilde{G}_F$, ϵ_H^A, ϵ_H^{RD}, ϵ_F^A, ϵ_F^{RD}

Exog. Variables: ϵ_H^G, ϵ_H, ϵ_F^G, ϵ_F

Appendix C: Algebra for Normalized Variables

Note: To match Romer (1990), we normalized all variables by $A_{H,t}$ and $A_{F,t}$ for the *home* and the *foreign* economy, respectively.

Home:

$A_{H,t+1} = \upsilon_{H,t} S_{H,t} + (1-\delta)A_{H,t} \Rightarrow \frac{A_{H,t}}{A_{H,t-1}} = \upsilon_{H,t}\frac{S_{H,t}}{A_{H,t-1}} + (1-\delta)\frac{A_{H,t-1}}{A_{H,t-1}} \Rightarrow$

$e^{\Delta a_{H,t}} = \upsilon_{H,t} s_{H,t-1} + (1-\delta)$

Foreign:

$A_{F,t+1} = \upsilon_{F,t} S_{F,t} + (1-\delta) A_{F,t} \Rightarrow \frac{A_{F,t}}{A_{F,t-1}} = \upsilon_{F,t} \frac{S_{F,t}}{A_{F,t-1}} + (1-\delta) \frac{A_{F,t-1}}{A_{F,t-1}} \Rightarrow$
$e^{\Delta a_{F,t}} = \upsilon_{F,t} s_{F,t-1} + (1-\delta)$

Home:

$$A_{HF,t+1} = A_{HF,t}(1-\delta) + \upsilon^A_{H,t}(A_{F,t+1} - A_{HF,t})$$
$$A_{HF,t} = A_{HF,t-1}(1-\delta) + \upsilon^A_{H,t-1}(A_{F,t} - A_{HF,t-1})$$
$$\frac{A_{HF,t}}{A_{H,t}} = \frac{A_{HF,t-1}}{A_{H,t}} + \upsilon^A_{H,t-1}\left(\frac{A_{F,t}}{A_{H,t}} - \frac{A_{HF,t-1}}{A_{H,t}}\right)$$
$$\frac{A_{HF,t}}{A_{H,t}} = \frac{A_{HF,t-1}}{A_{H,t-1}} \frac{1}{e^{\Delta a_{H,t}}} + \upsilon^A_{H,t-1}\left(\frac{A_{F,t}}{A_{H,t}} - \frac{A_{HF,t-1}}{A_{H,t-1}} \frac{1}{e^{\Delta a_{H,t}}}\right)$$

where $\Gamma_{F,t} = \frac{A_{F,t}}{A_{H,t}} = e^{\Delta a_{F,t} - \Delta a_{H,t}} \cdot \frac{A_{F,t-1}}{A_{H,t-1}}$ and $\Xi_{H,t} = \frac{A_{HF,t}}{A_{H,t}}$. It follows,

$$\Xi_{H,t} = \Xi_{H,t-1} \frac{1}{e^{\Delta a_{H,t}}} + \upsilon^A_{H,t-1}\left(\Gamma_{F,t} - \Xi_{H,t-1} \frac{1}{e^{\Delta a_{H,t}}}\right)$$

Foreign:

$$A_{FH,t+1} = A_{FH,t}(1-\delta) + \upsilon^A_{F,t}(A_{H,t+1} - A_{FH,t})$$
$$A_{FH,t} = A_{FH,t-1}(1-\delta) + \upsilon^A_{F,t-1}(A_{H,t} - A_{FH,t-1})$$
$$\frac{A_{FH,t}}{A_{F,t}} = \frac{A_{FH,t-1}}{A_{F,t}} + \upsilon^A_{F,t-1}\left(\frac{A_{H,t}}{A_{F,t}} - \frac{A_{FH,t-1}}{A_{F,t}}\right)$$
$$\frac{A_{FH,t}}{A_{F,t}} = \frac{A_{FH,t-1}}{A_{F,t-1}} \frac{1}{e^{\Delta a_{F,t}}} + \upsilon^A_{F,t-1}\left(\frac{A_{H,t}}{A_{F,t}} - \frac{A_{FH,t-1}}{A_{F,t-1}} \frac{1}{e^{\Delta a_{F,t}}}\right)$$

where $\Gamma_{H,t} = \frac{A_{H,t}}{A_{F,t}} = e^{\Delta a_{H,t} - \Delta a_{F,t}} \cdot \frac{A_{H,t-1}}{A_{F,t-1}}$ and $\Xi_{F,t} = \frac{A_{FH,t}}{A_{F,t}}$. It follows,

$$\Xi_{F,t} = \Xi_{F,t-1} \frac{1}{e^{\Delta a_{F,t}}} + \upsilon^A_{F,t-1}\left(\Gamma_{H,t} - \Xi_{F,t-1} \frac{1}{e^{\Delta a_{F,t}}}\right)$$

Home:

$$T_{H,t} = A_{H,t} + A_{HF,t}\gamma_{HF} \Longrightarrow \frac{T_{H,t}}{A_{H,t}} = \frac{A_{H,t}}{A_{H,t}} + \underbrace{\frac{A_{HF,t}}{A_{H,t}}}_{\Xi_{H,t}} \gamma_{HF} \Longrightarrow \frac{T_{H,t}}{A_{H,t}} = 1 + \Xi_{H,t} \cdot \gamma_{HF}$$

Foreign:

$$T_{F,t} = A_{F,t} + A_{FH,t}\gamma_{FH} \Longrightarrow \frac{T_{F,t}}{A_{F,t}} = \frac{A_{F,t}}{A_{F,t}} + \underbrace{\frac{A_{FH,t}}{A_{F,t}}}_{\Xi_{F,t}}\gamma_{FH} \Longrightarrow \frac{T_{F,t}}{A_{F,t}} = 1 + \Xi_{F,t} \cdot \gamma_{FH}$$