

Mandatory Disclosure and Financial Contagion*

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Abstract

The paper analyzes the welfare implications of mandatory disclosure of losses at financial institutions when it is common knowledge that some banks have incurred losses but not which ones. We develop a model that features “contagion,” meaning that banks not hit by shocks may still suffer losses because of their exposure to banks that are. In addition, banks in our model have profitable investment projects that require outside funding, but which banks will only undertake if they have enough equity. Investors thus value information about which banks were hit by shocks. We find that when the extent of contagion is large, it is possible for no information to be disclosed in equilibrium but for mandatory disclosure to increase welfare by allowing investment that would not have occurred otherwise. Absent contagion, however, mandatory disclosure will not raise welfare, even if markets are otherwise frozen. Our findings provide insight on when contagion is likely to be a concern, e.g. when banks are highly leveraged against other banks, and thus on when mandatory disclosure is likely to be desirable.

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1 Introduction

In trying to explain how the decline in U.S. house prices evolved into a financial crisis in which trade between financial intermediaries nearly ground to a halt, various analysts have singled out the prevailing uncertainty at the time regarding which entities incurred the bulk of the losses associated with the housing market. For instance, [Gorton \(2008\)](#) provides an early analysis of the crisis in which he argues

“The ongoing Panic of 2007 is due to a loss of information about the location and size of risks of loss due to default on a number of interlinked securities, special purpose vehicles, and derivatives, all related to subprime mortgages... The introduction of the ABX index revealed that the values of subprime bonds (of the 2006 and 2007 vintage) were falling rapidly in value. But, it was not possible to know where the risk resided and without this information market participants rationally worried about the solvency of their trading counterparties. This led to a general freeze of intra-bank markets, write-downs, and a spiral downwards of the prices of structured products as banks were forced to dump assets.”

Market participants emphasized the same phenomenon as the crisis was unfolding. Back in February 24, 2007, the *Wall Street Journal* attributed the following to former Salomon Brothers vice chairman Lewis Ranieri, the so-called “godfather” of mortgage finance:

“The problem ... is that in the past few years the business has changed so much that if the U.S. housing market takes another lurch downward, no one will know where all the bodies are buried. ‘I don’t know how to understand the ripple effects through the system today,’ he said during a recent seminar.”

In line with this view, some have argued that an important step in eventually stabilizing financial markets was the Fed’s decision to release the results of its stress tests on large U.S. banks. These tests required banks to report to Fed examiners how their respective portfolios would fare under various stress scenarios and thus the losses banks were vulnerable to. In contrast to the traditional confidentiality accorded to bank examinations, these results were publicly released. [Bernanke \(2013\)](#) summarizes the view that the public disclosure of the stress-test results played an important role in stabilizing financial markets:

“In retrospect, the [Supervisory Capital Assessment Program] stands out for me as one of the critical turning points in the financial crisis. It provided anxious investors with something they craved: credible information about prospective losses at banks. Supervisors’ public disclosure of the stress test results helped restore confidence in the banking system and enabled its successful recapitalization.”

In this paper, we examine whether uncertainty about which banks incurred losses – that is, uncertainty as to where the bad apples are located – can lead to market freezes that make it desirable for policymakers to intervene and force banks to disclose their financial position. The feature that turns out to be critical for such intervention to be beneficial in our model is contagion. By this, we mean that shocks that hit some banks lead to losses at other banks not directly hit by these shocks, e.g. losses of banks directly exposed to the subprime market may lead to losses at banks that hold few subprime mortgages.

In what follows, we focus on a model of balance sheet contagion in which banks that are hit by shocks end up defaulting on their obligations to other banks, so that banks not hit by shocks can still have their equity wiped out. We modify this model in two ways. First, we allow banks to raise additional funds from outside investors in order to finance profitable investment projects. However, we introduce an agency problem so that investors only want to invest in banks with sufficient equity. When investors are uncertain about which banks incurred losses, they may refuse to invest in banks altogether. Contagion exacerbates this problem, since investors worry not only that the banks they invest in were hit by shocks that wiped out their equity, but that these banks may be indirectly exposed to such shocks because they have financial obligations from banks that were directly hit. The greater the potential for contagion, the more likely are market freezes to occur.

Second, we allow banks to disclose whether they were hit by shocks. Thus, our model can speak to whether it might be desirable to mandate disclosure even though banks can hire an external auditor to conduct a stress test or directly release the information they provide examiners. We show that when the extent of contagion is small, mandatory disclosure cannot improve welfare when banks choose not to disclose in equilibrium, even if non-disclosure results in a market freeze where no bank can raise outside funds. But when contagion is large and the cost of disclosure is low, mandatory disclosure can improve welfare. Intuitively, contagion implies that information on the financial health of one bank is relevant for assessing the financial health of other banks. Since banks fail to internalize these informational spillovers, too little information will be revealed, creating a role for mandatory disclosure as a welfare improving intervention. Absent these spillovers, banks internalize the benefits of disclosure, and so if they choose not to disclose it must be because the cost of disclosure exceeds the benefits. In that case, forcing them to disclose will not be desirable.

Since our model is somewhat involved, an overview may be helpful. At the heart of our model is a set of banks arranged in a network that reflects financial obligations across banks. Some of these banks are hit with shocks that prevent them from paying their obligations to other banks in full. Since banks are interconnected, banks not hit by shocks are still vulnerable to losses. All banks, including those hit by a shock, have access to profitable

projects that require them to raise outside funds. However, because of an agency problem present at each bank, outside investors only want to invest in banks with enough equity. Banks that want to raise funds can disclose at a cost whether they were hit by a shock. This disclosure must be made before knowing which other banks were hit with shocks, and thus before knowing one's own equity. Outside investors see all the information that is disclosed and decide which banks if any to invest in and at what terms. If enough banks choose not to disclose their state, investors will be uncertain as to which banks were hit by shocks. If banks do raise funds, they learn their equity before deciding what to do with the funds they raised. Investors thus worry about whether the banks they fund will have adequate equity.

This framework allows us to draw the connection between contagion and the desirability of mandatory disclosure. It also allows us to show which features of the economic environment can give rise to contagion and market freezes, e.g. the degree of leverage banks have against other banks in the network, the magnitude of losses, and the relative and absolute number of banks hit by shocks. In addition, our approach leads us to derive expressions for contagion probabilities for a particular network with multiple bad banks, a result that may be of interest for researchers working on contagion independently of our results on disclosure.

The paper is structured as follows. Section 2 reviews the related literature. Section 3 develops the model of contagion we use in our analysis. In Section 4 we modify our model so that banks can raise additional funds, and we introduce an agency problem that makes investors leery of investing in banks with little equity. In Section 5, we introduce a disclosure decision. We then examine whether non-disclosure can be an equilibrium outcome, and if so whether mandatory disclosure can be welfare improving relative to that equilibrium. Section 6 considers more general network structures. Section 7 concludes.

2 Literature Review

Our paper is related to several different literatures, specifically work on i) financial contagion and networks, ii) disclosure, iii) market freezes, and iv) stress tests.

Turning first to the literature on contagion, various channels for contagion have been described in the literature. For a survey, see [Allen and Babus \(2009\)](#). We focus on models of contagion based on balance sheet effects in which a bank hit by a shock is unable to pay its obligations, making it difficult for other banks to meet their obligations. Examples of papers that explore this channel include [Kiyotaki and Moore \(1997\)](#), [Allen and Gale \(2000\)](#), [Eisenberg and Noe \(2001\)](#), [Gai and Kapadia \(2010\)](#), [Caballero and Simsek \(2012\)](#), [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2013\)](#), and [Elliott, Golub, and Jackson \(2013\)](#). These papers are largely concerned with how the pattern of obligations across banks affects the extent of

contagion, and whether certain network structures can reduce the extent of contagion. Our focus is quite different: Rather than exploring which policies might mitigate the extent of contagion, we examine whether policies can be used to mitigate the fallout from contagion once it occurs, e.g. restarting trade in markets that would otherwise remain frozen.

Since our model posits that banks connected via a network communicate information, we should point out that there is a literature on communication and networks, e.g. [De-Marzo, Vayanos, and Zwiebel \(2003\)](#), [Calvó-Armengol and de Martí \(2007\)](#), and [Galeotti, Ghiglino, and Squintani \(2013\)](#). However, these papers study environments in which agents communicate to others *on the network*. By contrast, we study an environment where agents communicate *about* the network, specifically the location of its bad nodes, *to outsiders*.

The other major literature our work relates to concerns disclosure. [Verrecchia \(2001\)](#) and [Beyer et al. \(2010\)](#) provide good surveys of this literature. A key result in this literature, established by [Milgrom \(1981\)](#) and [Grossman \(1981\)](#), is an “unravelling principle” which holds that all private information will be disclosed because agents with favorable information want to avoid being pooled with inferior types and receive worse terms of trade. [Beyer et al. \(2010\)](#) summarize the various conditions subsequent research has established as necessary for this unravelling result to hold: (1) disclosure must be costless; (2) outsiders know the firm has private information; (3) all outsiders interpret disclosure identically, i.e. outsiders have no private information (4) information can be credibly disclosed, i.e. information is verifiable; and (5) agents cannot commit to a disclosure policy ex-ante before observing the relevant information. Violating any one of these conditions can result in equilibria where not all relevant information is conveyed. In our model, non-disclosure can be an equilibrium outcome even when all of these conditions are satisfied. We thus highlight a distinct reason for the failure of the unravelling principle that is due to informational spillovers: In order to know whether a bank in our model is safe to invest in, outside investors need to know not just the bank’s own balance sheet, but also the balance sheets of other banks.

Ours is certainly not the first paper to explore disclosure in the presence of informational spillovers. One important predecessor is [Admati and Pfleiderer \(2000\)](#). Their setup also allows for informational spillovers and gives rise to non-disclosure equilibria. However, these equilibria rely crucially on disclosure being costly; when the cost of disclosure is zero in their model, all information will be disclosed. Our framework allows for non-disclosure even when disclosure is costless because it allows for informational complementarities that are not present in their model. In particular, disclosure by a bank in our model is not enough to establish whether that bank has positive equity, since this requires information about other banks in the network. This feature has no analog in their model. However, [Admati and Pfleiderer \(2000\)](#) are similar to us in showing that informational spillovers can make

mandatory disclosure welfare-improving.¹ Another difference between our model and theirs is that they assume agents commit to disclosing information before learning it, while in our model banks know their losses and then choose to disclose it. In addition, our setup allows us to study contagion and disclosure, something that cannot be deduced from their setup.

Our paper is also related to the literature on market freezes. As in our model, this literature emphasizes the role of informational frictions. Some of these papers emphasize private information, where agents are reluctant to trade with others for fear of being exploited by more informed agents. Examples include Rocheteau (2011), Guerrieri, Shimer, and Wright (2010), Guerrieri and Shimer (2012), Camargo and Lester (2011), and Kurlat (2013). Others have focused on uncertainty concerning each agent’s own liquidity needs and the liquidity needs of others which discourages trade. Examples include Caballero and Krishnamurthy (2008) and Gale and Yorulmazer (2013). One difference between our framework and these papers concerns the source of informational frictions. Since in our framework the uncertainty concerns information that can in principle be verified such as a bank’s balance sheet, it naturally focuses attention on the possibility that this information might be revealed. By contrast, previous papers have focused on private information on assets that may be more difficult to verify, or information that no agents are privy to and thus cannot be disclosed.

Finally, there is an emerging literature on stress tests. On the empirical front, Peristian, Morgan, and Savino (2010), Bischof and Daske (2012), Ellahie (2012), and Greenlaw et al. (2012) have looked at how the release of stress-test results in the US and Europe affected bank stock prices. These results are complementary to our analysis, which is more concerned with normative questions regarding the desirability of releasing stress-test results. There are also several recent theoretical papers on stress tests, e.g. Goldstein and Sapra (2013), Goldstein and Leitner (2013), Shapiro and Skeie (2012), Spargoli (2012), and Bouvard, Chaigneau, and de Motta (2013). In these papers, banks are not allowed to disclose information on their own. Thus, these papers sidestep the main question we are after, namely whether it is possible that banks voluntarily choose not to disclose but forcing banks to disclose is desirable.

3 A Model of Contagion

We begin with a bare-bones version of our model where banks make no decisions. This allows us to highlight how contagion works in our model and to motivate our measure of contagion.

Our approach to modelling contagion follows Allen and Gale (2000), Eisenberg and Noe (2001), Gai and Kapadia (2010), Caballero and Simsek (2012), and Acemoglu, Ozdaglar,

¹Foster (1980) and Easterbrook and Fischel (1984) also argue that spillovers may justify mandatory disclosure, although these papers do not develop formal models to study this.

and Tahbaz-Salehi (2013) in focusing on the role of balance sheet effects. Formally, there are n banks indexed by $i \in \{0, \dots, n-1\}$. Each bank i is endowed with a set of financial obligations $\Lambda_{ij} \geq 0$ to each bank $j \neq i$. Following Eisenberg and Noe (2001), we take these obligations as given without modelling where they come from. Kiyotaki and Moore (1997) and Zawadowski (2013) have shown that banks might choose to enter such obligations without insuring themselves, despite the potential for contagion under such arrangements.

For much of our analysis, we follow Caballero and Simsek (2012) in restricting attention to the special case in which

$$\Lambda_{ij} = \begin{cases} \lambda & \text{if } j = (i+1) \pmod{n} \\ 0 & \text{else} \end{cases} \quad (1)$$

This case is known as a ring network or circular network, since these obligations can be depicted graphically as if the n banks are located along a circle as shown in Figure 1. In Section 6, we show that our analysis can be extended to a larger class of networks. However, since the circular network is expositionally convenient, we begin by focusing on this case.

In addition to the obligations Λ_{ij} , each bank is endowed with some assets that can be liquidated if needed. We do not explicitly model the value of these assets, and simply set their value fixed at some value $\pi > 0$.

A fixed number of banks b are hit by negative net worth shocks, where $1 \leq b \leq n-1$. We refer to these as “bad” banks. We thus generalize Caballero and Simsek (2012), who assume $b = 1$. Each bad bank incurs a loss ϕ , where ϕ represents a claim on the bank by an outside sector, i.e. by an entity that is not any of the remaining banks in the network. We follow previous work in assuming ϕ is senior to the obligations to other banks in the network. Thus, a bank must use its available resources to pay its senior claimant before paying other banks in the network.² We shall refer to all remaining banks as “good.”

Let $S_j = 1$ if j is a bad bank and 0 otherwise. The vector $S = (S_0, \dots, S_{n-1})$ denotes the state of the banking network. By construction, $\sum_{j=0}^{n-1} S_j = b$. We assume shocks are equally likely to hit any bank, i.e. each of the $\binom{n}{b}$ possible locations of the bad banks within the network are equally likely. It follows that $\Pr(S_j = 1) = \frac{b}{n}$ for any bank j .

We now analyze the financial position of banks in the model. Banks can be either insolvent – i.e. unable to pay their obligation λ to another bank in full – or solvent, although they may have to liquidate some of their endowment to pay their debts. The feature we wish to highlight is that the equity of good banks may be hit because of their exposure to bad banks.

Let x_j denote the payment bank j makes to bank $j+1$, and y_j denote the payment bank

²We could have alternatively assumed these obligations have equal seniority as obligations to other banks on the network, although this setup is more cumbersome. We thank Fabrice Tourre for pointing this out.

j makes to the outside sector. Bank j has $x_{j-1} + \pi$ resources it can draw on to meet its obligations. Given our restrictions on the seniority, it must first pay the outside sector. Let $\Phi_j \equiv \phi S_j$ denote the obligation to the outside sector. Then the payment y_j must satisfy

$$y_j = \min \{x_{j-1} + \pi, \Phi_j\} \quad (2)$$

Bank j can then use any remaining resources to pay bank $j + 1$, to which it owes λ , and so

$$x_j = \min \{x_{j-1} + \pi - y_j, \lambda\} \quad (3)$$

Substituting in for y_j yields a system of equations involving only the payments between banks, $\{x_j\}_{j=0}^{n-1}$, that characterizes these payments:

$$x_j = \max \{0, \min (x_{j-1} + \pi - \Phi_j, \lambda)\}, \quad j = 0, \dots, n-1 \quad (4)$$

(4) involves n equations and n unknowns. Given a solution $\{x_j\}_{j=0}^{n-1}$, we can define the implied equity of bank j as the value of any residual resources after a bank settles all payments, i.e.

$$e_j = \max \{0, \pi - \Phi_j + x_{j-1} - x_j\} \quad (5)$$

Although e_j is redundant given the payments x_j , equity will turn out to be important later when we expand the model. While x_j and e_j depend on the state of the network S , i.e. $x_j = x_j(S)$ and $e_j = e_j(S)$, we omit the explicit reference to S when this dependence is not essential. Our first result establishes that (4) has a generically unique solution $\{x_j^*\}_{j=0}^{n-1}$.³

Proposition 1: For a given S , the system (4) has a unique solution $\{x_j^*\}_{j=0}^{n-1}$ if $\phi \neq \frac{n}{b}\pi$.

In the knife-edge case where the total losses of bad banks, $b\phi$, equal the total endowments of banks $n\pi$, multiple solutions are possible for large λ . However, these solutions are equivalent in the sense that the outside sector is paid in full across and bank equity is the same all solutions, i.e. $y_j = \Phi_j$ for all j , and equity $e_j = 0$ for all j . The only difference across solutions are the notional amounts banks default on to other banks.⁴

In what follows, we will initially restrict attention to the case of $\phi < \frac{n}{b}\pi$, so total losses incurred by bad banks $b\phi$ cannot be so large that they exceed the total resources of the banking system, $n\pi$. Although [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2013\)](#) argue that allowing for large losses can yield important insights on the nature of contagion, such shocks

³Our result is a special case of Theorem 2 in [Eisenberg and Noe \(2001\)](#) and Proposition 1 in [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2013\)](#). The latter establishes uniqueness for a generic network Λ_{ij} but does not provide exact conditions for non-uniqueness as we do for the particular network we analyze.

⁴[Eisenberg and Noe \(2001\)](#) also show in their Theorem 1 that $\{e_j\}_{j=0}^{n-1}$ is unique even if $\{x_j\}_{j=0}^{n-1}$ is not.

yields few insights for our purposes. In particular, when $\phi > \frac{n}{b}\pi$, two outcomes are possible, depending on the value of λ . When λ is small, the distribution of equity $\{e_j\}_{j=0}^{n-1}$ is independent of ϕ , and so this case can be understood even if we restrict $\phi < \frac{n}{b}\pi$. When λ is large, none of the n banks have equity when $\phi > \frac{n}{b}\pi$. Since we are interested in decisions when banks are unsure about their equity, this case offers little insight.

At the same time, we don't want the loss per bank ϕ to be too small, since as the next proposition shows, $\phi \leq \pi$ implies bad banks are solvent and so there is no contagion.

Proposition 2: If $\phi \leq \pi$, then $x_j^* = \lambda$ for all j and $e_j = \pi$ for any j for which $S_j = 0$.

The above insights suggest the following restriction on ϕ :

Assumption A1: Losses at bad banks ϕ satisfy $\pi < \phi < \frac{n}{b}\pi$.

When $\phi > \pi$, bad banks will be insolvent: Even if these banks receive the full amount λ from the bank that is indebted to them, they will have less than λ resources to pay their obligations. The equity of each bad bank must therefore be 0 under Assumption A1.

To understand the nature of contagion in this economy, it will help to begin with the case of one bad bank, i.e. $b = 1$, as in [Caballero and Simsek \(2012\)](#). Without loss of generality, let bank $j = 0$ be the bad bank. Given that bank 0 receives x_{n-1} from bank $n - 1$, the total amount of resources bank 0 can give to bank 1 is $\max\{x_{n-1} + \pi - \phi, 0\}$. We show in Proposition 3 below that under Assumption A1, there is at least one bank that is solvent and can pay its obligation λ in full. From this, it follows that bank $n - 1$ must be solvent, since if any bank $j \in \{1, \dots, n - 2\}$ were solvent, it would pay bank $j + 1$ in full, who in turn will pay bank $j + 2$ in full, and so on, until we reach bank $n - 1$.

Deriving the equity of each bank is straightforward. Bank 0 has $\pi + \lambda$ worth of resources and owes $\phi + \lambda$, so it will fall short on its obligation to bank 1 by

$$\Delta_0 = \min\{\phi - \pi, \lambda\}.$$

Since bank 1 is endowed with $\pi > 0$ resources, it can use them to make up some of the shortfall it inherits when it pays bank 2. If the shortfall $\Delta_0 > \pi$, bank 1 will also be insolvent, although its shortfall will be π less than shortfall it receives. The first bank that inherits a shortfall that is less than or equal to π will be solvent, with an equity position that is at least 0 but strictly less than π . Hence, we can classify banks into three groups: (1) Insolvent banks with zero equity, which includes both the bad bank and possibly several good banks; (2) Solvent banks whose equity is $0 \leq e_j < \pi$. When $b = 1$, there will be exactly one such bank; and (3) Solvent banks that are sufficiently far from the bad bank and have equity equal to π .

Since equity will figure prominently in our analysis below, it will be convenient to work with the case where e_j can take on only two values, 0 or π . For $b = 1$, this requires that

$\Delta_0 = \min \{\phi - \pi, \lambda\}$ be an integer multiple of π . For general values of b , we will need to impose that both ϕ and λ are integer multiples of π . Formally, we have

Assumption A2: ϕ and λ are both integer multiples of π .

For $b = 1$, Assumption A2 implies that the one solvent bank with equity less than π has exactly zero equity. The number of good banks with zero equity when $b = 1$ is thus

$$k = \frac{\Delta_0}{\pi} = \min \left\{ \frac{\phi}{\pi} - 1, \frac{\lambda}{\pi} \right\} \quad (6)$$

Caballero and Simsek (2012) refer to k as the size of the “domino effect” and use it as a measure of contagion. Two conditions are required for k to be large. First, the losses ϕ at each bad bank must be large. When ϕ is small, a bad bank will still be able to pay back a large share of its obligation λ , and so fewer banks will ultimately be affected by the loss. Second, a large k requires the obligation λ be large. Intuitively, when λ is small, banks are not very indebted to one another, and in the limit as $\lambda \rightarrow 0$, there will be no contagion to good banks regardless of how large losses ϕ at bad banks are. As λ rises, what matters is not so much that the bad bank’s obligation grows, but that more of the resources of the banking system flow across banks, including to bad banks where they end up being diverted to senior claimants. This starves the banking system of equity, leaving fewer resources for banks located downstream from the bad bank. A higher λ thus shifts resources from banks to senior claimants, implying more good banks will fall victim to contagion.⁵

Armed with this intuition, we can now move to the general case of an arbitrary number of banks, i.e. $1 \leq b \leq n - 1$. We begin with a preliminary result that under Assumption A1, at least one bank will be solvent and can pay its obligation in full.

Proposition 3: If $\phi < \frac{n}{b} \pi$, there exists at least one solvent bank j for which $x_j = \lambda$, and among solvent banks there exists at least one bank j with positive equity, i.e. $e_j > 0$.

As in the case with $b = 1$, there will be three types of banks when $b > 1$: (1) Insolvent banks with zero equity; (2) Solvent banks whose equity is $0 \leq e_j < \pi$; and (3) Solvent banks that are sufficiently far away from a bad bank whose equity $e_j = \pi$. Since we know there is at least one solvent bank j , we can start with this bank and move to bank $j + 1$. If bank $j + 1$ is good, it too will be solvent and its equity will be $e_{j+1} = \pi$. We can continue this way until we eventually reach a bad bank. Without loss of generality, we refer to this bad bank as bank 0. By the same argument as in the case where $b = 1$, Assumption A2 implies

⁵Per Elliott, Golub, and Jackson (2013), increasing λ in our setup implies more integration but not more diversification. However, unlike in their model where greater integration means firms swap their own equity for that of other firms, here greater integration implies greater exposure to shocks at other banks while leaving banks equally vulnerable to their own shocks. Hence, the effect of higher λ is (weakly) monotone.

that banks $1, \dots, k$ will have zero equity, where k is given by (6): Even if all of these banks are good, each will inherit a shortfall of at least π and will have to sell off its π assets. If any of these banks are bad themselves, the shortfall subsequent banks will inherit will be even larger, and so equity at the first k banks will be zero.

If bad banks are sufficiently spread out across the network, i.e. if there are at least k banks between any two bad banks, then exactly bk good banks will have zero equity while the rest will have equity π . But more generally, a bad bank may default on another bad bank. In this case, the number of good banks with zero equity may fall below bk . Hence, contagion can no longer be captured by the single parameter k . This leads us to introduce a different metric to capture contagion in the case with more than one bad bank. This measure will depend on k , but will in general depend on other parameters as well.

Before introducing this measure, we first need to discuss the nature of contagion in this more general case. In particular, we show that for large values of λ , the location of the bad banks within the network does not matter, since exactly bk good banks have zero equity regardless of which banks are bad. But when λ is small, the number of good banks with zero equity will depend on which banks are bad, i.e. on the exact realization of S .

As a first step, we argue that for sufficiently large λ , all banks will be able to pay some resources to another bank in the network regardless of where the bad banks are located.

Proposition 4: Under Assumption A1, $x_j(S) > 0$ for all j and all S iff $\lambda > b(\phi - \pi)$. When $\lambda \leq b(\phi - \pi)$, there exist realizations of S for which $x_j(S) = 0$ for at least one j .

If each bank j can pay some resources to bank $j + 1$, seniority implies each bank pays the outside sector in full. It follows that for large λ , senior claimants will be fully paid in all states. This in turn implies that the total amount of resources left within the banking network is the same regardless of where bad banks are located. Since Assumption A2 implies banks can have equity of either 0 or π and total equity is the same for all S , the number of banks with zero equity must be the same for all S whenever $\lambda > b(\phi - \pi)$. Formally:

Proposition 5: Under Assumptions A1 and A2, if $\lambda > b(\phi - \pi)$, the number of good banks with zero equity is equal to bk regardless of the state of the banking network S .

Next, consider the case where λ is small. In this case, bad banks may not be able to cover their obligation ϕ to senior claimants in full. As a result, some of the losses from bad banks will be borne by the outside sector rather than by banks on the network. The number of banks with zero equity will thus be a random variable whose value depends on the state S . For $\lambda < \phi - \pi$, we can explicitly characterize the distribution of the number of banks with zero equity. At such low values of λ , a bad bank will have no resources to pay its obligation to other banks, implying $x_j = 0$ for any bad bank j . This implies the domino effect from a

bad bank will be to wipe out the equity of the next $k = \frac{\lambda}{\pi}$ banks regardless of where other bad banks are located. The total number of banks with zero equity can thus range from $b + k$, when all b bad banks are located next to each other, to $b k + b$, when there are at least k good banks between any two bad banks. Figure 2 illustrates how the location of the bad banks can matter for the aggregate equity of the network in this case, and that the fraction of banks with zero equity can vary substantially across realizations. Allowing for more than one bad bank implies that uncertainty about the state S gives rise to uncertainty not just about which banks are bad, but also about the equity of the banking system as a whole.⁶

Denote the total number of banks with zero equity by ζ . To obtain the distribution for ζ when $\lambda < \phi - \pi$, we exploit the fact that our model in this case corresponds to a discrete version of a well-studied geometric problem in applied probability known as the circle-covering problem first introduced by [Stevens \(1939\)](#). In this problem, a fixed number of points are drawn at random locations from a circle of length 1, and then arcs of a fixed length less than 1 are drawn clockwise starting from each of these points. The only randomness is the location of the arcs. The circle-covering problem involves solving for the probability that the circle is covered by the arcs given the number of arcs and the length of each arc. In our setting, the number of bad banks is akin to the number of points drawn at random, while the potential for contagion k , expressed relative to the number of banks in the network, corresponds to the length of each arc. The region of the circle covered by arcs is akin to the fraction of banks with zero equity. The discrete version of this problem was analyzed in [Holst \(1985\)](#), [Ivchenko \(1994\)](#), and [Barlevy and Nagaraja \(2013\)](#). As [Holst \(1985\)](#) notes, the discrete version can be analyzed using Bose-Einstein statistics. This insight can be used to derive the distribution of ζ . However, we only require the expected value of $E[\zeta]$, which can be obtained using results in [Ivchenko \(1994\)](#) and [Barlevy and Nagaraja \(2013\)](#) and is summarized in the next lemma.

Lemma 1: Under Assumptions A1 and A2,

$$E[\zeta] = n - \frac{(n - b)!(n - k - 1)!}{(n - 1)!(n - b - k - 1)!}$$

where k is defined by (6) and is equal to $\frac{\lambda}{\pi}$ given $\lambda < \phi - \pi$.

Finally, for intermediate values of λ between $\phi - \pi$ and $b(\phi - \pi)$, the number of banks with zero equity ζ will again be random, with support ranging between $b + \frac{\lambda}{\pi} > b + k$ and $b\frac{\phi}{\pi} = b k + b$, where k is defined in (6). For these intermediate values of λ , the distribution of banks with zero equity is analogous to a circle covering problem in which the length of the

⁶Since the network we consider is symmetric, the location of the bad bank will not matter for aggregates when $b = 1$. But location can matter for aggregates with one bad bank for asymmetric networks. For more on aggregation in asymmetric networks, albeit in the context of production, see [Acemoglu et al. \(2012\)](#).

arcs is not fixed but rather depends on the location of the points drawn at random. As far as we know, this variation of the circle-covering problem case has yet to be studied. However, in Proposition 6 below we establish some comparative static results for $E[\zeta]$ for this case.

To recap, when $b > 1$, the number of good banks with zero equity can be random. To measure contagion in this case, consider what happens if we chose a good bank at random. The extent to which good banks are exposed to losses at bad banks will be reflected in the distribution of the equity of this good bank, i.e. how likely it will be to have to liquidate its endowment and end up with an equity below π . The smaller the probability that the equity value is equal to π , the more good banks that tend to have equity below π , and thus the greater the extent of contagion. Under Assumption A2, e_j can only take on two values. Hence, define p_g as the probability that a good bank can retain its endowment, i.e.,

$$p_g = \Pr(e_j = \pi | S_j = 0) \quad (7)$$

We will use p_g as our measure of contagion: A value of p_g close to 1 implies a good bank is likely to avoid liquidating its resources, while a value of p_g close to 0 means a good bank will be likely to be wiped out because of its exposure to bad banks. As we discuss in Section 6, more generally equity e_j can take on multiple values, and so we will need the distribution of e_j rather than a single parameter. For now, we can compute p_g as follows:

$$\begin{aligned} p_g &= \sum_{z=b+k}^{bk+b} \Pr(e_j = \pi | S_j = 0, \zeta = z) \Pr(\zeta = z) \\ &= \sum_{z=b+k}^{bk+b} \frac{n-z}{n-b} \Pr(\zeta = z) = \frac{n - E[\zeta]}{n-b}. \end{aligned}$$

Intuitively, the expected number of banks with positive equity is $n - E[\zeta]$. Since only good banks can have positive equity, and there are always $n - b$ good banks, the fraction of good banks with positive equity is just the ratio of the two. The next proposition summarizes how p_g varies in our model depending on the underlying parameters:

Proposition 6. Under Assumptions A1 and A2,

$$p_g = \begin{cases} \prod_{i=1}^{\lambda/\pi} \left(\frac{n-b-i}{n-i} \right) & \text{if } 0 < \lambda < \phi - \pi \\ \Psi\left(b, n, \frac{\phi}{\pi}, \frac{\lambda}{\pi}\right) & \text{if } \phi - \pi \leq \lambda \leq b(\phi - \pi) \\ 1 - \frac{b}{n-b} \left(\frac{\phi}{\pi} - 1 \right) & \text{if } b(\phi - \pi) < \lambda \end{cases} \quad (8)$$

where the function Ψ is weakly decreasing in ϕ/π and in λ/π .

Proposition 6 summarizes how p_g depends on the magnitude of the losses at bad banks ϕ , the depth of financial ties λ , the number of bad banks b , and the total number of banks n . One feature worth pointing out now is that the effect of bank losses ϕ on p_g depends on λ . For small λ , i.e. for $\lambda < \phi - \pi$, changes in ϕ have no effect on p_g : Increasing ϕ deepens losses for senior claimants but has no effect on other banks in the network. For larger λ , though, increasing ϕ lowers p_g given that losses at bad banks wipe out the equity of a larger number of good banks. For much of our analysis we treat p_g as fixed, although we will occasionally remark on the comparative statics of what drives p_g .

For $b = 1$, p_g reduces to $1 - \frac{k}{n-1}$ and reflects both the probability a good bank has zero equity and the fraction of good banks with zero equity. For $b > 1$, p_g reflects the probability a good bank has zero equity and the *average* fraction of good banks with zero equity.

Remark 1: The fact that the location of bad banks can matter for the aggregate equity of the network requires that the number of banks not be too large. To see this, suppose we hold the potential for contagion k in (6) fixed and keep the fraction of bad banks $\frac{b}{n}$ constant at some value θ , but let $n \rightarrow \infty$. Let ζ_n denote the (random) number of banks with zero equity when there are n banks in the network. When $\lambda < \phi - \pi$, by Theorem 4.2 in [Holst \(1985\)](#) it follows that $\frac{\zeta_n}{n}$ converges to a constant as $n \rightarrow \infty$. Likewise, the fraction of good banks with zero equity, $\frac{n - \zeta_n}{n - b}$, converges to a constant. This constant will equal p_g , which recall is just the *expected* fraction of good banks with zero equity. Taking the limit of (8) as $n \rightarrow \infty$ when $\lambda < \phi - \pi$ reveals that p_g converges to a simple expression:

$$\lim_{n \rightarrow \infty} p_g = (1 - \theta)^k \quad (9)$$

Intuitively, a good bank will only have positive equity if each of the k banks located clockwise to it are good. As $n \rightarrow \infty$, the probability any one bank is bad converges to θ independently of what happens to any finite collection of banks around it. Hence, the probability that all of the relevant k neighbor banks are good is $(1 - \theta)^k$. For any given θ , the limiting value of p_g can range between 0 and 1 as k varies from 0 to arbitrarily large integer values. Note that since $k = \min \left\{ \frac{\lambda}{\pi}, \frac{\phi}{\pi} - 1 \right\}$, values of k that exceed $\frac{1}{\theta} - 1$ will violate the second inequality in Assumption A1, which requires that $\frac{\phi}{\pi}$ be less than $\frac{n}{b} = \frac{1}{\theta}$. However, this restriction can be dispensed with for large n , since the probability that equity is wiped out at all banks becomes exceedingly small even without this assumption. While the limiting case as $n \rightarrow \infty$ rules out the empirically interesting case where aggregate bank equity is uncertain, it remains a useful benchmark. For example, it nicely illustrates that the contagion measure p_g in our setup can assume the full range of values, from no contagion ($p_g \rightarrow 1$) to full contagion ($p_g \rightarrow 0$). ■

Lastly, in some of our subsequent analysis we will need the unconditional probability that

a bank chosen at random has positive equity. Denote this probability by p_0 . Given b bad banks and $n - b$ good banks, and all bad banks have zero equity under Assumption A1, p_0 can be expressed directly in terms of p_g :

$$p_0 = \frac{n - b}{n} p_g + \frac{b}{n} \times 0 = \left(1 - \frac{b}{n}\right) p_g \quad (10)$$

4 Outside Investors and Bank Equity

We now build on our model of contagion by allowing banks to raise external funds in order to finance productive opportunities. Although all banks can use the funds they raise profitably, we introduce a moral hazard problem that implies only banks with enough equity will use the funds as intended. Specifically, we allow banks to divert funds for private gains, a temptation that is mitigated by the equity a bank would give up in that case. More generally, there are various actions banks can undertake when their equity is low that would be against the interests of outside investors, e.g. investing in risky projects or gambling for resurrection.

In this section, we focus on the full-information benchmark in which banks and outside investors know which banks are bad and thus the equity of each bank. In this case, allowing banks to raise funds has no impact on contagion. Since outside investors are only willing to finance banks with enough equity, banks that would have zero equity before raising funds will be unable to raise new funds, even though they could in principle invest these funds and generate income to pay their obligations. While allowing banks to raise funds leads to no new insights regarding contagion, it does introduce a reason for why bank equity can matter for the allocation of resources: Bank equity facilitates gains from trade that would not occur in its absence. This role for equity will become more important when we allow banks in the next section to withhold information about whether they were hit by shocks, since policy can affect what investors know and thus whether trade takes place.

Formally, suppose that outside investors – the same outsiders with senior claims against banks or a new group of outside investors – can choose to invest in any of the n banks in the network. Banks have profitable projects they can undertake, but funding these projects requires outside financing. We assume each bank has a finite number of profitable projects it can undertake. We set the capacity of the bank to 1 unit of resources. On their own, outside investors can earn a gross return of $\underline{r}$ per unit of resources. Banks can earn a gross return of R on the projects they undertake, where $R > \underline{r}$. Thus, there is scope for gains from trade.

We restrict banks and outside investors to transact through debt contracts that are junior to all of the bank's other obligations. Allowing equity contracts would not resolve the moral hazard problem we introduce below. Let r_j^* denote the equilibrium gross interest rate bank

j offers investors for the funds they invest. We assume the outside sector is large enough that r_j^* is set competitively, i.e. the expected gross returns from investing in a bank equal $\underline{r}$. Hence, $r_j^* \geq \underline{r}$, and the most a bank can earn by raising funds is $R - \underline{r}$.

After banks raise funds from outsiders, they can either invest these funds and earn R , or divert them to a project that accrues a purely private benefit v per unit invested. Private benefits cannot be seized by outsiders, and outsiders cannot monitor banks to prevent diversion. However, they can go after the bank's assets if it fails to pay its obligation r_j^* .

We want v to be large enough to ensure that banks with zero equity would choose to divert – so the moral hazard problem is binding – but not so large that even a bank that keeps its π worth of assets will be tempted to divert funds. The first condition requires $v > R - \underline{r}$, i.e. the private benefit v exceeds the most a bank can earn from undertaking the project. To ensure a bank with equity will not be tempted, we need to make sure that the payoff after undertaking the project, $\pi + R - r_j^*$, exceeds the payoff from diverting funds, $v + \max\{\pi - r_j^*, 0\}$, which reflects the fact that the bank would have to liquidate at least some of its assets to meet its obligation r_j^* . Thus, we need $v < R - \max\{r_j^* - \pi, 0\}$. Since a bank that can be trusted not to divert funds must only offer $\underline{r}$ to outsiders, the condition that ensures banks with assets worth π can credibly promise to invest the funds they raise is if $v < R - \max\{\underline{r} - \pi, 0\}$. The conditions on v we need can be summarized as follows:

Assumption A3: The private benefits v from diverting 1 unit of resources satisfy

$$R - \underline{r} < v < R - \max\{\underline{r} - \pi, 0\} \quad (11)$$

Note that the second inequality in (11) implies $v < R$, so diversion is socially wasteful.

We now show that in the full information benchmark, the same ζ banks that had no equity in the absence of investment will be unable to raise funds and will thus remain with zero equity, while the remaining $n - \zeta$ banks will be able to raise funds and raise their equity to $\pi + R - \underline{r}$. Toward this end, define $I_j \in [0, 1]$ as the amount outsiders invest in bank j . Since (11) involves strict inequalities, banks will either divert the funds they raise or invest. Let $D_j = 1$ if bank j decides to divert the funds and 0 otherwise. Recall that y_j denotes the obligation of bank j to its most senior creditors and x_j its payment to bank $j + 1$. Let w_j denote its payment to outsiders who invest in bank j . Then we have

$$\begin{aligned} y_j &= \min\{x_{j-1} + \pi + R(1 - D_j)I_j, \Phi_j\} \\ x_j &= \min\{x_{j-1} + \pi + R(1 - D_j)I_j - y_j, \lambda\} \\ w_j &= \min\{x_{j-1} + \pi + R(1 - D_j)I_j - y_j - x_j, r_j^*I_j\} \end{aligned}$$

Finally, the equity at each bank j is given by

$$e_j = \max \{0, x_{j-1} + \pi + R(1 - D_j) I_j - y_j - x_j - w_j\}$$

Let $\{\hat{y}_j, \hat{x}_j\}_{j=1}^n$ denote the payments to senior creditors and to banks, respectively, if outside investors could not fund any bank, i.e. if $I_j = 0$ for all j . Likewise, define $\{\hat{e}_j\}_{j=1}^n$ as the equity positions given $\{\hat{y}_j, \hat{x}_j\}_{j=1}^n$, i.e.

$$\hat{e}_j = \max \{0, \pi - \Phi_j + \hat{x}_{j-1} - \hat{x}_j\}$$

Note that $\hat{e}_j$ corresponds to the equity positions we solved for in the previous section. Our claim is that under full information, $e_j = 0$ whenever $\hat{e}_j = 0$, and $e_j > 0$ whenever $\hat{e}_j > 0$.

Proposition 7: Given Assumption A1-A3, with full information, $e_j = 0$ for any bank j for which $\hat{e}_j = 0$, and $e_j > 0$ if $\hat{e}_j > 0$. Moreover, $I_j = 0$ if and only if $\hat{e}_j = 0$.

Proposition 7 shows that even though bankrupt banks can try to raise funds to make up their shortfalls, under full information such banks would not be able to do so. Rather, with full information, contagion persists as before. However, we can now assign a social cost to contagion: When bank balance sheets are linked, shocks that drain more equity away from the banking system and redirect it to senior creditors reduce the scope for banks to create additional surplus. Since we take the network structure as given, we have nothing to say on ways to reduce contagion. Instead, our focus concerns what happens once we move away from the full information benchmark, e.g. because banks fail to disclose their losses. In that case, even if policymakers cannot do anything to alleviate contagion, they might still be able to minimize its consequences by affecting what outsiders know about the location of bad banks. To study this possibility, we need to allow banks to choose what information to disclose.

5 Disclosure

We now arrive at the final component into our model – allowing banks to decide whether to disclose their financial position before raising funds. If enough banks decide not to disclose, outsiders must decide whether to invest in banks not knowing exactly where all of the bad banks are located. This allows us to explore the main questions we are after: Under what conditions will market participants be unsure about which banks incurred losses, and in those cases would it be advisable to compel banks to reveal their financial position?

This section is organized as follows. We first describe how we model disclosure. We then provide conditions for the existence of a non-disclosure equilibrium where no bank discloses

its S_j . We then examine whether mandatory disclosure can improve welfare relative to this equilibrium. Our essential insight is summarized in Theorem 1, which shows that mandatory disclosure cannot improve welfare when contagion is small but can improve welfare when contagion is large and disclosure costs are small. Finally, we examine whether other equilibria can exist. While we provide conditions under which multiple equilibria exist, we argue that our main result reflects a tendency for contagion to produce insufficient disclosure rather than a need to help agents coordinate to a superior equilibrium.

5.1 Modelling Disclosure

To model disclosure, suppose that after nature chooses the location of the b bad banks, each bank j observes S_j , but not S_i for $i \neq j$. At this point, all banks simultaneously choose whether to incur a utility cost $c \geq 0$ and disclose their own S_j . The cost c is meant to capture the effort of conducting and documenting the result of stress-test exercises. In principle, c could reflect the cost of revealing information about trading strategies that rival banks can exploit. But it is not obvious whether we should treat these as costs a social planner would face, so we prefer to interpret c as the costs of running stress-tests.

Investors observe these announcements and decide what terms to offer each bank, if any. If banks accept such an offer, the outside investor must hand over his funds, giving up the outside option that would have earned $\underline{r}$. After outsiders choose whether to invest, the state of the network S is revealed and banks learn their equity. At this point, banks decide whether to invest the funds they raised or divert them. Finally, profits are realized and obligations are settled. Note that a bad bank with $S_j = 1$ will never want to disclose if $c > 0$. As such, we can describe each bank's decision by $a_j \in \{0, 1\}$, where $a_j = 1$ means bank j announces it is good and $a_j = 0$ means it announces nothing. Outside investors observe $a = (a_1, \dots, a_n)$ and choose whether to provide funds to any of the banks. Since we restrict attention to debt contracts, the terms offered to banks can be summarized as an amount of resources each bank j receives, $I_j^*(a)$, and an interest rate $r_j^*(a)$ bank j must repay its investors.

5.2 Existence of a Non-Disclosure Equilibrium

Our first question is under what conditions non-disclosure can be an equilibrium, i.e. where each bank sets $a_j = 0$ expecting $a_i = 0$ for $i \neq j$. This case is of interest because it implies outsiders will be uncertain as to the location of bad banks. For our equilibrium concept, we use the notion of sequential equilibria introduced by [Kreps and Wilson \(1982\)](#). This concept requires off-equilibrium beliefs to coincide with the limit of beliefs from a sequence in which players choose all strategies with positive probability but the weight on suboptimal actions

tends to zero. This restriction serves to rule out implausible off-equilibrium path beliefs. For example, without this restriction, off equilibrium outsiders could believe all banks that don't announce are bad, even though there are only b bad banks. Likewise, without this restriction outsiders can maintain particular beliefs about the neighbors of bank j if bank j deviates, even though bank j knows nothing about other banks when it decides on disclosure.

We now show that the existence of a non-disclosure sequential equilibrium depends on two parameters – the cost of disclosure c and the degree of contagion p_g . For non-disclosure to be an equilibrium, each good bank must be willing not to disclose, i.e. set $a_j = 0$, when it expects no other banks will disclose. To derive the optimal disclosure decision, we need to establish which banks outsiders fund and at what terms if no bank discloses and if single good bank discloses. Assumption A2 implies banks have equity of either 0 or π . If no bank discloses, the probability a randomly chosen bank has equity π is $p_0 = (1 - \frac{b}{n}) p_g$ as defined in (10). By Assumption A3, we know banks would divert funds and leave nothing for investors if that learn their equity is zero. If banks learn their equity is π , whether they invest or divert depends on the r_j^* they are charged. The next lemma summarizes when banks divert funds:

Lemma 2: Under Assumptions A1-A3, any bank j whose pre-investment equity is π will prefer $D_j = 0$ if and only if $r_j^*(a) \leq \bar{r} \equiv \pi + R - v$.

In other words, if outside investors charge a rate above some threshold $\bar{r}$, banks will divert funds regardless of their equity. In principle, outsiders might still fund banks at a rate above $\bar{r}$, since they can count on grabbing the equity of banks with positive equity. However, it turns out that the equilibrium interest rate charged to any bank never exceeds $\bar{r}$:

Lemma 3: Under Assumptions A1-A3, in any equilibrium, $r_j^*(a) \leq \bar{r}$ for any bank j that receives funding, i.e. for which $I_j^*(a) = 1$.

Assumption A3 ensures that the maximal rate $\bar{r}$ in Lemmas 2 and 3 exceeds the outside option of investors $\underline{r}$.⁷ We now argue that if p_0 is small, specifically if $p_0 < \underline{r}/\bar{r} < 1$, then outsiders will not finance any bank in a non-disclosure equilibrium, i.e. $I_j^* = 0$ for all j . Absent any information on S , the rate outside investors must charge to earn as much as their outside option is $\frac{\underline{r}}{p_0}$. From Lemma 3, banks cannot charge above $\bar{r}$ in equilibrium. Hence, the only possible non-disclosure equilibrium when $p_0 < \underline{r}/\bar{r}$ is if $I_j^* = 0$ for all j , or else outsiders must charge banks a rate above $\bar{r}$, which contradicts Lemma 3. Conversely, when $p_0 > \underline{r}/\bar{r}$, a non-disclosure equilibrium requires $I_j^* = 1$ for all j . Otherwise, there exists a rate $r_j \in (\frac{\underline{r}}{p_0}, \bar{r})$ that ensures an expected return above $\underline{r}$ to investors so both investors and banks would prefer this to no trade. Note that since p_0 is proportional to p_g from (10), the cutoff for p_0 can be expressed in terms of p_g , i.e. $I_j^* = 0$ if $p_g < \frac{n}{n-b} \underline{r}/\bar{r}$ and $I_j^* = 1$ if $p_g > \frac{n}{n-b} \underline{r}/\bar{r}$.

⁷Consider the two cases $\underline{r} > \pi$ and $\underline{r} \leq \pi$. If $\underline{r} > \pi$, the second inequality in (11) implies $\underline{r} < R + \pi - v \equiv \bar{r}$. If $\underline{r} \leq \pi$, the second inequality in (11) implies $v < R$, and hence $\bar{r} = \pi + R - v > \pi \geq \underline{r}$.

So far, we have shown that in a non-disclosure equilibrium, outsiders either invest in all banks or none, depending on the value of p_g . We now use this insight to verify whether a good bank would prefer not to disclose knowing that no other bank will disclose. Consider first the case where $p_g > \frac{n}{n-b} \frac{r}{\bar{r}}$, which implies $I_j^* = 1$ for all j if no disclosure is an equilibrium. Since a bank can attract funds even without disclosing, the only benefit to a good bank from disclosing is that it can pay outside investors less than it would have to otherwise. In particular, disclosure will increase the probability outsiders attach to the bank having positive equity from p_0 to p_g . This would allow a bank to borrow at a lower rate than the $\frac{r}{p_0}$ it must pay in equilibrium, the rate that ensures investors just earn their outside option in expectation. Formally, the payoff to a good bank from not disclosing is given by

$$p_g \left(\pi + R - \frac{r}{p_0} \right) + (1 - p_g) v \quad (12)$$

Since a good bank knows it is good, the payoff in (12) is computed using the conditional probability p_g , even though outsiders assign probability p_0 that the bank will have positive equity. If the bank opts to disclose it is good, it will be able to still attract funding if it offered any rate between $\frac{r}{p_g}$ and $\frac{r}{p_0}$. Hence, when no other good bank chooses to disclose, good banks will be willing not to disclose their own financial position if and only if the disclosure cost exceeds the maximal gain from lowering the rate they are charged, i.e.

$$c \geq p_g \left(\frac{r}{p_0} - \frac{r}{p_g} \right) = \frac{br}{n-b} \quad (13)$$

Hence, when $p_g > \frac{n}{n-b} \frac{r}{\bar{r}}$, a non-disclosure equilibrium exists if and only if $c > \frac{br}{n-b}$, i.e. when disclosure costs are large. In this case, the unique non-disclosure equilibrium is one where all banks receive funding. While this is the unique non-disclosure equilibrium, there may be other equilibria with partial or full disclosure given these values for p_g and c , an issue we return to below. For now, our only interest is in conditions under which there exists an equilibrium in which no information on S is revealed.

Next, consider the case where $p_g < \frac{n}{n-b} \frac{r}{\bar{r}}$. Recall that in this case, a non-disclosure equilibrium involves no investment in any of the banks, i.e. $I_j^* = 0$ for all j . We need to verify that no good bank would wish to disclose its position given no other bank discloses. Since $I_j^* = 0$ in equilibrium, the only way a bank could benefit from disclosure is if revealing it is good will induce outsiders to fund it. Hence, non-disclosure can be an equilibrium if either unilateral disclosure does not induce outsiders to invest in a bank, or if unilateral disclosure induces investment but the cost of disclosure exceeds the gains from attracting investment.

Given our restriction to sequential equilibria, a good bank that discloses unilaterally

should expect outside investors to assign probability p_g that it has equity π . Hence, outsiders will demand at least $\frac{r}{p_g}$ from it. From Lemma 2, we know that if $\frac{r}{p_g} > \bar{r}$, a bank will not be able to both pay enough to outsiders and credibly commit not to divert funds. Hence, if $p_g < r/\bar{r}$, a good bank will not be able to attract investment if it discloses unilaterally. In this case, non-disclosure is an equilibrium for any $c \geq 0$. The fact that non-disclosure is an equilibrium even when $c = 0$ is of particular interest, since it shows that our model gives rise to non-disclosure equilibria in cases not already encompassed in the survey of [Beyer et al. \(2010\)](#) we discussed above. That is, our model satisfies each of the conditions they identify for non-disclosure to unravel. Our non-disclosure is instead due to an informational spillover in which information from multiple agents is required to deduce whether a bank has sufficient equity to be worth investing in. This feature has no analog in previous work on disclosure, including work on informational spillovers such as [Admati and Pfleiderer \(2000\)](#), where non-disclosure equilibria only occur when disclosure is costly.⁸

The only remaining case is where $r/\bar{r} < p_g < \frac{n}{n-b}r/\bar{r}$. In this case, $p_0 < r/\bar{r} < p_g$. This means that outsiders will be too worried about default to invest when no bank discloses, but will invest in a bank if it alone reveals it is good. In particular, since $p_g > r/\bar{r}$, a bank that discloses can offer a rate below $\bar{r}$ that remains competitive with the return $\underline{r}$ outsiders can earn. By disclosing and attracting investment, the bank will achieve an expected gain of

$$p_g(R - \underline{r}/p_g) + (1 - p_g)v - c$$

Hence, non-disclosure is an equilibrium only when c makes disclosure unprofitable, i.e.

$$p_g(R - v) + v - \underline{r} < c$$

In short, non-disclosure is an equilibrium if either the probability of contagion p_g is small, enough to render unilateral disclosure ineffective, or if the cost of disclosure c is large. We can collect our findings into the following proposition:

Proposition 8. Assume that Assumptions A2 and A3 hold. Then

1. A non-disclosure equilibrium *with no investment* can only exist if $p_g \leq \min(1, \frac{n}{n-b}r/\bar{r})$.

⁸[Okuno-Fujiwara, Postlewaite, and Suzumura \(1990\)](#) obtain a result that is closer in spirit to ours. They provide several examples where non-disclosure can be an equilibrium. In one of these (Example 4), a firm can disclose information about another firm. The firm does not benefit from disclosing unfavorable information about its competitor because the competitor is at a corner and acts the same way if information is disclosed or not. This is similar to our result that unilateral disclosure isn't enough to induce investment.

Such an equilibrium exists if either

- (i) $p_g \leq \underline{r}/\bar{r}$; or
- (ii) $\underline{r}/\bar{r} < p_g \leq \frac{n}{n-b}\underline{r}/\bar{r}$ and $c \geq p_g(R - v) + v - \underline{r}$

2. A non-disclosure equilibrium *with investment* can exist only if $p_g \geq \frac{n}{n-b}(\underline{r}/\bar{r})$. Such an equilibrium exists if

- (i) $\frac{b}{n} \leq 1 - \underline{r}/\bar{r}$ to ensure $\frac{n}{n-b}(\underline{r}/\bar{r}) < 1$; and
- (ii) $c \geq \frac{b}{n-b}\underline{r}$

Figure 3 illustrates these results graphically. The shaded region in the figure corresponds to the region in non-disclosure equilibria exists. Since the thresholds for c are not generally comparable for $p_g < \frac{n}{n-b}\underline{r}/\bar{r}$ and $p_g > \frac{n}{n-b}\underline{r}/\bar{r}$, these two cases are shown separately.

Given that the degree of contagion as reflected in p_g depends on primitives that govern the financial network of banks, we can relate our existence results to features of the underlying network such as the losses ϕ at bad banks and the size of the obligations λ across banks. For example, when ϕ is small, p_g will be close to 1. If there is a non-disclosure equilibrium, then as long as b/n is small, it will be one in which all banks attract funds. Now, suppose news arrives that losses ϕ at bad banks increased. How this effects the non-disclosure equilibrium depends on λ . Recall that we showed in Section 3 that for $\lambda < \phi$, a change in ϕ has no effect on p_g . Thus, for small λ the news of large losses at some banks will have little observable effect: Banks will continue to attract funds. But if λ is large, p_g will fall with ϕ . If p_g falls sufficiently, the only possible non-disclosure equilibrium is one in which no bank attracts funds. Hence, the model suggests that large degrees of leverage against other banks as measured by λ allow shocks to give rise to market freezes that would not occur when λ is smaller. In the next subsection, we show that higher leverage may be related not only to the occurrence of market freezes but to whether mandating disclosure is desirable.

5.3 Mandatory Disclosure and Welfare

We now examine whether mandating disclosure can improve welfare relative to an equilibrium where no information is revealed. Forcing all banks to disclose is a natural benchmark given policymakers are often reluctant to discriminate among ex-ante similar banks. This does not imply mandatory disclosure is optimal; in general it is not in our model. But if mandatory disclosure can improve welfare, intervention will be useful. That said, achieving these welfare gains need not require a government authority. Historically, banks on their own have formed

mutual associations and clearinghouses to monitor and report on bank balance sheets.

Recall that in a non-disclosure equilibrium, outsiders will generically either invest in all banks or in none. Whether mandatory disclosure can improve upon a non-disclosure equilibrium depends on what investors do in equilibrium. We begin with the case of no investment, i.e. when $p_g < \frac{n}{n-b}r/\bar{r}$. In this case, mandatory disclosure can “unfreeze” markets, allowing those banks with positive equity to attract funds they wouldn’t have otherwise. However, the additional surplus this creates comes at the cost of forcing all banks to incur disclosure costs. To determine whether the additional surplus exceeds the cost, note that the expected number of banks with positive equity is $(n - b)p_g$. Each of these banks creates a surplus of $R - \underline{r}$. The cost of forcing all banks to produce information about their losses is cn . Hence, the expected surplus created exceeds the cost of disclosure iff

$$(n - b)p_g(R - \underline{r}) - cn > 0. \quad (14)$$

Observe that for $b > 1$, the number of banks with positive equity is random, so (14) represents an *ex-ante* criterion, before the location of bad banks is known. *Ex-post*, the gains from trade can turn out to be too low to justify the cost of disclosure.⁹ We will refer to mandatory disclosure as a *welfare improvement over no-disclosure* if (14) holds.

We can now examine whether the conditions that ensure the existence of a non-disclosure equilibrium are compatible with welfare improving mandatory disclosure. From Proposition 8, we know that when $p_g < r/\bar{r}$, a non-disclosure equilibrium exists for all $c \geq 0$. By contrast, (14) implies that forcing all firms to disclose will be valuable whenever the cost of disclosure c is not too large. Hence, the region in which no disclosure is an equilibrium but mandatory disclosure is welfare improving is non-empty. Formally,

Proposition 9. Under Assumptions A2 and A3, if $0 < p_g \leq r/\bar{r}$ and $c \leq (R - \underline{r}) \frac{n-b}{n} p_g$, mandatory disclosure is a welfare improvement over no-disclosure.

Intuitively, at low values of p_g , a good bank that unilaterally discloses its S_j will not be able to attract investment. It may therefore be individually optimal for each bank not to disclose even though all banks could be made better off if they coordinated to disclose.

Next, we turn to the case where $r/\bar{r} < p_g < \frac{n}{n-b}r/\bar{r}$. For these values of p_g , non-disclosure equilibria can exist, and necessarily involve no investment by outsiders, when c is large. By contrast, mandatory disclosure can only improve welfare when c is small. In contrast to the case where $p_g < r/\bar{r}$, now a good bank can attract funds if it discloses unilaterally. Thus,

⁹It is worth noting that one reason stress tests in the U.S. were viewed as successful is that they revealed the U.S. banking system to be relatively well-capitalized. Our model suggests that even if stress tests reveal a different state of affairs, as was arguably the case in Europe, a commitment to run stress-tests may still be a reasonable policy *ex-ante*, i.e. before the results of the tests are known.

it is not obvious that there exists a non-disclosure equilibrium that can be improved upon. However, since the private incentives to disclose need not coincide with a planner's incentives, the possibility of welfare improvement remains.

For $\underline{r}/\bar{r} < p_g < \frac{n}{n-b}\underline{r}/\bar{r}$, the conditions under which there exists a non-disclosure equilibrium that can be improved upon are summarized in Proposition 10 below. Two conditions prove to be necessary: v must be below $\underline{r}$, and the fraction of bad banks $\frac{b}{n}$ must not be too large. The first condition implies diversion is socially wasteful. To see this, recall that outsiders who invest in banks are assumed to give up the option to earn $\underline{r}$ on their funds. Hence, if a bank learns its equity is zero, the funds it raised will yield a (private) return of v . Since $v < \underline{r}$, identifying the banks with zero equity in advance and avoiding them from taking in funds would yield more total resources that could be used to make both the bank and investor better off. Banks do not take into account these benefits when they choose whether to disclose, and so disclosure may be inefficiently low. As for the role of the fraction of bad banks, recall that only good banks ever contemplate disclosure. The cost for a good bank of disclosure is c , while mandatory disclosure at *all* banks implies a cost of disclosure per good bank of $\frac{n}{n-b}c$. This higher cost may make mandatory disclosure undesirable despite the potential benefits from the release of information. Formally:

Proposition 10. Assume Assumptions A2 and A3 hold. If $\underline{r}/\bar{r} < p_g < \frac{n}{n-b}\underline{r}/\bar{r}$, then

1. If $v \geq \underline{r}$ and there exists a non-disclosure equilibrium, mandatory disclosure cannot be welfare improving over non-disclosure.
2. If $v < \underline{r}$, then

(a) If $\frac{b}{n} > \left(\frac{\bar{r}}{\underline{r}} - 1\right) \frac{\underline{r}-v}{R-\underline{r}}$, there exists no non-disclosure equilibrium that can be welfare improved via mandatory disclosure.

(b) If $\frac{b}{n} \leq \left(\frac{\bar{r}}{\underline{r}} - 1\right) \frac{\underline{r}-v}{R-\underline{r}}$, a non-disclosure equilibrium exists upon which mandatory disclosure is welfare improving for pairs (p_g, c) where

i. $\underline{r}/\bar{r} < p_g < \min \left\{ \frac{n}{n-b}\underline{r}/\bar{r}, \frac{\underline{r}-v}{(R-v)-(1-b/n)(R-\underline{r})} \right\}$, and

ii. $(R-v)p_g + (v-\underline{r}) \leq c \leq \frac{n-b}{n}p_g(R-\underline{r})$.

Moreover, $\min \left\{ \frac{n}{n-b}\underline{r}/\bar{r}, \frac{\underline{r}-v}{(R-v)-(1-b/n)(R-\underline{r})} \right\} < 1$, so condition (i) requires $p_g < 1$.

Note that Proposition 10 implies that a non-disclosure equilibrium can be improved upon only if p_g is strictly below 1. That is, mandatory disclosure will only be desirable if there is sufficiently high contagion from bad banks to good banks.

Finally, we turn to the case where $p_g > \frac{n}{n-b}\underline{r}/\bar{r}$. Recall from Proposition 8 that if a non-disclosure equilibrium exists in this case, all banks will be able to raise funds. This does

not mean that banks no longer have a reason to disclose: A bank that reveals it is good will be able to offer a lower interest to outside investors. This represents a purely private gain: A bank is able to keep more of the surplus it creates, but disclosure creates no new surplus. As Jovanovic (1982) points out, when disclosure is costly and yields only private gains, mandating disclosure is typically undesirable: It represents a costly activity with no social gains. Fishman and Hagerty (1989) similarly show that when disclosure is driven by rent-seeking, forcing more disclosure than occurs in equilibrium may not be desirable. By contrast, since our model exhibits informational spillovers, mandatory disclosure may be desirable even though each bank's decision to disclose is driven by rent-seeking. To see this, observe that in equilibrium, the available amount of resources is given by

$$(n - b) p_g (\pi + R) + (n - (n - b) p_g) v \quad (15)$$

That is, on average $(n - b) p_g$ banks have positive equity and invest the funds they raise, while the remainder divert their funds for private gains. By contrast, under mandatory disclosure, all banks with zero equity will be refused funding and outsiders deploy these funds on their own. Expected available resources are then equal to

$$(n - b) p_g (\pi + R) + (n - (n - b) p_g) \underline{r} \quad (16)$$

Although v represents private benefits that cannot be redistributed, comparing (15) and (16) still turns out to be the key to whether mandatory disclosure can be welfare improving. This is because if fewer resources are available under mandatory disclosure, it will be impossible to keep everyone as well off, so mandatory disclosure cannot improve welfare. But if more resources are available under mandatory disclosure, this will be without any resources used to obtain private benefits. Hence, as long as the additional resources exceed disclosure costs cn , there will be enough to leave outsiders equally well off but give more to banks. The welfare gain in this case is not due to unfreezing markets, but to preventing wasteful diversion. When a good bank contemplates the value of disclosure, it ignores the fact that its disclosure may help to identify which other banks have zero equity and prevent diversion. Comparing the difference between (15) and (16) to disclosure costs reveals that mandatory disclosure will be welfare improving when c satisfies

$$\frac{b\underline{r}}{n - b} < c < \left(1 - \frac{n - b}{n} p_g\right) (\underline{r} - v) \quad (17)$$

Once again, for this range to be non-empty, the same two conditions must be satisfied: diversion must be socially wasteful, i.e. $v < \underline{r}$, and the fraction of bad banks $\frac{b}{n}$ must not be

too large. Formally, for $p_g > \frac{n}{n-b} \frac{r}{\bar{r}}$ we have the following proposition:

Proposition 11. Assume Assumptions A2 and A3 hold. Suppose $p_g \geq \frac{n}{n-b} \frac{r}{\bar{r}}$. Then

1. If $v \geq \underline{r}$ and there exists a non-disclosure equilibrium, mandatory disclosure cannot be welfare improving over non-disclosure.
2. If $v < \underline{r}$, then

- (a) If $\frac{b}{n} > \frac{r-v}{(\underline{r}-v)(1-\underline{r}/\bar{r})+\underline{r}} (1 - \underline{r}/\bar{r})$, there exists no non-disclosure equilibrium that can be welfare improved via mandatory disclosure.
- (b) If $\frac{b}{n} \leq \frac{r-v}{(\underline{r}-v)(1-\underline{r}/\bar{r})+\underline{r}} (1 - \underline{r}/\bar{r})$, a non-disclosure equilibrium exists upon which mandatory disclosure is welfare improving whenever
 - i. $\frac{n}{n-b} \frac{r}{\bar{r}} \leq p_g \leq \frac{n}{n-b} \left(1 - \frac{b}{n-b} \frac{r}{\underline{r}-v}\right)$, and
 - ii. $\frac{b}{n-b} \underline{r} \leq c \leq (1 - \frac{n-b}{n} p_g)(\underline{r} - v)$.

Moreover, $\frac{n}{n-b} \left(1 - \frac{b}{n-b} \frac{r}{\underline{r}-v}\right) < 1$, so condition (i) only holds for $p_g < 1$.

Note the parallel with Proposition 10. Once again, mandatory disclosure can only be welfare improving if p_g is strictly below 1, i.e. when there is enough contagion.

We can combine Propositions 9-11 to form a result that captures how the desirability of mandatory disclosure varies with the degree of contagion p_g :

Theorem 1. Assume Assumptions A2 and A3 hold. For p_g close to 1, mandatory disclosure cannot improve upon a non-disclosure equilibrium. Conversely, for p_g close to but not equal to 0, if c is low, the non-disclosure equilibrium is improvable.

Remark 2: Note that neither Theorem 1 nor Propositions 8-11 require Assumption A1. In particular, our key results do not depend on being in what [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2013\)](#) describe as the “small shock” regime. This reinforces our observation in Remark 1 that Assumption A1 can be dispensed with, at least for some results. ■

We close with a few remarks. The first concerns comparative statics with respect to p_g . Although mandatory disclosure can be welfare improving when p_g is close to zero, this will not be true in the limit when $p_g = 0$. In that case, there are no banks worth investing in, and so disclosure serves no role. For values of p_g where mandatory disclosure can be welfare improving, the expected welfare gains from disclosure are not monotonic in p_g . For $p_g < \frac{n}{n-b} \frac{r}{\bar{r}}$, the expected gains are increasing in p_g , since a higher p_g implies a larger fraction of banks are expected to be solvent and benefit from unfreezing trade. This illustrates a tension in our model: Contagion makes it more likely that mandatory disclosure improves

welfare, but it also makes the gains from intervention smaller. When $p_g > \frac{n}{n-b} \frac{r}{\bar{r}}$, the expected gains from mandatory disclosure are decreasing in p_g . This is because for these values, investors finance all banks and the benefit of mandatory disclosure instead comes from avoiding socially wasteful diversion. A lower p_g implies more diversion. In this case, more contagion makes it both more likely that mandatory disclosure can be welfare improving and increases the gains from such intervention. But now there is a different tension: Although more contagion makes mandatory disclosure more desirable, it also makes non-disclosure equilibria in which investors invest in the absence of any information less likely.

Another implication of the model is that a decline in p_g can make mandatory disclosure desirable even if markets do not freeze up due to the gains from avoiding wasteful diversion. In fact, it is possible that a mild decline in p_g allows investment in banks to continue even as a role for mandatory disclosure emerges, while a larger decline leads to a suspension of trade and yet in this regime mandatory disclosure is only desirable at much larger degrees of contagion. When intervention is warranted may be a delicate issue.

Finally, we can use our analysis of contagion to relate our results to aspects of the underlying financial network. Recall that a low value of ϕ will imply p_g is close to 1. Thus, when losses at bad banks are small, there will be no need for mandatory disclosure. As losses rise, p_g will be unchanged if λ is small but will fall if λ is large. Higher leverage against other banks can lead to market freezes and justify intervention that would not be desirable otherwise. This implies that financial regulation that restricts leverage or reduces the degree of interconnectedness among banks may obviate the need for disclosing stress test results. Interestingly, current proposals for financial reform such as the Dodd-Frank Act view these policies as complements and argue that both be adopted simultaneously.

5.4 Multiple Equilibria

So far, we have argued it is possible for no bank to disclose its status in equilibrium even though forcing all banks to disclose improves welfare. This does not preclude the possibility that there may be other equilibria in which some or even all good banks disclose their status. Since our primary motivation is to determine whether it would ever be appropriate to mandate disclosure when uncertainty about the location of losses serves to paralyze markets, we have only focused on whether a non-disclosure equilibrium exists and whether intervention would be appropriate if we found ourselves in that equilibrium. However, the question of whether the model admits multiple equilibria is also of inherent interest. In particular, do our results correspond to a failure by agents to coordinate to an equilibrium they could achieve on their own, or does it reflect a tendency for inefficiently low disclosure?

We first argue that as long as the number of bad banks b is large enough, whenever

mandatory disclosure improves upon a non-disclosure equilibrium where no bank attracts funds, there exists another equilibrium in which all good banks reveal themselves to be good. In other words, for large b , whenever it is beneficial to unfreeze markets, good banks should be able to coordinate to all disclose their financial status.

Formally, recall that if markets are frozen in the absence of disclosure, forcing disclosure improves welfare if $(n - b)p_g(R - \underline{r}) > cn$, i.e. if the expected surplus created under full revelation exceeds the cost of forcing disclosure. We will now show that this condition ensures that all good banks disclosing must also be an equilibrium for sufficiently large b .

Proposition 12: Suppose $(n - b)p_g(R - \underline{r}) > cn$. Then given Assumption A3 and assuming $\phi > \pi$, $a_j = 1$ for all good banks j is an equilibrium if $b > \frac{\bar{r}}{\underline{r}} - 1$.

The reason the number of bad banks b must be large is that this allows outsiders to maintain more “pessimistic” beliefs about banks that fail to disclose. As is well known from previous work on disclosure, pessimistic beliefs can be used to sustain equilibria in which good agents disclose, since they expect that if they don’t disclose, outsiders will believe them to be the worst possible type. In our model, there is a limit to how negative beliefs can be. In particular, since there are exactly b bad banks, a good bank that deviates and chose not to disclose would be one of $b + 1$ that would fail to announce. Our restriction to sequential equilibria implies outsiders assign equal probability that a bank that doesn’t disclose is bad, namely $\frac{b}{b+1}$. For large b , this probability is close to 1 so beliefs are quite pessimistic. But for small values of b , a bank that deviates may still be perceived as having a high probability of being good, and so the penalty for deviating may not be severe enough to deter deviations.

Proposition 12 would seem to suggest that mandatory disclosure is desirable only when it helps agents coordinate on a superior equilibrium. However, this conclusion is not true in general, and it is misleading to read our results as implying that insufficient disclosure arises when agents coordinate on a bad equilibrium. To see this, we make two observations.

First, as our discussion suggests, Proposition 12 requires large values of b . For small values, it need not be the case that there is always another Pareto-superior equilibrium good banks can coordinate on whenever intervention is desirable. In Appendix B, we give an example where $b = 1$ for a slightly modified version of the model in which no disclosure is the unique equilibrium yet mandatory disclosure is welfare-improving. In that example, not disclosing is a dominant strategy for each good bank. Thus, mandatory disclosure can make agents better off even without another equilibrium they could coordinate on.¹⁰

¹⁰Since the existence of multiple equilibria is often related to strategic complementarities, we should note that disclosure in our model is not always a strategic complement: If we restrict other banks to a common disclosure probability, a bank’s incentive to disclose can fall with the probability others disclose. This is because there are two offsetting forces in our model: As more banks disclose, each remaining bank is perceived as more likely to be bad, encouraging disclosure. At the same time, if enough of the banks you are exposed to disclose, outsiders may invest in you even if you do not disclose, reducing your incentive to disclose.

Second, even when b is large, the divergence between private and social incentives implies a role for intervention beyond just addressing a coordination failure. Recall that Proposition 10 shows that mandatory disclosure can be beneficial at intermediate values of p_g when a bank could attract funds if it unilaterally disclosed. That is, intervention can be beneficial even when coordination is not a problem. The reason is that banks fail to take into account spillovers from disclosing their information. One way to get at this distinction would be to introduce independent private signals that prevent agents from coordinating, and then ask whether the unique equilibrium in that environment is efficient. For example, suppose banks and outsiders receive private signals on ϕ , the loss per bank. Good banks that believe ϕ is small would deduce p_g is close to 1 and prefer to disclose, while good banks that believe ϕ is large would prefer not to disclose if $c > 0$. We conjecture that since banks do not internalize all of the value of the information they disclose, banks would choose not to disclose at a threshold that is lower than is socially optimal.

6 Alternative Network Structures

So far, our model of financial contagion assumed a particular network structure in which the amount bank i owes some other bank j is given by $\Lambda_{ij} = \lambda$ for $j = i + 1 \pmod{n}$. We now argue that our result extends to a larger class of networks as defined by Λ_{ij} .

A general network corresponds to a specification of liabilities across banks that can be summarized by an $n \times n$ matrix Λ with zeros along the diagonal. We restrict attention to networks in which the pre-investment equity at any given bank in the absence of any shocks is the value π of the assets it owns. This requires that each bank have a zero net position with the remaining banks in the network, i.e.

$$\sum_{j \neq i} \Lambda_{ij} = \sum_{j \neq i} \Lambda_{ji} \quad (18)$$

Using network theory terminology, (18) implies Λ is a regular weighted directed network.

As in the case of the circular network, we assume the network is hit by a shock process governed by two parameters: b , the number of bad banks, and ϕ , the losses at each bad bank, where each of the $\binom{n}{b}$ possible locations of the bad banks within the network is equally likely.

Since each bank can in principle be obligated to any of the other $n - 1$ banks, the set of payments is now given by $\{x_{ij}\}_{i \neq j}$ as opposed to just n payments as before. Given that banks can have obligations to multiple banks, we need a priority rule for how available resources be divided if banks are unable to pay all of their obligations. We follow Eisenberg and Noe (2001) and Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) in assuming that an insolvent

bank pays the same pro-rata share of whatever resources it has to each of the banks it is indebted to. In particular, define Λ_i as bank i 's obligations, i.e.

$$\Lambda_i = \sum_{j=0}^{n-1} \Lambda_{ij} \quad (19)$$

If bank i is insolvent, it will pay each bank j it is obligated to a fraction $\frac{\Lambda_{ij}}{\Lambda_i}$ of the resources it has. This implies that the set of payments x_{ij} solve the system of equations

$$x_{ij} = \frac{\Lambda_{ij}}{\Lambda_i} \max \left\{ \min \left\{ \Lambda_i, \pi - S_i \phi + \sum_{r=0}^{n-1} x_{ri} \right\}, 0 \right\} \text{ for all } i \neq j \quad (20)$$

where recall $S_i = 1$ if bank i is bad. We can then define the pre-investment equity of bank i , meaning the equity of bank i if it did not raise any outside funds, as

$$e_i = \pi + \sum_{j=0}^{n-1} x_{ji} - S_j \phi - \sum_{j=0}^{n-1} x_{ij} \quad (21)$$

A convenient feature of the circular network we have analyzed thus far is that it implies a particular symmetry: Every good bank is equally likely to have its equity wiped out regardless of its identity. This allowed us to summarize contagion with a single statistic, p_g , the probability that a good bank will be unaffected by contagion, as opposed to requiring a vector of probabilities, one for each bank. We now argue that for networks that exhibit a strengthened version of this symmetry property, our main results regarding extreme degrees of contagion and the desirability of mandatory disclosure go through. This stronger symmetry property involves the distribution of pre-investment equity e_j :

Definition: A financial network Λ is *symmetrically vulnerable to contagion* given the shock process $\{b, \phi\}$ if the distribution of pre-investment equity for a good bank does not depend on its identity, i.e. $\Pr(e_j = x | S_j = 0)$ is independent of j for all $x \in [0, \pi]$.

One way to ensure that a network is symmetrically vulnerable to contagion is if the debt obligations that define the network are themselves symmetric. To motivate the exact symmetry that debt obligations must satisfy, suppose we have n distinct physical locations. Once we assign banks to different physical locations, the obligations between banks will give rise to a directed network across locations. Maintaining network theory terminology, we define a *symmetric network* as a network where obligations across banks are such that observing the links across physical locations does not reveal enough information to narrow down the location of any individual bank. That is, obligations across banks are such that any bank j can be located at any one of the possible locations and other banks can be arranged so that the implied network across physical locations remains the same. Formally,

Definition: A network Λ is *symmetric* if for any pair k and ℓ in $\{0, \dots, n-1\}$ there exists a bijective function $\sigma_{k,\ell} : \{0, \dots, n-1\} \rightarrow \{0, \dots, n-1\}$ such that (i) $\sigma_{k,\ell}(k) = \ell$ and (ii) for any pair i and j in $\{0, \dots, n-1\}$, $\Lambda_{\sigma_{k,\ell}(i), \sigma_{k,\ell}(j)} = \Lambda_{ij}$.

One example of a symmetric network is a circulant network, i.e. a network in which it is possible to order banks in such a way that the matrix of obligations Λ is a circulant, meaning that Λ_{ij} can be expressed solely as a function of the distance $i - j \pmod{n}$ between banks. The fact that obligations depend only on distance guarantees that observing links across locations reveals nothing about where individual banks are located, since rotating banks across physical locations would leave the pattern of obligations across locations unchanged. Circulant networks include the circular network, together with several other networks that have figured prominently in the literature on financial networks, e.g. complete financial networks where banks maintain equal liabilities with all other banks so $\lambda_{ij} = \lambda$ for all $i \neq j$, partially complete networks where banks have liabilities to some but not all other banks such as the interconnected ring network in [Acemoglu, Ozdaglar, and Tahbaz-Salehi \(2013\)](#), and multiple disconnected symmetric networks, e.g. isolated pairs of banks. While circulant networks are symmetric, not all symmetric networks are circulant; we give an example of a symmetric network that is not a circulant in Appendix C. Our results thus hold for a broader class of networks than circulant networks.

Our next result establishes that requiring the network to be symmetric is enough to ensure it will be symmetrically vulnerable to contagion.

Lemma 4: Any regular symmetric network Λ is symmetrically vulnerable to contagion.

Remark: It is not necessary for a network to be symmetric to be symmetrically vulnerable to contagion. In Appendix C, we give an example of an asymmetric network that is symmetrically vulnerable to contagion for a particular b and ϕ . That is, we give an example of a network where observing obligations across physical locations fully reveals where each bank is located, and yet the network can be symmetrically vulnerable to contagion. Thus, our results hold for a broader class of networks than just symmetric networks.

For the circular network we have focused on so far, Assumption A2 ensures that the pre-investment equity e_j for a good bank could only assume two values, 0 and π . Hence, this distribution can be summarized by a single parameter, $p_g = \Pr(e_j = \pi | S_j = 0)$. In the general case, the support of the distribution may contain more than two points. Even though contagion will now depend on a distribution function rather than the single parameter p_g , we can still derive generalizations of our previous results for the comparative statics of contagion with respect to the losses at bad banks and the size of obligations across banks, and for our results that relate the extent of contagion to the desirability of mandatory disclosure.

We start with results on comparative statics. We index the matrix of obligations across banks Λ by a scale factor λ so that

$$\Lambda(\lambda) = \lambda \Lambda(1) \tag{22}$$

That is, the scalar λ multiplies each entry of the baseline matrix $\Lambda(1)$. This is one way to generalize our comparative static in the circular network of simultaneously increasing the obligations between any two banks. As in the case of the circular network, increasing λ raises the leverage ratio for every bank. Recall that in Proposition 6, we showed that higher λ and higher ϕ led to greater contagion as measured by p_g . In the general case, an analog for lower p_g , and thus more contagion, is a first order stochastically lower distribution for equity. The next proposition establishes that higher λ and ϕ imply more contagion in this sense.

Proposition 13. Let Λ be a directed weighted *regular* network where the matrix Λ is indexed by λ as in (22). Then for each $i \in \{0, \dots, n-1\}$ and each $x \in [0, \pi]$ the probability $\Pr\{e_i \leq x \mid S_i = 0\}$ is weakly increasing in ϕ and λ .

Finally, we can still establish an analog to Theorem 1 which shows that the degree of contagion, as reflected in the likelihood of good banks having to liquidate assets and lowering their pre-investment equity to below π , is related to the desirability of mandatory disclosure:

Theorem 2. Suppose Λ is regular and symmetrically vulnerable to contagion, $\pi < \phi$, and Assumption A3 holds. If $\Pr(e_j = \pi \mid S_j = 0)$ is sufficiently close to 1, mandatory disclosure cannot improve upon non-disclosure. Conversely, there exists an equity level $e^* > 0$ with $0 < e^* < \pi$ such that if $\Pr(e_j \geq e^* \mid S_j = 0)$ is sufficiently close to but not equal to 0, mandatory disclosure will be welfare improving over non-disclosure for low enough c .

Theorem 2 strictly generalizes Theorem 1. However, it would be incorrect to conclude from this that the structure of the network is irrelevant for whether mandatory disclosure is desirable. This is because the network structure determines the extent of contagion. As an example, consider the complete network in which $\Lambda_{ij} = \lambda$ for all $j \neq 0$. In this case, the exact location of bad banks is irrelevant, since the equity of any good bank will be the same regardless of which banks are bad. Mandatory disclosure can serve no positive role in this case. Consistent with this, note that since the location of banks is irrelevant, the distribution of equity at good banks is degenerate. Hence, for a given e^* , the probability $\Pr(e_j \geq e^* \mid S_j = 0)$ jumps from 1 to 0 as we change e^* , a cutoff which depends on the network. But Theorem 2 tells us that mandatory disclosure is welfare improving only if $\Pr(e_j \geq e^* \mid S_j = 0)$ is close to but strictly above 0. Even though the conditions that ensure mandatory disclosure can be desirable do not depend on the particular network structure, whether these conditions can be satisfied does.

7 Conclusions and Future Work

This paper shows that when contagion is substantial and disclosure costs are not too high, mandatory disclosure may result in a Pareto improvement relative to an equilibrium without disclosure. This suggests that stress tests, at least insofar as they include mandatory disclosure of information clauses, are socially beneficial provided that there is enough dependence on their counterpart risk or enough “contagion.” These insights are arguably relevant for the recommendation that derivatives trade migrate from over-the-counter trading to centralized exchanges.¹¹ One of the reasons for this recommendation is the fragility due to chains of indirect exposure of counterparty risk our model aims to capture. While we do not model the equivalent to migrating to an exchange, we view the policy of mandatory disclosure of information as a potential substitute to the migration of trade to exchanges, i.e. we view it as a policy that can address some of the shortcomings of over-the-counter trade which motivate the policy of migration to centralized exchanges.

Since our model is relatively simple, which makes our arguments, we hope, transparent, it leaves out many features which we briefly mention here.

The simplicity of our game between banks and outside investors relies, in part, on our restrictions to networks that satisfy a particular symmetry property. This excludes several interesting cases. First, our set up excludes more realistic networks in which some banks are more centrally located than others. One might be able to gain some insights on how this asymmetry matters by looking at sparsely parameterized core-periphery networks as in [Babus and Kondor \(2013\)](#). For example, when is mandatory disclosure only desirable for core banks, in line with the fact that stress tests in practice were limited to large core banks, and when is it necessary to force the periphery to disclose as well? Second, we might allow the severity of the shocks to vary across banks, or the probability of a shock to hit a bank to vary across banks. This type of analysis may suggest better ways of performing stress tests.

More generally, one can use our framework to think about what optimal disclosure policy might be. Mandatory disclosure treats all banks fairly, but it is also inefficient; requiring only $n - 1$ banks to disclose in our model is equally informative but less costly. Still more targeted policies do even better, i.e. policies which pay banks as a function of the outcome of their disclosure, and then make the outcome public. For example, if less than half of all banks are bad, rewarding banks that disclose they are bad will be preferable to forcing all banks to disclose, as would rewarding banks that disclose they are good if less than half of banks are good. The optimal policy will thus depend on the exact details of the environment.

Another feature of our model that is worth investigating is the importance of our as-

¹¹For a discussion see, for example, [Duffie and Zhu \(2011\)](#) and [Duffie, Li, and Lubke \(2010\)](#) and the references therein.

sumption that disclosure is simultaneous. Allowing banks to move sequentially can potentially facilitate coordination. Since we show that our result cannot be entirely attributed to coordination failures, we suspect that some of our results would carry over to dynamic environments. However, sequential disclosure is likely to introduce new issues, such as informational cascades and herding where information gets “trapped” if banks that are exposed to bad banks choose not to reveal their own state, discouraging the banks exposed to them from disclosing their status.

Another assumption we impose that may be worth relaxing is that banks can provide incontrovertible proof of their state. A more realistic model would allow banks to give an informative yet imperfect signal. This opens new possibilities which may be relevant for the difference between the social and private value of information disclosure. Still another assumption in our model worth relaxing is that banks disclose actual losses. In practice, stress tests ask banks about potential future losses. Whether this matters for our results remains an open question.

Finally, our analysis focuses on stress tests as a source of information. But in the US, these tests were accompanied by capital injections for weak banks. The framework we propose here may be a useful start for exploring such questions.

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A Proofs

Proof of Proposition 1: We can rewrite the system of equations in (4) as

$$x_j = T_j(x_{j-1}) \equiv \max \{0, \min(x_{j-1} + \pi - \Phi_j, \lambda)\}$$

By repeated substitution, we can reduce this system of equations to a single equation

$$x_0 = T^*(x_0)$$

where

$$T^*(x_0) \equiv T_n \circ T_{n-1} \circ \cdots \circ T_1(x_0)$$

The mapping T^* is continuous, monotone, bounded. Moreover, for any x and y in $[0, \lambda]$, we have $|T^*(x) - T^*(y)| \leq |x - y|$. Let

$$\begin{aligned}\underline{x} &= \lim_{m \rightarrow \infty} (T^*)^m(0) \\ \bar{x} &= \lim_{m \rightarrow \infty} (T^*)^m(\lambda)\end{aligned}$$

These limits exist given T^* is monotone and bounded. By continuity, $\underline{x}$ and $\bar{x}$ must both be fixed points of T^* , i.e.

$$\underline{x} = T^*(\underline{x}) \text{ and } \bar{x} = T^*(\bar{x})$$

Moreover, by monotonicity, $(T^*)^m(0) \leq (T^*)^m(\lambda)$ for any m . Taking the limit, $\underline{x} \leq \bar{x}$. Hence, the set of fixed points of T^* is nonempty.

Suppose $\underline{x} < \bar{x}$. Then for any $\mu \in (0, 1)$, the value $x_\mu = \mu\underline{x} + (1 - \mu)\bar{x}$ must also be a fixed point of T^* , i.e.

$$x_\mu = T^*(x_\mu)$$

For suppose

$$x_\mu > T^*(x_\mu)$$

In this case, we have

$$\begin{aligned}x_\mu - \underline{x} &> T^*(x_\mu) - \underline{x} \\ &= T^*(x_\mu) - T^*(\underline{x}) \geq 0\end{aligned}$$

But this counterfactually implies

$$|x_\mu - \underline{x}| > |T^*(x_\mu) - T^*(\underline{x})|$$

Likewise, if

$$x_\mu < T^*(x_\mu)$$

then we can show that

$$\begin{aligned}\bar{x} - x_\mu &> \bar{x} - T^*(x_\mu) \\ &= T^*(\bar{x}) - T^*(x_\mu) \geq 0\end{aligned}$$

which again counterfactually implies

$$|x_\mu - \bar{x}| > |T^*(x_\mu) - T^*(\bar{x})|$$

We conclude that $T^*(x) = x$ for all $x \in [\bar{x}, \bar{x}]$. Next, we argue that for $x \in [\underline{x}, \bar{x}]$, for all $j \in \{1, \dots, n\}$,

$$T_j \circ \cdots \circ T_1(x) = T_{j-1}(x) + \pi - \Phi_j$$

For suppose not. That is, there exists some j such that either

- (i) $T_{j-1}(x) + \pi - \Phi_j > \lambda$
- (ii) $T_{j-1}(x) + \pi - \Phi_j < 0$

But then by continuity there must exist at least two values $x' \neq x''$ from $[\underline{x}, \bar{x}]$ such that

$$T_j(x') = T_j(x'')$$

and hence $T^*(x') = T^*(x'')$, which requires $x' = x''$, a contradiction. It follows that

$$T^*(x) = x + \sum_{j=1}^n (\pi - \Phi_j)$$

for all $x \in [\underline{x}, \bar{x}]$. But since $T^*(x)$ must equal x in this interval, we must have

$$\sum_{j=1}^n (\pi - \Phi_j) = 0$$

This implies that $\underline{x} = \bar{x}$, i.e. the fixed point of T^* is unique, whenever

$$\sum_{j=1}^n (\pi - \Phi_j) \neq 0$$

This completes the proof for the case where $n\pi \neq b\phi$. ■

Proof of Proposition 2: Since $\phi \leq \pi < \frac{n}{b}\phi$, we know from Proposition 1 that the (4) has a unique solution. It will suffice to verify that $x_j = \lambda$ is a solution. For any $j \in \{1, \dots, n\}$, we have

$$x_j = \max\{0, \min(\lambda + \pi - \Phi_j, \lambda)\}$$

Since $\pi - \Phi_j \geq 0$ whenever $\phi < \pi$, then $x_j = \lambda$ solves the system of equations (4). ■

Proof of Proposition 3: Suppose $e_j = 0$ for all j . By construction, $e_j \geq \pi - \Phi_j + x_{j-1} - x_j$. Summing up over all j yields

$$\begin{aligned} \sum_{j=1}^n e_j &\geq \sum_{j=1}^n (\pi - \Phi_j + x_{j-1} - x_j) \\ &= n\pi - b\phi \\ &> 0 \end{aligned}$$

This contradicts the fact that $e_j = 0$ for all j . Hence, there must exist at least one j for which $x_j = \lambda$.

Next, we argue that the fact that $e_j > 0$ for some j implies $x_j = \lambda$ for some j . For suppose not. Since $x_j = \max\{0, \min\{x_{j-1} + \pi - \Phi_j, \lambda\}\}$, it follows that $x_{j-1} + \pi - \Phi_j < \lambda$ for all j . Hence, $x_j = \max\{0, x_{j-1} + \pi - \Phi_j\}$. From this, it follows that $e_j =$

$\max\{\pi - \Phi_j + x_{j-1} - x_j, 0\} = 0$, since either $x_{j-1} + \pi - \Phi_j < 0$ in which case $x_j = 0$ and e_j is the maximum of a negative expression and 0, and thus equal to 0, or else $x_j = x_{j-1} + \pi - \Phi_j$ and so $e_j = \max\{\pi - \Phi_j + x_{j-1} - x_j, 0\} = \max\{0, 0\} = 0$. ■

Proof of Proposition 4: Define $\hat{S}$ as a state in which all the bad banks are located next to one another. Without loss of generality, we can order banks so that $\hat{S}_j = 1$ for $j = 0$ and $j \in \{n - b + 1, \dots, n - 1\}$. We now establish the claim through a sequence of steps. First, we argue that if the state of the network is given by $\hat{S}$, then for λ sufficiently large, all banks will transfer some positive resources to other banks on the network.

Result 1: Suppose $\lambda > b(\phi - \pi)$. Then if $S = \hat{S}$, the fixed point x_j that solves (4) satisfies $x_j > 0$ for all j .

Proof of Result 1: Suppose $x_0 = 0$. Then $x_j = \min\{j\pi, \lambda\}$ for all $j \in \{1, \dots, n - b\}$. Since $n\pi > b\phi$ under A1, we can subtract $b\pi$ from both sides to get

$$(n - b)\pi > b(\phi - \pi)$$

Set $\lambda = b(\phi - \pi) + \varepsilon$ where $\varepsilon > 0$. Choose ε sufficiently small so that

$$(n - b)\pi > b(\phi - \pi) + \varepsilon$$

Then $x_{n-b} = \lambda = b(\phi - \pi) + \varepsilon$. Since banks $n - b + 1$ through $n - 1$ are bad, we have

$$x_0 = \max\{0, x_{n-b} - b(\phi - \pi)\} = \varepsilon$$

Therefore, $x_0 > 0$, a contradiction. Since $T^*(x)$ is weakly increasing in λ , then if $T^*(0) > 0$ for $\lambda = b(\phi - \pi) + \varepsilon$, then $T^*(0) > 0$ for any $\lambda' > b(\phi - \pi) + \varepsilon$. ■

Let $T_j(x; S)$ denote the operator T_j for a particular state of the network S . Likewise, let $T^*(x; S)$ denote the composition of $T_j(x; S)$ for $j = 1, \dots, n$ for a particular S . The proof of Result 1 involves showing that for $\lambda > b(\phi - \pi)$, $T^*(0; \hat{S}) > 0$ whenever $\lambda > b(\phi - \pi)$. The next result establishes that as long as $\lambda > b(\phi - \pi)$, then for any vector S that corresponds to the possible location of the b bad banks, $T^*(0; S) > 0$. From this, it follows that as long as $\lambda > b(\phi - \pi)$, the fixed point x_j that solves (4) is positive for all x_j for all S .

Result 2: $T^*(0; S) \geq T^*(0; \hat{S})$ for all S and all x , including $x = 0$.

Proof: Observe that starting from $\hat{S}$, we can reach any state $S \neq \hat{S}$ with a finite number of steps where each step involves swapping a pair of adjacent banks, one good bank with a lower index and one bad bank with a higher index, so that after swapping them the bad bank has the lower index and the good bank has the higher index. Formally, there exists a sequence of vectors $(S^0, S^1, \dots, S^Q)$ where $Q < \infty$ such that $S^0 = \hat{S}$, $S^Q = S$, and for each q

$$S_i^{q+1} = \begin{cases} S_i^q & \text{if } i \notin \{j_q - 1, j_q\} \\ 1 - S_i^q & \text{if } i \in \{j_q - 1, j_q\} \end{cases}$$

for some j_q where $S_{j_q-1}^q = 1$. Intuitively, we can achieve any desired spacing between the bad banks by first moving bank 0 clockwise, then moving bank $n - 1$, and so on, until finally we move bank $n - b + 2$.

For each q and an initial x_0 , define x_j^q as the payment bank j makes if bank 0 pays x_0 to bank 1 and the state of the network is S^q . We can likewise define x_j^{q+1} when the state of the network is S^{q+1} . Formally,

$$\begin{aligned} x_j^q &= T_j \circ \cdots \circ T_1 (x_0; S^q) \\ x_j^{q+1} &= T_j \circ \cdots \circ T_1 (x_0; S^{q+1}) \end{aligned}$$

By construction, $S_j^q = S_j^{q+1}$ for $j \leq j_q - 2$, which implies $x_{j_q-2}^q = x_{j_q-2}^{q+1}$.

Let $G(\xi)$ denote the payment a good bank will make if it receives a payment ξ from its neighboring bank, and let $B(\xi)$ denote the payment a bad bank will make. Then

$$\begin{aligned} G(\xi) &\equiv \max\{0, \min\{\lambda, \xi + \pi\}\} \\ B(\xi) &\equiv \max\{0, \min\{\lambda, \xi + \pi - \phi\}\} \end{aligned}$$

By definition

$$B(\xi) = G(\xi - \phi) \tag{23}$$

Note that $G(\xi)$ is weakly increasing in ξ with slope bounded above by 1. We can now characterize the payment made by bank j_q when $S = S^q$ and $S = S^{q+1}$ using $G(\cdot)$ and $B(\cdot)$ as follows

$$\begin{aligned} x_{j_q}^q &= B\left(G\left(x_{j_q-2}^q\right)\right) \\ x_{j_q}^{q+1} &= G\left(B\left(x_{j_q-2}^{q+1}\right)\right) \end{aligned}$$

For any real number ξ , (23) implies

$$\begin{aligned} G(B(\xi)) &= G(G(\xi - \phi)) \\ B(G(\xi)) &= G(G(\xi) - \phi) \end{aligned}$$

Since $G(\cdot)$ has a slope bounded above by 1, then since $\phi > 0$,

$$G(\xi - \phi) \geq G(\xi) - \phi$$

Applying $G(\cdot)$ to both sides and using the fact that $G(\cdot)$ is monotone yields

$$G(G(\xi - \phi)) \geq G(G(\xi) - \phi)$$

or alternatively

$$G(B(\xi)) \geq B(G(\xi))$$

Setting $\xi = x_{j_q-2}^q = x_{j_q-2}^{q+1}$, we have

$$\begin{aligned} x_{j_q}^q &= B\left(G\left(x_{j_q-2}^q\right)\right) \\ &\leq G\left(B\left(x_{j_q-2}^q\right)\right) \\ &= G\left(B\left(x_{j_q-2}^{q+1}\right)\right) = x_{j_q}^{q+1} \end{aligned}$$

In other words, the state of the network that minimizes the resources bank 0 has at its disposal is when bank 0 and the $b - 1$ banks that come before it are bad. ■

Result 2 implies that for any S , a bank that pays nothing will be left with positive resources with which it can pay. This contradiction proves that if $\lambda > b(\phi - \pi)$, the fixed point of (4) must be strictly positive in all its terms.

Finally, we show that if $\lambda \leq b(\phi - \pi)$, there exists a state a fixed point with $x_j = 0$ for at least one j whenever $S = \hat{S}$.

Result 3: If $\lambda \leq b(\phi - \pi)$, then $x_j = 0$ for some j when $S = \hat{S}$.

Proof of Result 3: The proof is by construction. Suppose $S = \hat{S}$, and consider $x_0 = 0$. Then $x_j = \min\{j\pi, \lambda\}$ for all $j \in \{1, \dots, n - b\}$. Since $n\pi > b\phi$, subtracting $b\pi$ from both sides yields

$$(n - b)\pi > b(\phi - \pi)$$

Hence, $x_{n-b} = \lambda \leq b(\phi - \pi)$. Since the next b banks are bad, it follows that

$$x_0 = \min\{0, x_{n-b} - b(\phi - \pi)\} = 0$$

This confirms $x_0 = 0$ is a fixed point of (4). ■

Proof of Proposition 5: From Proposition 4, we know that $x_j > 0$ for all $j \in \{0, \dots, n - 1\}$. Hence,

$$x_j = \min\{\lambda, x_{j-1} + \pi - \Phi_j\}$$

Equity is then given by

$$e_j = \max\{0, x_{j-1} + \pi - \Phi_j - x_j\}$$

We consider each of the two cases for x_j . If $x_j = x_{j-1} + \pi - \Phi_j$, then

$$e_j = x_{j-1} + \pi - \Phi_j - x_j = 0$$

If instead $x_j = \lambda$, then $x_{j-1} + \pi - \Phi_j \geq \lambda$ and so

$$e_j = \max\{0, x_{j-1} + \pi - \Phi_j - \lambda\} = x_{j-1} + \pi - \Phi_j - \lambda$$

Either way, we have

$$e_j = x_{j-1} + \pi - \Phi_j - x_j$$

Summing up the equity values across banks yields

$$\sum_{j=1}^n e_j = n\pi - b\phi$$

Hence, the sum of equity values is the same, regardless of S . Assumption A2 implies $e_j \in \{0, \pi\}$. But this implies the cardinality of the set $\{j : e_j = 0\}$ is the same for all S . Let $\zeta \equiv \#\{j : e_j = 0\}$. Then we have

$$\sum_{j=1}^n e_j = (n - \zeta) \pi = n\pi - b\phi$$

Since $\lambda > b(\phi - \pi)$, then $\min\{\phi - \pi, \lambda\} = \phi - \pi$. From this, it follows that

$$k \equiv \frac{\min\{\phi - \pi, \lambda\}}{\pi} = \frac{\phi - \pi}{\pi}$$

and so $\phi = (k + 1) \pi$. Hence,

$$(n - \zeta) \pi = n\pi - b(k + 1) \pi$$

which gives

$$\zeta = b(k + 1)$$

as claimed. ■

Proof of the Proposition 6: For $0 < \lambda < \phi - \pi$, Lemma 1 implies

$$\begin{aligned} p_g &= \frac{n - E[\zeta]}{n - b} \\ &= \frac{(n - b)!(n - k - 1)!}{(n - b)(n - 1)!(n - b - k - 1)!} \\ &= \prod_{i=1}^k \left(\frac{n - b - i}{n - i} \right) \end{aligned}$$

Since $k = \frac{\min\{\phi - \pi, \lambda\}}{\pi} = \frac{\lambda}{\pi}$, we can rewrite p_g in this case as

$$p_g = \prod_{i=1}^{\lambda/\pi} \left(\frac{n - b - i}{n - i} \right)$$

For $\lambda > b(\phi - \pi)$, Proposition 5 implies $\zeta = bk + b$ with probability 1. Hence,

$$\begin{aligned} p_g &= \frac{n - bk - b}{n - b} \\ &= 1 - \frac{bk}{n - b} \end{aligned}$$

Since $b \geq 1$, from (6), $\lambda > b(\phi - \pi)$ implies $\lambda > \phi - \pi$, and so $k = \frac{\phi}{\pi} - 1$, and so

$$p_g = 1 - \frac{b}{n - b} \left(\frac{\phi}{\pi} - 1 \right)$$

Finally, our claim for the case of $\phi - \pi \leq \lambda \leq b(\phi - \pi)$ follows from Proposition 13. ■

Proof of Proposition 7: Our proof is by construction. We know from Proposition 3 that there exists at least one bank for which $\hat{e}_j > 0$. Start with this bank and move to bank $j + 1$, continuing on until reaching the first bad bank. Without loss of generality, we can refer to this as bank 1. Moreover, we know that $\hat{x}_0 = \lambda$, i.e. if outsiders did not invest in any of the banks, then bank 0 would be able to pay its obligation to bank 1 in full.

First, we argue that $x_0 = \lambda$, i.e. when banks can raise outside funds, it will still be the case that bank 0 will be able to pay its debt obligation to bank 1 in full. To see this, define

$$\begin{aligned} T_j(x) &= \max \{0, \min \{x + \pi + R(1 - D_j)I_j - \Phi_j, \lambda\}\} \\ &\geq \max \{0, \min \{x + \pi - \Phi_j, \lambda\}\} \equiv \hat{T}_j(x) \end{aligned}$$

As before, the payment x_0 must solve the fixed point

$$x_0 = T^*(x_0) = T_n \circ \cdots \circ T_1(x_0) \tag{24}$$

Since $T^*(x_0) \geq \hat{T}^*(x_0)$, then we know that

$$T^*(\lambda) \geq \hat{T}^*(\lambda) = \lambda$$

But $T^*(x) \leq \lambda$ for all x . Hence, $T^*(\lambda) = \lambda$, and so $x_0 = \lambda$ is a fixed point of (24).

Now, suppose bank 1 was able to raise funding, i.e. $I_1 = 1$. Let r_1 denote the rate bank 1 is charged. If bank 1 diverted the funds it obtained, its expected payoff would be v . If it invested the funds, it would get to keep

$$\max \{\lambda + \pi + R - y_1 - x_1 - w_1, 0\}$$

where

$$\begin{aligned} y_1 &= \min \{\phi, \lambda + \pi + R\} \\ x_1 &= \min \{\lambda, \lambda + \pi + R - y_1\} \\ w_1 &= \min \{r_1^*, \lambda + \pi + R - y_1 - x_1\} \end{aligned}$$

If $y_1 = \lambda + \pi + (R - r_1)$, bank 1 would get to keep 0, which is less than v . If $y_1 = \phi$, bank 1 would get to keep

$$\max \{ \lambda + \pi + (R - r_1) - \phi - x_1, 0 \}$$

which is 0 if $x_1 = \lambda + \pi + R - y_1$ and $\pi + (R - r_1) - \phi$ if $x_1 = \lambda$. Since $\phi > \pi$ under Assumption A1, this is less than $R - r_1$. Moreover, since $r_1 \geq \underline{r}$ in any equilibrium, $R - r_1 \leq R - \underline{r} < v$, where the last inequality follows from Assumption A3. Thus, bank 1 will not be able to raise outside funds, i.e. $I_1 = 0$. From this we can conclude that $e_1 = 0$, since bank 1's resources $\lambda + \pi - \phi$ are less than its obligation of λ to bank 2.

We now proceed by induction. Suppose $e_1 = \dots = e_{j-1} = 0$ and $I_1 = \dots = I_{j-1} = 0$. Assumption A2 implies $\hat{e}_j$ is equal to either 0 or π . We consider each case in turn.

Suppose first that $\hat{e}_j = 0$. We argue that $I_j = 0$, i.e. if bank j would have zero equity in the absence of investment, then bank j would be unable to raise funds when investment is allowed. For suppose not. Given $x_0 = \hat{x}_0 = \lambda$ and $I_1 = \dots = I_{j-1} = 0$, it follows that

$$x_{j-1} = \hat{x}_{j-1}$$

Since $\hat{e}_j = 0$, we know that under Assumptions A1 and A2, either $\hat{x}_{j-1} = \lambda$ or else $\hat{x}_{j-1} \leq \lambda - \pi$. In the first case, we can apply the same argument we used to establish $I_1 = 0$ to show that $I_j = 1$. In the second case, suppose bank j were able to raise funds. Then if bank j diverts the funds it obtains, its payoff would be v . In particular,

$$\begin{aligned} y_j &= \min \{ \Phi_j, x_{j-1} + \pi \} = \hat{y}_j \\ x_j &= \max \{ 0, \min \{ x_{j-1} + \pi - y_j, \lambda \} \} = \hat{x}_j \end{aligned}$$

and since

$$\hat{e}_j = \max \{ 0, \hat{x}_{j-1} + \pi - \hat{y}_j - \hat{x}_j \} = 0$$

then even before paying back outside investors w_j , the bank would have no resources left. By contrast, if the bank did not divert, then since $\hat{x}_{j-1} \leq \lambda - \pi$, its payoff will be at most $R - r_j \leq R - \underline{r} < v$, where $r_j \geq \underline{r}$ is the rate bank j will be charged by outside investors. Hence, $I_j = 0$ as claimed. Since $I_j = 0$ implies $x_j = \hat{x}_j$, it follows that $e_j = \hat{e}_j = 0$.

Next, suppose $\hat{e}_j = \pi$. Note that this implies $S_j = 0$, i.e. j must be a good bank. We want to show that $I_j = 1$ and $x_j = \lambda$. That is, if bank j would have full equity in the absence of investment, then bank j would raise funds when investment is allowed. To see this, observe that $\hat{e}_j = \pi$ implies $x_{j-1} = \hat{x}_{j-1} = \lambda$. Hence, we have

$$\begin{aligned} y_j &= \min \{ \Phi_j, \lambda + \pi \} = 0 \\ x_j &= \max \{ 0, \min \{ \lambda + \pi + R(1 - D_j)I_j, \lambda \} \} = \lambda \end{aligned}$$

If the bank obtained funds from outside investors, i.e. $I_j = 1$, and did not divert funds, it would earn $\pi + R - r_j$. If it chose to divert funds, it would receive $v + \min \{ \pi - r_j, 0 \}$. At $r_j = \underline{r}$, Assumption A3 ensures that the bank would prefer to invest than to divert the funds. Since outsiders can observe the state of each bank, it follows that the unique equilibrium is one where $r_j = \underline{r}$ and $I_j = 1$.

So far, we have established that starting from bank 1, continuing through all the consecutive banks for which $\widehat{e}_j = 0$ implies $I_j = 0$. Once we reach the first bank for which $\widehat{e}_j = \pi$, we know that $x_j = \lambda$, and we can keep going until we reach the next bad bank. Since this bank receives λ , the analysis would be the same as for bank 1. The claim then follows. ■

Proof of Lemma 2: If bank j has positive equity in equilibrium, it must be that $x_{j-1} = \lambda$, i.e. bank j is paid in full. This is because Assumptions A1 and A2 imply that if $x_{j-1} < \lambda$, then $\widehat{e}_j = 0$, i.e. such a bank would have no equity prior to raising any funds from outside investors. But we know from Assumption A3 that such a bank would divert funds, i.e. $D_j = 1$, and so such a bank would have no equity. Given this, a bank that receives outside funding would choose to invest the funds it raises rather than divert them iff

$$v + \max \{ \pi - r_j^*, 0 \} < \pi + R - r_j^* \quad (25)$$

Suppose $r_j^* < \pi$. In this case, $\max \{ \pi - r_j^*, 0 \} = \pi - r_j^*$. But then Assumption A3 tells us that (25) must hold, since it reduces to $v < R$. Next, suppose $r_j^* \geq \pi$. In this case, $\max \{ \pi - r_j^*, 0 \} = 0$. In that case, (25) only holds if $\pi \leq r_j^* \leq \pi + R - v$. Since $v < R$, this bound exceeds π . It follows that $D_j = 0$ if and only if $r_j^* \leq \pi + R - v$. ■

Proof of Lemma 3: From Lemma 2, the only scenario we have to explore is whether there exists an equilibrium with $r_j > \bar{r}$ in which a bank with positive equity chooses to divert, i.e. $D_j = 1$. Let p_j denote the probability that bank j has positive equity in equilibrium. Then the expected payoff to bank j is given by $p_j (r_j^* (1 - D_j) + \min \{ \pi, r_j^* \} D_j)$. When $D_j = 1$, this payoff collapses to $= p_j \pi$. But suppose an outside investor were to charge $r_j = \pi + \varepsilon$ where ε is sufficiently small so ensure that $r_j < \bar{r}$. In that case, the bank would be strictly better off since it is charged a lower rate. Moreover, since $\pi + \varepsilon < \bar{r}$, the bank will invest and pay $r_j = \pi + \varepsilon$ in full, so the investor that charges this amount will be better off. But then the original outcome with $r_j^* > \bar{r}$ could not have been an equilibrium. ■

Proof of Proposition 10: First, suppose $v \geq \underline{r}$. Then for any $p_g \in (0, 1)$, we have

$$\begin{aligned} (R - v) p_g + (v - \underline{r}) &= p_g (R - \underline{r}) + (1 - p_g) (v - \underline{r}) \\ &\geq p_g (R - \underline{r}) \\ &> p_g \frac{n-b}{n} (R - \underline{r}) \end{aligned}$$

Mandatory disclosure is preferable to no investment if

$$c < (R - \underline{r}) \frac{n-b}{n} p_g$$

But from above it follows that

$$c < (R - v) p_g + (v - \underline{r})$$

Since $p_g > \underline{r}/\bar{r}$ implies a good bank that unilaterally discloses will be able to raise funds, while the above inequality implies the benefits from attracting funds exceed the disclosure

cost, it follows that non-disclosure cannot be an equilibrium whenever mandatory disclosure is preferable to no investment.

Next, suppose $v < \underline{r}$. For any $p_g > \underline{r}/\bar{r}$, a non-disclosure equilibrium with no investment will exist if

$$p_g \leq \frac{n}{n-b} \underline{r}/\bar{r} \text{ and } c \geq (R-v)p_g + (v-\underline{r})$$

and mandatory disclosure will be preferable to no investment if

$$c \leq p_g \frac{n-b}{n} (R-\underline{r})$$

The only way both inequalities involving c can be satisfied is if

$$(R-v)p_g + (v-\underline{r}) \leq p_g \frac{n-b}{n} (R-\underline{r})$$

Rearranging, improveability on a non-disclosure equilibrium with no investment is possible only if

$$p_g \leq \frac{\underline{r}-v}{(R-v) - \frac{n-b}{n} (R-\underline{r})}$$

For this bound to exceed $\underline{r}/\bar{r}$ requires

$$\frac{(\underline{r}-v)}{(R-v) - \left(1 - \frac{b}{n}\right) (R-\underline{r})} > \underline{r}/\bar{r}$$

which, rearranging, implies

$$\frac{b}{n} < \left(\frac{\bar{r}}{\underline{r}} - 1\right) \frac{\underline{r}-v}{R-\underline{r}}$$

Finally, from A3,

$$\frac{\underline{r}-v}{(R-v) - \left(1 - \frac{b}{n}\right) (R-\underline{r})} = \frac{\underline{r}-v}{(\underline{r}-v) + \frac{b}{n} (R-\underline{r})} < 1$$

which completes the proof. ■

Proof of Proposition 11: First, suppose $v \geq \underline{r}$. The expected amount banks pay to investors is $\underline{r}$ both when there is no disclosure and when there is mandatory disclosure. For a good bank, then, the expected payoff under the non-disclosure equilibrium with investment is $p_g R + (1-p_g)v - \underline{r}$. Under mandatory disclosure, the expected payoff for a good bank is $p_g(R-\underline{r})$, which is strictly lower. This confirms some party will be made worse off with mandatory disclosure, so mandatory disclosure cannot be Pareto improving.

Next, suppose $v < \underline{r}$. A non-disclosure equilibrium with investment can only exist if $c > \frac{b\underline{r}}{n-b}$. At the same time, mandatory disclosure will be Pareto improving relative to an equilibrium where outsiders invest in all banks only if $c < \left(1 - \frac{n-b}{n} p_g\right) (\underline{r}-v)$. For mandatory disclosure to be Pareto improving and for there to exist a non-disclosure equilibrium with

investment, we need

$$\frac{b\underline{r}}{n-b} < \left(1 - \frac{n-b}{n}p_g\right)(\underline{r} - v)$$

or, rearranging, if

$$p_g \leq \frac{n}{n-b} \left(1 - \frac{b}{n-b} \frac{\underline{r}}{\underline{r}-v}\right)$$

If this inequality is violated at $p_g = \frac{n}{n-b} \frac{\underline{r}}{\underline{r}-v}$, then it will be violated for all $p_g \geq \frac{n}{n-b} \frac{\underline{r}}{\underline{r}-v}$. Hence, a necessary condition for the existence of a Pareto-improveable non-disclosure equilibrium is for

$$\frac{n}{n-b} \left(1 - \frac{b}{n-b} \frac{\underline{r}}{\underline{r}-v}\right) \geq \frac{n}{n-b} \frac{\underline{r}}{\bar{r}}$$

Rearranging, we have the condition

$$\frac{b}{n} \leq \frac{\underline{r}-v}{(\underline{r}-v)(1 - \underline{r}/\bar{r}) + \underline{r}} \left(1 - \frac{\underline{r}}{\bar{r}}\right)$$

Hence, without this condition, there exists no Pareto-improveable non-disclosure equilibrium with investment. With this condition, the interval $\left[\frac{n}{n-b} \frac{\underline{r}}{\bar{r}}, \frac{n}{n-b} \left(1 - \frac{b}{n-b} \frac{\underline{r}}{\underline{r}-v}\right)\right]$ will be non-empty. For any p_g in this interval, and so the as long as $c \in \left[\frac{b}{n-b} \underline{r}, (1 - \frac{n-b}{n} p_g)(\underline{r}-v) \frac{\underline{r}}{\bar{r}}\right]$, which is necessarily non-empty given the restriction on $\frac{b}{n}$, a non-disclosure equilibrium with investment is Pareto-improveable. Finally, observe that since $v < \underline{r}$, then

$$\frac{n}{n-b} \left(1 - \frac{b}{n-b} \frac{\underline{r}}{\underline{r}-v}\right) < \frac{n}{n-b} \left(1 - \frac{b}{n-b}\right)$$

But then we have

$$\begin{aligned} \frac{n}{n-b} \left(1 - \frac{b}{n-b} \frac{\underline{r}}{\underline{r}-v}\right) &< \frac{n}{n-b} \left(\frac{n-2b}{n-b}\right) \\ &= \frac{n^2 - 2nb}{n^2 - 2nb + b^2} < 1 \end{aligned}$$

Proof of Proposition 12: Suppose all good banks disclose. To confirm this is an equilibrium, we verify that no good bank wants to deviate. If a good bank discloses, its expected earnings rise by $p_g(R - \underline{r}) - c$. Given $(n-b)p_g(R - \underline{r}) - cn > 0$, this expected payoff is strictly positive. Next, suppose a good bank deviates and opts not to disclose. Under our refinement, the probability outsiders assign to this bank being good is $\frac{1}{b+1}$. The most optimistic scenario for this bank is that all other bad banks are sufficiently far away that outsiders believe this bank will have positive equity if it is good. In that case, it will be able to attract funding only if $\frac{1}{b+1} \bar{r} > \underline{r}$. If this condition is violated, i.e. if $b > \frac{\bar{r}}{\underline{r}} - 1$, then a bank that fails to disclose will be unable to raise funds in any state of the world, and so its payoff from not disclosing is 0. It follows that all good banks disclosing is an equilibrium. ■

Proof of Lemma 4. We want to show that for any pair j and k , the distribution of equity e_j for bank j conditional on bank j being good ($S_j = 0$) is the same as the distribution

of equity e_k for bank k conditional on k being good ($S_k = 0$).

Once again, let Ω denote the set of all realizations for S , i.e.

$$\Omega = \left\{ x \in \{0, 1\}^n : \sum_{j=1}^n x_j = b \right\}$$

Note that for any two realizations s and s' in Ω , $\Pr(S = s) = \Pr(S = s')$. Suppose we show that there exists a bijective mapping $\varphi : \Omega \rightarrow \Omega$ such that (i) $s_j = \varphi_k(s)$, and (ii) $e_j(s) = e_k(\varphi(s))$ for all $s \in \Omega$, i.e. the state and equity of bank j when $S = s$ is the same as the state and equity of bank k when $S = \varphi(s)$. Since all states have the same probability, it follows that $\Pr(e_j = x | S_j = 0) = \Pr(e_k = x | S_k = 0)$.

Heuristically, we can establish the existence of φ as follows. Suppose we place each bank j at the physical location j . We then construct a directed network across physical locations. Given a vector $s \in \Omega$ that implies which are the b bad banks, we can compute the equity of bank j when $S = s$.

Symmetry implies that for any bank k , we can rearrange banks across locations so that bank k lies in location j and the directed network across physical locations remains unchanged. Suppose we leave the shocks at the same physical locations implied by s . Since we have rearranged banks across locations, this implies the identity of the b bad banks is now generally different. In particular, the state of each bank will be given by $\varphi(s) = (s_{\sigma_{kj}}(0), \dots, s_{\sigma_{kj}}(n-1))$ for σ_{kj} as defined in the text.

By construction, $s_k = 1$ when $S = \varphi(s)$ iff $s_j = 1$ when $S = s$. Moreover, by construction, payments across physical locations are the same given payments depend only on flows across locations. This ensures that for any bank i , the equity e_i when $S = s$ is the same as the equity of bank $\sigma_{kj}(i)$ when $S = \varphi(s)$. In particular, the equity of bank k when $S = \varphi(s)$ is the same as the equity of bank j when $S = s$. This completes the proof. ■

Proof of Proposition 13: We first define the shortfall D_{ij} given the state of the network S as the difference between what bank i owes bank j and what bank i actually pays bank j :

$$D_{ij} = \Lambda_{ij} - x_{ij} \text{ for all } i, j \in \{0, \dots, n-1\} \quad (26)$$

We suppress the state of the network from the notation whenever seems clear. We can transform the operator in (20) defined over payments x_{ij} into an operator $F : \mathcal{D} \rightarrow \mathcal{D}$, where $\mathcal{D} \subset \mathbb{R}_+^n$ is the space of possible shortfalls given by

$$\mathcal{D} = \{D_{ij} \in [0, \Lambda_{ij}] , i, j \in \{0, \dots, n-1\}\}$$

This operator is defined by

$$(F)(D)_{ij} = \left(\frac{\Lambda_{ij}}{\Lambda_i} \right) \max \left\{ \min \left\{ \Lambda_i , \sum_{m \neq i} D_{mi} - \pi + S_i \phi \right\} , 0 \right\} \quad (27)$$

The set of fixed points of the shortfall operator corresponds to the set of fixed points of the operator defined over payments. Either of these can be used to derive equity, and hence the distribution of equity we wish to characterize.

Our proof now proceeds as follows. First, we show that for each S the shortfall $D(S)$ are weakly increasing in ϕ and in λ . Next we argue that this implies that the distribution of equity is stochastically decreasing with ϕ and in λ for each S . Then the result follows since the distribution of S is not a function of (ϕ, λ) .

We use the notation $F_{\phi, \lambda}$ to emphasize the dependence of the operator on the parameters (ϕ, λ) . It is easy to show that F is monotone, i.e. $F_{\phi, \lambda}(D') \geq F_{\phi, \lambda}(D)$ if $D' \geq D$, where the comparison is component by component. Thus, by Tarski's fixed point theorem, there exists a smallest fixed point, which is obtained as $D^*(\phi, \lambda) = \lim_{n \rightarrow \infty} F^n(0)$. Additionally, F is monotone on (ϕ, λ) , i.e. for each $D \in \mathcal{D}$, $F_{\phi', \lambda'}(D) \geq F_{\phi, \lambda}(D)$, whenever $(\phi', \lambda') \geq (\phi, \lambda)$. Then it follows that the smallest fixed point $D^*(\phi, \lambda)$ is increasing in (ϕ, λ) .

For any vector of shortfalls D , parameter (ϕ, λ) and state of the network S the implied equity of bank i is:

$$\begin{aligned} e_i(S) &= \max \left\{ 0, \pi - \phi S_i - \sum_{j=0}^{n-1} \Lambda_{ij} + \sum_{m=0}^{n-1} x_{mi}(S) \right\} \\ &= \max \left\{ 0, \pi - \phi S_i - \Lambda_i - \left(\sum_{m=0}^{n-1} D_{mi}(S) + \sum_{m=0}^{n-1} \Lambda_{mi} \right) \right\} \\ &= \max \left\{ 0, \pi - \phi S_i - \sum_{m=0}^{n-1} D_{mi}(S) \right\} \end{aligned} \quad (28)$$

where the last equality follows by regularity of the network, i.e. that $\Lambda_i = \sum_m \Lambda_{mi}$.

Consider the equity corresponding to $D = D^*(\phi, \lambda)$. Equity at bank i is given by

$$e_i(\phi, \lambda; S) = \max \left\{ 0, \pi - \phi S_i - \sum_{m=0}^{n-1} D_{mi}^*(\phi, \lambda; S) \right\} \quad (29)$$

where $D_{mi}^*(\phi, \lambda; S)$ is the amount bank m falls short on bank i for the smallest fixed point for the state S and parameters (ϕ, λ) . Using the monotonicity of $D^*(\phi, \lambda)$ it is immediate that $e_i(\phi, \lambda; S)$ is weakly decreasing in (ϕ, λ) for each S . While we have used the smallest fixed point in the definition (29), by Theorem 1 in [Eisenberg and Noe \(2001\)](#) every fixed point of $F_{\phi, \lambda}$ has the same implied equity values for each bank. Hence, the comparative static of equity must be the same for any fixed point.

Finally, the conditional probability of interest is given by

$$\Pr \{e_j \leq x \mid S_j = 0\} = \frac{\sum_{\{S' : S'_j = 0\}} \mathbb{I}\{e_j(\phi, \lambda; S') \leq x\} \Pr \{S'\}}{\sum_{\{S' : S'_j = 0\}} \Pr \{S'\}} \quad (30)$$

Since $\Pr \{S'\} = 1/\binom{n}{b}$ for all S' , it follows that $\Pr \{e_j \leq x \mid S_j = 0\}$ is decreasing in (ϕ, λ) . ■

Proof of Theorem 2: Suppose a bank is able to raise funds from outsiders at a rate r . Once the bank learns its pre-investment equity is e_j , it knows it will earn $e_j + R - r$ if it invests, and $v + \max \{e_j - r, 0\}$ if it diverts. We begin by observing that a bank charged r

will prefer to invest if $e_j > e^*(r)$ and to divert funds if $e_j < e^*(r)$, where

$$e^*(r) = v - R + r \quad (31)$$

Note that since $v < R$ from Assumption A3, $e^*(r) < r$. Now, suppose $e_j < e^*(r)$. Since $\max\{e_j - r, 0\} = 0$, the fact that $e_j < e^*(r)$ implies

$$e_j + R - r < e_j^* + R - r = v = v + \max\{e_j - r, 0\}$$

and so the bank would prefer to divert. Next, suppose $e^*(r) < e_j \leq r$. In that case, $\max\{e_j - r, 0\} = 0$. In this case, $e_j > e^*(r)$ implies

$$e_j + R - r > e_j^* + R - r = v = v + \max\{e_j - r, 0\}$$

and so the bank will prefer to invest. Finally, suppose $e_j > r > e^*(r)$. In that case, $\max\{e_j - r, 0\} = e_j - r$. Since $v < R$ under Assumption A3, we have

$$\begin{aligned} R + e_j - r &> v + e_j - r \\ &= v + \max\{e_j - r, 0\} \end{aligned}$$

and so the bank will prefer to invest in this case as well.

Note that under Assumption A3, $0 < e^*(r) \leq \pi$ for $r \in [\underline{r}, \bar{r}]$. In particular, the first inequality in (11) implies that for any $r \geq \underline{r}$,

$$e^*(r) = v + r - R \geq v + \underline{r} - R > 0$$

Since $r \geq \underline{r}$ in equilibrium, the inequality holds in equilibrium. In the other direction, the highest equilibrium rate charged to any bank is $\bar{r} = \pi + R - v$. For $r \leq \bar{r}$ we have

$$e^*(r) = v + r - R \leq v + \bar{r} - R = \pi$$

Given a network that is symmetrically vulnerable to contagion, $\Pr(e_j = x | S_j = 0)$ is the same for all j for any value of x . Hence, we can define

$$p_g^*(r) = \Pr(e_j \geq e^*(r) | S_j = 0)$$

That is, $p_g^*(r)$ is the probability that if bank j is good, it will have equity of at least $e^*(r)$, or alternatively the probability that a good bank that raises funds and is charged r will be willing to invest the funds after it learns its equity.

We now derive the analog to Propositions 8-11 to determine when a non-disclosure equilibrium exists and whether mandatory disclosure can improve upon it. The role of $\Pr(e_j = \pi | S_j = 0)$ is now replaced with $p_g^*(\tilde{r})$ where $\tilde{r} = \arg \max_r r p_g^*(r)$, i.e. the interest rate that maximizes the expected return to the lender.

First, if a non-disclosure equilibrium exists, we need to determine whether it will involve investment or not. Since $\phi > \pi$, bad banks would divert funds. Hence, outsiders only earn money from the $n - b$ good banks, and then only from those whose equity is high enough that they will choose not to divert the funds they raise. If the maximal expected amount

lenders expect to collect is below $\underline{r}$, a non-disclosure equilibrium must involve no investment. This condition is given by

$$\sup_{r \in [\underline{r}, \bar{r}]} \frac{n-b}{n} r p_g^*(r) < \underline{r} \quad (32)$$

If (32) is reversed, then a non-disclosure equilibrium would involve investment; otherwise, a lender and bank could enter a trading relationship that would make both of them better off.

Next, we want to derive conditions for when non-disclosure is an equilibrium. Suppose no other bank disclosed. If a good bank were to deviate and announce it was good, outsiders would expect that if they charged this bank r , the probability they would be paid r is $p_g^*(r)$. Hence, no disclosure is an equilibrium for any $c \geq 0$ if charging the r that maximizes the outside lender's expected return will not yield an expected return to the lender of at least $\underline{r}$, or

$$\sup_{r \in [\underline{r}, \bar{r}]} r p_g^*(r) < \underline{r} \quad (33)$$

When (33) is violated, a good bank could raise funds by disclosing it is good. In that case, a non-disclosure equilibrium exists if the cost of disclosure exceed the benefits. In particular, for values of $\sup_{r \in [\underline{r}, \bar{r}]} r p_g^*(r)$ such that

$$\underline{r} < \sup_{r \in [\underline{r}, \bar{r}]} r p_g^*(r) < \frac{n}{n-b} \underline{r} \quad (34)$$

non-disclosure can be an equilibrium only if

$$p_g^*(r^*)R + (1 - p_g^*(r^*))v - \underline{r} \leq c \quad (35)$$

where r^* is the equilibrium interest rate. The condition above makes use of the fact that in equilibrium, $r^* = \underline{r}/p_g^*(r^*)$. Finally, for

$$\sup_{r \in [\underline{r}, \bar{r}]} r p_g^*(r) > \frac{n}{n-b} \underline{r} \quad (36)$$

the only possible non-disclosure is one where outsiders invest in all banks. Let r^* denote the equilibrium rate charged to banks. If a bank were to deviate and reveal itself, it could lower the rate it was charged from r^* to $\frac{n-b}{n} r^*$. Given the cutoff $e^*(r)$ below which a bank would divert is less than r , we know that a bank that diverted would have no equity left. The expected payoff in equilibrium is given by

$$p_g^*(r^*)(R - r^*) + (1 - p_g^*(r^*))v \quad (37)$$

while the expected payoff from deviating is given by

$$p_g^*\left(\frac{n-b}{n} r^*\right) \left(R - \frac{n-b}{n} r^*\right) + \left(1 - p_g^*\left(\frac{n-b}{n} r^*\right)\right) v \quad (38)$$

For non-disclosure to be an equilibrium, the difference between the second and the first expression must be less the cost of disclosure c .

To establish the theorem, define $e^* = e^*(\underline{r})$. Consider the limit as $p_g^*(r) \rightarrow 0$. Since $p_g^*(r)$ is decreasing in r , it follows that

$$\sup_{r \in [\underline{r}, \bar{r}]} r p_g^*(r) < r p_g^*(\underline{r}) \quad (39)$$

Hence, in the limit, we have $\sup_{r \in [\underline{r}, \bar{r}]} r p_g^*(r) \rightarrow 0$ implying the only non-disclosure equilibrium is one where no investment takes place. Moreover, from (33), we know that non-disclosure will be an equilibrium for any $c \geq 0$.

Next, consider the limit as $p_g^*(\bar{r}) \rightarrow 1$, i.e. letting $Pr(e_j = \pi | S_j = 0)$ tend to 1. Since $p_g^*(r)$ is decreasing in r , then $p_g^*(r) \rightarrow 1$ for all $r \in [\underline{r}, \bar{r}]$, and the argument is identical to the one behind Theorem 1. ■

B An Example of Unique Equilibrium Dominated by Mandatory Disclosure

In this Appendix, we construct an example in which non-disclosure is the unique equilibrium but which can still be improved upon by requiring all banks to disclose.

To construct this example, we need to modify the model slightly, for reasons that will become clear below. In particular, we assume that if a *good* bank does not disclose its state S_j , there is still some probability that it will be able to reveal S_j to outsiders before they invest at no cost. That is, if bank j is good and chooses not to disclose, its S_j might still be revealed with probability ρ , independently of whether the signal is revealed at other banks and independently of the state of the network S . The model in the main paper is thus just a special case in which $\rho = 0$.

We want to find parameter values for which (1) there exists a non-disclosure equilibrium in which no firm chooses to incur the cost of disclosure; (2) mandatory disclosure nevertheless improves welfare relative to this equilibrium; (3) no equilibrium in which some good banks disclose with positive probability exists.

Consider the following values:

$$\begin{array}{ll} n = 10 & b = 1 \\ k = 8 & \rho = .2 \\ \underline{r} = 1.05 & R = 2 \\ v = 1 & \pi = 1.5 \end{array}$$

We now verify the three conditions in reverse order.
Detailed calculations to follow

C Examples of Symmetrically-Vulnerable-to-Contagion Networks

In this section, we provide some examples of networks that are symmetrically vulnerable to contagion to highlight the breadth of networks for which our results apply.

Example 1: A symmetric non-circulant network

Our first example demonstrates that the class of symmetric networks is larger than the class of circulant networks, i.e. networks in which we can order banks in such a way that Λ_{ij} is a function of $(i - j) \pmod{n}$, i.e. the distance between banks. Our example is a weighted directed cuboctahedral network. The financial obligations for this network are given by

$$\Lambda = \begin{bmatrix} 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda \\ 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 \\ 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 \end{bmatrix}$$

The implied network is shown graphically in Figure A1. The distinguishing feature of the cuboctahedral network is that each node belongs to exactly two triangular groups, as evident in Figure A1. None of the circulant networks with 12 nodes possess this feature. Essentially, the obligations Λ_{ij} depend not only on distance but also on whether bank i is even or odd.

Example 2: An asymmetric network that is symmetrically vulnerable to contagion

To show that symmetry is not necessary to satisfy symmetric vulnerability to contagion, we construct an example that uses a 4-regular asymmetric undirected graph, i.e. a graph where each node has exactly four vertices (4-regular) and whose automorphism group size is 1 (asymmetric). Gewirtz, Hill, and Quintas (1969) establish that the smallest such network involves 10 nodes. Starting with such a network, which we obtain using the algorithm by Meringer (1999) to compute automorphism group size, we impose equal directional flows of λ so that each node receives 2λ and pays 2λ . The asymmetry of the undirected graph must

carry over to the direct graph. The financial obligations for this network are given by

$$\Lambda = \begin{bmatrix} 0 & \lambda & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & \lambda & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & \lambda \\ 0 & 0 & 0 & \lambda & 0 & 0 & 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 & 0 & \lambda \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 \end{bmatrix}$$

The implied network is shown graphically in Figure A2.

Consider the case where $b = 1$, $\pi < \phi < 3\pi$, and $\lambda > \phi - \pi$. Although the network is asymmetric, we can easily confirm that e_j can only assume 3 values for each j : 0 , $\pi - \frac{\phi - \pi}{2}$, and π with probabilities $\frac{1}{10}$, $\frac{2}{10}$, and $\frac{7}{10}$, respectively.

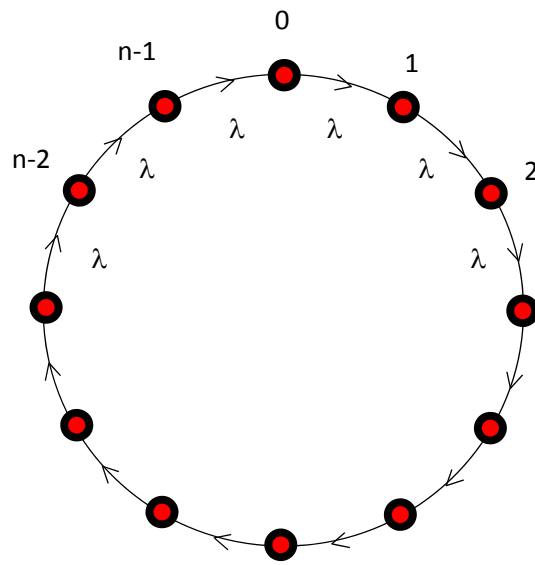


Figure 1: A Circular Network

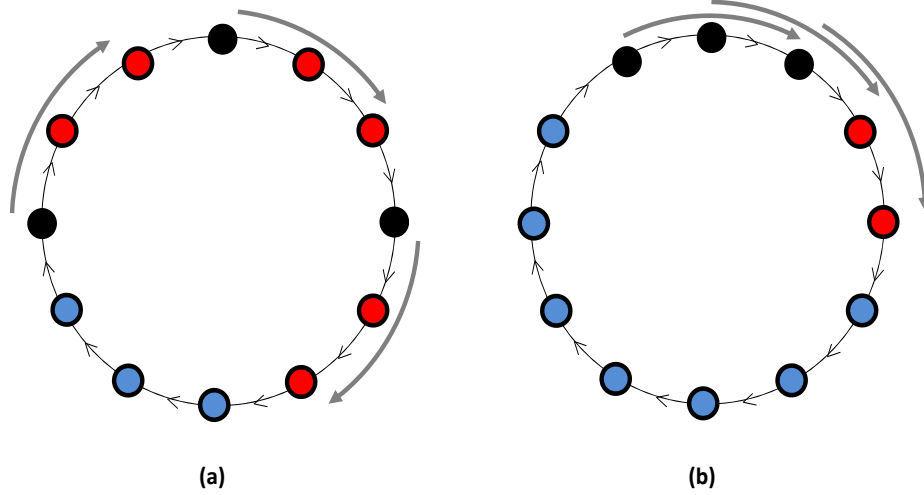
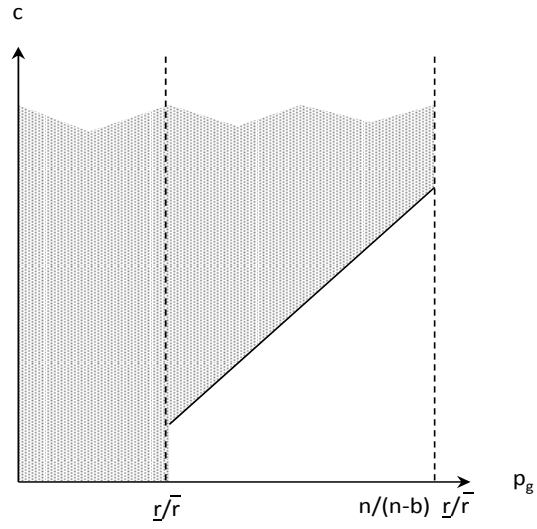
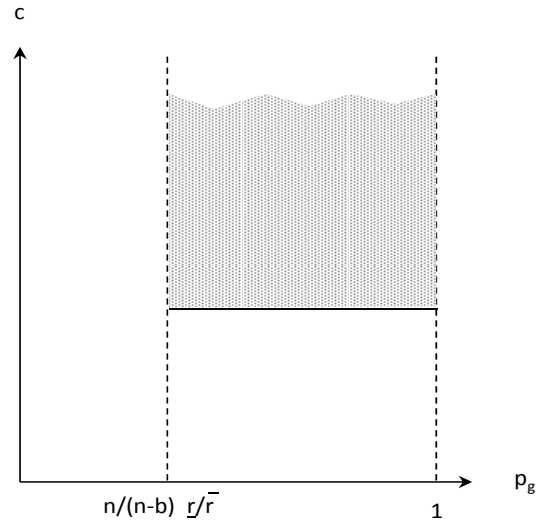


Figure 2: Aggregate Bank Equity Depends on Location of Bad Banks

The figure shows how the location of bad banks can lead to different aggregate bank equity. In the figure, the nodes colored black correspond to bad banks, the nodes colored red are good banks with zero equity, and the nodes colored blue are good banks with equity equal to π . Each circle shows a different realization for the location of the bad banks when $n = 12$, $b = 3$, $k = 2$, and assuming that $\lambda < \phi - \pi$. As seen in the figure, aggregate equity in the bank network is higher when the bad banks are concentrated together, as in part (b), than when bad banks are spaced apart, as in part (a).



a) $p_g < \frac{n}{(n-b)} \frac{\underline{r}}{\bar{r}}$



b) $p_g > \frac{n}{(n-b)} \frac{\underline{r}}{\bar{r}}$

Figure 3: Region where a non-disclosure equilibrium exists

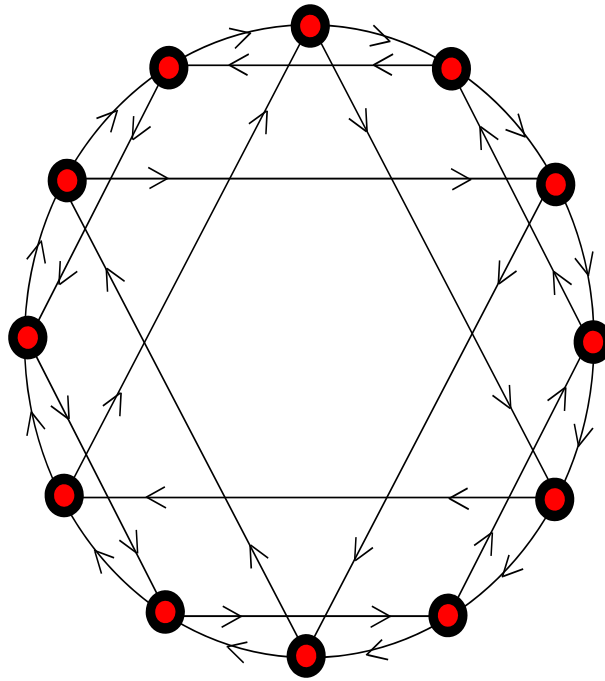


Figure A1: A Directed Cuboctahedral Network

An example of a symmetric network that cannot be represented as a circulant network

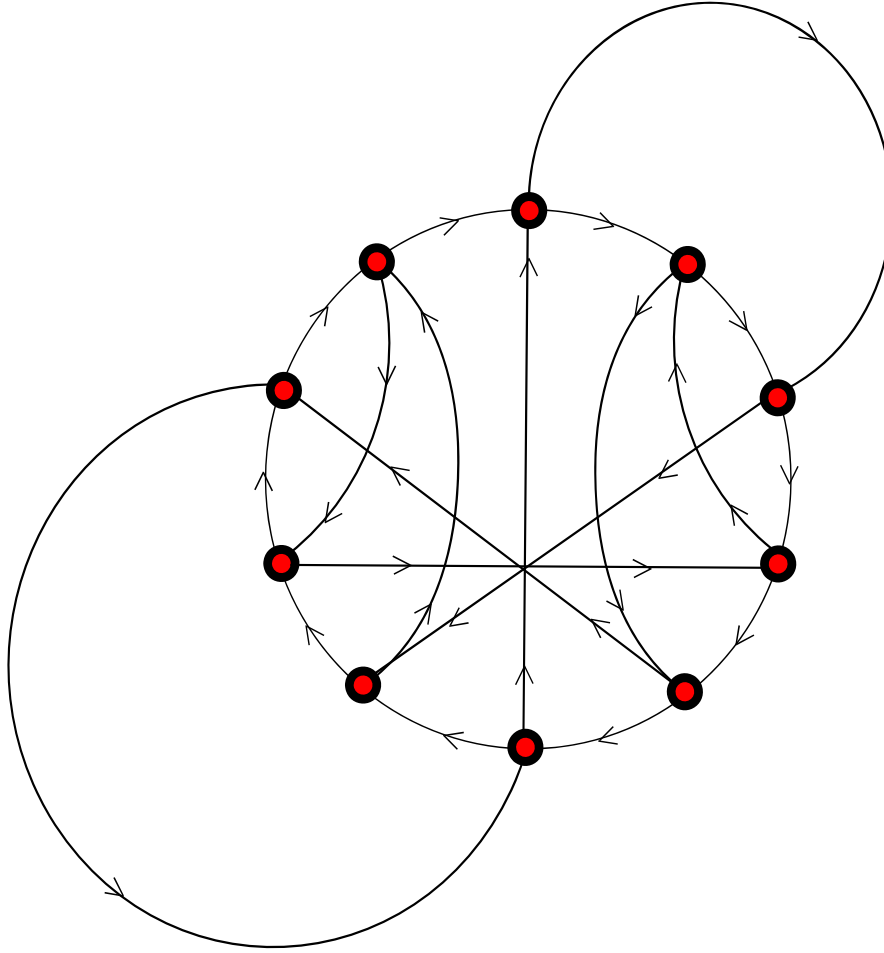


Figure A2: An asymmetric network that can satisfy Symmetric Vulnerability to Contagion