Self-Fulfilling Credit Cycles*

Costas Azariadis[†] Leo Kaas[‡] Yi Wen[§]

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Abstract

In U.S. data 1981–2012, unsecured firm credit moves procyclically and tends to lead GDP, while secured firm credit is at best acyclical. In this paper we develop a tractable dynamic general equilibrium model in which unsecured firm credit arises from self-enforcing borrowing constraints preventing an efficient capital allocation among heterogeneous firms. Capital from less productive firms is lent to more productive ones in the form of credit secured by collateral and also as unsecured credit based on reputation which is a forward-looking variable. We argue that self-fulfilling beliefs over future credit conditions naturally generate endogenously persistent business cycle dynamics. A dynamic complementarity between current and future borrowing limits permits uncorrelated sunspot shocks to trigger persistent aggregate fluctuations in debt, factor productivity and output. We show that sunspot shocks are quantitatively important, accounting for a substantial part of the volatility in firm credit and output.

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[†]Washington University in St. Louis and Federal Reserve Bank of St. Louis, E-mail: azariadi@artsci.wustl.edu

[‡]University of Konstanz, E-mail: leo.kaas@uni-konstanz.de

[§]Federal Reserve Bank of St. Louis and Tsinghua University, E-mail: yi.wen@stls.frb.org

1 Introduction

Over the past two decades, important advances in macroeconomic research illustrated how financial market conditions can play a key role in business cycle fluctuations. Starting with seminal contributions of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), much of this research shows how frictions in financial markets can amplify and propagate disruptions to macroeconomic fundamentals, such as shocks to total factor productivity (TFP) or to monetary policy.[1] More recently, and to some extent motivated by the events of the last financial crisis, several theoretical and quantitative contributions argue that shocks to the financial sector itself may not only lead to severe macroeconomic consequences but can also contribute significantly to business cycle movements. For example, Jermann and Quadrini (2012) develop a model with stochastic collateral constraints which they identify as residuals from aggregate time series of firm debt and collateral capital. Estimating a joint stochastic process for TFP and borrowing constraints, they find that both variables are highly autocorrelated and that financial shocks play an important role in business cycle fluctuations.[2] But what drives these shocks to financial conditions and to aggregate productivity? And what makes their responses so highly persistent?

This paper argues that unsecured firm credit is of key importance for answering these questions. We first document new facts about secured versus unsecured firm credit. Most strikingly, for the U.S. economy over the period 1981–2012, we find that unsecured debt is strongly procyclical, with some tendency to lead GDP, while secured debt is at best acyclical, thus not contributing to the well-documented procyclicality of total debt. This finding provides some challenge for business-cycle theories based on the conventional view of Kiyotaki and Moore (1997) that collateralized debt amplifies and even generates the business cycle. When credit is secured by collateral, a credit boom is associated with not only a higher leverage ratio but also a higher value of the collateralized assets. Conversely, an economic slump is associated with deleveraging and a decrease in the value of collateral. This suggests that secured debt, such as the mortgage debt, should be strongly correlated with GDP. But this is not what we find; to the contrary, based on firm-level data from Compustat and on aggregate data from the Flow of Funds Accounts, we show that it is the unsecured part of firm credit which strongly comoves with output.

[1]For recent surveys, see Quadrini (2011) and Brunnermeier et al. (2012).

[2]Other examples of financial shocks are Kiyotaki and Moore (2012) who introduce shocks to asset resaleability, Gertler and Karadi (2011) who consider shocks to the asset quality of financial intermediaries, and Christiano et al. (2010) who use risk shocks originating in the financial sector. These papers also impose or estimate highly persistent shock processes.

To examine the macroeconomic role of unsecured firm debt, we develop and analyze a parsimonious dynamic general equilibrium model with heterogeneous firms and limited credit enforcement. In the model, credit constraints and TFP are endogenous variables. Constraints on unsecured credit depend on the value that borrowers attach to future credit market conditions which is a forward-looking variable. TFP depends on the reallocation of existing capital among heterogeneous firms which, among others, depends on current credit constraints. When these constraints bind, they slow down capital reallocation between firms and push aggregate factor productivity below its frontier. We show that this model exhibits a very natural equilibrium indeterminacy which gives rise to endogenous cycles driven by self-fulfilling beliefs in credit market conditions (sunspot shocks). In particular, a one-time sunspot shock triggers an endogenous and persistent response of endogenous borrowing constraints and of TFP.

The model is a standard stochastic growth model which comprises a large number of firms facing idiosyncratic productivity shocks. In each period, productive firms wish to borrow from their less productive counterparts. Besides possibly borrowing against collateral, the firms exchange unsecured credit which rests on reputation. Building upon Bulow and Rogoff (1989) and Kehoe and Levine (1993), we assume that a defaulting borrower is excluded from future credit for a stochastic number of periods. As in Alvarez and Jermann (2000), endogenous forward–looking credit limits prevent default. These credit limits depend on the value that a borrower attaches to a good reputation which itself depends on future credit market conditions. An important contribution of this paper is the tractability of our framework which permits us to derive a number of insightful analytical results in Section 3. With standard and convenient specifications of preferences and technology, we characterize any equilibrium by one backward-looking and one forward-looking equation (Proposition 1).[3] With this characterization, we prove that unsecured credit cannot support first-best allocations, thereby extending related findings of Bulow and Rogoff (1989) and Hellwig and Lorenzoni (2009) to a growth model with idiosyncratic productivity (Proposition 2). We then prove the existence of multiple stationary equilibria for a range of parameter configurations (Proposition 3). While there is always an equilibrium without unsecured credit, there can also exist one or two stationary equilibria with a positive volume of unsecured credit. One of these equilibria supports an efficient allocation of capital between firms, and another one features a misallocation of capital. The latter equilibrium is the one that provides the most interesting insights, since unsecured credit is traded and yet factor productivity falls short of the efficient technology frontier.[4] We show that

[3]Much of the literature on limited enforceability of unsecured credit does not allow for such simple representations and therefore resorts to rather sophisticated computational techniques (see e.g. Kehoe and Perri (2002), Krueger and Perri (2006) and Marcet and Marimon (2011)).

[4]The other, determinate steady states of this model either do not sustain unsecured credit (and hence

this equilibrium is always locally indeterminate, and hence permits the existence of sunspot cycles fluctuating around the stationary equilibrium (Proposition 4). Moreover, output and credit respond persistently to a one-time sunspot shock.

In Section 4 we calibrate this model to the U.S. economy. While sunspot shocks are the main driving force for fluctuations in unsecured credit, we also introduce fundamental shocks to collateral and to aggregate technology. This allows us to analyze to which extent different financial shocks, separately affecting secured and unsecured credit, as well as independent aggregate productivity shocks, contribute to the observed movements in output and factor productivity in the recent business-cycle episodes. We find that sunspot shocks generate a significant share of total output volatility. We further demonstrate that an *uncorrelated* sunspot process generates highly persistent responses of key macroeconomic variables, with autocorrelation coefficients matching their empirical counterparts reasonably well. Similarly persistent responses are neither generated by fundamental financial shocks to collateral, nor by aggregate technology shocks. In fact, the estimated shock process requires a high autocorrelation of the shock series itself to match the data. Thus, the propagation of expectational shocks is an inherent feature of the endogenous model dynamics of unsecured credit.

Intuitively, the explanation for sunspot cycles and persistence is a dynamic complementarity in endogenous constraints on unsecured credit. Borrowers' incentives to default depend on their expectations of future credit market conditions, which in turn influence current credit constraints. If borrowers expect a credit tightening over the next few periods, their current default incentives become larger which triggers a tightening of current credit. This insight also explains why a one-time expectational shock *must be followed* by a long-lasting response of credit market conditions (and thus of macroeconomic outcomes): if market participants expect that a credit boom (or a credit slump) will die out quickly, these expectations could not be powerful enough to generate a sizable credit boom (or slump) today.

Another way to understand the role of expectations is that unsecured credit is like a bubble sustained by self-fulfilling beliefs, as has been argued by Hellwig and Lorenzoni (2009). Transitions from a "good" macroeconomic outcome with plenty of unsecured credit to a "bad" outcome with low volumes of unsecured credit can be triggered by widespread skepticism about the ability of financial markets to continue the provision of unsecured credit at the volume needed to support socially desirable outcomes, which is similar to the collapse of a speculative bubble.[5] The emergence and the bursting of rational bubbles in financially constrained economies

resemble similar dynamics as in a Kiyotaki–Moore–type model with binding collateral constraints) or they have an efficient allocation of capital (and hence exhibit the same business cycle properties as a frictionless model).

[5]Although we use a similar enforcement mechanism as Hellwig and Lorenzoni (2009), the existence of

has received attention in a number of recent contributions, e.g. Caballero and Krishnamurthy (2006), Kocherlakota (2009), Farhi and Tirole (2011) and Miao and Wang (2012). One difficulty with many of the existing macroeconomic models with bubbles is that the no-bubble equilibrium is an attracting steady state, so that they can only account for the bursting of bubbles but not for their buildup. Although there are no bubbles in our model, its equilibrium dynamics account for recurrent episodes of credit booms and busts which are solely driven by self-fulfilling beliefs. In a recent contribution, Martin and Ventura (2012) construct a model with permanent stochastic bubbles and they discuss the economy's response to belief shocks (investor sentiments), like we do. But in their model bubbles arise in an overlapping generations model with two-period lived investors for similar reasons as in Tirole (1985), whereas we consider a standard business cycle model with infinitely–lived households that permits a quantitative application.

Our work is also related to a literature on sunspot cycles arising from financial frictions. In an early contribution, Woodford (1986) shows that a simple borrowing constraint makes infinitely-lived agents behave like two-period-lived overlapping generations, so that endogenous cycles can occur with sufficiently strong income effects or with increasing returns in production (see e.g. Behabib and Farmer (1999) for a survey).[6] Harrison and Weder (2010) introduce a production externality in a Kiyotaki-Moore (1997) model and show that sunspots emerge for reasonable values of returns to scale. Other recent contributions find equilibrium multiplicity and indeterminacy in endowment economies with limited credit enforcement under specific assumptions about trading arrangements (Gu and Wright (2011)) and on the enforcement technology (Azariadis and Kaas (2012b)).[7] Perri and Quadrini (2011) develop a two-country model with financial frictions and show that self-fulfilling expectations of asset values may be responsible for the international synchronization of credit tightening.

The rest of this paper is organized as follows. In the next section, we document empirical evidence about secured and unsecured firm credit in the U.S. economy. Section 3, we lay out the model framework, we characterize all equilibria by a forward-looking equation in the reputation values of borrowers, and we derive our main results on equilibrium multiplicity,

multiple equilibria does not hinge on this specification. In fact, multiple equilibria with different levels of unsecured credit would also emerge if we used the stronger enforcement of Kehoe and Levine (1993) (i.e. two-sided market exclusion of defaulters in perpetuity).

[6]Although earlier work on indeterminacy has shown that sunspot shocks can induce persistent macroeconomic responses (e.g. Farmer and Guo (1994)), the adjustment dynamics are typically sensitive to the particular specifications of technologies and preferences. In our model, persistent responses arise necessarily due to the dynamic complementarity in endogenous credit constraints.

[7]Azariadis and Kaas (2012a) consider a multi-sector endogenous growth model with limited enforcement and also document equilibrium multiplicity due to a similar dynamic complementarity in credit constraints.

indeterminacy and sunspot cycles. In Section 4 we conduct a quantitative analysis to explore the separate impacts of sunspot shocks and fundamental shocks on business cycle dynamics. Section 5 concludes.

2 Unsecured versus Secured Firm Debt

This section summarizes evidence about firms' debt structure and its cyclical properties. We explore different firm-level data sets, covering distinct firm types, and we relate our findings to evidence obtained from the Flow of Funds accounts. In line with previous literature,[8] we show that unsecured debt constitutes a substantial part of firms' debt and is typically lower for samples including smaller firms. Time-series variation, whenever available, further indicates that unsecured debt plays a much stronger role for aggregate output dynamics than debt secured by collateral. We first describe the data and the variables measuring unsecured and secured debt and then report business cycle features.

2.1 Firm-level Data

We start with the publicly traded U.S. firms covered by Compustat for the period 1981–2012 for which Compustat provides the item "dm: debt mortgages and other secured debt". In line with Giambona and Golec (2012), we use this item to measure secured debt, with the residual debt attributed to unsecured debt. The (aggregate) unsecured debt share is then defined as the ratio between unsecured debt and total debt. To clean the data, we remove financial firms and utilities, and we also remove those firm-year observations where total debt is negative, where item "dm" is missing or where "dm" exceeds total debt. Since Compustat aggregates can easily be biased by the effect of the largest firms in the sample (cf. Covas and den Haan (2011), we also consider subsamples where we remove the largest 1% or 5% of the firms by their assets size.[9] To see the impact of the largest firms for unsecured borrowing, Figure 1 shows the series of the unsecured debt share for the three samples obtained from Compustat. The role of the largest firms is quite important for the level of the unsecured debt share, although much less for the time variation.[10] The very biggest firms are likely to have better access to

[8] This connects to a recent corporate finance literature examining heterogeneity in the debt structure across firms (e.g. Rauh and Sufi (2010), Giambona and Golec (2012) and Colla et al. (2013)), though not addressing business cycles.

[9] In the Appendix, we also consider series for which all firm-level variables are winsorized at the 1% and at the 99% level in order to remove the effects of outliers. We find that all results are robust to this adjustment.

[10] While the effect of the largest firms is also important for total debt *growth*, it is not important for its cyclicality, as we show in the Appendix.

bond markets and hence borrow substantially more unsecured. Removing the largest 1% (5%) of firms, however, cuts out 45% (75%) of the aggregate firm credit in the sample. Interestingly, in the years prior to the financial crisis of 2007-08, the unsecured debt share fell substantially, as firms expanded their mortgage borrowing relatively faster than other types of debt, with some reversal after 2008.

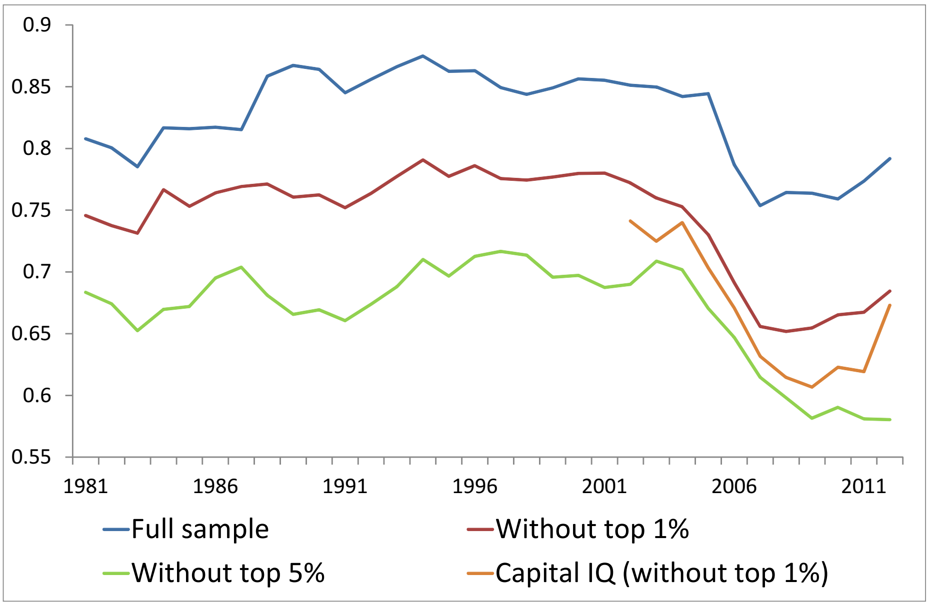

Figure 1: The share of unsecured debt in total debt for firms in Compustat and in Capital IQ.

While Compustat covers public firms, the vast majority of U.S. firms is privately owned. To complement the above evidence, we also explore two data sets so as to obtain debt structure information for private firms. We first look at firms included in the database of Capital IQ which is an affiliate of Standard and Poor's that produces the Compustat database but covers a broader set of firms. Since coverage by Capital IQ is comprehensive only from 2002 onwards, we report these statistics for the period 2002–2012. We clean the data in the same way as above and consider aggregates for the full sample (without financials and utilities) and for the sample without the 1% (5%) of the largest firms. Similar to the Compustat definitions, we use Capital IQ item "SEC: Secured Debt" and the residual "DLC+DLTT-SEC" to measure unsecured debt. The resulting unsecured debt shares show a similar cyclical pattern as those from Compustat for the same period. For visual clarity, Figure 1 only includes the series with the largest 1% of firms removed. We note that including larger firms or removing the top 5% of firms has similar effects as in Compustat, though it does not affect the U–shaped cyclical pattern in the figure. Relative to the corresponding series in Compustat, firms in Capital IQ

borrow more secured in all years, which is possibly explained by the fact that these firms have a lower market transparency and hence less access to bond markets.[11]
It is important to remark that even the private firms included in the Capital IQ database are relatively large firms with some access to capital markets, so they are also not fully representative for the U.S. business sector. To obtain evidence on the debt structure of small firms, we utilize the data collected in the Survey of Small Business Finances (SSBF) conducted by the Federal Reserve Board in 2003. Beyond similar surveys in the years 1987, 1993 and 1998, we cannot obtain evidence across time, however. Firms in this survey report their balances in different debt categories (and within each category for up to three financial institutions). For each loan, they report whether collateral is required and which type of collateral is used (real estate, equipment and others). We aggregate across firms for each debt category and measure as secured debt all the loans for which collateral is required, while unsecured debt comprises credit card balances and all loans without reported collateral requirements. We minimally clean the data by only removing observations with zero or negative assets or equity. Table 1 shows the results of this analysis. While mortgages and credit lines constitute the largest debt categories of small firms, accounting for almost three quarters of the total, significant fractions of the other three loan categories are unsecured. This results in an unsecured debt share of 19.3 percent for firms in the SSBF.[12]
The evidence presented in Figure 1 and in Table 1 suggests that the unsecured debt share lies somewhere between 0.2 (for the smallest firms) and 0.73 (for Compustat firms except the top 1%).[13] To obtain an aggregate measure of unsecured debt, we can further utilize the information in the Flow of Funds accounts in which firm debt is categorized into several broad categories. About 95% of all credit market liabilities of non-financial firms since 1981 are either attributed to mortgages (31%), loans (31%) or corporate bonds (33%). Among the non-mortgage loans in Table 1, 29.5% are unsecured, which is likely a higher share for bigger

[11] Firms in our Capital IQ sample are actually bigger than Compustat firms. In the period 2002-2012, the average asset size of Compustat firms in the full (bottom 99%; bottom 95%) samples are 2,602 (1,230; 550) Mio. Dollars, whereas Capital IQ firms in the full (bottom 99%; bottom 95%) samples have average asset size 3,391 (2,028 and 1,142). In total, there are about twice as many observations in Compustat than in Capital IQ in each year.

[12] Because collateral requirement is a dummy variable, only a fraction of these loans might actually be secured by collateral. This measure of unsecured credit should therefore be regarded as a lower bound.

[13] Note that the latter number is consistent with those found in two other studies about the debt structure of Compustat firms. Rauh and Sufi (2010) examine the financial footnotes of 305 randomly sampled non-financial firms in Compustat. Based on different measures, their unsecured debt share (defined as senior unsecured plus subordinated debt relative to total debt) is 70.3%. Giambona and Golec (2012) look at the distribution of unsecured debt shares for Compustat firms, reporting mean (median) values of 0.63 (0.75).

Table 1: Secured and unsecured debt in the Survey of Small Business Finances

Debt category	Share of debt	Secured by real estate/equipment	Secured by other collateral	Unsecured
Credit cards	0.6	0.0	0.0	100.0
Lines of credit	36.5	39.4	38.5	22.1
Mortgages	38.0	98.0	0.4	1.7
Motor vehicle loans	4.8	52.1	2.1	45.8
Equipment loans	6.5	62.0	1.7	36.4
Other loans	13.6	53.6	6.3	40.1
Total	100.0	65.4	15.2	19.3

Notes: All numbers are debt percentages.

firms. With corporate bonds attributed to unsecured credit and mortgages to secured credit, we end up with an unsecured credit share of at least 44%. In the calibration part of this paper we target this share at the value 0.5 and include some sensitivity analysis for higher and lower values.

2.2 Business Cycle Features

2.2.1 Compustat

We consider the time series from Compustat, deflate them by the price index for business value added, and linearly detrend the real series.[14] We label the detrended secured debt as d_t^s, unsecured debt as d_t^u, and real GDP as y_t. Table 2 reports the relative volatility as well as the contemporaneous correlations with output. Secured debt is weakly negatively correlated with GDP in the full sample, it becomes zero and weakly positive once we exclude the top 1% or 5% firms. In sharp contrast, unsecured debt is always strongly positively correlated with GDP. Thus, the well-known procyclicality of firm credit is driven by the independent role of unsecured debt. Both secured and unsecured debt are about three to four times as volatile as output.

Figure 3 graphs the correlations between current GDP and lagged (future) real debt levels. The top panel pertains to the full sample, the middle panel to the sample without the largest 1% of firms, and the bottom panel to the sample without the largest 5% of firms. Regardless of

[14] We use a linear trend to capture the low-frequency movements in credit and output that are quite significant over the period 1981–2012.

Table 2: Relative Volatility and Comovement with Output (Compustat)

	Relative Volatility			Correlation with GDP		
	full	w/o top 1%	w/o top 5%	full	w/o top 1%	w/o top 5%
Secured	3.61	3.39	2.76	-0.15	-0.05	0.15
Unsecured	4.19	3.73	4.43	0.70	0.70	0.75

the sample, unsecured debt (i) is positively correlated with GDP, (ii) leads GDP significantly by at least one year (the peak correlation is about 0.75 at one year lead). In sharp contrast, secured debt is (i) uncorrelated or negatively correlated with GDP, and (ii) tends to lag GDP even when the contemporaneous correlation is weakly positive (bottom panel).

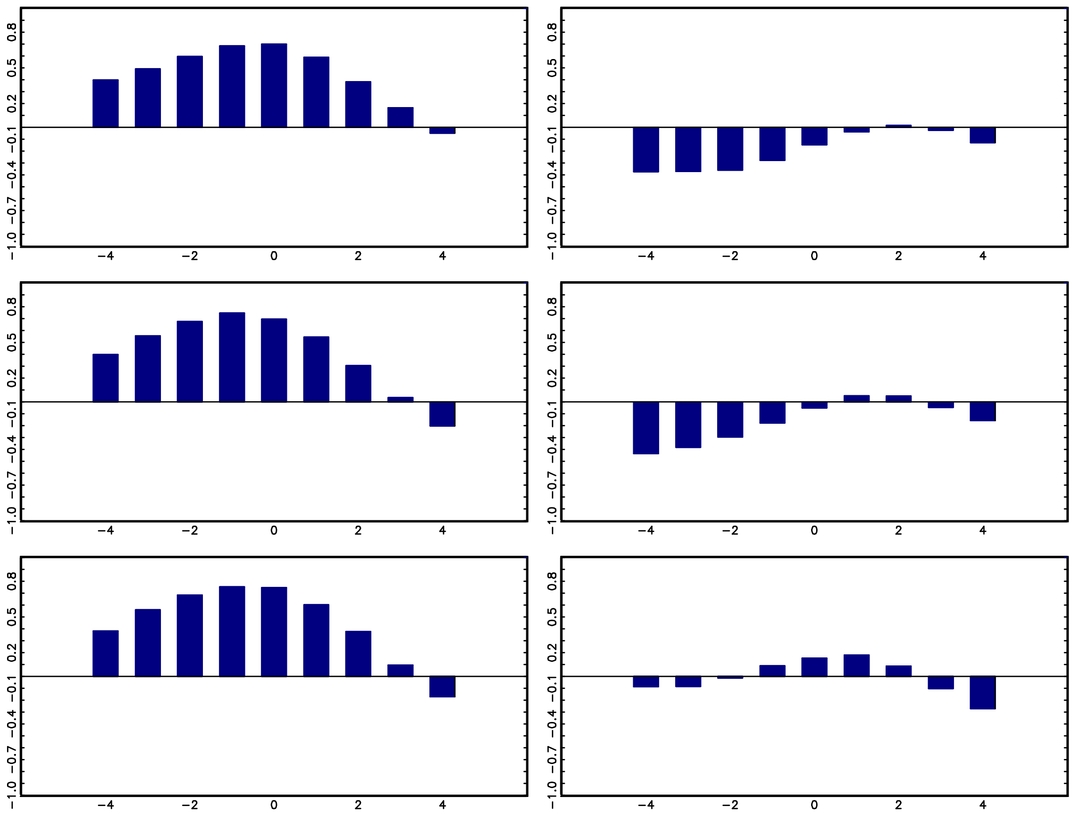

Figure 2: Correlations between y_t and d_{t+1} for $j \in [-4, 4]$ (left unsecured, right secured). The top (middle, bottom) graphs are for the full (bottom 99%, bottom 95%) Compustat samples.

To obtain some indication about causality, we conduct a Granger causality test to explore if secured or unsecured debt contain superior information to help predict output. To do so, we estimate equation

$$y_t = \alpha + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \gamma d^u_{t-1} + \tau d^s_{t-1} + \varepsilon_t$$

by OLS. We note that including two lags of GDP is good enough for the best fit in terms of R^2 which is equal to 0.835 when only lagged GDP is included as independent variables. We then add one-period lagged debt as independent variables into the regression. We find that the coefficient on unsecured debt is (weakly) significantly positive in all sample series, whereas that on secured debt is negative but insignificantly different from zero. We thus conclude that unsecured debt helps predict future GDP movements, while this is not the case for secured debt. This result suggests that in the Great Moderation period (including the recent financial crisis period), the so-called "credit cycle" and its intimate relation to the business cycle is not driven by movements in secured debt and the associated value of collateral, which much of the existing macro-finance literature often attribute to as the culprit of aggregate booms and busts. In Appendix A we complement these findings by a SVAR analysis showing how shocks to unsecured credit affect output significantly whereas shocks to secured credit are not.

Table 3: Granger Causality Test

	R^2	Unsecured debt	Secured debt
Benchmark	0.835	N/A	N/A
Full sample	0.845	0.025^{**}	-0.017
w/o top 1%	0.872	0.075^{***}	-0.046
w/o top 5%	0.889	0.093^{***}	-0.071^{***}

Notes: *** (**) indicates significance at the 5% (10%) level.

2.2.2 Flow of Funds Accounts

One potential weakness of applying evidence from Compustat in a macroeconomic context is that it only contains information about publicly-traded firms. Aggregate data from the Flow of Funds accounts, though covering the full non-financial business sector, are not completely informative regarding the distinction between secured and unsecured debt, as they only break the firms' credit market liabilities in several broad categories. Nonetheless, when we use those categories as proxies for secured and unsecured debt components, we confirm the main insights obtained above.

Since mortgages can be easily classified as secured debt, while corporate bonds add to unsecured debt, we use those series as proxies for the two debt categories.[15] While Table 4 indicates that corporate bonds are less volatile than the measure for unsecured debt obtained from Compustat data in Table 2, the contemporaneous correlations confirm our previous findings. While mortgages are acyclical, corporate bonds as a proxy for unsecured debt is strongly procyclical.[16]

Table 4: Relative Volatility and Comovement with Output (Flow of Funds)

	Relative Volatility	Correlation with GDP
Mortgages	3.52	0.00
Corporate bonds	1.58	0.53

We also obtain similar findings about lead-lag relations as for the Compustat series; see Figure 5 for the lead-lag correlations for the annualized series. Corporate bonds are strongly correlated with output, with a peak correlation of 0.6 at a one-year lead, while mortgages show much weaker cyclicality, lagging GDP by about two years.

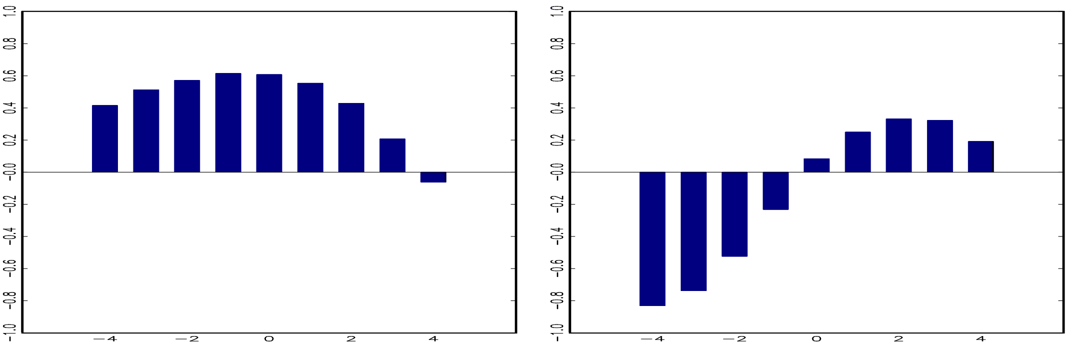

Figure 3: Lead-lag correlations between output and corporate bonds (left) and mortgages (right).

We briefly remark that those findings do not apply to the period before 1980 where the role of debt structure over the business cycle seems to be quite different. In fact in the period 1952-1980, mortgages appear to be strongly correlated with output, which is more consistent with

[15] As Table 1 suggests, there are significant fractions of both secured and unsecured non-mortgage loans, so we do not include loans in any of the proxy series.

[16] The table is based on qua62rterly data, deflated and detrended in the same way as for the Compustat series.

conventional macro-finance theories where the value of collateral determines firms' borrowing capacity over the cycle. At the same time, corporate bonds show a weaker (yet positive) correlation with output. Although we do not have more precise measures for secured and unsecured credit prior to 1981, this observation suggests that there is a structural break around this time, induced by regulatory changes that had a major impact on firms' debt policies.

3 The Model

In this section we develop a business-cycle model in which unsecured firm credit is exchanged between heterogeneous firms facing idiosyncratic productivity shocks. Self-enforcing limits on firm credit ensure that no firm opts for default in equilibrium. For expositional reasons, we present the model as simple as possible. Particularly, we abstract from any secured credit, we keep the labor supply fixed, and idiosyncratic productivity shocks are i.i.d. All three assumptions will be relaxed in the quantitative part of this paper. Tractability and the theoretical findings are preserved in these extensions, as we show in the Appendix.

3.1 The Setup

The model has a continuum $i \in [0,1]$ of firms, each owned by a representative entrepreneur, and a unit mass of workers. At any time t, all agents maximize expected discounted utility

$$\mathrm{E}_t(1-\beta)\sum_{\tau\geq t}\beta^{\tau-t}\ln(c_\tau) \tag{1}$$

over future consumption streams. Workers supply one unit of labor per period and have no capital endowment. Entrepreneurs own capital and have no labor endowment. They produce a consumption and investment good y_t using capital k_t' and labor ℓ_t with a common constant-returns technology $y_t = (k_t')^\alpha (A\ell_t)^{1-\alpha}$. Aggregate labor efficiency A is constant for now, which will be relaxed in Section 4.

Entrepreneurs differ in their ability to operate capital investment k_t. Some entrepreneurs are able to enhance their invested capital according to $k_t' = a^p k_t$; these entrepreneurs are labeled "productive". The remaining, "unproductive" entrepreneurs deplete some of their capital investment such that $k_t' = a^u k_t$. We assume that $a^p > a^u$ and write $\gamma \equiv a^u/a^p$ for the relative productivity gap.[17] All capital depreciates at common rate δ. Productivity realizations are independent across agents and uncorrelated across time; entrepreneurs are productive with

[17] This specification is similar to the (aggregate) shocks to capital quality considered by Gertler and Kiyotaki (2010) and is used for the purpose of tractability (see footnote 22 below).

probability π and unproductive with probability $1 - \pi$. Thus, a fraction π of the aggregate capital stock K_t is owned by productive entrepreneurs in any period. Uncorrelated productivity simplifies the model considerably; it also allows us to abstract from standard mechanisms inducing persistence of shocks under financial frictions due to sluggish adjustments in the net worth of borrowers (e.g. Kiyotaki and Moore (1997) and Bernanke and Gertler (1989)).[18]

Timing within each period is as follows. *First* entrepreneurs learn their productivity, they borrow and lend in a centralized credit market at gross interest rate R_t, and they hire labor in a centralized labor market at wage w_t. *Second* production takes place. *Third*, entrepreneurs redeem their debt; agents consume and save for the next period.

In the credit market, productive entrepreneurs borrow capital from unproductive entrepreneurs. All credit is unsecured and is only available to borrowers with a clean credit history.[19] If a borrower decides to default in some period, the credit record deteriorates and the entrepreneur is banned from unsecured credit for a stochastic number of periods. Defaulters are still allowed to stay in business; hence they are able to produce or to lend their assets to other firms. Each period after default, the entrepreneur's credit record is cleared with probability ψ in which case the entrepreneur regains full access to credit markets.

Since no shocks arrive during a credit contract (that is, debt is redeemed at the end of the period before the next productivity shock is realized), there exist default–deterring credit limits, defined similarly as in the pure–exchange model of Alvarez and Jermann (2000). These limits are the highest values of credit that prevent default. With permanent market exclusion ($\psi = 0$), this enforcement technology corresponds to the one discussed by Bulow and Rogoff (1989) and Hellwig and Lorenzoni (2009) who consider unsecured credit and assume that defaulters are excluded from future credit but are still allowed to save. Unsecured borrowing is founded on a producer's desire to maintain a clean credit record and hence continued access to future unsecured credit. Below we prove that credit constraints are necessarily binding in equilibrium (see Proposition 2).

Unlike entrepreneurs, workers have no access to credit. Further, as we see below, the loan yield R satisfies $R < 1/\beta$ in any stationary equilibrium, and hence also in any stochastic equilibrium near the steady state. Thus, workers are borrowing constrained and do not want to save; they simply consume their wage income in every period.

Let θ_t denote the constraint on a borrower's debt–equity ratio in period t. This value is common for all borrowers with a clean credit record and is endogenously determined in equilibrium to prevent default. A productive entrepreneur entering the period with equity e_t can borrow up

[18]See Appendix D for an extension to a framework with correlated productivity shocks.

[19]In the next section and in Appendix C, we relax this assumption by also allowing borrowers to borrow secured by providing collateral assets to their creditors.

to $b_t = \theta_t e_t$ and invest $k_t = e_t + b_t$. An unproductive entrepreneur lends out capital, so $b_t \leq 0$, and investment is $k_t = e_t + b_t \leq e_t$. The budget constraint for an entrepreneur with capital productivity $a^s \in \{a^p, a^u\}$ reads as

$$c_t + e_{t+1} = (a^s k_t)^\alpha (A_t \ell_t)^{1-\alpha} + (1-\delta) a^s k_t - w_t \ell_t - R_t b_t \ . \tag{2}$$

We are now ready to define equilibrium.

Definition: *For each realization of idiosyncratic productivities, a competitive equilibrium is a list of consumption, savings, and production plans for all entrepreneurs,* $(c_t^i, e_t^i, b_t^i, k_t^i, \ell_t^i)_{i\in[0,1]}$, *consumption of workers,* $c_t^w = w_t$, *factor prices for labor and capital* (w_t, R_t), *and debt-equity constraints* θ_t, *such that in every period* $t \geq 0$:[20]

(i) $(c_t^i, e_t^i, b_t^i, k_t^i, \ell_t^i)$ *maximizes entrepreneur* i*'s expected discounted utility (1) subject to budget constraints (2) and credit constraints* $b_t^i \leq \theta_t e_t^i$.

(ii) *Markets for labor and capital clear;*

$$\begin{aligned} \int_0^1 \ell_t^i \, di &= 1 \ , \\ \int_0^1 b_t^i \, di &= 0 \ . \end{aligned}$$

(iii) *If* $b_t^i \leq \theta e_t^i$ *is binding in problem (i), entrepreneur* i *is exactly indifferent between debt redemption and default in period* t, *where default entails exclusion from credit for a stochastic number of periods with readmission probability* ψ *in each period following default.*

3.2 Equilibrium Characterization

Since entrepreneurs hire labor so as to equate the marginal product to the real wage, all productive (unproductive) entrepreneurs have identical capital–labor ratios; these are linked by a no–arbitrage condition implied by perfect labor mobility:

$$\frac{k_t^p}{\ell_t^p} = \gamma \frac{k_t^u}{\ell_t^u} \ . \tag{3}$$

With binding credit constraints, a fraction $z_t \equiv \min[1, \pi(1+\theta_t)]$ of the aggregate capital stock K_t is operated by productive entrepreneurs. It follows from (3) and labor market clearing that

$$\frac{k_t^p}{\ell_t^p} = \frac{a_t K_t}{a^p} \leq K_t < \frac{a_t K_t}{a^u} = \frac{k_t^u}{\ell_t^u} \ ,$$

[20] In period $t = 0$, there is some given initial equity distribution $(e_0^i)_{i\in[0,1]}$.

where $a_t \equiv a^p z_t + a^u(1-z_t)$ is the average capital productivity. The gross return on capital for an entrepreneur with capital productivity $a^s \in \{a^u, a^p\}$ is then $R_t^s \equiv a^s[1-\delta+\alpha A^{1-\alpha}(a_t K_t)^{\alpha-1}]$ (see Appendix B for a derivation).

In any equilibrium, the gross interest rate cannot exceed the capital return of productive entrepreneurs R_t^p and it cannot fall below the capital return of unproductive entrepreneurs R_t^u. Thus it is convenient to write $R_t = \rho_t R_t^p$ with $\rho_t \in [\gamma, 1]$. When $\rho_t < 1$, borrowers are credit constrained. In this case the leveraged equity return $\tilde{R}_t \equiv [1+\theta_t(1-\rho_t)]R_t^p$ exceeds the capital return R_t^p. Unproductive entrepreneurs, on the other hand, lend out all their capital when $\rho_t > \gamma$; they only invest in their own inferior technology if $\rho_t = \gamma$. Therefore, credit market equilibrium is equivalent to the complementary-slackness conditions

$$\rho_t \geq \gamma \quad , \quad \pi(1+\theta_t) \leq 1 \ . \tag{4}$$

With this notation, the entrepreneurs' budget constraints (2) simplify to $e_{t+1} + c_t = \tilde{R}_t e_t$, when the entrepreneur is productive in t, and to $e_{t+1} + c_t = R_t e_t$ when the entrepreneur is unproductive. From logarithmic utility follows that every entrepreneur consumes a fraction $(1-\beta)$ of wealth and saves the rest.

To derive the endogenous credit limits, let $V_t(W)$ denote the continuation value of an entrepreneur with a clean credit record who has wealth W at the end of period t, prior to deciding consumption and saving. These values satisfy the recursive equation[21]

$$V_t(W) = (1-\beta)\ln[(1-\beta)W] + \beta\pi \mathrm{E}_t V_{t+1}(\tilde{R}_{t+1}\beta W) + \beta(1-\pi)\mathrm{E}_t V_{t+1}(R_{t+1}\beta W) \ .$$

The first term in this equation represents utility from consuming $(1-\beta)W$ in the current period. For the next period $t+1$, the entrepreneur saves equity βW which earns leveraged return $\tilde{R}_{t+1}$ with probability π and return R_{t+1} with probability $1-\pi$. It follows that continuation values take the form $V_t(W) = \ln(W) + V_t$ where V_t is independent of wealth, satisfying the recursive relation

$$V_t = (1-\beta)\ln(1-\beta) + \beta\ln\beta + \beta \mathrm{E}_t\Big[\pi \ln \tilde{R}_{t+1} + (1-\pi)\ln(R_{t+1}) + V_{t+1}\Big] \ . \tag{5}$$

For an entrepreneur with default flag and no access to credit, the continuation value is $V_t^d(W) = \ln(W) + V_t^d$, where V_t^d satisfies, analogously to equation (5), the recursion

$$V_t^d = (1-\beta)\ln(1-\beta) + \beta\ln\beta + \beta \mathrm{E}_t\Big[\pi\ln(R_{t+1}^p) + (1-\pi)\ln(R_{t+1}) + V_{t+1}^d + \psi(V_{t+1} - V_{t+1}^d)\Big] \ . \tag{6}$$

[21] In the absence of sunspot shocks, the expectations operator E_t could be dropped from this and from subsequent recursive equations, because all fundamental parameters are not uncertain in this section of the paper.

This entrepreneur is banned from unsecured credit in period $t+1$ so that the equity return is R^p_{t+1} with probability π and R_{t+1} with probability $1-\pi$. At the end of period $t+1$, the entrepreneur's credit record clears with probability ψ in which case continuation utility increases from V^d_{t+1} to V_{t+1}.

If a borrower has a clean credit record and enters period t with equity e_t, the debt-equity constraint θ_t makes him exactly indifferent between default and debt redemption if

$$\ln\Big[\tilde{R}_t e_t\Big] + V_t = \ln\Big[R^p_t(1+\theta_t)e_t\Big] + V^d_t \ .$$

Here the right-hand side is the continuation value after default: the entrepreneur invests $(1+\theta_t)e_t$, earns return R^p_t and does not redeem debt. The left-hand side is the continuation value under solvency, where the entrepreneur earns $\tilde{R}_t e_t$. Defining $v_t \equiv V_t - V^d_t \geq 0$ as the "value of reputation", this equation can be solved for the default-deterring constraint on the debt-equity ratio

$$\theta_t = \frac{e^{v_t}-1}{1-e^{v_t}(1-\rho_t)} \ . \tag{7}$$

This constraint is increasing in the reputation value v_t: a greater expected payoff from access to unsecured credit makes debt redemption more valuable, which relaxes the self-enforcing debt limit. In the extreme case when the reputation value is zero, unsecured credit cannot be sustained so that $\theta_t = 0$.

Using (5) and (6), reputation values satisfy the recursive identity

$$v_t = \beta \mathrm{E}_t\Big[\pi \ln \frac{\tilde{R}_{t+1}}{R^p_{t+1}} + (1-\psi)v_{t+1}\Big] = \beta \mathrm{E}_t\Big[\pi \ln\Big(\frac{\rho_{t+1}}{1-e^{v_{t+1}}(1-\rho_{t+1})}\Big) + (1-\psi)v_{t+1}\Big] \ . \tag{8}$$

We summarize this equilibrium characterization as follows.

Proposition 1 *Any solution $(\rho_t, \theta_t, v_t)_{t\geq 0}$ to the system of equations (4), (7) and (8) gives rise to a competitive equilibrium with interest rates $R_t = \rho_t R^p_t$ with $R^p_t = a^p[1-\delta+\alpha A^{1-\alpha}(a_t K_t)^{\alpha-1}]$ and $a_t = a^u + (a^p - a^u)\cdot \min[1, \pi(1+\theta_t)]$. The capital stock evolves according to*

$$K_{t+1} = \beta\Big[(1-\delta) + \alpha A^{1-\alpha}(a_t K_t)^{\alpha-1}\Big]a_t K_t \ . \tag{9}$$

An implication of this proposition is that any equilibrium follows two dynamic equations, the backward-looking dynamics of aggregate capital (equation (9)) and the forward-looking dynamics of reputation values, equation (8) or, equivalently, equation (10) below. Due to our modeling of the idiosyncratic productivity process, the latter identity is independent of the aggregate state K_t, and hence permits a particularly simple analysis of stationary and non-stationary equilibria.[22]

Using Proposition 1, we obtain two immediate results. First, there always exists an equilibrium where $v_t = 0$, $\theta_t = 0$ and $\rho_t = \gamma$ in all periods, so that unsecured credit is not enforceable. Intuitively, if no unsecured credit is available in future periods, there is no value to reputation, and hence any borrower would prefer to default on current unsecured credit so that constraints must be zero.[23] Second, we show that constraints on unsecured credit are necessarily binding. This is in line with earlier results by Bulow and Rogoff (1989) and Hellwig and Lorenzoni (2009) who show that the first best cannot be implemented by limited enforcement mechanisms which ban defaulting agents from future borrowing but not from future lending. It differs decisively from environments with two–sided exclusion, as in Kehoe and Levine (1993) and Alvarez and Jermann (2000), where first-best allocations can be sustained with unsecured credit under certain circumstances.[24] The intuition for this result is as follows. If borrowers were unconstrained, the interest rate would coincide with the borrowers' capital return. Hence there is no leverage gain, so that access to credit has no value for borrowers. In turn, every borrower would default on an unsecured loan, no matter how small. We summarize this finding in

Proposition 2 *Any equilibrium features binding borrowing constraints.*

It follows immediately that the equilibrium interest rate is smaller than the workers' marginal rate of intertemporal substitution, so that workers are indeed credit constrained (in steady state).

Corollary 1 *In any steady state equilibrium, $R < 1/\beta$.*

[22] On the one hand, reputation values are independent of aggregate capital since all returns are multiples of $1 - \delta + \alpha(A/(a_t K_t))^{1-\alpha}$ which follows from our assumption that firms differ in capital productivity, $k_t' = a^s k_t$ for $s = u, p$. On the other hand, if productivity shocks were autocorrelated, equation (10) has two lags and the capital distribution enters as an additional state variable; see Appendix D.

[23] A similar "autarky" equilibrium also obtains in the limited enforcement economy of Kehoe and Levine (1993) as decentralized by Alvarez and Jermann (2000); cf. Azariadis and Kaas (2007).

[24] In endowment economies with permanent exclusion of defaulters, it is well known that perfect risk sharing can be implemented if the discount factor is sufficiently large, if risk aversion is sufficiently strong or if the endowment gap between agents is large enough (see e.g. Kehoe and Levine (2001) and Azariadis and Kaas (2007)). Azariadis and Kaas (2012b) show that the role of the discount factor changes decisively if market exclusion is temporary.

3.3 Multiplicity and Cycles

Although borrowers must be constrained, the credit market may nonetheless be able to allocate capital efficiently. In particular, when the reputation value v_t is sufficiently large, credit constraints relax and the interest rate may exceed the capital return of unproductive entrepreneurs who then lend out all their capital to more productive entrepreneurs. Formally, when v_t exceeds the threshold value

$$\overline{v} \equiv \ln\left[\frac{1}{1-\gamma(1-\pi)}\right] > 0 \ ,$$

the equilibrium conditions (4) and (7) are solved by $\theta_t = (1-\pi)/\pi$ and $\rho_t = [1-e^{-v_t}]/(1-\pi) > \gamma$. Conversely, when v_t falls short of $\overline{v}$, credit constraints tighten, the interest rate equals the capital return of unproductive entrepreneurs ($\rho_t = \gamma$), who are then indifferent between lending out capital or investing in their own technology, so that some capital is inefficiently allocated. We can use this insight to rewrite the forward-looking equation (8) as

$$v_t = \mathrm{E}_t f(v_{t+1}) \ , \tag{10}$$

with

$$f(v) \equiv \begin{cases} \beta(1-\psi)v + \beta\pi\ln\left[\frac{\gamma}{1-e^{v}(1-\gamma)}\right] & , \text{ if } v \in [0,\overline{v}] \ , \\ \beta(1-\pi-\psi)v - \beta\pi\ln\pi & , \text{ if } v \in [\overline{v}, v_{max}] \ . \end{cases}$$

Here $v = v_{max} = -\ln\pi$ is the reputation value where the interest rate satisfies $\rho = 1$ and borrowers are unconstrained. It is straightforward to verify that f is strictly increasing if $\pi + \psi < 1$, convex in $v < \overline{v}$, and it satisfies $f(0) = 0$ and $f(v_{max}) < v_{max}$. This reconfirms that the absence of unsecured credit ($v = 0$) is a stationary equilibrium. Depending on economic fundamentals, there can also exist one or two steady states exhibiting positive trading of unsecured credit. Figure 4(a) shows a situation in which function f has three intersections with the 45-degree line: $v = 0$, $v^* \in (0,\overline{v})$ and $v^{**} \in (\overline{v}, v_{max})$. The steady states at $v = 0$ and at v^* have an inefficient capital allocation, whereas capital is efficiently allocated at $v^{**} > \overline{v}$. Figure 4(b) shows a possibility with only two steady states, at $v = 0$ and at $v^{**} > \overline{v}$. The following proposition describes how the set of stationary equilibria changes as the productivity ratio $\gamma = a^u/a^p$ varies.

Proposition 3 *For all parameter values $(\beta, \pi, \psi, \gamma)$ there exists a stationary equilibrium in which no unsecured credit is available and capital is inefficiently allocated. In addition, there are threshold values $\gamma_0 < \gamma_1 < 1$ of the productivity ratio such that:*

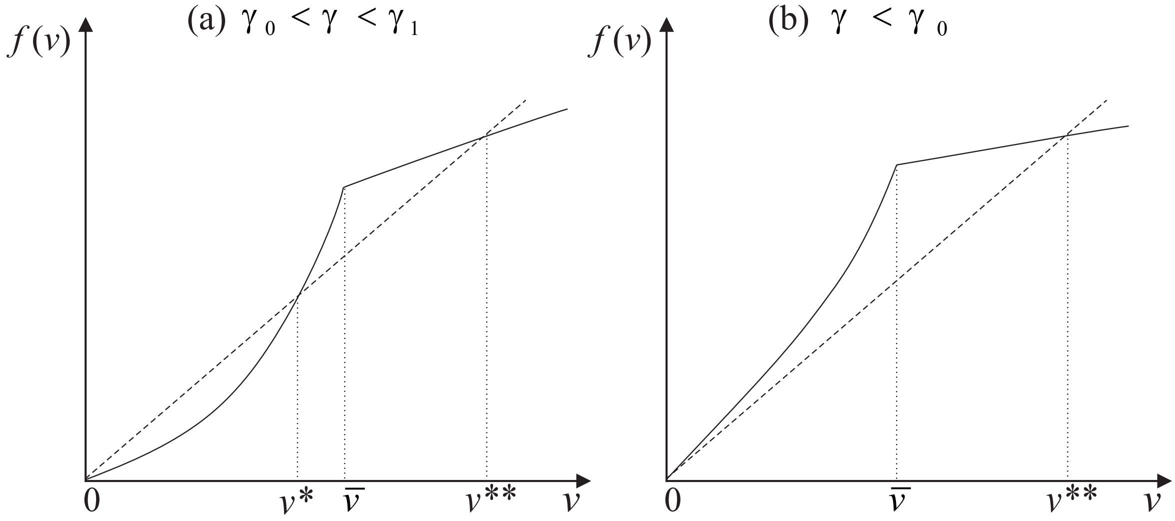

Figure 4: Steady states at $v = 0, v^*, v^{**}$.

(a) *For $\gamma \in (\gamma_0, \gamma_1)$, there are two stationary equilibria with unsecured credit: one at $v^* \in (0, \overline{v})$ with inefficient capital allocation and one at $v^{**} \in (\overline{v}, v_{max})$ with efficient capital allocation.*

(b) *For $\gamma > \gamma_1$, there is no stationary equilibrium with unsecured credit.*

(c) *For $\gamma \leq \gamma_0$, there exists a unique stationary equilibrium with unsecured credit and efficient capital allocation at the reputation value $v^{**} \in (\overline{v}, v_{max})$.*

For small enough idiosyncratic productivity fluctuations ($\gamma > \gamma_1$), unsecured credit is not enforceable because entrepreneurs value participation in credit markets too little. For larger idiosyncratic shocks, exclusion from future credit is a sufficiently strong threat so that unsecured credit is enforceable. With big enough shocks, the unique steady state with unsecured credit has an efficient factor allocation, while for intermediate values of γ, a third equilibrium with unsecured credit and some misallocation emerges.

The explanation for equilibrium multiplicity is a *dynamic complementarity* between endogenous credit constraints which are directly linked to reputation values. Borrowers' expectations of future credit market conditions affect their incentives to default which in turn determine current credit constraints. If future constraints are tight, the payoff of a clean credit record is modest so that entrepreneurs value access to unsecured credit only a little. In turn, current default–deterring credit limits must be small. Conversely, if entrepreneurs expect future credit markets to work well, a clean credit record has high value, and this relaxes current constraints.

As Figure 4 shows, multiplicity follows from a specific non-linearity between expected and current reputation values. To understand this non-linearity, it is important to highlight the different impact of market expectations on borrowing constraints and on interest rates. In the inefficient regime $v \le \overline{v}$, improvements in credit market expectations relax credit constraints without changes in the interest rate which leads to particularly large gains from participation and hence to a strong impact on the current value of reputation. Conversely, if $v > \overline{v}$, beliefs in better credit conditions also raise the interest rate which dampens the positive effect and hence mitigates the increase in the current reputation value.

Even when unsecured credit is available and possibly supports efficient allocations of capital, that efficiency rests upon the confidence of market participants in future credit market conditions. When market participants expect credit constraints to tighten rapidly, the value of reputation shrinks over time which triggers a self–fulfilling collapse of the market for unsecured credit. For instance, if $\gamma < \gamma_0$, the steady state at v^{**} is determinate and the one at $v = 0$ is indeterminate (see Figure 4(b)). That is, there exists an infinity of non–stationary equilibria $v_t = f(v_{t+1}) \to 0$ where the value of reputation vanishes asymptotically. These equilibria are mathematically similar to the bubble–bursting equilibria in overlapping–generation models. If $\gamma \in (\gamma_0, \gamma_1)$, the two steady states at $v = 0$ and at v^{**} are determinate, whereas the one at v^* is indeterminate. In that situation, a self-fulfilling collapse of the credit market would be described by an equilibrium with $v_t \to v^*$ where a positive level of unsecured credit is still sustained in the limit.

In both these events, a one-time belief shock can lead to a permanent collapse of the credit market. But in the latter case, indeterminacy also permits stochastic business cycle dynamics driven by self–fulfilling beliefs (sunspots).[25] Sunspot fluctuations vanish asymptotically if $\gamma < \gamma_0$, but they give rise to permanent volatility around the indeterminate steady state v^* if $\gamma \in (\gamma_0, \gamma_1)$.

Proposition 4 *Suppose that $\gamma \in (\gamma_0, \gamma_1)$ as defined in Proposition 3. Then there exist sunspot cycles featuring permanent fluctuations in credit, output and total factor productivity.*

The dynamic complementarity between endogenous credit constraints not only gives rise to expectations-driven business cycles, it also generates an endogenous propagation mechanism: a one-time expectational shock in period t triggers a persistent adjustment dynamics of reputation values v_k (and thus of credit constraints, investment and output) in subsequent periods $k > t$. Intuitively, a self-fulfilling credit boom (slump) in period t can only emerge if the boom (slump) lasts for several periods.

[25] To see this, consider any sequence of random variables $\varepsilon_{t+1} \in (-v_t, v^{**} - v_t)$, $t \ge 1$, satisfying $\mathrm{E}_t(\varepsilon_{t+1}) = 0$, and define the stochastic process $v_{t+1} = f^{-1}(v_t + \varepsilon_{t+1}) \in (0, v^{**})$ which solves equation (10).

Corollary 2 *A one-time sunspot shock $\varepsilon_t > 0$ ($\varepsilon_t < 0$) in period t induces a persistent positive (negative) response of firm credit and output.*

In the next section, we return to this finding by demonstrating how the autocorrelated response of credit depends on model parameters.

4 Quantitative Analysis

The previous section showed that this model can capture volatility of unsecured credit and output driven by expectational shocks, with potentially sluggish responses. In this section we calibrate an extended model to the U.S. economy in order to investigate the contribution of sunspot shocks relative to fundamental shocks to the financial and to the real sector.

4.1 Model extension

We extend the model in two directions. First, we include variable labor supply, modifying workers' period utility to $\ln(C_t - \frac{1}{1+1/\varphi} L_t^{1+1/\varphi})$, so that φ is the labor-supply elasticity. Second, we allow firms to also borrow secured by providing collateral. Specifically, we assume that fraction λ_t of a firm's assets serves as collateral and can be pledged to creditors. As this option is available to all firms, the relevant outside option of a defaulter is the exclusion from unsecured credit while retaining access to collateralized credit. As before, all credit is within the period and no default occurs in equilibrium, so secured and unsecured credit carry the same interest rate R_t. Besides sunspot shocks, we allow for shocks to λ_t and to aggregate technology A_t. The first type of shock directly affects secured credit and can easily be related to financial shocks on collateral constraints, such as those in Jermann and Quadrini (2012). Technology shocks help to account for those movements in output (or total factor productivity) which are not generated by the endogenous response of aggregate capital productivity due to shifts in credit.

In Appendix C, we solve this extended model in which the debt-equity constraint (7) becomes

$$\theta_t = \frac{e^{v_t} - 1 + \lambda_t}{1 - \lambda_t - e^{v_t}(1 - \rho_t)} \ . \tag{11}$$

Firm debt therefore contains a secured component $\theta_t^s = \frac{\lambda_t}{\lambda_t - \rho_t}$, which only depends on λ_t, and the unsecured one $\theta_t^u = \theta_t - \theta_t^s$. We also generalize (10) to an equation

$$v_t = \mathrm{E}_t f(v_{t+1}, \lambda_{t+1}) \ , \tag{12}$$

Thus, we obtain a similar dichotomy as before: the credit market equilibrium is independent of the level of the capital stock, of labor market variables or technology shocks.[26] We also confirm that any steady state with unsecured credit and some misallocation of capital is indeterminate so that self-fulfilling expectational shocks drive the dynamics of unsecured credit.[27] Note again that only the indeterminate steady state equilibrium allows for unsecured credit and inefficient capital allocations (Proposition 3). The other two determinate steady states of this model either feature efficient factor allocations or do not sustain unsecured credit. Hence their business-cycle properties would either resemble those of a standard frictionless model or those of an economy with exogenous, collateral-based credit constraints.[28]

4.2 Calibration

We calibrate this model to the U.S. economy with model parameters such that the indeterminate steady state equilibrium matches suitable long-run properties. The calibration targets correspond to statistics obtained for the U.S. business sector in the period 1981–2012. We set δ, α and β in a standard fashion to match plausible values of capital depreciation, factor income shares and the capital-output ratio of (5.95 per quarter). The Frisch elasticity is set to $\varphi = 1$. We normalize average capital productivity in steady state to $a = 1$, as well as steady-state labor efficiency to $A = 1$. We set the exclusion parameter $\psi = 0.025$ so that an average firm owner has difficulty obtaining unsecured credit for a period of 10 years after default.[29] We choose the remaining parameters π, λ and a^u to match the following three targets:[30] (1) Credit to non-financial firms is 0.82 of annual GDP; (2) the debt-equity ratio of constrained firms is set to $\theta = 3$; (3) unsecured credit is 50 percent of total firm credit.[31] Given that this

[26]While this property is useful to characterize equilibrium and to provide intuition for the main relationships, it is by no means essential for our theory. Alternative formulations of the collateral constraint, for example, would give rise to an equation $v_t = \mathrm{E}_t f(v_{t+1}, \lambda_{t+1}, K_{t+1}, A_{t+1})$, so that technology shocks feed (positively) into the credit market.

[27]Of course, there could also be fundamental shocks which affect unsecured credit independently of secured credit. For example, any shock that changes the default value would take an impact on θ^u_t. Sunspot shocks have the advantage to induce endogenous propagation (see below).

[28]Different from our model, collateral-based credit constraints may also be forward-looking as in Kiyotaki and Moore (1997) if collateral prices are endogenous, which may contribute to the amplification and propagation of shocks. Cordoba and Ripoll (2004) and Kocherlakota (2000) argue, however, that it is difficult to generate quantitatively significant amplification this way.

[29]For example, if a firm owner files for bankruptcy according to Chapter 11 of the U.S. Bankruptcy Code, bankruptcy remains on the credit record for a period of 10 years (see e.g. Chatterjee et al. (2007)).

[30]The normalization $a = a^u + \pi(1+\theta)(a^p - a^u) = 1$ then pins down parameter a^p.

[31](1) Credit market liabilities of non-financial business are 0.82 of annual GDP (average over 1952-2011, Flow of Funds Accounts of the Federal Reserve Board, Z.1 Table L.101). (2) Debt-equity ratios below 3 are usually

model has a two-point distribution of firm productivity (and hence of debt-equity ratios), the choice of target (2) is somewhat arbitrary. As a robustness check, we also calibrate the model with $\theta = 2$ and report the findings in Appendix E.[32] All parameters are listed in Table 5.

This model parameterization produces reasonable statistics along a few other dimensions. One of them is a plausible cross-firm dispersion of total factor productivity (TFP). With firm-level output equal to $y^i = (a^i - 1)k^i + (A\ell^i)^{1-\alpha}(a^i k^i)^\alpha$, we calculate a standard deviation of log TFP equal to 0.3, close to the within-industry average 0.39 reported in Bartelsman et al. (2013).[33]

Table 5: Parameter choices.

Parameter	Value	Description	Explanation/Target
δ	0.0194	Depreciation rate	Consumption of fixed capital
α	0.3	Production fct. elasticity	Capital income share
β	0.97	Discount factor	Capital-output ratio
φ	1	Frisch elasticity	$\varphi = 1$
ψ	0.025	Exclusion parameter	10-year punishment
π	0.184	Share of productive firms	Credit of non-financial firms
λ	0.569	Collateral share	Debt-equity ratio $\theta = 3$
a^u	0.936	Lowest productivity	Equity return dispersion
a^p	1.023	Highest productivity	Normalization $a = 1$

4.3 Persistence of sunspot shocks

In the absence of shocks to λ, the log-linearized dynamics of the credit-to-capital ratio follows

$$\hat{\theta}_{t+1} = \frac{1}{\varphi_2}\hat{\theta}_t + d_2 \varepsilon^s_{t+1} \ ,$$

where coefficients d_1, φ_2 are specified in the Appendix and ε^s_{t+1} is a sunspot shock. In particular, we find that the auto-correlation coefficient

$$\frac{1}{\varphi_2} = \frac{1}{\beta(1-\psi) + \beta\pi(1+\theta)\frac{a^p - a^u}{a^u}}$$

required to qualify for commercial loans (see Herranz et al. (2009)). In the SSBF (Capital IQ, Compustat), the mean debt-equity ratio is 3.04 (3.15, 2.43). For (3), see the discussion in Section 2.

[32]Departing from Table 5, this requires $a^u = 0.936$, $a^p = 1.013$, $\pi = 0.276$ and $\lambda = 0.462$.

[33]To calculate firm-level TFP z^i, write $z^i(k^i)^\alpha(\ell^i)^{1-\alpha} = y^i$, and use the labor demands of Appendix C to obtain (in steady state) $z^i = (a^i)^\alpha + \frac{K}{Y}\frac{a^i - 1}{(a^i)^{1-alpha}}$.

is 0.984 and close to the data analogue 0.99. If the model is calibrated with $\theta = 2$, the coefficient becomes even larger (0.987), but it is not too sensitive to the choice of θ.

4.4 Multiple shocks

4.4.1 Identification

Given this calibration of steady-state values, we identify sunspot shocks, as well as fundamental financial shocks (collateral parameter $\hat{\lambda}_t$) and technology shocks (labor efficiency parameter $\hat{A}_t$) as follows (see Appendix C for details). We use the time series of the credit-to-capital ratio from the Flow of Funds accounts to identify $\hat{\theta}_t$. We take the mortgages-to-capital ratio as a proxy for the unsecured-credit-to-capital ratio to measure $\hat{\theta}_t^s$. This we can use to identify both $\hat{\lambda}_t$ (shocks to secured credit) and sunspot shocks ε_t^s which are shocks altering the unsecured component of credit as captured in the reputation value $\hat{v}_t$. Labor efficiency $\hat{A}_t$ is identified so as to match the cyclical component of output. All three shocks together therefore generate the output dynamics of the data and we can measure how each of them contributes to the total volatility and how it accounts for output movements in specific episodes.
Note first that the three identified shocks can be correlated with each other. Because of this, we first show that even if we attribute all correlations to $(\hat{A}_t, \hat{\lambda}_t)$ shocks by ordering the sunspot shock as the last variable in a SVAR, sunspot shocks are still important in explaining output fluctuations. We conduct the analysis in two different ways.

4.4.2 SVAR Analysis

We run the following unrestricted SVAR with TFP ($\hat{A}_t$), collateral value ($\hat{\lambda}_t$), and unsecured credit (reputation value $\hat{v}_t$):

$$\begin{pmatrix} \hat{A}_t \\ \hat{\lambda}_t \\ \hat{v}_t \end{pmatrix} = B \begin{pmatrix} \hat{A}_{t-1} \\ \hat{\lambda}_{t-1} \\ \hat{v}_{t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{pmatrix}$$

with coefficient matrix B. We apply the Choleski decomposition such that $e_t = (e_{1t}, e_{2t}, e_{3t})' = C(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t})'$ with lower triangular matrix C, and we call ε_{3t} the sunspot shock. This identification method is equivalent to regressing e_{3t} on $\{e_{2t}, e_{1t}\}$, using the residual as our sunspot shock.
In our model, output $\hat{Y}_t$ is generated by contemporaneous and lagged values of the three variables $(\hat{A}, \hat{\lambda}, \hat{v})$. However, it seems that a linear combination of contemporaneous variables gives a very good approximation of $\hat{Y}_t$ (see Figure 5). Adding additional lags does not improve

the fit significantly. To recover the contributions of each shock to output dynamics, we simply assume that $\hat{Y}_t$ is a linear combination of the three variables in the VAR.

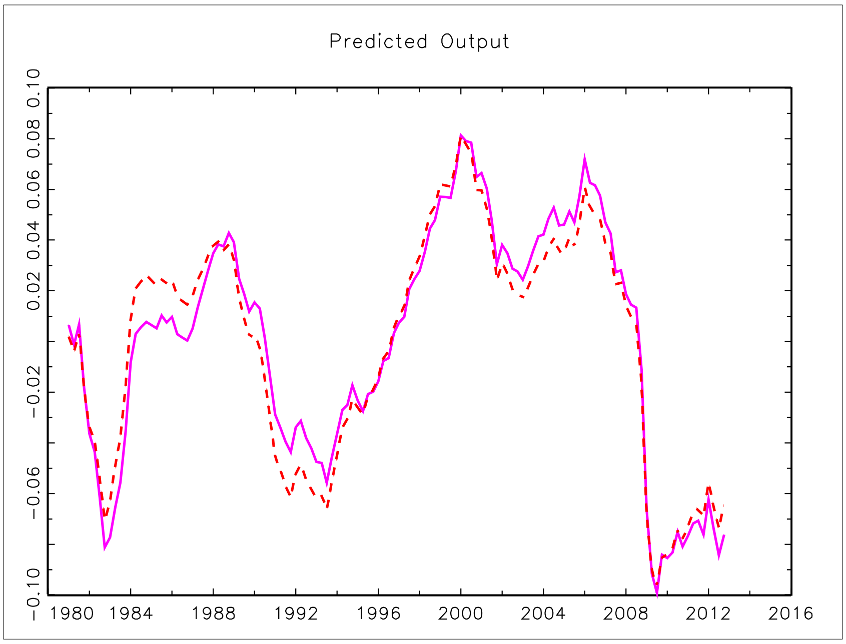

Figure 5: Actual output $\hat{Y}_t$ (dashed line) and predicted output (solid line) by $\overline{Y}_t = b \times [\hat{A}_t; \hat{\lambda}_t; \hat{v}_t]'$.

Notice that $\hat{A}_t$ in our VAR will pick up anything left unexplained by financial shocks (shocks to collateral $\hat{\lambda}$ and to unsecured credit $\hat{v}$). Hence we attribute all correlations to TFP shocks and no correlations in the residual vector e_t to sunspot shocks. In other words, we may be attributing too much influence to secured-credit shocks and TFP shocks, thus providing a lower bound on the contribution of sunspot shocks. Put differently, if the purified sunspot shock appears to be important, then it is even more important if we had attributed some of the correlations among e_t to the sunspot shocks. Hence our identification method goes against our null hypothesis that sunspots matter.

Figure 6 shows the time series decomposition of output into the three components associated with the three identified shocks $(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t})$, where the blue dashed line in each window represents the data output and the pink solid line represents the predicted output if only one single shock is active. The lower right graph puts all three shocks together, so that the dynamics of output is fully explained. TFP (shock 1) explains the 2009 recession episode quite well. This result holds regardless how we order the VAR variables. Secured credit (shock 2) contributes significantly to the 1990's credit expansion. Sunspot shocks ε_{3t} explain the broad business cycle features of output quite well (lower-left window) even though we have attributed all the correlations of the three shocks to TFP and to secured-credit shocks.

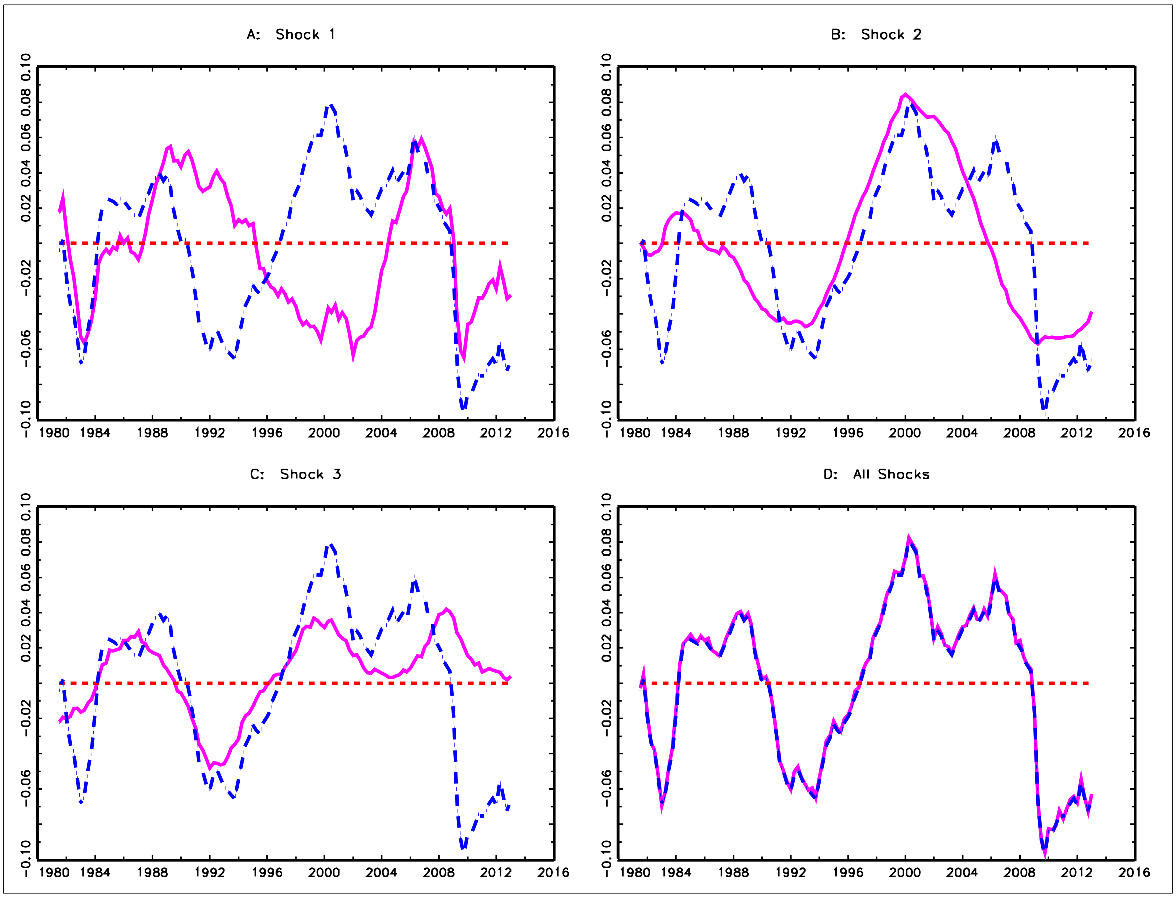

Figure 6: Decomposition of output in the three shocks: TFP (shock 1), secured credit (shock 2), sunspots (shock 3).

For a correlation analysis, we write the output series as $\hat{Y}_t = \hat{Y}_{1t} + \hat{Y}_{2t} + \hat{Y}_{3t}$, where each component is contributed from each of the three shocks respectively. We compute the following correlations $\text{corr}(\hat{Y}_{it}, \hat{Y}_t)$ for $i = 1, 2, 3$. Figure 7 shows that the part of output driven by sunspots (shock 3) is highly correlated with the data output, whereas the TFP shock is not

significantly correlated. This result is consistent with the top-left graph in Figure 6.
We can also decompose the total variance of output into the three components. We find that sunspot shocks explain about 40%-50% of total output variations.

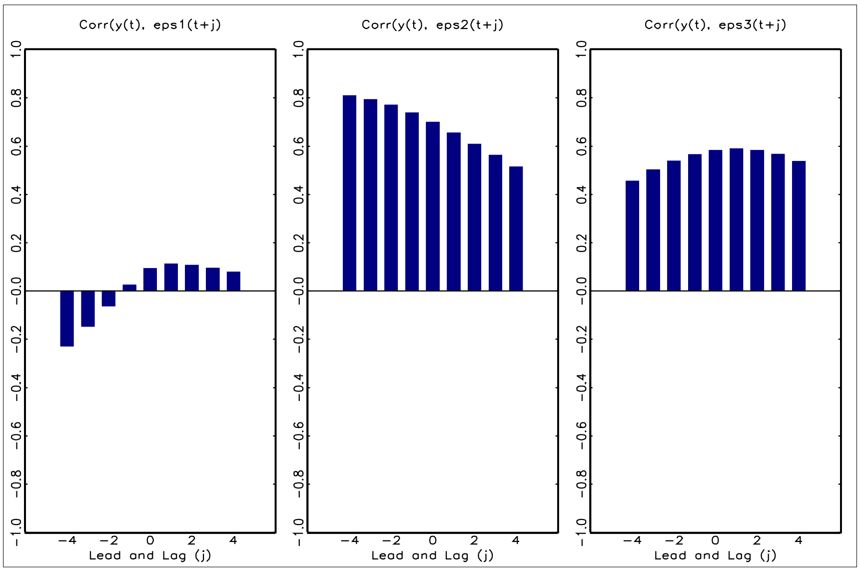

Figure 7: Lead-lag correlations between output and the three shock components.

We can also decompose the total variance of output (more specifically, the power spectrum) into the three components, with each contributed separately from the three identified shocks. We find that TFP shocks explain 55% of the output variance, λ-shocks explains 0.09%, and sunspot shocks explain the remaining 35%. Even though TFP shocks do not seem to be highly correlated with output, they still have enough variation to account for the bulk of output variance. Furthermore, even though the λ-shocks are highly correlated with output, the series does not have much high-frequency variation, hence does not contribute much to output fluctuations.

4.5 Alternative Specification

Since output is not perfectly co-linear with $\left[\hat{A}_t, \hat{\lambda}_t, \hat{v}_t\right]$, we consider a VAR with output included and ordered the last:

$$\begin{pmatrix} \hat{A}_t \\ \hat{\lambda}_t \\ \hat{v}_t \\ \hat{Y}_t \end{pmatrix} = B \begin{pmatrix} \hat{A}_{t-1} \\ \hat{\lambda}_{t-1} \\ \hat{v}_{t-1} \\ \hat{A}_{t-1} \end{pmatrix} + \begin{pmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \end{pmatrix},$$

We apply the Choleski decomposition such that $e_t = (e_{1t}, e_{2t}, e_{3t}, e_{4t})' = C(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, \varepsilon_{4t})'$ with lower triangular matrix C. In this case, if a linear combination of the three shock variables $\left[\hat{A}, \widehat{lambda}, \hat{v}\right]$ cannot perfectly explain output, then the residual will be picked up by the fourth shock ε_{4t}. But if the variance of ε_{4t} is small enough, then our results will not be affected by adding output. A nice feature of this new VAR system is that we can directly decompose output into the four shocks without suffering the same approximation errors in the previous three-variable VAR.

The results are broadly similar to those obtained above. Figure 8 shows that the first three shocks together can explain more than 99.9% of output variations (lower right graph entitled "all shocks"). In addition, the contribution of sunspot shocks (shock 3) is very similar to what we saw before. Again, TFP shocks (top left graph) appear to be important for the financial crisis, while shocks to secured credit (shock 2) are not very important overall.

Using spectral analysis, the total contribution of TFP shocks to output variance is 46.2%, that from λ_t shocks is 9.3%, and that from sunspot shocks is 44.5%, leaving only a tiny fraction of 0.05% to the fourth shock ε_{4t}.

5 Conclusions

Two enduring characteristics of the business cycle are the high autocorrelations of credit and output time series, and the strong cross-correlation between those two statistics. Understanding these correlations, without the help of persistent shocks to the productivity of financial intermediaries and of final goods producers, has been a long-standing goal of macroeconomic research and the motivation for the seminal contributions mentioned in the first paragraph of the introduction to this paper. Is it possible that cycles in credit, TFP and output are not the work of persistent productivity shocks that afflict all sectors of the economy simultaneously? Could these cycles instead come from small and temporary shocks to anticipated credit conditions?

This paper gives an affirmative answer to both questions within an economy in which part

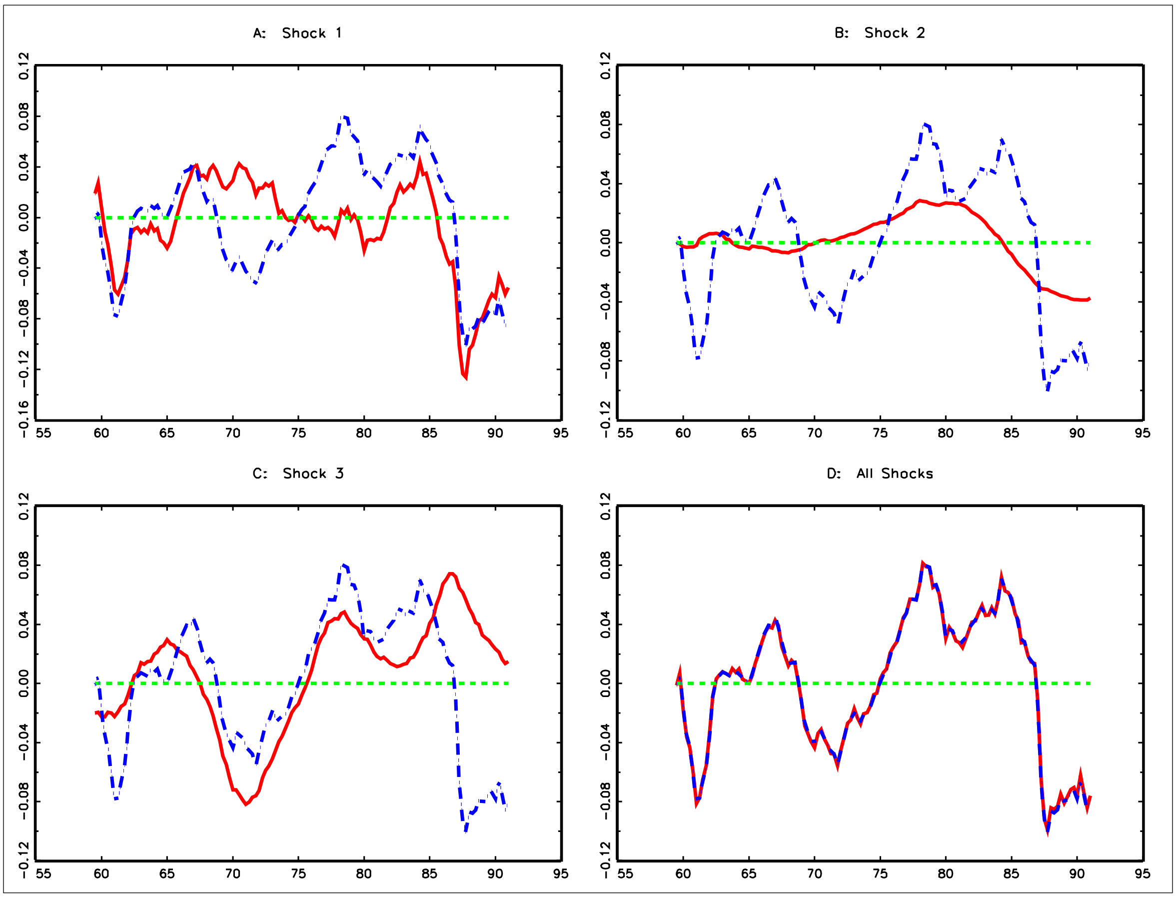

Figure 8: Decomposition of output in the four-variable VAR: TFP (shock 1), secured credit (shock 2), sunspots (shock 3).

of the credit firms require to finance investment is secured by collateral, and the remainder is based on reputation. Unsecured credit improves debt limits, facilitates capital reallocation and helps aggregate productivity, provided that borrowers expect plentiful unsecured credit in the future. Favorable expectations of future debt limits increase the value of remaining solvent and on good terms with one's lenders. Widespread doubts, on the other hand, about future credit will lead to long-lasting credit tightening with severe macroeconomic consequences.
It is this dynamic complementarity of current with future lending that connects macroeconomic performance over time and endows one-time expectational impulses with long lasting responses. A calibrated version of our economy, despite its apparent simplicity, matches well with observed features of the joint stochastic process governing U.S output, firm credit and investment and

illustrates the endogenous propagation of self-fulfilling belief shocks.

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Appendix A: Further Empirical Findings

We complement the business-cycle observations in Section 2 by a SVAR analysis to analyze impulse responses to different debt shocks. Constrained by our short sample, we use a two-variable VAR, assuming that shocks to debt have no contemporaneous impact on output. The findings shown in Figure 9 are consistent with those above. Shocks to unsecured debt account for significant impulse responses of output, while shocks to secured debt generate no significant output response. A variance decomposition further shows that shocks to unsecured debt explain about 35%-45% of the output variance, whereas shocks to secured debt can only account for 0%-10%.

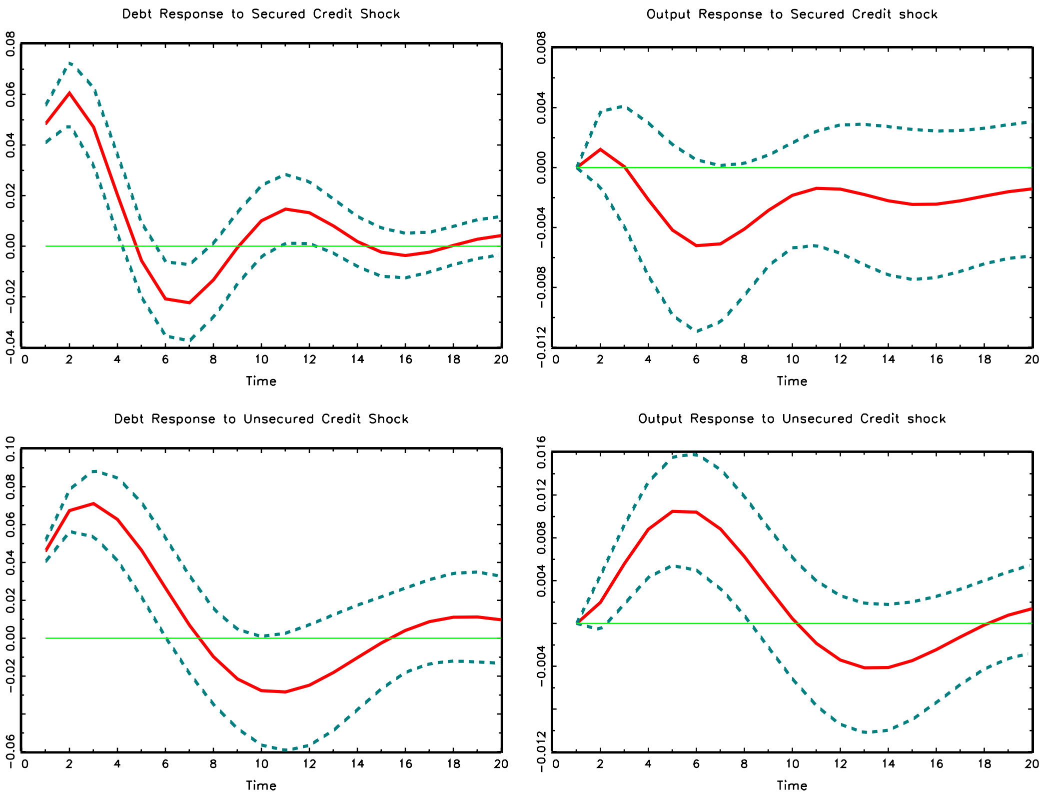

Figure 9: Impulse responses of output and debt to shocks to secured credit (top) and unsecured credit (bottom).

Appendix B: Proofs

Derivation of capital return R_t^s, $s = u, p$:
Consider a firm of type $s \in \{p, u\}$ with capital k_t^s. It employs ℓ_t^s workers satisfying the labor demand equation

$$(1-\alpha)A\Big(\frac{a^s k_t^s}{A\ell_t^s}\Big)^{\alpha} = w_t \ .$$

It follows for all firms

$$a^p \frac{k_t^p}{\ell_t^p} = a^u \frac{k_t^u}{\ell_t^u} \equiv c_t \ ,$$

where c_t is independent of firm type. Let L_t^s denote aggregate employment of type-s firms. Thus,

$$L_t^p = \frac{a^p}{c_t} z_t K_t \ , \ L_t^u = \frac{a^u}{c_t}(1-z_t)K_t \ .$$

Then labor market clearing $L_t^p + L_t^u = 1$ implies directly that

$$c_t = a_t K_t \text{ and } w_t = (1-\alpha)A^{1-\alpha}(a_t K_t)^{\alpha} \ ,$$

with $a_t = z_t a^p + (1-z_t)a^u$. Therefore, firm s employs $\ell_t^s = \frac{a^s}{a_t K_t} k_t^s$ workers, and its gross output net of labor costs is

$$(a^s k_t^s)^{\alpha}(A\ell_t^s)^{1-\alpha} + (1-\delta)a^s k_t^s - w_t \ell_t^s = a^s k_t^s\Big[\Big(\frac{A}{a_t K_t}\Big)^{1-\alpha} + 1 - \delta - \frac{w_t}{a_t K_t}\Big] = a^s k_t^s\Big[\alpha\Big(\frac{A}{a_t K_t}\Big)^{1-\alpha} + 1 - \delta\Big] \ .$$

This shows that $R_t^s = a^s[1-\delta+\alpha(A/(a_t K_t))^{1-\alpha}]$ is the capital return of a type-s firm.
Proof of Proposition 2: If borrowers were unconstrained in all periods, unproductive entrepreneurs lend out all their capital to productive entrepreneurs who borrow $(1-\pi)K_t$ in the aggregate, and the interest rate equals the capital return of productive entrepreneurs, $R_t = R_t^p$. It follows that there are no gains from leverage so that $\tilde{R}_t = \tilde{R}_t^p = R_t$ for all $t \geq 0$, and the only solution to equation (8) is $v_t = 0$ for all t. But then it follows from equation (7) that debt-equity constraints are $\theta_t = 0$, a contradiction. □

Proof of Corollary 1: In steady state, $K_{t+1} = K_t$ implies that $1/\beta = a(R^p/a^p)$ where $a = a^u + (a^p - a^u)\min(1, \pi(1+\theta))$ is average capital productivity. From Proposition 2, any steady state has binding credit-constraints, so that $R = \rho R^p < R^p$. Then either $\rho = \gamma = a^u/a^p$ implies $R = R^u = a^u(R^p/a^p) < a(R^p/a^p)$, or $\rho > \gamma$ and (4) implies $\pi(1+\theta) = 1$, so that $a = a^p$ and again $R < a(R^p/a^p)$. In any case, $R < 1/\beta$ follows. □

Proof of Proposition 3: Because of $f(v_{max}) < v_{max}$ and continuity, a solution $f(v) = v \in (\overline{v}, v_{max})$ exists iff $f(\overline{v}) > \overline{v}$. This condition is

$$[1-\gamma(1-\pi)]^{1+\Phi} > \pi^{\Phi} \ , \tag{13}$$

with $\Phi = \beta\pi/(1-\beta(1-\psi))$. The LHS in (13) is decreasing in γ, LHS<RHS at $\gamma = 1$, and LHS>RHS at $\gamma = 0$. Therefore there exists a solution $\gamma_1 \in (0,1)$ where LHS=RHS. It follows that the steady state $v^{**} \in (\overline{v}, v_{max})$ exists if $\gamma < \gamma_1$.
Since f is strictly convex in $v \in (0, \overline{v})$, a steady state $v^* \in (0, \overline{v})$ exists if $\gamma < \gamma_1$ (implying $f(\overline{v}) > \overline{v}$) and if $f'(0) < 1$. The latter condition is equivalent to $\gamma > \gamma_0 \equiv \frac{\Phi}{\Phi+1}$. This completes the proof. □

Appendix C: Extended Model and Log-Linearization

We first derive the dynamic equilibrium equations, for arbitrary stochastic processes for labor efficiency A_t and for the collateral share λ_t. Then we log-linearize the model at the indeterminate steady state, allowing for sunspot shocks.

Labor market equilibrium
Labor demand of firm i is

$$\ell_t^i = a_t^i k_t^i \Big(\frac{(1-\alpha)A_t^{1-\alpha}}{w_t}\Big)^{1/\alpha} ,$$

so that aggregate labor demand is $L_t^d = a_t K_t (\frac{(1-\alpha)A_t^{1-\alpha}}{w_t}\Big)^{1/\alpha}$ with average capital productivity

$$a_t = a^u + \pi(1+\theta_t)(a^p - a^u) \ . \tag{14}$$

With labor supply $L_t^s = w_t^{\varphi}$, the market-clearing wage is

$$w_t = (1-\alpha)^{\frac{1}{1+\varphi\alpha}} (a_t K_t)^{\frac{\alpha}{1+\varphi\alpha}} A_t^{\frac{1-\alpha}{1+\varphi\alpha}} \ .$$

This yields the equilibrium labor-to-capital ratio (in efficiency units)

$$\frac{A_t L_t}{a_t K_t} = (1-\alpha)^{\frac{\varphi}{1+\varphi\alpha}} (a_t K_t)^{-\frac{1}{1+\varphi\alpha}} A_t^{\frac{1+\varphi}{1+\varphi\alpha}} \ .$$

Credit market equilibrium
The return on capital for firm $i = p, u$ is

$$(a_t^i k_t^i)^{\alpha} (A_t \ell_t^i)^{1-\alpha} + a_t^i (1-\delta) k_t^i - w_t \ell_t^i = a_t^i [1-\delta+r_t] k_t^i \equiv R_t^i k_t^i \ ,$$

where

$$r_t \equiv \alpha \Big(\frac{A_t L_t}{a_t K_t}\Big)^{1-\alpha} , \tag{15}$$

is the average capital return. A productive firm that borrows θ_t per equity unit has leveraged equity return

$$\tilde{R}_t^p \equiv [R_t^p + \theta_t (R_t^p - R_t)] \ .$$

A firm that has no access to unsecured credit can only borrow secured, such that the debt does not exceed the value of collateral assets. Thus, the debt-equity constraint for secured borrowing θ_t^s is determined by $R_t \theta_t^s = \lambda_t R_t^p (1+\theta_t)$, and therefore

$$\theta_t^s = \frac{\lambda_t}{\frac{R_t}{R_t^p} - \lambda_t} \ . \tag{16}$$

As in Section 3, the continuation utility of a borrower with a clean credit record can be written $\ln(W) + V_t$ with end-of-period wealth W. Similarly, $\ln(W) + V_t^d$ is the continuation utility of a borrower with a default flag. Then a borrower with equity e decides not to default at the end of the period if

$$\ln[\tilde{R}_t^p e] + V_t \geq \ln[(1+\theta_t)R_t^p(1-\lambda_t)e] + V_t^d \ .$$

With $v_t = V_t - V_t^d$, the default-deterring debt-equity ratio can be solved from this equation as

$$\theta_t = \frac{\lambda_t + e^{v_t} - 1}{1 - \lambda_t - (1 - \frac{R_t}{\tilde{R}_t^p})} \ . \tag{17}$$

Clearly, θ_t increases in both the collateral share λ_t and in the reputation value v_t. Moreover, $\theta_t = \theta_t^s$ if $v_t = 0$ (so that unsecured credit cannot be sustained). We can also split the total credit constraint into a secured and into an unsecured component, $\theta_t = \theta_t^s + \theta_t^u$.

If a borrower decided to default, he is punished by exclusion from unsecured credit, retaining full access to secured credit. With probability ψ, the credit record clears, and the borrower can also borrow unsecured. Similar to the model without secured borrowing, we can derive a forward-looking equation for the reputation value:

$$v_t = \mathrm{E}_t \Big\{ \beta\pi \ln\Big[\frac{R_{t+1}^p + \theta_{t+1}(R_{t+1}^p - R_{t+1})}{R_{t+1}^p + \theta_{t+1}^s(R_{t+1}^p - R_{t+1})}\Big] + \beta(1-\psi)v_{t+1} \Big\} \ . \tag{18}$$

Here the expression in the term ln(.) is the excess leverage return that a borrower with a clean credit record enjoys relative to a defaulter who has only access to secured borrowing.

Credit market equilibrium with some misallocation of capital requires $R_t = R_t^u$ (so that unproductive firms are indifferent between lending and investing in their own technology). In such situations, we have $R_t/R_t^p = a^u/a^p = \gamma$, so that equation (18) simplifies to

$$v_t = \mathrm{E}_t \Big\{ \beta\pi \ln\Big[\frac{\gamma - \lambda_{t+1}}{1 - \lambda_{t+1} - (1-\gamma)e^{v_{t+1}}}\Big] + \beta(1-\psi)v_{t+1} \Big\} \ . \tag{19}$$

Capital accumulation and output

The aggregate capital stock evolves according to

$$K_{t+1} = \beta\Big[(1-\delta)a_t K_t + \alpha(A_t L_t)^{1-\alpha}(a_t K_t)^\alpha\Big] = \beta a_t K_t[1 - \delta + r_t] \ . \tag{20}$$

Since $a^p > 1 > a^u$, productive firms enhance their capital stock by $(a^p - 1)k_t^i$ while unproductive firms deplete capital $(1 - a^u)k_t^i$. In the aggregate, the term $(a_t - 1)K_t$, which may be positive or negative outside the steady state, therefore adds to or is subtracted from aggregate investment. Total output is thus written

$$Y_t = (a_t - 1)K_t + (A_t L_t)^{1-\alpha}(a_t K_t)^\alpha = (a_t - 1)K_t + a_t K_t \frac{r_t}{\alpha} \ . \tag{21}$$

Consumption and investment are

$$\begin{aligned} C_t &= w_t L_t + (1-\beta)a_t K_t[1 - \delta + r_t] = a_t K_t \Big[\frac{1-\alpha}{\alpha} r_t + (1-\beta)(1-\delta+r_t)\Big] \ , \\ I_t &= Y_t - C_t \ . \end{aligned}$$

Steady state and calibration of parameters

Given the calibration target for K/Y and $A = a = 1$, we can solve for steady-state values

$$K = (1-\alpha)^\varphi (K/Y)^{\frac{1+\varphi\alpha}{1-\alpha}} \ , \ r = \alpha Y/K \ ,$$

as well as for output, consumption and investment. Given the calibration target for the credit-to-capital ratio $\pi\theta$, we can solve for π (given any target value for θ, satisfying the requirement $\theta < (1-\pi)/\pi$ for misallocation of capital).

For any arbitrary choice of a^u and $a^p = a^u + \frac{1-a^u}{\pi(1+\theta)}$, and given the calibration target for the secured-credit-to-capital ratio $\theta^s\pi$, we obtain λ from $\theta^s = \lambda/(\gamma - \lambda)$, $\gamma = a^u/a^p$, and the reputation value in steady state from (17):

$$v = \ln\Big[\frac{(1-\lambda)(1+\theta)}{1+\theta(1-\gamma)}\Big] \ .$$

We then adjust a^u so that also equation (18) is satisfied in steady state, namely

$$v[1 - \beta(1-\psi)] = \beta\pi \ln\Big[\frac{\gamma - \lambda}{1 - \lambda - (1-\gamma)e^v}\Big] \ .$$

Log linearization

Log-linearize equations (14), (15), (16), (17), (19), (20), (21) to obtain

$$\hat{a}_t = \pi(1+\theta)(a^p - a^u)\hat{\theta}_t \ , \tag{22}$$

$$\hat{r}_t = r_1 \hat{A}_t + r_2[\hat{a}_t + \hat{K}_t] \ , \tag{23}$$

$$\hat{\theta^s}_t = \frac{\gamma}{\gamma - \lambda}\hat{\lambda}_t \ , \tag{24}$$

$$\hat{\theta}_t = d_1 \hat{v}_t + d_2 \hat{\lambda}_t \ , \tag{25}$$

$$\hat{v}_t = \mathrm{E}_t[\varphi_1 \hat{\lambda}_{t+1} + \varphi_2 \hat{v}_{t+1}] \ , \tag{26}$$

$$\hat{K}_{t+1} = \hat{K}_t + \frac{r}{1-\delta+r}[\hat{a}_t + \hat{r}_t\Big] \ , \tag{27}$$

$$\hat{Y}_t = \hat{K}_t + (1 + K/Y)\hat{a}_t + \hat{r}_t \ , \tag{28}$$

where

$$
\begin{aligned}
r_1 &= \frac{(1-\alpha)(1+\varphi)}{1+\varphi\alpha}\ , \\
r_2 &= -\frac{1-\alpha}{1+\varphi\alpha}\ , \\
d_1 &= \frac{e^v v}{\lambda+e^v-1} + \frac{(1-\gamma)e^v v}{1-\lambda-(1-\gamma)e^v}\ , \\
d_2 &= \frac{\lambda}{\lambda+e^v-1} + \frac{\lambda}{1-\lambda-(1-\gamma)e^v}\ , \\
\varphi_1 &= \frac{\beta\pi}{v}\frac{\lambda(1-\gamma)(e^v-1)}{(\gamma-\lambda)(1-\lambda-(1-\gamma)e^v)}\ , \\
\varphi_2 &= \beta(1-\psi) + \frac{\beta\pi(1-\gamma)e^v}{1-\lambda-(1-\gamma)e^v}\ .
\end{aligned}
$$

Because $\varphi_2 > 1$ at the indeterminate steady state, we obtain from (26) a stationary forward solution with sunspot shocks ε^s_{t+1} such that $\mathrm{E}_t(\varepsilon^s_{t+1}) = 0$:

$$
\hat{v}_{t+1} = \frac{1}{\varphi_2}\hat{v}_t - \frac{\varphi_1}{\varphi_2}\hat{\lambda}_{t+1} + \varepsilon^s_{t+1}\ . \tag{29}
$$

Identification of shocks

Given series for the credit-to-capital ratio and for the secured-credit-to-capital ratio (using mortgages as a proxy), we obtain $\hat{\theta}_t$ and $\hat{\theta^s}_t$. Then, we can solve for $\hat{\lambda}_t$ and ε^s_t from (24), (25) and (29). Finally, we choose $\hat{A}_t$ to match the output series $\hat{Y}_t$.

For comparison, it may also be interesting to look at TFP which is

$$
\hat{\mathrm{TFP}}_t = (\alpha + K/Y)\hat{a}_t + (1-\alpha)\hat{A}_t\ .
$$

Secured and unsecured credit are $D^s_t = \theta^s_t \pi K_t$, $D^u_t = (\theta_t - \theta^s_t)\pi K_t$, so

$$
\hat{D^s}_t = \hat{\theta^s}_t + \hat{K}_t\ ,\ \hat{D^u}_t = \frac{\theta}{\theta-\theta^s}\hat{\theta}_t - \frac{\theta^s}{\theta-\theta^s}\hat{\theta^s}_t + \hat{K}_t\ .
$$

Appendix D: Autocorrelated Productivity

This Appendix extends the model and Propositions 1–3 to an autocorrelated idiosyncratic productivity process. Specifically, suppose that productive entrepreneurs stay productive with probability π_p and become unproductive otherwise, whereas unproductive entrepreneurs become productive with probability π_u and stay unproductive otherwise. Assume that productivities are positively autocorrelated: $\pi_p > \pi_u$. The i.i.d. benchmark considered in the main text corresponds to the case $\pi_p = \pi_u = \pi$. We assume again that the collateral share is sufficiently low so as to ensure binding credit constraints and a capital misallocation in the absence of unsecured credit:

$$\lambda < \frac{\gamma(1-\pi_p)}{1-\gamma(\pi_p-\pi_u)} \ . \tag{30}$$

One major difference with the benchmark model is that the share of capital in the hands of productive entrepreneurs at the beginning of a period, denoted x_t, is a state variable which adjusts sluggishly over time (see Kiyotaki (1998)) according to

$$x_{t+1} = \frac{\pi_p \tilde{R}_t x_t + \pi_u R_t(1-x_t)}{\tilde{R}_t x_t + R_t(1-x_t)} \ , \tag{31}$$

where $R_t = \rho_t a^p R_t^*$ is again the gross interest rate (the equity return of unproductive entrepreneurs) and $\tilde{R}_t = [1+\theta_t(1-\rho_t)]a^p R_t^*$ is the equity return of productive entrepreneurs. Given x_t, fraction $z_t = \min(1, x_t(1+\theta_t))$ of capital is operated by productive entrepreneurs, $a_t = z_t a^p + (1-z_t)a^u$ is average capital productivity, and the capital return R_t^* is defined as in the main text. Capital market equilibrium boils down to the complementary slackness condition

$$\rho_t \geq \gamma \quad , \quad x_t(1+\theta_t) \leq 1 \ . \tag{32}$$

To derive the endogenous debt-equity ratio θ_t, define again $V_t(W)$ ($V_t^c(W)$) for the continuation values of a *productive* entrepreneur with a clean (bad) credit record who has wealth W at the end of period t. Similarly, define continuation values for *unproductive* entrepreneurs as $U_t(W)$ ($U_t^c(W)$). Because of logarithmic utility, all entrepreneurs save fraction β of wealth and continuation utilities can be written in the form $V_t(W) = \ln(W) + V_t$ etc. where V_t, V_t^c, U_t, U_t^c are independent of wealth and satisfy the recursive equations (with constant $C \equiv (1-\beta)\ln(1-\beta) + \beta \ln \beta$):

$$\begin{aligned}
V_t &= C + \beta \mathrm{E}_t \Big[\pi_p(\ln \tilde{R}_{t+1} + V_{t+1}) + (1-\pi_p)(\ln R_{t+1} + U_{t+1})\Big] \ , \\
V_t^c &= C + \beta \mathrm{E}_t \Big[\pi_p(\ln \tilde{R}_{t+1}^c + V_{t+1}^c + \psi(V_{t+1} - V_{t+1}^c)) \\
&\quad +(1-\pi_p)(\ln R_{t+1} + U_{t+1}^c + \psi(U_{t+1} - U_{t+1}^c))\Big] \ , \\
U_t &= C + \beta \mathrm{E}_t \Big[\pi_u(\ln \tilde{R}_{t+1} + V_{t+1}) + (1-\pi_u)(\ln R_{t+1} + U_{t+1})\Big] \ , \\
U_t^c &= C + \beta \mathrm{E}_t \Big[\pi_u(\ln \tilde{R}_{t+1}^c + V_{t+1}^c + \psi(V_{t+1} - V_{t+1}^c)) \\
&\quad +(1-\pi_u)(\ln R_{t+1} + U_{t+1}^c + \psi(U_{t+1} - U_{t+1}^c))\Big] \ .
\end{aligned}$$

Define $v_t \equiv V_t - V_t^c$ and $u_t \equiv U_t - U_t^c$ as reputation values for productive and unproductive entrepreneurs, satisfying

$$\begin{aligned} v_t &= \beta \mathrm{E}_t \Big[\pi_p \Big(\ln \frac{\tilde{R}_{t+1}}{\tilde{R}^c_{t+1}} + (1-\psi) v_{t+1} \Big) + (1-\pi_p)(1-\psi) u_{t+1} \Big] \ , \\ u_t &= \beta \mathrm{E}_t \Big[\pi_u \Big(\ln \frac{\tilde{R}_{t+1}}{\tilde{R}^c_{t+1}} + (1-\psi) v_{t+1} \Big) + (1-\pi_u)(1-\psi) u_{t+1} \Big] \ . \end{aligned}$$

These equations can be reduced to one in v_t with two forward lags, generalizing equation (8):

$$v_t = \beta \mathrm{E}_t \Big[\pi_p \ln \frac{\tilde{R}_{t+1}}{\tilde{R}^c_{t+1}} + (1-\psi)[\pi_p + 1 - \pi_u] v_{t+1} \Big] - \beta^2 (1-\psi)[\pi_p - \pi_u] \mathrm{E}_t \Big[\ln \frac{\tilde{R}_{t+2}}{\tilde{R}^c_{t+2}} + (1-\psi) v_{t+2} \Big] \ . \quad (33)$$

Default-deterring debt limits are linked to reputation values v_t according to the same equation (7) as in the main text. This generalizes Proposition 1 as follows: Any solution $(\rho_t, \theta_t, v_t, x_t)$ to the system of equations (31), (32), (33) and (7) defines a competitive equilibrium.

It is straightforward to check that credit constraints are binding if (30) holds, which generalizes Proposition 2. If constraints were slack in all periods, $\rho_t = 1$ and $\tilde{R}_t = \tilde{R}^c_t = R_t$ would imply that $v_t = 0$ in all periods t, so that default–deterring debt-equity ratios are $\theta_t = \lambda/(1-\lambda)$. On the other hand, because of (31), the capital share of productive entrepreneurs would converge to the stationary population share which is $x_t \to \overline{x}_{FB} \equiv \frac{\pi_h}{1+\pi_u-\pi_p}$. Capital market equilibrium with non-binding constraints requires however that the debt capacity of borrowers exceeds capital supply of lenders, $\theta_t x_t \geq 1 - x_t$ which boils down to $\lambda \geq (1-\pi_p)/(1-\pi_p+\pi_u)$, contradicting condition (30).

Condition (30) furthermore implies that there exists an equilibrium without unsecured credit ($v_t = 0$ for all t) where capital is misallocated. In this equilibrium, $\rho_t = \gamma$, $\theta_t = \overline{\theta} \equiv \frac{\lambda}{\lambda-\gamma}$, and the stationary capital share $\overline{x}$ solves the quadratic

$$\overline{x}\Big[(1-\gamma)\overline{x} + \gamma - \lambda\Big] = \pi_p(1-\lambda)\overline{x} + \pi_u(\gamma-\lambda)(1-\overline{x}) \ ,$$

which has a unique solution $\overline{x} \in (0,1)$. A credit market equilibrium with misallocated capital at $\rho = \gamma$ requires that $\overline{x}\overline{\theta} < 1 - \overline{x}$. It is straightforward to verify that this equivalent to condition (30).

Lastly, we generalize Proposition 3 as follows.

Proposition 5 *For all parameter values there exists a stationary equilibrium in which no unsecured credit is available and capital is inefficiently allocated. Provided that λ is sufficiently small, there are threshold values $\gamma_0 < \gamma_1 < 1$ such that:*

(a) *For $\gamma \in (\gamma_0, \gamma_1)$, there are two stationary equilibria with unsecured credit, one of them with inefficient capital allocation and the other one with efficient capital allocation.*

(b) *For* $\gamma > \gamma_1$*, there is no stationary equilibrium with unsecured credit.*

(c) *For* $\gamma \leq \gamma_0$*, there exists a unique stationary equilibrium with unsecured credit and efficient capital allocation.*

Proof: The existence of the equilibrium without unsecured credit has already been established above. Consider first a steady–state equilibrium with an inefficient capital allocation ($\theta x < 1 - x$ and $\rho = \gamma$) and unsecured credit ($v > 0$). Because of $\tilde{R}/\tilde{R}^c = \frac{\gamma-\lambda}{1-\lambda-e^v(1-\gamma)}$, equation (33) implies in steady state that

$$e^v = F(e^v) \equiv \left(\frac{\gamma - \lambda}{1 - \lambda - e^v(1-\gamma)}\right)^{\Phi} , \tag{34}$$

with parameter $\Phi \equiv \frac{\beta\pi_p - \beta^2(1-\psi)(\pi_p-\pi_u)}{1-\beta(1-\psi)[\pi_p+1-\pi_u]+\beta^2(1-\psi)^2(\pi_p-\pi_u)} > 0$. Redefine $\varphi = e^v > 1$ and note that F is increasing and strictly convex with $F(\varphi) \to \infty$ for $\varphi \to (1-\lambda)/(1-\gamma) > 1$. We also have that $F(1) = 1$ (which corresponds to the steady state $v = 0$ without unsecured credit). This implies that equation (34) has a solution $\varphi = e^v > 1$ if and only if $F'(1) < 1$ which is equivalent to $\gamma > \gamma_0 \equiv \frac{\lambda+\Phi}{1+\Phi}$. The stationary capital share x solves

$$x = H(x) \equiv \frac{\pi_p[1+\theta(1-\gamma)]x + \pi_u\gamma(1-x)}{[1+\theta(1-\gamma)]x + \gamma(1-x)} ,$$

where function H is increasing (because of $\pi_p > \pi_u$). This equation has a unique solution $x \in (0,1)$ which satisfies $\theta x < 1 - x$ if and only if $1/(1+\theta) > H(1/(1+\theta))$ which is equivalent to

$$\theta < \frac{1-\pi_p}{\pi_p(1-\gamma) + \pi_u\gamma} .$$

Using $\theta = \frac{\varphi-1+\lambda}{1-\lambda-\varphi(1-\gamma)}$, this is equivalent to

$$\varphi < \overline{\varphi} \equiv \frac{(1-\lambda)(1-\gamma(\pi_p-\pi_u))}{1-\gamma+\pi_u\gamma} .$$

Since F is increasing and convex with $F'(\varphi) > 1$, this holds if and only if $F(\overline{\varphi}) > \overline{\varphi}$ which is equivalent to

$$[1-\gamma(1-\pi_u)]^{1+\Phi} > (1-\lambda)^{1+\Phi}\left(\frac{\gamma}{\gamma-\lambda}\right)^{\Phi}[\pi_p - \gamma(\pi_p-\pi_u)]^{\Phi}[1-\gamma(\pi_p-\pi_u)] . \tag{35}$$

In this inequality, both the LHS and the RHS are decreasing functions of γ such that LHS(1) < RHS(1) (because of (30)) and LHS($\underline{\gamma}$) = RHS($\underline{\gamma}$) at $\underline{\gamma} \equiv \lambda/(1-\pi_p+\lambda(\pi_p-\pi_u)) < 1$. Moreover, we have $0 > \text{LHS}'(\underline{\gamma}) > \text{RHS}'(\underline{\gamma})$ if and only if

$$\lambda[\pi_p - \lambda(\pi_p-\pi_u)](1-\pi_u)(1+\Phi) < (1-\lambda)(1-\pi_p)\Phi[1-\pi_p+\lambda(\pi_p-\pi_u)] .$$

This inequality is true if λ is sufficiently small, so that we can conclude that there exists $\gamma_1 \in (\underline{\gamma}, 1)$ such that inequality (35) is satisfied for all $\gamma \in (\underline{\gamma}, \gamma_1)$ (see **??**). Since also $\gamma_0 \in (\underline{\gamma}, \gamma_1)$, we conclude that there exists a steady state with inefficient production and unsecured credit if and only if $\gamma \in (\gamma_0, \gamma_1)$.

Second, consider an equilibrium with unsecured credit and efficient production, so that $\rho > \gamma$ and $\theta = \frac{e^v - 1 + \lambda}{1 - \lambda - e^v(1-\rho)}$. The stationary capital share in such an equilibrium is $x = \frac{\pi_p(1-\rho) + \pi_u \rho}{1 - \rho(\pi_p - \pi_u)}$, and capital market equilibrium requires that $x\theta = 1 - x$. Combining these equations establishes the equilibrium interest rate at given reputation value v:

$$\rho = \frac{e^v - 1 + \lambda}{e^v(1 - \pi_u) - (1 - \lambda)(\pi_p - \pi_u)} \ . \tag{36}$$

On the other hand, equation (33) yields the stationary reputation value, analogously to (34),

$$e^v = \left(\frac{\rho - \lambda}{1 - \lambda - e^v(1 - \rho)} \right)^{\Phi} \ . \tag{37}$$

Solving (36) for e^v and substitution into (37) yields the following equation for the equilibrium value of ρ:

$$[1 - \rho(1 - \pi_u)]^{1+\Phi} = (1 - \lambda)^{1+\Phi} \left(\frac{\rho}{\rho - \lambda} \right)^{\Phi} [\pi_p - \rho(\pi_p - \pi_u)]^{\Phi} [1 - \rho(\pi_p - \pi_u)] \ . \tag{38}$$

In this equation, both sides (functions of ρ) are the same as both sides in inequality (35) (functions of γ). We conclude, again for λ sufficiently small, that $\rho = \gamma_1 < 1$ solves equation (38). In turn, for every $\gamma < \gamma_1 = \rho$, a steady-state equilibrium with efficient production and unsecured credit exists. This completes the proof. $\square$