

Providing Efficient Incentives to Work: Retirement Ages and the Pension System*

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Abstract

This paper provides a theoretical and quantitative analysis of efficient pension systems as integral parts of the overall tax code. We study lifecycle environments with active intensive and extensive labor margins. First, we analytically characterize Pareto efficient policies when the main tension is between redistribution and provision of incentives: while it may be more efficient to have highly productive individuals work more and retire older, earlier retirement may be needed to give them incentives to fully realize their productivity when they work. We show that, under plausible conditions, efficient retirement ages increase with productivity. We also show that this pattern is implemented by pensions that not only depend on the age of retirement but are designed to be actuarially *unfair*. Second, using individual earnings and retirement data for the U.S. as well as intensive and extensive labor elasticities, we calibrate policy models to simulate robust implications: it is efficient for individuals with higher lifetime earning to retire (*i*) older than they do in the data (at 69.5 *vs.* at 62.8 in the data, for the most productive workers) and (*ii*) older than their less productive peers (at 69.5 for the most productive workers *vs.* at 62.2 for the least productive ones), in sharp contrast to the pattern observed in the U.S. data. Finally, we compute welfare gains of between 1 and 5 percent and total output gains of up to 1 percent from implementing efficient work and retirement age patterns. We argue that distorting the retirement age decision offers a powerful novel policy instrument, capable of overcompensating output losses from standard distortionary redistributive policies.

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1 Introduction

Economic efficiency suggests that more productive individuals should work more and retire *later* than their less productive peers. However, if individuals can work with productivity below their maximum, *earlier* retirement may be needed to provide them with incentives to fully realize their productivities while they work. We study this tension in a class of lifecycle models. We emphasize active intensive and extensive margins of labor supply in the individual decisions of how much to work when not retired and when to retire. The paper provides a theoretical and quantitative analysis of the efficient distribution of retirement ages and examines how the interaction between the tax code and the pension system should be designed to implement the optimum.

Specifically, this paper studies lifecycle environment where individuals differ in two respects. First, individual workers are heterogeneous in their productivities. A worker's productivity changes over lifecycle and follows a privately known idiosyncratic hump-shaped productivity profile.¹ Second, individuals face privately known heterogeneous fixed costs of work.² Fixed utility cost of work introduces non-convexity into disutility of working. Combined with a hump-shaped productivity profile, this makes it optimal for a worker to choose to retire at some age while heterogeneity implies that retirement ages differ among workers. In other words, we study lifecycle environment that features both active intensive and extensive labor margins. A government in this environment reallocates resources across time and individuals to achieve efficiency and a certain level of redistribution. The government, however, cannot use policies contingent on productivity and fixed cost of work since productivity and fixed cost are private information available only to the individual.

Our first main result is to derive conditions on fundamentals under which efficient retirement ages are increasing in lifetime earnings. More generally, the analysis here clearly identifies factors that determine how efficient retirement ages change as a function of productivity. These factors are (i) *virtual* fixed costs of work, i.e., fixed cost of work plus rents from private information,³ and how they change with productivity, (ii) the distribution of productivities, and (iii) how redistributive the government is. The

¹At least since [Mincer \(1974\)](#), it has been known that productivity typically increases earlier in life and declines later leading some individuals to leave the labor force entirely, i.e. to retire. These changes in productivity do not happen to everyone at the same age or at the same rate. Some individuals experience significant decreases in their ability to produce rather early in life while others remain productive for many more years.

²In most of the paper we focus on cases where fixed cost is perfectly correlated with productivity type. We later study an extension that allows partial correlation between fixed cost and productivity profiles.

³The notion of virtual fixed costs here, or virtual types, is akin to Myerson's virtual types.

intuition behind virtual fixed costs driving the results is that the economy with private information is equivalent to an economy without frictions but with modified, or virtual, productivity profiles and fixed costs of work. We show that the virtual types depend on the distribution of productivities and on how redistributive the government is. To provide sharper focus on the underlying mechanisms, our baseline formulation is the case without income effects. A particularly tractable version, that abstracts from risk aversion and discounting, allows to derive closed form characterizations that sharply highlight the forces driving our results. We then reintroduce curvature into the utility function and conclude that, under plausible conditions, efficient retirement ages increase in lifetime earnings. That is, individuals with higher lifetime earnings should be given incentives to retire older than less productive individuals.

Our second main result is to show that a policy based on actuarially *unfair* pension benefits can implement the optimum. That is, the pension side of an optimal policy should be age dependent in a particular way - the present value of lifetime retirement benefits should rise with the age of retirement.⁴ We argue that distorting individual retirement age decision offers a powerful policy instrument. In particular, one important role of the retirement distortion is to undo part of the retirement incentives provided by a standard distortion of the consumption-labor margin, i.e., by a labor income tax. To demonstrate this, we provide a partial characterization of the distortions to both intensive and extensive labor margins, i.e., labor distortion and retirement distortion. We note that labor and retirement distortions both affect retirement incentives: income taxes decrease payoffs from an extra year in the labor force, while retirement benefits increase payoffs from staying out of the labor force. We show that, in the optimum, retirement distortion is lower than labor distortion at the time of retirement. Intuitively, this suggests that labor income taxes distort retirement decision too much. An implementation of the optimum thus requires a pension system with present value of retirement benefits increasing in retirement age to create benefits above and beyond income taxes.

Our third contribution is to provide a quantitative study of efficient work and retirement incentives. We use individual earnings and hours data from the U.S. Panel Study of Income Dynamics (PSID) in combination with retirement age data from the Health and Retirement Study (HRS) to calibrate and simulate efficient work and retirement choices and policy. We calibrate to also match the estimates of labor supply elasticity at the extensive margin.⁵ A quantitative result robust across calibrations is that individuals

⁴The design of the current pension system in the U.S., Social Security, is meant to be actuarially fair before age 65, i.e., benefits rise by 6.67% each year, but actuarially unfair after 65 with a decreasing present value of benefits, i.e., benefits rise by 5.5%. The actuarial fairness is of course affected by the actual life span.

⁵Chetty et al. (2011) emphasize the importance of calibrating the extensive margin elasticity as well

with higher lifetime earning in the U.S. should retire older than they do now and, more strikingly, older than less productive workers. We find in our benchmark calibration that in the optimum, the highest productivity types retire at 69.5, whereas in the data their average retirement age is 62.8. At the same time, individuals with lower lifetime earnings should retire younger than they do now, as well as younger than their more productive peers. In particular, the lowest productivity types retire in the optimum at 62.2 years compared to 69.5 for the highest productivity types. This pattern of retirement ages is in sharp contrast with the one found in the current individual data for the U.S., where average retirement age displays a predominantly decreasing pattern as a function of lifetime earnings. We summarize this contrast in Figure 1. The dashed line displays the average retirement ages for earnings deciles in the data while the solid line displays simulated efficient retirement ages.

Our quantitative study allows us to measure and decompose welfare gains and total output gains associated with inducing efficient retirement age distribution. We find that providing efficient incentives for both work and retirement results in large welfare gains. We compute welfare gains that range across calibrations between 1 and 5 percent of annual consumption equivalent. Notably, we also find a small but positive change in total output of up to 1 percent. We show that this increase in total output results from the meaningfully active intensive and extensive distortions of labor supply. Increasing standard distortions along the intensive margin generally leads to output losses in favor of redistribution and welfare gains. The additional policy instrument of distorting the retirement decision proves powerful enough to overcompensate by inducing more productive individuals to work more years and thus produce more.

Distorting individual retirement decisions efficiently is a compelling example of a policy reform that can produce perceivable benefits. A recent surge of research points towards evidence of significant effects of the incentives created by the interaction between tax and pension systems: from providing strong incentives to leave labor force at statutory retirement age (see, e.g., [French \(2005\)](#)) and resulting in significant amount of redistribution (see, e.g., [Feldstein and Liebman \(2002\)](#) and [Feldstein and Liebman \(2000\)](#)) to penalizing work after statutory retirement age regardless of how productive a worker is (e.g. [Gruber and Wise \(2007\)](#)), to cite just a few recent examples. A unifying theme that emerges from this evidence is a need to address the question of how to design these incentives to reap maximum welfare gains and what that implies about when individuals should retire.

The analysis in this paper contributes to several literatures. Most directly, it provides as the intensive one. We follow their review of the existing estimates and calibrate to match the range of estimates of the extensive elasticity from the individual studies they analyze.

vides a new and empirically-based policy application of the tools of a literature (see, e.g., Prescott, Rogerson, and Wallenius (2009) and Rogerson and Wallenius (2008)) reconciling macro and micro estimates of labor elasticities with meaningfully active intensive and extensive margins of labor supply.⁶ It also extends the literature on optimal distortionary policies with both margins of labor supply. That literature was reinvigorated with the contribution of Saez (2002), who studies optimal income transfer programs when labor responses are concentrated along the intensive responses or when labor responses are concentrated along the extensive responses. Numerous recent studies provide further theoretical extensions of that literature (see, e.g., Chone and Laroque (2010)). Finally, the analysis in this paper also contributes to the empirically-driven Mirrleesian literature that connects labor distortions to estimable distributions and elasticities, as do Diamond (1998), Saez (2001), and in dynamic environments Golosov, Troshkin, and Tsyvinski (2011). In particular, we provide elasticity-based expressions for labor distortions in the presence of both margins of labor supply. Our result about the increase in total output is related to analysis in Golosov and Tsyvinski (2006). Most modern studies of efficient redistributive policies largely result in increased distortions improving welfare but generally sacrificing total output (see, e.g., Fukushima (2010), Farhi and Werning (2010), Golosov, Troshkin, and Tsyvinski (2011), Weinzierl (2011)). Unlike much of the optimal tax literature, rather than focusing on a specific social welfare function, we characterize Pareto efficient allocations in the spirit similar to Werning (2007).

The questions we address and the policy implications we seek are also related to those in Conesa, Kitao, and Krueger (2009) as well as in Huggett and Parra (2010). Their approach differs from ours as they study policies within a set of parametrically restricted functions. One advantage of that approach is that it is computationally more feasible while allowing to study commonly used in practice policies. This paper examines a larger set of policies that are endogenously restricted by the information structure.

The rest of the paper proceeds as follows. The next section describes a lifecycle environment with active intensive and extensive labor margins. Section 3 makes precise the notions of distortions and of the tax and pension system in our environment. Section 4 provides analytic characterization of the baseline model, including efficient retirement age patterns. Section 5 theoretically examines policies that implement those patterns. Section 6 provides quantitative analysis based on the individual level U.S. data and intensive and extensive elasticities. Section 7 discusses extensions and generalizations. Section 8 concludes.

⁶For a review as well as international evidence of the effects of labor taxes see Rogerson (2010).

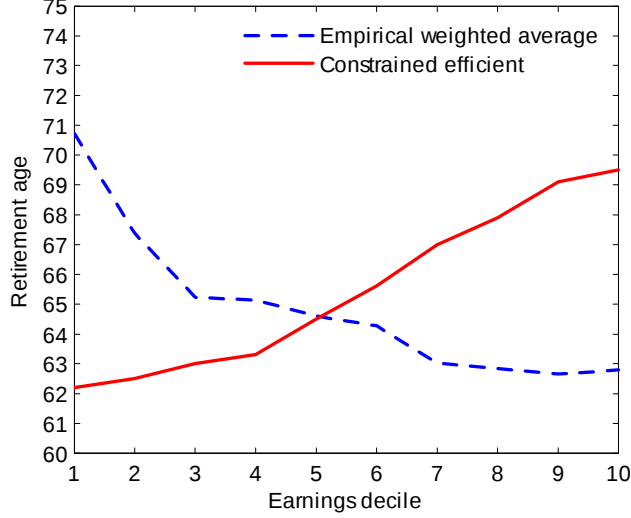


Figure 1: Empirical weighted average and simulated efficient retirement ages for the U.S., by lifetime earnings decile. Sources: HRS, PSID, and authors' calculations.

2 Environment

This section builds a lifecycle environment where intensive and extensive margins of labor supply are emphasized in the individual decisions about how much to work and when to retire. We use a baseline version of this environment in Section 4 to analytically study the tension between efficiency and equity and examine optimal allocations of consumption and output as well as the efficient distribution of retirement ages. In Section 5, we study how the interaction between the tax policy and the pension system should be designed to implement such distribution. We make precise what we mean by the interaction between the tax policy and the pension system in Section 3.

Time is continuous and runs from $t = 0$ to $t = 1$. The economy is populated by a continuum of individuals who are born at $t = 0$ and live until $t = 1$, at which point they all die. At any point in lifecycle, each individual chooses whether or not to work, and if so how much, and consumes a single consumption good and leisure. Individuals differ in two respects. First, individual workers are heterogeneous in their productivities. A worker's productivity changes over lifecycle and follows a privately known idiosyncratic hump-shaped productivity profile. Second, individuals face privately known heterogeneous fixed costs of work. Specifically, at time $t = 0$, each individual draws a type, θ , from a distribution of types, $F(\theta)$, where $F'(\theta) = f(\theta) > 0$ for all θ . Individual's type, θ , determines their productivity over lifecycle as well as preferences toward working, i.e., fixed utility cost the individual faces whenever they work. Individual lifetime income is

one way to interpret θ .

An individual's type θ determines this individual's productivity profile $\{\varphi(t, \theta)\}_{t \in [0,1]}$ over lifecycle. That is, when the individual works l hours at age t , then her income is $\varphi(t, \theta)l$. The productivity profile for the individual has the following two properties. First, the productivity profile, $\varphi(t, \theta)$, is continuous and twice continuously differentiable. Second, the productivity follows a hump-shape over the lifecycle, i.e. the profile exhibits an inverse U-shape. In other words, for any type θ , there exists an age t^* such that for all $t < t^*$, $\varphi_t(t, \theta) > 0$ and for all $t > t^*$, $\varphi_t(t, \theta) < 0$.

The latter property warrants a comment. The fact that wages or earnings are inverse U-shaped is a classic results in labor economics known at least since [Mincer \(1974\)](#). Without taking a stand on why this is the case, we take this stylized fact as given and study its implications for the efficient distribution of retirement ages and how the interaction between the tax policy and the pension system should be designed to implement such distribution.

In addition to productivity, the type θ determines individual preferences. In particular, a household that draws a type θ , has the following preferences

$$\int_0^1 e^{-\rho t} \left[u(c(t)) - v\left(\frac{y(t)}{\varphi(t, \theta)}\right) - \eta(\theta) \mathbf{1}[y(t) > 0] \right] dt \quad (1)$$

over the set of all allocations $\{c(t), y(t)\}_{t \in [0,1]}$ of consumption and income. Here, $\mathbf{1}[y(t) > 0]$ is an indicator function of positive output, $y(t)$. The utility function $u(\cdot)$ is strictly concave, increasing and satisfies standard Inada conditions. To facilitate intuition, we will consider linear $u(\cdot)$ cases. Moreover, $v(\cdot)$ is a strictly convex function with $v'(0) = 0$. These preferences exhibit fixed costs of working. This fixed utility cost of work can represent commute time, fixed costs of setting up jobs, etc.⁷ While we do not take a stand on the particular interpretation of this parameter, when we turn to a quantitative analysis of calibrated policy models in Section 6, we calibrate the fixed cost function $\eta(\theta)$ to match the observed patterns of retirement in the individual U.S. data. Intuitively, fixed utility cost of work introduces non-convexity into disutility of working. Combined with a hump-shaped productivity profile, this makes it optimal for a worker to choose to retire at some age while heterogeneity implies that retirement ages differ among workers. In other words, this environment features both active intensive and extensive margins in the form of decisions about how much to work and when to retire.

Given individual preferences and productivities, we define feasible allocations. An

⁷Note that alternatively, one can assume that firms have to pay fixed costs of setting up jobs and hence the fixed costs are in terms of consumption goods (see, e.g., [Rogerson and Wallenius \(2008\)](#)) Our formulation significantly simplifies the analysis.

allocation is defined as $\left(\{c(t, \theta), y(t, \theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}]}, K_t \right)_{t \in [0, 1]}$ where K_t is the aggregate asset holdings of all households. An Allocation is said to be feasible if

$$\int_{\underline{\theta}}^{\bar{\theta}} c(t, \theta) dF(\theta) + \dot{K}_t + H_t \leq \int_{\underline{\theta}}^{\bar{\theta}} y(t, \theta) dF(\theta) + rK_t$$

given K_0 where G_t is government expenditure. Throughout the paper, we will use the above budget constraint and its present value equivalent interchangeably:

$$\int_0^1 e^{-rt} \left[\int_{\underline{\theta}}^{\bar{\theta}} c(t, \theta) dF(\theta) \right] dt + H \leq \int_0^1 e^{-rt} \left[\int_{\underline{\theta}}^{\bar{\theta}} y(t, \theta) dF(\theta) \right] dt + (1 + r) K_0$$

where H is the time zero present value of government spending, i.e., $\int_0^1 e^{-rt} H_t dt$. A change in the order of the integrals leads to the following:

$$\int_{\underline{\theta}}^{\bar{\theta}} \int_0^1 e^{-rt} c(t, \theta) dt dF(\theta) + H \leq \int_{\underline{\theta}}^{\bar{\theta}} \int_0^1 e^{-rt} y(t, \theta) dt dF(\theta) + (1 + r) K_0$$

It can be shown that there exists a retirement age for each type, i.e., an age below which individuals work and above which they do not. Specifically, for each type θ , there exists $R(\theta)$ such that $y(t, \theta) > 0$ if and only if $t < R(\theta)$. We assume that this is the case from here on and provide a proof in the Appendix. When this is the case, an allocation is given by $\left(\{c(t, \theta)\}_{t \in [0, 1], \theta \in [\underline{\theta}, \bar{\theta}]}, \{R(\theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}]}, \{y(t, \theta)\}_{t \in [0, R(\theta)], \theta \in [\underline{\theta}, \bar{\theta}]}, \{K_t\}_{t \in [0, 1]} \right)$. Then, the above feasibility constraint can be written as

$$\int_{\underline{\theta}}^{\bar{\theta}} \int_0^1 e^{-rt} c(t, \theta) dt dF(\theta) \leq \int_{\underline{\theta}}^{\bar{\theta}} \int_0^{R(\theta)} e^{-rt} y(t, \theta) dt dF(\theta) + (1 + r) K_0 \quad (2)$$

Throughout the paper, we assume that θ is privately observed by the individuals and not the planner or the government. By appealing to the revelation principle we focus on direct mechanisms and emphasize incentive compatibility. An allocation is said to be *incentive compatible* if it satisfies the following condition for all $\theta, \hat{\theta}$:

$$\begin{aligned} \int_0^1 e^{-\rho t} u(c(t, \theta)) dt - \int_0^{R(\theta)} e^{-\rho t} \left[v \left(\frac{y(t, \theta)}{\varphi(t, \theta)} \right) + \eta(\theta) \right] dt \geq \\ \int_0^1 e^{-\rho t} u(c(t, \hat{\theta})) dt - \int_0^{R(\hat{\theta})} e^{-\rho t} \left[v \left(\frac{y(t, \hat{\theta})}{\varphi(t, \theta)} \right) + \eta(\theta) \right] dt \end{aligned} \quad (3)$$

We assume that the government desires to achieve some degree of redistribution and

to provide correct incentive for optimal working and retirement. That is, the planner has the following social welfare function

$$\int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dG(\theta) \quad (4)$$

where $U(\theta)$ is the life-time utility of a household of type θ given by expression (1). The function $G(\theta)$ is a cumulative density function, i.e., $G(\underline{\theta}) = 0$, $G(\bar{\theta}) = 1$, and $G'(\theta) = g(\theta) \geq 0$ and $G(\theta)$ is differentiable over interval $(\underline{\theta}, \bar{\theta}]$ ⁸. A redistributive motive for the planner implies that $G(\theta) \geq F(\theta)$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. The case with $F(\theta) = G(\theta)$ corresponds to the utilitarian planner, while the case with $G(\theta) = 1$, for all $\theta > \underline{\theta}$ corresponds to the Rawlsian social welfare function.

In this environment, an allocation is efficient if it maximizes social welfare function (4) subject to satisfying incentive compatibility (3) and feasibility (2). It will be later convenient to restate the mechanism design problem as

$$\max_{\left(\{c(t, \theta)\}_{t \in [0, 1], \theta \in [\underline{\theta}, \bar{\theta}]}, \{R(\theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}]}, \{y(t, \theta)\}_{t \in [0, R(\theta)], \theta \in [\underline{\theta}, \bar{\theta}]}\right)} \int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dG(\theta) \quad (5)$$

subject to incentive compatibility

$$\begin{aligned} \int_0^1 e^{-\rho t} u(c(t, \theta)) dt - \int_0^{R(\theta)} e^{-\rho t} \left[v\left(\frac{y(t, \theta)}{\varphi(t, \theta)}\right) + \eta(\theta) \right] dt \geq \\ \int_0^1 e^{-\rho t} u(c(t, \hat{\theta})) dt - \int_0^{R(\hat{\theta})} e^{-\rho t} \left[v\left(\frac{y(t, \hat{\theta})}{\varphi(t, \theta)}\right) + \eta(\theta) \right] dt \end{aligned}$$

and feasibility

$$\int_{\underline{\theta}}^{\bar{\theta}} \int_0^1 e^{-rt} c(t, \theta) dt dF(\theta) + H \leq \int_{\underline{\theta}}^{\bar{\theta}} \int_0^{R(\theta)} e^{-rt} y(t, \theta) dt dF(\theta) + (1 + r) K_0.$$

3 Distortions and policies

It is now useful to make precise the types of policies we will focus on. Using these policies, we define here the main margins that a policy choice distorts as well as the extent of these distortions. Then, in Section 4, we analytically characterize constrained efficient allocations in the environment described above and, in Section 5, we examine its policy

⁸In order to allow for extremes of redistribution, i.e., Rawlsian preferences, we restrict the differentiability to the open interval.

implications.

Consider a working individual as described above that pays age dependent income tax $T(t, y)$ at age t on income y . Upon retiring, the individual is entitled to the present value of lifetime pension benefits that depends on her retirement age as well as are a function of her income profile over working life, $b\left(R, Y\left(\{y(t)\}_{t=0}^R\right)\right)$. Here, $Y(\cdot)$ can be thought of as a measure of lifetime income or lifetime labor earnings. Facing this tax and pension benefit schedule, the individual solves the following problem:

$$\max_{c(t), R, y(t), a(t)} \int_0^1 e^{-\rho t} u(c(t)) dt - \int_0^R e^{-\rho t} \left[v\left(\frac{y(t)}{\varphi(t, \theta)}\right) + \eta(\theta) \right] dt$$

subject to

$$c(t) + \dot{a}(t) = (y(t) - T(t, y(t))) \mathbf{1}[t \leq R] + \mathbf{1}[t > R] \frac{rb\left(R, Y\left(\{y(t)\}_{t=0}^R\right)\right)}{e^{-rR} - e^{-r}} + ra(t)$$

where in the above formulation $\frac{rb\left(R, Y\left(\{y(t)\}_{t=0}^R\right)\right)}{e^{-rR} - e^{-r}}$ is the level of pension benefit that this individual receives at any point in time from age R to 1 and that will generate the present value of $b\left(R, Y\left(\{y(t)\}_{t=0}^R\right)\right)$; $a(t)$ is the level of asset holdings by the individual at date t .

Note that the above system of taxes and pension benefits resembles several features of the U.S. tax and social security system. In particular, the present value of pension benefits is a function of a measure of lifetime income analogous to the way social security benefits change with average indexed monthly earnings (AIME) in the U.S. Old Age, Survivors, and Disability Insurance program.⁹ However, the above system is significantly different from the U.S. tax code and social security system in other ways. In particular, contrary to the U.S. social security benefits formula, the present value of benefits in the system above potentially changes directly with the retirement age and the labor income taxes depend on age.

One can rewrite the date-by-date budget constraints above as the following present value budget constraint:

$$\int_0^1 e^{-rt} c(t) dt = \int_0^R e^{-rt} (y(t) - T(t, y(t))) dt + b\left(R, Y\left(\{y(t)\}_{t=0}^R\right)\right)$$

Using this budget constraint, the individual optimal choice of work and retirement im-

⁹ AIME is an inflation adjusted average of monthly earnings over the highest 35 years of earnings.

plies, slightly abusing notation, the following two optimality conditions:

$$\left[1 - T_y(t, y(t)) + e^{rt} \delta_{y(t)} Y \left(\{y(t)\}_{t=0}^R \right) b_Y \right] u'(c(t)) = v' \left(\frac{y(t)}{\varphi(t, \theta)} \right) \frac{1}{\varphi(t, \theta)} \quad (6)$$

$$\left[y(R) - T(R, y(R)) + e^{rR} b_R + e^{rR} b_Y \delta_R Y \left(\{y(t)\}_{t=0}^R \right) \right] u'(c(R)) = v \left(\frac{y(R)}{\varphi(R, \theta)} \right) + \eta(\theta), \quad (7)$$

where $\delta_{y(t)} Y$ is the Fréchet derivative of Y with respect to $y(t)$ and $\delta_R Y$ is the Fréchet derivative of Y with respect to R . Note that the above equations describe how and to what extents the intensive and extensive margins are distorted. To see this, notice that, in particular, in an undistorted allocation these conditions become:

$$u'(c(t)) = v' \left(\frac{y(t)}{\varphi(t, \theta)} \right) \frac{1}{\varphi(t, \theta)} \quad (8)$$

$$y(R) u'(c(R)) = v \left(\frac{y(R)}{\varphi(R, \theta)} \right) + \eta(\theta) \quad (9)$$

Given the above equations, we define in a natural way the extent to which each labor margin, intensive and extensive, is distorted. For any allocation, the *labor distortion*, $\tau_l(t, \theta)$, (also sometime referred to as labor wedge) is given by

$$(1 - \tau_l(t, \theta)) u'(c(t, \theta)) = v' \left(\frac{y(t, \theta)}{\varphi(t, \theta)} \right) \frac{1}{\varphi(t, \theta)}$$

That is, the labor wedge is a measure of how the intensive margin of labor supply is distorted. Analogously, in a natural way, we let *retirement distortion*, $\tau_r(\theta)$, (also referred to as retirement wedge) be defined by

$$(1 - \tau_r(\theta)) y(R(\theta), \theta) u'(c(R(\theta), \theta)) = v \left(\frac{y(R(\theta), \theta)}{\varphi(R(\theta), \theta)} \right) + \eta(\theta)$$

In other words, the retirement wedge measures how distorted the extensive margin of labor supply is, or how distorted the retirement decision is.

Given the above definitions of wedges and the notion of a system of taxes and pension benefits, we can relate the distortions of the intensive and extensive labor supply margins to the policy instruments by using optimality conditions (6) and (7):

$$\begin{aligned} \tau_l(t, \theta) &= T_y(t, y(t, \theta)) - e^{rt} \delta_{y(t)} Y \left(\{y(t, \theta)\}_{t=0}^R \right) b_Y \\ \tau_r(\theta) &= \frac{T(R(\theta), y(R(\theta), \theta))}{y(R(\theta), \theta)} - e^{rR(\theta)} \frac{b_R + \delta_R Y \cdot b_Y}{y(R(\theta), \theta)} \end{aligned}$$

This implies that characterizing the properties of the distortions of both the intensive and the extensive margins of labor supply can inform us about the properties of the system of policy instruments that we focus on. We show in Section 5 that a system of policy instruments we presented here can implement constrained efficient allocations. Before we do that, we turn in the next section to the characterization of a baseline form of our environment.

4 Characterization of efficient retirement

We now analytically study the tension between efficiency and equity in a baseline version of our lifecycle environment. Our analysis emphasizes active intensive and extensive margins of labor supply that represent themselves as the individual decisions to work and retire. In this section, we focus on a theoretical examination of the efficient distribution of retirement ages. In the next section, we examine how the interaction between the tax policy and the pension system described above should be designed to implement efficient retirement ages.

To provide sharper focus on the main underlying mechanisms and to build intuition, in the baseline formulation we start by temporarily abstracting from risk aversion. We also abstract for now from time discounting¹⁰. This allows us to derive closed form characterizations in this section. In Section 6 we reintroduce the curvature back into the utility function together with discounting and positive interest rates.

4.1 Retirement ages in a baseline model

We start by showing here that, under plausible conditions, efficient retirement ages increase in productivity. In other words, individuals with higher lifetime earnings should be given incentives to retire older than their less productive peers. We argue that distorting individual retirement decisions provides a novel and surprisingly powerful policy instrument. In particular, one important role of the retirement distortion is to undo part of the retirement incentives provided by a standard distortion of the consumption-labor margin, i.e., a labor income tax.

To build intuition, we first focus on a baseline case with linear $u(c)$ and no time discounting, i.e., $\rho = r = 0$. Assume that Frisch (intensive) elasticity of labor supply is constant, in particular $v(l) = \psi l^\gamma / \gamma$ with $\gamma > 1$. We provide closed form characterizations, in particular, of how the retirement age changes with type, or with lifetime income.

¹⁰We also abstract from government spending since with risk neutral households, it does not change our results about wedges.

Some of these results carry over in a straightforward way to the general case.

Assume in addition that the productivity profiles have the following property.

Assumption 1. *The productivity profile $\varphi(t, \theta)$ satisfies $\varphi_{t\theta}(t, \theta) \geq 0$.*

The above assumption about the productivity profiles ensures that in our mechanism design problem, individuals optimally retire at a certain age and do not re-enter the labor force. Multiple studies estimate heterogeneous productivity profiles over lifecycle and find similar patterns or at least patterns that do not deviate much from the property described in Assumption 1 (see, e.g., [Altig et al. \(2001\)](#) and [Nishiyama and Smetters \(2007\)](#)). In particular, higher earning individuals tend to have steeper growth in early ages and less steep decline in later years of their lives. We return to these properties in more detail in our quantitative analysis in Section 6.

Under these assumptions, individuals are indifferent between the timing of their consumption. Hence, we assume that consumption is constant over their lifecycle. Moreover, throughout this section, we will assume that providing incentives against local deviations is enough, i.e., we use the first-order approach. In the Appendix, we provide conditions on fundamentals so that this approach is valid. Under this assumption, the above incentive constraint becomes

$$U'(\theta) = \int_0^{R(\theta)} \psi \frac{\varphi_\theta(t, \theta)}{\varphi(t, \theta)} \frac{y(t, \theta)^\gamma}{\varphi(t, \theta)^\gamma} dt - \eta'(\theta) R(\theta) \quad (10)$$

Hence, the planner's problem can be rewritten as

$$\max_{\left(\{c(t, \theta)\}_{t \in [0, 1], \theta \in [\underline{\theta}, \bar{\theta}]}, \{R(\theta)\}_{\theta \in [\underline{\theta}, \bar{\theta}]}, \{y(t, \theta)\}_{t \in [0, R(\theta)], \theta \in [\underline{\theta}, \bar{\theta}]}\right)} \int_{\underline{\theta}}^{\bar{\theta}} U(\theta) dG(\theta) \quad (11)$$

subject to

$$\begin{aligned} c(\theta) - \int_0^{R(\theta)} \left[\psi \frac{1}{\gamma} \left(\frac{y(t, \theta)}{\varphi(t, \theta)} \right)^\gamma + \eta(\theta) \right] dt &= U(\theta) \\ \int_{\underline{\theta}}^{\bar{\theta}} c(\theta) dF(\theta) &\leq \int_{\underline{\theta}}^{\bar{\theta}} \int_0^{R(\theta)} y(t, \theta) dt dF(\theta) \\ U'(\theta) &= \int_0^{R(\theta)} \psi \frac{\varphi_\theta(t, \theta)}{\varphi(t, \theta)} \frac{y(t, \theta)^\gamma}{\varphi(t, \theta)^\gamma} dt - \eta'(\theta) R(\theta) \end{aligned}$$

As we noted above, the risk neutrality assumption significantly helps here in building intuition and highlighting the main economic mechanisms. In particular, we can fully characterize retirement age under some conditions. We start by fully characterizing

income, $y(t, \theta)$, by each individual at every age, and labor wedge, $\tau_l(t, \theta)$, in the following lemma.

Lemma 2. *The solution to planner's problem (11) satisfies the following two properties:*

1. *Working age income is given by*

$$y(t, \theta) = \psi^{\frac{1}{1-\gamma}} \left[1 + \gamma \frac{G(\theta) - F(\theta)}{f(\theta)} \frac{\varphi_\theta(t, \theta)}{\varphi(t, \theta)} \right]^{\frac{1}{1-\gamma}} \varphi(t, \theta)^{\frac{\gamma}{\gamma-1}}, \quad (12)$$

for type θ at all ages $t \leq R(\theta)$.

2. *The distortion to the intensive labor margin is given by the following wedge:*

$$\tau_l(t, \theta) = 1 - \frac{1}{1 + \gamma \frac{G(\theta) - F(\theta)}{f(\theta)} \frac{\varphi_\theta(t, \theta)}{\varphi(t, \theta)}}. \quad (13)$$

Proof. In the Appendix. □

The labor wedge in the above lemma is reminiscent of the formula derived by the empirically-driven literature that connects intensive labor distortions to productivity distributions, redistributive motives, and labor elasticities, as do, e.g., [Diamond \(1998\)](#), [Saez \(2001\)](#), and in dynamic environments [Golosov, Troshkin, and Tsyvinski \(2011\)](#). The difference here is that we provide elasticity-based expressions for labor distortions in the presence of both active margins of labor supply. In particular, to make this obvious one can rewrite the labor wedge formula (13) as

$$\frac{\tau_l(t, \theta)}{1 - \tau_l(t, \theta)} = \gamma \frac{G(\theta) - F(\theta)}{f(\theta)} \frac{\varphi_\theta(t, \theta)}{\varphi(t, \theta)}.$$

The formula illustrates that labor wedges are driven by the following forces: the intensive elasticity of labor supply, $1/(\gamma - 1)$, the distribution of productivities, $F(\theta)$, redistributive motives imbedded in Pareto weights, $G(\theta)$, and by the relative changes in productivity profiles over lifecycle, $\varphi_\theta(t, \theta)/\varphi(t, \theta)$. The first three forces are standard to the public finance literature and appear even when only the intensive margin is active. In particular, the higher the degree of redistribution, the higher the labor distortion. Analogously, the less elastic a particular type or the lower the measure of the workers of that type, the higher the labor distortion. An additional insight here is that the formula shows how labor wedges should evolve over lifecycle. In particular, when agents are past their highest productivity level, their labor distortion should increase with age, since the

additional force provided by the relative changes in productivity profiles over lifecycle, $\varphi_\theta(t, \theta) / \varphi(t, \theta)$ is increasing in t .

One variable of interest for our analysis in this environment is retirement age, $R(\theta)$. In particular, we are interested in how it changes with θ , or life-time earnings. In the following lemma, we provide a formula that characterizes retirement age. We then study examples of productivity profiles and their implications for efficient retirement age patterns.

Lemma 3. *The retirement age, $R(\theta)$, satisfies the following equation:*

$$\frac{\gamma - 1}{\gamma} y(R(\theta), \theta) = \eta(\theta) - \frac{G(\theta) - F(\theta)}{f(\theta)} \eta'(\theta) \quad (14)$$

where $y(t, \theta)$ is given by (12).

Proof. In the Appendix. □

Since $y(t, \theta)$ is known, the above formula pins down the retirement age. Moreover, it helps us characterize whether $R(\theta)$ is increasing in θ or not. In particular, suppose that $y(t, \theta)$ is increasing in θ and that $\eta(\theta)$ is constant and independent of θ . Then, we can see that retirement age must be increasing in θ . This is because $y(t, \theta)$ is decreasing in t and $y(t, \theta)$ is increasing in θ . Hence an increase in θ must be accommodated by an increase in $R(\theta)$. This would hold when the right hand side (14) is decreasing in θ . We summarize this discussion in the following proposition.

Proposition 4. *Suppose that $y(t, \theta)$ in (12) is weakly increasing in θ and that $\eta(\theta) - \frac{G(\theta) - F(\theta)}{f(\theta)} \eta'(\theta)$ is weakly decreasing in θ . Then the retirement age $R(\theta)$ is increasing in θ . In particular, when $\eta(\theta)$ is constant, $R(\theta)$ is increasing in θ .*

Proof. In the Appendix. □

To see the intuition for this result, notice that in an economy without information frictions, i.e., in full information model, economic efficiency implies that more productive individuals should retire at a later age provided that productivity profiles are increasing and fixed cost of work is weakly decreasing in lifetime productivity. With private information, this is not necessarily true. In order to provide incentives for truthful revelation of types, a planner may want to have more productive households retire earlier, i.e., by giving them higher utility through shorter working life. However, using an analogy with Myerson's virtual types, it can be shown that the economy with private information is equivalent to a full information economy with modified types, i.e., an economy with *virtual types*, where types are adjusted by their informational rents. Now if the virtual

types are so that fixed cost of working is weakly decreasing, then by the same efficiency argument, retirement age should be increasing in productivity.

In the model discussed above, virtual fixed cost of work for an agent of type θ is given by $\eta(\theta) - \frac{G(\theta) - F(\theta)}{f(\theta)} \eta'(\theta)$. To provide a partial intuition for this, consider a small increase – of size ε – in retirement age for agents of type θ . Virtual fixed cost is the effective utility cost of such a change¹¹. Note that this increase requires that the planner changes the utility of all the agents above θ , since it changes the right hand side of the incentive constraint (10) by $-\eta'(\theta) \varepsilon$. The planner can do so by increasing consumption for all types above θ by $-\eta'(\theta) \varepsilon$. Hence, the total cost of such a change is given by $\eta'(\theta) \varepsilon [1 - G(\theta) - (1 - F(\theta))] = -\eta'(\theta) \varepsilon [G(\theta) - F(\theta)]$. Therefore, the fixed cost of increasing R per unit of worker of type θ is given by $\eta(\theta) - \frac{G(\theta) - F(\theta)}{f(\theta)} \eta'(\theta)$.

The following example, provides more insight into the implications of the above proposition. That is, an example where we provide sufficient conditions on fundamentals under which $R(\theta)$ is increasing θ . Suppose that $G(\theta) = F(\theta)^\alpha$ with $0 < \alpha < 1$ and that $\varphi(t, \theta) = \theta \hat{\varphi}(t)$, i.e., parallel productivity profiles. Then, we must have

$$y(t, \theta) = \psi^{\frac{1}{1-\gamma}} \left[1 + \gamma \frac{F(\theta)^\alpha - F(\theta)}{\theta f(\theta)} \right]^{\frac{1}{1-\gamma}} (\theta \hat{\varphi}(t))^{\frac{\gamma}{\gamma-1}}$$

as well as

$$\psi^{\frac{1}{1-\gamma}} \left[1 + \gamma \frac{F(\theta)^\alpha - F(\theta)}{\theta f(\theta)} \right]^{\frac{1}{1-\gamma}} (\theta \hat{\varphi}(R(\theta)))^{\frac{\gamma}{\gamma-1}} = \frac{\gamma}{\gamma-1} \left[\eta(\theta) - \frac{F(\theta)^\alpha - F(\theta)}{f(\theta)} \eta'(\theta) \right]$$

and therefore, $R(\theta)$ is increasing in θ whenever the following conditions are satisfied:

$$\frac{d}{d\theta} \frac{F(\theta)^\alpha - F(\theta)}{\theta f(\theta)} < 0 \quad (15)$$

$$\frac{d}{d\theta} \left[\eta(\theta) - \frac{F(\theta)^\alpha - F(\theta)}{f(\theta)} \eta'(\theta) \right] < 0 \quad (16)$$

The following conditions imply that in the economy with virtual types: 1) virtual productivity profiles are increasing in type, condition (15); 2) virtual fixed cost of work are decreasing in type, condition (16). Furthermore, it establishes that there are two key determinants of the relationship between retirement age and lifetime earnings, or between R and θ . First, how $y(t, \theta)$ moves with θ and how (Myerson-like) virtual fixed cost of work $\eta(\theta) - \frac{G(\theta) - F(\theta)}{f(\theta)} \eta'(\theta)$ depends on θ .

¹¹Although this has an effect on total disutility from hours, we ignore that since we are interested in fixed cost of work.

4.2 Labor and retirement distortions

Next, we characterize efficient labor distortions and retirement distortions. In particular, we show how retirement and labor wedges are related to each other. The relationship between retirement wedge and labor wedge helps us in characterizing the policy system that implements the efficient retirement age pattern. Our main theoretical result here is to show that the retirement distortion is smaller than the labor distortion. In the appendix, we show this result for the general environment as well.

Proposition 5. *Suppose that $\eta'(\theta) = 0$. Then the retirement wedge $\tau_r(\theta)$ is lower than the labor wedge at retirement age, $\tau_l(R(\theta), \theta)$.*

Proof. In the Appendix. □

While the Appendix contains the proof, the result above follows from the following formula:

$$\tau_r(\theta) y(R(\theta), \theta) = \frac{1}{\gamma} \tau_l(R(\theta), \theta) y(R(\theta), \theta) - \frac{G(\theta) - F(\theta)}{f(\theta)} \eta'(\theta) \quad (17)$$

The above formula ties labor wedge, retirement wedge and the incentive cost of increasing retirement. For instance, it is clear from 17 that when $\eta'(\theta) = 0$, retirement wedge is lower than labor wedge.

The intuition for this result can be provided by focusing on the incentive cost of a unit increase in income through an increase in retirement age as opposed to an increase in hours worked. Consider a unit increase in $y(R(\theta), \theta)$. In addition to the effect that this increase has on resources and the utility of the household of type θ , it has an effect on the incentive constraint. In particular, it increases by

$$\gamma \psi \frac{\varphi_\theta(R(\theta), \theta) y(R(\theta), \theta)^{\gamma-1}}{\varphi(R(\theta), \theta)^\gamma}$$

On the other hand, an increase of size $\frac{1}{y(R(\theta), \theta)}$ in $R(\theta)$ increases income by a unit¹² and increases the RHS of the incentive constraint by $\psi \frac{\varphi_\theta(R(\theta), \theta) y(R(\theta), \theta)^{\gamma-1}}{\varphi(R(\theta), \theta)^\gamma}$. That is the incentive cost of an increase in $R(\theta)$ is lower than the incentive cost of an increase in $y(R(\theta), \theta)$ of comparable size. Hence, the distortions to retirement margin should be lower than the distortions to the intensive margin.

¹²To provide better intuition we use a loose argument here. These perturbations should be interpreted as (1) a change in $y(t, \theta)$ by 1 unit in an interval $[R(\theta) - \varepsilon, R(\theta)]$ for small $\varepsilon > 0$, (2) an increase in $R(\theta)$ by $\frac{\varepsilon}{y(R(\theta), \theta)}$.

Equation (17) also implies that when $\eta'(\theta)$ is positive the same equation holds¹³. Moreover, when the slope of $\eta'(\theta)$ is negative and low enough, the retirement wedge is lower than labor wedge.

The above result is helpful in characterizing whether labor income taxes distort retirement decision downward or upward, i.e., whether labor taxes provide additional incentives to retire younger or older. In other words, it helps in showing whether pension benefits should be designed to reward later or earlier retirement above and beyond the labor income tax schedule. As we show in the next section, in plausible cases, the above result would imply that retirement should be rewarded by benefit that increases with age in an actuarially *unfair* way.

5 Actuarially *unfair* pension system

In this section, we analytically study the types of policies we introduced in Section 3. Our goal here is the design of a pension system as an integral part of the tax code to implement efficient allocations studied above. We show that pension benefits depend on the age of retirement and, moreover, that the pension system should be designed to be actuarially *unfair*.

To provide a complete implementation of the constrained optimal allocation, we start from the baseline case studied in the previous section. Here, we show that a tax schedule of the form $\{T(t, y), b(R)\}$ can implement the allocations discussed above, where $T(t, y)$ is the income tax schedule at age t and $b(R)$ is the present value benefits.

We start by constructing the tax and benefits schedule as follows: Consider any incentive compatible allocation $\left(\{y(t, \theta)\}_{t \leq R(\theta)}, R(\theta), c(\theta)\right)_{\theta \in [\underline{\theta}, \bar{\theta}]}$ with the properties that $y(t, \theta)$ and $R(\theta)$ are both increasing functions of θ for all t . Let $T(t, y)$ be defined as a function that satisfies

$$\theta = \arg \max_{\hat{\theta}} y(t, \hat{\theta}) - T(t, y(t, \hat{\theta})) - \frac{\psi y(t, \hat{\theta})^\gamma}{\gamma \varphi(t, \theta)^\gamma} \quad (18)$$

The following lemma shows that this tax function exists and is unique.

Lemma 6. *Suppose that $y(t, \theta)$ is an increasing function θ . Then there must exist a function $T(t, y)$ that satisfies (18). Moreover, $T(t, y)$ is uniquely determined over the interval $[\min_{\theta} y(t, \theta), y(t, \bar{\theta})]$ up to a constant.*

¹³In the appendix, we show that $\eta'(\theta) \leq 0$ is a sufficient condition for the first order approach to work. Hence, when $\eta(\theta)$ is increasing, one should make sure (numerically) that the first order approach is valid.

Proof. In the Appendix. □

The idea for the above lemma is very intuitive. The static incentive compatibility of the allocation $(y(t, \theta) - T(t, y(t, \theta)), y(t, \theta))$ determines the slope of the tax function $T(t, \cdot)$ with respect to y . Hence, $T(t, y)$ should be uniquely determined over the mentioned interval up to a constant.

Using the tax function constructed above, we define the benefits. We define the function $\hat{b}(\theta)$ as

$$\hat{b}(\theta) = c(\theta) - \int_0^{R(\theta)} [y(t, \theta) - T(t, y(t, \theta))] dt \quad (19)$$

Since $R(\theta)$ is an increasing function of θ , there must exist an increasing function $b(R)$ such that $b(R(\theta)) = \hat{b}(\theta)$. For all $R \neq R(\theta)$ for some θ , we set $b(R)$ equal to big negative number so that agents would not choose those retirement ages. The following proposition shows that facing this tax and pension system, the allocation $\{y(t, \theta)\}_{t \leq R(\theta)}, R(\theta), c(\theta)$ is a local optimal for a household of type θ . We relegate the complete proof of optimality to the Appendix.

Proposition 7. *Consider an incentive compatible allocation*

$$\left(\{y(t, \theta)\}_{t \leq R(\theta)}, R(\theta), c(\theta) \right)_{\theta \in [\underline{\theta}, \bar{\theta}]}$$

such that $y(t, \theta)$ and $R(\theta)$ are both increasing in θ . Moreover, suppose that $\eta'(\theta) \leq 0$. Then the tax function $T(t, y)$ and the benefit schedule $b(R)$ constructed in (18), and (19) locally implement this allocation.

Proof. Given the above tax schedule, a household of type θ 's optimization problem is given by

$$\max_{R, y(t)} \int_0^R [y(t) - T(t, y(t))] dt + b(R) - \int_0^R \left[\frac{\psi}{\gamma} \frac{y(t)^\gamma}{\varphi(t, \theta)^\gamma} + \eta(\theta) \right] dt$$

We prove this claim in two steps. First, note that if an agent of θ works at age t , he will work to produce an income of $y(t, \theta)$. This is because of definition of $T(t, y)$ in (18). Now, we show that given this, picking $R(\theta)$ is locally optimal. Suppose on the contrary that the household chooses $R(\hat{\theta}) \leq R(\theta)$, then given the definition of b , the utility for

the household is given by

$$\begin{aligned} & \int_0^{R(\hat{\theta})} [y(t, \theta) - T(t, y(t, \theta))] dt - \int_0^{R(\hat{\theta})} \left[\frac{\psi y(t, \theta)^\gamma}{\gamma \varphi(t, \theta)^\gamma} + \eta(\theta) \right] dt \\ & + c'(\hat{\theta}) - \int_0^{R(\hat{\theta})} \left[y(t, \hat{\theta}) - T(t, y(t, \hat{\theta})) \right] dt \end{aligned} \quad (20)$$

Taking a derivative with respect to $\hat{\theta}$, we have

$$\begin{aligned} & \left[y(R(\hat{\theta}), \theta) - T(R(\hat{\theta}), y(R(\hat{\theta}), \theta)) - \frac{\psi y(R(\hat{\theta}), \theta)^\gamma}{\gamma \varphi(R(\hat{\theta}), \theta)^\gamma} - \eta(\theta) \right] R'(\hat{\theta}) \\ & + c'(\hat{\theta}) - \left[y(R(\hat{\theta}), \hat{\theta}) - T(R(\hat{\theta}), y(R(\hat{\theta}), \hat{\theta})) \right] R'(\hat{\theta}) \\ & - \int_0^{R(\hat{\theta})} \frac{\partial}{\partial \theta} y(t, \hat{\theta}) \left[1 - \frac{\partial}{\partial y} T(t, y(t, \hat{\theta})) \right] dt \end{aligned}$$

Evaluating the above expression when $\hat{\theta} = \theta$,

$$c'(\theta) - \left[\frac{\psi y(R(\theta), \theta)^\gamma}{\gamma \varphi(R(\theta), \theta)^\gamma} + \eta(\theta) \right] R'(\theta) - \int_0^{R(\theta)} \frac{\partial}{\partial \theta} y(t, \theta) \left[1 - \frac{\partial}{\partial y} T(t, y(t, \theta)) \right] dt$$

and by static incentive compatibility (18), the above expression becomes

$$c'(\theta) - \left[\frac{\psi y(R(\theta), \theta)^\gamma}{\gamma \varphi(R(\theta), \theta)^\gamma} + \eta(\theta) \right] R'(\theta) - \int_0^{R(\theta)} \psi \frac{y(t, \theta)^{\gamma-1}}{\varphi(t, \theta)^\gamma} \frac{\partial}{\partial \theta} y(t, \theta) dt$$

which is zero by incentive compatibility of the original allocation. This implies that $\hat{\theta} = \theta$ is a local extreme point of the function (20). In the appendix, we show that the second derivative of (20) at $\hat{\theta} = \theta$ is negative and hence $\hat{\theta} = \theta$ is the local maximizer of (20). Hence, the original allocation locally maximizes the utility of a household of type θ . $\square$

Intuitively, the proof of the above proposition shows that the local decision of changing retirement age coincides with the decision whether to lie about one's productivity type. Since the original allocation is incentive compatible, it is also optimal not to deviate and choose a different allocation of work and retirement ages.

6 Quantitative analysis

We now turn to the quantitative study of efficient work and retirement patterns. We use individual earnings and hours data in combination with individual retirement age data to calibrate variants of discrete time models in our general lifecycle environment described in Section 2. We calibrate to also match micro estimates of labor supply elasticity at the extensive margin. We simulate efficient work and retirement choices and policies that we analyze analytically above. To assess the importance of any potential differences between simulated efficient retirement patterns and the patterns in the data, we compute resulting welfare gains and total output gains.

6.1 Model parameters

For our quantitative study we consider discrete time version of the following functional form of $U(\theta)$:

$$\int_0^1 e^{-rt} \frac{c(t, \theta)^{1-\sigma} - 1}{1-\sigma} dt - \int_0^{R(\theta)} e^{-rt} \left[\frac{1}{\gamma} \left(\frac{y(t, \theta)}{\varphi(t, \theta)} \right)^\gamma + \eta(\theta) \right] dt$$

As a benchmark, we set $\sigma = 1$ so that we consider $\log(c(t, \theta))$ utility of consumption function. The intensive elasticity parameter, γ , is set to 3. This implies intensive Frisch elasticity of labor supply equal to $\alpha = 1/(\gamma - 1) = 0.5$, consistent with the evidence in [Chetty \(2011\)](#). We later study how robust the results are by also exploring Frisch elasticity of 0.3 and 3. We also explore risk aversion of 0.5 and 3, or alternatively intertemporal elasticity of substitution equal to 2 and 1/3. Individuals in our quantitative environment are born 25 years old, they experience changes in their productivities over discrete time, and they all expire at the same age of 85. Table 1 summarizes these parameter choices and the robustness ranges.

6.2 Empirical strategy

Our main sources of individual level data are individual earnings and hours data from the U.S. Panel Study of Income Dynamics (PSID) and individual retirement age data from the RAND files of the Health and Retirement Study (RAND HRS). Our general empirical approach is to treat individuals in the data as optimizing given the existing policies. That is, we observe individual decisions about how much to work and when to retire that are individually optimal given the existing income taxes and pension benefits those individuals faced when they made their decisions. In particular, we take individual retirement age decisions in the data as being individually optimal through the lens of our

Table 1: Parameters.

Parameter	Benchmark	Robustness	Notes
	Value	Range	
σ (relative risk aversion)	1	0.5, 1, 3	intertemporal elasticity of substitution: $1/\sigma$
α (intensive Frisch elasticity)	0.5	0.3, 0.5, 3	$\alpha = 1/(\gamma - 1)$
β (discount factor)	0.9804		discrete time
δ (marginal rate of transformation)	1.02		
age at $t = 0$	25		
age at $t = 1$	85		

environment, given the existing policies and the estimated productivity profiles. Taking that approach, we back out the unobservable fixed costs of work from the individual optimality conditions and calibrate to match the extensive elasticity estimates.

We start by estimating productivity profiles over lifecycle, $\varphi(t, \theta)$. For the benchmark quantitative case, we group individuals into ten equal sized groups (types) by their average annual labor earnings. Later, we use these types assigned to individual observations as proxies for lifetime earnings deciles. Our main data source for the productivity profiles is individual total labor earnings and total hours data from the PSID. We use the PSID data collection waves from 1990 onward to the latest currently available data wave of 2007 (containing data from 2006). The labor earnings are obtained directly from the PSID waves and are converted to constant 1990 dollars. We consider total labor earnings, which is a sum of a list of variables in the PSID that contain data on salaries and wages, separate bonuses, the labor portion of business income, overtime pay, tips, commissions, professional practice or trade payments, market gardening, additional job income, and other miscellaneous labor income. When using PSID waves, we treat heads of households and their spouses or long-term cohabitants as separate individuals. We restrict the sample to include only individuals with the total labor income of at least \$1,000 in 1990 dollars and with at least 250 total hours worked in a year resulting in a sample of 50,624 individuals total from all waves.

We follow a large part of the literature (see, e.g., [Nishiyama and Smetters \(2007\)](#) and [Altig et al. \(2001\)](#)) in using labor income per hour (computed hourly wage as a ratio of total labor earnings to total hours) as a proxy for working ability or productivity. We use this measure as a proxy for φ , which measures the return to effort.¹⁴

¹⁴In the future versions of the paper we will use constructed productivities that are directly implied by the data and the individual first-order conditions. The main challenge in that case is to correctly

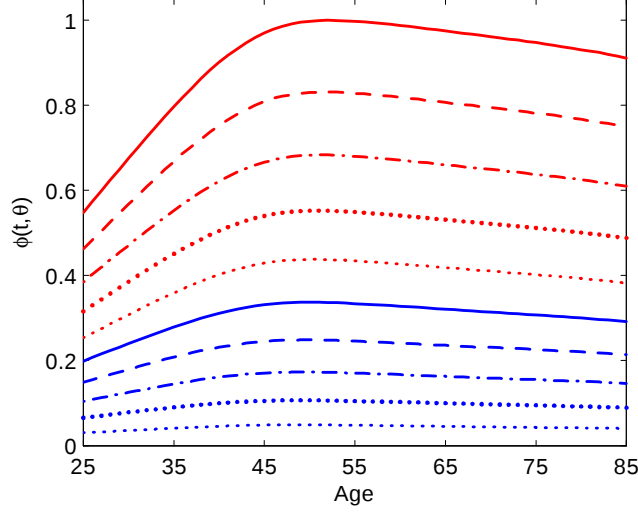


Figure 2: Estimated productivity profiles by type over lifecycle, $\varphi(t, \theta)$.

Table 2: Summary statistics.

Variable	Mean	Median	Std.Dev.	Min	Max
retirement age	64.53118	63	6.391109	47	94
average annual earnings	56,470.39	44,832.22	70,291.19	119.1287	1,994,460
Note: RAND HRS benchmark sample					
Number of observations: 2895					

We continue assuming that productivity profiles follow a potentially fanning out parametric form described in Section 2. In particular, we think of productivity profiles as given by

$$\varphi(t, \theta) = \theta \varphi(t) t^{\xi \theta}$$

To provide an interpretation of this functional form, we take logarithm of both sides to obtain

$$\log \varphi(t, \theta) = \log \theta + \log \varphi(t) + \xi \theta \log t$$

Here, the first term can be interpreted as lifetime earnings, while the second term represents an age component and the third term is an interaction term between age and lifetime earnings. Consequently, we regress log productivities on lifetime earnings, age, age squared, and interaction terms to estimate empirical productivity profiles. In this context, fanning out means that for high enough ages t , $\varphi(t)$ decreases faster than $t^{\xi \theta}$

account for private assets that appear in the individual optimality conditions when preferences are not without income effects.

increases. Figure 2 depicts the ten estimated productivity profiles, one for each lifetime earnings decile.¹⁵ Productivities of higher types are higher and generally increase faster for younger ages. The declines in productivities in later years of the lifecycle are not as pronounced, especially for higher lifetime earnings deciles.

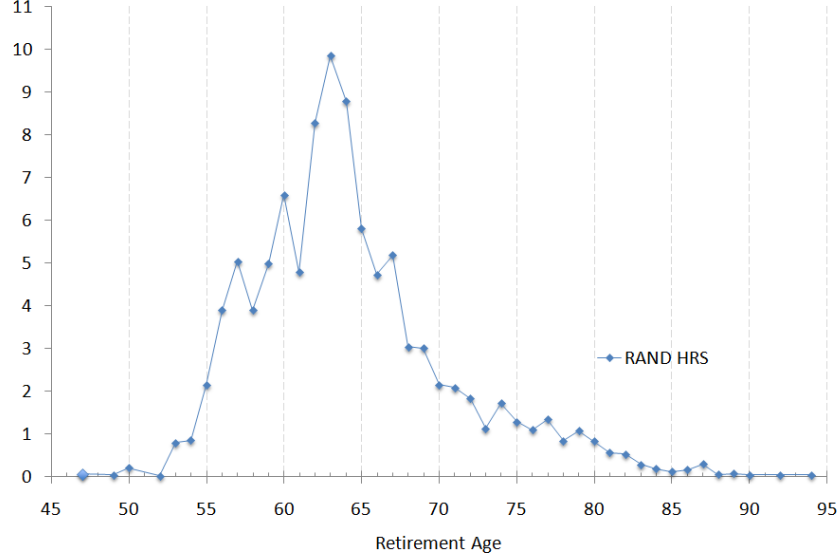


Figure 3: Unconditional distribution of retirement ages (as defined in the main text) in the HRS.

We also check our results for robustness by instead following closely the approach of [Nishiyama and Smetters \(2007\)](#) and grouping individual observations by type and place them into one of seven bins each for a ten year interval of ages - 25-35 years old, 34-45 years old, ..., 74-85 years old (the few remaining individuals older than 85 we put in the last group) - and extrapolate by using shape preserving cubic splines to obtain the productivity profiles. Another important check is to supplement our sample from the PSID with the individual observations from the HRS to increase the number of older age observations. The overall patterns of productivity profiles we find stay similar to the ones displayed in Figure 2.

The second important piece of empirical evidence is individual retirement ages provided by the HRS. In addition to labor earnings the HRS provides data on individual retirement decisions. Through the lens of our environment, retirement age means zero hours worked (excluding unemployment) from a given age onward. Figure 3 presents an unconditional distribution of retirement ages by this definition in our benchmark sample

¹⁵The overall shape of these profiles is similar to those obtained in the literature, see, e.g., [Altig et al. \(2001\)](#).



Figure 4: Retirement ages *vs.* average annual labor earnings.

of 2,895 males from all waves for whom we have full-time work observations before retirement observations. Table 2 provides summary statistics. We later expand the sample to check for robustness. We observe that the results are not changed dramatically when we take into account less than full-time work, allow for un-retirement or re-entry into the labor force, experiment with individual weights in the HRS, and include females in the sample.

As with the PSID observations above, we classify individuals by their average annual labor earnings deciles. A simple scatter plot reveals the relationship between the retirement age and the labor earnings presented in Figure 4. An alternative look at the retirement ages versus log earnings is displayed in Figure 5. Using our benchmark definitions of retirement and earnings, the relationship exhibits negative correlation coefficient of -0.158 and a regression coefficient -0.019 (see the regression line in Figure 4). To account for apparent heteroscedasticity, Table 3 also reports robust standard error of 0.00669 . This negative (or, at most, flat) relationship appears robust to various changes from how the retirement age is defined (e.g., allowing for coming out of retirement), to how the labor earnings are computed (e.g. considering only individuals who worked full time), and to including women in the sample. We connect this evidence with the productivity profiles by grouping individuals into deciles by labor earnings. This produces the pattern of retirement ages, for each of the ten types in our benchmark case, shown on the left panel of Figure 6.

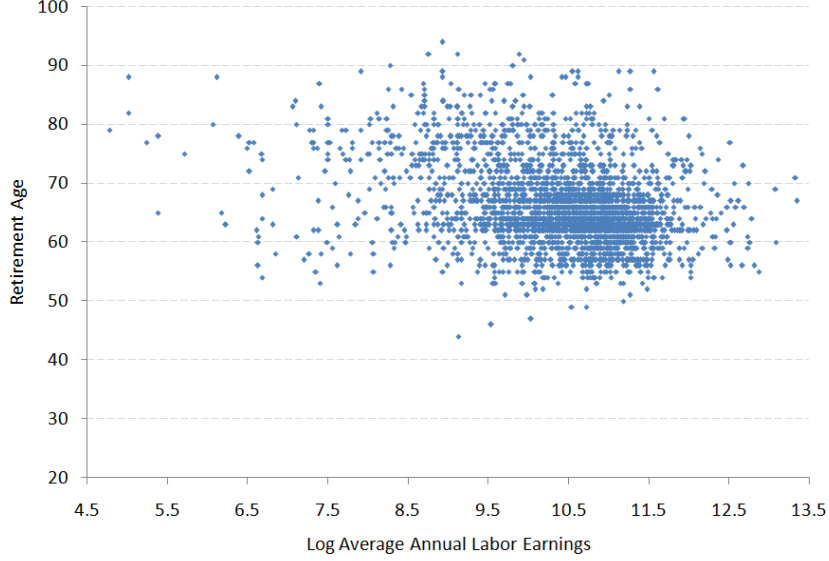


Figure 5: Retirement ages *vs.* logarithm of average annual labor earnings.

Table 3: Regression results.

Variable	Coefficient	Robust	
		Std.Err.	t-statistic
mean earnings / 1000	-.0188595**	.0066942	-2.82
constant	67.01545	.3436527	195.01

Note: ** indicates significance at the 1% level
Number of observations: 2895
F-statistic: 7.94
R-squared: 0.025

6.3 Calibration of the fixed costs

Given our preferences and the two pieces of empirical evidence above - a productivity profile and a retirement age for a given type - we calibrate the unobserved fixed cost of work to produce simulated extensive margin elasticity consistent with empirical estimates. To see the calibration procedure intuitively, notice that if, for simplicity, allocations that we observe in the data were undistorted, then individual optimality conditions (8) and (9) would pin down fixed costs of work for a given curvature in the utility of consumption. Specifically, for a given type θ at the time of retirement we have

$$y(R) - \varphi(R) \frac{v(y(R)/\varphi(R))}{v'(y(R)/\varphi(R))} = \frac{\eta}{u'(c)}, \quad (21)$$

which follows directly from combining conditions (8) and (9) discussed in Section 4.¹⁶ Since retirement optimality (21) only jointly identifies fixed cost of work and the curvature of utility of consumption (the right-hand side), we calibrate the pattern of fixed costs, $\eta(\theta)$, so that simulated extensive elasticity of labor supply falls in the range 0.13–0.43 of estimates from the individual studies analyzed in Chetty et al. (2011). To account for the distortions present in the allocations we observe in the data, for the actual calibration we use a version of individual optimality conditions (6) and (7) instead of the undistorted individual optimality conditions. We assume that in expressions (6) and (7) the existing labor income taxes are age independent and use TAXSIM¹⁷ to estimate effective tax rates. We use a stylized description of the core features of the U.S. Social Security to compute the derivatives of the pension benefits function that are present in conditions (6) and (7). In particular, if retirement age is less than 65, we let benefits decrease by 5/9 of 1% for each month up to 36 months before 65 and by 5/12 of 1% for each additional month down to age 62, as is done in the calculation of the Primary Insurance Amount (PIA) in the U.S. Social Security system. We also let the benefits increase if the retirement age is above 65 up to the age of 70. To compute the derivative of the benefits function with respect to a measure of lifetime earnings, we use the slopes of the PIA as a function of the AIME in the U.S. Social Security system. That is, 0.9 up to \$711 AIME, 0.32 up to \$4,288, and 0.15 after that.

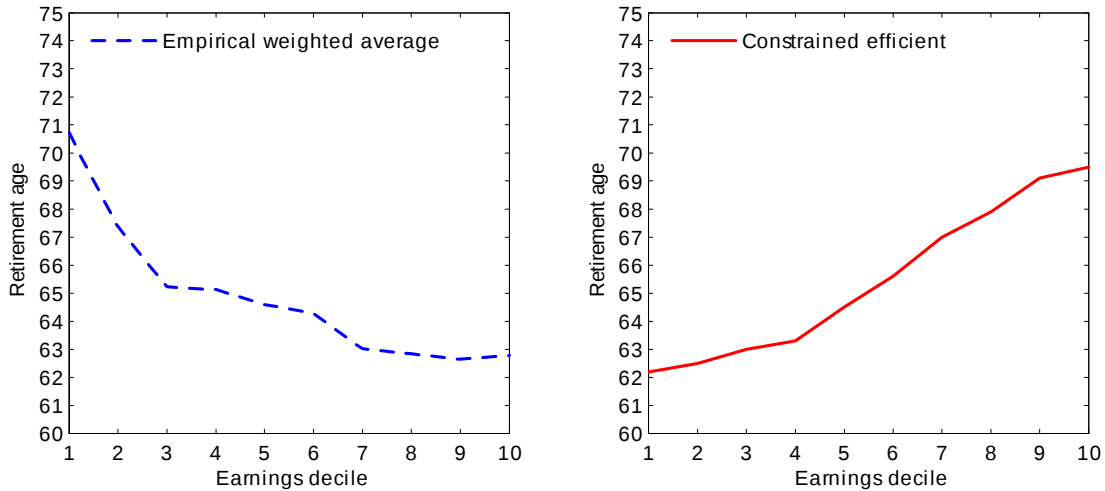


Figure 6: Empirical weighted average (left panel) and simulated efficient retirement ages (right panel) for the U.S., by lifetime earnings decile.

¹⁶In Appendix, we provide a graphical exposition of an intuitive example of this calibration.

¹⁷TAXSIM is a FORTRAN program of the National Bureau of Economic Research for estimating individual effective liabilities under U.S. Federal and State income tax laws from individual data. For more details and to use the program freely see <http://www.nber.org/~taxsim/>.

Table 4: Retirement ages for the U.S.

Earnings Decile	Retirement Age		
	Empirical	Simulated	
	Weighted Average	Efficient	Difference
<i>1st</i>	70.7	62.2	-8.5
<i>2nd</i>	67.4	62.5	-4.9
<i>3rd</i>	65.2	63.0	-2.2
<i>4th</i>	65.1	63.3	-1.8
<i>5th</i>	64.6	64.5	-0.1
<i>6th</i>	64.3	65.6	1.3
<i>7th</i>	63.0	67.0	4.0
<i>8th</i>	62.8	67.9	5.1
<i>9th</i>	62.7	69.1	6.4
<i>10th</i>	62.8	69.5	6.7

Note: Empirical ages are computed from RAND HRS.
Simulated ages are from the benchmark calibration.

6.4 Quantitative results

We use estimated productivity profiles, calibrated fixed costs, and the calibrated fixed costs to solve numerically the planning problem by direct optimization. We compute efficient allocations and analyze efficient retirement ages and their effects on welfare and total output.

The computed allocation that results in the benchmark case implies the retirement age pattern displayed on the right panel in Figure 6 (as well as summarized earlier in Figure 1). Figure 7 displays the labor distortions and retirement distortions associated with this allocation. The left panel displays labor distortions that are always positive, lower for low productive types, and generally increase through life. The right panel displays the difference between retirement distortion and labor distortion at the efficient age of retirement. The difference is everywhere negative implying that retirement distortion needs to undo part of the retirement incentives imbedded in the labor distortion, as discussed in Section 5.

Figure 6 displays a quantitative result largely robust across calibrations - individuals with higher lifetime earning in the U.S. should retire older than they do now (represented by the dashed line) and, importantly, older than less productive workers (represented by a positively sloped solid line). We find in our benchmark calibration that in the optimum, the highest productivity types retire at 69.5, whereas in the data their average retirement age is 62.8. Individuals with lower lifetime earnings retire younger than they do now, as well as younger than their more productive peers. In particular, the lowest

productivity types retire in the optimum at 62.2 years compared to 69.5 for the highest productivity types. This pattern of retirement ages is in sharp contrast with the one found in the current individual data for the U.S., where average retirement age displays a predominantly decreasing pattern as a function of lifetime earnings. Table 4 summarizes these differences for each earnings decile separately.

Our quantitative study also allows us to measure and decompose welfare gains and total output gains associated with inducing efficient retirement age distribution. We find that compared to the allocations with the existing system, providing efficient incentives for both work and retirement results in large welfare gains across calibrations of between 1 and 5 percent in annual consumption equivalent. Perhaps more notably, it also results in a small but positive change in total output across calibrations of up to 1 percent. The result about the increase in total output is in line with the analysis in Golosov and Tsyvinski (2006) as well as follows in spirit earlier contributions of Diamond and Mirrlees (1978) and even Diamond and Mirrlees (1986), although taking a more empirically driven approach. Note that at the same time, most modern studies of efficient redistributive policies largely result in increased distortions improving welfare but generally sacrificing total output (see, e.g., Fukushima (2010), Farhi and Werning (2010), Golosov, Troshkin, and Tsyvinski (2011), Weinzierl (2011)).

We also find that the increase in total output results from the meaningfully active intensive and extensive margins of labor supply. That is, even though increasing standard distortions of the intensive margin leads to output losses in favor of redistribution and welfare gains, the additional policy instrument of distorting the retirement decision proves powerful enough to overcompensate by inducing more productive individuals to work more years and thus produce more.

7 Productivity shocks over lifecycle

In this section we extend our analysis to investigate the importance of including idiosyncratic productivity shocks over lifecycle. This is in addition to the shocks at birth to the productivity profiles and fixed costs that we introduced in Section 2. That is, we extend the environment described in Section 2 to include shocks over time. We handle the additional complexity of the model by using a novel analytical approach to the recursive formulation of the planner's problem.

We now assume that labor productivity, θ_t , is stochastic over individual lifetime and has the following form: $a_t\theta_t$, where a_t is time dependent and non-stochastic with $a_0 = 1$.

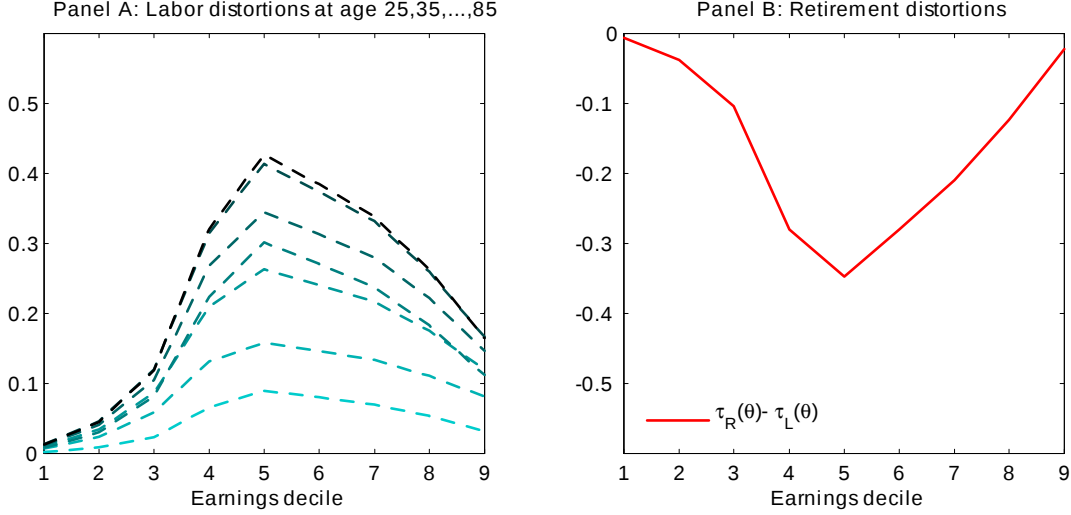


Figure 7: Labor distortions at ages 25, 35, 45, 55, 65, 75, and 85 (Panel A, generally higher distortions representing older ages) and retirement distortions (Panel B).

Moreover, θ_t is a stochastic process that satisfies

$$\frac{d\theta_t}{\theta_t} = \hat{\sigma}_t dW_t$$

where W_t is a standard Brownian motion. The instantaneous variance of innovations to productivity is potentially dependent on age. We let $\mathcal{F}_t^W$ be the filtration associated imposed by W_t . The above process for productivity is almost identical to the one estimated by [Storesletten, Telmer, and Yaron \(2004\)](#). The deterministic component of productivity, a_t , can be thought of as coming from a Mincer regression. That is productivity is assumed to be dependent on age through its dependence on education and experience. Note also that the environment studied above is a special case of the above environment; when $\hat{\sigma}_t = 0$ is set to zero for all $t \in [0, 1]$ and a_t is an inverse U-shape function of t . Furthermore, we assume that θ_0 is distributed according to a distribution $F(\theta_0)$ with density $f(\theta_0)$.

Given this environment, an allocation is a sequence of consumption and output $\{c_t, y_t\}_{t=0}^1$ that is progressively measurable with respect to W_t . The households have the following preferences over sequences of consumption and labor supply

$$E \int_0^1 e^{-\rho t} \left\{ u(c_t) - \frac{1}{1+\varepsilon} \left(\frac{y_t}{a_t \theta_t} \right)^{1+\frac{1}{\varepsilon}} - \eta \mathbf{1}[y_t > 0] \right\} dt$$

We assume that Frisch elasticity of labor supply is constant over household's life cycle.

In this environment, we represent the feasibility constraint by a present value resource constraint. The assumption is that the production technology is linear in capital and labor. If the gross rate of return on capital is given by $1 + r$, the aggregate feasibility constraint is given by

$$\int_{\underline{\theta}}^{\bar{\theta}} E \left[\int_0^1 e^{-rt} [c_t - y_t] dt \right] dF(\theta_0) \leq k_0$$

We assume that the planner maximizes a social welfare function with weights that depend on initial θ_0 . That is the planner maximizes the following objective function

$$\int_{\underline{\theta}}^{\bar{\theta}} \left\{ E \int_0^1 e^{-\rho t} \left[u(c_t) - \frac{1}{1 + 1/\varepsilon} \left(\frac{y_t}{a_t \theta_t} \right)^{1 + \frac{1}{\varepsilon}} - \eta \mathbf{1}[y_t > 0] \right] dt \right\} dG(\theta_0)$$

where $G'(\theta_0) = g(\theta_0) > 0$ is the welfare weight on a person of type θ_0 .

Recall that we assume that productivities as well as hours worked are private information and the planner can observe y_t . The individuals report their productivity types γ_t to the planner and the planner allocates consumption and output according to the history of reports. Since this is a continuous time setting, defining incentive compatibility over reports is less intuitive. We assume that reports are restricted such that if γ_t is the true productivity, $\gamma_t - \theta_t$ has finite variations.¹⁸ This is because, the planner can always make an inference about W_t based on the observed report γ_t , given by

$$dW_t^\gamma = \frac{d\gamma_t}{\hat{\sigma}_t}$$

If W_t^γ does not have the properties of a Brownian motion with probability one, the planner would know that the agent has deviated. Hence, the report γ_t has to be such that W_t^γ is a Brownian motion with respect to the filtration imposed by it, $\mathcal{F}_t^\gamma$. More formally, γ_t must be absolutely continuous with respect to θ_t . Then, an application of Girsanov's theorem would imply that there must exist an adapted process B_t so that

$$d\gamma_t = d\theta_t + B_t dt$$

or $\theta_t - \gamma_t$ cannot have instantaneous variations. We call the set of admissible reports Γ ; the set of all reports γ_t that satisfy the above condition:

$$\Gamma = \left\{ \{\gamma_t\}; \gamma_0 \in [\underline{\theta}, \bar{\theta}], d\gamma_t = d\theta_t + B_t dt \text{ for some } \mathcal{F}_t^W\text{-adapted } B_t \right\}$$

¹⁸We follow [De Marzo and Sannikov \(2006\)](#) and [Williams \(2011\)](#) in imposing these restrictions on the reports.

Now, we can define incentive compatibility. An allocation is incentive compatible if

$$\begin{aligned} & E^{\gamma^*} \int_0^1 e^{-\rho t} \left\{ u(c_t) - \frac{1}{1+1/\varepsilon} \left(\frac{y_t}{a_t \theta_t} \right)^{1+\frac{1}{\varepsilon}} - \eta \mathbf{1}[y_t > 0] \right\} dt \\ & \geq E^\gamma \int_0^1 e^{-\rho t} \left\{ u(c_t) - \frac{1}{1+1/\varepsilon} \left(\frac{y_t}{a_t \theta_t} \right)^{1+\frac{1}{\varepsilon}} - \eta \mathbf{1}[y_t > 0] \right\} dt, \quad \forall \{\gamma_t\} \in \Gamma \end{aligned}$$

where E^γ represents expectations under the measure induced by reporting strategy γ and γ^* represents truth telling strategy.

In the Appendix we show that the planning problem in this environment can be written recursively. Here, we state the discrete time version of the recursive formulation for $t \geq 1$ that can be analyzed quantitatively and refer the reader to the Appendix for explicit arguments:

$$\begin{aligned} v_t(w^c, w^h, w_1^h, \theta_-, \theta_0) = \\ \max \int [y(\varepsilon) - c(\varepsilon) + R^{-1} v_{t+1}(w^c(\varepsilon), w^h(\varepsilon), w_1^h(\varepsilon), \varepsilon \theta_-, \theta_0)] dF_t(\varepsilon) \end{aligned}$$

subject to

$$\begin{aligned} w^c &= \int [u(c(\varepsilon)) - \eta \mathbf{1}[y(\varepsilon) > 0] + \beta w^c(\varepsilon)] dF_t(\varepsilon) \\ w^h &= - \int \left[\frac{1}{1+\varepsilon^{-1}} \left(\frac{y(\varepsilon)}{a_t(\theta_0) \varepsilon \theta_-} \right)^{1+\varepsilon^{-1}} + \beta w^h(\varepsilon) \right] dF_t(\varepsilon) \\ w_1^h &= \int \left[\frac{a_s(\theta_0) + \theta_0 a'_s(\theta_0)}{\theta_0 a_s(\theta_0)} \left(\frac{y_s(\varepsilon)}{a_s(\theta_0) \theta_- \varepsilon} \right)^{1+\varepsilon^{-1}} + \beta w_1^h(\varepsilon) \right] dF(\varepsilon) \\ & u(c(\varepsilon)) - \eta \mathbf{1}[y(\varepsilon) > 0] - \frac{1}{1+\varepsilon^{-1}} \left(\frac{y(\varepsilon)}{a_t(\theta_0) \varepsilon \theta_-} \right)^{1+\varepsilon^{-1}} \\ & + \beta (w^c(\varepsilon) + w^h(\varepsilon)) \geq \\ & u(c(\hat{\varepsilon})) - \eta \mathbf{1}[y(\hat{\varepsilon}) > 0] - \frac{1}{1+\varepsilon^{-1}} \left(\frac{y(\hat{\varepsilon})}{a_t(\theta_0) \varepsilon \theta_-} \right)^{1+\varepsilon^{-1}} \\ & + \beta \left(w^c(\hat{\varepsilon}) + \frac{\hat{\varepsilon}^{1+\varepsilon^{-1}}}{\varepsilon^{1+\varepsilon^{-1}}} w^h(\hat{\varepsilon}) \right). \end{aligned}$$

Having a recursive formulation allows us to examine the importance of additional shocks over time quantitatively for an appropriately defined notion of retirement. Our preliminary quantitative analysis suggests that the overall results largely remain the same

in that more productive individuals efficiently retire older than their less productive peers. Compared to results in Section 6, the magnitudes of changes compared to the current retirement patterns are reduced as are total output gains.

8 Conclusion

This paper theoretically and quantitatively studies the efficient design of the pension system as an integral part of the tax code when both intensive and extensive labor margins are active. We analytically characterize Pareto efficient policies and derive efficient work and retirement age patterns and show that, under plausible conditions, efficient retirement age increases with lifetime earnings. We show that this pattern is implemented by pension benefits that depend on the age of retirement. Moreover, we show that this requires a pension system that is designed to be actuarially *unfair*. Using individual earnings and retirement data for the U.S. and, importantly, intensive and extensive labor elasticities, we calibrate and simulate the policy models to generate robust implications: first, it is efficient for individuals with higher lifetime earning to retire older than they do in the data, and second, older than less productive workers do. This is in sharp contrast with what is currently observed in the data. To assess the importance of this disparity, we quantify welfare and total output gains from implementing efficient work and retirement patterns. The main economic message of the paper is perhaps that distorting individual retirement decisions provides a novel and powerful policy tool, capable of overcompensating output losses from standard distortionary redistributive policies. One important role of that policy instrument is to undo part of the incentives to retire earlier that is imbedded in any labor income tax.

A Appendix

A.1 Proof of Lemma 2

The first order condition associated with the planning problem (11) is given by (we are suppressing θ)

$$g - \alpha - \mu' = 0 \quad (22)$$

$$\alpha - \lambda f = 0 \quad (23)$$

$$-\psi \frac{y(t)^{\gamma-1}}{\varphi(t)^\gamma} \alpha + \lambda f - \mu \psi \gamma \frac{y(t)^{\gamma-1}}{\varphi(t)^\gamma} \frac{\varphi_\theta(t)}{\varphi(t)} = 0, \forall t \leq R \quad (24)$$

$$\begin{aligned} - \left[\frac{\psi y(R)^\gamma}{\gamma \varphi(R)^\gamma} + \eta \right] \alpha + y(R) \lambda f - \left[\psi \frac{\varphi_\theta(R)}{\varphi(R)} \frac{y(R)^\gamma}{\varphi(R)^\gamma} - \eta' \right] \mu &= 0 \\ \mu(\underline{\theta}) = \mu(\bar{\theta}) &= 0 \end{aligned} \quad (25)$$

where $\alpha(\theta)$ is the multiplier on the first constraint, λ is the multiplier on feasibility, and $\mu(\theta)$ is the multiplier on local incentive constraint. Integrating over equation (22) and using the boundary conditions, we have

$$\int_{\underline{\theta}}^{\bar{\theta}} g(\theta) d\theta - \lambda \int_{\underline{\theta}}^{\bar{\theta}} f(\theta) d\theta = 0$$

and hence $\lambda = 1$, since $G(\bar{\theta}) = F(\bar{\theta}) = 1$. Moreover, we also have

$$\begin{aligned} \mu(\theta) &= \int_{\underline{\theta}}^{\theta} [g(\theta') - f(\theta')] d\theta' \\ &= G(\theta) - F(\theta) \end{aligned}$$

and we can rewrite the equation (24) as

$$\psi \frac{y(t)^{\gamma-1}}{\varphi(t)^\gamma} \left[1 + \gamma \frac{G(\theta) - F(\theta)}{f(\theta)} \frac{\varphi_\theta(t)}{\varphi} \right] = 1 \quad (26)$$

which implies the first result in Lemma 2. Moreover, since marginal utility of consumption is 1, labor wedge here is defined by $\tau_l = 1 - \psi \frac{y(t)^{\gamma-1}}{\varphi(t)^\gamma}$ and hence the second result in the lemma follows. $\square$

A.2 Proof of Lemma 3

If we replace equation (26) evaluated at $t = R(\theta)$ in (25), we have

$$\frac{y(R)}{\gamma} - y(R) = \eta(\theta) - \frac{G(\theta) - F(\theta)}{f(\theta)} \eta'(\theta)$$

which proves the lemma. $\square$

A.3 Proof of Proposition 5

It is sufficient to derive equation (17), the rest of the argument is described in the text. Note that wedges are defined as

$$\begin{aligned} \tau_r(\theta) y(R(\theta), \theta) &= y(R(\theta), \theta) - \left[\frac{\psi y(R(\theta), \theta)^\gamma}{\gamma \varphi(R(\theta), \theta)^\gamma} + \eta(\theta) \right] \\ \tau_l(R(\theta), \theta) y(R(\theta), \theta) &= y(R(\theta), \theta) - \psi \frac{y(R(\theta), \theta)^\gamma}{\varphi(R(\theta), \theta)^\gamma} \end{aligned}$$

and hence

$$\begin{aligned} \tau_r(\theta) y(R(\theta), \theta) - \frac{1}{\gamma} \tau_l(R(\theta), \theta) y(R(\theta), \theta) \\ = \frac{\gamma - 1}{\gamma} y(R(\theta), \theta) - \eta(\theta) \\ = -\frac{G(\theta) - F(\theta)}{f(\theta)} \eta'(\theta) \end{aligned}$$

where the last equality follows from Lemma 3. $\square$

A.4 Proof of implementation

In the body of the paper, we show that $\hat{\theta} = \theta$ satisfies the first order conditions. Here, we show that it also satisfies the second order condition and hence it is a local maximizer. Then, we show it is a global maximizer as well.

Lemma 8. *Suppose that $R'(\theta) \geq 0$ and $\eta'(\theta) \leq 0$. Then the choice of $\hat{\theta} = \theta$ is a local maximizer in (20).*

Proof. Note that given the proof of Proposition 7, the first order condition evaluated at $\hat{\theta} = \theta$, is given by

$$c'(\theta) - \left[\frac{\psi y(R(\theta), \theta)^\gamma}{\gamma \varphi(R(\theta), \theta)^\gamma} + \eta(\theta) \right] R'(\theta) - \int_0^{R(\theta)} \frac{\partial}{\partial \theta} y(t, \theta) \left[1 - \frac{\partial}{\partial y} T(t, y(t, \theta)) \right] dt = 0$$

Moreover, the second derivative of (20) at $\hat{\theta} = \theta$ is given by

$$\begin{aligned}
& \left[y(R(\hat{\theta}), \theta) - T(R(\hat{\theta}), y(R(\hat{\theta}), \theta)) - \frac{\psi y(R(\hat{\theta}), \theta)^\gamma}{\gamma \varphi(R(\hat{\theta}), \theta)^\gamma} - \eta(\theta) \right] R''(\hat{\theta}) \\
& + \left[\frac{\partial}{\partial t} y(R(\hat{\theta}), \theta) - \left[\frac{\partial}{\partial t} + \frac{\partial y(R(\hat{\theta}), \theta)}{\partial t} \times \frac{\partial}{\partial y} \right] T(R(\hat{\theta}), y(R(\hat{\theta}), \theta)) \right. \\
& \left. - \psi \frac{y(R(\hat{\theta}), \theta)^{\gamma-1}}{\varphi(R(\hat{\theta}), \theta)^\gamma} \left\{ \frac{\partial}{\partial t} y(R(\hat{\theta}), \theta) - \frac{y(R(\hat{\theta}), \theta)}{\varphi(R(\hat{\theta}), \theta)} \frac{\partial}{\partial t} \varphi(R(\hat{\theta}), \theta) \right\} \right] R'(\hat{\theta})^2 \\
& + c''(\hat{\theta}) - \left[y(R(\hat{\theta}), \hat{\theta}) - T(R(\hat{\theta}), y(R(\hat{\theta}), \hat{\theta})) \right] R''(\hat{\theta}) \\
& - \left[\frac{\partial}{\partial t} y(R(\hat{\theta}), \hat{\theta}) - \left[\frac{\partial}{\partial t} + \frac{\partial y(R(\hat{\theta}), \theta)}{\partial t} \times \frac{\partial}{\partial y} \right] T(R(\hat{\theta}), y(R(\hat{\theta}), \hat{\theta})) \right] R'(\hat{\theta})^2 \\
& - \left[\frac{\partial}{\partial \theta} y(R(\hat{\theta}), \hat{\theta}) - \frac{\partial}{\partial \theta} y(R(\hat{\theta}), \hat{\theta}) \frac{\partial}{\partial y} T(R(\hat{\theta}), y(R(\hat{\theta}), \hat{\theta})) \right] R'(\hat{\theta}) \\
& - \frac{\partial}{\partial \theta} y(R(\hat{\theta}), \hat{\theta}) \left[1 - \frac{\partial}{\partial y} T(R(\hat{\theta}), y(R(\hat{\theta}), \hat{\theta})) \right] R'(\hat{\theta}) \\
& - \int_0^{R(\hat{\theta})} \left\{ \frac{\partial^2}{\partial \theta^2} y(t, \hat{\theta}) \left[1 - \frac{\partial}{\partial y} T(t, y(t, \hat{\theta})) \right] - \left(\frac{\partial}{\partial \theta} y(t, \hat{\theta}) \right)^2 \frac{\partial^2}{\partial y^2} T(t, y(t, \hat{\theta})) \right\} dt
\end{aligned}$$

Evaluating the above at $\hat{\theta} = \theta$ gives us the following expression

$$\begin{aligned}
& \left[-\frac{\psi y(R(\theta), \theta)^\gamma}{\gamma \varphi(R(\theta), \theta)^\gamma} - \eta(\theta) \right] R''(\theta) \\
& + \left[-\psi \frac{y(R(\theta), \theta)^{\gamma-1}}{\varphi(R(\theta), \theta)^\gamma} \left\{ \frac{\partial}{\partial t} y(R(\theta), \theta) - \frac{y(R(\theta), \theta)}{\varphi(R(\theta), \theta)} \frac{\partial}{\partial t} \varphi(R(\theta), \theta) \right\} \right] R'(\theta)^2 \\
& + c''(\theta) \\
& - 2 \frac{\partial}{\partial \theta} y(R(\theta), \theta) \left[1 - \frac{\partial}{\partial y} T(R(\theta), y(R(\theta), \theta)) \right] R'(\theta) \\
& - \int_0^{R(\theta)} \left\{ \frac{\partial^2}{\partial \theta^2} y(t, \theta) \left[1 - \frac{\partial}{\partial y} T(t, y(t, \theta)) \right] - \left(\frac{\partial}{\partial \theta} y(t, \theta) \right)^2 \frac{\partial^2}{\partial y^2} T(t, y(t, \theta)) \right\} dt
\end{aligned}$$

If we take derivative of the incentive constraint in its local form with respect to θ , we

have

$$\begin{aligned}
& c''(\theta) - \left[\frac{\psi y(R(\theta), \theta)^\gamma}{\gamma \varphi(R(\theta), \theta)^\gamma} + \eta(\theta) \right] R''(\theta) \\
& + \left[-\psi \frac{y(R(\theta), \theta)^{\gamma-1}}{\varphi(R(\theta), \theta)^\gamma} \left\{ \frac{\partial}{\partial t} y(R(\theta), \theta) - \frac{y(R(\theta), \theta)}{\varphi(R(\theta), \theta)} \frac{\partial}{\partial t} \varphi(R(\theta), \theta) \right\} \right] R'(\theta)^2 \\
& + \left[-\psi \frac{y(R(\theta), \theta)^{\gamma-1}}{\varphi(R(\theta), \theta)^\gamma} \left\{ \frac{\partial}{\partial \theta} y(R(\theta), \theta) - \frac{y(R(\theta), \theta)}{\varphi(R(\theta), \theta)} \frac{\partial}{\partial \theta} \varphi(R(\theta), \theta) \right\} - \eta'(\theta) \right] R'(\theta) \\
& - \frac{\partial}{\partial \theta} y(R(\theta), \theta) \left[1 - \frac{\partial}{\partial y} T(R(\theta), y(R(\theta), \theta)) \right] R'(\theta) \\
& - \int_0^{R(\theta)} \left\{ \frac{\partial^2}{\partial \theta^2} y(t, \theta) \left[1 - \frac{\partial}{\partial y} T(t, y(t, \theta)) \right] - \left(\frac{\partial}{\partial \theta} y(t, \theta) \right)^2 \frac{\partial^2}{\partial y^2} T(t, y(t, \theta)) \right\} dt = 0
\end{aligned}$$

We can regroup the terms in the second order condition and use the above expression to further simplify

$$\begin{aligned}
& c''(\theta) + \left[-\frac{\psi y(R(\theta), \theta)^\gamma}{\gamma \varphi(R(\theta), \theta)^\gamma} - \eta(\theta) \right] R''(\theta) \\
& + \left[-\psi \frac{y(R(\theta), \theta)^{\gamma-1}}{\varphi(R(\theta), \theta)^\gamma} \left\{ \frac{\partial}{\partial t} y(R(\theta), \theta) - \frac{y(R(\theta), \theta)}{\varphi(R(\theta), \theta)} \frac{\partial}{\partial t} \varphi(R(\theta), \theta) \right\} \right] R'(\theta)^2 \\
& - 2 \frac{\partial}{\partial \theta} y(R(\theta), \theta) \left[1 - \frac{\partial}{\partial y} T(R(\theta), y(R(\theta), \theta)) \right] R'(\theta) \\
& - \int_0^{R(\theta)} \left\{ \frac{\partial^2}{\partial \theta^2} y(t, \theta) \left[1 - \frac{\partial}{\partial y} T(t, y(t, \theta)) \right] - \left(\frac{\partial}{\partial \theta} y(t, \theta) \right)^2 \frac{\partial^2}{\partial y^2} T(t, y(t, \theta)) \right\} dt \\
& = -\frac{\partial}{\partial \theta} y(R(\theta), \theta) \left[1 - \frac{\partial}{\partial y} T(R(\theta), y(R(\theta), \theta)) \right] R'(\theta) \\
& - \left[-\psi \frac{y(R(\theta), \theta)^{\gamma-1}}{\varphi(R(\theta), \theta)^\gamma} \left\{ \frac{\partial}{\partial \theta} y(R(\theta), \theta) - \frac{y(R(\theta), \theta)}{\varphi(R(\theta), \theta)} \frac{\partial}{\partial \theta} \varphi(R(\theta), \theta) \right\} - \eta'(\theta) \right] R'(\theta) \\
& = \left[-\psi \frac{y(R(\theta), \theta)^\gamma}{\varphi(R(\theta), \theta)^{\gamma+1}} \frac{\partial}{\partial \theta} \varphi(R(\theta), \theta) + \eta'(\theta) \right] R'(\theta)
\end{aligned}$$

Since $R'(\theta) \geq 0$, $\eta'(\theta) \leq 0$, and that $\frac{\partial}{\partial \theta} \varphi(R(\theta), \theta) \geq 0$, the above expression must be negative and therefore $\hat{\theta} = \theta$ is the local maximizer of (20). $\square$

Proposition 9. Suppose that $R'(\theta) \geq 0$ and $\eta'(\theta) \leq 0$. Then $\theta = \hat{\theta}$ is a global maximizer of (20).

Proof. Let $U(\theta, \hat{\theta})$ be given by (20). From lemma 8, we know that $\frac{\partial}{\partial \theta} U(\theta, \theta) = 0$.

Then, we want to show that $U(\theta, \theta) \geq U(\theta, \hat{\theta})$. One can rewrite this condition as

$$\int_{\hat{\theta}}^{\theta} \frac{\partial}{\partial \hat{\theta}} U(\theta, \tilde{\theta}) d\tilde{\theta} \geq 0$$

Here, we focus on the case where $\theta > \hat{\theta}$. In this case, it is sufficient to show that

$$\frac{\partial}{\partial \hat{\theta}} U(\theta, \hat{\theta}) \geq \frac{\partial}{\partial \hat{\theta}} U(\hat{\theta}, \hat{\theta}) = 0$$

Using the formula in (20), we have

$$\begin{aligned} \frac{\partial}{\partial \hat{\theta}} U(\theta, \hat{\theta}) &= R'(\hat{\theta}) \left[y(R(\hat{\theta}), \theta) - T(R(\hat{\theta}), y(R(\hat{\theta}), \theta)) \right] \\ &\quad - R'(\hat{\theta}) \left[\frac{\psi y(R(\hat{\theta}), \theta)^\gamma}{\gamma \varphi(R(\hat{\theta}), \theta)^\gamma} + \eta(\theta) \right] \\ &\quad + c'(\hat{\theta}) - R'(\hat{\theta}) \left[y(R(\hat{\theta}), \hat{\theta}) - T(R(\hat{\theta}), y(R(\hat{\theta}), \hat{\theta})) \right] \\ &\quad - \int_0^{R(\hat{\theta})} \frac{\partial}{\partial \theta} y(t, \hat{\theta}) \left[1 - T_y(t, y(t, \hat{\theta})) \right] dt \end{aligned}$$

As it can be seen, in order to show that $\frac{\partial}{\partial \hat{\theta}} U(\theta, \hat{\theta}) \geq \frac{\partial}{\partial \hat{\theta}} U(\hat{\theta}, \hat{\theta})$, we need to show that

$$\begin{aligned} &y(R(\hat{\theta}), \theta) - T(R(\hat{\theta}), y(R(\hat{\theta}), \theta)) - \frac{\psi y(R(\hat{\theta}), \theta)^\gamma}{\gamma \varphi(R(\hat{\theta}), \theta)^\gamma} - \eta(\theta) \geq \\ &y(R(\hat{\theta}), \hat{\theta}) - T(R(\hat{\theta}), y(R(\hat{\theta}), \hat{\theta})) - \frac{\psi y(R(\hat{\theta}), \hat{\theta})^\gamma}{\gamma \varphi(R(\hat{\theta}), \hat{\theta})^\gamma} - \eta(\hat{\theta}) \end{aligned}$$

since $R'(\hat{\theta}) \geq 0$. Note that by assumption $\eta(\theta) \leq \eta(\hat{\theta})$ and hence we need to show that

$$\begin{aligned} &y(R(\hat{\theta}), \theta) - T(R(\hat{\theta}), y(R(\hat{\theta}), \theta)) - \frac{\psi y(R(\hat{\theta}), \theta)^\gamma}{\gamma \varphi(R(\hat{\theta}), \theta)^\gamma} \geq \\ &y(R(\hat{\theta}), \hat{\theta}) - T(R(\hat{\theta}), y(R(\hat{\theta}), \hat{\theta})) - \frac{\psi y(R(\hat{\theta}), \hat{\theta})^\gamma}{\gamma \varphi(R(\hat{\theta}), \hat{\theta})^\gamma} \end{aligned}$$

The difference between the two sides of this inequality can be written as

$$\begin{aligned}
& \int_{\hat{\theta}}^{\theta} \frac{\partial}{\partial \theta} y(R(\hat{\theta}), \theta') \left[1 - \frac{\partial}{\partial y} T(R(\hat{\theta}), y(R(\hat{\theta}), \theta')) \right] d\theta' \\
& - \int_{\hat{\theta}}^{\theta} \psi \frac{y(R(\hat{\theta}), \theta')^{\gamma-1}}{\varphi(R(\hat{\theta}), \theta')^{\gamma}} \left[\frac{\partial}{\partial \theta} y(R(\hat{\theta}), \theta') - \frac{y(R(\hat{\theta}), \theta')}{\varphi(R(\hat{\theta}), \theta')} \frac{\partial}{\partial \theta} \varphi(R(\hat{\theta}), \theta') \right] d\theta' \\
& = \int_{\hat{\theta}}^{\theta} \left\{ \frac{\partial}{\partial \theta} y(R(\hat{\theta}), \theta') \left[1 - \frac{\partial}{\partial y} T(R(\hat{\theta}), y(R(\hat{\theta}), \theta')) \right] \right. \\
& \quad \left. - \psi \frac{y(R(\hat{\theta}), \theta')^{\gamma-1}}{\varphi(R(\hat{\theta}), \theta')^{\gamma}} \frac{\partial}{\partial \theta} y(R(\hat{\theta}), \theta') \right\} d\theta' \\
& + \int_{\hat{\theta}}^{\theta} \psi \frac{y(R(\hat{\theta}), \theta')^{\gamma}}{\varphi(R(\hat{\theta}), \theta')^{\gamma+1}} \frac{\partial}{\partial \theta} \varphi(R(\hat{\theta}), \theta') d\theta'
\end{aligned}$$

By the first order associated with (18), the first integral is zero and since $\varphi(t, \theta)$ is increasing in θ , the second term is positive and hence the whole difference is a positive number. $\square$

A.5 Existence of retirement age

Proposition 10. *In the solution to (11), a type θ prefers to work if and only if $t \leq R(\theta)$.*

Proof.

To show this, it is sufficient to show that in the solution to (11), $y(t, \theta)$ is decreasing in t when $t > t^*(\theta)$ where $t^*(\theta)$ is the point at which $\varphi(t, \theta)$ is maximized. To see this, consider equation (26)

$$y(t, \theta) = \psi^{\frac{1}{\gamma-1}} \frac{\varphi(t, \theta)^{\frac{\gamma}{\gamma-1}}}{\left[1 + \gamma \frac{G(\theta) - F(\theta)}{f(\theta)} \frac{\varphi_{\theta}(t, \theta)}{\varphi(t, \theta)} \right]^{\frac{1}{\gamma-1}}}$$

Note that when $t > t^*(\theta)$, $\varphi(t, \theta)$ is a decreasing function of t . Moreover, $\frac{\varphi_{\theta}(t, \theta)}{\varphi(t, \theta)}$ is an increasing function of t since

$$\frac{d}{dt} \frac{\varphi_{\theta}(t, \theta)}{\varphi(t, \theta)} = \frac{\varphi_{\theta t}(t, \theta)}{\varphi(t, \theta)} - \frac{\varphi_{\theta}(t, \theta) \varphi_t(t, \theta)}{\varphi(t, \theta)^2}$$

By assumption 1, $\varphi_{t\theta} \geq 0$. Moreover, $\varphi_t < 0$ and $\varphi_{\theta} \geq 0$. Hence, the above expression is positive. This means that $y(t, \theta)$ is given by a decreasing function divided by an

increasing function and therefore, when $t > t^*(\theta)$, $y(t, \theta)$ is decreasing in t and hence agents retire optimally. $\square$

A.6 Sufficiency of first-order approach

Proposition 11. *Suppose that $\eta(\theta)$ is a decreasing function of θ , $\varphi(t, \theta)$ is an increasing function of θ and that*

$$\left[1 + \gamma \frac{G(\theta) - F(\theta)}{f(\theta)} \frac{\varphi_\theta(t, \theta)}{\varphi(t, \theta)} \right]^{\frac{1}{1-\gamma}} \varphi(t, \theta)^{\frac{\gamma}{\gamma-1}}$$

is increasing in θ . Then the solution to the relaxed problem (11) is also a solution to the more restricted problem (5).

We first show the following lemma:

Lemma 12. *Suppose an allocation satisfies (10) and is such that $y(t, \theta)$ and $R(\theta)$ is increasing in θ . Then this allocation is incentive compatible.*

Proof. We want to show that an allocation that satisfies the above conditions satisfies the following

$$\begin{aligned} U(\theta, \theta) &= c(\theta) - \int_0^{R(\theta)} \left[\psi \frac{y(t, \theta)^\gamma}{\gamma \varphi(t, \theta)^\gamma} + \eta(\theta) \right] dt \geq \\ &c(\hat{\theta}) - \int_0^{R(\hat{\theta})} \left[\psi \frac{y(t, \hat{\theta})^\gamma}{\gamma \varphi(t, \theta)^\gamma} + \eta(\theta) \right] dt = U(\theta, \hat{\theta}); \forall \hat{\theta}, \theta \end{aligned}$$

To show this, we show that $U_2(\theta, \hat{\theta}) \geq 0$ whenever $\theta \geq \hat{\theta}$ and that $U_2(\theta, \hat{\theta}) \leq 0$ when $\theta \leq \hat{\theta}$. This would imply that

$$\begin{aligned} U(\theta, \theta) - U(\theta, \hat{\theta}) &= \int_{\hat{\theta}}^{\theta} U_2(\theta, \tilde{\theta}) d\tilde{\theta} \geq 0, \text{ if } \theta \geq \hat{\theta} \\ U(\theta, \hat{\theta}) - U(\theta, \theta) &= \int_{\theta}^{\hat{\theta}} U_2(\theta, \tilde{\theta}) d\tilde{\theta} \leq 0, \text{ if } \theta \leq \hat{\theta} \end{aligned}$$

To show that $U_2(\theta, \hat{\theta}) \geq 0$ whenever $\theta \geq \hat{\theta}$, we have the following

$$\begin{aligned} U_2(\theta, \hat{\theta}) &= c'(\hat{\theta}) - \int_0^{R(\hat{\theta})} \psi y_\theta(t, \hat{\theta}) \frac{y(t, \hat{\theta})^{\gamma-1}}{\varphi(t, \theta)^\gamma} dt \\ &\quad - \left[\psi \frac{y(R(\hat{\theta}), \hat{\theta})^\gamma}{\gamma \varphi(R(\hat{\theta}), \theta)^\gamma} + \eta(\theta) \right] R'(\hat{\theta}) \end{aligned}$$

Since the allocation satisfies (10),

$$c'(\hat{\theta}) = \int_0^{R(\hat{\theta})} \psi y_\theta(t, \hat{\theta}) \frac{y(t, \hat{\theta})^{\gamma-1}}{\varphi(t, \hat{\theta})^\gamma} dt + \left[\psi \frac{y(R(\hat{\theta}), \hat{\theta})^\gamma}{\gamma \varphi(R(\hat{\theta}), \hat{\theta})^\gamma} + \eta(\hat{\theta}) \right] R'(\hat{\theta})$$

Hence $U_2(\theta, \hat{\theta})$ can be written as

$$\begin{aligned} U_2(\theta, \hat{\theta}) &= \int_0^{R(\hat{\theta})} \psi y_\theta(t, \hat{\theta}) y(t, \hat{\theta})^{\gamma-1} \left[\varphi(t, \hat{\theta})^{-\gamma} - \varphi(t, \theta)^{-\gamma} \right] dt \\ &\quad + \left[\psi \frac{y(R(\hat{\theta}), \hat{\theta})^\gamma}{\gamma \varphi(R(\hat{\theta}), \hat{\theta})^\gamma} + \eta(\hat{\theta}) - \psi \frac{y(R(\hat{\theta}), \hat{\theta})^\gamma}{\gamma \varphi(R(\hat{\theta}), \theta)^\gamma} - \eta(\theta) \right] R'(\hat{\theta}) \end{aligned}$$

Given that $R'(\theta) \geq 0$, $\eta(\theta)$ is decreasing, $y(t, \theta)$ is increasing in θ and $\varphi(t, \theta)$ is increasing in θ , the above expression is positive when $\theta \geq \hat{\theta}$ and negative when $\theta \leq \hat{\theta}$. That completes the proof of the lemma. $\square$

Given the above lemma, and the formulas provided in the paper, under the provided assumptions above $y(t, \theta)$ is increasing in θ as well as $R(\theta)$. Hence the sufficient condition for the lemma are satisfied. $\square$

A.7 Recursive formulation

Here we provide arguments for arriving at the recursive formulation when in addition to one-time shocks to productivity and fixed cost we also include productivity shocks over

time. We proceed by defining the following two variables

$$\begin{aligned} w_t^c &= E_t^{\gamma^*} \left[\int_t^1 e^{-\rho\tau} \{u(c_\tau) - \eta \mathbf{1}[y_\tau > 0]\} d\tau \right] \\ w_t^h &= -E_t^{\gamma^*} \left[\int_t^1 e^{-\rho\tau} \frac{1}{1 + 1/\varepsilon} \left(\frac{y_\tau}{a_\tau \theta_\tau} \right)^{1+\frac{1}{\varepsilon}} d\tau \right] \end{aligned}$$

We later show that when time is discrete and the dynamic contracting problem has certain properties (private information follows geometric random walk and preferences satisfy a linearity condition), it is enough to keep track of the two promised utilities defined above. We also show how these two state variables are sufficient. In our environment, we derive the incentive constraint by taking the limit of a discrete time version of our model.¹⁹

In particular, suppose that time increment is given by τ and time is $t = 0, \tau, 2\tau, 3\tau, \dots, \lfloor \frac{1}{\tau} \rfloor \tau$. Moreover, let

$$\log \theta_{t+\tau}^\tau = \log \theta_t^\tau + \eta_{t+\tau}^\tau, \forall t = i\tau, i < \left\lfloor \frac{1}{\tau} \right\rfloor - 1$$

where $\eta_t \sim N\left(-\frac{1}{2}\tau\sigma_t^2, \sigma_t\sqrt{\tau}\right)$. Moreover, we let $\beta = e^{-\tau}$ be the discount factor across periods. By Donsker's invariance principle (see Billingsley (1968)), the stochastic process θ_t^τ converges to θ_t in distribution. Now consider an allocation $\{c_t, y_t\}_{t \in \mathbb{N} \times \tau}$ ²⁰. For this allocation, one can define promised utilities $\{w_t^c, w_t^h\}$ as above:

$$\begin{aligned} w_{t-\tau}^c &= E_{t-1} \left[\sum_{i=t/\tau}^{\lfloor 1/\tau \rfloor} \tau \beta^{i\tau} [u(c_{i\tau}) - \eta \mathbf{1}[y_{i\tau} > 0]] \right] \\ w_{t-\tau}^h &= -E_{t-1} \left[\sum_{i=t/\tau}^{\lfloor 1/\tau \rfloor} \tau \beta^{i\tau} \frac{1}{1 + \varepsilon^{-1}} \frac{y_{i\tau}^{1+\varepsilon^{-1}}}{(a_{i\tau} \theta_{i\tau}^\tau)^{1+\varepsilon^{-1}}} \right] \end{aligned}$$

¹⁹Our approach here closely follows the approach in [Farhi and Werning \(2010\)](#).

²⁰These allocations and other state variables are all separately dependent on τ but for simplicity we suppress that notation.

One-shot incentive constraints can be written as

$$\begin{aligned}
& \left\{ u(c_t(\theta^{t-\tau}, \theta_t)) - \eta \mathbf{1}[y_t(\theta^{t-\tau}, \theta_t) > 0] - \frac{1}{1+\varepsilon^{-1}} \frac{y_t(\theta^{t-\tau}, \theta_t)^{1+\varepsilon^{-1}}}{(a_t \theta_t)^{1+\varepsilon^{-1}}} \right\}^\tau \\
& + e^{-\rho\tau} (w_t^c(\theta^{t-\tau}, \theta_t) + w_t^h(\theta^{t-\tau}, \theta_t)) \geq \\
& \left\{ u(c_t(\theta^{t-\tau}, \hat{\theta}_t)) - \eta \mathbf{1}[y_t(\theta^{t-\tau}, \hat{\theta}_t) > 0] - \frac{1}{1+\varepsilon^{-1}} \frac{y_t(\theta^{t-\tau}, \hat{\theta}_t)^{1+\varepsilon^{-1}}}{(a_t \theta_t)^{1+\varepsilon^{-1}}} \right\}^\tau \\
& + e^{-\rho\tau} \left(w_t^c(\theta^{t-\tau}, \hat{\theta}_t) + \frac{\hat{\theta}_t^{1+\varepsilon^{-1}}}{\theta_t^{1+\varepsilon^{-1}}} w_t^h(\theta^{t-\tau}, \theta_t) \right)
\end{aligned}$$

To further simplify, let v_t^c and v_t^h be defined as the value of the contract to the agent at date t , after his type is realized:

$$\begin{aligned}
v_t^c(\theta^t) &= u(c_t(\theta^t)) - \eta \mathbf{1}[y_t(\theta^t) > 0] + \beta w_t^c(\theta^t) \\
v_t^h(\theta^t) &= -\frac{1}{1+\varepsilon^{-1}} \frac{y_t(\theta^t)^{1+\varepsilon^{-1}}}{(a_t \theta_t)^{1+\varepsilon^{-1}}} + \beta w_t^h(\theta^t)
\end{aligned}$$

Then, the above incentive constraints can be written as

$$v_t^c(\theta^t) + v_t^h(\theta^t) \geq v_t^c(\theta^{t-\tau}, \hat{\theta}_t) + \frac{\hat{\theta}_t^{1+\varepsilon^{-1}}}{\theta_t^{1+\varepsilon^{-1}}} v_t^h(\theta^{t-\tau}, \hat{\theta}_t)$$

In particular,

$$v_t^c(\theta^{t-\tau}, \theta_t) + v_t^h(\theta^{t-\tau}, \theta_t) \geq v_t^c(\theta^{t-\tau}, \theta_t - \delta\theta_t) + \frac{(\theta_t - \delta\theta_t)^{1+\varepsilon^{-1}}}{\theta_t^{1+\varepsilon^{-1}}} v_t^h(\theta^{t-\tau}, \theta_t - \delta\theta_t)$$

Rearranging terms, we have

$$\begin{aligned}
& \frac{v_t^c(\theta^{t-\tau}, \theta_t) - v_t^c(\theta^{t-\tau}, \theta_t - \delta\theta_t)}{\tau} + \frac{v_t^h(\theta^{t-\tau}, \theta_t) - v_t^h(\theta^{t-\tau}, \theta_t - \delta\theta_t)}{\tau} \\
& \geq \frac{(\theta_t - \delta\theta_t)^{1+\varepsilon^{-1}} - \theta_t^{1+\varepsilon^{-1}}}{\tau \times \theta_t^{1+\varepsilon^{-1}}} v_t^h(\theta^{t-\tau}, \theta_t - \delta\theta_t)
\end{aligned} \tag{27}$$

Note that as τ tends to zero v_t and w_t converge to the same limit since diffusions have continuous sample paths. Moreover, w_t^c and w_t^h must converge to diffusion processes such

that

$$\begin{aligned} dw_t^c &= (\rho w_t^c - u(c_t) + \eta \mathbf{1}[y_t > 0]) dt + \gamma_t^c \sigma_t dW_t \\ dw_t^h &= \left(\rho w_t^h + \frac{1}{1 + \varepsilon^{-1}} \frac{y_t^{1+\varepsilon^{-1}}}{(a_t \theta_t)^{1+\varepsilon^{-1}}} \right) dt + \gamma_t^h \sigma_t dW_t \end{aligned}$$

Now, the terms in the left-hand side of (27) can be thought of as instantaneous variations in w_t^c and w_t^h . Moreover, $\frac{(\theta_t - \delta \theta_t)^{1+\varepsilon^{-1}} - \theta_t^{1+\varepsilon^{-1}}}{\tau}$ can be thought of as instantaneous variations in $\theta_t^{1+\varepsilon^{-1}}$. By Ito's lemma

$$\begin{aligned} d(\theta_t^{1+\varepsilon^{-1}}) &= (1 + \varepsilon^{-1}) \theta_t^{\varepsilon^{-1}} \theta_t \sigma_t dW_t + \varepsilon^{-1} (1 + \varepsilon^{-1}) \theta_t^{\varepsilon^{-1}-1} \theta_t^2 \sigma_t^2 dt \\ &= (1 + \varepsilon^{-1}) \theta_t^{\varepsilon^{-1}+1} \sigma_t dW_t + \varepsilon^{-1} (1 + \varepsilon^{-1}) \theta_t^{\varepsilon^{-1}+1} \sigma_t^2 dt \end{aligned}$$

Hence, the instantaneous variation in $\theta_t^{1+\varepsilon^{-1}}$ is given by $(1 + \varepsilon^{-1}) \theta_t^{1+\varepsilon^{-1}} \sigma_t$. Hence, in the continuous time limit, the incentive constraint (27) becomes

$$\gamma_t^c + \gamma_t^h \geq (1 + \varepsilon^{-1}) w_t^h$$

Moreover, if we assume that the local downward incentive constraints are binding and are sufficient, incentive compatibility in the continuous time limit translates to:

$$\gamma_t^c + \gamma_t^h = (1 + \varepsilon^{-1}) w_t^h$$

Given this formulation, we can state our continuous time problem recursively. The HJB equation associated with the dual of this problem is given by

$$\begin{aligned} \rho J(w^c, w^h, \theta, t) &= \max_{\gamma^c, \gamma^h, c, y} y - c + J_c(\rho w^c - u(c) + \eta \mathbf{1}[y > 0]) \\ &\quad + J_h \left(\rho w^h + \frac{1}{1 + \varepsilon^{-1}} \frac{y^{1+\varepsilon^{-1}}}{(a_t \theta)^{1+\varepsilon^{-1}}} \right) + J_t \\ &\quad + \frac{1}{2} J_{cc} (\gamma^c \sigma_t)^2 + \frac{1}{2} J_{hh} (\gamma^h \sigma_t)^2 + \frac{1}{2} J_{\theta\theta} (\sigma_t \theta)^2 \\ &\quad + J_{hc} \gamma^c \gamma^h \sigma_t^2 + J_{c\theta} \gamma^c \theta \sigma_t^2 + J_{h\theta} \gamma^h \theta \sigma_t^2 \end{aligned}$$

subject to

$$\gamma^c + \gamma^h = (1 + \varepsilon^{-1}) w^h.$$

Having a recursive formulation allows us to examine the importance of additional shocks over time quantitatively for an appropriately defined notion of retirement. For

the quantitative analysis, it is useful to restate the problem in discrete time.

An allocation is $c_t(\theta_0, \varepsilon^t), y_t(\theta_0, \varepsilon^t)$. The relevant state variables are then

$$\begin{aligned} w_t^c(\theta_0, \varepsilon^{t-1}) &= E_t \left[\sum_{s=t}^T \beta^{s-t} \{u(c_s) - \eta \mathbf{1}[y_s > 0]\} \right] \\ w_t^h(\theta_0, \varepsilon^{t-1}) &= -E_t \left[\sum_{s=t}^T \beta^{s-t} \frac{1}{1 + \epsilon^{-1}} \left(\frac{y_s}{a_s(\theta_0) \theta_0 \varepsilon_0 \cdots \varepsilon_s} \right)^{1+\epsilon^{-1}} \right] \\ \hat{w}_t^h(\hat{\theta}_0, \varepsilon^{t-1}; \theta_0) &= -E_t \left[\sum_{s=t}^T \beta^{s-t} \frac{1}{1 + \epsilon^{-1}} \left(\frac{y_s(\hat{\theta}_0, \varepsilon^s)}{a_s(\theta_0) \theta_0 \varepsilon_0 \cdots \varepsilon_s} \right)^{1+\epsilon^{-1}} \right] \end{aligned}$$

where $\hat{w}_t^h$ is the continuation value of an individual with type $\hat{\theta}_0$ who has reported to be θ_0 in the first period and has told the truth ever since.

Then, the incentive constraint for $t \geq 1$ is given by

$$\begin{aligned} &u(c_t(\theta_0, \varepsilon^t)) - \eta \mathbf{1}[y_t(\theta_0, \varepsilon^t) > 0] - \frac{1}{1 + \epsilon^{-1}} \left(\frac{y_s(\theta_0, \varepsilon^t)}{a_s(\theta_0) \theta_0 \varepsilon_0 \cdots \varepsilon_s} \right)^{1+\epsilon^{-1}} \\ &+ \beta (w_{t+1}^c(\theta_0, \varepsilon^t) + w_{t+1}^h(\theta_0, \varepsilon^t)) \geq \\ &u(c_t(\theta_0, \varepsilon^{t-1}, \hat{\varepsilon}_t)) - \eta \mathbf{1}[y_t(\theta_0, \varepsilon^{t-1}, \hat{\varepsilon}_t) > 0] - \frac{1}{1 + \epsilon^{-1}} \left(\frac{y_s(\theta_0, \varepsilon^{t-1}, \hat{\varepsilon}_t)}{a_s(\theta_0) \theta_0 \varepsilon_0 \cdots \varepsilon_s} \right)^{1+\epsilon^{-1}} \\ &+ \beta \left(w_{t+1}^c(\theta_0, \varepsilon^{t-1}, \hat{\varepsilon}_t) + \frac{\hat{\theta}_t^{1+\epsilon^{-1}}}{\theta_t^{1+\epsilon^{-1}}} w_{t+1}^h(\theta_0, \varepsilon^{t-1}, \hat{\varepsilon}_t) \right) \end{aligned}$$

The incentive constraint at $t = 0$ is given by

$$\begin{aligned} U(\theta_0) &= u(c_0(\theta_0)) - \eta \mathbf{1}[y_0(\theta_0) > 0] - \frac{1}{1 + \epsilon^{-1}} \left(\frac{y_0(\theta_0)}{\theta_0} \right)^{1+\epsilon^{-1}} + \beta (w_1^c(\theta_0) + w_1^h(\theta_0)) \geq \\ &u(c_0(\hat{\theta}_0)) - \eta \mathbf{1}[y_0(\hat{\theta}_0) > 0] - \frac{1}{1 + \epsilon^{-1}} \left(\frac{y_0(\hat{\theta}_0)}{\theta_0} \right)^{1+\epsilon^{-1}} \\ &+ \beta (w_1^c(\hat{\theta}_0) + \hat{w}_1^h(\hat{\theta}_0; \theta_0)) \end{aligned}$$

The above incentive constraint in its local form is given by

$$U'(\theta_0) = \frac{y_s(\theta_0)^{1+\epsilon^{-1}}}{\theta_0^{\epsilon^{-1}}} + \frac{\partial}{\partial \theta_0} \hat{w}_1^h(\theta_0; \theta_0)$$

Note that by definition of $\hat{w}_1$,

$$\begin{aligned}\frac{\partial}{\partial \theta_0} \hat{w}_1^h(\hat{\theta}_0; \theta_0) &= -\frac{\partial}{\partial \theta_0} E_0 \left[\sum_{s=1}^T \beta^{s-1} \frac{1}{1+\epsilon^{-1}} \left(\frac{y_s(\hat{\theta}_0, \varepsilon^s)}{a_s(\theta_0) \theta_0 \varepsilon_0 \cdots \varepsilon_s} \right)^{1+\epsilon^{-1}} \right] \\ \frac{\partial}{\partial \theta_0} \hat{w}_1^h(\theta_0; \theta_0) &= E_0 \left[\sum_{s=1}^T \beta^{s-1} \frac{a_s(\theta_0) + \theta_0 a'_s(\theta_0)}{\theta_0 a_s(\theta_0)} \left(\frac{y_s(\theta_0, \varepsilon^s)}{a_s(\theta_0) \theta_0 \varepsilon_0 \cdots \varepsilon_s} \right)^{1+\epsilon^{-1}} \right]\end{aligned}$$

Hence, the version of the problem can be written as follows. For $t \geq 1$

$$v_t(w^c, w^h, w_1^h, \theta_-, \theta_0) = \max \int [y(\varepsilon) - c(\varepsilon) + R^{-1} v_{t+1}(w^c(\varepsilon), w^h(\varepsilon), w_1^h(\varepsilon), \varepsilon \theta_-, \theta_0)] dF_t(\varepsilon)$$

subject to

$$\begin{aligned}w^c &= \int [u(c(\varepsilon)) - \eta \mathbf{1}[y(\varepsilon) > 0] + \beta w^c(\varepsilon)] dF_t(\varepsilon) \\ w^h &= - \int \left[\frac{1}{1+\epsilon^{-1}} \left(\frac{y(\varepsilon)}{a_t(\theta_0) \varepsilon \theta_-} \right)^{1+\epsilon^{-1}} + \beta w^h(\varepsilon) \right] dF_t(\varepsilon) \\ w_1^h &= \int \left[\frac{a_s(\theta_0) + \theta_0 a'_s(\theta_0)}{\theta_0 a_s(\theta_0)} \left(\frac{y_s(\varepsilon)}{a_s(\theta_0) \theta_- \varepsilon} \right)^{1+\epsilon^{-1}} + \beta w_1^h(\varepsilon) \right] dF(\varepsilon) \\ &\quad u(c(\varepsilon)) - \eta \mathbf{1}[y(\varepsilon) > 0] - \frac{1}{1+\epsilon^{-1}} \left(\frac{y(\varepsilon)}{a_t(\theta_0) \varepsilon \theta_-} \right)^{1+\epsilon^{-1}} \\ &\quad + \beta (w^c(\varepsilon) + w^h(\varepsilon)) \geq \\ &\quad u(c(\hat{\varepsilon})) - \eta \mathbf{1}[y(\hat{\varepsilon}) > 0] - \frac{1}{1+\epsilon^{-1}} \left(\frac{y(\hat{\varepsilon})}{a_t(\theta_0) \varepsilon \theta_-} \right)^{1+\epsilon^{-1}} \\ &\quad + \beta \left(w^c(\hat{\varepsilon}) + \frac{\hat{\varepsilon}^{1+\epsilon^{-1}}}{\varepsilon^{1+\epsilon^{-1}}} w^h(\hat{\varepsilon}) \right)\end{aligned}$$

and at $t = 0$,

$$v_0(w^c, w^h) = \max \int \left[y(\theta_0) - c(\theta_0) + \frac{1}{R} v_1(w^c(\theta_0), w^h(\theta_0), w_1^h(\theta_0), \theta_0, \theta_0) \right] dF_0(\theta_0)$$

subject to

$$\begin{aligned}
w^c &= \int [u(c(\theta_0)) - \eta \mathbf{1}[y(\theta_0) > 0] + \beta w^c(\theta_0)] dF_0(\theta_0) \\
w^h &= - \int \left[\frac{1}{1 + \epsilon^{-1}} \left(\frac{y(\theta_0)}{\theta_0} \right)^{1 + \epsilon^{-1}} + \beta w^h(\theta_0) \right] dF_0(\theta_0) \\
U'(\theta_0) &= \frac{y(\theta_0)^{1 + \epsilon^{-1}}}{\theta_0^{\epsilon^{-1}}} + \beta w_1^h(\theta_0) \\
U(\theta_0) &= u(c(\theta_0)) - \eta \mathbf{1}[y(\theta_0) > 0] - \frac{1}{1 + \epsilon^{-1}} \left(\frac{y(\theta_0)}{\theta_0} \right)^{1 + \epsilon^{-1}} \\
&\quad + \beta (w^c(\theta_0) + w^h(\theta_0))
\end{aligned}$$

Furthermore, when θ_0 is discrete, the problem becomes for $t \geq 1$

$$v_t(w^c, w^h, w_1^h, \theta_-, \theta_0) = \max \int [y(\varepsilon) - c(\varepsilon) + R^{-1} v_{t+1}(w^c(\varepsilon), w^h(\varepsilon), w_1^h(\varepsilon), \varepsilon \theta_-, \theta_0)] dF_t(\varepsilon)$$

subject to

$$\begin{aligned}
w^c &= \int [u(c(\varepsilon)) - \eta \mathbf{1}[y(\varepsilon) > 0] + \beta w^c(\varepsilon)] dF_t(\varepsilon) \\
w^h &= - \int \left[\frac{1}{1 + \epsilon^{-1}} \left(\frac{y(\varepsilon)}{a_t(\theta_0) \varepsilon \theta_-} \right)^{1 + \epsilon^{-1}} + \beta w^h(\varepsilon) \right] dF_t(\varepsilon) \\
w_1^h &= \int \left[\frac{\theta_0^{1 + \epsilon^{-1}}}{(\theta_0 + 1)^{1 + \epsilon^{-1}}} \left(\frac{y_s(\varepsilon)}{a_s(\theta_0 + 1) \theta_- \varepsilon} \right)^{1 + \epsilon^{-1}} + \beta w_1^h(\varepsilon) \right] dF(\varepsilon) \\
&\quad u(c(\varepsilon)) - \eta \mathbf{1}[y(\varepsilon) > 0] - \frac{1}{1 + \epsilon^{-1}} \left(\frac{y(\varepsilon)}{a_t(\theta_0) \varepsilon \theta_-} \right)^{1 + \epsilon^{-1}} \\
&\quad + \beta (w^c(\varepsilon) + w^h(\varepsilon)) \geq \\
&\quad u(c(\hat{\varepsilon})) - \eta \mathbf{1}[y(\hat{\varepsilon}) > 0] - \frac{1}{1 + \epsilon^{-1}} \left(\frac{y(\hat{\varepsilon})}{a_t(\theta_0) \varepsilon \theta_-} \right)^{1 + \epsilon^{-1}} \\
&\quad + \beta \left(w^c(\hat{\varepsilon}) + \frac{\hat{\varepsilon}^{1 + \epsilon^{-1}}}{\varepsilon^{1 + \epsilon^{-1}}} w^h(\hat{\varepsilon}) \right)
\end{aligned}$$

and at $t = 0$

$$v_0(w^c, w^h) = \max \int \left[y(\theta_0) - c(\theta_0) + \frac{1}{R} v_1(w^c(\theta_0), w^h(\theta_0), w_1^h(\theta_0), \theta_0, \theta_0) \right] dF_0(\theta_0)$$

subject to

$$\begin{aligned}
w^c &= \int [u(c(\theta_0)) - \eta \mathbf{1}[y(\theta_0) > 0] + \beta w^c(\theta_0)] dF_0(\theta_0) \\
w^h &= - \int \left[\frac{1}{1 + \epsilon^{-1}} \left(\frac{y(\theta_0)}{\theta_0} \right)^{1 + \epsilon^{-1}} + \beta w^h(\theta_0) \right] dF_0(\theta_0) \\
&u(c(\theta_0)) - \eta \mathbf{1}[y(\theta_0) > 0] - \frac{1}{1 + \epsilon^{-1}} \left(\frac{y(\theta_0)}{\theta_0} \right)^{1 + \epsilon^{-1}} \\
&+ \beta (w^c(\theta_0) + w^h(\theta_0)) \\
&\geq u(c(\theta_0 - 1)) - \eta \mathbf{1}[y(\theta_0 - 1) > 0] - \frac{1}{1 + \epsilon^{-1}} \left(\frac{y(\theta_0 - 1)}{\theta_0} \right)^{1 + \epsilon^{-1}} \\
&+ \beta (w^c(\theta_0 - 1) + w^h(\theta_0 - 1)).
\end{aligned}$$

A.8 Intuitive calibration example

Here, we present a graphical depiction of an exceedingly simple example of a model in our environment. We use this example to provide an intuitive exposition of our strategy to calibrate fixed costs of work in terms of utility.

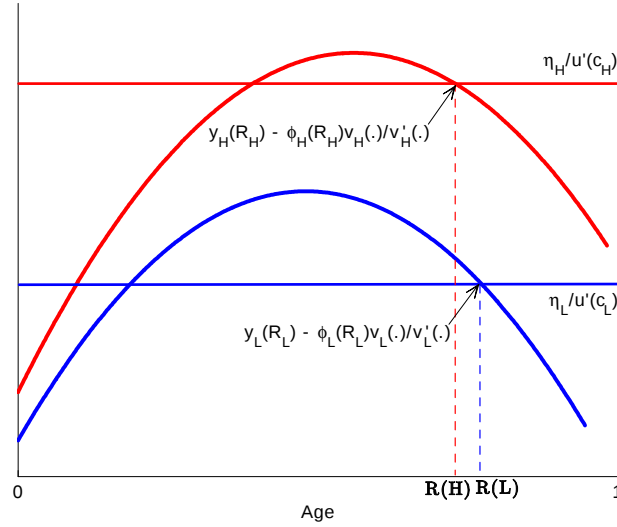


Figure 8: Intuitive calibration strategy in a simple example.

Consider a model in the general environment described in Section 2, but with only two types of individuals, H and L . Suppose that we already have estimated productivity profiles $\varphi_H(t)$ and $\varphi_L(t)$, such that $\varphi_H(t) > \varphi_L(t)$ at all t , as described in Section 6. Further, suppose we already have empirical retirement ages $R(H)$ and $R(L)$ for these

types, such that $R(H) < R(L)$, obtained as in Section 6. For simplicity, suppose for a moment that there are no distortions in the observed data described by the productivity profiles and the retirement ages. Then, individual optimality conditions (21) are graphically depicted in Figure 8 as an intersection of a hump-shaped curve (the left-hand side of (21)) and a horizontal line (the right-hand side of (21)) for both types. That is, Figure 8 intuitively shows that, for a given choice of curvature in the utility function, one can compute implied fixed costs of work, η_H and η_L .

However, the individual optimality conditions identify the ratio of fixed costs to marginal utility of consumption, not the fixed costs themselves. It is perceivable that preferences with much more curvature may result in a very different pattern of fixed costs across types. As we describe in Section 6, we deal with this identification issue by bringing in additional piece of empirical evidence - estimated elasticity of labor supply along the extensive margin. Our calibration strategy is then to iteratively choose implied fixed costs so that simulated extensive elasticity in our model falls in the range of estimates in the literature. In Section 6, we use the range of estimates rather than target a particular number because individual studies are generally not comprehensive and study a particular situation (e.g. low income females) where labor supply responds along the extensive margin.

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