

Risky, Lumpy Human Capital in Household Portfolios*

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Abstract

Illiquid human capital is typically the largest asset in a household's portfolio, for the bulk of working life. Financial assets, which are relatively more liquid, are accumulated throughout life as well, in preparation for both rainy days and life-cycle-related declines in earning ability. Optimal investment in these categories of wealth are necessarily made jointly, and together, determine the evolution of household portfolios over the life cycle. However, existing work has almost exclusively studied these investment decisions separately, in nearly all cases by positing a labor income flow and then deriving implications for household's financial portfolios (Cocco, Gomes, and Maenhout, 2005).

In this paper we aim to understand the evolution of household portfolios, defined broadly enough to include both human and financial wealth positions, over the life-cycle. A key feature of our approach is to include lumpy initial investments (formal education) and subsequent “on the job” training à la Ben-Porath (1967), where both are risky. To our knowledge we are the first to study human and financial investment decisions in such a setting. An important payoff of our approach is a unified view of household wealth over the life cycle. Quantitatively, a key finding is that our model is able to account for limited stock market participation with no appeal whatsoever to transactions costs. Instead, “corner solutions” to stock purchases emerge naturally from the optimality of front loading investment in human capital.

*The views expressed in this paper are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of Richmond or the Federal Reserve System.

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1 Introduction

Human capital is arguably the largest asset that households invest in over their lifetime. They also invest in financial assets, which, unlike human capital, are relatively liquid. The levels and stochastic properties of earnings, which are in turn determined by human capital investments, are also likely to influence household financial portfolios. Furthermore, there is a direct trade-off between investment in human capital, which requires forgoing current earnings, and financial investments. Optimal investments in human capital and financial assets are therefore necessarily made jointly and together determine the evolution of household portfolios over the life cycle.

In this paper we aim to understand the evolution of household portfolios, defined broadly enough to include both human and financial wealth positions, over the life-cycle. A key feature of our approach is to include lumpy initial investments (formal education) and subsequent “on the job” training à la Ben-Porath (1967), both of which are risky. Specifically, we allow young households to decide whether to attend and complete college, and we allow adult households accumulate human capital while working. To our knowledge we are the first to study human and financial investment decisions in such a setting. An important payoff of our approach is a unified view of household wealth over the life cycle.

Our results show that a rich characterization of human capital investment is important to match facts about household portfolios. One set of facts pertains to investment in stocks. Quantitatively, a key finding is that our model is able to account for limited stock market participation with no appeal whatsoever to transactions costs. Instead, “corner solutions” to stock purchases emerge naturally from the optimality of front loading investment in human capital.

Previous literature has examined various aspects of the relationship between human capital investment, returns to human capital and financial portfolios over the life cycle. One strand of the literature focuses on the role of labor income behavior in determining portfolio allocation. Jagannathan and Kocherlakota (1996) and Cocco, Gomes, and Maenhout (2005) point out that labor income usually acts as a substitute for holding a riskless asset, and as such, should encourage households to reduce the share of stocks in their portfolio as they age. Note, however, that this is not consistent with observed investment behavior. Others examine the role of labor supply. For example, Gomes, Kotlikoff, and Viceira (2008) endogenize the labor supply decision, thus allowing households who fare poorly on the stock market to hedge their losses by working more to increase their labor income. They also conclude that the optimal share of stocks in the household’s portfolio should decline with age. Yet others allow for shocks to labor income and risky assets returns to be correlated. For instance, Chai, Maurer, Mitchell, and Rogalla (2011) introduce countercyclical risky labor income dynamics. In contrast to the work cited earlier, these papers find household

portfolio allocation patterns to be consistent with empirical evidence. Finally, Roussanov (2010) focuses on the role of human capital investment. This paper endogenizes human capital investment through an indivisible option—agents can either work or study— and matches the life-cycle profile of household portfolios.

One aspect of human capital investment and its impact on household portfolios that has not been explicitly studied in the literature is the decision to invest in college. The investment itself, as well as subsequent labor market outcomes, has many facets that can potentially influence household portfolio choice. Hungerford and Solon (1987) find that returns to education are not a smooth function of the number of years of education; the return to a year of education that culminates in a diploma is higher than the return to a year that does not. In other words, college requires an investment commitment for certain number of years before it pays off. The risk of failure is high; non-completion rates in the U.S. are around 50 percent (Bowen, Chingos, and McPherson, 2009). Moreover, Ozdagli and Trachter (2011) calculate that the probability of dropping out is highest at the two- and three-year mark, implying that costly investments may already have been made for a number of years before the risk of failure is realized. Finally, college is costly, both in terms of explicit costs as well as the opportunity cost of foregone earnings. Paying for college implies reducing assets or taking on non-defaultable debt. All of these factors taken together are likely to put downward pressure on household investment in stocks. On the other hand, earnings are higher (Card, 1999) and unemployment risk is lower (Mincer, 1991) for those who are more highly educated. This may encourage investment in stocks among those who successfully complete college and realize these gains. Our model enables us to assess the resulting quantitative effect of these forces.

Before laying out the model in detail in Section 3, we describe some facts about household portfolios in the next section. The calibration is laid out in Section 4 and results are provided in Section 5.

2 Data

Our data on household portfolios comes from the Survey of Consumer Finances (SCF). The SCF is a survey of a cross section of U.S. families conducted every three years. It contains a wealth of detailed information on demographics and finances. In particular, it includes investments not just in directly held stocks but also stock investments in mutual funds and retirement accounts.

For comparison, we also include data from the Panel Study of Income Dynamics (PSID). Data on household portfolios comes from the wealth supplement. The supplemental data was collected

every five years between 1984 and 1999, and every other year since 1999. Respondents were asked whether anyone in the family held stock in publicly held corporations, mutual funds or investment trusts. Prior to 1999, respondents were asked to include stocks in employer pensions or Individual Retirement Accounts (IRAs) whereas from 1999 onwards, they were asked to exclude them. This may be one reason why participation is systematically lower in the PSID than in the SCF.

Figure 7 shows that participation is slightly hump-shaped as a function of age.¹ Participation rates are quite distinct by educational attainment, and generally increase with the level of education (Figure 8).

Conditional on participation, households hold a fairly constant share of their wealth in stocks over their life cycle, with the average being 43% (Figure 9). Interestingly, the share is quite similar across different levels of educational attainment (Figure 10).

With these facts in hand, we turn to the description of the model.

3 Model

3.1 Overview

Agents start life in the model as youth with a high-school diploma, endowed with a level of human capital, h^{HS} . At this stage in life, they decide whether or not they will attend college. They also decide how to allocate any wealth they have between a risky asset s_t and a risk-free asset b_t . If they choose to attend college, and have wealth, they can use it to finance their education. They can also borrow using non-defaultable debt to finance their education; this will be captured by $b_t < 0$. We believe that having nondefaultable debt is a good abstraction because individuals close to default will likely have not accumulated resources to have interesting portfolios, and therefore the option to default on consumer debt is not central for bond and stock market choices. This assumption is also consistent with the empirical fact that mortgage debt, which is collateralized, and student debt, which is non-defaultable, are typically the two largest forms of debt in household portfolios.

At the end of four years in college, the probability of completion, which depends on the agent's innate ability and human capital accumulation while in college, is realized. Those who complete college start their adult life with human capital h^{CG} while those who fail to complete accumulate human capital h^{SC} , where SC denotes "some college." Those who choose not to go to college start

¹In the current draft of the paper, we approximate life-cycle behavior by taking averages within age groups across all waves of the surveys. For example, stock market participation among 45-year-olds ranged from a low of 43% in the 1995 SCF to a high of 63% in the 2004 and 2007 percent, with an average participation rate of 55%.

with human capital h^{HS} .

Working adults divide their time between work and the accumulation of human capital, as in the model of Ben-Porath (1967). They consume and, as before, allocate any savings between stocks and bonds. As adults, they also have the option to borrow, that is $b_t \geq -\underline{b}$, with $\underline{b} > 0$, may be positive or negative. As before, they cannot default on this debt.

We allow for three potential sources of heterogeneity across agents — their immutable learning ability, a , human capital stock, h , and initial assets, x . The set of these characteristics are jointly drawn according to a distribution $F(a, h, x)$ on $A \times H \times X$ and we allow for and estimate correlations between returns to stocks, bonds, and human capital.

Time is discrete and indexed by $t = 1, \dots, T$ where $t = 1$ represents the first year after high school graduation. Agents work and accumulate human capital until $t = J - 1$ and start retirement in period $t = J$ when they face a simple consumption-savings problem.

3.2 Preferences

The general problem of an individual is to choose consumption over the life-cycle, $\{c_t\}_{t=1}^T$ to maximize the expected present value of utility over the life cycle

$$\max_{(\{c_t\} \in \Pi(\Psi_0))} E_0 \sum_{t=1}^T \beta^{t-1} \gamma_t u(c_t), \quad (1)$$

where $u(\cdot)$ is strictly concave and increasing. Also, $\Pi(\Psi_0)$ denotes the space of all feasible combinations $\{c_t\}_{t=1}^T$, given initial state Ψ_0 . Agents have a common discount factor, β and survival probability, γ_t . The latter is age specific. Preferences are represented by a standard time-separable CRRA utility function over consumption. Agents value consumption and do not value leisure.

3.3 Technology and financial markets

There are two financial assets in which the agent can invest, a risk-free asset, b_t , and a risky asset, s_t .

Risk-free assets

An agent can borrow or save using asset b_t which can be 0, positive, or negative. Savings will earn the risk-free interest rate, R_f . We assume that the borrowing rate, R_b , is higher than the savings rate: $R_b = R_f + \phi$. Debt is non-defaultable.

Risky assets

We call the risky assets stocks and we denote the agent's holdings of equity between period t and $t + 1$ by s_{t+1} . This amount entitles the owner to its stochastic return in period $t + 1$, $R_{s,t+1}$. This represents a gross real return and its excess return is given by:

$$R_{s,t+1} - R_f = \mu + \eta_{t+1}, \quad (2)$$

where η_{t+1} , the period $t + 1$ innovation to excess returns, is assumed to be independently and identically distributed (i.i.d.) over time and distributed as $N(0, \sigma_\eta^2)$. We allow innovations to excess returns to be correlated with innovations to the aggregate component of permanent labor income, as we explain in Section 2.5.

Given asset investments at age t , b_{t+1} and s_{t+1} , financial wealth at age $t + 1$ is given by $x_{t+1} = R_i b_{t+1} + R_{s,t+1} s_{t+1}$, with $R_i = R_f$ if $b \geq 0$ and $R_i = R_b$ if $b < 0$.

3.4 Human capital

Agents can invest in their human capital in two ways — by investing in a college education when young and by apportioning some of their time to acquiring human capital as adults, as on-the-job training. Human capital stock refers to “earning ability” and can be accumulated over the life cycle, while learning ability is fixed at birth and does not change over time. We assume that the technology for human capital accumulation is the same during and after college and that human capital is not productive until graduation.

3.4.1 Human capital investment as on-the-job training

College graduates, college dropouts and high school graduates who do not enroll in college optimally allocate time between market work and human capital accumulation as on-the-job training during the adult phase of their life.

Human capital evolves according to the human capital production, $H(a, h_t, l_t)$, which depends on the agent's immutable learning ability, a , human capital, h_t , and the fraction of available time put into human capital production, l_t . Human capital depreciates at a rate δ_i with $i \in \{cg, nc\}$ and $\delta_{cg} > \delta_{nc}$ where nc stands for both individuals with some college, SC and for high-school graduates who do not go to college, HS . The law of motion for human capital is given by

$$h_{t+1} = h_t(1 - \delta_i)H(h_t, l_t, a) \quad (3)$$

Following Ben-Porath (1967), the human capital production function is given by $H(h, l, a) = a(hl)^\alpha$ with $\alpha \in (0, 1)$.

3.4.2 College investment

Agents who wish to acquire a college degree optimally divide time between work and human capital while in college; they may invest in both risky and risk-free assets. In addition, they may choose to borrow to finance their college education. They face two types of risks: dropping out of college and uncertainty in their earnings after college. The college dropout risk depends on the human capital stock at the end of college, which in turn is determined by the agent's decision to allocate time to human capital accumulation during college. There are several sources of college financing: family contributions for college and need-based loans. Students may also use their labor income and savings during college to finance their college education.

During college, students may choose to work at the wage rate w_{col} , but their human capital is not productive until they leave college. Working during college diverts time from human capital accumulation and may increase students' chances of leaving college without acquiring a degree. At the same time, college students have jobs that pay a low wage and do not necessarily value students' human capital stock or contribute to human capital accumulation (Autor, Levy, and Murnane (2003)). However, students of high ability may be hired in better paid jobs than students of low ability.² Thus, we model a wage rate per time units worked in college, $w_{col}(a)$, instead of per efficiency units; this rate increases with the ability level of the student. This assumption prevents low-ability students from enrolling in college only to enjoy earnings during college that are much higher than the earnings they would have earned had they not enrolled in college. We assume that the growth rate in earnings during college is 0.

Agents are allowed to take out student loans up to $d(x) = \bar{d} - x$, which represents the full college cost, $\bar{d}$, minus the expected family contribution, x . They choose the loan amount, d , at the beginning of college, and they receive equal fractions of the loan each period in college. After college they will repay this loan in equal payments, p which are determined by the loan size, $d(x)$, interest rate on student loans, R_g and the duration of the loan, P . Consistent with the data, the interest rate on student loans is $R_f < R_g < R_b$.

College investment is risky. If a student with initial human capital h_1 decides to acquire a

²Our modeling is motivated by several important observations. Numerous studies show that people who choose to work while in school are more likely drop out of college (see Braxton, Hirschy, and McClendon (2003) and Stinebrickner and Stinebrickner (2008)). Thus, unlike in an environment where the possibility of dropping out from college is not modeled (or is completely exogenous), allowing for the choice to allocate time to work versus human capital accumulation during college is a key ingredient when accounting for the risk of investing in human capital.

college degree, the probability with which she succeeds is given by $\pi(h_5(h_1, a, l_{1,\dots,4}^*))$. This is a continuous, increasing function of the human capital stock after college years, h_5 , which in turn increases with the initial human capital stock, h_1 , the ability of the individual, a , and her choice of time devoted to human capital investment during college years, $l_{1,\dots,4}^*$. This formulation captures the idea that college preparedness, embodied in h_1 , student's learning capacity, captured in a , and effort to invest in human capital during college are important determinants of college completion. If the student completes college, she will walk into period 5 as a college graduate, i.e. $i = CG$, and if she does not complete college, she will walk into period 5 as a college dropout with some college education, i.e. $i = SC$.³

3.5 Labor Income

During the adult phase, $t = t_w, \dots, J-1$, with $t_w = 1$ for no college and $t_w = 5$ for college graduates and dropouts, human capital stock is valued each period in the labor market. Earnings are given by product of the stochastic component, z_t , the rental rate of human capital, w_t , the agent's human capital, h_t , and the time spent in market work, $(1 - l_t)$.

Therefore, agent's i earnings in period t are given by

$$\log(y_t) = G(w_t, h_t, l_t) + z_t \quad (4)$$

with $G(w_t, h_t, l_t)$ representing the deterministic component as a function of rental rate w_t , human capital stock at age t , h_t and labor effort, $1 - l_t$ and z_t representing the stochastic component. The rental rate of human capital evolves over time according to $w_t = (1 + g_i)^{t-1}$ with the growth rate, g_i with $i \in \{cg, nc\}$. Depending on whether agents have a college degree or not, they face different growth rates in the rental rate with $g_{cg} > g_{nc}$.⁴

The stochastic component, z_{it} consists of an idiosyncratic temporary shock $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$ and a persistent shock $u_{it} = \rho u_{i,t-1} + \nu_{it}$, with $\nu_{it} \sim N(0, \sigma_\nu^2)$ innovation process. The variables u_{it} and ϵ_{it} are realized at each period over the life cycle and are not correlated. The process for u_{it} is taken to be a random walk, following Gourinchas and Parker (2002) and Hubbard, Skinner, and Zeldes

³Modeling investment in four-year college and the risk of dropping out at the end of the fourth period in the model are justified by data: according to the BPS 1996/2001, 68.5% of students enroll in four-year colleges. Our findings also show that 89% of college dropouts are enrolled in college at least for three full years. Details are provided in the Appendix.

⁴The growth rates for wages are estimated from data. Evidence shows that wage growth rates for college dropouts and people with no college are similar and are lower than the wage growth rate for college graduates. Also, human capital depreciates faster for college graduates than for college dropouts and individuals who do not enroll in college. See Section 4.1 for details.

(1994). The latter estimate a general first-order autoregressive process and find the autocorrelation coefficient to be very close to one. We assume that the temporary shock ϵ_{it} is uncorrelated across households, but we decompose the permanent shock ν_{it} into an aggregate component $\xi_{it} \sim N(0, \sigma_\xi^2)$ and an idiosyncratic component $\omega_{it} \sim N(0, \sigma_\omega^2)$. This decomposition implies that the random component of aggregate labor income follows a random walk, an assumption made in the finance literature (see Fama and Schwert (1977) and Jagannathan and Wang (1996)).

Finally, we allow for the correlations between returns to human capital and financial assets through the aggregate component of the income process, ξ_t . Let Σ be the covariance matrix for the payoffs on all assets where

$$[R_f, R_s, \xi_t] \sim N(\mu, \Sigma) \quad (5)$$

3.6 Means-Tested Transfer and Retirement Income

In addition to labor income, agents receive means-tested transfers from the government, τ_t , which depend on age, t , income, y_t , and net assets, x_t . These transfers capture the fact that in the U.S. social insurance is aimed at providing a floor on consumption. Following Hubbard, Skinner, and Zeldes (1994) we specify these transfers by

$$\tau_t(t, y_t, x_t) = \max\{0, \underline{\tau} - (\max(0, x_t) + y_t)\} \quad (6)$$

Total pre-transfer resources are given by $\max(0, x_t) + y_t$ and the means-testing restriction is represented by the term $\underline{\tau} - \max((0, x_t) + y_t)$. These resources are deducted to provide a minimal income level $\underline{\tau}$. For example, if $x_t + y_t > \underline{\tau}$ and $x_t > 0$, then the agent gets no public transfer. By contrast, if $x_t + y_t < \underline{\tau}$ and $x_t > 0$, the the agent receives the difference, case in which he has $\underline{\tau}$ units of the consumption good at the beginning of the period. Agents do not receive transfers to cover debts, which requires the term $\max(0, x_t)$. Lastly, transfers are required to be nonnegative, which requires the “outer” max.

After period $t = J$ when agents start retirement, they get a constant fraction of their income in the last period as working adults, $\phi(y_J)$ which they divide between risky and risk-free investments.⁵

⁵We will relax this assumption in the quantitative part of the paper where we introduce an additional uncertainty during retirement (e.g., stochastic medical expenses along the lines of Hubbard, Skinner, and Zeldes (1995)), since this may slow down the pace at which wealth is being depleted.

3.7 Agent's Problem

The agent maximizes lifetime utility by choosing asset positions in bonds and stocks, time allocated to market work and to human capital, and borrowing.

We formulate the problem in a dynamic programming framework where any period t variable j_t is denoted by j and its period $t+1$ value by j' . The state vector is defined as follows. An individual's feasible set of consumption and savings is determined by his age, t , ability, a , beginning-of-period human capital, h and net worth, $x(b, s)$, current-period realization of the persistent shock to earnings, u , and current-period transitory shock, ν .

We solve the problem backwards starting with the last period of life when agents consume their savings. The value function in the last period of life is set to $V_T^R(a, h, x) = u(x)$. For the retirement phase, the value function is given by

$$V^R(t, a, h, b, s) = \sup_{b', s'} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \beta V^R(t+1, a, h', b', s') \right\} \quad (7)$$

where

$$c + b' + s' \leq \phi(y_J) + R_i b + R_s s$$

In the above, $R_i = R_f$ if $b \geq 0$ and $R_i = R_b$ if $b < 0$.

Retired agents do not accumulate human capital. They face a simple consumption-savings problem but may choose to invest in both risk-free and risky assets.

Given important differences between education groups during the remaining stages of the life cycle, we present the value functions and further details separately for the no-college and college paths.

3.7.1 No College

We use $V_J^{R,i}(t, a, h, b, s)$ from Equation 7 as a terminal node for the adult's problem on the no college path. Note that $V_R^{HS}(t, a, h, b, s, u, \nu) = V_R^R(a, h', b', s') \forall u, \nu$.⁶ We solve for the set of choices in the working phase, for which the value function is given by

$$V^{HS}(t, a, h, b, s, u, \nu) = \sup_{l, h', b', s'} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \beta E_{u'/u} V^{HS}(t+1, a, h', b', s', u', \nu') \right\} \quad (8)$$

where

$$c + b' + s' \leq w(1-l)hz + R_b b + R_s s + \tau(t, y, x) \text{ for } t = 1, \dots, J-1$$

⁶The same idea is used whenever we switch from one phase to another without explicitly stating this.

$$l \in [0, 1], h' = h_t(1 - \delta_{nc}) + \pi(a)a(hl)^\alpha$$

The value function $V^{HS}(t, a, h, b, s, u, \nu)$ gives the maximum present value of utility at age t from states h , b , and s , when learning ability is a and the realized shocks are u and ν . Solutions to this problem are given by optimal decision rules $l_j^*(t, a, h, b, s, u, \nu)$, $h^*(t, a, h, b, s, u, \nu)$, $b^*(t, a, h, b, s, u, \nu)$ and $s^*(t, a, h, b, s, u, \nu)$, which describe the optimal choice of the fraction of time spent in human capital production, the level of human capital, and risk-free and risky assets carried to the next period as a function of age, t , human capital, h , ability, a , and current assets, b and s when the realized state is (u, ν) . The value function, $V^{HS}(1, a, h, x)$, gives the maximum expected present value of utility if the agent chooses not to go to college from initial state h , when learning ability is a and initial assets are x .

3.7.2 College

As before, the problems for college graduates and dropouts are solved backwards, starting with the retirement phase for which the value function is given by Equation 7.

Working phase, $j = 5, \dots, J - 1$

We use $V_j^{R,i}(t, a, h, b, s)$ from the retirement phase as a terminal node and solve for the set of choices in this phase of the life-cycle. We further break down this phase into a post-repayment period and a repayment period. For the post-repayment period, $t = P + 1, \dots, J - 1$, the problem is identical to the one for working adults on the no-college path. Recall that during the repayment period after college, $t = 5, \dots, P$, agents have to repay their student loans with a per period payment $p = \frac{d(x)}{\sum_{t=1}^{P-5} R_g^t}$ with $d(x_1)$ the size of the loan which depends on initial assets x_1 and R_g the interest rate on student loans.

The value function is given by

$$V^i(t, a, h, b, s, u, \nu) = \sup_{l, h', b', s'} \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} + \beta E_{u'/u} V^i(t+1, a, h', b', s', u', \nu') \right\} \quad (9)$$

where

$$\begin{aligned} c + b' + s' &\leq w(1-l)hz + R_j b + R_s s + \tau(t, y, x) \text{ for } t = P+1, \dots, J-1 \\ c + b' + s' &\leq w(1-l)hz + R_j b + R_s s + \tau(t, y, x) - p(x_1) \text{ for } t = 5, \dots, P \\ l &\in [0, 1], h' = h_t(1 - \delta_i) + \pi(a)a(hl)^\alpha \end{aligned}$$

where $i = CG, SC$; $R_i = R_f$ if $b \geq 0$ and $R_i = R_b$ if $b < 0$.

College phase, $t = 1, \dots, 4$

For this stage of the life-cycle we first take into account the risk of dropping out from college and use $V^C(5, a, h, b, s, u, \nu) = \pi(h_5)V^{CG}(5, a, h, b, s, u, \nu) + (1 - \pi(h_5))V^{SC}(5, a, h, b, s, u, \nu)$ as the terminal node to solve for the optimal rules. Agents invest in their human capital during college and they may decide to work. Each period in college they pay direct college expenses, $\hat{d}$. In the first period, they also choose the loan amount for college education, d , which will be equally divided in four rounds of loans during college years. The value function is given by

$$\begin{aligned}
V^C(t, a, h, b, s) = & \max_{l, h', b', s', d} \left[\frac{c^{1-\sigma}}{1-\sigma} + \beta V^C(t, a, h, b, s) \right] \\
& c + b' + s' = w_{col}(1-l) + R_b b + R_s s + d/4 - \hat{d} \\
& l \in [0, 1], \quad h' = h(1 - \delta_c) + a(hl)^\alpha \\
& d \in D = [0, \bar{d}(x)] \text{ for } t = 1.
\end{aligned} \tag{10}$$

Solutions to this problem are given by optimal decision rules $l_j^*(t, a, h, b, s, u, \nu)$, $h^*(t, a, h, b, s, u, \nu)$, $b^*(t, a, h, b, s, u, \nu)$ and $s^*(t, a, h, b, s, u, \nu)$, which describe the optimal choice of the fraction of time spent in human capital production, the level of human capital, and risk-free and risky assets carried to the next period as a function of age, t , human capital, h , ability, a , and current assets, b and s when the realized state is (u, ν) . The value function, $V^C(1, a, h, x)$, gives the maximum expected present value of utility if the agent chooses to go to college from initial state h , when learning ability is a and initial assets are x .

Once the college and no-college paths are fully determined, agents then select between going to college or not by solving $\max[V^C(1, a, h, x), V^{HS}(1, a, h, x)]$.

4 Mapping the model to the data

There are four sets of parameters that we calibrate: 1) standard parameters, such as the discount factor and the coefficient of risk aversion; 2) parameters for the initial distribution of characteristics; 3) parameters specific to human capital and to the earnings process; and 4) parameters specific to asset markets and default. Our approach includes a combination of setting some parameters to values that are standard in the literature, calibrating some parameters directly to data, and jointly estimating the parameters that we do not observe in the data by matching moments for several observable implications of the model.

Adult age starts at age 20 for households without a college degree, and at age 24 for households with a college degree or with some college education. The per period utility function is CRRA,

$u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$, with the coefficient of risk aversion $\sigma = 5$, which is consistent with values chosen in the financial literature. Risk aversion is a key parameter and so we conduct robustness checks on it, in particular we consider higher values up to the upper bound of $\sigma = 10$ considered reasonable by (Mehra and Prescott, 1985). We also consider lower values, such as $\sigma = 3$. The discount factor ($\beta = 0.96$) chosen is also standard in the literature.

We find the joint distribution of unobserved characteristics by matching statistics of life-cycle earnings in the March Current Population Survey (CPS) for 1969-2002. Agents live 58 model periods, which corresponds to ages 21 to 78.⁷

Education groups are based on years of education completed with exactly 12 years for high school graduates who do not go to college, more than 12 years and less than 16 years of completed schooling for college dropouts, and with 16 and 17 years of completed schooling for college graduates.⁸ Life-cycle profiles of earnings are given in Figure 11.

The rental rate on human capital equals $w_t = (1 + g)^{t-1}$, and the growth rate is calibrated to match the PSID data on earnings as in Huggett, Ventura, and A.Yaron (2006). Given the growth rate in the rental rates, $g = 0.0014$, the depreciation rate is set to $\delta = 0.0114$, so that the model produces the rate of decrease of average real earnings at the end of the working life cycle. The model implies that at the end of the life cycle negligible time is allocated to producing new human capital and, thus, the gross earnings growth rate approximately equals $(1 + g)(1 - \delta)$. We set the elasticity parameter in the human capital production function, α , to 0.7. Estimates of this parameter are surveyed by Browning, Hansen, and Heckman (1999) and range from 0.5 to 0.9.

In the parametrization of the stochastic component of earnings, z_{it} , we follow Gallipoli et. al. (2010) who use the National Longitudinal Survey of Youth (NLSY) data using CPS-type wage measures to estimate the autoregressive coefficients for the transitory and persistent shocks to wages. For the persistent shock, $u_{it} = \rho u_{i,t-1} + \nu_{it}$, with $\nu_{it} \sim N(0, \sigma_\nu^2)$ innovation process and for idiosyncratic temporary shock $\epsilon_{it} \sim N(0, \sigma_\epsilon^2)$ they report the following values for high school graduates: $\rho = 0.955$, $\sigma_\omega^2 = 0.055$, and $\sigma_\nu^2 = 0.017$ and college graduates: $\rho = 0.945$, $\sigma_\omega^2 = 0.052$, and $\sigma_\nu^2 = 0.02$. We use the first set of values for people with no college and some college education, and the second set of values for those who complete four years of college. Recall that the temporary shock ϵ_{it} is uncorrelated across households, but the permanent shock ν_{it} is divided into an aggregate component ξ_{it} (distributed as $N(0, \sigma_\xi^2)$) and an idiosyncratic component

⁷For each year in the CPS, we use earnings of heads of households age 25 in 1969, age 26 in 1970, and so on until age 58 in 2002. We consider a five-year bin to allow for more observations, i.e., by age 25 at 1969, we mean high school graduates in the sample that are 23 to 27 years old. Real values are calculated using the CPI 1982-1984. For each year the following statistics are computed: mean, inverse skewness and the gini coefficient.

⁸Education groups in the model are identified by years of schooling in the CPS data since information on the type of the degree obtained is not available.

ω_{it} (distributed as $N(0, \sigma_\omega^2)$). We estimate the aggregate component of the income process, σ_ξ^2 , together with the returns to financial assets when estimating the covariance matrix for the payoffs, Σ in Equation 5. We set retirement income to be a constant fraction of labor income earned in the last year in the labor market. Following Cocco (2005) we set this fraction to 0.682 for high school graduates and for individuals with some college education and to 0.93 for college graduates.

We turn now to the last set of parameters in the model: those related to financial markets. We consider the mean equity premium $\mu = 0.06$. The risk-free rate is set equal to $R_f = 0.04$, consistent with values in the literature (McGrattan and Prescott (2000)) and the wedge between the borrowing and risk-free rate is $\phi = 0.07$ to match the average borrowing rate of $R_b = 0.11$ (according to the G 19 report of the Board of Governors). The interest rate on student loans is set to $R_g = 0.068$. The standard deviation of innovations to the risky asset is set to its historical value, $\sigma_\eta = 0.157$. Lastly, we estimate the covariance matrix for the payoffs on all assets to match key facts, such as the fraction of household portfolios held in each type of assets at each age t .

4.1 The Distribution of Assets, Ability and Human Capital

We estimate the joint distribution of initial assets, ability, and human capital by accounting for correlations between all these three characteristics in the following way. First, for the asset distribution, we use the SCF data as described in the Data section. Second, we calibrate the initial distribution of ability and human capital to match key properties of the life-cycle earnings distribution in the U.S. data. In order to carry out this procedure, we use the CPS 1969-2002 family files for heads of household aged 25 in 1969 and followed until 2002 for life-cycle earnings. Earnings distribution dynamics implied by the model are determined in several steps: i) we compute the optimal decision rules for human capital using the parameters described above for an initial grid of the state variable; ii) we simultaneously compute financial investment decisions and compute the life-cycle earnings for any initial pair of ability and human capital; and iii) we choose the joint initial distribution of ability and human capital to best replicate the properties of U.S. data.

Using a parametric approach, we search over the vector of parameters that characterize the initial state distribution to minimize the distance between the model and the data statistics. We restrict the initial distribution on the two dimensional grid in the space of human capital and learning ability to be jointly, log-normally distributed. This class of distributions is characterized by 5 parameters. In practice, the grid is defined by 20 points in human capital and ability. We find the vector of parameters $\gamma = (\mu_a, \sigma_a, \mu_h, \sigma_h, \rho_{ah})$ characterizing the initial distribution by solving the minimization problems $\min_\gamma \left(\sum_{j=5}^J |\log(m_j/m_j(\gamma))|^2 + |\log(d_j/d_j(\gamma))|^2 + |\log(s_j/s_j(\gamma))|^2 \right)$, where m_j , d_j , and s_j are mean, dispersion, and inverse skewness statistics constructed from the CPS

data on earnings, and $m_j(\gamma)$, $d_j(\gamma)$, and $s_j(\gamma)$ are the corresponding model statistics. Overall, we match 102 moments.⁹ Figure 1 illustrates the earnings profiles for individuals in the model versus CPS data when the initial distribution is chosen to best fit the three statistics considered. We obtain a fit of 8% (0% would be a perfect fit). The model performs well given riskiness of assets and stochastic earnings in the current paper.¹⁰ The model produces a correlation between ability and human capital of 0.77.

5 Results

First we study a version of the model where we restrict attention to human capital investment through on-the-job training as in Ben-Porath (1967). We focus on trade-offs between investments in financial assets and investments in human capital over the life cycle absent any college investment decision and college risk. Results are presented in Section 5.1. Next we add the college enrollment and dropout decisions to the model along with student loan debt and study the implications of college investment and financing on life-cycle portfolios. Results are presented in Section 5.2.

5.1 On-the-Job Human Capital Investment and Financial Investments

In this section we describe what our model predicts in terms of human capital investment and stock market participation. Note that we did not explicitly “match” these to the data, so they provide an independent metric of how well our model fits the facts.

The first set of figures (Figure 2) shows human capital accumulation over the life cycle. Households invest the greatest share of their time in human capital accumulation when young both because their opportunity cost of doing so is low and their absolute level of human capital is low. Human capital accumulates rapidly till agents are in their mid-30s after which it levels off as the time horizon shrinks and there is less time to collect the returns to human capital investment.

The incentives to invest in human capital over the life cycle dictate investments in risky assets. For young individuals human capital is a relatively high return investment. They would rather allocate more time to human capital and less to work. Consequently, they have lower participation rates in the stock market and invest a lower amount, on average, in risky assets. Once human

⁹For details on the calibration algorithm see Ionescu (2009).

¹⁰For instance, Huggett et. al. (2008) obtains a fit of 7% (for the same value of the elasticity parameter $\alpha = 0.7$) in a Ben Porath model where the main choice is investment in human capital to maximize lifetime earnings in a framework without investments in financial assets, debt, and earnings uncertainty. As a measure of goodness of fit, we use $\frac{1}{3J} \sum_{j=5}^J |\log(m_j/m_j(\gamma))| + |\log(d_j/d_j(\gamma))| + |\log(s_j/s_j(\gamma))|$. This represents the average (percentage) deviation, in absolute terms, between the model-implied statistics and the data.

Figure 1: Life-cycle earnings

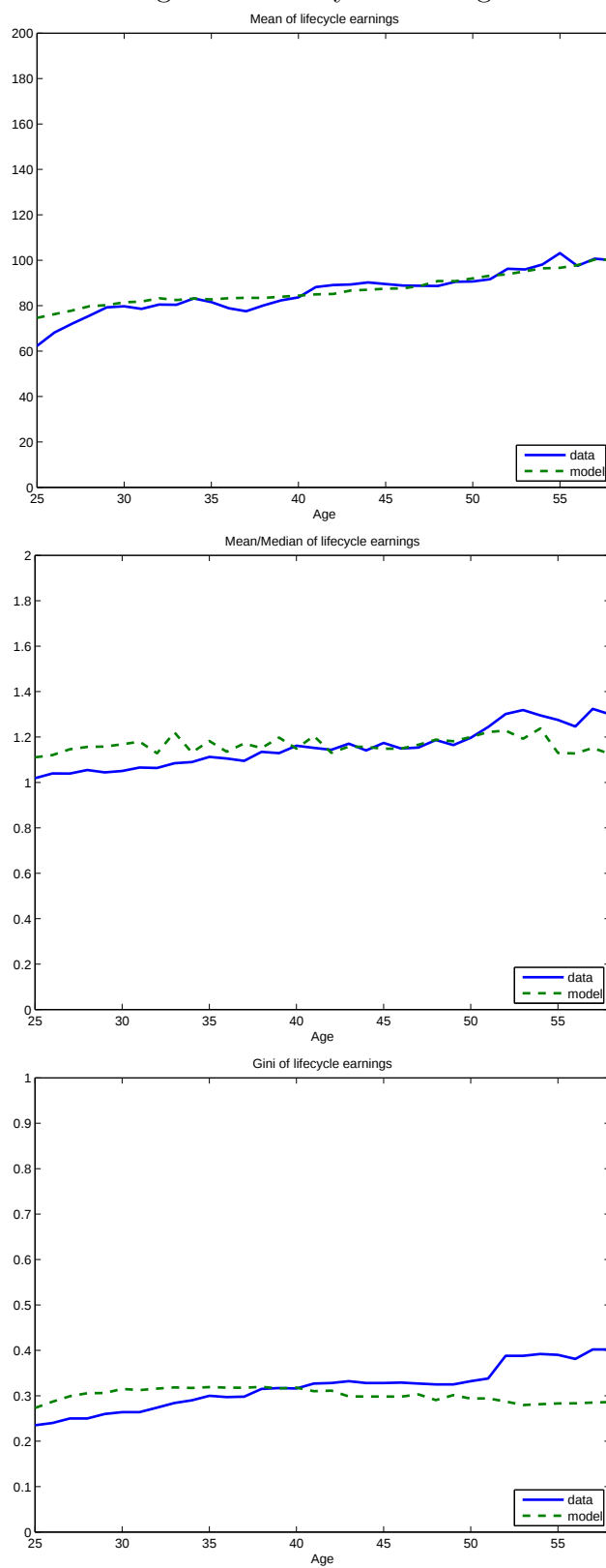
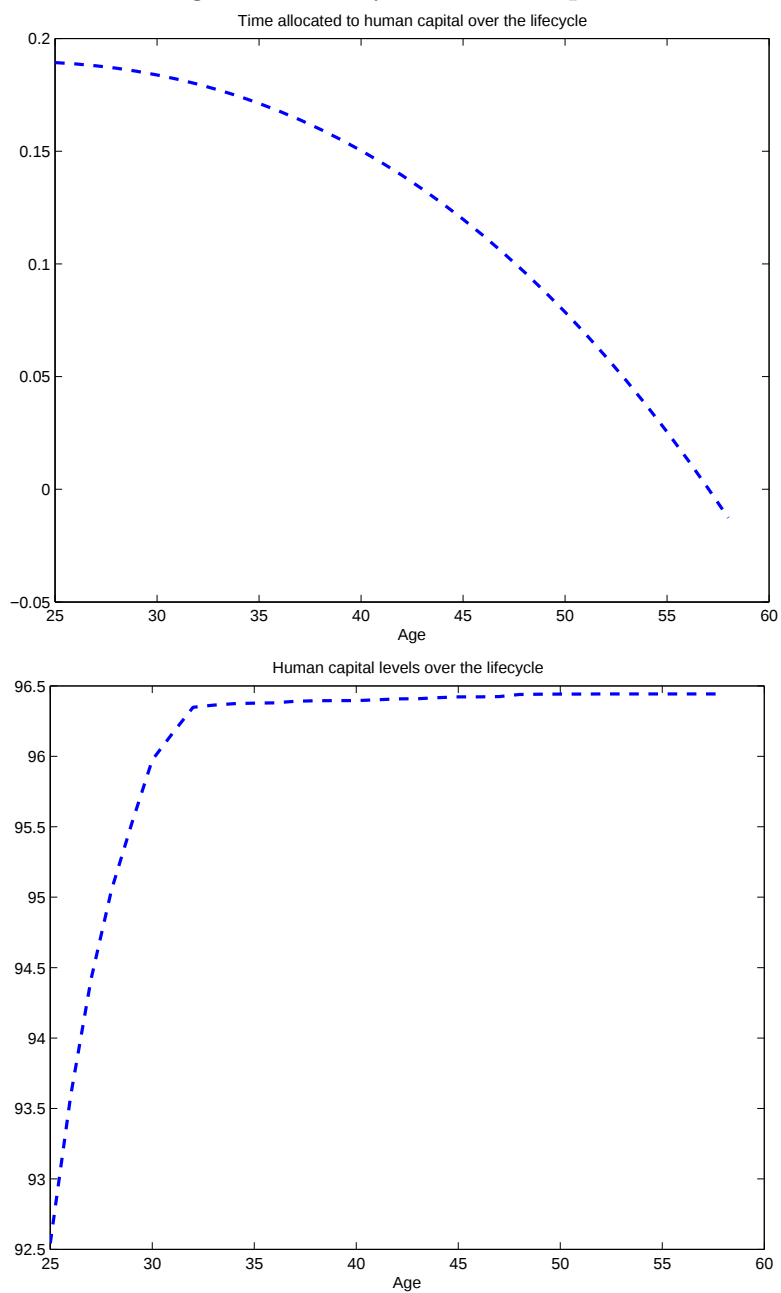


Figure 2: Life-cycle human capital



capital stock increases, however, human capital investment becomes more costly and delivers lower returns (higher opportunity cost and less time to collect). As a result, older individuals substitute away from human capital investment to financial investments. They choose to devote less time to human capital and instead increase their participation rates and the amount invested in risky assets. This trade-off is key in delivering life cycle financial investments in our model.

Our second set of results shows the model's predictions of stock market participation. This is illustrated in Figure 3, in which the results are compared with three sets of data - the SCF (averaged over cross sections), the 1992 SCF and the PSID. The profile of participation is qualitatively consistent with the data as well as quantitatively close. The model predicts that participation increases steadily with age and then starts to decline around age 50, as observed in the SCF.

Figure 3: Stock Market Participation over the Life Cycle

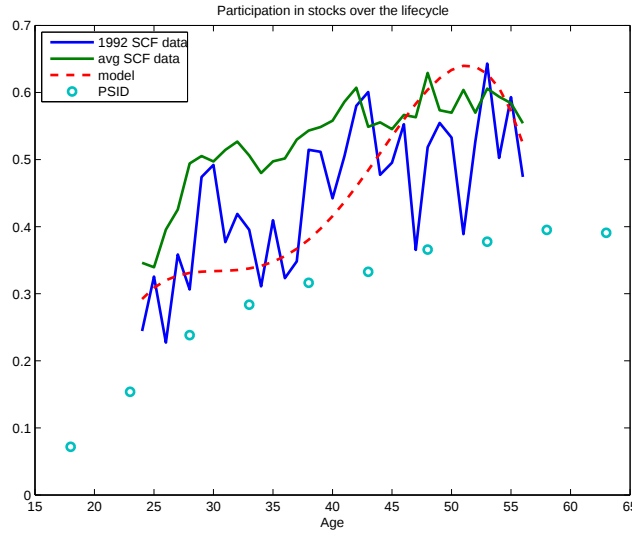
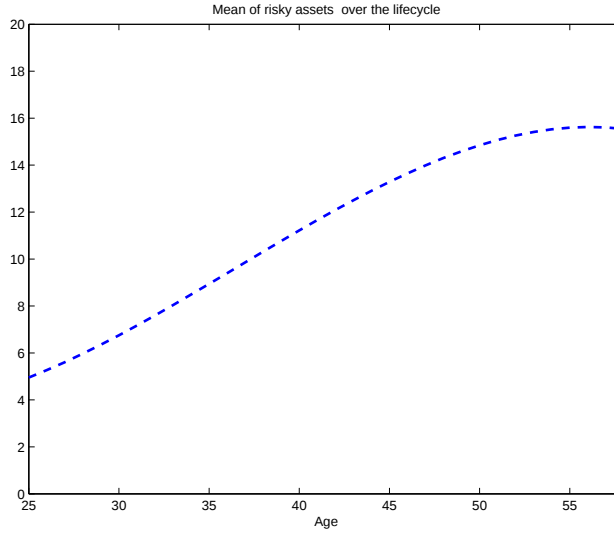


Figure 4 shows the life cycle profiles of new investments in risky assets each period in the model (year in the life cycle). The model predicts that investment in risky assets increases over the life cycle, which is also consistent with the data.

Figure 4: Life-cycle investments in risky financial assets



To summarize, our model does well overall in predicting patterns in life cycle household portfolios that are consistent with the data. Furthermore, the trade-offs between human capital and financial investments vary across individuals with different levels of initial human capital and ability. The next steps will be to see results disaggregated by initial human capital levels.

5.1.1 The importance of initial human capital

We now discuss results from the model disaggregated by initial human capital. The top panel of Figure 5 shows the model's predictions of life cycle earnings by quartile of initial human capital. The bottom two panels show levels of human capital and time invested in human capital accumulation, both of which follow a similar pattern across quartiles. Note that lifetime earnings are an increasing function of initial human capital levels and the differences across quartiles are quantitatively important. The incentives to invest in human capital are not monotone. While all groups invest more time in human capital at the beginning of their lives, individuals in the top quartile of initial human capital invest relatively less while individuals in the bottom quartiles devote relatively more time to human capital investment. For the bottom quartile, human capital investment has a low opportunity cost.

As a result, the participation in the stock market varies substantially across individuals with different levels of initial human capital. The participation of households in the top half of the distribution of initial human capital steadily increases with age, prediction which is consistent with the data. Households in the bottom half of the distribution of human capital follow a different pattern of participation. Around 30% of households age 25 participate in the stock market. This

figure reduces to only 10% by age 30, after which it increases again. Participation among 50-year-olds averages 50% after which age it declines again.

Lastly, individuals in the top half of the distribution invest higher amounts in risky assets relative to the bottom half of the distribution. This is a direct implication of the fact that individuals in the top quartiles have relatively high earning abilities and low incentives to invest in human capital.

5.2 College Investment: Implications for Life-Cycle Portfolios

In this section we study the interaction between college investment, on-the-job investment in human capital and financial assets over the life-cycle. We focus on the role of college risk and college debt for life-cycle portfolios. Results to be added.

6 Conclusion

We developed a heterogeneous life-cycle stochastic economy with complex life-cycle portfolio decisions that include both investments in risky human capital and financial assets. The predictions of our model are qualitatively and quantitatively consistent with observed household portfolio investments in human capital and financial assets. The model produces the observed profiles of earnings of high school graduates from the right distribution of initial characteristics. It also successfully characterizes investment behavior across individuals with different earning abilities.

Our model is able to account for limited stock market participation with no appeal whatsoever to transactions costs. Instead, behavior in the stock market emerges naturally from the optimality of front loading investment in human capital. Our results show the importance of accounting for the interaction between investments in human capital and financial assets when studying household portfolio decisions.

Figure 5: Life-cycle earnings and human capital by quartiles

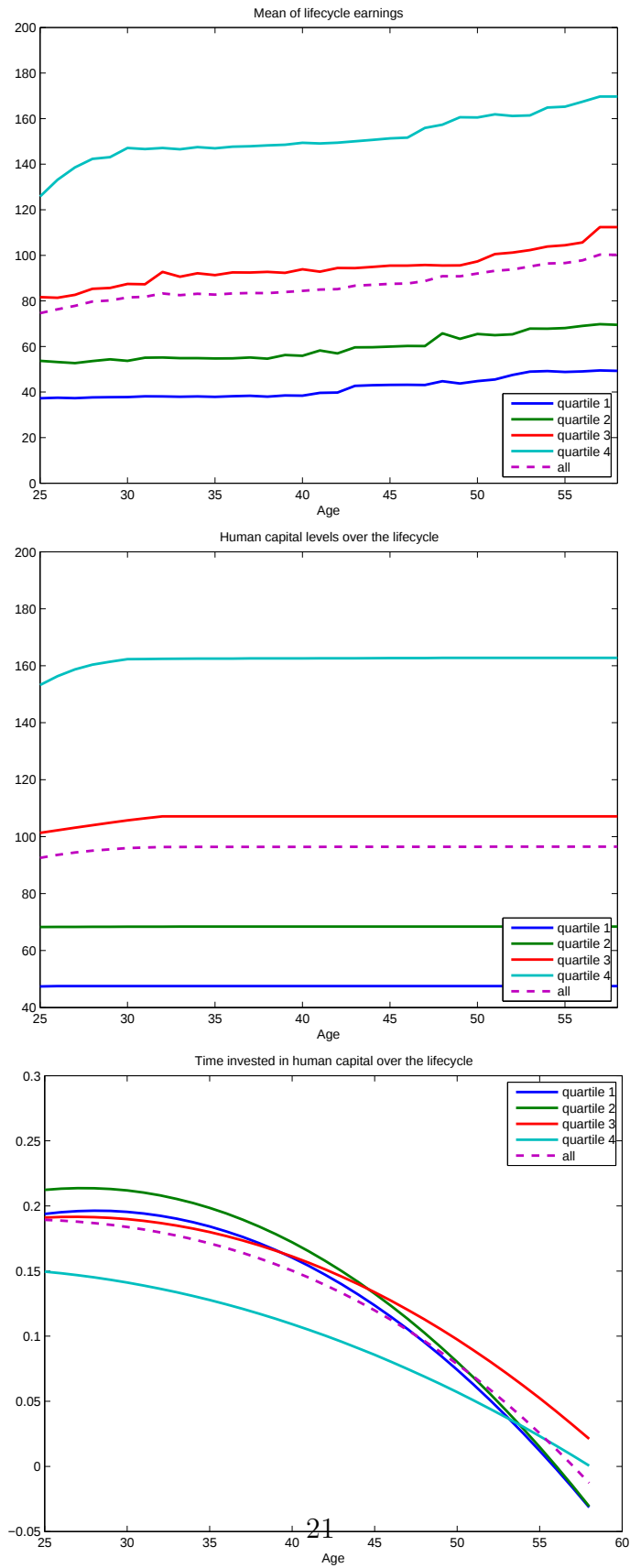
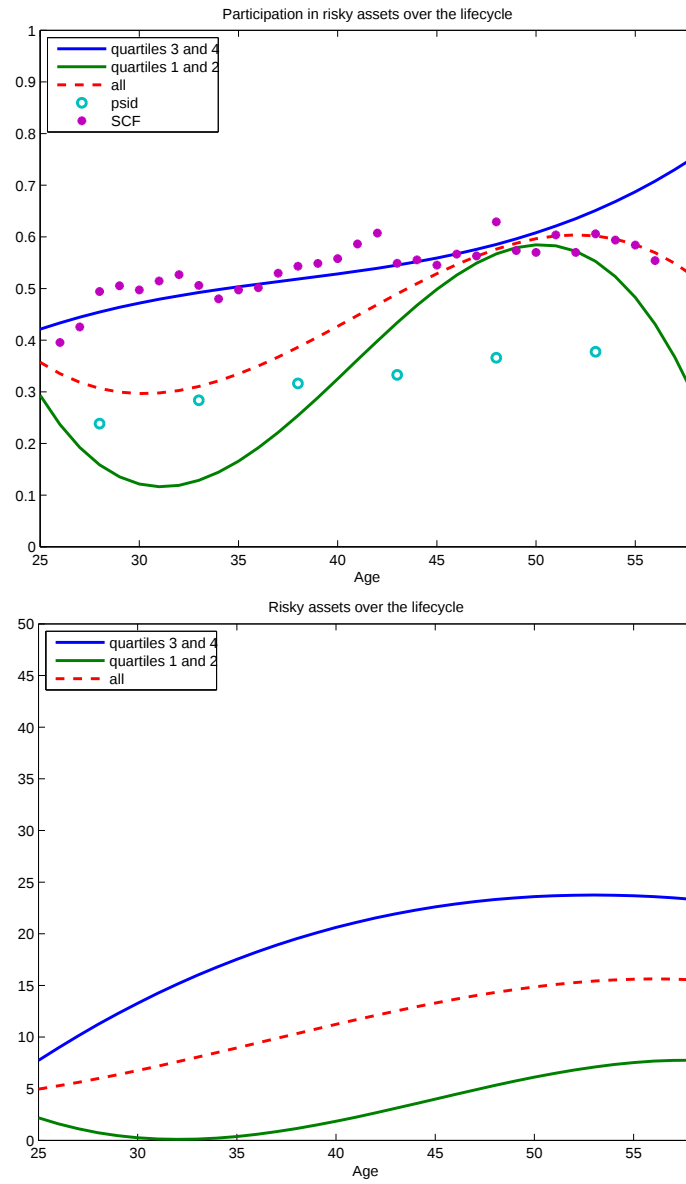


Figure 6: Life-cycle financial portfolios



A Figures

A.1 Financial portfolios

Figure 7: Household Stock Market Participation Rate by Age (SCF)

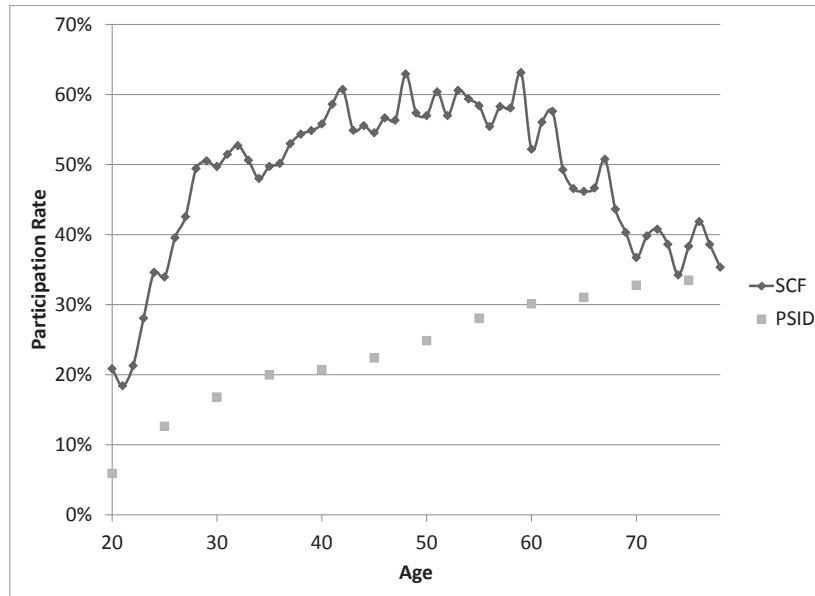


Figure 8: Household Stock Market Participation Rate by Age and Educational Attainment (SCF)

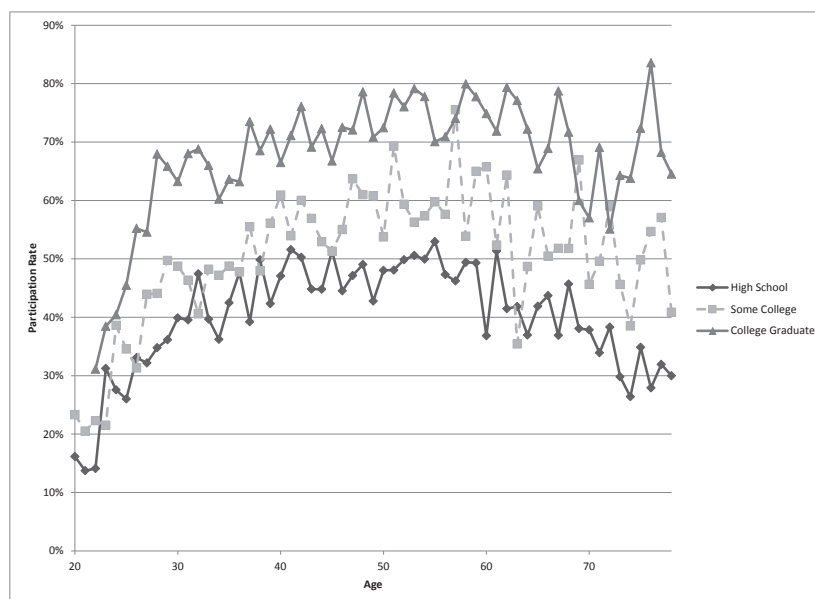


Figure 9: Share of Stocks in Household Portfolio by Age (SCF)

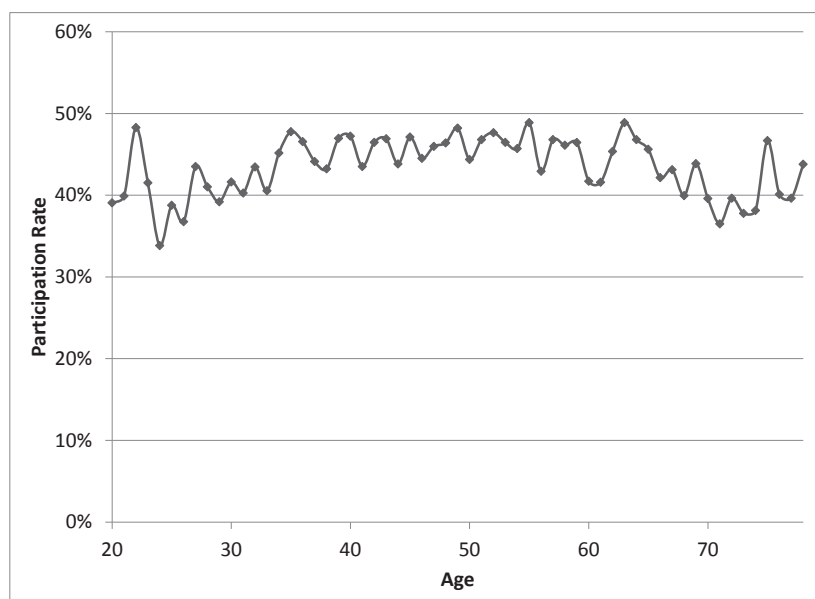
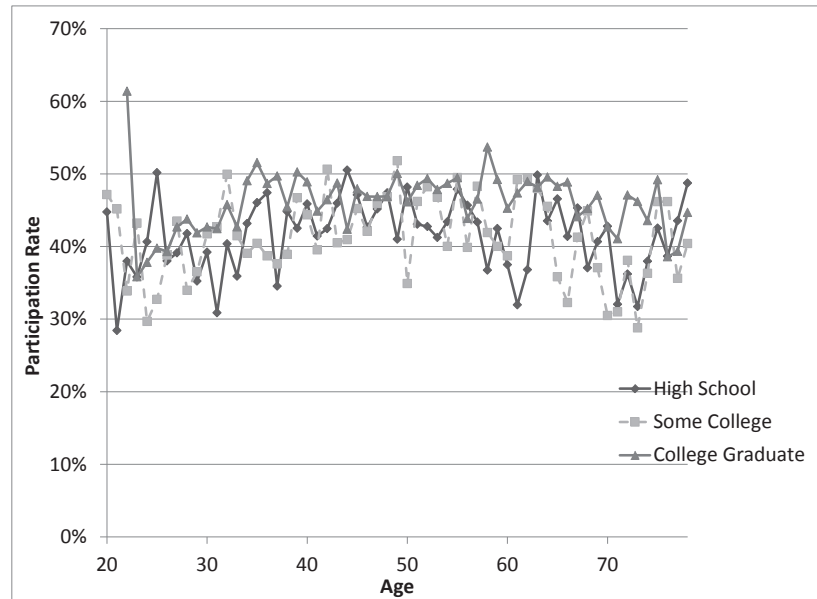
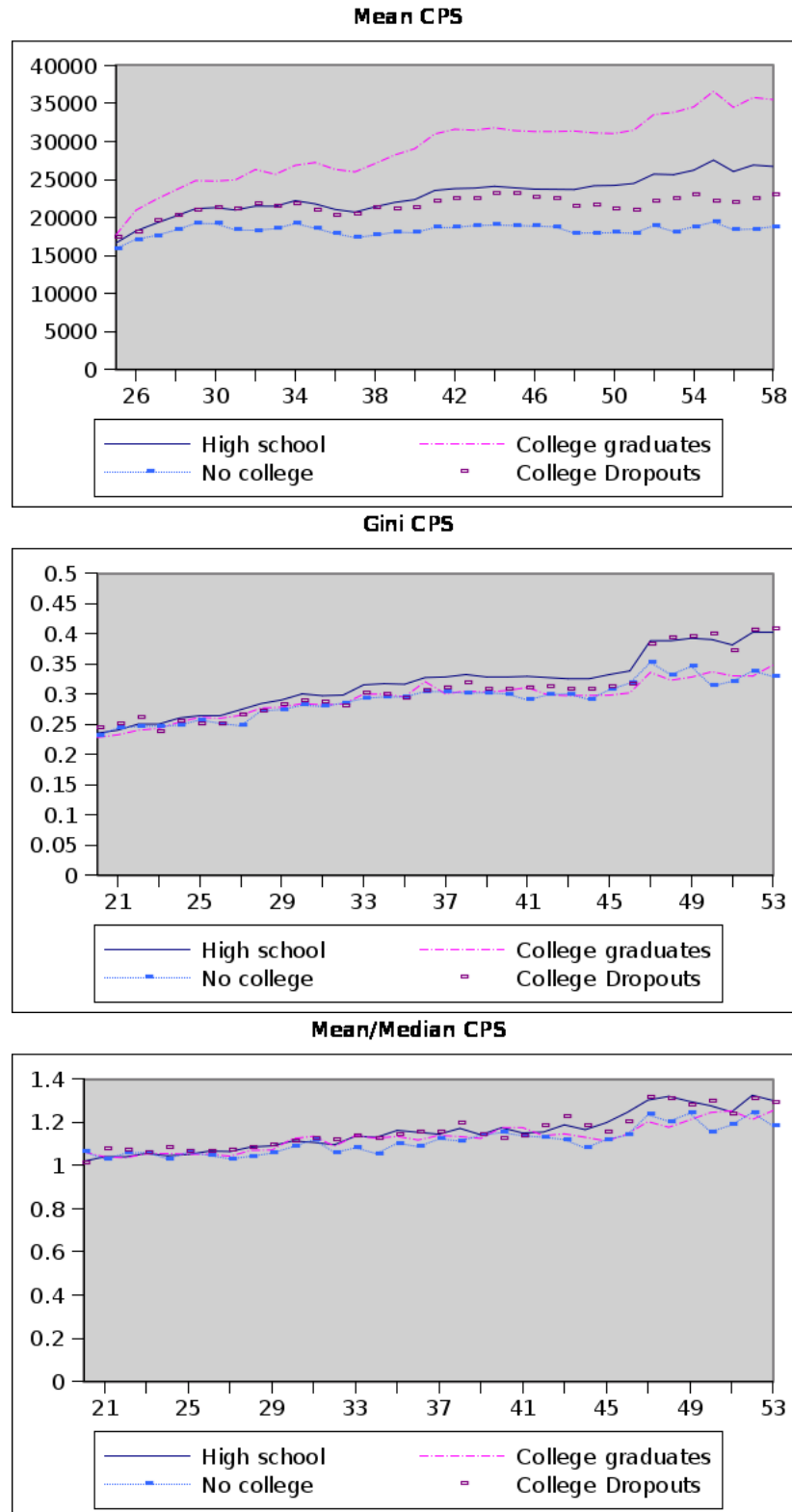


Figure 10: Share of Stocks in Household Portfolio by Age and Educational Attainment (SCF)



A.2 Earnings

Figure 11: Statistics of Earnings by Education Groups: Data CPS



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